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Analytic combinatorics in several variables, by Robin Pemantle and Mark C. Wilson, Cambridge Studies in Advanced Mathematics, Vol. 140, Cambridge University Press, Cambridge, 2013, xiv+380 pp., ISBN 978-1-107-03157-9

> Analytic combinatorics aims to enable precise quantitative predictions of the properties of large combinatorial structures. The theory has emerged over recent decades as essential both for the analysis of algorithms and for the study of scientific models in other disciplines, including statistical physics, computational biology, and information theory.

That is how my friend and colleague Philippe Flajolet, whose research in the 1980s, 1990s, and 2000s laid the groundwork for the publication of our book *Analytic combinatorics* in 2009, summarized the field. Philippe's untimely death in 2011 left a host of unanswered questions. This new book takes direct aim at one of the most important: How does the theory extend to cover multivariate problems?

The organization of the book is exemplary. A thorough and well-designed introduction provides full context and is worth rereading as one works through the book. A modern perspective on combinatorial enumeration sets the stage. Then two chapters on Fourier–Laplace asymptotics and two chapters on algebraic geometry cover the fundamental mathematical tools for the multivariate analysis to follow. The heart of the book, on new multivariate methods, is based on the research of the authors and a few other coauthors over a fifteen-year period starting around the turn of the new millennium. To make the book self-contained, three appendices cover necessary background material on manifolds, algebraic topology, and Morse theory.

The treatment of analytic methods for multivariate generating functions in this book is breathtaking. A detailed overview is followed by thorough chapters on smooth point asymptotics, multiple point asymptotics, and cone point asymptotics, then four worked examples, and extensions. The end result, a combination of analytic, Morse-theoretic, algebraic, topological, and asymptotic methods, is surprisingly effective. Indeed, it is astonishing that the authors have found relevant ways to exploit such a broad spectrum of mathematical tools to address the problem at hand. A full understanding of this material certainly does require proficiency in more branches of mathematics than the typical reader may have mastered, but the authors have committed to make the book self-contained and do not send the reader to the library or the internet for anything important.

This book is not for the faint of heart, nor is it likely to be suitable for finding a quick solution to a particular problem at hand. To paraphrase the comments of one researcher with decades of experience in the field at a recent conference: "Pemantle and Wilson have put everything they know into this book and the tools appear to be quite powerful, but it is not clear whether mere mortals can make effective use of them." I very much sympathize with that comment as the "mere mortal" who was Flajolet's coauthor, as some of the initial feedback for *Analytic combinatorics* was similar. After several years of teaching from the book, writing and grading exams

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and problem sets, identifying simple common-case corollaries to general theorems, expanding the pedagogy with web content and online video lectures, and watching as colleagues around the world have had great success extending and applying our work, I feel that *Analytic combinatorics* is an important and effective approach for the mere mortals of the 21st century and beyond. I look forward to a similar outcome for *Analytic combinatorics in several variables*, noting that the authors and their students and colleagues are maintaining a website chronicling applications and extensions since the publication of the book.

Whether or not one cares about applications, this book is an extremely well-written treatment of a relevant contemporary topic that many mathematicians will see as an opportunity to learn and appreciate new areas of mathematics and how they interact.

Incidentally, one excellent device used in this book that I wish more authors would use (including myself) is a list accompanying each bibliographic entry giving the pages where that entry is cited. As do many people, I keep an online version of the good books that I study (usually I have two hardcopies—one for home and one for the office), mainly to be able to perform searches. With the ability to quickly find the context for each reference, there is much less need for the online version.

As the author of many books, I cling to the conceit that "the reader" is someone who reads the book from cover to cover. Accordingly, I take pains to accommodate that person with some words of wisdom in the last sentence in the book. Pemantle and Wilson seem to cling to that same conceit, ending with this sentence: "This book is certainly not the last word on the subject, but rather an invitation to join the authors in further development of this research area, which combines beauty, utility, and tractability to a high degree and which has given us considerable enjoyment over many years." I hope and expect that many people will accept this invitation, further develop analytic combinatorics, and enjoy doing so.

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