CONTEMPORARY MATHEMATICS

739

Centre de Recherches Mathématiques Proceedings

Probabilistic Methods in Geometry, Topology and Spectral Theory

CRM Workshops

Probabilistic Methods in Spectral Geometry PDE
August 22–26, 2016

Probabilistic Methods in Topology
November 14–18, 2016
Centre de Recherches Mathématiques,
Université de Montréal, Québec, Canada

Yaiza Canzani
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2010 *Mathematics Subject Classification.* Primary 05C80, 11F72, 33C55, 35P20, 58J51, 58J65, 60C05, 60G15, 60G60, 81Q50.

Library of Congress Cataloging-in-Publication Data

Names: Canzani, Yaiza, 1987- editor. | Chen, Linan, 1984- editor. | Jakobson, Dmitry, 1970- editor.

Title: Probabilistic methods in geometry, topology, and spectral theory / Yaiza Canzani, Linan Chen, Dmitry Jakobson, editors.

Description: Providence, Rhode Island : American Mathematical Society ; Montreal, Quebec, Canada : Centre de Recherches Mathematiques, 2019. | Series: Contemporary mathematics, 0271-4132 ; volume 739 | "CRM workshop on probabilistic methods in spectral geometry and PDE, August 22-26, 2016 [and] probabilistic methods in topology, November 22-26, 2016." | Includes bibliographical references.

Identifiers: LCCN 2019023076 | ISBN 9781470441456 (paperback)

Subjects: LCSH: Mathematical physics–Congresses. | Probabilities–Congresses. | Geometric analysis–Congresses. | Topology–Congresses. | Spectral theory (Mathematics)–Congresses. | AMS: Combinatorics {For finite fields, see 11Txx} – Graph theory {For applications of graphs, see 68R10, 81Q30, 81T15, 82B20, 82C20, 90C35, 92E10, 94C15} – Random graphs [See also 60B20]. | Special functions (33-XX deals with the properties of functions as functions) {For orthogonal functions, see 42Cxx; for aspects of combinatorics see 05Axx; for number-theoretic aspects see 11-XX; for} | Partial differential equations – Spectral theory and eigenvalue problems [See also 47Axx, 47Bxx, 47F05] – Asymptotic distribution of eigenvalues and eigenfunctions. | Global analysis, analysis on manifolds [See also 32Cxx, 32Fxx, 32Wxx, 46-XX, 47Hxx, 53Cxx] {For geometric integration theory, see 49Q15} – Partial differential equations on manifolds; differential op | Probability theory and stochastic processes {For additional applications, see 11Kxx, 62-XX, 90-XX, 91-XX, 92-XX, 93-XX, 94-XX} – Combinatorial probability – Combinatorial probability. | Probability theory and stochastic processes {For additional applications, see 11Kxx, 62-XX, 90-XX, 91-XX, 92-XX, 93-XX, 94-XX} – Stochastic processes – Gaussian processes. | Probability theory and stochastic processes {For additional applications, see 11Kxx, 62-XX, 90-XX, 91-XX, 92-XX, 93-XX, 94-XX} – Stochastic processes – Random fields. | Quantum theory – General mathematical topics and methods in quantum theory – Quantum chaos [See also 37Dxx].

Classification: LCC QC20 .P7564 2019 | DDC 519.2–dc23

LC record available at https://lccn.loc.gov/2019023076

DOI: https://doi.org/10.1090/conm/739

10 9 8 7 6 5 4 3 2 1 24 23 22 21 20 19

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Preface

Thematic Semester on Probabilistic Methods in Geometry, Topology and Spectral Theory was held at Centre de Recherches Mathématiques in Montreal from August until December 2016. Probabilistic methods have played an increasingly important role in many areas of mathematics, from the study of random groups and random simplicial complexes in topology, to the theory of random Schrödinger operators in mathematical physics. The Thematic Semester at CRM (organized by the CRM Mathematical Analysis Laboratory and the CRM Probability Laboratory) included five intensive one-week workshops: *Frontiers in Mathematical Physics* workshop in Honour of Barry Simon's 70th Birthday; *Probabilistic Methods in Spectral Geometry and PDE* held on August 22–26, 2016; *Random Growth Problems and Random Matrices*; *Probabilistic Methods in Dynamical Systems and Applications*; and *Probabilistic Methods in Topology,* held on November 14-18. Workshops on Probabilistic Methods in Spectral Geometry and PDE, and on Probabilistic Methods in Topology are described below in more detail. The thematic semester featured an emphasis on interconnections and cross-fertilization of ideas between these topics, leading to new investigations and fruitful collaborations between participants. There were three Aisenstadt chairs during the semester: Yuval Peres, Scott Sheffield and Nalini Anantharaman. Nalini Anantharaman gave her lectures during the workshop on Probabilistic Methods in Spectral Geometry and PDE, whose participants contributed many papers to this volume. Other papers in this volume are related to the workshop on Probabilistic Methods in Topology, which included a series of introductory lectures by Matthew Kahle, and a lecture by Misha Gromov.

The workshop on Probabilistic Methods in Spectral Geometry and PDE was organized by Yaiza Canzani (Harvard), Linan Chen (McGill), Dmitry Jakobson (McGill), Armen Shirikyan (Cergy-Pontoise), Lior Silberman (UBC) and John A. Toth (McGill). The workshop brought together some of the leading researchers in quantum chaos, semi-classical theory, ergodic theory and dynamical systems, partial differential equations, probability, random matrix theory, mathematical physics, conformal field theory, and random graph theory. Its emphasis was on the use of ideas and methods from probability in different areas, such as quantum chaos (study of spectra and eigenstates of chaotic systems at high energy); geometry of random metrics and related problems in quantum gravity; solutions of partial differential equations with random initial conditions. The workshop was part of a series of related workshops held at the CRM and elsewhere, including a workshop on infinite-dimensional geometry (MSRI, December 2013), a workshop on the geometry of eigenvalues and eigenfunctions, and a workshop on manifolds of metrics and probabilistic methods in geometry and analysis (both held at the CRM in 2012).

Many of the talks concerned delocalization (equidistribution) of eigenfunctions at high energy, or the so-called quantum ergodicity. In particular this was the main topic of Nalini Anantharaman's first two Aisenstadt lectures. Quantum ergodicity is also the subject of three papers in the current volume.

In the paper *Quantum Limits of Eisenstein Series in* $\mathbb{H}^3$*,* Niko Laaksonen studies the quantum limits of Eisenstein series off the critical line for $\mathrm{PSL}_2(\mathcal{O}_K)$ where K is an imaginary quadratic field of class number one. He generalizes previous results of Petridis, Raulf and Risager on $\mathrm{PSL}_2(\mathbb{Z})\backslash\mathbb{H}^2$.

In the paper *Observability and Quantum Limits for the Schrödinger Equation on* $\mathbb{S}^d$*,* Fabricio Macià and Gabriel Rivière describe their recent results on semiclassical measures for the Schrödinger evolution on Zoll manifolds. The authors focus on the particular case of eigenmodes of the Schrödinger operator on the sphere endowed with its canonical metric. They also explore the relation of this problem with the observability question from control theory.

In the paper *Entropy Bounds and Quantum Unique Ergodicity for Hecke Eigenfunctions on Division Algebras,* Lior Silberman and Akshay Venkatesh prove the Arithmetic Quantum Unique Ergodicity conjecture for non-degenerate sequences of Hecke eigenfunctions on quotients $\Gamma\backslash G/K$, where $G \simeq PGL_d(\mathbb{R})$, K is a maximal compact subgroup of G and $\Gamma < G$ is a lattice associated to a division algebra over $\mathbb{Q}$ of prime degree d. More generally, they introduce a new method of proving positive entropy of quantum limits, which applies to higher-rank groups.

Several talks at the workshop were devoted to the study of "random" eigenfunctions; that subject was considered in three papers in this volume.

Linan Chen and Na Shu contributed the paper *A Geometric Treatment of Log-correlated Gaussian Fields.* One way to regularize a log-correlated Gaussian free field (GFF) is to consider (functionals of) its spherical averages. In even dimensions, this regularization approach has been adopted in the construction of the Liouville Quantum Gravity (LQG) measure and the proof of the Knizhnik-Polyakov-Zamolodchikov (KPZ) formula. In this article, the authors combine the Fourier-Bessel expansion with the spherical averages of the GFF to extend such a regularization approach to treat log-correlated GFFs in odd dimensions. In particular, the authors established the existence of the LQG measure and the KPZ formula under this setting.

In the paper *Tangent Nodal Sets for Random Spherical Harmonics,* Suresh Eswarathasan considers a fixed vector field V on $\mathbb{S}^2$ and studies the distribution of points which lie on the nodal set (of a random spherical harmonic) where V is also tangent. He shows that the expected value of the corresponding counting function is asymptotic to the eigenvalue with a leading coefficient that is independent of the vector field V. This demonstrates, in some form, a universality for vector fields up to lower order terms.

In the survey paper *Random Nodal Lengths and Wiener Chaos,* Maurizia Rossi collects some of the recent results on the "nodal geometry" of random eigenfunctions on Riemannian surfaces. She focuses on the asymptotic behaviour, for high energy levels, of the nodal length of Gaussian Laplace eigenfunctions on the torus (arithmetic random waves) and on the sphere (random spherical harmonics). She gives some insight on both Berry's cancellation phenomenon and the nature of nodal length second order fluctuations (non-Gaussian on the torus and Gaussian on the sphere) in terms of chaotic components. Finally she considers the general

case of monochromatic random waves, i.e. Gaussian random linear combination of eigenfunctions of the Laplacian on a compact Riemannian surface with frequencies from a short interval, whose scaling limit is Berry's Random Wave Model.

Another subject discussed at the workshop was the study of random metrics and random maps. This is related to two papers in the current volume: the paper of Linan Chen and Na Shu described earlier; and the paper contributed by V. Konakov, S. Menozzi and S. Molchanov, titled *The Brownian Motion on $Aff(\mathbb{R})$ and Quasi-Local Theorems.* This paper is concerned with Random walk approximations of the Brownian motion on the Affine group $Aff(\mathbb{R})$. The authors are in particular interested in the case where the innovations are discrete. In this framework, the return probabilities of the walk have fractional exponential decay in large time, as opposed to the polynomial one of the continuous object. The authors prove that integrating those return probabilities on a suitable neighbourhood of the origin, the expected polynomial decay is restored.

The workshop on Probabilistic Methods in Topology was the last workshop held during the thematic semester on Probabilistic Methods in Geometry, Topology and Spectral Theory at CRM. It was organized by D. Wise (McGill), M. Pichot (McGill), L. Silberman (UBC), P. Przytycki (McGill), I. Rivin (Temple), A. Nabutovsky (Toronto), M. Kahle (Ohio State). It brought together researchers working on random simplicial complexes and geometry of spaces of triangulations (with connections to manifold learning); topological statistics, and geometric probability; theory of random groups and their properties; random knots; and other problems.

Many talks at the workshop concerned the study of random graphs, random simplicial complexes and their properties, including the talks by Kahle, Bubenik, Addario-Berry, Abu Fraiha, Bobrowski, Behrstock, Lishak, Farber, Peled and Luczak. That was also the subject of the last Aisenstadt lecture by Nalini Anantharaman, who discussed asymptotic equidistribution of Laplace eigenvectors on large graphs. Two of the papers in the current volume are devoted to the study of random graphs.

In the paper *Formal Zeta Function Expansion and the Frequency of Ramanujan Graphs,* Joel Friedman shows that logarithmic derivative of the Zeta function of any regular graph is given by a power series about infinity whose coefficients are given in terms of the traces of powers of the graph's Hashimoto matrix. He then makes a formal calculation that suggests that for fixed d there is an $f(d) > 1/2$ such that a d-regular graph on n vertices is Ramanujan with probability at least $f(d)$ for n sufficiently large. A related argument for random covering graphs of degree n over a fixed, regular "base graph" suggests that for n large, a strict majority of random covering graphs are relatively Ramanujan.

The talks by Rivin and Even-Zohar concerned the study of random knots. An important tool in the study of knots are knot polynomials. The study of those polynomials is related to the study of Tutte (dichromatic) polynomials of graphs; and to the study of Bollobás-Riordan polynomials of *ribbon graphs* (graphs that are cellularly embedded into surfaces). In their paper *Rank and Bollobás-Riordan Polynomials: Coefficient measures and zeros*, D. Jakobson, T. Langsetmo, I. Rivin and L. Turner discuss some (numerical and theoretical) results about the coefficients and zeros of Tutte (dichromatic) polynomial of graphs of bounded degree whose size increases. In particular, certain natural coefficient measures converge to a delta-function, provided that the corresponding graphs converge in the sense

of Benjamini-Schramm. The authors also establish related results for Bollobás-Riordan polynomials for large ribbon graphs.

Yaiza Canzani
Linan Chen
Dmitry Jakobson

The Centre de Recherches Mathématiques (CRM) was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM receives funding from the Natural Sciences and Engineering Research Council (Canada), the FRQNT (Québec), the NSF (USA), and its partner universities (Université de Montréal, McGill, UQAM, Concordia, Université Laval, Université de Sherbrooke and University of Ottawa).

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This volume contains the proceedings of the CRM Workshops on Probabilistic Methods in Spectral Geometry and PDE, held from August 22–26, 2016 and Probabilistic Methods in Topology, held from November 14–18, 2016 at the Centre de Recherches Mathématiques, Université de Montréal, Montréal, Quebec, Canada.

Probabilistic methods have played an increasingly important role in many areas of mathematics, from the study of random groups and random simplicial complexes in topology, to the theory of random Schrödinger operators in mathematical physics.

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The workshop Probabilistic Methods in Topology brought together researchers working on random simplicial complexes and geometry of spaces of triangulations (with connections to manifold learning); topological statistics, and geometric probability; theory of random groups and their properties; random knots; and other problems.

This volume covers recent developments in several active research areas at the interface of Probability, Semiclassical Analysis, Mathematical Physics, Theory of Automorphic Forms and Graph Theory.

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ISBN 978-1-4704-4145-6

CONM/739

AMS/CRM