

CONTEMPORARY MATHEMATICS

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Quaternion Fusion Packets

Michael Aschbacher

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The book provides a characterization of the 2-fusion systems of the groups of Lie type and odd characteristic, a result analogous to the Classical Involution Theorem for groups. The theorem is the most difficult step in a two-part program. The first part of the program aims to determine a large subclass of the class of simple 2-fusion systems, while part two seeks to use the result on fusion systems to simplify the proof of the theorem classifying the finite simple groups.



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