

# CONTEMPORARY MATHEMATICS

766

## Geometry at the Frontier Symmetries and Moduli Spaces of Algebraic Varieties

2016–2018 Workshops  
Geometry at the Frontier  
November 2016, November 2017, November 2018  
Universidad de La Frontera, Pucón, Chile

Paola Comparin  
Eduardo Esteves  
Herbert Lange  
Sebastián Reyes-Carocca  
Rubí E. Rodríguez  
Editors

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## Preface

Geometry is one of the oldest branches of Mathematics, still vigorously alive. It has evolved from the axiomatic approach of Euclid to the study of complex spaces, and it has several applications today.

The Chilean Complex Geometry group has been important for the development of Mathematics in Chile. Its research production has proven to be fruitful, as shown by an important number of high impact publications, a wide network of collaborators, the organization of several international meetings and the fact that numerous former students of the group are now researchers and professionals in universities all over the country and abroad.

The workshops “Geometry at the Frontier” took place in November 2016, 2017 and 2018 in a local campus of the Universidad de La Frontera in the city of Pucón, located in the Chilean Patagonia. These workshops were organized by the members of the Chilean Complex Geometry Group and supported by one of the largest scientific grants funded by the Chilean government in pure mathematics, Anillo en Ciencia y Tecnología 1415 Conicyt-PIA.

The three conferences covered a spectrum of interrelated topics, with emphasis on algebraic varieties, automorphisms and moduli spaces. While algebraic geometry is a classical subject of study, it has recently witnessed a flurry of activity and important progress on many fundamental open questions. The workshops brought together Chilean and foreign researchers working in this and related areas to highlight the recent exciting progress and to explore future avenues of research.

The number of mathematicians working on algebraic geometry, and in particular on moduli spaces, has increased significantly in the last decades. We are truly convinced that this volume will be a useful and solid scientific source of recent developments on algebraic geometry. We believe that this volume will be also very useful for students and young researchers, giving them an overview and an insight on open questions in the area.

We cannot end these words without remembering a constant and very active participant of these meetings, and a permanent collaborator of the Chilean mathematical community, our friend José María Muñoz Porras (1956-2018), from the University of Salamanca. Always ready to discuss new mathematical (and other) ideas, to give new insights with a gentle smile, to stimulate and guide the younger mathematicians, with passion and joy. We miss him sorely.

The Editors



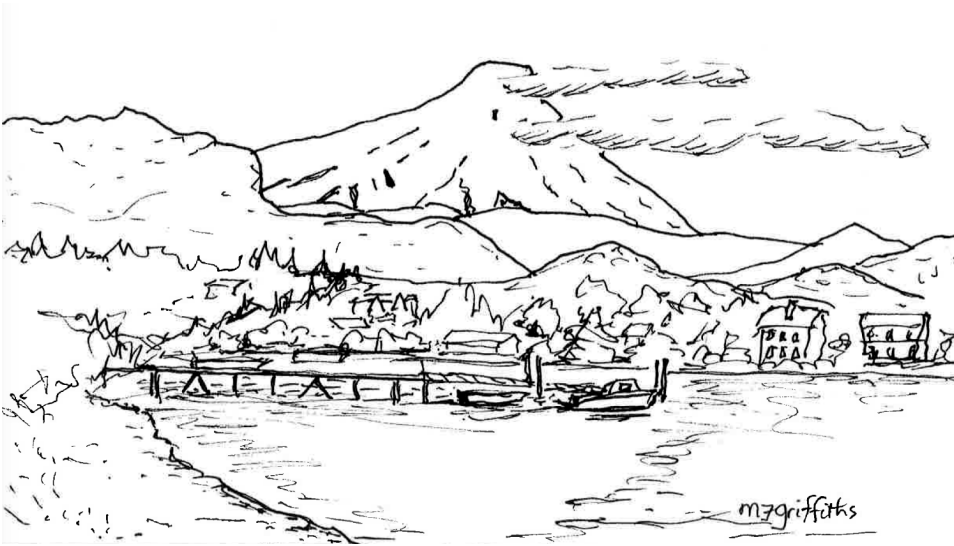


FIGURE 1. The conference venue, thanks to Marian (“Taffy”) Griffiths



FIGURE 2. Official Photo, Pucón, 2018. Standing, from right to left: Sebastián Reyes-Carocca, Ara Basmajian, Anita M. Rojas, Rubén A. Hidalgo, Perry Susskind, Abraham Bobadilla, Benjamín Baeza, Felipe Zingali, Ana Victoria Quedo, Eduardo Uribe, Arthur Newlands, Vinicius Marques da Costa, Andrea Tironi, Gian Pietro Pirola, Abel Castorena, Pablo Quezada, José Aburto, Rodrigo Quezada, Xavier Gómez-Mont, Yerko Torres, Juan Desimoni, Ageu Freire, João Paulo Figueredo, Sebastián Rahausen, Eduardo Silva, Giancarlo Lucchini Arteché, Yerika Marín, Claudio Bravo, Francisco Rubilar, Estefanía Bravo, Pietro Beri, Paola Comparin, Luca Ugaglia, Bruno Suzuki, Ana María Porto, Milagros Izquierdo, Juan Carlos Naranjo, Herbert Lange, Elizabeth Gasparim, Matthieu Calvez, Christian Pauly, Ernst Kani, John A. Velling, Rubí E. Rodríguez, Phillip Griffiths, Samuel Boissière, José María Muñoz Porrás, Alexis García-Zamora, Saúl Quispe, Víctor González-Aguilera, Alessandra Sarti, Francisco Plaza, Antonio Costa. Sitting, from left to right: Angel Carocca, Mauricio Godoy Molina, María Gioia Cifani, Irene Spelta, Ernesto Gironde, Leonardo Opazo, Kelyane Abreu, Danny Flores, María Paz Barría, Cristian Reyes, Yasmina Atarihuana, Juan Carlos García, Carolina Canales, Vicente Chicaiza.

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Articles in this volume are based on lectures given at three conferences on Geometry at the Frontier, held at the Universidad de La Frontera, Pucón, Chile in 2016, 2017, and 2018.

The papers cover recent developments on the theory of algebraic varieties—in particular, of their automorphism groups and moduli spaces. They will be of interest to anyone working in the area, as well as young mathematicians and students interested in complex and algebraic geometry.



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