CONTEMPORARY MATHEMATICS

753

Vertex Operator Algebras, Number Theory and Related Topics

International Conference
Vertex Operator Algebras, Number Theory and Related Topics
June 11–15, 2018
California State University, Sacramento, California

Matthew Krauel
Michael Tuite
Gaywalee Yamskulna
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Preface

An international conference titled "Vertex operator algebras, number theory, and related topics" was held June 11–15th, 2018, at California State University, Sacramento. The conference honored the 70th birthday of Geoffrey Mason, and brought together researchers from around the world to share and discuss recent advancements and problems surrounding automorphic forms, Hopf algebras, vertex algebras, and other topics intersecting Mason's illustrious career.

The history of modular forms can be traced back to the early 19th century, and includes significant contributions from such mathematical luminaries as Abel, Dirichlet, Eisenstein, Gauss, Jacobi, Klein, Kronecker, Poincaré, and Weierstrass; just to name a few from that century. Many others have left their mark since, and the study of modular—and more generally automorphic—forms continues to be an active area that attracts significant attention and interest.

Vertex algebras, on the other hand, were introduced in 1986 by Richard Borcherds, and the enhanced structure of a vertex operator algebra was brought about by Igor Frenkel, James Lepowsky, and Arne Meurman in 1988. One of the original motivations for developing the concept of a vertex operator algebra was its use in solving the Monstrous Moonshine conjecture, which mysteriously linked group theory with modular forms. This active area was of great interest to mathematicians, and came about just as the classification of finite simple groups was being completed.

Like others during this time, Monstrous Moonshine altered Mason's career substantially and led him to work in a number of seemingly unrelated areas. Originally a 'group theorist' in the 1970s, Mason expanded his research interests throughout the 1980s to include studying modular functions associated with finite simple groups, such as the Mathieu group. In the early 1990s Mason began working on problems surrounding vertex algebras, and this interest accelerated greatly and is witnessed in works with Chongying Dong, Haisheng Li, and others. In the early 2000s, Mason began collaborating with the late Marvin Knopp and initiated a systematic study of vector-valued modular forms. Mason's research in this area has grown greatly over the years, and continues to this day, along with his work on vertex operator algebras, group theory, and other relevant areas such as Hopf algebras.

The mathematics and interest in vertex operator algebras and vector-valued modular forms, including their deep intersection and use in other areas, continues to provide a rich and vibrant landscape in mathematics and physics. The resurgence of moonshine related to the Mathieu group and other groups, the increasing presence of algebraic geometry, and the 'irrational' theories of vertex operator algebras are just a few of the exciting and active areas in this present epoch. The

conference "Vertex operator algebras, number theory, and related topics" served as a timely opportunity to discuss the state of art of these areas, with a focus on topics surrounding vertex algebras and automorphic forms.

To this end, mathematicians from every continent were invited to attend, and presentations were given by researchers representing 15 countries. Overview talks were given in some areas, while recent developments were announced by others. Undergraduate students, graduate students, postdoctoral faculty, professors, and distinguished professors were present and able to attend talks and also discuss and collaborate on problems.

Speakers and participants were asked to submit papers for this volume, and an excellent collection of papers has been obtained representing diverse yet connected streams of research. The works presented here offer original contributions to the areas of vertex algebras and number theory, surveys on some of the most important topics pertaining to these fields, introductions to new fields related to these, and open problems from some of the leaders in these areas.

This conference could not have been possible without the support of various sources and institutions, and the help of many people.

The organizers would like to gratefully acknowledge support and funding provided by the National Science Foundation (NSF Grant DMS-1802478) and National Security Agency (NSA Grant Grant #H98230-18-1-0068). Additionally, the organizers would like to extend a sincere thank you for contributions and support to the Simons Foundation; the Department of Mathematics at Illinois State University; the Mathematics Department at University of California, Santa Cruz; and California State University, Sacramento (and in particular the Department of Mathematics and Statistics as well as the College of Natural Science and Mathematics).

Many people contributed to the success of this conference. Of particular importance, the organizers would like to thank the speakers for their excellent presentations as well as all participants for their engaging presence. A massive thank you is also extended to all authors who contributed a manuscript to this volume, as well as the anonymous referees who put substantial time into making this collection of papers possible.

Finally, the organizers would like to thank and congratulate Geoffrey Mason not only for his contributions in mathematical research, but also for his exceptional teaching, warm mentoring, collegiality and contagious affection for mathematics.

Matthew Krauel
Michael Tuite
Gaywalee Yamskulna

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University of Notre Dame

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VOAs and Number Theory • Krauel et al., Editors

This volume contains the proceedings of the International Conference on Vertex Operator Algebras, Number Theory, and Related Topics, held from June 11-15, 2018, at California State University, Sacramento, California.

The mathematics of vertex operator algebras, vector-valued modular forms and finite group theory continues to provide a rich and vibrant landscape in mathematics and physics. The resurgence of moonshine related to the Mathieu group and other groups, the increasing role of algebraic geometry and the development of irrational vertex operator algebras are just a few of the exciting and active areas at present.

The proceedings center around active research on vertex operator algebras and vector-valued modular forms and offer original contributions to the areas of vertex algebras and number theory, surveys on some of the most important topics relevant to these fields, introductions to new fields related to these and open problems from some of the leaders in these areas.

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