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- [2] N. Hochster and J. L. Roberts, *Rings of invariants of reductive groups acting on regular rings are Cohen-Macaulay*, *Adv. Math.* **13** (1974) 115–175.

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