TEACHING MATH ONLINE: THEORY INTO PRACTICE

Chad M. Topaz
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PRELIMINARIES
WHEN I GIVE RESEARCH TALKS
WHEN I GIVE RESEARCH TALKS
WHEN I GIVE TEACHING TALKS
WHEN I GIVE TEACHING TALKS
WHY AM I HERE?

- To critique your college/university
- To critique your department
- To find fault with what you do in the classroom
- To tell you what values you should have
- To share my current thinking
MY PRIORITIES IN LIFE
RIGHT NOW
MY PRIORITIES IN LIFE RIGHT NOW

• Stopping police killing of Black people
MY PRIORITIES IN LIFE RIGHT NOW

• Stopping police killing of Black people

• Fighting the spread of fascism
MY PRIORITIES IN LIFE RIGHT NOW

• Stopping police killing of Black people

• Fighting the spread of fascism

• Advocating for public health
MY PRIORITIES IN LIFE RIGHT NOW

• Stopping police killing of Black people

• Fighting the spread of fascism

• Advocating for public health

• Supporting my students
MY PRIORITIES IN LIFE RIGHT NOW

• Stopping police killing of Black people
• Fighting the spread of fascism
• Advocating for public health
• Supporting my students
• Maintaining my family's well-being
MY PRIORITIES IN LIFE RIGHT NOW

• Stopping police killing of Black people

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• Advocating for public health

• Supporting my students

• Maintaining my family's well-being

• Tending to my mental health
MY PRIORITIES IN LIFE RIGHT NOW

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- Tending to my mental health
- Tending to my students’ mental health
MY PRIORITIES IN LIFE
RIGHT NOW

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• Tending to my mental health
• Tending to my students’ mental health
• etc.
• etc.
• etc.
MY PRIORITIES IN LIFE RIGHT NOW

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- etc.
- etc.
- etc.
- Season 5 of Queer Eye
MY PRIORITIES IN LIFE RIGHT NOW

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• Tending to my students' mental health
• Season 5 of Queer Eye
• Teaching math
DOES IT EXIST?

Awesome Class  Disastrous Class

In-Person Class

Online Class
DOES IT EXIST?

Awesome Class  Disastrous Class

In-Person Class  ✓  ✓
Online Class     ✓  ✓
PLAN FOR THIS TALK

• Ideas from learning science that influence me
• Overview of my online Discrete Math course
A PEDAGOGICAL MODEL
THE DESIGN OF LEARNING ENVIRONMENTS

Expanded Edition

How People Learn

Brain, Mind, Experience, and School

Community Centered
Learner Centered
Knowledge Centered
Assessment Centered
KNOWLEDGE
A REVISION OF BLOOM’S TAXONOMY

Learning objectives:
Students will be able to **verb noun**.

**cognitive process**
**knowledge type**
A REVISION OF BLOOM’S TAXONOMY

<table>
<thead>
<tr>
<th>Factual knowledge</th>
<th>Conceptual knowledge</th>
<th>Procedural knowledge</th>
<th>Metacogn. knowledge</th>
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<tr>
<td>Recall</td>
<td>Interpret</td>
<td>Execute</td>
<td>Differentiate</td>
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<tr>
<td>Recognize</td>
<td>Classify</td>
<td>Implement</td>
<td>Organize</td>
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<td>Remember</td>
<td>Summarize</td>
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### A REVISION OF BLOOM’S TAXONOMY

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Students can construct the contrapositive of a given conditional statement.
LEARNING AND ENJOYMENT
“There is evidence that negative correlations between student achievement and their enjoyment of instructional methods exist under certain circumstances... students often reported enjoying the method from which they learned the least.”

MEASURING ACTUAL LEARNING VERSUS FEELING OF LEARNING IN RESPONSE TO BEING ACTIVELY ENGAGED IN THE CLASSROOM

Deslauriers et al., PNAS (2019)
# Chad’s Totally Made Up Matrix of Learning and Enjoyment

<table>
<thead>
<tr>
<th>Avoid</th>
<th>As much as possible</th>
<th>As much as possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avoid</td>
<td>Fine but useless</td>
<td>Totally fine</td>
</tr>
<tr>
<td>Avoid</td>
<td>Avoid</td>
<td>Be judicious</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Effect on student enjoyment</th>
<th>+</th>
<th>0</th>
<th>-</th>
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<tbody>
<tr>
<td>+</td>
<td>+</td>
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<td>0</td>
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<td>+</td>
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<tr>
<td>-</td>
<td></td>
<td>0</td>
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Effect on student learning
GOALS AND MOTIVATION
SOME IDEAS ABOUT GOALS AND MOTIVATION

• Posting the grade distribution for this exam will help motivate lower-performing students.
• Contests and competitions will engage and motivate students.
• My students would learn more if they cared less about grades.
• The best students are those who just love math.
• If I show my students the beauty of a topic they’ll learn it better.
• If I show my students the value of a mathematical technique they’ll learn it better.
GOAL ORIENTATION

Deshon and Gillespie (2005)
GOAL ORIENTATION

Performance-avoidance: Avoid getting one of the lowest scores in the class.

Mastery-approach: Understand the idea of proof by induction.

Deshon and Gillespie (2005)
GOAL ORIENTATION

Deshon and Gillespie (2005)
GOAL ORIENTATION

Deshon and Gillespie (2005)
MOTIVATION

ACTIVE CHOICE
Decision to pursue a goal

PERSISTENCE
Continuing in face of challenges/distractions

MENTAL EFFORT
Intellectual investment, willingness to try hard

Clark and Estes (2002)

EXPECTANCY
Belief that action will lead to an outcome

VALENCE
Importance placed on the outcome

Vroom (1964)
INCLUSIVE TEACHING
BELONGING

• Belonging
  ‣ “My teacher knows my name.”

• Peer acceptance and support
  ‣ “[My groupmate] told me I did a good job… she acknowledged what I was good at.”

• Instructor acceptance and support
  ‣ “[The professor] always tried to listen to your opinion and understand it.”
“[Inclusive] classroom environment perceptions predicted students’ belonging beliefs, which in turn predicted students’ motivation, engagement, and achievement in the course.”

STRATEGIES FOR INCLUSION

• Syllabus design
• Inclusive language
• Implicit bias mitigation
• Stereotype threat reduction
• Active learning
SYLLABUS DESIGN: WHAT TO DO

- Is explicit about everything
- Has fair policies
- Provides learning outcomes, prerequisites
- Explains assessment/evaluation scheme
- Serves as a resource
- Signals an inclusive environment
INCLUSIVE LANGUAGE:

• Give students (electronically) chance to tell pronouns
• When unsure, say “they” instead of “he/she”
• No more “you guys”
• Avoid “it’s clear,” “this one’s easy,” “obviously,” etc.
• Talk about yourself as a person, talk about your struggles
IMPLICIT BIAS

• Unconscious attribution of particular qualities to a member of a certain social group

• Everyone has it

• Why? Brain is trying to shortcut
SCIENCE FACULTY’S SUBTLE GENDER BIASES FAVOR MALE STUDENTS

Moss-Racusin et al., PNAS (2012)
COGNITIVE SOPHISTICATION DOES NOT ATTENUATE THE BIAS BLIND SPOT

“The so-called bias blind spot arises when people report that thinking biases are more prevalent in others than in themselves. In two studies, we found that none of these bias blind spots were attenuated by measures of cognitive sophistication. If anything, a larger bias blind spot was associated with higher cognitive ability.”

IMPLICIT BIAS:
WHAT TO DO

• Build new associations via evaluative conditioning and counterstereotypical exemplars

• Solicit feedback from students, observers

• Slow down / practice mindfulness

• Enhance objectivity (e.g., grade anonymously)

see, e.g., Fitzgerald et al., BMC Psych. (2019)
STEREOTYPE THREAT

- Feeling of risk of conforming to a stereotype

STEREOTYPE THREAT

- Feeling of risk of conforming to a stereotype

![Bar chart showing score on math test for alleged gender and no gender differences. Change in blue is significant, change in green is not.](chart.jpg)

Spencer, Steele, Quinn, J., Exp. Soc. Psych. (1999)
STEREOTYPE THREAT: WHAT TO DO

• Set high standards and communicate belief in students
• Have students affirm core values
• Increase representation of minority groups
• State that intelligence is malleable, achieved through effort
• Communicate that diversity is valued
• Construct environments and have the physical objects in the environment not reflect one majority group

https://en.wikipedia.org/wiki/Stereotype_threat#Mitigation
ACTIVE LEARNING

• What is active learning? Discuss.
ACTIVE LEARNING

• What is active learning? Discuss.

• When learners participate in the educational process by doing something besides listening
ACTIVE LEARNING INCREASES STUDENT PERFORMANCE IN STEM

Freeman et al., PNAS (2014)
PEER DISCUSSION IMPROVES SCORES ON IN-CLASS CONCEPT QUESTIONS

Mean items correct on GRE verbal

Easy
Q1
Q1 AD
Q2

Medium

Hard

Smith et al., Science (2009)
ACTIVE LEARNING DECREASES THE PERFORMANCE GAP

ACTIVE LEARNING: WHAT TO DO

• Think/pair/share
• One minute muddiest point paper
• Concept questions / peer instruction
• Group activities (worksheets, labs, etc.)
• Individual + group quizzes
• Peer feedback (exchange solutions, etc.)
• Many more!
DISCRETE MATHEMATICS
COURSE DESCRIPTION

This course will help you understand the world through a mathematical lens and will develop your powers of argumentation and critical thinking. We will explore and utilize diverse areas of discrete mathematics including logic, set theory, functions and relations, combinatorics, probability, networks, and more. We also will discuss methods and styles of mathematical proofs in order to prepare you for more advanced math courses. Finally, while mathematical knowledge is often perceived as being “pure,” we will highlight some ways in which it is socially constructed and hence subject to human limitations and biases.
COURSE FORMAT

To afford students flexibility during the COVID pandemic, this course is taught online. Students will watch lecture material asynchronously and will participate in a once-per-week synchronous small-group tutorial meeting with the instructor via video chat.
EVALUATION

Students will complete checkpoint quizzes on videos and reading assignments, regularly assigned homework problems, and reflective writing assignments. To move towards a non-hierarchical, transparent, and egalitarian grading system, the instructor follows the policy of “ungrading.” Over the course of the semester, students will develop a rubric to assess their own learning and will evaluate themselves according to this rubric.
Discrete Mathematics

MATH 200
Fall 2020, Online Course

Instructor
Chad Topaz
cmf6@williams.edu

Teaching Assistants
TBA

Office hours
Times TBD
Online (Zoom)

Help sessions
Times TBD
Online (Zoom)

COURSE DESCRIPTION

Understand the world through a mathematical lens and increase your powers of argumentation and critical thinking! We explore and utilize diverse areas of discrete math including logic, set theory, functions, relations, combinatorics, probability, networks, and more.
SYLLABUS

CLASS STRUCTURE

1. Reading
   Portions of textbook and ancillary materials

2. Videos
   Brief presentations of core material, posted online

3. Checkpoints
   Multiple choice questions on main ideas

4. Tutorial
   Synchronous small-group meeting with professor

5. Practice
   Problem sets to explore and solidify ideas

Ongoing instructor feedback and self-reflection
SYLLABUS

COURSE ENVIRONMENT

**Names/Pronouns**
You deserve to be addressed in the manner you prefer. To guarantee that I address you properly, you are welcome to tell me your pronoun(s) and/or preferred name at any time, either in person or via email.

**Diversity**
We embrace diversity of gender, gender expression, sex, sexual orientation, race, ethnicity, national origin, age, religion, disability status, family status, socioeconomic background, and other visible and non-visible categories. I do not tolerate discrimination.

**Accessibility**
I want you to succeed in this course. Contact glw3@williams.edu for learning accommodations. For personal issues, stress, health problems or life circumstances, contact the Dean’s office at x4171. Contact me if you have other special circumstances. I will find resources for you.

**Title IX**
You deserve a community free from discrimination, sexual harassment, a hostile environment, sexual assault, domestic violence, dating violence, and stalking. If you experience or know of a Title IX violation, you have many options for support and/or reporting; see titleix.williams.edu.

**Academic Integrity**
The honor code is a cornerstone of our learning community and of this course. It is your responsibility to know and follow academic integrity policies. I will gladly answer any questions you have.
# Pedagogical Design

## Knowledge Centered
- Reading: X
- Videos: X
- Checkpts: X
- Tutorial: X
- Problem Sets: X
- Asynch. Discuss.: X
- Office Hours: X
- Self Eval.: X

## Learner Centered
- Reading: X
- Videos: X
- Checkpts: X
- Tutorial: X
- Problem Sets: X
- Asynch. Discuss.: X
- Office Hours: X
- Self Eval.: X

## Assessment Centered
- Reading: X
- Videos: X
- Checkpts: X
- Tutorial: X
- Problem Sets: X
- Asynch. Discuss.: X
- Office Hours: X
- Self Eval.: X

## Community Centered
- Reading: X
- Videos: X
- Checkpts: X
- Tutorial: X
- Problem Sets: X
- Asynch. Discuss.: X
- Office Hours: X
- Self Eval.: X
READING

THE ART OF LOGIC IN AN ILLOGICAL WORLD

EUGENIA CHENG
A HISTORY OF INDIAN LOGIC

(ANCIENT, MEDIAEVAL AND MODERN SCHOOLS.)

BY

MAHARISHI SATIS CHANDRA VIDYABHUSANA,
M.A., Ph.D., M.R.A.S., F.A.S.B.,
Principal Sanskrit College, Calcutta. Joint Philological Secretary
Academy Society of Bengal. Fellow of the Calcutta
University, etc.

MOTILAL BANARSIDASS
DELHI :: PATNA :: VARANASI
Epistemic injustice in mathematics

Colin Jakob Rittberg1 · Fenner Stanley Tanswell2 · Jean Paul Van Bendegem1

Received: 31 January 2018 / Accepted: 8 October 2018 © Springer Nature B.V. 2018

Abstract

We investigate how epistemic injustice can manifest itself in mathematical practices. We do this as both a social epistemological and virtue-theoretic investigation of mathematical practices. We delineate the concept both positively—we show that a certain type of folk theorem can be a source of epistemic injustice in mathematics—and negatively by exploring cases where the obstacles to participation in a mathematical practice do not amount to epistemic injustice. Having explored what epistemic injustice in mathematics can amount to, we use the concept to highlight a potential danger of intellectual enculturation.

Keywords

Epistemic injustice · Virtues · Philosophy of mathematics · Folk theorems · Caramello · Royen · Ramanujan · Enculturation

1 Introduction

Desirable mathematics extends beyond the confines of correct calculations. Desirable mathematics also has to do with the ways in which mathematicians perform their craft: we value traits such as creativity, metic-
For practice
Rewrite the statement formally using quantifiers and variables. Then, write a negation for each statement.

- Everybody loves somebody.
- Somebody loves everybody.
- Any integer equals twice some integer.
- There is a program that gives the correct answer to any question that is posed to it.

Section 3.4: Arguments with quantified statements

Instantiation
- The rule of universal instantiation says that if some property is true of everything in a set, then it is true of any particular thing in the set.
- For example, Williams students work too hard. Anita is a Williams student. Therefore, Anita works too hard.

Universal Modus Ponens
- Universal instantiation can be combined with Modus Ponens to construct an argument called universal Modus Ponens:
  - \( \forall x \, (P(x) \rightarrow Q(x)) \)
  - \( P(a) \) for a particular \( a \)
  - \( \therefore Q(a) \)
- For example, if \( T \) is any right triangle with hypotenuse \( c \) and legs \( a \) and \( b \), then \( c^2 = a^2 + b^2 \). A given triangle is a right triangle with both legs equal to 1 and hypotenuse \( c \). Then \( c^2 = 1^2 + 1^2 = 2 \).

Universal Modus Tollens
- Similarly, universal instantiation can be combined with Modus Tollens to construct an argument called universal Modus Tollens:
  - \( \forall x \, (P(x) \rightarrow Q(x)) \)
  - \( \neg Q(a) \) for a particular \( a \)
  - \( \therefore \neg P(a) \)
- For example, all professors are absent-minded. Jude is not absent-minded. Therefore Jude is not a professor.
VIDEOS

Prof. Mo Omar (https://www.youtube.com/watch?v=zP9t001PXiU)
VIDEOS

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Dr. Sara Herke (https://www.youtube.com/watch?v=qZ736F8ljYU)
VIDEOS

Dr. Sara Herke (https://www.youtube.com/watch?v=qZ736F8ljYU)
CHECKPOINTS

1. The statement $p \lor (p \land q)$ is logically equivalent to $p \land (p \lor q)$.
   - True
   - False

2. The negation of “they tested positive for COVID and they are not infected” is...
   - They didn’t test positive for COVID and they are infected
   - They didn’t test positive for COVID or they are infected
   - They didn’t test positive for COVID or they are not infected
   - They tested positive for COVID and they are infected
   - They tested positive for COVID or they are not infected
TUTORIAL

- 75 min 2x/week
- 2 sections
- 30 students/section
- 5 students/tutorial
- 300 minutes
- 12 tutorials
- 25 min tutorials
WEEKLY SELF-EVALUATION

1. Why are you taking this course? What are your goals for yourself?

2. Articulate a plan for how you will “do” this course. In your plan, please include how much time you will allocate to this course each week, what you will do when you encounter challenges, and so forth. Be as specific and granular as you can.

3. Keeping in mind your goals for taking this course, what criteria/metrics/experiences/reflections will you use to evaluate yourself?

4. Now, using the criteria you discussed above, please evaluate your work in this course so far.
FINAL REFLECTIONS ON MY APPROACH

• Being human is more important than doing math.

• An online course is, first and foremost, a course.

• The learning science literature is valuable.

• You don’t have to do All The Things.