

---

VOLUME 35

NUMBER 2

APRIL 2022

---

# JOURNAL

OF THE

ISSN 0894-0347 (print)  
ISSN 1088-6834 (online)

---

A M E R I C A N M A T H E M A T I C A L S O C I E T Y

---

**EDITORS**

Laura G. DeMarco  
Simon Donaldson  
Pavel Etingof  
Michael J. Larsen  
Sylvia Serfaty  
Richard Taylor  
Shmuel Weinberger

**ASSOCIATE EDITORS**

Mohammed Abouzaid  
Ian Agol  
Jian Ding  
Dmitry Dolgopyat  
Lawrence Guth  
Ursula Hamenstadt  
Lars Hesselholt  
Richard Kenyon  
Bruce A. Kleiner  
Thomas Fun Yau Lam  
Ciprian Manolescu  
Lillian Pierce  
Anand Pillay  
Peter Scholze  
Amit Singer  
Gigliola Staffilani  
Benjamin Sudakov  
Ulrike Tillmann  
Burt Totaro  
Melanie Matchett Wood  
Zhiwei Yun



AMERICAN  
MATHEMATICAL  
SOCIETY

Providence, Rhode Island USA

## Journal of the American Mathematical Society

This journal is devoted to research articles of the highest quality in all areas of pure and applied mathematics.

**Submission information.** See **Information for Authors** at the end of this issue.

**Publication on the AMS website.** Articles are published on the AMS website individually after proof is returned from authors and before appearing in an issue.

**Subscription information.** The *Journal of the American Mathematical Society* is published quarterly and is also accessible electronically from [www.ams.org/journals/](http://www.ams.org/journals/). Subscription prices for Volume 35 (2022) are as follows: for paper delivery, US\$462 list, US\$369.60 institutional member, US\$415.80 corporate member, US\$277.20 individual member; for electronic delivery, US\$407 list, US\$325.60 institutional member, US\$366.30 corporate member, US\$244.20 individual member. Upon request, subscribers to paper delivery of this journal are also entitled to receive electronic delivery. If ordering the paper version, add US\$5.00 for delivery within the United States; US\$16 for surface delivery outside the United States. Subscription renewals are subject to late fees. See [www.ams.org/journal-faq](http://www.ams.org/journal-faq) for more journal subscription information.

**Back number information.** For back issues see [www.ams.org/backvols](http://www.ams.org/backvols).

Subscriptions and orders should be addressed to the American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904 USA. *All orders must be accompanied by payment.* Other correspondence should be addressed to 201 Charles Street, Providence, RI 02904-2213 USA.

**Copying and reprinting.** Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for permission to reuse portions of AMS publication content are handled by the Copyright Clearance Center. For more information, please visit [www.ams.org/publications/pubpermissions](http://www.ams.org/publications/pubpermissions).

Excluded from these provisions is material for which the author holds copyright. In such cases, requests for permission to reuse or reprint material should be addressed directly to the author(s). Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.

---

The *Journal of the American Mathematical Society* (ISSN 0894-0347 (print); ISSN 1088-6834 (online)) is published quarterly by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2213 USA and is mailed from Providence, Rhode Island. Periodicals postage is paid at Providence, Rhode Island. Postmaster: Send address changes to Journal of the AMS, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA.

© 2022 by the American Mathematical Society. All rights reserved.

This journal is indexed in *Mathematical Reviews*, *Zentralblatt MATH*, *Science Citation Index*<sup>®</sup>, *Science Citation Index*<sup>TM</sup>-*Expanded*, *ISI Alerting Services*<sup>SM</sup>, *CompuMath Citation Index*<sup>®</sup>, and *Current Contents*<sup>®</sup>/*Physical, Chemical & Earth Sciences*. This journal is archived in *Portico* and in *CLOCKSS*.

Printed in the United States of America.

⊗ The paper used in this journal is acid-free and falls within the guidelines established to ensure permanence and durability.

10 9 8 7 6 5 4 3 2 1      27 26 25 24 23 22

JOURNAL OF THE AMERICAN MATHEMATICAL SOCIETY  
CONTENTS

Vol. 35, No. 2

April 2022

Nicolas Ressayre, <a href="#">On the tensor semigroup of affine Kac-Moody lie algebras</a> .....	309
Semyon Dyatlov, Long Jin, and Stéphane Nonnenmacher, <a href="#">Control of eigenfunctions on surfaces of variable curvature</a> .....	361
Jan Reimann and Theodore A. Slaman, <a href="#">Effective randomness for continuous measures</a> .....	467
Pavel Galashin, Steven N. Karp, and Thomas Lam, <a href="#">Regularity theorem for totally nonnegative flag varieties</a> .....	513
Thorsten Kleinjung and Benjamin Wesolowski, <a href="#">Discrete logarithms in quasi-polynomial time in finite fields of fixed characteristic</a> .....	581

## Editors

Laura G. DeMarco  
Department of Mathematics  
Northwestern University  
Evanston, IL 60208-2730 USA  
demarco@northwestern.edu

Simon Donaldson  
Simons Center for Geometry and Physics  
Stony Brook University  
Stony Brook, NY 11794 USA  
sdonaldson@scgp.stonybrook.edu

Pavel Etingof  
Department of Mathematics  
Massachusetts Institute of Technology  
77 Massachusetts Avenue  
Cambridge, MA 02139 USA  
etingof@math.mit.edu

Michael J. Larsen  
Indiana University  
Department of Mathematics  
831 E 3rd Street  
Bloomington, IN 47405 USA  
mjlarson@indiana.edu

Sylvia Serfaty  
New York University-Courant Institute  
251 Mercer Street  
New York, NY 10012-1185 USA  
serfaty@cims.nyu.edu

Richard Taylor  
Stanford University  
Building 380  
450 Jane Stanford Way  
Stanford, CA 94305 USA  
rftaylor@stanford.edu

Shmuel Weinberger  
Department of Mathematics  
University of Chicago  
5734 S University Avenue  
Chicago, IL 60637 USA  
shmuel@math.uchicago.edu

## Associate Editors

Mohammed Abouzaid, Columbia University  
Ian Agol, University of California, Berkeley  
Jian Ding, University of Pennsylvania  
Dmitry Dolgopyat, University of Maryland  
Lawrence Guth, Massachusetts Institute of Technology  
Ursula Hamenstadt, University of Bonn  
Lars Hesselholt, Nagoya University  
Richard Kenyon, Yale University  
Bruce A. Kleiner, New York University-Courant Institute  
Thomas Fun Yau Lam, University of Michigan  
Ciprian Manolescu, Stanford University  
Lillian Pierce, Duke University  
Anand Pillay, University of Notre Dame  
Peter Scholze, Universität Bonn  
Amit Singer, Princeton University  
Gigliola Staffilani, Massachusetts Institute of Technology  
Benjamin Sudakov, ETH Zürich  
Ulrike Tillmann, University of Oxford  
Burt Totaro, University of California, Los Angeles  
Melanie Matchett Wood, Harvard University  
Zhiwei Yun, Massachusetts Institute of Technology

## Editorial Information

Information on the backlog for this journal can be found on the AMS website starting from <http://www.ams.org/jams>.

In an effort to make articles available as quickly as possible, articles are electronically published on the AMS website individually after proof is returned from authors and before appearing in an issue.

A Consent to Publish is required before we can begin processing your paper. After a paper is accepted for publication, the Providence office will send a Consent to Publish and Copyright Agreement to all authors of the paper. By submitting a paper to this journal, authors certify that the manuscript has not been submitted to nor is it under consideration for publication by another journal, conference proceedings, or similar publication.

### Information for Authors

**Initial submission.** All articles submitted to this journal are peer-reviewed. The AMS has a single blind peer-review process in which the reviewers know who the authors of the manuscript are, but the authors do not have access to the information on who the peer reviewers are. The AMS uses Centralized Manuscript Processing for initial submission. Authors should submit a PDF file using the Initial Manuscript Submission form found at [www.ams.org/submission/jams](http://www.ams.org/submission/jams) or should send one copy of the manuscript to the following address: Centralized Manuscript Processing, JOURNAL OF THE AMS, 201 Charles Street, Providence, RI 02904-2213 USA. If a paper copy is being forwarded to the AMS, indicate that it is for the *Journal of the AMS* and include the name of the corresponding author, contact information such as email address or mailing address, and the name of an appropriate Editor to review the paper (see the list of Editors above).

The first page must contain a *descriptive title* that is short, but informative; useless or vague phrases such as “some remarks about” or “concerning” should be avoided. Although an abstract is not required upon initial submission, upon acceptance authors will be requested to supply an abstract for the electronic version of this journal. The AMS offers free worldwide access to the electronic abstracts. An abstract should be at least one complete sentence and at most 300 words. No abstracts appear in the printed journal starting in 1998. Included with the footnotes to the paper should be the 2010 *Mathematics Subject Classification* representing the primary and secondary subjects of the article. The classifications are accessible from [www.ams.org/msc/](http://www.ams.org/msc/). The Mathematics Subject Classification footnote may be followed by a list of *key words and phrases* describing the subject matter of the article and taken from it. Journal abbreviations used in bibliographies are listed in the latest *Mathematical Reviews* annual index. The series abbreviations are also accessible from [www.ams.org/msnhtml/serials.pdf](http://www.ams.org/msnhtml/serials.pdf). To help in preparing and verifying references, the AMS offers MR Lookup, a Reference Tool for Linking, at [www.ams.org/mrlookup/](http://www.ams.org/mrlookup/).

**Electronically prepared manuscripts.** Manuscripts should be electronically prepared in  $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{L}\mathcal{A}\mathcal{T}\mathcal{E}\mathcal{X}$ . To this end, the Society has prepared  $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{L}\mathcal{A}\mathcal{T}\mathcal{E}\mathcal{X}$  author packages for each AMS publication. Author packages include instructions for preparing electronic manuscripts, samples, and a style file that generates the particular design specifications of that publication series. Articles properly prepared using the  $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{L}\mathcal{A}\mathcal{T}\mathcal{E}\mathcal{X}$  style file and the `\label` and `\ref` commands automatically enable extensive intra-document linking to the bibliography and other elements of the article for searching electronically on the Web.

Authors may retrieve an author package for *Journal of the AMS* starting from [www.ams.org/jams/jamsauthorpac.html](http://www.ams.org/jams/jamsauthorpac.html). The *AMS Author Handbook* is available in PDF format from the author package link. The author package can also be obtained free of charge by sending email to [tech-support@ams.org](mailto:tech-support@ams.org) or from the Publication Division, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA. When requesting an author package, please specify the publication in which your paper will appear. Please be sure to include your complete email address.

**After acceptance.** The source files for the final version of the electronic manuscript should be sent to the Providence office immediately after the paper has been accepted for publication. The author should also submit a PDF of the final version of the paper to the

Editor, who will forward a copy to the Providence office. Accepted electronically prepared manuscripts can be submitted via the Web at [www.ams.org/submit-book-journal/](http://www.ams.org/submit-book-journal/), sent via email to [pub-submit@ams.org](mailto:pub-submit@ams.org), or sent on CD to the Electronic Prepress Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA. When sending a manuscript electronically via email or CD, please be sure to include a message indicating in which publication the paper has been accepted. Complete instructions on how to send files are included in the author package.

**Electronic graphics.** Comprehensive instructions on preparing graphics are available from [www.ams.org/authors/journals.html](http://www.ams.org/authors/journals.html). A few of the major requirements are given here.

Submit files for graphics as EPS (Encapsulated PostScript) files. This includes graphics originated via a graphics application as well as scanned photographs or other computer-generated images. If this is not possible, TIFF files are acceptable as long as they can be opened in Adobe Photoshop or Illustrator.

Authors using graphics packages for the creation of electronic art should also avoid the use of any lines thinner than 0.5 points in width. Many graphics packages allow the user to specify a “hairline” for a very thin line. Hairlines often look acceptable when proofed on a typical laser printer. However, when produced on a high-resolution laser imagesetter, hairlines become nearly invisible and will be lost entirely in the final printing process.

Screens should be set to values between 15% and 85%. Screens which fall outside of this range are too light or too dark to print correctly. Variations of screens within a graphic should be no less than 10%.

Any graphics created in color will be rendered in grayscale for the printed version unless color printing is authorized by the Managing Editor and the Publisher. In general, color graphics will appear in color in the online version.

**AMS policy on making changes to articles after publication.** Articles are published on the AMS website individually after proof is returned from authors and before appearing in an issue. To preserve the integrity of electronically published articles, once an article is individually published to the AMS website, changes cannot be made in place in the paper. The AMS does not keep author-related information such as affiliation, current address, and email address up to date after a paper is electronically published.

Corrections of critical errors may be made to the paper by submitting an errata article to the Editor. The errata article will be published electronically, will appear in a future print issue, and will link back and forth on the Web with the original article.

**Secure manuscript tracking on the Web.** Authors can track their manuscripts through the AMS journal production process using the personal AMS ID and Article ID printed in the upper right-hand corner of the Consent to Publish form sent to each author who publishes in AMS journals. Access to the tracking system is available from [www.ams.org/mstrack/](http://www.ams.org/mstrack/). An explanation of each production step is provided on the Web through links from the manuscript tracking screen. Questions can be sent to [jams-query@ams.org](mailto:jams-query@ams.org).

**Inquiries.** Any inquiries concerning a paper that has been accepted for publication that cannot be answered via the manuscript tracking system mentioned above should be sent to [jams-query@ams.org](mailto:jams-query@ams.org) or directly to the Electronic Prepress Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA.



JOURNAL OF THE AMERICAN MATHEMATICAL SOCIETY  
CONTENTS

Vol. 35, No. 2

April 2022

Nicolas Ressayre, On the tensor semigroup of affine Kac-Moody lie algebras .....	309
Semyon Dyatlov, Long Jin, and Stéphane Nonnenmacher, Control of eigenfunctions on surfaces of variable curvature .....	361
Jan Reimann and Theodore A. Slaman, Effective randomness for continuous measures .....	467
Pavel Galashin, Steven N. Karp, and Thomas Lam, Regularity theorem for totally nonnegative flag varieties .....	513
Thorsten Kleinjung and Benjamin Wesolowski, Discrete logarithms in quasi-polynomial time in finite fields of fixed characteristic .....	581



0894-0347(202204)35:2;1-J

