

CONTEMPORARY MATHEMATICS

760

Centre de Recherches Mathématiques Proceedings

Characters in Low-Dimensional Topology

A Conference Celebrating the Work
of

Steven Boyer

June 2–6, 2018

Université du Québec à Montréal, Montréal,
Québec, Canada

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Olivier Collin
Stefan Friedl
Cameron Gordon
Stephan Tillmann
Liam Watson
Editors

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Preface

In June 2018, the ‘Boyerfest’ was held at the Université du Québec à Montréal. It was organised by the editors of this volume to celebrate Steven Boyer’s major contributions in the area of low-dimensional topology and his important role in the Canadian mathematical community on the occasion of his 65th birthday. The high standard of the 18 talks at the conference contributed greatly to its success. They covered a broad range of topics related to the topology and geometry of 3-manifolds, properties of their fundamental groups and associated representation varieties. The conference was attended by over a hundred mathematicians, and about a quarter of these were graduate students or postdoctoral fellows. It is the hope of the organisers that the conference inspired these young scientists to engage in exploring the many exciting research problems in this area of mathematics.

These proceedings mirror the spirit of the event. The papers in this volume were written by speakers and participants of the conference, as well as a number of contributors who could not attend. Included are papers by collaborators, former students of Steven Boyer and former postdoctoral fellows who have enjoyed his mentorship. We hope the proceedings capture the mathematical endeavours and recognition of Steve.

The editors would like to thank the authors for the work they have put into their contributions, and the referees for their commitment invested in the verification and improvement of these papers. The editors thank Christine Thivierge for her assistance and unrelenting patience in preparing this volume.

The organisers would like to acknowledge the generous support of the ‘Boyerfest’ by the Centre interuniversitaire de recherches en géométrie et topologie (CIRGET), the Centre de recherches mathématiques (CRM), the Fields Institute for Research in Mathematical Sciences, the National Science Foundation, the Unité Mixte Internationale CRM-CNRS, and the Université du Québec à Montréal. CIRGET and the Département de mathématiques at the Université du Québec à Montréal provided a wonderful conference environment, and the organisers are indebted to Alexandra Haedrich for precious and cheerful administrative support.

The Editors
March 2020

Talks at the Boyerfest

- Michel Boileau (Université d'Aix-Marseille)
Grothendieck rigidity of 3-manifold groups
- Jeff Danciger (University of Texas at Austin)
Exotic real projective Dehn surgery space
- Nathan Dunfield (University of Illinois at Urbana-Champaign)
An $SL(2, \mathbb{R})$ Casson-Lin invariant and applications
- Paolo Ghiggini (Université de Nantes)
Knot Floer homology from a dynamical viewpoint
- Bill Goldman (University of Maryland)
Topology, Geometry and Dynamics of Character Varieties
- Cameron Gordon (University of Texas at Austin)
Cyclic branched covers of L -space knots
- Josh Greene (Boston College)
On curves intersecting at most once
- Eli Grigsby (Boston College)
Braids, surfaces, and homological invariants
- Marc Lackenby (University of Oxford)
Every knot has characterising slopes
- Ana Lecuona (University of Glasgow)
Torus knots and rational homology balls
- Joseph Maher (CUNY Staten Island)
The compression body graph has infinite diameter
- Alan Reid (Rice University)
Profinite rigidity from representation rigidity
- Rachel Roberts (Washington University)
Nontrivial connected sums of fibered knots are persistently foliar
- Saul Schleimer (Warwick University)
From veering triangulations to pseudo-Anosov flows
- Genevieve Walsh (Tufts University)
Relatively hyperbolic groups with planar boundaries
- Liam Watson (Université de Sherbrooke)
Heegaard Floer homology via immersed curves
- Dani Wise (McGill University)
Orderability, Coherence, and Counting Cycles in Graphs
- Xingru Zhang (SUNY Buffalo)
On real characters

Biographical sketch of Steven Boyer

First steps. Steven Patrick Boyer was born in 1953 in South Charleston, West Virginia and moved to Montréal at the age of six. From a family of three boys, three girls, and many pets, he spent a good deal of his early years playing with friends and participating in organized sports. Access to a large and diverse home library coupled with lively family conversations nurtured his developing interests in a variety of subjects, among them history, natural sciences, and mathematics.

It was during his undergraduate studies at the University of New Brunswick in Fredericton that his passion for mathematics blossomed, and after completing a BA and MA in mathematics he joined the PhD program at Cornell University. Those formative years saw him shift his interests from geometric complex analysis to 4-dimensional geometric topology under the supervision of Peter Jack Kahn. His thesis, written in the post-Freedman era of 4-dimensional topology, investigated the representation of 2-dimensional homology classes by locally flat 2-spheres.

Steve met his wife, Maryse Camille Desrochers, at Cornell, where she was an algebra graduate student working under Stephen Chase. After obtaining their PhD's in 1983, they spent two years at the University of Cambridge as postdoctoral fellows. Steve's interest in 3-manifold topology was sparked by a Part III course given by Raymond Lickorish and solidified through daily discussions with Daniel Lines, who was also a postdoctoral fellow at Cambridge during this period. Interactions with visitors such as Tim Cochran, Mark Feighn, Rob Kirby, Bill Menasco, Marty Scharlemann, Hamish Short, and Abigail Thompson also contributed to his shift in research direction.

A mathematical correspondence with Kunio Murasugi led to an instructorship at the University of Toronto, which he took up on leaving Cambridge. The following two years were productive on various fronts. On the personal side, Steve and Maryse's daughter Stephanie was born soon after they arrived in Toronto. Mathematically, he produced a topological classification of compact, simply-connected 4-manifolds with connected boundaries, as well as a study of proper powers in free products of groups motivated by applications to problems in 3-dimensional surgery theory. He also collaborated with Andy Nicas, another young researcher at the University of Toronto, on generalisations of Casson's invariant to rational homology spheres.

The early years at UQAM. A chance meeting with François Lalonde led to a return to Montréal in 1987 to take up a tenure track position at Université du Québec à Montréal, one of a group of young academics recruited from mathematical centres around the world who aspired to create an international research pole in that city. Amongst other things, this led to the creation of the geometry-topology

research centre CIRGET (Centre Interuniversitaire de Recherche en Géométrie et Topologie).

Mathematically, Steve had started to focus on applications of representation-theoretic methods to 3-manifold topology and in the paper *Surgery formulae for Casson's invariant and extensions to homology lens spaces*, he and Daniel Lines showed how Dedekind sums could be used to produce an extension of Casson's invariant which was particularly well-suited to study problems in surgery theory. On the personal side, Steve's son Anthony was born soon after he returned to Montréal.

Between 1992 and 1994, Steve had the great fortune of supervising a Chinese postdoctoral fellow, Xingru Zhang, who has had a significant and lasting impact on him. They were heavily influenced by the work of Peter Shalen, especially his seminal contributions with Marc Culler to $SL(2, \mathbb{C})$ -character variety theory and the application of the theory to Dehn surgery problems with Culler, Cameron McA. Gordon, and John Luecke. Steve's collaboration with Xingru led to a sequence of articles published between 1996 and 2001 exploited, with increasing sophistication and strength, the $SL(2, \mathbb{C})$ -character variety theory of 3-manifold groups, culminating in the paper *A proof of the finite surgery conjecture*. Towards the end of the 1990's, Steve began a long-term collaboration with Cameron Gordon, whose camaraderie and complementary approach on mathematical problems has proved an unending source of inspiration.

CIRGET director years at UQAM. By 2001, Steve had established himself among the most influential topologists in Canada, having built an international network of collaborators and bringing much activity in the field to Montréal. When the founding director of CIRGET, François Lalonde, left UQAM for the Université de Montréal, Steve took on the challenge of pursuing CIRGET's development and increasing its international impact. The successes the centre has enjoyed over the years are due in no small part to Steve's dedication, patience, and pure tenacity. Few other colleagues were prepared to navigate the murky waters of university administration in the quest for recognition and resources. Much to the amusement of his colleagues, he did so with a determination and gusto undimmed by the inevitable disappointments. His efforts paid off though, as he secured long-term infrastructure support for the centre, the creation a research chair in geometry, and a suite of purpose-designed research rooms from the university. Under Steve's directorship, CIRGET's recruitment of postdoctoral fellows and research visitors has been transformed from the ad hoc to a stable program. Many of the young mathematicians who spent formative years in Montréal greatly benefited from Steve's non-prescriptive but responsible mentoring.

The last two decades have witnessed a constant expansion of methods and topics covered, and collaborators in Steve's research. In work with Culler, Shalen, and Zhang, and later with Gordon and Zhang, a powerful machine for studying exceptional surgery problems based on the Jaco-Shalen-Johannson theory of characteristic submanifolds was developed. With Zhang and Ian Agol he published a paper on the virtual fibering conjecture in 2008. A research collaboration with Michel Boileau was begun in the early 2000's which to date has resulted in a number of publications on a diverse list of topics including non-zero degree maps between 3-manifolds, commensurability between hyperbolic knot complements, the existence of foliations on graph manifold integer homology spheres, L -space branched covers

of links in the 3-sphere, and the Tits alternative for PD(3) groups. Much of this work was in collaboration with others including his doctoral student Radu Cebanu and postdoctoral fellow Genevieve Walsh, and colleagues Alan Reid, Dale Rolfsen, and Shicheng Wang.

Steve's interest in the orderability of the fundamental groups of 3-manifolds grew from discussions with fellow Canadian topologist Dale Rolfsen, and their 2005 article *Orderable 3-manifold groups*, written with Bert Wiest, was a great example of a work finely blending research aims with survey and reference purposes. Later discussions with his doctoral student Liam Watson and Cameron Gordon led to what in the early 2010's seemed like a very bold conjecture: an irreducible rational homology sphere is not a Heegaard Floer homology L -space if and only if its fundamental group is left-orderable. Inspired by a question of Ozsváth and Szabó, asking about a converse to their result that L -spaces do not admit taut foliations, András Juhász further contended that these conditions were equivalent to the existence of a co-oriented taut foliation on the 3-manifold, thus leading to what is now known as the *L -space conjecture*, one of the most important problems in contemporary 3-manifold topology

Work on the L -space conjecture gave Steve the opportunity to turn his attention to foliations on 3-manifolds. With Boileau he had constructed co-oriented taut foliations on graph manifold integer homology spheres with infinite fundamental groups, thus verifying the L -space conjecture for this class of manifolds. The 2017 paper *Foliations, orders, representations, L -spaces and graph manifolds*, with his postdoctoral fellow Adam Clay, introduced a relative form of the L -space conjecture and used it to establish the equivalence of the existence of a co-oriented taut foliation on a general graph manifold W with the left-orderability of its fundamental group. This relative approach was the model used by Jonathan Hanselmann, Jake Rasmussen, Sarah Rasmussen, and Liam Watson to formulate their Heegaard Floer gluing theorems, which allowed them to complete the proof of the L -space conjecture for graph manifolds. A recent collaboration with his postdoctoral fellow Ying Hu verified the L -space conjecture for families of branched covers by producing foliations with zero Euler class.

Approach to mathematics. Steve's approach to mathematics has been consistent since his early years. He is motivated primarily by the aesthetic pleasure it affords, the depths and unexpected connections it reveals, and the comradeship it engenders. He likes to say that intuition should lead technique, but not by too much, and finds that working on problems that allow him to assimilate new techniques is a great way to expand his mathematical horizons, though the choice of topics to explore is often a case of one thing leading to another. In the end, he sees that the research mathematician's lot is "to chase the shadows of that which interests, the challenge being in the choosing of the light."

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This volume contains the proceedings of a conference celebrating the work of Steven Boyer, held from June 2–6, 2018, at Université du Québec à Montréal, Montréal, Québec, Canada.

Boyer's contributions to research in low-dimensional geometry and topology, and to the Canadian mathematical community, were recognized during the conference.

The articles cover a broad range of topics related, but not limited, to the topology and geometry of 3-manifolds, properties of their fundamental groups and associated representation varieties.



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