# **CONTEMPORARY MATHEMATICS**

## 710

## Mathematical Analysis in Fluid Mechanics Selected Recent Results

International Conference on Vorticity, Rotation and Symmetry (IV)—Complex Fluids and the Issue of Regularity May 8–12, 2017 Centre International de Rencontres Mathématiques (CIRM), Luminy, Marseille, France

> Raphaël Danchin Reinhard Farwig Jiří Neustupa Patrick Penel Editors



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### Preface

Since the seminal paper by Jean Leray in 1934 on global in time existence of weak (turbulent) solutions to the Navier-Stokes system mathematical fluid mechanics has been a fast growing field based on the theory of partial differential equations, linear and nonlinear functional analysis and harmonic analysis. Some of the open problems of J. Leray's time could be solved because of his paper, but most of them are still open and led to one of the seven Millennium Prize problems of the Clay Mathematics Institute in 2000. Moreover, models describing the behaviour of non-Newtonian fluids such as blood and polymeric flows, mixtures of fluids, compressible and inviscid fluids and interaction with other physical impacts opened a fast field of problems requiring new analytical tools.

We selected 13 out of more than 35 talks that are representative of the great variety of topics that have been tackled during the international conference Vorticity, Rotation and Symmetry (IV)—Complex Fluids and the Issue of Regularity held at the Centre International de Rencontres Mathématiques (CIRM) in Luminy (Marseille, France), May 8–12, 2017. Four articles deal with the classical problem of regularity of solutions to the (Navier-)Stokes system, e.g., see the articles by K. Nakao & Y. Taniuchi (Serrin type regularity criterion using logarithmic Brezis-Gallouet-Wainger type inequalities), D. Chamorro, P.G. Lemarié-Rieusset & K. Mayoufi discussing an extension of the Caffarelli–Kohn–Nirenberg theory for weak solutions to the case with no control on the pressure, P. Maremonti proving an interpolation estimate for the  $L^{\infty}$ -norm for the Stokes problem, as well as H. Kozono & S. Shimizu on strong solutions and singular data in Lorentz/Besov spaces. Periodic solutions and their asymptotic profile are in the focus of an article by M. Kyed & G.P. Galdi, and D. Maity & M. Tucsnak extend the notion of maximal regularity to fluid-rigid body interaction problems. Two-phase flows with sharp interface governed by a convective mean curvature flow equation are analyzed by H. Abels & M. Moser; another important auxiliary regularity problem on elliptic equations with drift is investigated by N. Filonov and T. Shilkin. Two articles deal with compressible fluids. The first is by T. Piasecki and M. Pokorný on chemically reacting heat conducting compressible mixtures in the steady case and the second is by H. J. Choe and M. Yang on a blow up criterion for unsteady solutions involving the density and the divergence of the velocity field. A class of complex fluids with a simplified viscoelastic rate-type model with a stress-diffusion term is discussed by M. Bulíček, J. Málek, V. Průša and E. Süli; the authors prove the existence of a global in time weak solution. Electrical interation, magnetic and gravitational forces are crucial ingredients in the articles by Th. Holding & E. Miot on uniqueness and stability for the Vlasov-Poisson system, and by W. Deng & P. Zhang on

#### PREFACE

global smooth solutions to the incompressible MHD system with initial data close to an equilibrium state.

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The Editors

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This volume contains the proceedings of the International Conference on Vorticity, Rotation and Symmetry (IV)—Complex Fluids and the Issue of Regularity, held from May 8–12, 2017, in Luminy, Marseille, France.

The papers cover topics in mathematical fluid mechanics ranging from the classical regularity issue for solutions of the 3D Navier-Stokes system to compressible and non-Newtonian fluids, MHD flows and mixtures of fluids. Topics of different kinds of solutions, boundary conditions, and interfaces are also discussed.





