CONTEMPORARY MATHEMATICS

756

Geometry of Submanifolds

AMS Special Session on Geometry of Submanifolds
in Honor of Bang-Yen Chen's 75th Birthday
October 20–21, 2018
University of Michigan, Ann Arbor, Michigan

Joeri Van der Veken
Alfonso Carriazo
Ivko Dimitrić
Yun Myung Oh
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Dedicated to Professor Bang-Yen Chen, in honor of his 75th birthday

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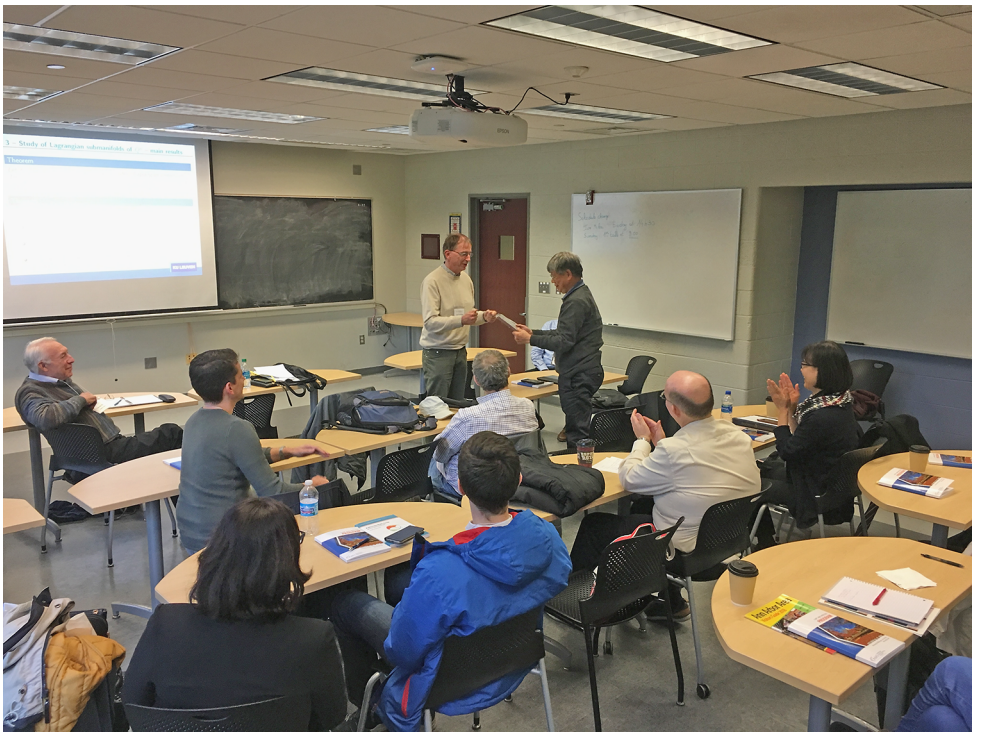
Foreword
by the Editors

About a century ago, the geometry of submanifolds gained a lot of momentum through the study of the Schläfli's conjecture, which stated that a real analytic Riemannian manifold of dimension n can be locally isometrically embedded into any real analytic Riemannian manifold of dimension $\frac{1}{2}n(n+1)$. M. Janet (1926), É. Cartan (1927) and C. Burstin (1931) made essential contributions to the understanding of the importance of the immersion problems and to a result that today bears their names.



On behalf of the organizers of the AMS Special Session in Bang-Yen Chen's honor, Luc Vrancken presents Bang-Yen Chen with a plaque. The text on the plaque reads: *A man is to be judged not just by his own accomplishments, but by those he has inspired in others.*

A major development for the theory was the much celebrated Embedding Theorem, proved by John Forbes Nash, Jr. (in a series of three papers published in 1954, 1956, and 1966). Nash's work received outstanding recognition from the mathematical community, including his election as a fellow of the American Mathematical Society in 2012, and the Abel Prize in 2015. After the important moment represented by the Nash's Embedding Theorem, many outstanding mathematicians, as e.g. Shiing-Shen Chern (2011–2004), Manfredo Perdigão do Carmo (1928–2018), or Shoshichi Kobayashi (1932–2012) directed some of their best efforts towards the geometry of submanifolds. To better shed a light on the geometry of submanifolds and its historical development, the editors of the present volume invited Leopold Verstraelen to write an essay describing from a broad perspective his views on this important chapter of mathematics.

The development of geometry of submanifolds benefited greatly from Bang-Yen Chen's contributions. Bang-Yen Chen is the author of the first monograph titled *Geometry of Submanifolds*, published in 1973 by Marcel Dekker, a work cited by many mathematicians since its publication. In 2019, the Dover paperback edition of this fundamental volume was published and is available today. The editors of the present volume share a common experience, as they have been attracted by this domain of differential geometry by reading and studying Bang-Yen Chen's *Geometry of Submanifolds*.

After 1973, many authors became interested by the geometry of submanifolds. Richard S. Palais and Chuu-Lian Terng wrote a monograph titled *Critical Point Theory and Submanifold Geometry*, published by Springer-Verlag in 1988, in their prestigious Lecture Notes in Mathematics series. In their *Introduction* to their volume titled *Submanifolds and Holonomy* (published in its first edition in 2003), Jürgen Berndt, Sergio Console, and Carlos Enrique Olmos wrote that "the study of submanifolds in Euclidean space has a long tradition, and many beautiful results and theories emerge from it".

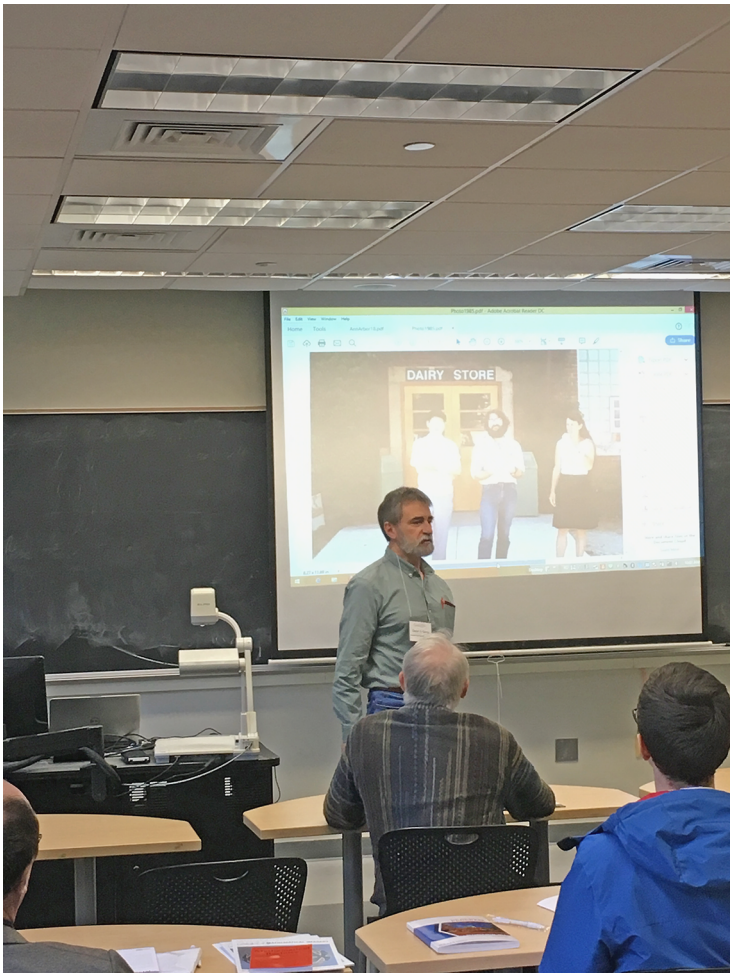
The present volume is, therefore, a celebration of the geometry of submanifolds and its connections with other areas of mathematics. It was a fortunate coincidence that in the month when Bang-Yen Chen turned 75, the American Mathematical Society scheduled their Fall Central Sectional Meeting with number 1143 in Ann Arbor, at the University of Michigan, in the proximity of East Lansing, where Bang-Yen Chen spent most of his academic career, with Michigan State University. Motivated by their gratitude towards a researcher who inspired their work, the editors of the present volume proposed an *AMS Special Session on Geometry of Submanifolds, in Honor of Bang-Yen Chen's 75th Birthday.* The event took place on October 20-21, 2018.

For a more complete perspective of the academic context in which this volume was prepared, it is useful to remind here a few biographical pieces of information. Bang-Yen Chen was born on October 3, 1943, in Toucheng Township, Yilan County, Taiwan. He received his B.S. from Tamkang University in 1965 and his M.S. from National Tsing Hua University in 1967. He obtained his Ph.D. degree from University of Notre Dame in 1970 under the supervision of Tadashi Nagano. Before his graduate years, Bang-Yen Chen taught at Tamkang University between 1966 and 1968, and at the National Tsing Hua University during the academic year 1967-1968. After his doctoral years (1968-1970) at University of Notre Dame, Bang-Yen Chen joined the faculty at Michigan State University as a research associate from

David E. Blair (left), Bang-Yen Chen (center), and Yun Myung Oh (right). Yun Myung Oh defended her doctoral dissertation under Bang-Yen Chen's supervision at Michigan State University in 2000. The title of her thesis was *Explicit Construction of Lagrangian Isometric Immersion of a Real Space Form* $M^n(c)$ *into a Complex Space Form* $M^n(4c)$.

1970-1972, then became associate professor in 1972, and full professor in 1976. He was presented with the title of University Distinguished Professor in 1990. After 2012, he became University Distinguished Professor Emeritus. Bang-Yen Chen is the author of over 500 works including 12 books, mainly in differential geometry and related subjects. His works have been cited over 25,000 times.

Three of the editors of the present volume, Ivko Dimitrić, Yun Myung Oh, and Bogdan Suceavă are his former doctoral students at Michigan State University. Bang-Yen Chen also directed Susumu Ishikawa's thesis at Kyushu University, as well as several other dissertations at Michigan State University, for Young Ho Kim, Yhuji Shibuya, Yoshihiko Tazawa, and Jie Yang.



Oscar Garay presenting his contribution titled *Binormal evolution surfaces swept out by elastica-like extremals*. During his intervention, he recalled his first academic visit at Michigan State University, at the beginning of the 1980s, when the geometry of submanifolds of finite type received a lot of attention from many geometers.

Knowing very well how valuable Dr. Bang-Yen Chen's personal and academic experience is, the editors extended him the invitation to contribute to this volume with an essay describing his most valuable experiences. Bang-Yen Chen wrote the essay *My Education in Differential Geometry and My Indebtedness*, hereby included, where he described his academic interactions with his mathematical mentors Tadashi Nagano, Shiing-Shen Chern, Tominosuke Otsuki, and Kentaro Yano.

Bang-Yen Chen is today known for several fundamental ideas in differential geometry, e.g., Chen inequalities, Chen invariants, Chen's conjectures, Chen surface, Chen-Ricci inequality, Chen submanifolds, Chen equality, submanifolds of finite type, the (M_+, M_-) method for compact symmetric spaces and the 2-numbers of Riemannian manifolds (this last concept based on a joint contribution with Tadashi

Bang-Yen Chen discussing with Ye-Lin Ou. In the background: Patrick J. Ryan (left) and Tim Buttsworth (right).

Nagano), and slant submanifolds, to mention just a few. In 1991, Bang-Yen Chen formulated the biharmonic conjecture, which claims that minimal submanifolds are the only biharmonic submanifolds in Euclidean spaces. Additionally, in 1996 he conjectured that every finite type spherical hypersurface is either of 1-type or of 2-type. At the time when the AMS Special Session took place, Bang-Yen Chen was working with Ye-Lin Ou on a volume titled *Biharmonic submanifolds and biharmonic maps in Riemannian geometry*. This work was published by World Scientific in 2020.

The editors would like to extend their thanks to all the scholars who participated in the AMS Special Session. Their expertise and their interactions have been particularly valuable and interesting. While their papers are not included in the present volume, the contributions of David E. Blair (Michigan State University), Alfonso Carriazo (University of Sevilla, Spain), Oscar J. Garay (University of the Basque Country, Spain), Elsa Ghandour (Université Polytechnique Hauts-de-France, France), Martin Magid (Wellesley College), Timothy James Buttsworth (The University of Queensland, Australia), have been extremely valuable to and tremendously appreciated by the editors. Also, many thanks to the co-authors of the contributors to the special sessions: Paul Baird (University of Brest, France), Martha Patricia Dussan Angulo and A. P. Franco Filho (both from Universidade de São Paulo), and Leonard M. Giugiuc (Colegiul National Traian, Drobeta-Turnu Severin, Romania).

Yusuf Doğru (left), Bang-Yen Chen (center), and Cihan Özgür (right), at the 6th Geometry Symposium in Uludağ University, in Bursa, Turkey, in July 2008

The editors of the present volume express their thanks to Georgia Benkart, who served as AMS Secretaries in the academic year 2018–2019, when the AMS Special Session from Ann Arbor was organized.

While the editors prepared the present volume, their work benefited from the outstanding support and expert consultations of several referees. Without their expertise the quality of the present volume would not be the same. Last, but not least, many thanks to Sergei Gelfand and Christine Thivierge for their editorial guidance and support during the preparation of the present volume.

The participants to the AMS Special Session on Geometry of Submanifolds, in Honor of Bang-Yen Chen's 75th Birthday. Photo taken at the end of the morning session on October 20, 2018

Oscar Garay (left), Bang-Yen Chen (center), and Ivko Dimitrić (right). Ivko Dimitrić defended his doctoral dissertation in 1989 at Michigan State University, under Bang-Yen Chen's coordination. The title of his work is *Quadric Representation and Submanifolds of Finite Type.* Photo taken on October 20, 2018 in Ann Arbor, MI.

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This volume contains the proceedings of the AMS Special Session on Geometry of Submanifolds, in honor of Bang-Yen Chen's 75th birthday, held from October 20–21, 2018, at the University of Michigan, Ann Arbor, Michigan.

The development of contemporary geometry of submanifolds benefited greatly from Bang-Yen Chen's contributions, as several interesting questions actively pursued today originate in his work. Chen is known for several fundamental ideas in differential geometry, including Chen inequalities, Chen invariants, Chen's conjectures, Chen surface, Chen-Ricci inequality, Chen submanifolds, Chen equality, submanifolds of finite type, and slant submanifolds.

The papers in this volume represent a celebration of the geometry of submanifolds and its connections with other areas of mathematics and cover themes rooted in Chen's work, from investigations on the spectrum of the Laplacian on complete Riemannian manifolds to the geometry of symmetric spaces. These contributions are written with the hope to inform and inspire.

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