

COMMENTARY ON
“ARITHMETIC ON CURVES”
BY BARRY MAZUR

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ABSTRACT. Immediately following the commentary below, this previously published article is reprinted in its entirety: B. Mazur, *Arithmetic on curves*, Bull. Amer. Math. Soc. (N.S.) **14** (1986), no. 2, 207–259.

This Editors’ Choice issue gives us the chance to showcase past articles with lasting value. It is usual for an editor’s introduction to describe later advances in the subject. Fortunately, Barry Mazur has contributed an afterword to this issue [2], surveying key developments since his article was published in 1986 [1]. As a result I will concentrate on the value the article continues to have, especially for young mathematicians.

The article gives a grand survey of some central themes in number theory going back thousands of years, leading up to Faltings’s proof of the Mordell conjecture in 1983. The article effectively gets across the idea that numbers have a reality; our elaborate modern theories have arisen from grappling with that reality. The article makes generous use of pictures and explicit calculations to show different approaches to numbers and, specifically, to integer solutions of polynomial equations.

I first read the article as a graduate student who was gradually being converted from Riemannian geometry to algebraic geometry. Even for mathematicians who do not want to be converted, the article can show how algebraic geometers and number theorists think, exploiting the interplay between powerful machines and rich examples. You have a tour guide with a distinctive point of view, who can draw out the small details that bring out the big picture.

All in all, this is one of the *Bulletin* articles that everyone should read. Enjoy!

REFERENCES

- [1] B. Mazur, *Arithmetic on curves*, Bull. Amer. Math. Soc. (N.S.) **14** (1986), no. 2, 207–259.
- [2] B. Mazur, *Afterword*, Bull. Amer. Math. Soc. (N.S.) **55** (2018), no. 3, 353–358.

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