CONTEMPORARY MATHEMATICS

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Zeta Functions in Algebra and Geometry

Second International Workshop
May 3–7, 2010
Universitat de les Illes Balears,
Palma de Mallorca, Spain

Antonio Campillo
Gabriel Cardona
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2000 *Mathematics Subject Classification.* Primary 11F66, 11G25, 11L07, 11G50, 14D10, 14E18, 14G40, 22E50, 32S40, 57M27.

Library of Congress Cataloging-in-Publication Data

International Workshop on Zeta Functions in Algebra and Geometry (2nd : 2010 : Universitat de Les Illes Balears)

Zeta functions in algebra and geometry : second International Workshop on Zeta Functions in Algebra and Geometry, May 3–7, 2010, Universitat de Les Illes Balears, Palma de Mallorca, Spain / Antonio Campillo ... [et al.], editors.

p. cm. — (Contemporary Mathematics ; v. 566)

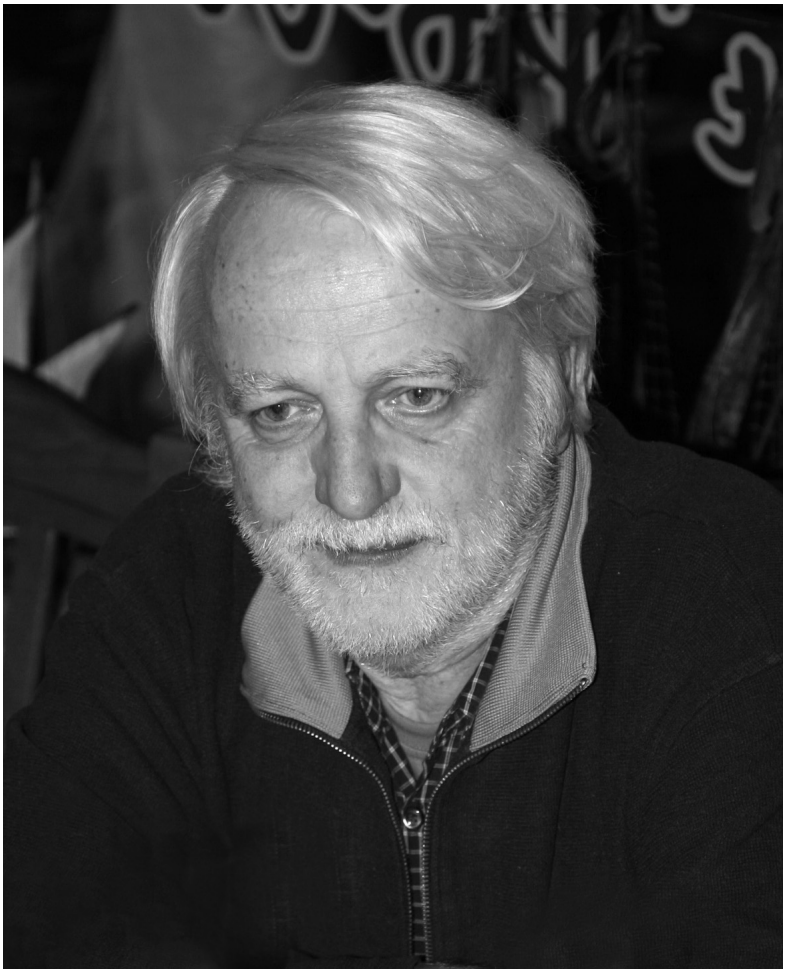
Includes bibliographical references.

ISBN 978-0-8218-6900-0 (alk. paper)

1. Functions, Zeta–Congresses. 2. Geometry, Algebraic–Congresses. 3. Algebraic varieties–Congresses. I. Campillo, Antonio, 1953– II. Title.

QA351.I58 2010
515′.56–dc23 2011050434

This volume is dedicated to Fritz Grunewald.



Fritz Grunewald 1949–2010

Contents

Preface

The present volume reflects the contents of the talks and some additional contributions given at the "*Second International Workshop on Zeta Functions in Algebra and Geometry*" held at Universitat de les Illes Balears, Palma de Mallorca, Spain, from May 3rd to May 7th, 2010.

Zeta functions can be attached to several mathematical objects like fields, groups, algebras, functions, and dynamical systems. Typically, zeta functions encode relevant arithmetic, algebraic, geometric or topological information about the original object. The conference was focused on the following topics:

(1) Arithmetic and geometric aspects of local, topological and motivic zeta functions,
(2) Poincaré series of valuations,
(3) Zeta functions of groups, rings and representations,
(4) Prehomogeneous vector spaces and their zeta functions,
(5) Height zeta functions.

Local zeta functions were introduced by A. Weil in the sixties and have been extensively studied by J.-I. Igusa, J. Denef and F. Loeser, among others. More recently, using ideas of motivic integration due to M. Kontsevich, a generalization of these functions, called motivic zeta functions, was introduced by Denef and Loeser. All these functions contain geometric, topological, and arithmetic information about mappings defined over local (and other) fields. In close terms, recently T. Hales discovered a motivic nature on integrals which play a central role in the Langlands program.

Using integration over spaces of functions in a spirit similar to motivic integration, A. Campillo, F. Delgado and S. M. Gusein-Zade study Poincaré series of some filtrations on the ring of germs of holomorphic functions of a singularity and its geometric and topological applications. In particular, unexpected connections relating valuation theory with zeta functions have been obtained.

M. du Sautoy and F. Grunewald, among others, have studied extensively zeta functions of groups which were introduced originally as potentially new invariants in attempts to understand the difficult problem of classifying infinite nilpotent groups. Recently du Sautoy has found that these zeta functions are an important tool in trying to understand the problem of classifying the wild class of finite p-groups.

Prehomogeneous vector spaces and their zeta functions were introduced by M. Sato, and have been studied extensively by T. Shintani, M. Kashiwara, F. Sato, T. Kimura, and A. Gyoja, among others. These spaces play a central role in the stunning generalization of Gauss's composition laws obtained by M. Bhargava.

The distribution of rational points of bounded height of smooth varieties over global fields is related to convergence properties of height zeta functions and estimated by the Batyrev-Manin conjecture and refinements. Current work by E. Peyre, Y. Tschinkel and A. Chambert-Loir, among others, provides extensive study and progress on the subject.

We organized the contributed papers into four parts. Part I, "*L-functions of varieties over finite fields and Artin L-functions*", contains the contributions of Pilar Bayer, Anne Frühbis-Krüger and Shabnam Kadir, and Antonio Rojas-León. Part II, "*Height zeta functions and arithmetic*", contains the contributions of Driss Essouabri, Yuri I. Manin and Sho Tanimoto and Yuri Tschinkel. Part III, "*Motivic zeta functions, Poincaré series, complex monodromy and knots*", contains the contributions of Nero Budur, Carlos Galindo and Francisco Monserrat, Evgeny Gorsky, Lars Halvard Halle and Johannes Nicaise, and András Némethi and Fernando Román. Part IV, "*Zeta functions for groups and representations*", contains the contributions of Nir Avni, Benjamin Klopsch, Uri Onn, and Christopher Voll, and Aner Shalev.

We now describe briefly the content of the articles forming this volume. There are contributions which are expository papers in each of the parts. Pilar Bayer's article discusses Artin L-functions of Galois representations of dimension 2 which is perfectly inserted in a complete historical context of the Artin conjecture.

The article of Tanimoto and Tschinkel surveys recent partial progress towards a proof of the Manin conjecture for equivariant compactifications of solvable algebraic groups. They use height zeta functions to study the asymptotic distribution of rational points of bounded height on projective equivariant compactifcations of semi-direct products.

The article of Budur is an excellent survey of some analytic invariants of singularities, mostly those ones which are related with the log canonical threshold, spectra associated with the mixed Hodge structure on the vanishing cohomology of Milnor fibers, multiplier ideals and jumping numbers, different zeta functions (monodromy zeta function, topological zeta function, Denef-Loeser motivic zeta function), and different versions of the Bernstein-Sato b-polynomial.

Related with this problem are the *monodromy conjectures*. What are *monodromy conjectures*? The answer to this question is the heart of the survey article of Halle and Nicaise. They give a very readily guide on new directions opened by a still mysterious conjecture formulated by Jun-Ichi Igusa (on p-adic integrals), that however seems quite natural from the point of view of the developments of algebraic geometry (conjecture of Borevich-Shafarevich, Weil conjectures, ...). They also give a new definition for motivic zeta functions of Calabi-Yau varieties over a complete discretely valued field in terms of analytic rigid geometry and base changes and proved that they verify a very precise global version of the monodromy conjecture.

Valuations are considered here in the context of singularity theory, which is one of the main sources of valuation theory as well as a research area in which valuations are an essential tool. The article of Galindo and Montserrat provides a concise survey of some aspects of the theory plane valuations offering a valuable view of the whole set and the current status of some of the top research problems.

Shalev's survey article gives an overview over a number of results on applications of certain zeta functions associated with groups to several topics including random generation, random walks on groups and commutator width.

The aim of the article of Frühbis-Krüger and Kadir is to give numerical examples to the conjectured change in degree of the zeta function for singular members of families of Calabi-Yau varieties over finite fields which are deformations of Fermat varieties.

Rojas-Leon's article contains some interesting and significant improvements to the classical Weil estimates for trigonometric sums associated to polynomials in one variable by utilizing Deligne-Katz-Laumon methods based on the local Fourier transform.

The main goal of Essouabri's article is to understand the asymptotic behavior of the number of rational points on Zariski open subsets of toric varieties in $\mathbb{P}^n(\mathbb{Q})$.

Manin's article lies at the interface of Diophantine geometry and model theory. The Manin's goal is: given certain combinatorial data about the set of K-rational points on a projective cubic surface defined over K, is to reconstruct the definition field K and the equation of the surface. The approach of the paper is based on Zilber's well-known reconstructions of algebraic geometry using model theory, but here one is not working over algebraically closed fields.

Heegard–Floer homology was introduced by Ozsváth and Szabó as a tool to understand Seiberg-Witten invariants of 3-manifold. Némethi and Román present a computation of the lattice cohomology of a special, but very important for singularity theory applications, class of 3-manifolds: they are obtained by surgery on an algebraic link in the 3-dimensional sphere. Lattice cohomology is a combinatorial construction starting from the plumbing graph of a manifold, which leads to certain cohomology groups. The connections with Seiberg–Witten invariants and Heegard–Floer theory are also presented.

Gorsky's article offers several very interesting conjectures related with homologies of torus knots $T_{n;m}$ using the combinatorics of $q;t$-Catalan numbers and their (several) generalizations.

The article of Avni, Klopsch, Onn and Voll is focused on the study of zeta functions associated to representations of some compact p-adic analytic groups by means of the Kirillov's orbit method, Clifford theory and p-adic integration.

The sponsors of Palma de Mallorca's Workshop include the Fundation for Scientific Research - Flanders (FWO), the Spanish Ministerio de Ciencia e Innovación, the local Govern de les Illes Balears, the Junta de Castilla y León, the Consell de Mallorca, the Ajuntament de Palma, the Caixa de Balears, the program Ingenio Mathematica, the Unversities Complutense de Madrid (UCM), Illes Balears (UIB) and Valladolid (UVA) and the Departament de Ciències Matemàtiques i Informàtica (UIB). We thank all of them and we also thank the American Mathematical Society (AMS) and the Real Sociedad Matemática Española (RSME) for agreeing to publish this volume as one of their common publications.

We finally want to thank all organizations and people that helped in organizing the conference and editing the proceedings, among others, the members of the Local Organizing Committee: Ll. Huguet (Chair), A. Campillo, G. Cardona, M. González–Hidalgo, A. Mir (Spain), the members of the Organizing Committee A. Melle-Hernández (Spain), W. Veys (Belgium), W. A. Zúñiga-Galindo (México)

and the members of the Scientific Committee: A. Campillo (Spain), J. Denef (Belgium), F. Grunewald (Germany), S. M. Gusein-Zade (Russia), M. Larsen (USA), I. Luengo (Spain), Y. Tshinkel (USA), A. Yukie (Japan).

A short time before our workshop in Palma de Mallorca started, we heared the unexpected and sad news that Fritz Grunewald passed away. As leading specialist in the study of zeta functions in algebra, he was a distinguished speaker at the first edition of our "*International Workshop on Zeta Functions in Algebra and Geometry*" held in Segovia, Spain, in June 2007. Actually, he was very enthusiastic about that initiative and he immediately accepted to be a member of the scientific committee for the second edition in Palma. In that role, he was a great help for us. In fact, we still exchanged mails about the organization of the workshop few days before his decease.

During the first day of the workshop, the lectures of Dan Segal and Alex Lubotzky were in honour of Fritz: outstanding mathematician, extraordinary person and fantastic friend. This is indeed how we will remember him.

Antonio Campillo
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CONM 566

Zeta Functions in Algebra and Geometry • Campillo et al., Editors

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ISBN 978-0-8218-6900-0

9 780821 869000

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