

# CONTEMPORARY MATHEMATICS

754

## 75 Years of Mathematics of Computation

Symposium

Celebrating 75 Years of Mathematics of Computation

November 1–3, 2018

The Institute for Computational and Experimental Research  
in Mathematics (ICERM)

Susanne C. Brenner

Igor Shparlinski

Chi-Wang Shu

Daniel B. Szyld

Editors

# 75 Years of Mathematics of Computation



# CONTEMPORARY MATHEMATICS

---

754

## 75 Years of Mathematics of Computation

Symposium

Celebrating 75 Years of Mathematics of Computation

November 1–3, 2018

The Institute for Computational and Experimental Research  
in Mathematics (ICERM)

Susanne C. Brenner

Igor Shparlinski

Chi-Wang Shu

Daniel B. Szyld

Editors

## EDITORIAL COMMITTEE

Dennis DeTurck, Managing Editor

Michael Loss      Kailash Misra      Catherine Yan

2010 *Mathematics Subject Classification*. Primary 11Mxx, 11Gxx,  
16Zxx, 35Rxx, 41Axx, 65Fxx, 65Mxx, 65Nxx.

---

### Library of Congress Cataloging-in-Publication Data

Names: Celebrating 75 Years of Mathematics of Computation (2018: Providence, R.I.), author. | Brenner, Susanne C., editor. | Shparlinski, Igor E., editor. | Shu, Chi-Wang, editor. | Szyld, Daniel, editor.

Title: 75 years of Mathematics of computation : symposium on Celebrating 75 Years of Mathematics of Computation, November 1-3, 2018, the Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island / Susanne C. Brenner, Igor Shparlinski, Chi-Wang Shu, Daniel B. Szyld, editors.

Other titles: Mathematics of computation.

Description: Providence, Rhode Island : American Mathematical Society, [2020] | Series: Contemporary mathematics, 0271-4132 ; volume 754 | Includes bibliographical references.

Identifiers: LCCN 2019056241 | ISBN 9781470451639 (paperback) | ISBN 9781470456375 (ebook)

Subjects: LCSH: Numerical analysis--Congresses. | Number theory--Data processing--Congresses. | Computational complexity--Congresses. — AMS: Number theory – Zeta and  $L$ -functions: analytic theory. | Number theory – Arithmetic algebraic geometry (Diophantine geometry). | Associative rings and algebras – Computational aspects of associative rings. | Partial differential equations – Miscellaneous topics. | Approximations and expansions | Numerical analysis – Numerical linear algebra. | Numerical analysis – Partial differential equations, initial value and time-dependent initial-boundary value problems. | Numerical analysis – Partial differential equations, boundary value problems.

Classification: LCC QA297 .C393 2020 | DDC 518--dc23

LC record available at <https://lcn.loc.gov/2019056241>

DOI: <https://doi.org/10.1090/conm/754>

---

**Color graphic policy.** Any graphics created in color will be rendered in grayscale for the printed version unless color printing is authorized by the Publisher. In general, color graphics will appear in color in the online version.

**Copying and reprinting.** Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy select pages for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for permission to reuse portions of AMS publication content are handled by the Copyright Clearance Center. For more information, please visit [www.ams.org/publications/pubpermissions](http://www.ams.org/publications/pubpermissions).

Send requests for translation rights and licensed reprints to [reprint-permission@ams.org](mailto:reprint-permission@ams.org).

© 2020 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights  
except those granted to the United States Government.

Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines  
established to ensure permanence and durability.

Visit the AMS home page at <https://www.ams.org/>

10 9 8 7 6 5 4 3 2 1      25 24 23 22 21 20

## Contents

Preface	vii
Invited presentations at the symposium	ix
Adaptive low-rank approximations for operator equations: Accuracy control and computational complexity MARKUS BACHMAYR and WOLFGANG DAHMEN	1
The Prager–Synge theorem in reconstruction based a posteriori error estimation FLEURIANNE BERTRAND and DANIELE BOFFI	45
Linear and nonlinear fractional elliptic problems JUAN PABLO BORTHAGARAY, WENBO LI, and RICARDO H. NOCHETTO	69
Lattice algorithms for multivariate approximation in periodic spaces with general weight parameters RONALD COOLS, FRANCES Y. KUO, DIRK NUYENS, and IAN H. SLOAN	93
Multiscale modeling, homogenization and nonlocal effects: Mathematical and computational issues QIANG DU, BJORN ENGQUIST, and XIAOCHUAN TIAN	115
The Stokes complex: A review of exactly divergence–free finite element pairs for incompressible flows MICHAEL NEILAN	141
Finite element methods for Maxwell’s equations PETER MONK and YANGWEN ZHANG	159
Partial differential equation regularization for supervised machine learning ADAM M. OBERMAN	177
Iterative methods for linear systems of equations: A brief historical journey YOUSEF SAAD	197
The IEQ and SAV approaches and their extensions for a class of highly nonlinear gradient flow systems JIE SHEN and XIAOFENG YANG	217

A class of bound-preserving high order schemes: The main ideas and recent developments CHI-WANG SHU	247
BDDC domain decomposition algorithms OLOF B. WIDLUND	261
Computing modular polynomials and isogenies of rank two Drinfeld modules over finite fields PERLAS CARANAY, MATTHEW GREENBERG, and RENATE SCHEIDLER	283
A new ranking function for polynomial selection in the number field sieve NICOLAS DAVID and PAUL ZIMMERMANN	315
A nilpotent quotient algorithm for finitely presented associative $\mathbb{Z}$ -algebras and its application to integral group rings BETTINA EICK and TOBIAS MOEDE	327
A tale of two omegas MICHAEL J. MOSSINGHOFF and TIMOTHY S. TRUDGIAN	343

## Preface

The year 2018 marked the 75th anniversary of the founding of *Mathematics of Computation*, one of the four primary research journals published by the American Mathematical Society and the oldest research journal devoted to computational mathematics. To celebrate this milestone, the symposium “Celebrating 75 Years of Mathematics of Computation” was held during November 1–3, 2018, at the Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island. (A symposium for the 50th anniversary of the journal took place in Vancouver in 1993.)

The sixteen papers in this volume, written by the symposium speakers and editors of the journal, include both survey articles and new contributions.

On the discrete side, there are four papers covering topics in computational number theory and computational algebra. On the continuous side, there are twelve papers covering topics in machine learning, high dimensional approximations, non-local and fractional elliptic problems, gradient flows, hyperbolic conservation laws, Maxwell’s equations, Stokes’s equations, a posteriori error estimation, and iterative methods. Together they provide a snapshot of significant achievements in the past quarter century in computational mathematics and also in important current trends.

We are thankful to the administration and staff members at ICERM for everything they did to make the symposium a great success. Funding for the event by the National Science Foundation is gratefully acknowledged.

Last but not least, we thank the speakers at the symposium and the authors of the papers in this volume for their excellent contributions. A list of the nineteen invited lectures at the symposium in the order that they were presented is provided on the following page.

Susanne C. Brenner  
Igor Shparlinski  
Chi-Wang Shu  
Daniel B. Szyld





## Invited presentations at the symposium

- Gunter Malle (Technische Universität Kaiserslautern), Character tables of finite groups.
- Bettina Eick (Technische Universität Braunschweig), The classification of groups of prime-power order.
- Paul Zimmermann (Institut National de Recherche en Informatique et en Automatique), What if Gauss had had a computer?
- Renate Scheidler (University of Calgary), Dan Shanks' CUFFQI algorithm resurrected.
- Andrew Odlyzko (University of Minnesota), The Riemann Hypothesis and computers.
- Carl Pomerance (Dartmouth College), Primality testing, then and now.
- Chi-Wang Shu (Brown University), Bound-preserving high order schemes for hyperbolic equations- survey and recent developments.
- Wolfgang Dahmen (University of South Carolina), Accuracy controlled computation - the merit of residuals.
- Peter Monk (University of Delaware), Finite element methods for Maxwell's equations.
- Andrea Bertozzi (University of California, Los Angeles), Graphical models in machine learning, networks and uncertainty quantification.
- Adam M. Oberman (McGill University), Partial differential equations approaches to optimization and regularization of deep neural networks.
- Qiang Du (Columbia University), Nonlocal modeling, analysis and computation.
- Yousef Saad (University of Minnesota), A brief journey to the past of iterative methods for solving sparse linear systems.
- Bjorn Engquist (University of Texas, Austin), Fast solvers for frequency domain wave propagation.
- Ricardo Nochetto (University of Maryland, College Park), Numerical methods for fractional diffusion.
- Olof B. Widlund (New York University), BDDC domain decomposition algorithms.

- Frances Kuo (University of New South Wales), High dimensional integration - the Quasi-Monte Carlo way.
- Jie Shen (Purdue University), A new and robust approach to construct energy stable schemes for gradient flows.
- Douglas Arnold (University of Minnesota), Mathematics of Computation meets geometry.

The full program for the ICERM symposium, including the names and affiliations of all the participants and the slides for most of the invited talks, can be found online at

[https://icerm.brown.edu/topical\\_workshops/tw18-7-ymc/](https://icerm.brown.edu/topical_workshops/tw18-7-ymc/)

The slides of a brief presentation of the history of *Mathematics of Computation* from the welcome reception is also available there.

## Selected Published Titles in This Series

- 754 **Susanne C. Brenner, Igor Shparlinski, Chi-Wang Shu, and Daniel B. Szyld, Editors**, 75 Years of Mathematics of Computation, 2020
- 752 **Samuel Cosque and Grigor Sargsyan, Editors**, Trends in Set Theory, 2020
- 751 **Ashish K. Srivastava, André Leroy, Ivo Herzog, and Pedro A. Guil Asensio, Editors**, Categorical, Homological and Combinatorial Methods in Algebra, 2020
- 750 **A. Bourhim, J. Mashreghi, L. Oubbi, and Z. Abdelali, Editors**, Linear and Multilinear Algebra and Function Spaces, 2020
- 749 **Guillermo Cortiñas and Charles A. Weibel, Editors**, *K*-theory in Algebra, Analysis and Topology, 2020
- 748 **Donatella Danielli and Irina Mitrea, Editors**, Advances in Harmonic Analysis and Partial Differential Equations, 2020
- 747 **Paul Bruillard, Carlos Ortiz Marrero, and Julia Plavnik, Editors**, Topological Phases of Matter and Quantum Computation, 2020
- 746 **Erica Flapan and Helen Wong, Editors**, Topology and Geometry of Biopolymers, 2020
- 745 **Federico Binda, Marc Levine, Manh Toan Nguyen, and Oliver Röndigs, Editors**, Motivic Homotopy Theory and Refined Enumerative Geometry, 2020
- 744 **Pieter Moree, Anke Pohl, L'ubomír Snoha, and Tom Ward, Editors**, Dynamics: Topology and Numbers, 2020
- 743 **H. Garth Dales, Dmitry Khavinson, and Javad Mashreghi, Editors**, Complex Analysis and Spectral Theory, 2020
- 742 **Francisco-Jesús Castro-Jiménez, David Bradley Massey, Bernard Teissier, and Meral Tosun, Editors**, A Panorama of Singularities, 2020
- 741 **Houssam Abdul-Rahman, Robert Sims, and Amanda Young, Editors**, Analytic Trends in Mathematical Physics, 2020
- 740 **Alina Bucur and David Zureick-Brown, Editors**, Analytic Methods in Arithmetic Geometry, 2019
- 739 **Yaiza Canzani, Linan Chen, and Dmitry Jakobson, Editors**, Probabilistic Methods in Geometry, Topology and Spectral Theory, 2019
- 738 **Shrikrishna G. Dani, Surender K. Jain, Jugal K. Verma, and Meenakshi P. Wasadikar, Editors**, Contributions in Algebra and Algebraic Geometry, 2019
- 737 **Fernanda Botelho, Editor**, Recent Trends in Operator Theory and Applications, 2019
- 736 **Jane Hawkins, Rachel L. Rossetti, and Jim Wiseman, Editors**, Dynamical Systems and Random Processes, 2019
- 735 **Yanir A. Rubinstein and Bernard Shiffman, Editors**, Advances in Complex Geometry, 2019
- 734 **Peter Kuchment and Evgeny Semenov, Editors**, Differential Equations, Mathematical Physics, and Applications, 2019
- 733 **Peter Kuchment and Evgeny Semenov, Editors**, Functional Analysis and Geometry, 2019
- 732 **Samuele Anni, Jay Jorgenson, Lejla Smajlović, and Lynne Walling, Editors**, Automorphic Forms and Related Topics, 2019
- 731 **Robert G. Niemeyer, Erin P. J. Pearse, John A. Rock, and Tony Samuel, Editors**, Horizons of Fractal Geometry and Complex Dimensions, 2019
- 730 **Alberto Facchini, Lorna Gregory, Sonia L'Innocente, and Marcus Tressl, Editors**, Model Theory of Modules, Algebras and Categories, 2019
- 729 **Daniel G. Davis, Hans-Werner Henn, J. F. Jardine, Mark W. Johnson, and Charles Rezk, Editors**, Homotopy Theory: Tools and Applications, 2019

For a complete list of titles in this series, visit the  
AMS Bookstore at [www.ams.org/bookstore/conmseries/](http://www.ams.org/bookstore/conmseries/).

The year 2018 marked the 75th anniversary of the founding of *Mathematics of Computation*, one of the four primary research journals published by the American Mathematical Society and the oldest research journal devoted to computational mathematics. To celebrate this milestone, the symposium “Celebrating 75 Years of Mathematics of Computation” was held from November 1–3, 2018, at the Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island.

The sixteen papers in this volume, written by the symposium speakers and editors of the journal, include both survey articles and new contributions.

On the discrete side, there are four papers covering topics in computational number theory and computational algebra. On the continuous side, there are twelve papers covering topics in machine learning, high dimensional approximations, nonlocal and fractional elliptic problems, gradient flows, hyperbolic conservation laws, Maxwell’s equations, Stokes’s equations, a posteriori error estimation, and iterative methods. Together they provide a snapshot of significant achievements in the past quarter century in computational mathematics and also in important current trends.



ISBN 978-1-4704-5163-9



9 781470 451639

CONM/754