CONTEMPORARY MATHEMATICS

797

Recent Developments in Fractal Geometry and Dynamical Systems

AMS Special Session
Fractal Geometry and Dynamical Systems
May 14–15, 2022
Virtual

Sangita Jha
Mrinal Kanti Roychowdhury
Saurabh Verma
Editors

AMS American Mathematical Society

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We dedicate this volume to our beloved parents

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Preface

This book is a theme issue based on the papers presented virtually at an AMS Special Session on "Fractal Geometry and Dynamical Systems", May 14–15, 2022. The session was attended virtually by 40 researchers. There were total 24 invited talks. The list of participants and conference program can be obtained from $http://www.ams.org/meetings/sectional/2294\_special.html$. The organizing committee consisted of Sangita Jha (National Institute of Technology Rourkela, India), M.K. Roychowdhury (University of Texas Rio Grandey Valley, US) and Saurabh Verma (Indian Institute of Information and Technology Allahabad, India). We would like to thank all participants and speakers in the conference, who made possible such a successful and fruitful event.

Fractal geometry has recently expanded into a vast field that encompasses practically all fields of science and engineering. It is the study of the properties of fractal objects, which are commonly referred to as fractals. The notion of fractal naturally arises in the study of non-linear functions. We have been fortunate to have eminent speakers in the field and dynamic enthusiastic researchers contributed to our volume with significant advances including some expository articles. The talks in the conference covered a large range of topics: some of them are concerned with geodesics in generalizations of the Sierpiński carpet by Berkove, Karangozishvili, and Smith and this work is relevant to the isoperimetric problem for fractals. The book includes the non-autonomous dynamics of complex polynomials by Comerford, Radulescu, and Cavanagh. It also includes the theory and applications of polymorphisms by Jorgensen and Tian. In particular, they covered areas of probability theory, analysis of Markov maps, transition operators, joinings of measure spaces, and applications to harmonic analysis of self-similar (fractal) measures. Others deal with the random beta-transformations on fat Sierpiński gasket by Dajani, Li, and Zhang; the connections between the shape and size of a period lattice for an elliptic function and its dynamics by Hawkins and Koss; the correspondence between unicritical polygons and maximally multicritical polygons in a lamination modeling a degree d polynomial with connected Julia set by Burdette, Hale, and Mayer; the theory of IFS measures associated with Bratteli diagrams by Bezuglyi and Jorgensen. Others deal with topological and geometric problems related to dynamical systems: unmating of certain expanding Thurston maps by Wilkerson, ergodicity of falling particle systems by Simanyi, problem of computability in dynamical systems by Burr and Wolf. Applications include pure ergodic theory, combinatorial number theory by Cömez. Fractal interpolation over curves was introduced by Massopust, the construction of fractal functions using weak construction was studied by Verma, Priyadarshi and the dimensional analysis of non-stationary fractal function was discussed by Chandra, Abbas.

Last but not least, we would like to express our gratitude to Christine Thivierge for her assistance with the edition and to the American Mathematical Society for continuing to support our proposal. Without the contributions of the authors and the anonymous referees, who nonetheless did good work, this collection would not have been conceivable. They were given incredibly strict deadlines. Our sincere appreciation to each of you.

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Fractal Geometry and Dynamical Systems • Jha et al., Editors

This volume contains the proceedings of the virtual AMS Special Session on Fractal Geometry and Dynamical Systems, held from May 14–15, 2022.

The content covers a wide range of topics. It includes nonautonomous dynamics of complex polynomials, theory and applications of polymorphisms, topological and geometric problems related to dynamical systems, and also covers fractal dimensions, including the Hausdorff dimension of fractal interpolation functions. Furthermore, the book contains a discussion of self-similar measures as well as the theory of IFS measures associated with Bratteli diagrams. This book is suitable for graduate students interested in fractal theory, researchers interested in fractal geometry and dynamical systems, and anyone interested in the application of fractals in science and engineering. This book also offers a valuable resource for researchers working on applications of fractals in different fields.

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