BOOK REVIEWS

Group actions in ergodic theory, geometry, and topology: Selected papers, by Robert
   J. Zimmer, with David Fisher (editor) and with a Foreword by David Fisher,
   Alexander Lubotzky, and Gregory Margulis, University of Chicago Press, Chi-
   cago, IL, 2020, 672 pp., ISBN 978-0-226-56827-0

One of the central themes that recurs in discussions of Robert Zimmer's work
and its wide-ranging influence—indeed a theme that unites the three topics that
David Fisher, Alex Lubotzky, and Gregory Margulis have chosen to illustrate this
influence in their foreword to the volume under review—is that of rigidity. The term
"rigidity" is commonly used to refer to the situation in which an a priori weaker
notion of equivalence or homology between mathematical objects of a certain type
implies a formally much stronger one. One may also desymmetrize this setup and
speculate about individual objects which fail to have any but the most simply
structured or even trivial relations with members of an exhaustive class of objects
of the same kind, a sociological condition of extreme isolation which in its most
pronounced manifestations has acquired the name "superrigidity".

While rigid behaviour is something that pervades mathematics, and indeed is
at the basis of many classification theorems, the theory that is generally invoked
through the marquee use of the terms rigidity and superrigidity has a distinctive
flavour to it. It is a theory that is essentially analytic in nature, in the sense that
it involves structures of a noncompact or infinite asymptotic character (in fact the
notion of boundary as a "compactification" of asymptotic information underpins
many rigidity results), and it employs tools and arguments that can often be con-
ceptualized by means of a local-to-global principle in which nonuniform estimates or
conditions get automatically upgraded to uniform ones. This automatic upgrading
has the effect of blocking the existence of examples or relations of an "interesting" or
nonobvious nature, and it precludes phenomena of higher complexity that in more
flexible circumstances could be engineered by piecing together finitary information
into an infinitary whole through an asymptotic process (one can think for instance
of mixing properties or entropy in dynamics, or of inductive limit constructions in
operator algebras). Simply put, rigidity drives a wedge between the finite and the
infinite, thwarting passage from one to the other.

While the conclusion of a rigidity theorem may be peremptory, there is nothing
perfunctory in what it takes to prove such results, which are hard by nature. Be-
yond the fact that anything of this kind can be established at all, what has been

628   BOOK REVIEWS

absolutely remarkable, starting with the dramatic groundbreaking work of Mostow and Margulis in the 1960s and early 1970s, is how deeply the theory of rigidity has intertwined many branches of mathematics (Lie groups, representation theory, topology, differential geometry, ergodic theory, geometric group theory, hyperbolic dynamics, PDEs, number theory, harmonic analysis, conformal geometry, operator algebras) in a wide variety of novel and frequently unexpected ways. This syncretism has become a hallmark of the subject, indeed one that can make it hard to penetrate. Those with a philosophical inclination may recognize something almost Hegelian in the whole endeavor, a kind of "negation of the negation" in which the degrees of freedom that get locked out of the objects of the theory reappear in sublated form, at the level of the theory itself, as a dizzying complex of ideas and techniques crisscrossing different parts of mathematics.

What types of objects, then, are we actually talking about, or not talking about? To a large extent the theory we have been describing here is, in one way or another, a theory about groups, both infinite discrete groups and noncompact Lie groups, and often the two are considered in tandem via the notion of lattice (i.e., discrete subgroups for which the associated quotient admits a finite invariant measure). Groups, as it turns out, serve as a kind of universal parameter or coordinate space, a kind of Goldilocks structure, for rigidity results across a broad network of different settings.

One possibility is for the group to arise as a functorial skeleton for a fleshier object, such as in the geometric formulation of Mostow rigidity, which says that the fundamental group determines a closed manifold with constant negative curvature and dimension at least three up to isometry [19]. This picture may also be turned around so that we start instead with the group and pass to a canonically defined enveloping object with an a priori higher degree of homogeneity, as in the question of when a group can be recovered from its associated von Neumann algebra [13].

Another possibility is to regard the group as more of an external object and investigate the rigidity properties of maps or embeddings from the group into other spaces. This viewpoint goes back to the historical origins of the subject, to Hilbert's fifth problem in the case of Lie groups and, in the case of their discrete subgroups, to work of Selberg, Calabi, Vesentini, and Weil from the late 1950s and early 1960s [5, 6, 23, 25, 26]. The local rigidity theorem that emerged from the latter flurry of activity, due to Weil in its general form, asserts that cocompact discrete subgroups of most semisimple Lie groups are deformation rigid in the sense that any embedding of the subgroup which is path-connected to the canonical one is in fact conjugate to it within the ambient Lie group. A similar local rigidity phenomenon had been observed to occur in smooth dynamics, and in the 1960s Anosov established a pivotal result in this context showing that if a diffeomorphism of a compact manifold has hyperbolic structure (is "Anosov"), then its perturbations are topologically conjugate to it [1]. Efforts to improve the regularity of such a conjugacy under various conditions would eventually lead, remarkably enough, to a certain conjunction between hyperbolic dynamics and the rigidity theory of Lie groups and their lattices, and indeed some of the recent striking advances in the latter subject revolve around a significant deepening of this connection, as we will see.

While the expression "local rigidity" may initially sound like an oxymoron, it usefully distinguishes this type of result from the "global" or "strong" ones which, from the viewpoint of Weil's local rigidity theorem, free the embeddings of the subgroup from any hypothesized internal relations within the ambient Lie group.

The archetype of such a global result is the algebraic formulation of Mostow rigidity, which asserts that, for $n \geq 3$, every isomorphism from one cocompact discrete subgroup of $\mathrm{PSO}(1, n)$ to another extends to an automorphism of $\mathrm{PSO}(1, n)$. The key to Mostow's approach was to examine the ergodic properties of an asymptotic boundary, an idea that was also used by Furstenberg in the 1960s to study the question of when a given discrete group embeds as a lattice in a Lie group. In Furstenberg's case the instrument in question was his probabilistic notion of Poisson boundary, which was to become a ubiquitous tool in the rigidity business [11]. Later in the early 1970s, as part of his deep work on lattices and arithmeticity, Margulis established his superrigidity theorem, demonstrating among other things that every finite-dimensional representation of an irreducible lattice in a connected semisimple Lie group G with finite centre and real rank at least two has a simple description in terms of the structure of G itself [17].

Although based on different technology, Margulis's normal subgroup theorem from the same period has a similar superrigidity flavour [16]. One form of the statement is that the normal subgroups of an irreducible lattice in a semisimple real Lie group of real rank at least two are either finite or of finite index, i.e., such lattices are "almost" simple. This is a particularly stark illustration of the principle of sociological separation up to finitary information, in this case within the category of discrete groups. While simplicity itself may not sound like an exceptional property from an abstract group theory point of view, what is remarkable here is the analytic-dynamical underpinning of the result, which reflects in a rather direct way the dichotomy between amenability and property (T) on which the proof hinges. This dichotomy represents a distillation of the deformation-versus-rigidity paradigm in its most fundamental group-theoretic terms: an amenable group, according to the characterization of Følner, is one which, like the prototypical $\mathbb{Z}^n$, admits finite sets which are approximately invariant under translation, a property which leads to a rich structure theory based on finite approximation (from Ornstein–Weiss quasitower decompositions and entropy theory in dynamics to hyperfiniteness and classification in operator algebras), while property (T), originally introduced by Kazhdan to show that many lattices (e.g., $\mathrm{SL}(3, \mathbb{Z})$) are finitely generated, requires that every unitary representation of the group admitting approximately invariant unit vectors has a genuinely invariant unit vector, a universal rigidity condition that has been discovered over the years to have many powerful and astonishing ramifications. The finiteness in Margulis's theorem boils down to the fact that only finite groups can both be amenable and have property (T). Underscoring the somewhat subtle analytic nature of the conclusion is the fact that many examples of the lattices in question, like $\mathrm{SL}(3, \mathbb{Z})$, admit subgroups of arbitrarily large finite index.

One of the great novelties in the work of Mostow and Margulis was the use of ideas and methods from ergodic theory, above all via boundary theory but also, for example, in the dynamical verification of amenability in the proof of the normal subgroup theorem. Zimmer's profound and far-reaching contribution to the conceptual development of this whole line of research was to have made ergodic theory itself the object of analysis, most decisively in the cocycle superrigidity theorem that he established in the late 1970s as a versatile generalization of Margulis's superrigidity [29]. An important part of this program was Zimmer's introduction of a version of amenability for actions, which, despite extending the ordinary notion of amenability for groups, becomes quite a different creature when applied to

actions of nonamenable groups, with crucial connections to Furstenberg's Poisson boundary and many applications outside of the rigidity framework [27, 28].

Given a measure-preserving action $G \curvearrowright (X, \mu)$ of a group on a standard probability space (*p.m.p. action* for short), a *cocycle* is a measurable map $\rho : G \times X \to H$ into another group satisfying the identity $\rho(g_1 g_2, x) = \rho(g_1, g_2 x)\rho(g_2, x)$. One natural source of cocycles are orbit equivalences between two free p.m.p. actions, i.e., measure isomorphisms that send orbits to orbits. Zimmer's cocycle superrigidity theorem concerns cocycles for ergodic actions when, among more general possible situations, the source and target groups are simple centreless Lie groups of real rank at least two. The conclusion in this case is that the cocycle, under most circumstances of interest, will be cohomologous in a natural sense to one that takes the form of an isomorphism between the groups, with no space dependence ("untwisting"). There is also a version for lattices, with the untwisting occurring within the ambient Lie groups. As a part of a more general pair of statements, Zimmer showed that from cocycle superrigidity one can derive orbit equivalence rigidity for the class of ergodic p.m.p. actions of the above mentioned Lie groups (i.e., orbit equivalence among two such actions implies isomorphism of the groups and conjugacy of the actions), as well as for certain families of lattice actions like the canonical ones of $\mathrm{SL}(n, \mathbb{Z})$ on $\mathbb{R}^n/\mathbb{Z}^n$ for $n \geq 2$ [29]. This should be contrasted with a theorem of Ornstein and Weiss which asserts that all free p.m.p. actions of countably infinite amenable groups are orbit equivalent [20]. Zimmer's orbit equivalence rigidity can also be framed in more geometric terms as a statement about the measure-theoretic structure of a foliation determining the Riemannian structure along the leaves, much in the spirit of Mostow rigidity.

As explained in the preface to the book, it took quite a bit of time before the impact of this work of Zimmer became fully manifest in the form of orbit equivalence superrigidity results, first by Furman in the late 1990s [8] and then a bit later by Monod and Shalom in the context of bounded cohomology [18] and by Popa in a framework inspired by von Neumann algebra theory that enabled a direct use of property (T) and related spectral gap conditions [21, 22]. Together with neighbouring streams of research in ergodic theory exploring orbit equivalence and related phenomena for actions of amenable and nonamenable groups (stemming from work of Dye in the 1950s and of Ornstein and Weiss in the 1970s and involving invariants like cost and ℓ^2-Betti numbers), a full-throttled embrace of the groupoid viewpoint advocated early on by Zimmer's doctoral advisor George Mackey in his virtual group formalism, and constantly replenishing connections to operator algebras that cycle all the way back to von Neumann, this has all coalesced into an area of mathematics that has been dubbed "measured group theory" [9, 12, 24].

In his introduction to the book, Zimmer gives an illuminating personal account of the gestation of his research program, driven as it was by his sense as a student that there was a whole domain of investigation waiting to be tapped at the interface of ergodic theory and the study of noncompact Lie groups and their discrete subgroups. On the heels of his cocycle superrigidity theorem, and motivated by its tantalizing geometric implications, Zimmer began to direct his attention to actions of Lie groups and their discrete subgroups on compact manifolds, which became the focus of a series of theorems and conjectures in the 1980s that launched what has come to be known as the "Zimmer program". This is a voluminous subject with many surprising twists and various tentacles to other spheres of activity,

including the rigidity theory of Anosov actions of higher rank Abelian groups pioneered by Katok and Spatzier [15]. The connection to cocycle superrigidity is that, in the context of volume-preserving actions on compact manifolds, it furnishes a mechanism for producing (in conjunction with other tools like property (T)) an invariant measurable Riemannian metric, whose existence implies subexponential derivative growth through the vanishing of Lyapunov exponents [30]. In the search for smooth rigidity results the focus thereby shifts towards the problem of finding invariant measures on the one hand and, on the other, promoting measurable metrics to smooth ones by leveraging the slow growth condition. As Zimmer discovered, the latter can be accomplished using property (T) when there is some control on the growth, and in particularly favourable circumstances one can even get property (T) to function as a replacement for cocycle superrigidity [31].

Encouraged by a theorem he had obtained on perturbations of isometric actions, Zimmer conjectured that volume-preserving actions of higher rank semisimple Lie groups and their lattices on compact manifolds should generally behave in an algebraically structured way, and in particular that actions of lattices should be isometric or even factor through a finite group when the dimension of the manifold is low enough. This served to lay out the basic coordinates of the Zimmer program, and can be viewed as a nonlinear version of the picture that issues from Margulis's superrigidity describing the finite-dimensional representation theory of lattices. In the 1980s Zimmer verified the conjecture in the presence of various additional geometric structures, like the smooth distal ones treated in [32]. As another example of progress from a somewhat different angle, Zimmer proved in [33] that, for smooth volume-preserving actions of a property (T) group on a compact manifold, the existence of an invariant measurable Riemannian metric implies that the action is compact (or, in older terminology, has discrete spectrum), which means that the associated unitary representation on the Hilbert space of L^2 functions decomposes into finite-dimensional subrepresentations. Combined with cocycle superrigidity technology, one could then infer that the actions appearing in the conjecture are compact, which can be interpreted as a measurable version of the conjecturally expected isometric behaviour, one that is strong enough to rule out mixing properties. Similar dynamical statements were established by Furman and Monod for a large class of products of property (T) groups [10], part of a trend starting in the late 1990s and early 2000s that has expanded the study of rigidity beyond lattices to groups of a more general nature. These and many other results in and around the Zimmer program, including influential ideas and results of Gromov concerning rigid geometric structures, are discussed in a comprehensive survey by David Fisher that was published a decade ago and is reprinted in this volume.

Despite all of the advances, strategies for resolving the general form of Zimmer's conjecture remained elusive at the time of Fisher's survey. The situation has changed dramatically, however, with a recent breakthrough of Brown, Fisher, and Hurtado, who proved among more general statements that, for $n > 2$, every smooth action of a cocompact lattice in $SL(n, \mathbb{R})$ on a compact manifold of dimension less than $n - 1$ factors through a finite quotient [2,3]. This achievement at the heart of the Zimmer program has given occasion to the additional text that Fisher has prepared as an afterward to the volume, a highly engaging personal narrative chronicling the pollination, percolation, and cross-fertilization of ideas that led to the results. The Brown–Fisher–Hurtado argument uses cocycle superrigidity in accord with Zimmer's basic template but makes an important shift of perspective

toward the problem of uniformizing estimates of subexponential derivative growth across invariant measures, a reorientation inspired by advances in the rigidity theory of Anosov $\mathbb{Z}^d$-actions. To produce invariant measures, the proof elaborates on the "nonresonance implies invariance" principle that had recently been developed by Brown, Rodriguez Hertz, and Wang [4], making use of ideas and results from measure rigidity that concern the algebraic nature of invariant measures in higher rank situations, including Ratner's theorems on unipotent flows and the deployment of Ledrappier and Young's work on entropy. The smooth metric whose existence leads to the conclusion (via a standard appeal to Margulis superrigidity) is obtained by a novel application of Lafforgue's strong property (T). The latter is defined in terms of an exponentially convergent averaging procedure and is used to absorb the subexponential derivative growth descending from superrigidity, mirroring a maneuver that had been made earlier in the ostensibly quite different framework of Anosov $\mathbb{Z}^d$-actions [7]. One of the most intriguing aspects of the whole theory surrounding the Zimmer program is the role of entropy-type growth conditions like those on display here. Entropy often serves as a kind of gauge for rigid behaviour, and indeed Zimmer ends his introduction with some speculative thoughts on the complementary relationship between entropy and arithmeticity for actions of higher rank simple Lie groups and the factor maps between them, a topic that he explored in the 2000s [34] and that seems ripe for revisiting in the wake of Brown, Fisher, and Hurtado.

The 38 papers of Zimmer collected in the volume under review are organized into eight thematic categories which, to some degree, are also chronological. Section 1 contains early work on the structure theory of p.m.p. actions. This includes the structure theorem itself for ergodic actions, which was inspired by the topological-dynamical structure theorem of Furstenberg and was also subsequently developed by Furstenberg to give his celebrated proof of Szemerédi's theorem on arithmetic progressions in positive density subsets of the integers. Another important result here is a dichotomy concerning the question of when the ergodicity of an action of a simple Lie group is inherited by a discrete subgroup, notable for its use of the Howe–Moore–Sherman mixing phenomenon for unitary representations of such groups. Section 2 is devoted to papers treating amenable actions and some of their diverse applications to Lie groups, operator algebras, and foliation theory. In Section 3 one finds the papers establishing cocycle superrigidity and some of its consequences for orbit equivalence rigidity among actions of Lie groups. The nine papers in Section 4 lay out the foundations of the Zimmer program on actions of Lie groups and their lattices on compact manifolds. Section 5 is devoted to a single paper with Stuck which shows that, for many semisimple Lie groups with property (T), the stabilizers of every faithful irreducible properly ergodic p.m.p. action are almost everywhere trivial, another expression of rigidity which is closely related to Margulis's normal subgroup theorem and has gained much currency over the last few years within the framework of invariant random subgroups. Section 6 bundles several papers on holonomy groups, fundamental groups, and arithmeticity, representing a branch of the Zimmer program that takes inspiration from Margulis's arithmeticity theorem and investigates, for example, the situation in which the fundamental group of a manifold on which a Lie group acts relates back to the acting group in a rigidly algebraic way. The articles in Section 7 explore rigidity in manifolds endowed with geometric structure, as expressed for example through their automorphism groups. Finally, the papers in Section 8, written in collaboration with Nevo, pursue the

idea of using stationary measures to study actions of semisimple Lie groups on compact manifolds, a setting in which the invariant measures guaranteed to exist for amenable acting groups are often absent.

This collection has been conceived and prepared with a considerable amount of care, and is a pleasure to peruse. If there is anything to fault the book with, although this really speaks to one of its virtues, it is that one is left yearning for additional commentary, in the same spirit and style, covering other aspects of the context and impact of this distinguished body of research. For those who have benefited in any number of ways from the insights and expansive reach of Zimmer's work, or those wishing to delve into a fascinating branch of mathematics at the crossroads of ergodic theory, Lie groups, topology, geometry, smooth dynamics, and operator algebras, this is a perfect opportunity to dig into some of the key sources. The book is also a compelling testament to the value of cultivating a personal vision, of having one's "own garden", to quote Alain Connes [14]. In Zimmer's case one is even tempted to see a teleology at work, but, to use Zimmer's own words, "this would be retrojection too grandiose and deterministic". What is ultimately on display here are the rewards of venturing bold questions about the way that fundamental, and even fundamentally different, structures interact with and illuminate each other, a pursuit that could even be said to epitomize what has become, through the revolutionary transformations that reset the basic parameters of the subject a little more than century ago, the modern mathematical enterprise.

References

[1] D. V. Anosov, *Geodesic flows on closed Riemannian manifolds of negative curvature* (Russian), Trudy Mat. Inst. Steklov. **90** (1967), 209. MR0224110

[2] A. Brown, D. Fisher, and S. Hurtado, *Zimmer's conjecture: subexponential growth, measure rigidity, and strong property (T)*, `arXiv:1608.04995` (2016).

[3] A. Brown, D. Fisher, and S. Hurtado, *Zimmer's conjecture for actions of* $\mathrm{SL}(m, \mathbb{Z})$, Invent. Math. **221** (2020), no. 3, 1001–1060, DOI 10.1007/s00222-020-00962-x. MR4132960

[4] A. Brown, F. Rodriguez Hertz, and Z. Wang, *Global smooth and topological rigidity of hyperbolic lattice actions*, Ann. of Math. (2) **186** (2017), no. 3, 913–972, DOI 10.4007/annals.2017.186.3.3. MR3702679

[5] E. Calabi, *On compact, Riemannian manifolds with constant curvature. I*, Proc. Sympos. Pure Math., Vol. III, American Mathematical Society, Providence, R.I., 1961, pp. 155–180. MR0133787

[6] E. Calabi and E. Vesentini, *On compact, locally symmetric Kähler manifolds*, Ann. of Math. (2) **71** (1960), 472–507, DOI 10.2307/1969939. MR111058

[7] D. Fisher, B. Kalinin, and R. Spatzier, *Global rigidity of higher rank Anosov actions on tori and nilmanifolds*, J. Amer. Math. Soc. **26** (2013), no. 1, 167–198, DOI 10.1090/S0894-0347-2012-00751-6. With an appendix by James F. Davis. MR2983009

[8] A. Furman, *Orbit equivalence rigidity*, Ann. of Math. (2) **150** (1999), no. 3, 1083–1108, DOI 10.2307/121063. MR1740985

[9] A. Furman, *A survey of measured group theory*, Geometry, rigidity, and group actions, Chicago Lectures in Math., Univ. Chicago Press, Chicago, IL, 2011, pp. 296–374. MR2807836

[10] A. Furman and N. Monod, *Product groups acting on manifolds*, Duke Math. J. **148** (2009), no. 1, 1–39, DOI 10.1215/00127094-2009-018. MR2515098

[11] H. Furstenberg, *Poisson boundaries and envelopes of discrete groups*, Bull. Amer. Math. Soc. **73** (1967), 350–356, DOI 10.1090/S0002-9904-1967-11748-8. MR210812

[12] D. Gaboriau, *Orbit equivalence and measured group theory*, Proceedings of the International Congress of Mathematicians. Volume III, Hindustan Book Agency, New Delhi, 2010, pp. 1501–1527. MR2827853

[13] A. Ioana, S. Popa, and S. Vaes, *A class of superrigid group von Neumann algebras*, Ann. of Math. (2) **178** (2013), no. 1, 231–286, DOI 10.4007/annals.2013.178.1.4. MR3043581

[14] A. Jackson, *Interview with A. Connes*, Celebratio Mathematica, 2021.

[15] A. Katok and R. J. Spatzier, *First cohomology of Anosov actions of higher rank abelian groups and applications to rigidity*, Inst. Hautes Études Sci. Publ. Math. **79** (1994), 131–156. MR1307298

[16] G. A. Margulis, *Factor groups of discrete subgroups and measure theory* (Russian), Funktsional. Anal. i Prilozhen. **12** (1978), no. 4, 64–76. MR515630

[17] G. A. Margulis, *Discrete subgroups of semisimple Lie groups*, Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)], vol. 17, Springer-Verlag, Berlin, 1991, DOI 10.1007/978-3-642-51445-6. MR1090825

[18] N. Monod and Y. Shalom, *Orbit equivalence rigidity and bounded cohomology*, Ann. of Math. (2) **164** (2006), no. 3, 825–878, DOI 10.4007/annals.2006.164.825. MR2259246

[19] G. D. Mostow, *Quasi-conformal mappings in n-space and the rigidity of hyperbolic space forms*, Inst. Hautes Études Sci. Publ. Math. **34** (1968), 53–104. MR236383

[20] D. S. Ornstein and B. Weiss, *Ergodic theory of amenable group actions. I. The Rohlin lemma*, Bull. Amer. Math. Soc. (N.S.) **2** (1980), no. 1, 161–164, DOI 10.1090/S0273-0979-1980-14702-3. MR551753

[21] S. Popa, *Cocycle and orbit equivalence superrigidity for malleable actions of w-rigid groups*, Invent. Math. **170** (2007), no. 2, 243–295, DOI 10.1007/s00222-007-0063-0. MR2342637

[22] S. Popa, *On the superrigidity of malleable actions with spectral gap*, J. Amer. Math. Soc. **21** (2008), no. 4, 981–1000, DOI 10.1090/S0894-0347-07-00578-4. MR2425177

[23] A. Selberg, *On discontinuous groups in higher-dimensional symmetric spaces*, Contributions to function theory (Internat. Colloq. Function Theory, Bombay, 1960), Tata Institute of Fundamental Research, Bombay, 1960, pp. 147–164. MR0130324

[24] Y. Shalom, *Measurable group theory*, European Congress of Mathematics, Eur. Math. Soc., Zürich, 2005, pp. 391–423. MR2185757

[25] A. Weil, *On discrete subgroups of Lie groups*, Ann. of Math. (2) **72** (1960), 369–384, DOI 10.2307/1970140. MR137792

[26] A. Weil, *On discrete subgroups of Lie groups. II*, Ann. of Math. (2) **75** (1962), 578–602, DOI 10.2307/1970212. MR137793

[27] R. J. Zimmer, *Amenable ergodic actions, hyperfinite factors, and Poincaré flows*, Bull. Amer. Math. Soc. **83** (1977), no. 5, 1078–1080, DOI 10.1090/S0002-9904-1977-14392-9. MR460598

[28] R. J. Zimmer, *Amenable ergodic group actions and an application to Poisson boundaries of random walks*, J. Functional Analysis **27** (1978), no. 3, 350–372, DOI 10.1016/0022-1236(78)90013-7. MR0473096

[29] R. J. Zimmer, *Strong rigidity for ergodic actions of semisimple Lie groups*, Ann. of Math. (2) **112** (1980), no. 3, 511–529, DOI 10.2307/1971090. MR595205

[30] R. J. Zimmer, *Volume preserving actions of lattices in semisimple groups on compact manifolds*, Inst. Hautes Études Sci. Publ. Math. **59** (1984), 5–33. MR743815

[31] R. J. Zimmer, *Kazhdan groups acting on compact manifolds*, Invent. Math. **75** (1984), no. 3, 425–436, DOI 10.1007/BF01388637. MR735334

[32] R. J. Zimmer, *Lattices in semisimple groups and distal geometric structures*, Invent. Math. **80** (1985), no. 1, 123–137, DOI 10.1007/BF01388551. MR784532

[33] R. J. Zimmer, *Spectrum, entropy, and geometric structures for smooth actions of Kazhdan groups*, Israel J. Math. **75** (1991), no. 1, 65–80, DOI 10.1007/BF02787182. MR1147291

[34] R. J. Zimmer, *Entropy and arithmetic quotients for simple automorphism groups of geometric manifolds*, Geom. Dedicata **107** (2004), 47–56, DOI 10.1023/B:GEOM.0000049088.17279.57. MR2110753

DAVID KERR

MATHEMATISCHES INSTITUT
WWU MÜNSTER, EINSTEINSTR. 62
48149 MÜNSTER, GERMANY