AMERICAN MATHEMATICAL SOCIETY
Advancing research. Creating connections.

December 2020

Priorities for the National Science Foundation

Founded in 1888, the American Mathematical Society (AMS) is dedicated to advancing the interests of mathematical research and scholarship and connecting the diverse global mathematical community. We do this through our book and journal publications, meetings and conferences, database of research publications[1] that goes back to the early 1800s, professional services, advocacy, and awareness programs.

The AMS has 30,000 individual members worldwide and supports mathematical scientists at every career stage.

The AMS advocates for increased and sustained funding for the National Science Foundation (NSF). The NSF supports more fundamental research in the mathematical sciences—and done at colleges and universities—than any other federal agency.[2] A significant increase in Congressional appropriations would help address the effects of years of high-quality grant proposals that go unfunded due to limited funding. Those unmet needs continue. A 2019 National Science Board report[3] stated that in fiscal year 2018, "approximately $3.4 billion was requested for declined proposals that were rated Very Good or higher in the merit review process." This accounts for about 5440 declined proposals at the NSF. The U.S. is leaving potentially transformative scientific research unfunded, while other countries are making significant investments.

The applications of advances in theoretical science, including theory of mathematics, occur on a time scale that means the investment is often hard to justify in the short run.

In the next section we give an overview of our two priorities. The second, and final section offers a discussion of existing funding mechanisms for mathematicians.

AMS Priorities for the National Science Foundation

1. **Invest in fundamental research.**
2. **Invest in the next generation of research scientists.**

---

1 https://mathscinet.ams.org/mathscinet

2 See Figures 5B-13 and 5B-15 at https://ncses.nsf.gov/pubs/nsb20202/academic-r-d-in-the-united-states

3 https://www.nsf.gov/nsb/publications/2020/nsb202013.pdf




Photo by Kate Awtrey, Atlanta Convention Photography

Invest in fundamental research

Fundamental research in mathematics touches on all of the scientific priorities of the Biden-Harris agenda. As examples, mathematicians model the spread of pandemics and help assess the efficacy of vaccine programs; we produce basic research needed for advances in artificial intelligence and machine learning; and—as described below—mathematicians' theoretical work underpins imaging technologies used to detect diseases, including cancer.

Mathematics research is at the extreme for long-term payoff. Correct mathematical results are as valid today as they will be in 30 and even 300 years. Equally, correct results from 300 years ago are still valid.

The applications of advances in theoretical science, including theory of mathematics, occur on a timescale that means the investment is often hard to justify in the short run.

And yet if we look back to the success, as opposed to ahead to when we expect success, the investment in fundamental research has had huge payoffs.

According to a recent study on journal usage, mathematics is at the extreme for the life of journal articles. Across all subject disciplines, journal half-lives peaked between two and four years. Seventeen percent of all journals had usage half-lives that exceeded six years, however, with mathematics journals at the extreme—36% of the mathematics journals examined had usage half-lives exceeding six years.[4]

Correct mathematics is never *replaced* by newer, correct mathematics. Instead, it is *augmented* by deeper mathematics.

Here are two examples of NSF mathematics investments whose benefits to society were not known at time of investment:

1. Public-key cryptography was initially based on the question of factoring numbers into their prime factors. Number theory is central to commerce and defense.

   Diffie and Hellman's groundbreaking 1976 paper created the concepts of public key cryptography and digital signatures and solved the key exchange problem.[5] Their key exchange protocol was the first and remains one of the most frequently used in a range of today's different security protocols. This work was partially supported by NSF Grant ENG 10173. According to Google scholar, this paper has been cited almost 20,000 times, and the number of citations to it has grown over the decades since it was published; in each year since 2005, over 700 authors have cited this paper.

---

[4] See the AMS Open Access Primer: http://www.ams.org/government/AMSPrimerOnOpenAccessGlossary.3-13-19RMH.pdf

[5] Diffie, W. and Hellman, M. "New Directions in Cryptography." IEEE Trans. Info. Th. 22, 644-654, 1976.

It took until the 1990s to realize the applications of the Diffie-Hellman paper and subsequent work on public key cryptography to the internet, e-commerce, and finance.

Today, research in public key cryptography remains vital to national security as industries and governments around the world compete to build a high-functioning quantum computer. National Institute of Standards and Technology (NIST) has organized a competition to develop and standardize the post-quantum public key cryptography that will have to replace the systems currently in use.[6]

2 MRI scans are crucial tools in modern medicine: 40 million scans are performed yearly in the U.S. MRIs are essential in some fields:

- Neurologists seek to pinpoint brain tumors or study demyelinating diseases and dementia.
- MRIs play an important role in the diagnosis of cancer and the planning of treatment; they are one of the most effective tools for early detection of cancer.

MRI technology underwent a revolution accelerated in 2017 when the FDA approved two new MRI devices which dramatically speed up important MRI applications, from 8x to 16x. Siemens' technology (CS Cardiac Cine) allows movies of the beating heart; GE's technology (HyperSense) allows rapid 3D imaging, for example of the brain. Pediatric MRI scan times can be reduced in representative tasks from 8 minutes to 70 seconds, while preserving the diagnostic quality of images. Children with conditions that require repeated imaging benefit tremendously, as they can thus be imaged successfully and comfortably with far less frequent use of sedation.

In the mid-2000s, David Donoho (Stanford) and his NSF-funded postdoc Jared Tanner (now at Oxford) studied this problem. Their NSF funding enabled fundamental research in high-dimensional geometry, which is at the heart of the mathematics revolutionizing MRI and other imaging—compressed sensing. Their work developed over the next decade, culminating in the aforementioned 2017 FDA approval. The work continues, and continues to have impact in many image reconstruction areas, including MRI imaging. NSF-funded mathematics research and NIH funding of cognate disciplines played a key role in these developments.

---

6 https://www.nist.gov/news-events/news/2020/07/nists-post-quantum-cryptography-program-enters-selection-round

The opening years of the twenty-first century have been remarkable ones for the mathematical sciences. The imaging advancement described above is just one success. For many more examples, as well as an overview of the vitality of the discipline more broadly, we recommend:

National Research Council. 2013. *The Mathematical Sciences in 2025*. Washington, DC: The National Academies Press. https://doi.org/10.17226/15269.

National Research Council. 2012. *Fueling Innovation and Discovery: The Mathematical Sciences in the 21st Century*. Washington, DC: The National Academies Press. https://doi.org/10.17226/13373. (This is a summary of the findings which appear, in full, in the first resource.)

Invest in the next generation of research scientists

We are at a critical time for building and ensuring a stable STEM workforce of the future, a challenge exacerbated by the COVID-19 pandemic. Becoming a PhD STEM researcher requires focus and dedication; the work is demanding. The public perception of higher education has become increasingly negative, while respect for science is eroding. And the COVID-19 pandemic has deferred and even derailed students' dreams and plans. It is harder than ever to choose and pursue a career in STEM.

The AMS is deeply concerned about workforce development and the U.S. ability to support new PhDs and other early career scientists. As we consider our own programs, and anticipate a renewed interest and investment in the STEM workforce from the Biden-Harris administration, we believe it is important to:

- **attract new students to STEM fields;**
- **continue to engage the ones already in the pipeline;**
- **diversify STEM fields along all axes, including race, gender, and geography;**
- **work toward racial equity in science;**
- **train students for a wide variety of careers;**

and, at this point in time,

- **support new PhDs in the current depressed job market.**

Investment in the mathematical sciences benefits all STEM fields. Students from all sciences and engineering fields take multiple mathematics courses as undergraduates. Social scientists study statistics and, increasingly, more additional mathematics. Efforts to diversify our field will thus have payoff in all fields, as math teachers and professors teach many more students across fields than do those from the other sciences.

Investment in training mathematicians directly benefits the U.S. government; the National Security Agency (NSA) is said to be the largest single employer of PhD mathematicians in the country.[7] NSA mathematicians work on research in a wide variety of math fields, not only in ones we might identify as related to the NSA mission.

It is vitally important that the NSF is able to support graduate students, post-doctoral fellows, and other early career scientists, who are disproportionately affected by the COVID-19 pandemic and are most likely to have had their career goals deferred or derailed. At this challenging time, we cannot risk losing a generation of scientists who leave the field and never return.

It might be useful to reflect on what happened in 2009. Using regular appropriations, NSF's Division of Mathematical Sciences (DMS) created 45 new postdoctoral fellowships.[8] For these 45 new positions created, almost 800 math PhDs applied. Further, using funds delivered by the American Recovery and Reinvestment Act of 2009, the NSF was successful at increasing the number of Faculty Early Career Development Program (CAREER) and Graduate Research Fellowship Program (GRFP) awardees. We are hoping to have similar funding soon, to help early career scientists bridge the gap between graduate school and tenure-track positions.

Increased funding, in the longer term, for Mathematical Sciences Postdoctoral Research Fellowships (MSPRF), is recommended.[9]

The AMS hosts MathJobs.org.[10] This is *the* place where PhD mathematicians look for jobs. We have good data, including monthly and weekly postings of new jobs. Here are a few data points, that tell a story:

In 2019, a total of 2019 jobs were posted on MathJobs.org. As of the end of October 2020, the number of new job postings was approximately 1200. We would expect most positions to be posted at that point in the year. But, one could argue that the year isn't yet over. We can, then, consider some comparisons of monthly postings, showing that postings over the past few months are down sharply from what they were a year ago:

# of new jobs posted, 2019		# of new jobs posted, 2020
August	185	93
September	287	158
October	434	270

These are the months we expect most jobs to be posted. Our hiring cycle works on the academic cycle—most jobs begin in September, with hiring done throughout the previous fall and during the first few months of the year.

---

7 https://www.careerrookie.com/company/National-Security-Agency--NSA-/chm0q5741bfmcp80jcf

8 https://www.msri.org/web/msri/about-msri/news/msri-in-the-media/177https:/www.msri.org/web/msri/about-msri/news/msri-in-the-media/177

9 https://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5301&org=DMS&from=home

10 https://mathjobs.org/

We are at a critical time for building and ensuring a stable STEM workforce of the future, a challenge exacerbated by the COVID-19 pandemic.

Almost all (but not all) of the positions posted on MathJobs.org are for academic positions. To put these numbers in context, 1960 individuals received PhDs in the mathematical sciences (mathematics, applied mathematics, statistics, biostatistics) at U.S. institutions during the period July 1, 2017 through June 30, 2018. About 67% of the cohort took jobs in academia (33% in government, business, industry). About 30% of the cohort entered postdoc positions.

The pandemic is disproportionately affecting women. We are seeing evidence that the productivity of women has been drastically affected because women tend to be the ones that take care of things with family (both children and elderly people).[11] Even if we stop the tenure clock and make other accommodations, this research slowdown is going to delay the progress of women in their careers, while most men will move forward. It will negatively affect already existing salary disparities. The NSF may be able to offer programs to offset this problem.

In addition to negative impacts on already established researchers and new PhDs, the pandemic is negatively affecting students. Many graduate programs are freezing or at least "pausing" graduate admissions in mathematics. Columbia University provides one example.[12] Graduate admissions are a moving target, and it is impossible to get hard statistics to capture the situation. However, there are crowd-sourcing efforts in the community, trying to collect information about and understand the landscape. To adopt to the current situation, many graduate programs are not requiring the Graduate Record Examinations (GRE) for incoming students; Brown University graduate student Emily Winn is curating a Google doc with information about which programs have GRE requirements, including which programs are waiving requirements this year.[13] The AMS maintains a listserv for Directors of Graduate Programs. On that listserv there has been discussion of this topic. Here is one comment, from the Director of Graduate Studies at an east-coast university: "*We are not suspending admissions, however our department did extend all of deadlines for graduate students by one year last Spring and the current job market for students hoping to finish PhD this year looks terrible, so it is very unclear how many spots we will have available for new admissions*." The listserv comments are not vetted, but we get the clear impression that this is the case in many math departments. And, each spot taken by a student who would otherwise graduate and leave for a job is one that cannot be offered to a first-year graduate student.

---

[11] https://www.nytimes.com/2020/10/06/science/covid-universities-women.html?referringSource=articleShare

[12] https://www.math.columbia.edu/2020/09/21/pause-in-graduate-admissions/

[13] See: https://docs.google.com/spreadsheets/d/1hmdO7af3-ILvtJQO-szayG6bITvAYBQ1JcYXFZ_6apE/edit#gid=0. Emily is an NSF-funded graduate student: http://emilytwinn.com/

Specific Funding Mechanisms

The AMS urges increased funding for the Mathematical Sciences Research Institutes, a funding mechanism of the Division of Mathematical Sciences (DMS). These institutes play a critical role advancing research and building and sustaining the mathematical sciences community. Mathematics is one of the few NSF-funded fields in which the majority of PhD scientists work in academia, and programming at the institutes often includes the opportunity for us to interact with industry; indeed, some focus programming on this interaction. The institutes play a significant and important role broadening participation in the mathematical sciences, and institute programming includes many workshops aimed at doing so. These observations, and the hope for a greater number of institutes, echo the 2020 DMS Committee of Visitors report.[14] Additionally, Congress has voiced its support—the Senate Committee on Appropriations, in their FY2021 CJS Explanatory Statement, "recognizes the importance of the NSF Mathematical Sciences Institutes across the country, which provide important basic research in multiple fields."[15]

We note that DMS award size is the smallest of any division at the NSF.[16] The single most important mechanism for NSF support of graduate students is—indirectly—through research grants (including individual investigator awards). This support is essential to advancing mathematical sciences. Here are some quotes from three notable AMS members:

> "*I have used my NSF grant to support my graduate students in the summer. They each usually get one, maybe two summers worth of funding, so it's not a huge portion of the budget. But it is essential for their graduate study. If they do not get the support, they end up teaching during the summer. On the other hand, with my NSF support, they have an entire three months to focus solely on their research, including the opportunity to attend a conference or two where they meet peers and senior members of the field who can give them guidance about their research and their career. For my students, the summer support has been invaluable.*"
>
> —**Tara Holm**, *Cornell University, past chair of the AMS Committee on Education*

---

[14] Presented at the November 30, 2020 Mathematical & Physical Sciences Advisory Committee (MPSAC) meeting.

[15] https://www.appropriations.senate.gov/news/committee-releases-fy21-bills-in-effort-to-advance-process-produce-bipartisan-results

[16] https://dellweb.bfa.nsf.gov/awdfr3/default.asp

"The teaching load for graduate students can be high–especially given the economics and teaching scale at state schools. Being supported from grants, even for a summer, provides students the opportunity to focus on research, especially for students already in candidacy. Post candidacy is a critical time for graduate education. The number of GRFs is modest as compared to the total graduate enrollment at even Research I universities. Grant support opportunities serve a larger population and so support a more diverse population."

—**Scott Wolpert**, *University of Maryland, past chair of the AMS Committee on Science Policy*

"Graduate student funds in my NSF grants supported graduate students by offering them occasional release time during the academic year. This was usually a half-time RA for a semester. Typically, this support was given during students' 4th year (when they are achieving progress on their thesis research) and/or in the Fall semester of their 5th year (when they prepare job applications). Graduate student funds in my grants also supported graduate student travel. These funds were very helpful and constructive, helping my students complete their thesis in a timely manner and obtain their first post-PhD position."

—**Eric Friedlander**, *University of Southern California, past AMS President*

Support for graduate students through research grants can increase the diversity of the pool of PhD mathematicians. Because there are relatively few Graduate Research Fellowship Program (GRFP) awards, and these are only available to students early in graduate school, only those who know to apply will, and only those graduate students who look strong early on have a chance of getting a GRFP award. Prospective graduate students (and those in their first years of graduate school) who attend(ed) a small undergraduate institution, those without strong undergraduate training, and those without a good understanding of "the system" may get into graduate school and only after a year or two begin to shine. These individuals are completely left out of the GRFP pool, as their undergraduate professors may not know about available funding. Even if they do apply, their applications may look weaker than those who have access to more advanced coursework during their undergraduate years.

Prepared by

Bryna Kra, *Sarah Rebecca Roland Professor of Mathematics at Northwestern University, AMS Trustee* and *past chair of Board of Trustees*

Kasso Okoudjou, *Professor of Mathematics at Tufts University, AMS Executive Committee of the Council*

Jill Pipher, *Elisha Benjamin Andrews Professor of Mathematics, Founding Director of the NSF-funded ICERM*, and *Vice President for Research at Brown University, AMS President*

Karen Saxe, *PhD, AMS Associate Executive Director, Government Relations*
Contact: kxs@ams.org