

# ABSTRACTS

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**NOTATIONS IN THIS JOURNAL** are the following:

\* Indicates who will present the paper at the meeting.

# SUBSCRIPTION INFORMATION

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## PAPERS PRESENTED AT MEETINGS

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**THIS CALENDAR** lists meetings of the Society which have been approved by the Council at which papers may be presented. Programs of Annual Meetings appear in the *Notices* and on the AMS website; programs for sectional meetings appear on the AMS Web pages in the Meetings & Conferences section, and are electronically archived in the *Notices* section on the AMS website.

MEETING #	DATE	PLACE	ABSTRACT DEADLINE	ABSTRACT ISSUE
1136	March 17–18, 2018	Columbus, OH	January 22	Vol 39, No. 2
1137	April 14–15, 2018	Portland, OR	February 6	Vol 39, No. 2
1138	April 14–15, 2018	Nashville, TN	February 13	Vol 39, No. 2
1139	April 21–22, 2018	Boston, MA	February 20	Vol 39, No. 2
1140	June 11–14, 2018	Shanghai, Peoples Rep of China	TBA	NONE
1141	September 29–30, 2018	Newark, DE	July 31	Vol 39, No. 3
1142	October 6–7, 2018	Fayetteville, AR	August 7	Vol 39, No. 3
1143	October 20–21, 2018	Ann Arbor, MI	August 21	Vol 39, No. 4
1144	October 27–28, 2018	San Francisco, CA	August 28	Vol 39, No. 4
1145	January 16–19, 2019	Baltimore, MD	TBA	TBA
1146	March 15–17, 2019	Auburn, AL	TBA	TBA
1147	March 22–24, 2019	Honolulu, HI	January 22	TBA

## SAN DIEGO, CA, January 10–13, 2018

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Abstracts of the 1135th Meeting.

### 00 ► General

1135-00-27     **Dana Randall\***, Georgia Institute of Technology, Atlanta, GA 30332. *Emergent phenomena in random structures and algorithms*. Preliminary report.

Markov chain Monte Carlo methods have become ubiquitous across science and engineering to model dynamics and explore large combinatorial sets. Over the last 20 years there have been tremendous advances in the design and analysis of efficient sampling algorithms for this purpose. One of the striking discoveries has been the realization that many natural Markov chains undergo phase transitions whereby they abruptly change from being efficient to inefficient as some parameter of the system is modified, also revealing interesting properties of the underlying stationary distributions.

We will explore valuable insights that phase transitions provide in three settings, First, they allow us to understand the limitations of certain classes of sampling algorithms, potentially leading to faster alternative approaches. Second, they reveal statistical properties of stationary distributions, giving insight into various interacting models, such as colloids, segregation models and interacting particle systems. Third, they predict emergent phenomena that can be harnessed for the design of distributed algorithms for certain asynchronous models of programmable active matter. We will see how these three research threads are closely interrelated and inform one another. (Received June 22, 2017)

1135-00-33     **Gunnar Carlsson\*** ([carlsson@stanford.edu](mailto:carlsson@stanford.edu)), Stanford University, Stanford, CA 94305. *Topological Modeling of Complex Data* Preliminary report.

One of the fundamental problems faced by science and industry is that of making sense of large and complex data sets. To approach this problem, we need new organizing principles and modeling methodologies. One such approach is through topology, the mathematical study of shape. The shape of the data, suitably defined, is an important component of exploratory data analysis. In this talk, we will discuss the topological approach, with numerous examples, and consider some questions about how it will develop as mathematics. (Received June 22, 2017)

1135-00-34 **André Neves\***, University of Chicago, Chicago, IL 60637. *Wow, so many minimal surfaces*. Preliminary report.

Minimal surfaces are ubiquitous in Geometry but they are quite hard to find. For instance, Yau in 1982 conjectured that any 3-manifold admits infinitely many closed minimal surfaces but the best one knows is the existence of at least two.

In a different direction, Gromov conjectured a Weyl Law for the volume spectrum that was proven last year by Liokumovich, Marques, and myself.

I will cover a bit the history of the problem and then talk about recent work with Irie, Marques, and myself: we combined Gromov's Weyl Law with the Min-max theory Marques and I have been developing over the last years to prove that, for generic metrics, not only there are infinitely many minimal hypersurfaces but they are also dense. (Received November 2, 2017)

1135-00-53 **Rebecca Lea Morris\*** ([email@rebeccaleamorris.com](mailto:email@rebeccaleamorris.com)). *Alternative proof presentations*.

The way a proof is presented can raise or lower the entry barrier required to understand it. As both students and researchers alike want to learn new mathematics, proof presentation is thus important both in teaching and research contexts. In this talk, I will consider different ways of presenting the same geometric proof to illustrate more precisely how presentation can make it easier or more difficult to understand. In particular, I will focus on how decisions about notation, diagrams and argument structure affect how efficiently the resulting proof presentation manages information. Good information management can make it much easier to understand, while bad information management can make it much harder. As part of this discussion, I will also consider different senses of mathematical understanding. (Received July 09, 2017)

1135-00-139 **David Hestenes\*** ([hestenes@asu.edu](mailto:hestenes@asu.edu)), 2416 S. Palm Dr., Tempe, AZ 85282. *Quaternions in Geometric Algebra and Physics*.

Geometric Algebra and Calculus has emerged as a unified mathematical language for the whole of physics — a language that simplifies formulation and solution of all fundamental equations while providing new insights into the geometric structure of physics [1]. In this talk I discuss how quaternions fit into Geometric Algebra with emphasis on rotational dynamics, spinors and Hopf fibrations in electrodynamics.

[1] Geometric Calculus website: <http://geocalc.clas.asu.edu>. Most papers and books on GC can be accessed or traced from here and links to other websites, especially one at Cambridge University. (Received August 02, 2017)

1135-00-152 **Jill Pipher\*** ([jpipher@math.brown.edu](mailto:jpipher@math.brown.edu)). *Nonsmooth boundary value problems*.

The regularity properties of solutions to linear partial differential equations in domains depend on the structure of the equation, the degree of smoothness of the coefficients of the equation, and of the boundary of the domain. Quantifying this dependence is a classical problem, and modern techniques can answer some of these questions with remarkable precision. For both physical and theoretical reasons, it is important to consider partial differential equations with non-smooth coefficients. We'll discuss how some classical tools in harmonic and complex analysis have played a central role in answering questions in this subject at the interface of harmonic analysis and PDE. (Received August 04, 2017)

1135-00-186 **Erica Walker\*** ([ewalker@tc.edu](mailto:ewalker@tc.edu)). *Hidden in Plain Sight: Mathematics Teaching and Learning Through a Storytelling Lens*.

What are the essential components from historical and contemporary narratives about mathematicians' paths to mathematics success? In this address, we uncover 'hidden' insights gained from my 20+ years of research and practice in mathematics and mathematics education. Through the stories of mathematicians, I will share three key themes and their implications of these themes for teaching, learning, and building empowered and engaged communities of mathematics students of all ages. (Received September 05, 2017)

1135-00-187 **Ronald Mickens\*** ([rmickens@cau.edu](mailto:rmickens@cau.edu)). *Nonstandard Finite Difference Schemes: Impact, Importance, and Dynamical Consistency*.

Nonstandard finite difference (NSFD) schemes are an alternative methodology for constructing discretizations of differential equations for the purpose of calculating numerical solutions. The genesis of this technique arose from earlier attempts to formulate for classical mechanics a structure based on discrete time rather the assumption of a continuous time independent variable. In this talk, we will discuss the impact of NSFD schemes in the research community as well as in the how they achieve dynamic consistency by incorporating specific features of a given system of differential equations. (Received September 05, 2017)

1135-00-189 **Antonina Kolokolova\*** (kol@mun.ca). *Power of reasoning over richer domains.*

How does the richness of underlying concepts affects the power of reasoning systems built on them? This question arises in many different settings, from automated reasoning to mathematical proofs.

Does it help to augment propositional, resolution-based reasoning used in modern Satisfiability solvers with non-propositional reasoning in underlying theory such as arithmetic? We show that even a theory of uninterpreted functions, decidable in near-linear time, helps enormously: resolution over that theory can simulate a much more powerful Frege (natural deduction) system.

Then, we look at complexity of proofs of existence of expander graphs, pseudorandom objects widely used in theoretical computer science. Surprisingly, we show that it is possible to prove existence their existence by purely combinatorial and probabilistic reasoning, using only concepts definable by polynomial-size formulas. An interesting corollary is that monotone natural deduction reasoning is just as powerful as its non-monotone counterpart, in stark contrast to circuit complexity.

Based on joint work with Vijay Ganesh and Robert Robere, and Samuel R. Buss, Michal Koucký and Valentine Kabanets. (Received September 26, 2017)

1135-00-373 **Laura M Smith\*** (lausmith@fullerton.edu), 800 N State College Blvd, Fullerton, CA 92831. *Developing Applied Mathematics Research Projects for Undergraduates.* Preliminary report.

Developing an interesting and accessible research project for undergraduates can be quite challenging. We first will discuss methods of defining problems. These include investigating current events and newly available datasets or collaborating with industry and faculty in other fields on interdisciplinary work. Other possibilities involve looking into the advisor's current research projects and taking smaller pieces or applications and modifying them into a project at an appropriate level. Another approach is to allow students to search through recent papers on a topic of interest until they find one that is well-written with mathematical techniques they are able to understand. We will examine some open problems in applied mathematics that are appropriate for undergraduate students. (Received August 29, 2017)

1135-00-879 **Ursula H Martin\*** (ursula.martin@cs.ox.ac.uk). *The social machine of mathematics.*

How does mathematics come about? Formal proof is only part of the story, and in this paper I present the results of highly interdisciplinary work, using philosophy, social science and history alongside computer science research in artificial intelligence, argumentation theory and verification, to show the scope for new techniques to support concept formation and argument finding, while highlighting the roles that risk, doubt, error, explanation and group knowledge play in the human production and use of mathematics. (Received September 16, 2017)

1135-00-1214 **Ana-Maria Croicu\*** (acroicu@kennesaw.edu), 1000 Chastain Rd, MD# 1102, Kennesaw, GA 30144. *NSF Scholarship Program in Science and Mathematics at Kennesaw State University.*

The NSF Scholarship Program in Science and Mathematics at Kennesaw State University (KSU) has the following objectives: (1)To improve the efforts to recruit more high-school students into STEM disciplines at KSU; (2)To increase participation, retention and completion of an undergraduate degree in one of the STEM disciplines by academically talented, but financially needy students; (3) To provide opportunities to attract more academically talented, underrepresented (minority and women) or first-generation low-income students;(4) To increase the number of students enrolling in STEM graduate programs and to help students enter workplace after graduation; (5)To broaden the student academic experience through participation in STEM related cohorts, mentoring and research activities, internship sand conference presentations. The evaluation of the NSF grant indicates that the program has been successful in supporting, promoting and retaining undergraduate students in STEM disciplines. NSF Scholarship recipients were found to have higher GPAs on average, they were more likely to have a GPA above 3.0, exhibited higher retention rates and were more likely to engage in research with faculty relative to STEM students who did not receive the NSF Scholarship. (Received September 20, 2017)

1135-00-1243 **Johannes C. Familton** (jfamilton@bmcc.cuny.edu), 199 Chambers Street, New York,, NY , and **Lucio Prado\*** (lprado@bmcc.cuny.edu). *Rodrigues, Euler, and Hamilton: The problem of Rotations.* Preliminary report.

In 1840 Rodrigues wrote a paper about the laws of geometry that control the displacement of a solid system in space. This work was a precursor to Hamilton's quaternions. Rodrigues' work went unnoticed until 1846 when Cayley acknowledged Euler's and Rodrigues' priority describing orthogonal transformations in a letter to the Editors of the Philosophical Magazine.

In this talk, we want discuss the history of Rodrigues' work, and his 1840 paper. We will analyze its connection with Euler's rotation theorem, and Hamilton's quaternions. (Received September 21, 2017)

1135-00-1261 **Amanda E Francis\*** ([afrancis@carroll.edu](mailto:afrancis@carroll.edu)), Amanda E Francis, Department of Mathematics, 1601 N Benton Ave, Helena, MT 59625. *Comparing two scoring options for link prediction in networks*. Preliminary report.

Given a network of known relationships between people in a social network, can you predict the most likely new relationship to form? There are various known methods for link prediction, for example, the Katz method counts the number of "paths" between people, and assigns a weight based on these counts. Alternately, we can use the mathematics of electrical resistor networks, assigning scores based on effective resistance. In this talk I give results comparing these two scoring methods on simple networks. (Received September 20, 2017)

1135-00-1266 **Yunfan Zhao\*** ([yzhao58@jhu.edu](mailto:yzhao58@jhu.edu)) and **Thomas G Marge** ([thomas.g.marge@jhu.edu](mailto:thomas.g.marge@jhu.edu)). *Efficient and Perfect Domination on Archimedean Lattices*. Preliminary report.

A set of vertices  $S$  is said to dominate a graph  $G=(V,E)$  if every vertex in  $V$  is either in set  $S$  or is adjacent to a vertex in set  $S$ . A dominating set is a perfect dominating set if every vertex not in the dominating set is dominated exactly once. For a finite graph  $G$ ,  $T$  is a minimal dominating set if it dominates  $G$  and has a cardinality less than or equal to every other dominating set of  $G$ . Similarly,  $T_p$  is a minimal perfect dominating set if it is a perfect dominating set of  $G$  and has a cardinality less than or equal to every other perfect dominating set of  $G$ . For finite graphs, the minimal domination ratio of  $G=\frac{|T|}{|V|}$  and the minimal perfect domination ratio of  $G=\frac{|T_p|}{|V|}$ . We extend the notion of domination ratio and perfect domination ratio to infinite graphs composed of replicable subgraphs. Archimedean lattices are vertex-transitive infinite graphs formed by regular polygons. We calculate upper and lower bounds for domination ratios of Archimedean lattices. Furthermore, we have solved exact domination ratios for most Archimedean lattices. (Received September 20, 2017)

1135-00-1269 **Maria-Jesus Munoz-Lopez\***, [munoz172@umn.edu](mailto:munoz172@umn.edu), and **Pedro Caro, Luca Gerardo-Giorda** and **Andoni Garcia**. *Numerical Reconstruction for Electrical Impedance Tomography*.

Electrical impedance tomography (EIT) refers to a non-invasive medical imaging technique in which the electrical properties of conductive tissues are inferred from surface electrode measurements. This technique is commonly modeled by the inverse Calderon problem. This is a mathematical problem, which consists of reconstructing the coefficient of the conductivity equation in a domain from the knowledge of a set of its measurements on the boundary. Ideally, these measurements determine the Dirichlet-to-Neumann map and, therefore, one usually assumes the data to be given by such a map. This situation corresponds to having access to infinite-precision measurements, which is unrealistic. We check numerically the theoretical results of Caro and Garcia (2017), where the Calderon problem is studied assuming the data to contain measurement errors, and formulas are provided to reconstruct the conductivity and its normal derivative on the surface. (Received September 20, 2017)

1135-00-1331 **Helge Holzmann**, Appelstr. 9a, 30167 Hannover, Germany, and **Mila Runnwerth\*** ([information@tib.eu](mailto:information@tib.eu)), Welfengarten 1 B, 30167 Hannover, Germany. *Preserving information on mathematical software via web archives*.

Software is an essential ingredient in mathematical research especially in numerical analysis, mathematical modelling, and statistics. However, the traditional publication process has not come up with a satisfyingly neat solution to reference software yet, taking into account its dynamics in order to allow for reproducibility. While software on GitHub might be "cited" by linking to the repository including the precise commit given by its SHA signature, commercial software can only be referenced by its web presence most of the time. In 2016, we evaluated swMath, a database for information on mathematical software, on how a software's archived web site reflects its development. We found, although some web sites are already archived and indeed provide a temporal overview of their corresponding software's development including documentation, there is need for improvement. In 2017, we presented a demo framework to archive and cite software homepages adding a time stamp. Our vision is a service to archive semantically linked web sites of mathematical software on the basis of swMATH. The resulting archive contains all information on the software at a specific time and can be cited in a traditional publication via DOI. (Received September 21, 2017)

1135-00-1362 **Tegan H Emerson\*** ([tegan.emerson@nrl.navy.mil](mailto:tegan.emerson@nrl.navy.mil)). *Persistence Images for Differentiating Class Based Network Structures.*

The human brain, and many other complex structures, can be studied using networks. One can define the nodes and edges of the graph in a multitude of ways. In functional networks modeling mental processing structures the nodes correspond to different brain regions and the edges between them are weighted by a measure of similarity of time series measured at the brain regions. These networks can then be studied using ideas from computational topology: a set of algorithmic methods that characterize topological invariants such as connectedness, loops, or holes in high-dimensional data structures. These methods go beyond pairwise connections and enable one to understand global low-dimensional structures in networks, which is difficult for existing methods. In particular, persistent homology, a method that consists of a mathematical formalism to explore the persistence of such structures, has led to promising results in many applications including dynamical systems, neuronal networks, and others. Persistence Images are a stable vector representation of homological features identified from persistent homology. Persistence images can detect differences between networks associated to different classes more strongly than other representations of homological information in many settings. (Received September 21, 2017)

1135-00-1384 **Renee Martin\*** ([mpowell@stfrancis.edu](mailto:mpowell@stfrancis.edu)), 103A West Edson, Poplar Grove, IL 61065. *Analyzing NFL Overtime.* Preliminary report.

In this talk, we consider the National Football League's rules for overtime. We use Markov chain models to represent sudden death, modified sudden death 15-minute overtime, the newly changed modified sudden death 10-minute overtime, a theoretical alternative modified sudden death where each team is required to possess the ball at least once. Through our model analysis, we find the average length of overtime and the probability of the team possessing the ball first during overtime winning the game. Furthermore, we predict how the new 10-minute overtime length may affect game outcomes. (Received September 21, 2017)

1135-00-1411 **Katja D Vassilev\*** ([kdv@princeton.edu](mailto:kdv@princeton.edu)), 5073 Frist Center, Princeton University, Princeton, NJ 08544, and **Lily Seitelman, Maria Warns and Lawrence Seminario-Romero.** *Bifurcation Properties for Navigation Constellation Design.*

The Aerospace Corporation continually advises the Air Force on satellite improvement and maintenance for the Global Positioning System (GPS). The performance of GPS-like satellite constellations, referred to as Walker or Modified Walker Constellations, are of particular interest to the Aerospace Corporation. A Walker Constellation is a system of symmetrically spaced satellites designed for even coverage. They can be modified by altering the number and spacing of satellites and part of our research involves providing methods for modification of Walker Constellations. Our research involves analyzing how adjusting the semi-major axis and inclination of these GPS-like constellations affects the visibility and geometry of each constellation. A cost analysis determines the number of satellites that were viable at each semi-major axis. By varying the semi-major axes of these constellations, we find the bifurcation points corresponding to the number of satellites in a constellation required to achieve adequate visibility, geometry, and cost effectiveness. (Received September 22, 2017)

1135-00-1456 **Michael Kohlhase\*** ([michael.kohlhase@fau.de](mailto:michael.kohlhase@fau.de)). *Virtual research environments for computational mathematics: The OpenDreamKit project and the Math-in-the-Middle Approach to System Interoperability.*

OpenDreamKit – “Open Digital Research Environment Toolkit for the Advancement of Mathematics” – is an H2020 EU Research Infrastructure project that aims at supporting, over the period 2015–2019, the ecosystem of open-source mathematical software systems. OpenDreamKit will deliver a flexible toolkit enabling research groups to set up Virtual Research Environments, customised to meet the varied needs of research projects in pure mathematics and applications.

An important step in the OpenDreamKit endeavor is to foster the interoperability between a variety of systems, ranging from computer algebra systems over mathematical databases to front-ends. This is the mission of the integration work package. We report on the Math-in-the-Middle approach to system (computer algebra systems) interoperability. This architecture consists of a central mathematical ontology that documents the domain and fixes a joint vocabulary, or even a language, going beyond existing systems such as OpenMath, combined with specifications of the functionalities of the various systems. Interaction between systems can then be enriched by pivoting around this architecture. (Received September 22, 2017)

1135-00-1980 **Jan J. Blom\*** ([harry.blom@springer.com](mailto:harry.blom@springer.com)), 233 Spring Street, New York, NY 10013.  
*Trends and Topics in Mathematics Publishing.* Preliminary report.

In this talk we present a brief overview of recent developments in mathematics publishing. Topics discussed will include the building of a public knowledge graph, options to share and read articles legally without a subscription, and supporting persistent identifiers such as ORCID. (Received September 25, 2017)

1135-00-2009 **Gwen Spencer\*** ([gwen Spencer@gmail.com](mailto:gwen Spencer@gmail.com)) and **Greg Clark.** *How Low Can You Go? On the Biplanar Crossing Number of the Hypercube.*

Suppose that for  $G = (V, E)$ , the edge set  $E$  is partitioned into two disjoint subsets,  $E_1$  and  $E_2$ , and let  $G_i = (V, E_i)$ . Each  $G_i$  has some crossing number  $cr(G_i)$ . The *Biplanar Crossing Number* of  $G$  is the minimum of  $cr(G_1) + cr(G_2)$  over all partitions of  $E$ . Crossing Numbers for hypercubes are poorly understood (for  $k \geq 5$ , the crossing number of the  $k$ -cube is unknown), and the best biplanar drawings known for hypercubes rely on highly-symmetric partitions of  $E$  into smaller hypercubes (or modified hypercubes). I will mention some new results on the *Biplanar Crossing Number* of low-dimensional hypercubes. (Received September 25, 2017)

1135-00-2081 **Yena Kim\*** ([rednblue1004@uf1.edu](mailto:rednblue1004@uf1.edu)), Department of Mathematics, 1400 stadium rd., University of Florida, gainesville, FL 32611, and **Maia Martcheva** ([maia@uf1.edu](mailto:maia@uf1.edu)), Department of Mathematics, 1400 Stadium rd., University of Florida, gainesville, FL 32611.  
*One the principle of host evolution in immuno-epidemiological models.*

Over the last 20 or so years, scientists have been interested in linking within host and between host models of infectious diseases, thus studying these diseases on multiple scales. Here, we are using the so called nested approach introduced first by Gilchrist and Sasaki. The main question that Gilchrist and Sasaki addressed was the co-evolution of the pathogen and the host. Because of the simplicity of their model, they used for the evolution of the host the maximization of the life span of the infected individuals. Using a simple SI model, Pugliese confirmed analytically the above principle host evolution in nested immuno-epidemiological models. In this talk, we argue for the first time that the host evolves not to maximize its life span in the infectious class but to minimize the case fatality proportion that is the number of death of all cases. The principle of minimization of case fatality proportion is equivalent to the principle of maximization of host life span in the infectious class for diseases without recovery, that is diseases of SI type. However, if the host have the option to recover, it does not make sense to maximize its life span in the infectious class. In this case, only the principle of minimization of case fatality proportion is valid. (Received September 25, 2017)

1135-00-2109 **Prairie Wentworth-Nice\*** ([pwentwo1@swarthmore.edu](mailto:pwentwo1@swarthmore.edu)). *A Relaxation of the Total Coloring Game on Trees.* Preliminary report.

We define a new version of the graph coloring game, the *1-relaxed total coloring game with complete defect*. In this game, two players, Alice and Bob, take turns coloring uncolored elements (vertices and edges) of a graph from a set of colors such that any element in the graph is adjacent or incident to at most one element that is the same color as it. Alice wins if at the end of the game all elements of the graph are colored. Bob wins, if at some point in the game there is an uncolored element that cannot be colored. The *1-relaxed total game chromatic number* of  $G$ , denoted  ${}^1\chi_g''(G)$ , is the least number of colors for which Alice has a winning strategy on  $G$ . We provide a strategy for Alice which shows that  ${}^1\chi_g''(T) \leq \max\{\Delta + 2, 7\}$  for all trees  $T$ . (Received September 25, 2017)

1135-00-2147 **Patrick D. F. Ion\*** ([pion@umich.edu](mailto:pion@umich.edu)) and **Olaf Teschke.** *Continuing toward a Global Digital Mathematics Library.*

In 2006 International Mathematical Union endorsed the notion of a global digital mathematics library (GDML), and after a 2012 US National Research Council report, the GDML WG was set up at ICM 2014. There are difficulties mobilizing the mathematical community for worthwhile infrastructure building when times are both perilous and well off, depending on where you stand. An IMKT (International Mathematical Knowledge Trust) has been founded in Waterloo ON, Canada. Specific initiatives are commencing: 1. Special Function Concordance. 2. FABSTRACTS 3. Formal Harmonization 4. Bibliographic and text analysis.

The presentation describes what we have and then concentrates on traditional GDML concerns, especially desirable content aggregation. What must a GDML include? How big is our literature? What progress is there in digitization, both in quality and quantity? Where are digital math collections today, whether records of print, newer document types or software and data? What are copyright and other restrictions on our mathematical legacy?



The GDML will create a distributed framework of resources. While some connected information exists already, a lot of work remains to be done. We discuss examples, open questions, their contexts and possible answers. (Received September 25, 2017)

1135-00-2166 **Andrew Gillette, Tyler Kloefkorn and Victoria Sanders\***, victoriasanders@email.arizona.edu. *Using Sage to Create Lists of Shape Functions for Trimmed Serendipity and Serendipity Finite Elements*. Preliminary report.

The finite element method is a numerical method used to approximate solutions to partial differential equations. These methods work by dividing the computational domain into smaller, more manageable pieces, and associating a space of piecewise-defined functions. There are a number of finite element methods, and some are more efficient for computation than others. A specific finite element features a list of shape functions (or polynomial functions attached to differential forms). In this project, we build lists of the shape functions for the trimmed serendipity and (non-trimmed) serendipity families of finite elements. Along the way, we introduce code in Sage to evaluate the Koszul operator and generate lists of shape functions. (Received September 25, 2017)

1135-00-2302 **Steinn Sigurdsson\*** (ss3783@cornell.edu), 525 Davey Laboratory, University Park, PA 16802. *The arXiv: current status and future plans*.

I will discuss the current status of the arXiv, the automated electronic archive and distribution server for research article preprints and related materials. After 25 years, the arXiv is the dominant provider of preprints of research articles to the mathematical and physical sciences. I will also discuss plans for extending the scope of arXiv and the implementation of the arXiv Next Generation, and have a conversation about where the arXiv ought to go next. (Received September 25, 2017)

1135-00-2311 **Joshua M Schroeder\*** (schro252@msu.edu), 221 Spartan Ave, East Lansing, MI 48823, and **Prairie Wentworth-Nice, James Zak and Charles Dunn**. *The total coloring game on  $k$ -bounded graphs*.

The *total coloring game* involves two players taking turns coloring the elements (vertices and edges) of a graph  $G$  such that no two adjacent or incident elements of the graph share a color. The first player (Alice) wins if all elements can be colored, while the second player (Bob) wins if some element cannot be colored. The *total game chromatic number* of  $G$ , denoted  $\chi_g''(G)$ , is the least number of colors for which Alice has a winning strategy on  $G$ . Recall that a graph is said to be  *$k$ -bounded* if it allows an orientation such that the maximum outdegree is  $k$ . We show for any  $k$ -bounded graph  $G$  such that the maximum degree is  $\Delta$ , that  $\chi_g''(G) \leq \Delta + 3k + 2$ , by providing a winning strategy for Alice. This establishes bounds for the total game chromatic number of outerplanar graphs, and trees, as well as providing a bound for planar graphs, for which no bound had been previously established. (Received September 25, 2017)

1135-00-2331 **Rebin A Muhammad\*** (rm775311@ohio.edu), 24 home street apt 104, athens, OH 45701, and **Sergio López-Permouth**. *On Module over infinite-dimensional Algebras*. Preliminary report.

The talk will begin by reviewing topics introduced in a 2017 JLMA paper by L.M. Al-Essa, Sergio López-Permouth and N. M. Muthana. Let  $A$  be an infinite-dimensional  $K$ -algebra, where  $K$  is a field and let  $\mathcal{B}$  be a basis for  $A$ . A basis  $\mathcal{B}$  is called an amenable basis when  $K^{\mathcal{B}}$  become an  $A$ -module in a natural way. It has been shown that different basis induce different modules. Should the induced modules be isomorphic we say the two basis are congenial. Basic setting example in previous study of amiability and congeniality was related to polynomial algebras of a single variable. We investigate other algebras such as  $k[x, y]$  and Kite Algebra. We found that if the algebra is non-commutative then there may exist a basis that is left amenable but not right amenable and vice versa. We give an example in the Kite Algebra for each of these cases. We also discuss the simplicity of basis in polynomial algebras of  $n$  variables and show that there is always a simple basis. (Received September 25, 2017)

1135-00-2456 **Colin Adams, Frank Morgan\*** (fmorgan@williams.edu) and **Cesar E. Silva**. *The Williams College SMALL Program and undergraduate mathematics research*. Preliminary report.

The authors will discuss the history of the SMALL program including research done by undergraduates. (Received September 26, 2017)

1135-00-2528 **Edward Dunne, Fabian Müller\*** ([fabian.mueller@fiz-karlsruhe.de](mailto:fabian.mueller@fiz-karlsruhe.de)), **Norman Richert** and **Olaf Teschke**. *Organizing the mathematical literature: On the road to MSC 2020*.

The Mathematical Subject Classification has been used for decades to organize the mathematical literature, and it still remains an efficient tool due to—or in spite of—the immense growth of the number of publications since its introduction. Currently, the MSC 2020 is under joint revision by Mathematical Reviews and zbMATH: At <https://msc2020.org/>, the mathematical community is invited to contribute enhancements and additions. We report on the status of the revision and outline the next steps. Furthermore, we present some data analysis obtained in collaboration with the Heidelberg Institute for Theoretical Studies, which may underscore that the classification system is surprisingly reliable, and human classification still outperforms current state-of-the-art tools in data mining and machine learning. We indicate some ideas on how such tools can nevertheless be used to contribute to the revision, while at the same time renewing the invitation to contribute to the community input phase, which will stay open until the ICM 2018. (Received September 26, 2017)

1135-00-2804 **Victor Reiner** and **Hugh Thomas\*** ([hugh.ross.thomas@gmail.com](mailto:hugh.ross.thomas@gmail.com)). *Open Access and the Journal of Algebraic Combinatorics*.

The editors-in-chief and the editorial board of the Journal of Algebraic Combinatorics informed Springer in June that they would be resigning effective December 31, 2017, in order to start a new journal to be called Algebraic Combinatorics, and to be run on Fair Open Access principles. One of the co-presenters (V.R.) was an interim editor-in-chief at Algebraic Combinatorics; the other (H.T.) was an editor-in-chief of the Journal of Algebraic Combinatorics and is now (as of January 1) an editor-in-chief at Algebraic Combinatorics. We will speak about what prompted the decision, and how things are working out. We will also explain and motivate the Fair Open Access principles. Questions from the audience are very welcome. (Received September 26, 2017)

1135-00-2946 **Samir Chowdhury\*** ([chowdhury.57@osu.edu](mailto:chowdhury.57@osu.edu)), Department of Mathematics, The Ohio State University, 231 W 18th Ave, Columbus, OH 43210, and **Facundo Mémoli** ([memoli@math.osu.edu](mailto:memoli@math.osu.edu)), Department of Mathematics, The Ohio State University, 231 W 18th Ave, Columbus, OH 43210. *Distances between Networks and the Stability of Network Invariants*.

Techniques from the shape matching and metric geometry literature have recently been applied to perform stable hierarchical clustering and persistent homology on directed, weighted network data (sets of nodes with possibly asymmetric pairwise relations, i.e. finite real-valued square matrices). The main workhorse in this approach is a network distance  $d_{\mathcal{N}}$  resembling the Gromov-Hausdorff distance between metric spaces that induces a pseudometric structure on the collection of all finite square matrices. Previous work using  $d_{\mathcal{N}}$  does not address the “continuous limits” of sequences of finite networks, which are important for modeling very large networks. We prove that the limit of a  $d_{\mathcal{N}}$ -convergent sequence of finite networks is a compact topological space with a continuous edge-weight function. We develop the notion of sampling finite networks from such spaces and use these results to precisely characterize the networks satisfying  $d_{\mathcal{N}}(\cdot, \cdot) = 0$ . For practical use, we identify readily computable network invariants by drawing analogies with the shape matching literature. Using these invariants, we provide a simple polynomial-time algorithm that computes a lower bound on the  $d_{\mathcal{N}}$ -distance between two networks. (Received September 26, 2017)

1135-00-2978 **Alessandra Pantano\*** ([apantano@uci.edu](mailto:apantano@uci.edu)). *Increasing scope and quality of a math circle program for minority students*. Preliminary report.

Over the past three years, faculty at UCI have piloted an innovative form of Math Circle - the UCI Math CEO program, which specifically targets students from disadvantaged backgrounds. Approximately 100 middle-school students come to the UCI campus each week throughout the academic year to explore interesting and challenging mathematical investigations, puzzles and games with the help of undergraduate mentors. These meetings provide a setting in which children do not only learn math; they acquire familiarity with college and STEM careers. The program also features STEM-focused field trips for participating middle-school students, and bilingual college-orientation workshops for their parents. Almost all the kids in the program are Latinx. In this talk, we describe efforts to increase the scope and the quality of Math CEO. (Received September 26, 2017)

1135-00-3109 **Jordan Schettler\***, [jordan.schettler@sjsu.edu](mailto:jordan.schettler@sjsu.edu), and **Mitchell Chavarria**. *The Well-Tempered Guitar: An Undergraduate Research Project Inspired by Curved Frets*.

Anders Thidell has designed guitars with “curved frets” (i.e., not parallel line segments) that model irregular tuning systems which differ from the classical 12 tone equal temperament. These non-equally spaced scales were inspired by a tuning system of Bach for the harpsichord. We explain how one can quantify the efficacy of these

designs from a mathematical perspective. This work was part of an undergraduate research group in the NSF funded PUMP program (Preparing Undergraduates through Mentoring towards PhDs). (Received September 26, 2017)

1135-00-3190 **B I Mahler\***, Hertford College, Catte Street, Oxford, OX1 3BW, United Kingdom.  
*Flooding filtration on directed networks.*

We describe the notion of energy landscape and critical nodes of a complex network as developed by Weinan E. et al. We explain how persistent homology can be used to find critical nodes. We note that these methods in their original form are only suitable to be used on undirected unweighted networks and explore ways to generalise them to be applicable in the setting of directed weighted networks. (Received September 27, 2017)

1135-00-3201 **Islam Faisal\*** (islamfm@aucegypt.edu), **Andrew Nguyen** (atn015@ucsd.edu), **Surabhi Desai** (sd207@st-andrews.ac.uk), **Prem Talwai** (pmt55@cornell.edu) and **Shantanu Joshi** (s.joshi@ucla.edu). *Survey on Triplet Mining for Facial Recognition Convolutional Neural Networks.* Preliminary report.

A convolutional neural network can be used to map face images to a feature embedding space. This feature embedding space, equipped with a metric can be used to determine how similar two face images are, which in turn is used for recognition and verification tasks. Triplet loss is a loss function used to train convolutional neural networks for such tasks. A bottleneck for this optimization problem is selecting triplets good enough to result in fast convergence and high accuracy. In this talk, we survey different triplet mining techniques used in training convolutional neural networks for the task of facial recognition and verification. (Received September 27, 2017)

1135-00-3223 **Tim Chartier\*** (tichartier@davidson.edu), Department of Mathematics and Computer Sci., P.O. Box 6908, Davidson, NC 28035. *Thinking like a Data Scientist In and Out of Class.*

In our data deluge, gaining actionable insights from data is highly desired in fields that include government, retail, health care, finance and sports. Mathematics classes tool students with methods and a mindset to tackle analytics questions. In this talk, we will discuss ways to engage students in data science in mathematics classes for majors and nonmajors. We will also discuss opportunities outside the classroom that have been successful at Davidson College, particularly in the realm of sports analytics that include aiding college coaches, consulting businesses and professional sports teams, and fielding questions from national media. Important questions can be considered in a variety of settings. How does one approach creating robust datasets that in themselves become valuable assets? What mathematical techniques can be used to analyze data and of what pitfalls should we beware? How do we communicate results so others can make decisions from our work? Whether in or out of the classroom, cultivating the mindset of a data scientist can prepare students for the ever evolving field of data analytics. (Received September 27, 2017)

## 01 ► History and biography

1135-01-96 **Susan E. Kelly\*** (skelly@uwlax.edu), Mathematics and Statistics Department, UW-La Crosse, 1725 State Street, La Crosse, WI 54601, and **Carly Shinnors** and **Katherine Zoroufy**. *Martha Euphemia Lofton Haynes: First African American Woman to Earn a PhD in Mathematics.*

Euphemia Lofton Haynes was born in Washington DC in 1890, and her education began in segregated schools. She earned a bachelor's degree in mathematics in 1914 and a master's in education in 1930. She became the first African American woman to earn a doctorate in mathematics when she received her degree in 1943 from The Catholic University of America. She spent her career teaching in segregated and later desegregated schools in Washington DC. She later served on the DC school board, serving as president from 1966-67. She was a tireless fighter for equal education for all public-school students, and was instrumental in the 1967 Hobson v Hansen court case that ended the tracking system in Washington DC public schools. This talk will present significant aspects of Haynes' life and her thoughts on mathematics, education, gender and race. (Received September 26, 2017)

1135-01-100 **Donald A. Sokol\*** (3471das@gmail.com), Donald, A., Sokol, Burr Ridge, IL 60527.  
*Plimpton 3222: Rosetta Stone Part deux.*

The ancient world tablet created by the Babylonians and identified as Plimpton 322 in the museum at Columbia University introduced the people of that period (circa 1800 B.C.) to the algorithm used to calculate a specific

form of what was later presented by the Greeks as the Pythagorean Theorem. Both the Babylonians and Greeks were reluctant to recognize square roots and negative numbers as legitimate in mathematics and left much about them unsaid in the Tablet and Theorem. Euclid finished the task by eliminating them from further discussion in his treatise *The Elements*. This presentation begins where the Babylonian tablet (Plimpton 322) ends and addresses scale and orientation. Several other algorithms not covered in the earlier work are also discussed. (Received July 26, 2017)

1135-01-234 **M.E. Tobin\*** (m.tobin@gmail.com), 1700 Bolton Street, Baltimore, MD 21217. *Advances in Transcendental Number Theory since the Proof of the Gelfond-Schneider Theorem.*

A survey of the literature of transcendence theory since the proof of the Gelfond-Schneider Theorem (1934) reveals two divergent tendencies manifested in the work of Alan Baker and Boris Zilber and presaged by the writings of Hermite and Cantor, respectively. The first "school," embodied in the work of Baker, uses auxiliary functions to approximate transcendental numbers to natural numbers. The second, represented by Zilber and, distantly, Cantor, seeks to approximate transcendental numbers to complex numbers. The evolution of these somewhat divergent techniques is observed in the ensuing literature and analyzed. The synthetic ramifications of a turn toward computer science modeling and thinking are accounted for and evaluated, especially the research that was sparked by the appearance of Daniel Richardson's (1968) "Some Undecidable Elementary Functions of a Real Variable." An ambient assessment of the current state of transcendence theory is made on this basis, specifically in regard to the syncretistic implications of computer science applications in the field. (Received August 14, 2017)

1135-01-302 **Jemma Lorenat\*** (jlorenat@pitzer.edu). *Algebraic symbols and geometrical reality: the algebraic geometry of Charlotte Angas Scott.*

Through the 1890s the geometer Charlotte Angas Scott motivated her research as building a geometric understanding of algebraically given curves. In her publications, Scott referred to a historical lineage of analytic/algebraic geometers, who had all addressed the subject with a range of analytic and geometric techniques. While drawing upon this tradition, Scott revisited the algebraic treatments of her contemporaries to provide graphical analyses and show geometrical reality. These expressions suggest a visually oriented approach, which can also be observed in Scott's extensive and creative use of figures and diagrams. These drawn objects reflected Scott's language choices, and frequent decision to explicitly forgo algebraic terminology in favor of geometrically suggestive names. This talk will proceed roughly chronologically to illustrate Scott's conception of geometrical existence through her practice of alternative proofs beginning with her research on singularities, then considering her writings on intuition, and concluding with the pedagogical writings of her first textbook. (Received August 21, 2017)

1135-01-378 **Adrian Rice\*** (arice4@rmc.edu), Department of Mathematics, Randolph-Macon College, Ashland, VA 23005. *Partnership, Partition, and Proof: The Path to the Hardy-Ramanujan Partition Formula.*

This year marks one hundred years since the publication of one of the most startling results in the history of mathematics: Hardy and Ramanujan's asymptotic formula for the partition function. To celebrate the centenary, this paper looks at the creation of their remarkable theorem: where it came from, how it was proved, and how the assistance of a third contributor helped to influence its ultimate form. (Received August 29, 2017)

1135-01-438 **Ursula H Martin\*** (ursula.martin@cs.ox.ac.uk), **Christopher D Hollings** (christopher.hollings@maths.ox.ac.uk) and **Adrian Rice** (arice4@rmc.edu). *What does Ada Lovelace's correspondence with Augustus De Morgan tell us about her ability?*

Ada Lovelace (1815-1852) is famed as the author of a paper explaining the workings and potential of Charles Babbage's unbuilt analytical engine. She learned most of the mathematics she needed in a remarkable correspondence course that she took with Augustus De Morgan. She worked through his textbook on differential calculus, supplemented by patching the gaps in her knowledge through more elementary textbooks. Discussions with De Morgan show her grappling with material at the frontier of current knowledge, for example divergent series, and the subtleties of Peacock's Permanence Principle. We pinpoint Lovelace's keen eye for detail, fascination with big questions, and flair for deep insights, which enabled her to challenge some deep assumptions in her teacher's work, and suggest that her ambition, in time, to do significant mathematical research was entirely credible, though sadly curtailed by her ill-health and early death. (Received September 03, 2017)

1135-01-458 **Lawrence D’Antonio\*** ([ldant@ramapo.edu](mailto:ldant@ramapo.edu)). *Newton’s Headache: the Motion of the Lunar Apse*. Preliminary report.

Newton remarked to Halley that lunar theory gave him a headache. In particular the calculation of the motion of the lunar apse frustrated Newton (the lunar apse is an endpoint of the major axis of the ellipse defining the lunar orbit). The apse rotates approximately  $3^\circ$  per month due to solar perturbations, but Newton’s calculations only showed half of this amount, leading Newton to say that the problem was “too complicated and cluttered with approximations.” We examine the work of Newton on this problem and the later solution of Clairaut, Euler, and d’Alembert. (Received September 04, 2017)

1135-01-465 **Christopher D Hollings\*** ([christopher.hollings@maths.ox.ac.uk](mailto:christopher.hollings@maths.ox.ac.uk)), Mathematical Institute, University of Oxford, Woodstock Road, Oxford, OX2 6GG, United Kingdom. *Abstraction and axioms: some parallels between 19th-century British and 20th-century American mathematics*. Preliminary report.

During the nineteenth century, we see early examples of both abstract and axiomatic approaches to algebra in the works of several British mathematicians, most notably Augustus De Morgan. We also see criticisms of the associated methods. In the early decades of the twentieth century, similar ideas took a prominent place in the American mathematical community, though apparently largely independently of the prior British work. In this talk, I will look at the similarities that are present in the algebraic works of these two communities, and compare the points upon which each was criticised. (Received September 05, 2017)

1135-01-512 **David R Bellhouse\*** ([bellhouse@stats.uwo.ca](mailto:bellhouse@stats.uwo.ca)), Dept of Statistics and Actuarial Science, University of Western Ontario, London, Ontario N6A5B7, Canada. *A Decade in the Life of William Playfair*.

William Playfair (1759 – 1823) is best known in mathematics and statistics for his invention of some graphical techniques for understanding trends in data. To provide a flavour for the range of his abilities and some of the vicissitudes of his life, I will focus on the decade 1800 – 1810. At the beginning of this decade, Playfair wrote *The Statistical Breviary* which contains some of his graphs. In the middle of the decade he produced the first posthumous edition of Adam Smith’s *Wealth of Nations* and at the end of the decade he was in the middle of publishing an enormous genealogical work, *British Family Antiquity*. During the same decade he was sent twice to the Fleet Prison for debt and once to Newgate Prison for his involvement in a questionable scheme to conceal information from the creditors of one of his colleagues. (Received September 06, 2017)

1135-01-683 **Stephan Ramon Garcia\*** ([stephan.garcia@pomona.edu](mailto:stephan.garcia@pomona.edu)), Department of Mathematics, Pomona College, 610 N College Ave, Claremont, CA 91711. *G.H. Hardy: mathematical biologist*.

G.H. Hardy, the great analyst who “discovered” the enigmatic Ramanujan and penned *A Mathematician’s Apology*, is most widely known outside of mathematics for his work in genetics. Hardy’s fame stems from a condescending one-page letter to the editor in *Science* concerning the stability of genotype distributions from one generation to the next. His result is now known as the Hardy–Weinberg Law, which every biology student learns today. How did Hardy, who his colleague C.P. Snow described as “the purest of the pure,” become one of the founders of modern genetics? What would Hardy say if he knew that he had earned scientific immortality for something so mathematically simple? (Received September 12, 2017)

1135-01-695 **Mohammad K. Azarian\*** ([azarian@evansville.edu](mailto:azarian@evansville.edu)), Department of Mathematics, University of Evansville, 1800 Lincoln Avenue, Evansville, IN 47722. *The Famous Cubic Equation of Ghiyath al-Din Jamshud Mas’ud al-Kashi [Jamshud Kashani] and Mathematica*. Preliminary report.

Ghiyath al-Din Jamshud Mas’ud al-Kashi [Jamshud Kashani] (1380-1429) discussed the calculation of  $\sin 1^\circ$  in his *Risala al-watar wa’l jaib* (“The Treatise on the Chord and Sine”). Al-Kashi applied Ptolemy’s theorem to an inscribed quadrilateral to obtain his famous cubic equation, and then he invented an iteration algorithm to calculate  $\sin 1^\circ$ . For his calculations he used the value of  $(1/3)\sin 3^\circ$  with 9 sexagesimal places of accuracy, and he obtained  $\sin 1^\circ$  with nine correct sexagesimal places as well. Our goal in this presentation is to use  $(1/3)\sin 3^\circ$  with higher accuracies, and apply Mathematica to obtain approximations for  $\sin 1^\circ$ . As expected, there will be significant improvement on the accuracy of the value of  $\sin 1^\circ$ . Surprisingly, this improvement does not continue indefinitely as we use more accurate values for  $(1/3)\sin 3^\circ$ . (Received September 13, 2017)

1135-01-738 **Tony Crilly\*** ([t.crilly@btinternet.com](mailto:t.crilly@btinternet.com)). *Thomas P. Kirkman – a life in mathematics*. Who was Thomas P. Kirkman (1806-1895)? Apart from fame gained from the combinatorial fifteen schoolgirls problem, what else did he do in mathematics? Just as importantly I seek to understand the conditions of his life.

A lone researcher, he made incisive discoveries while serving as an Anglican minister in a remote parish in the north of England, but felt himself ignored. The paper will highlight aspects of his character, his mathematics, his philosophy and his spiritual convictions, all of which contributed to a fascinating life. (Received September 13, 2017)

1135-01-803 **Jacqueline Feke\*** ([jfeke@uwaterloo.ca](mailto:jfeke@uwaterloo.ca)), Department of Philosophy, 200 University Avenue West, Hagey Hall, Room 365, Waterloo, Ontario N2L 3G1, Canada. *Ancient Greek Geographical Maps vs. Geometrical Diagrams*.

Claudius Ptolemy, the second-century mathematician, is remembered most of all for his contributions in astronomy, but just as influential was his Geography, a lengthy treatise, the majority of which consists of a catalogue of approximately eight thousand localities and their coordinates, which he intended to be drawn on a map of the known part of the world. Ptolemy deliberates on the proper structure of the map, which should maintain the proper ratios of distances between localities on the earth. Indeed, the principal aim of the Geography is the production of an image, a mathematical representation and likeness of the known part of the earth. In this talk, I investigate how Ptolemy's maps compare to the predominant type of image in the ancient Greek mathematical tradition: the geometrical diagram. I will explore the style of image, the utility, and function of ancient Greek geographical maps in contrast to geometrical diagrams. (Received September 14, 2017)

1135-01-852 **Kenneth L Manders\***, [mandersk@pitt.edu](mailto:mandersk@pitt.edu). *On Proofs of the Quadratic Reciprocity Theorem*. Preliminary report.

Few mathematical results have drawn as many distinct proofs as Gauss' Quadratic Reciprocity theorem. It nonetheless remains mysterious, lying at the origin of extensions ranging from Gauss' work to the frontiers of contemporary number theory. Notably, it presents a simple case of regularities in the factorization of primes in algebraic number fields.

We will discuss several of the families of proofs in light of this connection. (Received September 15, 2017)

1135-01-944 **Maria Zack\*** ([mzack@pointloma.edu](mailto:mzack@pointloma.edu)), MICS Department, Point Loma Nazarene University, 3900 Lomaland Drive, San Diego, CA 92106. *Where Did They Learn That? Mathematical Knowledge in 18th Century Portugal*.

In 1750's, the mathematics taught at the universities in Portugal was much less advanced than what was being taught in neighboring countries. However, there were traces of sophisticated ideas from the mathematics of materials evidenced in Portuguese buildings, particularly those erected after the 1755 earthquake the leveled Lisbon. These buildings show evidence of seismic engineering. This talk discusses the role that manuscript material used in military engineering schools may have played in transmitting mathematical information from francophone Europe to Portugal. The author had been working with a little-known 1742 manuscript of a Portuguese translation of a French engineering text written by Belidor. This manuscript appears to provide one of the "missing links" in explaining how the mathematics of materials became known in Portugal well before the 1772 reformation of the mathematics curriculum taught in Portuguese universities. (Received September 17, 2017)

1135-01-1013 **Gizem Karaali\*** ([gizem.karaali@pomona.edu](mailto:gizem.karaali@pomona.edu)), Claremont, CA 91711. *Ada's Poetic Science: Correspondences of Ada Lovelace and Charles Babbage*. Preliminary report.

Augusta Ada, Countess of Lovelace, is today viewed as the first person to recognize the power of algorithmic machines and a pioneer in computer programming. Even though her father (the famed poet Lord Byron) abandoned her mother when Ada was a mere baby, and as a result her mother worked tirelessly to steer her away from poetry, in Ada's life, poetry and mechanics, literature and mathematics seem to coexist peacefully. In this talk we aim to explore the interplay between mathematics and poetry in Ada Lovelace's life, and seek clues in her correspondences with Charles Babbage, the inventor of the Analytical Engine. (Received September 18, 2017)

1135-01-1061 **christine proust\*** ([christine.proust@orange.fr](mailto:christine.proust@orange.fr)), Laboratoire SPHERE (UMR 7219), Case 7093, 5 rue Thomas Mann, 75205 Paris, France. *Floating sexagesimal arithmetic in Antiquity (Mesopotamia, early second millennium BCE)*.

Sophisticated computation methods were developed 4000 years ago in Mesopotamia in the context of scribal schools. These computation methods are based on the use of a floating sexagesimal place value notation. They rely on original notions of numbers, quantity, measurement unit, order, divisibility, algorithm, sexagesimality, etc. This presentation explores some aspects of these original notions through cuneiform texts dealing with reciprocals, factorization, and the generation of "Pythagorean" triples. (Received September 19, 2017)

1135-01-1113 **Michael J. Barany\*** ([michael@barany.com](mailto:michael@barany.com)), 6107 Carson Hall, Hanover, NH 03755.

*Invidious Comparisons: The social and political shaping of the Fields Medal, 1936-1966.*

First presented in 1936, the Fields Medal quickly became one of mathematicians' most prestigious, famous, and in some cases notorious prizes. Because its deliberations are confidential, we know very little about the early Fields Medals: how winners were selected, who else was considered, what values and priorities were debated—all these have remained locked in hidden correspondence. Until now.

My talk will analyze newly discovered letters from the 1950 and 1958 Fields Medal committees, which I claim demand a significant change to our understanding of the first three decades of medals. I will show, in particular, that the award was not considered a prize for the very best mathematicians, or even for the very best young mathematicians. Debates from those years also shed new light on how the age limit of 40 came about, and what consequences this had for the Medal and for the mathematics profession. I argue that 1966 was the turning point that set the course for the Fields Medal's more recent meaning. (Received September 19, 2017)

1135-01-1180 **David E Peifer\*** ([dpeifer@unca.edu](mailto:dpeifer@unca.edu)). *Max Dehn's Black Mountain College Students.*

Max Dehn was a member of the Black Mountain College (BMC) faculty the last seven years of his life, from 1945 until 1952. At the beginning of his career, Dehn had done ground breaking work in topology and infinite group theory. It is surprising that at the end of his career he taught at this small liberal arts college with an emphases on the arts. What influence did Dehn have on this unique community? As part of an answer to this question, we will examine the careers of a few of the BMC students whose lives were influenced by their time with Dehn. Both Trueman MacHenry and Peter Nemenyi were BMC graduates who later received PhD's in mathematics and statistics. Dorothea Rockburne, a New York based contemporary artist, says that Dehn was the BMC faculty who had the most influence on her painting career. Ruth Asawa, who was a San Francisco based sculptor and art educator, took several courses from Dehn and kept a lifelong relationship with his family. (Received September 20, 2017)

1135-01-1251 **Johann D. Gaebler\*** ([johann.gaebler@wolfson.ox.ac.uk](mailto:johann.gaebler@wolfson.ox.ac.uk)) and **W. Hugh Woodin.**

*"Maybe from this profusion of formal logic . . . some useful idea will come": Lebesgue, Borel, Baire, and the Birth of Descriptive Set Theory.*

Set theory has played a crucial role in laying the foundations of mathematics for more than a century and a half. Nevertheless, from the beginning, unique technical challenges and controversies ranging from the Burali-Forti paradox to the ubiquity of independence have beset the discipline.

We can understand many developments in set theory as attempts to respond constructively to those obstacles. In this talk, we home in on three French mathematicians at the turn of the century: Henri Lebesgue, Émile Borel, and René-Louis Baire. Their careful mediation between the traditional values of mathematical analysis and Georg Cantor's subversive new theory secured a place for sets in the mathematical mainstream. At the same time, theirs was an uneasy truce between Cantor's higher transfinite and what they saw as the demands of mathematics proper. Their forays into descriptive set theory shine a light on the more general trend of *dephilosophication* in mathematics, and how successful mathematical revolutions often travel in the guise of the establishment they replace. (Received September 20, 2017)

1135-01-1271 **Jim Hoste\*** ([jhoste@pitzer.edu](mailto:jhoste@pitzer.edu)), Pitzer College, 1050 N Mills Ave, Claremont, CA

91711, and **Jozef Przytycki.** *Charles Newton Little: America's first knot theorist.* Preliminary report.

The modern theory of knots, a subfield of topology, arose in the latter half of the 1800s after Lord Kelvin proposed that atoms were "knotted vortices in the ether." This led the Scottish physicist Peter Guthrie Tait to begin tabulating knots, a laborious task in which he was later joined by C.N. Little and Thomas P. Kirkman. Over a period of about 40 years, the three men created a list of all alternating knots with 11 or less crossings and all non-alternating knots to 10 crossings. While they could be sure that their tables listed, in theory, all possibilities, they had no proof whatsoever that their tables did not contain duplications. This would have to wait until well into the 20th century with the development of algebraic topology. In this talk I will review the early history of knot theory with a focus on the life and work of C.N. Little. (Received September 20, 2017)

1135-01-1355 **Elizabeth Cornwall\*** ([ecornwall@dixie.edu](mailto:ecornwall@dixie.edu)). *Shadow Tables and the Origins of the Tangent Function.*

Measuring shadows was of critical importance to many early cultures and were used to keep track of time, determine the altitude of the sun, among other practical applications. Their use in time keeping was particularly vital to Islamic near eastern cultures who created tables of shadows using different gnomon lengths which were included in their astronomical handbooks, or *zījes*.

Modern scholars have found similarities between these shadow tables and our modern-day tangent and cotangent functions. Some even call them tangent tables. My talk will survey various key Arabic sources written in the 9<sup>th</sup> – 13<sup>th</sup> centuries which include tables and discussions of shadow lengths. In particular, I will examine a table of shadow lengths and investigate some of its features and the information we can draw from it. From that, I will explore some approaches modern historians have used to explain these table values and whether or not it is appropriate to label them as the tangent values resulting from the tangent function. (Received September 21, 2017)

1135-01-1372 **Paul R Wolfson\*** ([pwolfson@wcupa.edu](mailto:pwolfson@wcupa.edu)), Dept of Mathematics, West Chester University, West Chester, PA 19383. *Riemann's Twofold Path to Curvature*. Preliminary report.

Riemann's 1854 habilitation address leaves puzzles for historians of mathematics, because it lays out fundamental features of what we now call Riemannian geometry but offers few details of either the steps by which Riemann reached his conclusions or the insights which motivated them. Olivier Darrigol [2] demonstrated a very plausible path directly from Gauss's work on surfaces. Others have suggested a strong connection between Riemann's physical researches and the starting point of Riemann's geometry. (See [1].) This talk traces a path from that starting point to Riemann's curvature via some natural geometric developments. In doing so, it explains some puzzles about his work and also something of the structure of the habilitation address. [1] Bottazzini, U. and R. Tazzioli, *Naturphilosophie and Its Role in Riemann's Mathematics*, *Revue d'histoire des mathématiques* 1 (1995), pp. 3-38. [2] Darrigol, O. *The Mystery of Riemann's Curvature*, *Historia Mathematica* 42 (2015), pp. 47-83. (Received September 21, 2017)

1135-01-1417 **June Barrow-Green\*** ([june.barrow-green@open.ac.uk](mailto:june.barrow-green@open.ac.uk)), School of Mathematics & Statistics, The Open University, Walton Hall, Milton Keynes, MK7 6AA, United Kingdom. *The mathematical education of George Gabriel Stokes*. Preliminary report.

In 1835 George Gabriel Stokes left Ireland to study at Bristol College. From there he went up to Pembroke College, Cambridge. Coached by William Hopkins, he graduated as Senior Wrangler and first Smith's prizeman in 1841. He stayed in Cambridge for the rest of his career, being elected to the Lucasian chair in 1849, a post he held for fifty-four years until his death in 1903, his research being largely focussed on fluid mechanics and optics. In this talk I shall examine the educational environment in which Stokes' mathematical talents developed, and look at the extent to which it provided him with a platform to make a career as a mathematical physicist. (Received September 22, 2017)

1135-01-1581 **Robert E. Bradley\*** ([bradley@adelphi.edu](mailto:bradley@adelphi.edu)), Adelphi University, Department of Mathematics & Computer Science, 1 South Ave., Garden City, NY 11530. *D'Alembert and the Case for Limits*.

Jean Le Rond d'Alembert (1717-1783) mastered the differential and integral calculus as it was practiced in Continental Europe during the first half of the 18th century and went on to introduce a number of important innovations of his own to the field. However, one of his most valuable and lasting contributions to the development of analysis was his role as an early champion of the limit concept, as opposed to the doctrine of infinitely small quantities, in providing "the true metaphysics of the differential calculus." We consider d'Alembert's arguments for this approach to the foundations of calculus, as given in Diderot's *Encyclopédie* and in his other writings. (Received September 23, 2017)

1135-01-1600 **Walter Meyer\*** ([meyer1@adelphi.edu](mailto:meyer1@adelphi.edu)). *The History of Mathematics Curricula: The Case of Probability*. Preliminary report.

There is no fixed pattern for how the American undergraduate mathematics curriculum has changed. New courses may arrive slowly or in a rush. Contemplating the reasons for the pace helps us to understand our profession. This talk considers upper division (calculus required) probability. This is a common entry in the catalogs of colleges today, but it was not always so. The history of research in probability will be briefly reviewed. Laplace's remarks on the centrality of probability in life and his dominant textbook will be mentioned. But a good deal of the talk will be devoted to the 20th century in America where we have better information, in particular that from the Cajori Two survey of mathematics curricula in the 20th century. (Received September 23, 2017)

1135-01-1613 **John McCleary\***, Box 69 Vassar College, Poughkeepsie, NY 12604. *Dehn and Hilbert's Third Problem*. Preliminary report.

Among Hilbert's celebrated Paris problems, the third of the published list was the first to be solved, by Hilbert's student Max Dehn. In this presentation I will consider Dehn's solution in the context of research into the foundations of geometry that was part of Hilbert's work and the work of earlier researchers. I will consider work



of Bricard and of Sforza who are cited in Dehn's Annalen paper on the third problem. Dehn's solution will also be put into relief against the reformulations that followed on the heels of his work. (Received September 23, 2017)

1135-01-1640 **Caroline Ehrhardt\*** ([caroline.ehrhardt@univ-paris8.fr](mailto:caroline.ehrhardt@univ-paris8.fr)). *The editor as a scientific entrepreneur: Emile Borel and the promotion of new fields of investigation in mathematics.*

French mathematician Emile Borel (1871-1956) was the editor of several scientific journals, book series and textbooks throughout his career. IN this talk, I will focus on two of them, the "collection de monographies sur la théorie des fonctions" and the Revue du mois, which he launched respectively in 1895 and 1906. In both cases, Borel used his editing function to promote new areas of research on which he worked, namely integration theory in the collection and probabilities in the Revue du mois. As he edited this two publications, Borel took advantage of his position to make cutting edge research available to a large group of readers beyond the world of mathematics. Indeed, as a series the collection offered short textbooks while the revue was a journal aimed at showcasing recent scientific work to the general public. The Revue du Mois deserves special emphasis, for it allow us to glimpse at the way Borel highlighted new research questions and provided explicit mathematical explanations to show that the probabilities had a role to play in the society of the early 20th century. (Received September 24, 2017)

1135-01-1706 **G. Arthur Mihram\*** ([dmihram@usc.edu](mailto:dmihram@usc.edu)), 301 N. Harrison St., Ste 9F-200, Princeton, NJ 08540, and **Danielle Mihram** ([dmihram@usc.edu](mailto:dmihram@usc.edu)), University of Southern California, LVL 113, MC 2571, Los Angeles, CA 90089. *The Historical and Educational Role of Mathematics in the Arts and Humanities.* Preliminary report.

Science is that human activity devoted to the search for the very explanation for (i.e., for the truth about) any particular naturally occurring phenomenon. Our Modern Science's 'Method' [Teorema 28(2):35,2009] requires of any prospective scientist both observation and mental reflexion thereon [mathematician Cotes's Preface, Newton's Mathematical Principles for Natural Philosophy, 2nd ed., ca. 1713]. Yet, mathematician Quinn [AMS Notices 59(1):31,2012] notes that mathematics is not science, their respective validity criteria differing: internal vs. external, resp. One can add that mathematics is neither sufficient (e.g., pure mathematics) nor necessary (e.g. Darwin, Nobel Laureate Lorenz, sociobiologist Wilson) for Science. Why, then, has mathematics been recognized as a necessary curricular requirement in the arts and humanities (A+H)? Of course, we two have noted that mathematical faculty are rather unique, publishing statements which are irrefutably true, thereby providing an academic challenge. British 19th-Century university Calendars list mathematics as an art, a language, not in the Faculty of Science. But, most importantly, the graduate's mental discipline acquired in mathematics classes elicits him/her, throughout life, to reach conclusions, more likely impeccable. (Received September 24, 2017)

1135-01-1708 **Andreas Blass, Joerg Brendle, Will Brian, Joel David Hamkins, Michael Hardy and Paul B. Larson\*** ([larsonpb@miamioh.edu](mailto:larsonpb@miamioh.edu)), Department of Mathematics, Miami University, Oxford, OH 45056. *Cardinal characteristics related to permutations of conditionally convergent series.*

We consider the smallest cardinality of a set of permutations of the natural numbers with the property that that every conditionally convergent series of real numbers can be rearranged by a member of the set to no longer converge to the same sum. We call this smallest cardinality the rearrangement number. We show that the rearrangement number is uncountable, and that whether or not it equals the cardinal of the continuum is independent of the usual axioms of set theory. We compare the rearrangement number with several natural variants, for example one obtained by requiring the rearranged series to still converge but to a new, finite limit. We also compare the rearrangement number with several well-studied cardinal characteristics of the continuum. We present some new forcing constructions designed to add permutations that rearrange series from the ground model in particular ways, thereby obtaining consistency results going beyond those that follow from comparisons with familiar cardinal characteristics. One simple consequence of these results is the following : for any countable set of conditionally converging real series, there is a permutation which makes them all converge to different values. (Received September 24, 2017)

1135-01-1715 **Abdullah M. Abu-Rqayiq\*** ([rqayiq83@nmsu.edu](mailto:rqayiq83@nmsu.edu)), 1423 Sweet Ave., Zip 88001, Las Cruces, NM 88001. *On An Arabic Mathematical Manuscript From Early 19th Century.*

In the Middle Ages, Arabic scholars provided a significant contribution to many fields of knowledge such as astronomy, mathematics, philosophy, chemistry, etc. For example, Algebra was Originally developed in a few Arabic treatises written in the 9th century. In this talk, I will present one of the very rare mathematical manuscripts written in the early 19 th century in Egypt. Written in 1824 by the Egyptian instructor, Ahmad At-Tabbakh,

Al-Bahja Assaneya Fi Mabda' Al-Oloum Arryadeya (translated as Fundamentals of Mathematical Sciences) is evidence of the scientific interaction between Egypt and European countries at that time. At-Tabbakh translated an Italian mathematical book that existed in the library of the school that he attended. This 145-page manuscript is a translation of an Italian text of an unknown author. While many studies examined Arabic mathematical texts, hardly any researchers explored mathematical texts of this era. My research tries to correct this imbalance by studying this manuscript and bringing it to light. In my talk, I will provide a historical background, analyze the content and explain how this manuscript shows a scientific cross-cultural interaction between the Arabic and western civilizations in the early 19th century. (Received September 24, 2017)

1135-01-1880     **Craig Fraser\*** ([craig.fraser@utoronto.ca](mailto:craig.fraser@utoronto.ca)), IHPST, University of Toronto, Victoria College, 91 Charles St. West, Toronto, Ontario M5S 1K7, Canada. *Euler and Mathematical Rigor*. Preliminary report.

Euler is sometimes seen as a mathematician who was motivated primarily by mathematical discovery and exploration. While there were implicit assumptions and principles that informed his work, he was not a critical mathematician in the modern sense. On the other it would be inaccurate to say that he was simply naive as a mathematical thinker. There are places in his vast corpus where he explicitly considers questions of mathematical rigor, and the nature and value of proof. The paper looks at some examples and considers what we can infer from them concerning his understanding of rigor. (Received September 25, 2017)

1135-01-2391     **Toke Lindegaard Knudsen\*** ([toke.knudsen@hum.ku.dk](mailto:toke.knudsen@hum.ku.dk)), Dept. of Cross-Cultural and Regional Studies, University of Copenhagen, Karen Blixens Plads 8, Building 10, 2300 Copenhagen S, Denmark. *Johannes Hjelmslev and the Didactics of Geometry*. Preliminary report.

The first decades of the 20th century saw an intense discussion in Denmark of the didactic method of geometry. It became clear early on that there were only two viable paths for the teaching of geometry. One was to uphold the Euclidean ideal and teach geometry according to the axiomatic method, the other was to consider geometry as a natural science in which connections are seen through experiments. In the “experimental method,” outlined in textbooks already from 1904, the pupils go as far as they can through experiments, then switch to deduce new results from the set of “axioms” brought forth by the experiments. Johannes Hjelmslev (1873–1950), who was professor of mathematics at the University of Copenhagen, considered classical geometry a crude and poor approximation to the physical world and constructed what he called “the geometry of reality” as a better model for the physical world. Some of Hjelmslev’s claims, including that a tangent of a circle has a line segment in common with the circle, were rejected by some, but others took to his ideas. In particular, his followers wrote school textbooks according to his geometry. The talk will trace the discussion of the didactics of geometry in Denmark with an emphasis on the contributions by Hjelmslev. (Received September 26, 2017)

1135-01-2843     **Eunsoo Lee\*** ([eunsoo@stanford.edu](mailto:eunsoo@stanford.edu)), Stanford, CA 94305. *Diagrams for Dummies: Visual Auxiliaries in printed diagrams of Euclid’s Elements*.

The printed Elements in the sixteenth century presented more concise and practical diagrams than those of previous manuscripts. While conventional diagrams were limited to implementing the description of the text, the new diagrams introduced more practical constructions absent from the text, deviating from the tradition and also from the text.

This change into more practical diagrams reflects the increased emphasis on the pedagogical value of the diagram. As is evident from the compass arcs upon the diagram, readers of the Elements were invited to draw their own diagrams. This backdrop of increased engagement with the diagram facilitated learning the Elements for mathematical novices (Matheseos tyrones). These tool-based diagrams were more effective for teaching beginners than the earlier, less-functional diagrams.

This paper traces a brief history of these tool-based diagrams, which I call Tyronian diagrams. Closer scrutiny is needed to determine when Tyronian diagrams first appeared and how it was circulated together with the formal version of the Elements. To this end, the paper investigates diagrams in early printed editions of the Elements in the sixteenth century. This investigation provides us with a snapshot of a key shift in diagram implementation in mathematics. (Received September 26, 2017)

1135-01-2848     **Dominic Klyve\*** ([klyved@cwu.edu](mailto:klyved@cwu.edu)), Dept of Mathematics, 400 E University Way, Ellensburg, WA 98926. *A Guilty Euler Searches for Large Primes*.

Euler’s first paper in number theory, written when he was 25 years old, was a *tour de force* of new ideas and connections in the study of primality and factoring. In the paper, he established several new lines of inquiry that he and others would spend centuries following. He also disproved the claim that Fermat numbers, integers of

the form  $F_n := 2^{2^n} + 1$ , are all prime, and thereby removed from the mathematical world what was believed to be the easy possibility of generating arbitrarily large prime numbers. This talk will use Euler's desire to expiate his "guilt" over the factorization of  $F_5$  as a lens to read much of his later work in number theory. (Received September 26, 2017)

## 03 ► *Mathematical logic and foundations*

1135-03-2      **John Stillwell\*** ([stillwell@usfca.edu](mailto:stillwell@usfca.edu)). *The Brouwer Fixed Point Theorem*. Preliminary report.

The Brouwer fixed point theorem is a fundamental theorem of topology, which underlies theorems ranging from Brouwer's theorem on the invariance of dimension to the Nash equilibrium theorem. So, not surprisingly, various proofs of it have been proposed.

This talk will discuss some of the proofs, with particular attention to the proof of Sperner 1928, which is in a certain sense the most elementary possible. The amount of "nonconstructivity" required to prove the theorem is of interest because Brouwer himself rejected the theorem when he lectured on intuitionism in Berlin in 1927.

The recent development of reverse mathematics has identified the so-called "weak König lemma" as the essential nonconstructive ingredient in the proof of the Brouwer fixed point theorem. This development puts us in a better position to understand, for example, why the fixed point theorem is easier in dimension 1 than in higher dimensions. (Received September 06, 2017)

1135-03-190      **Aristotelis Panagiotopoulos\*** ([panagio@caltech.edu](mailto:panagio@caltech.edu)), California Institute of Technology, Mathematics Department, 253-37, Pasadena, CA 91125, and **Martino Lupini**. *Games orbits play*.

Classification problems occur in all areas of mathematics. Descriptive set theory provides methods to assign complexity to such problems. Using a technique developed by Hjorth, Kechris and Sofronidis proved for example, that the problem of classifying all unitary operators  $\mathcal{U}(\mathcal{H})$  of an infinite dimensional Hilbert space up to unitary equivalence  $\simeq_U$  is strictly more difficult than classifying graph structures with domain  $\mathbb{N}$  up to isomorphism.

We present a game-theoretic approach to anti-classification results for orbit equivalence relations and use this development to reorganize conceptually the proof of Hjorth's turbulence theorem. We also introduce a dynamical criterion for showing that an orbit equivalence relation is not Borel reducible to the orbit equivalence relation induced by a CLI group action; that is, a group which admits a complete left invariant metric (recall that, by a result of Hjorth and Solecki, solvable groups are CLI). We deduce that  $\simeq_U$  is not classifiable by CLI group actions.

This is a joint work with Martino Lupini. (Received September 21, 2017)

1135-03-192      **Simon Thomas\*** ([simon.rhys.thomas@gmail.com](mailto:simon.rhys.thomas@gmail.com)), Mathematics Department, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854. *The isomorphism and bi-embeddability relations for countable torsion abelian groups*.

In this talk, I will discuss the isomorphism  $\cong_{TA}$  and bi-embeddability  $\equiv_{TA}$  relations on the space of countable torsion abelian groups. As I will explain, the bi-embeddability relation has a strictly simpler complete invariant than the isomorphism relation. Thus it is somewhat counterintuitive that  $\cong_{TA}$  and  $\equiv_{TA}$  turn out to be incomparable with respect to Borel reducibility. However, under a relatively mild large cardinal assumption, we obtain the intuitively correct result if we replace Borel reducibility by  $\Delta^1_2$  reducibility. This is joint work with Filippo Calderoni. (Received September 06, 2017)

1135-03-193      **Sebastien Vasey\*** ([sebv@math.harvard.edu](mailto:sebv@math.harvard.edu)). *Non-elementary classification theory*.

The classification theory of elementary classes was started by Michael Morley in the early sixties, when he proved that a countable first-order theory with a single model in some uncountable cardinal has a single model in all uncountable cardinals. The proof of this result, now called Morley's categoricity theorem, led to the development of forking, a joint generalization of linear independence in vector spaces and algebraic independence in fields, which is now a central pillar of modern model theory.

Lately, it has become apparent that forking also exists in several non-elementary contexts. Prime among those is the axiomatic framework of abstract elementary classes (AECs), encompassing the class of models of any  $\mathbb{L}_{\infty, \omega}$ -theory and closely connected to the more general accessible categories. A test question to judge progress in this direction is the forty year old eventual categoricity conjecture of Shelah, which says that a version of Morley's categoricity theorem should hold of any AEC. I will survey recent developments, including

the connections with category theory and large cardinals, as well as my resolution of the eventual categoricity conjecture for classes of models axiomatized by a universal  $\mathbb{L}_{\infty, \omega}$ -theory. (Received September 25, 2017)

1135-03-194 **Keita Yokoyama\*** ([y-keita@jaist.ac.jp](mailto:y-keita@jaist.ac.jp)). *Ramsey's theorem in arithmetic.*

Calibrating the strength of various combinatorial principles is one of the important topics in the study of reverse mathematics. Especially, deciding the strength of Ramsey's theorem for pairs is hard and it is precisely studied from the viewpoints of both of computability theory and proof theory. In this talk, I will focus on the first-order consequences of infinite Ramsey's theorem and overview the recent developments. (Received September 22, 2017)

1135-03-274 **Simon Cho, Cory M. Knapp, Clive Newstead and Liang Ze Wong\*** ([wonglz@uw.edu](mailto:wonglz@uw.edu)). *Weak equivalences between categories of models of type theory.* Preliminary report.

Kapulkin and Lumsdaine have outlined a specific program by which to precisely state and prove the conjecture that intensional type theory is the internal language of higher categories. We build upon their progress on this program by establishing notions of weak equivalence in various categories of models of type theory, and proving that the comparison functors between these categories of models are Dwyer-Kan equivalences. We show that (the categories of) contextual categories, categories with attributes, comprehension categories, and type-theoretic fibration categories are all equivalent in this sense, assuming the existence of  $\text{Id}$ ,  $\Sigma$  and  $\Pi$  types in each of these contexts. (Received August 17, 2017)

1135-03-387 **Peter LeFanu Lumsdaine\*** ([p.l.lumsdaine@math.su.se](mailto:p.l.lumsdaine@math.su.se)). *A general definition of dependent type theories.* Preliminary report.

(Includes joint work with A. Bauer, P. Haselwarther, and T. Winterhalter)

We propose a general definition of syntactic type theories which yields as instances Martin-Löf's intensional and extensional type theories, and their various later extensions by further logical constructs — inductive families, inductive-recursive types à la Dybjer/Setzer, coinductive types, higher inductive types, and so on.

The novelty compared to established approaches (logical frameworks, pure type systems, etc.) is that it yields not some embedded counterpart of a theory, whose equivalence with the original might be non-trivial, but literally the original type theory itself as it might typically be presented.

The need for such a framework has been sorely felt recently, since without one, important meta-theorems and constructions can be given only for specific type theories, even when believed to hold in more generality.

Complete definitions of type theories are so large and intricate as to be almost impossible to get right without extensive debugging. To this end, we are formalising the present definition in Coq, to give confidence in its correctness and fitness-for-purpose. (Received August 30, 2017)

1135-03-406 **Daniel J Hathaway\*** ([daniel.hathaway@du.edu](mailto:daniel.hathaway@du.edu)). *Ramsey Theory on Generalized Baire Space.*

It is well known that every Borel subset of  $[\omega]^\omega$  is Ramsey. On the other hand, open (in the standard topology) subsets of  $[\kappa]^\kappa$  fail to be Ramsey. We examine various weaker topologies. For example, given  $\gamma < \kappa$ , every clopen set in the  $< \gamma$ -box topology is Ramsey. It is unknown whether clopen can be replaced by open. We consider open sets in other topologies, assuming  $\kappa$  is a large cardinal. (Received September 20, 2017)

1135-03-584 **Wilfried Sieg\*** ([sieg@cmu.edu](mailto:sieg@cmu.edu)), Carnegie Mellon University, Department of Philosophy, Pittsburgh, PA 15213. *The Cantor-Bernstein Theorem: How many proofs?*

The Cantor-Bernstein Theorem states: if there is an injection  $f$  from  $A$  to  $B$  and an injection  $g$  from  $B$  to  $A$ , then there is a bijection  $h$  from  $A$  to  $B$ . Dedekind was the first to prove the Theorem – without appealing to Cantor's well-ordering principle, but rather by using his chain theory; the proof was only published in 1932. A careful analysis of his proof reveals a basic structure of argumentation that can be seen in the many other proofs. (A full account of proofs and their history is given in Hinkis' book, *Proofs of the Cantor-Bernstein Theorem*, 2013.) My contention is that there is essentially a single proof, in two minor variants. The second variant is due to Zermelo in 1908 who used, not knowing about Dedekind's proof, the latter's chain theory. It is of interest to note that the various proofs obtain one of two bijections. The proofs have been fully and “naturally” formalized, by Sieg and Walsh, in Zermelo Fraenkel set theory ZF. That allows us to pinpoint exactly where proof differences appear. (Received September 09, 2017)

1135-03-642 **Aaron W Anderson\*** (awanders@caltech.edu) and **Martino Lupini**. *The Fraïssé Limit of Finite Dimensional Matrix Algebras with the Rank Metric.*

We show that a certain ring,  $M(\mathbb{F}_q)$ , constructed by von Neumann and realized as the coordinatization of a continuous geometry, can also be realized as the metric Fraïssé limit of the class of finite-dimensional matrix algebras over a finite field  $\mathbb{F}_q$ , equipped with the rank metric. Von Neumann constructed  $M(\mathbb{F}_q)$  as the completion of the direct limit of an inductive sequence of matrix rings, and showed that the resulting ring does not depend on the choice of sequence. We provide an alternate proof of the latter by the uniqueness of the Fraïssé limit. We show that the automorphism group of this metric structure is extremely amenable, implying (by the metric Kechris-Pestov-Todorćević correspondence) an approximate Ramsey Property. We also provide an explicit bound for the approximate Ramsey Property. Both the extreme amenability result and the Ramsey Property bound rely on work by Carderi and Thom, who proved that  $M(\mathbb{F}_q)$ 's unit group is extremely amenable. (Received September 11, 2017)

1135-03-840 **Shay Allen Logan\*** (salogan@ncsu.edu), Department of Philosophy, North Carolina State University, Raleigh, NC 27695. *Constant domain semantics for contractionless first-order relevance logics.*

Quantified relevance logics are incomplete for the naïve constant-domain semantics. But they're complete for the varying-domain semantic theory known as *stratified* semantics. In this talk I give a *constant-domain* stratified semantics for contractionless relevance logics. I do so by blending together Fine's stratified semantics and Restall's four-valued semantics for contractionless relevance logics. In the resulting semantic theory, the domain of a model comes in two pieces:  $D$  and  $\Omega$ .  $D$  contains objects that can be named by individual constants.  $\Omega$  contains 'arbitrary objects' (AOs). AOs are 'arbitrary' in the following two senses:

- First, at any level  $X$  of the stratification, almost every AO is featureless in all the  $X$ -setups.
- Second, if  $\omega$  is an AO that is featureless at level  $X$ , there is a higher level  $Y$  where, for any  $d \in D \cup \Omega$  that *isn't* featureless at  $X$ ,  $\omega$  is indistinguishable from  $d$  throughout some fragment of the level- $Y$  model.

$\forall x\phi(x)$  is true in a setup  $s$  at a level  $X$  just when there is an AO  $\omega$  and a level  $Y$  above  $X$  such that  $\phi(\omega)$  is true in all situations at level  $Y$  that are extensions of  $s$ . (Received September 15, 2017)

1135-03-841 **Ermek Nurkhaidarov\*** (esn1@psu.edu). *The Automorphism Group of a Recursively Saturated Model of Peano Arithmetic.*

Let  $M$  to be a countable recursively saturated model of Peano Arithmetic. If  $M$  has an element which is bigger than the standard cut  $\omega$  but smaller than any non-standard definable elements, we call such  $M$  *wide*. Pointwise stabilizers are the basic open subgroups of the automorphism group of  $M$ .

A countable recursively saturated model of Peano Arithmetic is characterized by two invariants: its first order theory and its standard system. We show that the automorphism group of a wide, countable recursively saturated models of Peano Arithmetic codes its standard system. From which we obtain:

**Theorem 1.** *Suppose that  $M_1$  and  $M_2$  are wide, countable, recursively saturated models of Peano Arithmetic such that their automorphism groups are topologically isomorphic. Then  $\text{SSy}(M_1) = \text{SSy}(M_2)$ .*

This theorem is an improvement on a result Kossak-Schmerl 95, who proved it for arithmetically saturated models. (Received September 15, 2017)

1135-03-842 **Cameron Donnay Hill\*** (cdhill@wesleyan.edu). *0,1-Laws and pseudofiniteness of  $\aleph_0$ -categorical theories.*

I will survey some recent results around questions of the form "What are pseudofinite  $\aleph_0$ -categorical theories like?" and "What are almost-sure  $\aleph_0$ -categorical theories like?" Ideally, we would like to answer such questions solely in terms of structural properties the theories' models — without reference to ultraproducts or probability theory. I will present evidence that certain higher amalgamation properties and model-theoretic properties like super-simplicity are sufficient to answer our questions completely. (Received September 15, 2017)

1135-03-851 **Nam Trang\*** (ntrang@uci.edu), University of California, Irvine, Irvine, CA 92697. *Models of the Axiom of Determinacy and Their Generic Extensions.* Preliminary report.

Forcing and elementary embeddings are central topics in set theory. Most of what set theorists have focused on are the study of forcing and elementary embeddings over models of ZFC.

In this talk, we focus on forcing and elementary embeddings over models of the Axiom of Determinacy (AD). In particular, we focus on answering the following questions: work in  $V$  which models AD. Let  $\mathbb{P}$  be a forcing poset and  $g \subseteq \mathbb{P}$  be  $V$ -generic.

- 1) Does  $V[g]$  model AD?
- 2) Is there an elementary embedding from  $V$  to  $V[g]$ ?

Regarding question 1, we want to classify what forcings preserve AD. We show that forcings that add Cohen reals, random reals, and many other well-known forcings do not preserve AD.

Regarding question 2, an analogous statement to the famous Kunen's theorem for models of ZFC, can be shown: suppose  $V = L(X)$  for some set  $X$  and  $V$  models AD, then there is no elementary embedding from  $V$  to itself. We conjecture that there are no elementary embeddings from  $V$  to itself.

We present some of the results discussed above. There is still much work to do to completely answer questions 1 and 2. This is an ongoing joint work with D. Ikegami. (Received September 15, 2017)

1135-03-867 **Michael Gene McGrady\*** ([michael.mcgrady@gmail.com](mailto:michael.mcgrady@gmail.com)), 223 Taylor Ave. S., North Bend, WA 98045. *The molecular structure of mathematical proof.*

Finite classes are decidable. The validity of all mathematical logic formulas are equivalent to the validity of the members of finite classes ordered by their number of dyadic predicates. There is a mathematical **proof procedure** in the first-order logic that allows for a **discovery procedure** for the **decision procedure**. Hence, this proof procedure, based on Herbrand's fundamental theorem of logic, is itself at least a heuristic solution to **an alternative version of the Entscheidungsproblem (decision problem) for finite sets**. The keystone of the proof procedure is that the dyadic predicate atomic formulas  $P_{\alpha,\beta}^1 \dots P_{\alpha,\beta}^i$  have a ubiquitous **molecular structure**  $\Sigma = (P_{m-1,m}^1 \vee P_{m-2,m}^1 \vee \dots \vee P_{0,m}^1) \vee \dots \vee (P_{m-1,m}^i \vee P_{m-2,m}^i \vee \dots \vee P_{0,m}^i)$  such that  $m, i \in \mathbb{Z}^+$  underlying first-order logic proofs seen through the lens of a closed, prenex normal form, highly restricted reduction class for validity  $\exists x \exists y M_{xx'y}$  where  $x' = x + 1$ , and which allows the discovery procedure to find the decision procedure. (Received September 15, 2017)

1135-03-871 **Farshad Nahangi\*** ([f\\_nahangi@yahoo.com](mailto:f_nahangi@yahoo.com)), Fremont, CA. *Omitting Types in Temporal Logic.*

We present and investigate a version of Omitting Types Theorem in first-order temporal logic with connectives U(ntil) and S(ince). For achieving this purpose, we introduce the notions of realization and omitting types at a moment of time and we show that Omitting Types Theorem holds but not in the general case. (Received September 15, 2017)

1135-03-1015 **Rachel Epstein\*** ([rachel.epstein@gcsu.edu](mailto:rachel.epstein@gcsu.edu)) and **Karen Lange** ([klange2@wellesley.edu](mailto:klange2@wellesley.edu)). *Computable reducibility and equality on a given set.* Preliminary report.

An equivalence relation  $E$  on the set of all computably enumerable (c.e.) sets is *computably reducible* to an equivalence relation  $F$  on the c.e. sets, written  $E \leq F$ , if there is a computable function  $f$  such that  $W_n E W_m$  if and only if  $W_{f(n)} F W_{f(m)}$ . Coskey, Hamkins, and R. Miller have explored the hierarchy of equivalence relations on the c.e. sets. Here we look at a natural class of equivalence relations and fit them into the hierarchy. The equivalence relation  $E_A$  on the c.e. sets is given by  $W_n E_A W_m$  if and only if  $W_n \cap A = W_m \cap A$ . If  $A$  is c.e., then it is not hard to show that  $E_A$  is computably bireducible to the equality equivalence relation on the class of c.e. sets, which we call  $=^{ce}$ . If  $A$  is co-c.e., then  $E_A \leq^{ce}$  and the reduction is strict if and only if  $A$  is hyper-hyper-immune. We also construct sets  $A$  and  $B$  such that  $E_A$  and  $E_B$  are incomparable under computable reducibility. (Received September 18, 2017)

1135-03-1100 **Jeremy Avigad\*** ([avigad@cmu.edu](mailto:avigad@cmu.edu)), Department of Philosophy, Baker Hall 161, Carnegie Mellon University, Pittsburgh, PA 15213. *Mathematical Language from a Design Perspective.*

Among the sciences, mathematics is characterized by the level of precision used in making claims and justifying them. This fact has made it possible for logicians to develop idealized descriptions of mathematical language and rules of inference, in order to model key aspects of mathematical reasoning. That, in turn, makes it possible to develop computational tools to support such reasoning with better means of computation, verification, and search.

But there is a gap between informal mathematical language and these idealized versions. When we read an ordinary mathematical text, we rely on a deep-seated understanding of mathematical norms and conventions, as well as the subject matter before us, to disambiguate expressions and fill in information that is only implicit in the written text. For many computational applications, this information has to be made fully explicit.

The goal, then, is to design formal mathematical languages that approximate the power, efficiency, and flexibility of informal language, while at the same time providing fully explicit and unambiguous representations of mathematical knowledge. In this talk, I will describe some of the challenges that brings, and some of the

mechanisms that logicians and computer scientists have developed to address them. (Received September 19, 2017)

1135-03-1194 **Chris Lambie-Hanson\*** ([lambiec@macs.biu.ac.il](mailto:lambiec@macs.biu.ac.il)) and **Philipp Lücke.** *Squares, ascent paths, and chain conditions.*

Using a variety of square principles, we obtain results on the consistency strengths of the non-existence of  $\kappa$ -Aronszajn trees with narrow ascent paths and of the infinite productivity of strong  $\kappa$ -chain conditions. In particular, we show that, if  $\kappa$  is an uncountable regular cardinal that is not weakly compact in  $L$ , then:

- (1) for every  $\lambda < \kappa$ , there is a  $\kappa$ -Aronszajn tree with a  $\lambda$ -ascent path;
- (2) there is a  $\kappa$ -Knaster poset  $\mathbb{P}$  such that  $\mathbb{P}^\omega$  does not have the  $\kappa$ -chain condition;
- (3) there is a  $\kappa$ -Knaster poset that is not  $\kappa$ -stationarily layered.

This answers questions of Cox and Lücke. (Received September 20, 2017)

1135-03-1222 **Daniel J Hathaway\*** ([daniel.hathaway@du.edu](mailto:daniel.hathaway@du.edu)). *Disjoint Infinity-Borel Functions.*

It is well known that sets of reals that have logically simple definitions satisfy desirable regularity properties, such as Lebesgue measurability, etc. We define a family  $F$  of continuum many Borel functions from  $\mathbb{R}$  to  $\mathbb{R}$ , and call a function  $g$  from  $\mathbb{R}$  to  $\mathbb{R}$  “nice” iff it is disjoint from only countably many of the functions in  $F$ . We first show that ZFC proves every Borel function is nice. We then show that that statement that every  $\Delta_1^1$  function is nice is equivalent to  $\omega_1$  being inaccessible in  $L[r]$  for every real number  $r$ . Next, we show that  $AD^+$  implies every  $g$  is nice. Finally, although a well-ordering of  $\mathbb{R}$  implies some function is not nice, we show that it is consistent with ZF that every function is nice and there is a non-principal ultrafilter on  $\omega$ . (Received September 20, 2017)

1135-03-1231 **Johann D. Gaebler\*** ([johann.gaebler@wolfson.ox.ac.uk](mailto:johann.gaebler@wolfson.ox.ac.uk)) and **W. Hugh Woodin.**  *$\mathbf{I}_0$  Implies  $\mathbf{PD}$ .*

For several decades, connecting determinacy axioms to large cardinals was a core goal for many set theorists. Martin and Steel’s 1989 proof of projective determinacy from the existence of infinitely many Woodin cardinals with a measurable cardinal above marked a critical advance in this effort. However the first proof of  $\mathbf{PD}$  from large cardinal axioms was known several years earlier, but was never published.

The purpose of this talk is to outline Woodin’s 1984 proof of  $\mathbf{PD}$  from  $\mathbf{I}_0$ . Of central importance is the construction of different representations of sets of reals from trees of certain elementary embeddings  $j : L_\lambda(V_{\lambda+1}) \rightarrow L_\lambda(V_{\lambda+1})$ , for  $\lambda$  the supremum of the critical sequence of  $j$ . We focus in particular on the techniques as they differ from those employed in Martin-Steel. Time permitting, we shall discuss how the same techniques extend to a proof of  $\mathbf{AD}_{L(\mathbb{R})}$  and the determinacy of the universally Baire sets. (Received September 22, 2017)

1135-03-1279 **Iian B. Smythe\*** ([i.smythe@rutgers.edu](mailto:i.smythe@rutgers.edu)), Department of Mathematics, Rutgers, The State University of New Jersey, Piscataway, NJ 08854. *Madness in vector spaces, round 2.*

A collection of infinite-dimensional subspaces of a vector space is *maximal almost disjoint*, or *mad*, if distinct elements have finite-dimensional intersection and the collection is maximal with respect to this property. We consider questions regarding the possible sizes of such families of subspaces. Of particular interest is the minimum (infinite) size of such a family, and the spectrum of their possible sizes. We establish that the former is small in certain models of set theory, while the latter can be made arbitrarily large. This talk is in part a sequel to that given by the author at the JMM in 2017. (Received September 20, 2017)

1135-03-1369 **Francisco Guevara Parra\*** ([guevara.guevaraparra@mail.utoronto.ca](mailto:guevara.guevaraparra@mail.utoronto.ca)), Bahen Centre, Room 6290, 40 St. George Street, Toronto, Ontario M5S2E4, Canada. *Sequential- $M$ -separable spaces.*

A topological space  $Z$  is said to be  $M$ -separable if for each sequence  $\{D_n : n \in \omega\}$  of dense subsets of  $Z$ , there is a selection  $\{E_n \in [D_n]^{<\omega} : n \in \omega\}$  of finite sets with dense union. We will study the productivity of this property on countable spaces. It is known that countable Fréchet spaces are  $M$ -separable, and if we assume the Proper Forcing axiom, the product of two countable Fréchet spaces is again  $M$ -separable. We will prove a ZFC version of this result: the product of two analytic spaces that are  $M$ -separable and sequential is  $M$ -separable. (Received September 21, 2017)

1135-03-1643 **Jin Du\*** ([jdu8@uic.edu](mailto:jdu8@uic.edu)), Chicago, IL. *Diamond, Scales and SCH down to  $\aleph_{\omega_2}$ .*

I will discuss a paper I submitted last year.

Gitik and Rinot proved assuming the existence of a supercompact that it is consistent to have a strong limit cardinal  $\kappa$  such that  $2^\kappa = \kappa^+$ , there is a very good scale at  $\kappa$ , and  $\diamond$  fails along some reflecting stationary subset

of  $\kappa^+ \cap \text{cof}(\omega)$ . In this paper, we force over Gitik and Rinot's model but with a modification of Gitik-Sharon diagonal Prikry forcing to get this result for  $\kappa = \aleph_{\omega_2}$ . (Received September 24, 2017)

1135-03-1699 **Paul B. Larson\*** ([larsonpb@miamioh.edu](mailto:larsonpb@miamioh.edu)), Department of Mathematics, Miami University, Oxford, OH 45056, and **Jindrich Zapletal**. *Canonical models for fragments of the Axiom of Choice*.

We develop technology for investigation of natural forcing extensions of the model  $L(\mathbb{R})$  which satisfy such statements as “there is a nonprincipal ultrafilter on the integers”, “there is a total selector for the Vitali equivalence relation”, “there exists an infinite MAD family” or “there exists a Hamel basis”. The technology reduces many questions about ZF implications between consequences of the Axiom of Choice to natural ZFC forcing problems. (Received September 24, 2017)

1135-03-1828 **Marion Scheepers\*** ([mscheepe@boisestate.edu](mailto:mscheepe@boisestate.edu)), Department of Mathematics, Boise State University, Boise, ID 83725. *Ramsey Theory, Topology and Forcing*.

In this talk we give a brief survey of equivalences between infinitary Ramsey theoretic statements, covering properties of topological spaces, and properties of certain forcing posets. (Received September 25, 2017)

1135-03-1857 **Carlos A. Di Prisco\*** ([ca.di@uniandes.edu.co](mailto:ca.di@uniandes.edu.co)). *Ultrafilters and forcing on the space of block sequences of finite sets*.

We examine Ramsey properties of subsets of the space  $FIN^\infty$  of infinite block sequences of finite sets of natural numbers. We study ultrafilters and forcing notions related to these properties, following the lines started by Blass and Hindman and continued by Matet, Eisworth and García Ávila. We consider the localization of these properties to subsets of the space that are selective or semiselective coideals, and discuss some properties of the corresponding forcing notions. We place the results presented in the context of topological Ramsey spaces. (Received September 25, 2017)

1135-03-1926 **Thorsten Altenkirch\*** ([txa@cs.nott.ac.uk](mailto:txa@cs.nott.ac.uk)). *The Joy of QIITs*.

Quotient inductive-inductive types (QIITs) are set-truncated mutually inductive higher inductive types. Applications include the definitions of the Cauchy Reals in the HoTT book and the partiality monad. In both cases the use of QIITs avoids having to use instances of the axiom of choice (here countable choice). Another example is the definition of the internal syntax of type theory avoiding untyped preterms.

In my talk I want to address the following topics:

- What is a general definition of QIITs?
- Can we construct QIITs from inductive types and quotients?
- What are the issues with set truncation?

(Received September 25, 2017)

1135-03-2038 **Natasha Dobrinen\***, University of Denver, Department of Mathematics, C.M. Knudson Hall, Room 300 2390 S. York St., Denver, CO 80209. *The triangle-free homogeneous graph had finite big Ramsey degrees*.

It is a central question in the theory of homogeneous relational structures as to which structures have finite big Ramsey degrees. A homogeneous structure  $\mathbf{S}$  is said to have *finite big Ramsey degrees* if for each finite substructure  $A$  of  $\mathbf{S}$ , there is a number  $n(A)$  such that any coloring of the copies of  $A$  in  $\mathbf{S}$  into finitely many colors can be reduced down to no more than  $n(A)$  colors on some substructure  $\mathbf{S}'$  isomorphic to  $\mathbf{S}$ . This is interesting not only as a Ramsey property for infinite structures, but also because of its implications for topological dynamics. Prior to the work in this paper, finite big Ramsey degrees were proved for a handful of homogeneous structures, each of which do not omit a non-trivial substructure. Each of those results were critically tied to a Ramsey theorem on trees due to Milliken. Our work provides new tools to represent the homogeneous triangle-free graph and develops the necessary Ramsey theory, using the technique of forcing, to deduce finite big Ramsey degrees in ZFC. The methods developed seem robust enough that modifications should likely apply to a large class of homogeneous structures omitting some finite substructure. (Received September 25, 2017)

1135-03-2040 **Thomas Hales\***, Math Department, University of Pittsburgh, Pittsburgh, PA 15260. *Formal Abstracts in Mathematics*. Preliminary report.

Formal proof technology has been under development for decades. With sufficient effort, it is now possible to give a complete formal verification of long and complex mathematical proofs.



This talk will describe recent efforts to extend formal proof technology so that all mathematicians might benefit. A formal abstract service gives the statement of the main theorem and definitions of each published mathematical paper. The abstracts are written in a language that is readable by both humans and machines. The definitions are grounded in a foundational system for doing mathematics. (Received September 25, 2017)

1135-03-2050 **Michael Pinsker\*** (marula@gmx.at). *Canonical functions and the Ramsey property, revisited.*

A function from one first-order structure into another first-order structure is called *canonical* if it sends tuples of the same type in the first structure to tuples of the same type in the second structure. This regularity notion for functions has found numerous applications in model theory, universal algebra, and theoretical computer science since its invention 7 years ago. In particular, it facilitates the understanding of self-embeddings, endomorphisms, and polymorphisms of structures, and has been applied, for example, in the classification of reducts of structures and in the study of certain computational problems related to them.

Any function between two countable structures gives rise to canonical functions in a natural way, provided its domain structure has the Ramsey property, and its goal structure is  $\omega$ -categorical. We outline a new proof of this fact using the framework of topological dynamics, and present the recent discovery that under certain conditions also the converse holds, i.e., the possibility of obtaining canonical functions in that way implies the Ramsey property for the domain. We moreover outline the main applications mentioned above, and the most important open problems connected to canonical functions. (Received September 26, 2017)

1135-03-2221 **Caleb Davis, Jeffrey Hirst and Jake Pardo\*** (pardojj@appstate.edu), 339 Walker Hall, 121 Bodenheimer Drive, Boone, NC 28607, and **Tim Ransom**. *Reverse Mathematics of Hypergraph Colorings*. Preliminary report.

Reverse mathematics essentially seeks to break results from all areas of mathematics down into their most basic axioms. Graph theory has proven to be a topic replete with fascinating reverse mathematical results, and so it only makes sense to ask similar questions about what happens when we extend our scope to include hypergraphs. I will discuss a bit of what is currently known about the reverse mathematics of hypergraph colorings as well as explain some recent joint work with Davis, Hirst, and Ransom. (Received September 25, 2017)

1135-03-2249 **Timothy Trujillo\*** (trujillo@shsu.edu). *Hypersaturated numbers in ultra-Ramsey theory*. We present an alternative formalism to deal with ultrafilters grounded on the use of the infinite hypersaturated numbers of nonstandard analysis, which - in a precise sense - replace the use of ultrafilters. We then use the formalism to prove a new infinite-dimensional extension of Ramsey's theorem for ultrafilter trees. We conclude by extending the result to the abstract setting of topological Ramsey spaces. (Received September 25, 2017)

1135-03-2395 **Michael Shulman\*** (shulman@sandiego.edu). *Homotopical trinitarianism: a perspective on homotopy type theory*.

As promoted by Harper, "computational trinitarianism" emphasizes the unity of the three faces of type theory: category theory, logic, and computation. That is, type theory is at once a syntax for categories, a foundation for mathematics, and a programming language. This talk will be a high-level overview of *homotopy* type theory from the triune perspective: it is at once a syntax for higher categories, a foundation for higher-categorical / homotopical mathematics, and a homotopical programming language. Individual homotopy type theorists may prefer one perspective or another, depending on their background and interests; but I believe everyone can benefit from a respect and appreciation for all three. Various threads in the past and present of homotopy type theory have been motivated by one or another of the three perspectives, and often seem to pull the theory in different directions; but a trinitarian has faith that beneath this apparent conflict lies a deeper unity waiting to be understood in the future. (Received September 26, 2017)

1135-03-2416 **Clinton T Conley\***, clintonc@andrew.cmu.edu. *Unfriendly colorings: a descriptive set-theoretic view*.

Given a graph, a red/blue coloring of its vertices is *unfriendly* if every red vertex has at least as many blue neighbors as red neighbors, and vice-versa. Such colorings always exist for finite graphs, but for infinite graphs their existence quickly becomes quite subtle. We investigate certain descriptive set-theoretic analogs of these colorings with various definability constraints.

This talk may include work with Alexander Kechris, Andrew Marks, Robin Tucker-Drob, and Spencer Unger. (Received September 26, 2017)

1135-03-2506 **Francis Adams\*** (fadams@gsu.edu), Georgia State University, Atlanta, GA 30306. *The Loose Number of Graphs on Topological Spaces.*

Given a graph  $G$  on a topological space  $X$ , we define the loose number of  $G$ ,  $\lambda(G)$ . This invariant depends on the graph-theoretic properties of  $G$  as well as the topology on the vertex set  $X$ . When  $X$  is separable metric,  $\lambda(G)$  lies between two well-known graph invariants, the chromatic number and the coloring number. Evaluating this cardinal leads to interesting connections with forcing, infinitary combinatorics, descriptive set theory, and topology. We discuss these connections and provide many examples. Much of this work is joint with Jindrich Zapletal. (Received September 26, 2017)

1135-03-2533 **Monica VanDieren\*** (vandieren@rnu.edu). *Symmetry in Abstract Elementary Classes.*

In this presentation we describe how understanding symmetry in abstract elementary classes contributed to the convergence of two separate research strands: good frames and tameness. (Received September 26, 2017)

1135-03-2545 **Joachim Mueller-Theys\*** (mueller-theys@gmx.de). *The Unprovability of Unprovability.*

Unprovable consistency formulæ seem to be exceptional singularities. We found out that, given “sufficient strength”, “derivability conditions”, and consistency, *all*  $\neg\iota(\ulcorner\sigma\urcorner)$  are unprovable. This follows from Löb’s Theorem in connection with our Universalisation Lemma:  $\Sigma \not\vdash \neg\iota(\ulcorner\perp\urcorner) \Rightarrow \Sigma \not\vdash \neg\iota(\ulcorner\sigma\urcorner)$ , and it makes certain issues more or less trivial: *total negative self-irrepresentability* and *unprovability of all consistency sentences* (particularly, when induced by disprovable sentences (Gödel’s 2nd Incompleteness Theorem)), whereby consistency sentences state in the standard model that  $\Sigma$  is consistent. The ASL-abstract provides the details. (Received September 26, 2017)

1135-03-2618 **Joshua A. Cole\*** (colej@wabash.edu). *The Two Quantifier Theory of the Effectively Closed Muchnik Degrees.* Preliminary report.

Over the past 15-20 years, many computability theorists have studied the  $\Pi_1^0$  subsets of  $2^{\mathbb{N}}$  as mass problems. Under both Medvedev (strong) and Muchnik (weak) reducibility, a corresponding lattice structure arises (these are denoted by  $\mathcal{E}_s$  and  $\mathcal{E}_w$ , respectively). In addition to being natural objects of mathematical interest, these lattices are important to computability theory and provide insight into the foundations of mathematics. It is natural (especially in computability theory) to ask questions about the structure of such objects, such as whether these lattices are dense. It turns out both lattices are dense, Binns, Shore, and Simpson having recently shown the density of  $\mathcal{E}_w$ .

Adding new techniques to the priority argument used to prove the density of  $\mathcal{E}_w$ , we show that its  $\forall\exists$ -theory in the language of partial orders is decidable. In fact, we show that  $\mathcal{E}_w$  and  $\mathcal{E}_s$  have the same  $\forall\exists$ -theory (in the language of partial orders). In our proof we also use many of the ideas (including solving the multi-extension of embeddings problem) used by Cole and Kihara (independently) for deciding the  $\forall\exists$ -theory of  $\mathcal{E}_s$ . (Received September 26, 2017)

1135-03-2686 **Peter Komjath\*** (kope@cs.elte.hu), P. O. Box 50 1583, Budapest, Hungary. *Some Ramsey theorems on metric spaces.*

We survey some older and recent Ramsey type results on metric spaces. (Received September 26, 2017)

1135-03-2697 **Athar Abdul-Quader\*** (aabdulquader@gradcenter.cuny.edu). *Classifying Enayat models of Peano Arithmetic.*

Simpson used arithmetic forcing to show that every countable model  $\mathcal{M} \models \text{PA}$  has an undefinable, inductive subset  $X \subseteq M$  such that the expansion  $(\mathcal{M}, X)$  is pointwise definable. Enayat later showed that there are many models with the property that every expansion upon adding a predicate for an undefinable class is pointwise definable. We refer to models with this property as Enayat models. That is, a model  $\mathcal{M} \models \text{PA}$  is Enayat if for each undefinable class  $X \subseteq M$ , the expansion  $(\mathcal{M}, X)$  is pointwise definable. In this talk we show that a model is Enayat if it is countable, has no proper cofinal submodels and is a conservative extension of each of its elementary cuts. (Received September 26, 2017)

1135-03-3046 **Kristina Sojakova\*** (ksojakov@cs.cmu.edu). *Higher Inductive Types as Homotopy-Initial Algebras.*

Homotopy type theory is a new field of mathematics based on the recently-discovered correspondence between constructive type theory and abstract homotopy theory.

Higher inductive types, which form a crucial part of this new system, generalize ordinary inductive types such as the natural numbers to higher dimensions. We will look at a few different examples of higher inductive types such as suspensions, quotients, and the torus, indicating how we can use their associated induction principle to reason about them.

We will introduce a general class of higher inductive types which we call  $W$ -quotients as a combination of well-founded trees and quotients, and indicate how the particular examples of higher inductive types we saw previously - of different structure and dimension - all arise as special cases of  $W$ -quotients.

Finally, we show that just like the natural numbers arise as initial algebras of a certain form, a  $W$ -quotient (and hence a suspension, a quotient, and the torus) arises naturally as a homotopy-initial algebra of a certain form. (Received September 26, 2017)

1135-03-3142 **Rachid Atmai\*** ([ratmai@miracosta.edu](mailto:ratmai@miracosta.edu)), 4320 Cassanna Way apt#1809, Oceanside, CA 92057. *Definable counterexamples to the Continuum Hypothesis and consistency results from determinacy axioms.* Preliminary report.

We present some partial results which point in the direction of obtaining new definable counterexamples to the Continuum Hypothesis. These results have the potential of leading to the identification of a canonical generic extension of  $L(\mathbb{R})$ , in which  $\Theta^{L(\mathbb{R})} > \omega_3$ , the Axiom of Choice holds and which settles combinatorial questions about  $\omega_2$ . In the context of Woodin's  $\Omega$ -logic, the goal of this investigation is to identify axioms which mirror forcing axioms and which maximize the theory of initial segments of the universe  $V$ , but under which the Continuum Hypothesis fails. These partial results build on previous work of Jackson and Woodin. (Received September 26, 2017)

1135-03-3167 **Andre Kornell\*** ([kornell@math.ucdavis.edu](mailto:kornell@math.ucdavis.edu)), Department of Mathematics, One Shields Avenue, Davis, CA 95616. *Reasoning about incomplete structures.*

Tarski's theorem expresses the incompatibility of classical reasoning with the use of truth as a predicate. This difficulty is commonly addressed by introducing a hierarchy of languages, each of which formalizes the semantics of the languages below it. Many accounts of the mathematical universe ascribed to it a similar hierarchical or indefinitely extensible character. We will formalize a system of reasoning appropriate to the consideration of structures that are indefinitely extendable or indefinitely expandable. This system of reasoning allows theories that establish Tarski's truth axioms for its formula, and that establish the correctness of their own inferences. I will sketch such a theory describing a universe of sets that is complete both horizontally and vertically, which we take to be the standard structure for ZFC. I will also sketch such a theory describing a universe of sets that is incomplete both horizontally and vertically. In the former case, we expand the standard structure of ZFC by a primitive truth predicate, which is not taken to satisfy the principle of bivalence; in the latter case, the burden of Tarski's theorem is shifted to the incompleteness of the universe. I may also mention a natural completeness principle appropriate to this latter conception. (Received September 26, 2017)

## 05 ► Combinatorics

1135-05-24 **Joshua D Brakensiek\***, Department of Mathematical Sciences, Wean Hall 6113, Carnegie Mellon University, Pittsburgh, PA 15213. *Vertex Isoperimetry and Independent Set Stability for Tensor Powers of Cliques.*

The  $n$ th tensor power of  $t$ -clique,  $K_t^n$ , is the graph on vertex set  $[t]^n = \{1, \dots, t\}^n$  such that two vertices  $x, y \in [t]^n$  are connected iff  $x_i \neq y_i$  for all  $i \in [n]$ . Let the density of a subset  $S$  of  $K_t^n$  be  $\mu(S) = |S|/t^n$ . Let the vertex boundary of a set  $S$  be the vertices, including those of  $S$ , which are incident to some vertex of  $S$ .

First, given  $\nu \in [0, 1]$ , what is the smallest possible density of the vertex boundary of a subset of density  $\nu$ ? Let  $\Phi_t(\nu)$  be the infimum of these minimum densities as  $n \rightarrow \infty$ . We find a recursive relation to compute  $\Phi_t(\nu)$  efficiently to arbitrary precision.

Second, given an independent set  $I$  of density  $(1 - \epsilon)/t$ , how close it is to a maximum density independent set  $J$ ? We show that  $\mu(I \setminus J) \leq 4e^{(\log t)/(\log t - \log(t-1))}$  as long as  $\epsilon < 1 - 3/t + 2/t^2$ . This substantially improves on results of Alon, Dinur, Friedgut, and Sudakov (2004) and Ghandehari and Hatami (2008) which had an  $O(\epsilon)$  upper bound. We also show the exponent is optimal as  $n \rightarrow \infty$  and  $\epsilon \rightarrow 0$ . The methods have similarity to recent work by Ellis, Keller, and Lifshitz (2016) in the context of Kneser graphs and other settings. (Received June 21, 2017)

1135-05-38 **Federico Ardila\***, San Francisco State University, San Francisco, CA 94110, and **Marcelo Aguiar**, Cornell University, Ithaca, NY 14853. *Algebraic structures on polytopes.*

Generalized permutahedra are a beautiful family of polytopes with a rich combinatorial structure and strong connections to optimization. We study their algebraic structure: we prove they are the universal family of polyhedra with a certain "Hopf monoid" structure. This construction provides a unifying framework to organize and study many combinatorial families: 1. It uniformly answers open questions and recovers known results

about graphs, posets, matroids, hypergraphs, and simplicial complexes. 2. It reveals that three combinatorial reciprocity theorems of Stanley and Billera–Jia–Reiner on graphs, posets, and matroids are really the same theorem. 3. It shows that permutahedra and associahedra “know” how to compute the multiplicative and compositional inverses of power series. The talk will be accessible to undergraduates and will not assume previous knowledge of these topics. (Received June 27, 2017)

1135-05-44 **Eva Czabarka** and **Laszlo A. Szekely\*** (szekely@math.sc.edu), szekely@math.sc.edu, and **Zoltan Toroczka**i and **Shanise Walker**. *Algebraic Monte-Carlo algorithms for the bipartite Partition Adjacency Matrix existence and construction problems*. Preliminary report.

Given a set  $W$  and numbers  $d(w)$  associated with  $w \in W$ , and a  $W_i : i \in I$  partition of  $W$ , with numbers  $c(W_i, W_j)$  associated with unordered pairs of partition classes, the Partition Adjacency Matrix *existence problem* asks whether there is a simple graph  $G$  on the vertex set  $W$  with degrees  $d_G(w) = d(w)$  for  $w \in W$ , with exactly  $c(W_i, W_j)$  edges with endpoints in  $W_i$  and  $W_j$ ; and the Partition Adjacency Matrix *construction problem* asks for such a graph, if they exist. (These problems are motivated by the concept of *assortativity* of network science, and the problems are conjectured to be NP-hard.) The *bipartite* version of these problems are more restricted:  $I = I_1 \cup I_2$  and  $c(W_i, W_j) = 0$  whenever  $i, j \in I_1$  or  $i, j \in I_2$ .

We provide algebraic Monte-Carlo algorithms for the bipartite Partition Adjacency Matrix existence and construction problems, which run in polynomial time, say, when  $|I|$  is fixed. When the algorithms provide a positive answer, they are always correct, and when the truth is positive, the algorithms fail to report it with small probability. (Received July 02, 2017)

1135-05-45 **Eva Czabarka\*** (czabarka@math.sc.edu), **Laszlo A. Szekely** and **Stephan Wagner**. *A tanglegram Kuratowski theorem*. Preliminary report.

A tanglegram  $(L, R, M)$  in  $n$  leaves is a pair of rooted binary trees  $L, R$ , each with  $n$  leaves and a matching  $M$  between the leaves. Two tanglegrams are the same if there is a root-preserving tree isomorphism between the left trees and a root preserving isomorphism between the right trees that maps matching edges to matching edges. A tanglegram layout is a drawing where the two trees are drawn as plane trees such that the leaves face each other, and matching edges are drawn as straight lines (so only matching edges are allowed to cross). The crossing number of a layout is the number of (unordered) pairs of edges that cross, the tangle crossing number of a tanglegram is the minimum crossing number over all of its layouts. Biologists use the tangle crossing number to estimate relevant quantities (e.g. number of times when a parasite switched hosts, where the two trees are the phylogenetic trees of host and parasite and the matching is given by which parasite exploits which host); determining the tangle crossing number is known to be NP hard. We show a Kuratowski-type theorem for tanglegrams, namely, that a tanglegram is nonplanar precisely when it contains one of two nonplanar 4-leaf tanglegrams as induced subtanglegrams. (Received July 02, 2017)

1135-05-56 **Ellen Veomett\*** (erv2@stmarys-ca.edu) and **Emmanuel Tsukerman**. *A General Method to Determine Limiting Optimal Shapes for Edge-Isoperimetric Inequalities*.

For a general family of graphs on  $\mathbb{Z}^n$ , we translate the edge-isoperimetric problem into a continuous isoperimetric problem in  $\mathbb{R}^n$ . We then solve the continuous isoperimetric problem using the Brunn-Minkowski inequality and Minkowski’s theorem on Mixed Volumes. This translation allows us to conclude, under a reasonable assumption about the discrete problem, that the shapes of the optimal sets in the discrete problem approach the shape of the optimal set in the continuous problem as the size of the set grows. The solution is the zonotope defined as the Minkowski sum of the edges of the original graph.

We demonstrate the efficacy of this method by revisiting some previously solved classical edge-isoperimetric problems. We then apply our method to some discrete isoperimetric problems which had not previously been solved. The complexity of those solutions suggest that it would be quite difficult to find them using discrete methods only. (Received July 12, 2017)

1135-05-114 **Allison Joan Ganger\*** (ganger@allegheny.edu), **Shannon N. Golden** (sgold940@live.kutztown.edu) and **Carter A. Lyons** (clyons@nebrwvlesleyan.edu). *Classification of Algebraically Defined Graphs by Girth*. Preliminary report.

An algebraically defined graph  $\Gamma_{\mathcal{R}}(f(x, y))$  is constructed using a specific ring  $\mathcal{R}$  and function  $f(x, y)$ . These graphs are bipartite with each partite set consisting of all coordinate pairs in  $\mathcal{R}^2$ . We denote the vertices of the first partite set by  $(a_1, a_2)$  and of the second by  $[x_1, x_2]$ . In order for two vertices  $(a_1, a_2)$  and  $[x_1, x_2]$  to be adjacent, their coordinates must satisfy the equation  $a_2 + x_2 = f(a_1, x_1)$ . The focus of our study is the girth, or length of a shortest cycle, of these graphs. In this talk, we will use incidence geometry to motivate our study

of algebraically defined graphs. We will also discuss the effect that changing the ring  $\mathcal{R}$  and function  $f(x, y)$  has on the girth of the algebraically defined graph  $\Gamma_{\mathcal{R}}(f(x, y))$ , with particular emphasis on the case  $\mathcal{R} = \mathbb{R}$ . (Received July 28, 2017)

1135-05-118 **Hector Miranda\***, School of Mathematical Sciences, 85 Lomb Memorial Drive, Rochester Institute of Technology, Rochester, NY 14623-5604, and **Heather Newman**, Mathematics Department, Fine Hall, 304 Washington Rd, Princeton University, Princeton, NJ 08544. *Classifications of Edge-Transitive and Edge-Betweenness-Uniform Graphs.*

A graph is edge-transitive (vertex-transitive) if its automorphism group acts transitively on its edges (vertices). We are the first to completely determine all (connected) edge-transitive graphs on 12 through 20 vertices. Furthermore, we identified infinite classes of edge-transitive graphs, including all graphs of the form  $K_m \times K_n$ ; several classes of 3-circulants; and a construction for an edge-transitive and connected biregular (non-trivial) subgraph of  $K_{m,n}$  for every pair  $(m, n)$  with  $\gcd(m, n) > 2$ . Finally, we investigate uniform edge betweenness centrality, a necessary condition for edge-transitivity. The betweenness centrality of an edge is the fraction of shortest paths between all pairs of vertices passing through that edge. A graph is said to have uniform edge betweenness centrality if the edge betweenness centrality is the same value for all edges. Graphs that have uniform edge betweenness centrality but are not edge-transitive appear to be rare; of the over 11.9 million (connected) graphs on less than or equal to 10 vertices, only 4 have uniform edge betweenness centrality but not edge-transitivity. However we have been able to identify infinite classes of circulant graphs that have uniform edge betweenness centrality but are not edge-transitive. (Received July 29, 2017)

1135-05-120 **Connor Mattes\***, Applied Mathematics and Statistics, Colorado School of Mines, Golden, CO 80401-1887, and **Marika Witt**, Mathematics and Computer Science Department, 300 W. Hawthorne Road, Whitworth University, Spokane, WA 99251.  *$L(h, k)$  labeling of graphs.*

$L(h, k)$  labeling is a generalization of the  $L(2, 1)$  labeling, which was introduced by Griggs and Yeh and motivated by the channel assignment problem. In  $L(h, k)$  labeling, labels of adjacent vertices differ by at least  $h$  and labels of vertices that are at distance two differ by at least  $k$ . The span of an  $L(h, k)$  labeling is the difference between the largest and smallest labels of a graph, while the  $L(h, k)$  span of a graph is the smallest span of all  $L(h, k)$  labelings of a graph. The decision problem of whether the  $L(2, 1)$  span of a general graph is less than or equal to  $t$  is shown to be NP-complete. We determined the  $L(h, k)$  labeling and span of some subgraphs of complete graphs and complete bipartite graphs for all positive integer values of  $h$  and  $k$ , obtained by removing a maximum matching and removing the edges in an arbitrary path. We also determined the  $L(2, 1)$  span of the complete bipartite graph minus the edges of an arbitrary path by giving a lower bound and a construction. (Received July 29, 2017)

1135-05-121 **Caroline Accurso\***, Dept. of Mathematics and Computer Science, DeSales University, Center Valley, PA 18034, and **Leaha Hand**, Department of Mathematics, Boise State University, Boise, ID 83725. *Weak dynamic coloring of graphs.*

The  $k$ -weak dynamic number of a graph  $G$  is the smallest number of colors we can use to color the vertices of  $G$  in such a way that each vertex  $v$  of degree  $d(v)$  sees at least  $\min\{r, d(v)\}$  colors on its neighborhood. The chromatic number of a hypergraph  $H$  is the smallest number of colors we can use to color the vertices of  $H$  in such a way that each edge of size at least 2 sees at least 2 different colors.

2-weak dynamic coloring of graphs is a well-studied subject, as it has a close relation to proper coloring of hypergraphs. Here we study  $k$ -weak dynamic coloring of graphs when  $k \geq 3$ . We use the discharging method to prove that all planar graphs have 3-weak dynamic number at most 6. We also use the Probabilistic methods to determine upper bounds for  $k$ -weak dynamic number of  $d$ -regular graphs. (Received July 29, 2017)

1135-05-130 **Darren Narayan\***, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623. *Graph Theoretical Centrality Properties for Analyzing Functional Connectivity of the Human Brain.*

We will show how two different graph theoretical centrality properties can be used analyze to functional connectivity of the human brain. Edge betweenness centrality measures the ratio of shortest paths between two distinct vertices that contain a particular edge. Edge clustering centrality measures the frequency at which an edge appears across all local subgraphs induced by each vertex and its neighbors. The latter metric is tied to a problem from structural graph theory in which we seek the largest subgraph that is a Cartesian product of two complete bipartite graphs  $K_{1,m}$  and  $K_{1,1}$ . Here the most central edge is the one appearing in the largest set of overlapping triangles. We use these centrality properties to analyze functional MRI data from a study involving

the viewing and pantomiming of tools at the Rochester Center for Brain Imaging at the University of Rochester. (Received August 01, 2017)

1135-05-208 **Zhanar Berikkyzy\*** (zhanar@ucr.edu), **Ryan Martin** and **Chelsea Peck**. *On the Edit Distance of Powers of Cycles*. Preliminary report.

The edit distance between two graphs on the same labeled vertex set is defined to be the size of the symmetric difference of the edge sets. The edit distance function of a hereditary property  $\mathcal{H}$  is a function of  $p \in [0, 1]$  that measures, in the limit, the maximum normalized edit distance between a graph of density  $p$  and  $\mathcal{H}$ .

In this work, we address the edit distance function for  $\text{forb}(H)$ , where  $H = C_h^t$ , the  $t^{\text{th}}$  power of the cycle of length  $h$ . For  $h \geq 2t(t+1) + 1$  and  $h$  not divisible by  $t+1$ , we determine the function for all values of  $p$ . For  $h \geq 2t(t+1) + 1$  and  $h$  divisible by  $t+1$ , the function is obtained for all but small values of  $p$ . We also obtain some results for smaller values of  $h$ . (Received August 10, 2017)

1135-05-222 **Eric Rowland\*** (eric.rowland@hofstra.edu). *Enumeration of binomial coefficients by their  $p$ -adic valuations*.

In 1947 Nathan Fine obtained a product formula for the number of integers  $m$  in the range  $0 \leq m \leq n$  such that  $\binom{n}{m}$  is not divisible by  $p$ . Subsequently, many authors found formulas counting binomial coefficients with  $p$ -adic valuation  $\nu_p(\binom{n}{m}) = \alpha$  for particular values of  $p^\alpha$ , but a general formula remained elusive. We give a matrix product, generalizing Fine's result, for the generating function

$$T_p(n, x) := \sum_{m=0}^n x^{\nu_p(\binom{n}{m})}$$

which simultaneously counts binomial coefficients with  $p$ -adic valuation  $\alpha$  for all  $\alpha \geq 0$ . The polynomial  $T_p(n, x)$  was recently identified by Spiegelhofer and Wallner as a central object in the structure of formulas for the number of binomial coefficients with  $p$ -adic valuation  $\alpha$ . We also give a further generalization to multinomial coefficients. (Received August 13, 2017)

1135-05-228 **Libby Taylor\*** (libbytaylor@gatech.edu). *Characterizations of regularity in matroids*. Preliminary report.

A regular matroid is a matroid which can be represented by a matrix over any field. Regular matroids enjoy many elegant properties that ordinary matroids do not, so it is often of interest to determine when a given matroid is regular. We give three distinct characterizations of regular matroids among orientable ones. The main result is stated in terms of lattice theory: an orientable matroid is regular if and only if the fundamental circuits of each basis generate the same lattice. Several enumerative results follow as corollaries to this lattice-theoretic characterization. The main such corollary is that a matroid is regular if and only if the number of integrally independent cocircuits is exactly equal to the rank of the matroid. In addition, we prove that a matroid is regular if and only if all of its circuits and cocircuits are orthogonal with respect to the standard inner product on  $\mathbb{R}^n$ . Each of these characterizations gives a parameter measuring “how far” the matroid is from being regular, so they seem to provide richer combinatorial information than previous characterizations. (Received August 14, 2017)

1135-05-253 **Timothy Sun\*** (tim@cs.columbia.edu). *Face distributions of embeddings of complete graphs*. Preliminary report.

A longstanding open question of Archdeacon and Craft asks whether or not every complete graph has a minimum genus embedding with at most one nontriangular face. We exhibit such an embedding for each complete graph except  $K_8$ , the complete graph on 8 vertices, and we go on to prove that no such embedding can exist for this graph. Our approach also solves a more general problem, giving a complete characterization of the possible face distributions (i.e. the number of faces of each length) realizable by minimum genus embeddings of each complete graph. We also tackle analogous questions for nonorientable and maximum genus embeddings. (Received August 16, 2017)

1135-05-258 **Sam Spiro\*** (sspiro@ucsd.edu). *Polynomial Relations of Matrices of Graphs*.

Let  $A_G$  and  $L_G$  denote the adjacency matrix and Laplacian matrix of the graph  $G$ . If  $G$  is  $d$ -regular, then  $A_G = dI - L_G$ , and it is easy to use this relationship to translate from eigenvalues of  $A_G$  to eigenvalues of  $L_G$ , and vice versa. If  $G$  is  $(d_1, d_2)$ -regular, then  $A_G^2 = (L_G - d_1I)(L_G - d_2I)$ . Remarkably, it also turns out that this relationship allows one to translate between the eigenvalues of  $A_G$  and  $L_G$ .

Given these results, it is natural to ask which graphs  $G$  have the property that there exists a polynomial  $f$  and positive integer  $r$  such that  $A_G^r = f(L_G)$ . In this talk we shall prove that the only graphs with this property are the regular and biregular graphs. We shall also briefly discuss more general relations of the form

$f(A_G) = g(L_G)$  for polynomials  $f$  and  $g$ , and relations involving the signless and normalized Laplacian of a graph. (Received September 25, 2017)

1135-05-272 **John Asplund** and **Thao Do\***, thaodo@mit.edu, and **Arran Hamm**, **László Székely**, **Libby Taylor** and **Zhiyu Wang**. *Biplanar Crossing Numbers: The Probabilistic Method*.

Given a graph  $G$ , its crossing number  $cr(G)$  is the minimum number of pairs of crossing edges in a drawing of  $G$  in the plane. The biplanar crossing number of  $G$ , denoted by  $cr_2(G)$ , is the minimum of  $cr(G_1) + cr(G_2)$  among all edge partition  $G = G_1 \cup G_2$ . Probabilistic method has been used by Czabarka, Sýkora, Székely, Vrto to prove  $cr_2(G) \leq \frac{3}{8} cr(G)$  for any graph  $G$  and by Spencer to prove with high probability the biplanar crossing number of the Erdős-Renyi random graph  $G(n, p)$  is asymptotically largest possible. In this talk we shall present those results and explain explicitly how Spencer's method implies similar results for the  $k$ -planar crossing number (the minimum sum of crossing numbers when drawing  $G$  in  $k$  planes) of  $G(n, p)$ . We also extend the result to  $d$ -regular random graph  $G(n, d)$ . (Received August 17, 2017)

1135-05-275 **Robert Davis\*** (davisr@math.msu.edu), Department of Mathematics, Michigan State University, 619 Red Cedar Rd., East Lansing, MI 48824, and **Bruce Sagan**. *A Polyhedral Proof of a Wreath Product Identity*. Preliminary report.

In 2013, Beck and Braun proved and generalized multiple identities involving permutation statistics via discrete geometry. Namely, they recognized the identities as specializations of integer point transforms for certain polyhedral cones. They extended many of their proof techniques to obtain identities involving wreath products, but some identities were resistant to their proof attempts. In this talk, we provide an entirely geometric justification of the identity

$$\sum_{k \geq 0} ([k+1]_q + u[r-1]_u [k]_q)^n t^k = \frac{\sum_{(\epsilon, \pi) \in \mathbb{Z}_{r,t} \otimes_n} q^{\text{maj}(\epsilon, \pi)} t^{\text{des}(\epsilon, \pi)} u^{\text{col}(\epsilon, \pi)}}{\prod_{j=0}^n (1 - tq^j)}$$

for fixed positive integers  $r$  and  $n$ , first established by Biagioli and Zeng. (Received August 17, 2017)

1135-05-290 **Marcus Schaefer\*** (mschaefer@cdm.depaul.edu). *Taking a Detour: Gioan's Theorem, and Pseudolinear Drawings of Complete Graphs*.

We give new and short proofs of two known results including Gioan's theorem, stating that any two good drawings of a complete graph with the same rotation system are isomorphic up to Reidemeister moves of type 3, and a characterization of pseudolinear drawings of the complete graph via an excluded configuration: a bad  $K_4$ . (Received August 19, 2017)

1135-05-310 **Pamela E. Harris\*** (peh2@williams.edu), 33 Stetson Ct, Bascom House Room 106C, Williamstown, MA 01267. *Kostant's Partition Function*.

In this talk we introduce Kostant's partition function, which counts the number of ways to represent a particular weight (vector) as a nonnegative integral sum of positive roots of a Lie algebra (a finite set of vectors). This partition function is involved in the computation of weight multiplicities in the representation theory of finite-dimensional classical Lie algebras. However, the properties of this function can be studied independent of this setting via a combinatorial perspective. We will provide some recent results from undergraduate students at Williams College stemming from such a view and end with some open problems for further investigation. (Received August 23, 2017)

1135-05-313 **Joshua P Swanson\*** (jps314@uw.edu), **Connor Ahlback** (ahlback@math.washington.edu) and **Brendon Rhoades** (bprhoades@ucsd.edu). *Refined Cyclic Sieving on Words and Tableaux*.

The *cyclic sieving phenomenon* (CSP) introduced by Reiner–Stanton–White encodes the orbit structure of cyclic actions on finite sets in univariate polynomials. Such polynomials are typically combinatorial generating functions—e.g.  $q$ -binomial coefficients. We will introduce the recent, natural notion of *refinement* of a CSP. Initial examples arose from the *cyclic descent type* of a word. More recently, further examples have been uncovered for many skew tableaux, resulting in what we have termed “Euler–Mahonian” refined cyclic sieving. We will explain these results as time allows.

Based on joint work with Connor Ahlback and Brendon Rhoades. (Received August 22, 2017)

1135-05-372 **Mark Ellingham\*** (mark.ellingham@vanderbilt.edu), **D. Christopher Stephens** and **Xiaoya Zha**. *The orientable genus of complete tripartite graphs*. Preliminary report.

In 1969 White conjectured that the orientable genus of the complete tripartite graph  $K_{\ell, m, n}$ , with  $\ell \geq m \geq n \geq 2$ , is  $g(K_{\ell, m, n}) = \lceil (\ell - 2)(m + n - 2)/2 \rceil$ . In 1976, Stahl and White similarly conjectured that the nonorientable

genus is  $\tilde{g}(K_{\ell,m,n}) = \lceil (\ell - 2)(m + n - 2)/4 \rceil$ . The authors verified the nonorientable conjecture (which is true with three small exceptions) in 2006. This involved examining four cases based on the parity of  $m$  and  $n$ . We have recently completed the proof of the orientable conjecture. This involved examining sixteen cases, depending on the values of  $m$  and  $n$  modulo 4. Some of the cases required significantly more work than any of the four nonorientable cases. We give an overall outline of the proof and discuss some of the techniques used. (Received August 29, 2017)

1135-05-381 **Ira M. Gessel and Yan Zhuang\*** (zhuangy@brandeis.edu). *Shuffle-compatible permutation statistics.*

We call a permutation statistic *st shuffle-compatible* if for any two disjoint permutations  $\pi$  and  $\sigma$ , the distribution of *st* over the shuffles of  $\pi$  and  $\sigma$  depends only on  $\text{st}(\pi)$ ,  $\text{st}(\sigma)$ , and the lengths of  $\pi$  and  $\sigma$ . We associate to every shuffle-compatible permutation statistic *st* an algebra, called the *shuffle algebra* of *st*, whose multiplication describes the distribution of *st* over shuffles of permutations. In this talk, we present a shuffle-compatibility criterion which shows that the shuffle algebra of every shuffle-compatible descent statistic (i.e., a statistic which is a coarsening of the descent set) is a quotient of the algebra  $\text{QSym}$  of quasisymmetric functions, as well as a dual criterion which exploits the duality between  $\text{QSym}$  and the coalgebra structure of noncommutative symmetric functions. These results are used to give explicit descriptions of the shuffle algebras of many shuffle-compatible descent statistics. (Received August 29, 2017)

1135-05-395 **Axel Brandt\*** (axbrandt@davidson.edu), **Michael Ferrara, Nathan Graber, Stephen Hartke and Sarah Loeb.** *Entire Colorability for a Class of Plane Graphs.*

A plane graph  $G$  is *entirely  $k$ -colorable* if every element in the set of vertices, edges, and faces of  $G$  can be colored from  $\{1, 2, \dots, k\}$  so that every two adjacent or incident elements have distinct colors. In 2011, Wang and Zhu asked if every simple plane graph  $G$ , other than  $K_4$ , is entirely  $(\Delta(G) + 3)$ -colorable. In 2012, Wang, Mao, and Miao answered in the affirmative for simple plane graphs with  $\Delta(G) \geq 8$ . We show that every loopless plane multigraph with  $\Delta(G) = 7$ , no 2-faces, and no two 3-faces sharing an edge is entirely 10-colorable. (Received August 31, 2017)

1135-05-421 **Yichao Chen and Jonathan L Gross\*** (gross@cs.columbia.edu). *Genus polynomials of ring-like graphs with spiders.*

An  $H$ -linear sequence of graphs is obtained by binding increasingly many copies of a fixed graph  $H$  into a chain. An  $H$ -ring-like sequence is formed by binding the two end-copies of  $H$  in each such chain, so as to form cycles of copies of  $H$ . Within this context, we introduce subgraphs called spiders. Beyond their use in extending a linear graph family to other families whose genus polynomials are amenable to calculation with feasible effort, the spiders allow for a multiplicity of ways to bind the two end-copies of chain. Our main objective herein is to describe a unified method for constructing a homogeneous linear recursion for the genus polynomials of ring-like sequences or linear sequences, with or without spiders. The prevalent approach to genus polynomials heretofore has been to solve a system of simultaneous recursions of partial genus polynomials. Additionally, we prove here that the modes of the genus distribution sequences of several types of ladder-graphs are either the upper-rounding or the lower-rounding of their average genera. (Received September 01, 2017)

1135-05-429 **Georgia Benkart, Laura Colmenarej, Pamela E Harris, Rosa Orellana, Greta Panova and Anne Schilling\*** (anne@math.ucdavis.edu), Department of Mathematics, University of California, One Shield Avenue, Davis, CA 95616, and **Martha Yip.** *A minimaj-preserving crystal on ordered multiset partitions.*

We provide a crystal structure on the set of ordered multiset partitions, which recently arose in the pursuit of the Delta Conjecture. This conjecture was stated by Haglund, Remmel and Wilson as a generalization of the Shuffle Conjecture. Various statistics on ordered multiset partitions arise in the combinatorial analysis of the Delta Conjecture, one of them being the minimaj statistic, which is a variant of the major index statistic on words. Our crystal has the property that the minimaj statistic is constant on connected components of the crystal. In particular, this yields another proof of the Schur positivity of the graded Frobenius series of the generalization  $R_{n,k}$  due to Haglund, Rhoades and Shimozono of the coinvariant algebra  $R_n$ . The crystal structure also enables us to demonstrate the equidistributivity of the minimaj statistic with the major index statistic on ordered multiset partitions. (Received September 01, 2017)



1135-05-441

**K. Kappagantula, M. Cutimanco, P. Labelle and V. Shramchenko\***(vasilisa.shramchenko@usherbrooke.ca), 2500 boul. de l'Université, Sherbrooke, Quebec J1K2R1, Canada. *Topological recursion of Eynard-Orantin, ribbon graphs, and Feynman diagrams.*

The Chekhov-Eynard-Orantin topological recursion applied to the algebraic curve corresponding to the quantum harmonic oscillator produces a hierarchy of multi-differentials defined on that curve. By rearranging these differentials in an appropriate way, one can reproduce the WKB series for the wave function of the quantum harmonic oscillator. Using the same multi-differentials, one generates naturally the numbers of  $N$ -rooted graphs, which in their turn count Feynman diagrams in a certain quantum field theory. (Received September 03, 2017)

1135-05-446

**David Jordan and Monica Vazirani\***, Department of Mathematics, One Shields Ave, Davis, CA 95616. *An elliptic Schur-Weyl construction of the rectangular representation of the DAHA.*

Building on the work of Calaque-Enriquez-Etingof, Lyubashenko-Majid, and Arakawa-Suzuki, Jordan constructed a functor from quantum  $D$ -modules on general linear groups to representations of the double affine Hecke algebra (DAHA) in type  $A$ . When we input quantum functions on  $GL(N)$  the output is  $L(k^N)$ , the irreducible DAHA representation indexed by an  $N \times k$  rectangle. For the specified parameters  $L(k^N)$  is  $Y$ -semisimple, i.e. one can diagonalize the Dunkl operators. We give an explicit combinatorial description of this module via its  $Y$ -weight basis.

This is joint work with David Jordan. (Received September 04, 2017)

1135-05-473

**Bret J. Benesh, Dana C. Ernst and Nandor Sieben\*** (nandor.sieben@nau.edu).*Impartial Avoidance and Achievement Games For Generating Finite Groups.*

We study two impartial games introduced by Anderson and Harary and further developed by Barnes. Both games are played by two players who alternately select previously unselected elements of a given finite group. The first player who builds a generating set from the jointly selected elements wins the achievement game. The first player who cannot select an element without building a generating set loses the avoidance game. We determine the nim-numbers of these games for some finite group families. A key tool is a computer program that can determine these numbers for fairly large groups. The algorithm uses the lattice of intersection subgroups, which are intersections of maximal subgroups. (Received September 05, 2017)

1135-05-478

**Shira Zerbib\*** (zerbib@umich.edu), MSRI, Berkeley, CA 94720. *Improving the known bound in Vizing's conjecture.*

A well-known conjecture of Vizing [?] is that  $\gamma(G \square H) \geq \gamma(G)\gamma(H)$  for any pair of graphs  $G, H$ , where  $\gamma$  is the domination number and  $G \square H$  is the Cartesian product of  $G$  and  $H$ . Suen and Tarr, improving a result of Clark and Suen, showed  $\gamma(G \square H) \geq \frac{1}{2}\gamma(G)\gamma(H) + \frac{1}{2}\min(\gamma(G), \gamma(H))$ . We further improve their result by showing  $\gamma(G \square H) \geq \frac{1}{2}\gamma(G)\gamma(H) + \frac{1}{2}\max(\gamma(G), \gamma(H))$ . (Received September 05, 2017)

1135-05-488

**Jin Ho Kwak\*** (jinkwak@postech.ac.kr), **Jean-Bing Liu** and **Ja Eun Lee**. *Enumeration of Regular Graph Coverings; Some recent results.*

One of the basic enumeration problems for a graph  $G$  is

- (1) enumerate all  $n$ -fold regular coverings of  $G$ , or
- (2) enumerate all connected  $n$ -fold regular coverings of  $G$ .

In this talk, we enumerate the isomorphism classes of connected graph coverings when the covering transformation group is a generalized quaternion or a semi-dihedral group.

*This is a joint work with Jian-Bing Liu and Ja Eun Lee.* (Received September 06, 2017)

1135-05-493

**Ada Nicole Morse\*** (ada.morse@uvm.edu), Department of Mathematics and Statistics, 16 Colchester Ave., Burlington, VT 05401. *DNA Origami and Knots in Graphs.*

We discuss a new connection between knot and graph theory arising from the origami method of nanoscale DNA self-assembly design, in which a long unknotted scaffolding strand of DNA is folded into the shape of an embedded graph by smaller staple strands. For graphs embedded on surfaces, optimal routes for the scaffolding strand correspond to unknotted  $A$ -trails, i.e. Eulerian circuits with certain turning restrictions. We show that every Eulerian graph has an embedding containing an unknotted  $A$ -trail and that there exist embedded Eulerian graphs with no unknotted  $A$ -trails. On the torus, we characterize when checkerboard-colorable embeddings have unknotted  $A$ -trails and apply this characterization to certain regular triangulations. In closing, we discuss new directions arising from the case of graphs in space but not on surfaces. (Received September 06, 2017)

1135-05-513 **Mark E. Watkins\*** ([mewatkin@syr.edu](mailto:mewatkin@syr.edu)), Mathematics Department, 215 Carnegie, Syracuse University, Syracuse, NY 13244-1150. *A Survey of Infinite Graphs of Connectivity I.*

Let  $\Gamma$  be an infinite, locally finite, connected graph having a cut-vertex. We present necessary and sufficient conditions for the action of the automorphism group  $\text{Aut}(\Gamma)$  to be, respectively, vertex-transitive, edge-transitive, primitive, regular, the group of a Cayley graph, the group of a planar primitive graph, or a Frobenius group. [Some of the older results in this list had been obtained jointly with H.A. Jung and J.E. Graver.] We prove that a locally finite graph having a cut-vertex has a Frobenius automorphism group if and only if every vertex is incident with exactly two isomorphic lobes that are themselves a graphical regular representation (GRR). (Received September 06, 2017)

1135-05-537 **Anastasia M Chavez\***, [a.chavez@berkeley.edu](mailto:a.chavez@berkeley.edu), and **Nicole Yamzon**. *The Dehn-Sommerville relations and the Catalan matroid.*

The  $f$ -vector of a  $d$ -dimensional polytope  $P$  stores the number of faces of each dimension. When  $P$  is simplicial the Dehn-Sommerville relations condense the  $f$ -vector into the  $g$ -vector, which has length  $\lfloor (d+1)/2 \rfloor$ . Thus, to determine the  $f$ -vector of  $P$ , we only need to know approximately half of its entries. This raises the question: Which  $\lfloor (d+1)/2 \rfloor$  subsets of the  $f$ -vector of a general simplicial polytope are sufficient to determine the whole  $f$ -vector? We prove that the answer is given by the Catalan matroid. This is joint work with Nicole Yamzon. (Received September 07, 2017)

1135-05-551 **Yichao Chen\*** ([ycchen@hnu.edu.cn](mailto:ycchen@hnu.edu.cn)), Department of Mathematics, Hunan University, Yuelv Shan, ChangSha, Hunan 410082, Peoples Rep of China, and **Jonathan L. Gross** ([gross@cs.columbia.edu](mailto:gross@cs.columbia.edu)), Department of Computer Science, Columbia university, New York city, NY 10027. *An Euler-genus approach to the calculation of the crosscap-number polynomial.*

In 1994, J Chen, J Gross, and R. Rieper demonstrated how to use the rank of Mohar's overlap matrix to calculate the crosscap-number distribution, that is, the distribution of the embeddings of a graph in the non-orientable surfaces. That has ever since been by far the most frequent way that these distributions have been calculated. This paper introduces a way to calculate the Euler-genus polynomial of a graph, which combines the orientable and the non-orientable embeddings, without using the overlap matrix. The crosscap-number polynomial for the non-orientable embeddings is then easily calculated from the Euler-genus polynomial and the genus polynomial. (Received September 07, 2017)

1135-05-565 **Amanda Lohss\*** ([aglohss@gmail.com](mailto:aglohss@gmail.com)). *Tableaux and the ASEP.*

Several variations of Young tableaux have recently been introduced due to connections with the asymmetric simple exclusion process (ASEP), an important particle model which has been studied extensively in physics, biology, and other sciences. In particular, tableaux provide a combinatorial formula for the steady state distribution of the ASEP. This talk will introduce the ASEP and its connection with tableaux. In addition, several results regarding tableaux and the ASEP will be discussed that confirm previous conjectures. (Received September 08, 2017)

1135-05-576 **Tair Akhmejanov\***, Cornell University, Ithaca, NY 14850. *Growth Diagrams from Polygons in the Affine Grassmannian.*

We introduce a new type of growth diagram, arising from the geometry of the affine Grassmannian for  $GL_m$ . These affine growth diagrams are in bijection with the  $c_{\vec{\lambda}}$  many components of the polygon space  $\text{Poly}(\vec{\lambda})$  for  $\vec{\lambda}$  a sequence of minuscule weights and  $c_{\vec{\lambda}}$  the Littlewood–Richardson coefficient. Unlike Fomin growth diagrams, they are infinite periodic on a staircase shape, and each vertex is labeled by a dominant weight of  $GL_m$ . Letting  $m$  go to infinity, a dominant weight can be viewed as a pair of partitions, and we recover the RSK correspondence and Fomin growth diagrams within affine growth diagrams. The main combinatorial tool used in the proofs is the  $n$ -hive of Knutson-Tao-Woodward. (Received September 08, 2017)

1135-05-579 **Thomas Hameister**, **Sujit Rao**, **Victor Reiner\*** ([reiner@umn.edu](mailto:reiner@umn.edu)) and **Connor Simpson**. *Hilbert functions for Chow rings of uniform matroids and their  $q$ -analogues.*

In their celebrated recent proof of the Rota-Heron-Welsh and Mason conjectures, Adiprasito, Huh, and Katz showed that the Chow ring of a matroid satisfies Poincare duality, the hard Lefschetz theorem, and the Hodge-Riemann inequalities. In particular, the Hilbert function of such a Chow ring always forms a symmetric, unimodal sequence.

We report here on REU work of three of the authors interpreting these Hilbert functions combinatorially for all uniform matroids and for their finite field vector space  $q$ -analogues. In particular, for the full vector space

matroid, one recovers a q-Eulerian polynomial studied by Shreshian and Wachs. (Received September 09, 2017)

1135-05-591 **Jeske Glenn, Christopher O'Neill\*** (coneill@math.ucdavis.edu), **Vadim Ponomarenko** and **Benjamin Sepanski**. *Augmented Hilbert series of numerical semigroups.*

A numerical semigroup  $S$  is a subset of the natural numbers that is closed under addition, and a factorization of  $n \in S$  is an expression of  $n$  as a sum of generators of  $S$ . The Hilbert series of  $S$  is the rational generating function  $\sum_{n \in S} t^n$ , and there are several characterizations of the numerator in terms of key properties of  $S$ . In this talk, we characterize the numerator of several “augmented” Hilbert series, where the coefficient of each  $t^n$  is some arithmetic quantity derived from the factorizations of  $n$ , such as the maximum factorization length of  $n$  or number of distinct factorization lengths of  $n$ . The results presented here are from an undergraduate research project from the 2017 SDSU REU. (Received September 09, 2017)

1135-05-592 **Jeske Glenn, Christopher O'Neill\*** (coneill@math.ucdavis.edu), **Vadim Ponomarenko** and **Benjamin Sepanski**. *Numerical semigroup invariants and eventually quasipolynomial behavior.*

A numerical semigroup  $S$  is a subset of the natural numbers that is closed under addition, and a factorization of  $n \in S$  is an expression of  $n$  as a sum of generators of  $S$ . In this talk, we examine several factorization invariants, which are arithmetic quantities assigned to each semigroup element  $n$ , such as the maximum factorization length of  $n$  or number of distinct factorization lengths of  $n$ . A surprisingly large collection of factorization invariants coincide with a quasipolynomial (that is, a polynomial with periodic coefficients) for sufficiently large semigroup elements; we explore structural reasons for this phenomenon, as well as its implications on computation. Several of the results presented here are from an undergraduate research project from the 2017 SDSU REU. (Received September 09, 2017)

1135-05-597 **Lara Pudwell\*** (lara.pudwell@valpo.edu), Department of Mathematics and Statistics, 1900 Chapel Drive, Valparaiso, IN 46383. *Patterns in trees.*

One interesting enumeration problem in combinatorics asks how many trees  $T$  contain a prescribed smaller tree  $t$ . During the summers of 2010, 2011, and 2012, my teams of REU students studied this question for specific families of trees using two different definitions of what it means for one tree to contain another tree. In this talk, I'll discuss what we know about this problem and describe other variations that are open for exploration. (Received September 09, 2017)

1135-05-600 **Stanisław Radziszowski\*** (spr@cs.rit.edu), Department of Computer Science, Rochester Institute of Technology, Rochester, NY 14623. *Computations in Ramsey Theory.*

Ramsey theory is often regarded as the study of how order emerges from randomness. While originated in mathematical logic, it has applications in geometry, number theory, game theory, information theory, approximation algorithms, and other areas of mathematics and theoretical computer science.

Ramsey theory studies the conditions of when a combinatorial object necessarily contains some smaller given objects. The central concept in Ramsey theory is that of arrowing, which in the case of graphs describes when colorings of larger graphs necessarily contain monochromatic copies of given smaller graphs. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey theory, always involving arrowing. The determination of whether this arrowing holds is notoriously difficult, and thus it leads to numerous computational challenges concerning various types of Ramsey numbers.

This talk will overview how computers are increasingly used to study the bounds on Ramsey, and properties of Ramsey arrowing in general. This is happening in the area where traditional approaches typically call for classical computer-free proofs. It is evident that now we understand Ramsey theory much better than a few decades ago, increasingly due to computations. (Received September 09, 2017)

1135-05-603 **John R Greene\*** (jgreene@d.umn.edu), Department of Mathematics and Statistics, University of Minnesota Duluth, 1117 University Drive, Duluth, MN 55812. *Combinatorial properties of traces of matrix products.*

It might come as a surprise that there are no  $2 \times 2$  real matrices  $A$  and  $B$  for which  $\text{Tr}(A^2 B^4) < \text{Tr}(AB^2 AB^2) < \text{Tr}(ABAB^3)$ . More generally, suppose  $x_1 = \text{Tr}(A^2 B^{2n-2})$ ,  $x_2 = \text{Tr}(ABAB^{2n-3})$ ,  $\dots$ ,  $x_n = \text{Tr}(AB^{n-1} AB^{n-1})$ . If  $\sigma$  is a permutation of  $n$  we may ask if there are matrices  $A$  and  $B$  for which  $x_{\sigma(1)} < x_{\sigma(2)} < \dots < x_{\sigma(n)}$ . It turns out that for most permutations (when  $n$  is large), the answer is no. Call a permutation **trace order consistent** if  $2 \times 2$  matrices  $A$  and  $B$  exist for which  $x_{\sigma(1)} < x_{\sigma(2)} < \dots < x_{\sigma(n)}$ . Using elementary properties

of the zeros of Chebyshev polynomials of the second kind, an exact count is given for the number of trace order consistent permutations. (Received September 10, 2017)

1135-05-615 **Luisa Carini\*** (lcarini@unime.it). *On the Multiplicity-Free Plethysms  $p_2[s_\lambda]$ .*

Given two Schur functions  $s_\mu(x)$  and  $s_\lambda(x)$ , where  $x = (x_1, x_2, \dots)$  is an infinite sequence of variables,  $\mu$  and  $\lambda$  are partitions of weight  $m$  and  $n$ , respectively, the plethysm  $s_\mu[s_\lambda(x)]$  is the symmetric function obtained by substituting the monomials of  $s_\lambda(x)$  for the variables of  $s_\mu(x)$ . Littlewood introduced this operation in the context of the representations of the general linear group and showed that for any partition  $\mu$  of  $m$ ,

$$s_\mu[s_\lambda(x)] = \sum_{\gamma \vdash mn} c_{\mu, \lambda}^\gamma s_\gamma(x)$$

where the sum runs over all partitions  $\gamma$  of  $mn$  and  $c_{\mu, \lambda}^\gamma$  are non negative integers.

The problem of computing the coefficients  $c_{\mu, \lambda}^\gamma$  is one of the fundamental open problems in the theory of symmetric functions and has proved to be very difficult. Essentially there are explicit formulas for  $c_{\mu, \lambda}^\gamma$  in a few special cases.

Here we will describe all the shapes  $\lambda$  such that the plethysms  $p_2[s_\lambda](x)$  of the power symmetric function  $p_2(x)$  and the Schur function  $s_\lambda(x)$  are multiplicity-free. (Received September 11, 2017)

1135-05-617 **Aaron Robertson\*** (arobertson@colgate.edu). *On the Distribution of Ramsey Objects.*

Given the intractable calculations for Ramsey-type numbers, we approach the problem from a statistical viewpoint. In addition to finding astoundingly good approximations for the distributions enumerating monochromatic objects, by presenting strong evidence, we hope to convince the viewer that statistical Ramsey theory could prove to be a fruitful endeavor. This is joint work with Will Cipolli and Maria Dascalu (undergraduate). (Received September 11, 2017)

1135-05-636 **Jackson Autry, Christopher O'Neill, Vadim Ponomarenko\***

(vponomarenko@mail.sdsu.edu) and **Samuel Yih**. *Graph Realization for Atomic Simplexes.*

A numerical semigroup is a cofinite subset of  $\mathbb{N}_0$ , closed under addition, including 0. These are well-studied, and we propose a new construction lending insight into their structure. Among their properties is a finite set of atoms, out of which each semigroup element may be built. Given numerical semigroup  $S$  and element  $n \in S$ , we build a simplicial complex on the set of atoms, based on which subsets of semigroup atoms may have their sum subtracted from  $n$  with the result in  $S$ . This work, done in an REU program primarily by undergraduates, considers the question of which ordinary graphs can appear as simplicial complexes in the manner described above. (Received September 13, 2017)

1135-05-657 **Akiyoshi Tsuchiya\*** (a-tsuchiya@ist.osaka-u.ac.jp). *Normal Gorenstein Fano polytopes arising from perfect graphs.*

Gorenstein Fano polytopes form one of the distinguished classes of lattice polytopes. Especially normal Gorenstein Fano polytopes are of interest. In this talk, we will present a new class of normal Gorenstein Fano polytopes arising from perfect graphs. (Received September 12, 2017)

1135-05-658 **Thomas Tucker\*** (ttucker@colgate.edu), **Jonathan Gross** (gross@cs.columbia.edu) and **Toufik Mansour** (toufik@math.haifa.ac.il). *Imbedding statistics for linear families via Markov chains.* Preliminary report.

The *genus polynomial* for a finite graph  $G$  is the generating function  $g_G(z) = \sum a_i z^i$ , where  $a_i$  is the number of imbeddings of  $G$  in the surface of genus  $i$ . A *linear family*  $G_n$  of graphs is formed by taking  $n$  copies of the same graph  $G$  and forming a path of them by adding edges in the same way between one copy of  $G$  and the next. For any such linear family there is a *production* or *transfer* matrix  $M(z)$  and initial vector  $v(z)$  (all entries are polynomials in  $z$  with non-negative integer coefficients) such that the genus polynomials for the imbedding types of  $G_n$  are given by  $M^n(z)v(z)$ . The columns of  $M(1)$  have constant column sum  $s$  so  $(1/s)M(1)$  is a matrix for a Markov chain whose states are the imbedding types of the linear family. We show how to use the Jordan normal form for  $M(1)$  to find the average genus of each imbedding type for each member of a linear family. (Received September 12, 2017)

1135-05-665 **Chris Godsil, Krystal Guo, Mark Kempton\*** (mkempton@cmsa.fas.harvard.edu) and **Gabor Lippner**. *Perfect state transfer in perturbations of strongly regular graphs.* Preliminary report.

Algebraic graph theory has found great success in applications to quantum computing, particularly to the problem of perfect transfer of a quantum state through a network of interacting qubits. Perfect state transfer

has been studied in numerous classes of graphs, and has been examined in graphs with weights on edges or vertices. We will examine perfect state transfer on strongly regular graphs that have been perturbed by a weight on an edge and a pair of vertices. We will give infinite families of new examples where perfect state transfer can occur. (Received September 12, 2017)

1135-05-685 **Lowell Abrams\*** (labrams@gwu.edu), 11505 Gilsan Street, Silver Spring, MD , and **Daniel Slilaty**. *Symmetric Spherical Grids*.

If an embedding of a graph  $G$  in the sphere is a quadrangulation of the sphere, then  $G$  is necessarily bipartite. Assuming that  $G$  has minimum vertex degree 3 and that all vertices in one block of  $V(G)$  have degree 4, we refer to  $G$  as a spherical grid. We discuss general structural properties in spherical grids, then use these to completely characterize rotationally symmetric spherical grids having two vertices of degree  $n$ ,  $2n$  vertices of degree 3, and all other vertices of degree 4. Furthermore, we show how to represent all possible examples as nets. (Received September 12, 2017)

1135-05-688 **Enes Ozel\*** (eozel@usc.edu), 1026 W 22nd Street Apt 16, Los Angeles, CA 90007. *Permutations with Restricted Positions and Asymptotic Distributions of Numbers of Cycles of Fixed Sizes*.

In this study we consider different restrictions defined on the symmetric group,  $S_n$ , and on a specific class of one-sided restrictions prove that for any fixed  $k$ , the number of  $k$ -cycles of a uniformly chosen permutation adhering to this restriction has a Normal asymptotic distribution. (Received September 13, 2017)

1135-05-703 **William J. Keith\*** (wjkeith@mtu.edu), 316 Fisher Hall, 1400 Townsend Drive, Houghton, MI 49931. *Refinements of the  $q$ -analogue of the Frame-Robinson-Thrall formula by descents*.

The  $q$ -analogue of the Frame-Robinson-Thrall formula gives  $\sum q^{maj(T)}$  where  $T$  ranges over all standard Young tableaux of a given shape  $\lambda$ . In pursuit of a conjecture of Sagan and collaborators on 321-avoiding permutations, a refinement of this formula was proved for tableaux of shapes  $\lambda = (n, k)$  with a specified number of descents. In this talk we pursue this refinement further for its own interest, producing formulas for three-part partitions. We discuss the goal of a refinement for general partition shapes. (Received September 13, 2017)

1135-05-712 **Shalosh B. Ekhad** and **Doron Zeilberger\***, DoronZeil@gmail.com. *Using Symbol-Crunching to find ALL Sucker's Bets (with given deck sizes)*.

In 1970, Statistics giant, Bradley Efron, amazed the world by coming up with a set of four dice, let's call them A,B,C,D, whose faces are marked with [0,0,4,4,4,4], [3,3,3,3,3,3],[2,2,2,2,6,6],[1,1,1,5,5,5] respectively, where die A beats die B, die B beats die C, die C beats die D, but, surprise surprise, die D beats die A! This was an amazing demonstration that "being more likely to win" is not a transitive relation. But that was only one example, and of course, instead of dice, we can use decks of cards, where they are called (by Martin Gardner, who popularized this way back in 1970) , "sucker's bets".

Can you find all such examples, with a specified number of decks, and sizes? If you have a computer algebra system (in our case Maple), you sure can! (Received September 13, 2017)

1135-05-714 **Megan Bernstein, Matthew Fahrback\*** (matthew.fahrback@gatech.edu) and **Dana Randall**. *Analyzing Boltzmann samplers for Bose-Einstein condensates with Dirichlet generating functions*.

Boltzmann sampling is commonly used to uniformly sample objects of a particular size from large combinatorial sets. For this technique to be effective, one needs to prove that (1) the sampling procedure is efficient and (2) objects of the desired size are generated with sufficiently high probability. We use this approach to give a provably efficient sampling algorithm for a class of weighted integer partitions related to Bose-Einstein condensation from statistical physics. Our sampling algorithm is a probabilistic interpretation of the ordinary generating function for these objects, derived from the symbolic method of analytic combinatorics. Using the Khintchine-Meinardus probabilistic method to bound the rejection rate of our Boltzmann sampler through singularity analysis of Dirichlet generating functions, we offer an alternative way to analyze Boltzmann samplers for objects with multiplicative structure. (Received September 13, 2017)

1135-05-739 **Bennet Goeckner, Corbin Groothuis, Cyrus Hettle, Brian Kell, Pamela Kirkpatrick, Rachel Kirsch\*** (rkirsch@huskers.unl.edu) and **Ryan Solava**. *Universal partial words*.

Chen, Kitaev, Mütze, and Sun recently introduced the notion of universal partial words, a generalization of universal words and de Bruijn sequences. Universal partial words allow for a wild-card character  $\diamond$ , which is a

placeholder for any letter in the alphabet. We extend results from the original paper and develop additional proof techniques to study these objects. For non-binary alphabets, we show that universal partial words have periodic  $\diamond$  structure and are cyclic, and we give number-theoretic conditions on the existence of universal partial words. In addition, we provide an explicit construction for an infinite family of universal partial words over non-binary alphabets. (Received September 13, 2017)

1135-05-762 **Alexander Mednykh\*** ([smedn@mail.ru](mailto:smedn@mail.ru)), Sobolev Institute of Mathematics, Novosibirsk, Russia. *Jacobians of circulant graphs*. Preliminary report.

The notion of the Jacobian group of a graph, which is also known as the Picard group, the critical group, and the dollar or sandpile group, was independently introduced by many authors R. Bacher, P. de la Harpe and T. Nagnibeda (1997), N. L. Biggs (1999), D. Lorenzini (2008), B. Baker and S. Norine (2009) and others. We define Jacobian of a graph as the maximal Abelian group generated by the flows obeying two Kirchhoff's laws. This notion arises as a discrete version of the Jacobian in the classical theory of Riemann surfaces. It also admits a natural interpretation in various areas of physics, coding theory, and financial mathematics. The Jacobian group is an important algebraic invariant of a finite graph. In particular, its order coincides with the number of spanning trees of the graph.

The purpose of this report is to determine the structure of the Jacobian for circulant graphs, the generalized Petersen graph,  $I$ -,  $Y$ -,  $H$ - graphs and their generalizations. We also present new formulas for the number of spanning trees and investigate arithmetical properties of these numbers. In many important cases, we describe the Jacobian group explicitly. In the general case, we propose an effective algorithm for its calculation. (Received September 14, 2017)

1135-05-764 **Roman Nedela\*** ([nedela@ntis.zcu.cz](mailto:nedela@ntis.zcu.cz)), Univerzity 8, Pilsen, Czech Rep. *Complete regular dessins and skew-morphisms of cyclic groups*.

A dessin is a 2-cell embedding of a connected bipartite graph into a closed orientable surface with a fixed 2-colouring determining the bi-partition. The concept of dessins was introduced by Grothendieck as a combinatorial counterpart of algebraic curves. An automorphism of a dessin is a colour- and orientation-preserving map automorphism. A dessin is regular if its automorphism group is regular on the edges. If the underlying graph of a regular dessin is the complete bipartite graph  $K_{m,n}$ , we call it an  $(m,n)$ -complete regular dessin.

We introduce a correspondence between  $(m,n)$ -complete regular dessins and admissible pairs of skew-morphisms of the cyclic groups of orders  $m$  and  $n$ . A skew-morphism  $\varphi$  of a finite group  $A$  is a permutation on  $A$  such that  $\varphi(1) = 1$  and  $\varphi(xy) = \varphi(x)\varphi^{\pi(x)}(y)$  for all  $x, y \in A$  where  $\pi : A \rightarrow \mathbb{Z}_{|\varphi|}$  is an integer function. In particular, every group automorphism is a skew morphism. We determine the pairs  $(m,n)$  for which there exists exactly one dual pair of  $(m,n)$ -complete regular dessins thus generalising an elder result by Jones, Nedela and Škovič (2008). This is a joint work with Kan Hu, M. Škovič and Y.Q.Feng. (Received September 14, 2017)

1135-05-768 **Alexander Diaz-Lopez\*** ([diazlopezalexander@gmail.com](mailto:diazlopezalexander@gmail.com)), 135 Meadowbrook Dr, Huntingdon Valley, PA 19006, and **Pamela Harris, Erik Insko, Mohamed Omar** and **Bruce Sagan**. *Positivity Property of Descent Polynomials*. Preliminary report.

We say that a permutation  $\pi = \pi_1 \pi_2 \cdots \pi_n \in S_n$  has a descent at index  $i$  if  $\pi_i > \pi_{i+1}$ . Let  $Des(\pi)$  denote the set of indices where  $\pi$  has a descent. Given a set  $I$  of positive integers, we define  $D(I; n) = \{\pi \in S_n \mid Des(\pi) = I\}$  and  $d(I; n) := |D(I; n)|$ . We say  $d(I; n)$  is the descent polynomial of  $I$ . In this talk we will show that descent polynomials, like peak polynomials, can be written in a binomial basis with (strictly) positive coefficients. We will then describe the coefficients combinatorially, and explore some of their properties. (Received September 14, 2017)

1135-05-809 **Vincent Coll, Matt Hyatt, Andrew W. Mayers** and **Nicholas W. Mayers\*** ([nwm215@lehigh.edu](mailto:nwm215@lehigh.edu)). *Counting Seaweed Algebras of Index  $k$* . Preliminary report.

The *index* of a Lie algebra is an important algebraic invariant of the Lie algebra. Here we investigate the index of seaweed (or biparabolic) subalgebras of  $\mathfrak{sl}(n)$ . Standard "seaweeds" may be parametrized by a pair of compositions of the positive integer  $n$  and for all  $n$ , and certain  $k(n)$ , we provide closed-form formulas and the generating functions for  $C(n, k)$  – the number of parametrizing pairs which yield a seaweed subalgebra of  $\mathfrak{sl}(n)$  of index  $k$ . These formulas were conjectured by discerning Pascal-type relations among the  $C(n, k)$  and finding a pattern. Though our results are combinatorial in nature, we make use of new Lie algebraic techniques. (Received September 14, 2017)

1135-05-813 **Alex Cameron\*** (acamer4@uic.edu), and **Emily Heath**, eheath3@illinois.edu. *A (5, 5)-coloring of  $K_n$  with few colors.*

For fixed integers  $p$  and  $q$ , let  $f(n, p, q)$  denote the minimum number of colors needed to color all of the edges of the complete graph  $K_n$  such that no clique of  $p$  vertices spans fewer than  $q$  distinct colors. Any edge-coloring with this property is known as a  $(p, q)$ -coloring. We construct an explicit  $(5, 5)$ -coloring that shows that  $f(n, 5, 5) \leq n^{1/3+o(1)}$  as  $n \rightarrow \infty$ . This improves upon the best known probabilistic upper bound of  $O(n^{1/2})$  given by Erdős and Gyárfás, and comes close to matching the best known lower bound  $\Omega(n^{1/3})$ . (Received September 14, 2017)

1135-05-831 **Venkata Raghu Tej Pantangi\*** (pvrt1990@ufl.edu) and **Peter Sin**. *Smith groups and Critical groups of Polar Graphs.*

The Smith group and Critical group are interesting invariants of a graph. The Smith group of a graph is the abelian group whose cyclic decomposition is given by the Smith normal form of the adjacency matrix of the graph. The *critical* group is the finite part of the abelian group whose cyclic decomposition is given by the Smith normal form of the Laplacian matrix of a graph. The order is the number of the *critical* group is the number of spanning forests of the graph. There are very few families of graphs with known Smith Groups and *critical* Groups. It is therefore of interest to calculate these invariants for some well known families of graphs. In this presentation, we shall compute the elementary divisors of the adjacency and Laplacian matrices of families of polar graphs. These graphs have as vertices the isotropic one-dimensional subspaces of finite vector spaces with respect to non-degenerate forms, with adjacency given by orthogonality. This is joint work with Peter Sin. (Received September 15, 2017)

1135-05-837 **Anthony Bonato\*** (abonato@ryerson.ca), Department of Mathematics, 350 Victoria St, Toronto, Ontario M6R1T5, Canada. *Enemies of enemies are friends: the ILAT model for complex networks.*

In structural balance theory, edges emerge in networks via transitivity (friends of friends are friends) and anti-transitivity (enemies of enemies are friends). While transitivity has received ample attention by complex network researchers, much less work has been done on modeling anti-transitivity.

We present a dynamic, deterministic model based on the principle of anti-transitivity. In the Iterated Local Anti-Transitivity (ILAT) model, for every node  $u$  in a given time-step, we add a node that is joined to the complement of the closed neighborhood of  $u$ . The ILAT model satisfies several properties observed in complex networks, such as densification power laws, constant diameter, and high local clustering. (Received September 15, 2017)

1135-05-845 **Esmeralda Nastase** (akmanf@ilstu.edu), Xavier University, Mathematics Department, Cincinnati, OH 45207, and **Papa Amar Sissokho\*** (nastasee@xavier.edu), Illinois State University, Mathematics Department, Normal, IL 61790. *The maximum size of a partial spread in a finite projective space.*

Let  $V = V(n, q)$  denote the vector space of dimension  $n$  over the field with  $q$  elements. A partial  $t$ -spread of  $V$  is a collection of  $t$ -dimensional subspaces of  $V$  whose pairwise intersection is trivial. In a recent paper, we determined the maximum cardinality of a partial  $t$ -spread for almost all values of the parameters  $n$ ,  $t$ , and  $q$ . We will talk about this result and its relevance to coding theory. (Received September 26, 2017)

1135-05-849 **Stephen Melczer\*** (smelczer@sas.upenn.edu). *Analytic Combinatorics in Several Variables: Applications and Effective Methods.*

Recent work in the field of analytic combinatorics in several variables has shown how techniques from real and complex analysis, algebraic geometry, and topology can be combined to create robust and rigorous tools for the analysis of a large variety of combinatorial problems. This talk introduces some of these methods, discusses the extent to which they can be automated, and highlights examples from number theory, combinatorics, and physics. (Received September 15, 2017)

1135-05-853 **Benjamin Sepanski\*** (ben\_sepanski@baylor.edu), 841 Eagles Nest Dr., Hewitt, TX 76643, and **Jeske Glenn**, **Chris O'Neill** and **Vadim Ponomarenko**. *Augmented Hilbert series of numerical semigroups.*

Several new explicit formulas for certain augmented Hilbert Series measuring maximal and minimal factorization lengths for all numerical semigroups are presented. A weighted type of Euler characteristic of an associated simplicial complex is a key element of some of these formulas. (Received September 15, 2017)

1135-05-860 **Christine E. Heitsch\*** ([heitsch@math.gatech.edu](mailto:heitsch@math.gatech.edu)), School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30332. *Meanders and RNA Folding.*

An RNA molecule is a biochemical chain which folds in 3D via noncrossing base pairings called a secondary structure. Abstractly, two maximally different RNA secondary structures form a single closed loop known as a meander. Although meanders occur in a variety of mathematical settings, much remains unknown including their exact enumeration. Efforts to understand the geometry of RNA configuration landscapes lead naturally to a local move transformation on meanders. The resulting meander graphs have some interesting characteristics, which may yield new counting approaches. Additionally, MCMC sampling of meanders can provide insight into RNA branching configurations at viral genome length scales. (Received September 15, 2017)

1135-05-865 **Kara Stasikelis\*** ([stasike@clermson.edu](mailto:stasike@clermson.edu)), Department of Mathematics, Clemson University, O-110 Martin Hall, Clemson, SC 29634, and **Svetlana Poznanovic.** *Properties of the Promotion Markov Chain on Linear Extensions.*

The Tsetlin library is a model for the way an arrangement of books on a library shelf evolves over time. It assumes that, given  $n$  books, one book is read and returned at the end of the shelf before another one is picked up. Suppose the probability that a book  $i$  is picked up is  $x_i$ . An interesting property of this Markov chain is that its eigenvalues can be computed exactly and they are linear in the  $x_i$ 's. This result has been generalized in various ways by various people. In this work, we investigate the extended promotion Markov Chain introduced by Ayer, Klee, and Schilling in 2014. They showed that for a poset that is a rooted forest, the transition matrix has eigenvalues that are linear in  $x_1, \dots, x_n$ . We show the same result for a larger class of posets. (Received September 15, 2017)

1135-05-889 **Steven Schluchter\*** ([steven.schluchter@gmail.com](mailto:steven.schluchter@gmail.com)), Department of Mathematical Sciences, George Mason University, 4400 University Drive, MS 3F2, Fairfax, VA 22312. *Applications of ordinary voltage graph theory and intersection theory to graph embeddability.*

We will discuss the extendability of free actions of groups on graphs to embeddings of graphs in surfaces. We will make use of ordinary voltage graph embeddings, which encode the (derived) embeddings we seek in the form of algebraic data assigned to other (base) embeddings. We will develop and apply our homologically driven matrix analysis of ordinary voltage graph embeddings and the derived embeddings they encode. We will show that for each prime  $p > 5$ , the generalized Petersen graph  $GP(2p, 2)$  can be embedded in the torus, but not as a derived embedding. Furthermore, we will show that for each prime  $q > 3$ , there exists an ordinary voltage graph that has no derived embedding in the nonorientable surface of Euler characteristic  $2 - 2q$ , yet the corresponding derived graph does have an embedding in this surface. (Received September 16, 2017)

1135-05-890 **Katherine Moore\*** ([moorek@math.dartmouth.edu](mailto:moorek@math.dartmouth.edu)). *Permutations Realized by Dynamical Systems.*

Recently developed methods for estimating the entropy of time series are based on properties of the set of permutations that represent the short-term behavior of dynamical systems. Specifically, it has been shown that for piecewise monotone interval maps, the number of distinct permutations realized in this way encodes the system's topological entropy. In this talk, I will present a method for understanding the permutations realized by signed shifts, a class of maps which is particularly amenable to a combinatorial interpretation, and provide enumerative results in certain cases. Additionally, I will connect these ideas to properties of periodic points and permutation statistics for cycles to further motivate methods for understanding time series complexity. (Received September 16, 2017)

1135-05-892 **Jinha Kim, Ryan R. Martin, Tomáš Masařík, Warren Shull, Heather C. Smith\*** ([heather.smith@math.gatech.edu](mailto:heather.smith@math.gatech.edu)), **Andrew Uzzell** and **Zhiyu Wang.** *Local Dimension and Size of a Poset.* Preliminary report.

The original notion of poset dimension is due to Dushnik and Miller (1941). Very recently, Uerckerdt (2016) proposed a variant, called local dimension, which has garnered considerable interest. A local realizer of a poset  $P$  is a collection of partial linear extensions of  $P$  that cover the comparabilities and incomparabilities of  $P$ . The local dimension of  $P$  is the minimum frequency of a local realizer where frequency is the maximum multiplicity of an element of  $P$ .

Hiraguchi (1955) proved that any poset with  $n$  points has dimension at most  $n/2$ , which is sharp. We prove that the local dimension of a poset with  $n$  points is  $O(n/\log n)$ . To show that this bound is best possible, we use probabilistic methods to prove the following stronger result which extends a theorem of Chung, Erdős, and



Spencer (1983): There is an  $n$ -vertex bipartite graph in which each difference graph cover of the edges also covers one of the vertices  $\Omega(n/\log n)$  times. (Received September 22, 2017)

1135-05-912 **Mehtaab Sawhney\*** (msawhney@mit.edu) and **David Stoner**. *On a Conjecture Regarding Permutations which Destroy Arithmetic Progressions.*

Hegarty conjectured for  $n \neq 2, 3, 5, 7$  that  $\mathbb{Z}/n\mathbb{Z}$  has a permutation which destroys all arithmetic progressions mod  $n$ . For  $n \geq n_0$ , Hegarty and Martinsson demonstrated that  $\mathbb{Z}/n\mathbb{Z}$  has an arithmetic-progression destroying permutation. However  $n_0 \approx 1.4 \times 10^{14}$  and thus resolving the conjecture in full remained out of reach of any computational techniques. However, this project using constructions modeled after those used by Elkies and Swaminathan for the case of  $\mathbb{Z}/p\mathbb{Z}$  with  $p$  being prime, establishes the conjecture in full. Furthermore our results do not rely on the fact that it suffices to study when  $n < n_0$  and thus our results completely independent of the proof given by Hegarty and Martinsson. (Received September 16, 2017)

1135-05-935 **Sylvie Corteel, Olya Mandelshtam** and **Lauren Williams\***, williams@math.berkeley.edu. *Particle processes and Markov chains.*

I'll give an overview of the connection between various versions of the exclusion process (a Markov chain consisting of particles hopping on a 1D lattice, either with open boundaries or on a circle) and orthogonal polynomials, including Askey-Wilson polynomials, Koornwinder polynomials, and Macdonald polynomials. (Received September 17, 2017)

1135-05-936 **Alexander Diaz-Lopez\*** (diazlopezalexander@gmail.com), 800 Lancaster Ave (SAC 305), Department of Mathematics, Villanova University, Villanova, PA 19085, and **Pamela Harris, Erik Insko** and **Mohamed Omar**. *A proof of the peak polynomial positivity conjecture.*

We say that a permutation  $\pi = \pi_1\pi_2 \cdots \pi_n \in \mathfrak{S}_n$  has a peak at index  $i$  if  $\pi_{i-1} < \pi_i > \pi_{i+1}$ . Let  $\mathcal{P}(\pi)$  denote the set of indices where  $\pi$  has a peak. Given a set  $S$  of positive integers, we define  $\mathcal{P}(S; n) = \{\pi \in \mathfrak{S}_n : \mathcal{P}(\pi) = S\}$ . In 2013 Billey, Burdzy, and Sagan showed that for subsets of positive integers  $S$  and sufficiently large  $n$ ,  $|\mathcal{P}(S; n)| = p_S(n)2^{n-|S|-1}$  where  $p_S(x)$  is a polynomial depending on  $S$ . They gave a recursive formula for  $p_S(x)$  involving an alternating sum, and they conjectured that the coefficients of  $p_S(x)$  expanded in a binomial coefficient basis centered at  $\max(S)$  are all nonnegative. In this talk we introduce a new recursive formula for  $|\mathcal{P}(S; n)|$  without alternating sums and we use this recursion to prove that their conjecture is true. (Received September 17, 2017)

1135-05-942 **Aida Abiad, Boris Brimkov, Aysel Erey, Lorinda Leshock, Xavier Martinez-Rivera\*** (xaviermr@auburn.edu), **Suil O, Sung-Yell Song** and **Jason Williford**. *On the Wiener index, distance cospectrality and transmission-regular graphs.*

Let  $G$  be a simple, loopless, connected graph. The distance matrix  $D$  of  $G$  is the (symmetric) matrix whose rows and columns are indexed by the ordered vertices of  $G$ , and whose  $(i, j)$ -entry is the distance between the  $i$ th and  $j$ th vertices. Two graphs are  $D$ -cospectral if their distance matrices have the same spectrum. The Wiener index of  $G$ , a topological index used in theoretical chemistry as a structural descriptor for organic molecules, is half of the sum of all the entries of  $D$ . The Laplacian matrix of  $G$  is  $L = \Delta - A$ , where  $A$  and  $\Delta$  are the adjacency and degree matrices of  $G$ , respectively. The eigenvalues of  $L$  are called the Laplacian eigenvalues of  $G$ .

A graph is  $k$ -transmission-regular if the sum of each of the rows of its distance matrix is equal to  $k$ , and  $k$  is called the transmission regularity of the graph.

In this talk, a new construction of  $D$ -cospectral graphs is presented, as well as tight bounds on the transmission regularity of a transmission regular graph. Particular attention will be paid to a certain class of tree-like graphs, for which a result establishing a link between their Wiener index and Laplacian eigenvalues will be given, which generalizes a result of Mohar and Merris (whose result only holds for trees). (Received September 17, 2017)

1135-05-946 **Thomas C. Hull\*** (thull@wne.edu). *Combinatorial geometry in origami: the Kawasaki-Maekawa connection.*

The study of flat origami involves the properties and processes of folding two-dimensional materials using straight crease lines so that the image of the folding is also two-dimensional. That is, the folded object is flat and can be pressed in a book without crumpling or adding new creases. There are two fundamental results that pertain to the interior vertices in a flat origami crease pattern. One is geometrical: Kawasaki's Theorem states that a vertex will fold flat if and only if the alternating sum of the sector angles around the vertex is zero. The other is combinatorial: Maekawa's Theorem states that the difference between the number of mountain (convex) and valley (concave) at a flat-foldable vertex is always 2. It turns out that these results, Maekawa's Theorem and the necessary direction of Kawasaki's Theorem, are equivalent if we generalize their domains from folding flat

paper to folding cones. Doing this also expands the two Theorems to statements about the geometry of the folded vertex. We will explain and prove this more general result and discuss the possibility of generalizing to multiple-vertex crease patterns. (Received September 17, 2017)

1135-05-967 **Daryl R DeFord\*** ([daryl.r.deford.gr@dartmouth.edu](mailto:daryl.r.deford.gr@dartmouth.edu)), Dartmouth College, Kemeny Hall, 27 N. Main St., Hanover, NH 03755. *Dynamical Modeling for Multiplex Networks.*

Multiplex analyses of networks offer an important tool for leveraging the additional structure available in disaggregated data. In this talk, we will discuss the matched sum, a standard model for multiplex networks, describing the combinatorial and asymptotic properties of these graphs as the number of layers increases. These results allow us to characterize the pathological behavior that occurs when inter-layer connections overwhelm the intra-layer content. As an alternative to this structural approach, we present a family of dynamically-motivated models that avoids these pathologies and provide related spectral results. We apply these new methods to centrality and clustering problems for several data sets from the social sciences. (Received September 18, 2017)

1135-05-972 **Yuan Yuan Zheng\***, [yyz22@math.utoronto.ca](mailto:yyz22@math.utoronto.ca). *Parametrizing topological Ramsey spaces.* We present a moderately abstract parametrized Ellentuck theorem, which implies that essentially all infinite-dimensional Ramsey properties proven using topological Ramsey space theory can be parametrized by products of infinitely many perfect sets. (Received September 18, 2017)

1135-05-975 **Alexander Engström, Christian Go and Matthew T. Stamps\*** ([matt.stamps@yale-nus.edu.sg](mailto:matt.stamps@yale-nus.edu.sg)). *Betti numbers and anti-lecture hall compositions of random threshold graphs.*

This talk examines the one-to-one-to-one correspondence between threshold graphs, Betti numbers of quotients of polynomial rings with 2-linear resolutions, and anti-lecture hall compositions. We present explicit combinatorial mappings between each of these families of objects and give formulas for the expected values of the Betti numbers and anti-lecture hall composition corresponding to a random threshold graph. (Received September 18, 2017)

1135-05-980 **Evan Chen\*** ([evanchen@mit.edu](mailto:evanchen@mit.edu)). *Schur-Concavity for Avoidance of Increasing Subsequences in Block-Ascending Permutations.*

For integers  $a_1, \dots, a_n \geq 0$  and  $k \geq 1$ , let  $\mathcal{L}_{k+2}(a_1, \dots, a_n)$  denote the set of permutations of  $\{1, \dots, a_1 + \dots + a_n\}$  whose descent set is contained in  $\{a_1, a_1 + a_2, \dots, a_1 + \dots + a_{n-1}\}$ , and which avoids the pattern  $12 \dots (k+2)$ . We exhibit some bijections between such sets, most notably showing that  $\#\mathcal{L}_{k+2}(a_1, \dots, a_n)$  is symmetric in the  $a_i$  and is in fact Schur-concave. This generalizes a set of equivalences observed by Mei and Wang. (Received September 18, 2017)

1135-05-986 **Heather D Jordon\*** ([hjordon@albion.edu](mailto:hjordon@albion.edu)), **Benjamin Abels, Nichole Brown and Alexandria Pokorny.**  *$\pm$ Skolem-Type Difference Sets.* Preliminary report.

For a positive integer  $t$ , a Skolem-type difference set of order  $t$  is a partition of the set  $\{1, 2, \dots, 3t\}$  into triples  $\{a_i, b_i, c_i\}$  such that  $a_i + b_i = c_i$ . Skolem-type difference sets and their many generalizations are well studied, and necessary and sufficient conditions for Skolem difference sets are known. A practical application of Skolem-type difference sets involves rewriting the triple  $\{a_i, b_i, c_i\}$  as  $a_i + b_i + c_i = 0$  where  $c_i$  is necessarily negative. Hence, we study  $\pm$ Skolem-type difference sets of order  $2t$  which are partitions of the set  $\{\pm 1, \pm 2, \dots, \pm 3t\}$  into triples  $\{a_i, b_i, c_i\}$  such that  $a_i + b_i + c_i = 0$ . We also study a Langford-type generalization of this concept, namely, partitions of the set  $\{\pm d, \pm(d+1), \dots, \pm(d+3t-1)\}$  into  $2t$  triples  $\{a_i, b_i, c_i\}$  such that  $a_i + b_i + c_i = 0$  for  $d = 2$  and  $d = 3$ . (Received September 18, 2017)

1135-05-988 **Jacob Fox, Janos Pach and Andrew Suk\*** ([asuk@ucsd.edu](mailto:asuk@ucsd.edu)). *Approximating the rectilinear crossing number.*

A *straight-line* drawing of a graph  $G$  is a mapping which assigns to each vertex a point in the plane and to each edge a straight-line segment connecting the corresponding two points. The *rectilinear crossing number* of a graph  $G$ ,  $\overline{\text{cr}}(G)$ , is the minimum number of pairs of crossing edges in any straight-line drawing of  $G$ . Determining or estimating  $\overline{\text{cr}}(G)$  appears to be a difficult problem, and deciding if  $\overline{\text{cr}}(G) \leq k$  is known to be NP-hard. In fact, the asymptotic behavior of  $\overline{\text{cr}}(K_n)$  is still unknown.

In this talk, we present a deterministic  $n^{2+o(1)}$ -time algorithm that finds a straight-line drawing of any  $n$ -vertex graph  $G$  with  $\overline{\text{cr}}(G) + o(n^4)$  pairs of crossing edges. Together with the well-known Crossing Lemma due to Ajtai et al. and Leighton, this result implies that for any dense  $n$ -vertex graph  $G$ , one can efficiently find a straight-line drawing of  $G$  with  $(1 + o(1))\overline{\text{cr}}(G)$  pairs of crossing edges. This is joint work with Jacob Fox and Janos Pach. (Received September 18, 2017)

1135-05-1024 **Corey Vorland\*** ([corey.vorland@ndsu.edu](mailto:corey.vorland@ndsu.edu)), Minard 408. 1210 Albrecht Boulevard, Fargo, ND 58108. *Homomesy in products of three chains and multidimensional recombination.*

The actions of rowmotion and promotion on order ideals of a poset have generated a significant amount of interest in recent algebraic combinatorics research. One property associated to rowmotion and promotion that suggests a poset is “nice” is the homomesy property, which occurs when a statistic on a set has the same average value over any orbit as the global average of that statistic. In this talk, we will discuss homomesy on the product of chains poset using a beautiful technique called recombination. We will also expand from the product of chains setting to a more general class of posets. (Received September 18, 2017)

1135-05-1025 **Elanor West\*** ([ewest13@jhu.edu](mailto:ewest13@jhu.edu)), 9 E. 33rd St. #401, Baltimore, MD 21218, and **Xiao Xie, Hayden S Helm** and **Anan Zhang**. *Rendezvous Search on the Edges of Platonic Solids*. Preliminary report.

The classic Rendezvous Search problem involves two players moving along the same line at random until they meet. We were inspired by the Astronaut Problem rendition in which two players are on a sphere moving until they meet. We have simplified the model to discrete units of time and to take place along the edges of platonic solids. We assume the search ends when the two players can see each other. We have compared the mean times to end on all five solids, and have altered assumptions and strategies in various versions to see how certain changes affect the mean time to end. Some variations we have tried are: simultaneous versus sequential versions, varying lines of sight, multi-step strategies, and additional edges and nodes. Most of these variations we have explored more closely on either the dodecahedron, icosahedron, or both, since they are closer approximations to a sphere than the octahedron, cube, or tetrahedron. (Received September 18, 2017)

1135-05-1032 **Yesim Demiroglu Karabulut\*** ([yesim.demiroglu@rochester.edu](mailto:yesim.demiroglu@rochester.edu)). *Waring’s Problem in Finite Rings*.

In this talk we apply some classical mathematics to extend results for Waring’s problem to the context of general finite rings. Whereas there has been an extensive literature devoted to obtaining Waring type results over  $\mathbb{Z}/n\mathbb{Z}$ , over finite fields, and over certain kinds of matrix rings, we are not aware of any other such results in the context of general finite rings. We will start by explaining some fundamental Artin-Wedderburn theory and how one can use it together with a variant of Hensel’s lemma to deduce sharp results for the Waring’s problem over general (not necessarily commutative) finite rings by building on analogous results over finite fields. We will also present some elementary new proofs (using Cayley digraphs and spectral graph theory) for Waring results over finite fields, and explain how in the process of re-proving these finite field results, we obtain an original result providing an analogue of Sárközy’s theorem in the finite field setting (showing that any subset  $E$  of a finite field  $\mathbb{F}_q$  for which  $|E| > \frac{q^k}{\sqrt{q-1}}$  must contain at least two distinct elements whose difference is a  $k^{\text{th}}$  power). (Received September 18, 2017)

1135-05-1057 **Igor Pak** and **Damir Yeliussizov\*** ([damir@math.ucla.edu](mailto:damir@math.ucla.edu)), 520 Portola Plaza Math Sci 6363 UCLA, Los Angeles, CA 90095. *Ehrhart polynomial of some Schläfli simplices*.

We show that Ehrhart polynomial of certain Schläfli simplices can be computed in polynomial time. We apply this result to the problem of counting integer partitions. In particular, we prove that the number of binary partitions of  $N$  can be computed in  $\text{poly}(\log N)$  time. (Received September 19, 2017)

1135-05-1069 **Mohamed Omar\*** ([omar@g.hmc.edu](mailto:omar@g.hmc.edu)), 301 Platt Boulevard, Claremont, CA 91711. *Incidence Problems in Convex Geometry*.

Recently, much attention has been given to the use of algebraic techniques in understanding questions on incidence problems in convex geometry. We report on recent advances that exploit the neural ring and toric ideals. (Received September 19, 2017)

1135-05-1082 **Franklin H. J. Kenter\*** ([kenter@usna.edu](mailto:kenter@usna.edu)) and **Jephian C.-H. Lin**. *On the error of a priori sampling: zero forcing sets and propagation time*.

Zero forcing is an iterative process on a graph used to bound the maximum nullity. The process begins with some colored, and the other vertices can become colored under a specific color change rule. The goal is to find a minimum set of vertices such that after iteratively applying the rule, all of the vertices become colored. Of particular interest is the propagation time of a chosen set which is the number of steps the rule must be applied in order to color all the vertices of a graph.

We give a linear algebraic interpretation of zero forcing: Find a set of vertices  $S$  such that for any weighted adjacency matrix  $\mathbf{A}$ , whenever  $\mathbf{Ax} = \mathbf{0}$ , all of  $\mathbf{x}$  can be recovered using only the entries corresponding to  $S$ . Here,  $S$  must be chosen before  $\mathbf{A}$  is known. In this light, any error in  $\mathbf{x}_S$  effects the error of  $\mathbf{s}$  exponentially

in the propagation time. This error can be quantitatively measured using newly defined zero forcing-related parameters: the error polynomial vector and the variance polynomial vector. Hence, quality of two zero forcing sets can objectively be compared even if the sets have the same size and propagation time. Examples and constructions are given. (Received September 19, 2017)

1135-05-1095 **Kristina Marotta\*** (kmarott@cedarcrest.edu), 927 Dorset Rd, Allentown, PA 18104, **James Hammer** (jmhammer@cedarcrest.edu), 100 College Dr, Allentown, PA 18104, and **Joshua Harrington** (jsharrin@cedarcrest.edu), 100 College Dr, Allentown, PA 18104. *(a, b)-Sudoku Latin Squares Forbidding Distance One.*

Let  $n$  be a positive integer and let  $(a, b)$  be an ordered pair such that  $ab = n$ . An  $(a, b)$ -Sudoku Latin square is an  $n \times n$  array partitioned into  $a \times b$  regions in the natural way so that every row, column, and  $a \times b$  region contains every symbol  $\{1, 2, \dots, n\}$  exactly once. An  $n \times n$  array has property K if no two adjacent cells, that is cells that share an edge, contain consecutive integers. We will investigate the necessary and sufficient conditions for the existence of  $(a, b)$ -Sudoku Latin squares that have property K. (Received September 19, 2017)

1135-05-1096 **George E Andrews\*** (gea1@psu.edu). *Partitions with even parts below odd parts and mock theta functions.*

We begin considering all partitions in which each even part is smaller than each odd part. The full generating function is of only minor interest. However, of much greater interest is the subclass consisting of those partitions where in addition the largest even part is the only part that appears an odd number of times. There is a natural "crank" for these partitions and a nice Ramanujanesque congruence modulo 5. There is also a natural relation with the third order mock theta function  $\nu(q)$ . (Received September 19, 2017)

1135-05-1105 **Joshua Fallon\*** (jfall03@lsu.edu), jfall03@lsu.edu, and **Kirsten Hogenson, Lauren Keough, Mario Lomeli, Marcus Schaefer and Pablo Soberon Bravo.** *On The Maximum Rectilinear Crossing Number of Subdivided Stars.*

Given a graph  $G$ , its maximum rectilinear crossing number  $\overline{\text{mcr}}(G)$  is the maximum number of edge crossings that can appear in a drawing of  $G$  in the plane with each edge a line segment. It is trivially bounded above by the graph's thrackle bound, which is the number of non-consecutive edge pairs. For a caterpillar  $T$ ,  $\overline{\text{mcr}}(T)$  and the thrackle bound are equal. If a tree is not a caterpillar, it must contain a subgraph  $S$  isomorphic to  $K_{1,3}$  with each edge subdivided once. Woodall has shown that no rectilinear drawing of  $S$  achieves the thrackle bound. We present some results on more general subdivided stars to contribute to the understanding of maximum rectilinear crossing number of trees. (Received September 22, 2017)

1135-05-1156 **Michael Joseph\*** (mjoseph@daltonstate.edu). *Toggling antichains of posets.*

Toggles are simple involutions that generate permutation groups on various combinatorial sets. First introduced by Cameron and Fon-Der-Flaass, the toggle group on the set of order ideals of a poset has been well-studied for years, particularly for studying a map called *rowmotion*. Recently Striker has motivated the study of toggle groups on general families of subsets, including antichains. We will examine the relationship between the toggle groups on antichains and order ideals, including an explicit isomorphism between the two groups. We will also describe a piecewise-linear generalization of toggling for Stanley's chain polytope, especially the connections with an established generalization of order ideal toggles to the order polytope of a poset. (Received September 20, 2017)

1135-05-1174 **Alex Happ\*** (alex.happ@uky.edu), University of Kentucky, 715 Patterson Office Tower, Lexington, KY 40506, and **Richard Ehrenborg.** *Sums of powers of the descent set statistic.*

We study the sum of the  $r$ th powers of the descent set statistic and how many small prime factors occur in these numbers. The results will depend upon the base  $p$  expansion of  $n$  and  $r$ . (Received September 20, 2017)

1135-05-1177 **Michael Tait\*** (mtait@cmu.edu). *Applications of projective plane graphs in combinatorial number theory.*

A sum-product theorem says roughly: a subset of an algebraic structure with certain properties cannot look like both an arithmetic progression and a geometric progression at the same time. In this talk, we discuss how to use graphs coming from projective planes to prove sum-product theorems in a variety of settings. Using similar techniques, we can prove a theorem about the structure of product sets in extra special groups. This is joint work with Thang Pham, Craig Timmons, Le Anh Vinh, and Robert Won. (Received September 20, 2017)

1135-05-1178 **Michael Tait\*** (mtait@cmu.edu). *7 theorems in spectral extremal graph theory.*

Theorems in extremal graph theory ask to optimize a combinatorial invariant over a fixed family of graphs. In this talk, we discuss how to prove several theorems in this area where the graph invariant in question is a function of the eigenvalues or eigenvectors of the adjacency matrix of the graph. A representative result is a proof of a conjecture of Boots and Royle from 1991: the planar graph of maximum spectral radius (of its adjacency matrix) is the join of an edge and a path. This is joint work with Josh Tobin. (Received September 20, 2017)

1135-05-1216 **Federico Castillo and Fu Liu\*** (fuliu@math.ucdavis.edu). *Combinatorics of nested Braid fan.*

Generalized permutohedra are defined as polytopes whose normal fan coarsens the Braid fan  $Br_d$ . We wanted to generalize the construction so that edges of direction  $e_i + e_j - e_k - e_l$  may appear. In joint work with Castillo, we introduce the nested Braid fan  $Br_d^2$ , which is a refinement of  $Br_d$  by considering the first difference of ordered coordinates. We show that  $Br_d^2$  is the normal fan of usual nested permutohedra, a subfamily of which is regular nested permutohedra, and thus is a projective fan. We then determine the combinatorics of  $Br_d^2$  or regular nested permutohedra by giving a one-to-one correspondence between cones of  $Br_d^2$  and chains in the poset of ordered partition. (Received September 20, 2017)

1135-05-1228 **Elizabeth Bailey Matson\*** (eab0052@auburn.edu), **Charles Camacho, Silvia Fernández-Merchant, Marija Jelic, Rachel Kirsch, Linda Kleist and Jennifer White.** *Bounding The Tripartite-Circle Crossing Number of Complete Tripartite Graphs.* Preliminary report.

A  $k$ -circle drawing of a graph  $G$  is a drawing of  $G$  in the plane where the vertices are placed on the boundary of  $k$  disjoint circles with the requirement that the edges of  $G$  do not cross the boundary of any circle. The  $k$ -circle crossing number of a graph  $G$  is the minimum number of edge-crossings in any  $k$ -circle drawing of  $G$ . For the special case when  $G$  is a  $k$ -partite graph, it is additionally required that the  $k$  sets of disjoint vertices each be placed on separate circles and thus there are no edges in the interior of any circle. The corresponding crossing number is called the  $k$ -partite-circle crossing number. We extend recent work on the bipartite-circle crossing number of complete bipartite graphs to introduce and explore the tripartite-circle crossing number of complete tripartite graphs. (Received September 20, 2017)

1135-05-1230 **Aida Abiad, Boris Brimkov\*** (boris.brimkov@rice.edu), **Xavier Martinez-Rivera, Suil O and Jingmei Zhang.** *Spectral bounds for the connectivity of regular graphs with given order.*

The second-largest eigenvalue and second-smallest Laplacian eigenvalue of a graph are measures of the graph's connectivity. These parameters can be used to analyze the robustness, resilience, and synchronizability of networks, and are related to other connectivity attributes such as the vertex- and edge-connectivity, isoperimetric number, and characteristic path length. In this talk, we give upper bounds for the second-largest eigenvalues of regular graphs and multigraphs which guarantee a desired vertex- or edge-connectivity. The given bounds are in terms of the order and degree of the graphs, and hold with equality for infinite families of graphs. (Received September 20, 2017)

1135-05-1255 **Elizabeth Bailey Matson\*** (eab0052@auburn.edu) and **Chris Rodger.** *Extreme Equitable Block-Colorings of  $K_v$  and  $K_v - F$ .*

An  $H$ -decomposition of a graph  $G$  is a partition  $P$  of  $E(G)$  into blocks, each element of which induces a copy of  $H$ . An  $(s, p)$ -equitable  $H$ -coloring of  $G$  is a coloring of the blocks in  $P$  with exactly  $s$  colors such that each vertex  $v$  is incident with blocks colored with exactly  $p$  colors, the blocks containing  $v$  being shared out as evenly as possible among the  $p$  color classes. The smallest value of  $s$  for which there exists an  $(s, p)$ -equitable  $H$ -coloring of  $G$ , denoted  $\chi'_p(v)$ , is considered for  $C_4$ -colorings of  $K_v - F$  where  $F$  is a 1-factor of  $K_v$ ; this will follow from suitable  $K_2$ -colorings of  $K_{v/2}$ . Of particular interest is when  $\chi'_p(v) > p$ , in which case traditional edge-coloring proof techniques are rendered useless. The color vector  $V(E)$  of an  $(s, p)$ -equitable  $H$ -coloring  $E$  of  $G$  is defined to be  $(c_1(E), c_2(E), \dots, c_s(E))$ , arranged in non-decreasing order, where  $c_i(E)$  is the number of vertices in  $G$  incident with a block of color  $i$ . In all cases where  $\chi'_p(v) > p$ , the extreme values of  $V(E)$  are considered, namely  $c_1(E)$  and  $c_s(E)$ . An overview of recent findings is presented, utilizing in some cases the powerful proof technique of graph amalgamations. (Received September 20, 2017)

1135-05-1256 **Michael Ferrara\*** ([michael.ferrara@ucdenver.edu](mailto:michael.ferrara@ucdenver.edu)). *Some Results on Ramsey-Type Problems for Graph Saturation.*

For a family  $\mathcal{F}$  of graphs, we say that a graph  $G$  is  $\mathcal{F}$ -saturated if  $G$  contains no element of  $\mathcal{F}$  as a subgraph, but for any edge  $e \notin G$ ,  $G + e$  contains some member of  $\mathcal{F}$ . We let  $\text{sat}(n, \mathcal{F})$ , the *saturation number* of  $\mathcal{F}$ , denote the minimum number of edges in an  $\mathcal{F}$ -saturated graph of order  $n$ .

In this talk, we will discuss several Ramsey-type problems in the setting of graph saturation. In 1987, Hanson and Toft conjectured the saturation number for the family of graphs that are Ramsey-minimal for a collection of cliques. We will present some prior progress toward this challenging problem, and some newer results for general families of graphs that we hope will provide some insight into the conjecture. From there, we will consider a related class of saturation problems for edge-colored graphs, with particular attention on some interesting differences between monochromatic and rainbow target graphs. (Received September 20, 2017)

1135-05-1260 **Sam Hopkins\*** ([shopkins@mit.edu](mailto:shopkins@mit.edu)). *The coincidental down-degree expectations (CDE) property for posets and homomesy.*

Reiner, Tenner, and Yong recently introduced the coincidental down-degree expectations (CDE) property for posets. A poset  $P$  is CDE if the expected down-degree is the same for the uniform distribution on  $P$  and for the distribution on  $P$  that weights each element proportional to the number of maximal chains passing through that element. Reiner, Tenner, and Yong showed that many posets familiar to algebraic combinatorialists, such as certain intervals of the weak Bruhat order, are CDE. In earlier independent work with Chan, Haddadan, and Moci (which was motivated by some surprising combinatorial results arising from a new approach to Brill-Noether theory) we found a large family of CDE intervals of Young's lattice. In more recent work I extended these results to the shifted setting, thereby completing the case-by-case proof that all minuscule lattices are CDE, as was conjectured by Reiner-Tenner-Yong (a uniform proof was subsequently given by Rush). I will conclude by explaining how, thanks to a result of Striker, these results also establish that the antichain cardinality statistic is homomesic with respect to rowmotion or gyration acting on sets of order ideals of certain "balanced" shapes, extending a result of Propp and Roby. (Received September 20, 2017)

1135-05-1283 **Maria Monks Gillespie\*** ([mgillespie@math.ucdavis.edu](mailto:mgillespie@math.ucdavis.edu)), **Jake Levinson** and **Kevin Purbhoo**. *Crystallizing the Schur  $Q$ -functions.*

Crystal base theory, which first appeared in the study of the representation theory of quantum groups, has recently provided new methods for understanding the combinatorics of symmetric functions having a positive Schur expansion. In particular, there is a crystal structure - a certain weighted directed graph - on the set of all semistandard Young tableaux of a given shape, whose connected components have total weight equal to Schur functions.

We provide a similar crystal-like structure on shifted tableaux such that the weights of the connected components are the Schur  $Q$ -functions, an important class of symmetric functions that arises naturally in the orthogonal Grassmannian, in the projective representation theory of  $S_n$ , and as a certain specialization of the Hall-Littlewood polynomials. (Received September 20, 2017)

1135-05-1350 **David Galvin\*** ([dgalvin1@nd.edu](mailto:dgalvin1@nd.edu)), Department of Mathematics, University of Notre Dame. *The non-locality of graph coloring.*

Graph coloring — assigning colors to vertices of a graph, with adjacent vertices receiving different colors — seems local (the color of a vertex is influenced only by those of its immediate neighbors), but turns out to be highly non-local. A classical result illustrating this is Erdős' construction of locally tree-like (so 2-colorable) graphs that require an arbitrarily large palette to color completely.

I'll talk about some work (of and/or with Engbers, Feldheim, Kahn, Peled, Randall, Sorkin and Spinka) on the non-locality of coloring bipartite graphs in general and portions of the hypercubic lattice in particular, with ramifications for some problems in statistical physics and theoretical computer science.

A theme will be our precipitous drop in knowledge in going from 3 colors to 4 or more. (Received September 21, 2017)

1135-05-1358 **Steven Klee\*** ([klees@seattleu.edu](mailto:klees@seattleu.edu)), **Eran Nevo**, **Isabella Novik** and **Hailun Zheng**. *A lower bound theorem for centrally symmetric simplicial polytopes.*

The Lower Bound Theorem for simplicial polytopes states that among all simplicial  $d$ -polytopes on  $n$  vertices, a stacked polytope has the minimum number of faces of each dimension. In this talk, we will discuss an extension of this result to the world of centrally symmetric simplicial polytopes. Our approach employs tools from graph rigidity to understand stresses on the graph of a centrally symmetric polytope. (Received September 21, 2017)

1135-05-1373 **Andries E. Brouwer, Sebastian M. Cioabă, Ferdinand Ihringer and Matt McGinnis\*** (mamcginn@udel.edu). *The smallest eigenvalues of Hamming and Johnson graphs.*

We prove a conjecture by Van Dam & Sotirov on the smallest eigenvalue of (distance- $j$ ) Hamming graphs and a conjecture by Karloff on the smallest eigenvalue of (distance- $j$ ) Johnson graphs. (Received September 21, 2017)

1135-05-1376 **Tad P. White\*** (tad@super.org). *Quota Trees.*

Given a nonnegative integer “quota” for each vertex of a directed multigraph  $G$ , a **quota tree** is an immersed rooted tree which hits each vertex of  $G$  the prescribed number of times. When the quotas are all one, the tree is actually embedded and we recover the usual notion of a spanning arborescence (directed spanning tree). The usual algorithms which produce spanning arborescences with various properties typically have (sometimes more complicated) “quota” analogues.

One context in which quota trees arise is in characterizing the sizes of the Myhill-Nerode equivalence classes in a connected deterministic finite-state automaton recognizing a given regular language. The obstruction to realizing a given set of M-N class sizes is precisely the existence of a suitable quota tree.

We give necessary and sufficient conditions for the existence of a quota tree (or forest) over a given directed graph with specified quotas, solving the M-N class size problem as a special case. We derive a formula for the number of such quota trees, which generalizes both the matrix-tree theorem and Cayley’s formula for counting labeled trees. We show how to sample uniformly from the set of forests with given quotas, and how to compute a minimum-weight quota forest. (Received September 25, 2017)

1135-05-1385 **Carolina Benedetti V.\*** (c.benedetti@uniandes.edu.co). *Combinatorial aspects of Non-crossing and  $R$ -polytopes.* Preliminary report.

N. Thieme defined unipotent polytopes  $U(\beta, P)$  in connection to certain representations of the unipotent group of upper triangular matrices over a finite field. In this talk we will define two subpolytopes of  $U(\beta, P)$  where  $P$  is the line poset on  $[n]$  and  $\beta$  is the composition  $(1^n)$ . These two subpolytopes denoted  $NC$  and  $NN$  arise as the convex hull of non-crossing and non-nesting partitions of  $[n]$ , respectively. The  $NN$  polytope corresponds to Stanley’s chain polytope of the root lattice of type  $A$ . As such, some of its combinatorics is understood but not all. In particular we will provide a description of its 1-skeleton and extend this description to  $NC$ . Moreover, we will show how  $NC$  and  $NN$  can be thought of as  $R$ -polytopes (i.e. as polytopes arising from relations on a finite set) and characterize different types of faces for  $R$ -polytopes in general. This is joint work with F. Alinieifard, N. Bergeron, S. Li, F. Saliola. (Received September 21, 2017)

1135-05-1387 **Carolina Benedetti V.\*** (c.benedetti@uniandes.edu.co). *A parking function model for flow polytopes.* Preliminary report.

We reinterpret a Lidskii formula for computing volumes of flow polytopes in terms of a family of combinatorial objects. This approach proves to be useful to compute the volumes in several cases. In particular, we recover classical results of Pitman-Stanley and show that the volume of a certain new flow polytope (called Caracol polytope) is the product of the Catalan number  $C_{n-2}$  times  $n^{n-2}$ , that is the number of parking functions of length  $n-1$ . This is joint work with R. Gonzalez, C. Hanusa, P. Harris, A. Khare, A. Morales, M. Yip. (Received September 21, 2017)

1135-05-1390 **Sean J English\*** (sean.j.english@wmich.edu), **Nathan Graber** (nathan.graber@ucdenver.edu), **Pamela Kirkpatrick** (prk213@lehigh.edu), **Abhishek Methuku** (abhishekmethuku@gmail.com) and **Eric Sullivan** (eric.2.sullivan@ucdenver.edu). *Saturation for Berge Hypergraphs.*

Let  $H$  be a  $k$ -uniform hypergraph, and  $F$  be a simple graph on the same vertex set. We say  $H$  is *Berge- $F$*  if there exists a bijection  $f : E(F) \rightarrow E(H)$  such that for each  $e \in E(F)$ , we have  $e \subset f(e)$ . If there exists a subhypergraph of  $H$  that is *Berge- $F$*  we say that  $H$  contains *Berge- $F$* . A hypergraph,  $H$  is *Berge- $F$ -saturated* if  $H$  does not contain *Berge- $F$*  but  $H + e$  contains *Berge- $F$*  for every edge  $e \in E(\overline{H})$ . The  $k$ -uniform *saturation number* of *Berge- $F$* , denoted  $\text{sat}_k(n, \text{Berge-}F)$ , is the minimum number of edges in a  $k$ -uniform hypergraph  $H$  such that  $H$  is *Berge- $F$ -saturated*. In this talk we will explore the saturation numbers of many *Berge* hypergraphs. (Received September 21, 2017)

1135-05-1395 **Colin Defant** and **Sumun Iyer\*** (ssi1@williams.edu). *Domination of unitary Cayley Graphs.*

The unitary Cayley graph of  $\mathbb{Z}/n\mathbb{Z}$ , denoted  $X_{\mathbb{Z}/n\mathbb{Z}}$ , has vertices  $0, 1, \dots, n-1$  with  $x$  adjacent to  $y$  if  $x-y$  is relatively prime to  $n$ . We observe that every unitary Cayley graph is a direct product of balanced, complete multipartite graphs. We present results on the quality of known upper bounds for the domination number

of  $X_{\mathbb{Z}/n\mathbb{Z}}$ . Extending work of Mekiš, we give lower bounds for the domination numbers of direct products of complete graphs. We also present a conjecture for the exact values of the upper domination numbers of direct products of balanced, complete multipartite graphs, prove the conjecture in certain cases, and pose the remaining cases as an open question. (Received September 21, 2017)

1135-05-1439 **Frederick Butler\*** (fbutler@ycp.edu), Department of Physical Sciences, York College of Pennsylvania, 441 County Club Rd., York, PA 17403. *Cycle-Euler-Mahonian Triples of Permutation Statistics*. Preliminary report.

A triple of permutation statistics is called cycle-Euler-Mahonian if it is equidistributed with the number of left-to-right minima (lrmin), the number of descents (des), and the major index (maj). In this talk we give examples of some cycle-Euler-Mahonian triples involving well-known Mahonian statistics, including the number of inversions (inv) and Denert's statistic (den). (Received September 22, 2017)

1135-05-1449 **Amy Grady\*** (agrady@clemson.edu) and **Svetlana Poznanovic**. *Sorting Index and Mahonian-Stirling pairs for labeled forests*.

Björner and Wachs defined a major index for labeled plane forests and showed that it has the same distribution as the number of inversions. We will define and study the distributions of a few other natural statistics on labeled forests. Specifically, we introduce the notions of bottom-to-top maxima, cyclic bottom-to-top maxima, sorting index and cycle minima. Then we show that the pairs (inv, Btmax), (sor, Cyc), and (maj, Cbtmax) are equidistributed. Our results extend the result of Björner and Wachs and generalize results for permutations. (Received September 22, 2017)

1135-05-1470 **Gregg Musiker\*** (musiker@math.umn.edu), Minneapolis, MN 55455, and **Tom Roby** (tom.robby@uconn.edu), Storrs, CT 06269. *A path formula for birational rowmotion on the product of two chains*. Preliminary report.

We give a formula in terms of families of non-intersecting lattice paths for iterated actions of the birational rowmotion map on a product of two chains. Birational rowmotion is an action on the space of assignments of rational functions to the elements of a poset. It is lifted from the well-studied *rowmotion* map on order ideals (equivariantly on antichains) of a partially ordered set  $P$ , which when iterated on special posets has unexpectedly nice properties in terms of periodicity, cyclic sieving, and homomesy (constant averages for each orbit). Grinberg and Roby showed that birational rowmotion exhibits the same periodicity as ordinary rowmotion on the product of two chains via an indirect argument, but our formula sheds additional insight on the underlying dynamics.

This work began at the 2015 AIM (American Institute of Mathematics) Workshop on *Dynamical Algebraic Combinatorics*, and we thank AIM for its hospitality. (Received September 22, 2017)

1135-05-1502 **Kevin M. Grace\*** (kgrace3@lsu.edu). *All that glitters is not golden-mean: Structure of golden-mean matroids*.

Three closely related classes of  $\text{GF}(4)$ -representable matroids are the golden-mean matroids, the matroids representable over all fields of size at least 4, and the matroids representable over  $\text{GF}(4)$  as well as fields of all characteristics. Subject to announced results of Geelen, Gerards, and Whittle, we have characterized the highly connected matroids in each of these classes. As a direct consequence of this characterization, we give the extremal functions for these classes of matroids, including the golden-mean matroids. The extremal function indicates the maximum number of elements of a simple matroid of rank  $r$ . This implies that a conjecture made by Archer in 2005 holds for matroids of sufficiently high rank. (Received September 22, 2017)

1135-05-1508 **Wasin So\*** (wasin.so@sjsu.edu). *Distance energy change of graphs due to edge deletion*. Preliminary report.

The distance matrix of a connected graph records the distance between each and every pair of vertices. It is a well studied concept in literature. Recently, much attention is given to the distance energy of a connected graph, which is defined as the sum of the absolute values of the eigenvalues of its distance matrix. We are interested in the problem of studying how distance energy changes when edges are deleted. The goal is to characterize those connected graphs and their edges such that the deletion of such edges increases the distance energy. The full characterization is still not known. In this talk, we report some known results in literature and the new result about complete bipartite graphs. Conjectures related to computational experiments will also be mentioned. (Received September 22, 2017)



1135-05-1520 **Vladimir Bercov\*** (v.bercov@yahoo.com), 153 Tysens Lane, Staten Island, NY 10306.  
*Connected dominating sets and a new graph invariant.*

We define a new invariant  $\eta(G)$  of a graph  $G = (V, E)$  as a maximum length of a sequence of subsets of vertices  $V_1, V_2, V_3, \dots$ , where  $V_i \cap V_j = \emptyset$  ( $i \neq j$ ), every vertex of  $V_1 \cup V_2 \cup \dots \cup V_{(k-1)}$  is adjacent to a vertex in  $V_k$ , and subgraph induced by  $V_k$  ( $k \geq 2$ ) is connected. From the definition it follows that  $\omega(G) \leq \eta(G) \leq h(G)$ , where  $\omega(G)$  is clique number and  $h(G)$  is number of Hadwiger. If chromatic number  $\chi(G) \leq 4$  then  $\eta(G) \geq \chi(G)$ . The Nordhaus-Gaddum inequalities for the new invariant are:  $\eta(G)\eta(G) \geq n(G)$ ,  $\eta(G) + \eta(G) \leq 6n(G)/5$ , where  $n(G) = |V|$ . For graphs  $G$  with independence number three without induced chordless cycles of length seven, connected dominating number  $\gamma_c(G) \leq 4$  and  $\eta(G) \geq n(G)/4$ . Conjecture: for every graph  $G$  :  $\eta(G) \geq \chi(G)$ . (Received September 22, 2017)

1135-05-1524 **Aras Erzurumluoglu\*** (aerzurum@uottawa.ca). *Hamiltonicity in 2-Block-Intersection Graphs of Twofold Triple Systems.*

A *balanced incomplete block design* (BIBD( $v, k, \lambda$ ))  $(V, \mathcal{B})$  is a combinatorial design in which (i)  $|V| = v$ , (ii) for each block  $B \in \mathcal{B}$ ,  $|B| = k$ , and (iii) each 2-subset of  $V$  occurs in precisely  $\lambda$  blocks of  $\mathcal{B}$ . A BIBD( $v, 3, 2$ ) is a *twofold triple system* (TTS( $v$ )).

Given a combinatorial design  $\mathcal{D}$  with block set  $\mathcal{B}$ , the *block-intersection graph* (BIG) of  $\mathcal{D}$  is the graph having  $\mathcal{B}$  as its vertex set, and in which two vertices  $B_1 \in \mathcal{B}$  and  $B_2 \in \mathcal{B}$  are adjacent if and only if  $|B_1 \cap B_2| \neq 0$ . Similarly, the *i-block-intersection graph* (*i*-BIG) of  $\mathcal{D}$  is the graph having  $\mathcal{B}$  as its vertex set, and in which two vertices  $B_1 \in \mathcal{B}$  and  $B_2 \in \mathcal{B}$  are adjacent if and only if  $|B_1 \cap B_2| = i$ .

Recently David Pike and I completed the spectrum for TTSs with (respectively, without) Hamilton cycles in their 2-BIGs. In this talk I will present some of the constructions that collectively lead to these results. (Received September 22, 2017)

1135-05-1535 **Thotsaporn "Aek" Thanatipanonda\*** (thotsaporn@gmail.com), 87/99 Radjaprug Road, Bangwa, Bangkok, 10160, Thailand. *The Curious Bounds of Floor Function Sums.*

We consider the sums involving floor functions of the forms: for a fixed positive integer  $m$ ,

$$S_m(\{a, b\}, K) = \sum_{k=0}^K \left( \left\lfloor \frac{a+b+k}{m} \right\rfloor - \left\lfloor \frac{a+k}{m} \right\rfloor - \left\lfloor \frac{b+k}{m} \right\rfloor + \left\lfloor \frac{k}{m} \right\rfloor \right),$$

where  $0 \leq a, b, K \leq m-1$  and

$$S_m(\{a, b, c\}, K) = \sum_{k=0}^K \left( \left\lfloor \frac{a+b+c+k}{m} \right\rfloor - \left\lfloor \frac{a+b+k}{m} \right\rfloor - \left\lfloor \frac{b+c+k}{m} \right\rfloor - \left\lfloor \frac{a+c+k}{m} \right\rfloor + \left\lfloor \frac{a+k}{m} \right\rfloor + \left\lfloor \frac{b+k}{m} \right\rfloor + \left\lfloor \frac{c+k}{m} \right\rfloor - \left\lfloor \frac{k}{m} \right\rfloor \right),$$

where  $0 \leq a, b, c, K \leq m-1$ . The bounds of the above sums are studied by Jacobsthal (1957), Carlitz (1960), Grimson (1974), and Tverberg (2012) and have been known. However, the bounds for the general sums  $S_m(\{a_1, a_2, \dots, a_n\}, K)$  with  $n \geq 4$  defined analogously have only been done partially by Onphaeng and Pongsriiam (2017). The other bounds are still left open. In this talk, we discuss these results and give the conjectures of the complete bounds. (Received September 23, 2017)

1135-05-1558 **Jie Han\*** (jhan@ime.usp.br) and **Andrew Treglown**. *The complexity of perfect matchings and packings in dense hypergraphs.*

Given two  $k$ -graphs  $H$  and  $F$ , a perfect  $F$ -packing in  $H$  is a collection of vertex-disjoint copies of  $F$  in  $H$  which together cover all the vertices in  $H$ . In the case when  $F$  is a single edge, a perfect  $F$ -packing is simply a perfect matching. For a given fixed  $F$ , it is generally the case that the decision problem whether an  $n$ -vertex  $k$ -graph  $H$  contains a perfect  $F$ -packing is NP-complete. In this talk we describe a general tool which can be used to determine classes of (hyper)graphs for which the corresponding decision problem for perfect  $F$ -packings is polynomial time solvable. We then give applications of this tool. For example, we give a minimum  $l$ -degree condition for which it is polynomial time solvable to determine whether a  $k$ -graph satisfying this condition has a perfect matching (partially resolving a conjecture of Keevash, Knox and Mycroft). We also answer a question of Yuster concerning perfect  $F$ -packings in graphs. (Received September 23, 2017)

1135-05-1560 **Holly Mandel and Oliver Pechenik\***, Department of Mathematics, University of Michigan, Ann Arbor, MI 48109. *Orbits of plane partitions of exceptional Lie type.*

We study the dynamics of the rowmotion action of Brouwer and Schrijver on plane partitions over minuscule posets of exceptional Lie type. We resolve some cyclic sieving conjectures of Rush and Shi (2013) by using ideas

of Dilks, Pechenik and Striker to relate these dynamics to a  $K$ -theoretic analogue of Schützenberger’s promotion action on linear extensions of posets. (Received September 23, 2017)

1135-05-1565 **Younghwan Kim\*** ([younghwan.kim.2@asu.edu](mailto:younghwan.kim.2@asu.edu)). *Cyclic Sieving Phenomenon on Matchings*. Preliminary report.

Reiner-Stanton-White defined the cyclic sieving phenomenon (CSP) associated to a finite cyclic group action on a finite set and a polynomial. Sagan observed the CSP on the set of non-crossing matchings on  $[2n] := \{1, 2, \dots, 2n\}$  using the cyclic group  $C_{2n}$  generated by a cyclic shift of order  $2n$  and the  $q$ -Catalan polynomial  $X(q) = \frac{1}{[n+1]_q} \left[ \begin{matrix} 2n \\ n \end{matrix} \right]_q$ . Bowling-Liang presented a similar result on the set of one-crossing matchings with a completely different proof. In this talk, we focus on the set  $P_n$  of all matchings on  $[2n]$ . We find recursions for the number of elements in  $P_n$  fixed by  $\frac{2\pi}{d}$  rotations for  $d|2n$ . We find the polynomial  $X_n(q)$  such that  $P_n$  together with  $X_n(q)$  and  $C_{2n}$  exhibits the CSP. (Received September 23, 2017)

1135-05-1582 **Josh Hallam\*** ([hallamjw@wfu.edu](mailto:hallamjw@wfu.edu)), **Jeremy Martin** and **Bruce Sagan**. *Increasing Spanning Forests in Graphs and Simplicial Complexes*.

Let  $T$  be a tree with vertices labeled by distinct integers. We say  $T$  is *increasing* if the labels along any path from the smallest vertex to any other vertex are increasing. Now suppose  $G$  is a graph with vertex set  $\{1, 2, \dots, n\}$ . An *increasing spanning forest* of  $G$  is a spanning forest such that each connected component is increasing. We will discuss some properties of the generating function for the increasing spanning forests including connections with the chromatic polynomial of the graph. We will also discuss the generalization of this work to simplicial complexes and multigraphs. (Received September 23, 2017)

1135-05-1584 **Sheila Sundaram\*** ([shsund@comcast.net](mailto:shsund@comcast.net)), Pierrepont School, One Sylvan Road North, Westport, CT 06880. *Variations on the  $S_n$ -module  $Lie_n$ : results and conjectures*. Preliminary report.

For each prime  $q$ , we define an  $S_n$ -module  $Lie_n^{(q)}$  which allows us to establish the Schur-positivity of the sum of power-sum symmetric functions indexed by partitions  $\lambda$  whose parts are (a) all powers of a fixed prime  $q$ , or (b) all relatively prime to a fixed prime  $q$ . These results are generalised by replacing the prime  $q$  with an arbitrary subset  $S$  of primes. The representations  $Lie_n(q)$  admit simple descriptions in terms of the Lie module, and are also related to the conjugacy action on the long cycles of  $S_n$ . These constructions lead to new conjectures on Schur-positivity. (Received September 23, 2017)

1135-05-1656 **Jang Soo Kim\*** ([jangsookim@skku.edu](mailto:jangsookim@skku.edu)), Department of Mathematics, Sungkyunkwan University, Suwon, South Korea. *Combinatorics of  $q$ -integrals*.

In this talk we give a combinatorial interpretation for  $q$ -integrals over order polytopes in terms of linear extensions or  $P$ -partitions. As applications, we give a combinatorial interpretation of Askey’s  $q$ -Selberg integral, product formulas for reverse plane partitions and another proof of the hook length property of the  $d$ -complete posets due to Proctor. (Received September 24, 2017)

1135-05-1681 **Ira M. Gessel**, **Sean T. Griffin** and **Vasu V. Tewari\*** ([vasut@math.washington.edu](mailto:vasut@math.washington.edu)). *Schur-positivity of Gessel’s tree symmetric function and the Catalan arrangement*.

Gessel introduced a multivariate formal power series tracking the distribution of ascents and descents in labeled binary trees. In addition to showing that it was a symmetric function, he conjectured that it was Schur-positive. I will discuss connections between specializations of Gessel’s tree symmetric function and Frobenius characteristics of symmetric group actions on certain deformations of Coxeter arrangements. I will also describe briefly an interpretation of this function in terms of the Stanley-Reisner ring of the Boolean complex. (Received September 24, 2017)

1135-05-1688 **Fan Chung\*** ([fan@ucsd.edu](mailto:fan@ucsd.edu)), 1555 Coast Walk, La Jolla, CA. *Clustering in sequences of points and graphs*. Preliminary report.

In this talk, we will examine several notions of clustering for sequences of points and graphs. We will discuss a number of old and new results and also describe various open problems. (Received September 24, 2017)

1135-05-1691 **Sebastian Donoso** and **Anh Le\***, Northwestern University, 2033 Sheridan Road, Evanston, IL, and **Joel Moreira**, **Martina Neuman** and **Wenbo Sun**. *Good lower bounds for multiple recurrence*.

In 2005, Bergelson, Host and Kra showed that if  $(X, \mu, T)$  is an ergodic probability preserving system and  $A \subset X$ , then for every  $\epsilon > 0$  there exist “many”  $n \in \mathbb{N}$  such that  $\mu(A \cap T^{-n}A \cap \dots \cap T^{-kn}A) > \mu^{k+1}(A) - \epsilon$  for  $k \leq 3$ ,

extending Khintchine's theorem which deals with  $k = 1$ . They also showed that this estimate is false in general for  $k \geq 4$ .

This phenomenon is called multiple recurrence with good lower bounds. Good lower bounds for certain polynomial expressions was studied by Frantzikinakis in 2008, but several tantalizing questions remained open. In this talk we briefly survey this topic and answer some natural questions regarding polynomial expressions, commuting transformations, and configurations involving the prime numbers. We also present evidence that some other questions are probably very hard. (Received September 24, 2017)

1135-05-1718 **Murong Xu\*** ([xumurong@math.wvu.edu](mailto:xumurong@math.wvu.edu)), 320 Armstrong Hall, P.O. Box 6310, West Virginia University, Morgantown, WV 26506, and **Hong-Jian Lai** and **Xuezheng Lv**. *On  $r$ -hued coloring of graphs without short induced paths.*

For integers  $k, r > 0$ , a  $(k, r)$ -coloring of a graphs  $G$  is a proper coloring on the vertices of  $G$  with  $k$  colors such that every vertex  $v$  of degree  $d(v)$  is adjacent to vertices with at least  $\min\{d(v), r\}$  different colors. The  $r$ -hued chromatic number, denoted by  $\chi_r(G)$ , is the smallest integer  $k$  for which a graph  $G$  has a  $(k, r)$ -coloring. Some of the recently achieved results on  $r$ -hued coloring of  $P_4$ -free graphs and  $P_5$ -free graphs will be presented. (Received September 24, 2017)

1135-05-1719 **Adam Giambrone\*** ([adam.giambrone@uconn.edu](mailto:adam.giambrone@uconn.edu)). *Ribbon Graphs, Partial Duality, and Homogeneously Adequate States of a Link Diagram.*

The fields of graph theory and knot theory are deeply connected. As observed back in the late 1800s, every (checkerboard-colored) link diagram corresponds to an edge-signed planar graph called the *Tait graph*. Alternatively, each state for a given link diagram can be used to construct a *state ribbon graph*. In this talk, we describe how the Tait graph, state ribbon graphs, and partial duality for state ribbon graphs can be used to provide a method for finding all homogeneously adequate states of a link diagram. (Received September 24, 2017)

1135-05-1733 **Glenn Hurlbert\*** ([ghurlbert@vcu.edu](mailto:ghurlbert@vcu.edu)). *Algorithmic Aspects of Graph Pebbling.*

Graph pebbling originated as a model proposed by Lagarias and Saks to solve a number theoretic problem of Erdős and Kleitman. It has since been used to solve zero-sum problems on abelian groups and  $p$ -adic solutions in additive forms, and has also grown into an area of its own. Here we discuss some of the complexity issues surrounding graph pebbling, and present some of the algorithmic approaches involved. (Received September 24, 2017)

1135-05-1737 **Patricia Hersh\*** ([plhersh@ncsu.edu](mailto:plhersh@ncsu.edu)) and **Karola Meszaros**. *Posets from polytopes.* Preliminary report.

Given a polytope  $P$  and a generic cost vector  $\mathbf{c}$ , one obtains a directed graph  $G(P, \mathbf{c})$  on the 1-skeleton of  $P$  by orienting edge  $e_{u,v}$  from  $u$  to  $v$  if and only if  $c(u) < c(v)$ . Since  $G(P, \mathbf{c})$  is acyclic, it gives rise to a partially ordered set. We give conditions on such  $G(P, \mathbf{c})$  which guarantee that every open interval of the associated poset has order complex homotopy equivalent to a ball or a sphere, as well as examples meeting these conditions. In addition to briefly discussing this, we may also mention some possible connections to the diameter of the 1-skeleton of  $P$ . (Received September 24, 2017)

1135-05-1738 **Florian Kohl** and **McCabe Olsen\*** ([mccabe.olsen@uky.edu](mailto:mccabe.olsen@uky.edu)). *Level algebras and lecture hall polytopes.* Preliminary report.

For  $\mathbf{s} = (s_1, s_2, \dots, s_n) \in \mathbb{Z}_{\geq 1}^n$ ,  $\mathbf{s}$ -lecture hall polytopes are a well studied family of simplices which admit many nice properties. In this work, we investigate which  $\mathbf{s}$ -lecture hall polytopes have Gorenstein, or more generally level, semigroup algebras. We provide a characterization of the Gorenstein property for a large family of  $\mathbf{s}$ -lecture hall polytopes and additionally provide a complete characterization of the level property in terms of  $\mathbf{s}$ -inversion sequences. (Received September 24, 2017)

1135-05-1743 **Josh Carlson\*** ([jmsdg7@iastate.edu](mailto:jmsdg7@iastate.edu)). *Throttling for Cops and Robbers.* Preliminary report.

The color change rule for zero forcing in graph  $G$  is that a blue vertex  $v$  can force a white vertex  $w$  blue if  $w$  is only white neighbor of  $v$ .  $B_0$  is initial set of blue vertices and  $B_{i+1}$  is set of blue vertices after the color change rule is applied to every vertex in the set  $B_i$ . Set  $B_0$  is a zero forcing set if there is a  $t$  such that  $B_t = V(G)$ . The zero forcing number of  $G$  is minimum size of a zero forcing set. The propagation time for  $B_0$ ,  $pt(G, B_0)$ , is smallest  $t$  such that  $B_t = V(G)$ . The zero forcing throttling number of  $G$  is the minimum of  $|B_0| + pt(G, B_0)$  where  $B_0$  ranges over all zero forcing sets of  $G$ . Zero forcing throttling was introduced and studied by Butler and Young in 2013. PSD zero forcing is a variant in which the color change rule is applied to each component of  $G - B_i$  separately. Recently results were obtained for the PSD zero forcing throttling number  $th_+$ . Cops and

robbers is a game played on a graph in which cops and a robber alternate turns moving along the edges of the graph. Cops win if a cop moves to the vertex where the robber is. Results on throttling for cops and robbers and its connection to  $th_+$  will be presented. Joint work with J. Breen, B. Brimkov, D. Ferrero, L. Hogben, K. Perry, C. Reinhart. (Received September 24, 2017)

1135-05-1747 **Laura Escobar\***, DEPARTMENT OF MATHEMATICS, 273 Altgeld Hall, 1409 W. Green Street, Urbana, IL 61801. *Bott-Samelson varieties and combinatorics.*

In the 1970's Hansen and Demazure independently constructed resolutions of singularities for Schubert varieties: the Bott-Samelson varieties. In this talk I will describe their relation with associahedra. I will also discuss joint work with Pechenick-Tenner-Yong linking Magyar's construction of these varieties as configuration spaces with Elnitsky's rhombic tilings. Finally, based on joint work with Wyser-Yong, I will give a parallel for the Barbasch-Evens desingularizations of certain families of linear spaces which are constructed using symmetric subgroups of the general linear group. (Received September 24, 2017)

1135-05-1748 **Ju Zhou\*** ([zhou@kutztown.edu](mailto:zhou@kutztown.edu)). *Perfect Matching Transitivity of Graphs.*

An automorphism of a graph  $G = (V(G), E(G))$  is a permutation of the vertex set  $V(G)$ , such that the pair of vertices  $(u, v)$  form an edge if and only if the pair  $(\sigma(u), \sigma(v))$  also form an edge. A perfect matching transitive graph is a graph such that for any two perfect matching  $M$  and  $N$  of  $G$ , there exists an automorphism of  $G$  such that  $f(M) = N$ . What kinds of graphs are perfect matching transitive? What is the relationship between perfect matching transitive and vertex transitive? What is the relationship between perfect matching transitive and edge transitive? In this talk, the author will talk about some research results about perfect matching transitive graphs. (Received September 26, 2017)

1135-05-1788 **Peter Doyle, Jay Pantone\*** ([jaypantone@dartmouth.edu](mailto:jaypantone@dartmouth.edu)) and **Everett Sullivan**. *Patterns and Colorability in Chord Diagrams.*

A chord diagram with  $n$  chords is a set of  $2n$  points in a line connected in  $n$  pairs. Chord diagrams, sometimes called matchings, play an important role in mathematical biology, knot theory, and combinatorics, and as a result they have been intensely studied by mathematicians, computer scientists, and biologists alike. In this talk we'll examine two interesting families: 2-colorable chord diagrams, and chord diagrams without short chords. Using a combination of symbolic, analytic, and experimental methods we find counting sequences, generating functions, and asymptotics for each family. (Received September 24, 2017)

1135-05-1789 **Torin Greenwood\*** ([greenwood@math.gatech.edu](mailto:greenwood@math.gatech.edu)), School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30332-0160. *Using Experimental Data to Deconvolve Structural Signals.*

The combinatorial arrangement of RNA base pairings encodes functional information, and a sequence is typically predicted to fold to a single minimum free energy conformation. But, an increasing number of RNA molecules are now known to fold into multiple stable structures. Discrete optimization methods are commonly used to predict foldings, and adding experimental data as auxiliary information improves prediction accuracy when there is a single dominant conformation. In this talk, we describe the challenges of extending the thermodynamic prediction approaches with experimental data to multimodal structural distributions. We also analyze the experimental data independently with a statistical framework, determining how the data can be improved to successfully deconvolve multimodal signals. (Received September 24, 2017)

1135-05-1791 **Daniel Johnston\*** ([johndan@gvsu.edu](mailto:johndan@gvsu.edu)), **Cory Palmer** and **Amites Sarkar**. *Rainbow Turán Numbers for Paths and Forests of Stars.*

For a fixed graph  $F$ , we consider the maximum number of edges in a properly edge-colored graph on  $n$  vertices which does not contain a rainbow copy of  $F$ , that is, a copy of  $F$  all of whose edges receive a different color. This maximum, denoted by  $ex^*(n; F)$ , is the rainbow Turán number of  $F$ , and its systematic study was initiated by Keevash, Mubayi, Sudakov and Verstráte [*Combinatorics, Probability and Computing* **16** (2007)]. In this talk, we look at  $ex^*(n; F)$  when  $F$  is a forest of stars, and consider bounds on  $ex^*(n; F)$  when  $F$  is a path with  $l$  edges, disproving a conjecture in the aforementioned paper for  $l = 4$ . (Received September 24, 2017)

1135-05-1792 **N. Alon, A. Kostochka, B. Reiniger\*** ([breiniger@iit.edu](mailto:breiniger@iit.edu)), **D. B. West** and **X. Zhu**. *Augmented trees and constructing hypergraphs with large girth and chromatic number.*

An  $r$ -augmented tree is a rooted tree with  $r$  edges added from each leaf to ancestors. For natural numbers  $d, g, r$ , we construct a bipartite  $r$ -augmented complete  $d$ -ary tree having girth at least  $g$ . The resulting graphs can be used to give several constructions of sparse graphs with large girth and large (list-)chromatic number. In

particular, a concise explicit construction of uniform hypergraphs with large girth and large chromatic number will be presented. (Received September 24, 2017)

1135-05-1802 **Michael Dairyko, Michael Ferrara, Bernard Lidický, Ryan R Martin, Florian Pfender and Andrew J Uzzell\*** (uzzellan@grinnell.edu). *Degree conditions for small contagious sets in bootstrap percolation.*

Bootstrap percolation is a cellular automaton that was introduced in 1979 by Chalupa, Leath, and Reich. Let  $r \geq 2$ . In  $r$ -neighbor bootstrap percolation on a graph  $G$ , all vertices are either “infected” or “uninfected.” The initially infected set  $A \subseteq V(G)$  grows by iteratively infecting all uninfected vertices with at least  $r$  infected neighbors. If all vertices eventually become infected, we say that the initial set  $A$  is  $r$ -contagious.

Let  $m(G, r)$  denote the minimum size of an  $r$ -contagious set in  $G$ . It is easy to see that  $m(G, r) \geq \min\{|V(G)|, r\}$ . What conditions on  $G$  imply that  $m(G, r) = r$ ? Let  $\sigma_2(G) = \min\{d(x) + d(y) : xy \notin E(G)\}$ . Freund, Poloczek, and Reichman showed that if  $\sigma_2(G) \geq n$ , then  $m(G, 2) = 2$ , and that this bound is best possible. We show that  $\sigma_2(G) \geq n - 2$  nearly ensures that  $m(G, 2) = 2$ : if  $\sigma_2(G) \geq n - 2$  and  $m(G, 2) > 2$ , then either  $G$  is a member of one of four infinite families of graphs or  $G$  is one of nine exceptional graphs. We also show that if  $G$  is a graph with degree sequence  $d_1 \leq \dots \leq d_n$  such that for all  $1 \leq i < n/2$ , either  $d_i \geq i + 1$  or  $d_{n-i} \geq n - i - 1$ , then either  $m(G, 2) = 2$ ,  $G \cong C_5$ , or  $G$  is a member of one of two infinite families of graphs. (Received September 24, 2017)

1135-05-1811 **John Sinkovic\*** (johnsinkovic@gmail.com) and **Zachary Dockstader**. *The inertia bound for a graph.* Preliminary report.

The inertia bound is an upperbound on the independence number of a graph. Attributed to D.M. Cvetković, and sometimes referred to as the Cvetković bound, it uses eigenvalues of adjacency-like matrices. It was shown recently that graphs exist for which this bound is not tight. We share some results from a recent undergraduate research project focused on finding such graphs. (Received September 25, 2017)

1135-05-1840 **Aida Abiad\*** (a.abiadmonge@maastrichtuniversity.nl), Tongersestraat 53, 6211LM Maastricht, Netherlands, and **Willem Haemers**, , Netherlands. *Switched symplectic graphs and their 2-ranks.*

We apply Godsil-McKay switching to the symplectic graphs over  $\mathbb{F}_2$  with at least 63 vertices and prove that the 2-rank of (the adjacency matrix of) the graph increases after switching. This shows that the switched graph is a new strongly regular graph with the same parameters. For the symplectic graph on 63 vertices we investigate repeated switching by computer and find many new strongly regular graphs with the above parameters for  $\nu = 3$  with various 2-ranks. Using these results and a recursive construction method for the symplectic graphs from Hadamard matrices, we obtain several graphs with the above parameters, but different 2-ranks. (Received September 25, 2017)

1135-05-1861 **Sarah Wolff\*** (wolffs@denison.edu). *The Ballots are Missing, Who Won the Election? Inferring Rankings from First Order Marginals.*

Motivated by applications in rankings-based elections we consider the question of recovering election results from first order marginals. For example, in an election between three candidates, a voter ranks the candidates in her preferred order, and a function  $f : S_3 \rightarrow [0, 1]$  gives the normalized count of the votes for each ranking. The first-order marginals in this situation tell how many people chose each candidate in first place, second place, and third place. Given this information, can one uniquely recover the original function  $f$ ? First considered by Shah and Jagabathula, they determined that one can recover a  $k$ -sparse function  $f$  so long as it satisfies three constraints: the nonzero values of  $f$  are distinct, each permutation  $\rho$  in the support of  $f$  has a unique edge, and no subset of function values sums to another function value. We relax the third condition to find that recovery is still possible if the function satisfies only the first two conditions. Capturing the first order marginals using a ‘first-order matrix’ allows us to rephrase the question in terms of the combinatorial structure of the matrix. We give an explicit characterization of the structure, which in turn allows us to determine the original function. (Received September 25, 2017)

1135-05-1881 **Sara Solhjem and Jessica Striker\*** (jessica.striker@ndsu.edu). *Sign matrix polytopes from Young tableaux.*

Motivated by the study of polytopes formed by the convex hull of permutation matrices and alternating sign matrices, we define two new families of polytopes as convex hulls of sign matrices, which are certain  $\{0, 1, -1\}$  matrices in bijection with semistandard Young tableaux. We investigate various properties of these polytopes, including their inequality descriptions, vertices, facets, and face lattice. (Received September 25, 2017)

1135-05-1899 **Michael Ferrara, Daniel Johnston, Sarah Loeb\*** (s1loeb@wm.edu), **Florian Pfender, Alex Schulte, Heather Smith, Eric Sullivan, Michael Tait and Casey Tompkins.** *Edge-colored saturation for complete graphs.* Preliminary report.

Let  $\mathcal{C}$  be a family of edge-colored graphs. A  $t$ -edge-colored graph  $G$  is  $(\mathcal{C}, t)$ -saturated if  $G$  does not contain any element of  $\mathcal{C}$ , but for any edge  $e \notin G$  and any color  $i \in [t]$ , the addition of  $e$  to  $G$  in color  $i$  creates some element of  $\mathcal{C}$  in  $G$ . Let  $\text{sat}_t(n, \mathcal{C})$  denote the minimum number of edges in a  $(\mathcal{C}, t)$ -saturated graph on  $n$  vertices.

Let  $\mathcal{C}_r(H)$  be the family consisting of every edge-colored copy of  $H$  in which exactly  $r$  colors are used on  $E(H)$ . We identify  $\text{sat}_t(n, \mathcal{C}_2(K_3))$  when  $t \geq 2$ . For  $\mathcal{R}(K_k) = \mathcal{C}_{\binom{k}{2}}(K_k)$ , we show  $\text{sat}_t(n, \mathcal{R}(K_k)) \geq c \log n$ , which improves a lower bound from Barrus, Ferrara, Vandenbushe, and Wenger, and matches their upper bound. We give additional results for  $\text{sat}_t(n, \mathcal{C}_r(K_k))$  for  $t \geq r$  as  $r$  ranges from 1 to  $\binom{k}{2}$ . (Received September 25, 2017)

1135-05-1928 **Michael Chmutov, Max Glick\*** (glick.107@osu.edu) and **Pavlo Pylyavskyy.** *The Berenstein-Kirillov group and cactus groups.*

Berenstein and Kirillov have studied the action of Bender-Knuth moves on semistandard tableaux. Losev has studied a cactus group action in Kazhdan-Lusztig theory; in type A this action can also be identified in the work of Henriques and Kamnitzer. We establish the relationship between the two actions. We show that the Berenstein-Kirillov group is a quotient of the cactus group. We use this to derive previously unknown relations in the Berenstein-Kirillov group. This is joint work with M. Chmutov and P. Pylyavskyy. (Received September 25, 2017)

1135-05-1935 **Robin Anderson**, Saint Louis University, **Shuliang Bai**, University of South Carolina, **Fidel Barrera-Cruz\*** (fidelbc@math.gatech.edu), Georgia Institute of Technology, **Éva Czabarka**, University of South Carolina, **Giordano Da Lozzo**, University of California, Irvine, **Natalie L. F. Hobson**, Sonoma State University, **Jephian C.H. Lin**, Iowa State University, **Austin Mohr**, Nebraska Wesleyan University, **Heather C. Smith**, Georgia Institute of Technology, **László A. Székely**, University of South Carolina, and **Hays Whitlatch**, University of South Carolina. *Crossing number and the tangle crossing number analogies.*

A tanglegram consists of a pair of rooted binary plane trees with the same number of leaves, and a perfect matching between the two leaf-sets. These structures are of use in phylogenetics and are represented with straightline drawings where the leaves of the two plane binary trees are on two parallel lines and only the matching edges can cross. The tangle crossing number of a tanglegram is the minimum crossing number over all such drawings and is related to biologically relevant quantities, such as the number of times a parasite switched hosts.

In this talk we present results for tanglegrams which parallel known theorems for the crossing number of graphs. We show that removing any matching edge from a tanglegram of size  $n$  results in a drop of the tangle crossing number by at most  $n - 3$ , and this is sharp. We also show that the maximum tangle crossing number of a tanglegram of size  $n$ , denoted  $\gamma(n)$ , satisfies  $\frac{1}{2} \binom{n}{2} (1 - o(1)) \leq \gamma(n) < \frac{1}{2} \binom{n}{2}$ . Finally we present an algorithm for computing non-trivial lower bounds on the tangle crossing number in  $O(n^4)$  time. (Received September 25, 2017)

1135-05-1939 **Brendan Pawlowski and Brendon Rhoades\*** (bprhoades@math.ucsd.edu). *Line configurations and the Delta Conjecture.*

The *Delta Conjecture* of Haglund-Remmel-Wilson predicts an extension of the Shuffle Theorem of Macdonald theory. In joint work with Haglund and Shimozono, the second author defined a graded  $S_n$ -module which plays the role of a ‘coinvariant ring’ for the Delta Conjecture. It is well known that the coinvariant ring presents the cohomology of the manifold of complete flags in  $\mathbb{C}^n$ . Given two positive integers  $k \leq n$ , we describe a variety  $X_{n,k}$  of ‘line configurations’ which serves as a generalization of the flag manifold for the Delta Conjecture. (Received September 25, 2017)

1135-05-1942 **Emily Barnard\*** (e.barnard@northeastern.edu), Northeastern University, 567 Lake Hall, Boston, MA 02115. *The biCatalan Kreweras Complement.* Preliminary report.

In Coxeter-biCatalan combinatorics, we study the combinatorics of a certain “doubling” or “twinning” of classical Catalan objects. In this talk, we discuss the biCatalan analogue to the classical noncrossing partition lattice and the Kreweras complement. (Received September 25, 2017)

1135-05-1955 **David Burstein\*** ([dburste1@swarthmore.edu](mailto:dburste1@swarthmore.edu)). *Tools for constructing graphs with fixed degree sequences.*

To detect statistical anomalies in networks, we compare empirically observed networks with realizations from a realistic random graph model. Since many real world networks exhibit degree heterogeneity, we consider some challenges in randomly constructing graphs with a given bidegree sequence in an unbiased way. In particular, we propose a novel method for the asymptotic enumeration of directed graphs that realize a bidegree sequence,  $d$ , with maximum degree  $d_{max} = O(S^{\frac{1}{2}-\tau})$  for an arbitrarily small positive number  $\tau$ , where  $S$  is the number of edges specified by  $d$ ; the previous best results allow for  $d_{max} = o(S^{\frac{1}{3}})$ . Our approach is based on two key steps, graph partitioning and degree preserving switches. The former allows us to relate enumeration results to degree sequences that are easy to handle, while the latter facilitates expansions based on numbers of shared neighbors of pairs of nodes. (Received September 25, 2017)

1135-05-1957 **Marie Meyer\*** ([marie.meyer@uky.edu](mailto:marie.meyer@uky.edu)) and **Benjamin Braun**. *Laplacian Simplices.*

There are many advantageous ways to associate a polytope to a graph  $G$ . A recent construction is to consider the convex hull of the rows of the Laplacian matrix of  $G$  to form what is called the Laplacian simplex,  $T_G$ . In this talk we focus on properties of  $T_G$  including reflexivity, the integer decomposition property, and unimodality of Ehrhart  $h^*$ -vectors according to graph type. (Received September 25, 2017)

1135-05-1960 **Lindsey-Kay Lauderdale\*** ([llauderdale@uttyler.edu](mailto:llauderdale@uttyler.edu)). *On the fixing sets of finite groups.* Preliminary report.

The *fixing number* of a graph  $\Gamma$  is the minimum number of labeled vertices that, when fixed, remove all nontrivial automorphisms from the automorphism group of  $\Gamma$ . The *fixing set* of a finite group  $G$  is the set of all fixing numbers of graphs whose automorphism groups are isomorphic to  $G$ . Previously, authors have studied the fixing sets of both abelian groups and symmetric groups. In this talk, we will discuss the fixing sets of dihedral groups and symmetric groups. (Received September 25, 2017)

1135-05-1971 **Franklin H. J. Kenter\*** ([kenter@usna.edu](mailto:kenter@usna.edu)). *Partitioning with Eigenvector Norms.*

We investigate the role of eigenvector norms in spectral graph theory to various combinatorial problems including the densest subgraph problem, the Cheeger constant, among others.

We introduce randomized spectral algorithms that produce guarantees which, in some cases, are better than the classical spectral techniques. In particular, we will give an alternative Cheeger “sweep” (graph partitioning) algorithm which provides a linear spectral bound for the Cheeger constant at the expense of an additional factor determined by eigenvector norms.

Finally, we apply these ideas and techniques to problems and concepts unique to directed graphs. (Received September 25, 2017)

1135-05-1987 **William Gasarch\*** ([gasarch@cs.umd.edu](mailto:gasarch@cs.umd.edu)), **Guangqi Cui**, **John Dickerson**, **Naveen Durvasula**, **Erik Metz**, **Naveen Raman** and **Sung Hyun Yoo**. *The Muffin Problem.*

Consider the following problem:

You have 5 muffins and 3 students. You want to divide the muffins evenly so that everyone gets  $5/3$  of a muffin. You can clearly divide each muffin in 3 pieces and give each person  $5/3$ . Note- the smallest piece is of size  $1/3$ .

Is there a procedure where the smallest piece is bigger than  $1/3$ ? (Hint: You Can!)

More generally: What is the best you can do with  $m$  muffins and  $s$  students? We will talk about many of our General Theorems we have on this problem. And more concretely we will:

- (1) Establish what happens when the number of students is 1,2,3,4.
- (2) What about  $s=5$ ? Hmm. That's when things get weird. How weird? Come and find out!

(Received September 25, 2017)

1135-05-1998 **Breann Flesch**, **Jessica Kawana**, **Joshua D Laison\*** ([jlaison@willamette.edu](mailto:jlaison@willamette.edu)), **Dana Lapidés** and **Stephanie Partlow**. *Veto Interval Graphs and Variations.*

We introduce a variation of interval graphs, called veto interval (VI) graphs. A VI graph is represented by a set of closed intervals, each containing a point called a veto mark. The edge  $ab$  is in the graph if the intervals corresponding to the vertices  $a$  and  $b$  intersect, and neither contains the veto mark of the other. We find families of graphs which are VI graphs, and separating examples of graphs which are not VI graphs. We also introduce related graph families, including unit VI graphs and midpoint unit VI (MUVI) graphs. (Received September 25, 2017)

1135-05-2023 **Benjamin Steinhurst\*** ([bsteinhurst@mcdaniel.edu](mailto:bsteinhurst@mcdaniel.edu)), 2 College Hill, McDaniel College, Westminster, MD 21157. *Fibonacci Fractals: From Global to Local Structure*. Preliminary report.

Fibonacci words are constructed by taking  $f_1 = 0$  and  $f_2 = 1$  and defining  $f_2 = 01$  by concatenation. So  $f_3 = f_2f_1 = 010$ . This is a fine example of a global, or large scale, construction rule. On the other hand  $L$ -systems create a sequence of drawing instructions by using a local substitution and then parsing the instructions symbol by symbol to create an image. As coming from Monnerot-Dumaine, a Fibonacci word can be drawn symbol by symbol using a drawing rule as one would for an  $L$ -system. There is a self-similarity in the Fibonacci word that becomes evident when these drawings are properly rescaled.

To even prove the existence of the scaling limit one has to create a local replacement rule to complement to global concatenation rule. Any understanding of the fractal geometry of the scaling limits also relies completely on this global-to-local transfer. The Fibonacci fractal is the simplest example of such a global sequence defining rule being transformed into a local substitution rule that can be studied. I will present the current status of the problem and point out several open questions that remain. (Received September 25, 2017)

1135-05-2028 **Bruno Benedetti\*** ([bruno@math.miami.edu](mailto:bruno@math.miami.edu)), Department of Mathematics, 1365 Memorial Drive, Coral Gables, FL 33146. *A random model for algebraic curves, with a threshold for Cohen-Macaulayness*. Preliminary report.

A problem in introducing random models in commutative algebra, is that natural properties like Cohen-Macaulayness end up being unlikely. We sketch a curious random model in which there is a natural probabilistic threshold for Cohen-Macaulayness. Given a graph  $G$ , one can construct an algebraic curve  $C$  with dual graph  $G$ , with the property that  $G$  is connected if and only if  $C$  is arithmetically Cohen-Macaulay. So if we start with an Erdős-Rényi graph  $G=G(n,p)$ , the construction gives a "random curve  $C=C(n,p)$ " dual to it; and this curve has an obvious threshold for arithmetic Cohen-Macaulayness, given by Erdős-Rényi's theorem. If time permits, we sketch how the Linial-Meshulam model of random 2-complexes yields a notion of "random algebra" with a threshold for having depth at least 3. (This relies on work with Di Marca and Varbaro). (Received September 25, 2017)

1135-05-2044 **Chris A Rodger** ([rodgecr1@auburn.edu](mailto:rodgecr1@auburn.edu)), 221 Parker Hall, Auburn University, Auburn, AL 36849, and **Bin Yeh\*** ([bzy0006@auburn.edu](mailto:bzy0006@auburn.edu)), 221 Parker Hall, Auburn University, Auburn, AL 36849. *Decomposing Graphs With Two Associate Classes Into Paths Of Length 3*.

For nonnegative integers  $n, p, \lambda_1$ , and  $\lambda_2$ , the graph of two associate classes  $G(n, p, \lambda_1, \lambda_2)$  is the graph with  $np$  vertices, partitioned into  $p$  parts  $V_1, \dots, V_p$ , each of size  $n$ , in which two vertices are joined by  $\lambda_1$  edges if they are in the same part, and by  $\lambda_2$  edges if they are in different parts. In this paper, we completely settled the decomposition problem of graphs with two associate classes into paths of length 3. (Received September 25, 2017)

1135-05-2048 **Evangelos Kranakis, Danny Krizanc and Yun Lu\***, Department of Mathematics, Kutztown University of PA, Kutztown, PA 19530. *Reconstructing Cactus Graphs from Shortest Path Information*.

In this paper, we investigate the problem of reconstruct graphs based on the given information. In particular, if the underlying network is a cactus graph, then we are able to provide an algorithm that returns all node-labelled cacti consistent with a given set shortest path information. (Received September 25, 2017)

1135-05-2051 **Joshua A. Grochow\*** ([jgrochow@colorado.edu](mailto:jgrochow@colorado.edu)), 1111 Engineering Drive, ECOT 717, 430 UCB, Boulder, CO 80309-0430. *The Cap Set Conjecture, the polynomial method, and applications (after Croot-Lev-Pach, Ellenberg-Gijswijt, and others)*.

The card game Set asks players to find lines in a subset - drawn from a deck of cards - of the four-dimensional vector space over the integers mod 3. In  $n$ -dimensional generalized Set, we get the seemingly-innocuous cap set question: how large can a subset of  $(\mathbb{Z}_3)^n$  be and still contain no lines? This question stood open for 30 years; in this talk, we'll see the beautiful, elementary, and wonderfully short proof of the Cap Set Conjecture (that the largest subset is exponentially smaller than the whole space), due to Ellenberg and Gijswijt, following on Croot-Lev-Pach. We will also see some of the many applications of the result and its proof, not only in combinatorics, but also in commutative algebra, the geometry of tensors, and computational complexity. Little background will be assumed beyond linear algebra over the integers modulo a prime. (Received September 25, 2017)



1135-05-2070 **Nathan F Williams\*** ([nathan.f.williams@gmail.com](mailto:nathan.f.williams@gmail.com)) and **Marko Thiel**.  
*Winnie-the-Pooh and the Strange Expectations*. Preliminary report.

We previously generalized D. Armstrong’s conjecture (and P. Johnson’s proof) for the expected number of boxes in a simultaneous core to simply-laced type (we now have a generalization to all affine Weyl groups). By analogy, we were led to consider the expected norm of a weight in a highest weight representation  $V_\lambda$  of a complex semisimple Lie algebra  $\mathfrak{g}$ . We give a proof that this expectation is  $\frac{1}{n+1}(\lambda + 2\rho, \lambda)$  by relating it to the “Winnie-the-Pooh problem.” Our proof works for all types except  $A$  and  $C$ ; the same formula holds in these two remaining types, but we are forced to provide a direct computation. This is joint work with Marko Thiel. (Received September 25, 2017)

1135-05-2093 **Shira Viel\*** ([shira\\_viel@ncsu.edu](mailto:shira_viel@ncsu.edu)), North Carolina State University, Department of Mathematics, Box 8205, Raleigh, NC 27695. *Surfaces, orbifolds, and dominance*. Preliminary report.

The associahedron is a well-known  $n$ -dimensional polytope whose vertices are labeled by triangulations of an  $(n+3)$ -gon with edges given by diagonal flips. The cyclohedron is defined analogously using centrally-symmetric triangulations of a  $(2n+2)$ -gon, or, modding out by the symmetry, triangulations of an  $(n+1)$ -gon with a single two-fold branch point in the center. The polytopes can be realized in such a way that their normal fans are the “ $\mathfrak{g}$ -vector fans” or “mutation fans” for certain cluster algebras, and the normal fan of the cyclohedron refines that of the associahedron. In this talk, we will justify and generalize this mutation fan refinement relationship as a consequence of a simple combinatorial operation which maps the  $(n+1)$ -gon with one “orbifold point” to the  $(n+3)$ -gon and more generally maps a triangulated orbifold to a triangulated surface while preserving the number of diagonals and respecting adjacencies. This induces a relationship between the signed adjacency matrices associated to the triangulations, called dominance, which gives rise to many interesting phenomena which we will discuss as time permits. (Received September 25, 2017)

1135-05-2100 **Leslie Hogben\*** ([hogben@aimath.org](mailto:hogben@aimath.org)). *On crossing numbers of complete multipartite graphs*.

The crossing number  $\text{cr}(G)$  of a graph  $G$  is the minimum number of crossings in a nondegenerate planar drawing of  $G$ . There has been extensive study of crossing numbers of complete bipartite graphs and complete graphs since Turán posed the question for the complete bipartite graph. Much less is known for complete tripartite graphs. Zarankiewicz established an upper bound for  $\text{cr}(K_{n,n})$ . We establish an upper bound for crossing number of the complete tripartite graph analogous to Zarankiewicz’s bound.

A complete multipartite graph is balanced if the partite sets all have the same cardinality. Richter and Thomassen proved in 1997 that the limit as  $n \rightarrow \infty$  of  $\text{cr}(K_{n,n})$  over the maximum number of crossings in a drawing of  $K_{n,n}$  exists and is at most  $\frac{1}{4}$ . We show that for a fixed  $r$  and the balanced complete  $r$ -partite graph,  $\zeta(r) := \frac{3(r^2-r)}{8(r^2+r-3)}$  is an upper bound to the limit superior of the crossing number divided by the maximum number of crossings in a drawing.

Joint work with E. Gethner, B. Lidický, F. Pfender, A. Ruiz, Michael Young (Received September 25, 2017)

1135-05-2113 **Amanda Ruiz\*** ([alruiz@sandiego.edu](mailto:alruiz@sandiego.edu)). *Realization Spaces of Phased Matroids*.

Consider a finite set of vectors  $V \in \mathbb{C}^r$ . The phased matroid of  $V$  is a tool for keeping track of some of the geometric information of  $V$ . Phased matroids are a new field which do for the complex numbers what oriented matroids do for phased matroids.

More recently, Baker and Bowler have introduced matroids over hyperfields, which oriented matroids and phased matroids are special cases of, drawing attention to the properties they have in common.

In this talk we will discuss a the realization space of phased matroids, a property that is quite different for phased matroids than oriented matroids. (Received September 25, 2017)

1135-05-2124 **Sinan G Aksoy\***, Pacific Northwest National Laboratory, Richland, WA 99352, and **Eduardo Cotilla-Sanchez**, **Mahantesh Halappanavar** and **Emilie Purvine**. *A Generative Graph Model for Electrical Infrastructure Networks*.

We propose a generative graph model for electrical infrastructure networks that accounts for heterogeneity in both vertex and edge type. To inform the design of this model, we analyze the properties of power grid graphs derived from the real-world Eastern Interconnect, Texas Interconnect, and Polish power grids. Across these datasets, we find that subgraphs induced by vertices of the same voltage level exhibit shared structural properties atypical to small-world networks, while subgraphs induced by so-called transformer edges linking vertices of different voltage types contain a more limited structure, consisting mainly of small, disjoint star graphs. We propose a

two-phase graph model, based on the Chung-Lu model, that is designed to match both these inter and intra-network properties. Our model may be used to generate synthetic graphs, test algorithms and hypotheses at different scales, and serve as a baseline model on top of which further information about the electrical network may be appended. Lastly, we demonstrate the model's effectiveness on the aforementioned data. (Received September 25, 2017)

1135-05-2137 **Colin Starr\*** ([cstarr@willamette.edu](mailto:cstarr@willamette.edu)), **Erin McNicholas**, **Will Agnew-Svoboda**, **Alana Huszar**, **Jeff Schreiner-McGraw** and **Corrine Yap**. *Unipancyclic Binary Matroids*.

Analogous to the concept of uniquely pancyclic graphs, we define a uniquely pancyclic (UPC) matroid of rank  $r$  to be a (simple) rank- $r$  matroid containing exactly one circuit of each length  $\ell$  for  $3 \leq \ell \leq r + 1$ . Our discussion addresses the existence and properties of UPC matroids. We consider properties of binary UPC matroids in general, and prove that all binary UPC matroids have a connectivity of 2. (Received September 25, 2017)

1135-05-2142 **Oscar Levin\*** ([oscar.levin@unco.edu](mailto:oscar.levin@unco.edu)), 501 20th Street, Campus Box 122, Greeley, CO 80639. *Prime labelings of infinite graphs with connections to computability theory*.

A prime labeling of a graph is a function which maps vertices to positive integers such that adjacent vertices have relatively prime labels. This notion of graph labeling has been well studied for finite graphs; here we extend the definition to infinite graphs. We will give a simple necessary and sufficient condition for an infinite graph to have a prime labeling, and then use ideas from computability theory to prove that our condition is as simple as possible. The talk will conclude with some ideas for future research into infinite graph labelings suitable for undergraduate students. (Received September 25, 2017)

1135-05-2155 **Justin M. Troyka\*** ([jmtroyka@math.dartmouth.edu](mailto:jmtroyka@math.dartmouth.edu)) and **Sergi Elizalde**. *Exact and asymptotic enumeration of cyclic permutations according to descent set*.

We use a bijection of Gessel & Reutenauer (1993) to find a simple formula for the number of cyclic permutations with a given descent set, by expressing this number in terms of ordinary descent numbers (*i.e.* those counting all permutations with a given descent set). This formula has several consequences, including the theorem that, for almost all sets  $I \subseteq [n - 1]$ , the fraction of size- $n$  permutations with descent set  $I$  which are cycles is asymptotically  $1/n$ . As a special case, we recover a result of Stanley (2007) for alternating cycles. We also compute the quasisymmetric generating function for permutations avoiding  $k - 1$  consecutive descents, and in the case of  $k = 3$  we use this to find a simple formula for the number of cyclic permutations avoiding two consecutive descents. (Received September 25, 2017)

1135-05-2187 **R. W. R. Darling** and **Dev R Phulara\*** ([phulara@comcast.net](mailto:phulara@comcast.net)). *The combinatorial data fusion problem: A Gomory-Hu cut tree approach*.

Two different data sources may place incompatible relational structures on the same set  $V$  of objects. A connected weighted graph  $(V, E, w)$  arises from one data source and other data source introduces an independence system  $\mathcal{S}$  on  $V$ , which may be characterized by its circuits, called forbidden sets. The *combinatorial data fusion problem* seeks a subgraph  $(V, E_1)$  of  $(V, E)$  of maximum edge weight so that no vertex component of  $(V, E_1)$  contains any forbidden set. In this talk we will introduce the combinatorial data fusion problems and show how it generalizes many well known combinatorial optimization problems. We will also discuss some approximation algorithms based on Gomory-Hu cut tree to solve those problems. (Received September 25, 2017)

1135-05-2206 **K. A. S. Factor** and **S. K. Merz\*** ([smerz@pacific.edu](mailto:smerz@pacific.edu)), Department of Mathematics, University of the Pacific, Stockton, CA 95219. *Split Domination in Tournaments*.

In a strongly connected digraph, we consider the problem of finding a set of minimum size that is both dominating and separating. A set of vertices,  $S$ , is dominating provided for all  $v$  in the digraph, either  $v \in S$  or  $(w, v) \in A(D)$  for some  $w \in S$ . In a strongly connected digraph, a set of vertices is separating provided removing this set of vertices results in a digraph that is either trivial or not strongly connected. Let  $D$  be a strongly connected digraph. Then  $\gamma_s(D)$  denotes the split domination number of the digraph, that is, the minimum size of a subset of  $V(D)$  that is both dominating and separating. We consider  $\gamma_s(T)$  where  $T$  is a particular type of tournament. (Received September 26, 2017)

1135-05-2207 **Joel Nishimura\*** (joel.nishimura@gmail.com), **Bailey K Fosdick** (bailey.fosdick@colostate.edu), **Daniel B Larremore** (daniel.larremore@colorado.edu) and **Johan Ugander** (jugander@stanford.edu). *Random graph models with fixed degree sequences: choices, consequences and irreducibility proofs for sampling.*

It is surprisingly difficult to determine which features of an empirical graph are noteworthy—a task that requires choosing an appropriate null model against which to compare. Since empirical graphs have distinctive degree sequences, one of the most popular null models is the configuration model: a uniform distribution over graphs with a fixed degree sequence. While it is commonly treated as though there is only a single configuration model, one sampled via stub-matching, there are many, depending on whether self-loops and multiedges are allowed and whether edge stubs are labeled or not. We show, these different configuration models can lead to drastically, sometimes opposite, conclusions. In order to sample from these different configuration models, we review and develop the underpinnings of Markov chain Monte Carlo methods based upon double-edge swaps. Namely, we present new results on the irreducibility of the Markov chain for graphs with self-loops, either proving irreducibility or exactly characterizing the degree sequences for which the Markov chain is reducible. This work completes the study of the irreducibility of double edge-swap Markov chains (and the related Curveball Markov chain) for all combinations of allowing self-loops, multiple self-loops and/or multiedges. (Received September 25, 2017)

1135-05-2224 **Thomas A McConville\*** (thomasmcconvillea@gmail.com), 77 Massachusetts Ave, Cambridge, MA 02139, and **Pavel Galashin, Sam Hopkins** and **Alexander Postnikov**. *Root system chip-firing.*

We introduce a variation on the classical chip-firing game called the vector-firing process. The chip-firing game, also known as the Abelian Sandpile Model, is a famous example of a confluent system. That is, given an initial configuration of chips on a graph, there may be many ways to "play" the game, but the ending configuration of chips does not depend on the choices made. Recently, Jim Propp introduced labeled chip-firing on a line, which exhibits partial confluence in the sense that it is confluent from some initial configurations but not for others. With an aim to generalize and explain the partial confluence of the labeled chip-firing game, we introduce the vector-firing process on points in a real vector space. In this talk, I will define this process, focusing on the root system case, and present several results and conjectures on the confluence of these systems. This is based on joint work with Pavel Galashin, Sam Hopkins, and Alexander Postnikov. (Received September 25, 2017)

1135-05-2233 **Omar A. AbuGhneim, Dylan Peifer\*** (djp282@cornell.edu) and **Ken W. Smith**. *All (96, 20, 4) Difference Sets.*

A  $\langle v, k, \lambda \rangle$  difference set is a nonempty proper subset  $D$  of a finite group  $G$  such that  $|G| = v$ ,  $|D| = k$ , and each nonidentity element of  $G$  can be written as  $d_i d_j^{-1}$  for  $d_i, d_j \in D$  in exactly  $\lambda$  different ways. In 1978, Robert Kibler at the National Security Agency in Fort Meade, Maryland published a description of all noncyclic difference sets with  $k < 20$ . Kibler's decision to stop his extensive computer search for difference sets at block size 19 was motivated partly by the difficult barrier at  $k = 20$ , the difference sets with parameters (96, 20, 4). In this talk, we announce the completion of the search for all (96, 20, 4) difference sets in the 231 groups of order 96, relying on the computer software GAP and the work of numerous authors over the last few decades. We also introduce a GAP package that implements a general algorithm for exhaustively enumerating all difference sets in any group, and discuss its strengths and limitations. (Received September 25, 2017)

1135-05-2262 **Tomas E. Guardia\*** (guardia@gonzaga.edu), 728 East Mission Avenue, Spokane, WA 99202. *Fiboquadratic Numbers as a Solution of a Recurrence Relation*. Preliminary report. Fiboquadratic numbers were introduced as a consequence of an infinite extension of the board of Rithmomachia. A first approach was to express these numbers as quadratic terms of Fibonacci Numbers. Now, we will see that Fiboquadratic numbers are a solution of a linear recurrence relation which is a combination of Lucas Numbers. (Received September 25, 2017)

1135-05-2280 **Thomas M McKenzie\*** (mckenzie@gonzaga.edu), Gonzaga University, Department of Mathematics, 502 East Boone Avenue, Spokane, WA 99258. *Book Embeddings and Matroids*. Preliminary report.

We develop the concept of an outerplanar matroid. Then we look at examples of such matroids from graph theory, linear algebra, and commutative algebra. Next we use outerplanar matroids to develop the concept of a matroid book embedding. Finally, we look at examples of outerplanar book embeddings in the three areas mentioned above. (Received September 25, 2017)

1135-05-2284 **Emily Gunawan\*** ([emily.gunawan@uconn.edu](mailto:emily.gunawan@uconn.edu)), Department of Mathematics, University of Connecticut, 341 Mansfield Road U1009, Storrs, CT 06269-1009. *Cluster algebras and binary words*. Preliminary report.

Cluster algebras, introduced in 2000, are a class of commutative rings with rich combinatorial structure. They are connected to many areas of mathematics and physics, including representation theory, dynamical systems, geometry, and string theory.

We establish a connection between cluster algebras from triangulated surfaces and binary words. Given a cluster variable, we associate to it a finite binary word. The subwords of this binary word are in bijection with the terms of the cluster variable. (Received September 25, 2017)

1135-05-2286 **Lindsay A Erickson\*** ([lindsay.erickson@augie.edu](mailto:lindsay.erickson@augie.edu)), 2001 S Summit Avenue, Sioux Falls, SD 57197. *Machine learning applied to edge-Nim on graphs*. Preliminary report.

In order to further the scope of combinatorial research in games on graphs, we created an online computer vs human version of edge-Nim on graphs. This version, called “Trap-tactic” (available online at [traptactic.augie.edu](http://traptactic.augie.edu)), captures move information in an effort to sift through move sequences and isolate winning conditions, specifically on the  $K_{3,3}$  graph. Upon capturing enough iterations of the game, we then apply machine learning algorithms to the data to predict the winner of the game under given initializations. This paper will discuss the range and scope of machine learning algorithms and the results generated from their application. (Received September 25, 2017)

1135-05-2287 **James Zak\*** ([jameszak@live.unc.edu](mailto:jameszak@live.unc.edu)). *The Total Coloring Game on Forests*.

In the *total coloring game* two players, Alice and Bob, take turns coloring the vertices and edges (elements) of a graph from a given set of colors subject to the constraint that no two adjacent or incident elements can be the same color. If all elements of the graph are eventually colored, then Alice wins the game. Otherwise, Bob wins, meaning there is an uncolored element of the graph with no remaining legal color. The *total game chromatic number* of  $G$ , denoted  $\chi_g''(G)$ , is the least number of colors for which Alice has a winning strategy on  $G$ . We have shown that on any forest  $F$ ,  $\chi_g''(F)$  can be bounded above by  $\Delta(F) + 3$ , and that there exists a family of forests  $\mathcal{F}$ , for which  $\chi_g''(F) \geq \Delta(F) + 2$  for all  $F \in \mathcal{F}$ . (Received September 25, 2017)

1135-05-2308 **Carlos A Alfaro** ([alfaromontufar@gmail.com](mailto:alfaromontufar@gmail.com)), , Mexico, **Walter Carballosa\*** ([waltercarb@gmail.com](mailto:waltercarb@gmail.com)), Miami, FL, **Silvia Fernandez-Merchant** ([silvia.fernandez@csun.edu](mailto:silvia.fernandez@csun.edu)), Northridge, CA, **Alain Olavarrieta** ([alainolavarrieta@gmail.com](mailto:alainolavarrieta@gmail.com)), Austin, TX , and **Yakov Sapozhnikov** ([yakov.sapozhnikov.473@my.csun.edu](mailto:yakov.sapozhnikov.473@my.csun.edu)), Northridge, CA. *On the rectilinear crossing numbers of complete 4-partite graphs*.

In this work we study the rectilinear crossing number of complete 4-partite graphs. We prove that the rectilinear crossing number of complete 4-partite graph  $K_{2,2,2,2}$  is equal to 8, which shows that topological crossing number of the complete 4-partite  $K_{n_1, n_2, n_3, n_4}$  is not always equal to its rectilinear crossing number since  $cn(K_{2,2,2,2}) = 6$ . Besides, using an optimal rectilinear drawing of  $K_{2,2,2,2}$ , we give a general rectilinear construction of  $K_{n_1, n_2, n_3, n_4}$  that provides an upper bound of the rectilinear crossing number of complete 4-partite graphs. This upper bound improves some previous known upper bounds. For instance, our general construction improves the upper bound for  $cr(K_{2,2,2,3})$  from 15 to 12, settling  $cr(K_{2,2,2,3}) = \overline{cr}(K_{2,2,2,3}) = 12$ . This upper bound establishes an inequality about the crossing number of complete 4-partite graphs similar to the Zarankiewicz’s conjecture. This inequality provides a beneficial result for a large class of complete 4-partite graphs respect to an extended kind of the Zarankiewicz’s conjecture for complete 4-partite graphs, like the well-known one for complete tripartite graphs. (Received September 25, 2017)

1135-05-2324 **Ryan Schneider** and **John Shareshian\*** ([shareshi@math.wustl.edu](mailto:shareshi@math.wustl.edu)). *Regular Hessenberg varieties and parabolic characters of Hecke algebras*.

I will discuss our joint work relating the Betti numbers of regular Hessenberg varieties to character values of Hecke algebras at Kazhdan-Lusztig basis elements. (Received September 25, 2017)

1135-05-2332 **Jineon Baek**, **Hayan Nam\*** ([hayann@uci.edu](mailto:hayann@uci.edu)) and **Myungjun Yu**. *Johnson’s bijections and their application to counting simultaneous core partitions*.

A partition with no hook lengths divisible by  $a$  is called an  $a$ -core partition. For two coprime numbers  $a$  and  $b$ , a partition is called an  $(a, b)$ -core partition if it is both  $a$ -core and  $b$ -core partition. Johnson recently proved Armstrong’s conjecture which states that the average size of an  $(a, b)$ -core partition is  $(a + b + 1)(a - 1)(b - 1)/24$ . He uses various coordinate changes and one-to-one correspondences that are useful for other counting problems about simultaneous core partitions. We give an expression for the number of  $(a, b_0, b_1, \dots, b_n)$ -core partitions

where  $a$  and  $b_0$  are coprime. We also evaluate the largest size of a self-conjugate  $(s, s + 1, s + 2)$ -core partition. (Received September 26, 2017)

1135-05-2333 **Ae Ja Yee\*** (aey2@psu.edu), University Park, PA 16802, and **Seunghyun Seo**. *Singular overpartitions and partitions with prescribed hook difference conditions*. Preliminary report. Singular overpartitions, which were defined and studied by George Andrews, are overpartitions whose Frobenius symbols have at most one overlined entry in each row. This new subclass of overpartitions naturally arises from the construction of overpartition analogues of Rogers–Ramanujan type theorems for ordinary partitions with restricted successive ranks. Hook differences are more generalized notion of successive ranks. In this talk, I will discuss singular overpartitions associated with a theorem for partitions with prescribed hook differences by Andrews, Baxter, Bressoud, Burge, Forrester and Viennot. This is joint work with S. Seo from Kangwon National Univeristy, Korea. (Received September 26, 2017)

1135-05-2363 **Jacques Verstraete\***, 9500 Gilman Drive, La Jolla, CA 92093-0111, and **Zoltan Furedi, Tao Jiang, Alexandr Kostochka** and **Dhruv Mubayi**. *Paths and matching in convex geometric hypergraphs*.

A convex geometric hypergraph is a hypergraph whose vertex set comprises the vertices of a convex  $n$ -gon. We extend earlier results of a number of authors from convex geometric graphs to convex geometric uniform hypergraphs, with focus on the case of tight paths and matching shoes. Consequently we obtain a number of asymptotically sharp results, and the current best upper bounds on the Turán problem for tight paths. (Received September 26, 2017)

1135-05-2380 **Helene Barcelo\*** (hbarcelo@msri.org), **Curtis Greene, Abdul Salam Jarrah** and **Volkmr Welker**. *Discrete cubical homology groups*. Preliminary report.

Discrete cubical homotopy theory is a (refined) discrete analogue of homotopy theory, associating a (bigraded) sequence of groups to a simplicial complex, capturing some of its combinatorial and topological structure. It can be defined for graphs, resulting in algebraic invariants that differ substantially from the classical homotopy groups. One can also define discrete cubical homology groups in analogy to the continuous case. We will review these notions for graphs and compare them to the classical groups as well as to the recently defined notion of path homology of graphs (Grigor'yan et al). (Received September 26, 2017)

1135-05-2383 **Art Duval\*** (aduval@utep.edu) and **Amy Wagler** (awagler2@utep.edu). *Matroids and statistical dependency*. Preliminary report.

What does it mean for a set of more than two variables to be statistically dependent, even if no two of them are pairwise dependent? We compare different ways of determining this dependency and show they are consistent. We show that, if we make common statistical assumptions on data, then the resulting structure of dependencies may be described by a matroid. We use real examples from biology to demonstrate how the description with matroids helps simplify the presentation of complex variable dependency. (Received September 26, 2017)

1135-05-2396 **Soojin Cho\*** (chosj@ajou.ac.kr), Department of Mathematics, Ajou University, South Korea, **Suyoung Choi** (schoi@ajou.ac.kr), Department of Mathematics, Ajou University, South Korea, and **Shizuo Kaji** (skaji@yamaguchi-u.ac.jp), Department of Mathematical Sciences, Yamaguchi University, Japan. *Geometric representations of finite groups on real toric spaces*.

We develop a framework to construct geometric representations of finite groups  $G$  through the correspondence between real toric spaces  $X^{\mathbb{R}}$  and simplicial complexes with characteristic matrices. We give a combinatorial description of the  $G$ -module structure of the homology of  $X^{\mathbb{R}}$ . As applications, we make explicit computations of the Weyl group representations on the homology of real toric varieties associated to the Weyl chambers of type  $A$  and  $B$ , which show an interesting connection to the topology of posets. We also realize a certain kind of Foulkes representation geometrically as the homology of real toric varieties. (Received September 26, 2017)

1135-05-2404 **Dongsu Kim\*** (dongsu.kim@kaist.edu) and **Zhicong Lin**. *A combinatorial bijection on  $k$ -noncrossing partitions*.

For any integer  $k \geq 2$ , we prove combinatorially the following Euler (binomial) transformation identity,

$$NC_{n+1}^{(k)}(t) = t \sum_{i=0}^n \binom{n}{i} NW_i^{(k)}(t),$$

where  $NC_m^{(k)}(t)$  (resp.  $NW_m^{(k)}(t)$ ) is the enumerative polynomial on partitions of  $\{1, \dots, m\}$  avoiding  $k$ -crossings (resp. enhanced  $k$ -crossings) by number of blocks. The special  $k = 2$  and  $t = 1$  case, asserting the Euler

transformation of Motzkin numbers are Catalan numbers, was discovered by Donaghey 1977. The result for  $k = 3$  and  $t = 1$ , arising naturally in the recent study of pattern avoidance in ascent sequences and inversion sequences, was proved only analytically. (Received September 26, 2017)

1135-05-2409 **Michael A. Jackson\*** ([majackson@gcc.edu](mailto:majackson@gcc.edu)). *Open Questions in the Forest of Polytope Number Sequences*. Preliminary report.

Polytope number sequences are sequences of positive integers arising from the geometry of polytopes in any dimension. This talk will begin by introducing the topic of polytope numbers and giving a sampling of results my students have found over the past seven years. The remainder of the talk will discuss open questions about polytope numbers including ones related to Fermat's Last Theorem and Waring's problem. We will also discuss how moving into the forest of polytope numbers continues to show new paths to follow. (Received September 26, 2017)

1135-05-2446 **Kevin Dilks\*** ([kevin.dilks@ndsu.edu](mailto:kevin.dilks@ndsu.edu)), **Jessica Striker** and **Corey Vorland**. *Increasing Labelings and Generalized Promotion*.

Previous work of Dilks, Pechenik, and Striker established a connection between rowmotion on plane partitions and  $K$ -promotion on increasing tableaux of square shape, and also showed that  $K$ -promotion could be described as a series of local involutions in addition to the usual jeu-de-taquin box sliding definition. In this talk, we generalize to establish a connection between a generalized promotion operation on increasing labelings of any poset, and rowmotion on order ideals of an associated poset. (Received September 26, 2017)

1135-05-2453 **Martha Precup\*** ([mprecup@math.northwestern.edu](mailto:mprecup@math.northwestern.edu)) and **Megumi Harada**. *The cohomology of abelian Hessenberg varieties*.

Hessenberg varieties are subvarieties of the flag variety with important connections to representation theory, algebraic geometry, and combinatorics. These varieties have gained recent attention due to a conjecture of Shareshian and Wachs relating the chromatic quasisymmetric function of the incomparability graph of a unit interval order to the dot action representation on the cohomology of an associated regular semisimple Hessenberg variety. In this talk, we will discuss an inductive formula for the Betti numbers of certain regular Hessenberg varieties called abelian Hessenberg varieties. Using a theorem of Brosnan and Chow, this formula yields an inductive description of the corresponding chromatic quasisymmetric function. This formula also gives another proof of the Stanley-Stembridge conjecture in this case, and generalizes a result of Stanley. (Received September 26, 2017)

1135-05-2458 **Guantao Chen**, **Zhicheng Gao**, **Songling Shan\*** ([songling.shan@vanderbilt.edu](mailto:songling.shan@vanderbilt.edu)) and **Xingxing Yu**. *Circumferences of 3-connected graphs with bounded maximum degrees*. Preliminary report.

In 1993 Jackson and Wormald conjectured that for any positive integer  $d$  with  $d \geq 4$ , there exists a positive real number  $\alpha$  depending only on  $d$  such that if  $G$  is a 3-connected  $n$ -vertex graph with maximum degree at most  $d$ , then  $G$  has a cycle of length at least  $\alpha n^{\log d - 1}$ . They showed that the exponent in the bound is best possible if the conjecture is true. We confirm the conjecture for  $d \geq 370$ . (Received September 26, 2017)

1135-05-2469 **Robert W Bell\*** ([rbell@math.msu.edu](mailto:rbell@math.msu.edu)), 919 E Shaw Ln, Rm E35, Lyman Briggs College, East Lansing, MI 48864. *Weak cop number of infinite graphs*. Preliminary report.

A classical pursuit and evasion game on graphs is that of cops and robbers. One studies the minimal number of cop pawns needed to capture a robber pawn. This two player game with complete information is played in turns on a graph, where pawns occupy vertices and all of a player's pawns may move on a turn by sliding along an edge to an adjacent vertex. We study the weak cop number of Lehner: the minimal number of cop pawns needed to either capture the robber or prevent the robber from visiting any vertex infinitely often. For instance, the cop number of a ray is infinite, but its weak cop number is one. We extend results on products to this setting, establish that isometric lines can be guarded, and show how a locally finite connected graph that admits a planar embedding without vertex accumulation points has weak cop number at most three. This is joint work with Jordan DuBeau and Elizabeth Matys. (Received September 26, 2017)

1135-05-2508 **Robert Jajcay\*** ([robert.jajcay@fmph.uniba.sk](mailto:robert.jajcay@fmph.uniba.sk)), Katedra algebrý, Fakulta matematiky, fyziky a informatiky UK, Mlynska dolina, 84248 Bratislava, Slovak Rep. *Generalized Cayley maps and Petrie duals*.

Cayley maps are orientable embeddings of Cayley graphs that admit a group of orientation preserving automorphisms acting regularly on the vertices of the map. They proved repeatedly useful and constitute a large part

of the class of orientably regular maps (orientable maps that admit a group of orientation preserving automorphisms acting regularly on the darts of the map). It is therefore natural to try to generalize this concept. We propose a generalization that requires the existence of an automorphism group acting regularly on the vertices, while dropping the requirement for this group to be orientation preserving. Namely, a generalized Cayley map is an orientable or non-orientable map that admits a group of automorphisms acting regularly on its vertices (in case of orientable maps, the automorphisms are allowed to include the orientation reversing reflections). We investigate this new class of highly symmetric maps, and show a surprising connection to the concept of Petrie duality. (Received September 26, 2017)

1135-05-2510 **Cristopher Moore\*** ([moore@santafe.edu](mailto:moore@santafe.edu)), Santa Fe Institute, 1399 Hyde Park Rd., Santa Fe, NM 87501. *Topological defects and entropic forces in tilings and colorings.*

Many discrete models on two-dimensional lattices, such as domino tilings, 3-colorings of the square lattice, and the triangular antiferromagnetic, have "height functions" that allow us to view states as random surfaces. Topological defects in these models, such as lattice sites not covered by a domino or neighboring sites with the same color, then act like vortices or charged particles, with entropically-driven forces between them. I will describe these defects and forces heuristically, from a physics point of view, and suggest some open problems. (Received September 26, 2017)

1135-05-2525 **Patrick Mackey, Kathleen Nowak\*** ([kathleen.nowak@pnnl.gov](mailto:kathleen.nowak@pnnl.gov)), **Jennifer Webster and Stephen J. Young.** *A New Influence-Based Genetic Algorithm for Community Detection.*

We introduce a new influence-based genetic algorithm for maximizing modularity. It uses Dijkstra's algorithm to improve the quality of the candidate partitions in the genetic pool. We compare our new genetic algorithm against six different algorithms, three state of the art advanced algorithms which use a genetic algorithm primer and three widely used deterministic algorithms. Our results suggest that advanced non-deterministic algorithms should benefit from using our new genetic algorithm as their base. (Received September 26, 2017)

1135-05-2537 **Michael Kural\*** ([mkural@mit.edu](mailto:mkural@mit.edu)). *Fuss-Schröder Paths and Rooted Plane Forests.*

We describe a bijection between  $(k, k)$ -Fuss-Schröder paths of type  $\lambda$  and certain rooted plane forests with  $n(k+1)+2$  vertices. This yields a recursion which allows us to analytically enumerate the number of large  $(k, r)$ -Fuss-Schröder paths of type  $\lambda$ , solving an open question posed by An, Jung, and Kim. Furthermore, we generalize the concept of  $(k, r)$ -Fuss-Schröder paths to  $(k, S)$ -Fuss-Schröder paths, in which  $r$  can take any value in a given set  $S$ , and enumerate these paths as well. (Received September 26, 2017)

1135-05-2544 **Joungmin Song\*** ([songj@gist.ac.kr](mailto:songj@gist.ac.kr)), Gwangju Institute of Science and Technology, Cheomdan Gwagiro 123, GIST College Bldg A-410, Gwangju, 61005, South Korea. *Characteristic polynomial of hyperplane arrangements via enumerative combinatorics and finite field method.*

A hyperplane arrangement is a finite set of affine hyperplanes in a real affine space. Let  $\mathcal{J}_n$  be the hyperplane arrangement consisting of all hyperplanes (or walls)  $H_{ij}$ ,  $0_k$ ,  $1_l$ , where, for each  $i, j, k$ , and  $l \in \{1, 2, \dots, n\}$ ,

$$H_{ij} := \{\mathbf{x} \in \mathbb{R}^n \mid x_i + x_j = 1\} = H_{ji}$$

and

$$0_k := \{\mathbf{x} \in \mathbb{R}^n \mid x_k = 0\}, \text{ and } 1_l := \{\mathbf{x} \in \mathbb{R}^n \mid x_l = 1\}.$$

The number of the *regions*, i.e., the connected components of  $\mathbb{R}^n \setminus \bigcup_{H \in \mathcal{J}_n} H$  is given by the characteristic polynomial  $\chi_n(t)$ . We formulate  $\chi_n(t)$  via enumerative combinatorics and finite field method. We give a direction forward generalizing this process to  $\mathcal{H}_n$ , whose walls are of the form

$$w_S = \left\{ \mathbf{x} \in \mathbb{R}^n \mid \sum_{i \in S} x_i = 1 \right\}.$$

(Received September 26, 2017)

1135-05-2625 **Bernardo M. Abrego\*** ([bernardo.abrego@csun.edu](mailto:bernardo.abrego@csun.edu)), Department of Mathematics, California State University, Northridge, 18111 Nordhoff Street, Northridge, CA 91330-8313, and **Silvia Fernandez** ([silvia.fernandez@csun.edu](mailto:silvia.fernandez@csun.edu)), Department of Mathematics, California State University, Northridge, 18111 Nordhoff Street, Northridge, CA 91330-8313. *On the rectilinear local crossing number of complete graphs.*

The local crossing number of a drawing of a graph is the largest number of crossings in any edge of the drawing. In a rectilinear drawing of a graph, the vertices are points in the plane in general position and the edges are

drawn as line segments. The rectilinear local crossing number of a graph  $G$ , denoted  $\overline{\text{lcr}}(G)$ , is the minimum local crossing number over all rectilinear drawings of  $G$ .

In this talk, we present recent results when  $G$  is a complete graph or a bipartite complete graph. In particular, the parameter  $\text{lcr}(G)$  is completely determined for all complete graphs and for all complete bipartite graphs with one class having at most 4 vertices.

Some of the results presented were obtained in collaboration with Kory Dondzila, Evgeniya Lagoda, Alain Olavarrieta, Seyed Sajjadi, and Yakov Sapozhnikov. We thank the members of the CSUN Crossing Numbers Research Group for valuable discussions, and we acknowledge partial support from NSF project DMS-1400653. (Received September 26, 2017)

1135-05-2639 **Jessica McDonald** and **Gregory J. Puleo\*** (gjp0007@auburn.edu). *t-Cores for  $(\Delta + t)$ -edge-colouring.*

We extend the edge-coloring notion of *core* (subgraph induced by the vertices of maximum degree) to *t-core* (subgraph induced by the vertices  $v$  with  $d(v) + \mu(v) > \Delta + t$ ), and find a sufficient condition for  $(\Delta + t)$ -edge-coloring. In particular, we show that for any  $t \geq 0$ , if the  $t$ -core of  $G$  has multiplicity at most  $t + 1$ , with its edges of multiplicity  $t + 1$  inducing a multiforest, then  $\chi'(G) \leq \Delta + t$ . This extends previous work of Ore, Fournier, and Berge and Fournier. More generally, we prove bounds on the *fan number* of a graph  $G$ , a parameter introduced by Scheide and Stiebitz as an upper bound on the edge chromatic number. We give an exact characterization of the multigraphs  $H$  such that  $\text{Fan}(G) \leq \Delta(G) + t$  for all graphs  $G$  having  $H$  as their  $t$ -core. We show how this characterization implies a theorem of Hoffman and Rodger about cores of  $\Delta$ -edge-colourable simple graphs. (Received September 26, 2017)

1135-05-2708 **Katherine Burke** (kburke@smith.edu), **Haley Hoech** (hhoech@smith.edu), **Joan Kim** (jkim32@smith.edu) and **Nicole Magill\*** (nmagill1@smith.edu), Department of Mathematics and Statistics, Smith College, Northampton, MA 01063. *Generalized splines and geometry.*

Given an edge-labeled graph, a generalized spline is a way of labeling the vertices so that each pair of adjacent vertex-labels differs by a multiple of the label on the edge between them. When edges are labeled by polynomials, this definition is a natural extension of the definition of splines from applied mathematics and analysis. We describe several new results on generalized splines, including connections to geometry. (Received September 26, 2017)

1135-05-2719 **Adriano Garsia\*** (garsia@math.ucsd.edu) and **G. Xin**. *Proof of the Functional Equation Conjecture.*

In a (2008) seminal paper T. Haglund, J. Morse and M. Zabrocki conjectured a Compositional extension of the Shuffle Conjecture. In 2015 E. Carlsson and A. Mellit proved this conjecture. What is not widely known is that in a 2013 thesis A. Hicks discovered that there is a quasi symmetric sharpening of the Compositional Shuffle Conjecture. More precisely, in the quasi-symmetric version, the combinatorial side is predicted to yield a polynomial in the variables  $x, q$  which satisfies a certain Functional Equation. In joint work, the presenter and G. Xin proved this latter conjecture. This talk gives the outline of this proof and describes how this result leads to a proof of a sharpening of the Compositional Shuffle Conjecture. (Received September 26, 2017)

1135-05-2721 **Roman Nedela** and **Martin Skoviera\*** (skoviera@dcs.fmph.uniba.sk), Department of Computer Science, Comenius University, Bratislava, 842 48. *AmPLY upper embeddable graphs*. Preliminary report.

A connected graph  $G$  is upper embeddable if it has a 2-cell embedding into an orientable surface of genus  $\lfloor \beta/2 \rfloor$  where  $\beta$  is the Betti number (cycle rank) of  $G$ ; it is amPLY upper embeddable, if there is a pair of adjacent vertices in  $G$  whose removal leaves an upper embeddable graph. AmPLY upper embeddable graphs have been introduced by the authors as a tool for the construction of Hamilton cycles and paths in embedded cubic graphs (in particular cubic vertex-transitive graphs) via surface duality. We explain the basic idea of the method and prove that every cyclically 5-connected cubic graph is amPLY upper embeddable. We conjecture that the same is true also for cyclically 4-connected cubic graphs. (Received September 26, 2017)

1135-05-2733 **Kirk Boyer**, **Boris Brimkov**, **Sean English**, **Daniela Ferrero**, **Ariel Keller\*** (ariel.keller@gmail.com), **Rachel Kirsch**, **Michael Phillips** and **Carolyn Reinhart**. *Zero Forcing Polynomial for Cycles and Singly-Chorded Cycles.*

Zero forcing in a graph is an iterative process where, at each step, any colored vertex with a single uncolored neighbor forces its neighbor to become colored. We define the zero forcing polynomial of a graph  $G$  of order  $n$  by  $Z(G, x) = \sum_{i=1}^n z(G, i)x^i$ , where  $z(G, i)$  is the number of zero forcing sets of  $G$  of size  $i$ . Many different graphs



can have the same zero forcing polynomial; however, certain families of graphs can be recognized by their zero forcing polynomial. In this talk, we consider one such family. In particular, we show that cycles, singly-chorded cycles, and for  $n = 6$  a graph  $\mathcal{G}$  consisting of a doubly-chorded cycle with one pendant vertex share the same zero forcing polynomial and that no other graphs have this same polynomial.

Keywords: zero forcing set, zero forcing polynomial, cycles (Received September 26, 2017)

1135-05-2749 **Hsien-Chih Chang\*** (hchang17@illinois.edu), **Axel Brandt** (axbrandt@davidson.edu), **Tanya Jeffries** (neferzippy@gmail.com), **Sarah Loeb** (sarah1347@gmail.com) and **Marcus Schaefer** (mschaefer@cdm.depaul.edu). *On Local Crossing Numbers of Complete Graphs and Hypercubes*. Preliminary report.

The local crossing number of a graph is the smallest number  $k$  required to draw the graph in which every edge is crossed at most  $k$  times. Such notion has been proven useful in the graph drawing community in pursuing a definition suited to model graphs that are closed to planar, as well as a tool to understand other related graph parameters.

In this talk, we report some progress on local crossing numbers, followed by discussion and future directions. Specifically,

1. Deciding the local crossing numbers of complete graphs is hard in general even for complete graphs; we prove that for complete graph on 8 vertices the value is 3. The upper bound comes from explicit construction and the lower bound follows from a case-by-case analysis.

2. We provide upper and lower bounds on the local crossing numbers of the hypercubes. The upper bound comes from a recursive construction; as for lower bound we use the graph-embedding strategy similar to the one for (standard) crossing numbers on sparse graphs with good expansion properties. Our bounds are tight up to logarithmic factors.

This is a joint work with Axel Brandt, Tanya Jeffries, Sarah Loeb, and Marcus Schaefer. The research was conducted during the MRC workshop "Beyond Planarity: Crossing Numbers of Graphs" in June 2017. (Received September 26, 2017)

1135-05-2806 **Caroline Daugherty, Josh Laison, Rebecca Robinson and Kyle Salois\***, kjsalois@willamette.edu. *Intersection Graphs of Maximal Convex Sub-Polygons of  $k$ -Lizards*. Preliminary report.

A  $k$ -lizard is a simply connected polygon with sides parallel to a regular  $2k$ -gon. For a  $k$ -lizard  $P$ , let  $S$  be the set of all maximal sub-polygons contained in  $P$ . A graph  $G$  is a  $k$ -maximal sub-polygon graph (or  $k$ -MSP graph) if there exists a  $k$ -lizard  $P$  and a one-to-one correspondence between vertices of  $G$  and polygons in  $S$  such that two vertices are adjacent in  $G$  if and only if their corresponding polygons in  $S$  intersect. We find separating examples of graphs that are  $k$ -MSP graphs but not  $j$ -MSP graphs for  $j \neq k$ . (Received September 26, 2017)

1135-05-2817 **Shadisadat Ghaderi\*** (shadisadat.ghaderi@guttman.cuny.edu), 50 West 40th Street, Room 605, Guttman Community College, New York, NY 10018, and **Jerzy Wojciechowski**. *The Matroid Intersection Conjecture for Singular Matroids*.

The Matroid Intersection Conjecture, proposed by Nash-Williams in 1990, is the extension of the well-known finite matroid intersection theorem of Edmonds to infinite sets. The conjecture is still open; however, some special cases were proved to be true. We introduce a new methodology to approach the conjecture that is inspired by the proof of the general version of König's Theorem for bipartite graphs. We develop new techniques that generalize the critical set approach used in the proof of the countable version of König's Theorem. Our results enable us to prove that the Matroid Intersection Conjecture is true for a pair of singular matroids on a set that is infinitely countable. Note that a matroid is singular when it is a direct sum of matroids such that each term of the sum is a uniform matroid either of rank one or of co-rank one. (Received September 26, 2017)

1135-05-2825 **James J. Madden\*** (madden@math.lsu.edu), 222 Prescott Hall, Louisiana State Univ., Baton Rouge, LA 70803. *On the Distribution of Maximal Runs in Binary Words*. Preliminary report.

Let  $R$  be a finite set of positive integers and let  $K$  be a function from  $R$  to the natural numbers. Let  $W(n, R, K)$  denote the set of binary words of length  $n$  that, for all  $r \in R$ , contain exactly  $K(r)$  maximal subwords of  $r$  identical consecutive symbols. We exhibit the generating function for the cardinality of  $W(n, R, K)$ . We also express, by means of generating functions, the probability that a binary word of length  $n$  produced by a Markov process (with specified parameters) belongs to  $W(n, R, K)$ . All these results generalize a famous result of DeMoivre, *Doctrine of Chances*, Second Edition, Problem LXXXVIII: "To find the Probability of throwing

a Chance assigned a given number of times without intermission, in any given number of Trials.” (Received September 26, 2017)

1135-05-2842 **Arthur L Gershon\*** ([arthur.gershon@case.edu](mailto:arthur.gershon@case.edu)). *A Path Through the StArrs: Using the Transfer Matrix Method to Compute Generating Functions for Strip Arrangements on Chessboards*. Preliminary report.

For positive integers  $m$  and  $n$ , let  $T(m, n)$  be the number of ways to arrange strips on an  $m \times n$  chessboard with at most one horizontal strip in each row and at most one vertical strip in each column. We show how to use the transfer matrix method to compute, for a fixed positive integer  $m$ , the generating function for the sequence  $\{T(m, n) : n \geq 0\}$ . This technique can be generalized to strip arrangements that allow up to  $h$  horizontal strips in a row and  $k$  strips in a column; the case  $h = k = 1$  that we will focus on is of interest due to an apparent connection with the eight-vertex model of statistical physics. Time permitting, we will also discuss results for strip arrangements that allow any number of strips in each row and column (i.e., the case  $h = k = \infty$ ). (Received September 26, 2017)

1135-05-2882 **Ewin N Tang\*** ([ewin@utexas.edu](mailto:ewin@utexas.edu)), **Neeraja R Kulkarni** ([kulkarnin@carleton.edu](mailto:kulkarnin@carleton.edu)) and **Joe Suk** ([ybjosuk@gmail.com](mailto:ybjosuk@gmail.com)). *Factorizations of  $k$ -nonnegative matrices*.

A matrix is *totally nonnegative* if all of its minors are nonnegative. Totally nonnegative matrices have long attracted attention because of their applications in combinatorics, dynamics and probability, as well as their interesting topological structure. In particular, the semigroup of invertible totally nonnegative matrices can be partitioned, based on their factorizations into Chevalley generators and diagonal matrices, into cells that form a CW-complex. The closure poset of this CW-complex is described by the Bruhat order, as seen by considering subwords of factorizations.

Our work considers  $k$ -nonnegative matrices, in which all minors of order at most  $k$  are nonnegative. We give a minimal set of generators in two special cases:  $(n - 1)$ -nonnegative invertible matrices, and  $(n - 2)$ -nonnegative triangular matrices with 1s on the diagonal. We describe how these semigroups can also be partitioned into cells that are homeomorphic to open balls. By describing the subwords that arise naturally from our new generators, we extend the Bruhat order on totally nonnegative matrices to these cases. As a result, the topological closure of the cells of  $k$ -nonnegative matrices induces an easily describable, analogous poset. (Received September 26, 2017)

1135-05-2883 **Sarah Peluse\*** ([speluse@stanford.edu](mailto:speluse@stanford.edu)). *Finding three-term progressions in subsets*.

How big can a subset of  $\{1, 2, \dots, N\}$  or an abelian group be if it does not contain any three-term progression  $x, x + y, x + 2y$  with  $y \neq 0$ ? How big can a subset of the alternating group be if it does not contain any three-term progression  $x, xy, xy^2$  with  $y$  not equal to the identity permutation? How big can a subset of  $\{1, 2, \dots, N\}$  or  $\mathbb{F}_q$  be if it does not contain any three-term progression  $x, x + y^{1060}, x + y^{31} + y^{26} + y^{23} + y^{14} + y^{10}$  with  $y \neq 0$ ? In this talk, I will discuss the relationship between these three questions, the techniques used in trying to answer them, and the different challenges encountered when we consider longer progressions. (Received September 26, 2017)

1135-05-2891 **Hunter Dane Rehm\*** ([rehm.hunter@uwlax.edu](mailto:rehm.hunter@uwlax.edu)), 1502 State st., La Crosse, WI 54601. *Rainbows in Graphs*. Preliminary report.

Ramsey Theory is a branch of mathematics whose main questions are of the form: If a system is chaotic or random and we partition the system into smaller pieces can we guarantee that the smaller pieces are no longer chaotic or have some nice structure? Anti-Ramsey Theory type problems ask the opposite questions: If a system is structured and we partition the system into smaller pieces can we guarantee that the smaller pieces break the structure? The open question that we focused on was an anti-Ramsey type question. We wanted to find out exactly how much structure we could give a system before we could guarantee that a smaller piece of our system must break the structure. Our structure breaking pieces are called rainbow 3-term arithmetic progression. We developed tools (theorems) to study the system of graphs called grid graphs (think of an array or a well-planned city with a Google maps view). We eventually answered the open question for any  $m$  by  $n$  grid graph and more! (Received September 26, 2017)

1135-05-2907 **Sam Bardwell-Evans\*** ([sbardwel@caltech.edu](mailto:sbardwel@caltech.edu)). *A Generalization of the Carlson-Simpson Theorem Analogous to Gowers' Theorem*. Preliminary report.

We prove a generalization of Carlson and Simpson's Dual Ramsey Theorem that is analogous to Gowers' theorem for  $\text{FIN}_k$ . Instead of considering the tetris operation  $T : \{0, \dots, k\} \rightarrow \{0, \dots, k - 1\}$ , we fix a finite, non-empty alphabet  $A$  and consider analogous functions  $f : A \cup \{0, \dots, k\} \rightarrow A \cup \{0, \dots, k - 1\}$ , and instead of finding an

infinite sequence of functions  $b_n$  in  $\text{FIN}_k$  such that the set of all finite sums  $\sum T^{d_n}(b_n)$  is monochromatic, we find an infinite sequence of (not necessarily finitely supported) functions  $b_n : \omega \rightarrow A \cup \{0, \dots, k-1\}$  such that the set of all (not necessarily finite) sums  $\sum f_n(b_n)$  is monochromatic. The natural multidimensional generalization of this theorem implies the Carlson-Simpson Theorem. (Received September 26, 2017)

1135-05-2913 **Luke Juusola\*** (ljuusola@caltech.edu) and **Martino Lupini**. *The Borel Quantum Chromatic Number*. Preliminary report.

The idea of graph colorability has proven very useful in many aspects of combinatorics and has achieved broad impacts over many fields. This has led to the study of graph colorings in a large variety of contexts. Of these, we concern ourselves primarily with "quantum mathematics," where the notion can be defined in terms of a quantum graph coloring game. We add to this theory by considering two variants. First, we consider standard Borel graphs and study the quantum analogue of the Borel chromatic numbers. Then we do the same for measurable graphs and colorings, and finally, we consider connections among them, such as separation. (Received September 26, 2017)

1135-05-2950 **Diljit Singh\***, diljit.singh@macaulay.cuny.edu. *Combinatorial Models of Quantum Matrices*.

Quantum matrices are matrices with entries that almost commute, e.g., they commute up to a constant. These structures arise naturally when studying Hopf algebras, representation theory, and knot invariants. Much recent progress in the theory of quantum determinants has been done using combinatorial and algebraic methods revolved around path systems over Cauchon Diagrams. We introduce and bound two growth functions: one counting the expected number of legal path systems in a diagram with  $m$  squares colored black,  $B_n(x)$ ; the other counting the expected number of path systems if our matrix (with a fixed coloring) increases in dimension,  $D_b(x)$ .

Using these diagrams, we also provide a simple and new proof for a case of the quantum analog of Sylvester's Determinant Identity. In addition to this we give a recursive method to grow our proof to cover infinitely more cases of the identity. (Received September 26, 2017)

1135-05-2955 **Huseyin Acan\***, huseyin.acan@rutgers.edu, and **Jeff Kahn**. *Disproof of a conjecture of Alon and Spencer*.

Let  $\nu_k$  denote the size of a largest family of edge-disjoint  $k$ -cliques that can be packed into the random graph  $G = G_{n,1/2}$ . Alon and Spencer conjectured the expected value of  $\nu_k$  to be  $\Omega(n^2/k^2)$  when  $k$  is slightly smaller than the clique number of  $G$ . We disprove this conjecture by showing that the expected value in question is  $O(n^2/k^3)$ .

Our main interest lies in answering the following more general question. Let  $k \ll \sqrt{n}$  and  $A_1, \dots, A_t$  be random  $k$ -subsets of  $[n]$ , chosen uniformly and independently. Then what can we say about the probability

$$\mathbb{P}(|A_i \cap A_j| \leq 1 \forall i \neq j) ?$$

We provide upper bounds for this probability that almost agree with the values obtained by pretending the events  $\{|A_i \cap A_j|\}_{i < j}$  are independent. (Received September 26, 2017)

1135-05-3050 **Paul K Horn\*** (paul.horn@du.edu). *Graph curvature and the geometry and eigenvalues of graphs*.

In this talk, I'll discuss some recent works about notions of graph curvature and how they can be used to extract information about geometrical properties of graphs. We'll discuss, in particular, some ways that graph curvature notions can be used to understand mixing of random walks on graphs as well as Laplacian eigenvalues of graphs. (Received September 26, 2017)

1135-05-3052 **Louis Deaett\*** (louis.deaett@quinnipiac.edu). *Matroids and the minimum rank of matrix patterns*. Preliminary report.

The *zero-nonzero pattern* of a matrix specifies precisely which of its entries are nonzero. One problem of interest is to determine the smallest possible rank of a matrix given only this combinatorial description. Here we show how to generalize this problem to the setting of matroids, and observe that some known lower bounds still apply in this setting. Moreover, we exploit the matroid-theoretic context to give a new perspective on some known results, improve on some others a bit, and establish a few new results as well. Ultimately, however, the potential of this approach seems largely untapped; we outline directions in which the connections with matroid theory could be strengthened so as to bring more powerful tools to bear on the original matrix-theoretic problem. (Received September 26, 2017)

1135-05-3071 **Kassie Archer** and **Christina Graves\*** (cgraves@uttyler.edu). *Enumerating acyclic digraphs by descents.*

A descent of a labeled acyclic digraph is a directed edge  $x \rightarrow y$  with  $x > y$ . In this talk, we find a recurrence for the number of labeled acyclic digraphs with a given number of descents. (Received September 26, 2017)

1135-05-3086 **Jessalyn Bolkema\*** (jessalyn.bolkema@huskers.unl.edu) and **Corbin Groothuis**. *Hunting Invisible Rabbits on the Hypercube.*

The pursuit game “Hunters and Rabbits” is a variation on “Cops and Robbers” featuring a single rabbit trapped on a graph. The rabbit is invisible to the group of hunters seeking it, so the hunters need a strategy guaranteed to capture the rabbit in finite time without any knowledge of the rabbit’s current or former position. The *hunter number* for a given graph is the least number of hunters necessary to achieve this goal. We determine the hunter number for the discrete hypercube, and along the way, we establish a winning strategy for any graph satisfying certain isoperimetric nesting properties, applicable to grids, hypercubes, and other similar objects. (Received September 26, 2017)

1135-05-3091 **Yuval Roichman** and **Michelle Wachs\*** (wachs@math.miami.edu). *On  $r$ -inversions and symmetric functions.* Preliminary report.

The  $r$ -inversion number is a statistic on words of length  $n$  (over the positive integers), which interpolates between the descent number ( $r = 2$ ) and the inversion number ( $r = n$ ). We consider a symmetric function  $U_{n,r}$  that enumerates words of length  $n$  by this statistic. The symmetric function  $U_{n,r}$  is an example of an LLT polynomial. The LLT polynomials were shown to be Schur-positive by Grojnowski and Haiman by means of Kazhdan-Lusztig theory. It is an open question to give a combinatorial description of the coefficients in the Schur basis expansion. For  $r = 2$  and  $r = n$ , such descriptions are well known. For  $r = 3$ , a description (in a more general setting) was conjectured by Haglund and was proved by Blasiak using noncommutative Schur functions and Lam’s algebra of ribbon Schur operators. Here we give a more elementary proof for the  $r = 3$  case, which uses classical RSK theory, and we give some consequences. (Received September 26, 2017)

1135-05-3099 **Alexander B Kunin\*** (abk170@psu.edu), **Zvi H Rosen** and **Vladimir Itskov**. *The polar complex of hyperplane codes.*

Feed-forward codes, studied by Giusti et al., arise as the firing patterns of the output of a one layer feed-forward neural network, and are a special case of convex codes. We introduce hyperplane codes, a natural generalization of feed-forward codes. We construct a new object, the polar complex of a code, to extend results from Giusti et al. regarding obstructions to convexity. Moreover, we demonstrate several obstructions particular to hyperplane codes. Using results from discrete geometry, we demonstrate that the polar complex of a hyperplane code is extendably shellable, which subsumes the other properties thus far considered. We demonstrate that the polar complex arises naturally via the polarization of the neural ideal, as discussed in Jeffries et al., and draw some connections to oriented matroids for future work. (Received September 26, 2017)

1135-05-3106 **John Shareshian** and **Michelle Wachs\*** (wachs@math.miami.edu). *Gamma-positivity of variations of Eulerian polynomials.*

An identity of Chung, Graham and Knuth involving binomial coefficients and Eulerian numbers motivates our study of a class of polynomials that we call binomial-Eulerian polynomials. These polynomials share several properties with the Eulerian polynomials. For one thing, they are  $h$ -polynomials of simplicial polytopes, which gives a geometric interpretation of the fact that they are palindromic and unimodal. A formula of Foata and Schützenberger shows that the Eulerian polynomials have a stronger property, namely  $\gamma$ -positivity, and a formula of Postnikov, Reiner and Williams does the same for the binomial-Eulerian polynomials. We obtain  $q$ -analogs of both the Foata-Schützenberger formula and an alternative to the Postnikov-Reiner-Williams formula, and we show that these  $q$ -analogs are specializations of analogous symmetric function identities. Algebro-geometric interpretations of these symmetric function analogs are presented. (Received September 26, 2017)

1135-05-3164 **Bryan L. Williams\*** (bryan.l.williams1@navy.mil), **Lucas A. Overbey** and **Lakeisha D. Williams**. *Variants of Competition Graphs in Information Networks.*

Joel E. Cohen introduced the notion of a competition graph (CG) as a way to analyze relationships between a variety of organisms, predators and prey alike, within an ecological system. In an ecosystem, a CG is a mapping of a directed food web onto an undirected graph such that nodes represent predators/prey and edges represent resource competition between organisms. CG variants include common enemy graphs (CEGs), competition-common enemy graphs (C-CEGs), and  $m$ -step competition graphs ( $m$ -CGs). Each of these variants maps an initial set of directed relationships onto new, undirected graphs that, depending on the application, can reveal

competitors for resources or attention, potential adversaries and allies, behaviorally connected communities, and structural relationships therein. In this work, we investigate those applications by deriving the aforementioned CGs and variants in an effort to explore characteristics of relationships between the spread of information and/or misinformation and nodes of interest in social networks. (Received September 26, 2017)

1135-05-3193 **Jessica De Silva\*** ([jessica.desilva@huskers.unl.edu](mailto:jessica.desilva@huskers.unl.edu)), 1617 F St., Apt. 27, Lincoln, NE 68508. *Laplacian Polynomials of Threshold Graphs*. Preliminary report.

Graphs with extremal properties are often found in the class of threshold graphs. Threshold graphs can be obtained from any graph by repeated application of the Kelmans transformation. This transformation has been shown to decrease the number of spanning trees and increase the number of cliques in a graph. In this talk, we show that the Kelmans transformation also decreases (in absolute value) the coefficients of the Laplacian polynomial. This result implies that the coefficients of the Laplacian polynomial of threshold graphs are minimal. (Received September 27, 2017)

1135-05-3213 **Sylvie Corteel\*** ([corteel@irif.fr](mailto:corteel@irif.fr)), IRIF, CNRS et Universite Paris Diderot, Case 7014, 75205 Paris Cedex 13, France. *Combinatorics of Koornwinder polynomials*.

In this talk, I will explain how to build Koornwinder polynomials at  $q = t$  from moments of Askey-Wilson polynomials. I will use the classical combinatorial theory of Viennot for orthogonal polynomials and their moments. An extension of this theory allows to build multivariate orthogonal polynomials. The key steps for this construction are a Cauchy identity for Koornwinder polynomials due to Mimachi and a Jacobi-Trudi formula for the 9th variation of Schur functions due to Nakagawa, Noumi, Shirakawa and Yamada. This is joint work with Olya Mandelsham, Dennis Stanton and Lauren Williams. (Received September 27, 2017)

1135-05-3222 **Jordan Olliver Tirrell\*** ([jtirrell@mtholyoke.edu](mailto:jtirrell@mtholyoke.edu)). *Combinatorics of orthogonal polynomials and skew tableaux*.

Viennot and Flajolet developed the combinatorial theory of orthogonal polynomials, which interprets a moment sequence as an enumeration of weighted Motzkin paths. We will show how this can be used to obtain elementary combinatorial proofs for certain enumerative formulas. Our main example extends a bijection between Motzkin paths and Young tableaux with at most three rows to the case of skew tableaux, which we enumerate using Chebyshev polynomials. (Received September 27, 2017)

1135-05-3240 **Laura Escobar** and **Benjamin J. Wyser\*** ([bwyser@okstate.edu](mailto:bwyser@okstate.edu)), Department of Mathematics, Oklahoma State University, 401 MSCS, Stillwater, OK 74078, and **Alexander Yong**. *K-orbit closures and Barbasch-Evens-Magyar varieties*. Preliminary report.

The closures of orbits of a symmetric subgroup on the flag manifold are important objects in representation theory, and are a sort of generalization of Schubert varieties. In some cases, such orbit closures admit explicit resolutions of singularities (first considered by Barbasch and Evens) very similar in spirit to the well-known Bott-Samelson resolutions of Schubert varieties. Roughly speaking, such a resolution corresponds to the choice of a closed orbit and a reduced word for a certain Weyl group element. If one drops the requirement that the word be reduced, one obtains a larger class of smooth varieties which map finite-to-one to  $K$ -orbit closures. I will describe joint work with Laura Escobar and Alexander Yong where we consider these varieties (which we call Barbasch-Evens-Magyar varieties, or BEM varieties) in some detail. Among the results I will discuss are a description of BEM varieties as iterated fiber products, which leads to a very concrete diagrammatic description of them in type  $A$ . I will also discuss some combinatorial results on the moment polytopes of BEM varieties, particularly in the case where the symmetric subgroup in question is  $GL_p \times GL_q$ . (Received September 27, 2017)

## 06 ► *Order, lattices, ordered algebraic structures*

1135-06-143 **Joshua D Nolan\*** ([nolanjd@lafayette.edu](mailto:nolanjd@lafayette.edu)), 111 Quad Drive, Lafayette College, Easton, PA 18042-1784, and **Clifford A Reiter**. *Visualizing 3-dimensional Quasi-crystals*. Preliminary report.

It is known that 3-dimensional quasi-crystals have essentially two dimensional structure or have icosahedral symmetry. Those with icosahedral symmetry appear in three forms. We create 3-dimensional point sets with quasi-crystalline structure using canonical projection. We enhance our visual understanding of these quasi-crystals by adding edges, faces and/or 3-facets. (Received August 03, 2017)

1135-06-1763 **Jean S. Joseph\*** (jjose107@fau.edu). *Order Type of the Rational Numbers.*

In 1895, Georg Cantor characterized the order type of  $\mathbb{Q}$ . A nice consequence of that characterization is that  $\mathbb{Q} \times \mathbb{Q}$  with the lexicographic order is isomorphic to  $\mathbb{Q}$ . In this talk, we will present an algorithmic version of Cantor's theorem that has a constructive proof. (Received September 24, 2017)

1135-06-2468 **Hugh Thomas\*** (hugh.ross.thomas@gmail.com) and **Nathan Williams**. *Rowmotion on trim lattices.*

There are two different definitions of rowmotion on a distributive lattice, which coincide, as first shown by Cameron and Fon-der-Flaass. (One is the familiar toggling definition; the other is based on labelling the edges of the Hasse diagram, and sending the element whose lower covers' labels form the set  $X$  to the element whose upper covers' labels form the set  $X$ .) We show that the two definitions can both be extended to lattices which are trim (i.e. left modular and extremal), and that they still coincide. (Received September 26, 2017)

1135-06-2759 **Martha Lee Hollist Kilpack\*** (mlhkilpack@mathematics.byu.edu). *Loop representation for closure operator lattices.* Preliminary report.

Will loops be more effective than groups as a algebraic representative for closure operator lattices?

A theorem of Birkhoff and Frink we know every algebraic lattice can be represented as a subalgebra lattice of a finite algebraic structure. In work by the presenter and Magidin we know for a finite lattice  $L$ , the generated closure operator lattice is isomorphic to the lattice of subgroups of a group if and only if  $L$  is a chain.

What about loops? As loops are groups without the associative there are many lattices which are subloop but not subgroup lattices. We will look at some preliminary finding showing many types of closure operator lattices are isomorphic to a subloop lattices. For example the closure operator lattice generated from the non-chain 4 element lattices is a subloop lattice but not a subgroup lattice. (Received September 26, 2017)

## 08 ► General algebraic systems

1135-08-989 **Ian Hill\*** (hillim@dukes.jmu.edu), 57 S High St, Harrisonburg, VA 22801, and **Johua Ducey** and **Peter Sin**. *The Critical of the Kneser Graphs on 2-subsets of an  $n$ -set.*

Let  $KG(n,2)$  be the Kneser graph on the 2-subsets of an  $n$  element set. We use methods stemming from the representation theory of the symmetric group and algebraic graph theory to find a cyclic decomposition of the critical group for this family of graphs. (Received September 18, 2017)

1135-08-2872 **Karl Schmidt\*** (karls@uoregon.edu). *Factorizable Module Algebras.*

The aim of this talk is to introduce and study a large class of  $\mathfrak{g}$ -module algebras which we call factorizable, generalizing the Gauss factorization of (square or rectangular) matrices. This class includes appropriate localizations of coordinate algebras of corresponding reductive groups  $G$ , their parabolic subgroups, basic affine spaces, and many others. It turns out that tensor products of certain factorizable algebras are also factorizable. We also have quantum versions of all these constructions in the category of  $U_q(\mathfrak{g})$ -module algebras. Quite surprisingly, our quantum factorizable algebras are naturally acted on by the quantized enveloping algebra  $U_q(\mathfrak{g}^*)$  of the dual Lie bialgebra  $\mathfrak{g}^*$  of  $\mathfrak{g}$ . (Received September 26, 2017)

## 11 ► Number theory

1135-11-10 **Benjamin Linowitz\*** (benjamin.linowitz@oberlin.edu), 10 North Professor St, Department of Mathematics, Oberlin College, Oberlin, OH 44145. *Geodesics on hyperbolic surfaces, quaternion algebras, and the Chebotarev density theorem in short intervals.*

In this talk we will discuss some results at the crossroads of number theory (both algebraic and analytic) and hyperbolic geometry. Geometrically, we will discuss the extent to which arithmetic hyperbolic surfaces (e.g., Shimura curves) which are not commensurable may share arbitrarily large portions of their geodesic length spectra. This problem may be recast in terms of counting the number of quaternion algebras over a number field  $K$  admitting embeddings of a fixed set of quadratic extensions of  $K$ . Additionally, we will show how the Chebotarev density theorem may be used to address problems involving the geometry of geodesics on arithmetic hyperbolic surfaces. (Received June 01, 2017)

1135-11-62 **Alina Ostafe\*** (alina.ostafe@unsw.edu.au), University of New South Wales, School of Mathematics and Statistics, Sydney, NSW 2052, Australia. *Multiplicatively dependent points in orbits of algebraic dynamical systems.*

Bombieri, Masser and Zannier (1999) proved that the intersection of a curve defined over a number field with the union of all proper algebraic subgroups of the multiplicative group  $\mathbb{G}_m^n$  is a set of bounded height (unless this is false for an obvious reason).

Based on this result, in this talk we present recent finiteness results on multiplicative relations of values of polynomials or rational functions defined over a number field. As an application, we obtain new results on multiplicative dependence in the orbits of a univariate polynomial dynamical system. We also obtain a broad generalisation of the Northcott theorem replacing the finiteness of preperiodic points from a given number field by the finiteness of initial points with two multiplicatively dependent elements in their orbits. (Received July 16, 2017)

1135-11-136 **Tessa Cotron\*** (tessa.cotron@emory.edu), **Anya Michaelsen** (anm1@williams.edu), **Emily Stamm** (emstamm@vassar.edu) and **Weitao Zhu** (wz1@williams.edu). *Lacunary Eta-quotients Modulo Powers of Primes.*

An integral power series is called lacunary modulo  $M$  if almost all of its coefficients are divisible by  $M$ . Motivated by the parity problem for the partition function,  $p(n)$ , Gordon and Ono studied the generating functions for  $t$ -regular partitions, and determined conditions for when these functions are lacunary modulo powers of primes. We generalize their results in a number of ways by studying infinite products called Dedekind eta-quotients and generalized Dedekind eta-quotients. We then apply our results to the generating functions for the partition functions considered by Nekrasov, Okounkov, and Han. (Received August 01, 2017)

1135-11-173 **Roger B. Nelsen\*** (nelsen@lclark.edu), Lewis & Clark College, Portland, OR. *Alternative Proofs in Number Theory.*

Some time ago I was looking at several textbooks for the undergraduate number theory course. I was struck by how few illustrations were included in many of those textbooks. A number—specifically a positive integer—can represent many things: the cardinality of a set; the length of a line segment; or the area of a plane region. Such representations naturally lead to a variety of visual arguments for topics in elementary number theory. Since the undergraduate number theory course usually begins with properties of the positive integers, the texts should have more pictures. In this talk I will present some visual proofs useful in the study of perfect numbers. (Received August 07, 2017)

1135-11-174 **Skye Binegar\*** (binegars@reed.edu), **Randy Dominick**, **Meagan Kenney**, **Jeremy Rouse** and **Alex M. Walsh**. *An Elliptic Curve Analogue to the Fermat Numbers.*

The classic Fermat numbers, which take the form  $2^{2^n} + 1$ , have several notable properties, including order universality, coprimality and definition by a recurrence relation. For an arbitrary elliptic curve and rational point of infinite order, we define a sequence analogous to the classic Fermat numbers, which we call an *elliptic Fermat sequence*. We demonstrate that these sequences have many of the same properties as the classic Fermat numbers, and we identify significant patterns within the sequence generated by a specific elliptic curve and point. (Received August 07, 2017)

1135-11-175 **Christopher V. Donnay\*** (christopherdonnay@gmail.com), **Havi E. H. Ellers**, **Kate A. O'Connor**, **Katherine E. Thompson** and **Erin K. Wood**. *Numbers Represented by a Finite Set of Binary Quadratic Forms.*

Every quadratic form represents 0; therefore, if we take any number of quadratic forms and ask which integers are simultaneously represented by all members of the collection, we are guaranteed a nonempty set. But when is that set more than just the “trivial”  $\{0\}$ ? We address this question in the case of reduced, positive-definite, primitive, integral binary quadratic forms. For forms of the same discriminant, we can use the structure of the underlying class group. If, however, the forms have different discriminants, we must apply class field theory. (Received August 07, 2017)

1135-11-263 **Wade Hindes\*** (whindes@gc.cuny.edu) and **Rafe Jones**. *Riccati equations and polynomial dynamics over function fields.*

Given  $d \geq 2$ , Odoni conjectured that there is a polynomial  $\phi(x) \in \mathbb{Z}[x]$  whose arboreal Galois representation is surjective. By adapting a method of Loeper on the Galois theory of iterated trinomials, we prove some cases of a generalization of this conjecture obtained by replacing  $\mathbb{Z}$  with  $\mathbb{F}_p[t]$  or  $\mathbb{Q}[t]$ . Along the way, we establish a fairly general primitive prime divisor theorem for polynomial orbits over function fields (in any characteristic).

Interestingly, the main step in our proof is to rule out “Riccati differential equations” in backwards orbits. (Received August 16, 2017)

1135-11-269 **Richard Moy\*** ([rmoy@willamette.edu](mailto:rmoy@willamette.edu)), Willamette University, Department of Mathematics, 900 State Street, Salem, OR 97301, and **Michael Filaseta** ([filaseta@mailbox.sc.edu](mailto:filaseta@mailbox.sc.edu)), Department of Mathematics, University of South Carolina, Columbia, SC 29208. *On the Galois group over  $\mathbb{Q}$  of a truncated binomial expansion.*

The polynomial  $p_{r,t}(x) = \sum_{j=0}^r \binom{t+j}{j} x^j$  is the normalized  $t^{\text{th}}$  derivative of  $1 + x + \dots + x^{r+t}$ . This polynomial family has appeared in investigations of the Schubert calculus of Grassmannians as well as the Belyi maps of various families of dessin d’enfant. It was previously shown that for fixed  $r$ , the polynomial  $p_{r,t}(x)$  is irreducible for all but finitely many  $t$ . Using the theory of Newton polygons and Pell equations, we show that for  $r \neq 6$ ,  $p_{r,t}(x)$  has Galois group  $S_r$  for all but finitely many  $t$ . When  $r = 6$ , we show there are  $O(\log(T))$  values of  $t \leq T$  for which the Galois group of  $p_{6,t}$  is  $PGL_2(5)$  instead of  $S_6$ . (Received August 16, 2017)

1135-11-277 **Jared Duker Lichtman\*** ([jd1.18@dartmouth.edu](mailto:jd1.18@dartmouth.edu)), 5020 Cloister Drive, North Bethesda, MD 20852-3364. *Explicit estimates for the distribution of numbers free of large prime factors.*

A number is called  $y$ -smooth if all its prime factors are at most  $y$ . There is a large literature on the distribution of  $y$ -smooth numbers up to  $x$ . Asymptotic estimates for this distribution are quite useful in many applications, including the analysis of factorization and discrete logarithm algorithms. However, very little is known about this distribution that is numerically explicit. We generalize the saddle-point method of Hildebrand and Tenenbaum, giving explicit and fairly tight intervals in which the true count lies. For example, we determine that the count of integers at most  $10^{500}$  free of prime factors exceeding  $10^{35}$  lies within 0.65% of  $1.505 \times 10^{482}$ . This interval is so tight we can exclude the famous Dickman–de Bruijn asymptotic estimate as too small ( $1.472 \times 10^{482}$ ) and the Hildebrand–Tenenbaum main term as too large ( $1.513 \times 10^{482}$ ). One of the essential ideas in the proof is to relate the desired count of smooth numbers to a contour integral over a vertical line in the complex plane. The integrand includes an oscillatory sum over the primes, which requires an explicit form of the Prime Number Theorem. Joint work with Carl Pomerance. (Received August 17, 2017)

1135-11-289 **Stephan Ramon Garcia\*** ([stephan.garcia@pomona.edu](mailto:stephan.garcia@pomona.edu)), 610 N College Ave, Claremont, CA 91711. *Primitive root biases for prime pairs.*

We highlight remarkable biases displayed by prime pairs that were discovered by Pomona undergraduates in 2017. We study the difference between the number of primitive roots modulo  $p$  and modulo  $p+k$  for prime pairs  $p, p+k$ . Assuming the Bateman–Horn conjecture, we prove the existence of strong sign biases for such pairs and we prove that for a small positive proportion of prime pairs  $p, p+k$ , the dominant inequality is reversed. (Received August 19, 2017)

1135-11-303 **Jie Sun** and **Kyle N. Sutela\*** ([knsutela@mtu.edu](mailto:knsutela@mtu.edu)). *Prime Partitions.*

Prime partitions are partitions of integers into prime parts. In this paper, we first consider prime partitions with distinct parts. By using generating functions, we obtain some inductive formulas to calculate the number of prime partitions with distinct parts. Our formulas give two generalizations of the Euler’s formula for the integer partition case. Then we consider general prime partitions with not necessarily distinct parts. By keeping track of the recurrence of primes in a partition and finding bijections between different prime partitions, we get some inductive formulas to calculate the number of general prime partitions. Finally by numerical experimentation we find an approximation of some analytical formulas for the number of general prime partitions. (Received August 21, 2017)

1135-11-307 **Alan Haynes\*** ([haynes@math.uh.edu](mailto:haynes@math.uh.edu)) and **Jens Marklof**. *Higher dimensional Steinhaus and Slater problems via homogeneous dynamics.*

The three gap theorem, also known as Steinhaus conjecture or three distance theorem, states that the gaps in the fractional parts of  $\alpha, 2\alpha, \dots, N\alpha$  take at most three distinct values. Motivated by a question of Erdős, Geelen and Simpson, we explore a higher-dimensional variant, which asks for the number of gaps between the fractional parts of a linear form. Using the ergodic properties of the diagonal action on the space of lattices, we prove that for almost all parameter values the number of distinct gaps in the higher dimensional problem is unbounded. Our results in particular improve earlier work by Boshernitzan, Dyson and Bleher et al. We furthermore discuss a close link with the Littlewood conjecture in multiplicative Diophantine approximation.



Finally, we also demonstrate how our methods can be adapted to obtain similar results for gaps between return times of translations to shrinking regions on higher dimensional tori. (Received August 22, 2017)

1135-11-309 **Edray H. Goins** and **Pamela E. Harris\*** ([peh2@williams.edu](mailto:peh2@williams.edu)), 33 Stetson Ct, Bascom House, Rm 106C, Williamstown, MA 01267, and **Bethany Kubik** and **Aba Mbirika**.  
*Lattice point visibility on generalized lines of sight.*

For a fixed  $b \in \mathbb{N} = \{1, 2, 3, \dots\}$  we say a point  $(r, s)$  in the integer lattice  $\mathbb{Z} \times \mathbb{Z}$  is  $b$ -visible from the origin if it lies on a power function  $f(x) = ax^b$  with  $a \in \mathbb{Q}$  and no other integer lattice point lies on this line of sight between  $(0, 0)$  and  $(r, s)$ . We prove that the proportion of  $b$ -visible integer lattice points is given by  $\frac{1}{\zeta(b+1)}$ , where  $\zeta(s)$  denotes the Riemann-zeta function. We also show that even though the proportion of  $b$ -visible lattice points approaches 1 as  $b$  approaches infinity, there exist arbitrarily large  $b$ -invisible rectangular arrays of lattice points for any fixed  $b$ . This work specialized to  $b = 1$  recovers original results from the classical lattice point visibility setting where the lines of sight are given by linear functions with rational slope through the origin. (Received August 23, 2017)

1135-11-317 **Stephan Ramon Garcia\*** ([stephan.garcia@pomona.edu](mailto:stephan.garcia@pomona.edu)), Department of Mathematics, Pomona College, 610 N. College Ave, Claremont, CA 91711. *Quotient sets.*

Given a subset  $A \subseteq \mathbb{N}$ , what can be said of the set of quotients obtained from  $A$ ? We discuss a host of interesting and surprising results about quotient sets in both the real and  $p$ -adic settings. (Received August 23, 2017)

1135-11-336 **Terrence Richard Blackman\*** ([tblackman@mec.cuny.edu](mailto:tblackman@mec.cuny.edu)), 1650 Bedford Avenue, Brooklyn, NY 11225. *Spectral correspondences for Maass waveforms on quaternion groups.*

We prove that in most cases the Jacquet-Langlands correspondence between newforms for Hecke congruence groups and newforms for quaternion orders is a bijection. Our proof covers almost all cases where the Hecke congruence group is of cocompact type, i.e., when a bijection is possible. The proof uses the Selberg trace formula. (Received August 25, 2017)

1135-11-355 **Shiang Tang\***, 1400 east, 155 south, Salt Lake City, UT 84112. *Inverse Galois Theory and  $p$ -adic Galois representations.*

The classical Inverse Galois Problem asks whether every finite group can be realized as a Galois group over the rationals. In this talk, we introduce a variant of this problem: for a prime number  $p$  and a connected reductive algebraic group  $G$ , are there continuous homomorphisms from  $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$  to  $G(\mathbb{Q}_p)$  with Zariski-dense images? There is a complete answer to this question. The method we use is Galois deformation theory, which became one of the central tools in modern number theory when Andrew Wiles announced his celebrated proof of Fermat's Last Theorem. (Received August 27, 2017)

1135-11-380 **Ellina Grigorieva\*** ([egrigorieva@twu.edu](mailto:egrigorieva@twu.edu)), PO BOX 425262, Denton, TX 76204.  
*Teaching Methods of Solving Number Theory Problems.*

It is known that students have a hard time when trying to solve math problems involving integers, perhaps due to the fact that they study numbers in elementary school and basically never touch the topic again throughout the entire math curriculum. Many don't find arithmetic problems interesting or of much use in our everyday life since we don't feel that we need to know number theory for such fields as engineering or programming. However, number theory is important, gives us a power over the physical world we live in, and has been studied since the beginning of time. Carl Friedrich Gauss called it {The queen of mathematics}. Solving number theory problems helps us to think as mathematicians. We no longer view math as a disjointed collection of formulas and facts to memorize; instead, we learn to approach abstract problems in a systematic and creative manner. Additionally, we lose any inhibitions and fear of failure from our faulty memories. Solving problems empowers us. Methods presented in this talk are based on the author's own math Olympiad experience and over 25 years of teaching at a university level. (Received August 29, 2017)

1135-11-389 **Brandt Kronholm\*** ([brandt.kronholm@utrgv.edu](mailto:brandt.kronholm@utrgv.edu)), University of Texas Rio Grande Valley, Department of Mathematics, 1201 W. University Dr., Edinburg, TX 78539, **Stephanie Flores** ([stephanie.flores01@utrgv.edu](mailto:stephanie.flores01@utrgv.edu)), University of Texas Rio Grande Valley, Department of Mathematics, 1201 W. University Dr., Edinburg, TX 78539, and **Anabel Hernandez** ([anabel.hernandez01@utrgv.edu](mailto:anabel.hernandez01@utrgv.edu)), University of Texas Rio Grande Valley, Department of Mathematics, 1201 W. University Dr., Edinburg, TX 78539. *A complete characterisation of the maximal coefficients of Gaussian polynomials of the form  $\begin{bmatrix} M \\ 3 \end{bmatrix}$  and an extension of a theorem on first differences of partitions into at most three parts.* It is well known that the coefficients of Gaussian polynomials are unimodal. Working from recent results of H. Hahn, we provide a complete characterization of the maximal coefficients of Gaussian polynomials  $\begin{bmatrix} M \\ 3 \end{bmatrix}$ . Our general results come from a novel manipulation of the  $q$ -series informed by polyhedral geometry in which we establish a quasipolynomial for  $\begin{bmatrix} M \\ 3 \end{bmatrix}$ . Additionally, we extend a theorem on first differences of partitions into at most three parts to the coefficients of Gaussian polynomials  $\begin{bmatrix} M \\ 3 \end{bmatrix}$ . (Received August 30, 2017)

1135-11-568 **Laura S. Walton\*** ([laura@math.brown.edu](mailto:laura@math.brown.edu)), 151 Thayer Street, Brown University, Box 1917, Providence, RI 02912. *Idempotent relations for periodic point counts over finite fields.* Let  $V$  be a quasiprojective variety defined over  $\mathbb{F}_q$ , and let  $\phi : V \rightarrow V$  be an endomorphism of  $V$  that is also defined over  $\mathbb{F}_q$ . Let  $G$  be a finite subgroup of  $\text{Aut}_{\mathbb{F}_q}(V)$  with the property that  $\phi$  commutes with every element of  $G$ . Then, idempotent relations in the group ring  $\mathbb{Q}[G]$  give relations between the periodic point counts for the maps induced by  $\phi$  on the quotients of  $V$  by the various subgroups of  $G$ . This is a generalization of work of Ernst Kani and Michael Rosen. (Received September 08, 2017)

1135-11-585 **Carl Pomerance\*** ([carl.pomerance@dartmouth.edu](mailto:carl.pomerance@dartmouth.edu)), Mathematics Department, H.B. 6188, Dartmouth College, Hanover, NH 03755. *New results on an ancient function.* I refer to the function of Pythagoras that sends  $n$  to  $s(n)$ , the sum of the divisors of  $n$  that are less than  $n$ . A still-open conjecture of Catalan & Dickson: each orbit in the  $s$ -dynamical system (i.e.,  $n, s(n), s(s(n)), \dots$ ) is bounded. Modeling such a sequence as a random geometric progression, Bosma & Kane showed that the average of  $\log(s(n)/n)$  for  $n$  even is a constant  $\lambda < 0$  (and for  $n$  odd, it's  $-\infty$ ). A new result: the average of  $\log(s(s(n))/s(n))$  for  $n$  even is also  $\lambda$ .

Pythagoras noted 2-cycles in the  $s$ -dynamical system, the so-called amicable numbers. It's been known since 1981 that the reciprocal sum of the amicable numbers is finite, and in 2011 Bayless & Klyve showed this sum is  $< 656,000,000$ . Recently with Nguyen we improved the bound to 222.

I also report on some new results with Pollack & Thompson on fibers of  $s$ . (Received September 09, 2017)

1135-11-675 **Emerald Tatiana Stacy\*** ([stacye@math.oregonstate.edu](mailto:stacye@math.oregonstate.edu)), 715 Grouse Pl, Philomath, OR 97370. *On Small Heights of Abelian Totally  $p$ -adic Numbers.* The height of an algebraic number  $\alpha$  is a measure of how arithmetically complicated  $\alpha$  is. We say an algebraic number is totally real (or totally  $p$ -adic) if its minimal polynomial splits completely over  $\mathbb{R}$  (or  $\mathbb{Q}_p$ ). Further,  $\alpha$  is abelian if  $\mathbb{Q}(\alpha)$  is Galois with abelian Galois group over  $\mathbb{Q}$ . Let  $d \geq 1$  and  $p$  be a prime. Then there exists a smallest nontrivial height of an abelian totally  $p$ -adic algebraic number of degree  $d$ , which we will call  $\tau_{d,p}^{ab}$ . This talk will include the results describing aspects of the set  $\{\tau_{d,p}^{ab} : p \text{ is a prime}\}$ , and why it can be determined by a congruence condition on  $p$ . (Received September 12, 2017)

1135-11-680 **John Cullinan\*** ([cullinan@bard.edu](mailto:cullinan@bard.edu)), Department of Mathematics, Annandale-On-Hudson, NY 12401. *Arithmetic Properties of Jacobians of Curves Defined by the Generalized Laguerre Polynomials.* Let  $L_n^{(\alpha)}(x) = \sum_{j=0}^n \binom{n+\alpha}{n-j} \frac{(-x)^j}{j!}$  be the  $n$ th Generalized Laguerre Polynomial. In this talk we explore the arithmetic of the algebraic curves  $\mathcal{L}(n)/\mathbb{Q}$  defined by  $L_n^{(\alpha)}(x) = 0$ , viewed as a two-variable polynomial over  $\mathbb{Q}$ , and their Jacobians  $\mathcal{J}(n)/\mathbb{Q}$ . We introduce a conjecture for the endomorphism ring, Mordell-Weil group, and image of the  $\ell$ -adic representations of the  $\mathcal{J}(n)$  for all  $n \geq 4$ . (Received September 12, 2017)

1135-11-689 **David Krumm\*** ([dkrumm@colby.edu](mailto:dkrumm@colby.edu)). *A finiteness theorem for specializations of dynatomic polynomials.* Preliminary report. Let  $\Phi_n(c, x)$  denote the  $n$ -th dynatomic polynomial of the quadratic polynomial  $x^2 + c$ , viewed as a polynomial in two variables, and let  $G$  be the Galois group of  $\Phi_n$  over the function field  $\mathbb{Q}(c)$ . It is known that  $\Phi_n$  is irreducible and that  $G$  is isomorphic to a wreath product of a cyclic group and a symmetric group. Let  $E_n$  denote the set of rational numbers  $c$  such that the specialized polynomial  $\Phi_n(c, x)$  is either reducible or has Galois group different

from  $G$ . For  $n \leq 4$  the set  $E_n$  is infinite but well understood. In this talk we will discuss the conjecture that  $E_n$  is finite for  $n > 4$ , in particular proving this for  $n = 5$  and  $6$ . (Received September 13, 2017)

1135-11-706 **Jen Paulhus\*** (paulhus@math.grinnell.edu). *Decomposable Jacobian Varieties.*

Curves whose Jacobian varieties can be factored into many elliptic curves have interesting applications to rank and torsion questions. Given a group acting on a curve, we have developed several techniques to decompose the corresponding Jacobian variety. We will briefly discuss the techniques and give numerous interesting examples, such as Jacobians which factor completely into elliptic curves, and families of curves where there is a fixed factor across all Jacobians in that family. (Received September 13, 2017)

1135-11-711 **Colin Defant\*** (cdefant@princeton.edu), Fine Hall, Washington Road, Princeton, NJ 08544-1000. *Iterates of Schemmel Totient Functions.*

For each nonnegative integer  $r$ , the  $r^{\text{th}}$  Schemmel totient function is the multiplicative arithmetic function satisfying

$$S_r(p^\alpha) = \begin{cases} p^{\alpha-1}(p-r), & \text{if } p \geq r; \\ 0, & \text{if } p < r \end{cases}$$

for all primes  $p$  and positive integers  $\alpha$ . In particular,  $S_1 = \varphi$  is Euler's totient function. We make the additional convention  $S_r(0) = 0$ . After discussing results of Pillai, Shapiro, and Erdős–Granville–Pomerance–Spiro concerning iterates of Euler's totient function, we consider analogous results for the Schemmel totient functions. For example, it turns out that the function  $H_r: \mathbb{N} \rightarrow \{0, 1\}$  given by  $H_r(n) = \lim_{k \rightarrow \infty} S_r^k(n)$  is completely additive. We will discuss a conjecture due to Harrington and Jones and pose some additional open problems and conjectures. (Received September 13, 2017)

1135-11-727 **Harold G Diamond\*** (diamond@math.uiuc.edu), Prof. Harold G. Diamond, UIUC Department of Math, 1409 W. Green St., Urbana, IL 61801. *The Convolution Square Root of 1.*

The square root of the Riemann zeta function is the generating function of an arithmetic function  $\nu$  that we call the convolution square root of 1, by analogy with  $\zeta(s)^2$  and the divisor function  $1 * 1$ . It is not hard to see that  $\nu$  is multiplicative. We show how knowing its summatory function  $D_{1/2}(x)$  with a modest error term implies the prime number theorem, and we indicate how to approximate  $D_{1/2}(x)$ . (Received September 13, 2017)

1135-11-734 **Jeffrey Hatley\*** (hatleyj@union.edu) and **Antonio Lei**. *Comparing anticyclotomic Selmer groups of positive coranks for congruent modular forms.*

We study the variation of Iwasawa invariants of the anticyclotomic Selmer groups of congruent modular forms under the Heegner hypothesis. In particular, we show that even if the Selmer groups we study may have positive coranks, the  $\mu$ -invariant vanishes for one modular form if and only if it vanishes for the other, and that their  $\lambda$ -invariants are related by an explicit formula. This generalizes results of Greenberg–Vatsal and Emerton–Pollack–Weston for the cyclotomic extension, as well as results of Pollack–Weston and Castella–Kim–Longo for the anticyclotomic extension when the Selmer groups in question are cotorsion. (Received September 13, 2017)

1135-11-745 **Amanda Folsom\***, Amherst College, Amherst, MA 01002. *Quantum modular forms and quantum Jacobi forms.*

Quantum modular forms were defined by Zagier in 2010; they are similar to mock modular forms in that they feign modularity in some way, with the notable exception that their domain is not the upper half-plane, but rather the rational numbers. Questions of interest to many have been to understand spaces of quantum modular forms, determine explicit examples of and sources of quantum modular forms, and to understand the relationship – if any – between quantum modular and mock modular forms. In this talk we discuss these problems and the developing theory of quantum modular forms. In particular, we discuss the notion of a quantum Jacobi form as defined by the author and Bringmann in 2016. We also offer compelling examples with combinatorial origins, and corollaries pertaining to the evaluation of Eichler integrals. (Received September 13, 2017)

1135-11-757 **Stephen Choi\*** (schoia@sfu.ca), **Jean-Paul Allouche**, **Tamas Erdelyi**, **Alain Denise** and **Bahman Saffari**. *Bounds on Autocorrelation Coefficients of Rudin-Shapiro Polynomials.*

In this talk, we will discuss lower and upper bounds on the autocorrelation coefficients of Rudin-Shapiro polynomials. This is a joint work with J-P. Allouche, T. Erdélyi, A. Denise and B. Saffari. (Received September 14, 2017)

1135-11-825 **Robert Schneider\*** ([robert.schneider@emory.edu](mailto:robert.schneider@emory.edu)). *Toward an arithmetic of partitions.* Much like the positive integers  $\mathbb{Z}^+$ , the set  $\mathcal{P}$  of integer partitions ripples with interesting patterns and relations. Now, as Alladi-Erdős point out, the prime decompositions of integers are in bijective correspondence with the set of partitions into prime parts, if we associate 1 to the empty partition. Might some number-theoretic theorems arise as images in  $\mathbb{Z}^+$  (i.e. in prime partitions) of greater algebraic and set-theoretic structures in  $\mathcal{P}$ ? In the 1970s, Andrews developed a beautiful theory of partition ideals using ideas from lattice theory, teasing the possibility of a universal algebra of partitions. Looking in a similar direction for arithmetic structures in the partitions, we show that many objects from elementary and analytic number theory are special cases of general partition-theoretic and  $q$ -series theorems: a multiplicative arithmetic of partitions that specializes to classical cases; a class of “partition zeta functions” containing  $\zeta(s)$  and other Dirichlet series as well as exotic non-classical cases; partition formulas for arithmetic densities of subsets of  $\mathbb{Z}^+$  such as  $k$ th-power-free integers; and other phenomena at the intersection of the additive and multiplicative branches of number theory. (Received September 14, 2017)

1135-11-926 **Robert L. Benedetto\*** ([rlbenedetto@amherst.edu](mailto:rlbenedetto@amherst.edu)). *Computing arboreal Galois groups of some PCF polynomials.*

Let  $K$  be a number field, let  $f \in K(x)$  be a rational function of degree  $d \geq 2$ , and let  $a \in K$ . The roots of  $f^n(z) - a$  are the  $n$ -th preimages of  $a$  under  $f$ , and they have the natural structure of a  $d$ -ary rooted tree  $T$ . The resulting representation of the absolute Galois group of  $K$  in the automorphism group of  $T$  is called an arboreal Galois representation. In many cases, it is expected that its image has finite index in the full automorphism group, but in some cases, such as when  $f$  is postcritically finite (PCF), the image has infinite index. In this talk, we present some new PCF examples where the arboreal Galois group can be computed completely. (Received September 17, 2017)

1135-11-949 **Michael A. Bennett\*** ([bennett@math.ubc.ca](mailto:bennett@math.ubc.ca)), 1984 Mathematics Road, Vancouver, BC V6T 1Z2, Canada, and **Samir Siksek**. *Erdos-Selfridge and supersingularity.*

If  $k$  is a sufficiently large positive integer, we show that the Diophantine equation

$$n(n+d) \cdots (n+(k-1)d) = y^\ell$$

has at most finitely many solutions in positive integers  $n, d, y$  and  $\ell$ , with  $\gcd(n, d) = 1$  and  $\ell \geq 2$ . Our proof relies upon Frey-Hellegouarch curves and results on supersingular primes for elliptic curves without complex multiplication, derived from upper bounds for short character sums and sieves, analytic and combinatorial. (Received September 17, 2017)

1135-11-970 **Emanuel Carneiro\*** ([carneiro@impa.br](mailto:carneiro@impa.br)). *Fourier optimization and primes gaps.*

Fourier optimization problems appear naturally within several different fields of mathematics, particularly in analysis and number theory. These are problems in which one imposes certain conditions on a function and its Fourier transform, and then wants to optimize a certain quantity. A recent example is given in the proof of the optimal sphere packing in dimensions 8 and 24. In this talk I want to show how certain optimization problems of this sort appear naturally in connection to the question of bounding the maximal gap between consecutive primes, under the Riemann hypothesis. In particular, we improve the best known bounds for this problem, that dates back to the works of Cramer in the 1920's. This is a joint work with M. Milinovich (Univ. of Mississippi) and K. Soundararajan (Stanford Univ.) (Received September 18, 2017)

1135-11-996 **Lenny Fukshansky\*** ([lenny@cmc.edu](mailto:lenny@cmc.edu)) and **Nikolay Moshchevitin**. *An effective variation of Kronecker's approximation theorem.*

The classical Kronecker's approximation theorem asserts that the image of an integer lattice under a linear map is dense in the multiplicative torus in every dimension. We will discuss an effective version of such a statement for algebraic lattices, as well as a variation of this result for points of the lattice avoiding a fixed algebraic set. (Received September 18, 2017)

1135-11-1078 **Xander Faber\*** ([awfaber@super.org](mailto:awfaber@super.org)). *The arithmetic of Newton's method.*

Fix a squarefree polynomial  $f(z)$  with rational coefficients. The familiar Newton map

$$N(z) = z - \frac{f(z)}{f'(z)}$$

is a rational function with super-attracting fixed points at the roots of  $f$ . For a general choice of rational number  $x$ , consider the set of primes  $p$  for which the Newton sequence  $x, N(x), N^2(x), \dots$  converges  $p$ -adically to a root of  $f$ . Felipe Voloch and I conjectured that this set has density zero in 2010. I will sketch a recent proof of the

first nontrivial case of this conjecture, namely for the polynomial  $f(z) = -2z^3 + 3z^2$ . (Received September 19, 2017)

1135-11-1112 **Jordan S. Ellenberg** (ellenber@math.wisc.edu) and **Daniel Rayor Hast\*** (hast@math.wisc.edu). *Rational points on solvable curves over  $\mathbb{Q}$  via non-abelian Chabauty.*

We study the Selmer varieties of smooth projective curves of genus at least two defined over  $\mathbb{Q}$  which geometrically dominate a curve with CM Jacobian. We extend a result of Coates and Kim to show that Kim's non-abelian Chabauty method applies to such a curve. By combining this with results of Bogomolov–Tschinkel and Poonen on unramified correspondences, we deduce that any cover of  $\mathbf{P}^1$  with solvable Galois group, and in particular any superelliptic curve over  $\mathbb{Q}$ , has only finitely many rational points over  $\mathbb{Q}$ . (Received September 19, 2017)

1135-11-1136 **Kim Klinger-Logan\*** (kling202@umn.edu), 127 Vincent Hall, 206 Church Street, Minneapolis, MN 55455. *A spectral interpretation of zeros of certain functions.* Preliminary report.

We prove that all the zeros of a certain family of meromorphic functions are on the critical line  $\operatorname{Re}(s) = 1/2$  and are simple (except for possibly  $s = 1/2$ ), by relating the zeros to the discrete spectrum of unbounded self-adjoint operators. For example, for  $h(s)$  a meromorphic function with no zeros in  $\operatorname{Re}(s) > 1/2$  with  $h(s)$  real-valued on  $\mathbb{R}$ , and  $\frac{h(1-s)}{h(s)} \ll |s|^{1-\epsilon}$  in  $\operatorname{Re}(s) > 1/2$ , the only zeros of  $h(s) \pm h(1-s)$  are on the critical line. One such instance of this result is  $h(s) = \xi_k(2s)$  the completed zeta-function of a number field  $k$  or, more generally, many self-dual automorphic  $L$ -functions. We use spectral theory suggested by results of Lax-Phillips and ColinDeVerdière. This simplifies ideas of W. Müller, J. Lagarias, M. Suzuki, H. Ki, O. Velásquez Castañón, D. Hejhal, L. de Branges and P.R. Taylor. (Received September 19, 2017)

1135-11-1147 **Djordje Milićević\*** (dmilicevic@brynmawr.edu), Bryn Mawr College, Department of Mathematics, 101 North Merion Avenue, Bryn Mawr, PA 19010, and **Valentin Blomer**, **Gergely Harcos** and **Péter Maga**. *The sup-norm problem for  $\operatorname{GL}(2)$  over number fields.*

Eigenfunctions of the Laplacian are basic building blocks of harmonic analysis on Riemannian manifolds. The sup-norm problem asks for bounds on the pointwise values of an  $L^2$ -normalized eigenfunction in terms of its Laplacian eigenvalue or other increasing parameters. Exciting progress in arithmetic cases means that this question, which is particularly interesting from the point of view of quantum mechanics, now occupies a prominent position at the interface of automorphic forms, analytic number theory, and analysis.

In this talk, we will present our recent bounds solving the sup-norm problem for spherical Hecke–Maaß newforms of square-free level for the group  $\operatorname{GL}(2)$  over a number field, with a power saving over the local geometric bound simultaneously in the eigenvalue and the level aspect. Our bounds feature a Weyl-type exponent in the level aspect, they reproduce or improve upon all known special cases, and over totally real fields they are as strong as the best known hybrid result over the rationals.

The talk will emphasize several new features and difficulties that the number field setting (and specifically complex places) introduces and new techniques we developed to address them, which are also of independent interest. (Received September 19, 2017)

1135-11-1149 **Andrew Kwon\*** (akwon@andrew.cmu.edu), 946 Sherbourne Circle, Lake Mary, FL 32746. *Minimal Additive Complements.*

Let  $C, W \subseteq \mathbb{Z}$ . If  $C + W = \mathbb{Z}$ , then the set  $C$  is called an additive complement to  $W$  in  $\mathbb{Z}$ . If no proper subset of  $C$  is an additive complement to  $W$ , then  $C$  is called a minimal additive complement. We provide a partial answer to a question posed by Kiss, Sándor, and Yang regarding the minimal additive complement of sets of the form  $W = (n\mathbb{N} + A) \cup F \cup G$ , where  $|F| < \infty$ ,  $(F \bmod n) \subseteq (A \bmod n)$  and  $(G \bmod n) \cap (A \bmod n) = \emptyset$ . (Received September 19, 2017)

1135-11-1150 **Marc Chamberland\*** (chamberl@grinnell.edu), 1116 8th. Ave., Grinnell, IA 50112. *Analytic Formulas for the Euler Totient Function.* Preliminary report.

Starting with the formula

$$\sum_{k=1}^n ka_k = \sum_{j=1}^n \phi(j) \sum_{l=1}^{\lfloor n/j \rfloor} a_{lj}$$

which holds for all sequences  $\{a_k\}_1^n$ , we construct known and unknown formulas involving the totient function. These include finite and infinite sums involving the harmonic numbers,  $L$ -series, polylogarithms and Dirichlet characters. (Received September 19, 2017)

1135-11-1170 **Alok B Shukla\*** ([a.ok.shukla@ou.edu](mailto:a.ok.shukla@ou.edu)), 1012 Drake Dr, Apt 7, Norman, OK 73071. *On Klingen Eisenstein Series.*

We give a representation theoretic approach to the Klingen lift generalizing the classical construction of Klingen Eisenstein series to arbitrary levels for both paramodular and Siegel congruence subgroups. (Received September 20, 2017)

1135-11-1173 **Edward B. Burger\*** ([burger@southwestern.edu](mailto:burger@southwestern.edu)), Office of the President, P.O. Box 770, Southwestern University, Georgetown, TX 78627. *Applications of Orthogonality within Non-Archimedean and Human Contexts.*

Here we offer a brief overview of J.D. Vaaler's well-known work on Siegel's Lemma and his work on orthogonal projections over non-Archimedean spaces. We describe how orthogonality, together with the geometry of numbers over the Adele space, established the existence of a primitive basis version of Siegel's Lemma over an arbitrary number field. This result generalized and extended work of K. Mahler, A. Schinzel, and H. Weyl. We conclude with some reflections and observations on orthogonality within our mathematical culture and how it can lead to improved results. (Received September 20, 2017)

1135-11-1213 **Heidi Goodson\*** ([hgoodson@haverford.edu](mailto:hgoodson@haverford.edu)). *Hypergeometric Properties of Genus 3 Generalized Legendre Curves.*

Inspired by a result of Manin, we study the relationship between certain period integrals and the trace of Frobenius of genus 3 generalized Legendre curves. We show that both of these properties can be computed in terms of "matching" classical and finite field hypergeometric functions, a phenomenon that has also been observed in elliptic curves and many higher dimensional varieties. (Received September 20, 2017)

1135-11-1238 **Wolfgang M. Schmidt\*** ([wolfgang.schmidt@colorado.edu](mailto:wolfgang.schmidt@colorado.edu)). *On parametric geometry of numbers.*

We study successive minima of a given lattice with respect to a symmetric convex body  $K(q)$  depending on the parameter  $q$ . More precisely,  $K(q)$  is the image of a given body  $K$  when applying the  $q$ -th power  $T^q$  of a symmetric linear map  $T$ . The subject arose from Diophantine Approximation where (in the simplest case) one deals with approximation to a given irrational by rationals with denominator at most  $e^q$ . After a brief review of basic facts we will discuss some current work and conjectures. (Received September 20, 2017)

1135-11-1240 **Khoa Dang Nguyen\*** ([dangkhoa.nguyen@ucalgary.ca](mailto:dangkhoa.nguyen@ucalgary.ca)), 2500 University Drive NW, University of Calgary, Math Sciences Building, MS542, Calgary, Alberta T2N 4T4, Canada. *Bounded height in families of dynamical systems.*

Let  $a, b \in \bar{\mathbb{Q}}$  be such that exactly one of  $a$  and  $b$  is an algebraic integer, and let  $f_t(z) = z^2 + t$  be a family of quadratic polynomials parametrized by  $t \in \bar{\mathbb{Q}}$ . We prove that the set of all  $t \in \bar{\mathbb{Q}}$  for which there exist  $m, n \geq 0$  such that  $f_t^m(a) = f_t^n(b)$  has bounded height. This is a special case of a more general result supporting a new bounded height conjecture in arithmetic dynamics. This is joint work with DeMarco, Ghioca, Krieger, Tucker, and Ye. (Received September 20, 2017)

1135-11-1262 **Trevor Hyde** ([tghyde@umich.edu](mailto:tghyde@umich.edu)) and **Jeffrey C. Lagarias\*** ([lagarias@umich.edu](mailto:lagarias@umich.edu)). *Polynomial Splitting Measures and Cohomology of the Pure Braid Group.*

Work of the second author and Benjamin L. Weiss (2015) studied the probability of particular factorization types of monic square-free polynomials of degree  $n$  over the finite field  $F_q$ . These probabilities are labeled by partitions of  $n$ , so define class functions on the symmetric group  $S_n$ . It showed these probabilities on each class interpolate in the parameter  $q$  to rational functions of  $q$  that are Laurent polynomials. The second author (J. Number Theory 2016) found that at  $q = 1$  rescaled versions of these probabilities had a representation theoretic meaning as characters of an explicitly described representation of  $S_n$ . The present work (Arnold Math. J. 2017) studies the individual coefficients in the Laurent polynomials and shows they also have a representation-theoretic interpretation. The coefficients are each given to a subrepresentation of the  $S_n$ -action acting on a particular cohomology group of the pure braid group. The results were discovered by computer experiments. (Received September 20, 2017)

1135-11-1320 **Byungchul Cha\*** ([cha@muhlenberg.edu](mailto:cha@muhlenberg.edu)), Muhlenberg College, 2400 W. Chew st, Allentown, PA 18104, and **Dong Han Kim**. *Berggren trees, Romik system and Lagrange Theorem.*

An old result of Berggren says that there exist three  $3 \times 3$  matrices  $M_1, M_2, M_3$  such that, for every triple  $(x, y, z)$  of positive coprime integers (with  $y$  even) satisfying  $Q(x, y, z) := x^2 + y^2 - z^2 = 0$ , there exists a unique sequence

$d_1, d_2, \dots, d_k$  with  $d_j \in \{1, 2, 3\}$  such that

$$(x, y, z)^\top = M_{d_1} \cdots M_{d_k}(3, 4, 5)^\top.$$

If  $y$  is odd, the same is true with  $(4, 3, 5)$  replacing  $(3, 4, 5)$ . That is, the set of all such triples  $(x, y, z)$  forms infinite ternary trees rooted at  $(3, 4, 5)$  and  $(4, 3, 5)$ . We study the generalizations of these trees arising from other quadratic forms  $Q(x, y, z)$ . Also, we investigate the properties of Romik's dynamical system, which closely resembles the classical dynamical system of continued fractions on the unit interval defined by Gauss map. In particular, we present an analogue of Lagrange Theorem for Romik system which concerns eventually periodic digit expansions for points over a real quadratic field. This work originated from the collaboration with Emily Nguyen ('16) and Brandon Tauber ('16), funded by Center for Undergraduate Research in Mathematics. (Received September 21, 2017)

1135-11-1321 **Hugh L. Montgomery\*** (hlm@umich.edu), Department of Mathematics, University of Michigan, 530 Church Street, Ann Arbor, MI 48109-1043. *New Trigonometric Extremal Problems Related to Pair Correlations*. Preliminary report.

Assume the Riemann Hypothesis (RH), and for real  $\alpha$  and  $T \geq 2$ , put

$$F(\alpha, T) = \left(\frac{T}{2\pi} \log T\right)^{-1} \sum_{\substack{0 < \gamma \leq T \\ 0 < \gamma' \leq T}} T^{i\alpha(\gamma - \gamma')} w(\gamma - \gamma')$$

where  $w(u) = 4/(4 + u^2)$ . We know that  $F$  is real valued, even, nonnegative, and that

$$F(\alpha) = (1 + o(1))T^{-2\alpha} \log T + \alpha + o(1)$$

uniformly for  $0 \leq \alpha \leq 1$ . If  $R \in L^1(\mathbb{R})$ , then

$$\sum_{\substack{0 < \gamma \leq T \\ 0 < \gamma' \leq T}} \widehat{R}\left(\frac{\gamma - \gamma'}{2\pi} \log T\right) w(\gamma - \gamma') = \left(\frac{T}{2\pi} \log T\right) \int_{\mathbb{R}} R(\alpha) F(\alpha) d\alpha.$$

This has been used to obtain results concerning the spacing of the zeta zeros. In the past it was required that  $R(\alpha) = 0$  when  $|\alpha| > 1$ . We note that this is overly severe: For many purposes it suffices to constrain the sign of  $R$  in this range. (Received September 21, 2017)

1135-11-1336 **Nicholas J. Newsome\*** (nnews001@ucr.edu), **Maria S. Nogin** and **Adnan H. Sabuwala**. *Symmetry of the Power Sum Polynomials*.

Sums of powers of integers have been studied extensively for many centuries. The Pythagoreans, Archimedes, Fermat, Pascal, Bernoulli, Faulhaber, and other mathematicians have discovered formulas for sums of powers of the first  $n$  natural numbers. Among these is Faulhaber's well-known formula which expresses the power sums as polynomials whose coefficients involve Bernoulli numbers.

In this talk, we give an elementary proof that for each natural number  $p$ , the sum of  $p$ th powers of the first  $n$  natural numbers can be expressed as a polynomial in  $n$  of degree  $p + 1$ . We also prove a novel identity involving Bernoulli numbers and use it to show symmetry of these polynomials. In addition, we make a few conjectures regarding the roots of these polynomials, and speculate on the asymptotic behavior of their graphs. (Received September 21, 2017)

1135-11-1346 **Kate Petersen\*** (petersen@math.fsu.edu) and **Christopher Sinclair**. *Equidistribution of Elements of Norm*.

Upon quotienting by units, the elements of norm 1 in a number field  $K$  form a countable subset of a torus of dimension  $r+s-1$  where  $r$  and  $s$  are the numbers of real and pairs of complex embeddings. When  $K$  is Galois with cyclic Galois group we demonstrate that this countable set is equidistributed in this torus with respect to a natural partial ordering. (Received September 21, 2017)

1135-11-1370 **Shabnam Akhtari** and **Jeffrey Vaaler\*** (vaaler@math.utexas.edu), PO Box 495, Yachats, OR 97498. *Diophantine inequalities for the Weil height*.

We will describe two Diophantine problems with bounds on the Weil height of the solutions. A unit  $\beta$  in a finite Galois extension  $l/\mathbb{Q}$  is a *Minkowski unit* if its Galois conjugates generate a subgroup of maximum rank. Minkowski proved that such units exist. We show that such units exist with Weil height bounded by a natural expression depending on the full group of units.

Let  $l/k$  be a finite extension of number fields and  $F$  in  $l[x_1, x_2, \dots, x_N]$  a norm form associated to a full module in  $l$ . If  $\gamma \neq 0$  is an algebraic integer in  $k$ , the set of algebraic integers  $\xi_1, \xi_2, \dots, \xi_N$  in  $l$  that satisfy the norm form equation  $F(\xi_1, \xi_2, \dots, \xi_N) = \gamma$  is naturally a union of certain disjoint equivalence classes. We show that each equivalence class has a representative with Weil height bounded by an expression depending on the full module. (Received September 21, 2017)

1135-11-1371 **Cameron Stewart\*** (cstewart@uwaterloo.ca), **Francesco Pappalardi** (pappa@mat.uniroma3.it), **Min Sha** (shamin2010@gmail.com) and **Igor Shparlinski** (igor.shparlinski@unsw.edu.au). *Multiplicatively dependent vectors of algebraic numbers.*  
 In this talk, we estimate the number of multiplicatively dependent vectors of algebraic numbers of fixed degree, or within a fixed number field, and bounded height. (Received September 21, 2017)

1135-11-1471 **Erik R. Tou\*** (etou@uw.edu), University of Washington – Tacoma, Interdisciplinary Arts & Sciences, 1900 Commerce St., Campus Box 358436, Tacoma, WA 98402. *The Prime Number Theorem For Juggling Patterns.*

The mathematics of juggling emerged after the development of siteswap notation in the 1980s. Consequently, much work was done to establish a mathematical theory that describes and enumerates the patterns that a juggler can execute. More recently, mathematicians have provided a broader picture of juggling *sequences* as an infinite set possessing properties similar to the set of positive integers. This theoretical framework moves beyond the physical possibilities of juggling and instead seeks more general mathematical results, such as an enumeration of juggling patterns with a fixed period and arbitrary number of balls. One problem unresolved until now is the enumeration of *primitive* juggling sequences, those fundamental juggling patterns that are analogous to the set of prime numbers. By applying analytic techniques to previously known generating functions, we give asymptotic counting theorems for primitive juggling sequences, much as the prime number theorem gives asymptotic counts for prime rational integers. (Received September 22, 2017)

1135-11-1568 **NicoleLooper\*** (nlooper@math.northwestern.edu). *A lower bound on the canonical height for polynomials.*

Let  $K$  be a number field. The canonical height  $\hat{h}_\phi$  associated to a rational function  $\phi: \mathbb{P}^1(K) \rightarrow \mathbb{P}^1(K)$  measures arithmetic information about the forward orbits of points under  $\phi$ . Silverman conjectured that for a given number field  $K$  and  $d \geq 2$ , there exist constants  $\kappa_1 > 0$  and  $\kappa_2$  such that for all degree  $d$  rational maps  $\phi \in K(x)$  and all  $\alpha \in K$ , either  $\alpha$  is preperiodic under  $\phi$ , or  $\hat{h}_\phi(\alpha) \geq \kappa_1 h_{\mathcal{M}_d}(\phi) + \kappa_2$ , where  $h_{\mathcal{M}_d}$  is a height on the moduli space  $\mathcal{M}_d$  of degree  $d$  rational functions. We will discuss a proof of such a uniform lower bound across large classes of polynomials. (Received September 23, 2017)

1135-11-1595 **Ryan Patrick Wood\*** (rpw54@nau.edu), **Jeff Rushall** and **Pauline Gonzalez.** *On Cubic Residue Matrices.*

In 2001, R. Chapman conjectured that a special infinite class of matrices, constructed using quadratic residue symbols, had constant determinant values. This conjecture, known as Chapman's Evil Determinant Problem was resolved by in 2014 M. Vsemirnov. In this talk, we will present a generalization of Chapman's problem involving cubic residues. (Received September 23, 2017)

1135-11-1648 **Frank Garvan\*** (fgarvan@uf1.edu), Department of Mathematics, University of Florida, Gainesville, FL 32611-8105. *New Ramanujan mock theta function identities.* Preliminary report.

In his last letter to Hardy, Ramanujan defined ten mock theta functions of order 5 and three of order 7. He stated that the three mock theta functions of order 7 are not related. We show how we discovered new identities for the fifth order functions and there are actually surprising relationships between the order 7 functions. (Received September 24, 2017)

1135-11-1672 **Michael J. Mossinghoff\*** (mimossinghoff@davidson.edu), Davidson College, Davidson, NC 28035, **Dilum De Silva**, Bowling Green State University Firelands, Huron, OH 44839, **Vincent Pigno**, California State University, Sacramento, CA 95819, and **Christopher Pinner**, Kansas State University, Manhattan, KS 66506. *The Lind-Lehmer constant for certain  $p$ -groups.*

In 2005, Lind formulated an analogue of Lehmer's well-known problem regarding the Mahler measure for general compact abelian groups, and defined a Lehmer constant for each group. Since then, this Lind-Lehmer constant has been determined for many finite abelian groups, including all but a thin set of finite cyclic groups. We establish some new congruences satisfied by the Lind Mahler measure for finite  $p$ -groups, and use them to determine the Lind-Lehmer constant for many groups of this form. We also develop an algorithm that determines a small set of possible values for a given  $p$ -group of a particular form. This method is remarkably effective, producing just one permissible value in all but a handful of trials. This leads to some questions regarding the size of the smallest nontrivial  $(p-1)$ st root of unity mod  $p^r$ . (Received September 24, 2017)



1135-11-1674 **Michael J. Mossinghoff\*** (mimossinghoff@davidson.edu), Davidson College, Davidson, NC 28035, and **Timothy S. Trudgian**, University of New South Wales, Canberra. *The Liouville function and the Riemann hypothesis.*

The Liouville function  $\lambda(n)$  is the completely multiplicative arithmetic function defined by  $\lambda(p) = -1$  for each prime  $p$ . Pólya investigated its summatory function  $L_0(x) = \sum_{n \leq x} \lambda(n)$  and Turán studied a weighted relative  $L_1(x) = \sum_{n \leq x} \lambda(n)/n$ . Each noted that the Riemann hypothesis would follow if one of these functions never changed sign for large  $x$ . While it has been known since the work of Haselgrove in 1958 that these functions do change sign infinitely often, oscillations in these functions and their relatives remain of interest, since the Riemann hypothesis would follow if their oscillations could be bounded in a particular way. We describe a method involving substantial computation that establishes improved bounds on the magnitude of oscillations in the generalized family of functions  $L_\alpha(x) = \sum_{n \leq x} \lambda(n)/n^\alpha$  with  $\alpha \in [0, 1/2) \cup (1/2, 1]$ . (Received September 24, 2017)

1135-11-1701 **Eva G Goedhart\*** (goedhart@lvc.edu), 101 N. College Ave, Annville, PA 17003, and **Helen G Grundman**. *Solving the Diophantine Equation*  
 $(a^2cX^k - 1)(b^2cY^k - 1) = (abcZ^k - 1)^2$ .

Given positive integers  $a, b, c, k$  with  $k \geq 7$ , the equation  $(a^2cX^k - 1)(b^2cY^k - 1) = (abcZ^k - 1)^2$  has no integer solutions  $x, y, z > 1$  with  $a^2x^k \neq b^2y^k$ . I will present the proof of this result using results on continued fractions and a Diophantine approximation theorem due to M.A. Bennett. (Received September 24, 2017)

1135-11-1722 **Paul Pollack** (pollack@uga.edu), Athens, GA, and **Enrique Treviño\*** (trevino@lakeforest.edu), Lake Forest, IL. *Finding the four squares in Lagrange's Theorem.*

In 1986, Rabin and Shallit presented three random algorithms to compute, given a positive integer  $n$ , integers  $X, Y, Z, W$  with  $X^2 + Y^2 + Z^2 + W^2 = n$ . The fastest of the three has expected runtime  $O((\log n)^2)$ , but this runtime analysis assumes the truth of the Extended Riemann Hypothesis. The other two algorithms admit slightly worse runtime estimates but are unconditional, in the sense that no unproved hypotheses are used in the proof of correctness or the running-time analysis. In this talk we explain how to modify their algorithms to do better unconditionally. (Received September 24, 2017)

1135-11-1735 **Cristina M Ballantine\*** (cballant@holycross.edu) and **Mircea Merca**. *Bisected theta series, least  $r$ -gaps in partitions, and polygonal numbers.*

The least  $r$ -gap,  $g_r(\lambda)$ , of a partition  $\lambda$  is the smallest part of  $\lambda$  appearing less than  $r$  times. In this article we introduce two new partition functions involving least  $r$ -gaps. We consider a bisection of a classical theta identity and prove new identities relating Euler's partition function  $p(n)$ , polygonal numbers, and the new partition functions. To prove the results we use an interplay of combinatorial and  $q$ -series methods. (Received September 24, 2017)

1135-11-1782 **Jacqueline Anderson\*** (jacqueline.anderson@bridgew.edu), Bridgewater, MA 02325. *Dynamical Belyi Maps.*

We study the dynamical properties of a large class of rational maps on  $\mathbb{P}^1$  with exactly three ramification points. We explicitly construct two infinite families of such maps and in doing so, we answer a question of Silverman about the number of conservative polynomials of degree  $d$  defined over  $\mathbb{Q}$ . We also prove a result about the reduction of such maps that yields strong information about their rational preperiodic points. This is joint work with Irene Bouw, Ozlem Ejder, Neslihan Girgin, Valentijn Karemaker, and Michelle Manes. (Received September 24, 2017)

1135-11-1801 **Niki Myrto Mavraki\*** (myrtomav@math.ubc.ca) and **Hexi Ye**. *Equidistribution and preperiodic points for families of rational maps.*

The arithmetic equidistribution theorem for small points with respect to an adelic measure, proved independently by Baker–Rumely and Favre–Rivera-Letelier has had many applications in the study of dynamics of families of rational maps. In this talk we show that most 1-parameter families of rational maps fail to satisfy the adelic hypothesis in the aforementioned equidistribution theorem. We present a more general equidistribution theorem for quasi-adelic measures and give some applications of our theorem. (Received September 24, 2017)

1135-11-1842 **David Lowry-Duda\*** ([d.lowry@warwick.ac.uk](mailto:d.lowry@warwick.ac.uk)), Warwick Mathematics Institute, Zeeman Building, University of Warwick, Coventry, CV4 7AL, United Kingdom. *Counting lattice points on hyperboloids.*

In this talk, we discuss improved asymptotics for the number of lattice points  $x = (x_1, \dots, x_{d+1})$  with  $\|x\| < R$  and lying on the one-sheeted hyperboloid  $x_1^2 + \dots + x_k^2 = x_{d+1}^2 + h$ . Counting these lattice points is a problem very similar in flavor to the generalized Gauss circle problem, which concerns counting all lattice points lying within a  $d$ -dimensional sphere of radius  $R$ . We describe ideas and techniques from shifted convolution sums and modular forms leading to improved results on both sharp and smoothed asymptotics. (Received September 25, 2017)

1135-11-1848 **Lillian B. Pierce, Caroline L. Turnage-Butterbaugh\*** ([ctb@math.duke.edu](mailto:ctb@math.duke.edu)) and **Melanie Matchett Wood.** *An effective Chebotarev density theorem for families of fields, with applications to class groups.*

This talk will present a new effective Chebotarev theorem that holds for all but a possible zero-density subfamily of certain families of number fields of fixed degree. For certain families, this work is unconditional, and in other cases it is conditional on the strong Artin conjecture and certain conjectures on counting number fields. As an application, we obtain nontrivial average upper bounds on  $\ell$ -torsion in the class groups of the families of fields. (Received September 25, 2017)

1135-11-1946 **Levent Hasan Ali Alpöge\*** ([lalpoge@math.princeton.edu](mailto:lalpoge@math.princeton.edu)). *The average number of rational points on genus 2 curves is bounded.*

We prove that, when genus two curves  $C/\mathbb{Q}$  with a marked Weierstrass point are ordered by height, the average number of rational points  $\#|C(\mathbb{Q})|$  is bounded. (Received September 25, 2017)

1135-11-1954 **Robert Grizzard\*** ([grizzarr@lafayette.edu](mailto:grizzarr@lafayette.edu)), 217 Spring Garden St #101, Easton, PA 18042. *Counting reducible polynomials.* Preliminary report.

Building off of the work of Chern-Vaaler, in 2007 Masser and Vaaler gave an asymptotic estimate for the number of algebraic numbers of given degree and bounded height. Part of establishing this result is to estimate how many polynomials of given degree and bounded height are reducible. While general forms of Hilbert's Irreducibility Theorem give a power savings of (about)  $1/2$  compared to the total number of polynomials, Masser and Vaaler used an argument originating (I think) from W. M. Schmidt to get a full power savings. Counting other types of algebraic numbers (as in recent joint work with J. Gunther) leads to issues of counting reducible polynomials with various coefficients specified. In this talk, we'll explore how far we can push Schmidt's argument to get a full power savings for these other cases. This leads to an improvement in many cases of an old result of S. D. Cohen on Hilbert's Irreducibility Theorem, although, unlike Cohen, we can say nothing about the Galois Group. (Received September 25, 2017)

1135-11-1961 **Sharon Frechette\*** ([sfrecheth@holycross.edu](mailto:sfrecheth@holycross.edu)), **Holly Swisher** ([swisherh@math.oregonstate.edu](mailto:swisherh@math.oregonstate.edu)) and **Fang-Ting Tu** ([tu@math.lsu.edu](mailto:tu@math.lsu.edu)). *Appell-Lauricella Hypergeometric Functions over Finite Fields.* Preliminary report.

We define a finite-field version of Appell-Lauricella hypergeometric functions built from period functions in several variables, paralleling the development by Fuselier, et. al in the single variable case. We develop geometric connections between these functions and the family of generalized Picard curves, establishing a finite-field analogue of Koike and Shiga's cubic transformation for the Appell hypergeometric function  $F_1$ , proving a conjecture of Ling Long. We also count the number of  $\mathbb{F}\mathbb{F}_p$ -points on the generalized Picard curves, and give several transformation and reduction formulas satisfied by these hypergeometric functions in several variables. (Received September 25, 2017)

1135-11-1965 **Peter Paule\*** ([peter.paule@risc.jku.at](mailto:peter.paule@risc.jku.at)), Research Inst for Symbolic Computation (RISC), Johannes Kepler University Linz, A-4040 Linz, Austria, and **Cristian-Silviu Radu.** *The Rogers-Ramanujan Functions and Computer Algebra.*

Combinatorially, the first Rogers-Ramanujan function can be defined as the generating function of the number of partitions of  $n$  in which the differences between parts are at least two. The second Rogers-Ramanujan function is defined similarly. The talk discusses various aspects of these functions under the light of recent computer algebra developments. Topics include modular transformations, the Rogers-Ramanujan continued fraction and its connection to Klein's icosahedral equation. (Received September 25, 2017)

1135-11-1966 **Paul Fili\*** ([paul.fili@okstate.edu](mailto:paul.fili@okstate.edu)), 532 Mathematical Sciences Bldg, Oklahoma State University, Stillwater, OK 74078. *Effective unlikely intersections and a metric of mutual energy.*

We introduce a metric of mutual energy for adelic measures associated to the Arakelov-Zhang pairing. Using this metric and potential theoretic techniques involving discrete approximations to energy integrals, we prove an effective bound on a problem of Baker and DeMarco on unlikely intersections of dynamical systems, specifically, for the set of complex parameters  $c$  for which  $z = 0$  and  $1$  are both preperiodic under iteration of  $f_c(z) = z^2 + c$ . (Received September 25, 2017)

1135-11-1973 **Paul Fili\*** ([paul.fili@okstate.edu](mailto:paul.fili@okstate.edu)) and **Lukas Pottmeyer.** *Height bounds under splitting conditions.*

In earlier work, Fili and Petsche used potential theoretic techniques to establish a lower bound for the height of algebraic numbers that satisfy splitting conditions, such as being totally real or  $p$ -adic, improving on earlier work of Bombieri and Zannier in the totally  $p$ -adic case. These bounds applied as the degree of the algebraic number over the rationals tended towards infinity. We discuss how one can make the dependence on the degree of the algebraic number explicit. In particular, we demonstrate lower bounds on the energy of an open neighborhood of the real line in the complex plane. (Received September 25, 2017)

1135-11-2034 **Emily Gullerud\*** ([gullerej@uwec.edu](mailto:gullerej@uwec.edu)) and **aBa Mbirika** ([mbirika@uwec.edu](mailto:mbirika@uwec.edu)). *An Euler phi function for the Eisenstein integers and some applications.* Preliminary report.

The Euler phi function on a given integer  $n$  yields the number of positive integers less than and relatively prime to  $n$ . Equivalently, it gives the order of the group of units in the quotient ring  $\mathbb{Z}/(n)$  for a given integer  $n$ . We generalize the Euler phi function to the Eisenstein integer ring  $\mathbb{Z}[\rho]$  where  $\rho$  is the primitive third root of unity  $e^{2\pi i/3}$  by finding the order of the group of units in the ring  $\mathbb{Z}[\rho]/(\theta)$  for any given Eisenstein integer  $\theta$ . As one application, we prove that the celebrated Euler-Fermat theorem holds for the Eisenstein integers. Time permitting, we discuss the structure of certain unit groups  $(\mathbb{Z}[\rho]/(\gamma^n))^\times$  where  $\gamma$  is prime in  $\mathbb{Z}[\rho]$  and  $n \in \mathbb{N}$ , thereby generalizing well-known results of similar applications in the integers and some lesser known results in the Gaussian integers. (Received September 25, 2017)

1135-11-2077 **Nina V Zubrilina\*** ([nina57@stanford.edu](mailto:nina57@stanford.edu)), 531 Lasuen Mall, P.O. Box 17601, Stanford, CA 94309. *Bounds on the Number of Connected Components of the Sigma Divisor Function.*

For  $r \in \mathbb{R}, r > 1$  and  $n \in \mathbb{N}$ , the generalized sigma divisor function  $\sigma_{-r}$  is defined by  $\sigma_{-r}(n) = \sum_{d|n} d^{-r}$ . We show the number  $C_r$  of connected components of  $\overline{\sigma_{-r}(\mathbb{N})}$  satisfies

$$\pi(r) + 1 \leq C_r \leq \frac{1}{2} \exp \left[ \frac{1}{2} \frac{r^{20/9}}{(\log r)^{29/9}} \left( 1 + \frac{\log \log r}{\log r - \log \log r} + \frac{\mathcal{O}(1)}{\log r} \right) \right],$$

where  $\pi(t)$  is the prime counting function. We also show that  $C_r$  does not take all integer values, namely that it cannot be equal to 4. (Received September 26, 2017)

1135-11-2118 **Jeffrey Lin Thunder\*** ([jthunder@niu.edu](mailto:jthunder@niu.edu)), Mathematical Sciences Department, Northern Illinois University, DeKalb, IL 60115. *Heights in Function Fields: An Application to the Geometry of Curves.*

Given a hyperelliptic curve defined over a finite field, there is a quantity called the a-number attached to the curve. This a-number (a non-negative integer) encapsulates certain geometric information about the curve. It turns out that computing these a-numbers can be accomplished via counting solutions of unexpectedly small height to certain Diophantine equations over function fields.

In this talk we'll briefly discuss the connection alluded to above and then demonstrate the counting arguments that completely answer the question of how many hyperelliptic curves of given genus have a particular a-number. This is joint work with Derek Garton and Colin Weir. (Received September 25, 2017)

1135-11-2130 **Nicholas J. Newsome** ([nnews001@ucr.edu](mailto:nnews001@ucr.edu)), **Maria S. Nogin** ([mnogin@csufresno.edu](mailto:mnogin@csufresno.edu)) and **Adnan H. Sabuwala\*** ([asabuwala@csufresno.edu](mailto:asabuwala@csufresno.edu)), 5245 N Backer Ave, M/S PB 108, Fresno, CA 93740. *Congruences of Power Sums.*

The following congruence for power sums,  $S_n(p)$ , is well known and has many applications:

$$1^n + 2^n + \cdots + p^n \equiv \begin{cases} -1 \pmod{p}, & \text{if } p-1 \mid n; \\ 0 \pmod{p}, & \text{if } p-1 \nmid n, \end{cases}$$

where  $n \in \mathbb{N}$  and  $p$  is prime. We extend this congruence, in particular, to the case when  $p$  is any power of a prime. We also show that the sequence  $(S_n(m) \bmod k)_{m \geq 1}$  is periodic and determine its period. (Received September 25, 2017)

1135-11-2144 **Sidney W. Graham\*** ([graha1sw@cmich.edu](mailto:graha1sw@cmich.edu)). *Iteration in the Ideal Sieve*. Preliminary report.

Modern sieve methods began with the work of Brun in 1915 on twin primes. In the years since, sieves have developed into a large subject with many important applications to the distribution of primes. Sieve constructions can be very complicated and involve extensive notation. However, the underlying principle is simple, for a sieve problem is an inclusion-exclusion problem with incomplete information.

In the usual formulation of the sieve, one starts with a set of integers and removes all multiples of a set of sifting primes up to a certain limit  $z^\beta$ . In this talk, we consider a simplified sieving problem in which all the sifting primes  $p$  lie in an interval  $z^{1/(R+1)} < p \leq z^{1/R}$ . In particular, we discuss an iteration procedure that allows one to use a lower bound sieve for  $z^{1/(R+1)} < p \leq z^{1/R}$  to derive an upper bound sieve for  $z^{1/(R+2)} < p \leq z^{1/(R+1)}$ , and *vice versa*. (Received September 25, 2017)

1135-11-2148 **Kelly A Emmrich\*** ([emmrich.kelly@uwlax.edu](mailto:emmrich.kelly@uwlax.edu)), 1725 State Street, La Crosse, WI 54601, and **Clark Lyons** ([clark2718@berkeley.edu](mailto:clark2718@berkeley.edu)), Berkeley, CA. *Norm-Euclidean Ideal Classes in Galois Cubic Fields*.

Lenstra introduced the notion of a norm-Euclidean ideal class as a generalization of norm-Euclideanity of a number field. He classified all quadratic number fields possessing a norm-Euclidean ideal class. We investigate the Galois cubic case. We show that up to discriminant  $10^{11}$  at most two such number fields possess a nontrivial norm-Euclidean ideal class, and we conjecture no more exist. In an attempt to settle our conjecture, we prove explicit bounds on the first few non-residues of cubic characters under the generalized Riemann hypothesis. (Received September 25, 2017)

1135-11-2239 **Surjeet Kaushik, Narasimha Kumar and Naomi Tanabe\*** ([ntanabe@bowdoin.edu](mailto:ntanabe@bowdoin.edu)). *Sign of Fourier Coefficients for Hilbert Modular Forms*.

Signs of Fourier coefficients have been studied extensively in various settings due to rich arithmetic and algebraic properties they encompass. In this talk, I will survey some results on sign changes for Hilbert modular forms and discuss our recent result concerning a half-integral weight form. (Received September 25, 2017)

1135-11-2285 **Renee Bell\*** ([rbell@mit.edu](mailto:rbell@mit.edu)), MIT Department of Mathematics, 77 Massachusetts Ave., Bldg. 2-239A, Cambridge, MA 02139, USA, Cambridge, MA 02139. *Local-to-Global Principles for Galois Covers of Curves in Characteristic  $p$* .

Given a Galois cover of curves  $X \rightarrow Y$  with Galois group  $G$  which is totally ramified at a point  $x$  and unramified elsewhere, restriction to the punctured formal neighborhood of  $x$  induces a Galois extension of Laurent series rings  $k((u))/k((t))$ . If we fix a base curve  $Y$ , we can ask when a Galois extension of Laurent series rings comes from a global cover of  $Y$  in this way. Harbater proved that over a separably closed field, this local-to-global principle holds for any base curve if  $G$  is a  $p$ -group, and gave a condition for the uniqueness of such an extension. Using a generalization of Artin-Schreier theory to non-abelian  $p$ -groups, we characterize the curves  $Y$  for which an extension to a global cover of curves is unique over a more general ground field. (Received September 25, 2017)

1135-11-2288 **Carrie E. Finch-Smith\*** ([finchc@wlu.edu](mailto:finchc@wlu.edu)). *Sierpiński and Riesel numbers: computations and coverings*. Preliminary report.

In 1956, Riesel showed that there are infinitely many odd positive integers  $k$  such that  $k \cdot 2^n - 1$  is composite for all positive integers  $n$ . In honor of Riesel's work, such a  $k$  is known as a Riesel number. Sierpiński numbers are similar, with the difference that  $k \cdot 2^n + 1$  is composite for all positive integers  $n$ . In this talk, we discuss the use of coverings of the integers to construct Sierpiński numbers and Riesel numbers. In addition, we also consider modifications of covering methods to produce Sierpiński or Riesel numbers with special properties. (Received September 25, 2017)

1135-11-2328 **Peng Zhao\*** ([peng.2.zhao@uconn.edu](mailto:peng.2.zhao@uconn.edu)), Department of Mathematics, University of Connecticut, 341 Mansfield Road U1009, Storrs, CT 06269. *A Large Sieve Type Inequality and Its Applications on Waring-Goldbach Problem*. Preliminary report.

In this report, we present a large sieve type of inequalities under the mean value conjecture of Riemann-zeta function. Combined with the circle method, we obtain an upper bound for the exceptional set on Waring-Goldbach Problem. (Received September 25, 2017)

1135-11-2337 **Andrew Shallue\*** ([ashallue@iwu.edu](mailto:ashallue@iwu.edu)), Illinois Wesleyan University, 1312 Park St, Bloomington, IL 61701. *New work on the existence of challenge pseudoprimes.*

A PSW challenge pseudoprime is a composite number congruent to 2 or 3 modulo 5 which is a base-2 Fermat pseudoprime and a Fibonacci pseudoprime. Famously, there are no known examples, and no proof that such numbers either exist or do not exist. I will discuss recent progress on this question, including both a positive result and a negative result. The positive result is joint work with Andrew Fiori, where we prove that a similar class of Frobenius pseudoprimes (in the sense of Grantham) exist on average. The negative result is joint work with Jonathan Webster, where we develop a new algorithmic approach to tabulation that pushes out the lower bound on the smallest PSW challenge pseudoprime. (Received September 25, 2017)

1135-11-2386 **Edward Voskanian\*** ([evosk001@ucr.edu](mailto:evosk001@ucr.edu)), 7020 Estepa Dr, Tujunga, CA 91042. *On the Quasiperiodic Structure of the Complex Roots of a Nonlattice Dirichlet Polynomial.* Preliminary report.

In the book "Fractal Geometry, Complex Dimensions and Zeta Functions", by Michel L. Lapidus and Machiel van Frankenhuysen, there is an open problem asking if there is a natural way in which the quasiperiodic pattern of the complex roots of a nonlattice Dirichlet polynomial can be understood in terms of a suitable generalized quasicrystal. We approximate the roots of a given nonlattice Dirichlet polynomial using an explicit procedure developed by Lapidus and van Frankenhuysen, allowing one to see the emergence of a quasiperiodic pattern. To use this procedure, we need a way to find good simultaneous Diophantine approximations, and a way to approximate the roots of a polynomial of very high degree. As suggested by Lapidus and van Frankenhuysen, we have implemented the original LLL algorithm to obtain good simultaneous Diophantine approximations. And we use a suitable QR algorithm for companion matrices to approximate the roots of a polynomial of very high degree. We also address Conjecture 4.1 from the paper "On the Complex Dimensions of Nonlattice Fractal Strings in Connection with Dirichlet Polynomials", by J.M. Sepulcre and E. Dubon, and provide more numerical evidence to support this conjecture. (Received September 27, 2017)

1135-11-2389 **Kubra Nari\*** ([nari15@itu.edu.tr](mailto:nari15@itu.edu.tr)), Informatics Institute, Istanbul Technical University, 34469 Istanbul, Turkey, and **Enver Ozdemir** and **Ergun Yaraneri**. *An Integer Factorization Algorithm.*

In this research, we develop a new integer factorization algorithm which has polynomial running time for certain integers. The algorithm is based on binary quadratic forms of a positive discriminant. A binary quadratic form is an integer valued function  $F(x, y) = ax^2 + bxy + cy^2$  such that  $\gcd(a, b, c) = 1$ . The discriminant of  $F(x, y)$  is  $\Delta = b^2 - 4ac$ . There is an equivalence relation between binary quadratic forms of the same discriminant. For the set of equivalence classes of binary quadratic forms, Gauss defined a group operation called composition of binary quadratic forms. Groups behave distinctly for negative and positive discriminants. For a negative discriminant, each equivalence class has a unique representative which is called a reduced form. On the other hand, for a positive discriminant each class does not have unique representative of reduced form. However, each reduced form in a class belongs to a unique cycle. We show that for certain kind of integers  $n$  which are especially being used in RSA crypto system, if we start with a reduced form  $(1, b, c)$  of discriminant  $n$  and if we proceed on the cycle, we end up with a form,  $(k_1p, k_2p, d)$  such that  $p$  is a nontrivial divisor of  $n$ . (Received September 26, 2017)

1135-11-2417 **Nicolas Allen Smoot\*** ([nsmoot@risc.uni-linz.ac.at](mailto:nsmoot@risc.uni-linz.ac.at)), Julius Raab Strasse 10/5/2523, 4040 Linz, Austria. *The Computation of Ramanujan–Kolberg Identities.* Preliminary report.

In 1919 Ramanujan discovered a pair of beautiful identities relating the generating functions for  $p(5n + 4)$  and  $p(7n + 5)$  to rational sums and quotients of Dedekind's eta function. These were shown by Kolberg to be special cases of a more general class of identities, now called Ramanujan–Kolberg identities. The identities of this type are notable for illustrating a remarkable connection between complex analysis and number theory, and are especially useful for studying congruences in various partition functions studied by Andrews, Paule, and Radu. Recently, an algorithm by Radu was developed which uses the theory of modular functions to produce the Ramanujan–Kolberg identities inherent in various arithmetic functions. It takes as input an arithmetic sequence  $a(n)$  which has an eta quotient as a generating function, along with a pair of integers  $m, t$ , with  $0 \leq t < m$ , and a given integer  $N$ . Radu's algorithm returns the Ramanujan–Kolberg identities for  $a(mn + t)$  over a space of eta quotients modular with respect to  $\Gamma_0(N)$ . We give an implementation of this algorithm. As examples, we will show some newly discovered identities, including recent results in broken diamond partitions and some identities involving  $p(11n + 6)$ . (Received September 26, 2017)

1135-11-2498 **Tyler Melton\*** (tm3746@stu.armstrong.edu), 11935 Abercorn St, Savannah, GA 31419, and **Rosina Campbell** (rc2283@stu.armstrong.edu) and **Duc Van Huynh** (duc.huynh@armstrong.edu). *Constructing irreducible polynomials over finite field.*

Let  $p$  be a rational prime and let  $D$  be a negative discriminant. By the Chebotarev Density Theorem, the Hilbert class polynomial  $H_D(X)$  has a high probability of being irreducible over  $\mathbb{F}_p$  when the class group  $\text{Cl}(D)$  is cyclic. For each integer  $n$ , we will combine this idea with isogeny volcanoes to describe an algorithm to construct irreducible polynomials over  $\mathbb{F}_p$  of degree at least  $n$ . (Received September 26, 2017)

1135-11-2523 **Rafe Jones\***, rfjones@carleton.edu, and **Jordan Cahn** and **Jacob Spear**. *Perfect powers in orbits of rational functions over number fields.*

One of the primary aims of the field of arithmetic dynamics is to study number-theoretic questions involving dynamical objects. One such object is the orbit of a point under iteration of a rational function: the sequence  $P, f(P), f(f(P)), \dots$  where  $f$  is a rational function with complex coefficients and  $P$  is a complex number. If we require  $f$  to have coefficients in a number field  $K$ , and  $P$  also to belong to  $K$ , then number-theoretic questions abound. For instance, for a given  $f$ , does there exist  $P$  whose orbit contains infinitely many algebraic integers? Does there exist  $P$  whose orbit contains infinitely many distinct squares in  $K$ ? I'll discuss how to solve this last problem using Faltings' theorem and other tools from arithmetic geometry. (Received September 26, 2017)

1135-11-2551 **Christopher D Sinclair\*** (csinclai@uoregon.edu), Department of Mathematics, University of Oregon, Eugene, OR 97403. *Mahler measure, volumes of polynomials and random matrix theory.*

In 2001, Chern and Vaaler demonstrated an amazing formula for the volume of the set of polynomials of fixed degree and bounded Mahler measure. My thesis problem, working under Vaaler, was to explain and generalize this formula to other multiplicative distance functions on polynomials. In this talk I will explain a general framework which explains Chern and Vaaler's remarkable formula and show how their discovery leads to a deeper connection between number theory and random matrix theory. (Received September 26, 2017)

1135-11-2713 **Sarah Arpin** (sarah.arpin@colorado.edu), **Joel Ornstein** (joel.ornstein@colorado.edu) and **Michael Wheeler\*** (michael.wheeler@colorado.edu). *Geometry of Lattices of Cyclotomic Number Rings Under the Minkowski Embedding.*

Cryptographers are pursuing the use of hard problems on lattices of rings of integers, such as Ring Learning With Errors, as an approach to post-quantum cryptography. Lattice based cryptosystems are one of the most promising avenues for post-quantum security. Our research looks into the geometry of the lattices formed by cyclotomic number rings, looking for structure that could potentially lead to vulnerability. Cyclotomic number rings are the most likely number rings to be used in cryptographic applications due to ease of computation and lack of known vulnerabilities. We look at the geometry of the lattice that arises from the Minkowski Embedding, for any  $n$ . Using number theory and Galois theory, we completely describe the inner products of pairs of basis vectors in this lattice. Research supervised by Katherine E. Stange, University of Colorado, Boulder. (Received September 26, 2017)

1135-11-2767 **Thái Hoàng Lê\*** (leth@olemiss.edu) and **Pierre-Yves Bienvenu**. *Uniformity of the Möbius function in function fields.* Preliminary report.

The Möbius randomness principle states that the Möbius function does not correlate with “structured”, or “low complexity” sequences. We study a function field instance of this principle. Let  $\mathbb{F}_p[t]$  be the ring of polynomials over  $\mathbb{F}_p$  and  $\mu$  be the Möbius function defined on  $\mathbb{F}_p[t]$ . We are interested in non-trivial, uniform bounds for exponential sums  $\sum_{\deg f=n} \mu(f) e_p(Q(f))$ , where  $Q$  is any multivariate polynomial in  $n+1$  variables and  $Q(f)$  is  $Q$  evaluated at the coefficients of  $f$ . I will talk about our result in the linear case and progress in the quadratic case. This is joint work with Pierre-Yves Bienvenu. (Received September 26, 2017)

1135-11-2784 **Victoria Jakicic\*** (gkwt@iup.edu). *Zeros of Newform Eisenstein Series on  $\Gamma_0(q_1 q_2)$ .*

During a Research Experience for Undergraduates at Texas A&M in the summer of 2017, we examined the zeros of newform Eisenstein series  $E_{\chi_1, \chi_2, k}(z)$  of weight  $k$  on  $\Gamma_0(q_1 q_2)$ , where  $\chi_1$  and  $\chi_2$  are primitive characters modulo  $q_1$  and  $q_2$ , respectively. In this talk, we will use the Fourier expansion to examine the zeros where the imaginary part is greater than  $\sqrt{k}$ . (Received September 26, 2017)

1135-11-2809 **TJ Combs** and **Michelle Manes\*** ([mmanes@math.hawaii.edu](mailto:mmanes@math.hawaii.edu)), University of Hawaii, Department of Mathematics, 2565 McCarthy Mall, Keller 401A, Honolulu, HI 96822. *Cubic and higher degree arboreal Galois representations*. Preliminary report.

We provide an infinite family of cubic polynomials in  $\mathbb{Q}[x]$  for which the image of the arboreal Galois representation is full. We will also discuss more recent work on Galois representations for higher degree polynomials (Received September 26, 2017)

1135-11-2828 **Cezar Lupu\*** ([lupucezar@gmail.com](mailto:lupucezar@gmail.com)), Pittsburgh, PA 15260. *Another look at Zagier's formula for multiple zeta values involving 2's and 3's*. Preliminary report.

Zagier's formula is a remarkable example of both strength and the limits of the motivic formalism used by Brown in proving Hoffman's conjecture where the motivic argument does not give us a precise value for the special multiple zeta values  $\zeta(2, 2, \dots, 2, 3, 2, 2, \dots, 2)$  as rational linear combinations of products  $\zeta(m)\pi^{2n}$  with  $m$  odd. The formula is proven indirectly by computing the generating functions of both sides in closed form and then showing that both are entire functions of exponential growth and that they agree at sufficiently many points to force their equality. By using the Taylor series of integer powers of arcsin function and a related result about expressing rational zeta series involving  $\zeta(2n)$  as a finite sum of  $\mathbb{Q}$ -linear combinations of odd zeta values and powers of  $\pi$ , we derive a new and direct proof of Zagier's formula in the special case  $\zeta(2, 2, \dots, 2, 3)$ . Towards the end of the talk, we present a more general formula for  $\zeta(2, 2, \dots, 2, m)$ , with  $m \geq 3$  integer. We also discuss similar results for the multiple t-values. (Received September 26, 2017)

1135-11-2863 **Ángel Chávez** ([angel.chavez@pomona.edu](mailto:angel.chavez@pomona.edu)) and **George Todd\*** ([toddg@union.edu](mailto:toddg@union.edu)). *Supercharacters and Mixed Moments of Kloosterman Sums*.

Recent work has expressed many exponential sums as values of various supercharacter theories arising from the action of subgroups of  $\mathrm{GL}_d(\mathbb{Z}/n\mathbb{Z})$  on  $(\mathbb{Z}/n\mathbb{Z})^d$ . We investigate the supercharacter theory arising from the action of  $\mathrm{GL}_2(\mathbb{F}_p)$  on  $(\mathbb{F}_p)^2$  whose values correspond to Kloosterman sums. We then use the resulting supercharacter table and a result of Williams to express fourth degree mixed Kloosterman moments in terms of the trace of Frobenius of an elliptic curve. (Received September 27, 2017)

1135-11-3014 **Rafael Ceja Ayala\*** ([rceja127@gmail.com](mailto:rceja127@gmail.com)), 7960 La Rivera Dr apt 138, Sacramento, CA 95826, and **Jason Turner**, **Erik Knutsen** and **Alejandra Alvarado**. *The Binary and Ternary Interpretation of the Collatz Conjecture*.

Consider the function  $C(x) : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ , where  $C(x) = 3x + 1$  if  $x$  is odd and  $C(x) = \frac{x}{2}$  if  $x$  is even. The Collatz conjecture states that regardless of the initial value of  $x$ ,  $C^k(x) = 1$  for some finite  $k \in \mathbb{N}$ . We study the behavior of analogous functions over  $\mathbb{Z}[i]$  and  $\mathbb{Z}[\omega]$ . In particular, we show that the binary interpretation of  $C(x)$  extends to a binary interpretation of our Gaussian Collatz function and a ternary interpretation of our Eisenstein Collatz function. (Received September 26, 2017)

1135-11-3131 **Melissa Emory\*** ([mleaq2@mail.missouri.edu](mailto:mleaq2@mail.missouri.edu)). *Generalizing the Gan-Gross-Prasad Conjecture to a Similitude Group*.

In the 1990s, Benedict Gross and Dipendra Prasad formulated an intriguing conjecture connected with restriction laws for automorphic representations of a particular group. More recently, Gan, Gross, and Prasad extended this conjecture, now known as the *Gan-Gross-Prasad Conjecture*, to the remaining classical groups. Roughly speaking, they conjectured the non-vanishing of a certain period integral is equivalent to the non-vanishing of the central value of a certain  $L$ -function. Ichino and Ikeda refined the conjecture to give an explicit relationship between this central value of a  $L$ -function and the period integral. We propose a similar conjecture for a non-classical group, the general spin group, and prove the first two cases. (Received September 26, 2017)

## 12 ► Field theory and polynomials

1135-12-702 **Moises R Delgado\*** ([moises.delgado@upr.edu](mailto:moises.delgado@upr.edu)). *Transversal intersection for absolute irreducibility of hyperplane sections of generalized Fermat varieties and on the conjecture on exceptional APN Functions*.

The decomposition of a Fermat variety  $X$  defined by the multivariate polynomial of degree  $n$ ,  $\phi(x, y, z) = f(x) + f(y) + f(z)$  in  $P^3(\mathbb{F}_2)$ , where  $f$  is a function on a finite field  $F$ , plays a crucial part in the study of APN functions and exceptional APN functions. For the exceptional numbers,  $n = 2^k + 1$  and  $n = 2^{2k} - 2^k + 1$  (Gold and Kasami-Welch numbers, respectively) very important results have been obtained. In this talk we explore  $X$  related to the Kasami-Welch degree monomials and its decomposition into absolutely irreducible components. We show that, in this decomposition, the components intersect transversally at a singular point. This structural

fact implies that the corresponding Fermat varieties, related to Kasami-Welch degree polynomial families, are absolutely irreducible. Consequently, these new families contribute substantially to the proof of the conjecture of exceptional APN functions on the hardest case: The Kasami-Welch degree case. (Received September 13, 2017)

1135-12-1705 **Dawn C. Nelson\*** ([dnelson@saintpeters.edu](mailto:dnelson@saintpeters.edu)). *Permutation polynomial research with beginning undergraduates*. Preliminary report.

Lidl and Mullen challenged: “Consider the binomial  $f(x) = x^k + ax^j$  with  $k > j \geq 1$ ,  $\gcd(k, j) = 1$ , and  $a \in F_q^*$ . Determine conditions on  $k, j$  and  $q$  so that  $f(x)$  permutes  $F_q$ .” By considering the question for  $F_p$  (with  $p$  prime), this challenge can be tackled by beginning undergraduate students.

In this talk, I describe how a student (whose highest level math class was AP Calculus) used Mathematica to study permutation binomials. The student was able to enumerate a comprehensive list of conditions on  $k, j$  and  $p$ . I explain her results and conjectures. The talk finishes with a list of several level-appropriate open questions. (Received September 24, 2017)

1135-12-1941 **Carlos E. Arreche\*** ([arreche@utdallas.edu](mailto:arreche@utdallas.edu)). *Differential Galois theory for difference equations and hypertranscendence*.

There is a differential Galois theory that associates to a given linear difference equation a geometric object, called the Galois group, that encodes the differential equations satisfied by the solutions. I will describe this Galois theory and some algorithms that have been used recently to prove that certain generating series arising in combinatorics are hypertranscendental. (Received September 25, 2017)

1135-12-2122 **Chad Awtrey\*** ([cawtre@elon.edu](mailto:cawtre@elon.edu)). *Some open problems in computational Galois theory*.

The work of 19th century mathematician Evariste Galois shows that roots of polynomials have inherent symmetries, which are encoded as permutations of the roots. In the language of abstract algebra, these symmetries form a group under function composition, called the polynomial’s Galois group. Properties of the polynomial’s Galois group reveal arithmetic about the polynomial. For example, a polynomial’s roots using radicals if and only if the Galois group has a certain property. Naturally, a fundamental task in computational algebra is to determine the Galois group of a polynomial. In this talk, we will discuss the history and context of Galois’ work, recent computational results, as well as some potential open problems suitable for investigation by undergraduate students. (Received September 25, 2017)

## 13 ► Commutative rings and algebras

1135-13-60 **Michael DiPasquale\*** ([mdipasq@okstate.edu](mailto:mdipasq@okstate.edu)), Department of Mathematics, 401 Math Sciences Building, Stillwater, OK 74078. *The toric ring of a two-borel ideal is Koszul*.

Let  $M$  and  $N$  be two monomials of the same degree, and let  $I$  be the smallest Borel ideal containing  $M$  and  $N$ . We show that the toric ring of  $I$  is Koszul by constructing a quadratic Gröbner basis for the associated toric ideal. Our proofs use the construction of graphs corresponding to fibers of the toric map. As a consequence, we deduce that the Rees algebra is also Koszul, giving a positive answer to a question posed by Conca. We conclude with remarks on the normality and the Cohen-Macaulay property of the toric ring. This is joint work with Francisco, Mermin, Schweig, and Sosa. (Received July 14, 2017)

1135-13-209 **Chloe I. Avery, Caitlyn Booms\*** ([cbooms1@nd.edu](mailto:cbooms1@nd.edu)), **Timothy M. Kostolansky, S. Loepp** and **Alex Semendinger**. *Completions of Noncatenary Local Integral Domains*. Preliminary report.

Let  $A$  be a local ring with unique maximal ideal  $M$ , and let  $\widehat{A}$  be the  $M$ -adic completion of  $A$ . We say that  $A$  is *catenary* if, given any pair of prime ideals  $P \subseteq Q$ , the length of every saturated chain of prime ideals between them is the same, and we say that  $A$  is *noncatenary* otherwise. In this talk we characterize the completions of noncatenary local integral domains. (Received August 11, 2017)

1135-13-210 **Chloe I. Avery, Caitlyn Booms, Timothy M. Kostolansky, S. Loepp** and **Alex Semendinger\*** ([aps5@williams.edu](mailto:aps5@williams.edu)). *How Noncatenary Can a Local UFD Be?* Preliminary report.

We say a ring  $R$  is *catenary* if its prime ideal structure satisfies a nice property: given any two prime ideals  $P \subset Q$  of  $R$ , all saturated chains of prime ideals between  $P$  and  $Q$  have the same length. In general, it is hard to construct examples of rings that are not catenary, and it is especially hard to do so if the rings satisfy other “nice” properties. For instance, it was not until 1993 that Raymond C. Heitmann constructed an example of a



Noetherian local unique factorization domain. It was unknown whether there was a limit to “how noncatenary” a Noetherian local UFD could be. In this talk, we answer this question, based on research done at the SMALL REU at Williams College in 2017. (Received August 11, 2017)

1135-13-211 **Ya. S. Krylyuk\*** (krylioukiaroslav@fhda.edu). *Ya. S. Krylyuk "The Poisson Conjecture and its connection to the classical invariant theory" (Preliminary Report)*. Preliminary report.

In the talk will be discussed the reformulation of the Poisson conjecture on the polynomial symplectomorphisms bounded by the given degree and the connection of the algebra of the defining equations of such set to the classical invariant theory. A special attention will be paid to the case of polynomial symplectomorphisms in two variables. (Received August 11, 2017)

1135-13-213 **Chloe I. Avery\*** (cia@umail.ucsb.edu), **Caitlyn Booms**, **Timothy M. Kostolansky**, **S. Loepf** and **Alex Semendinger**. *Completions of Noncatenary Local UFDs*.

Let  $A$  be a local ring with maximal ideal  $M$  and let  $\hat{A}$  be the completion of  $A$  with respect to  $M$ . We call  $A$  catenary if, for every pair of prime ideals  $P$  and  $Q$ , all saturated chains of prime ideals between  $P$  and  $Q$  have the same length. It was conjectured in 1956 that noncatenary local UFDs do not exist. It was not until 1993 that Raymond C. Heitmann constructed an example of a noncatenary local UFD by considering the relationship between local UFDs and their completions. In this talk, we characterize completions of noncatenary local UFDs, and give illustrative examples. (Received August 11, 2017)

1135-13-365 **Robin Baidya\*** (rbaidya1@gsu.edu), Department of Mathematics and Statistics, Georgia State University, Atlanta, GA 30303. *Capacities and cancellation*. Preliminary report.

Let  $R$  be a commutative Noetherian ring, and let  $M$  and  $N$  be finitely generated  $R$ -modules. We give a lower bound on the global surjective (respectively, splitting) capacity of  $M$  with respect to  $N$  over  $R$  in terms of local surjective (respectively, splitting) capacities and dimensions of prime ideals in  $j\text{-Spec}(R)$ . In the case that  $M$  is projective and  $N$  is  $R$ , this result fully recovers Serre’s Splitting Theorem from algebraic  $K$ -theory. In the case that  $R$  is a hereditary ring, we can also provide conditions equivalent to having a given global surjective, splitting, or injective capacity. Finally, we obtain an analogue of Bass’s Cancellation Theorem that replaces  $R$  with a homothetic  $R$ -module. All of these results generalize recent work by De Stefani, Polstra, and Yao. (Received August 28, 2017)

1135-13-533 **Adam Boocher\*** (boocher@math.utah.edu). *Bounds on Betti Numbers of Graded Rings*.

If  $I$  is a homogeneous ideal in a polynomial ring  $S$  there are many interesting invariants we can associate to the quotient  $R = S/I$ . One is the collection of Betti numbers  $\beta_i(R)$  which are a measure of the complexity and symmetry of the defining polynomials in  $I$ . In this talk I’ll discuss some classical conjectures concerning upper and lower bounds for the Betti numbers and some recent progress for Koszul rings and for rings defined by monomials. (Received September 07, 2017)

1135-13-550 **Shuenn Siang Ng\*** (sng40@gsu.edu), 25 Park Place, Room 1338, Atlanta, GA 30303. *Characterization of  $F$ -rationality when  $R$  is Cohen Macaulay via its canonical module*.

Let  $R$  be a  $F$ -finite Cohen Macaulay ring (not necessarily local) of prime characteristic with canonical module  $\omega$ . We try to characterize  $F$ -rationality of  $R$  by studying the module structure via the Frobenius homomorphism of  ${}^e M$  and homomorphisms from  ${}^e M$  to  $\omega$ , in which  $M$  is a finitely generated faithful module. We obtained some conditions involving  ${}^e M$  and  $\omega$  that are equivalent to  $R$  being  $F$ -rational. For example, one of such equivalent conditions is that  $R$  is normal and for every  $P \in \text{Spec}(R)$  with height at least 2, there exists  $e > 0$  such that  $\omega_P$  is a homomorphic image of a direct sum of  ${}^e(P\omega)_P$ . (Received September 07, 2017)

1135-13-659 **Alessandro De Stefani**, **Eloísa Grifo** and **Jack Jeffries\*** (jackjeff@umich.edu). *The Zariski-Nagata theorem in all characteristics*.

One version of a classical result by Zariski and Nagata describes symbolic powers in polynomial rings over the complex numbers in terms of differential operators. Namely, the  $n$ -th symbolic power of a prime consists of the elements such that each differential operator of order at most  $n-1$  sends the element into the prime ideal. This was extended to polynomial rings over perfect fields by Dao, De Stefani, Grifo, Huneke, and Núñez-Betancourt. However, this description fails in mixed characteristic. In this paper, we use  $p$ -derivations, a notion due to Buium and Joyal, to define a new kind of differential powers in mixed characteristic, and prove that this new object does coincide with the symbolic powers of prime ideals. This seems to be the first application of  $p$ -derivations to Commutative Algebra.

This is joint work with Alessandro De Stefani and Eloísa Grifo. (Received September 12, 2017)

1135-13-743 **Thomas Polstra** and **Ilya Smirnov\*** ([ismirnov@umich.edu](mailto:ismirnov@umich.edu)), Department of Mathematics, University of Michigan, 2074 East Hall, 530 Church Street, Ann Arbor, MI 48109. *Hilbert-Kunz multiplicity, F-signature, and finite determinacy of singularities.*

Given a singularity defined by power series in  $F[[x_1, \dots, x_n]]$ , we may ask whether it is possible to truncate the defining equations so that the resulting singularity in  $F[x_1, \dots, x_n]$  will be equivalent to the original one.

We approach this problem in positive characteristic, where we may use Hilbert-Kunz multiplicity and F-signature to compare singularities. Provided that the original singularity was a complete intersection, we prove that the singularity of a truncation can be made arbitrarily close to the original if we leave sufficiently many terms. (Received September 13, 2017)

1135-13-966 **Robert M. Walker\*** ([robmarsw@umich.edu](mailto:robmarsw@umich.edu)), 530 East Church Street, ANN ARBOR, MI 48109. *Uniform Symbolic Topologies in Normal Toric Rings.*

A Noetherian ring  $R$  has the uniform symbolic topology property (USTP) if there's an integer  $D := D(R) > 0$  such that the symbolic power  $P^{(DN)} \subseteq P^N$  for all prime ideals  $P$  in  $R$  and all integers  $N > 0$ . For instance, all excellent finite-dimensional regular rings have USTP, and a large class of isolated singularities also have USTP (Ein-Lazarsfeld-Smith, Hochster-Huneke, Huneke-Katz-Validashti, Ma-Schwede). A toric ring is a domain of finite type over a field, generated by Laurent monomials. In this talk, we present a formula for the multiplier  $D(R)$  such that any normal toric ring  $R$  has USTP on the set of monomial primes: this is one of the conditional USTP results my dissertation affords for rings whose singular locus may have positive dimension. (Received September 18, 2017)

1135-13-1353 **Eloisa Grifo\***, [eloisa.grifo@virginia.edu](mailto:eloisa.grifo@virginia.edu). *Symbolic powers: the Containment Problem and Harbourne's conjecture.* Preliminary report.

The Containment Problem for ordinary and symbolic powers of ideals asks when the containment  $I^{(a)} \subseteq I^b$  holds. If  $I$  is a radical ideal in a regular ring, a famous result by Ein-Lazarsfeld-Smith, Hochster-Huneke and Ma-Schwede partially answers this question. Harbourne proposed an improvement on this result, which unfortunately does not hold in full generality. In this talk, we will discuss versions of Harbourne's Conjecture that do hold. (Received September 21, 2017)

1135-13-1498 **Raymond C Heitmann\*** ([heitmann@math.utexas.edu](mailto:heitmann@math.utexas.edu)) and **Linquan Ma**. *Extended Plus Closure in Complete Local Rings.*

The (full) extended plus closure was introduced by the first author in order to attack the homological conjectures in the hope that it would play the same role in mixed characteristic that tight closure does in characteristic  $p$ . Here, by adapting André's perfectoid algebra techniques, we show that this closure has the colon-capturing property for complete local domains. In fact, more generally, if  $R$  is a (possibly ramified) complete regular local ring of mixed characteristic,  $I$  and  $J$  are ideals of  $R$ , and the local domain  $S$  is a finite  $R$ -module, then  $(IS : J) \subseteq (I : J)S^{epf}$ . A consequence is that all ideals in regular local rings are closed, a fact which implies the validity of the direct summand conjecture and the Briançon-Skoda theorem in mixed characteristic. (Received September 22, 2017)

1135-13-1516 **Daniel Smolkin\*** ([smolkin@math.utah.edu](mailto:smolkin@math.utah.edu)). *Subadditivity of test ideals.*

Given a ring  $R$  of positive characteristic, an ideal  $\mathfrak{a}$  in  $R$ , and a positive number  $t$ , one can construct what's called the *test ideal* of this data, denoted  $\tau(\mathfrak{a}^t)$ . This notion was introduced by Hara and Yoshida in 2003, based on work in tight closure by Hochster and Huneke, and it measures the singularities of  $R$  and  $R/\mathfrak{a}$ . Hara and Yoshida also showed that on regular rings, test ideals obey a *subadditivity* formula, namely  $\tau(\mathfrak{a}^s \mathfrak{b}^t) \subseteq \tau(\mathfrak{a}^s) \tau(\mathfrak{b}^t)$ , and Takagi generalized this formula to the affine case, using the Jacobian ideal as a correction term. This formula has a number of important applications, such as bounding the growth of symbolic powers of ideals.

In this talk, I will discuss a subadditivity formula that holds for non-regular rings using the formalism of cartier algebras. We will compare this formula to earlier subadditivity formulas and see how to compute the correction term in the toric case. (Received September 22, 2017)

1135-13-1517 **Denise A Rangel Tracy\*** ([d.a.rangeltracy@ccsu.edu](mailto:d.a.rangeltracy@ccsu.edu)). *Totally Reflexive Modules.*

In this talk for non-experts, we will discuss the notion of totally reflexive modules, also known as Gorenstein projective modules, which can be precisely defined using tools from homological algebra. The questions of their existence and abundance will be address. In addition, for a special case, we can give a complete description of these modules via matrices. (Received September 22, 2017)

1135-13-1717 **Eric Canton\*** (ecanton2@math.unl.edu). *Log canonical thresholds of graded sequences of ideals in positive characteristic*. Preliminary report.

The log canonical threshold (LCT) of an ideal  $\mathfrak{a}$  on a smooth variety  $X$  is a fundamental measure of the singularities of the associated subscheme, defined via the coefficients that appear on certain divisors on smooth varieties resolving the singularities of  $\mathfrak{a}$ . In positive characteristics, these resolutions are not known to exist, and so one considers all possible normal varieties having a proper morphism to  $X$ , providing a potentially infinite collection of conditions to check. Even in characteristic zero, if we consider a multiplicatively graded sequence of ideals, there may be an infinite number of resolutions to check. Jonsson and Mustata used the valuation space of  $X$  (in characteristic zero) to prove that there exists a valuation computing the LCT of a graded sequence, and make several related conjectures regarding the numerical properties of these computing valuations. The speaker presents some of his recent work that applies Frobenius splitting techniques to study these LCTs in positive characteristics using valuation spaces, proving the theorems of Jonsson and Mustata. (Received September 24, 2017)

1135-13-1902 **Rebecca R.G.\*** (rيرهbuh@syr.edu) and **Felipe Pérez**. *Test Ideals for all Characteristics*.

We define the test ideal of a general closure operation  $\text{cl}$ , and give some of its properties. We show that if the closure  $\text{cl}$  comes from a big Cohen-Macaulay module or algebra, or a family of such modules or algebras, then the test ideal shares many properties of the tight closure test ideal used in characteristic  $p > 0$ . In particular, it is connected to the singularities of the ring. We also apply our results to a mixed-characteristic closure operation based on work of Ma and Schwede. (Received September 25, 2017)

1135-13-1991 **Cory H Colbert\*** (cc6@williams.edu). *Partially Ordered Sets and Noetherian Rings*. Preliminary report.

The question of which partially ordered sets are isomorphic to the spectrum of some commutative Noetherian ring is an important open question within commutative ring theory. We highlight classical results in this area and we present new classifications of partially ordered sets that are Noetherian spectra, up to order-isomorphism. (Received September 25, 2017)

1135-13-2002 **Cory H Colbert\*** (cc6@williams.edu). *Enlarging Noetherian Rings While Preserving Their Spectrum*.

If  $M$  is a maximal ideal of height at least two in a Noetherian ring  $R$ , a well-known inequality asserts that  $|R/M| + \aleph_0 \leq |\text{Spec } R| \leq |R| \leq |R/M|^{\aleph_0}$ . Given that many naturally encountered local rings  $R$  of dimension at least two satisfy  $|\text{Spec } R| = |R|$ , it is natural to wonder if equality can ever fail. Using a technique inspired by the works of Nagata and Heitmann, we prove that there are examples of local rings  $R$  such that  $|\text{Spec } R| < |R|$  in every Krull dimension. The technique, which may be of independent interest, essentially allows one to greatly enlarge a Noetherian ring while completely preserving its prime ideal structure. We conclude by showing that such curious examples of Noetherian rings with  $|\text{Spec } R| < |R|$  contain Noetherian subrings  $S$  such that  $|S| = |\text{Spec } R|$  and  $\text{Spec } S \cong \text{Spec } R$ . (Received September 25, 2017)

1135-13-2011 **Cory H Colbert\*** (cc6@williams.edu). *Results concerning the spectrum of Noetherian rings*. Preliminary report.

Given a commutative ring  $R$ , it is easy to produce a partially ordered set from it by considering  $\text{Spec } R$  with the inclusion relations. We are concerned with the reverse direction: if  $U$  is a partially ordered set, when is  $U$  the spectrum of some Noetherian commutative ring  $R$ ? We present some classical results in this area as well as some new classifications. (Received September 25, 2017)

1135-13-2104 **David F. Anderson** and **Darrin Weber\*** (dw238@evansville.edu). *Zero-Divisor Graphs on Commutative Rings Without Identity*.

A zero-divisor graph is a graph, denoted  $\Gamma(R)$ , whose vertices are the nonzero zero-divisors of a ring  $R$  and two vertices are connected by an edge if and only if their product is 0. Most of the work on zero-divisor graphs have focused on commutative rings with identity, although several results happen to apply to rings without identity. We focus on the zero-divisor graphs of commutative rings without identity, as well as classify all commutative rings without identity that have zero-divisor graphs on 14 or fewer vertices. (Received September 26, 2017)

1135-13-2165 **Ranthonny A.C. Edmonds\*** ([ranthonny-edmonds@uiowa.edu](mailto:ranthonny-edmonds@uiowa.edu)),  
 Ranthonny-Edmonds@uiowa.edu. *Unique Factorization in Polynomial Rings with Zero Divisors.*

In an integral domain, it is well known that  $R$  is a unique factorization domain (UFD) if and only if  $R[X]$  is a unique factorization domain. This result does not hold if we generalize to polynomial rings with zero divisors. In this talk, we will discuss factorization in commutative rings with zero divisors in the context of polynomial rings, with particular attention paid to unique factorization. We discuss when a polynomial ring is a UFR in the sense of Bouvier and Galovich and when it is a UFR in the sense of Fletcher.

Though both Galovich and Fletcher established structure theorems to characterize unique factorization in arbitrary rings, the inherit structure of polynomial rings affords surprisingly straightforward proofs of standard results without the use of such powerful machinery. Specifically, we will show an elementary proof that a polynomial ring  $R[X]$  is a unique factorization ring in the sense of Bouvier or Galovich if and only if  $R$  is a UFD. We also show that the following are equivalent: (1)  $R[X]$  is a UFR in the sense of Fletcher (2) each (regular) nonzero nonunit has unique factorization into irreducible elements (3) each (regular) nonunit element is a product of principal primes (4)  $R$  is a finite direct product of UFD's. (Received September 26, 2017)

1135-13-2170 **William D Taylor\*** ([wdtaylor@uark.edu](mailto:wdtaylor@uark.edu)) and **Lance Edward Miller**. *Bounds for and Computations of  $s$ -multiplicity.*

In this talk, we will define  $s$ -multiplicities, a family of multiplicity functions that continuously deforms between Hilbert-Samuel and Hilbert-Kunz multiplicity, and discuss some of their most important properties. Included in those are several important lower bounds on  $s$ -multiplicity which allow us to attack a generalization of the Watanabe-Yoshida conjecture on Hilbert-Kunz multiplicity. We will also discuss some recent work on the polynomial-like behavior this function sometimes displays. (Received September 25, 2017)

1135-13-2220 **Thomas M Ales\*** ([tales@masonlive.gmu.edu](mailto:tales@masonlive.gmu.edu)), 4400 University Drive, MS: 3F2,  
 Exploratory Hall, room 4400, Fairfax, VA 22030. *Tight closure invariants in combinatorial rings of characteristic 0.* Preliminary report.

The  $*$ -core of an ideal  $I$  is the intersection of all  $*$ -reductions of the ideal, a  $*$ -reduction being any ideal  $J \subseteq I$  such that the tight closure of  $J$  contains  $I$ . The  $*$ -spread of  $I$  is defined to be the number of generators of a minimal such reduction. Though difficult to compute in general, certain theorems enable us to compute the tight closure of ideals of certain combinatorial structured rings. We demonstrate that the  $*$ -spread of the maximal ideal  $\mathfrak{m}$  in a Stanley-Reisner ring over a field of characteristic 0 is invariant with respect to the dimension of the defining simplicial complex of the ring. We also describe the minimal  $*$ -reductions of  $\mathfrak{m}$  in monomial and binomial rings. It is also possible to describe the polynomials that generate the  $*$ -cores of these ideals and we provide examples. (Received September 25, 2017)

1135-13-2491 **George Whelan\***, [gwhelan@gmu.edu](mailto:gwhelan@gmu.edu). *Generalized Depth and Associated Primes in the Perfect Closure  $R^\infty$ .* Preliminary report.

Let  $(R, \mathfrak{m})$  be a reduced Noetherian local ring of characteristic  $p > 0$ . If we consider a finitely generated  $R$ -module  $M$ , we can study the notions of depth and associated primes of both  $M$  and its Frobenius iterates  $F^e(M)$ . We can then extend  $R$  to its perfect closure  $(R^\infty, \mathfrak{m}^\infty)$ , which will in general no longer be Noetherian. These notions then become more subtle when we extend scalars to the  $R^\infty$ -module  $R^\infty \otimes_R M$ .

In this talk, we will define these more subtle measures of  $R^\infty \otimes_R M$  over  $R^\infty$ , and establish some relationships with depth and associated primes of the iterates  $F^e(M)$  over  $R$ . Specifically if  $R$  is an  $F$ -pure ring, then the depth of  $F^e(M)$  will stabilize for  $e \gg 0$ , and we call this corresponding value the stabilizing depth of  $M$  over  $R$ . As we will show, this measure of  $M$  will coincide with some of these non-Noetherian depth measures of  $R^\infty \otimes_R M$  over  $R^\infty$ . (Received September 26, 2017)

1135-13-2642 **Patricia Klein\*** ([triciajk@umich.edu](mailto:triciajk@umich.edu)). *Recovering and exploring Lech's inequality.* Lech's inequality states that for every  $d$ -dimensional local ring  $(R, \mathfrak{m})$  and every  $\mathfrak{m}$ -primary ideal  $I$  of  $R$ ,

$$\frac{e_I(R)}{\ell(R/I)} \leq d! \cdot e_{\mathfrak{m}}(R).$$

We will give a new proof of Lech's inequality that utilizes characteristic  $p > 0$  techniques and yet yields the result in all characteristics. We will also discuss special cases when  $R$  can be replaced by a  $d$ -dimensional  $R$  module  $M$  in Lech's inequality, either retaining the upper bound  $d! \cdot e_{\mathfrak{m}}(R)$  or replacing it by another explicit upper bound. (Received September 26, 2017)

1135-13-2644 **Janet Page\*** ([jpage8@uic.edu](mailto:jpage8@uic.edu)). *Frobenius Complexity for Pairs*. Preliminary report.

Central to the study of singularities in characteristic  $p$  is the Frobenius morphism and its splittings. Given a commutative ring  $R$  of positive characteristic, the total Cartier algebra is the ring of all potential Frobenius splittings of  $R$ , or all  $p^{-e}$  linear maps on  $R$ . This ring need not be finitely generated over  $R$ , which led Enescu and Yao to define Frobenius complexity as a measure of its non-finite generation. In this talk, I will review some results on Frobenius complexity, and introduce a notion of Frobenius complexity for pairs. I will give some examples which are a part of ongoing work. Many of the examples we discuss will be on a certain class of toric rings called Hibi rings, which are defined in terms of finite posets. (Received September 26, 2017)

1135-13-2677 **Jesse Gerald Smith\*** ([jesse.smith@maryvillecollege.edu](mailto:jesse.smith@maryvillecollege.edu)), 13105 Buttermilk Rd, Knoxville, TN 37932. *Isomorphisms of Idea-Based Zero-Divisor Graphs*. Preliminary report.

Let  $R$  be a commutative ring with nonzero identity and  $I$  a proper ideal of  $R$ . The *ideal-based zero-divisor graph* of  $R$  with respect to the ideal  $I$ , denoted by  $\Gamma_I(R)$ , is the graph on vertices  $\{x \in R \setminus I \mid xy \in I \text{ for some } y \in R \setminus I\}$ , where distinct vertices  $x$  and  $y$  are adjacent if and only if  $xy \in I$ . In this work, we consider properties graph isomorphisms of ideal-based zero-divisor graphs. In particular, we seek to understand how isomorphisms of  $\Gamma_I(R)$  relate to isomorphisms restricted to  $\Gamma(R/I)$ , where  $\Gamma(R/I)$  is the zero-divisor graph of the factor ring  $R/I$ . (Received September 26, 2017)

1135-13-2683 **Nursel Erey\*** ([nursel.erey@ndsu.edu](mailto:nursel.erey@ndsu.edu)). *Powers of ideals associated to graphs*.

Let  $G$  be a finite simple graph with vertex set  $V(G) = \{x_1, \dots, x_n\}$ . Let  $k$  be a field. The edge ideal  $I(G) \subseteq k[x_1, \dots, x_n]$  of  $G$  is generated by the monomials  $x_i x_j$  such that  $x_i$  is adjacent to  $x_j$  in  $G$ . In this talk we will discuss some questions and results regarding the minimal free resolutions of powers of  $I(G)$ . (Received September 26, 2017)

1135-13-2688 **Mark N. Blumstein\*** ([blumstei@math.colostate.edu](mailto:blumstei@math.colostate.edu)), 841 Oval Dr., Fort Collins, CO 80523. *Commutative Algebra in the Graded Category with an Application to Equivariant Cohomology Rings*.

The *Degree* of a graded  $R$ -module  $M$  is defined as  $\lim_{t \rightarrow 1} (1-t)^{\dim(M)} PS_M(t)$ , where  $PS_M(t)$  is the Hilbert-Poincare series of the module, and  $\dim$  is Krull dimension. It turns out that this number has a decomposition from commutative algebra as follows:  $\deg(M) = \sum_{P \in \mathcal{D}(M)} \ell_R(M) \deg(R/P)$ , where  $\mathcal{D}(M)$  is the set of minimal primes of  $M$  which have maximal dimension.

The main result of this talk gives a re-interpretation of the sum formula above for equivariant cohomology rings. In this setting,  $G$  is a compact Lie group acting on a nice enough topological space  $X$ , and cohomology coefficients are taken in  $\mathbb{Z}/p$ . The main result is:

$$\deg(H_G^*(X)) = \sum_{[A,c] \in \mathcal{B}_{\max}(G,X)} \frac{1}{|W_G(A,c)|} \deg(H_{C_G(A,c)}^*(c)),$$

where  $A$  is an elementary abelian  $p$ -group of maximal rank,  $c$  is a component of  $X^A$  (the fixed point set,) and  $C_G(A,c)$ ,  $W_G(A,c)$  refer to the centralizer and Weyl group respectively of  $A$  in  $G$ .

This work is in the spirit of Daniel Quillen's series of Annals papers from 1972, *The Spectrum of an Equivariant Cohomology Ring I and II*. An example from the cohomology of groups for an extra special 5-group will be presented. (Received September 26, 2017)

1135-13-2790 **Haydee Lindo\*** ([08hml@williams.edu](mailto:08hml@williams.edu)). *Trace ideals and their applications in commutative algebra*.

Given a commutative ring  $R$ , the trace ideal of an  $R$ -module  $M$  is the ideal generated by the homomorphic images of  $M$  in  $R$ . I will speak on some recent developments in the theory of trace ideals and some applications of trace ideals in understanding endomorphism rings and rigid modules over commutative rings. (Received September 26, 2017)

1135-13-2851 **Kevin Bombardier\*** ([kevin-bombardier@uiowa.edu](mailto:kevin-bombardier@uiowa.edu)). *Atoms in Quasi-local Atomic Domains*. Preliminary report.

We consider quasi-local integral domains  $(R, M)$  in which every nonzero nonunit is a finite product of irreducible elements, also called atoms, and usually with a finite number of non-associate atoms. We discuss several results concerning the number of non-associate atoms, the number of non-associate atoms not in  $M^2$ , and the number of generators of  $M$ . (Received September 26, 2017)

1135-13-3255 **Craig L. Huneke\***, 400 Emmet Street South, Charlottesville, VA 22904. *How complicated are polynomials in many variables?*

The title question refers to systems of polynomial equations in many variables over a field. It can be made precise in many ways, for example, through the complexity of detecting whether a given polynomial can be expressed as a linear combination (with polynomial coefficients) of other polynomials.

Another sense in which it can be made precise is through comparisons of numerical data about the ideal generated by the polynomial equations, which generalize the numbers of generators and relations. This additional numerical data was originally introduced in the 1890's by David Hilbert to "count" the number of polynomial invariants of the action of a group (this was the work that "killed" invariant theory for a brief time!). In the last two years, three long-standing problems about these numerical invariants have been solved.

This talk will introduce the main players in this story: Hilbert functions, free resolutions, projective dimension, Betti numbers, and regularity. The first part of the talk will be historical and introductory, and the second half will focus on the solution of Ananyan and Hochster of Stillman's conjecture. (Received October 12, 2017)

## 14 ► Algebraic geometry

1135-14-146 **Perry Robert Strahl\*** (perrystrahl@gmail.com). *The Picard Group of Genus Zero Stable Quotients to Flag Varieties*. Preliminary report.

We compute the Picard group of the moduli space of genus zero stable quotients to projective space, Grassmannians, and any flag variety in the case of more than 2 markings. Furthermore, in the case of exactly 2 markings, we calculate the Picard group of the moduli space of genus zero stable quotients to projective space, Grassmannians, and to partial flag varieties where the ranks of the subspaces differ by more than 1. The first two moduli spaces mentioned are due to Alina Marian, Dragos Oprea, and Rahul Pandharipande. The latter is a generalization of this theory, originally due to Ionuț-Ciocan Fontanine, Bumsig Kim, and Daves Maulik. Projectivity of the coarse moduli space is proved first. The Picard rank is obtained using a torus action on the moduli space to perform tangent space calculations. When the number of markings is  $\geq 3$ , generators are determined by a geometric analysis of the interior of the moduli space. When the number of markings is 2, generators and relations are found by intersecting with curves. (Received August 03, 2017)

1135-14-161 **Changho Keem\*** (ckeem1@gmail.com), Department of Mathematics, College of Natural Sciences, Seoul National University, Seoul, 151-742, South Korea. *Hilbert scheme of smooth curves of genus  $g$  in  $\mathbb{P}^3$  of degree  $g - 1$* . Preliminary report.

We denote by  $\mathcal{H}_{d,g,r}$  the Hilbert scheme of smooth curves, which is the union of components whose general point corresponds to a smooth irreducible and non-degenerate curve of degree  $d$  and genus  $g$  in  $\mathbb{P}^r$ . In this preliminary report, we show that any non-empty  $\mathcal{H}_{g-1,g,3}$  is irreducible and generically smooth for almost all genus  $g$ . We also discuss several sporadic cases where  $\mathcal{H}_{g-1,g,3}$  become reducible. (Received August 06, 2017)

1135-14-254 **Luke Oeding\*** (oeding@auburn.edu), AL, and **Elina Robeva** (erobeva@mit.edu) and **Bernd Sturmfels** (bernd@mis.mpg.de). *Decomposing Tensors into Frames*.

A symmetric tensor of small rank decomposes into a configuration of only few vectors. We study the variety of tensors for which this configuration is a unit norm tight frame. (Received August 16, 2017)

1135-14-410 **Sameera Vemulapalli\*** (sameera.vemulapalli@berkeley.edu), 10225 Orange Avenue, Cupertino, CA 95014. *Uniform bounds for the number of rational points on symmetric squares of curves with low Mordell-Weil rank*.

A central problem in Diophantine geometry is to uniformly bound the number of  $K$ -rational points on a smooth curve  $X/K$  in terms of  $K$  and its genus  $g$ . A recent paper by Stoll proved uniform bounds for the number of  $K$ -rational points on a hyperelliptic curve  $X$  provided that the rank of the Jacobian of  $X$  is at most  $g - 3$ . Katz, Rabinoff and Zureick-Brown generalized his result to arbitrary curves satisfying the same rank condition.

In this paper, we prove conditional uniform bounds on the number of rational points on the symmetric square of  $X$  outside its algebraic special set, provided that the rank of the Jacobian is at most  $g - 4$ . We also find rank-favorable uniform bounds in the hyperelliptic case. (Received August 31, 2017)

1135-14-440 **Jose Luis Gonzalez\*** (jose.gonzalez@ucr.edu) and **Kalle Karu** (karu@math.ubc.ca). *Examples of non-finitely generated Cox rings*.

We bring examples of toric varieties blown up at a point in the torus that do not have finitely generated Cox rings. These examples are generalizations of previous work where toric surfaces of Picard number 1 were studied. In this article we consider toric varieties of higher Picard number and higher dimension. In particular, we bring

examples of weighted projective 3-spaces blown up at a point that do not have finitely generated Cox rings. (Received September 03, 2017)

1135-14-554 **Sean Howe\*** ([seanpkh@gmail.com](mailto:seanpkh@gmail.com)). *Motivic random variables: Examples and applications.* In algebraic geometry over finite fields, there are many natural statistics to investigate. For example, we could ask:

\* How many points are there on a random smooth plane curve? Or,

\* How many lines are there on a random smooth cubic surface?

The answer will typically differ depending on the finite field, but often has a uniform description using geometry: for example as the degree goes to infinity, the average number of points on a smooth plane curve over the finite field with  $q$  elements approaches  $q+1$  – which is also the number of points on the projective line! Using the language of motivic random variables, we can give precise meaning to funny looking probabilistic statements like “the average smooth plane curve is the projective line,” strengthening point counting statistics over finite fields. The same statements make sense over the complex numbers, where they can be interpreted using Hodge theory, and this leads to some surprising new connections – for example, between sieving theory and representation stability. In this talk we’ll introduce the language of motivic random variables with some simple examples, then highlight some applications of the theory. (Received September 08, 2017)

1135-14-834 **David Wen\***, [dwen@math.ucsb.edu](mailto:dwen@math.ucsb.edu). *Towards Minimal Models of Elliptic Fourfolds.*

Fiber spaces play an important role in the minimal model program as the possible end results reduce the down to the cases of Mori fiber spaces, Iitaka fibrations over canonical models and varieties of general type. A natural problem to consider would be, if we started with an algebraic fiber space, how might it behave under the birational transformations in the minimal model program. More specifically and concretely, with a few extra assumptions we can ask, does an elliptic fibration establish a relationship between minimal models of the total space and base space? The case of elliptic threefolds was established by Grassi relating minimal models of elliptic threefolds to log minimal models of the base surface. This talk will present some ideas towards understanding the elliptic fourfold case and higher dimensions. (Received September 15, 2017)

1135-14-859 **Elizabeth Drellich\*** ([edrelli1@swarthmore.edu](mailto:edrelli1@swarthmore.edu)), 500 College Ave, Swarthmore, PA 19081. *Spline models for algebra and geometry.*

When engineers or computer graphics people use the word “spline” they are talking about matching (polynomial) curves to smoothly model complicated surfaces. Dual to this idea is the concept of an “algebraic spline.” Given a graph whose edges are labeled by ideals of a given ring  $R$ , an algebraic spline is a function  $f$  from the vertices to  $R$  such that if two vertices  $v$  and  $w$  are adjacent, then  $f(v) - f(x)$  is in the ideal labeling edge  $vw$ . Splines can be used to explicitly compute compute (equivariant) cohomology rings of algebraic varieties called GKM spaces. This talk will present recent work extending the spline model to new settings, both geometric and purely algebraic. (Received September 15, 2017)

1135-14-902 **Ashvin Anand Swaminathan\*** ([ashvins@math.princeton.edu](mailto:ashvins@math.princeton.edu)), 1 Lawrence Drive, Apartment #808, Princeton, NJ 08540, and **Anand Patel** ([anand.patel@okstate.edu](mailto:anand.patel@okstate.edu)). *Inflectionary Invariants for Plane Curve Singularities.*

Let  $k$  be an algebraically closed field of characteristic 0, and let  $f \in k[[x, y]]$  be the germ of an isolated plane curve singularity. We study the role of the singularity germ  $f$  in the analysis of inflectionary behavior of curves specializing to a curve with a singularity cut out by  $f$ . We introduce a numerical function  $m \mapsto \text{AD}^m(f)$ , an invariant canonically associated to the isomorphism class of the singularity germ  $f$ , which arises as an error term in the problem of enumerating  $m^{\text{th}}$ -order inflection points in a 1-parameter family of curves acquiring a singular member with singularity given by  $f = 0$ . For an ordinary nodal singularity  $f = xy$ , we explicitly compute  $\text{AD}^m(f) = \binom{m+1}{4}$ , and we deduce as a corollary that  $\text{AD}^m(f) \geq \mu_f \cdot \binom{m+1}{4}$  for an arbitrary  $f$ , where  $\mu_f$  is the Milnor number of  $f$ . The numerical function  $m \mapsto \text{AD}^m(f) - \mu_f \cdot \binom{m+1}{4}$  is thus also an invariant of the singularity type, and it measures the extent to which the singularity counts as an  $m^{\text{th}}$ -order inflection point. Our main results can be applied to address a broad range of enumerative questions concerning inflection points in families of curves. (Received September 16, 2017)

1135-14-1525 **Han-Bom Moon\*** ([hmoon@ias.edu](mailto:hmoon@ias.edu)), 217 Simonyi Hall, 1 Einstein Dr, Princeton, NJ 08540, and **Sang-Bum Yoo** ([sangbum.yoo@gmail.com](mailto:sangbum.yoo@gmail.com)), Division of General Studies, Ulsan, 44919, South Korea. *Birational geometry of moduli spaces of parabolic bundles.*

I will describe a project on birational geometry of the moduli space of parabolic bundles on the projective line in the framework of Mori's program, and its connection with classical invariant theory and conformal blocks. (Received September 22, 2017)

1135-14-1725 **Desmond Coles\*** ([coles.69@buckeyemail.osu.edu](mailto:coles.69@buckeyemail.osu.edu)), 2015 Summit Street, Columbus, OH 43201, and **Neelav Dutta**, **Sifan Jiang**, **Ralph Morrison** and **Andrew Scharf**. *Most Planar Graphs do not Admit Faithful Tropicalizations in the Plane.*

Let  $k$  be a nonarchimedean field, such as the  $p$ -adics. A powerful tool for studying an algebraic curve  $X$  over  $k$  is its *minimal Berkovich skeleton*  $\Gamma(X)$ , a metric graph that encodes geometric information about  $X$ ; such as the genus. To study  $\Gamma(X)$ , we use *tropical geometry*, which transforms  $X$  into a piecewise linear subset of  $\mathbb{R}^n$ , denoted  $\text{Trop}(X)$ . This set contains some subgraph of  $\Gamma(X)$ , and if it contains the entirety of  $\Gamma(X)$ , we call  $\text{Trop}(X)$  *faithful*.

In 2015, Baker and Rabinoff proved that every curve has a faithful tropicalization in  $\mathbb{R}^3$ . The next question is: which curves have a faithful tropicalization in  $\mathbb{R}^2$ ? In 2015, Brodsky, Joswig, Morrison and Sturmfels answered this question up to genus  $g = 5$ . We extend their results to genus 6, including the important case of smooth plane quintic curves, and prove an important restriction on such graphs: as  $g$  goes to infinity, the percentage of trivalent planar graphs with  $g$  loops that appear in a planar faithful tropicalization goes to zero. Our proof uses discrete combinatorial methods, including estimates of the number of triangulations of lattice polygons, combined with previous graph theoretical work. (Received September 25, 2017)

1135-14-1893 **Derek Tomlin\*** ([derek.tomlin@mavs.uta.edu](mailto:derek.tomlin@mavs.uta.edu)) and **M. Vanciliff**. *Discussing a few Quadratic Quantum  $\mathbb{P}^3$ s.*

In this talk, we investigate some geometric data associated to a few families of quadratic quantum  $\mathbb{P}^3$ s. In particular, we analyze the *line schemes* of these algebras. Some of the algebras discussed are graded skew-Clifford algebras; for those that are, we relate the intersection points of the line scheme to normal, degree-two elements in the algebra (Received September 27, 2017)

1135-14-2092 **Mahrud Sayrafi\*** ([mahrud@berkeley.edu](mailto:mahrud@berkeley.edu)) and **Michael Stillman**, Malott Hall, Mathematics Department, Cornell University, Ithaca, NY 14853. *Computations over Local Rings in Macaulay2.*

Local rings are ubiquitous in algebraic geometry and commutative algebra. Not only are they naturally meaningful in a geometric sense, but also they are extremely useful as many problems can be attacked by first reducing to the local case and taking advantage of their nice properties.

We present a software package for performing computations over localizations of polynomial rings with respect to prime ideals. The main tools and procedures here involve homological properties, such as the flatness property of localization and the existence of minimal free resolutions for finitely generated modules over local rings, which follows from Nakayama's lemma. The procedures presented here are described as pseudocodes and implemented in Macaulay2, a computer algebra software specializing in algebraic geometry and commutative algebra.

The main motivation for this work is enabling mathematicians to computationally study the local properties of algebraic varieties near irreducible components of higher dimension, such as the intersection multiplicity of higher dimensional varieties. (Received September 26, 2017)

1135-14-2317 **James Phillips\*** ([jp5ay@virginia.edu](mailto:jp5ay@virginia.edu)). *Good reduction of covers of elliptic curves.*

Given a branched cover of curves defined over a mixed-characteristic discretely valued field, one asks whether this cover has good reduction to characteristic  $p$ . This arises from a question concerning tame fundamental groups: we know that one has a surjection from the tame fundamental group of a curve to that of its reduction, and finding the kernel of this map amounts to determining which covers have good reduction. We show that we indeed have good reduction in the case of a cover of an elliptic curve defined over a sufficiently small field and branched over a single point whose Galois group has a cyclic Sylow  $p$ -subgroup. (Received September 25, 2017)

1135-14-2403 **Bradley Weaver\*** ([brw4sz@virginia.edu](mailto:brw4sz@virginia.edu)), Department of Mathematics, University of Virginia, 141 Cabell Drive, Charlottesville, VA 22904. *The Local Lifting Problem for  $D_4$ .*

For a prime  $p$ , an algebraically closed field  $k$  of characteristic  $p$ , a cyclic-by- $p$  group  $G$  and a  $G$ -extension  $L|K$  of complete discrete valuation fields of characteristic  $p$  with residue field  $k$ , the local lifting problem asks whether the extension  $L|K$  lifts to characteristic zero. If every such  $G$ -extension  $L|K$  lifts to characteristic zero, then  $G$



is denominated a local Oort group for  $k$ . In this talk we shall motivate the local lifting problem (via the global lifting problem for curves), and discuss briefly why  $D_4$  (the dihedral group of order eight) is a local Oort group for every algebraically closed field of characteristic two. (Received September 26, 2017)

1135-14-2530 **Jonathan Mboyo Esole\*** ([mboyoesole@gmail.com](mailto:mboyoesole@gmail.com)), Northeastern University, Department of Mathematics, 360 Huntington Avenue, Boston, MA 02115. *Hyperplane arrangements and birational geometry of elliptic fibrations.*

Elliptic fibrations obtained by crepant resolutions of Weierstrass models resulting from Tate's algorithm are often used by physicists to model gauge theories characterized by a reductive Lie group  $G$  with Lie algebra  $\mathfrak{g}$  and a representation  $\mathbf{R}$  of  $G$ . I will explain how the birational geometry of such elliptic fibrations is controlled in many ways by a hyperplane arrangement defined from the data of  $\mathfrak{g}$  and  $\mathbf{R}$ . (Received September 26, 2017)

1135-14-2615 **Andrew Scharf\*** ([als6@williams.edu](mailto:als6@williams.edu)), **Ralph Morrison**, **Sifan Jiang**, **Desmond Coles** and **Neelav Dutta**. *Degrees of freedom in constructing algebraic and tropical curves in the plane.*

In algebraic geometry, one of the most important objects of study is  $M_g$ , the space of algebraic curves of genus  $g$ . In 2009, Castryck and Voigt introduced  $M_P$ , the locus inside of  $M_g$  of all curves with a given Newton polygon  $P$ , where  $P$  has  $g$  interior lattice points. This space has dimension at most  $2g + 1$ , although this upper bound is not achieved for all polygons.

In 2015, Brodsky et al introduced the space  $\mathbb{M}_P$  as a tropical analog of  $M_P$ ; this polyhedral space parametrizes the metric graphs with  $g$  loops that appear in tropical curves with Newton polygon  $P$ . We prove that  $\dim(M_P) = \dim(\mathbb{M}_P)$  for maximal polygons, positively answering an open question by Brodsky et al. Our proof is constructive, and involves an explicit method of triangulating polygons to give rise to this equality. We also draw on previous work of Koelman (1991) on toric varieties to connect the algebraic and tropical dimensions. We apply our result to completely characterize which polygons  $P$  give the maximal  $2g + 1$  degrees of freedom when constructing a curve, answering another open question of Brodsky et al. The key players here are trigonal curves, curves of genus 6, and sextics with a decreasing number of nodal singularities. (Received September 26, 2017)

1135-14-2624 **T. Alden Gassert\*** ([gassert@hws.edu](mailto:gassert@hws.edu)), 300 Pulteney St, Geneva, NY 14456, and **Caleb Shor** ([caleb.shor@wne.edu](mailto:caleb.shor@wne.edu)), 1215 Wilbraham Rd, Springfield, MA 01119. *Sylvester sums of compound sequence semigroup complements.*

In this talk, we consider the complement  $NR(G)$  of a numerical semigroup generated by a compound sequence  $G$ . By generalizing a result of Tuenter, we are able to compute sums of powers of elements in  $NR(G)$ , also known as Sylvester sums. As an application, we compute the weights of higher-order Weierstrass points on suitable towers of complex algebraic curves. (Received September 26, 2017)

1135-14-2791 **King Leung Lee\*** ([kingleung.lee@rutgers.edu](mailto:kingleung.lee@rutgers.edu)), 27 Mulock Place, 2nd Floor, East Newark, NJ 07029, and **Zhiyuan Li**, **Jacob Sturm** and **Xiaowei Wang**. *GIT Stability of some toric surfaces.*

In this talk we will discuss the GIT and K-stability of some explicit toric surfaces (Received September 26, 2017)

1135-14-2960 **Neriman Tokcan\*** ([tokcan@umich.edu](mailto:tokcan@umich.edu)), 530 Church Street, Ann Arbor, MI 48109. *On the Waring rank of binary forms.*

The  $K$ -rank of a binary form  $f$  in  $K[x, y]$ ,  $K \subseteq \mathbb{C}$ , is the smallest number of  $d$ -th powers of linear forms over  $K$  of which  $f$  is a  $K$ -linear combination. We provide lower bounds for the  $\mathbb{C}$ -rank (Waring rank) and for the  $\mathbb{R}$ -rank (real Waring rank) of binary forms depending on their factorization. We completely classify binary forms of Waring rank 3. (Received September 26, 2017)

1135-14-2964 **Thomas B Hill\*** ([thomasbhill@comcast.net](mailto:thomasbhill@comcast.net)), **Shantel Spatig** ([shantel.black@aggiemail.usu.edu](mailto:shantel.black@aggiemail.usu.edu)) and **Andreas Malmendier** ([andreas.malmendier@usu.edu](mailto:andreas.malmendier@usu.edu)). *Relations Between Theta Functions in Genus One and Two from Geometry.*

Genus-two curves with special symmetries are related to pairs of genus-one curves by two and three-sheeted coverings. This classical work dates back to early 20th century and is known as Jacobi and Hermite reduction. In turn, Jacobians of genus-two curves can be used to construct complex two-dimensional algebraic manifolds known as Kummer surfaces. On the other hand, the defining coordinates and parameters of both elliptic curves and Kummer surfaces can also be understood as Theta functions of genus one and genus two, respectively. This result goes back to the seminal work of Mumford in the 1980s. We use the geometric relation between elliptic

curves and Kummer surfaces to derive functional relations between Theta functions of genus one and genus two along Humbert varieties of low discriminant. (Received September 26, 2017)

1135-14-3126 **Malgorzata A Marciniak\*** ([mmarciniak@lagcc.cuny.edu](mailto:mmarciniak@lagcc.cuny.edu)), 4266 Phlox Place, Apt C7, Flushing, NY 11355, and **Shenglan Yuan**. *Hessian Dynamics on the Hesse pencil*.

Our ultimate goal is to answer the question of whether the group of rational points on an elliptic curve  $C$  is related to the group of rational points on its Hessian  $H(C)$ . Prior to attacking this problem, we considered the Hessian dynamics on the pencil of cubic curves (called the Hesse pencil). Here we describe and analyze the Hessian dynamics in terms of projective coordinates on the space of parameters on the Hesse pencil. (Received September 26, 2017)

## 15 ► *Linear and multilinear algebra; matrix theory*

1135-15-70 **Trung Hoa Dinh\*** ([trunghoa.math@gmail.com](mailto:trunghoa.math@gmail.com)), Department of Mathematics and Statistics, University of North Florida, 1 UNF Drive, Jacksonville, FL 32224. *Some open problems in the theory of matrix means and matrix functions*.

The problem to find a matrix analog of the geometric mean of non-negative numbers was a long standing since the product of two positive semi-definite matrices is not a such one. In 1975, Pusz and Woronowitz solved this problem and showed that the geometric mean  $A\sharp B := A^{1/2}(A^{-1/2}BA^{-1/2})^{1/2}A^{1/2}$  of two positive definite matrices  $A$  and  $B$  is the unique solution of the matrix Riccati equation  $XA^{-1}X = B$ . Consequently, Ando and Kubo (1980) developed an axiomatic theory of operator means on the set of positive semi-definite matrices and showed that there exists an affine order-isomorphism from the class of operator means onto the class of positive operator monotone functions which were introduced by Loewner (1930). In 1996, Petz proved that there is a correspondence between monotone metrics and operator means in the sense of Kubo and Ando, and hence, connected three important theories in matrix analysis and quantum information theory.

In this talk we introduce some picture of matrix means and a general approach of matrix convexity. The problem of characterization of matrix functions by inequalities is also discussed. The topics and open problems are well-suited for undergraduate students with background in Linear Algebra and Analysis. (Received July 18, 2017)

1135-15-200 **Wei Gao\*** ([wzg0021@auburn.edu](mailto:wzg0021@auburn.edu)), Parker Hall 221, Department of Mathematics and Statistics, Auburn University, Auburn, AL 36830, and **Zhongshan Li** and **Lihua Zhang**. *Tree Sign Patterns that Require  $\mathbb{H}_n$* .

A sign pattern (matrix)  $\mathcal{A}$  is a matrix whose entries are from the set  $\{+, -, 0\}$ . The qualitative class of  $\mathcal{A}$ , denoted  $Q(\mathcal{A})$ , is defined as  $Q(\mathcal{A}) = \{B \in M_n(\mathbb{R}) \mid \text{sgn}(B) = \mathcal{A}\}$ . The refined inertia of a square real matrix  $B$ , denoted  $\text{ri}(B)$ , is the ordered 4-tuple  $(n_+(B), n_-(B), n_z(B), 2n_p(B))$ , where  $n_+(B)$  (resp.,  $n_-(B)$ ) is the number of eigenvalues of  $B$  with positive (resp., negative) real part,  $n_z(B)$  is the number of zero eigenvalues of  $B$ , and  $2n_p(B)$  is the number of pure imaginary eigenvalues of  $B$ . The set of refined inertias  $\mathbb{H}_n = \{(0, n, 0, 0), (0, n - 2, 0, 2), (2, n - 2, 0, 0)\}$  is important for the onset of Hopf bifurcation in dynamical systems. An  $n \times n$  sign pattern  $\mathcal{A}$  is said to require  $\mathbb{H}_n$  if  $\mathbb{H}_n = \{\text{ri}(B) \mid B \in Q(\mathcal{A})\}$ . In this talk, we discuss the star and path sign patterns that require  $\mathbb{H}_n$ . It is shown that for each  $n \geq 5$ , a star sign pattern requires  $\mathbb{H}_n$  if and only if it is equivalent to one of the five sign patterns identified in the talk. It is also shown that no path sign pattern of order  $n \geq 5$  requires  $\mathbb{H}_n$ . (Received August 08, 2017)

1135-15-294 **Yang Qi\*** ([yangqi@galton.uchicago.edu](mailto:yangqi@galton.uchicago.edu)), University of Chicago, Department of Statistics, 5747 South Ellis Avenue, Jones 114, Chicago, IL 60637, and **Lek-Heng Lim** ([lekheng@galton.uchicago.edu](mailto:lekheng@galton.uchicago.edu)), University of Chicago, Department of Statistics, 5747 South Ellis Avenue, Jones 122B, Chicago, IL 60637. *Secant varieties and tensor ranks*. Preliminary report.

Secant varieties are closely related to tensor ranks. In this talk, we first recall the relations between secant varieties and tensor ranks, and then study some basic properties of secant varieties. Finally we show how these properties imply interesting phenomena of tensor ranks. This is a joint work with Lek-Heng Lim. (Received August 20, 2017)

1135-15-993 **Akram Aldroubi** and **Keaton Hamm\***, [keaton.hamm@vanderbilt.edu](mailto:keaton.hamm@vanderbilt.edu), and **Ahmet Bugra Koku** and **Ali Sekmen**. *CUR Matrix Decomposition and Subspace Clustering*.

The subspace clustering problem seeks to classify, or cluster, data in a high-dimensional space that is drawn from the union of much smaller dimensional subspaces. For example, images of a single face under different

illuminations can be well-modeled to lie in a small linear subspace of the ambient space. One method of attack for this problem is to find a similarity matrix from the data which identifies the clusters. This talk will discuss an intriguing matrix decomposition method called CUR decomposition, and describe how some of the known similarity matrix methods are special cases of this general decomposition in the case that the subspaces are independent. (Received September 18, 2017)

1135-15-1043 **Ben Z Webb\*** (bwebb@mathematics.byu.edu), 1895 N. 1450 E., Provo, UT 84604, and **Leonid Bunimovich**. *Isospectral Graph Reductions and Improved Estimates of Matrices' Spectra*.

Via an isospectral graph reduction the adjacency matrix of a graph can be reduced to a smaller matrix while its spectrum is maintained up to some known (and possibly empty) set. It is then possible to estimate the spectrum of the original matrix by considering Gershgorin-type estimates associated with the reduced matrix. Our main result is that the eigenvalue estimates associated with Gershgorin, Brauer, Brualdi, and Varga improve as the matrix is reduced. Given that such estimates improve with each successive reduction, it is also possible to estimate the eigenvalues of a matrix with increasing accuracy by repeated use of the isospectral reduction process. (Received September 18, 2017)

1135-15-1117 **Shuliang Bai\*** (sbai@math.sc.edu) and **Linyuan Lu**. *On the spectral radius of hypergraph with  $e$  edges and  $\{0, 1\}$ -tensor with  $e$  ones*. Preliminary report.

Let  $r \geq 2$ , let  $f_r: [0, \infty) \rightarrow [1, \infty)$  be the unique analytic function such that  $f_r\left(\binom{k}{r}\right) = \binom{k-1}{r-1}$  for any  $k \geq r-1$ . We prove that the spectral radius of an  $r$ -uniform hypergraph  $H$  with  $e$  edges is at most  $f_r(e)$ . The equality holds if and only if  $e = \binom{k}{r}$  for some positive integer  $k$  and  $H$  is the union of a complete  $r$ -uniform hypergraph  $K_k^r$  and some possible isolated vertices. This result generalizes the classical Stanley's theorem on graphs. We extend the problem to general  $\{0, 1\}$ -tensors and prove that the spectral radius of an  $r$ -th order  $\{0, 1\}$ -tensor  $A$  with  $e$  ones is at most  $e^{\frac{r-1}{r}}$  with the equality holds if and only if  $e = k^r$  for some integer  $k$  and all ones lies in a principle sub-tensor  $\mathbf{1}_{k \times \dots \times k}$ . We also prove a stabilistic result for general  $A$  with  $e$  ones where  $e = k^r + l$  with relatively small  $l$ . Using the stabilistic result, we completely characterize the maximum tensors among all  $r$ -th order  $\{0, 1\}$ -tensor  $A$  with  $k^r + l$  ones, with  $-r - 1 \leq l \leq r$ , for sufficiently large  $k$ . (Received September 19, 2017)

1135-15-1215 **Wayne Barrett, Steve Butler, Shaun M. Fallat, H. Tracy Hall, Leslie Hogben, Jephian C.-H. Lin\*** (chinhunglin@uvic.ca), **Bryan L. Shader** and **Michael Young**. *The inverse eigenvalue problem of a graph: Multiplicities and minors*.

Given a simple graph  $G$ , the family  $\mathcal{S}(G)$  collects all real symmetric matrices whose off-diagonal  $i, j$ -entry is nonzero whenever  $\{i, j\}$  is an edge in  $G$  and zero otherwise. The diagonal entries can be any real number. The inverse eigenvalue problem of a graph is asking to characterise the possible spectra among matrices in  $\mathcal{S}(G)$ . Recently, many new tools for this problem are developed through the Implicit Function Theorem, including the Strong Spectral Property, the Strong Multiplicity Property, and the matrix liberation lemma. In this talk, we will show how to use these techniques to construct matrices of a given graph and with the desired spectrum. (Received September 20, 2017)

1135-15-1258 **Amanda E Francis\*** (afrancis@carroll.edu), Dr Amanda E Francis, Department of Mathematics, 1601 N Benton Ave, Helena, MT 59625, and **Ben Webb** and **Dallas Smith**. *Equitable Decompositions of Graphs: Using any symmetry of a graph (or matrix) to simplify eigenvalue and eigenvector computations*.

In this talk I will discuss connections between the symmetries (automorphisms) of a graph and its spectral properties. Whenever a graph has a symmetry, i.e. a nontrivial automorphism  $\phi$ , it is possible to use  $\phi$  to decompose any matrix  $M$  appropriately associated with the graph. The result of this decomposition is a number of strictly smaller matrices whose collective eigenvalues are the same as the eigenvalues of the original matrix  $M$ . Some of the matrices that can be decomposed are the graph's adjacency matrix, Laplacian matrix, etc. Because this decomposition has connections to the theory of equitable partitions it is referred to as an equitable decomposition. I will also discuss a new (sharp) bound on the number of simple eigenvalues of undirected graphs. (Received September 20, 2017)

1135-15-1263 **Xavier Martinez-Rivera\*** (xaviermr@auburn.edu). *A new principal rank characteristic sequence*.

The necessity to know certain information about the principal minors of a given/desired matrix is a situation that arises in several areas of mathematics; as a result, researchers associated a sequence with a symmetric (or complex Hermitian) matrix, which they defined as follows: The *enhanced principal rank characteristic sequence*

(*epr-sequence*) of an  $n \times n$  symmetric (or complex Hermitian) matrix  $B$  is  $\ell_1 \ell_2 \cdots \ell_n$ , where  $\ell_k$  is  $\mathbf{A}$  (respectively,  $\mathbf{N}$ ) if all (respectively, none of) the principal minors of order  $k$  are nonzero; if some (but not all) are nonzero, then  $\ell_k = \mathbf{S}$ .

Known results about *epr*-sequences will be discussed, concluding with the introduction of a new principal rank characteristic sequence for Hermitian matrices: The *signed enhanced principal rank characteristic sequence* (*sepr-sequence*), which was recently introduced by the present speaker as a refinement of the *epr*-sequence. Results regarding the attainability of *sepr*-sequences will be presented. Particular attention will be paid to results forbidding certain subsequences from appearing in the *sepr*-sequence of a Hermitian matrix. (Received September 20, 2017)

1135-15-1288 **J. William Helton, Igor Klep and Jurij Volčič\*** ([jurij.volcic@auckland.ac.nz](mailto:jurij.volcic@auckland.ac.nz)), University of Auckland, Department of Mathematics, Auckland, New Zealand. *Free singularity loci of noncommutative polynomials.*

The free singularity locus of a (matrix-valued) noncommutative polynomial  $f$  is defined to be the union of hypersurfaces

$$\mathcal{Z}(f) = \bigcup_{n \in \mathbf{N}} \mathcal{Z}_n(f), \quad \mathcal{Z}_n(f) = \{X \in M_n(k)^g : \det f(X) = 0\}.$$

Such sets naturally arise in free analysis and free real algebraic geometry, for example as the free algebraic closure of the boundary of a free spectrahedron or the complement of the domain of a noncommutative rational function.

In this talk we will first establish an algebraic certificate for the inclusion of free loci  $\mathcal{Z}(f_1) \subseteq \mathcal{Z}(f_2)$  to hold. Then the correspondence between irreducible components of  $\mathcal{Z}_n(f)$  for large  $n$  and irreducible factors of  $f$  in the free algebra will be explained. Lastly, smooth points on  $\mathcal{Z}(f)$  will be related to one-dimensional kernels of images of  $f$ , which implies the quantum version of Kippenhahn's conjecture and pertains to Positivstellensätze on free semialgebraic sets. (Received September 21, 2017)

1135-15-1354 **Elina Robeva and Anna Seigal\*** ([seigal@berkeley.edu](mailto:seigal@berkeley.edu)). *Duality of graphical models and tensor networks.*

We give a duality between graphical models and tensor networks. Graphical models are a statistical tool for representing multivariate probability distributions in terms of conditional independence relations. Tensor networks are a graphical way to represent tensors. We also explore how certain algorithms translate under the duality correspondence. (Received September 21, 2017)

1135-15-1430 **Harm Derksen\*** ([hderksen@umich.edu](mailto:hderksen@umich.edu)), Ann Arbor, MI 48109-1043. *Singular Values for Tensors.*

The Singular Value Decomposition (SVD) for matrices has many applications. It can be used for finding low rank approximations of a matrix. In Principal Component Analysis (PCA) it is used for dimension reduction of data. There are generalizations of the SVD to higher order tensors. One generalization is the low rank tensor decomposition (known as the CP, CANDECOMP or PARAFAC decomposition). This decomposition is not numerically stable which can be problematic in applications. Another generalization is the Tucker decomposition (or Higher Order Singular Value Decomposition) which is numerically stable but does not typically give low rank decompositions. We will discuss other, new generalizations of the SVD, the Diagonal Singular Value Decomposition and the Slope Decomposition, and how such decompositions can be used to generalize PCA. (Received September 22, 2017)

1135-15-1670 **Thomas R Cameron\*** ([thcameron@davidson.edu](mailto:thcameron@davidson.edu)). *On Descartes' rule of signs for matrix polynomials.* Preliminary report.

In this talk, we discuss an open problem regarding the generalization of Descartes' rule of signs for matrix polynomials

$$P(\lambda) = A_m \lambda^m + A_{m-1} \lambda^{m-1} + \cdots + A_1 \lambda + A_0,$$

where  $A_i$  are  $n \times n$  Hermitian and positive/negative definite matrices, or otherwise null, and  $A_m \neq 0$ . Specifically, we conjecture that

$$z^+(P) \leq n \cdot v(P) \quad \text{and} \quad z^-(P) \leq n \cdot c(P),$$

where  $z^+(P)$  denotes the number of positive eigenvalues of  $P$ ,  $z^-(P)$  the number of negative eigenvalues,  $v(P)$  the number of alternations of signs of  $P$ , and  $c(P)$  the number of permanences of signs.

In the process of building a case for our conjecture, we will gain perspective for the historical proofs of Descartes' rule and its extensions, which will lead to a better understanding of what makes this open problem both interesting and difficult. (Received September 24, 2017)

1135-15-1673 **X. Sherry Li\*** (xsli@lbl.gov), **C. Gorman**, **P. Ghysels**, **G. Chavez** and **F.-H. Rouet**. *Matrix-free construction of HSS representations using adaptive randomized sampling.*

An active research area in recent years is the development of fast hierarchical matrix tools for linear and eigen solvers. Although the theoretical foundation for the hierarchical matrices has been solidified, there is a lack of robust algorithms and software that can handle many practical issues. For example, randomized sampling has been shown to be an effective tool to reveal the low rank structures, but choosing the right number of samples is difficult. In this talk we present new results for HSS compression using an Adaptive Randomized Sampling strategy. In particular, we developed a robust stopping criterion based on a new stochastic relative error estimation. We present results for situations when the dense matrix is given and in the matrix-free setting. We show some practical difficulties about implementation and present some solutions that get around the problems. (Received September 24, 2017)

1135-15-1675 **David Miller\*** (davmiller@hartford.edu), CT. *A fast inversion algorithm for the Laurent-Vandermonde matrix.*

Although Gaussian elimination uses  $\mathcal{O}(n^3)$  operations to invert an arbitrary matrix, matrices with a special Vandermonde structure can be inverted in only  $\mathcal{O}(n^3)$  operations using a fast, Traub-like algorithm. The Traub algorithm has been extended from Vandermonde matrices involving monomials to polynomial-Vandermonde matrices involving real orthogonal polynomials, and the Szegő polynomials.

In this talk we consider extending the properties of the original Traub algorithm to a special class of polynomials, called the Laurent polynomials. We derive a fast  $\mathcal{O}(n^2)$  Traub-like algorithm to invert the associated Laurent-Vandermonde matrix. (Received September 24, 2017)

1135-15-1687 **Fan Chung\*** (fan@ucsd.edu), 1555 Coast Walk, La Jolla, CA. *A generalized Alon-Boppana bound and weak Ramanujan graphs.* Preliminary report.

We will discuss some recent developments in several directions of spectral graph theory, including spectral bounds for graphs with general degree distribution and some variations of Ramanujan graphs, satisfying vertex and edge expansion properties. (Received September 24, 2017)

1135-15-1841 **Uladzimir L Shtukar\*** (vshtukar@yahoo.com), 1906 Raj Drive, Durham, NC 27703. *Reduced Row Echelon Forms of Matrices, Canonical Bases for Subspaces of vector Spaces, and Subalgebras of Lie Algebras.* Preliminary report.

Canonical bases for subspaces of vector spaces are introduced to be such that generate the matrix in reduced row echelon form. They are classified for (n-1)-dimensional and (n-2)-dimensional subspaces. It is proved that the total number of all nonequivalent canonical bases is equal the number of m-combinations from n given numbers. Subalgebras of two important Lie algebras are found utilizing canonical bases. One of them is 6-dimensional Lie algebra of Lorentz group. (Received September 25, 2017)

1135-15-2012 **Ryan Gabrys\*** (ryan.gabrys@navy.mil), 4573 Monroe Ave., San Diego, CA 92115. *Reconciling Similar Sets of Data.*

Suppose we have some data (a file for instance) on Host A and some related data on Host B, and it is desired that Host B obtains all the information on Host A. One naive approach would be for Host A to simply transmit all of its data to Host B; however, if the sets are similar, this approach could be wasting valuable network resources. Another approach is to compute hashes on the data and then compare hashes to iteratively determine the difference. With this approach, many rounds of communication are required. In adverse network conditions, the more rounds of communication required, the greater the stress placed on already limited network resources. As a result, we are interested schemes for synchronizing data that require a small number of communication rounds and have small communication overhead.

In this work, we consider the problem of synchronizing two sets of data where the size of the symmetric difference between the sets is small and, in addition, the elements in the symmetric difference are related through the Hamming distance metric. Upper and lower bounds are derived on the minimum amount of information exchange. Furthermore, explicit encoding and decoding algorithms are provided for many cases. (Received September 25, 2017)

1135-15-2068 **Mohammad Adm\*** (mjamathe@yahoo.com), Department of Mathematics and Statistics, University of Konstanz, 78464 Konstanz, Germany, and **Shaun Fallat** (shaun.fallat@uregina.ca), Department of Mathematics and Statistics, University of Regina, Regina, SK S4S0A2, Canada. *On the eigenvalues of acyclic matrices.*

In this talk, we will present some recent work concerning the maximum nullity of acyclic matrices with a prescribed number of negative eigenvalues. Particular interest is paid to the case of one negative eigenvalue,

where characterizations of the optimal matrices attaining the largest nullity will be shown. As a by-product cut-vertex type inequalities along with a new derivation of the maximum nullity for such matrices will also be demonstrated. (Received September 25, 2017)

1135-15-2190 **Dallas Smith\*** (dallas.smith@mathematics.byu.edu), Provo, UT 84606, and **Ben Webb** (bwebb@mathematics.byu.edu), **Amanda Francis** (afrancis@carroll.edu) and **Derek Sorensen** (derek.sorensen@maths.ox.ac.uk). *Applications of Equitable Decompositions for Graphs with Symmetries.*

The symmetries of a graph are characterized by the graph's set of automorphisms. If a graph  $G$  has a symmetry, it is possible to decompose any automorphism compatible matrix  $M$  associated with  $G$ , such as its adjacency and Laplace matrices, into a number of smaller matrices  $M_1, \dots, M_n$ . These smaller matrices collectively have the same eigenvalues as the original matrix  $M$  including multiplicities. This process is referred to as an equitable decomposition. Here we discuss a number of applications of this decomposition. First we demonstrate that not only can a matrix  $M$  be decomposed but that the eigenvectors of  $M$  can also be equitably decomposed. Additionally, we prove under mild conditions that if a matrix  $M$  is equitably decomposed the resulting divisor matrix, which is the divisor matrix of the associated equitable partition, will have the same spectral radius as the original matrix  $M$ . Finally, we describe how an equitable decomposition effects the Gershgorin region of a matrix  $M$ , which can be used to localize the eigenvalues of  $M$ . (Received September 25, 2017)

1135-15-2205 **Zhuoheng He** (jz10073@auburn.edu), 221 Parker Hall, Auburn University, Auburn, AL 36849, **Jianzhen Liu\*** (jz10073@auburn.edu), 221 Parker Hall, Auburn University, Auburn, AL 36849, and **Tin-Yau Tam**, 221 Parker Hall, Auburn University, Auburn, AL 36849. *The general  $\phi$ -Hermitian solution to mixed pairs of quaternion matrix Sylvester equations.*

Let  $\mathbb{H}^{m \times n}$  be the space of  $m \times n$  matrices over  $\mathbb{H}$ , where  $\mathbb{H}$  is the real quaternion algebra. Let  $A_\phi$  be the  $n \times m$  matrix obtained by applying  $\phi$  entrywise to the transposed matrix  $A^T$ , where  $A \in \mathbb{H}^{m \times n}$  and  $\phi$  is a nonstandard involution of  $\mathbb{H}$ . We first give some properties of the Moore-Penrose inverse of the quaternion matrix  $A_\phi$ . Then we consider two systems of mixed pairs of quaternion matrix Sylvester equations  $A_1X - YB_1 = C_1$ ,  $A_2Z - YB_2 = C_2$  and  $A_1X - YB_1 = C_1$ ,  $A_2Y - ZB_2 = C_2$ , where  $Z$  is conditions for the existence of a solution  $(X, Y, Z)$  to those systems in terms of the ranks and Moore-Penrose inverses of the given coefficient matrices will be presented. Moreover, the general solutions to these systems are explicitly given when they are solvable. (Received September 26, 2017)

1135-15-2219 **Kristin A. Camenga** and **Patrick X Rault\*** (rault@email.arizona.edu), 9040 S Rita Rd, Ste 2260, Tucson, AZ 85747. *A Proof of a Generalized Lax Conjecture for Numerical Ranges.* Preliminary report.

Let  $A$  denote an  $n \times n$  unitarily irreducible complex matrix. Let  $H_1$  and  $iH_2$  be the hermitian and skew-hermitian parts of  $A$ , and define the polynomial  $F_A(x, y, t) := \det(xH_1 + yH_2 + tI)$ . We define  $\Gamma_F : F(x, y, t) = 0$  in  $\mathbb{P}^2(\mathbb{R})$ , whose dual  $\Gamma_{\hat{F}}$  is called the boundary generating curve. The convex hull of the latter is the numerical range of  $A$ , denoted  $W(A)$ . This process thus gives a method for going from  $A$  to  $W(A)$ . Lax conjectured in 1958 that that we can also do the reverse: for a certain type of curve  $\mathcal{C}$ , there exist symmetric matrices  $B$  and  $C$  for which  $G(x, y, t) := \det(xB + yC + tI)$  satisfies  $\mathcal{C} = \Gamma_G$ . This conjecture was proved in 2005 by Lewis, Parrilo, and Ramana. In 2012, Helton and Spitzkovsky completed the connection by proving that there exists a symmetric matrix  $S$  for which  $F_A = F_S$ ; in particular  $W(A) = W(S)$ . In this talk we give a concise proof of the following generalization: For any factor  $g$  of  $F_A$ , there exists a symmetric matrix  $T$  for which  $F_T = g$ . We will analyze examples of these factors and how the curves they define can intersect.

This is joint work with L. A. Deaett, T. Sendova, I.M. Spitzkovsky, and R. Yates. (Received September 27, 2017)

1135-15-2318 **Louis Deaett**, **Jonathan Fischer**, **Colin Garnett** and **Kevin Vander Meulen\*** (kvanderm@redeemer.ca), Department of Mathematics, Redeemer University College, Ancaster, ON L9K 1J4, Canada. *Non-sparse companion matrix constructions.* Preliminary report.

A companion matrix is a matrix template for obtaining a prescribed characteristic polynomial. The Frobenius companion matrix is the most familiar companion matrix. It is an example of a sparse companion matrix, having exactly  $n^2 - 2n + 1$  zero entries. Recently all the sparse companion matrices were classified in terms of intercylic digraphs. In this presentation, we develop constructions for non-sparse companion matrices, noting that some are superpatterns of sparse companion matrices, but some are not, leading to some interesting open questions. (Received September 25, 2017)

1135-15-2320 **Feng Bill Shi\*** (bill110@email.unc.edu). *Community Structures in Hypergraph Networks*. Preliminary report.

Networks have become a standard tool in representing and analyzing systems consisting of complex interactions. An important advancement in network methods is the ability to detect large-scale structures-communities-that often correspond to functional or organizational modules of the systems. Consequently, applications in diverse domains have facilitated the growth of network science; substantial progress has been made for temporal or multiplex networks.

However, most of the work is on pairwise interactions, and relatively little progress is made on structures due to higher-order interactions. While pairwise interactions are the fundamental structural units and their patterns reveal important information about the networks, considering higher-order interactions adds not only marginal knowledge but also potentially new insights [Benson et al., Science 353,163]. E.g., higher-order interactions between species are found to be a possible mechanism to maintain a stable coexistence of diverse competitors in nature [Grilli et al., Nature 548,210].

We develop a formal treatment for higher-order interactions using hypergraphs, and extend the stochastic block model via tensor decompositions. We apply the method to a set of real networks and compare the results with pairwise networks. (Received September 25, 2017)

1135-15-2476 **Guangming Jing\*** (gjing1@gsu.edu), 2697 Shetland Ln, Duluth, GA 30096, and **Zhongshan Li, Yubin Gao, Yanling Shao and Wei Fang**. *The index set of scrambling indices*.

Recall a nonnegative square matrix  $A$  is called primitive if there exist a positive integer  $k$  such that  $A^k > 0$ . Let  $A$  be a primitive matrix and  $D$  be its corresponding digraph. The scrambling index of  $A$  is the smallest positive integer  $k$  such that for every pair of vertices  $u$  and  $v$ , there exists a vertex  $w$  such that  $u \xrightarrow{k} w$  and  $v \xrightarrow{k} w$  in  $D$ . In this talk we will present our result on the index set of the scrambling index for all primitive matrices. (Received September 26, 2017)

1135-15-2482 **Shaun Fallat\*** (shaun.fallat@uregina.ca), Department of Mathematics and Statistics, University of Regina, Regina, Sask. s4s 0a2, Canada, and **Karen Meagher, Abolghasem Soltani and Boting Yang**. *Compressed Cliques Graphs and Positive Zero Forcing*.

In connection with positive definite zero forcing, we consider an object called the compressed cliques graph resulting for examination of particular clique coverings. We will discuss a number of properties associated with this graph including uniqueness, forbidden subgraphs, and explore its association with positive definite zero forcing of a graph. (Received September 26, 2017)

1135-15-2534 **Jillian E McLeod\***, US Coast Guard Academy, Mathematics Department, 15 Mohegan Blvd, New London, CT 06320. *Partition Regular Matrices as Cryptographic Primitives*. Preliminary report.

Kernel partition regular matrices are characterized by Rado's *Columns Condition* and have been used to describe combinatorial properties that are preserved under passage to finite partitions. It is known that the problem of determining whether an arbitrary matrix satisfies the columns condition is at least  $NP$ -hard. We investigate possible ways to exploit the difficulty of this problem for purposes of information security. These preliminary explorations indicate that matrices which satisfy the columns condition may in fact make good cryptographic primitives in asymmetric key cryptographic systems. (Received September 26, 2017)

1135-15-2659 **Leonard Stevenson\***, ls879@drexel.edu, and **Dmitry Kaliuzhnyi-Verbovetskyi**. *Integration of noncommutative functions*.

A brief review of noncommutative functions and the difference-differential operator will pave the way for a discussion of the theory of integration (or antiderivation) for zero order nc functions as well as some comments about integrating higher order nc functions. (Received September 26, 2017)

1135-15-2824 **Sima Ahsani\*** (sza0043@auburn.edu), 221 Parker Hall, Auburn University, Auburn, AL 36849. *Inequalities Related to Geometric Mean of Matrices*. Preliminary report.

An  $n \times n$  complex matrix  $A$  is called positive definite if it is hermitian and all its eigenvalues are positive. The set of all positive definite matrices denoted by  $\mathbb{P}_n$  and form a Riemannian manifold. The geodesic connecting  $A, B \in \mathbb{P}_n$ , is  $\gamma(t) = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^tA^{\frac{1}{2}}, t \in [0, 1]$ , and the midpoint of it for  $t = \frac{1}{2}$  is called the geometric mean of  $A$  and  $B$  and denoted as  $A\sharp B$ . Audenaert recently proved that for commuting pairs of positive definite

matrices  $A_i$  and  $B_i, i = 1, \dots, m$  and for any unitary invariant norm  $\|\cdot\|$

$$\left\| \sum_{i=1}^m A_i B_i \right\| \leq \left\| \left( \sum_{i=1}^m A_i^{1/2} B_i^{1/2} \right)^2 \right\| \leq \left\| \left( \sum_{i=1}^m A_i \right) \left( \sum_{i=1}^m B_i \right) \right\|.$$

We will talk about similar inequalities in the non-commuting case when matrix multiplication is replaced with the geometric mean. Also, Numerical counterexamples will be given for some related inequality questions. (Received September 26, 2017)

1135-15-2876 **Brian Camara\*** (bcamara@student.bridgew.edu), 63 Wing Road, Acushnet, MA 02743, and **John Pike**. *Algebraic Voting Theory*.

My research over the summer of 2017 through Bridgewater State University's Adrian Tinsley Program focused on the mathematical theory of voting, its connections with representation theory, and ways to explain some results in this area using linear algebra. Voting is of immense practical significance in our society, so understanding it from a mathematical point of view is of utmost importance. I will begin by discussing a ubiquitous class of voting systems and their connections with representation theory as detailed in a paper by Daugherty, Eustis, Minton, and Orrison. I will then sketch an alternate proof of the main theorem in that paper using less sophisticated mathematics and introduce a novel result of my own. Time permitting, I will comment on an intriguing open problem. (Received September 26, 2017)

1135-15-2961 **Emily Gullerud, aBa Mbirika\*** (mbirika@uwec.edu) and **Rita Post**. *Characteristic polynomials and eigenvalues for a family of tridiagonal real symmetric matrices and a tantalizing connection to Pascal's triangle*. Preliminary report.

We explore the family  $\{A_n\}_{n=1}^{\infty}$  of  $n \times n$  tridiagonal real symmetric matrices with zeroes on the diagonal and ones on the subdiagonal and superdiagonal. After deriving a three-term recurrence relation for the characteristic polynomials of this family, we find a closed form solution. The coefficients of these characteristic polynomials turn out to involve the diagonal entries of Pascal's triangle in an attractively inviting manner. Lastly, we explore a relation between the eigenvalues of various members of the family. More specifically, we give a sufficient condition for when  $\text{spec}(A_m)$  is contained in  $\text{spec}(A_n)$ . (Received September 26, 2017)

1135-15-2980 **Christopher Logan Hambric\*** (clhambric@email.wm.edu), **Chi-Kwong Li** (math.ckli@gmail.com), **Diane Pelejo** (dcpelajo@math.upd.edu.ph) and **Junping Shi** (junpingshi@gmail.com). *Potential Stability of Matrix Sign Patterns*.

The study of stable matrices has several applications in predicting the long-term behavior of a system. We consider matrix sign patterns which have stable realizations in order to find general restrictions on the structure of an  $n$ -by- $n$  stable matrix. In our research, we expand upon previous results regarding the minimum number of non-zero entries necessary for an irreducible  $n$ -by- $n$  matrix to be stable, and time allowing we will discuss the problem of completely characterizing the set of potentially stable 3-by-3 sign patterns. Our approach uses a combination of graph and matrix theory in order to examine the structure and properties of these sign patterns. (Received September 26, 2017)

1135-15-3169 **Charles R Johnson** (crjohn@wm.edu), P.O. Box 8795, Williamsburg, VA 23187, and **Pietro Paparella\*** (pietrop@uw.edu), 18115 Campus Way NE, Bothell, WA 98011. *A new proof of the Gauss-Lucas, Siebeck, and Bôcher-Grace theorems*.

The Gauss-Lucas, Siebeck, and Bôcher-Grace theorems are classical results in the geometry of polynomials. Proofs of these results can be found in many textbooks and articles, but the approaches are seemingly disparate. In this talk, we demonstrate that these theorems can be shown via a simple, single argument utilizing the field of values of a matrix. (Received September 26, 2017)

## 16 ► Associative rings and algebras

1135-16-240 **Nicholas R Baeth\*** (baeth@ucmo.edu) and **Daniel Smertnig**. *A structure theorem for length sets in local quaternion orders*.

If  $R$  is a Noetherian ring, every non-zero-divisor can be written as the product of finitely many irreducible elements. However, this representation may not be unique. If  $r$  is a non-zero-divisor of a ring  $R$ , the length set of  $r$  is  $\mathbf{L}_R(r) = \{n : r = a_1 \cdots a_n \text{ with each } a_i \text{ irreducible}\}$ . Length sets and related invariants provide a measure of how non-unique factorization in  $R$  can be. In this talk we introduce some of these invariants and show that, like classical orders in number fields, even though factorization can be highly non-unique, length sets of elements



in local quaternion orders possess a great deal of structure. In particular we give a structure theorem for unions of sets of lengths in terms of almost arithmetical progressions. (Received August 15, 2017)

1135-16-383 **Jonathan Beardsley\*** ([jbeards1@uw.edu](mailto:jbeards1@uw.edu)). *Toward Derived Hopf-Galois Extensions*. Preliminary report.

I will describe how, in the derived setting, the quotient map  $\hat{\mathbb{Z}}_2 \rightarrow \mathbb{Z}/2\mathbb{Z}$  is a Hopf-Galois extension, with associated Hopf-algebra  $B\mathbb{Z} \simeq S^1$ . Here, the algebra structure is the group structure on  $S^1$  and the coalgebra structure is the diagonal map  $\Delta: S^1 \rightarrow S^1 \times S^1$  (which makes  $S^1$  into a bialgebra, since it is an algebra object in the category of topological coalgebras). In particular, we regard  $\mathbb{Z}/2\mathbb{Z}$  as the quotient of the group action  $\mathbb{Z} \rightarrow \text{Aut}(\mathbb{Z})$  given by  $1 \mapsto -1$ . Describing this in full detail requires methods from stable homotopy theory and the theory of operads, which I will omit. However, the above Hopf-Galois extension suggests that, more generally, given an action of a group  $G$  on a ring  $R$ , we can think of  $R \rightarrow R/G$  as a Hopf-Galois extension with ‘‘Hopf-algebra’’ the space  $BG$ . This is a form of generalized Koszul duality. (Received August 29, 2017)

1135-16-445 **Akira Masuoka\*** ([akira@math.tsukuba.ac.jp](mailto:akira@math.tsukuba.ac.jp)), Ten-ou-dai 1-1-1, Tsukuba, Ibaraki 305-8571. *Survey on Hopf crossed products*.

Hopf crossed products in their fully generalized form were introduced by R. Blattner, M. Cohen and S. Montgomery (1986). I will review the progress of Hopf-algebra Theory related with those crossed products, which includes various examples of closed embeddings of quantum groups as well as the construction of quantized enveloping algebras using 2-cocycle deformation. (Received September 04, 2017)

1135-16-450 **Chelsea Walton\*** ([notlaw@temple.edu](mailto:notlaw@temple.edu)), Philadelphia, PA 19122, **Xingting Wang**, Philadelphia, PA 19122, and **Milen Yakimov**, Baton Rouge, LA. *Poisson geometry of PI elliptic algebras*.

I will present joint work with Xingting Wang and Milen Yakimov on the Poisson geometry and representation theory of various elliptic algebras that are module-finite over their center.

The subject (title, abstract, and coauthors) of this talk are subject to change. (Received September 04, 2017)

1135-16-451 **Adriana Mejia Castano** ([sighana25@gmail.com](mailto:sighana25@gmail.com)), Florianopolis, Brazil, **Susan Montgomery** ([smontgom@usc.edu](mailto:smontgom@usc.edu)), Los Angeles, CA, **Sonia Natale** ([natale@famaf.unc.edu.ar](mailto:natale@famaf.unc.edu.ar)), Cordoba, Argentina, **Maria Vega** ([maria.vega@usma.edu](mailto:maria.vega@usma.edu)), West Point, NY, and **Chelsea Walton\*** ([notlaw@temple.edu](mailto:notlaw@temple.edu)), Philadelphia, PA 19122. *Deformations of semisimple Hopf algebras*.

I will discuss the bi-Galois objects and cocycle deformations of the noncommutative, noncocommutative, semisimple Hopf algebras of odd dimension  $p^3$  and of dimension  $pq^2$ . This is joint work with Adriana Mejia Castano, Susan Montgomery, Sonia Natale, and Maria Vega. (Our project began at the BIRS WINART workshop in March 2016.) (Received September 04, 2017)

1135-16-543 **Jieru Zhu\*** ([jieru.zhu-1@ou.edu](mailto:jieru.zhu-1@ou.edu)), Department of Mathematics, University of Oklahoma, Norman, OK 73019-3103, and **Jonathan Kujawa**, University of Oklahoma. *Presenting cyclotomic Schur algebras*.

A classical result states that the action of  $\mathfrak{gl}(V)$  and the symmetric group on  $d$  letters mutually centralize each other on the  $d$ -fold tensor of  $V$ . If  $V$  admits an action by  $\mathbb{Z}/r\mathbb{Z}$ , it induces an action of the wreath product of  $\mathbb{Z}/r\mathbb{Z}$  and the symmetric group on  $d$  letters. A Levi Lie subalgebra  $\mathfrak{g}$  of  $\mathfrak{gl}(V)$  gives the full centralizer of this action, and we further showed a presentation for the cyclotomic Schur algebra as a quotient of the enveloping algebra of  $\mathfrak{g}$ . This also provides a PBW type basis and a second presentation with idempotent generators. These results extend to the quantum setting and yield similar presentations and a basis for the the cyclotomic  $q$ -Schur algebra. When  $r = 2$ , they become presentations for the Type B hyperoctahedral Schur algebra defined by Richard Green. (Received September 07, 2017)

1135-16-593 **Jason Gaddis\*** ([gaddisj@maimioh.edu](mailto:gaddisj@maimioh.edu)) and **Daniel Rogalski**. *Quivers supporting graded Calabi-Yau algebras*.

A graded Calabi-Yau algebra of global dimension 3 is necessarily the path algebra of a quiver modulo relations determined by a superpotential on the quiver. In this talk, I will present a classification of quivers with two or three vertices that support such algebras under the additional hypothesis that the algebras have finite GK dimension. (Received September 09, 2017)

1135-16-622 **Karin Baur, Eleonore Faber, Sira Gratz, Khrystyna Serhiyenko\*** (khrystyna.serhiyenko@berkeley.edu) and **Gordana Todorov**. *sl<sub>k</sub> friezes from Grassmannians*. Preliminary report.

Conway-Coxeter frieze is a lattice of shifted rows of positive integers satisfying the diamond rule: the determinant of any 2x2 matrix formed by the neighboring entries is 1. It is known that cluster-tilted algebras of type A are in bijection with such friezes. In particular, given an Auslander-Reiten quiver of such algebra  $B$  we can apply the specialized Caldero-Chapoton map to every indecomposable  $B$ -module and obtain a frieze.

Morier-Genoud et al. studied generalized friezes called  $sl_k$  friezes, which are lattices of positive integers where the determinant of any  $k \times k$  matrix is 1. In a similar manner, we investigate how  $sl_k$  friezes can be obtained from cluster categories  $C$  associated to the Grassmannian  $\text{Gr}(k, n)$ . In particular, we determine a finite collection of objects in  $C$  that can be arranged to produce a frieze. If  $\text{Gr}(k, n)$  is of finite type we can view the corresponding frieze inside the Auslander-Reiten quiver of  $C$  such that 2x2 diamonds in a frieze arise from triangles. (Received September 11, 2017)

1135-16-673 **Siân Fryer\*** (sianfryer@math.ucsb.edu), **Tina Kanstrup, Ellen Kirkman, Anne Shepler and Sarah Witherspoon**. *Color Lie rings and PBW deformations of skew group algebras*.

We examine color Lie rings arising from finite groups of diagonal matrices acting linearly on finite dimensional vector spaces, and show that (under certain conditions) their enveloping algebras are quantum Drinfeld orbifold algebras, i.e. PBW deformations of certain skew group algebras. Conversely, every quantum Drinfeld orbifold algebra of a particular type arising from the action of an abelian group can be realized as the universal enveloping algebra of a color Lie ring. Special cases of these results yield more familiar objects: for example, a Lie superalgebra is simply a color Lie ring with only two colors and base ring  $\mathbb{K}$ . This approach lends itself well to direct computation, and many concrete examples will be given. (Received September 12, 2017)

1135-16-747 **Ellen E. Kirkman\*** (kirkman@wfu.edu), Department of Mathematics and Statistics, Wake Forest University, Box 7388, Winston-Salem, NC 27109, and **Kenneth Chan, Chelsea Walton and James J. Zhang**. *The McKay Correspondence for Semisimple Hopf Actions on Regular Graded Algebras*.

Let  $\mathbb{k}$  be an algebraically closed field of characteristic zero. Let  $H$  be a semisimple Hopf algebra acting on an Artin-Schelter regular algebra  $A$  of dimension 2, where  $A$  is a graded  $H$ -module algebra, and the  $H$  action on  $A$  is inner-faithful with trivial homological determinant. We extend many of the results of the classical McKay correspondence, when  $A = \mathbb{k}[u, v]$  and  $G$  is a finite subgroup of  $\text{SL}_2(\mathbb{k})$  acting on  $A$  naturally, to this non(cocommutative) setting. (Received September 13, 2017)

1135-16-848 **Natasha Rozhkovskaya\*** (rozhkovs@math.ksu.edu). *Explicit constructions of Segal-Sugawara vectors*.

The center of an affine vertex algebra at the critical level of a simple finite-dimensional Lie algebra is a commutative algebra. In this talk we will report on the explicit formulas for generators, the Segal-Sugawara vectors, in the case of exceptional Lie algebra of type  $G_2$  (which is a joint work with A. I. Molev and E. Ragoucy). (Received September 15, 2017)

1135-16-864 **Gordana G Todorov\*** (g.todorov@northeastern.edu), **Van Nguyen** (nguyen@hood.edu) and **Shijie Zhu** (zhu.shi@husky.neu.edu). *Preprojective algebras*.

In this talk I will discuss several descriptions of Preprojective Algebras associated to tree-type quivers. In the paper we combine results given by Baer-Geigle-Lenzing, Crawley-Boevey, Ringel, and others, and we show that for any tree-type quiver, these different descriptions of preprojective algebra are all equivalent.

*Preprojective algebras of tree-type quivers*, (with Van Nguyen and Shijie Zhu), arXiv: 1612.01585. (Received September 15, 2017)

1135-16-878 **James J Zhang\*** (zhang@math.washington.edu), Department of Mathematics, Box 354350, Seattle, WA 98195. *Hopf algebras of Gelfand-Kirillov dimension 1 and 2*.

We survey some recent results about infinite dimensional Hopf algebras of GK-dimension one and two. (Received September 15, 2017)

1135-16-907 **Rad Dimitric\***, City University of New York, Department of Mathematics, 1S, Staten Island, NY 10314. *Kappa-Slender Modules*.

We are working within the category  $R\text{Mod}$  of (left)  $R$ -modules. For an arbitrary finite cardinal  $\kappa$ , we define classes of coordinate  $\kappa$ -slender and tailwise  $\kappa$ -slender modules as well as related classes of  $h\kappa$ -modules and initiate a study of these classes. The first two classes are generalizations of the classical notions of slender modules, by way of relaxing the requirement of the image the coordinate vectors or tails to be  $< \kappa$ -generated, instead of finite. (Received September 16, 2017)

1135-16-916 **Daniel Yee\*** ([dyee@fsmail.bradley.edu](mailto:dyee@fsmail.bradley.edu)), 925 N. Rebecca Pl., Peoria, IL 61606. *On the structure of Connected Hopf Algebras containing a Semisimple Lie Algebra*. Preliminary report.

In a Hopf Algebra  $H$ , the vector space of primitive elements  $P(H)$  is naturally a Lie subalgebra of the Lie algebra associated to  $H$ . Since a nontrivial connected Hopf algebra implies  $P(H) \neq 0$ , we study the case where  $P(H)$  is a finite dimensional semisimple Lie algebra and how it effects the structure of the connected Hopf algebra  $H$ . We also compare results with the case when  $P(H)$  is a solvable Lie algebra. (Received September 19, 2017)

1135-16-923 **Miriam Cohen\*** ([mia@math.bgu.ac.il](mailto:mia@math.bgu.ac.il)), Department of Mathematics, Beer Sheva, Israel, and **Sara Westreich** ([swestric@biu.ac.il](mailto:swestric@biu.ac.il)), Department of Management, Bar Ilan University, Ramat Gan, Israel. *From group theory to Hopf algebras via integrals*.

Left coideal subalgebras are the appropriate analogue in Hopf algebras of subgroups in groups. They both have integrals that play a central role in their structure theory and give rise to Harmonic analysis for Hopf algebras. Hopf quotients which are analogues of group quotients are realized by integrals. We use this point of view to give intrinsic definitions of nilpotency and solvability of Hopf algebras. (Received September 17, 2017)

1135-16-1065 **Anne V. Shepler** and **Sarah Witherspoon\*** ([sjw@math.tamu.edu](mailto:sjw@math.tamu.edu)). *Twisted tensor product algebras and resolutions*.

Twisted tensor products generalize tensor products of algebras, semi-direct products with groups, and crossed products with Hopf algebras. We will introduce twisted tensor products and give a general construction of projective resolutions of modules arising from a twisted tensor product of complexes. We will mention several recent applications, ranging from understanding algebraic deformations to determining finiteness properties of cohomology. (Received September 19, 2017)

1135-16-1110 **Jacob Laubacher\*** ([jacob.laubacher@snc.edu](mailto:jacob.laubacher@snc.edu)), 100 Grant Street, De Pere, WI 54115. *Properties of the secondary Hochschild homology*.

It is well known that, in dimension one, Hochschild homology corresponds to Kähler differentials whenever  $A$  is a commutative algebra. In this talk we present a similar result for a generalization of the Hochschild homology, as well as show there is an exact sequence connecting the two homologies in low dimension. (Received September 19, 2017)

1135-16-1132 **Alexandru Chirvasitu\*** ([achirvas@buffalo.edu](mailto:achirvas@buffalo.edu)), Buffalo, NY, and **Issan Patri**, Chennai, India. *Dynamics on quantum groups*.

Compact quantum groups are dual objects to appropriately well-behaved cosemisimple complex Hopf algebras. The talk is aimed at discussing various parallels, in the quantum context, of the rich theory of dynamics and ergodic theory on classical compact groups.

We present notions of inner / outer automorphisms for compact quantum groups. For compact matrix quantum groups (the quantum geometer's analogues of compact Lie groups) it turns out that, just as is the case classically, the inner automorphism group is a compact Lie group and the outer automorphism group is discrete. This has non-trivial implications on the behavior of automorphisms of a compact quantum group when such automorphisms are regarded as permutations of the representation ring of the latter.

We also give examples of compact matrix quantum groups with infinitely-generated fusion rings, in stark contrast with the classical situation. This has links to the invariant theory of finite group actions on free Laurent rings, to be discussed as time permits.

(joint w/ Issan Patri) (Received September 19, 2017)

1135-16-1151 **Kendra E Pleasant\*** ([kendra.pleasant@morgan.edu](mailto:kendra.pleasant@morgan.edu)). *Central Sets Theorem in Adequate Partial Semigroups*.

Let  $S$  be a nonempty set and  $*$  be an associate partial operation on  $S$  in the sense that for all  $x, y, z$  in  $S$  if either  $x*(y*z)$  or  $(x*y)*z$  is defined then so is the other and they are equal. We define the pair  $(S, *)$  to be a partial semigroup. A partial semigroup is said to be *adequate* if for any finite subset,  $F$ , of  $S$  there exists  $x$  in  $S$

such that  $x * y$  is defined for all  $y$  in  $F$ . We will prove the Central Sets Theorem in this new setting. (Received September 19, 2017)

1135-16-1206 **Kenneth L Price\*** ([pricek@uwosh.edu](mailto:pricek@uwosh.edu)), Department of Mathematics, University of Wisconsin Oshkosh, 800 Algoma Boulevard, Oshkosh, WI 4901-8631. *Counting Elementary Gradings of Matrix Rings*. Preliminary report.

The speaker will present an explicit formula for the number of nonequivalent elementary gradings by a finite group on the ring of square matrices of size  $n$  over a field. (Received September 20, 2017)

1135-16-1208 **Van Cat Nguyen** ([nguyen@hood.edu](mailto:nguyen@hood.edu)), Dpartment of Mathematics, Hood College, Frederick, MD 21701, **Xingting Wang\*** ([xingting@temple.edu](mailto:xingting@temple.edu)), Department of Mathematics, Temple University, Philadelphia, PA 19122, and **Sarah Witherspoon** ([sjw@math.tamu.edu](mailto:sjw@math.tamu.edu)), Department of Mathematics, Texas A&M University, Collega station, TX 77843. *Finite generation of cohomology rings of some pointed Hopf algebras in positive characteristic*. Preliminary report.

It is a long standing conjecture that the cohomology ring of a finite-dimensional Hopf algebra is always finitely generated. So far affirmative answers for noncommutative and noncocommutative Hopf algebras are given in a case by case basis. In this talk, over a base field of characteristic  $p > 2$ , we prove the cohomology rings of the bosonization of the rank 2 Nichols algebra of Jordan type over a cyclic group of order  $p$  and their liftings in  $p=3$  are finitely generated. We will apply the twisted tensor product and Anick resolutions to achieve that goal. (Received September 20, 2017)

1135-16-1219 **Holly E Attenborough\***, [attenborough@uwplatt.edu](mailto:attenborough@uwplatt.edu). *Classification of Weakly Azumaya Algebras Using Weak Galois Cohomology*. Preliminary report.

Weak Galois cohomology can be used to classify all  $F$ -algebras  $A$  containing  $B$  such that  $A$  is weakly Azumaya with respect to  $B$ , where  $K/F$  is a finite Galois extension and  $B$  a central simple  $K$ -algebra. There is a one-to-one correspondence between weakly Azumaya algebras with respect to  $B$  and weakly Azumaya algebras with respect to  $\tilde{B}$  for any  $\tilde{B}$  equivalent to  $B$  in the Brauer Group of  $K$ . Moreover, it will be shown that if there exists a weakly Azumaya algebra with respect to  $B$  then there exists a Frobenius algebra  $A$  which is weakly Azumaya with respect to  $B$ .

This paper is an extension of the work on weakly Azumaya algebras by D. Haile and L. Rowen in *Weakly Azumaya Algebras* ( J. Algebra, 250:134-177, 2002). (Received September 20, 2017)

1135-16-1239 **Simon Lentner** ([simon.lentner@uni-hamburg.de](mailto:simon.lentner@uni-hamburg.de)), **Svea Nora Mierach** ([snmierach@mun.ca](mailto:snmierach@mun.ca)), **Christoph Schweigert** ([christoph.schweigert@uni-hamburg.de](mailto:christoph.schweigert@uni-hamburg.de)) and **Yorck Sommerhäuser\*** ([sommerh@mun.ca](mailto:sommerh@mun.ca)). *Hochschild Cohomology and the Modular Group*.

It is a known fact that the modular group acts projectively on the center of a factorizable ribbon Hopf algebra. As the center is the zeroth Hochschild cohomology group, it is natural to ask whether this action can be extended to the higher Hochschild cohomology groups. As we will explain in this talk, this is indeed the case. In fact, we will see that, up to homotopy, the modular group acts projectively on the entire Hochschild cochain complex. (Received September 20, 2017)

1135-16-1274 **Daniel P. Bossaller\*** ([db684513@ohio.edu](mailto:db684513@ohio.edu)) and **Sergio R. Lopez-Permouth**. *Algebras Having Bases Consisting Solely of Strongly Regular Elements*. Preliminary report.

Various recent papers deal with the so-called “invertible algebras”, those algebras over arbitrary (not necessarily commutative) unital rings which have bases that consists solely of invertible elements. Somewhat surprisingly, many familiar algebras satisfy this property, including all finite dimensional algebras over fields other than  $\mathbb{F}_2$ . It is also known precisely which finite-dimensional algebras over  $\mathbb{F}_2$  are invertible. We introduce the concept of a locally invertible algebra,  $A$ , and we show that this property is equivalent to the property that  $A$  has a basis consisting solely of strongly von Neumann regular elements. Among other results, we show that this family of algebras is strictly larger than that of invertible algebras. In particular, we show that it includes all finite dimensional algebras over arbitrary fields, as well as all clean algebras. Most importantly, the new notion opens this type of inquiry to the consideration of non-unital algebras; we will show various examples of non-unital locally invertible algebras and, if time permits, we will will announce current results in our work of determining which Leavitt Path Algebras that have locally invertible bases. (Received September 20, 2017)

1135-16-1375 **Robert Laugwitz\***, Department of Mathematics, Rutgers University, Hill Center for the Mathematical Sciences, 110 Frelinghuysen Rd, Piscataway, NJ 08854-8019. *Comodule algebras and 2-cocycles over the Drinfeld double.*

Given a comodule algebra over a bialgebra, a crossed product with its dual naturally becomes a comodule algebra over the Drinfeld double. As an application, a morphism of second non-abelian cohomology spaces of dually paired Hopf algebras to their Drinfeld double is obtained. These construction generalize to the double bosonizations of Majid. (Received September 21, 2017)

1135-16-1396 **Robert Won\*** (wonrj@wfu.edu) and **Calum Spicer**. *Simple  $\mathbb{Z}$ -graded domains of Gelfand-Kirillov dimension 2.* Preliminary report.

Let  $k$  be an algebraically closed field of characteristic zero and let  $A_1$  be the first Weyl algebra over  $k$ . Smith proved that the category of  $\mathbb{Z}$ -graded  $A_1$ -modules is equivalent to the category of quasicoherent sheaves on a quotient stack. Won extended this result to certain  $\mathbb{Z}$ -graded generalizations of the Weyl algebra. Here, we study simple  $\mathbb{Z}$ -graded domains  $A$  of Gelfand-Kirillov dimension 2. Bell and Rogalski classified these rings up to graded Morita equivalence. We show that the category of  $\mathbb{Z}$ -graded  $A$ -modules is equivalent to the category of quasicoherent sheaves on a quotient stack. (Received September 21, 2017)

1135-16-1409 **Ann E. Rogers\*** (arogers1936@gmail.com), DePaul University, Department of Mathematical Sciences, 2320 N. Kenmore Ave., Chicago, IL 60614. *The Jacobian Conjecture: A survey of some recent results.*

For nearly 80 years, legions of attempts to prove the Jacobian Conjecture have failed. Deceptively simple to state, the Jacobian Conjecture posits that any locally injective polynomial mapping of  $n$ -dimensional complex affine space shall always be globally invertible. Although the Conjecture has been proven for limited cases, it has yet to be solved in general. This survey will include some recent progress towards solving what S. Smale considers to be one of the most important open problems for 21st century mathematicians. Particular attention will be given to work which has proven that the Jacobian Conjecture is equivalent to other statements, including the Dixmier Conjecture, which asserts that any endomorphism of the Weyl algebra  $A_n(k)$  over a field  $k$  of characteristic zero must be an automorphism. (Received September 22, 2017)

1135-16-1440 **Xin Tang\*** (xtang@uncfsu.edu), 1200 Murchison Road, Fayetteville, NC 28301. *Quantum Weyl-Polynomial Algebras and Their Localizations.* Preliminary report.

In this presentation, we present some preliminary results on a family of quantum Weyl-Polynomial algebras, which can be considered as the extensions of both quantum Weyl algebras and quantum polynomial algebras. We will establish some standard properties for these algebras. In particular, we will establish a criterion for the existence of simple localizations and study its implications. We will study the endomorphisms and automorphisms for these algebras and their localizations; as a matter of fact, we will identify some invariants for the purpose of characterizing bijective endomorphisms (i.e. automorphisms). Both the non-root of unity and the root of unity cases will be addressed. In the root of unity case, the method of discriminant is expected to play a critical role. Some generalizations and applications will be discussed as well. (Received September 22, 2017)

1135-16-1467 **Brett Collins\*** (bacwvf@mail.missouri.edu). *Generalized Littlewood-Richardson coefficients for branching rules and extremal weight crystals.*

Littlewood-Richardson coefficients are certain multiplicities that arise naturally in many areas of representation theory and algebraic combinatorics. Derksen and Weyman used quiver invariant theory to identify these coefficients with the dimension of a weight space of semi-invariants. In this talk, I will describe how their methods can be used to give a similar interpretation for the multiplicities of certain branching rules for  $GL_n$  and extremal weight crystals. This allows a proof of their saturation, a polytopal description of the nonzero coefficients, and that their positivity can be determined in polynomial time. (Received September 22, 2017)

1135-16-1599 **Yuri Bahturin\*** ([bahturin@mun.ca](mailto:bahturin@mun.ca)), Department of Mathematics and Statistics, Memorial University of Newfoundland, St. Johns, NL A1C5S7, Canada. *Group Gradings on Matrix Algebras and Actions of Taft Algebras.*

The study of group graded simple algebras is greatly facilitated if we view them as comodules over group algebras or Hopf module algebras over the dual group algebras. In this talk we would like to show how the developed theory of gradings can help to study the actions of Hopf algebras on the matrix algebras. This is joint work with Susan Montgomery. (Received September 23, 2017)

1135-16-1751 **Ellen E. Kirkman\*** ([kirkman@wfu.edu](mailto:kirkman@wfu.edu)), Department of Mathematics and Statistics, Wake Forest University, Box 7388, Winston-Salem, NC 27109. *Reflection Hopf Algebras.* Preliminary report.

The Shephard-Todd-Chevalley Theorem states that when a finite group  $G$  acts linearly on a commutative polynomial ring  $A = k[x_1, \dots, x_n]$  over a field  $k$  of characteristic zero, the invariant subring  $A^G$  is a commutative polynomial ring if and only if  $G$  is generated by reflections. More generally, let  $H$  be a finite dimensional semi-simple Hopf algebra that acts on an Artin-Schelter regular algebra  $A$  so that  $A$  is an  $H$ -module algebra, the grading on  $A$  is preserved, and the action of  $H$  on  $A$  is inner faithful. When  $A^H$  is Artin-Schelter regular, we call  $H$  a reflection Hopf algebra for  $A$ . We present some examples of such pairs  $(A, H)$ . (Received September 24, 2017)

1135-16-1989 **Elizabeth Wicks\*** ([lizwicks@uw.edu](mailto:lizwicks@uw.edu)). *Frobenius-Perron Theory of Modified ADE Bound Quiver Algebras.* Preliminary report.

The Frobenius-Perron dimension for an abelian category was recently introduced. We apply this theory to the category of representations of the finite dimensional radical squared zero algebras associated to certain modified ADE graphs. In particular, we take an ADE quiver with arrows in a certain orientation and an arbitrary number of loops at each vertex. We show that the Frobenius-Perron dimension of this category is equal to the maximum number of loops at a vertex. Along the way, we introduce a lemma which can be applied in general to calculate the Frobenius-Perron dimension of a radical squared zero quiver algebra with loops. (Received September 25, 2017)

1135-16-1997 **Maria D. Vega\*** ([maria.vega@usma.edu](mailto:maria.vega@usma.edu)), West Point, NY 10996. *Cocycle deformations and Galois objects for semisimple Hopf algebras of dimension  $p^3$ .*

Let  $p$  be a prime numbers. In this talk will discuss Galois objects and cocycle deformations of the noncommutative, noncocommutative, semisimple Hopf algebras of odd dimension  $p^3$ . We obtain that the  $p+1$  non-isomorphic self-dual semisimple Hopf algebras of dimension classified by Masuoka have no non-trivial cocycle deformations, extending his previous results for the 8-dimensional Kac-Paljutkin Hopf algebra. This is joint work with A. Castaño, S. Montgomery, S. Natale, and C. Walton. (Received September 25, 2017)

1135-16-2231 **Geoffrey T Glover\*** ([glovergt7775@uwec.edu](mailto:glovergt7775@uwec.edu)), 5708 Curtis St, McFarland, WI 53558, and **Austin Holmes**, **Tennie Jacobson** and **Jared Johnson**. *Determining algebras of Hasse graphs associated with the 600-cell.* Preliminary report.

We can construct a graded algebra associated to a directed Hasse graph of a regular polytope by taking the quotient of the free algebra on the set of edges of the graph by the relations given by equating two directed paths having the same initial and final vertices. Previous work has studied the finite Coxeter groups  $A_n$ ,  $B_n$ ,  $D_n$ ,  $I_2(p)$  and  $H_3$  and their related polytopes. The current goal of our project is to determine the structure of a graded algebra,  $A(\Gamma_{H_4})$ , that is associated to the Hasse graph,  $\Gamma_{H_4}$ , obtained from the 600-cell. The symmetry group of the 600-cell, and thus the automorphism group of the graphs, is isomorphic to the  $H_4$  Coxeter group. For each unique symmetry, we consider the Hasse subgraph consisting of fixed  $k$ -faces of the polytope under the action. From each Hasse subgraph, we determine the graded dimension of the related subalgebras of  $A(\Gamma_{H_4})$  by counting the directed paths between each pair of levels in the graph. We have created programs to produce the generating functions that will in turn describe  $A(\Gamma_{H_4})$  under the action of each symmetry class. This then allows us to describe the complete algebraic structure using representation theory. (Received September 25, 2017)

1135-16-2279 **Luigi Ferraro**, **Ellen Kirkman** and **W. Frank Moore\*** ([moorewf@wfu.edu](mailto:moorewf@wfu.edu)), PO Box 7388, Department of Mathematics, Wake Forest University, Winston-Salem, NC 27109, and **Robert Won**. *Hopf algebra actions on some AS-regular algebras of small dimension.* Preliminary report.

The classical Chevalley-Shephard-Todd Theorem gives a characterization of when a group acting linearly on the commutative polynomial ring has a ring of invariants that is isomorphic to a polynomial ring. Understanding

when group actions (or more generally, Hopf actions) on AS-regular algebras give AS-regular invariant rings has proven to be a difficult problem. In this preliminary report, we provide some new examples of Hopf actions of the algebras  $\mathcal{A}_{4m}$  and  $\mathcal{B}_{4m}$  of Masuoka on some AS-regular algebras such that the ring of invariants is also AS-regular. (Received September 25, 2017)

1135-16-2375 **David C. Meyer, Roberto C. Soto\*** (rcsoto@fullerton.edu) and **Daniel J. Wackwitz**. *Versal Deformation Rings and Symmetric Special Biserial Algebras*.

Let  $k$  be an algebraically closed field and let  $\Lambda$  be a symmetric special biserial algebra over  $k$ . We apply methods from representation theory to study versal deformation rings of  $\Lambda$ -modules, which are complete local commutative Noetherian  $k$ -algebras with residue field  $k$  which can be expressed as a quotient ring of power series algebras over  $k$  in finitely many commuting variables. (Received September 26, 2017)

1135-16-2649 **Igor Klep\*** (igor.klep@auckland.ac.nz), **Špela Špenko** and **Jurij Volčič**. *Positive trace polynomials and the Procesi-Schacher conjecture*.

Positivstellensatz is a fundamental result in real algebraic geometry providing algebraic certificates for positivity of polynomials on semialgebraic sets. In this talk Positivstellensätze for trace polynomials positive on semialgebraic sets of  $n \times n$  matrices are provided. A Krivine-Stengle-type Positivstellensatz is proved characterizing trace polynomials nonnegative on a general semialgebraic set  $K$  using weighted sums of hermitian squares with denominators. The weights in these certificates are obtained from generators of  $K$  and traces of hermitian squares. For compact semialgebraic sets  $K$  Schmüdgen- and Putinar-type Positivstellensätze are obtained: every trace polynomial positive on  $K$  has a sum of hermitian squares decomposition with weights and without denominators. The methods employed are inspired by invariant theory, classical real algebraic geometry and functional analysis. (Received September 26, 2017)

1135-16-2694 **Maria D Vega\*** (maria.vega@usma.edu), West Point, NY 10996. *Categorical twisted Frobenius-Schur indicators*.

Linchenko and Montgomery defined Frobenius-Schur indicators for semisimple Hopf algebras. More recently, they have been generalized to appropriate tensor categories by Mason, Ng, and Schauenberg. In this talk, I will discuss Frobenius-Schur indicators twisted by Hopf automorphisms. For groups, these were introduced by Bump and Ginzburg, building on earlier work of Mackey. I will also explain how the notion can be generalized to pivotal categories. This is joint work with D. Sage. (Received September 26, 2017)

1135-16-2816 **Cris Negron\*** (negronc@mit.edu). *Drinfeld twists of small quantum groups and Belavin-Drinfeld triples*.

I will discuss some recent work on producing, and classifying, Drinfeld twists of small quantum groups. In the case of the (full) small quantum group associated to a simple Lie algebra  $\mathfrak{g}$  over  $\mathbb{C}$ , I will describe how so-called Belavin-Drinfeld triples on the Dynkin diagram of  $\mathfrak{g}$  produce twists of the small quantum group. One can read off the properties of the corresponding twisted algebra from the given Belavin-Drinfeld triple. If time permits I will also discuss a complete classification of Drinfeld twists for quantum Borel algebras. (Received September 26, 2017)

1135-16-2904 **Alexander H Sistko\*** (alexander-sistko@uiowa.edu), 14 MacLean Hall, Iowa City, IA 52242-1419, and **Miodrag C Iovanov**. *Maximal Subalgebras of Finite-Dimensional Algebras*.

We present classification theorems for maximal subalgebras of finite-dimensional algebras over a field  $k$ . This is done by classifying maximal subalgebras of semisimple algebras, and then lifting to the general case. When  $k$  is nice (ex. algebraically closed), the classification can be understood in terms of the ideal structure of the Jacobson radical. For bound quiver algebras, this gives us nice presentations of subalgebras. Trivial extensions and separable extensions feature prominently in the classification, allowing us to relate representation-theoretic properties of an algebra to those of its subalgebras via induction and restriction. If time permits, we discuss potential applications, ex. to determining isomorphism classes of subalgebras, and minimal generating sets of algebras. (Received September 26, 2017)

1135-16-2917 **Yevgenia Kashina\***, Department of Mathematical Sciences, DePaul University, Chicago, IL 60614. *On classification of semisimple Hopf algebras of dimension 32*. Preliminary report.

In this talk we will discuss the classification of semisimple Hopf algebras of dimension 32 with commutative Hopf subalgebra of index 2. These Hopf algebras can be obtained as abelian extensions of a group algebra of dimension 2 and a dual of a group algebra of dimension 16. We will also consider categorical invariants of

these Hopf algebras, such as their Grothendieck ring structures and their (higher) Frobenius-Schur indicators. (Received September 26, 2017)

1135-16-2951 **Grant James Keane\*** ([keanegj2903@uwec.edu](mailto:keanegj2903@uwec.edu)), Grant Keane, Department of Mathematics, 105 Garfield Avenue, Eau Claire, WI 54702, **Haotian Wu** ([wuh6709@uwec.edu](mailto:wuh6709@uwec.edu)), Wu Haotian, Department of Mathematics, 105 Garfield Avenue, Eau Claire, WI 54729, and **Alice Ching** ([chinga1167@uwec.edu](mailto:chinga1167@uwec.edu)), Alice Ching, Department of Mathematics, 105 Garfield Avenue, Eau Claire, WI 54702. *The Moduli Space of 5-dimensional Non Nilpotent Complex Associative Algebras.*

In this talk, we discuss the construction of the moduli space of 5-dimensional complex associative algebras which are not nilpotent. This consists of the equivalence classes of such algebras under isomorphism, and the construction uses a type of bootstrap method, by considering extensions of semisimple algebras by nilpotent algebras of lower dimension. We find that the space is stratified by some simple projective orbifolds of the form  $\mathbb{C}P^1/G$  where  $G$  is a subgroup of the symmetric group  $\Sigma_2$  acting on  $\mathbb{P}^1$  by permuting the projective coordinates. This stratification is consistent with deformation theory in the sense that every deformation of one of these algebras is either a smooth deformation along the stratum, or factors through a jump deformation to another stratum. This verifies a conjecture of Fialowski-Penkava for this moduli space. (Received September 26, 2017)

1135-16-2956 **Tyler Jules Gonzales\*** ([gonzaltj9215@uwec.edu](mailto:gonzaltj9215@uwec.edu)), Tyler Gonzales, Department of Mathematics, 105 Garfield Avenue, Eau Claire, WI 54702, and **Jory Lee Wagner** ([wagnerjl@uwec.edu](mailto:wagnerjl@uwec.edu)), Jory Wagner, Department of Mathematics, 105 Garfield Avenue, Eau Claire, WI 54702. *The Moduli Space of Complex  $\mathbb{Z}_2$ -graded 3|2-dimensional associative algebras.*

$\mathbb{Z}_2$ -graded associative algebras arise both in physics and mathematics. We study here the 3|2-dimensional complex associative algebras, constructing the moduli space of isomorphism classes of such algebras using the notion of extensions of algebras of lower dimension, which means we can use our knowledge of lower dimensional algebras to construct the higher dimensional ones. We also study the deformations of these algebras, which we analyze by computing a special type of deformation called a versal deformation. This moduli space is stratified by some projective orbifolds of a very simple type, and we describe this stratification. (Received September 26, 2017)

1135-16-3059 **Mustafa Hajij**, Department of Computer Science, University of South Florida, Tampa, FL 33647, and **Jesse S. F. Levitt\***, USC Dornsife, Department of Mathematics, Los Angeles, CA 90089. *The colored Jones polynomial as an invariant of  $q$ -Weyl algebras.* Preliminary report.

The colored Jones polynomial is a knot invariant that plays a central role in low dimensional topology. We consider a walk along a braid model of the colored Jones polynomial that was developed by Armond, Huynh and Lê. The walk model gives rise to ordering a word in a  $q$ -Weyl algebra which is addressed and studied from multiple perspectives, with applications to the Jones unknot conjecture. (Received September 26, 2017)

1135-16-3082 **Miodrag C Iovanov\*** ([miodrag-iovannov@uiowa.edu](mailto:miodrag-iovannov@uiowa.edu)), IA. *The tame-wild dichotomy for infinite dimensional algebras and Brauer-Thrall 3 conjectures.* Preliminary report.

A classical result of Drozd which sits at the foundations of representation theory of finite dimensional algebras states that every such algebra is either tame or wild, but not both. Whether this dichotomy holds for infinite dimensional algebras is an important open question, which is formulated more generally in the language of coalgebras, and is conjectured true by D. Simson, who proved this in special cases. We present an approach and solution to this question, as well as to another conjecture called the Brauer Thrall 3 conjecture, on existence of indecomposables of arbitrarily large infinite dimension, also due to Simson. We will present the basic definitions and setup, and give the results; time permitting, we also present some related questions on the representation type of Hopf algebras. (Received September 26, 2017)

1135-16-3175 **Chris A Magyar\*** ([magyarca@uwec.edu](mailto:magyarca@uwec.edu)), 106 Spruce Street, Eau Claire, WI 54703. *Mapping the moduli space of complex graded associative algebras through computer algebra systems.*

The moduli space of algebras on a certain vector space consists of all the isomorphism classes of algebras. We have been studying moduli spaces of complex associative  $\mathbb{Z}_2$ -graded algebras, or algebras that have a decomposition into a direct sum of two subspaces, one called the *even* elements and the other the *odd* elements.



The approach we use to construct all the algebras is to consider extensions of an algebra on a lower dimensional space by an algebra structure on another lower dimensional space. This allows us to build moduli spaces of algebras from the knowledge of the structure of lower dimensional moduli spaces.

We use computer algebra systems to construct these graded algebras through representing them by *codifferentials*. These objects can easily be represented as lists that encode the entire multiplication structure of a particular algebra. Once we have all the codifferentials for a given space, we use code we have developed to calculate each algebra's deformations, thus mapping the moduli space.

Currently we are mapping the moduli space of the  $2|3$  graded space and working on converting our code from Maple to Sage. (Received September 26, 2017)

1135-16-3189 **Bach Nguyen\***, bnguy38@lsu.edu. *Quantum cluster algebra and quantum foldings*. Preliminary report.

As a noncommutative analog of cluster algebra, quantum cluster algebra was defined by Berenstein and Zelevinsky in 2005. Since then, it has been an active research area with important applications such as in the study of canonical bases, combinatorics and representation theory. Recently, Goodearl and Yakimov gave a construction of quantum cluster structure for algebras which are CGL extensions. We apply their theory to the setting of quantum foldings and more general situations than that of CGL extensions. This is a work in progress. (Received September 27, 2017)

1135-16-3210 **Anne V Shepler\***, ashepler@unt.edu, and **Victor Reiner**. *Invariant Theory of Reflection Groups*.

By the Chevalley, Shephard-Todd Theorem, the invariants in a polynomial algebra under the action of a reflection group, real or complex, form their own polynomial algebra. Solomon showed that a similar phenomenon occurs with exterior algebras: the invariants in the exterior algebra of differential forms again form their own exterior algebra. We apply techniques of Gutkin and Opdam on characters to extend Solomon's Theorem in classical invariant theory and examine combinatorial conjectures motivated by W-Catalan combinatorics. Joint work with Victor Reiner. (Received September 27, 2017)

## 17 ► *Nonassociative rings and algebras*

1135-17-67 **Florin Panaite** (florin.panaite@imar.ro), **Paul T. Schrader\*** (stpaul@bgsu.edu) and **Mihai Staic** (mstaic@bgsu.edu). *Hom-Tensor Categories and the Hom-Yang-Baxter Equation*.

It is known that tensor categories provide the appropriate categorical framework for Hopf algebras. *Hom-algebras* (coalgebras) are algebraic structures that satisfy a generalized associativity (coassociativity) condition. In this presentation we introduce a new type of category called a *hom-tensor category* and show how it provides the appropriate setting for the category of modules over a hom-bialgebra. We then study the notion of a *hom-braided category* and argue that this is the right setting for the category of modules over quasitriangular hom-bialgebras. We also discuss how the hom-Yang-Baxter equation fits into this framework. (Received July 18, 2017)

1135-17-320 **Pamela E. Harris\*** (peh2@williams.edu), 33 Stetson Ct, Bascom House Room 106C, Williamstown, MA 01267. *Computing weight multiplicities*.

Central to the study of the representation theory of Lie algebras is the computation of weight multiplicities, which are the dimensions of vector subspaces called weight spaces. The multiplicity of a weight can be computed using a well-known formula of Kostant that consists of an alternating sum over a finite group and involves a partition function. In this talk, we present some recent results related to questions regarding the number of terms contributing nontrivially to Kostant's weight multiplicity formula along with some formulas to compute  $q$ -weight multiplicities for certain finite-dimensional Lie algebras. The work presented is in collaboration with a group of undergraduate research students at Williams College: Kevin Chang, Edward Lauber, Haley Lescinsky, Grace Mabie, Gabriel Ngwe, Cielo Perez, Aesha Siddiqui, and Anthony Simpson. (Received August 23, 2017)

1135-17-363 **Karina H Batistelli\***, khatistelli@gmail.com, and **Carina Boyallian**. *QHWM of the "orthogonal" and "symplectic" types Lie subalgebras of the matrix quantum pseudodifferential operators*.

In this talk we will characterize the irreducible quasifinite highest weight modules of some subalgebras of the Lie algebra of matrix quantum pseudodifferential operators  $N \times N$ .

In order to do this, we will first give a complete description of the anti-involutions that preserve the principal gradation of the algebra of  $N \times N$  matrix quantum pseudodifferential operators and we will describe the Lie subalgebras of its minus fixed points. We will obtain, up to conjugation, two families of anti-involutions that show quite different results when  $n = N$  and  $n < N$ . We will then focus on the study of the “orthogonal” and “symplectic” type subalgebras found for case  $n = N$ , specifically the classification and realization of the quasifinite highest weight modules. (Received August 28, 2017)

1135-17-367 **Corinne A. Manogue\*** ([corinne@physics.oregonstate.edu](mailto:corinne@physics.oregonstate.edu)), Department of Physics, Oregon State University, Corvallis, OR 97331, and **Tevian Dray** ([tevian@math.oregonstate.edu](mailto:tevian@math.oregonstate.edu)), Department of Mathematics, Oregon State University, Corvallis, OR 97331. *Quaternionic Spin*.

The standard Dirac operator in physics involves representations of the Clifford algebra in  $3 + 1$  spacetime dimensions, usually expressed in terms of  $4 \times 4$  complex matrices. Rewriting these representations over the quaternions naturally leads to a formalism that treats the massive and massless Dirac equations on an equal footing, and that naturally lives in  $5 + 1$  spacetime dimensions. Extending to the octonions in turn extends the spacetime dimension to  $9 + 1$ ; remarkably, solutions to the octonionic Dirac equation must be quaternionic. Some applications to physics will be briefly discussed if time permits. (Received August 29, 2017)

1135-17-368 **Lida Bentz** ([blida@math.vt.edu](mailto:blida@math.vt.edu)), Department of Mathematics, Virginia Tech, Blacksburg, VA 24061, and **Tevian Dray\*** ([tevian@math.oregonstate.edu](mailto:tevian@math.oregonstate.edu)), Department of Mathematics, Oregon State University, Corvallis, OR 97331. *Subalgebras of the Split Octonions*.

The proper subalgebras of the octonions  $\mathbb{O}$  are well-known: the reals  $\mathbb{R}$ , the complex numbers  $\mathbb{C}$ , and the quaternions  $\mathbb{H}$ . The Cayley–Dickson process also yields split cousins of these division algebras, denoted  $\mathbb{C}'$ ,  $\mathbb{H}'$ , and  $\mathbb{O}'$ , but each of these algebras admits additional subalgebras that are not, however, Cayley–Dickson algebras. We present a complete classification of such subalgebras, including examples in dimensions 3, 5, and 6. (Received August 29, 2017)

1135-17-677 **Julia Pevtsova\*** ([julia@math.washington.edu](mailto:julia@math.washington.edu)), [julia@math.washington.edu](mailto:julia@math.washington.edu), and **David J Benson, Srikanth B Iyengar and Henning Krause**. *Detecting projectivity of modules and nilpotence in cohomology for finite dimensional (graded) Hopf algebras*. Preliminary report.

For a finite group  $G$ , classical theorems of Quillen and Chouinard tell us how to detect whether a class in mod  $p$  cohomology is nilpotent or whether a module is projective: one has to restrict to elementary abelian subgroups of  $G$ . For connected finite group schemes, the detecting family consists of one-parameter subgroups as shown by Suslin, Friedlander, and Bendel. I’ll give an overview of these results and describe some new developments in this direction for finite supergroup schemes. (Received September 13, 2017)

1135-17-874 **Nehemiah Kuhns\*** ([nehemiahkuhns@my.unt.edu](mailto:nehemiahkuhns@my.unt.edu)). *Uniserial representations of  $\text{Vec}(R)$  with a single Casimir eigenvalue*.

The cohomology classes of  $\text{Vec}(R)$  taking values in homomorphism spaces between tensor density modules were computed by Feigin and Fuchs. The cup products of these classes are also known. In this talk, we solve the cup equation to classify the uniserial (completely indecomposable) extensions of arbitrary length of representations of  $\text{Vec}(R)$  having a single Casimir eigenvalue and weights bounded below. (Received September 15, 2017)

1135-17-1223 **Connor O’Dell\*** ([connorodell@my.unt.edu](mailto:connorodell@my.unt.edu)). *Non-resonant uniserial representations of  $\text{Vec}(\mathbb{R})$* .

The non-resonant bounded uniserial representations of  $\text{Vec}(\mathbb{R})$  form a certain class of infinite dimensional representations, all of whose subquotients are indecomposable. Feigin and Fuks classified all bounded indecomposable extensions of length two, and Conley completed a similar classification in length three. We classify all non-resonant bounded uniserial extensions of  $\text{Vec}(\mathbb{R})$  up to length 6. Beyond this length, all such extensions appear to arise as subquotients of extensions of arbitrary length. Some examples may be explained by the pseudodifferential operator cocycle discovered by Khesin and Kravchenko. (Received September 20, 2017)

1135-17-1323 **S. McKay Sullivan\*** ([mckay.sullivan@dixie.edu](mailto:mckay.sullivan@dixie.edu)) and **Bojko Bakalov**. *Inhomogeneous Supersymmetric Bilinear Forms*.

We consider inhomogeneous supersymmetric bilinear forms, i.e., forms that are neither even nor odd. We provide a classification of an important class of such forms up to dimension seven. We show that these forms can be used to construct a new type of oscillator Lie superalgebra. (Received September 21, 2017)

1135-17-1356 **Irfan Bagci** and **Samuel Chamberlin\***, 8700 NW River Park Dr. Box 30, Parkville, MO 64152. *Integral bases for the universal enveloping algebras of cartan type map superalgebras.*

Let  $\mathfrak{g}$  be a finite dimensional complex simple Lie superalgebra of Cartan type and  $A$  be a commutative associative algebra with unity over  $\mathbb{C}$ . We define an integral form for the universal enveloping superalgebra of the map superalgebra  $\mathfrak{g} \otimes A$ , and exhibit an explicit integral basis for this integral form. (Received September 21, 2017)

1135-17-2412 **Michael Reeks\*** ([mar3nf@virginia.edu](mailto:mar3nf@virginia.edu)) and **Can Oguz**. *Trace of the twisted Heisenberg category.*

Categorification is a process by which structures can be lifted to higher categorical levels, often revealing new information. The original structure can be recovered through an inverse process, decategorification. Decategorification is typically accomplished by taking the Grothendieck group. Several recent works have shown that an alternative decategorification functor, the trace (or zeroth Hochschild homology), can reveal additional rich algebraic structures. In this talk, we shall describe the trace of a categorification of the twisted Heisenberg algebra and connect it to a certain infinite dimensional Lie algebra known as a W-algebra. (Received September 26, 2017)

1135-17-2500 **Lisa Schneider\*** ([lmschneider@salisbury.edu](mailto:lmschneider@salisbury.edu)). *Weight Functions and Multiplicities Associated to Demazure Flags.* Preliminary report.

In this talk, we will present results concerning weighted multiplicities associated to Demazure flags of Demazure modules for the current algebra  $\mathfrak{sl}_2[t]$ . While we know exactly when a Demazure flag of a Demazure module exists in this setting, the appearance of combinatorial objects (in generating functions of multiplicities and specializations of those generating functions) portrays an interesting connection between representation theory and number theory. We will first briefly introduce the notion of a Demazure flag and the associated  $q$ -multiplicities. Then, we will define a weight function and weighted multiplicities. For small levels, we will discuss some known closed forms for generating functions of these weighted multiplicities. Observing patterns in small levels, we will conjecture a generalization of the closed forms as well as relationships to interesting combinatorial objects. (Received September 26, 2017)

1135-17-2735 **Matthew Lee\*** ([mlee@math.ucr.edu](mailto:mlee@math.ucr.edu)). *Global Weyl Modules for Maximal Parabolics of Twisted Affine Lie Algebras.*

In this talk we will discuss the structure of non standard maximal parabolics of twisted affine Lie algebras, global Weyl modules and the associated commutative associative algebra,  $A_\lambda$ . Since the global Weyl modules associated with the standard maximal parabolics have found many applications the hope is that these non-standard maximal parabolics will lead to different, but equally interesting applications. (Received September 26, 2017)

## 18 ► Category theory; homological algebra

1135-18-191 **Emily Riehl\*** ([eriehl@math.jhu.edu](mailto:eriehl@math.jhu.edu)), Department of Mathematics, 3400 N Charles Street, Baltimore, MD 21218, and **Michael Shulman**. *A synthetic theory of  $\infty$ -categories in homotopy type theory.*

Homotopy type theory grew out of a new model for intentional type theory in a category whose objects represent homotopy types or  $\infty$ -groupoids rather than sets and a new axiom asserting that “identity is equivalent to equivalence,” which implies that all constructions are homotopy invariant. In this talk, we propose foundations for a synthetic theory of  $\infty$ -categories in homotopy type theory motivated by a particular model of homotopy type theory, which contains a well-known model of  $\infty$ -categories whose category theory can be developed synthetically — the “Rezk” or “complete Segal spaces.” We introduce simplices to probe the internal categorical structure of types, and define Segal types, in which binary composites exist uniquely up to homotopy, and Rezk types, in which the categorical isomorphisms are equivalent to the type-theoretic identities — a “local univalence” condition. We then develop the synthetic theory of  $\infty$ -categories including functors, natural transformations, co- and contravariant type families with discrete fibers, a “dependent” Yoneda lemma that looks like “directed identity-elimination,” and the theory of coherent adjunctions. (Received September 15, 2017)

1135-18-273 **Jonathan Beardsley** ([jbeards1@uw.edu](mailto:jbeards1@uw.edu)) and **Liang Ze Wong\*** ([wonglz@uw.edu](mailto:wonglz@uw.edu)). *The enriched Grothendieck construction.* Preliminary report.

Fibrations, or fibered categories, were introduced by Grothendieck in order to define stacks and descent theory. They have since found numerous applications outside of algebraic geometry, such as to algebraic topology and type theory. The Grothendieck construction and its inverse show that the category of fibrations over a base

category  $\mathcal{B}$  is equivalent to the category of pseudofunctors  $\mathcal{B}^{op} \rightarrow \mathbf{Cat}$ , also known as graded or indexed categories. In this talk, we develop the theory of fibrations for categories enriched over a semi-cartesian monoidal category  $\mathcal{V}$ , along with enriched versions of the Grothendieck construction and its inverse. We then mention some modifications required when enriching over  $\mathbf{Vect}_k$ , which is not semi-cartesian. (Received August 17, 2017)

1135-18-470 **Joanna Meinel** and **Van C. Nguyen\*** (nguyen@hood.edu), Department of Mathematics, Hood College, Frederick, MD 21701, and **Bregje Pauwels**, **Maria Julia Redondo** and **Andrea Solotar**. *Gerstenhaber structure of a class of special biserial algebras.*

For any integer  $N \geq 1$ , we consider a class of self-injective special biserial algebras  $A_N$  given by quiver and relations over a field  $k$ . We study the Gerstenhaber structure of its Hochschild cohomology ring  $HH^*(A_N)$ . This Hochschild cohomology ring is a finitely generated  $k$ -algebra, due to the results by Snashall and Taillefer. We employ their cohomology computations and Suárez-Álvarez's approach to compute all Gerstenhaber brackets of  $HH^*(A_N)$ . Furthermore, we study the Lie algebra structure of the degree-1 cohomology  $HH^1(A_N)$  as embedded into a direct sum of Virasoro algebras and provide a decomposition of  $HH^n(A_N)$  as a module over  $HH^1(A_N)$ . This joint project was started at the WINART workshop at BIRS in April 2016. (Received September 05, 2017)

1135-18-526 **Anastasios Stefanou\*** (astefanou@albany.edu), **Vin de Silva** (vin.desilva@pomona.edu) and **Elizabeth Munch** (muncheli@egr.msu.edu). *Interleavings on categories with coherent  $[0, \infty)$ -action.*

The interleaving distance is a powerful tool in TDA which has been shown to provide a metric for such topological signatures as persistence diagrams and Reeb graphs. In this talk we generalize the idea of interleavings to a broader class of objects, namely categories with coherent  $[0, \infty)$ -action. This allows us to show that many commonly used distances, such as the  $L_\infty$ -distance and the Hausdorff metric, are in fact special cases of interleaving distances. In addition, there is a natural way to define morphisms between these categories that generalizes the stability results of TDA to a broad class of objects by showing that the morphisms are 1-Lipschitz. (Received September 07, 2017)

1135-18-646 **Bhanumati Dasgupta\*** (bhanumati@msn.com) and **Soura Dasgupta**. *A simple hermitian analogue of the Morita theorems.* Preliminary report.

The two Morita theorems state that a functor between categories of modules over algebras (CMAs) is an equivalence iff it is given by a single object  $P$  where this object with connecting maps forms a Morita context. Frohlich et al generalized it partially for algebras with involutions. Algebras with anti-structures include algebras with involutions. Our theorem claims that a functor between categories of modules with hermitian forms over algebras with antistructure, which agrees with an equivalence between the underlying CMAs on the underlying modules and morphisms, is an equivalence implies it is given by a single object  $(P, \Phi)$  where  $P$  is a module and  $\Phi$  is a non-singular form where this object with connecting maps forms a hermitian Morita context. The converse is Hahn's hermitian analogue of Morita I while ours is a simple hermitian analog of Morita II. It has been an open question since 1985, despite several attempts to solve it. E.g. Verhaege et al, Hernandez. This theorem is important because all such equivalences preserve non-singular, split and hyperbolic modules, give rise to isomorphic Witt groups of the underlying rings, and induce generalized Brauer groups and Azumaya algebras with antistructure (Received September 23, 2017)

1135-18-799 **Julia Plavnik\*** (julia@math.tamu.edu), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77840. *On classification of modular categories by dimension.*

The problem of classifying modular categories is motivated by applications to topological quantum computation as algebraic models for topological phases of matter. These categories have also applications in different areas of mathematics like topological quantum field theory, representation theory, among others.

A complete classification of modular categories seems to be out of reach at the moment. Therefore a lot of efforts are done in advance in the classification of these categories under certain restrictions. Different directions have been considered: classification by rank, and by dimension, by dimension of the simple objects, among others.

In this talk, we will focus on the classification of modular categories by dimension. We will start by introducing some of the basic definitions and properties of these categories. We will also present different examples to understand better the structure and the notion of dimension in this setting.

The idea of the talk is to give a panorama of the current situation of the classification program for modular categories based on their dimension. (Received September 14, 2017)

1135-18-817 **Parsa Bonderson\***, Microsoft Station Q, Santa Barbara, CA 93106, and **Meng Cheng, Roger Mong** and **Alan Tran**. *Fermionic Topological Phases and Modular Transformations*.

The effective theories describing 2+1D bosonic topological phases of matter and their modular transformations has been well-understood in terms of modular tensor categories for many years. In contrast, a similar understanding of 2+1D fermionic topological phases remained unspecified. We detail the algebraic structure of the universal properties of fermionic topological phases, including the braiding statistics of the quasiparticles and vortices. We explain how these are related to the the spinor structures of the physical fermions and modular transformations of the system. (Received September 14, 2017)

1135-18-1199 **Michael Norman Crumley\*** ([crumley@findlay.edu](mailto:crumley@findlay.edu)), The University of Findlay, 1000 N. Main St., Department of Mathematics, Findlay, OH 45840. *Ultraproducts of Tannakian Categories*.

We consider neutral tannakian categories as first-order structures in an appropriately chosen language, and show that while the property of being tannakian is first-order, being neutral is not. We therefore introduce the restricted ultraproduct of neutral tannakian categories, which is itself a neutral tannakian category, and compute its underlying Hopf algebra. It is often then possible to do computations (e.g. cohomological) in the restricted ultraproduct which then hold “almost everywhere” in the factors, leading to generic results in the representation theory of algebraic groups. (Received September 20, 2017)

1135-18-1404 **John M Zhang\*** ([jmzhang@math.ucla.edu](mailto:jmzhang@math.ucla.edu)), UCLA Department of Mathematics, Box 951555, Los Angeles, CA 90095. *An introduction to derivators*.

We introduce the theory of derivators, which can be seen as an enhancement of traditional derived categories. We then discuss how we can use derivators to extend Happel’s result on derived equivalences of certain quivers. (Received September 21, 2017)

1135-18-2052 **Emily Riehl\*** ([eriehl@math.jhu.edu](mailto:eriehl@math.jhu.edu)), Department of Mathematics, Johns Hopkins University, Baltimore, MD 21218, and **Evan Cavallo** and **Christian Sattler**. *On the directed univalence axiom*. Preliminary report.

The bisimplicial sets model of homotopy type theory has, in addition to an interval type  $I$ , a directed interval type 2, which respectively parametrize “paths” and “arrows” inside a general type. Sattler’s *directed univalence axiom* asserts that arrows in the universe are equivalent to spans of types. This talk will contain a progress report on (i) efforts to verify the directed univalence axiom in the bisimplicial sets model and (ii) explorations of its consequences in the type theory for synthetic  $\infty$ -categories introduced by Riehl and Shulman. (Received September 25, 2017)

1135-18-2401 **Peter Schauenburg\***, Institut de Mathématiques de Bourgogne, Faculté des Sciences Mirande, 9 avenue Alain Savary, 21000 Dijon, France, and **Michaël Mignard**. *Modular categories are not determined by their modular data*.

A modular category affords a representation of the modular group on its complexified Grothendieck ring, the S- and T-matrices (also known as the modular data) being the images of two generators of the modular group. The most easily accessible modular categories are perhaps the module categories of the Drinfeld doubles of finite groups, and more generally twisted Drinfeld doubles, which are quasi-Hopf algebras associated to a group and a three-cocycle. The modular data is at the basis of the definition of modular categories; it is central to applications in mathematical physics and low-dimensional topology; from a purely algebraic viewpoint, the modular data are simply an all-important structural element of a modular category, of key importance for example for any effort to classify them. We settle the question whether the modular data might in fact already contain all the information on a modular category: It turns out that this can already fail for the module categories of twisted Drinfeld doubles of finite groups; such categories can share the same modular data without being equivalent as modular categories. Apart from these counterexamples, we will also report on the large number of inequivalent twisted Drinfeld doubles which we classified by computer algebraic methods. (Received September 26, 2017)

1135-18-2414 **John C. Baez, John D. Foley\*** ([foley@metsci.com](mailto:foley@metsci.com)), **Joseph Moeller** and **Blake S. Pollard**. *Modeling the composition of networks with operads*. Preliminary report.

Networks can be combined in many ways including by overlaying one on top of another or sitting one next to another. We encode these two ways of combining networks as a specific kind of functor and prove that the application of a novel general construction to these functors results in typed operads. The class of operads we construct—which we call network operads—contains a wealth of examples whose many to one operations serve as a syntax for designing complex networks by composing simpler networks. We give examples of concrete

ways to compose networks with our setup by matching specific kinds of networks to actions of network operads. Remarkably network operads can provide a unified treatment of the structural design and behavioral tasking of dynamic networks. (Received September 26, 2017)

1135-18-2553 **Henry J Tucker\*** ([hjtucker@ucsd.edu](mailto:hjtucker@ucsd.edu)), La Jolla, CA. *Extra special fusion categories*. Preliminary report.

The objects of fusion categories generalize properties of complex representations of finite groups. Tambara and Yamagami classified a large family of fusion categories defined by having only one non-invertible object (i.e. dimension 1) and being  $\mathbb{Z}/2$ -graded with the goal of distinguishing the representation categories of the dihedral group of order 8 and the quaternion group. It was shown later by Evans and Gannon that if a category of group representations is to be Tambara-Yamagami then the group must be an extra-special 2-group. The present work is motivated similarly; we wish to understand the fusion categories whose fusion rule is like that for extra special 3-groups. This family instead has 2 non-invertible objects and is  $\mathbb{Z}/3$ -graded. We begin this classification program by establishing a classification of fusion categories with fusion rules like the complex representations of the Heisenberg group over the finite field of order 3. We also consider the case where the group of invertible objects in the category forms the Klein four-group; an example of such a fusion category comes from the theory of conformal inclusions, however no such fusion category can be realized by group representations. This work is joint with Hans Wenzl and Daniel Copeland. (Received September 26, 2017)

1135-18-2707 **Daniel Creamer\*** ([dan1010c@math.tamu.edu](mailto:dan1010c@math.tamu.edu)), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843, and **Eric Rowell**. *Computational Approach to Classifying Rank 6 Modular Categories*. Preliminary report.

Each modular category has a pair of matrices, called the modular data, associated to it. We use the known relations of the modular data to classify the possible modular data of all rank six modular categories. (Received September 26, 2017)

1135-18-2839 **Paul Bruillard, Cesar Galindo, Siu-Hung Ng, Julia Plavnik\*** ([julia@math.tamu.edu](mailto:julia@math.tamu.edu)), **Eric Rowell** and **Zhenghan Wang**. *On the classification of super-modular categories by rank*.

Super-modular categories are unitary premodular categories that are almost modular. These categories are important for both mathematical and physical reasons. For example, they are used to model fermionic topological phases of matter. Also, the general structure of premodular categories is reduced to that of modular or super-modular categories, which is another motivation to study super-modular categories.

In this talk, we will start by introducing the basic definitions, examples, and properties of super-modular categories. We will mention the similarities and differences between the theory of super-modular and modular categories. We will also present the classification of super-modular categories up to rank 6, and if time allows, we will mention some of the techniques used to pursue this classification. (Received September 26, 2017)

1135-18-2884 **Paige North\*** ([paigenorth@gmail.com](mailto:paigenorth@gmail.com)). *Weak factorization systems as models of dependent type theory*. Preliminary report.

In this talk, I will describe the foundational relationship between homotopy theory and dependent type theory: that is, I will describe those (non-functional, non-algebraic) weak factorization systems which form display map categories with fairly weak  $\Sigma$ ,  $\Pi$ , and  $\text{Id}$  types. If time permits, I will also touch on the relationship between weak factorization systems and comprehension categories. (Received September 26, 2017)

1135-18-2972 **Kathleen Lee\*** ([klee6@poets.whittier.edu](mailto:klee6@poets.whittier.edu)), 10117 Gunn Ave, Apt B1, Whittier, CA 90605. *On classification of (weakly integral) modular categories by dimension*.

We look at classing strictly weakly integral modular categories of dimension  $4q^2$  and  $2^5$ . Motivation to classify these categories comes from their importance in various fields of mathematics, including topological quantum field theory, conformal field theory, representation theory of quantum groups, vertex operator algebras and applications in physics. To classify these categories we look at the possible number of invertible object and then look at the trivial component  $\mathcal{C}_{ad}$ . From there we can look at the cases for the non-integral component and classify the possibilities. (Received September 26, 2017)

1135-18-2990 **Marc Keilberg\*** ([keilberg@usc.edu](mailto:keilberg@usc.edu)). *Investigating invariants of, and new categories obtained from,  $\text{Rep}(D(G))$  via change of braidings*.

The fusion category  $\text{Rep}(D(G))$ , where  $D(G)$  is the Drinfeld double of the finite group  $G$ , admits canonical braidings which are well-studied and known to yield factorizable modular categories. We describe all other possible braidings in group theoretical terms, and determine when they yield factorizable modular categories.

We then consider the question of when such braidings determine the same modular data as the canonical braidings. This can happen in non-trivial ways, and we demonstrate that it is possible for a braiding to determine inequivalent modular data. In this fashion we obtain new examples of factorizable modular categories.

We conclude by applying the results to a few invariants which can be expressed in terms of the modular data. This yields a number of new identities for the invariants, as well as new relations among the modular data and fusion ring. (Received September 26, 2017)

## 19 ► *K-theory*

1135-19-176 **Seth Baldwin\*** ([seth.baldwin@unc.edu](mailto:seth.baldwin@unc.edu)) and **Shrawan Kumar**. *Positivity in  $T$ -equivariant  $K$ -theory of flag varieties associated to Kac-Moody groups II.*

We prove sign-alternation of the structure constants in the basis of the structure sheaves of opposite Schubert varieties in the torus-equivariant Grothendieck group of coherent sheaves on the flag varieties  $G/P$  associated to an arbitrary symmetrizable Kac-Moody group  $G$ , where  $P$  is any parabolic subgroup. This generalizes the work of Anderson-Griffeth-Miller from the finite case to the general Kac-Moody case, and affirmatively answers a conjecture of Lam-Schilling-Shimozono regarding the signs of the structure constants in the case of the affine Grassmannian. (Received August 07, 2017)

## 20 ► *Group theory and generalizations*

1135-20-69 **Ruth Charney\*** ([charney@brandeis.edu](mailto:charney@brandeis.edu)), Department of Mathematics, MS 050, Brandeis University, 415 South St., Waltham, MA 02453. *Searching for Hyperbolicity.*

While groups are defined as algebraic objects, they can also be viewed as symmetries of geometric objects. This viewpoint gives rise to powerful tools for studying infinite groups. The work of Max Dehn in the early 20th century on groups acting on the hyperbolic plane was an early indication of this phenomenon. In the 1980's, Dehn's ideas were vastly generalized by Mikhail Gromov to a large class of groups, now known as hyperbolic groups. In recent years there has been an effort to push these ideas even further. If a group fails to be hyperbolic, might it still display some hyperbolic behavior? Might some of the techniques used in hyperbolic geometry still apply? The talk will begin with an introduction to some basic ideas in geometric group theory and Gromov's notion of hyperbolicity, and conclude with a discussion of recent work on finding and encoding hyperbolic behavior in more general groups. (Received July 18, 2017)

1135-20-170 **Keith Jones** and **Gregory A. Kelsey\*** ([gkelsey@bellarmine.edu](mailto:gkelsey@bellarmine.edu)). *The Star Geometry of Diestel-Leader Groups.*

Given a bordified space, Karlsson defines an incidence geometry of stars at infinity. These stars and their incidence are closely related to well-understood objects when the space is hyperbolic,  $CAT(0)$ , or a bounded convex domain with the Hilbert metric. It is not clear what star geometries are possible when the space is a finitely generated group that is not negatively-curved. We compute the star geometry of the Diestel-Leader graph that is the Cayley graph of the lamplighter group bordified by its horofunction boundary. We also discuss the generalization of this result to other Diestel-Leader groups. (Received August 07, 2017)

1135-20-295 **Joseph Kirtland\*** ([joe.kirtland@marist.edu](mailto:joe.kirtland@marist.edu)), Department of Mathematics, 3399 North Road, Poughkeepsie, NY 12601. *Finite Groups with a Frattini Subgroup Property Satisfied by Nilpotent Groups.* Preliminary report.

Given a finite nilpotent group  $G$  and a Sylow  $p$ -subgroup  $P$  of  $G$ , then  $P \cap \Phi(G) = \Phi(P)$ . This talk will investigate finite groups that share this property. (Received August 21, 2017)

1135-20-332 **Laxmi K Chataut\*** ([laxmi.chataut@uwc.edu](mailto:laxmi.chataut@uwc.edu)) and **Martyn R Dixon** ([mdixon@ua.edu](mailto:mdixon@ua.edu)). *Groups with the weak maximal condition on non-permutable subgroups.*

Let  $H$  be a subgroup of a group  $G$ . Then  $H$  is said to be *permutable* if it permutes with every subgroup of  $G$ , that is,  $HK = KH$  for every subgroup  $K$  of  $G$ . Let  $\mathcal{P}$  be a subgroup theoretical property or class of groups, then  $\bar{\mathcal{P}}$  is the class of all groups that either are not- $\mathcal{P}$  groups or are trivial. A group  $G$  is said to satisfy the *weak maximal condition on  $\mathcal{P}$ -subgroups* (denoted by  $\max\text{-}\infty\text{-}\mathcal{P}$ ) if for every ascending chain  $H_1 < H_2 < H_3 < \dots < H_n < \dots$  of  $\mathcal{P}$  subgroups of  $G$ ,  $|H_{i+1} : H_i|$  is infinite for only finitely many  $i$ . Thus, for example, on letting  $\mathcal{P}$  denote the class of permutable subgroups, we may speak of groups satisfy  $\max\text{-}\infty\text{-}\bar{\mathcal{P}}$ , the weak maximal condition on non-permutable subgroups. Groups with this property are the subject of our interest. The main result is; Let  $G$

be a group with the weak maximal condition on non-permutable subgroups. We prove that if  $G$  is a generalized radical group then  $G$  is either quasihamiltonian or a soluble-by-finite minimax group. (Received August 24, 2017)

1135-20-342 **Samuel DeHority, Xavier Gonzalez, Neekon Vafa and Roger Van Peski\*** ([rpeski@princeton.edu](mailto:rpeski@princeton.edu)), 3792 Frist Center, Princeton University, Princeton, NJ 08544.  
*Moonshine for all finite groups.*

In recent literature, moonshine has been explored for some groups beyond the Monster, for example the sporadic O’Nan and Thompson groups. This collection of examples may suggest that moonshine is a rare phenomenon, but a fundamental and largely unexplored question is how general the correspondence is between modular forms and finite groups. For every finite group  $G$ , we give constructions of infinitely many graded infinite-dimensional  $\mathbb{C}[G]$ -modules where the McKay-Thompson series for a conjugacy class  $[g]$  is a weakly holomorphic modular function properly on  $\Gamma_0(\text{ord}(g))$ . As there are only finitely many normalized Hauptmoduln, groups whose McKay-Thompson series are normalized Hauptmoduln are rare, but not as rare as one might naively expect. We give bounds on the powers of primes dividing the order of groups which have normalized Hauptmoduln of level  $\text{ord}(g)$  as the graded trace functions for any conjugacy class  $[g]$ , and completely classify the finite abelian groups with this property. In particular, these include  $(\mathbb{Z}/5\mathbb{Z})^5$  and  $(\mathbb{Z}/7\mathbb{Z})^4$ , which are not subgroups of the Monster. (Received August 26, 2017)

1135-20-344 **Sahana Hassan Balasubramanya\*** ([hbsahana@gmail.com](mailto:hbsahana@gmail.com)). *Acylindrical group actions on quasi-trees.*

A group  $G$  is acylindrically hyperbolic if it admits a non-elementary acylindrical action on a hyperbolic space. This class is broad enough to include many examples of interest, e.g., non-elementary hyperbolic and relatively hyperbolic groups, most mapping class groups, most fundamental groups of 3-manifolds,  $\text{Out}(F_n)$ , etc. One of the goals of my research was to answer the following : Which groups admit non-elementary cobounded acylindrical actions on quasi-trees ? (By a quasi-tree I mean a connected graph quasi-isometric to a tree, which form a subclass of hyperbolic spaces.)

I prove that every acylindrically hyperbolic group  $G$  has a generating set  $X$  such that the corresponding Cayley graph  $\Gamma$  is a non-elementary quasi-tree and the action of  $G$  on  $\Gamma$  is acylindrical. The proof utilizes the notions of hyperbolically embedded subgroups and projection complexes. As an application, I obtain some new results about hyperbolically embedded subgroups and quasi-convex subgroups of acylindrically hyperbolic groups. (Received August 26, 2017)

1135-20-391 **Michael Hull and Ilya Kapovich\*** ([kapovich@math.uiuc.edu](mailto:kapovich@math.uiuc.edu)), 1409 West Green Street, UIUC Department of Mathematics, Urbana, IL 61801. *Counting conjugacy classes in  $\text{Out}(F_N)$ .*

We show that if a f.g. group  $G$  has a non-elementary WPD action on a hyperbolic metric space  $X$ , then the number of  $G$ -conjugacy classes of  $X$ -loxodromic elements of  $G$  coming from a ball of radius  $R$  in the Cayley graph of  $G$  grows exponentially in  $R$ . As an application we prove that for  $N \geq 3$  the number of distinct  $\text{Out}(F_N)$ -conjugacy classes of fully irreducibles  $\phi$  from an  $R$ -ball in the Cayley graph of  $\text{Out}(F_N)$  with  $\log \lambda(\phi)$  on the order of  $R$  grows exponentially in  $R$ . Here  $\lambda(\phi)$  is the dilatation or the stretch factor of  $\phi$ . (Received August 30, 2017)

1135-20-422 **David Pengeley\*** ([davidp@nmsu.edu](mailto:davidp@nmsu.edu)), Mathematics, Oregon State University, Corvallis, OR 97331, and **Daniel Ramras** ([dramras@iupui.edu](mailto:dramras@iupui.edu)), Mathematics, Indiana University - Purdue University Indian, Indianapolis, IN 46202. *A quaternionic unraveling of the double-twist in three-space.*

Physicists and mathematicians have long known it is possible to unravel a double-twist in three space, embodied in motions like the Dirac belt trick, Feynman plate trick, or Philippine candle dance. Quaternions can reveal how efficiently and beautifully this can be done, providing both theoretical constraints on the minimal required complexity, and insights into the geometry and level of simplicity possible. You will emerge knowing how to perform the quaternionic unraveling with your hand. (Received September 01, 2017)

1135-20-428 **Charles A Matthews\*** ([cmatthews@se.edu](mailto:cmatthews@se.edu)). *The Permutations Project.*

It is well-known that the disjoint cycle decomposition of the permutation  $(12 \cdots n)^k$  consists of cycles all of the same length  $n/\text{gcd}(n, k)$ . It has been proven that the disjoint cycle decomposition of the permutation  $(12 \cdots n_1)^{k_1} \cdots (12 \cdots n_g)^{k_g}$  consists of cycles of at most  $g$  distinct lengths, but the proof involves the topology of multiple curves on a surface of genus  $g$  along with the use of the Euler Characteristic, and no explicit formula for the cycle lengths is known when  $g > 1$ . Southeastern began a project with faculty and undergraduate students



from mathematics and computer science to find explicit formulas for the multiplicities  $m_1, m_2, \dots, m_g$  and the lengths  $\ell_1, \ell_2, \dots, \ell_g$  such that the disjoint cycle decomposition of the permutation  $(12 \cdots n_1)^{k_1} \cdots (12 \cdots n_g)^{k_g}$  consists of  $m_i$  cycles of length  $\ell_i$  for  $1 \leq i \leq g$ . This open problem is quickly accessible by undergraduate students at every level, and our students have discovered and proven some new theorems and stated about fifty conjectures still waiting to be proven. Methods have included linear algebra, number theory, graph theory, abstract algebra, programming with Mathematica, and supercomputing. (Received September 01, 2017)

1135-20-563 **MurphyKate Montee\*** ([murphykate.montee@gmail.com](mailto:murphykate.montee@gmail.com)). *Random Groups and Cube Complexes.*

In 1993, Gromov introduced the density model of random groups. In this model, we are interested in groups with relators of equal length  $\ell$ . The number of relators is determined by the density  $d$ . Properties satisfied by such groups with probability 1 as  $\ell$  tends to infinity are said to hold with overwhelming probability. In this model, many properties exhibit a phase shift at a specific density. For example, random groups are, with overwhelming probability, infinite hyperbolic for  $d < \frac{1}{2}$  and trivial for  $d > \frac{1}{2}$ . Recent work of Ollivier-Wise and Mackay-Przytycki has shown that, with overwhelming probability, random groups at density  $d < \frac{5}{24}$  admit a non-trivial action on a CAT(0) cube complex. On the other hand, Żuk and Kotowski-Kotowski have shown that, with overwhelming probability, at density  $d > \frac{1}{3}$ , a random group satisfies Property (T). This provides an upper bound on the maximal density at which a random group admits a non-trivial action on a CAT(0) cube complex. Recent work hopes to improve the lower bound to  $d < \frac{1}{4}$ . (Received September 08, 2017)

1135-20-660 **Eduard Einstein\*** ([ece02006@gmail.com](mailto:ece02006@gmail.com)), 310 Malott Hall, Ithaca, NY 14853. *Hierarchies for Relatively Hyperbolic Virtually Special Groups.* Preliminary report.

Wise's malnormal special quotient theorem (MSQT) is a key ingredient in Agol's proof of the Virtual Haken Conjecture. The most important step in proving the MSQT is the construction of a hierarchy for hyperbolic compact special non-positively curved cube complexes. In this talk, I will explain how to extend these results from the hyperbolic setting to the relatively hyperbolic setting. (Received September 12, 2017)

1135-20-666 **Martino Garonzi** and **Luise-Charlotte Kappe\***, [menger@math.binghamton.edu](mailto:menger@math.binghamton.edu), and **Eric Swartz.** *On covering numbers of groups.*

A set of proper subgroups is a cover for a group if its union is the whole group. The minimal number of subgroups needed to cover a group is called its covering number. No group is the union of two proper subgroups. Tomkinson showed that the covering number of a solvable group has the form prime-power-plus-one and for each such integer there exists a solvable group having this integer as a covering number. In addition he showed that 7 is not a covering number. So far it has been shown that the integers  $< 27$ , which are not covering numbers, are 2, 7, 11, 19, 21, 22 and 25. We extend this list by determining all integers  $< 129$  which are covering numbers. (Received September 12, 2017)

1135-20-718 **Brendan Burns Healy\*** ([burns.healy@tufts.edu](mailto:burns.healy@tufts.edu)), 503 Boston Avenue, Medford, MA 02155. *Acyindrical Hyperbolicity and CAT(0) Groups.*

Acyindrical hyperbolicity makes up a broad class of groups classically studied using geometric group theory. We take tools that have been developed for these groups and apply them to the class of CAT(0) groups to answer the question: "When is a group simultaneously acylindrically hyperbolic and CAT(0)? In doing so, we obtain a result about the kinds of CAT(0) spaces acylindrically hyperbolic groups can act on. We also give some applications. (Received September 13, 2017)

1135-20-769 **Daniel Groves\*** ([groves@math.uic.edu](mailto:groves@math.uic.edu)), **Jason Fox Manning** and **Alessandro Sisto.** *Boundaries of relatively hyperbolic groups under Dehn filling.*

We begin an investigation into the behavior of Bowitch and Gromov boundaries under the operation of Dehn filling. In particular, we show many Dehn fillings of a toral relatively hyperbolic group with 2-sphere boundary are hyperbolic with 2-sphere boundary. As an application, we show that the Cannon Conjecture implies a relatively hyperbolic version of the Cannon Conjecture. (Received September 14, 2017)

1135-20-900 **Matt Larson\*** ([matthew.larson@yale.edu](mailto:matthew.larson@yale.edu)). *Power maps in finite groups.*

Motivated by algorithmic number theory, Pomerance and Shparlinski have obtained results on the number of cycles in the functional graph of the map  $x \mapsto x^a$  in  $\mathbb{F}_p^*$ . We prove similar results for other families of finite groups. We also show that the cyclic group of order  $n$  minimizes the number of cycles among all nilpotent groups of order  $n$ . (Received September 16, 2017)

1135-20-1055 **Avantha Kodithuwakku\*** ([kaa0006@auburn.edu](mailto:kaa0006@auburn.edu)), Department of Mathematics, Auburn University, Auburn, AL 36849. *Structure of Symmetry Classes of Tensors Associated with Characters of a Group.*

Certain properties and the structure of symmetry classes of tensors associated with different types of characters of a finite group will be discussed, emphasizing the importance of idempotent property of certain symmetrizers in studying the symmetry classes. Furthermore I will also discuss the ongoing work on geometry of standard symmetrized tensors. (Received September 19, 2017)

1135-20-1204 **Arturo Magidin\*** ([magidin@louisiana.edu](mailto:magidin@louisiana.edu)), Department of Mathematics, University of Louisiana at Lafayette, P.O. Box 43568, Lafayette, LA 70504-3568, and **Martha L.H. Kilpack**, Department of Mathematics, 275 TMCB Brigham Young University, Provo, UT 84602. *The lattice of algebraic closure operators on an infinite subgroup lattice.*

We consider the lattice of algebraic closure operators on the lattice of all subgroups of an infinite group  $G$ , and ask when this is isomorphic to the lattice of subgroups of some group  $K$ . We had previously proven that in the finite case, this occurs if and only if  $G$  is cyclic of prime power order. We show that in the infinite case, this occurs if and only if  $G$  is the Prüfer  $p$ -group. (Received September 20, 2017)

1135-20-1205 **Matt Sunderland\*** ([msunderland@gradcenter.cuny.edu](mailto:msunderland@gradcenter.cuny.edu)), 365 Fifth Avenue, New York, NY 10016. *Linear Progress in Weakly Hyperbolic Groups.*

A random walk  $w_n$  on a separable, geodesic hyperbolic metric space  $X$  converges to the boundary  $\partial X$  with probability one when the step distribution supports two independent loxodromics. In particular, the random walk makes positive linear progress:

$$\liminf_{n \rightarrow \infty} \frac{1}{n} d_X(x_0, w_n x_0) > 0 \text{ almost surely.}$$

When (1) the step distribution has exponential tail and (2) the action on  $X$  is acylindrical, it is known that progress is linear with exponential decay, *i.e.*, there exists  $C > 0$  such that for all integers  $n > 0$ ,

$$\mathbb{P}(d_X(x_0, w_n x_0) \leq n/C) \leq C e^{-n/C}.$$

We extend the exponential decay result to the non-acylindrical case. (Received September 20, 2017)

1135-20-1245 **Derrick Wigglesworth\*** ([diggles@math.utah.edu](mailto:diggles@math.utah.edu)) and **Radhika Gupta**. *Loxodromics for the cyclic splitting complex.*

The outer automorphism group of a finite rank free group,  $\text{Out}(F_n)$ , has strong parallels with the mapping class group of a closed surface and with  $\text{SL}_n(\mathbb{Z})$ . In this talk, I will build on this analogy, discussing some recent results concerning the cyclic splitting complex, which can be thought of as an  $\text{Out}(F_n)$  analog for the curve complex. In particular, I will discuss recent work with R. Gupta in which we use the boundary of the cyclic splitting complex to identify the loxodromic elements for this action. We will also discuss some results which indicate that this action may be acylindrical. (Received September 20, 2017)

1135-20-1248 **Robert M Guralnick\*** ([guralnic@usc.edu](mailto:guralnic@usc.edu)), 3620 S. Vermont Ave, Department of Mathematics, University of Southern California, Los Angeles, CA 90089-2532. *Quivers, Modules and Group Theory.* Preliminary report.

We will discuss several problems in group theory in which module theory from noncommutative rings and quiver theory play a crucial role. We will particularly discuss the problem of classifying closed irreducible subgroups of  $\text{GL}_n(k)$ ,  $k$  algebraically closed which have only finitely many orbits and obtaining an analogue of the singular value decomposition for matrices over finite fields. (Received September 20, 2017)

1135-20-1290 **Catherine Pfaff\*** ([cpfaff@math.ucsb.edu](mailto:cpfaff@math.ucsb.edu)) and **Daryl Cooper**. *Compactifying Teichmüller Space Using the Right Trousers.*

We introduce a new method for assigning to a marked hyperbolic metric on a closed surface  $S$  of genus  $g \geq 2$  a measured foliation on  $S$ . These foliations provide a new naturality to the compactification of the Teichmüller space  $\mathcal{T}(S)$  by projective measured foliations and allow one to decompose  $\mathcal{T}(S)$  into copies of the Teichmüller space of a once-punctured torus. (Received September 21, 2017)

1135-20-1304 **Robert Kropholler, Ian J. Leary and Ignat Soroko\*** ([ignat.soroko@ou.edu](mailto:ignat.soroko@ou.edu)). *Uncountably many quasi-isometry classes of groups of type FP.*

An interplay between algebra and topology goes in many ways. Given a space  $X$ , we can study its homology and homotopy groups. In the other direction, given a group  $G$ , we can form its Eilenberg-MacLane space  $K(G, 1)$ . It is natural to wish that it is “small” in some sense. If  $K(G, 1)$  space has  $n$ -skeleton with finitely many cells, then  $G$  is said to have type  $F_n$ . Such groups act naturally on the cellular chain complex of the universal cover for

$K(G, 1)$ , which has finitely generated free modules in all dimensions up to  $n$ . On the other hand, if the group ring  $ZG$  has a projective resolution  $(P_i)$  of length  $n$  where each module  $P_i$  is finitely generated, then  $G$  is said to have type  $FP_n$ . There have been many intriguing questions on whether classes  $F_n$  and  $FP_n$  are different, and some of them are still open. Bestvina and Brady gave first examples of groups of type  $FP_2$  which are not finitely presentable (i.e. not of type  $F_2$ ). In his recent paper, Ian Leary has produced uncountably many of such groups. Using Bowditch's concept of taut loops in Cayley graphs, we show that Ian Leary's groups actually form uncountably many classes up to quasi-isometry. (Received September 21, 2017)

1135-20-1351 **Talia Fernós\*** ([t\\_fernos@uncg.edu](mailto:t_fernos@uncg.edu)). *CAT(0) Cube Complexes and their Boundaries.*

The universe of CAT(0) cube complexes is rich and diverse thanks to the ease by which they can be constructed and the variety of natural structures they admit. As a consequence, they admit several boundaries, such as the visual boundary, the Roller boundary, and Hagen's simplicial boundary. In this talk we will discuss some relationships between these boundaries, together with the boundaries of groups acting nicely on them. (Received September 21, 2017)

1135-20-1377 **Geoffrey – Mason\*** ([gem@ucsc.edu](mailto:gem@ucsc.edu)). *New Families of Quasihopf algebras and tensor Categories.*

The twisted double of a finite group is a construction that has been very influential in both orbifold conformal field theory and Hopf algebra theory. This is mainly because such objects have a module category that is modular. In this talk we will present a generalization of the twisted quantum double whose module category is also modular, and explain the group-theoretic background, which is cohomological in nature. We describe a number of examples, which give rise to new classes of quasihopf algebras and modular tensor categories. (Joint work with Siu-Hung Ng.) (Received September 21, 2017)

1135-20-1382 **Emily J Evans\*** ([ejevans@mathematics.byu.edu](mailto:ejevans@mathematics.byu.edu)), 275 TMCB, Provo, UT 84602, and **Wayne Barrett** and **Amanda E Francis**. *Resistance Distance in Linear 2-Trees.*

We consider the problem of determining the maximal resistance distance, also called effective resistance, on the family of linear 2-trees. In particular we explicitly determine the maximal resistance distance on a graph  $G_n$  with vertex set  $V(G) = \{1, 2, \dots, n\}$  and  $\{i, j\} \in E(G_n)$  if and only if  $0 < |i - j| \leq 2$ . We obtain an explicit formula for the resistance distance  $r_n(j, k)$  between any two vertices  $j$  and  $k$  of  $G_n$ . To our knowledge  $\{G_n\}_{n=3}^\infty$  is the first nontrivial family with diameter going to  $\infty$  for which all resistance distances have been explicitly calculated. Moreover, we show that  $r_{n+1}(1, n+1) - r_n(1, n) \rightarrow \frac{1}{5}$  as  $n \rightarrow \infty$ , establishing that  $r_n(1, n) \rightarrow \infty$  as  $n \rightarrow \infty$ . We also obtain similar results for linear 2-trees with different vertex-edge structures but also have the diameter going to  $\infty$ . This gives preliminary evidence that resistance distance may yet be a viable method for link prediction, machine learning, etc. for certain subclasses of geometric graphs. (Received September 21, 2017)

1135-20-1465 **Hoang Thanh Nguyen\*** ([nguyen36@uwm.edu](mailto:nguyen36@uwm.edu)). *Distortion of surfaces in 3-manifolds.* Preliminary report.

We give a large scale geometric classification of immersed surfaces in 3-manifolds by using distortion of the fundamental group of an immersed surface in the fundamental group of a 3-manifold. (Received September 22, 2017)

1135-20-1685 **Ruth Charney, Matt Cordes** and **Devin Murray\***, [dmurray@brandeis.edu](mailto:dmurray@brandeis.edu). *Quasi-Möbius maps on the Morse boundary.*

The Morse boundary,  $\partial_* X$  is a quasi-isometry invariant for proper metric spaces, and thus can be used to distinguish non quasi-isometric groups. An interesting question to explore is, when can the Morse boundary serve as a tool to show two groups are quasi-isometric? In analogy with the hyperbolic manifold case, it is possible to define quasi-Möbius structures on the Morse boundary. Quasi-Möbius maps give an answer to this question. Given two proper metric spaces with non-empty Morse boundary, say  $X$  and  $Y$ , there is a quasi-Möbius map  $h : \partial_* X \rightarrow \partial_* Y$  if and only if  $f$  extends to a quasi-isometry  $f : X \rightarrow Y$ . (Received September 24, 2017)

1135-20-1806 **Christopher Cashen** and **Pallavi Dani\***, Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803, and **Anne Thomas**. *The quasi-isometry classification of certain hyperbolic right-angled Coxeter groups.*

A fundamental question in geometric group theory is that of classifying finitely generated groups up to quasi-isometry. When one is considering hyperbolic groups, a useful tool for this purpose is the (Gromov) boundary,

which is a quasi-isometry invariant. I will describe what has been established about the quasi-isometry classification of hyperbolic right-angled Coxeter groups through studying boundaries. I will focus on recent progress made jointly with C. Cashen and A. Thomas. (Received September 24, 2017)

1135-20-1871 **Bena Tshishiku\*** ([bona@math.harvard.edu](mailto:bona@math.harvard.edu)) and **Genevieve Walsh**. *Groups with Bowditch boundary a 2-sphere.*

Bestvina-Mess showed that the duality properties of a group are encoded in any boundary that gives a Z-set compactification; for example, a hyperbolic group with Gromov boundary an n-sphere is a PD(n+1) group. For relatively hyperbolic pairs (G,P), the natural boundary – the Bowditch boundary – does not give a Z-set compactification. Nevertheless we show that if the Bowditch boundary of (G,P) is a 2-sphere, then (G,P) is a PD(3) pair. This is joint work with Genevieve Walsh. (Received September 25, 2017)

1135-20-1877 **Bena Tshishiku\*** ([bona@math.harvard.edu](mailto:bona@math.harvard.edu)) and **Nick Salter**. *The lifting problem for braid groups.*

The mapping class group Mod(S) of a surface S is the group of diffeomorphisms of S, modulo isotopy. The Nielsen realization problem asks, “Can a given subgroup  $G < \text{Mod}(S)$  be lifted to the diffeomorphism group?” The answer is “yes” for finite G (by work of Kerckhoff) and “no” for  $G = \text{Mod}(S)$  (by work of Morita). For most infinite  $G < \text{Mod}(S)$ , we don’t know. I will discuss a special case of this problem, where G is the braid group. This is joint work with Nick Salter. (Received September 25, 2017)

1135-20-1904 **Risto Atanasov\*** ([ratanasov@email.wcu.edu](mailto:ratanasov@email.wcu.edu)). *Nilpotency class of powerful p-subgroups.* Preliminary report.

A finite p-group G is called *powerful* if either p is odd and  $[G, G] \subseteq G^p$  or  $p = 2$  and  $[G, G] \subseteq G^4$ . We will discuss results that bound the nilpotency class of a powerful p-group in terms of the exponent of a quotient by a normal subgroup. This is a joint work with Ilir Snopce and Slobodan Tanushevski. (Received September 25, 2017)

1135-20-2026 **Masayoshi Kaneda\*** ([mkaneda@uci.edu](mailto:mkaneda@uci.edu)), American University of Kuwait, P.O. Box 3323, Safat, 13034 Kuwait, Kuwait. *Structure and a duality of binary operations on monoids and groups.*

In this paper we introduce novel views of monoids and groups. More specifically, for a given set S, let  $S^{S \times S}$  be the set of binary operations on S. We equip  $S^{S \times S}$  with canonical binary operations induced by the elements of S. Let  $S_{mn}^{S \times S}$  (respectively,  $S_{gr}^{S \times S}$ ) be the set of binary operations that make S monoids (respectively, groups). Then we have the following “duality”: for each  $z \in S_{mn}^{S \times S}$  a certain subset of  $S^{S \times S}$ , denoted by  $S_z^*$ , is a monoid with a canonical binary operation and is isomorphic to  $(S, z)$ . If  $z \in S_{gr}^{S \times S}$ , then  $S_z^{S \times S}$  can be partitioned into copies of  $S_z^*$ . We also give a new characterization of group binary operations which distinguishes them from the other binary operations. These results give us new insights into monoids and groups, and will provide new tools and directions in studying these objects. (Received September 25, 2017)

1135-20-2174 **Dan Rossi\*** ([drossi@math.arizona.edu](mailto:drossi@math.arizona.edu)), 617 N Santa Rita Ave, Tucson, AZ 85721. *Fields of Character Values in Finite Groups.*

I will discuss two results involving fields of character values in finite groups. The first relates the number of rational-valued irreducible characters and the number of rational conjugacy classes in a finite group. The second, for any prime p and any field F, relates the F-valued p-Brauer characters and the p-regular F-conjugacy classes (those classes whose value at any p-Brauer character lies in F) of G to those of  $G/N$ , for certain normal subgroups N of the finite group G. These results extend work of Navarro-Tiep and Isaacs-Navarro, respectively. (Received September 25, 2017)

1135-20-2214 **Carolyn Abbott\***, [c\\_abbott@math.berkeley.edu](mailto:c_abbott@math.berkeley.edu), and **Francois Dahmani**. *Property  $P_{naive}$  for acylindrically hyperbolic groups.*

We show that under mild hypotheses, an acylindrically hyperbolically group satisfies a strong ping-pong condition called property  $P_{naive}$ . Roughly speaking, this property allows one to construct many free subgroups in such a group. I will describe this property and discuss various consequences of it. This is joint work with Francois Dahmani. (Received September 25, 2017)

1135-20-2226 **Bryan Jacobson\*** ([bryan.j.jacobson@vanderbilt.edu](mailto:bryan.j.jacobson@vanderbilt.edu)). *Algebraic subgroups of acylindrically hyperbolic groups.*

A subgroup of a group G is called algebraic if it can be expressed as a finite union of solution sets to systems of equations. We prove that a non-elementary subgroup H of an acylindrically hyperbolic group G is algebraic if and only if there exists a finite subgroup K of G such that  $C_G(K) \leq H \leq N_G(K)$ . We provide some applications of

this result to free products, torsion-free relatively hyperbolic groups, and ascending chains of algebraic subgroups in acylindrically hyperbolic groups. (Received September 25, 2017)

1135-20-2426 **Eric Carson Rowell\*** (rowell@math.tamu.edu). *Braiding Necklaces and Loops.*

Unitary representations of the braid group arise as quantum gates in the topological model for quantum computation via particle exchange in 2-dimensional phases of matter. In 3 spacial dimensions, exchanging point-like particles is not very promising from this perspective since the topology of  $\mathbb{R}^3$  with points deleted is not interesting. However loop-like excitations can have non-trivial exchange statistics. I will describe some recent explorations on representations of the motion groups of free loops as well as loops confined concentrically to an auxiliary loop in  $\mathbb{R}^3$ . This is based on joint work with several sets of authors. (Received September 26, 2017)

1135-20-2448 **Nathan Corwin, Gili Golan, Susan Hermiller, Ashley Johnson\*** (ajohnson18@una.edu) and **Zoran Sunic.** *An Algorithmic Property of Thompson's Group  $F$ .*

The algorithmic property autostackable was introduced in 2014 by Brittenham, Hermiller and Holt as a way to solve the word problem, find normal forms and build Van Kampen diagrams. The class of autostackable groups is known to include, to list a few, all groups with finite complete rewriting systems, all groups with an asynchronously automatic structure with a prefix-closed normal form set, a non- $FP_3$  group, and the fundamental group of every closed 3-manifold. Thompson's Group  $F$ , introduced in 1965 by Richard Thompson, is the set of orientation-preserving, piecewise linear homeomorphisms of the closed unit interval with slopes of the form  $2^n$  for  $n \in \mathbb{Z}$  and finitely many break points at dyadic rational numbers. In this talk, we will discuss the autostackability of Thompson's Group  $F$ . (Received September 26, 2017)

1135-20-2599 **Kropholler\***, Department of Mathematics, Bromfield-Pearson Hall, 503 Boston Avenue, Medford, MA 02155, and **Gilman** and **Schleimer.** *Groups whose word problem is not multiple context free.*

We study the class of groups with multiple context free word problem (MCF). This contains the class of context free groups which, by work of Lyndon and Schupp, is equivalent to the class of virtually free groups.

In recent work Salvati has shown that  $\mathbb{Z}^2$  is in MCF. This result has been recently strengthened by Meng-Che "Turbo" Ho to show that all finitely generated abelian groups are in MCF.

The class is also closed under taking subgroups, finite index overgroups, and free products.

We show that this class is not closed under direct products by considering which RAAGs are in the class. We also show that hyperbolic 3-manifold groups are not in the class and that the only virtually nilpotent groups in the class are virtually abelian. (Received September 26, 2017)

1135-20-2652 **Kim Ruane\*** (kim.ruane@tufts.edu), 503 Boston Avenue, Department of Mathematics, Tufts University, Medford, MA 02155, and **Chris Hruska.** *Semistability of  $CAT(0)$  IFP Groups.*

We will discuss how to show  $CAT(0)$  groups with the Isolated Flats Property (IFP) are semistable at infinity. It is an open question as to whether all finitely presented groups are semistable at infinity - it is even open as to whether all  $CAT(0)$  groups are satisfy this property. This result is a corollary of arecent theorem concerning topology of the boundaries of  $CAT(0)$  IFP groups (joint work with C. Hruska). (Received September 26, 2017)

1135-20-2704 **Gabrielle Melamed\*** (gmelamed@hawaii.edu), **Austin Wei** (abw@22014@mymail.pomona.edu) and **Jonathan Pham** (jonatdp1@uci.edu). *An extension of cleaning for the Dessin d'Enfant.* Preliminary report.

The Dessin d'Enfant (hereafter dessin) is in correspondence with permutation triples  $(\sigma_0, \sigma_1, \sigma_\infty) \in S_n^3$  where  $\sigma_0\sigma_1\sigma_\infty = \text{Id}$  and the subgroup  $\langle \sigma_0, \sigma_1 \rangle \leq S_n$  is transitive: this group is called the monodromy group (hereafter MG). We define the composition of two dessins, by inserting a copy of the first dessin in place of each edge in the second dessin. The cleaning of a dessin is a canonical example of this composition process in which all original white vertices are colored black and a white vertex is inserted for each edge so that each new white vertex has degree two.

The MG of the cleaned dessin embeds nicely into the group  $G \wr C_2$  where  $G$  is the MG of the pre-cleaned dessin.

We define and study a generalization of cleaning, called  $k$ -cleaning, which similarly induces a nice embedding into the group  $G \wr C_k$ . In particular we show that given a dessin with monodromy group  $G \geq A_n$  the MG of the corresponding  $k$ -cleaned dessin must contain  $A_n \wr C_k$  for  $k > 3$ , and  $k > 2$  provided that  $|\sigma_0| \neq |\sigma_1|$ . This result is used to compute the MG for the two families of trees of diameter six with passport size 1. (Received September 26, 2017)

1135-20-2732 **Stephen M Gagola, Jr\*** ([gagola@math.kent.edu](mailto:gagola@math.kent.edu)), Department of Mathematics, Kent State University, 1300 University Esplanade, Kent, OH 44242. *When the mapping from characters to kernels is injective.* Preliminary report.

Let  $G$  be a finite group. We completely characterize those groups for which the mapping  $\chi \mapsto \ker \chi$ , from irreducible characters to normal subgroups, is injective. Also considered is the situation when this mapping is injective when restricted to non-linear characters. (Received September 26, 2017)

1135-20-2772 **Saikat Das\*** ([saikat.das@rutgers.edu](mailto:saikat.das@rutgers.edu)), 101 Warren Street, Room 216, Newark, NJ 07003. *Coarse Geometric Properties of  $\text{Out}(G)$ .*

Given  $G = A_1 * A_2 * \dots * A_n$ , a finite free product of finite groups; we are interested in understanding the coarse geometric properties of  $\text{Out}(G) := \text{Aut}(G)/\text{Inn}(G)$ . One such coarse geometric property is relative hyperbolicity of  $\text{Out}(G)$ .

$\text{Out}(G)$  acts geometrically on the spine of the deformation space of  $G$ -trees. Guirardel and Levitt showed that the deformation space is contractible and so is its spine. So, I am also interested in investigating the coarse geometric properties of the spine of the deformation space.

Behrstock, Drutu and Mosher showed that an algebraically thick group is non relatively hyperbolic. In our context, if the spine of the deformation space is metrically thick then  $\text{Out}(G)$  is non relatively hyperbolic.

In this presentation we will investigate the algebraic thickness of  $\text{Out}(G)$  and the metric thickness of the spine of the deformation space of  $G$ -trees to understand relative hyperbolicity of  $\text{Out}(G)$  (Received September 26, 2017)

1135-20-3042 **Anisah Nabilah Nu'Man\*** ([anuman@ursinus.edu](mailto:anuman@ursinus.edu)). *Tame Filling Functions for Diestel-Leader Groups.* Preliminary report.

Tame filling functions for finitely presented groups are a quasi-isometry invariant refinement of the intrinsic diameter (isodiametric) function and the extrinsic diameter function. In this talk we will discuss intrinsic tame filling functions for the family of groups  $\Gamma_d(q)$  whose Cayley graphs, with respect to a given generating set  $S_{d,q}$ , are the Diestel-Leader graphs  $\text{DL}_d(q)$ . (Received September 26, 2017)

1135-20-3200 **Zachary Gates\*** ([zg8bf@virginia.edu](mailto:zg8bf@virginia.edu)). *Finite presentability of Kac-Moody groups over finite fields.*

Kac-Moody groups are groups associated to a class of infinite-dimensional Lie algebras and exist over any ground field. They come with an associated Weyl group and hence Coxeter diagram. A Kac-Moody group  $\$G\$$  over  $\$F_q\$$  is always finitely generated, so it is natural to ask if it is also finitely presented. If all labels in the Coxeter diagram are finite, then Abramenko and Mühlherr showed that  $\$G\$$  is finitely presented with a few exceptions if  $\$q=2\$$  or  $\$3\$$ . It has been conjectured that  $\$G\$$  is never finitely presented if there is one infinite label in the Coxeter diagram. We show that this is the case for at least a few classes of such diagrams by utilizing a theorem of Gandini and choosing an appropriate space on which we let  $\$G\$$  act. (Received September 27, 2017)

## 22 ► Topological groups, Lie groups

1135-22-298 **Firas Y Hindeleh\*** ([hindelef@gvsu.edu](mailto:hindelef@gvsu.edu)), 1 Campus Dr, Allendale, MI 49401. *On the Classification of Seven Dimensional solvable Lie Algebras With six-dimensional Nilradical.* Preliminary report.

Low dimensional solvable Lie Algebras were completely classified up to dimension six. A general theorem asserts that if  $g$  is a solvable Lie Algebra of dimension  $n$ , then the dimension of its nilradical is at least  $\frac{n}{2}$ . For the seven dimensional algebras, the nilradical's dimension could be 4, 5, 6 or 7. The four and seven-dimensional nilradical cases were classified. We examine the six-dimensional nilradical case. We first looked for the six-dimensional nilpotent algebras and found 32 algebras. In this talk, we give an update on our progress with this project and shed light on the latest six completed sub-cases. (Received August 21, 2017)

1135-22-415 **Bob Palais\*** ([bob.palais@uvu.edu](mailto:bob.palais@uvu.edu)), 800 W University Parkway, Orem, UT 84058. *Computational advantages and historical insights from viewing quaternion multiplication as geodesic vector addition on  $S^2$ .*

We will develop geodesic vector addition on the ordinary two-sphere,  $S^2$ , a group operation isomorphic to unit quaternion multiplication. This description offers insight into algorithms that implement and interpolate rotations, the belt, plate, and tangle tricks from physics, and several constructions from Lie groups. We will use this geometric perspective to understand the algebraic structure of the product and the mathematical inevitability of its discovery in Rodrigues' 1840 paper on  $SO(3)$  as a continuous group. (Received September 01, 2017)

1135-22-447 **Peter Loth\*** (lothp@sacredheart.edu), Sacred Heart University, Department of Mathematics, 5151 Park Avenue, Fairfield, CT 06825. *Invariants of the duals of Warfield groups*. Preliminary report.

Warfield groups are direct summands of simply presented abelian groups or, alternatively, are abelian groups possessing a nice decomposition basis with simply presented cokernel. They were classified up to isomorphism in terms of cardinal invariants by Warfield in the local case, and by Stanton and Hunter-Richman in the global case. In this talk, we classify up to topological isomorphism the (compact) Pontrjagin duals of Warfield groups, dualizing Stanton's invariants. (Received September 04, 2017)

1135-22-1010 **Rachelle C DeCoste** and **Lisa DeMeyer\*** (demey11a@cmich.edu), Department of Mathematics, Pearce 214, Central Michigan University, Mount Pleasant, MI 48858, and **Meera Mainkar**. *Geodesics properties of two-step nilmanifolds constructed from graphs*.

Dani and Mainkar (2005) introduced a method for constructing a simply connected 2-step nilpotent Lie group  $N$  from a simple directed graph  $G$ . The construction gives rise to a natural left-invariant metric on  $N$ . Working with the corresponding metric Lie algebra, we will discuss joint work on geometric properties of these groups including geodesic properties, and describe the interplay between the graph  $G$  and the group  $N$ . Following results of Mast (1994), Eberlein (1994) and Lee-Park (1996), we will discuss results on the density of closed geodesics in the compact quotient  $N$  by a lattice, where  $N$  is constructed from a simple graph. (Received September 18, 2017)

1135-22-1103 **Meera Mainkar\*** (maink1m@cmich.edu) and **Jonas Dere**. *Anosov diffeomorphisms on infra-nilmanifolds associated to graphs*. Preliminary report.

Anosov diffeomorphisms play an important and beautiful role in dynamics as the notion give examples of structurally stable dynamical systems. The only known examples of Anosov diffeomorphisms are on infra-nilmanifolds which can be thought as a non-abelian generalization of tori. In this talk we will consider Anosov diffeomorphisms on a class of infra-nilmanifolds constructed from simply connected two-step nilpotent Lie groups associated with simple graphs. We give a full characterization of these type of infra-nilmanifolds admitting Anosov diffeomorphisms. We will also discuss some new examples. This work is still in progress and is joint with Jonas Deré. (Received September 19, 2017)

1135-22-1485 **Johnathan R Marquardt\*** (marquardtjohnathan@gmail.com), 322 Marble street, Joliet, IL 60435. *A Partial Characterization of the Symmetric Spaces of the Unitary Group of Degree Two and Three*. Preliminary report.

This work presents a characterization of specific symmetric spaces of the 2nd degree unitary group,  $U(2)$ , as well as a characterization of all matrices in  $U(2)$ . Moreover, the generalized and extended symmetric spaces of  $U(2)$  given by inner automorphisms are characterized. Structure and conditions for unitary matrices to be in these symmetric spaces are established through a parameterization of  $U(2)$ . This work is contributes to the ongoing process of characterizing these symmetric spaces. (Received September 22, 2017)

1135-22-2167 **Jonathan Epstein\*** (jepstein@math.ou.edu). *When does a solvable Lie group fail to admit a maximally symmetric metric?* Preliminary report.

The existence of maximally symmetric left-invariant metrics on a Lie group has strong connections to the existence of distinguished metrics, e.g. Ricci soliton and Einstein. After recalling the definition of a maximally symmetric metric and some known results about their existence, we will discuss conditions under which a solvable Lie group fails to admit such metrics. (Received September 25, 2017)

## 26 ► Real functions

1135-26-18 **George A Anastassiou\*** (ganastss@memphis.edu), Dept. Mathematical Sciences, 3725 Norriswood Avenue, Memphis, TN 38016. *Principles of General Fractional Analysis for Banach space valued functions*. Preliminary report.

Here we present a general fractional analysis theory for Banach space valued functions of real domain. A series of general Taylor formulae with Bochner integral remainder is presented. We discuss the continuity of general Riemann-Liouville Bochner fractional integrals and we prove their semigroup property. Then we introduce the right and left generalized Banach space valued fractional derivatives and we establish the corresponding fractional Taylor formulae with Bochner integral remainders. Furthermore we study the iterated generalized left and right fractional derivatives and we establish Taylor formulae for the case, and we find interesting Bochner integral representation formulae for them. We study the differentiation of the left and right Riemann-Liouville

fractional Bochner integrals. At the end we give Ostrowski type inequalities on this general setting, plus other interesting applications. (Received June 14, 2017)

1135-26-1419 **Laramie Paxton\*** (realtimemath@gmail.com) and **Kevin Vixie** (vixie@speakeasy.net). *Moving from Simple Analysis to Research through Exploration.*

The focus of this talk is to start with a rather simple analysis problem and explore the different directions that it can take us, in the process demonstrating how these kinds of explorations can bring us to the edge of research-level problems. Suppose that for  $f : \mathbb{R} \rightarrow \mathbb{R}$ , we have  $|f'(x)| \leq \lambda|f(x)|$  for some  $\lambda > 0$  and for all  $x$ ;  $f$  is continuous and differentiable; and  $f(0) = 0$ . We will show that  $f(x) = 0$  everywhere, and after exploring the one-dimensional version, we will generalize the problem to  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$  and to the case in which  $f$  is merely Lipschitz continuous (which implies that  $f$  is only differentiable almost everywhere). Lastly, we will examine this problem from a more intuitive geometric perspective and briefly discuss our current research direction stemming from this simple problem. (Received September 22, 2017)

1135-26-1486 **Jun Tao\*** (jtao68@yahoo.com), 14207 Eagle Mine Dr, Poway, CA 92064. *Proof of the Formula for Arc Length in a New Way.*

We approximate the length of a curve using tangent lines that touch the curve at an arbitrary point and get the approximation formula  $L \approx \sum_{i=1}^n \sqrt{1 + [f'(x_i^*)]^2} \Delta x$ , where  $x_i^*$  represents an arbitrary point in the  $i$ th subinterval. Then we prove  $L = \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{1 + [f'(x_i^*)]^2} \Delta x$ . Since  $x_i^*$  is an arbitrary point, the proved formula fully satisfies the requirement of the definition of a definite integral and can be converted into the formula  $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$ .  $L = \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{1 + [f'(x_i^*)]^2} \Delta x$  is a generic formula and covers the formula  $L = \lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{1 + [f'(x_i^c)]^2} \Delta x$ , where  $x_i^c$  represents a certain point, derived by approximating the length of a curve using secant lines. (Received September 22, 2017)

1135-26-1886 **Eze R Nwaeze\***, 560 Perry St., Apt. 113, Auburn, AL 36830. *Generalized weighted trapezoid and Grüss type inequalities on time scales.*

In this talk, some new generalized weighted trapezoid and Grüss type inequalities on time scales, with a parameter function  $\psi : [0, 1] \rightarrow [0, 1]$ , are presented. Our results give a broader generalization of the results due to Pachpatte. In addition, we discuss the cases when the time scale is  $\mathbb{R}$  and  $\mathbb{Z}$ . (Received September 25, 2017)

1135-26-3081 **Fernando Lopez-Garcia\*** (fal@cpp.edu). *Orthogonal decomposition of functions and applications.*

Sobolev spaces are fundamental to study the existence and uniqueness of variational solutions of differential equations. In particular, the validity of the so-called Korn inequality, which is strongly related to the geometry of the domain where the functions are defined, is basic in the analysis of the linear elasticity equations. In this talk, we will describe a local-to-global technique to transfer the validity of certain Korn type inequalities from cubes to more general domains (John domains). This technique is based on the possibility of decomposing an arbitrary function  $f$ , orthogonal to the Kernel of the linearized strain tensor, into the sum of a collection of functions, with the same property, supported on Whitney cubes. (Received September 26, 2017)

## 28 ► *Measure and integration*

1135-28-1060 **Dong Hyun Cho\*** (j94385@kyonggi.ac.kr), Department of Mathematics, Kyonggi University, Suwon, Kyonggido 16227, South Korea. *A measurable function similar to the Itô integral on a generalized analogue of Wiener space.*

Let  $C[0, T]$  denote a generalized analogue of Wiener space, that is, the space of continuous real-valued functions on the interval  $[0, T]$ . On the space  $C[0, T]$ , we introduce a finite measure  $w_{\alpha, \beta; \varphi}$  with its properties, where  $\varphi$  is an arbitrary finite measure on the Borel class of  $\mathbb{R}$ . Using the measure  $w_{\alpha, \beta; \varphi}$ , we also introduce a measurable function on  $C[0, T]$  which is similar to the Itô integral. We also establish the fact that if  $\varphi(\mathbb{R}) = 1$ , then  $w_{\alpha, \beta; \varphi}$  is a probability measure with the mean function  $\alpha$  and the variance function  $\beta$ , and the measurable function is reduced to the Paley-Wiener-Zygmund integral on the space  $C[0, T]$ . As an application of the integral, we derive a generalized Paley-Wiener-Zygmund theorem which is useful to calculate the integrals on  $C[0, T]$ . (Received September 19, 2017)



1135-28-1418 **Laramie Paxton\*** (realtimemath@gmail.com) and **Kevin R. Vixie**. *A Singular Integral as a Boundary Measure*.

Singular integrals comprise a rich area of analysis, the most well known example being the Hilbert Transform. In this talk, we will discuss a singular integral that also intersects geometric measure theory. For functions  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  that are  $C^{1,1}$  (i.e. the first derivative is Lipschitz continuous), for which 0 is a regular value (i.e. the gradient  $\nabla f$  does not vanish on the 0-level set), and whose 0-level set is bounded, there is a not too hard proof that our singular integral computes  $\mathcal{H}^{n-1}(\{f^{-1}(0)\})$ , the  $(n-1)$ -dimensional Hausdorff measure of the 0-level set of  $f$ . We will also briefly mention less regular 0-level sets for which this result holds – for example, distance functions for sets of finite perimeter. (Received September 22, 2017)

1135-28-2385 **Linda A. Ness\*** (nesslinda@gmail.com), 154 S Alward Ave, Basking Ridge, NJ 07920. *Exploiting Algorithmizable Mathematical Structure Theorems To Represent Data*. Preliminary report.

A fundamental issue in data science is reproducibility of analytical results. Exploiting algorithmizable mathematical structure theorems to represent data promises to improve reproducibility. In this short talk I'll present this idea by illustrating use of the Singular Value Decomposition Theorem from Linear Algebra to represent multi-scale geometric structure and use of a Lemma originally due to Fefferman, Kenig and Pipher to represent data as multiscale non-parametric measures. Since this second representation is a recursive binary tree structure it can be easily accessible to undergraduates. I'll summarize examples of undergraduate and short masters level projects and a recent project with a Women in Science of Data and Mathematics group. (Received September 26, 2017)

1135-28-2437 **Bobby Wilson\*** (blwilson@mit.edu), Department of Mathematics, 77 Massachusetts Ave., Simons Building (Building 2), Room 2-171, Cambridge, MA 02139. *Orthogonal Projections and Regularity of Sets*.

We will discuss the interplay between two concepts regarding sets with dimension strictly less than that of the ambient vector space: the qualitative properties of the projections of sets and the rectifiability of those same sets. The history of these connections date back to the work of Besicovitch in 1928. We will focus on work linking the theory of projections to infinite-dimensional Banach space geometry and probability. (Received September 26, 2017)

## 30 ► *Functions of a complex variable*

1135-30-48 **Stephen M Deterding\*** (sde243@g.uky.edu). *Bounded point derivations on  $R(X)$  and  $R^p(X)$* .

Let  $X$  be a compact subset of the complex plane and let  $R(X)$  be the uniform closure of rational functions with poles off  $X$ . A bounded point derivation on  $R^p(X)$  at a point  $x_0$  is a bounded linear functional  $D$  on  $R(X)$  such that  $D(fg) = D(f)g(x_0) + D(g)f(x_0)$  for all functions  $f, g$  belonging to  $R(X)$ . Bounded point derivations generalize the concept of the derivative to functions which may not be differentiable. A related space is  $R^p(X)$ . For  $1 \leq p < \infty$ ,  $R^p(X)$  is the closure of rational functions with poles off  $X$  in the  $L^p$  norm. Bounded point derivations can also be defined for  $R^p(X)$ ; however, the definition needs to be altered a bit. In this talk, we present some results concerning bounded point derivations on both spaces and how they relate to the classical definition of the derivative. (Received July 05, 2017)

1135-30-354 **David Cardon, Tamas Forgacs\*** (tforagacs@csufresno.edu), **Andrzej Piotrowski, Evan Sorensen and Jason White**. *On sector reducing operators*. Preliminary report.

Let  $\{\gamma_k\}_{k=0}^\infty$  be a sequence of real numbers, and let  $T: \mathbb{R}[x] \rightarrow \mathbb{R}[x]$  be defined by  $T[\sum_{k=0}^n a_k x^k] = \sum_{k=0}^n a_k \gamma_k x^k$ . If  $\pi_{S_\delta}$  denotes the set of all real polynomials whose zeros lie in the sector  $S_\delta := \{z \mid |\text{Arg}(z)| < \delta\}$ , and  $T(\pi_{S_\delta}) \subset \pi_{S_{\delta'}}$  for some  $0 < \delta' < \delta$ , we say that  $T$  is a sector reducing operator. In this talk we address some properties sector reducing operators must possess. In particular, we show that in order for  $T$  to be a sector reducer, the sequence  $\{\gamma_k\}_{k=0}^\infty$  must satisfy  $\gamma_n \gamma_{n+2} / \gamma_{n+1}^2 < 1$  for all  $n$ , and  $\limsup_{n \rightarrow \infty} \gamma_n \gamma_{n+2} / \gamma_{n+1}^2 < 1$ . We also discuss connections to the works of de Bruijn, Pólya and the first author vis á vis complex zero strip decreasing operators. (Received August 27, 2017)

1135-30-439 **Dmitry Khavinson\*** ([dkhavins@usf.edu](mailto:dkhavins@usf.edu)), 4202 E. Fowler Ave, University of South Florida, Department of Mathematics, Tampa, FL 33620. *What is an "inner function"?* Preliminary report.

The concept of an inner function has been a focal point of function theoretic operator theory since the celebrated Beurling theorem characterizing invariant (with respect to the unilateral shift) subspaces in Hardy spaces in the unit disk. In the 1990s it was extended to Bergman spaces where however Beurling's theorem fails. Since 1960s many plausible ways of extending the notion of an inner function were pursued in the context of Hardy, Bergman and other spaces of analytic functions in a more general setting than the unit disk: multiply connected domains, Riemann surfaces and several variables.

We shall discuss some of the previous results and more recent viewpoints and developments. (Received September 03, 2017)

1135-30-490 **Santosh B Joshi\*** ([santosh.joshi@walchandsangli.ac.in](mailto:santosh.joshi@walchandsangli.ac.in)), Department Of Mathematics, Walchand College Of Engineering, Sangli, Vishrambag, Sangli, India. *Recent Trends in Bi-univalent functions.*

Let  $A$  denote the class of functions of the form :

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad (1)$$

which are analytic in the unit disc  $U = \{z \in \mathbb{C} : |z| < 1\}$ . Let  $S$  denote the subclass of  $A$ , which is analytic univalent and normalized in  $U$  and is of the form given by (1)

A function  $f \in A$  is said to be bi-univalent in  $U$  if both  $f(z)$  and  $f^{-1}(z)$  are univalent in  $U$ . Let  $\Sigma$  denote the class of bi-univalent functions in  $U$  given by (1).

In 1967, Lewin first investigated the class  $\Sigma$  of bi-univalent functions and showed that  $|a_2| < 1.51$ . Subsequently, Brannan and Clunie in 1967 conjectured that  $|a_2| \leq \sqrt{2}$ . In 1969 Netanyahu, on the other hand showed that  $\max_{f \in \Sigma} |a_2| = \frac{4}{3}$ . The coefficient estimate problem for each of the Taylor-Maclaurin coefficients  $|a_n|$  ( $n \geq 3; n \in \mathbb{N}$ ) for each  $f \in \Sigma$  given by (1) is still an open problem. We will present survey of aforementioned work during lecture.

(Received September 06, 2017)

1135-30-577 **Nobutaka Nakazono\*** ([nobua.n1222@gmail.com](mailto:nobua.n1222@gmail.com)), Department of Physics and Mathematics, Aoyama Gakuin University, Sagamihara, Kanagawa 229-8558, Japan. *Reduction from ABS equations to the Painlevé equations.*

In this talk, we show a reduction of a system of ABS equations to the Painlevé equations. Using this relation we construct the Lax representations of the Painlevé equations. This work has been supported by a JSPS KAKENHI Grant Number JP17J00092. (Received September 09, 2017)

1135-30-582 **Javad Mashreghi\*** ([javad.mashreghi@mat.ulaval.ca](mailto:javad.mashreghi@mat.ulaval.ca)), Dept Math, Pav. vachon, Quebec, QC G1S 1M8, Canada. *Approximation in "strange" function spaces.*

If  $X$  is a function space on the open unit disc  $D$ , and  $f \in D$ , then the dilation  $f_r(z) := f(rz)$  usually exhibits good properties. For example, in most function spaces  $f_r$  tends to  $f$ , as  $r \rightarrow 1$ , in the norm-topology of  $X$ . In fact, it was believed that this is always the case since  $f_r$  is analytic on a disc larger than  $D$ . Recently, we found a counter-example among the de Branges-Rovnyak spaces in which the dilations behave quite differently. E.G., we can construct an explicit example  $f$  such that  $\|f_r\| \rightarrow \infty$ . Approximation in these spaces has still unknown features. (Received September 09, 2017)

1135-30-583 **Faruk F. Abi-Khuzam\*** ([farukakh@aub.edu.lb](mailto:farukakh@aub.edu.lb)), Department of Mathematics, American Universit, Beirut, Lebanon, and **Florian J. Bertrand** and **Giuseppe A. Della Sala**. *Star functions in several complex variables.*

Given a function  $f$  meromorphic in the plane, its star-function is a subharmonic function defined in the upper half-plane and encoding many of the important functionals associated with  $f$ , such as the counting functions for the zeros and poles, the Nevanlinna characteristic, and the maximum modulus. It has been used to solve some extremal problems, and to unify several results in the theory. In many instances, the extremal functions in such problems correspond to a harmonic star-function. In this paper we define a star function for a meromorphic function of several complex variables, characterize those  $f$  with harmonic star function, and give applications to growth problems. (Received September 09, 2017)

1135-30-730

**Rajitha Ranasinghe\*** (rajitha13@knights.ucf.edu), Department of Mathematics, University of Central Florida, 4393 Andromeda Loop N, Orlando, FL 32816, and **Xin Li** (xin.li@ucf.edu), Department of Mathematics, University of Central Florida, 4393 Andromeda Loop N, Orlando, FL 32816. *Some New Inequalities for Rational Functions with Restricted Zeros*. Preliminary report.

Let  $\mathcal{P}_n$  be the class of polynomials of degree at most  $n$  and  $\|\cdot\|$  be the *sup norm* on the unit circle. For  $p \in \mathcal{P}_n$ , S. N. Bernstein proved

$$\|p'\| \leq n\|p\|; \quad (1)$$

and the *Bernstein lemma* is

$$\max_{|z|=R>1} |p(z)| \leq R^n \|p\|. \quad (2)$$

By restricting the zeros of  $p(z)$  to  $|z| > 1$ , (1) and (2) both can be improved. The improvement of (1) is due to Erdős and Lax and that of (2) is due to Ankeny and Rivlin. We generalize latter's result for the space  $\mathcal{R}_n$  of rational functions with prescribed poles.

Moreover, for  $p \in \mathcal{P}_n$  with  $\|p\| = 1$ , by restricting zeros of  $p(z)$  to  $|z| < 1$ , Turán obtained a reverse inequality to that of Erdős and Lax. By further restricting zeros of  $p(z)$  to regions  $|z| \geq K$  with  $K \geq 1$ , Malik and Govil had proved Erdős-Lax and Turán type inequalities. We present our preliminary results of rational analogues to these inequalities.

Furthermore, in working towards our main results, we established several auxiliary results which may be of interest in their own. (Received September 13, 2017)

1135-30-781

**Michael Dorff** and **Samaneh Hamidi\*** (s.hamidi\_61@yahoo.com), Department of Mathematics, Brigham Young University, Provo, UT 84602, and **Jay Jahangiri** and **Elif Yasar**. *Convolution of univalent harmonic strip mappings*.

Let  $f_k = h_k + \overline{g_k}$  be the family of harmonic univalent functions with  $h_k(z) + g_k(z) = \frac{z}{1-z}$  for  $k = 1, 2$ . Earlier it was shown that if  $f_1 * f_2$  is locally univalent and sense-preserving, then  $f_1 * f_2$  is univalent and convex in the direction of the real axis. This resulted in several papers determining condition under which  $f_1 * f_2$  is locally univalent and sense-preserving. In this paper we consider the family of harmonic univalent functions  $f_k = h_k + \overline{g_k}$  (where  $k = 1, 2$ ) that are shears of the analytic map  $h_k - g_k = \frac{1+z}{1-z}$  with dilatation  $\omega_k = e^{i\theta k} z^k$ . We prove that if the convolution  $f_1 * f_2$  is locally one-to-one and sense-preserving, then  $f_1 * f_2$  is univalent and convex in the direction of the real axis. (Received September 14, 2017)

1135-30-810

**Zair Ibragimov\*** (zibragimov@fullerton.edu), 800 N. State College Blvd., 154 McCarthy Hall, Fullerton, CA 92831. *Quasi-Isometric Extensions of Quasisymmetric Mappings of the Real Line Compatible with Composition*.

In this talk I will discuss construction and some basic properties of an operator that extends each quasi-symmetric homeomorphism of the real line to a quasi-isometry of the upper-half plane in a homomorphic fashion. Epstein and Markovic have shown that a homomorphic extension to quasiconformal homeomorphisms of the upper-half plane is not possible. (Received September 14, 2017)

1135-30-1021

**Meredith Sargent\*** (meredithsargent@wustl.edu). *Carlson's theorem for different measures*.

We use an observation of Bohr connecting Dirichlet series in the right half plane  $\mathbb{C}_+$  to interpret Carlson's theorem about integrals in the mean as a special case of the ergodic theorem by considering any vertical line in the half plane as an ergodic flow on the polytorus. Of particular interest is the imaginary axis because Carlson's theorem for Lebesgue measure does not hold there. In this note, we construct measures for which Carlson's theorem does hold on the imaginary axis for functions in the Dirichlet series analog of the disk algebra  $\mathcal{A}(\mathbb{C}_+)$ . (Received September 18, 2017)

1135-30-1031

**Krishna Subedi\*** (krshna.subedi@rockets.utoledo.edu), 1425 Oak Hill Ct Apt #32, Toledo, OH 43614, and **Zeljko Cuckovic** (zeljko.cuckovic@utoledo.edu), 2801 Bancroft, Toledo, OH 43606. *Necessary condition for the hyponormality of Toeplitz operators on the weighted Bergman space*.

Recently, Cuckovic and Curto have proved that for  $\varphi \equiv \alpha z^n + \beta z^m + \gamma \bar{z}^p + \delta \bar{z}^q$  where  $\alpha, \beta, \gamma, \delta \in \mathbb{C}$  and  $m, n, p, q \in \mathbb{Z}, m < n$  and  $p < q$ . By letting  $T_\varphi$  act on vectors of the form

$$z^k + cz^l + dz^r \quad (k < l < r).$$

They studied the asymptotic behavior of a suitable matrix of inner products, as  $k \Rightarrow \infty$ . And they obtain

$$|\alpha|^2 n^2 + |\beta|^2 m^2 - |\gamma|^2 p^2 - |\delta|^2 q^2 \geq 2|\bar{\alpha}\beta mn - \bar{\gamma}\delta pq|.$$

In this talk I will show this conjecture is true in weighted Bergman space too. (Received September 18, 2017)

1135-30-1091 **Khang D Tran\*** ([khangt@csufresno.edu](mailto:khangt@csufresno.edu)), Department of Mathematics, California State University, Fresno, Fresno, CA 93740. *Zeros of polynomials and their generating functions.*

We discuss an approach which shows that the zeros of various sequences of polynomials lie on fixed curves on the complex plane. In particular, we study connections between the zeros of a sequence of polynomials and its generating function. As an example, we characterize sequences of hyperbolic polynomials satisfying four-term recurrences with constant coefficients. In another example, we analyze connections between the hyperbolicity of polynomials forming the denominator of the generating function and the hyperbolicity of the generated sequence. (Received September 19, 2017)

1135-30-1155 **Myrto Manolaki\*** ([mmanolaki@usf.edu](mailto:mmanolaki@usf.edu)). *The problem of existence of universal Taylor series.*

The theory of universal Taylor series has connections with various branches of Analysis, ranging from Operator Theory to Potential Theory. This talk is concerned with one of the most fundamental questions of the area: Given a planar domain  $G$  and a point  $w$  in  $G$ , is it possible to find a holomorphic function on  $G$  with the property that the partial sums of its Taylor expansion about  $w$  can approximate every plausible function outside  $G$ ? It is known that, in the case where  $G$  is simply connected, most holomorphic functions possess such universal Taylor expansions. However, the multiply connected case is not properly understood. We will discuss some results which aim to shed some light on the above open problem. (Received September 19, 2017)

1135-30-1338 **Koushik Ramachandran\*** ([koushik.ramachandran@okstate.edu](mailto:koushik.ramachandran@okstate.edu)). *Convexity of level curves of Martin functions and associated Maximum principles.*

Let  $\Omega$  be an unbounded domain in  $\mathbb{R} \times \mathbb{R}^d$ . A positive harmonic function  $u$  in  $\Omega$  which vanishes on the boundary,  $\partial\Omega$ , of  $\Omega$  is called a Martin function on  $\Omega$ . In this talk, we will prove that when  $\Omega$  is convex, the super-level sets of a Martin function are also convex. As a consequence we obtain that if in addition  $\Omega$  is also rotationally symmetric, then the maximum of any Martin function along a slice  $\Omega \cap (\{x\} \times \mathbb{R}^d)$  is attained at  $(x, 0)$ . This talk is based on joint work with Gallagher, A.-K., and Lebl, J. (Received September 21, 2017)

1135-30-1360 **Andrew Starnes\*** ([starnes@math.utk.edu](mailto:starnes@math.utk.edu)), 227 Ayres Hall, Knoxville, TN 37996-1320. *The Multiple Loewner Equation with Rapidly and Randomly Oscillating Functions.*

Kager, Nienhuis, and Kadanoff conjectured that the hull generated from the Loewner equation driven by two constant functions with constant weights could be generated by a single rapidly and randomly oscillating function. We will discuss the proof of their conjecture and a generalization to multiple continuous driving functions with varying weights. Lastly, we investigate a simulation method which comes from the proof for hulls generated by the multiple Loewner equation. (Received September 21, 2017)

1135-30-1378 **Jonathan Holland\*** ([jehsma@rit.edu](mailto:jehsma@rit.edu)). *Hawking temperature and elliptic curves.*

In a series of papers in the 1970s, Gibbons and Hawking observed that the Euclidean Schwarzschild solution in four dimensions admits a natural complexification that is regular at the orbifold singularity of the event horizon. In that complexification, time becomes periodic with the (imaginary) period  $8\pi i M$ , whose modulus is the reciprocal of the black hole temperature. This talk describes a novel way to obtain the same period, using the classical Weierstrass theory of elliptic curves, which avoids the Euclidean ansatz. The construction is elementary. (Received September 25, 2017)

1135-30-1416 **See Keong Lee\*** ([sklee@usm.my](mailto:sklee@usm.my)), School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia. *The Janowski starlikeness of a solution of second order differential equation.* Preliminary report.

Let  $\mathbb{D} = \{z : |z| < 1\}$  be the unit disc on the complex plane  $\mathbb{C}$ . Let  $a(z)$ ,  $b(z)$  and  $c(z)$  be analytic functions on  $\mathbb{D}$  such that the equation

$$c(z)F''(z) + a(z)F'(z) + b(z)F(z) = 0,$$

has unique solution  $F(z)$  satisfying  $F(0) = 0 = F'(0) - 1$ . In this preliminary work, conditions on  $c(z)$ ,  $a(z)$  and  $b(z)$  will be investigated so that the solution  $F(z)$  is subordinate to  $\frac{1+Az}{1+Bz}$ ,  $-1 \leq B < A \leq 1$ . (Received September 22, 2017)

1135-30-1532 **Tamas Erdelyi\*** ([terdelyi@math.tamu.edu](mailto:terdelyi@math.tamu.edu)), Department of Mathematics, Texas A&M University, College Station, TX 77843. *The asymptotic value of the Mahler measure of the Rudin-Shapiro polynomials.*

In this talk we show that the Mahler measure of the Rudin-Shapiro polynomials of degree  $n - 1$  with  $n = 2^k$  is asymptotically  $(2n/e)^{1/2}$ , as it was conjectured by B. Saffari in 1985. Our approach is based heavily on the Saffari and Montgomery conjectures proved recently by B. Rodgers. (Received September 22, 2017)

1135-30-1771 **Michael J. Miller\***, Department of Mathematics, Le Moyne College, Syracuse, NY 13214. *Extending the Grace-Heawood theorem to minimal regions.* Preliminary report.

If  $P$  is a complex polynomial of degree  $n$  such that  $P(-1) = P(1)$ , then the Grace-Heawood theorem guarantees that  $P$  has a critical point in every disk or half-plane containing both points  $\pm i \cot(\pi/n)$ . In this paper, we examine how to extend this theorem to minimal regions in the complex plane. (Received September 24, 2017)

1135-30-1818 **Eric Chang\*** ([changer@bu.edu](mailto:changer@bu.edu)). *The Sierpinski Mandelbrot Arc.*

We identify a structure that lies in the parameter plane of the map  $F(z) = z^2 + \lambda/z^3$  where  $z$  and  $\lambda$  are complex. A “Sierpindelbrot arc” consists of infinitely many alternating Mandelbrot sets and Sierpinski holes. This arc lies in the parameter plane for  $F(z) = z^4 + \lambda/z^3$  as well as another type of arc. These two types of arcs together comprise a “Sierpinski Mandelbrot spiral.” (Received September 26, 2017)

1135-30-1844 **Daniel Seco\*** ([dseco@mat.uab.cat](mailto:dseco@mat.uab.cat)). *A characterization of Dirichlet inner functions.*

Inner functions have played a mayor role in the complex function theory of the Hardy space. In previous work, we have been concerned with inner functions defined in a similar fashion in other spaces (including Bergman and Dirichlet). In this talk, I will present a characterization of Dirichlet inner functions in terms of their multiplication properties. (Received September 25, 2017)

1135-30-2185 **Tyler C Bongers\*** ([bongerst@msu.edu](mailto:bongerst@msu.edu)). *Stretching and Rotation Sets of Quasiconformal Maps.*

Quasiconformal maps in the plane are orientation preserving homeomorphisms that satisfy certain distortion inequalities; infinitesimally, they map circles to ellipses of bounded eccentricity. Such maps have many useful geometric distortion properties. In this work, we study the size of the sets where a quasiconformal map can exhibit given stretching and rotation behavior. We improve results by Astala-Iwaniec-Prause-Saksman and Hitruhin to give examples of stretching and rotation sets with non-sigma-finite measure at the appropriate Hausdorff dimension. (Received September 25, 2017)

1135-30-2668 **Stacey Muir\***, Mathematics Department, Scranton, PA 18510. *Properties of a Generalized Harmonic Bernardi Integral Operator.*

The Bernardi integral operator generalizes the Alexander and Libera operators on analytic mappings of the open unit disk. It preserves the families of convex and starlike analytic mappings and has connections with subordination. We extend this operator to complex-valued harmonic mappings of the open unit disk and discuss various geometric properties and a weak subordination result for this extended operator. (Received September 26, 2017)

1135-30-2678 **David Nielsen Horton\*** ([horton@math.utk.edu](mailto:horton@math.utk.edu)), 227 Ayres Hall, 1403 Circle Drive, Knoxville, TN 37996-1320. *A Bound for Loewner-Weierstrass Hulls.* Preliminary report.

The Loewner Equation gives a one-to-one correspondence between continuous  $\mathbb{R}$ -valued functions and certain simply connected subsets of the upper half-plane,  $\mathbb{H}$ . It has been shown that the hulls generated by the Weierstrass function exhibit a phase change from simple curves to something else. However, these results focused on showing the existence of this phase change and gave only rough estimates of the bounds used. By leveraging the scaling property of the Weierstrass function, I have developed the means to tighten these bounds and I have a much sharper bound on the Lip-1/2 norm of the Weierstrass function. (Received September 26, 2017)

1135-30-2979 **M Chasse, T Forgacs and A Piotrowski\*** ([apiotrowski@alaska.edu](mailto:apiotrowski@alaska.edu)), 11066 Auke Lake Way, Juneau, AK 99801. *Polynomially Interpolated Legendre Multiplier Sequences.*

We prove that if a multiplier sequence for the Legendre basis can be interpolated by a polynomial, then the polynomial must have the form  $\{h(k^2 + k)\}_{k=0}^{\infty}$ , where  $h \in \mathbb{R}[x]$ . We also prove that a non-trivial collection of polynomials of a certain form interpolate multiplier sequences for the Legendre basis, and we state conjectures on how to extend these results. (Received September 26, 2017)

1135-30-3006 **Kouros Tavakoli\*** (ktavakoli@okcu.edu). *Comparison of Conformal-Like Metrics.*

In this talk I study geometric and analytic properties of some conformal-like metrics. In addition, I'll explain how they are related to the hyperbolic metric. Finally, I present a few interesting examples to clarify the comparison. (Received September 26, 2017)

1135-30-3115 **Trailokya Panigrahi** (trailokyap06@gmail.com), Department of Mathematics, KIIT University, Bhubaneswar, Orissa, India, and **Ram N Mohapatra\***

(ramm1627@gmail.com), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Role of Integral operators and associated class of univalent functions in Geometric Function Theory.*

There are many integral operators like Dizok-Srivastava Operator, Srivastava-Atiya Operators have been considered in geometric function theory which gives rise to distinguished class of univalent functions. In this talk we will make a short review of some such operators and use Komatu Integral Operator to obtain a new class of analytic functions. We obtain an upper bound of second Hankel determinant. At the end we shall mention about a method of solving differential equations in the complex domain through univalent function approach. (Received September 26, 2017)

## 31 ► Potential theory

1135-31-475 **Guilherme Silva\*** (silvag@umich.edu), East Hall 1859, 530 Church Street, Ann Arbor, MI 48108. *Vector critical measures and the hermitian plus external source random matrix model.*

It has been known for some time that the average characteristic polynomial of the hermitian random matrix model with external source is a multiple orthogonal polynomial (MOP) on the real line, with weights whose difference is linear and connected to the eigenvalues of the source.

Although this connection has been explored in the literature, the results available so far are always obtained under strong symmetry assumptions on the potential and on the source.

In this talk we plan to shed some light on the hermitian plus external source matrix model for arbitrary (that is, non-symmetric) potentials. Starting from the existence of an appropriate algebraic equation, known as the spectral curve of the matrix model, we construct a vector critical measure that should ultimately describe the limiting zero distribution of the aforementioned MOP's. This vector critical measure is the solution of an electrostatic model that involves three measures and interactions of both Nikishin and Angelesco types. The first two measures live on the real line and the third measure lives on the so-called S-contour, whose existence is one of our main results.

This is a joint work with Andrei Martinez-Finkelshtein (Universidad de Almeria, Spain) (Received September 05, 2017)

1135-31-1343 **Erik Lundberg\*** (elundber@fau.edu). *The Khavinson-Shapiro conjecture: a call to arms.*

Ellipsoids possess many surprising potential theoretic properties. One nice property is that the solution to the classical Dirichlet problem for Laplace's equation posed on an ellipsoid with polynomial data turns out to be a polynomial. In 1992, D. Khavinson and H.S. Shapiro further showed that the solution is entire (that is extends as an entire function of several complex variables) if the data is entire. They conjectured that these two properties (polynomial solvability for polynomial data and entire solvability for entire data) each characterize ellipsoids. We will speculate on how to approach the cases that remain open after the progress made by H. Render in 2008. (Received September 21, 2017)

1135-31-2700 **Lucio M-G Prado\*** (lprado@bmcc.bmcc.cuny.edu), Department of Mathematics, BMCC, CUNY, 199 Chambers Street, New York, NY 10007.  *$p$ -Laplacian's Surjectivity on Infinite Graphs.*

Consider a connected locally finite simplicial graph  $G$  with vertex set  $V$ , we study the problem of the discrete version of the  $p$ -Laplacian adapted from Riemannian manifold, and its surjectivity if  $G$  is infinite. I will give some overview of some concepts that play central role as  $p$ -capacity, infinite  $p$ -hyperbolic graphs, and the existence and uniqueness of solution in  $p$ -Dirichlet space for  $p$ -Poisson equation with finite support source on infinite graphs, and how, in general, we can study surjectivity of the  $p$ -Laplacian. It is important to point out that surjectivity of the  $p$ -Laplacian has direct relation to the existence of solution of  $p$ -Poisson equations. (Received September 26, 2017)

## 32 ► *Several complex variables and analytic spaces*

1135-32-519 **Mourad E.H. Ismail\***, Department of Mathematics, University of, Central Florida, Orlando, FL 32816, and **Chun-Kong Law**, Department of Applied Mathematics, National Sun Yat-sen University, Kaohsiung, Taiwan. *Towards a turning point theory for  $q$ -orthogonal polynomials*. Preliminary report.

We develop an intuitive approach to the Plancherel-Rotach asymptotics around the largest zero of a polynomial satisfying a linear functional equation. We first treat the toy problem of Hermite polynomials in order to explain the process by which we determine the correct Plancherel-Rotach asymptotics. We then treat the Stieltjes-Wigert polynomials,  $q$ -Laguerre polynomials and the Al-Salam-CHihara polynomials. Our approach does not use any refined properties of the orthogonal polynomials we treat but we only uses the second operator operator whether it is differential, difference,  $q$ -difference, or involves the Askey-Wilson operator. (Received September 06, 2017)

1135-32-545 **John E. McCarthy\*** (mccarthy@wustl.edu), Dept. of Mathematics, Washington University, 1, Brookings Drive, St. Louis, MO 63130, and **James E. Pascoe**. *A non-commutative Julia inequality*.

Julia's inequality is a boundary version of Schwarz's lemma. We will discuss a version of this for non-commutative functions on polynomial polyhedra. (Received September 07, 2017)

1135-32-1421 **K Bickel**, Bucknell University, Lewisburg, PA, **J E Pascoe**, Washington University, St Louis, MO, and **A A Sola\***, Department of Mathematics, Stockholm University, 10691 Stockholm, Sweden. *Singularities of rational inner functions on the bidisk*. Preliminary report.

I will discuss analytic and geometric aspects of the behavior of rational inner functions at singular points on the two-torus. The results we have obtained will be illustrated by examining several examples in detail. (Received September 22, 2017)

1135-32-2522 **Peter Ebenfelt\*** (pebenfelt@ucsd.edu), Department of Mathematics, University of California, San Diego, La Jolla, CA 92093-0112. *The obstruction function and deformations of three-dimensional strictly pseudoconvex CR manifolds*. Preliminary report.

The obstruction function on the (smooth) boundary of a strictly pseudoconvex domain in  $\mathbb{C}^n$  arises as the obstruction to smoothness up to the boundary of the Cheng-Yau solution to a particular Dirichlet problem. Graham showed that the obstruction function is actually a local CR invariant on the boundary. In  $\mathbb{C}^2$ , this function occurs also as the lowest order term in the Bergman kernel (and the Szegő kernel, properly defined). Moreover, it arises in variational formulas for global invariants such as  $Q^1$ -curvature and the Burns-Epstein invariant under deformations. In this talk, we will explain this, and discuss what can be gleaned from the vanishing of the obstruction function. (Received September 26, 2017)

1135-32-2654 **Jerry R. Muir, Jr.\***, Department of Mathematics, The University of Scranton, Scranton, PA 18510. *Hardy-type spaces arising from a vector-valued Cauchy kernel*.

An integral formula of Cauchy type was recently developed that reproduces any  $\mathbb{C}^n$ -valued holomorphic mapping of the open unit ball of  $\mathbb{C}^n$  that extends continuously to the boundary using a fixed vector-valued kernel and a scalar transform of the boundary values of the function. We consider Hardy-type spaces associated with this vector-valued kernel. In particular, we introduce and study spaces of vector-valued holomorphic mappings properly containing the vector-valued Hardy spaces that are reproduced through the process described above and isomorphic spaces of scalar-valued non-holomorphic functions that satisfy many of the familiar properties of Hardy space functions. (Received September 26, 2017)

## 33 ► *Special functions*

1135-33-140 **Rosihan M Ali\*** (rosihan@usm.my), School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Penang, Malaysia, and **See Keong Lee** and **Saiful R Mondal**. *On monotonicity and starlikeness of the generalized Bessel function*.

The generalized Bessel function

$${}_a\mathbf{B}_{b,p,c}(z) := \sum_{k=0}^{\infty} \frac{(-c)^k}{k! \Gamma\left(ak + p + \frac{b+1}{2}\right)} \left(\frac{z}{2}\right)^{2k+p}$$

is studied for a fixed  $a \in \{1, 2, 3, \dots\}$ . Representation formulations for  ${}_a\mathbb{B}_{b,p,c}$  are derived in terms of the parameters  $a, b$ , and  $p$ , and the function  ${}_a\mathbb{B}_{b,p,c}$  is shown to be a solution of a differential equation of order  $a + 1$ . Monotonicity properties of  ${}_a\mathbb{B}_{b,p,c}$  are also investigated for  $c \leq 0$ . Finally, the radius of starlikeness of positive order is obtained for the normalized analytic function

$$f_{a,\nu}(z) := \left( 2^{a\nu-a+1} a^{-\frac{a(a\nu-a+1)}{2}} \Gamma(a\nu+1) {}_a\mathbb{B}_{2a-1, a\nu-a+1, 1}(a^{a/2}z) \right)^{\frac{1}{a\nu-a+1}}$$

in the unit disk. When  $a = 1$ , the results obtained reduced to earlier known results. (Received August 02, 2017)

1135-33-223 **Jae-Ho Lee\*** ([jaeho.lee@unf.edu](mailto:jaeho.lee@unf.edu)), Department of Mathematics & Statistics, University of North Florida, 1 UNF Drive, Jacksonville, FL 32224, and **Hajime Tanaka** ([htanaka@tohoku.ac.jp](mailto:htanaka@tohoku.ac.jp)), Graduate School of Information Sciences, Tohoku University, 6-3-09 Aramaki-Aza-Aoba, Aoba-ku, Sendai, 980-8079, Japan. *Dual polar graphs, a nil-DAHA of rank one, and non-symmetric dual  $q$ -Krawtchouk polynomials.*

Let  $\Gamma$  be a dual polar graph with diameter  $D \geq 3$ . From every pair of a vertex of  $\Gamma$  and a maximal clique containing it, we construct a  $2D$ -dimensional irreducible module for a nil-DAHA of type  $(C_1^\vee, C_1)$ . Using this module, we define non-symmetric dual  $q$ -Krawtchouk polynomials and describe their orthogonality relations. (Received August 13, 2017)

1135-33-399 **Richard Askey\*** ([raaskey@hotmail.com](mailto:raaskey@hotmail.com)), 5815 American Parkway, Apt. 228, Madison, WI 53718. *Interesting inequalities.*

Two types of inequalities will be discussed. One is an old set of inequalities due to Newton and Maclaurin which contains special cases which should be in a first course in calculus. All that is needed to prove them is Rolle's theorem and when a quadratic has real roots. The three variable cases have geometric consequences which high school students can appreciate. The second types of inequalities are variations on inequalities introduced by Lehmus and Schur, including one in four variables. (Received August 31, 2017)

1135-33-402 **Howard S Cohl\*** ([hcohl@nist.gov](mailto:hcohl@nist.gov)), Applied & Computational Mathematics Division, National Institute of Standards & Technology, Mission Viejo, CA 92694, and **Roberto S Costas-Santos, Philbert R. Hwang** and **Tanay V. Wakhare**. *Generalizations of generating functions for basic hypergeometric orthogonal polynomials.*

We derive generalized generating functions for basic hypergeometric orthogonal polynomials by applying connection relations with one free parameter to them. In particular, we generalize generating functions for Askey-Wilson, continuous  $q$ -ultraspherical/Rogers, little  $q$ -Laguerre/Wall, and  $q$ -Laguerre polynomials. Depending on what type of orthogonality these polynomials satisfy, we derive corresponding definite integrals, infinite series, bilateral infinite series, and  $q$ -integrals. Using the generating function for continuous  $q$ -ultraspherical/Rogers polynomials and a recent result by Ismail & Simeonov, we derive corresponding generalized expansions for continuous  $q$ -Jacobi, Askey-Wilson and Wilson polynomials with two, four, and four free parameters respectively. The coefficients of these expansions are given in terms of  ${}_8\phi_7$ . Using the specialization of this analysis for continuous  $q$ -ultraspherical/Rogers polynomials, we derive a new quadratic transformation for basic hypergeometric series connecting  ${}_2\phi_1$  and  ${}_8\phi_7$ . (Received August 31, 2017)

1135-33-463 **Neelam Singha\*** ([neelam.singha1990@gmail.com](mailto:neelam.singha1990@gmail.com)), Department of Mathematics, IIT KGP, Kharagpur, 721302, India, and **Chandal Nahak**. *Applications of orthogonal polynomials for solving a class of fractional optimal control problems.*

The concept of fractional calculus emerged with the possibility of defining non-integer order derivatives. The modernization of classical problems and properties in fractional calculus sense, motivate us to inspect admissible applications and physical properties unattended by integer order operators. The need to investigate optimal control problems with dynamical constraints involving fractional order operators give rise to the problem of fractional optimal control. In literature, orthogonal polynomials like Chebyshev polynomials, Jacobi polynomials, Legendre polynomials, have already practiced while handling FOCPs. In this report, we discuss a class of fractional optimal control problems, where the system dynamical constraint involves a combination of classical and fractional derivatives. We have designed a well-organized algorithm to obtain the numerical solution of the proposed problem by exercising Laguerre orthogonal polynomials. The key motive associated with the present approach is to convert the concerned fractional optimal control problem to an equivalent standard quadratic programming problem with linear equality constraints. Given examples and graphically representations analyze the computational technique of the method together with its efficiency and accuracy. (Received September 05, 2017)



1135-33-540 **Ronald E. Mickens\*** ([rmickens@cau.edu](mailto:rmickens@cau.edu)), Clark Atlanta University, Department of Physics, Atlanta, GA 30314. *The Imani Periodic Functions.*

The Imani functions are special classes of periodic solutions to the Leah function equation (LFE)

$$L(x, y) : \frac{y^2}{2} + \left(\frac{3}{4}\right)x^{4/3} = \frac{3}{4}$$

$$x(0) = 1, \quad y(0) = 0,$$

where  $x = x(t)$  and  $y = y(t)$ . These functions correspond to generalizations of the standard trigonometric cosine and sine functions and, by explicit construction, have analogous mathematical properties. We also demonstrate that a similar class of periodic functions may be expressed in terms of the Jacobi cosine and sine functions [1]. For background to this topic, the paper of Mickens [2] provides a broad discussion of periodic solutions to the functional equation  $x^2 + y^2 = 1$ .

#### References

- [1] P. F. Byrd and M. D. Friedman, Handbook of Elliptic Integrals for Engineers and Physicists (Springer-Verlag, Berlin, 1954).  
 [2] R. E. Mickens, Periodic solutions of the functional equation  $f(t)^2 + g(t)^2 = 1$ , Journal of Difference Equations and Applications, 22 (2016), 67–74. (Received September 07, 2017)

1135-33-1022 **T. M. Dunster\*** ([mdunster@mail.sdsu.edu](mailto:mdunster@mail.sdsu.edu)), Department of Mathematics and Statistics, College of Sciences, San Diego State University, San Diego, CA 92182-7720. *On the asymptotics of Airy functions and their zeros.*

Asymptotic expansions for Airy functions and their zeros are well known, and have a multitude of applications. Here we present new forms of expansions for these functions, and use these to study their zeros. In particular, we obtain explicit representations for the coefficients in the asymptotic expansions of their zeros, and also derive explicit error bounds for these expansions. As a consequence we verify the main aspects of the Fabijonas-Olver conjecture (SIAM Review, 41, (1999), 762-773). (Received September 18, 2017)

1135-33-1063 **Roderick Wong\*** ([mawong@cityu.edu.hk](mailto:mawong@cityu.edu.hk)) and **Zhitao Wen.** *Asymptotics of Orthogonal Polynomials Associated with a Generalized Freud Weight.* Preliminary report.

In this talk, we discuss the asymptotic behavior of the monic polynomials orthogonal with respect to the weight function  $w(x) = |x|^{2\alpha} e^{-(x^4 + tx^2)}$ , where  $\alpha > -\frac{1}{2}$  and  $t$  is any real number. We consider this problem in three separate cases: (i)  $c > -2$ , (ii)  $c = -2$ , and (iii)  $c < -2$ , where  $c := tN^{-1/2}$  is a constant,  $N = n + \alpha$  and  $n$  is the degree of the polynomial. In the first two cases, the support of the associated equilibrium measure  $\mu_t$  is a single interval, whereas in the third case the support of  $\mu_t$  consists of two intervals. In each case, globally uniform asymptotic expansions are obtained in several different regions. These regions together cover the whole complex plane. (Received September 19, 2017)

1135-33-1311 **Bruce R Miller\*** ([bruce.miller@nist.gov](mailto:bruce.miller@nist.gov)), 100 Bureau Drive, Gaithersburg, MD 20899. *Cataloging of DLMF's Special Functions.* Preliminary report.

The initial efforts in NIST's Digital Library of Mathematical Functions (DLMF; <http://dlmf.nist.gov/>) project focused on an informal view of the mathematics, with an eye to human engineers and scientists. And yet these efforts were taken with an acute awareness of the underlying semantics and its ultimate utility.

We will discuss ongoing efforts to (at least in part) formalize the DLMF's data, in particular leading towards machine-readable representations of the information. A fundamental stage in this evolution is the development of an effective catalog of the special functions covered in DLMF, in particular their definitions, domains of applicability and most significant properties. This effort is in support of the 'Special Function Concordance' which will lead to the interoperability of different libraries and software systems which deal with these functions. (Received September 21, 2017)

1135-33-1397 **Zilong Song\***, Department of Mathematics and Statistics, York University, Toronto, ON M3J 1P3, Canada, and **R Wong**, Department of Mathematics, 83 Tat Chee Avenue, Kowloon, Hong Kong. *Asymptotics of Pseudo-Jacobi Polynomials with Varying Parameters.*

In this talk, we present the asymptotic behavior of the Pseudo-Jacobi polynomials  $P_n(z; a, b)$  as  $n \rightarrow \infty$  for  $z$  in the whole complex plane. These polynomials are also known as the Romanovski-Routh polynomials. They occur in quantum mechanics, quark physics and random matrix theory. When the parameter  $a$  is fixed or  $a > -n$ , there is no real-line orthogonality. Here, we consider the case when the parameters  $a$  and  $b$  depend on  $n$ ; more precisely, we assume  $a = -(An + A_0)$ ,  $A > 1$  and  $b = Bn + B_0$ , where  $A, B, A_0, B_0$  are real constants. Our main

tool is the asymptotic method developed for differential equations with a large parameter. (Received September 21, 2017)

1135-33-1843 **Dan Dai\*** ([dandai@cityu.edu.hk](mailto:dandai@cityu.edu.hk)), Department of Mathematics, City University of Hong Kong, Hong Kong, Peoples Rep of China, **Mourad E.H. Ismail** ([mourad.eh.ismail@gmail.com](mailto:mourad.eh.ismail@gmail.com)), Department of Mathematics, University of Central Florida, Orlando, FL, and **Xiang-Sheng Wang** ([xswang@louisiana.edu](mailto:xswang@louisiana.edu)), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA. *Doubly infinite Jacobi matrices revisited: resolvent and spectral measure.*

We study the resolvent and spectral measure of certain doubly infinite Jacobi matrices via asymptotic solutions of two-sided difference equations. By finding the subdominant (or minimal) solutions or calculating the continued fractions for the difference equations, we derive explicit formulas for the matrix entries of resolvent of doubly infinite Jacobi matrices corresponding to Lommel polynomials, associated ultraspherical polynomials, and Al-Salam-Ismail polynomials. The spectral measures are then obtained by inverting Stieltjes transformations. (Received September 25, 2017)

1135-33-2419 **Ahmad El-Guindy\*** ([ahmad.el-guindy@qatar.tamu.edu](mailto:ahmad.el-guindy@qatar.tamu.edu)), Texas A&M University at Qatar, Doha, Qatar, and **Mourad E.H. Ismail**. *q-analogues of 2-D Hermite polynomial and partition statistics.*

We study some arithmetic results connected to q-analogues of 2-D Hermite polynomials which were introduced by Ismail and Zhang. In particular, specializing the generating function for those polynomials we deduce refinements and higher analogs of the crank partition statistic, and we explore analogs of the rank-crank PDE for them. (Received September 26, 2017)

1135-33-3100 **Ira M. Gessel\*** ([gessel@brandeis.edu](mailto:gessel@brandeis.edu)). *Rational functions with nonnegative power series coefficients.* Preliminary report.

Mourad E. H. Ismail and M. V. Tamhankar [A combinatorial approach to some positivity problems, SIAM J. Math. Anal. 10 (1979), 478–485] proved, by applying MacMahon's Master Theorem to a  $3 \times 3$  matrix, that for  $0 \leq \lambda \leq 1$  the coefficient of  $x^l y^m z^n$  in

$$\frac{1}{1 - (1 - \lambda)x - \lambda y - \lambda xz - (1 - \lambda)yz + xyz}$$

is nonnegative. They expressed this coefficient, when nonzero, as a positive number times the square of an alternating sum. Easy consequences of this result are the nonnegativity of the coefficients of  $(1 - x - y - z + 4xyz)^{-1}$  and  $(1 - 2x - 2y - 2z + 3xy + 3xz + 3yz)^{-1}$ . The nonnegativity of the latter series was proved by G. Szegő in 1933.

I will discuss a multivariable generalization of Ismail and Tamhankar's result that applies MacMahon's Master Theorem to matrices of arbitrary size. (Received September 26, 2017)

1135-33-3148 **Constanze Liaw, Jessica Stewart Kelly and John M. Osborn\*** ([john\\_osborn@baylor.edu](mailto:john_osborn@baylor.edu)). *Moment Representations of Type I  $X_2$  Exceptional Laguerre Polynomials.*

The  $X_m$  exceptional orthogonal polynomials (XOP) form a complete set of eigenpolynomials to a differential equation. Despite being complete, the XOP set does not contain polynomials of every degree. Thereby, the XOP escape the Bochner classification theorem.

In literature two ways to obtain XOP have been presented. When  $m = 1$ , Gram-Schmidt orthogonalization of a so-called "flag" was used. For general  $m$ , the Darboux transform was applied.

Here, we present a possible flag for the  $X_m$  exceptional Laguerre polynomials. There is a large degree of freedom in doing so. Further, we derive determinantal representations of the  $X_2$  exceptional Laguerre polynomials involving certain adjusted moments of the exceptional weights. We find a recursion formula for these adjusted moments. The particular canonical flag we pick keeps both the determinantal representation and the moment recursion manageable. (Received September 26, 2017)

1135-33-3214 **Harish Nagar\*** ([drharishngr@gmail.com](mailto:drharishngr@gmail.com)), 2-A-16, Nagar Villa, Bapu Nagar, Bhilwara, Bhilwara, RI 311001, India, and **Ajay Kumar Tripathi**. *On Unified Integrals involving the product of General Classes of Polynomials of Several Variables with Applications.*

This paper presents three unified integrals involving the product of general classes of polynomials of several variables with generalized Humbert type polynomial. The above generalized Humbert type polynomial generalizes the well-known class of polynomials by Gegenbauer, Legendre, Pincherle, Horadan Gould and Milovarivic-Dordevic, Sinha and Shrestha. (Received September 27, 2017)

## 34 ► Ordinary differential equations

1135-34-37 **Wandi Ding\*** ([wding@mtsu.edu](mailto:wding@mtsu.edu)), 1301 E. Main Street, MTSU Box 34, Murfreesboro, TN 37132, **Ryan J. Florida**, 1301 E. Main Street, MTSU Box 34, Murfreesboro, TN 37132, **Jeffery Summers**, 1301 E. Main Street, MTSU Box 34, Murfreesboro, TN 37132, and **Puran Nepal**, 1301 E. Main Street, MTSU Box 34, Murfreesboro, TN 37132. *Experience and Lessons Learned from Using SIMIODE Modeling Scenarios.*

In this talk, we will share our experience and lessons learned from using Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations (SIMIODE) modeling scenarios. Specific examples will be presented. Open questions are asked, new insights will be brought forth and discussions are encouraged on how to “best” teach Differential Equations using modeling approaches. (Received June 26, 2017)

1135-34-52 **Lianwen Wang\*** ([lwang@ucmo.edu](mailto:lwang@ucmo.edu)), School of Computer Science and Mathematics, University of Central Missouri, Warrensburg, MO 64093. *On Continuity, Boundedness, and Monotonicity of Solutions of Nonlinear Systems of Differential Equations.*

Continuity, boundedness, and monotonicity are studied for a nonlinear system of differential equations. It is proved that all solutions are eventually monotonic and can be extended to infinity under natural assumptions. Moreover, necessary and sufficient conditions for boundedness of all solutions are established. (Received July 08, 2017)

1135-34-107 **Joseph P Previte** and **Joseph E Paultet\*** ([jep7@psu.edu](mailto:jep7@psu.edu)), Penn State Behrend, 4205 College Drive, Erie, PA 16563. *A BVP nonexistence proof using Green's Theorem.*

Several recent papers investigate the boundary value problem

$$\phi''(t) + \lambda\phi'(t) + \phi(t)^2 = 0, \quad t \geq 0$$

subject to

$$\phi(0) = 1, \quad \phi(\infty) = 0,$$

which arises in certain situations of boundary layer flow. Previous work on the problem established the existence of a  $\lambda_{\min} \in [1, 2/\sqrt{3}]$  such that solutions exist for  $\lambda \geq \lambda_{\min}$ . It has been conjectured that for  $\lambda < \lambda_{\min}$  no solution exists. We improve existing results by proving that for  $\lambda < \lambda_1 \approx .96105$  no solution to the boundary value problem exists. The proof employs a novel application of Green's Theorem and is applicable to other boundary value problems. (Received August 04, 2017)

1135-34-151 **Corban Harwood\*** ([rharwood@georgefox.edu](mailto:rharwood@georgefox.edu)). *Managing Project Logistics in Teaching Differential Equations.*

Projects provide tangible connections to course content and can motivate students to learn at a deeper level. This talk focuses on the efficient implementation of projects in a Differential Equations course which develop and analyze mathematical models of a problem based upon known data and real-life situations. Logistical pitfalls and insights are highlighted as well as several key implementation resources, such as SIMIODE and other community support. Student feedback demonstrates a positive correlation between the use of projects and an enhanced understanding of the course topics when logistical issues are reduced. Best practices learned over the years will be shared along with example projects. (Received August 04, 2017)

1135-34-207 **Bhuvaneswari Sambandham\*** ([buna.sambandham@dixie.edu](mailto:buna.sambandham@dixie.edu)), Dixie State University, Saint George, UT 84770, and **Aghalaya S Vatsala** and **Vinodh Kumar Chellamuthu**. *Numerical results for Linear Sequential Caputo Fractional Boundary Value problem with mixed boundary conditions.* Preliminary report.

In our earlier work, we have developed the generalized monotone iterative technique for sequential Caputo fractional boundary value problems with mixed boundary conditions. As a byproduct, we have obtained a representation form for the linear nonhomogeneous sequential Caputo fractional boundary value problem in terms of the Green's function. In this work, we have developed the numerical simulations for a linear nonhomogeneous sequential Caputo fractional boundary value problem with mixed boundary conditions. This in turn, will be used as a tool to develop the numerical results for the nonlinear sequential Caputo fractional boundary value problem via generalized monotone iterative technique. (Received August 11, 2017)

1135-34-243 **Brian Christopher Pennington\*** ([brian\\_pennington@baylor.edu](mailto:brian_pennington@baylor.edu)), Department of Mathematics, Baylor University, Waco, TX 76798-7328. *Boundary data smoothness for solutions of nonlocal boundary value problems for nth order differential equations.*

Under certain conditions, solutions of the  $n$ th order boundary value problem,  $y^{(n)} = f(x, y, y', \dots, y^{(n-1)})$ ,  $y(a) - \sum_{k=1}^p a_k y(\xi_k) = y_1$ ,  $y^{(i-1)}(\gamma) = y_i$ , for  $2 \leq i \leq n-1$ , and  $y(b) - \sum_{l=1}^q b_l y(\eta_l) = y_n$ , are differentiated

with respect to boundary conditions, where  $c < a < \xi_1 < \dots < \xi_p < \gamma < \eta_1 < \dots < \eta_q < b < d$ ,  $a_1, \dots, a_p, b_1, \dots, b_q \in \mathbb{R}$ , and  $y_1, \dots, y_n \in \mathbb{R}$ . The method involves application of a Peano Theorem for initial value problems. (Received August 15, 2017)

1135-34-523 **Petr Zemánek\*** ([zemanekp@math.muni.cz](mailto:zemanekp@math.muni.cz)), Kotlářská 2, 61137 Brno, Czech Rep. *On square integrable solutions and principal solutions for second order Sturm–Liouville dynamic equations.*

In the study of linear differential, difference, and dynamic equations or systems we can identify, among others, two substantial directions: the oscillation theory and the spectral theory. In the first one, roughly speaking, the numbers of zeros of real-valued solutions are investigated and a crucial role is played by the *principal solution*, which existence is equivalent to the nonoscillatory behavior. In the second one, associated operators are investigated and a crucial role is played by square integrable/summable complex-valued solutions. Especially, the square integrability/summability of the *Weyl solution* yields a lower bound for their number. In this talk, we show a close connection between the principal solution and the Weyl solution of the second order Sturm–Liouville dynamic equation

$$-[p(t)y^\Delta(t, \lambda)]^\Delta + q(t)y^\sigma(t, \lambda) = \lambda w(t)y^\sigma(t, \lambda), \quad t \in [a, \infty) \cap \mathbb{T}, \quad (\text{E})$$

where  $\mathbb{T}$  is a time scale and the coefficients are real-valued. In addition, we present also further results concerning square integrable solutions of equation (E), which were derived by using the principal solution. Finally, we discuss a generalization of these results to linear Hamiltonian differential systems. (Received September 07, 2017)

1135-34-616 **Zhivko S. Athanassov\*** ([zhivko@math.bas.bg](mailto:zhivko@math.bas.bg)), Institute of Mathematics, Bulgarian Academy of Sciences, G. Bonchev Str. 8, 1113 Sofia, Bulgaria. *Perron-Type Existence Theorems for First Order Initial Value Problems.* Preliminary report.

Upper (major) and lower (minor) functions were first introduced by de La Vallée Poussin in 1916 in his study of Lebesgue integral. Evidently equivalent notions were introduced independently by Perron in 1914 who used them to define a new integral. In his famous paper of 1915, Perron proved by the use of upper and lower functions the existence of a solution of the initial value problem for the differential equation  $dx/dt = f(t, x)$ . Our theorems generalize and extend Perron's theorem. Perron's requirement that  $f$  be continuous on a specified compact subset on the plane is significantly modified. (Received September 11, 2017)

1135-34-648 **Dina Yagodich\*** ([dyagodich@frederick.edu](mailto:dyagodich@frederick.edu)), Frederick, MD. *Sneaking in LaTeX while teaching by modeling.*

While doing initial modeling scenarios at the beginning of the semester, I took the opportunity to teach basic LaTeX techniques. Students turned in work using ShareLaTeX, a web site that allows students to share with one collaborator for free. I was able to see their code and make comments to help students master using this important typesetting language. (Received September 11, 2017)

1135-34-732 **Neha Murad\*** ([nmurad@ncsu.edu](mailto:nmurad@ncsu.edu)), 2810 Brigadoon Drive, Apt 21, Raleigh, NC 27606, and **Harvey Thomas Banks** and **Rebecca Everett**. *The iterative process of quantitative modeling of infection dynamics in renal transplant recipients.* Preliminary report.

Mathematical models play a significant role in providing a numerical and analytical perspective to biological models. In the presence of data, inverse problems are performed to estimate unknown parameters for these models. Statistical error models used during inverse problem formulations help quantify the uncertainty and variability that arises with using experimental data. This process of applying mathematical and statistical techniques for modeling physical processes is an iterative one and often leads to new insights with every new iteration. There is a relatively recent research effort in modeling the mechanisms of solid organ transplants, specifically kidney transplants. We present mathematical and statistical models to illustrate the iterative process of modeling for renal transplant recipients infected by BK virus by improving the current mathematical model to be more biologically accurate. Using a second order difference-based method to eliminate statistical error model misspecification, we also show how modified residuals from the inverse problem can be used to detect discrepancies in mathematical model formulation. (Received September 13, 2017)

1135-34-735 **Patrice G Tiffany\*** ([patrice.tiffany@manhattan.edu](mailto:patrice.tiffany@manhattan.edu)), Manhattan College, Department of Mathematics, Riverdale, NY 10471, and **Rosemary C Farley** ([rosemary.farley@manhattan.edu](mailto:rosemary.farley@manhattan.edu)), Manhattan College, Department of Mathematics, Riverdale, NY 10471. *It's Time to Change our Traditional Differential Equations Course.*

We have been using a modeling first approach in our DE classes while others have used a more traditional approach. The course is required of every engineering student. There is a common cumulative final that typically includes traditional methods of solving DE by hand. Last spring there were 9 sections with a total of

244 students and 7 faculty members. In the fall, there are only 2 sections and we are the only faculty involved in teaching differential equations.

We are using modeling first approach together with the extensive use of a computer algebra system. We have found that the modeling first approach works best when students actually solve problems and get answers that make sense. This semester we are going to add more modeling scenarios and more qualitative analysis. We will be working with a member of the assessment team at our institution to collect the information to evaluate our progress. Hopefully these results convince others to join us in this modeling first approach. Perhaps we will even be able to convince the department that it is time to make significant changes in the course description.

We will report on the progress made during this semester including the modeling activities that worked best, how our students reacted, and what the assessment revealed. (Received September 13, 2017)

1135-34-746 **Erdal Imamoglu\*** (eimamog@ncsu.edu), Department of Mathematics, North Carolina State University, Box 8205, Raleigh, NC 27695, and **Mark van Hoeij**. *On Standard Forms of Linear Ordinary Differential Equations with Rational Function Coefficients.*

In this presentation we will introduce the standard form of a linear ordinary differential equation (LODE) with rational function coefficients and will give a fast algorithm to find it. The crucial step to find the standard form of a LODE is to compute an integral basis, which is also normalized at the point at infinity, for the LODE. Since the standard form of a LODE is unique up to a differential isomorphism, our algorithm proposes a new method to classify LODEs with rational function coefficients. (Received September 13, 2017)

1135-34-773 **John R. Graef\*** (john-graef@utc.edu), Department of Mathematics, University of Tennessee at Chattanooga, Chattanooga, TN. *A boundary value problem for a Hadamard fractional differential equation with nonlocal multi-point boundary conditions.*

Results on the existence and uniqueness of solutions to a boundary value problem for a nonlinear Hadamard fractional differential equation with nonlocal multi-point boundary conditions are proved. The results are obtained by using a variety of fixed point theorems. Some illustrative examples are given. (Received September 14, 2017)

1135-34-775 **John R. Graef\*** (john-graef@utc.edu), Department of Mathematics, University of Tennessee at Chattanooga, Chattanooga, TN, and **N. Guerraiche** and **S. Hamani**. *Initial value problems for fractional functional differential inclusions with Hadamard type derivatives in Banach spaces.*

The authors establish sufficient conditions for the existence of solutions to boundary value problems for fractional differential inclusions involving the Hadamard type derivatives of order  $\alpha \in (0, 1]$  in Banach spaces. (Received September 14, 2017)

1135-34-801 **Zuzana Dosla\*** (dosla@math.muni.cz), Department of Mathematics and Statistics, Masaryk University, Brno, Czech Rep, **Mauro Marini** (mauro.marini@unifi.it), Department of Mathematics and Informatics, University of Florence, Florence, Italy, and **Serena Matucci** (serena.matucci@unifi.it), Department of Mathematics and Informatics, University of Florence, Florence, Italy. *Boundary value problems on a half-line for differential equations with mean curvature operator.*

Motivated by searching for positive radially symmetric solutions in a fixed exterior domain in  $\mathbb{R}^N$  for partial differential equations involving the curvature operator, we deal with the asymptotic and boundary value problems on the half-line for equations with the Euclidean mean curvature operator

$$\left( a(t) \frac{x'}{\sqrt{1+x'^2}} \right)' + b(t)F(x) = 0, \quad (1)$$

and with the Minkowski mean curvature operator

$$\left( a(t) \frac{x'}{\sqrt{1-x'^2}} \right)' + b(t)F(x) = 0, \quad (2)$$

(Received September 14, 2017)

1135-34-863 **Heidi A. Berger\*** (heidi.berger@simpson.edu), 701 N. C Street, Indianola, IA 50125, and **Clinton K. Meyer** and **Anna Mummert**. *Compartmental model of land use dynamics within the Tallgrass Prairie ecosystem: the case for the Conservation Reserve Program (CRP).* Preliminary report.

Tallgrass prairie has been reduced to a fraction of its original extent, due to rapid conversion to other land use types, especially agricultural and urban. Restoration is a relatively new process to convert agricultural land back to communities dominated by native vegetation. We used a modified susceptible-infectious-recovered

(SIR) compartmental model to quantify changes between land use types, incorporating the impact of human population within the Midwestern tallgrass prairie ecoregion. We considered transitions between native prairie, prairie restored through the Conservation Reserve Program (CRP), agricultural, and urban land. We used historical data to determine parameter ranges. Sensitivity analyses emphasize the importance of increasing incentives for CRP enrollment as a means to restoring the tallgrass prairie ecoregion. (Received September 15, 2017)

1135-34-931 **Christina M. Alvey\*** ([christina.alvey@msmc.edu](mailto:christina.alvey@msmc.edu)). *Qualitative Analysis of Differential Equations and the Zombie Apocalypse in a Mathematical Biology Course.*

In a special topics course on mathematical biology focusing on qualitative analysis of differential equations, we introduced models to describe the transmission of both real and fictional diseases, among other topics. Modeling scenarios included diseases such as influenza, varicella, greyscale from Game of Thrones, and zombie infection. In this talk, we will explore the effectiveness of using these phenomena as motivating models for studying differential equations, as well as discuss the advantages and disadvantages of using the zombie apocalypse as an extended example throughout the course. (Received September 17, 2017)

1135-34-945 **Prof. Florin Diacu** and **Sawsan Alhowaity\*** ([salhowaity@su.edu.sa](mailto:salhowaity@su.edu.sa)), Victoria, BC , Canada. *Relative equilibria in the 4- and 5-body problem.* Preliminary report.

We explore the equation of motion in curved and flat space. We first derive the bifurcation of the integrals of motion when the curvature parameter passes through zero. Then we provide a method to find the relative equilibria in the curved 4-and 5-body problem. We seek the relative equilibria on the equator for 4-body problem. In particular, we study the case of two negligible masses and two equal masses. As a result, we prove that square relative equilibria always exist on the 2-sphere of constant Gaussian curvature. Also, we consider the case of three equal masses and one negligible mass, and will put into the evidence some kite-shaped solutions. Finally, we prove the existence of relative equilibria on the equator in the 5-body problem. (Received September 17, 2017)

1135-34-1016 **Bill Skerbitz\*** ([william.skerbitz@wayzata.k12.mn.us](mailto:william.skerbitz@wayzata.k12.mn.us)), 4955 Peony Ln N, Plymouth, MN 55446. *Experiencing the “Logic of Logistics”.*

An approach to developing an introductory and intuitive understanding of logistic models will be presented. The model is built via in-class simulation and data collection, adherence to assumptions, and qualitative reasoning. Best of all, instead of students “creating” a model, they ARE the model! (Received September 26, 2017)

1135-34-1129 **Jesica Bauer\*** ([jbauer@carroll.edu](mailto:jbauer@carroll.edu)) and **Eric Sullivan** ([esullivan@carroll.edu](mailto:esullivan@carroll.edu)). *Brewing the Perfect Cup of Tea with Differential Equations.*

What makes a great cup of tea? What do differential equations have to do with it? We designed a differential equations modeling scenario where students model various aspects of the preparation of a perfect cup of tea. Students were given photographs of the steeping process, temperature data, and volume data for the dissolution of a sugar cube and were asked to estimate parameters and generate differential equation models for all three processes. We will present solutions to the models, common student approaches, suggestions for running the modeling scenario in your class, and logistical issues and concerns. This work is published on the open access resource SIMIODE (Received September 19, 2017)

1135-34-1357 **Vincenzo Michael Isaia\*** ([vincenzo.isaia@indstate.edu](mailto:vincenzo.isaia@indstate.edu)), Dept. of Mathematics and Computer Science, Terre Haute, IN 47809. *Simple Simulation of Nonlinear Differential Equations with (Fairly) General Deviating Arguments.* Preliminary report.

A simple approach (well suited for modelers, scientists and engineers) for the numerical simulation of DEs in a single independent variable with deviating arguments will be presented. Key features besides simplicity: user controlled order, approximation is a continuous function on its domain, discontinuities are not integrated over and don't pose difficulties, the same method applies to retarded, neutral and advanced equations, freedom from root finding techniques and convergence results are readily obtained. The method is applicable to stiff equations as well.

The types of deviating arguments (which are assumed to be delays) include, nonlinear in time, non-monotone in time, state dependent (including the possibility of dependence on derivatives of the state), and distributed (in the form of integration with respect to a kernel). Initial history may be non-analytic, but the delay/kernel should be analytic with respect to time. Although not to be presented here, the method extends to the PDE case as well. (Received September 21, 2017)

1135-34-1400 **Aghalaya S. Vatsala\*** (vatsala@louisiana.edu), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504, and **Yunxiang Bai**. *Fractional Impulsive Differential Equations and Comparison Results*. Preliminary report.

In this work, we have considered fractional impulsive differential equation with initial conditions and the impulses are introduced as non-homogeneous terms. Using Laplace transform method we have obtained a closed form of the solution for the linear impulsive differential equation. This will enable us to prove the uniqueness of the solution of the linear fractional impulsive differential equation. We have developed a comparison result relative to coupled lower and upper solutions of the nonlinear fractional impulsive differential equations with initial conditions. We also present some numerical results. (Received September 21, 2017)

1135-34-1425 **Miguel Castro-Rivera\*** (miguelcr1997@gmail.com), 500 Dr. John Will Harris Street, Bayamón, PR 00957, **Pradyuta Padmanabhan** (pradyuta@gmail.com), 22407 Foxhound Ln, Middleburg, VA 20117, **Carmen Caiseda** (ccaiseda@bayamon.inter.edu), 500 Dr. John Will Harris Street, Bayamón, VA, and **Padhu Seshaiyer** (pseshaiy@gmu.edu), 4400 University Drive, MS 3F2, Fairfax, VA 22030. *Mathematical Modeling, Analysis and Simulation of the spread of gangs in the youth population of Puerto Rico*. Preliminary report.

Crime fighting and gang activity are controversial social issues. To prevent and minimize gang spread in the youth of Puerto Rico, a new mathematical model has been developed. Due to gang membership being treated as an infection, the SIR model (used in infectious disease modeling) by Kermack and McKendrick (1927) was employed to develop the governing differential equations. This new model accounts for the possibility of determining how members of the youth community interact with the infected adult (gang) community using 3 different mixing patterns: proportionate, preferred and like-with-like mixing. The numerical results of the implementation of the three different patterns were studied. Regardless of the mixing pattern, it was found that the greatest reduction of gang infection occurred when parameters such as gang activity, recruitment, conviction and rehab/recidivism rates, were varied in combination. A Graphical User Interface (GUI), integrated with the new gang mathematical model, was also developed that allows users to enter new data and do predictive analysis. (Received September 22, 2017)

1135-34-1428 **Joe Latulippe\*** (jlatulip@norwich.edu), Department of Mathematics, 158 Harmon Dr., Northfield, VT 05663, and **Joseph Minicucci**. *A mathematical model for Amyloid beta influenced calcium signaling through the IP<sub>3</sub> receptors*. Preliminary report.

Amyloid beta (A $\beta$ ) peptides have been linked to the pathogenesis of Alzheimer's disease by altering intracellular calcium regulatory mechanisms. In this study we use mathematical modeling and analysis to study the effects of A $\beta$  on calcium regulation by specifically tracking the contribution through the 1,4,5-Inositol-triphosphate (IP<sub>3</sub>) receptor. Our goal is to better understand how various levels of A $\beta$  affect the IP<sub>3</sub> production cascade and leads to subsequent calcium release through IP<sub>3</sub> receptors on the endoplasmic reticulum. We develop a closed-cell calcium model and show how we calibrated our model to existing experimental data. We also use global sensitivity analysis to understand how model parameters affect solutions. We further investigate the behavior of the model under various parameter regimes and explore how different levels of A $\beta$  affect model solutions. By linking experimental data with model solutions, we seek to precisely identify the physiological processes in the IP<sub>3</sub> production mechanism that are directly influenced by A $\beta$ . These results can then be used to better understand the role that A $\beta$  plays in altering intracellular calcium signals. (Received September 22, 2017)

1135-34-1433 **Michael A. Karls\*** (mkarls@bsu.edu), Department of Mathematical Sciences, Ball State University, Muncie, IN 47306. *Incorporating a Modeling First Approach into a Traditional ODE Course*.

In summer 2017, after teaching an introductory ODE course in a traditional format for many years, I decided to try incorporating a "modeling first" approach into my fall 2017 course, using projects from SIMIODE. In this talk I will outline the projects chosen and look at how they fit into the overall course structure, how they were introduced to the students, student reactions and responses to the projects, and the impact of the changes to the course on the students. I will also discuss what worked and what did not work and how the course could be modified for subsequent offerings. (Received September 22, 2017)

1135-34-1476 **Alessandro Arsie\*** (alessandro.arsie@utoledo.edu), The University of Toledo, 2801 W. Bancroft St, Toledo, OH 43606, and **Christian Ebenbauer**. *A generalization of LaSalle's invariance principle and applications*.

In this talk, I will present a generalization of LaSalle's invariance principle. In particular, I will show how it can be used to locate the  $\omega$ -limit set of a bounded solution of a given autonomous vector field  $f$  on a Riemannian

manifold. Assuming to know that the  $\omega$ -limit set  $\Omega$  is contained in an embedded submanifold  $S$  and using an auxiliary function that we call *height function*  $W$  for  $f$  and  $S$ , we show how to obtain a better estimate of the location of  $\Omega$  under mild assumptions. Several consequences and an application to a type of polynomial vector fields will be presented. (Received September 22, 2017)

1135-34-1501 **Renato Carlos Calleja\*** ([calleja@mym.iimas.unam.mx](mailto:calleja@mym.iimas.unam.mx)), Depto. Matemáticas y Mecánica, IIMAS, Admon. No. 20, Delegación Alvaro Obregón, 04510 Mexico City, Mexico, and **Eusebius Doedel** and **Carlos García Azpeitia**. *Symmetries and choreographies in families that bifurcate from the polygonal relative equilibrium of the n-body problem.*

In my talk I will describe numerical continuation and bifurcation techniques in a boundary value setting used to follow Lyapunov families of periodic orbits. These arise from the polygonal system of  $n$  bodies in a rotating frame of reference. When the frequency of a Lyapunov orbit and the frequency of the rotating frame have a rational relationship, the orbit is also periodic in the inertial frame. We prove that a dense set of Lyapunov orbits, with frequencies satisfying a Diophantine equation, correspond to choreographies. We present a sample of the many choreographies that we have determined numerically along the Lyapunov families and bifurcating families, namely for the cases  $n=4,6,7,8$  and, 9. We also present numerical results for the case where there is a central body that affects the choreography, but that does not participate in it. This is joint work with Eusebius Doedel and Carlos García Azpeitia. (Received September 22, 2017)

1135-34-1578 **Anca Radulescu\*** ([radulesa@newpaltz.edu](mailto:radulesa@newpaltz.edu)), **Caitlin Kennedy**, **Johanna Herron** and **Annalisa Scimemi**. *A mathematical model of coupled excitation and inhibition in a neuronal network for movement and reward.*

The neuronal circuit that relays information between the cortex, the basal ganglia nucleus of the striatum and the thalamus controls the execution of learnt motor tasks and stereotyped behaviors, anxiety and reward. This circuit, known as the cortico-striatal-thalamo-cortical (CSTC) pathway, is composed of a series of excitatory glutamatergic and inhibitory GABAergic synaptic connections. Hyperactivity in the CSTC pathway is implicated with the onset of neuropsychiatric disorders like obsessive compulsive disorder and Tourette's syndrome. Here we use a mathematical modeling approach to determine how generalized and local changes in balanced excitation and inhibition control the network dynamics of the CSTC pathway. Our findings indicate that local changes in inhibition of distinct clusters of striatal neurons can powerfully control the activity of the CSTC pathway. In contrast, local changes in the excitation of these cells do not exert a similarly powerful control of the activity of this network. These findings provide new insights into the network mechanisms that control the activity of neuronal circuits implicated with the onset of neuropsychiatric disorders. (Received September 23, 2017)

1135-34-1620 **Kenneth T.-R. McLaughlin**, **Robert Jenkins** and **Kyle Pounder\*** ([kpounder@math.arizona.edu](mailto:kpounder@math.arizona.edu)), Department of Mathematics, University of Arizona, 617 N Santa Rita Ave, Tucson, AZ 85721. *Nearly singular Jacobi matrices and applications to the finite Toda lattice.*

In this talk, we consider a singular limit for the inverse spectral problem for Jacobi matrices. The main results of the analysis are quite general estimates for the entries of a Jacobi matrix under certain assumptions about the relative sizes of the weights (norming constants). We apply these estimates to provide a detailed long time asymptotic analysis of the finite Toda lattice. The formulas we obtain improve upon the classical results of Moser by giving precise estimates of the associated error. Moreover, the Riemann-Hilbert techniques allow one, if they should so desire, to compute the complete asymptotic expansions for the various dynamical quantities. Finally, we apply our general estimates to study the time evolution of nearly singular Jacobi matrices under the Toda flow. (Received September 23, 2017)

1135-34-1712 **Yangyang Wang\*** ([wang.9737@mbi.osu.edu](mailto:wang.9737@mbi.osu.edu)), Jennings Hall 3rd Floor, 1735 Neil Ave., Columbus, OH 43210, and **Jeffery Gill**, **Hillel Chiel** and **Peter Thomas**. *Analysis for piecewise smooth models of a biological motor control system.* Preliminary report.

Motor systems must adapt to perturbations and changing conditions both within and outside the body. We refer to the ability of a system to maintain performance despite perturbations as "robustness". In this work, we explore how sensory feedback could alter a neuromechanical trajectory to enhance robustness for motor control. The underlying mechanism would be to adjust the timing of a particular phase of the trajectory and to vary the path of neuromechanical trajectories. As a concrete example, we focus on a piecewise smooth neuromechanical model of triphasic motor patterns in the feeding apparatus of the marine mollusk, *Aplasia californica*. To understand the mechanism of the robust sensory feedback control, we generalize variational and phase response analysis developed for stable limit cycle systems to the piecewise smooth neuromechanical system with hard boundary



conditions. Based on our analysis, we quantify the robustness of the Aplysia model to the applied perturbation and compare it to the experimental observations. (Received September 24, 2017)

1135-34-1762 **Leila Mirsaleh Kohan\*** (lmirsalehkohan@twu.edu) and **Ellina Grigorieva.**  
*Autoimmune Diseases Induced by Exposure to Radiation.* Preliminary report.

Autoimmune diseases can be developed by exposure to radiation. Ionizing radiation modifies the immune system and diminishing its normal ability to fight diseases. The extents of the modifications depend on the dose rate and duration of radiation exposure. This work employs mathematical simulations of autoimmune process dynamics under chronic irradiation. We have constructed a mathematical model consisting of four non-linear differential equations. The variables used in the modeling are the concentration of target cells of the tissue, concentration of cytotoxic T-lymphocytes against given cells, the concentration of tissue-specific antigen formed during the destruction of the target cells, and the concentration of T-suppressor cortical thymus. Utilizing the MAPLE program, we will illustrate that autoimmune processes could be accelerated by low dose rate in long chronic irradiation. This model can be expanded to simulate the dynamic of autoimmunity in mammals exposed to radiation. In this presentation, further results of our simulations and future work will be discussed. (Received September 24, 2017)

1135-34-1764 **Leila Mirsaleh Kohan\*** (lmirsalehkohan@twu.edu) and **Ellina Grigorieva.** *Modeling and Control of the Development of Autoimmune Disorder.*

Autoimmune diseases can be developed by exposure to radiation. Ionizing radiation modifies the immune system and diminishing its normal ability to fight diseases. The extents of the modifications depend on the dose rate and duration of radiation exposure. This work employs mathematical simulations of autoimmune process dynamics under chronic irradiation. We have constructed a mathematical model consisting of four non-linear differential equations. The variables used in the modeling are the concentration of target cells of the tissue, concentration of cytotoxic T-lymphocytes against given cells, the concentration of tissue-specific antigen formed during the destruction of the target cells, and the concentration of T-suppressor cortical thymus. Utilizing the MAPLE program, we will illustrate that autoimmune processes could be accelerated by low dose rate in long chronic irradiation. This model can be expanded to simulate the dynamic of autoimmunity in mammals exposed to radiation. In this presentation, further results of our simulations and future work will be discussed. (Received September 24, 2017)

1135-34-1790 **Yiyang Deng\*** (2017050@ctbu.edu.cn), Chongqing Nan'an District, No.19 Xuefu Avenue, Chongqing Chongqing Technology and Business U, Chongqing, and **Bingyu Li** and **Shingqing zhang.** *Some Notes on Four-body Co-circular Central Configurations.*

There are two symmetric families of four-body co-circular central configurations, namely the kite and isosceles trapezoid. Using mutual distances as coordinates, we prove that if the four-body central configuration is an isosceles trapezoid, then the diagonals of the isosceles trapezoid cannot be perpendicular to each other. Furthermore, we show that for any four-body co-circular central configuration, the diagonals of the quadrilateral cannot be perpendicular except that the configuration is a kite. For any convex non-collinear central configuration of the planar Newtonian 4-body problem with adjacent equal masses  $m_1 = m_2 \neq m_3 = m_4$ , with equal lengths for the two diagonals, we prove it must possess a symmetry and must be an isosceles trapezoid; furthermore, which is also an isosceles trapezoid when the length between  $m_1$  and  $m_4$  equals the length between  $m_2$  and  $m_3$ . (Received September 24, 2017)

1135-34-1815 **Marisabel Rodriguez Rodriguez\*** (marisabel@asu.edu), **Yun Kang** and **Robert E. Page Jr..** *Population and vitellogenin dynamics of a honeybee colony influencing division of labor.*

The complexity of honeybees has provided an ample study of different mechanisms affecting their population dynamics. Our goal is to provide further understanding of honeybees and underlying mechanisms of nutritional regulation influencing their age-based division of labor and sudden or gradual changes in the populations within a colony. These mechanisms are affected not only by intracolony processes but also outside factors such as weather or change of season. In this study, we present a non-linear differential equation system that models the population dynamics of brood, nurse bees, and foragers within a colony. The dynamics of these populations are influenced by the available stored pollen in cells and the current levels of vitellogenin (Vg) in the fat body of nurse bees. Analytical analysis and numerical simulations provide: (1) a honeybee colony can survive if the brood's feeding rate is less than the efficiency of pollen collection rate by foragers for Vg production; (2) decreasing the queen's feeding rate decreases population size of brood, nurse bees, and foragers; (3) synthesis of

Vg and low mortality rate of nurses are critical for colony survival; (5) high transition rate from nurse to forager is not colony sustainable in the long term. (Received September 25, 2017)

1135-34-1963 **Mohsen Razzaghi\*** ([razzaghi@math.msstate.edu](mailto:razzaghi@math.msstate.edu)), Department of Mathematics and Statistics, Mississippi State University, 175 President's Circle, Mississippi State, MS 39762.  
*An approximate method for solving fractional differential equations.*

Fractional differential equations (FDEs) are generalizations of ordinary differential equations to an arbitrary (non-integer) order. FDEs have attracted considerable interest because of their ability to model complex phenomena. Due to the extensive applications of FDEs in engineering and science, research in this area has grown significantly all around the world. Generally speaking, most of the FDEs do not have exact analytic solutions. Therefore, seeking numerical solutions of these equations is becoming more and more important. In this talk, an approximate method for solving fractional differential equations is presented. The method is based upon the fractional Taylor series approximations. The operational matrix for the fractional Taylor series is given. This matrix is then utilized to reduce the solution of the FDEs to a system of algebraic equations. The method is computationally very attractive and gives very accurate results. The numerical solutions are compared with available exact or approximate solutions in order to assess the accuracy of the proposed method. (Received September 25, 2017)

1135-34-2069 **Vladimir Dragovic\*** ([vladimir.dragovic@utdallas.edu](mailto:vladimir.dragovic@utdallas.edu)) and **Vasilisa Shramchenko.**  
*Algebraic-geometric approach to the Schlesinger systems: from Poncelet to Painlevé VI.*

A new method of construction of algebra-geometric solutions of rank two Schlesinger systems is presented. For an elliptic curve represented as a ramified double covering of  $CP^1$ , a meromorphic differential is constructed with the following property: the common projection of its two zeros on the base of the covering, regarded as a function of the only moving branch point of the covering, is a solution of a Painlevé VI equation. This differential provides an invariant formulation of a classical Okamoto transformation for the Painlevé VI equations. A generalization of this differential to hyperelliptic curves is also constructed. The corresponding solutions of the rank two Schlesinger systems associated with elliptic and hyperelliptic curves are constructed in terms of these differentials. The initial data for the construction of the meromorphic differentials include a point in the Jacobian of the curve, under the assumption that this point has non-variable coordinates with respect to the lattice of the Jacobian while the branch points vary. This method is motivated by an observation of Hitchin, who related the Poncelet polygons to algebraic solutions of a Painlevé VI equation. The research has been partially supported by the NSF grant 1444147. (Received September 25, 2017)

1135-34-2117 **Khanh Ngoc Dinh\*** ([kdinh@crimson.ua.edu](mailto:kdinh@crimson.ua.edu)), 2330 University Boulevard, Tuscaloosa, AL 35401, and **Roger Blaine Sidje.** *A comparison of the adaptive Magnus expansion method and other ODE solvers for the chemical master equation with time-dependent propensities.*

There has been increasing interest in recent years to solve the Chemical Master Equation (CME) directly to analyze the stochasticity of biochemical reacting systems, providing an alternative to Monte Carlo methods such as the Stochastic Simulation Algorithm (SSA) or First Reaction Method (FRM). The Finite State Projection (FSP) provides a framework for solving the CME, upon which different approaches have been developed to improve efficiency. These approaches are only applicable when the reaction rates remain constant. However, the reaction rates can change over time for some biological problems. One approach is to use the Magnus expansion to approximate the CME solution as a matrix exponential, for which the Krylov-based method EXPOKIT can be applied. We consider various adaptive time-step Magnus schemes, based on analysis of error and residual, and compare the adaptive Magnus-based methods with some current approaches, such as Adams-Basforth, Runge-Kutta and BDF, to compute the transient probability distributions of a transcriptional regulatory system in *E. coli*, where reaction rates change due to cell volume increase. The results prove the Magnus-based numerical methods to be an efficient ODE solver for large stiff time-dependent CME problems. (Received September 25, 2017)

1135-34-2201 **Yun Kang\*** ([yun.kang@asu.edu](mailto:yun.kang@asu.edu)), **K. Messan**, **G. DeGrandi-Hoffman** and **C. Castillo-Chavez.** *Population dynamics of honeybee: Effects of Disease, Parasites, and Dispersal Behavior.*

Honeybees are an amazing and highly beneficial insect species that play important roles in undisturbed and agricultural ecosystems. Unfortunately, honeybees are increasingly threatened by numerous factors including disease, parasite, dispersal behavior. In this talk, we will present a couple of models to explore: (1) What are the synergistic effects of parasitism and virus infections on honeybee population dynamics and its persistence. And (2) how foraging behavior of honeybees in the presence of varroa mite infestations affect the population dynamics of honeybees and mites, respectively. (Received September 25, 2017)

- 1135-34-2204 **Rachel Elizabeth TeWinkel\*** (tewinke2@uwm.edu), **Istvan Lauko** and **Gabriella Pinter**. *Equilibria Analysis for a Two-Population Epidemic Model with One Population Being a Reservoir for Infection.*

We consider a system of non-linear differential equations describing the spread of an epidemic between two populations. The model assumes one population spreads infection among itself and acts as a reservoir of infection for the second population. We explore the conditions under which the epidemic is endemic in both populations and discuss the global asymptotic stability of this endemic equilibrium using a Lyapunov function and the results of some limit theorems. We will present the results of numerical simulations and explore Monkeypox as an example of a disease that can be modeled in this way. (Received September 25, 2017)

- 1135-34-2296 **Yaqin Feng, John R Graef, Lingju Kong and Min Wang\*** (wangmin@rowan.edu). *The forward and inverse problems for a fractional boundary value problem.*

In this talk, we consider the forward and inverse problems for a fractional boundary value problem with Dirichlet boundary conditions. The existence and uniqueness of solutions for the forward problem is first proved. Then an inverse source problem is considered. (Received September 25, 2017)

- 1135-34-2581 **Jennifer A Anderson\*** (janderson34@twu.edu), Denton, TX, and **Ellina Grigorieva**. *Modeling Autoimmune Disease with Differential Equations.* Preliminary report.

We will build a mathematical model of a developed autoimmune process considering cell autoimmunity that plays the main role in any autoimmune disorder using a system of three non-linear differential equations. As model variables, we will use the concentration of target cells not bearing damage, concentration of cytotoxic T lymphocytes against given cells, and the concentration of the tissue-specific antigen formed because of the destruction of the target cells. All concentrations will be expressed in the moles per liter. We will investigate the model over the time interval  $[0, T]$  given either by months or days. (Received September 26, 2017)

- 1135-34-2657 **Benito Chen-Charpentier, Maria Leite\*** (mcleite@mail.usf.edu), **Orou Gaoue** and **Folashade Agosto**. *Plants Persistence under Multiple Stressors: Mathematical Modeling Approach.* Preliminary report.

Population viability analysis measures the likelihood of a population going extinct. It is often used to determine the impact of different endangered species management and their potential impacts of habitat loss. Plants, especially endangered species are usually affected by multiple stressors, including insects, and herbivores, environmental factors such as drought and heat, and other species. Here we present models based on systems of ordinary differential equations of two herbivores feeding (stressors) on the same plant species. The models incorporate several features: no competition between the herbivores, only intra-species competition, and intra as well as inter-species competition. In addition, the models integrate distinct types of multiple stressors: synergistic or antagonistic.

We investigate conditions for coexistence of the tree. Our results suggest that to obtain stable coexistence solutions the model needs to incorporate significant herbivore species interaction: inter- or intra-species competition or both. (Received September 26, 2017)

- 1135-34-2667 **Ravi P. Agarwal** (agarwal@tamuk.edu), Texas A&M University-Kingsville, Department of Mathematics, MSC 172, 700 University Blvd., Kingsville, TX 78363-8202, **Turker Ertem\*** (turkerertem@gmail.com), Toros University, Department of Industrial Engineering, Bahcelievler M., 16. Cadde, No:77, 33140 Mersin, Yenisehir, Turkey, and **Agacik Zafer** (agacik.zafer@aum.edu.kw), American University of the Middle East, College of Engineering and Technology, Department of Mathematics & Statistics, Egalia, Kuwait. *On asymptotic integration of higher order delay differential equations.*

We study the asymptotic integration problem for higher order nonlinear delay differential equations of the form  $L_n x(t) = f(t, x(g(t)))$ , where  $L_n$  is a linear operator. It is shown that solutions are asymptotic to prescribed solutions of  $L_n x = 0$  at infinity. (Received September 26, 2017)

- 1135-34-2725 **David J. Gerberry** and **Hem Raj Joshi\*** (joshi@xavier.edu), Department of Mathematics, Xavier University, Cincinnati, OH 45207-4441. *HIV epidemiology; mass incarceration and HIV incidence; male-female ratio;  $R_0$ .*

In this work, we develop a mathematical model of HIV epidemiology to explore a possible mechanism by which mass incarceration can lead to increased HIV incidence. The results are particularly relevant for the African American community in the United States that represents only 12% of the total population but accounts for 45% of HIV diagnoses and 40% of the incarcerated population. While most explanations of the link between mass

incarceration (or anything else that leads to a population with a low ratio of males to females) and higher HIV burden are based on the complicated idea of sexual concurrency,

we propose a much simpler mechanism based on the idea of sexual activity compensation. (Received September 26, 2017)

1135-34-2779 **Daniel H Burkow\*** (dburkow@asu.edu), Arizona State University, P.O. Box 873901, Tempe, AZ 85287. *Intermyocellular Lipids and the Progression of Muscular Insulin Resistance*. Preliminary report.

Diabetes is a disease characterized by reduced insulin action and secretion, leading to elevated blood glucose. In the 1990s, studies showed that intravenous injection of fatty acids led to a sharp negative response in insulin action that subsided hours after the injection. This dissertation is focused on the formulation of a model built around the known mechanisms of glucose and fatty acid storage and metabolism within myocytes. Data from euglycemic-hyperinsulinemic clamp with fatty acid infusion studies are used to validate the qualitative behavior of the model and estimate parameters. Analysis and numerical simulation of the model demonstrates critical bifurcations implicating nutrient excess as a driver of muscular insulin resistance. Finally, clinical applications of this class of models are discussed. (Received September 26, 2017)

1135-34-2926 **Katie Elliott\*** (katie\_elliott@baylor.edu), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76798-7328. *The Bessel Differential Equation and Left Definite Theory*.

The classical second-order Bessel differential equation in Sturm-Liouville form is  $-y''(x) + (\nu^2 - \frac{1}{4})x^{-2}y(x) = \lambda y(x)$  for all  $x \in (0, \infty)$ . We discuss the case when  $\nu = 0$  and explore the maximal and minimal operators generated by the Bessel differential expression. We also study the square of the Bessel differential expression through it's Friedrichs extension. We look at the Second Left-Definite Theory of the Bessel differential expression and show that for  $f \in H_2$ , the second left-definite Hilbert space associated with the Friedrichs extension of the Bessel differential expression on  $L^2(0, \infty)$ , and for  $\varepsilon > 0$ , we have  $\int_0^\infty |f''(x)|^2 dx \geq 2\varepsilon \int_0^\infty \left| \frac{f'(x)}{x} \right|^2 dx - \varepsilon(\varepsilon + 6) \int_0^\infty \left| \frac{f(x)}{x} \right|^2 dx$ . In particular, the choice of  $\varepsilon = 1/4$  gives the classic Bessel-Hardy inequality, thus arriving at this classic inequality from an alternate direction. (Received September 26, 2017)

1135-34-2957 **Jones M Mutua\***, Department of Mathematics and Statistics, 206 Haag Hall, 5100 Rockhill Rd, Kansas City, MO 64110, and **Alan S Perelson, Anil Kumar and Naveen K Vaidya**. *Modeling Immune Responses to HIV infection Under Drugs of Abuse*.

Drugs of abuse, such as opiates, have been widely associated with enhancing HIV replication, accelerating disease progression and diminishing host immune responses, thereby making it harder to effectively manage HIV infections. In this talk, I will present a mathematical model incorporating antibody responses to HIV infection under morphine conditioning. We further extend the model to study the effects of pharmacodynamic properties of morphine on HIV dynamics. Finally, we present analysis of our model to study how periodic morphine intake influences HIV dynamics within a host. (Received September 26, 2017)

1135-34-3017 **Vardayani Ratti\*** (vardayani.ratti@dartmouth.edu), **Peter G Kevan** and **Hermann J Eberl**. *Using Mathematical Model to Study How the Interplay Between Various Stressors Affects the Dynamics of Honeybee Colonies*.

The western honeybees are vanishing. Recent years have seen honeybees in distress, with up to 35% of colonies breaking down annually. There is no single cause that is believed to be responsible for the colony losses. In this talk, a mathematical model for the honeybee-varroa mite-virus complex is presented in which, based on division of labour, the bee population is divided into two categories: hive bees and forager bees. The model consists of ordinary differential equations for the dependent variables: uninfected hive bees, uninfected foragers, infected hive bees, virus-free mites, and virus-carrying mites. The main objective of the model is to study the interplay between disease propagation and division of labour in a honeybee colony. The model focuses on Acute Bee Paralysis Virus and is studied using a combination of analytical and computational techniques. (Received September 26, 2017)

1135-34-3068 **Daniel Brumley\***, dbrumley1@uco.edu, and **Britney Hopkins**. *The Existence of Solutions to an Even-Order Boundary Value Problem*.

We outline a method for proving the existence of multiple positive solutions to an even-order differential equation with nonhomogeneous boundary conditions. Beginning with a transformation of the problem into a system of second order differential equations satisfying homogeneous boundary conditions, our method culminates in

successive applications of the Guo-Krasnosel'skii Fixed Point Theorem to produce the desired result. (Received September 26, 2017)

1135-34-3151 **Veysel Fuat Hatipoglu\*** (veyselkuat.hatipoglu@mu.edu.tr), Mugla Sitki Kocman University, Fethiye Faculty of Management, Department of Business Administration, 48300 Mugla, Turkey, and **Martin Bohner**. *Cobweb Model with Fractional Order Derivatives*.

In this paper, the cobweb model in terms of fractional order derivatives reformulated. In particular, we describe a linear cobweb model by using fractional order derivatives. In the developed model we deal with conformable derivative as fractional derivative. Then the general solution as well as the stability criteria of the proposed model is given. Finally the developed model is illustrated with some examples. (Received September 26, 2017)

1135-34-3196 **Ted Theodosopoulos\*** (ttheodosopoulos@nuevaschool.org), 131 E. 28th Ave., San Mateo, CA 94403. *Modeling challenges for a high school course on differential equations*.

We present an advanced semester-long elective course for high school students that introduces elements from the theory of ordinary differential equations in an inquiry-driven environment through a sequence of modeling challenges. The challenges are drawn from across the physical, life and social sciences, and engineering. The topics investigated include first order equations, linear higher order equations and dynamical systems. The techniques that we explore include power series approximations, transform methods and phase diagrams. (Received September 27, 2017)

1135-34-3241 **Deborah Shutt\*** (shutt\_d@hotmail.com), Virginia Military Institute, **Sara Y Del Valle** (shutt\_d@hotmail.com), Los Alamos National Laboratory, **Carrie A Manore** (shutt\_d@hotmail.com), Los Alamos National Laboratory, **Stephen Pankavich** (shutt\_d@hotmail.com), Colorado School of Mines, and **Aaron T Porter** (shutt\_d@hotmail.com), Colorado School of Mines. *A Stochastic Model and Bayesian Estimation of Important Outbreak Characteristics for Zika in South and Central America*

As South and Central American countries prepare for increased birth defects from Zika virus outbreaks and plan for mitigation strategies to minimize ongoing and future outbreaks, understanding important characteristics of Zika outbreaks and how they vary across regions is important. We developed a mathematical model for the 2015/2016 Zika outbreak in South and Central America. We fit the model to publicly available data, using Approximate Bayesian Computation to estimate parameter distributions and provide uncertainty quantification. An important model input is the at-risk susceptible population, which can vary with a number of factors including climate, elevation, population density, and socio-economic status. We informed this initial condition using the highest historically reported dengue incidence. We then estimated the basic reproduction number, or the expected number of new human infections arising from a single infected human, to range between 4 and 6 in most South American regions. We estimated the reporting rates and the number of pregnant women that were infected with Zika. The uncertainty in parameter estimates highlights a need for research and data collection that will better constrain parameter ranges. (Received October 02, 2017)

1135-34-3242 **Carrie Manore\*** (cmanore@lanl.gov), Los Alamos National Laboratory, **Todd Graham** (cmanore@lanl.gov), University of Portland, **Alexa Carr** (cmanore@lanl.gov), University of Portland, **Alicia Feryn** (cmanore@lanl.gov), University of Portland, **Hannah Calendar** (cmanore@lanl.gov), University of Portland, **Harshini Mukundan** (cmanore@lanl.gov), Los Alamos National Laboratory, and **Shailja Jakhar** (cmanore@lanl.gov), University of Portland. *Point of Care Diagnostic Deployment and Targeted Treatment of Antimicrobial Resistant Non-Typhoidal Salmonella: a New Mathematical Model and Cost-Benefit Analysis*

Invasive non-typhoidal Salmonella (NTS) is among the leading causes of blood stream infections in sub-Saharan Africa and other developing regions. Invasive NTS can be difficult to treat and have high case-fatality rates, in part due to emergence of strains resistant to broad-spectrum antibiotics. Furthermore, improper treatment contributes to increased antibiotic resistance and death. Point of care (POC) diagnostic tests that identify infection within a short time and differentiate between resistant and non-resistant NTS strains may improve patient outcomes and decrease resistance at the community level. We developed the first model for NTS dynamics in high-risk populations to analyze the potential advantages and disadvantages of POC diagnostic deployment and resulting impact for patients. Our analysis strongly supports the use of POC diagnostics coupled with targeted antibiotic use for all patients upon arrival in the clinic for optimal patient and public health outcomes. We show that even the use of imperfect POC diagnostics can significantly reduce both total costs and number of deaths as compared to no diagnostics. (Received October 02, 2017)

## 35 ► *Partial differential equations*

1135-35-14

**Jerome Goddard II, Quinn A Morris\*** (qmorris1@swarthmore.edu), **Catherine Payne, R Shivaji and Byungjae Song.** *Steady States of Reaction-Diffusion Equations with U-Shaped Density Dependent Dispersal on the Boundary.*

We consider positive solutions to equations of the form

$$\begin{cases} -\Delta u = \lambda u(1-u), & x \in \Omega, \\ \frac{\partial u}{\partial \eta} + \gamma \sqrt{\lambda} g(u)u = 0, & x \in \partial\Omega, \end{cases}$$

where  $\lambda, \gamma > 0$  are parameters,  $g : [0, 1] \rightarrow [0, \infty)$  is a nonlinear function, and  $\Omega$  is a bounded domain in  $\mathbb{R}^N$ ,  $N \geq 1$ . Such problems arise in the study of population dynamics in a habitat  $\Omega$  when the population exhibits nonlinear density dependent dispersal on the boundary. We analyze the persistence of the population (existence, non-existence, uniqueness, and multiplicity of positive steady states) as the patch size ( $\lambda$ ) and hostility of the outside matrix ( $\gamma$ ) vary. We obtain our results via the method of sub-super solutions. (Received August 25, 2017)

1135-35-66

**Viktoria Savatorova\*** (viktoria.savatorova@unlv.edu) and **Aleksei Talonov.** *Multi-scale modeling of high frequency wave propagation in heterogeneous medium with cracks.*

In this work we perform multi-scale modeling of small amplitude wave propagation in pre-loaded heterogeneous media with cracks. We suppose the existence of several length scales: the smallest microscale defining the characteristic size of cracks, the mesoscale defining the characteristic size of periodic distribution of heterogeneities, and the macroscale which can be defined as a global characteristic size. We assume cracks to be isolated, randomly oriented, with periodic distribution of concentration of cracks. We start with studying open cracks, and then consider closed cracks taking into account the Coulomb friction between crack faces. When the wavelength of a travelling signal becomes comparable with the mesoscale's characteristic size, successive reflections and refractions of the waves lead to the formation of a complicated sequence of the pass and stop frequency bands. We present an asymptotic procedure based upon a two-scale approach to derive wave dispersion, identify pass and stop frequencies, and study how displacements and wave velocities depend on averaged concentration and distribution of cracks, direction of wave propagation, and an external load. (Received July 17, 2017)

1135-35-80

**Ugur G. Abdulla, Jian Du and Adam L. Prinkey\*** (aprinkey2009@my.fit.edu), 150 W. University Blvd., Melbourne, FL 32901. *Evolution of Interfaces for the Nonlinear Double Degenerate Parabolic Equation of Turbulent Filtration with Absorption.*

This paper presents a full classification of the short-time behavior of the interfaces and local solutions to the nonlinear double degenerate reaction-diffusion equation of turbulent filtration with strong absorption

$$u_t = \left( |(u^m)_x|^{p-1} (u^m)_x \right)_x - bu^\beta, \quad mp > 1, \beta > 0$$

The interface may expand, shrink or remain stationary, as a result of the competition of the diffusion and reaction terms near the interface, expressed in terms of the parameters  $m, p, \beta, \text{sign} b$  and asymptotics of the initial function near its support. In all cases we prove explicit formula for the interface and local solution with accuracy up to constant coefficients. The methods of the proof are based on nonlinear scaling laws, and a barrier technique using special comparison theorems in irregular domains with characteristic boundary curves. Numerical analysis using a weighted essentially nonoscillatory (WENO) scheme is pursued and comparison of numerical and analytical results is presented. (Received July 24, 2017)

1135-35-82

**Xiaolong Han\*** (xiaolong.han@csun.edu), California State University, Northridge, Northridge, CA 91330, and **Michael Murray and Chuong Tran.** *Nodal curves of eigenfunctions in the disc.*

In this talk, we will discuss the geometric properties of nodal curves of Laplacian eigenfunctions in the disc, particularly their relations with the geodesics. (Received July 24, 2017)

1135-35-85

**Ugur G. Abdulla** (abdulla@fit.edu), 150 W University Blvd, Melbourne, FL 32901, and **Roqia A. Jeli\*** (rjeli2011@my.fit.edu), 506 Golden dove AVE NE, Palm Bay, FL 32907. *Evolution of Interfaces for the Nonlinear Parabolic p-Laplacian Type Reaction-Diffusion Equations.*

We present a full classification of the short-time behavior of the interfaces and local solutions to the nonlinear parabolic p-Laplacian type reaction-diffusion equation of non-Newtonian elastic filtration

$$u_t - \left( |u_x|^{p-2} u_x \right)_x + bu^\beta = 0, \quad p > 1, \beta > 0$$

The interface may expand, shrink, or remain stationary as a result of the competition of the diffusion and reaction terms near the interface, expressed in terms of the parameters  $p, \beta$ , *sign*  $b$ , and asymptotic of the initial function near its support. In all cases, we prove the explicit formula for the interface and the local solution with accuracy up to constant coefficients. The methods of the proof are based on nonlinear scaling laws, and a barrier technique using special comparison theorems in irregular domains with characteristic boundary curves. The results are published in *European Journal of Applied Mathematics, Volume 28, 5(2017)*. (Received July 25, 2017)

1135-35-103 **Ugur Abdulla, Evan Cosgrove and Curtis Earl\*** (cear12013@my.fit.edu), 150 W. University Blvd, Melbourne, FL 32901, and **Jonathan Goldfarb**. *On the optimal control of the Stefan type parabolic free boundary problems with state constraints.*

We consider the inverse Stefan type free boundary problem for the general second order parabolic PDE under state constraints in the form of the upper bound for the temperature. This problem arises in bioengineering problem about the laser ablation of the skin. We implement optimal control framework following a variational formulation developed in *U. G. Abdulla, Inverse Problems and Imaging, 7,(2013),307-340 & 10(2016),869-898*, with penalty functional added due to state constraint. We pursue space-time discretization and prove the convergence of the sequence of discrete optimal control problems to the continuous optimal control both with respect to functional and control. We prove the Frechet differentiability in Besov spaces and derive the formula for the Frechet differential under minimal assumptions on the data. (Received July 27, 2017)

1135-35-105 **Ugur G. Abdulla and Habeeb Aal Rkhais\*** (haalrkhais2014@my.fit.edu). *On the Qualitative Theory of the Nonlinear Degenerate Parabolic Equations of the Reaction-Diffusion-Convection Type.*

We apply the methods developed in papers *U.G. Abdulla, J. Diff. Eq., 164, 2000, 321-354; U.G. Abdulla & J.King, SIAM J. Math. Anal., 32, 2(2000), 235-260*, to solve the problem on the initial development and asymptotics of the interfaces and local solutions near the interfaces for the reaction-diffusion-convection equation

$$u_t - (u^m)_{xx} + a(u^\gamma)_x + bu^\beta, \quad m > 1, \gamma, \beta > 0, a, b \in \mathbb{R}$$

with compactly supported initial function. Depending on the relative strength of three competing forces such as diffusion, convection, and reaction, the interface may expand, shrink or remain stationary. We pursue full classification of the asymptotics of the interfaces and local solutions in terms of parameters  $m, \gamma, \beta$ , *sign*  $a$ , *sign*  $b$ . The methods used are rescaling and blow-up techniques for the identification of the asymptotics of the solution along the class of interface type curves, construction of the barriers and application of the comparison theorem in non-cylindrical domains with characteristic boundary curves. (Received July 27, 2017)

1135-35-106 **Ugur G. Abdulla and Lamees Alzaki\*** (lalzaki2013@my.fit.edu). *Analysis of Interfaces for the Nonlinear Degenerate Second Order Parabolic Equations Modeling Diffusion-Convection Processes.*

We consider Cauchy problem with compactly supported initial function for the nonlinear degenerate second order parabolic PDE

$$u_t = (u^m)_{xx} + b(u^\gamma)_x, \quad m > 1, \gamma > 0, b \in \mathbb{R}$$

modeling diffusion-convection processes arising in fluid or gas flow in a porous media, plasma physics, population dynamics in mathematical biology and other applications. Due to the property of the finite speed of propagation the problem develops interfaces or free boundaries separating the region where solution is positive from the region where it vanishes. We present full classification of the short-time behaviour of the interfaces and local solutions near the interfaces. The interface may expand, shrink, or remain stationary as a result of the competition of the diffusion and convection forces near the interface, expressed in terms of the parameters  $m, \gamma$ , *sign*  $b$ , asymptotics of the initial function near its support, and whether interface is the right or left boundary curve. In all cases, we prove the explicit formula for the interface and the local solution with accuracy up to constant coefficients. The methods of the proof are based on nonlinear scaling laws, and a barrier technique using special comparison theorems in irregular domains with characteristic boundary curves. (Received July 27, 2017)

1135-35-126 **Ugur G Abdulla and Amna Ali.S Abu weden\*** (aabuweden2014@my.fit.edu). *Some Free Boundary Problems for the Nonlinear Degenerate Multidimensional Parabolic Equations Modeling Reaction-Diffusion Processes.*

This paper presents a full classification of the short-time behavior of the interface in the Cauchy problem for the nonlinear second order degenerate parabolic PDE

$$u_t - \Delta u^m + bu^\beta = 0, \quad x \in \mathbb{R}, t > 0$$

with nonnegative and radially symmetric initial function  $u_0$  such that

$$\text{supp } u_0 \subset \{|x| < R\}, \quad u_0 \sim C(R - |x|)^\alpha, \quad \text{as } |x| \rightarrow R - 0,$$

where  $m > 1, C, \alpha, \beta > 0, b \in \mathbb{R}$ . Interface surface  $t = \eta(x)$  may shrink, expand or remain stationary depending on the relative strength of the diffusion and reaction terms near the boundary of support, expressed in terms of the parameters  $m, \beta, \alpha, \text{sign } b$  and  $C$ . In all cases we prove explicit formula for the interface asymptotics, and local solution near the interface. (Received July 30, 2017)

1135-35-177 **Eric C. Stachura\*** ([estachura@haverford.edu](mailto:estachura@haverford.edu)). *Existence of weak solutions to refraction problems in negative refractive index materials.*

Weak solutions to refraction problems are studied when one material has a negative refractive index. It is shown how to construct an optical interface between two media such that, given a fixed direction  $m \in S^2$ , all monochromatic light rays emanating from the first medium are refracted into rays parallel to  $m$ . Methods from optimal mass transport are used to prove existence of weak solutions. Finally, the PDE associated with the surface is shown to be of Monge-Ampère type. (Received August 07, 2017)

1135-35-182 **Erhan Bayraktar** and **Christian Keller\*** ([ckell@umich.edu](mailto:ckell@umich.edu)), Department of Mathematics, University of Michigan, 2074 East Hall, 530 Church Street, Ann Arbor, MI 48109. *Path-dependent Hamilton-Jacobi equations in infinite dimensions.*

We propose notions of minimax and viscosity solutions for a class of fully nonlinear path-dependent PDEs with nonlinear, monotone, and coercive operators on Hilbert space. Our main result is well-posedness (existence, uniqueness, and stability) for minimax solutions. A particular novelty is a suitable combination of minimax and viscosity solution techniques. Thereby, we establish a comparison principle for path-dependent PDEs under conditions that are weaker even in the finite-dimensional case. In contrast to most of the related works on PDEs in infinite dimensions, perturbed optimization is entirely avoided. The path-dependent setting itself enables us to circumvent the lack of compactness in infinite-dimensional Hilbert spaces. As an application, our theory makes it possible to employ the dynamic programming approach to study optimal control problems for a fairly general class of (delay) evolution equations in the variational framework. Furthermore, differential games associated to such evolution equations can be investigated following the Krasovskii-Subbotin approach similarly as in finite dimensions. (Received August 07, 2017)

1135-35-204 **Anthony Guzman\***, Department of Mathematics and Statistics, 111 Cummington Mall, Boston University, Boston, MA 02215, and **Ryan Vogt**, School of Mathematics, 206 Church St SE, University of Minnesota, Minneapolis, MN 55455. *Observability Analysis and Estimator Design for a Cardiac Cell Model.*

Certain cellular variables, such as ionic concentrations and gating states, are thought to be critical to the formation of dangerous cardiac arrhythmias, but some of these quantities may be difficult or impossible to measure directly during in vitro experiments. We examined the Luo-Rudy dynamic (LRd) model, which is a 17th-order nonlinear ODE model of the action-potential dynamics of a cardiac cell, as a basis for reconstructing important cellular variables. To determine whether measurements of any individual dynamical variable were sufficient to estimate the remaining variables, we used a Matlab-based numerical linearization approach to analyze a model property called observability. The observability results showed that membrane potential was sufficient for estimating the other dynamical variables in the model, and that strength of observability tended to increase with increasing heart rate. We adjusted several LRd parameters to amplify the alternans already present in the model, and showed that this parametric variation had a minimal impact on the observability results. For selected scenarios, Kalman filters were designed and tested with simulated data to check the effectiveness of reconstructing unmeasured variables from membrane potential and other quantities. (Received August 09, 2017)

1135-35-323 **Daniel G Marvin\*** ([danielmarvin09@gmail.com](mailto:danielmarvin09@gmail.com)), 8614 Manchester Rd #7, Silver Spring, MD 20901. *An Analysis of Traffic Flow with Laplacian Constraints in Bounded Space.* Preliminary report.

With the population continuing to increase in the metropolitan area of the District of Columbia (D.C.), traffic reduction will remain a top priority for regional policy makers in the foreseeable future. One solution has resulted in an incorporation of traffic circles to minimize stand-still time while also minimizing negative traffic incidents, such as collisions and fatalities.

A mathematical approach which offers an interesting perspective on the system is modeling the traffic flow as an incompressible and irrotational fluid as defined by two of Laplace's equations often used in Fluid Dynamics.



We pose several questions regarding optimizing efficiency versus safety and study how variations in the boundary conditions, given by variations in geometry and physical constraints, affect the resulting traffic flow.

Traffic circles and traffic flow were studied as a conservative system modeled as an incompressible fluid within a finite-dimensional, bounded, vector space. Through analysis of the space, the notion that symmetry is the key factor in optimization with respect to both efficiency and safety was mathematically yielded given defined constraints. (Received August 23, 2017)

1135-35-334 **Michael E Filippakis\*** (mfilip@unipi.gr), Department of Digital Systems, 122 Grigoriou Labraki Str, 18532 Piraeus-Greece, 18532 Piraeus, Greece. *Nodal solutions for Robin problems.*

We consider a semilinear Robin problem driven by the negative Laplacian plus an indefinite, unbounded potential. The reaction term is a Caratheodory function of arbitrary structure outside an interval  $[-c, c]$  ( $c > 0$ ), odd on  $[-c, c]$  and concave near zero. Using a variant of the symmetric mountain pass theorem, together with truncation, perturbation and comparison techniques, we show that the problem has a whole sequence  $\{u_n\}_{n \geq 1}$  of distinct nodal solutions converging to zero in  $C^1(\overline{\Omega})$ . The publication of this paper has been partly supported by the University of Piraeus Research Center. (Received August 24, 2017)

1135-35-352 **Mihaela Ifrim\*** (ifrim@wisc.edu) and **John Hunter** (hunter@math.ucdavis.edu). *A normal form flow for two dimensional water waves.*

This article is concerned with the infinite depth water wave equations in two space dimensions. We consider this problem expressed in Eulerian coordinates. Viewing this problem as a quasilinear dispersive equation, we establish two results: (i) we produce a Hamiltonian flow which acts like a normal form that modifies the water wave equations into a new equivalent system with only cubic and higher order terms. (ii) further, using this idea, we establish local well-posedness and cubic lifespan bounds for solutions with small localized data. Neither of these results are new; they have been recently obtained by Wu, Alazard-Burq-Zuily, Hunter-Ifrim-Tataru using different coordinates and methods. Instead our goal is improve the understanding of this problem by providing a different approach for normal forms for water waves. (Received September 26, 2017)

1135-35-418 **Avner Peleg\*** (avpeleg@gmail.com), **Quan M Nguyen**, **Debananda Chakraborty** and **Toan T Huynh**. *Using analysis of fast collisions between solitons of the nonlinear Schrödinger (NLS) equation for mathematical modeling of pulse propagation in broadband optical waveguide systems.*

I will present a method for calculating the effects of weak perturbations on a single fast collision between two fundamental solitons of the NLS equation. The method is based on obtaining an equation for the collision-induced change in the envelopes of the colliding solitons and then projecting both sides of the equation on the eigenfunctions of the linear operator describing weak perturbations about the NLS soliton. I will illustrate how the method is used for calculating the collision-induced changes in the soliton parameters and shape. Furthermore, I will show how the results of the single-collision analysis can be used in constructing deterministic and stochastic mathematical models for pulse propagation in broadband optical waveguide systems. (Received September 01, 2017)

1135-35-430 **Peter Hintz** and **Andras Vasy\*** (andras@math.stanford.edu), 450 Serra Mall, Bldg 380, Stanford, CA 94305-2125. *Microlocal analysis for Kerr-de Sitter black holes.*

In this lecture I will describe a framework for the Fredholm analysis of non-elliptic problems both on manifolds without boundary and manifolds with boundary, with a view towards wave propagation on Kerr-de Sitter spaces, which is the key analytic ingredient for showing the stability of black holes. This lecture focuses on the general setup such as microlocal ellipticity, real principal type propagation, radial points and generalizations, as well as (potentially) normally hyperbolic trapping, as well as the role of resonances. (Received September 01, 2017)

1135-35-432 **Dung Le\***, Department of Mathematics, One UTSA Circle, San Antonio, TX 78249. *On multi-species and cross diffusion.*

Reaction diffusion systems have been used in biological/ecological models and ignored the diffusion rates of individuals of species depend on the concentration of individuals of the same species (self-diffusion), or of the other species (cross diffusion). We will discuss the introduction of cross diffusion, the models, their analysis and open problems. (Received September 01, 2017)

1135-35-457 **Peter Hintz\***, University of California, Berkeley, Department of Mathematics, 805 Evans Hall, Berkeley, CA 94720, and **Andras Vasy**, Stanford University, Department of Mathematics, 380 Serra Mall, Stanford, CA 94305. *The stability of Kerr-de Sitter black holes.*

I will discuss Kerr-de Sitter black holes, which are rotating black holes in a universe with a positive cosmological constant, i.e. they are explicit solutions (in 3+1 dimensions) of Einstein’s equations of general relativity. They are parameterized by their mass and angular momentum.

I will discuss the geometry of these black holes, and then talk about the stability question for these black holes in the initial value formulation. Namely, appropriately interpreted, Einstein’s equations can be thought of as quasilinear wave equations, and then the question is if perturbations of the initial data produce solutions which are close to, and indeed asymptotic to, a Kerr-de Sitter black hole, typically with a different mass and angular momentum. In this talk, I will emphasize geometric aspects of the stability problem, in particular showing that Kerr-de Sitter black holes with small angular momentum are stable in this sense. (Received September 04, 2017)

1135-35-468 **Stephen C Preston** and **Alejandro Sarria\*** (asarria@ung.edu), 175 Nugget Rd, Dahlonoga, GA 30533. *Lagrangian aspects of the axisymmetric Euler equation.*

We discuss new blowup results for the axisymmetric three-dimensional Euler equations with swirl on a cylinder. Working with the equations in Lagrangian form along either the axis or the boundary and imposing oddness on the vertical component of the flow, we extend some blowup criteria due to Chae, Constantin and Wu related to assumptions on the sign of the pressure Hessian. (Received September 05, 2017)

1135-35-489 **Sukjung Hwang** (sukjung\_hwang@yonsei.ac.kr), 50 Yonsei-ro, Seodaemun-gu, Department of Mathematics, Yonsei University, Seoul, 03722, South Korea, and **Seick Kim\*** (kimseick@yonsei.ac.kr), 50 Yonsei-ro, Seodaemun-gu, Department of Mathematics, Yonsei University, Seoul, 03722, South Korea. *Green’s function for second order elliptic equations in non-divergence form.*

We consider second order elliptic equations in non-divergence form with the coefficients whose mean oscillation satisfies Dini’s condition. We construct the Green’s function for the operator in bounded  $C^{1,1}$  domains in  $\mathbb{R}^n$  with  $n \geq 3$ , and show that the Green’s function and its first and second derivatives have the natural pointwise bounds. (Received September 06, 2017)

1135-35-497 **Vincent R Martinez\*** (vmartin6@tulane.edu), Mathematics Department, Tulane University, 6823 St. Charles Ave, New Orleans, LA 70118, **Nathan Glatt-Holtz**, Mathematics Department, Tulane University, 6823 St. Charles Ave, New Orleans, LA 70118, and **Geordie Richards**, 3900 Old Main Hill, Logan, UT 84322. *Asymptotic Coupling and Uniqueness of Invariant Measures in Hydrodynamic Equations.* Preliminary report.

In their 1967 seminal paper, Foias and Prodi captured precisely a notion of finitely many degrees of freedom in the context of the two-dimensional (2D) incompressible Navier-Stokes equations (NSE). In particular, they proved that if a sufficiently large low-pass filter of the difference of two solutions converge to 0 asymptotically in time, then the corresponding high-pass filter of their difference must also converge to 0 in the infinite-time limit. In other words, the high modes are “eventually enslaved” by the low modes. One could thus define the number of degrees of freedom to be the smallest number of modes needed to guarantee this convergence for a given flow, insofar as it represents a solution to the NSE. This property has since led to several developments in the long-time behavior of solutions to the NSE, such as, for instance, to data assimilation, and existence of determining forms. In this talk, we will discuss this asymptotic enslavement property with regards to the issue of uniqueness of invariant measures for stochastically forced equations, specifically those in the context of hydrodynamic and related equations. (Received September 06, 2017)

1135-35-498 **Vincent R Martinez\*** (vmartin6@tulane.edu), New Orleans, LA 70118, and **Michael Jolly**, **Tural Sadigov** and **Edriss Titi**. *Existence of a determining form for the subcritical surface quasi-geostrophic equation.*

The existence of an inertial manifold for the 2D incompressible Navier-Stokes equations (NSE) remains an outstanding open problem. When restricted to this manifold, the dynamics of the original system reduces to that of a finite-dimensional ordinary differential equation (ODE), known as an “inertial form,” in a finite-dimensional phase space. In a series of works by C. Foias, M. Jolly, R. Kravchenko, and E. Titi, it was nevertheless shown that one can embed the global attractor of the 2D NSE into that of an ODE, known as a “determining form,” but over an infinite-dimensional phase space. That determining forms exist for some other weakly dissipative

systems, e.g., 1D damped-driven Korteweg de Vries or nonlinear Schrodinger equations, for which the existence of an inertial manifold is still unresolved, has also been proved recently by M. Jolly, T. Sadigov, and E. Titi. In this lecture, we show that this is the case for the subcritically dissipative surface quasi-geostrophic equation as well. To do so, we establish appropriate a priori bounds and a “Foias-Prodi” type phenomenon, in which high modes of the solution are asymptotically enslaved to its low modes. This is accomplished via De Giorgi techniques and elementary harmonic analysis. (Received September 27, 2017)

1135-35-502 **Roman Shvydkoy\***, 851 S Morgan St, M/C249, Chicago, IL 60521. *On flocking and global well-posedness of singular models of self organized dynamics.*

We introduce a new class of models arising in studies of self-organized dynamics of agents. These models belong to the class of fractional parabolic equations with (critical) drift. (Received September 06, 2017)

1135-35-504 **Roman Shvydkoy\***, 851 S Morgan St, M/C 249, Chicago, IL 60607. *Energy law for the type I blowup of solutions to the 3D Navier-Stokes system.*

We prove that the natural energy balance relation survives a type I in time blowup of solutions to the Navier-Stokes system. The proof uses an interaction scheme based on local energy inequality. This is a joint work with Trevor Leslie. (Received September 06, 2017)

1135-35-507 **M Grillakis** and **M Machedon\*** (mxm@math.umd.edu), Department of Mathematics, University of Maryland, College Park, MD 20742. *Title: On the pair excitation function.*

We will review the history of the rigorous theory of the pair excitation function approximating the evolution of a coherent state, and describe the most recent results of M. Grillakis and M. Machedon for a Hamiltonian with two body interaction potential  $N^{3\beta-1}v(N^\beta(x-y))$ , with  $\beta < 2/3$ . (Received September 06, 2017)

1135-35-548 **Irene M. Gamba\*** (gamba@math.utexas.edu), Department of Mathematics and, Institute of Computational Engineering and Sc, The University of Texas at Austin, Austin, TX 78712-1082. *The Landau equation: Analysis and Approximations to collisional plasmas.*

We will discuss recent progress on the rigorous analytical issues of constructing solutions for the non-linear Landau equation as limiting ones for the Boltzmann equation for elastic binary particle interactions, in the grazing collision limit, for Coulomb potentials. Approximations and computational models for this problem are at the core of collisional plasma theories. In particular we will discuss several aspects of conservative solvers for the kinetic transport equations of particle interactions, that involve both the Boltzmann and its approximate Landau equation, by means of operator splitting that hybridize energy conservative Discontinuous Galerkin schemes for the transport advective part, and a fast energy conservative Lagrangian spectral solver for the collisional part, i.e. the Boltzmann or Landau operators. The algorithm links both schemes by a projection method. The evolution of the energy conservative Lagrangian spectral solver is shown to satisfy error estimates and to converge to the expected Maxwellian equilibrium density in the long time limit. (Received September 07, 2017)

1135-35-558 **Janak R Joshi\*** (janakraj.joshi@my.unt.edu), 2301 West Prairie Street, Apt#2, Denton, TX 76201, and **Joseph Iaia**. *Existence of Solutions for Semilinear Neumann problems with prescribed number of zeros on exterior domains.*

In this paper we prove the existence of an infinite number of radial solutions of  $\Delta u + K(r)f(u) = 0$  with Neumann-like boundary conditions on the exterior of the ball of radius  $R$  centered at the origin in  $\mathbb{R}^N$  such that  $\lim_{r \rightarrow \infty} u(r) = 0$  with prescribed number of zeros where  $f : \mathbb{R} \rightarrow \mathbb{R}$  is odd and there exists a  $\beta > 0$  with  $f < 0$  on  $(0, \beta)$ ,  $f > 0$  on  $(\beta, \infty)$  with  $f$  superlinear for large  $u$ , and  $K(r) \sim r^{-\alpha}$  with  $0 < \alpha < 2(N-1)$ . (Received September 08, 2017)

1135-35-571 **Tarek M. Elgindi** and **In-Jee Jeong\*** (in-jee\_jeong@alumni.brown.edu), 304 Fine Hall, Princeton, NJ 08544. *Finite-time singularity formation for the Boussinesq system.*

We consider the 2D Boussinesq system on sectors with angle less than  $\pi$ , and show that there exists Lipschitz continuous velocity field and density pair  $(u_0, \rho_0)$  which becomes singular in finite time. The initial data can be compactly supported and in particular the solution has finite energy. The proof consists of three parts: local well-posedness for the Boussinesq equation in critical spaces, the analysis of exactly scale-invariant solutions, and finally a cut-off argument. Moreover, when the angle of the sector is less than  $\pi/2$ , we show that the singularity can occur for  $C^\infty$  and compactly supported initial data. We remark that the 2D Euler equation is globally well-posed in all of these situations, so that the singularity is not coming from the domain or the lack of smoothness on the data. We then discuss implications of these results to the issue of singularity formation for the 3D Euler equations in the axisymmetric geometry. (Received September 08, 2017)

1135-35-574 **Pablo Raúl Stinga\*** ([stinga@iastate.edu](mailto:stinga@iastate.edu)). *How to approximate the fractional Laplacian by the fractional discrete Laplacian.*

We use the solution to the semidiscrete heat equation in combination with the language of semigroups to define and obtain the pointwise formula for the fractional powers of the discrete Laplacian on a mesh of size  $h > 0$ . It is shown that solutions to the continuous fractional Poisson equation  $(-\Delta)^s U = F$  can be approximated by solutions to the fractional discrete Dirichlet problem  $(-\Delta_h)^s u = f$  in  $B_R^h$ ,  $u = 0$  in  $\mathbb{Z}_h \setminus B_R^h$ . We obtain error estimates in the strongest possible norm, namely, the  $L^\infty$  norm, under minimal natural Hölder regularity assumptions. Key ingredients for the analysis are the regularity estimates for the fractional discrete Laplacian, which are of independent interest. (Received September 08, 2017)

1135-35-601 **Suncica Canic\***, Department of Mathematics, University of Houston, 4800 Calhoun Road, Houston, TX 77025, and **Boris Muha** and **Martina Bukac**. *Existence of weak solutions to a class of fluid-structure interaction problems involving composite structures.*

Fluid-structure interaction problems with composite structures arise in many applications. One example is the interaction between blood flow and arterial walls. Arterial walls are composed of several layers, each with different mechanical characteristics and thickness. No mathematical results exist so far that analyze existence of solutions to nonlinear, fluid-structure interaction problems in which the structure is composed of several layers. In this talk we will summarize the main difficulties in studying this class of problems, and present a computational scheme based on which a proof of the existence of a weak solution was obtained. Our results reveal a new physical regularizing mechanism in FSI problems: inertia of the thin fluid-structure interface with mass regularizes evolution of FSI solutions. Implications of our theoretical results on modeling the human cardiovascular system will be discussed. (Received September 10, 2017)

1135-35-668 **Dashan Fan** and **Jingchun Chen\*** ([jingchun@uwm.edu](mailto:jingchun@uwm.edu)). *Long time decay estimates in Triebel-Lizorkin spaces for evolution equations with structural dissipation.* Preliminary report.

In this paper, we employ the generalized multiplier theorem and duality theorem to derive the estimates in Triebel-Lizorkin spaces for the Cauchy problem for evolution operators with structural dissipation. The obtained estimates are a natural extension of the known results in real Hardy spaces for these models. (Received September 12, 2017)

1135-35-705 **Dipendra Regmi\*** ([dipendra.regmi@ung.edu](mailto:dipendra.regmi@ung.edu)), 3820 Mundy Mill Rd, Oakwood, GA 30566. *Global regularity criteria for 2D micropolar equations with partial dissipation.* Preliminary report.

In this presentation, we discuss the global regularity (in time) issue of two dimensional incompressible micropolar equations with various partial dissipation. Micropolar fluids represent a class of fluids with non-symmetric stress tensor (called polar fluids) such as fluids consisting of suspending particles, dumbbell molecules, etc. Whether or not its classical solutions of 2D micropolar equations without velocity dissipation and micro-rotational viscosity develop finite time singularities is a difficult problem, and remains open. We mainly focus on two types of partial dissipation cases, and we present the global regularity condition. (Received September 13, 2017)

1135-35-717 **Alexander Pankov\*** ([alexander.pankov@morgan.edu](mailto:alexander.pankov@morgan.edu)). *Gap solitons in periodic nonlinear Schrödinger equations on periodic metric graphs.*

We consider stationary periodic nonlinear Schrödinger equations on periodic metric graphs. The main assumption is that zero is not in the spectrum of the linear part of the equation. Under natural assumption on linear and nonlinear potentials, we prove the existence of nontrivial finite energy solutions that decay exponentially fast at infinity. The proof is based on the generalized Nehari manifold approach in combination with periodic approximations. (Received September 13, 2017)

1135-35-755 **Peter Constantin**, **Tarek Elgindi** and **Huy Nguyen\***, Fine Hall, Washington Road, Princeton University, Princeton, NJ 08544, and **Vlad Vicol**. *On singularity formation in a Hele-Shaw model.*

We will discuss a lubrication approximation model of the interface between two immiscible fluids in a Hele-Shaw cell, derived in [Constantin et al, Physical Review E, 1993] and widely studied since. The model consists of a single one dimensional fourth-order nonlinear degenerate parabolic equation for the thickness of a thin neck of fluid, and two boundary conditions fixing the neck height and pressure jump. We prove that starting from a smooth interface with positive neck height, no singularity can arise as long as the neck height remains positive. As a consequence, we show that if the pressure of the less viscous fluid is larger than some explicit number then

pinching happens in either finite or infinite time. This is joint work with P. Constantin, T. Elgindi and V. Vicol. (Received September 14, 2017)

1135-35-818 **Susan Friedlander\*** ([susanfri@usc.edu](mailto:susanfri@usc.edu)), **Juraj Foldes**, **Nathan Glatt-Holtz** and **Geordie Richards**. *Asymptotics for magnetostrophic turbulence in the Earth's fluid core.*

We consider the 3-D MHD equations in the presence of stochastic forcing. We discuss results concerning the asymptotics of these PDE in the limit of vanishing parameters. In particular, we establish that the system sustains ergodic statistically steady states. (Received September 14, 2017)

1135-35-883 **Huicong Li\*** ([hli17@tulane.edu](mailto:hli17@tulane.edu)), Center for Partial Differential Equations, East China Normal University, 500 Dongchuan Road, Minhang, Shanghai, 200241, Peoples Rep of China. *Dynamics and asymptotic profiles of endemic equilibrium for SIS epidemic models with cross-diffusion.*

We consider two diffusive SIS epidemic models in heterogeneous environment, with a cross-diffusion term modeling the effect that susceptible individuals tend to move away from higher concentration of infected individuals. It is first shown that the corresponding Neumann initial-boundary value problem in a bounded smooth domain possesses a unique global classical solution which is uniformly-in-time bounded regardless of the strength of the cross-diffusion and the spatial dimension. It is further shown that, even in the presence of cross-diffusion, the models still admit the threshold-type dynamics in terms of the basic reproduction number  $\mathcal{R}_0$ ; that is, the unique disease free equilibrium is globally stable if  $\mathcal{R}_0 < 1$ , while if  $\mathcal{R}_0 > 1$ , the disease is uniformly persistent and there is an endemic equilibrium which is globally stable in some special cases. Our results on the asymptotic profiles of endemic equilibrium illustrate that restricting the motility of susceptible population may eliminate the infectious disease entirely for the first model with constant total population but fails for the second model with varying total population. (Received September 16, 2017)

1135-35-919 **Qi Han\*** ([qhan@tamusa.edu](mailto:qhan@tamusa.edu)), Assistant Professor, Department of Mathematics, Texas A&M University at San Antonio, San Antonio, TX 78224. *Positive ground states for nonlinear static Schrödinger equations that have potentials vanishing at infinity.*

We consider the existence of positive ground states for the nonlinear Schrödinger elliptic equations

$$-\Delta u + V(x)u = K(x)g(u) \quad (1)$$

that have potentials  $V(x), K(x) \geq 0$  both vanishing at infinity in a measure-theoretical sense when  $N \geq 3$ . (Received September 16, 2017)

1135-35-938 **Jiahong Wu\*** ([jiahong.wu@okstate.edu](mailto:jiahong.wu@okstate.edu)), Department of Mathematics, Oklahoma State University, 401 Mathematical Sciences, Stillwater, OK 74078. *Stability and regularity results for the 2D Boussinesq equations with partial dissipation.*

This talk presents recent work on the stability problem concerning several equilibrium solutions to the 2D Boussinesq equations with partial dissipation. First, we describe the linear and nonlinear stability results on the hydrostatic equilibrium of the 2D Boussinesq equations with only velocity dissipation. This is a joint work with C. Doering, K. Zhao and X. Zheng. Second, we explain the work with L. Tao on the linear stability of the Couette flow for the 2D Boussinesq equations with only vertical dissipation. In addition, we will briefly report work in progress on the large-time behavior of perturbations of the hydrostatic equilibrium and on the nonlinear stability of the Couette flow. (Received September 17, 2017)

1135-35-960 **Anna Ghazaryan**, **Yuri Latushkin** and **Xinyao Yang\*** ([xinyao.yang@xjtu.edu.cn](mailto:xinyao.yang@xjtu.edu.cn)), 411 Annex Ave, Apt. B4, Nashville, TN 37209. *Stability of multi-dimensional fronts in exponentially weighted norms.*

We consider a class of systems of reaction diffusion equations that frequently appears in combustion theory and chemical modeling. We study stability of traveling fronts in multi-dimensional cases. The essential spectrum of the operator obtained by linearizing the system about the front touches the imaginary axis, and thus we have to work in the intersection of the spaces of functions with and without exponential weights. We extend the stability theorems to the case of exponentially weighted spaces, and prove algebraic decay of perturbations of the front. (Received September 17, 2017)

1135-35-987 **Hung Duc Le\*** ([hdlgw3@mail.missouri.edu](mailto:hdlgw3@mail.missouri.edu)). *Elliptic Equations with Transmission and Wentzell Boundary Conditions and an Application to Steady Water Waves in the Presence of Wind.*

In this talk, we present results about the existence and uniqueness of solutions of elliptic equations with transmission and Wentzell boundary conditions. We provide Schauder estimates in Hölder and Sobolev spaces. As an

application, we develop an existence theory for small-amplitude two-dimensional traveling waves in an air-water system with surface tension. The water region is assumed to be irrotational and of finite depth, and we permit a general distribution of vorticity in the atmosphere. (Received September 18, 2017)

1135-35-991 **Pradeep Godar Chhetri\*** ([pgc9509@louisiana.edu](mailto:pgc9509@louisiana.edu)), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504, and **Aghalaya S Vatsala**. *Generalized Monotone Method for Riemann-Liouville Fractional Reaction Diffusion Equation with Applications*. Preliminary report.

In this work, initially we have obtained the integral representation form for the solution of the linear Riemann-Liouville fractional reaction diffusion equation of order  $q$ , where  $0 < q < 1$ . We have proved a comparison result relative to coupled lower and upper solutions of the nonlinear Riemann-Liouville reaction diffusion equation when the forcing term is the sum of a non-decreasing and non-increasing functions. Finally, we have proved generalized monotone method for the nonlinear Riemann-Liouville fractional reaction diffusion equation where the monotone sequences converge uniformly and monotonically to the coupled minimal and maximal solutions of the nonlinear problem. We have also proved that the solution is unique for the non-linear Riemann-Liouville fractional reaction diffusion equation. (Received September 22, 2017)

1135-35-1009 **F. Linares, G. Do Nascimento** and **G. Ponce\*** ([ponce@math.ucsb.edu](mailto:ponce@math.ucsb.edu)), Department of Mathematics, UCSB, Santa Barbara, CA 93106. *Special solution for the generalized Schrödinger equation*.

We consider the initial value problem associated to the generalized derivative Schrödinger equations

$$\partial_t u = i\partial_x^2 u + \lambda|u|^\alpha \partial_x u, \quad x, t \in \mathbb{R}, \quad |\lambda| = 1, \quad 0 < \alpha \leq 1, \quad (1)$$

and

$$\partial_t u = i\partial_x^2 u + \lambda\partial_x(|u|^\alpha u), \quad x, t \in \mathbb{R}, \quad |\lambda| = 1 \quad 0 < \alpha \leq 1. \quad (2)$$

Following an argument introduced by Cazenave and Naumkin we shall establish the local well-posedness for a class of small data in an appropriate weighted Sobolev space.

The other main tools in the proof include the homogeneous and inhomogeneous versions of Kato smoothing effect for the linear Schrödinger equation obtained by Kenig-Ponce-Vega.

(Received September 18, 2017)

1135-35-1037 **Ioan Bejenaru\*** ([ibejenaru@ucsd.edu](mailto:ibejenaru@ucsd.edu)) and **Sebastian Herr**. *Global well-posedness for the Dirac-Klein-Gordon system*.

We prove global well-posedness and scattering for the massive Dirac-Klein-Gordon system with small initial data of subcritical regularity in dimension three. To achieve this, we impose a non-resonance condition on the masses.

(Received September 18, 2017)

1135-35-1039 **Ioan Bejenaru\*** ([ibejenaru@ucsd.edu](mailto:ibejenaru@ucsd.edu)) and **Sebastian Herr**. *Global Well-Posedness and Scattering for the cubic Dirac equation*.

Global well-posedness and scattering for the cubic Dirac equation with small initial data in the critical space is established. The proof is based on a sharp endpoint Strichartz estimate for the Klein-Gordon equation which is captured by constructing an adapted systems of coordinate frames. (Received September 18, 2017)

1135-35-1040 **David M. Ambrose\*** ([dma68@drexel.edu](mailto:dma68@drexel.edu)), 3141 Chestnut St., Philadelphia, PA 19104. *Well-posedness and ill-posedness results for equations with nonlinear and/or degenerate dispersion*.

Equations with nonlinear and/or degenerate dispersion, such as the Dym equation and Rosenau-Hyman compacton equations, have been investigated for their integrable structures and coherent structures such as compactly supported traveling waves. With J. Douglas Wright, Gideon Simpson, and Dennis Yang, we have complemented the results in the literature by proving ill-posedness and well-posedness results. These equations may be ill-posed when the data can change sign, and may be well-posed for initial data of a fixed sign. (Received September 18, 2017)

1135-35-1050 **Li Wang\*** ([lwang46@buffalo.edu](mailto:lwang46@buffalo.edu)). *Singular shocks in particle laden flow*.

In this talk, we will consider singular shock dynamics for gravity-driven thin film flow with a suspension of particles down an incline. This flow is described by a system of conservation laws equipped with an equilibrium theory for particle settling and resuspension. Singular shock appears in the high particle concentration case that relates to the particle-rich ridge observed in the experiments. We analyze the formation of the singular shock as well as its local structure, and extend to the case with surface tension, an effect in the form of fourth order diffusion which regularizes the singular shock. (Received September 18, 2017)

1135-35-1053 **Jeremy L Marzuola\*** ([marzuola@math.unc.edu](mailto:marzuola@math.unc.edu)). *Small data global solutions for the Camassa-Choi equations.*

In joint work with Benjamin Harrop-Griffiths, we consider solutions to the Cauchy problem for an internal-wave model derived by Camassa-Choi (1996). This model is a natural generalization of the Benjamin-Ono and Intermediate Long Wave equations in the case of weak transverse effects. We prove the existence and long-time dynamics of global solutions from small, smooth, spatially localized initial data on  $\mathbb{R}^2$ . (Received September 18, 2017)

1135-35-1054 **Jeremy L. Marzuola\*** ([marzuola@math.unc.edu](mailto:marzuola@math.unc.edu)). *Symmetry Breaking in Density Functional Theory due to Dirac Exchange for a Hydrogen Molecule.*

We study symmetry breaking of a hydrogen molecule system under the density functional theory with Dirac exchange (the XLDA model). The constrained variational problem undergoes a spontaneous symmetry breaking, as the strength of the non-convex exchange term increases. We further characterize the limiting behavior of the minimizer when the strength of the exchange term goes to infinity. (Received September 18, 2017)

1135-35-1059 **Udita N Katugampola\*** ([dkatugampola@floridapoly.edu](mailto:dkatugampola@floridapoly.edu)), Department of Mathematical Sciences, Florida Polytechnic University, Lakeland, FL 33809. *New fractional integral with applications to time-fractional Porous Medium Equation (fPME).* Preliminary report.

In this talk we study some properties of a generalized fractional integral which unifies six familiar fractional integrals into a unique form given by

$$\left({}^{\rho}\mathcal{I}_{a+;\eta,\kappa}^{\alpha,\beta} f\right)(x) = \frac{\rho^{1-\beta} x^{\kappa}}{\Gamma(\alpha)} \int_a^x \frac{\tau^{\rho\eta+\rho-1}}{(x^{\rho}-\tau^{\rho})^{1-\alpha}} f(\tau) d\tau, \quad 0 \leq a < x < b \leq \infty.$$

We obtain a series representation of this integral along with asymptotic expansion and use them to solve a *time-fractional porous medium equation* in a subdiffusive case ( $0 < \alpha < 1$ ) of the form

$$\begin{cases} \partial_t^{\alpha} u(x, t) = (D_0 u^m(x, t) u_x(x, t))_x, \\ u(0, t) = C, \quad u(x, 0) = 0 \end{cases}$$

where  $\partial_t^{\alpha}$  is the Riemann-Liouville fractional derivative of order  $\alpha$ . We show that *Erdélyi-Kober* type integral naturally appears in the model equation which then be replaced by the integral in question. We study several cases of  $m$  to illustrate the applicability of this approach. Also, we derive two new identities of Taylor-type which can be used in broader sense. (Received September 19, 2017)

1135-35-1062 **Kim B. Hoewoon\*** ([kimho@math.oregonstate.edu](mailto:kimho@math.oregonstate.edu)), Department of Mathematics, Oregon State Univ, Corvallis, OR 97331. *Fourier Transforms and Generalized Hyers-Ulam Stability of Laplace Equations in a Half Space.*

In 1940, a Polish-American mathematician, S. M. Ulam proposed the stability problem of the linear functional equation  $f(x + y) = f(x) + f(y)$  that can be generalized as ‘‘Under what conditions a mathematical object satisfying a certain property approximately must be close to an object satisfying the property exactly?’’. For the last decades, stability problems of various functional equations, not only linear case, have been extensively investigated and generalized by many mathematicians. An extension of the Ulam’s stability problems in terms of differential equations was recently proposed: the differential equation  $\phi(f, y, y', \dots, y^{(n)}) = 0$  has Hyers-Ulam stability provided for given  $\epsilon > 0$  and a function  $y$  satisfying  $|\phi(f, y, y', \dots, y^{(n)})| \leq \epsilon$ , there exists the solution  $y_0$  to the differential equation such that  $|y(x) - y_0(x)| \leq K(\epsilon)$  and  $\lim_{\epsilon \rightarrow 0} K(\epsilon) = 0$ . In this presentation, we will talk about the generalized Hyers-Ulam stability of the Laplace equation  $\Delta u = 0$  in a half space in terms of Fourier transforms as an open and accessible problem for undergraduate research. (Received September 19, 2017)

1135-35-1090 **Jashmon Rana\*** ([jr937963@cameron.edu](mailto:jr937963@cameron.edu)), 2800 West Gore Blvd, Lawton, OK 73505, and **Narayan Thapa** ([nthapa@cameron.edu](mailto:nthapa@cameron.edu)), 2800 West Gore Blvd, Lawton, OK 73505. *Time Dependent Parameter Identification Problem in Parabolic Partial Differential Equation Through Over-Specification of Data.* Preliminary report.

Coefficients and source identification problems in parabolic type of partial differential equations have received notable attention in several fields such as quantum mechanics, finance, chemical diffusion, and fluid dynamics recently. In this presentation, we discuss on time dependent parameter  $a(t)$  associated with parabolic type of partial differential equation. Over-specification of data  $u(x^*, t) = H(t)$ ,  $0 \leq t \leq T$ , initial and boundary data are used to estimate the parameter. A number of numerical examples will be presented to validate the proposed computational method. (Received September 19, 2017)

1135-35-1159 **Eitan Tadmor\*** (tadmor@cscamm.umd.edu), CSCAMM, CSIC Bldg 406, University of Maryland, Paint Branch Drive, University of Maryland, College Park, MD 20742.  
*Regularity and emergence of flocking in PDE models with a commutator structure.*

We discuss the question of global regularity for a general class of Eulerian dynamics driven by a forcing with a commutator structure. The study of such systems is motivated by the hydrodynamic description of agent-based models for flocking driven by alignment. For commutators involving bounded kernels, existence of strong solutions follows for initial data which are sub-critical, namely – the initial divergence is “not too negative” and the initial spectral gap is “not too large”. Singular kernels, corresponding to fractional Laplacian of order  $0 < s < 1$ , behave better: global regularity persists and flocking follows. Singularity helps! (Received September 20, 2017)

1135-35-1201 **Maya Chhetri, Pavel Drabek and Ratnasingham Shivaji\*** (r\_shivaj@uncg.edu), Dept of Mathematics and Statistics, University of North Carolina at Greensboro, 116 Petty, 317 College Ave, Greensboro, NC 27412. *S-shaped bifurcation diagrams in exterior domains.*

We study a nonlinear eigenvalue problem on the exterior to a simply connected bounded domain in  $\mathbb{R}^N$  containing the origin. We consider positive weak solutions satisfying Dirichlet boundary conditions on the compact boundary and decaying to zero at infinity. We discuss multiplicity and uniqueness results of solutions with respect to a bifurcation parameter and conjecture an *S*-shaped bifurcation diagram for positive reaction terms which are singular at the origin and sublinear at infinity. As a by-product, on regions exterior to a ball with radially symmetric weight functions, we obtain radial symmetry of solutions when uniqueness holds. (Received September 20, 2017)

1135-35-1285 **Jinping Zhuge\*** (jinping.zhuge@uky.edu). *Quantitative analysis of boundary layer in periodic homogenization.*

In this talk, we present a quantitative analysis for Neumann problem of the second-order elliptic system with first-order periodically oscillating data:

$$\begin{cases} -\frac{\partial}{\partial x_i} \left\{ a_{ij} \left( \frac{x}{\varepsilon} \right) \frac{\partial}{\partial x_j} u_\varepsilon(x) \right\} = 0 & \text{in } \Omega, \\ \frac{\partial}{\partial \nu_\varepsilon} u_\varepsilon(x) = T_{ij}(x) \cdot \nabla \left\{ g_{ij} \left( x, \frac{x}{\varepsilon} \right) \right\} & \text{on } \partial\Omega, \end{cases}$$

where  $\partial/\partial \nu_\varepsilon$  is the conormal derivative and  $T_{ij} = n_i e_j - n_j e_i$  are tangent vector fields on  $\partial\Omega$ . The domain  $\Omega$  is supposed to be bounded and uniformly convex (or of finite type). We identify the corresponding homogenized system and establish the nearly sharp rate of convergence in  $L^2$  for both Dirichlet and Neumann problem. The regularity of the homogenized boundary data will also be emphasized. This is a joint work with Zhongwei Shen. (Received September 20, 2017)

1135-35-1313 **Isaac Harris\*** (iharris@tamu.edu) and **William Rundell**. *Direct Method for Reconstructing Inclusions from Electrostatic Data.*

In this talk, we will discuss the use of the Linear Sampling Method to reconstruct impenetrable inclusions from Electrostatic Cauchy data. We consider the case of a perfectly conducting and impedance inclusion. In either case we see that the Dirichlet to Neumann mapping can be used to reconstruct impenetrable sub-regions. We also propose a non-iterative method to reconstruct the impedance parameter from the knowledge of multiple Cauchy pairs which can be computed from Dirichlet to Neumann mapping. Some numerical reconstructions will be presented in two dimensions. (Received September 21, 2017)

1135-35-1315 **Isaac Harris\*** (iharris@tamu.edu), **Fioralba Cakoni** and **Housseem Haddar**. *Homogenization of the Transmission Eigenvalue Problem for a Periodic Media.*

In this talk we consider the inverse acoustic/electromagnetic scattering problem of determining information about the macro and micro-structure of a periodic media where the period is characterized by a small parameter. To this end, we study the transmission eigenvalue problem as the small parameter tends to zero to obtain the homogenized eigenvalue problem. This is a non-linear and non-selfadjoint eigenvalue problem which makes its investigation mathematically challenging. We prove (weak) convergence of the eigenvalues/functions as well as show that the effective material properties can be determined by the measured homogenized transmission eigenvalues. (Received September 21, 2017)



- 1135-35-1361 **David J Wollkind, Richard A Cangelosi, Bonni J Dichone\*** ([dichone@gonzaga.edu](mailto:dichone@gonzaga.edu)) and **Mitchell G David**. *A Stuart-Watson Nonlinear Stability Analysis of a Generalized Matkowsky Heat Equation*.

The Matkowsky model heat equation originally analyzed through terms of third-order to illustrate his two-time method of nonlinear stability theory is extended through terms of fifth-order for a generalized source. This formulation can be interpreted as a model population interaction-dispersion equation as well. When analyzed by the Stuart-Watson method of nonlinear stability theory, it is shown that under the proper conditions the two subcritical cases behave in exactly the same manner as the two supercritical ones unlike the outcome for the truncated system. Further there also exists a region of metastability allowing for the possibility of population outbreaks. These results are then used to offer an explanation for the occurrence of isolated vegetative patches and sparse homogeneous distributions in the relevant ecological parameter range where there is subcriticality for a plant-groundwater model system as opposed to periodic rhombic patterns and dense homogeneous distributions occurring in its supercritical regime. (Received September 21, 2017)

- 1135-35-1380 **Wei Feng\*** ([fengw@uncw.edu](mailto:fengw@uncw.edu)), Department of Mathematics and Statistics, 601 South College Road, Wilmington, NC 28409, and **Weihua Ruan** and **Xin Lu**. *Traveling Wave Solutions in Mixed Monotone Models of Population Biology, with Time Delays and Density-Dependent Diffusions*.

We study the existence of traveling wave solutions in a general class of mixed quasi-monotone reaction-diffusion systems (of quasi-linear type and with time delays). First, by applying the Schauder Fixed Point Theorem, we prove the existence of a traveling wave solution between classically defined upper and lower solutions. For better applications of the upper-lower solutions method on various real-life models, the existence result is further extended under weak form or piecewise smooth upper-lower solutions. In several population biology models with time delays or density-dependent diffusions (single-species logistic growth, N-species competition, food chain, and ratio-dependent predator-prey with Gompertz growth), we apply our main result to establish the existence of traveling wave solutions flowing towards the positive or coexistent states under reasonable conditions on ecological parameters. (Received September 21, 2017)

- 1135-35-1393 **Robert L Pego\*** ([rpego@cmu.edu](mailto:rpego@cmu.edu)), Pittsburgh, PA 15213. *Merging-splitting models of group dynamics by Bernstein function theory*. Preliminary report.

We study coagulation-fragmentation equations inspired by a simple model derived in fisheries science to explain data on the size distribution of schools of pelagic fish. The model lacks detailed balance and admits no H-theorem, but we are able to study equilibria, self-similarity and large-time behavior using complex function theory for Bernstein and Pick functions. A useful tool is the characterization of the generating functions of completely monotone sequences as those Pick functions analytic and nonnegative on  $(-\infty, 1)$ . This is joint work with Jian-Guo Liu and Pierre Degond. (Received September 21, 2017)

- 1135-35-1405 **Nicholas M Ercolani\*** ([ercolani@math.arizona.edu](mailto:ercolani@math.arizona.edu)). *Yang-Mills-Higgs Equations on Riemann Surfaces of Higher Genus*.

We study the Yang-Mills-Higgs equations for vector bundles of rank 2 on a Riemann surface of genus 2 making use of the explicit knowledge of the moduli space of rank 2 holomorphic bundles on such a surface that is provided by the classical work of Narasimhan and Ramanan. We expect to be able to construct explicit solutions in the neighborhood of a constant curvature branch of solutions along the same lines as our earlier work for the Ginzburg-Landau equations on Riemann surfaces of higher genus. This talk will be a progress report on that effort. This is joint work with I.M. Sigal. (Received September 21, 2017)

- 1135-35-1407 **Doyoon Kim\*** ([doyoon\\_kim@korea.ac.kr](mailto:doyoon_kim@korea.ac.kr)), Department of Mathematics, Korea University, 145 Anam-ro, Seongbuk-gu, Seoul, 02841, South Korea. *Boundary value problems for stationary Stokes system in weighted Sobolev spaces*.

We present the unique solvability of solutions in Sobolev spaces to the stationary Stokes system with variable coefficients on a bounded Reifenberg flat domain. We discuss both the Dirichlet boundary condition and the conormal derivative boundary condition. The solution spaces are Sobolev spaces with Muckenhoupt type weights. The coefficients are in the class of partially BMO functions so that, locally, they have no regularity conditions in one direction. This is based on joint work with Hongjie Dong and Jongkeun Choi. (Received September 21, 2017)

1135-35-1422 **Pierre Germain, Benjamin Harrop-Griffiths\*** ([benjamin.harrop-griffiths@cims.nyu.edu](mailto:benjamin.harrop-griffiths@cims.nyu.edu)) and **Jeremy L Marzuola**. *Degenerate dispersive equations.*

In this talk we discuss recent work on some quasilinear Hamiltonian dispersive equations that act as toy models for the phenomenon of degenerate dispersion, where the dispersion relation may degenerate at a point in space. This is joint work with Pierre Germain and Jeremy Marzuola. (Received September 22, 2017)

1135-35-1521 **Mathew Gluck\*** ([mathew.r.gluck-1@ou.edu](mailto:mathew.r.gluck-1@ou.edu)), University of Oklahoma, Department of Mathematics, 601 Elm Ave., Norman, OK 73019-3103. *Embedding Inequalities for a Family of Integral Operators and Applications.*

In this talk embedding inequalities for a family of integral operators  $E_{\alpha,\beta} : L^p(\partial\mathbb{R}_+^n) \rightarrow L^q(\mathbb{R}_+^n)$  that include the classical Poisson extension operator and a Riesz-potential-type operator will be discussed. For the conformally invariant parameters the extremal functions are classified and the value of the optimal embedding constant is computed. As an application, for certain parameters we prove the embedding inequalities for bounded subdomains  $\Omega \subset \mathbb{R}^n$  and show that if the optimal embedding constant for  $\Omega$  is strictly larger than the optimal embedding constant for the unit ball, then the optimal constant is attained. We show that this criterion is satisfied by an annular domain whose hole is sufficiently small. Motivated by this fact we prove the existence of a domain  $\Omega$  equipped scalar-flat metric  $g$  in the conformal class of the Euclidean metric whose isoperimetric constant is strictly larger than that of the Euclidean ball. Some of this work is joint work with Meijun Zhu. (Received September 22, 2017)

1135-35-1522 **Zilong Song\***, Department of Mathematics and Statistics, York University, 4700 Keele street, Toronto, ON, Canada, **Xiulei Cao**, Department of Mathematics and Statistics, York University, 4700 Keele street, Toronto, ON, Canada, and **Huaxiong Huang**, Department of Mathematics and Statistics, York University, 4700 Keele street, Toronto, ON, Canada. *On connection conditions through boundary layer of Poisson-Nernst-Planck system.*

We consider the dynamic Poisson-Nernst-Planck (PNP) system, with assumptions of local electro-neutrality (LEN) and near global electro-neutrality (GLN), i.e., without  $O(1)$  extra charge. There is boundary layer near domain boundary, which requires more computational efforts to resolve. A natural question is that, can we replace the boundary layer by some effective boundary/connection conditions and solve the bulk region directly? In this talk, we present connection conditions for various boundary conditions, including Dirichlet and flux conditions for ion concentration and Dirichlet and Robin conditions for electric potential. Some numerical examples will be discussed to verify the connection formula. Also, we investigate a PNP system combined with Hodgkin-Huxley model, which has important applications in neural cells. (Received September 22, 2017)

1135-35-1536 **Abdullah S Erdogan\*** ([aserdogan@gmail.com](mailto:aserdogan@gmail.com)), [aserdogan@gmail.com](mailto:aserdogan@gmail.com), and **Ali Ugur Sazaklioglu** and **Allaberen Ashyralyev**. *Unique solvability of a source identification problem for a semilinear equation with a final overdetermination.*

In this work, unique solvability of a source identification inverse problem for a semilinear equation with a final overdetermination in a Banach space is investigated. Moreover, the first and second order of accuracy difference schemes are presented for numerically solving this problem. The existence and uniqueness results for these difference schemes are given. The efficiency of the proposed method is evaluated by means of computational experiments. (Received September 23, 2017)

1135-35-1541 **Yakov Shlapentokh-Rothman\***, Fine Hall, Washington Road, Princeton, NJ 08544. *The asymptotically self-similar regime for the Einstein vacuum equations.*

We will dynamically construct singular solutions to the Einstein vacuum equations which are asymptotically self-similar in that successive rescalings around the singularity converge to a self-similar solution. Connections both to Christodoulou's bounded variation solutions of the spherically symmetric Einstein-scalar field system and to the ambient metric construction of Fefferman and Graham will be elaborated on. This is joint work with Igor Rodnianski. (Received September 23, 2017)

1135-35-1559 **Tim Laux** and **Nung Kwan Yip\*** ([yip@math.purdue.edu](mailto:yip@math.purdue.edu)). *Convergence of a thresholding scheme to motion by mean curvature of filaments in  $\mathbb{R}^3$ .*

We prove the convergence of a thresholding scheme to the motion by mean curvature of curves (filaments) in space ( $\mathbb{R}^3$ ). The scheme was first proposed by Ruuth-Merriman-Xin-Osher (2001). It generalizes to higher codimensions (codimension equal two) the classical thresholding scheme for hypersurfaces (codimension equal one) which was initiated by Merriman-Bence-Osher in the 90's. The algorithm is essentially a time-splitting

scheme, alternating two very simple steps: (1) linear diffusion and (2) projection to functions with norm one. Almost all the proofs of convergence in the hypersurface case make use of maximum principle. Such a technique is not available for higher codimensions. We will formulate the scheme using the gradient flow structure found by Esedoglu-Otto (2015) together with a computation by Lin for an energy estimate for Ginzburg-Landau functional. (Received September 23, 2017)

1135-35-1618 **R Mikulevicius and Chukiat Phonsom\*** (phonsom@usc.edu), 3620 S Vermont Ave KAP104, Los Angeles, CA 90089. *On the Cauchy problem for integro-differential equations in the scale of spaces of generalized smoothness.*

Parabolic integro-differential model Cauchy problem is considered in the scale of  $L_p$ -spaces of functions whose regularity is defined by a scalable Lévy measure. Existence and uniqueness of a solution is proved by deriving a priori estimates. Some rough probability density function estimates of the associated Levy process are used as well. Precisely we consider the following equation with  $\lambda \geq 0$

$$\begin{aligned}\partial_t u(t, x) &= Lu(t, x) - \lambda u(t, x) + f(t, x) + \int_U \Phi(t, x, \nu) q(dt, d\nu), \\ u(0, x) &= g(x) \text{ in } E = [0, T] \times \mathbf{R}^d\end{aligned}$$

where an integro-differential operator with Lévy measure  $\pi$  is defined by

$$L\varphi(x) = L^\pi \varphi(x) = \int [\varphi(x+y) - \varphi(x) - \chi_\sigma(y) y \cdot \nabla \varphi(x)] \pi(dy), \varphi \in C_0^\infty(\mathbf{R}^d),$$

and  $q(dt, d\nu)$  is a compensated point measure. (Received September 23, 2017)

1135-35-1686 **Marcelo M Disconzi\*** (marcelo.disconzi@vanderbilt.edu), 1326 Stevenson Center, Nashville, TN 37240, and **Chenyun Luo**. *The incompressible limit of the compressible free-boundary Euler equations with surface tension.*

We consider the compressible free-boundary Euler equations with surface tension. We prove that its solutions converge to solutions of the incompressible free-boundary Euler equations when the sound speed tends to infinity. (Received September 24, 2017)

1135-35-1693 **Isabelle Gallagher\*** (gallagher@math.ens.fr), Ecole Normale Supérieure, 45, rue d'Ulm, Paris, France. *From Newton to Navier-Stokes, or how to connect fluid mechanics equations from microscopic to macroscopic scales.*

The question of deriving Fluid Mechanics equations from deterministic systems of interacting particles obeying Newton's laws, in the limit when the number of particles goes to infinity, is a longstanding open problem suggested by Hilbert in his 6th problem. One step in the program consists in deriving Fluid Mechanics Equations from the Boltzmann equation on the one hand, and the Boltzmann equation from particle systems on the other. In this talk we shall show how to answer Hilbert's question at a formal level, and why it is very difficult, and actually an open problem to this day, to make the argument rigorous. We shall also discuss a few successful attempts in this program, in particular the works of Golse and Saint Raymond which provide a rigorous derivation of the incompressible Navier-Stokes equations from the Boltzmann equation. (Received September 24, 2017)

1135-35-1726 **Hantaek Bae, Elaine Cozzi, Gung-Min Gie and James P Kelliher\*** (kelliher@math.ucr.edu). *Techniques of 2D fluids applied to the aggregation equation.*

The viscous and inviscid aggregation equation with Newtonian potential models a number of different physical systems and has close analogs in 2D incompressible fluid mechanics. We give an overview of how techniques from 2D fluid mechanics can be adapted to analyze both the viscous and inviscid aggregation equation, including well-posedness, the inviscid limit, and the propagation of striated regularity. (Received September 24, 2017)

1135-35-1739 **Jiancheng Lyu** (jianchel@uci.edu), **Jack Xin** (jack.xin@uci.edu) and **Yifeng Yu\*** (yyu1@math.uci.edu). *Curvature effect in shear flow: slowdown of turbulent flame speeds with Markstein number.*

It is well-known in the combustion community that curvature effect in general slows down flame propagation speeds because it smooths out wrinkled flames. In this talk, I will present a joint work with Jiancheng Lyu and Jack Xin. We have proved that the turbulent flame speed (an effective burning velocity) is decreasing with respect to the curvature diffusivity (Markstein number) for shear flows in the well-known G-equation model. (Received September 24, 2017)

1135-35-1742 **Hung Tran** (hung@math.wisc.edu) and **Yifeng Yu\*** (yyu1@math.uci.edu). *A rigidity result for effective Hamiltonians with 3-mode periodic potentials.*

Let  $\overline{H}_1, \overline{H}_2$  be the effective Hamiltonians associated with the Hamiltonians  $\frac{1}{2}|p|^2 + V_1, \frac{1}{2}|p|^2 + V_2$ , respectively. In this talk, I will present a rigidity result which says that if the dimension  $n = 2$  and each of  $V_1, V_2$  contains exactly 3 mutually non-parallel Fourier modes, then

$$\overline{H}_1 \equiv \overline{H}_2 \iff V_1(x) = V_2\left(\frac{x}{c} + x_0\right) \quad \text{for all } x \in \mathbb{R}^2$$

for some  $c \in \mathbb{Q} \setminus \{0\}$  and  $x_0 \in \mathbb{R}^2$ . This is a joint work with Hung Tran. (Received September 24, 2017)

1135-35-1752 **Alicia Machuca\***, amachuca@twu.edu. *A Method for Explicit Solution Formulas for Integrable Evolution Equations in 2+1 Dimensions.*

A systematic method is developed to obtain solution formulas for certain explicit solutions to integrable nonlinear partial differential equations in two spatial variables and one time variable. The method utilizes an underlying Marchenko integral equation that arises in the corresponding inverse scattering problem. The method is demonstrated for the Kadomtsev-Petviashvili and the Davey-Stewartson equations. Such solution formulas are expressed in a compact form in terms of matrix exponentials, by using a set of four constant matrices as input. The formulas hold for any sizes of the matrix quadruplets and hence yield a large class of explicit solutions. (Received September 24, 2017)

1135-35-1765 **Lorena Bociu, Lucas Castle\*** (lcastle@ncsu.edu) and **Irena Lasiecka**. *Well-Posedness and Control in a Free Boundary Fluid-Structure Interaction.* Preliminary report.

We consider a system of partial differential equations modeling motion of an elastic solid inside of an incompressible fluid with a force applied to the body of the system. The fluid is modeled by the incompressible Navier-Stokes equations while the structure is given by a damped linear wave equation. Given sufficiently regular initial data, we establish global well-posedness in time and consider an optimal control for the problem of minimizing turbulence in the fluid flow. We establish the existence of an optimal control and discuss the derivation of the first order necessary optimality conditions that characterize the control. (Received September 24, 2017)

1135-35-1781 **Subhash Subedi\*** (sxs2754@louisiana.edu), Lafayette, LA 70503, and **Aghalaya s. Vatsala**. *Quenching problem for one dimensional fractional reaction diffusion equation.*

We study the quenching problems for ordinary caputo fractional differential equation and time dependent caputo-fractional reaction diffusion equation in one dimensional finite and infinite domain with a nonlinear source. We also compare the solution with the solution of the standard reaction diffusion equation. (Received September 24, 2017)

1135-35-1787 **Julia Anderson-Lee\*** (julia.andersonlee@gmail.com), **Scott Hansen** and **Sri Sritharan**. *A mathematical approach to understanding the vibrations within a block subjected to freezing rocking.*

The study of seismic engineering is extremely interesting not only from an engineering perspective but also within other disciplines which have applications in engineering and modeling of dynamical systems. In particular, rocking systems—made popular as a research area by G.W. Housner with his paper regarding modeling the displacement of block-shaped structures—involve interesting mathematics in the area of coupled partial differential equations. In this talk, a coupled system of partial differential equations incorporating the strain energy of the block, and internal longitudinal vibrations is presented. Then, using only the internal vibrations and the angle of rocking, a coefficient of restitution is formulated. These results are used to construct the expected motion of the block and compared to the motion of the block predicted by Housner. (Received September 24, 2017)

1135-35-1819 **Ivan A Blanki\*** (blanki@math.ksu.edu), Manhattan, KS 66502, and **Niles Armstrong, Ashok Aryal, Brian Benson, Zheng Hao and Jeremy LeCrone**. *Mean Value Theorems and the Geometry of Mean Value Sets for Divergence Form Elliptic PDEs.*

One basic property of harmonic functions is the fact that at any point the value of the function is exactly equal to the average value of the function over a ball (or a sphere) centered at that point. This theorem can be used to establish other basic properties of harmonic functions including the maximum principle, the Harnack inequality, and some important a priori estimates. Standard proofs of the mean value theorem rely on smoothness and symmetry properties of the Laplacian and therefore do not generalize nicely to non-Euclidean spaces or more general elliptic operators. However, in his Fermi lectures on the obstacle problem in 1998, Luis Caffarelli indicated how one could use solutions of specific obstacle problems to construct an elegant proof of the mean value theorem which does not depend on either the smoothness or symmetry properties of the Laplacian. In this talk, I will

explain that proof and show how my collaborators and I have used it to generalize the mean value theorem to other settings. (Received September 25, 2017)

1135-35-1847 **Zheb Lei** and **Qi S. Zhang\*** ([qizhang@math.ucr.edu](mailto:qizhang@math.ucr.edu)), Math. Dept. UC Riverside, Riverside, CA 92521. *A Liouville theorem for the axially symmetric Navier-Stokes equations with some super-critical conditions.*

We present a Liouville theorem for ancient solutions of the axially symmetric Navier-Stokes equations with some super-critical conditions under the standard scaling. (Received September 25, 2017)

1135-35-1855 **Peiyong Wang\*** ([pywang@math.wayne.edu](mailto:pywang@math.wayne.edu)), 656 W.Kirby, 1150 FAB, Department of Mathematics, Wayne State University, Detroit, MI 48202. *Symmetry and approximate symmetry for a singularly perturbed free boundary problem.*

In this talk, we study the symmetric properties of a problem arising from energy industry. In the first step, we apply the moving plane method over a ring in a new way with an idea of adding a dominating symmetric function to overcome the lack of radial monotonicity that we thought of. In the second step, we prove the approximate symmetric in a ring-like domain. In this part, the main difficulty is the absence of the elliptic comparison principle. However, we prove a parabolic comparison principle so that we can abstract comparable radially symmetric functions through an evolution to approximate the solution over a ring-like domain. Thus proves the free boundary of the solution is restricted in a thin ring. Our two main methods as well as the results are new and seem promising in application in other related problems. (Received September 25, 2017)

1135-35-1938 **Patrick Guidotti\***, Department of Mathematics, 340 Rowland Hall, University of California, Irvine, CA 92697-3875. *Domain Variations and Moving Boundary Problems.*

In recent decades maximal regularity techniques have been successfully applied to the analysis to a variety of moving boundary problems. A common feature of this approach is the use of a domain fixing transformation, known as the Hanzawa transformation. In this talk, a geometrically motivated alternative transformation is proposed. Among its benefits are more transparent representations for the linearizations required for the applications of maximal regularity results. The description of the new transformation will be complemented by the discussion of a prototypical application. (Received September 25, 2017)

1135-35-1958 **Nicholas J Kass\*** ([nkass@huskers.unl.edu](mailto:nkass@huskers.unl.edu)) and **Mohammad A Rammaha** ([mrammaha1@unl.edu](mailto:mrammaha1@unl.edu)). *Existence of solutions for strongly damped wave equations of the  $p$ -Laplacian type with boundary sources.* Preliminary report.

Results on the existence of suitably defined weak solutions of a damped wave equation

$$u_{tt} - \Delta_p u - \Delta u_t = f(u)$$

in a bounded domain  $\Omega \subset \mathbb{R}^3$  with smooth boundary  $\partial\Omega$  subject to a generalized Robin boundary condition with nonlinear boundary source terms. Here,  $-\Delta_p$  with  $2 < p < 3$  denotes the classical  $p$ -Laplacian. We provide a rigorous proof of the existence of local solutions to this problem given suitable restrictions on the feedback terms using a Galerkin scheme, with particular attention given to the convergence of the nonlinearities arising from the  $p$ -Laplacian. It is shown that these solutions must satisfy an appropriate energy inequality, and from this sufficient conditions for global existence are obtained. (Received September 27, 2017)

1135-35-1979 **Mihaela Ifrim** and **Daniel Tataru\*** ([tataru@math.berkeley.edu](mailto:tataru@math.berkeley.edu)). *The Benjamin-Ono equation.*

The aim of this talk is to present recent joint work with Mihaela Ifrim on the short and long time dynamics for the Benjamin-Ono equation. (Received September 25, 2017)

1135-35-2027 **Vera Mikyoung Hur\***, 1409 W Green St, Urbana, IL 61801. *Instabilities of Stokes waves.*

In the 1980s, MacKay and Saffman made a signature argument to determine when the collision of a pair of purely imaginary eigenvalues for the linear water wave problem is avoided or lead to loss of stability of a Stokes wave. I will use a spectral perturbation approach to rigorously justify this. I will discuss results from numerical computation. (Received September 25, 2017)

1135-35-2033 **Parul Verma\*** ([verma38@purdue.edu](mailto:verma38@purdue.edu)), **Reshma Kalyan Sundaram** and **Doraiswami Ramkrishna**. *A population balance model to optimize chemotherapy infusion rate.*

Vincristine (VCR) is a core chemotherapeutic drug administered to pediatric Acute Lymphoblastic Leukemia (ALL) patients. It is administered via an IV bolus injection or via a minibag for approximately 10 minutes. In a subgroup of population, it leads to Vincristine Induced Peripheral Neuropathy (VIPN), which is the dose-limiting toxicity. Even though VCR has been used as a chemotherapeutic drug for more than 50 years now, predictors

and mechanism of VIPN induction are unclear. VIPN incidence seems to be associated with VCR cumulative dosage. Hence, we are interested in finding an optimal infusion time that would lead to a reduction in VCR dosage. A population balance model, which is a set of partial differential equations, is developed to describe the mechanism of VCR in cells in different phases. The model is a function of time and cell age. Infusion time was optimized to maximize number of cancer cells being killed and minimize bone marrow toxicity by the end of induction phase, with VCR being administered weekly. Model was solved using method of characteristics and successive generations. Analysis showed that an infusion time of 60 minutes produced a better efficacy than the usual 10 minutes. Lesser amount of drug with same efficacy as before may reduce the chances of VIPN. (Received September 25, 2017)

1135-35-2063 **Jessica Lin\*** ([jessica.lin@mcgill.ca](mailto:jessica.lin@mcgill.ca)), Department of Mathematics, Burnside Hall, Montreal, QC H2J2M8, Canada. *An Introduction to Stochastic Homogenization.*

One way of modeling phenomena in “typical” physical settings is to study partial differential equations (PDEs) in random environments. The subject of stochastic homogenization is concerned with identifying the asymptotic behavior of solutions to PDEs with random coefficients. Specifically, we are interested in the following: if the random effects are microscopic compared to the lengthscale at which we observe the phenomena, can we predict the behavior which takes place on average? For certain models of PDEs and under suitable hypotheses on the environment, the answer is affirmative. In this talk, I will introduce this subject area and present a few models for which we know homogenization occurs. (Received September 25, 2017)

1135-35-2079 **Alex Plotkowski\***, [plotkowskij@ornl.gov](mailto:plotkowskij@ornl.gov), and **M. M. Kirka, R. R. Dehoff, Y. Lee, N. Raghavan, M. Haines, A. Prabhu, H. Rezayat and S. S. Babu.** *Applications of PDEs to Process Modeling of Metal Additive Manufacturing at mm- and sub-micron Length Scales.*

Mechanical properties and component level performance are intimately linked to microstructure, which is in turn dictated by the spatial (nm to mm) and temporal (10-6 to 10-1 seconds) response of the material to the manufacturing conditions under which it was produced. Understanding these linkages is necessary for ensuring quality while minimizing cost and maximizing reliability. Process modeling relies on solving highly coupled partial differential equations under complex boundary conditions. While extensive progress has been made in the last three decades to improve the fundamental understanding of the relevant physical phenomena, recent interest in metal additive manufacturing has posed new challenges in spanning disparate length scales and considering complex interactions between geometry, process, and material. This presentation will introduce approaches that have been successfully used for modeling of conventional metals processing and their recent applications to additive manufacturing. Special attention will be paid to emerging challenges and opportunities in additive manufacturing, including state of the art computational modeling, reduced-order approaches, and the use of advanced modeling capabilities to described integrated component-structure-process optimization. (Received September 25, 2017)

1135-35-2084 **Timothy E. Faver\*** ([tef36@drexel.edu](mailto:tef36@drexel.edu)) and **J. Douglas Wright.** *Traveling waves in mass and spring dimer Fermi-Pasta-Ulam-Tsingou lattices.*

A Fermi-Pasta-Ulam-Tsingou lattice is a one-dimensional chain of particles connected to their nearest neighbors by nonlinear springs. It is well-established that monatomic lattices, which consist of identical particles and springs, possess solitary traveling wave solutions to their equations of motion; these are waves that decay exponentially fast to zero at spatial infinity. We study two species of more general, heterogeneous lattices: mass dimer lattices, in which the masses of the particles alternate between two values but all springs exert the same force, and spring dimer lattices, in which the spring forces alternate but the masses are all the same. For each species, we prove the existence of nanopteron traveling wave solutions. These waves are the sum of an exponentially decaying term and a periodic term, so that, in contrast to the solitary wave, the nanopteron may asymptote to a nonvanishing oscillation. Our existence proof relies on a quantitative contraction mapping argument that incorporates a natural singular perturbation found in the dimer traveling wave equations. (Received September 25, 2017)

1135-35-2103 **Chong Wang\*** ([chongwang@gwu.edu](mailto:chongwang@gwu.edu)), 532 20th St NW, Apt 705, Washington, DC 20006, and **Yanxiang Zhao and Xiaofeng Ren.** *Pattern formation - on the modeling of multi-constituent inhibitory systems.*

Skin pigmentation, animal coats and block copolymers can be considered as multi-constituent inhibitory systems and studied as a nonlocal geometric variational problem. The free energy of the system is the sum of two terms: the local part - the total size of the interfaces separating the three components, and the nonlocal part - a longer

ranging interaction energy that inhibits micro-domains from unlimited growth. We establish that in different parameter ranges there are corresponding assemblies of certain patterns that exist as the stationary sets of the free energy functional. Numerically, a diffusive interface model is proposed and many self-assembly processes, which form various patterns, are showed here. Different numerical schemes are compared and a new technique is introduced to be consistent with the Euler-Lagrange equation in the sharp interface model. (Received September 25, 2017)

1135-35-2116 **Tam Do\*** ([tando@usc.edu](mailto:tando@usc.edu)). *Vorticity Gradient Growth for the Axisymmetric 3D Euler Equations Without Swirl.*

In the 2D Euler Equations, it is known that the  $L^\infty$  norm of the gradient of vorticity can grow at most double exponentially in time. This bound has been proven to be sharp by Kiselev and Sverak on the unit disc. We examine the possibility of gradient growth in the 3D axisymmetric setting for flows without swirl component. (Received September 25, 2017)

1135-35-2127 **Johnny Corbino\*** ([jcorbino@mail.sdsu.edu](mailto:jcorbino@mail.sdsu.edu)), 5500 Campanile Drive, GMCS bldg, Computational Science Research Center, San Diego, CA 92182. *SubFlow: Modeling Geological Sequestration of Carbon Dioxide with Mimetic Discretization Methods.*

We introduce a reliable software package to simulate the long-term storage of CO<sub>2</sub> injected into geological formations. Depleted oil fields and subsurface saline aquifers are examples of this type of formations. Our software (SubFlow) considers all the required parameters to predict whether a preselected injection site is in fact suitable for CO<sub>2</sub> sequestration. SubFlow is an open-source, sustainable, and reliable software element that allows its users to model the transport of reactive chemical compounds (CO<sub>2</sub>, H<sub>2</sub>S, etc.) in porous media. What makes SubFlow different from the rest is its numerical core (the module that solves the problem's governing equations). In our software, we use a novel numerical approach to solve the main partial differential equations (PDE) that governs the migration of reactive compounds in porous media at typical injection depths. We use mimetic discretization methods (MDM) to attain higher-order accurate simulations without compromising the physical constraints inherent to the problem. For this type of problem, MDM have proven to be a versatile and competitive alternative to the widely used standard finite-difference (FDM) and finite element (FEM) methods. (Received September 25, 2017)

1135-35-2140 **Kelsey Wells\*** ([kelsey.wells@huskers.unl.edu](mailto:kelsey.wells@huskers.unl.edu)) and **Petronela Radu.** *Properties and convergence of state-based Laplacians.*

Motivated by the state-based peridynamic framework we introduce a new nonlocal Laplacian operator. This operator is a double integral operator with two kernels with interaction horizons of radius  $\delta$  and  $\epsilon$  respectively. We study the connections between this state-based Laplacian and the nonlocal bond-based and classical Laplacians. We also show convergence of the operator to the classical Laplacian with rates dependent on  $\delta$  and  $\epsilon$ . (Received September 25, 2017)

1135-35-2151 **Keith J Galvin\***, [kgalvin@aphysci.com](mailto:kgalvin@aphysci.com). *A Synthetic Turbulence Generation Model for Wall-Bounded Shear Flows Based on Resolvent Analysis.*

This talk will focus on some preliminary work that has been undertaken with the aim of developing a new space-time representation of large scale inflow turbulence at the stern of a body of revolution. This new turbulence reconstruction model is largely based on the resolvent analysis framework proposed by Dr. Beverly McKeon of Caltech and Dr. Ati Sharma of University of Southampton, which provides a linear approach to determining the "shapes" of the largest eddies in a wall-bounded turbulent flow, based only on the incompressible NSE and a mean turbulent velocity profile. Once the shapes are determined, the amplitudes of the various eddies are chosen such that the total synthetic flow field satisfies constraints on the mean distribution of turbulent velocity components in the wall-normal direction, as obtained from a general two-equation turbulence model used in a steady RANS code. An initial numerical experiment will be presented indicating good agreement between spectral and correlation properties of the reconstructed turbulent field and experimental data. (Received September 25, 2017)

1135-35-2178 **Daniel Onofrei\*** ([onofrei@math.uh.edu](mailto:onofrei@math.uh.edu)), University of Houston, Dept. of Mathematics, 4800 Calhoun Road, 641 Philip G. Hoffman Hall, Houston, TX 77294-3008, and **Eric Platt.** *Field Control Through Manipulation of Surface Sources.*

In this talk we will discuss our past and recent results regarding the problem of acoustic and electromagnetic field manipulation with the help of active sources situated on exterior surfaces. We will first present a series of smooth scalar control results for the Helmholtz equation and based on these results we will describe our strategy for electromagnetic field control. Applications to scattering cancellation, field synthesis (in free space and waveguides) and inverse source problems will be discussed as well. (Received September 25, 2017)

1135-35-2183 **Jianjun Paul Tian\*** ([jtian@nmsu.edu](mailto:jtian@nmsu.edu)), 3625 Santa Sabina Ave, Las Cruces, NM 88012. *Mathematical modeling of PDGF-driven glioma reveals the infiltrating dynamics of immune cells into tumors.* Preliminary report.

Although tumor-infiltrated immune cells consist of a significant component of many cancers and their role remains elusive, an understanding of how the infiltration of immune cells into tumors is regulated is important. Based on our recent experimental results about mutant IDH1 gliomas, we propose a mathematical model for the infiltrating dynamics of immune cells into tumors. Our model distinguishes wtIDH1 gliomas and muIDH1 gliomas by different values of the maximum of chemoattractant production rate. Our model reveals how wtIDH1 tumors reach death volume earlier than muIDH1 tumors do, and shows that as tumor volume increases in both types of gliomas in time, the net increasing rate of immune cells infiltrated into the tumor increases while the percentage of immune cells infiltrated into the tumor decreases. Our model predicts that the wtIDH1 tumor mice will survive longer if the immune cells are blocked, and for more aggressive glioma mice, there is little difference in their survivals between wtIDH1 and muIDH1 tumor mice. Our computation shows if the chemotactic coefficient and the chemoattractant production rate decrease, tumor mice will gain longer survivals. This is a joint work with B. Nin, X. Zeng, F. Szulzewsky, S. Holte, P. Maini, E. Holland. (Received September 25, 2017)

1135-35-2234 **Melissa Guemo Ngamini\*** ([melissa.ngamini@morehouse.edu](mailto:melissa.ngamini@morehouse.edu)), 170 Northside Dr SW, 109, Atlanta, GA 30313. *Nonlinear Filtering Problems for systems governed by PDEs.*

The Kalman filter is one of the very essential discovery for the last century and provides estimate of state and parameters. It works well if the dynamics are nearly linear but it requires improvements for significantly nonlinear and non-Gaussian system.

In this presentation we introduce an improved variation of the Kalman filter for large scale nonlinear stochastic systems based on the Gaussian filter. This is an approach based on the optimal filtering theory; i.e., the optimal filter based on the Bayes' formula for discrete time dynamics and the Zakai equation for continuous time.

That is, we develop the filtering update via the assumed Gaussian density filter. A key step is that we update the covariance in the square root factors form and thus we update the square root factors of the Gaussian covariance. This evolves into the reduced Gaussian filter based on the reduced factor updates. For dissipative system, we develop an alternative to the reduced Gaussian filter, by the assumed covariance filter. For systems that are time reversible, we use the time reversal filter and use the quasi reversible method for mildly diffusive system which are systems that are not time reversible.

(Received September 25, 2017)

1135-35-2243 **Di Kang\*** ([di.kang@cgu.edu](mailto:di.kang@cgu.edu)), **Ali Nadim** and **Marina Chugunova.** *Modeling of the pulmonary surfactant dynamics in alveoli.*

We derive a new model of the alveoli compartment, taking into account pulmonary surfactant production and recycling by type II cells as well as its degradation. As the thickness of alveoli coating is much smaller than the average radius of the alveoli, we employ the classical lubrication approximation to describe the thin liquid film dynamics in the presence of the pulmonary surfactant, which is a surface tension reducing agent and thus prevents the lungs from collapse. In the lubrication limit, we derive a degenerate system of two coupled parabolic partial differential equations that describe the time evolution of the thickness of the coating film inside the alveoli together with that of the surfactant concentration at the interface. We present numerical simulations using parameter values consistent with experimental measurements. (Received September 25, 2017)

1135-35-2252 **David M Ambrose** and **Gideon Simpson\*** ([grs53@drexel.edu](mailto:grs53@drexel.edu)), Department of Mathematics, Korman Center, 33rd & Market Streets, Philadelphia, PA 19104, and **J Douglas Wright** and **Dennis G Yang.** *Existence theory for magma equations in dimension two and higher.*

We examine a degenerate, dispersive, nonlinear wave equation related to the evolution of partially molten rock in dimensions two and higher. This simplified model, for a scalar field capturing the melt fraction by volume, has been studied by direct numerical simulation where it has been observed to develop stable solitary waves. In this work, we prove local in time well-posedness results for the time dependent equation, on both the whole space and the torus, for dimensions two and higher. We also prove the existence of the solitary wave solutions in dimensions two and higher. (Received September 25, 2017)



1135-35-2272 **Feng Rao\*** ([raofeng2002@163.com](mailto:raofeng2002@163.com)) and **Yun Kang** ([yun.kang@asu.edu](mailto:yun.kang@asu.edu)). *The complex dynamics of a diffusive prey-predator model with an allee effect in prey*. Preliminary report.

This paper investigates complex dynamics of a predator-prey interaction model that incorporates: (a) An Allee effect in prey; (b) the Michaelis-Menten type functional response between prey and predator; and (c) Diffusion in both species. We provide rigorous mathematical results of the proposed model including: 1) the stability of non-negative constant steady states; 2) sufficient conditions that lead to Hopf/Turing bifurcations; 3) a priori estimates of positive steady states; 4) the non-existence and existence of non-constant positive steady states when the model is under zero-flux boundary condition. We also perform completed analysis of the corresponding ODE model to obtain a better understanding on effects of diffusion on the stability. Our analytical results show that the small values of the ratio of the prey's diffusion rate to the predator's diffusion rate are more likely to destabilize the system, thus generate Hopf-bifurcation and Turing instability that can lead to different spatial patterns. Through numerical simulations, we observe that our model, with or without Allee effect, can exhibit extremely rich pattern formations that include but not limit to strips, spotted patterns, symmetric patterns. (Received September 25, 2017)

1135-35-2322 **Jae Min Lee\*** ([jlee10@gradcenter.cuny.edu](mailto:jlee10@gradcenter.cuny.edu)), 365 Fifth Avenue, Room 4208, New York, NY 10016, and **Stephen Carl Preston**. *On the global Lagrangian solutions of some Euler-Arnold equations*.

In this paper, we prove that the Lagrangian solutions of two PDEs are global in both space and time: the mu-Hunter-Saxton equation and the Camassa-Holm equation. The method is to express the equations in Lagrangian coordinates and transform them via a change of variable. Consequently, we obtain both the global weak conservative solutions of Bressan-Constantin and global spatial smoothness of Lagrangian trajectories due to McKean in a much simpler way. The method is inspired by the geometric approach of Arnold and Lenells. (Received September 25, 2017)

1135-35-2323 **Xiaoqian Xu\***, Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, and **Alexander Kiselev**, Durham, NC 27708. *Suppression of chemotactic explosion by mixing*.

Chemotaxis plays a crucial role in a variety of processes in biology and ecology. One of the most studied PDE models of chemotaxis is given by Keller-Segel equation, which describes a population density of bacteria or mold which attract chemically to substance they secrete. However, solution of Keller-Segel equation can exhibit dramatic collapsing behavior. In other words, there exist initial data leading to finite time blow up. In this talk, we will discuss the possible effects resulting from interaction of chemotactic and fluid transport processes, namely we will consider the Keller-Segel equation with additional advection term modeling ambient fluid flow. We will prove that the presence of fluid can prevent the singularity formation. We will discuss two classes of flows that have the explosion arresting property. Both classes are known as very efficient mixers. (Received September 25, 2017)

1135-35-2379 **Daniele Garrisi\*** ([daniele.garrisi@gmail.com](mailto:daniele.garrisi@gmail.com)), Namgu Inharo 100, Inha University, Department of Mathematics, 5N211, Incheon, 22212, South Korea. *Symmetric rearrangements and the Concentration-Compactness of minimizing sequences for variational problems with coupled non-linearities*.

Variational problems with coupled non-linearities arise in the study of the stability of standing-wave solutions to systems of non-linear equations, as the coupled non-linear Schrödinger equation. The stability follows from the Concentration-Compactness of minimizing sequences of a certain functional  $J$  on a constraint  $S(\lambda)$ . Such compactness can be obtained from the sub-additivity property of  $I(\lambda) := \inf\{J(u) \mid u \in S(\lambda)\}$  defined for every  $\lambda$  in  $\mathbb{R}^n$ . The sub-additivity property rules out the dichotomic case in the Lemma of Concentration-Compactness of P. L. Lions. In scalar equations, it can be obtained with a simple rescaling argument. In coupled equations ( $n \geq 2$ ) the problem is more challenging, although rescalings still work in some pure-power cases. In this presentation, we illustrate an approach that we introduced for a coupled non-linear Klein-Gordon equation (G., *Advanced Nonlinear Studies*, 2012). It is based on a symmetric decreasing rearrangement type inequality, which follows from an argument of J. Byeon (*Journal of Differential Equations*, 2000). Such approach allows us to broaden the class of coupled non-linearities where one can prove the sub-additivity of  $I$  and thus the Concentration-Compactness. (Received September 26, 2017)

1135-35-2388 **Tarek M Elgindi\***, 9500 Gilman Drive, La Jolla, CA 92023, and **In-Jee Jeong**.  
*Singularity formation in incompressible fluids.*

We will discuss some recent results on singularity formation for finite-energy strong solutions to the 2D Boussinesq and 3D Euler systems based on the analysis of scale-invariant solutions. The work consists of three parts: local well-posedness in critical spaces, the analysis of scale-invariant solutions (which belong to those spaces), and then a cut-off argument to ensure finite energy. This is a joint work with I. Jeong. (Received September 26, 2017)

1135-35-2397 **Sedar Ngoma\*** (ngoma@geneseo.edu), Department of Mathematics, 330D South Hall, State University of New York at Geneseo, Geneseo, NY 14454. *Recovering a source function in a parabolic equation.*

We investigate an inverse source problem for a parabolic partial differential equation with an integral constraint. The source function depends only on the space variable. We show the existence and uniqueness of classical solutions and establish the continuous dependence of the solution on the data. Our proof yields a numerical algorithm that we implement using the finite element method. The numerical results we present show the accuracy of the proposed scheme. (Received September 26, 2017)

1135-35-2410 **James M. Haley\***, 259 Maple Avenue, Pittsburgh, PA 15218. *Forecasting Chaotic Business Cycles.*

In order to create business cycle chaos, a dynamical system must exist that has at least three dimensions in continuous time, which include real stock returns, inflation, and real economic output. This system of differential equations requires at least one nonlinear term to capture the model's sensitive dependence on initial conditions. By applying a Sprott dynamical system, it becomes possible to make short-term forecasts of the economy. It should not be surprising that economists, who use orthodox methods that ignore chaos, failed to predict the Great Recession, which was the worst recession since the Great Depression. When the short-term interest rate is targeted to be 3%, the economy mean-reverts, like a Langevin equation perturbed by noise. Then business cycles no longer exist, because forecasts become normal with decreasing variance. (Received September 26, 2017)

1135-35-2478 **Danny Arrigo\*** (darrigo@uca.edu), Department of Mathematics, 201 Donaghey Ave., Conway, AR 72035, and **Andrea Weaver** (aweaver3@cub.uca.edu), Department of Mathematics, 201 Donaghey Ave., Conway, AR 72035. *Nonclassical Symmetries of a Generalized KdV Equation.* Preliminary report.

It is generally known the classical Lie symmetries of partial differential equations (PDEs) and equivalent systems can be different. Recently it has been shown for a general class of nonlinear Convection-Diffusion equation and nonlinear Wave equation, that their nonclassical symmetries are contained within the nonclassical symmetries of the equivalent system. It is natural to ask if this is true in general. Here, we consider a general class of KdV equation (i.e. K(m,n) equation) and an equivalent system. We calculate the nonclassical symmetries of both and contrast similarities and differences to determine whether the conjecture true. (Received September 26, 2017)

1135-35-2485 **Luiz Gustavo Farah, Justin Holmer\*** (jholmer@nsf.gov) and **Svetlana Roudenko**.  
*Blow-up for the 2D cubic Zakharov-Kuznetsov equation.*

The generalized Zakharov-Kuznetsov (ZK) equation is a higher dimensional version of the generalized Korteweg-de Vries (gKdV) equation. The two-dimensional focusing cubic case of ZK is  $L^2$  critical, and analogous to the quintic case of gKdV, blow-up is expected for negative energy solutions with mass slightly above that of the ground state solitary wave. We prove that such solutions do in fact blow-up in either finite or infinite time. This is accomplished by reducing the problem to a nonlinear Liouville theorem stating that any solution that remains close a modulation of the ground state and satisfies a uniform-in-time  $L^2$  compactness property, must in fact be equal to the ground state. (Received September 26, 2017)

1135-35-2493 **Alfonso Castro** (castro@g.hmc.edu) and **Ivan Ventura\*** (ibventura@cpp.edu).  
*Existence of solutions to a semilinear elliptic boundary value problem with augmented Morse index bigger than two.*

Building on the construction of least energy sign-changing solutions to variational semilinear elliptic boundary value problems introduced in the work of Castro-Cossio-Neuberger, we prove the existence of a solution with *augmented Morse index* at least three when a sub level of the corresponding action functional has nontrivial topology. We provide examples where the set of least energy sign changing solutions is disconnected, hence has nontrivial topology. (Received September 26, 2017)

1135-35-2504 **Roger Thelwell\*** (thelwerj@jmu.edu). *Coefficient Recovery in a Quasilinear Parabolic PDE.*

A numerical recovery algorithm is presented of the simultaneous recovery of two important soil property parameters in the Richards Equation, a quasilinear parabolic PDE. The numerical method is based on an integral identity realized by an adjoint approach. Because the integral identity allows a direct view of the sensitivity of recovery, and tangible ingredients in the identity, it permits a clear and intuitive process by which to understand recovery and features which make recovery difficult. It also allows the sensitivity of the recovery to be explored. Several manufactured examples are considered to highlight indicators that might prove useful. The method is then applied to simulated data (generated by HYDRUS-1D) and finally to data from a physical laboratory. (Received September 26, 2017)

1135-35-2527 **Salomé M Martínez\***, samartin@dim.uchile.cl. *On a competitive system with ideal free dispersal.*

In this article we study long term behavior of the following competitive system

$$\begin{cases} \frac{\partial u}{\partial t} = \nabla \cdot \left[ \alpha(x) \nabla \frac{u}{m} \right] + u(m(x) - u - bv) & \text{in } \Omega, t > 0, \\ \frac{\partial v}{\partial t} = \nabla \cdot [\beta(x) \nabla v] + v(m(x) - cu - v) & \text{in } \Omega, t > 0, \\ \nabla \frac{u}{m} \cdot \hat{n} = \nabla v \cdot \hat{n} = 0 & \text{on } \partial\Omega, t > 0, \end{cases}$$

which supports for the first species an *ideal free distribution*, that is a positive steady state which matches the per-capita growth rate and therefore there is no movement. Previous results have stated that when  $b = c = 1$  the ideal free distribution is an evolutionary stable strategy, that is  $v$  always becomes extinct. Thus, of particular interest will be to study the interplay between the inter-specific competitions  $b, c$  and the diffusion coefficients  $\alpha(x)$  and  $\beta(x)$  in the existence of positive steady states, and obtaining critical values for which bifurcation from semi-trivial steady states arises as well as establish the existence of multiple positive steady states. (Received September 26, 2017)

1135-35-2550 **Alexander Kiselev, Yao Yao and Andrej Zlatos\*** (zlatos@ucsd.edu). *Local regularity for the modified SQG patch equation.*

We show local regularity for the patch dynamics version of the modified SQG equation, which interpolates between the two-dimensional Euler and SQG equations as a parameter  $\alpha$  increases from 0 to  $\frac{1}{2}$ . The result holds for all  $\alpha < \frac{1}{2}$  for the PDE on the whole plane, and for all small enough  $\alpha$  on the half-plane. The latter case is a precursor to our proof of finite time blow-up for this model, while the question of global regularity remains open on the whole plane. (Received September 26, 2017)

1135-35-2556 **Russell W Schwab\***, Department of Mathematics, Michigan State University, 619 Red Cedar Rd, East Lansing, MI 48824. *A min-max formula for Lipschitz operators that satisfy the global comparison principle and some applications.*

We investigate Lipschitz maps,  $I$ , mapping  $C^2(D) \rightarrow C(D)$ , where  $D$  is an appropriate domain. The global comparison property (GCP) simply states that whenever two functions are ordered in  $D$  and touch at a point, i.e.  $u(x) \leq v(x)$  for all  $x$  and  $u(z) = v(z)$  for some  $z \in D$ , then also the mapping  $I$  has the same order, i.e.  $I(u, z) \leq I(v, z)$ . It has been known since the 1960's, by Courrège, that if  $I$  is a linear mapping with the GCP, then  $I$  must be represented as a linear drift-jump-diffusion operator that may have both local and integro-differential parts. It has also long been known and utilized that when  $I$  is both local and Lipschitz it will be a min-max over linear and local drift-diffusion operators, with zero nonlocal part. In this talk we discuss some recent work that bridges the gap between these situations to cover the nonlinear and nonlocal setting for the map,  $I$ . These results open up the possibility to study Dirichlet-to-Neumann mappings for fully nonlinear equations as integro-differential operators on the boundary, and well as to possibly apply the integro-differential results to some Hele-Shaw type free boundary problems. We also discuss these applications of the min-max. This is joint work with Nestor Guillen. (Received September 26, 2017)

1135-35-2562 **Mary P Thomas** (mthomas@mail.sdsu.edu), 5500 Campanile Drive, San Diego, CA 92024, and **Mariangel Garcia** and **Neelam Patel\*** (neelam\_patel1990@yahoo.com), 5500 Campanile Drive, Encinitas, CA 92024, and **Manuel Valera** and **Jose E Castillo**. *Development and optimization of a PETSc-based parallel GCCOM model.*

The General Curvilinear Coastal Ocean Model (GCCOM) is a high-resolution (sub-km), non-hydrostatic, large eddy simulation CFD model that uses a full, 3D curvilinear coordinate system. This results in increased accuracy,

resolution, and reduced times to solution. GCCOM is a petascale model: it requires significant memory (over 100 double precision arrays); communication along all 3 dimensions at each iteration; and simulations generating TBytes of data. The MPI-based parallel model needed improvements in the libraries used for the multi-grid pressure solver used in the serial model. The PETSc scientific programming package was chosen because it has a rich suite of non-linear solvers, and the PETSc Data Management Distributed (DMDAs) have build-in communication and halo cell management. The performance improvement of the pressure solver offset the level of effort required to port the model to the PETSc framework: the entire model needed to be migrated to using all PETSc objects. In this talk we discuss our experiences in developing and testing the PETSc-based model, the parallel framework developed for automating test case jobs, and present results that validate model results and demonstrate scaling of the model. (Received September 26, 2017)

1135-35-2578 **Timur Akhunov\*** (4400 Vestal Pkwy, Binghamton, NY 13902, and **Lyudmilla Korobenko** and **Cristian Rios**. *When solutions of the Laplace equation are not smooth. How much degeneracy is needed?*

Pierre-Simon marquis de Laplace in his 5 volume *Traité de mécanique céleste* unified mathematical physics by placing calculus front and center. Among Laplace's chief innovations, was an introduction of a differential equation that now bears his name. Fast forward to the 20th century. Laplacian is crucial to understand the properties of light, heat, sound and atomic phenomena. Moreover, Einstein has pushed us to understand the Laplacian in the curved world, where some of the mathematicians' favorite toys no longer play as advertised. Come hear the story, where calculus, geometry and physics come together. (Received September 26, 2017)

1135-35-2583 **Deniz Bilman** (bilman@umich.edu) and **Peter D. Miller\*** (millerpd@umich.edu). *A Robust Inverse Scattering Transform for the Focusing Nonlinear Schrödinger Equation.*

We propose a modification of the standard inverse scattering transform for the focusing nonlinear Schrödinger equation (also other equations by natural generalization). The purpose is to deal with arbitrary-order poles and potentially severe spectral singularities in a simple and unified way. As an application, we use the modified transform to place the Peregrine solution and related "rogue wave" solutions in an inverse-scattering context for the first time. This allows one to directly study the stability of such solutions. The modified transform method also allows rogue waves to be generated on top of other structures by elementary Darboux transformations, rather than the generalized Darboux transformations in the literature. (Received September 26, 2017)

1135-35-2604 **Colin Klaus\*** (klaus.68@mbi.osu.edu), **Emmanuele DiBenedetto** and **Ugo Gianazza**. *A Necessary and Sufficient Condition for the Continuity of Local Minima of Parabolic Variational Integrals with Linear Growth.*

For proper minimizers of parabolic variational integrals with linear growth with respect to  $|Du|$ , a necessary and sufficient condition for  $u$  to be continuous at a point  $(x_0, t_0)$  in terms of a sufficient fast decay of the total variation of  $u$  about  $(x_0, t_0)$  is given. These minimizers arise also as proper solutions to the Total Variation Flow (parabolic 1-Laplacian). Hence, the continuity condition is in force for such solutions. (Received September 26, 2017)

1135-35-2626 **Angel A Boada\*** (aboadavelazco@sdsu.edu) and **Jose E Castillo**. *High Order Mimetic Methods on Overlapping Grids.*

Overture is a portable and flexible object-oriented framework for solving partial differential equations (PDEs). One of its features is the composite overlapping grid generation for solving problems that involve the simulation of complex moving geometry. Overlapping grids are a type of block structured body-fitted conforming grids that are used to resolve fine-scale features in a particular domain. One of the most prominent advantages of using these grids is the high efficiency for high-order methods. In this talk, we examine the viability of mimetic operators on overlapping grids by solving representative PDEs on grids generated by Overture, while exploring different interpolation techniques on these grids (both implicitly and explicitly). Examples will be presented to demonstrate the effectiveness of the schemes. (Received September 26, 2017)

1135-35-2638 **Cecilia F Mondaini\*** (cmondaini@math.tamu.edu), Dept. of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843, and **Animikh Biswas**, **Ciprian Foias** and **Edriss S. Titi**. *An ensemble data assimilation algorithm via feedback-control.*

We consider a feedback-control (nudging) approach for ensemble data assimilation that works for a general class of dissipative dynamical systems and observables. More specifically, given a probability distribution associated to uncertainties of spatially coarse measurements, we construct a time-dependent family of probability distributions which converge asymptotically in time, in a suitable sense, to the statistics of the true physical system. This result holds under a suitable assumption on the relaxation parameter, depending, in particular, on the spatial

resolution of the measurements and the forcing term. This is a joint work with A. Biswas, C. Foias and E. S. Titi. (Received September 26, 2017)

1135-35-2655 **Changfeng Gui** and **Amir Moradifam\*** ([amirm@ucr.edu](mailto:amirm@ucr.edu)). *The Sphere Covering Inequality and Its Applications.*

We show that the total area of two distinct Gaussian curvature 1 surfaces with the same conformal factor on the boundary, which are also conformal to the Euclidean unit disk, must be at least  $4\pi$ . In other words, the areas of these surfaces must cover the whole unit sphere after a proper rearrangement. We refer to this lower bound of total areas as the Sphere Covering Inequality. This inequality and its generalizations are applied to a number of open problems related to Moser-Trudinger type inequalities, mean field equations and Onsager vortices, etc, and yield optimal results. In particular we confirm the best constant of a Moser-Trudinger type inequality conjectured by A. Chang and P. Yang in 1987. This is a joint work Changfeng Gui. (Received September 26, 2017)

1135-35-2658 **Guher Camliyurt** and **Igor Kukavica\*** ([kukavica@usc.edu](mailto:kukavica@usc.edu)). *On quantitative uniqueness for solutions of elliptic and parabolic PDEs.*

We first briefly review the results on the size of the nodal (zero) set for solutions of partial differential equations elliptic and parabolic type. These are intimately connected to the quantitative uniqueness, i.e., to the question of the order of vanishing of solutions. We present several recent results for elliptic and parabolic type equations. The results are joint with Guher Camliyurt. (Received September 26, 2017)

1135-35-2672 **Michael Capps\*** ([cappsm@rams.colostate.edu](mailto:cappsm@rams.colostate.edu)), Department of Mathematics, 1874 Campus Delivery, Fort Collins, CO 80523. *Sparse conductivity reconstructions from electrical impedance tomography data using complex geometrical optics solutions and prior structural information.*

In many applications of EIT some prior information about the spatial structures of the conductivity distribution is known, and the conductivity is expected to be piecewise constant. Here a fast method of reconstructing such sparse conductivity distributions from the CGO solutions is presented. Results are presented for several types of experimental data. (Received September 26, 2017)

1135-35-2682 **Ali Behzadan\*** ([abehzadan@ucsd.edu](mailto:abehzadan@ucsd.edu)), 9500 Gilman Drive, Dept. 0112, La Jolla, CA 92093, and **Michael Holst** ([mholst@ucsd.edu](mailto:mholst@ucsd.edu)), 9500 Gilman Drive, Dept. 0112, La Jolla, CA 92093. *Multiplication in Sobolev Spaces.*

We will review some of the classical pointwise multiplication theorems in Sobolev-Slobodeckij spaces, and along the way we discuss a counterexample that illustrates how certain multiplication theorems fail in Sobolev-Slobodeckij spaces when a bounded domain is replaced by  $\mathbb{R}^n$ . We identify the source of the failure, and examine why the same failure is not encountered in Bessel potential spaces. We will also mention a particularly important variation of one of the multiplication theorems that is relevant to the study of nonlinear PDE systems arising in general relativity and other areas. (Received September 26, 2017)

1135-35-2710 **Tahir Bachar Issa\*** ([itb0004@auburn.edu](mailto:itb0004@auburn.edu)), 230 Opelika Road Apt 201, Auburn, AL 36830, and **Wenxian Shen**. *Dynamics in chemotaxis models of parabolic-elliptic type on bounded domain with time and space dependent logistic sources.*

We consider the dynamics a chemotaxis system of parabolic-elliptic type with local as well as nonlocal time and space dependent logistic source on bounded domains. We first prove the local existence and uniqueness of classical solutions for various initial functions. Next, under some explicit conditions on the coefficients, the chemotaxis sensitivity and the space dimension, we prove the global existence and boundedness of classical solutions with certain given integrable or uniformly continuous nonnegative initial functions. Then, under the same conditions for the global existence, we show that the system has an entire positive classical solution. Moreover, if the coefficients are periodic in  $t$  with period  $T$  or are independent of  $t$ , then the system has a time periodic positive solution with periodic  $T$  or a steady state positive solution. If the coefficients are independent of  $x$ , then the system has a spatially homogeneous entire positive solution. Finally, under some further explicit assumptions, we prove that the system has a unique entire positive solution which is globally stable. Furthermore, if the coefficients are periodic or almost periodic in  $t$ , then the unique entire positive solution is also periodic or almost periodic in  $t$ . (Received September 26, 2017)

1135-35-2769 **Guher Camliyurt\*** ([camliyur@usc.edu](mailto:camliyur@usc.edu)), 3620 S. Vermont Ave., USC Mathematics Dep. KAP 104, Los Angeles, CA 90089, and **Igor Kukavica** ([kukavica@usc.edu](mailto:kukavica@usc.edu)), 3620 S. Vermont Ave., USC Math Dep. KAP 104, Los Angeles, CA 90089. *A Lagrangian analyticity result for the Euler equations.*

We revisit preservation of analyticity and Gevrey regularity for the Euler equation. We provide a result on preservation of Gevrey norm and analyticity in Lagrangian formulation and discuss the validity of the result in the Eulerian variables. (Received September 26, 2017)

1135-35-2776 **Evelyn M Lunasin\*** ([lunasin@usna.edu](mailto:lunasin@usna.edu)), U. S Naval Academy, 572C Holloway Road, Math Dept Stop 9E, Annapolis, MD 21402. *Subgrid Scale Models for Large Eddy Simulation.*

In this talk, I will present global well-posedness results and numerical results for various  $\alpha$  subgrid scale turbulence models. I will also discuss the global well-posedness results for the inviscid simplified Bardina model in three dimensions; a result which is currently not available for the other inviscid  $\alpha$ -models. The inviscid simplified Bardina model is a globally well-posed model which approximates the Euler equation without the addition of viscous or hyperviscous terms. (Received September 26, 2017)

1135-35-2796 **Dhanapati Adhikari\*** ([dadhikari@marywood.edu](mailto:dadhikari@marywood.edu)), 2300 Adams Avenue, Scranton, PA 18509. *Remarks on the global regularity of two-dimensional Boussinesq equations.*

The Boussinesq equations is a system of non-linear partial differential equations which model thermal convection and geostrophic flows. The 2D Boussinesq equations have received significant attentions in recent years because of their close connections to 3D hydrodynamics equations. Numerous studies have been done on a global regularity of the 2D Boussinesq equations with partial dissipations or no dissipations. This talk presents some of the recent results on these equations with partial dissipation in horizontal or vertical direction. (Received September 26, 2017)

1135-35-2836 **Emel Bolat\*** ([emel.bolat@mavs.uta.edu](mailto:emel.bolat@mavs.uta.edu)), 1624 Ridge Haven Dr. #713, Arlington, TX 76011, and **Yue Liu** ([yliu@uta.edu](mailto:yliu@uta.edu)), The University of Texas at Arlington, Department of Mathematics Box 19408, 1225 West Mitchell, Arlington, TX 76019. *Local Well-Posedness and Blow-up Criteria for the Rotational b-family Equation.* Preliminary report.

In this study, a mathematical model of long-crested water waves propagating mainly in one direction with the effect of Earth's rotation is derived by following the formal asymptotic procedures. This model equation is corresponding to the Camassa-Holm ( $b=2$ ) and Degasperis-Procesi ( $b=3$ ) approximation of the two-dimensional incompressible and irrotational Euler equations. This new model equation is called the rotational  $b$ -family equation. First of all, we establish the local well-posedness of the rotational  $b$ -family equation. Then we determine the consequences of the Coriolis force caused by the Earth rotation and nonlocal higher nonlinearities on blow up criteria. (Received September 26, 2017)

1135-35-2871 **Philip Isett\*** ([isett@math.utexas.edu](mailto:isett@math.utexas.edu)). *A Proof of Onsager's Conjecture for the Incompressible Euler Equations.*

In an effort to explain how anomalous dissipation of energy occurs in hydrodynamic turbulence, Onsager conjectured in 1949 that weak solutions to the incompressible Euler equations may fail to exhibit conservation of energy if their spatial regularity is below  $1/3$ -Hölder. I will discuss a proof of this conjecture that shows that there are nonzero,  $(1/3 - \epsilon)$ -Hölder Euler flows in 3D that have compact support in time. The construction is based on a method known as "convex integration," which has its origins in the work of Nash on isometric embeddings with low codimension and low regularity. A version of this method was first developed for the incompressible Euler equations by De Lellis and Székelyhidi to build Hölder-continuous Euler flows that fail to conserve energy, and was later improved by Isett and by Buckmaster-De Lellis-Székelyhidi to obtain further partial results towards Onsager's conjecture. The proof of the full conjecture combines convex integration using the "Mikado flows" introduced by Daneri-Székelyhidi with a new "gluing approximation" technique. The latter technique exploits a special structure in the linearization of the incompressible Euler equations. (Received September 26, 2017)

1135-35-2874 **Changhui Tan\*** ([ctan@rice.edu](mailto:ctan@rice.edu)). *Global regularity for Burgers equation with density-dependent fractional dissipation.*

Fractional Burgers equations are a family of equations which connect inviscid and viscous Burgers equations. It is well-known that if the dissipation is strong, the solution is globally regular. On the other hand, if the dissipation is weak (called supercritical case), the solution can lose regularity at a finite time. In this talk, I will introduce a model where the dissipation depends on density. The model is motivated by self-organized dynamics in math biology. Despite that the equation shares a lot of similarities to fractional Burgers equation,

the solution is globally regular, even in the supercritical case. I will explain the regularization mechanism that is due to the nonlocal nonlinear modulation of dissipation. This is a joint work with T. Do, A. Kiselev and L. Ryzhik. (Received September 26, 2017)

1135-35-2924 **Daniel Spirn\*** ([spirn@math.umn.edu](mailto:spirn@math.umn.edu)), 206 Church St. S.E., School of Mathematics, Minneapolis, MN 55455. *Detection of a quench in a superconducting cavity.*

Superconducting radio frequency cavities are used to efficiently accelerate particles in high energy experimental labs. Small defects on the cavity surface can cause rapid heating of the cavity, leading to a thermal quench and potential damage the device. We present a method for quench detection in SRF cavities cooled in liquid helium. A rigorous mathematical formula is derived to localize the quench position from dynamical data over a finite time interval at a second sound detector. This is joint work with Ru-Yu Lai (U. Minnesota). (Received September 26, 2017)

1135-35-2932 **Terrance L Pendleton\*** ([terrance.pendleton@drake.edu](mailto:terrance.pendleton@drake.edu)), IA , and **Hailiang Liu.** *On Invariant-Preserving Finite Difference Schemes for the Camassa-Holm Equation and the Two-Component Camassa-Holm System.*

The purpose of this talk is to develop and test novel invariant-preserving finite difference schemes for both the Camassa-Holm (CH) equation and one of its 2-component generalizations (2CH). The considered PDEs are strongly nonlinear, admitting soliton-like peakon solutions which are characterized by a slope discontinuity at the peak in the wave shape, and therefore suitable for modeling both short wave breaking and long wave propagation phenomena. The proposed numerical schemes are shown to preserve two invariants, momentum and energy, hence numerically producing wave solutions with smaller phase error over a long time period than those generated by other conventional methods. We first apply the scheme to the CH equation and showcase the merits of considering such a scheme under a wide class of initial data. We then generalize this scheme to the 2CH equation and test this scheme under several types of initial data. (Received September 26, 2017)

1135-35-2947 **Elisabeth MM Brown\*** ([embrown5@ncsu.edu](mailto:embrown5@ncsu.edu)) and **Michael Shearer.** *A Scalar Conservation Law for Plume Migration in Carbon Sequestration.*

A quasi-linear hyperbolic partial differential equation with a discontinuous flux models geologic carbon dioxide (CO<sub>2</sub>) migration and storage. Two flux functions characterize the model, giving rise to flux discontinuities. One convex flux describes the invasion of the plume into pore space, and the other captures the flow as the plume leaves CO<sub>2</sub> bubbles behind, which are then trapped in the pore space. We investigate the method of characteristics, the structure of shock and rarefaction waves, and the result of binary wave interactions. The dual flux property introduces unexpected differences between the structure of these solutions and those of a scalar conservation law with a convex flux. During our analysis, we introduce a new construction of cross-hatch characteristics in regions of the space-time plane where the solution is constant, and there are two characteristic speeds. This construction is used to generalize the notion of the Lax entropy condition for admissible shocks, and is crucial to continuing the propagation of a shock wave if its speed becomes characteristic. (Received September 26, 2017)

1135-35-2949 **Ramesh Karki\*** ([rkarki@iue.edu](mailto:rkarki@iue.edu)), 1110 Barrington Ridge, Richmond, IN 47374, and **Roza Aceska** and **Alessandro Arsie.** *Optimal reconstruction of initial data in some evolutionary PDEs via finite discrete samplings.* Preliminary report.

We study about how we can optimally reconstruct initial data for some evolutionary PDEs using only finite discrete measurements at later times. Mainly, we discuss our method in the case of a linear evolutionary PDE with constant coefficients and even order partial derivatives with respect to space variable. We also discuss this approach in the case of some PDEs with time dependent coefficients and its possibility to deal with some non-linear PDEs. (Received September 26, 2017)

1135-35-2959 **Rachidi B Salako\*** ([rbs0016@auburn.edu](mailto:rbs0016@auburn.edu)), Auburn University, Auburn, AL 36849, and **Wenxian Shen** ([wenixsh@auburn.edu](mailto:wenixsh@auburn.edu)), Auburn University, Auburn, AL 36849. *Parabolic-elliptic chemotaxis model with space-time dependent logistic sources on  $\mathbb{R}^N$ . I. Persistence and asymptotic spreading.*

We consider a parabolic-elliptic chemotaxis system with space -time logistic source function on  $\mathbb{R}^N$ , and study : (i) Pointwise and Uniform persistence of solutions with an explicit uniform lower bound in the later case. (ii) Asymptotic spreading of solutions with compactly supported initial functions and prove that there are two positive constant  $0 < c_-^* < c_+^* < \infty$  such that for every  $t_0 \in \mathbb{R}$  and every nonnegative initial function

$u_0 \in C_{\text{unif}}^b(\mathbb{R}^N)$  with nonempty compact support, we have that

$$\lim_{t \rightarrow \infty} \sup_{|x| \geq ct} u(x, t + t_0; t_0, u_0) = 0, \quad \forall c > c_+^*,$$

and

$$\liminf_{t \rightarrow \infty} \sup_{|x| \leq ct} u(x, t + t_0; t_0, u_0) > 0, \quad \forall 0 < c < c_-^*.$$

We also discuss the spreading properties of solutions with front-like initial functions. (Received September 26, 2017)

1135-35-3019 **Igor Kukavica, Walter Rusin\*** ([walter.rusin@okstate.edu](mailto:walter.rusin@okstate.edu)) and **Nabil Ziane**. *Local regularity conditions for the Navier-Stokes equations.*

In this talk, we address the interior regularity of suitable weak solutions of the 3D incompressible Navier-Stokes equations. We present partial regularity criteria based on only one component of the velocity. Furthermore, the uniform estimate obtained in this criterion allows us to prove a generalization of the Leray regularity condition. Finally, we present a local extension of the  $L_t^\infty L_x^3$  regularity due to Escauriaza-Serëgin-Šverák. (Received September 26, 2017)

1135-35-3033 **Saulo Orizaga\*** ([sorizaga@math.duke.edu](mailto:sorizaga@math.duke.edu)) and **Karl Glasner**. *Diblock copolymers and their aligning properties due to electric fields.*

The nonlinear partial differential equation describing phase segregation incorporating polymer stretching is studied in the context of the strong segregation limit. This results in a free boundary problem which is analyzed for a physical relevant structure. The instabilities associated with the an electric field and the lamellar structure are discussed and presented. (Received September 26, 2017)

1135-35-3060 **Maxim Zyskin\*** ([maxim.zyskin@nottingham.ac.uk](mailto:maxim.zyskin@nottingham.ac.uk)), School of math, University of Nottingham, Nottingham, Nottingham NG7 2RD. *Mott moving boundary problem of metal oxidation.*

We describe a moving boundary type model of metal oxidation, and present local existence as well as numerical results. This problem has important applications, including nanopropellants (Received September 26, 2017)

1135-35-3083 **Ryan M. Evans\*** ([ryan.evans@nist.gov](mailto:ryan.evans@nist.gov)), NIST, 100 Bureau Drive, Gaithersburg, MD 20899, **Arvind Balijepalli**, NIST, 100 Bureau Drive, Gaithersburg, MD 20899, and **Anthony J. Kearsley**, NIST, 100 Bureau Drive, Gaithersburg, MD 20899. *Nonlinear PDEs and Precision Medicine.*

Tailoring therapies to individuals or specific subsets of a population in order to deliver personalized care could fundamentally remake healthcare delivery. However, widespread use of personalized care is currently limited by our ability to routinely measure pathology in individuals. Moreover, existing clinical diagnostics are in many cases prohibitively expensive. This has led to the development of Biological Field Effect Transistors (Bio FETs)—microscale instruments in which ligand molecules diffuse through a solution-well onto a surface to bind with receptors. Ligand binding with receptors modulates current flow through the device, and produces a signal used to study the receptor ligand dynamics of interest. A nonlinear PDE model for Bio-FET experiments will be presented. Analyzing this model is a challenge, owing to multiple disparate time scales for reaction and diffusion. It will be shown that in this set of equations reduces to a nonlinear integrodifferential equation (IDE) with a singular convolution kernel. Numerical approximations to the solution to this equation will be presented. These results give experimentalists novel way of estimating binding affinities in biomolecule interactions; this is essential for identifying effective drug targets. (Received September 26, 2017)

1135-35-3110 **Mihaela Ignatova, Igor Kukavica\*** ([kukavica@usc.edu](mailto:kukavica@usc.edu)), **Irena Lasiecka** and **Amjad Tuffaha**. *The global existence of a fluid-structure system with small data.*

We address a system of partial differential equations modeling motion of an elastic body inside an incompressible fluid. The fluid is modeled by the incompressible Navier-Stokes equations while the structure is represented by the damped wave equation with interior damping. We will review the local for large and global existence results for small data. The global existence result is obtained for small initial data in a suitable Sobolev space. (Received September 26, 2017)

1135-35-3136 **Yanni Zeng\*** ([ynzeng@uab.edu](mailto:ynzeng@uab.edu)).  *$L^p$  Asymptotic Behavior of Solutions to General Hyperbolic-Parabolic Systems of Balance Laws in Multi Space Dimensions.*

We study time asymptotic behavior of solutions for a general system of hyperbolic-parabolic balance laws in  $m$  space dimensions,  $m \geq 2$ . The system has physical viscosity matrices. Besides, there is a lower order term to account for relaxation, damping or chemical reaction. The viscosity matrices and the Jacobian matrix of the



lower order term are rank deficient. We study Cauchy problem around a constant equilibrium state. Under a set of reasonable assumptions, existence of solution global in time is established, and  $L^p$  decay rates ( $p \geq 2$ ) of the solution to the constant equilibrium state are obtained. We may further study the large time behavior of the solution. We show that it is time-asymptotically approximated by the solution of the corresponding linear system with the same initial data. For  $p \geq 2$ , optimal  $L^p$  convergence rates to the asymptotic solution are obtained. These rates are faster by  $(t+1)^{-1/2}$  (or  $(t+1)^{-1/2} \ln(t+2)$  if  $m = 2$ ) when comparing to the convergence rates to the constant equilibrium state. Our results are general and apply to physical models such as gas flows with translational and vibrational non-equilibrium. The result on asymptotic behavior is new even for the special case of hyperbolic balance laws. (Received September 26, 2017)

1135-35-3153 **Slim IBRAHIM\*** ([ibrahims@uvic.ca](mailto:ibrahims@uvic.ca)), Victoria, BC V8Z6L7, Canada, **Yasunori Maekawa** ([maekawa@math.kyoto-u.ac.jp](mailto:maekawa@math.kyoto-u.ac.jp)), Kyoto, Japan, and **Nader Masmoudi** ([masmoudi@courant.nyu.edu](mailto:masmoudi@courant.nyu.edu)), Courant Institute, New York University, New York, NY.  
*Stability of Kolmogorov flows.*

The aim of this talk is to show enhanced dissipation in view of the stability analysis of the steady Kolmogorov flows. The proof relies on some resolvent estimate on the imaginary axis of the resolvent parameters for the linearized problem around the steady Kolmogorov flows. In particular, our result gives an affirmative answer to numerical conjecture about the size of the basin of attraction. (Received September 26, 2017)

1135-35-3154 **Martina Hofmanova** and **James-Michael Leahy\*** ([leahyj@usc.edu](mailto:leahyj@usc.edu)), USC Dornsife, Department of Mathematics, 3620 S. Vermont Ave., KAP 104, Los Angeles, CA 90089, and **Torstein Nilssen**. *On the Navier Stokes Equation with Rough Transport Noise.*

In this talk, we present some results on the existence of weak-solutions of the Navier-Stokes equation perturbed by transport-type rough path noise with periodic boundary conditions in dimensions two and three. The noise is smooth and divergence free in space, but rough in time. We will also discuss the problem of uniqueness in two dimensions. The proof of these results makes use of the theory of unbounded rough drivers developed by M. Gubinelli et al.

As a consequence of our results, we obtain a pathwise interpretation of the stochastic Navier-Stokes equation with Brownian and fractional Brownian transport-type noise. A Wong-Zakai theorem and support theorem follow as an immediate corollary. This is joint work with Martina Hofmanová and Torstein Nilssen. (Received September 26, 2017)

1135-35-3206 **Alexandru Tamasan\*** ([tamasan@math.ucf.edu](mailto:tamasan@math.ucf.edu)), Department of Mathematics, 4000 Central Florida Blvd., Orlando, FL 32816, and **Alexander Timonov** ([atminonov@uscupstate.edu](mailto:atminonov@uscupstate.edu)), Division of Mathematics and Computer Science, 800 Univ. Way, Spartanburg, SC 29302. *Current density impedance imaging without boundary information.* Preliminary report.

We consider the inverse problem of recovering the electrical conductivity of a body from interior measurements of the magnitude of the current density field. Mathematically, we need to solve a boundary value problem for the 1-Laplacian with Complete Electrode Model (CEM) boundary conditions. This problem has nonunique solutions. We presented the analysis of a regularized method which recovers an approximate conductivity with a given precision. (Received September 27, 2017)

1135-35-3239 **Edriss S. Titi\*** ([titi@math.tamu.edu](mailto:titi@math.tamu.edu)), Department of Mathematics, Texas A&M University, 3368 TAMU, College Station, TX 77843-3368. *The Navier-Stokes, Euler and Related Equations.*

In this talk I will present the most recent advances concerning the questions of global regularity of solutions to the three-dimensional Navier-Stokes and Euler equations of incompressible fluids. Furthermore, I will also present recent global regularity (and finite time blow-up) results concerning certain three-dimensional geophysical flows, including the three-dimensional viscous (non-viscous) “primitive equations” of oceanic and atmospheric dynamics. (Received September 27, 2017)

## 37 ► *Dynamical systems and ergodic theory*

1135-37-5 **Maleafisha J.P.S. Tladi\*** (maleafisha.tladi@gmail.com), Department of Mathematics & Applied Mathemati, Sovenga, Limpopo 0727, South Africa. *Lagrangian Chaos and Transport in Geophysical Fluid Flows.*

Advances in our understanding of the dynamics of mesoscale eddies and vortex rings have been based on modons which are nonlinear Rossby solitary wave solutions of quasi-geostrophic equations. This article focuses primarily on geometric singular perturbation (GSP) theory and Melnikov techniques to address the problem of adiabatic chaos and transport for translating and rotating modons to the quasi-geostrophic potential vorticity system. A very general and central question is what hypotheses on the equations and singular solutions guarantee that the solutions approximate some solutions for the perturbed quasi-geostrophic potential vorticity system. We present a geometric approach to the problem which gives more refined a priori energy type estimates on the position of the invariant manifold and its tangent planes as the manifold passes close to a normally hyperbolic piece of a slow manifold. We apply Melnikov technique to show that the Poincare map associated with modon equations has transverse heteroclinic orbits. We appeal to the Smale-Birkhoff Homoclinic Theorem and assert the existence of an invariant hyperbolic set which contains a countable infinity of unstable periodic orbits, a dense orbit and infinitely many heteroclinic orbits. (Received May 18, 2017)

1135-37-11 **Mrinal K Roychowdhury\*** (mrinal.roychowdhury@utrgv.edu), University of Texas Rio Grande Valley, 1201 West University Drive, Edinburg, TX. *An overview of optimal quantization.*

The basic goal of quantization for probability distribution is to reduce the number of values, which is typically uncountable, describing a probability distribution to some finite set and thus approximation of a continuous probability distribution by a discrete distribution. Mixed distributions are an exciting new area for optimal quantization. Recently, in the paper “An overview of the quantization for mixed distributions”, available in arXiv, I have determined the optimal sets of  $n$ -means, the  $n$ th quantization error, and the quantization dimensions of different mixed distributions. Besides, I have discussed whether the quantization coefficients for the mixed distributions exist. The results in this paper will give a motivation and insight into more general problems in quantization of mixed distributions. I will talk about it. (Received June 05, 2017)

1135-37-77 **Muhammad U. Abdulla\***, 150 West University Blvd., Melbourne, FL 32901, **Ugur G. Abdulla**, 150 West University Blvd., Melbourne, FL 32901, **Naveed H. Iqbal**, 150 West University Blvd., Melbourne, FL 32901, and **Jake Barrett** (barrettj2015@my.fit.edu), 150 West University Blvd., Melbourne, FL 32901. *Minimal Orbits, Sharkovski Ordering and Universality in Chaos.*

We prove the outstanding conjecture on the number of third minimal odd periodic orbits of continuous endomorphisms on the real line. In a recent paper *Abdulla et al., International Journal of Bifurcation and Chaos*, **27**, 5, 2017, it is proved that there are  $4k - 3$  types of second minimal  $2k + 1$ -orbits,  $k \geq 3$ , each characterized with unique cyclic permutations and directed graphs of transitions with accuracy up to inverses. In this paper, we prove that there are  $8k^2 + 32k - 110$  types of third minimal  $2k + 1$  periodic orbits,  $k \geq 4$ , each characterized with unique cyclic permutations and digraphs with accuracy up to inverses. The primary application of this result is to the problem of identifying and classifying the distribution of superstable periodic windows within the chaotic regime of bifurcation diagrams of the one-parameter family of unimodal maps. It is revealed in the referred article, that by fixing the maximum number of appearances of periodic windows, a universal pattern of distribution arises. In particular, the second (or third) appearance of all orbits in the bifurcation diagrams were always a second (or third) minimal orbit, with both a Type 1 cyclic permutation (and respective digraph), and a unimodal topological structure. (Received July 23, 2017)

1135-37-119 **Daniel Olszewski\***, Department of Mathematics, Carroll College, Helena, MT 59625, and **Mariya Savinov**, Department of Mathematics, University of Pittsburgh, Pittsburgh, PA 15260. *Controlling period-2 electrical activity in a cardiac cell model.*

Ordinary differential equations allow us to model the behavior of single cardiac cells in response to stimuli. Extending these equations into a cable of cells, we get a set of partial differential equations that describe the flux of voltage and ions across the cable. We study a specific dynamic exhibited by these systems called alternans. Electrical alternans, the beat-to-beat alternations of cellular action potential duration (APD) and/or intracellular calcium concentration amplitude (peak  $Ca$ ), is a dynamical state that often precedes life-threatening arrhythmia, which is characterized by the irregular propagation of electrical waves and is the leading cause of sudden cardiac death. Previous efforts aimed at controlling alternans have utilized mathematical models that

primarily exhibit voltage-driven alternans; here, we consider the impact of intracellular calcium mechanisms. We utilize the Shiferaw-Fox et al. cardiac action potential model, which is capable of both voltage- and calcium-driven alternans, for a single cell (0D) and for a cable of cells (1D). Our results indicate that the success of strategies to control cardiac alternans in experimental and clinical settings may depend on the mechanism underlying alternans. (Received July 29, 2017)

1135-37-181 **Joseph H Silverman\*** ([jhs@math.brown.edu](mailto:jhs@math.brown.edu)), Mathematics Department, Brown University, 151 Thayer Street, Providence, RI 02912, and **Adriana Salerno**, Department of Mathematics, Bates College, Lewiston, ME 04240. *Böttcher coordinates for one-dimensional superattracting germs over nonarchimedean fields*. Preliminary report.

Let  $R$  be a ring of characteristic 0 with field of fractions  $K$ , and let  $m \geq 2$ . The Böttcher coordinate of a power series  $\varphi(x) \in x^m + x^{m+1}R[[x]]$  is the unique power series  $f_\varphi(x) \in x + x^2K[[x]]$  satisfying  $\varphi \circ f_\varphi(x) = f_\varphi(x^m)$ . In this paper we study integrality properties of the coefficients of  $f_\varphi(x)$ , both for their intrinsic interest and for potential applications to  $p$ -adic dynamics. (Received August 07, 2017)

1135-37-183 **Andrew Bridy\*** ([andrewbridy@tamu.edu](mailto:andrewbridy@tamu.edu)) and **Thomas J. Tucker** ([thomas.tucker@rochester.edu](mailto:thomas.tucker@rochester.edu)). *Arboreal finite index for cubic polynomials*.

Let  $K$  be a global field of characteristic 0. Let  $f \in K[x]$  and  $b \in K$ , and set  $K_n = K(f^{-n}(b))$ . The projective limit of the groups  $\text{Gal}(K_n/K)$  embeds into the automorphism group of an infinite rooted tree. A major problem in arithmetic dynamics is to find conditions that guarantee the index is finite; a complete answer would give a dynamical analogue of Serre's celebrated open image theorem. We solve the finite index problem for cubic polynomials over function fields by proving a complete list of necessary and sufficient conditions. For number fields, the proof of sufficiency is conditional on both the abc conjecture and a form of Vojta's conjecture. (Received August 07, 2017)

1135-37-270 **Israel Ncube\*** ([ncube.israel@gmail.com](mailto:ncube.israel@gmail.com)). *Stability of equilibria in the presence of distributed time delays in neuronal networks*. Preliminary report.

Adopting general delay distributions, we consider the stability of equilibria of some common mathematical models of artificial neuronal micro-circuits. In particular, we characterise the linearised stability of equilibria and how it is impacted by model parameters. (Received August 17, 2017)

1135-37-358 **Joshua P. Bowman\*** ([joshua.bowman@pepperdine.edu](mailto:joshua.bowman@pepperdine.edu)), Natural Science Division, Pepperdine University, 24255 Pacific Coast Highway, Malibu, CA 90263, and **Slade Sanderson** ([slade.sanderson@pepperdine.edu](mailto:slade.sanderson@pepperdine.edu)). *Angels' staircases and trajectories on homothety surfaces*.

A homothety surface is an orientable surface equipped with an atlas whose transition maps are homotheties – that is, compositions of translations and scaling. A discrete set of cone-type singularities is allowed. Homothety surfaces enjoy many of the same properties as translation surfaces have, including a well-defined notion of direction (slope) everywhere except at the singularities and an action by  $GL(2, \mathbb{R})$ . They have so far appeared only sporadically in the literature, and many basic questions about their dynamical properties remain open.

In this work, we study a certain one-parameter family of genus 2 homothety surfaces. We show that they all have essentially the same behavior for their linear trajectories, which have a surprisingly simple relation to trajectories on the square torus. There is a dense open set of directions in which the surface has a periodic trajectory. In an uncountable set of directions, the closure of a critical trajectory has a Cantor set cross-section. We give explicit formulas for these directions and show that all the Cantor sets involved have Hausdorff dimension zero.

This is joint work with Slade Sanderson. (Received August 27, 2017)

1135-37-376 **Izabel Aguiar\***, [izabel.p.aguiar@gmail.com](mailto:izabel.p.aguiar@gmail.com), and **Paul Constantine**. *Dynamic Active Subspaces*. Preliminary report.

This research explores the concepts of parameter space dimension reduction for dynamical systems using by extending active subspaces to dynamical systems. Analysis, dynamic mode decomposition (DMD), sparse identification for nonlinear dynamic systems (SINDy), and other methods are implemented to discover and reconstruct active subspaces for time dependent systems. Such methods allow us to build dynamical systems that model parameter sensitivity over time for a specific state of the original system. (Received August 29, 2017)

1135-37-443 **Gerardo Chowell\*** ([gchowell@gsu.edu](mailto:gchowell@gsu.edu)). *Epidemic Growth Scaling: Implications for Disease Forecasting and Estimation of the Reproduction Number.*

The increasing use of mathematical models for epidemic forecasting has highlighted the importance of designing reliable models that capture the baseline transmission characteristics of specific pathogens and social contexts. Here, we review recent progress on modeling and characterizing early epidemic growth patterns from infectious disease outbreak data, and survey the types of mathematical formulations that are most useful for capturing a diversity of early epidemic growth profiles, ranging from sub-exponential to exponential growth dynamics. Specifically, we review mathematical models that incorporate spatial details or realistic population mixing structures, including meta-population models, individual-based network models, and simple SIR-type models that incorporate the effects of reactive behavior changes or inhomogeneous mixing. In this process, we also analyze simulation data stemming from detailed large-scale agent-based models previously designed and calibrated to study how realistic social networks and disease transmission characteristics shape early epidemic growth patterns, general transmission dynamics, and control of international disease emergencies such as the 2009 A/H1N1 influenza pandemic and the 2014-2015 Ebola epidemic in West Africa. (Received September 03, 2017)

1135-37-456 **De-Jun Feng, Chiu-Hong Lo and Shuang Shen\*** ([shuangshen@nwpu.edu.cn](mailto:shuangshen@nwpu.edu.cn)). *Lyapunov Exponents for products of matrices.*

Let  $\mathbf{M} = (M_1, \dots, M_k)$  be a tuple of real  $d \times d$  matrices. Under certain irreducibility assumptions, we give checkable criteria for deciding whether  $\mathbf{M}$  possesses the following property: there exist two constants  $\lambda \in \mathbb{R}$  and  $C > 0$  such that for any  $n \in \mathbb{N}$  and any  $i_1, \dots, i_n \in \{1, \dots, k\}$ , either  $M_{i_1} \cdots M_{i_n} = \mathbf{0}$  or  $C^{-1}e^{\lambda n} \leq \|M_{i_1} \cdots M_{i_n}\| \leq Ce^{\lambda n}$ , where  $\|\cdot\|$  is a matrix norm. The proof is based on symbolic dynamics and the thermodynamic formalism for matrix products. As applications, we are able to check the absolute continuity of a class of overlapping self-similar measures on  $\mathbb{R}$ , the absolute continuity of certain self-affine measures in  $\mathbb{R}^d$  and the dimensional regularity of a class of sofic affine-invariant sets in the plane. (Received September 04, 2017)

1135-37-496 **Gregory Varner\*** ([gvarner@jbu.edu](mailto:gvarner@jbu.edu)). *Unique Measure for the Time-Dependent Navier-Stokes Equations.*

An extension of results concerning the uniqueness of measure for time-homogeneous Markov processes to the time-inhomogeneous case is applied to the random Navier-Stokes equations. In particular, we will discuss the necessary conditions for the associated measures to be exponentially mixing. A specific example that will be considered is the two-dimensional Navier-Stokes equations on the sphere under time-dependent deterministic force and a random kick-force activated by a dirac or an indicator function. The necessary conditions for the existence and uniqueness of the measure for the system will be discussed. (Received September 06, 2017)

1135-37-511 **Michael E. Taylor\*** ([met@math.unc.edu](mailto:met@math.unc.edu)), Mathematics Dept., Phillips Hall, University of North Carolina, Chapel Hill, NC 27599. *Variations on quantum ergodic theorems.*

We discuss some quantum ergodic theorems, related to local behavior of eigenfunctions of the Laplace operator and related operators on a compact Riemannian manifold, emphasizing results that hold without the hypothesis that the classical Hamiltonian flow (e.g., the geodesic flow) associated to the operator be ergodic. Cases treated include both integrable Hamiltonians and some associated with "soft chaos." (Received September 06, 2017)

1135-37-580 **Henk Bruin\*** ([henk.bruin@univie.ac.at](mailto:henk.bruin@univie.ac.at)), Faculty of Mathematics, University of Vienna, Oskar Morgensternplatz 1, 1090 Vienna, Austria. *Matching for translated  $\beta$ -transformations.*

This talk is about the parameter space of a family of translated  $\beta$ -transformations  $T(x) = \beta x + a \pmod{1}$  in regard to the property of "matching". This means that at some iterate  $T^n(0) = T^n(1)$ , and this has a bearing on the invariant density of  $T$ . The prevalence of matching is proved under specific number-theoretic conditions: the slope  $\beta$  is a (specifically quadratic) Pisot number. (This is joint work with Carlo Carminati, Pisa, and Charlene Kalle, Leiden) (Received September 09, 2017)

1135-37-698 **Daniel M Look\*** ([dlook@stlawu.edu](mailto:dlook@stlawu.edu)), SLU Dept of Mathematics, 23 Romoda Drive, Canton, NY 13617. *The Dynamics of Two Circle Inversions.*

Previous results demonstrate a large amount of dynamical variability within the family of  $n$ -circle inversions with  $n > 2$  when complex radii are allowed. We will concentrate on the anti-holomorphic family of maps

$$z \mapsto \frac{r^2 \bar{z}}{\bar{z}^2 - 1}$$

that arise from a particular 2-circle inversion.

The dynamics possible for this family differ in many ways from the  $n > 2$  case; in particular, for  $n = 2$  we see the appearance of parameter values for which the Julia set is the entire Riemann sphere. We show that this

family does not allow Herman rings and use this to discuss the topology of the connectedness locus. (Received September 13, 2017)

1135-37-729 **Diana Davis\*** ([dianajdavis@gmail.com](mailto:dianajdavis@gmail.com)), 500 College Avenue, Swarthmore, PA 19081.

*Tiling billiards and interval exchange transformations.*

*Tiling billiards* is a fascinating new dynamical system where beams of light refract through planar tilings. I'll explain how, with a group of undergraduates in the SMALL REU in 2016, we used a simple geometric observation to transform this two-dimensional problem into a one-dimensional problem, which reduces it to understanding interval exchange transformations. (Received September 13, 2017)

1135-37-830 **Mrinal K Roychowdhury\*** ([mrinal.roychowdhury@utrgv.edu](mailto:mrinal.roychowdhury@utrgv.edu)), 1201 West Univesity Drive, University of Texas Rio Grande Valley, Edinburg, TX 78539. *Optimal quantization.*

Quantization for probability distributions refers to the idea of estimating a given probability by a discrete probability supported by a finite number of points. Quantization dimension gives the speed how fast the  $n$ th quantization error goes to zero as  $n$  approaches to infinity. It has broad applications in signal processing and data compression. I will talk about it. (Received September 15, 2017)

1135-37-997 **Weam M Al-Tameemi\*** ([weam.altameemi@tamiu.edu](mailto:weam.altameemi@tamiu.edu)), 5201 University Blvd., LBV 313, Laredo, TX 78041. *Strange Attractors and Some Dynamical Properties of a Unimodal Map.*

It is clear that sensitivity to initial conditions is a special case of expansive sensitivity. In 1993, MacEachern and Berliner considered the converse of the previous statement and showed that the answer is negative applied on a discrete dynamical system generated by the Tent map, which is a piecewise unimodal linear map on the interval  $[0, 1]$ . The purpose of this presentation is to generalize their analysis to continuous unimodal maps on the unit interval  $[0, 1]$ , in particular on the Logistic map  $f_r(x) = rx(1-x)$  when  $r = 4$ . In this direction we will first have a brief review of strange attractors and provide a classification according to the dynamical system which generates them and their geometrical properties. Then, we will classify the points of the unit interval of the Logistic map when  $r = 4$  according to their shadows and will prove that the set of all shadows of a point is negligible and the set of all points having shadows has Lebesgue measure one. (Received September 18, 2017)

1135-37-1017 **David M McClendon\*** ([mccleend2@ferris.edu](mailto:mccleend2@ferris.edu)), Department of Mathematics, ASC 2021, Big Rapids, MI 49307, and **Aimee S.A. Johnson** ([aimee@swarthmore.edu](mailto:aimee@swarthmore.edu)), Department of Mathematics & Statistics, 500 College Ave., Swarthmore, PA 19081. *Speedups of  $\mathbb{Z}^d$ -odometers.* Preliminary report.

A speedup of dynamical system  $(X, T)$  is a system  $(X, T^p)$  where  $p : X \rightarrow \{1, 2, 3, \dots\}$ . The big-picture question surrounding this talk is whether, given two dynamical systems, there exists a speedup of the first system which is isomorphic to the second. In the measure-preserving category, Arnoux, Ornstein and Weiss showed that given any two ergodic transformations, there exists a speedup of one system (where the  $p$  is measurable) which is isomorphic to the second. In the topological category, things are more interesting; Ash proved that given two Cantor minimal systems, whether or not such a speedup exists depends on the dimension groups of the systems. Furthermore, Alvin, Ash and Ormes recently showed that any bounded speedup of an odometer is an odometer that is conjugate to the original odometer.

In the early 2010s, Johnson and McClendon extended the notion of speedup to actions of  $\mathbb{Z}^d$ , and proved an analogue of the Arnoux et al. result. In this talk, we discuss speedups of  $\mathbb{Z}^d$ -odometers; we show that as with  $\mathbb{Z}$ -odometers, a speedup of a  $\mathbb{Z}^d$ -odometer must be an odometer, but unlike the  $\mathbb{Z}$  case, the speedup need not be topologically conjugate to the original system. (Received September 18, 2017)

1135-37-1028 **Hal L Smith\*** ([halsmith@asu.edu](mailto:halsmith@asu.edu)), School of mathematical & Statistical Sciences, Arizona State University, Tempe, AZ 85287. *Persistence versus extinction for a class of discrete-time structured population models.*

We provide sharp conditions distinguishing persistence and extinction for a class of discrete-time dynamical systems on the positive cone of an ordered Banach space generated by a map which is the sum of a positive linear contraction  $A$  and a nonlinear perturbation  $G$  that is compact and differentiable at zero in the direction of the cone. Such maps arise as year-to-year projections of population age, stage, or size-structure distributions in population biology where typically  $A$  has to do with survival and individual development and  $G$  captures the effects of reproduction. The threshold distinguishing persistence and extinction is the principal eigenvalue of  $(I - A)^{-1}DG(0)$  provided by the Krein-Rutman Theorem, and persistence is described in terms of associated eigen-functionals. Our results are illustrated by application to a plant model with a seed bank. (Received September 18, 2017)

1135-37-1118 **Thomas Silverman\*** (thomas\_silverman@brown.edu). *A non-archimedean  $\lambda$ -lemma and  $J$ -stability.*

In a celebrated paper published in 1983, R. Mañé, P. Sad, and D. Sullivan prove a result about holomorphic families of injections called the  $\lambda$ -Lemma with impressive applications to the complex dynamics of families of one-variable rational functions. In this talk, I will discuss a framework for studying the dynamics of families of one-variable rational functions parametrized by Berkovich spaces over a complete non-archimedean field and a suitable non-archimedean analogue of the  $\lambda$ -Lemma. I will also explain how this can be used to prove the equivalence of two stability conditions in non-archimedean dynamics. (Received September 19, 2017)

1135-37-1127 **Cameron Bishop** (cbishop01@wesleyan.edu) and **David Hughes\*** (drh272@psu.edu), 729 Tussey Lane, State College, PA 16801, and **Kurt Vinhage** (kvinhage@uchicago.edu) and **Yun Yang** (yyang@gc.cuny.edu). *Entropy Rigidity and Flexibility for Suspension Flows of Anosov Diffeomorphisms.*

We prove that for any smooth suspension flow over Anosov diffeomorphisms of two dimensional torus, the suspension flow is smoothly conjugate to a constant-time suspension flow over a hyperbolic automorphism of the two torus if and only if the volume measure is the measure with maximal entropy. (Received September 19, 2017)

1135-37-1142 **Mark C McClure\*** (mcmccclur@unca.edu). *Computational adventures in complex dynamics.* Preliminary report.

Complex dynamics has long been a field filled with vivid illustrations. In this presentation, we'll discuss some of the images I've generated through my collaboration with Jane, as well as some of the computational issues that can arise when generating images arising in original research. Examples include the iteration of Weierstrass Elliptic functions and the iteration of rational functions with neutral orbits but no attractive orbits. Generation of these types of images is not a simple matter of numerical power but involve challenges for computer algebra as well. (Received September 19, 2017)

1135-37-1224 **Trevor Hyde\*** (tghyde@umich.edu), 609 Lawrence St. Apt. 4, Ann Arbor, MI 48104, and **Michael Zieve**. *An arithmetic dynamical Mordell-Lang theorem.* Preliminary report.

Suppose  $u(x), f(x)$  are rational functions with coefficients in a finitely generated field  $K$  of characteristic 0 such that  $\deg(f) \geq 2$ . We show that for any  $p \in K$  the set  $\{n \in \mathbb{N} : f^n(p) \in u(K)\}$  is the union of finitely many arithmetic progressions. This result was conjectured by Cahn, Jones, and Spear; it may be viewed as an arithmetic version of the dynamical Mordell-Lang conjecture in one dimension. Our proof involves a new type of dynamical situation, namely that of iterated fibered products, which leads to further questions and results. (Received September 20, 2017)

1135-37-1399 **Susmita Sadhu\*** (susmita.sadhu@gcsu.edu) and **Christian Kuehn**. *Noise induced mixed-mode oscillations in a predator-prey system with two time-scales near the onset of a limit cycle.*

We study the effect of stochasticity, in the form of Gaussian white noise, in a three-species predator-prey model with two distinct time-scales. The interactions between the three species is modeled by a system of slow-fast Itô stochastic differential equations. For a suitable parameter regime, the deterministic drift part of the model admits a folded node singularity and exhibits a singular Hopf bifurcation. We transform the stochastic model into its normal form near the folded node, which can be then used to understand the interplay between deterministic and stochastic small amplitude oscillations. The stochastic model admits several kinds of noise driven mixed-mode oscillations that capture the intermediate dynamics between two cycles of population outbreaks of the prey. We perform numerical simulations to study the distribution of the random number of small oscillations between two large oscillations, which can be related to the return time between the outbreaks. Depending on the noise intensity and the distance to the Hopf bifurcation, we find that the distributions of the small oscillations resemble the 1200 years record on the return times of larch budmoth outbreak events in the subalpine larch forests in the European Alps. (Received September 21, 2017)

1135-37-1403 **Melike Sirlanci\*** (sirlanci@usc.edu), University of Southern California, Department of Mathematics, Los Angeles, CA 90089, **Susan E Luczak** (luczak@usc.edu), University of Southern California, Department of Psychology, 90089, and **I G Rosen** (grosen@usc.edu), University of Southern California, Department of Mathematics, Los Angeles, CA 90007. *Estimating the Distribution of Random Parameters in an Abstract Parabolic System Modelling Transdermal Alcohol Biosensor.*

We estimate the distribution of random parameters based on aggregate population data in a family of infinite dimensional dynamical systems with unbounded input and output operators. The systems that we consider are governed by regularly dissipative, or abstract parabolic, operators in a Gelfand triple setting. Our approach synthesizes some recent results on random systems of this form with an abstract approximation framework for the estimation of distributions of random parameters in dynamical systems. We are able to rewrite the system in such a way that the random parameters can be treated as additional space variables and the system becomes amenable to application of results from standard (deterministic) linear semigroup theory. In stating our approximation results we make use of well-known convergence results for sequences of linear semigroups of operators. Our work is motivated by the problem of estimating blood alcohol concentration (BAC) based on the aggregate data in the form of transdermal alcohol levels in a population collected using a biosensor that detects and counts ethanol molecules on the surface of the skin. Numerical results involving actual clinical and field data will be presented and discussed. (Received September 21, 2017)

1135-37-1427 **Pavel Galashin\*** (galashin@mit.edu) and **Pavlo Pylyavskyy**. *R-systems.* Birational toggling on Gelfand-Tsetlin patterns appeared first in the study of geometric crystals and geometric Robinson-Schensted-Knuth correspondence. Based on these birational toggle relations, Einstein and Propp introduced a discrete dynamical system called *birational rowmotion* associated with a partially ordered set. We generalize birational rowmotion to the class of arbitrary strongly connected directed graphs, calling the resulting discrete dynamical system *the R-system*. We study its integrability from the points of view of singularity confinement and algebraic entropy. We show that in many cases, singularity confinement in an *R*-system reduces to the Laurent phenomenon either in a cluster algebra, or in a Laurent phenomenon algebra, or beyond both of those generalities, giving rise to many new sequences with the Laurent property possessing rich groups of symmetries. Some special cases of *R*-systems reduce to Somos and Gale-Robinson sequences. (Received September 22, 2017)

1135-37-1432 **Gunog Seo\*** (gseo@colgate.edu) and **Gail S.K. Wolkowicz** (wolkowicz@mcmaster.ca). *Pest Control by Generalist Parasitoids: a Bifurcation Theory Approach.* Megal et al. (Math. Med. Biol. 25, 1-20; 2008) studied a spatial and non-spatial host-parasitoid model motivated by biological control of horse-chestnut leaf miners, microlepidoptera (micro moths) which have spread through Europe. Here we focus on the non-spatial model. They considered predation of leaf miners by a generalist parasitoid with Holling type II functional response. They showed that there can be at most six equilibrium points and discussed local stability. In my talk, I revisit their non-spatial model identifying cases missed in their investigation and discuss the possible consequences with respect to pest control strategies. I study both the local stability of equilibria and global properties. I use a bifurcation theoretical approach and provide analytical expressions for transcritical, fold, and Hopf bifurcations. My numerical results show very interesting dynamics, which include multiple limit cycles, homoclinic orbits, and codimension one bifurcations including: transcritical, fold, Hopf, and cyclic-fold bifurcations as well as codimension two bifurcations including: Bautin and Bogdanov-Takens bifurcations. (Received September 22, 2017)

1135-37-1458 **John R. Doyle\*** (jdoyle@latech.edu), Mathematics & Statistics, Louisiana Tech University, Ruston, LA 71272. *Dynamical modular curves and uniform boundedness of preperiodic points.* Preliminary report. Morton and Silverman's uniform boundedness conjecture for preperiodic points of rational maps, which was originally stated over number fields, has a natural analogue over function fields. I will state the function field version of the conjecture and discuss a recent proof of this conjecture for the family  $f_c(z) = z^d + c$  with  $d \geq 2$ . The proof involves giving lower bounds on the gonality of (the geometric components of) the associated dynamical modular curves. This is joint work with Bjorn Poonen. (Received September 22, 2017)

1135-37-1483 **Natalie Priebe Frank** and **E Arthur Robinson, Jr.\*** (robinson@gwu.edu), Department of Mathematics, George Washington University, Washington, DC 20052. *A family of infinite local complexity tiling flows with countable Lebesgue spectrum.* Preliminary report.

For  $R > 1$ , we consider a 1-parameter family of 1-dimensional tiling substitutions generalizing the well known Fibonacci substitution  $a \rightarrow b; b \rightarrow ba$ . Here  $a$  and  $b$  are 'short' and 'long' tiles respectively, whose lengths, also

denoted  $a$  and  $b$ , satisfy  $a \in [1, R]$  and  $b \in [R, R + 1]$ . We first expand each tile by  $\lambda = (R + 1)/R$ . Then  $\lambda a$  becomes  $b$ , but  $\lambda b$  becomes two adjacent tiles  $ba$  with length ratio  $b/a = R$ . We study the resulting tiling dynamical system.

The choice of  $R$  determines whether the tiling space has finite or infinite local complexity. It is infinite except for a countable set of  $R$ , and in fact  $R = (1/2)(1 + \sqrt{5})$  gives the usual Perron-Frobenius Fibonacci substitution flow, which has pure discrete spectrum as a consequence of the Pisot property. Other  $R$  giving finite local complexity give weakly mixing flows which, because of finite local complexity, are not strongly mixing. But we show that the infinite local complexity cases all have countable Lebesgue spectrum, and are thus strongly mixing. The entropy, however, is zero. (Received September 22, 2017)

1135-37-1499      **Jason Atnip\*** ([jason.atnip@unt.edu](mailto:jason.atnip@unt.edu)). *Dimensions of Non-autonomous Meromorphic Functions of Finite Order.*

In this talk we study two classes of meromorphic functions previously studied by Mayer and by Kotus and Urbański. In particular we estimate a lower bound for the Julia set and the set of escaping points for non-autonomous additive and affine perturbations of functions from these classes. For particular classes we are able to calculate these dimensions exactly. We accomplish this by constructing non-autonomous graph directed Markov systems, which sit inside of the aforementioned non-autonomous Julia sets. We also give estimates for the eventual and eventual hyperbolic dimensions of these non-autonomous perturbations. (Received September 22, 2017)

1135-37-1537      **Renato Feres\*** ([feres@math.wustl.edu](mailto:feres@math.wustl.edu)) and **Timothy Chumley**. *Entropy production in random billiards and the second law of thermodynamics.* Preliminary report.

A random dynamical system is said to be time-reversible if the statistical properties of orbits do not change after reversing the arrow of time. The degree of irreversibility is captured by the notion of entropy production rate. A general formula for entropy production will be presented that applies to a class of random billiard systems on Riemannian manifolds with boundary for which it is meaningful to talk about energy exchange between billiard particle and boundary. This formula establishes a relation between the purely mathematical concept of entropy production rate and the physical concept of thermodynamic entropy. In particular, it recovers Clausius formulation of the second law of thermodynamics: the system must evolve so as to transfer energy from hot to cold. This is joint work with Tim Chumley. (Received September 23, 2017)

1135-37-1572      **Leonard John Carapezza\*** ([carapezza@math.utah.edu](mailto:carapezza@math.utah.edu)). *Unique Equilibrium States and Equilibrium States with Positive Entropy.*

It was proved by Bowen that for expansive homeomorphisms with the specification property, a certain class of continuous potential functions all have unique equilibrium states. Since then it has been shown that there are cases for which these hypotheses can be weakened and the conclusion still holds. The question of when these equilibrium states have strictly positive entropy has also been studied. In this talk I will summarize some known results and the methods by which they were obtained, as well as discuss work initiated at the MRC towards results of this kind for a particular class of piecewise monotone interval maps. (Received September 23, 2017)

1135-37-1575      **Kelsey Butera\*** ([buterak2@hawkmail.newpaltz.edu](mailto:buterak2@hawkmail.newpaltz.edu)) and **Anca Radulescu** ([radulesa@newpaltz.edu](mailto:radulesa@newpaltz.edu)). *Template iterations of quadratic maps and hybrid Mandelbrot sets.*

As a particular problem within the field of non-autonomous discrete systems, we consider iterations of two quadratic maps  $f_{c_0} = z^2 + c_0$  and  $f_{c_1} = z^2 + c_1$ , according to a prescribed binary sequence, which we call *template*. We study the asymptotic behavior of the critical orbits, and define the Mandelbrot set in this case as the locus for which these orbits are bounded. Unlike in the case of single maps, this concept can be understood in several ways. For a fixed template, one may consider this locus as a subset of the parameter space in  $\mathbb{C}^2$ ; for fixed quadratic parameters, one may consider the set of templates which produce bounded critical orbits. We consider both situations, as well as *hybrid* combinations of them, and we investigate basic topological properties of these sets.

We use this framework to study the effect of errors in copying mechanisms (such as DNA replication). Viewing one of the functions as the correct one, and the other as an erroneous perturbation – we consider problems that a sustainable replication system may have to solve when facing the potential for errors. We find that it is possible to tell which specific errors are more likely to affect the system's dynamics, in absence of prior knowledge of their timing. (Received September 23, 2017)



1135-37-1622 **Marco Antonio López\*** ([marco.lopez@unt.edu](mailto:marco.lopez@unt.edu)). *Dimension of shrinking target sets arising from nonautonomous dynamical systems.*

For a dynamical system on a metric space  $X$  we define a shrinking target set consisting of those points  $x \in X$  whose orbits hit a ball of shrinking radius infinitely often. In special cases, such sets arise from Diophantine approximation. One aspect of such sets that is often studied is their Hausdorff dimension. We will talk about how thermodynamic formalism can be applied to characterize the Hausdorff dimension of such sets for a certain class of nonautonomous iterated function systems. (Received September 23, 2017)

1135-37-1795 **Mojtaba Moniri\*** ([m-moniri@wiu.edu](mailto:m-moniri@wiu.edu)) and **Saman Moniri** ([moniri@umich.edu](mailto:moniri@umich.edu)). *Attracting and repelling cycles, Gröbner bases, and Brouwer's FPT.*

In approximating the onset and the first bifurcation points of the parameters for logistic maps for various short periodic orbits, calculation of Gröbner bases via computer algebra systems such as Mathematica have shown useful. If one picks a parameter rather close to such a bifurcation point, direct approximation of the limiting cycle and its length determination could be problematic (this is due to accumulation of round-off error at each iteration step). We use Gröbner bases to deeper approximate some attracting and repelling limiting cycles of interest beyond what is achievable by iteration. We also provide examples where the iterative cycle approximation, while is still valid, may (and for some other examples may not) be supplied to Brouwer's fixed point theorem to establish the period length. (Received September 24, 2017)

1135-37-1905 **Mónica Moreno Rocha\*** ([mmoreno@cimat.mx](mailto:mmoreno@cimat.mx)), Centro de Investigación en Matemáticas, CIMAT, Callejón Jalisco s/n, 36023 Guanajuato, Gto, Mexico. *Elliptic functions and Herman rings.* Preliminary report.

Consider a dynamical system on the Riemann sphere defined by the iterates of a rational function. The maximal connected components where the iterates form a normal family are known as Fatou components. Periodic Fatou components are classified into five types: super-attracting, attracting and parabolic basins, Siegel disks and Herman rings. In contrast to other type of periodic components, Herman rings are not associated to a periodic orbit, hence it is not a simple problem to determine when a rational function has a cycle of Herman rings.

In the context of iteration of elliptic functions, it was shown by J. Hawkins and L. Koss (2004) that for any lattice  $\Lambda$ , the Weierstrass  $\wp_\Lambda$  function has no cycle of Herman rings. Similar results have been obtained for other elliptic functions, mostly of even order and over special lattices. In this talk we describe a quasiconformal procedure between an elliptic function of order  $n \geq 2$  and a rational function of degree  $d \geq 2$  in order to obtain a new elliptic function of order  $n + d - 1 \geq 3$  that exhibits a cycle of Herman rings. (Received September 25, 2017)

1135-37-2046 **Daniel Cuzzocreo\*** ([dcuzz@math.northwestern.edu](mailto:dcuzz@math.northwestern.edu)). *Cantor Julia Sets and Automorphisms of Shift Quotients.* Preliminary report.

In 1991, Blanchard, Devaney and Keen proved the existence of a surjective homomorphism from the fundamental group of the shift locus in the space of monic and centered degree  $d$  polynomials onto the automorphism group of the one-sided  $d$ -shift. In this talk, we look at some generalizations concerning polynomials with Cantor Julia sets which are outside the shift locus and automorphisms of quotients of the  $d$ -shift. (Received September 25, 2017)

1135-37-2159 **Elizabeth Sattler\*** ([1sattler@carleton.edu](mailto:1sattler@carleton.edu)) and **Ben Matson**. *S-limited shifts.* Preliminary report.

In this talk, we will explore the construction and dynamical properties of  $S$ -limited shifts. An  $S$ -limited shift is a subshift defined on a finite alphabet  $\mathcal{A} = \{1, \dots, p\}$  by a set  $\mathcal{S} = \{S_1, \dots, S_p\}$ , where  $S_i \subseteq \mathbb{N}$  describes the allowable lengths of blocks in which the corresponding letter may appear. We will discuss conditions for which an  $S$ -limited shift is a subshift of finite type or sofic. We will also discuss conjugacy conditions and a formula for calculating the entropy of an  $S$ -limited shift. (Received September 25, 2017)

1135-37-2172 **David Darmon\*** ([david.darmon.ctr@usuhs.edu](mailto:david.darmon.ctr@usuhs.edu)). *Information Theoretic Model Selection for Reconstruction of Stochastic Dynamical Systems from Data.* Preliminary report.

In the absence of mechanistic or phenomenological models of real world systems, data-driven models become necessary. The discovery of various embedding theorems in the 1980s and 1990s motivated a powerful set of tools for analyzing deterministic dynamical systems via delay-coordinate embeddings of observations of their component states. However, in many branches of science, the condition of operational determinism is not satisfied, and stochastic models must be brought to bear. For such stochastic models, the tool set developed for delay-coordinate embedding is no longer appropriate, and a new toolkit must be developed. We present

an information theoretic criterion for data-driven modeling of stochastic dynamical systems: the negative log-predictive likelihood. We develop a non-parametric estimator for the negative log-predictive likelihood, and demonstrate its performance on stochastic maps and flows. Finally, we show how the output of the model selection procedure can be used to compare candidate predictors for a stochastic system to an information theoretic lower bound. (Received September 25, 2017)

1135-37-2208 **Micah Brame\*** ([mbrame@butler.edu](mailto:mbrame@butler.edu)), 4600 Sunset Ave, Indianapolis, IN 46208, and **Scott Kaschner** ([skaschne@butler.edu](mailto:skaschne@butler.edu)), 4600 Sunset Ave, Indianapolis, IN 46208. *Limits of Julia sets for sums of power maps and polynomials.*

For maps of one complex variable,  $f$ , given as the sum of a degree  $n$  power map and a degree  $d$  polynomial whose constant term has modulus less than one, we provide necessary and sufficient conditions that the geometric limit as  $n$  approaches infinity of the set of points that remain bounded under iteration by  $f$  is the closed unit disk. (Received September 25, 2017)

1135-37-2223 **Hamid Al-Saqban, Paul Apisa, Alena Erchenko, Osama Khalil\*** ([khalil.37@osu.edu](mailto:khalil.37@osu.edu)), **Shahriar Mirzadeh** and **Caglar Uyanik**. *Hausdorff Dimension of Exceptional Directions for The Teichmüller Geodesic Flow.*

Chaika and Eskin proved that the conclusions of Birkhoff's and Oseledets' theorems for the Teichmüller geodesic flow hold at every translation surface in almost every direction. We extend their results by showing that the Hausdorff dimension of the set of directions exhibiting a definite amount of deviation from the correct limit is not full. Our approach consists of 3 components: proving that Chaika-Eskin's results hold uniformly in large sets, reducing our problem to one of recurrence to these good sets and finally controlling directions with bad recurrence properties via a technique originally due to Margulis.

This is joint work with Hamid Al-Saqban, Paul Apisa, Alena Erchenko, Shahriar Mirzadeh and Caglar Uyanik. This work grew out of the MRC program "Dynamical Systems: Smooth, Symbolic, and Measurable". (Received September 25, 2017)

1135-37-2225 **Kathryn A Lindsey\*** ([kathryn.a.lindsey@gmail.com](mailto:kathryn.a.lindsey@gmail.com)), Maloney Hall, Dept. of Mathematics, Boston College, Chestnut Hill, MA 02467. *3D shapes associated to Julia sets.* Any planar "shape"  $P$  can be embedded isometrically as part of a convex surface  $S$  in  $\mathbb{R}^3$  such that the boundary of  $P$  is the support of the curvature of  $S$ . In particular, if  $P$  is a connected filled Julia set of a polynomial, this can be done so that the curvature distribution of the convex surface is proportional to the measure of maximal entropy on the Julia set. What would the associated convex subset of  $\mathbb{R}^3$  look like? What can it tell us about the dynamics of the polynomial? This talk is based on joint work with Laura DeMarco. (Received September 25, 2017)

1135-37-2229 **Simone Evans\*** ([evanss3@hawkmail.newpaltz.edu](mailto:evanss3@hawkmail.newpaltz.edu)) and **Anca Radulescu** ([radulesa@newpaltz.edu](mailto:radulesa@newpaltz.edu)). *Dynamics of Quadratic Networks.*

Many natural systems are organized as self-interacting networks composed of coupled quadratic nodes. Because these nodes receive functional input from not only themselves but also the other nodes in the network, they have ensemble behavior different from that of isolated functional nodes. Our objective is to study how the architecture of a network affects asymptotic dynamics. We extend accepted theorems and results from systems with isolated quadratic nodes to networks of quadratic nodes. (Received September 25, 2017)

1135-37-2250 **Rodrigo Treviño\*** ([rodrigo@math.umd.edu](mailto:rodrigo@math.umd.edu)) and **Scott Schmieding** ([schmiedi@math.northwestern.edu](mailto:schmiedi@math.northwestern.edu)). *Graph iterated function systems, Bratteli diagrams, and random substitution tilings of  $\mathbb{R}^d$ .*

We study tilings obtained by performing random substitutions. These tilings can be seen as given by graph iterated function systems obtained by extending the blowup construction of M. Barnsley and A. Vince. We use Bratteli diagrams to organize the hierarchical structure of the tilings and introduce renormalization dynamics through the shift of the Bratteli diagrams. The Lyapunov spectrum of the induced cocycle on the cohomology bundle then controls the behavior of ergodic averages of the  $\mathbb{R}^d$  action on the tiling space. The associated invariant distributions help explain questions physical nature such as behavior of diffraction measures as well as the spectral properties of Schrodinger operators associated to these tilings. This generalizes several works focused on self-similar tilings as this can be seen as a non-stationary version of those tilings. (Received September 25, 2017)

1135-37-2264 **Anthony Sanchez\*** ([asanch33@uw.edu](mailto:asanch33@uw.edu)). *W-measurable sensitivity of semigroup actions*. Preliminary report.

We study the notions of measurable sensitivity and  $W$ -measurable sensitivity in the context of semigroup actions. We also consider the restriction of  $W$ -measurably sensitive semigroup actions to sub-semigroups and ask when does this restriction remain  $W$ -measurably sensitive. (Received September 25, 2017)

1135-37-2269 **Khoa D Tran\*** ([kdtran3@illinois.edu](mailto:kdtran3@illinois.edu)), 205 E Stoughton St, Unit 22, Champaign, IL 61820. *Stability of Periodic Fixed Points and Invariant Sets of the Modulated Logistic Map*.

In this talk, we are studying the stability of the 1- and 2-periodic fixed points and the invariant sets of the two-dimensional dynamical system proposed by Elhadj and Sprott in [1]. The map exemplifies a slave-master relationship where the first, master component is defined by the logistic map and the second is also a logistic map regulated by the first component. We begin by finding the fixed points of the map and determine their stability as a function of its three parameters  $a$ ,  $b$ , and  $c$ . In certain parametric regimes, we further analyze the basin of attraction of stable fixed points and fully describe the dynamics of arbitrary initial data. Lastly we want to numerically and analytically determine the invariant sets of the second component in the domain  $[0, 1]$  as it usually represents the validity region for the underlying models. We will detail our analysis using computer graphics and simulations to demonstrate our results.

[1]Elhadj, Zeraoulia, and J. C. Sprott. "The effect of modulating a parameter in the logistic map." *Chaos: An Interdisciplinary Journal of Nonlinear Science*. 18.2 (2008): 023119. (Received September 25, 2017)

1135-37-2294 **Stephen G. Eubank\*** ([eubank@vt.edu](mailto:eubank@vt.edu)), BioInformatics Institute, 1015 Life Sciences Circle, Blacksburg, VA 24061, and **Madhurima Nath** ([mmath@vt.edu](mailto:mmath@vt.edu)) and **Yihui Ren** ([yren2@vt.edu](mailto:yren2@vt.edu)). *An extension of networked dynamical systems suitable for perturbative analysis*. Preliminary report.

Networked dynamical systems contain an inherently discrete parameter – the interaction network itself. Deriving macroscopic behavior for networked systems from microscopic interactions is known to be intractable in general, but approximations with provably good error bounds are needed for solving parameter estimation, prediction, and control problems. Unfortunately, discrete problems do not satisfy the continuity constraints required for perturbative analysis. Here, we show how to extend any particular instance of a networked dynamical system  $\mathcal{D}$  into a one-parameter family of systems  $D(\xi)$  that is amenable to perturbative analysis. By construction,  $D(\xi)$  is well-defined for all  $\xi \in [0, 1]$ ;  $D(1/2) = \mathcal{D}$ ;  $D(0)$  is a simpler system of non-interacting nodes; and  $D(\xi)$  is self-dual under  $\xi \leftrightarrow 1 - \xi$ . Solutions for  $D(1)$  can be derived from  $D(0)$ . Solutions for finite systems are finite-degree polynomials in  $\xi$  with bounded coefficients. Taylor expansions around the solutions at 0 and 1 can be developed by analyzing minimal cuts and flows in configuration space. Although determining all coefficients exactly requires solving an intractable satisfiability problem, Monte Carlo sampling provides estimates that suffice for most purposes. (Received September 25, 2017)

1135-37-2303 **Nick Mendler\*** ([nickmendler101@gmail.com](mailto:nickmendler101@gmail.com)). *Connectedness of affine Iterated Function Systems*.

Special cases, conjectures, and open questions regarding; the parameter-space of affine Iterated Function Systems with connected attractors; 'just-touching' connected attractors; notions of shortest path giving pathwise-connection in 'code-space' vs the attractor. (Received September 25, 2017)

1135-37-2319 **Emily G. Burkhead\***, CB #3250, Phillips Hall, Chapel Hill, NC 27599, and **Jane M. Hawkins**. *Modeling Virus Dynamics with Stochastic Cellular Automata*.

A cellular automaton (CA) is a continuous shift commuting map of a symbolic shift space on a finite alphabet and is a deterministic dynamical system of great interest in mathematics, physics, and computer science. While CAs are used to model complex dynamics, stochastic CA (SCA) are closely related to these and provide better models of physical phenomena when great uncertainty or extreme complexity is involved. SCA are dynamical systems that are homogeneous in time and space, modeling parallel processes like CA, but at each point in time and space, there are multiple local rules to choose from to update a state (always from the same finite list of rules). We discuss topological dynamical properties of SCA in the context of virus dynamics models. The models originally arose using clinical data, but we are able to articulate the differences in their viral dynamics using some topological dynamical properties. We consider surjectivity and topological transitivity, and we apply our definitions and results to existing models of dynamics that exhibit different behavior and capture properties of HIV and Ebola virus, labelling the behavior as H-dynamics (surjective and topologically transitive) and E-dynamics (neither). (Received September 25, 2017)

1135-37-2494 **Vidhu S Prasad\*** (vidhu\_prasad@um1.edu), Dept of Mathematics, One University Ave., University of Massachusetts Lowell, Lowell, MA 01824. *Parametrized Homeomorphic measures theorems on cubes*. Preliminary report.

Every “good” measure  $\mu$  (nonatomic Borel probability measure positive on open sets and zero on boundary), is homeomorphic to product Lebesgue measure  $\lambda$  (for some homeomorphism  $h$  of  $I^n$ ,  $\mu = \lambda h$ .)

For the space of good measures with the weak topology and the space of homeomorphisms with uniform convergence, the mapping  $\pi : H(I^n) \rightarrow M(I^n)$  defined by  $\pi(h) = \lambda h$  is continuous onto. If  $\pi$  has continuous cross section over  $\mathcal{K} \subset M(I^n)$  then we say the measures  $\mathcal{K}$  are represented by homeomorphisms.

We show that for finite  $n$ ,  $2 \leq n < \infty$ , the  $n$ -dimensional good measures in  $I^{n+1}$  (measures of the form  $\mu \times \lambda_1$  where  $\mu \in M(I^n)$  and  $\lambda_1$  is one dimensional Lebesgue measure on the unit interval  $I$ ) can be represented by homeomorphisms of  $I^{n+1}$ .

For the case of the Hilbert cube  $I^\infty$  the infinite dimensional good measures (those in  $M(I^\infty) \times \lambda \subset M(I^\infty \times I^\infty)$  where  $\lambda \in M(I^\infty)$  is infinite product Lebesgue measure) can be represented by homeomorphisms of  $I^\infty \times I^\infty$ .

This represents joint work with Vincent C. Peck. (Received September 26, 2017)

1135-37-2532 **e. H. el Abdalaoui** and **Cesar E. Silva\*** (csilva@williams.edu). *Möbius random law and infinite rank one maps*. Preliminary report.

We discuss Sarnak’s conjecture on the Möbius-Liouville randomness law in the context of infinite measure-preserving rank-one maps. (Received September 26, 2017)

1135-37-2542 **Debra Mimbs\*** (dmimbs@leeuniversity.edu), 761 Abby Glen Drive NE, Cleveland, TN 37312. *Circle Maps Dynamics and Their Relationship to Julia Sets*.

In order to study the Julia set of a given function, one may study the corresponding (Thurston) invariant lamination, i.e. a collection of chords of the unit circle  $\mathbb{S}$  (called leaves) that are pairwise disjoint inside the open unit disk and satisfy a few dynamical properties. By generating an equivalence relation with certain properties, one then arrives at a space that is topologically equivalent to a Julia set. Therefore, the dynamics of maps on the unit circle directly yield information about the dynamics within the Julia set itself. Certain dynamical properties and interesting open questions that may yield themselves nicely to an undergraduate audience will be introduced. (Received September 26, 2017)

1135-37-2554 **Christopher L Cox\*** (clcox@udel.edu). *Obstructions to Ergodicity in No-slip Billiards*.

No-slip billiards are dynamical systems based on a model in which linear and angular momentum may be exchanged at boundary collisions. We give an analytic proof of an experimentally verified conjecture that a large class of two dimensional billiards are not ergodic. New numerical results consider examples involving multiple body systems and three dimensional systems. (Received September 26, 2017)

1135-37-2614 **Dmitry Kleinbock, Ioannis Konstantoulas\*** (ykonstant@gmail.com) and **Florian Richter**. *Eventually always hitting sets in rapid mixing systems*. Preliminary report.

In this work, joint with Dmitry Kleinbock and Florian Richter, we are studying a non-conventional shrinking target problem in the presence of rapid mixing. We consider the set of points whose long orbit segments eventually hit the corresponding shrinking targets for all future times and ask for criteria to determine whether it is a null or conull set. Even in the presence of total independence this set can behave erratically due to the involvement of extremal rather than mean statistics. For this reason we have focused on product and Bernoulli systems which exhibit perfect or near perfect mutual independence; our eventual goal is to get these conditions for the Gauss map and extract applications to a constrained Diophantine approximation problem. (Received September 26, 2017)

1135-37-2631 **Shahriar Mirzadeh\*** (shahmir@brandeis.edu), MS:050 415 South St., Waltham, MA 02453. *Exceptional sets in the moduli space of translation surfaces and Hausdorff dimension*.

Moduli space of translation surfaces is a non-compact space, so it is natural to ask questions about the largeness of divergence geodesics. We prove that for every translation surface  $(X, \omega)$ , the set of angles  $\theta \in [0, 2\pi)$  so that the orbit of  $(e^{i\theta} X, e^{i\theta} \omega)$  under the Teichmüller geodesic flow is divergent on average has Hausdorff dimension at most  $1/2$ . We use a function that controls the excursions into the compact part of the space. This function was introduced by Eskin, Magulis and Mozes in the setting of homogeneous spaces and later was developed for translation surfaces by Athreya. Also, it turns out that some of the methods used by Kadyrov, Kleinbock, Lindenstrauss and Margulis for solving a similar problem in the context of homogeneous spaces can be applied to the setting of translation surfaces as well. This is joint work with Hamid Al-Saqban, Paul Apisa, Alena

Erchenko, Osama Khalil and Caglar Uyanik. This work grew out of the MRC program “Dynamical Systems: Smooth, Symbolic, and Measurable”. (Received September 26, 2017)

1135-37-2684 **Scott Schmieding, Kitty Yang, May Mei\*** (meim@denison.edu) and **Claire Merriman**. *The automorphism group of the square of a subshift*. Preliminary report.

Let  $\mathcal{A}$  be a finite alphabet and  $X$  a closed subset of  $\mathcal{A}^{\mathbb{Z}}$  invariant under the shift map  $\sigma: \mathcal{A}^{\mathbb{Z}} \rightarrow \mathcal{A}^{\mathbb{Z}}$ . The automorphism group  $\text{Aut}(X, \sigma)$  consists of homeomorphisms  $\phi: X \rightarrow X$  such that  $\phi\sigma = \sigma\phi$ , and has been intensely studied over the years. We consider the the following question: when are  $\text{Aut}(X, \sigma)$  and  $\text{Aut}(X, \sigma^2)$  isomorphic? We give examples where they are known to be isomorphic, and examples where they are not. We then focus on the case where  $(X, \sigma)$  is minimal, and state various results, primarily in the case where  $(X, \sigma)$  has linear complexity. (Received September 26, 2017)

1135-37-2692 **Caleb J Ziegler\*** (caleb.ziegler@unt.edu), Department of Mathematics, University of North Texas, 1155 Union Circle #311430, Denton, TX 76203. *On Mixing Properties of Rank One Subshifts*.

Rank one subshifts are dynamical systems generated by a regular combinatorial process based on sequences of positive integers called the cut and spacer parameters. Despite the simple process that generates them, rank one subshifts comprise a generic set and are the source of many counterexamples. As a result, measure theoretic rank one subshifts have been extensively studied and the topological analogue has been the basis of much recent work.

We examine the factor structure for topological rank one subshifts. First, we classify the finite factors of the system based on the spacer parameters. Furthermore, we show that we can determine the maximal equicontinuous factor based on the presence of finite factors. These methods also lead to a classification of the weak mixing and mixing properties as well. (Received September 26, 2017)

1135-37-2738 **Harrison Bray, Diana Davis, Kathryn Lindsey and Chenxi Wu\*** (wuchenxi2013@gmail.com). *Dynamics of generalized beta-transformations*. Preliminary report.

We study the dynamical properties of the topological generalized beta transformations, which generalizes the concept of generalized beta transformations defined by Gora. In particular, we generalize the result on admissible sequence for unimodular maps to the case of generalized beta maps, and also study the properties of the topological entropy and its Galois conjugates, generalizing some results by Tiozzo and others. This talk represents an ongoing collaboration with Diana Davis, Kathryn Lindsey and Harry Bray. (Received September 26, 2017)

1135-37-2753 **Russell Lodge\*** (russell.lodge@stonybrook.edu). *Boundary values of Thurston’s pullback map*.

Since its inception by mathematicians such as Fatou and Julia, complex dynamics has sought to understand rational maps as dynamical systems on the Riemann sphere, together with their parameter spaces. Postcritically-finite maps (those rational maps where each critical point has finite forward orbit) play a central role in the structure of parameter space, and much effort has gone towards giving a combinatorial classification of such maps. To this end, a groundbreaking theorem of W. Thurston characterizes those postcritically-finite topological self-covers of the sphere that are equivalent to rational maps. The characterization is given in terms of the preimages of multicurves under the cover, but since little is known about such preimages, Thurston’s theorem is notoriously difficult to apply. I will show how the global dynamics of multicurves can be well understood in terms of group theory and the Weil-Petersson extension of Thurston’s pullback map on Teichmueller space. (Received September 26, 2017)

1135-37-2799 **Daniel Cuzzocreo, Joanna Furno\*** (jfurno@math.uh.edu), **Kenneth Jacobs** and **Scott Kaschner**. *Seeing Repelling Dynamics*. Preliminary report.

Many algorithms for drawing Julia sets in complex dynamics rely on detecting attracting behavior to draw Fatou components. When a rational map has no attracting cycles, we must use another algorithm, such as calculating random preimages. Many current algorithms for drawing bifurcation loci in parameter space rely on detecting attracting behavior. In joint work with Daniel Cuzzocreo, Kenneth Jacobs, and Scott Kaschner, we discuss how these algorithms can miss repelling behavior. (Received September 26, 2017)

1135-37-2820 **Petko M. Kitanov\*** (pkitanov@gmail.com), 142 Golden Rod Lane, Apt. 5, Rochester, NY 14623. *Huygens’ Clocks Revisited*.

This paper presents an analysis of the behavior of a large class of coupled identical oscillators, including Huygens clocks, using methods of equivariant bifurcation theory. The equivariant normal form for such systems is

developed and the possible solutions are characterized. The transformation of the physical system parameters to the normal form parameters is given explicitly and applied to the physical values appropriate for Huygens' clocks, and to those of more recent studies. It is shown that Huygens' physical system could only exhibit anti-phase motion, explaining why Huygens observed exclusively this. In contrast, some more recent researchers have observed in-phase or other more complicated motion in their own experimental systems. Here it is explained which physical characteristics of these systems allow for the existence of these other types of stable solutions. The present analysis not only accounts for these previously observed solutions in a unified framework, it also introduces behavior not classified by other authors, such as a "toroidal breather" and a chaotic toroidal breather. (Received September 26, 2017)

1135-37-2881 **Cara Donovan\*** ([cmdono18@g.holycross.edu](mailto:cmdono18@g.holycross.edu)). *A Dynamical Systems Approach To Climate Modeling.*

The goal of this project is to use low-dimensional mathematical models to better understand current climate, historical climate, and the drivers of climate change. In 1968, Mikhail Budyko developed one of the first Energy Balance Models, a differential equation that expresses the global average surface temperature of the Earth as a function of latitude. Budyko incorporated several relevant climate parameters such as albedo and the greenhouse gas effect. The latitude of interest in trying to model climate conditions of the past and present is that of the ice line. Using Budyko's model with a new albedo function that incorporates land and the fact that glaciers form at -10 degrees C, we can see how the ice line moves when varying certain parameters. Changes in the parameter values can cause qualitative changes in the equilibrium solutions also known as bifurcations. Climate scientists refer to these as tipping points because they often indicate major shifts in the climate system. So far, we have produced several bifurcation diagrams for different parameters to discern the possibility of extreme climate conditions in both the Neoproterozoic Era and the current climate state. Our model supports the theory of Snowball Earth and a subsequent ice-free state in the Neoproterozoic Era. (Received September 26, 2017)

1135-37-2903 **Subhadip Chowdhury\*** ([subhadip@math.uchicago.edu](mailto:subhadip@math.uchicago.edu)). *Rotation Number, Ziggurat Fringes, and Fractal Boundary.*

In this talk, we discuss some interesting rigidity and rationality properties of Calegari-Walker ziggurats – i.e. the graphs of extremal rotation numbers associated to positive words in free groups. Specifically, we give an explicit formula for fringe length, revealing integral projective self-similarity in Ziggurat fringes, which are low-dimensional projections of characteristic polyhedra on the bounded cohomology of free groups. This explains phenomena observed experimentally by Gordenko, Calegari and Walker. (Received September 26, 2017)

1135-37-2940 **Boyan Xu\*** ([xu3369@gmail.com](mailto:xu3369@gmail.com)), 412 W Washington ST, Urbana, IL 61801. *Delay embeddings and topological time series analysis.* Preliminary report.

In a recent paper, J. Perea and C. Tralie introduce a method of detecting quasiperiodic behavior in video data using persistent homology. Their technique is motivated by Takens delay embedding theorem: generically, one can reconstruct a dynamical system by a uniform finite sampling of an observation function along trajectories. In our work we characterize precisely quasiperiodic functions with delay embedding yielding Tori and Klein bottles. Our methods involve studying infinitesimal behavior of observation functions along integral curves and generalize to arbitrary compact manifolds. (Received September 26, 2017)

1135-37-2945 **Joshua J Clemons\*** ([jclemons@vt.edu](mailto:jclemons@vt.edu)), Mathematics Department, 460 McBryde Hall, Blacksburg, VA 24061-0123. *An exploration of Mandelbrot sets in unexpected places.* Preliminary report.

The Mandelbrot set is defined to be the set  $\{c : p_c^n(0) \text{ is bounded}\}$  where  $p_c(z) = z^2 + c$ . Douady and Hubbard's work proved that by showing a parameterized family of maps is quadratic like for a sufficient set of parameters one can conclude that the parameter space of that family of maps contains a homeomorphic copy of the Mandelbrot set described above. By looking at different quadratic families, we will see some strikingly beautiful parameter planes and be able to conclude that many of the structures that look like Mandelbrot sets are indeed what they appear to be. Lastly, we will analyze the amount of distortion of the Mandelbrot sets (from the original) in these unexpected locations. (Received September 26, 2017)

1135-37-2981 **Ayse A Sahin\*** ([ayse.sahin@wright.edu](mailto:ayse.sahin@wright.edu)) and **Ilie Ugarcovici**. *Odometer actions of more general groups.* Preliminary report.

We discuss the dynamics and spectrum of odometer actions of general groups, explaining the connection between the geometric properties of the group and the spectrum of the actions. (Received September 26, 2017)

1135-37-3021 **Tiancheng Ouyang\*** (ouyang@math.byu.edu), Department of Mathematics, Brigham Young University, Provo, UT 84602. *Variational methods and numerical simulation on the  $N$ -body problem of celestial mechanics.*

According to Newton's Second Law, the motion of  $N$  point bodies with positive masses  $m_1, m_2, \dots, m_N$  located at positions  $x_1, x_2, \dots, x_N \in \mathbb{R}^3$  is governed by the system of second-order nonlinear vector differential equations

$$m_i \ddot{x}_i = \sum_{\substack{j=1, \\ j \neq i}}^N \frac{Gm_i m_j (x_j - x_i)}{\|x_i - x_j\|^3},$$

where the derivative is with respect to the time variable  $t$ , and  $G$  is the universal gravitational constant.

In this talk, a brief introduction of the variational methods of  $N$ -body Problem from 2006–2017 with emphasize on the work of boundary value problems for 3–5 body problems in 2D and 3D will be given.

Some numerical simulations of periodic and quasi-periodic orbits will be demonstrated. Very interested phenomenons of 3D orbits of solar system will be discussed. (Received September 26, 2017)

1135-37-3027 **David Cospser\***, 402 N Blackford St., LD270, Indianapolis, IN 46202, and **Michal Misiurewicz**. *Entropy Locking.*

In certain one-parameter families of piecewise continuous piecewise linear interval maps with two laps, topological entropy stays constant as the parameter varies. In this talk we will provide a simple geometric explanation for this phenomenon as well as mention some results regarding periodic points of these families. (Received September 26, 2017)

1135-37-3049 **Thomas Bellsky\***, Department of Mathematics & Statistics, University of Maine, Orono, ME 04469. *Stalking methods for ensemble Kalman filter covariance inflation.*

The artificial inflation of ensembles is a common technique in ensemble data assimilation whereby the ensemble variance is increased in order to prevent deviation of the ensemble from the truth. Various techniques for inflating ensembles exist in the literature. This talk will discuss shadowing and stalking methods and our implementation of stalking techniques as a method of ensemble inflation. We will also offer results from a low order chaotic system supporting the use of stalking methods. (Received September 26, 2017)

1135-37-3055 **Eric Chang\*** (changer@bu.edu). *The Sierpinski Mandelbrot Spiral.*

We identify three new structures that lie in the parameter plane of family of maps  $F(z) = z^n + \lambda/z^d$ , where  $z$  and  $\lambda$  are complex,  $n \geq 4$  is even and  $d \geq 3$  is odd. There exists in the parameter plane a “Sierpindelbrot arc” of infinitely many alternating Mandelbrot sets and Sierpinski holes. In fact, there are two types of SM arcs, and there are infinitely many of each in the parameter plane. One can picture  $\lambda$  in the parameter plane moving along infinitely many arcs of one type and passing through a single arc of the other type in a spiraling fashion, comprising the third structure, a “Sierpinski Mandelbrot spiral.” Furthermore, there are infinitely many of each type of arc, so there are infinitely many SM spirals in the parameter plane as well. (Received September 26, 2017)

1135-37-3065 **Daniel B Larremore\*** (daniel.larremore@colorado.edu), **Vidit Agrawal**, **Andrew B Cowley**, **Woodrow L Shew** and **Juan G Restrepo**. *Estimating the entropy of activity in excitable networks.*

Networks of excitable nodes can be used to model the collective behavior of biological neuronal networks in the mammalian cortex. Such model networks have been used to investigate the role of network topology and synapse strength in characterizing phenomena observed in experiments with brain tissues, like stability of activity, and dynamic range. While all nodes in such models are excitable, some nodes emit excitatory signals to their network neighbors, while others emit inhibitory signals. Surprisingly, this approach of modeling inhibitory nodes explicitly (as opposed to modeling inhibition as a field) creates a secondary regime in the dynamics that guarantees ceaseless network activity. As a result, the ongoing dynamics fluctuates ceaselessly through a wide variety of states whose richness can be described using entropy measures. Here I will present progress in estimating the entropy of ceaseless activity in excitable networks as a function of model parameters, with an eye toward making biologically relevant predictions for future experiments. (Received September 26, 2017)

1135-37-3080 **Saulo Orizaga\*** (sorizaga@math.duke.edu). *Modeling and computation of phase separation models.* Preliminary report.

In this talk we discuss the computational challenges involved while solving phase field models. We present a new efficient numerical algorithm that has the energy decreasing property. The numerical method is then modified

to treat gradient flow problems with variable mobility. The effectiveness of the methods for solving thin film type of problems is presented discussed. (Received September 26, 2017)

## 39 ► *Difference and functional equations*

1135-39-115 **Johnny Henderson\*** ([johnny\\_henderson@baylor.edu](mailto:johnny_henderson@baylor.edu)), Department of Mathematics, Baylor University, Waco, TX 76798-7328, and **Jeffrey Neugebauer** ([jeffrey.neugebauer@eku.edu](mailto:jeffrey.neugebauer@eku.edu)), Department of Mathematics and Statistics, Eastern Kentucky University, Richmond, KY 40475-3133. *Smallest eigenvalues for a fractional difference equation with right focal boundary conditions.*

The theory of  $u_0$ -positive operators is applied to obtain smallest eigenvalue comparison results for right focal boundary value problems for Atici-Elloe fractional linear difference equations. (Received July 28, 2017)

1135-39-123 **Chris D Lynd\*** ([clynd@bloomu.edu](mailto:clynd@bloomu.edu)) and **James Wright Sharpe**. *The Asymptotic Behavior of the Solutions of a  $k$ -th-order Difference Equation.*

When trying to prove the convergence of the solutions of a  $k^{\text{th}}$ -order difference equation, the M&M Theorem is one of the few tools available. We analyze the solutions of the  $k^{\text{th}}$ -order difference equation  $x_{n+1} = (x_{n-1} + x_{n-2} + \dots + x_{n-k})^p$  where  $p$  is a real number between 0 and 1 and the initial terms  $x_1, \dots, x_k$  are positive. We use the M&M Theorem to prove that every positive sequence generated by the  $p^{\text{th}}$  power of the  $k^{\text{th}}$ -order Fibonacci recurrence relation converges to  $k^{\frac{p}{1-p}}$ . (Received July 29, 2017)

1135-39-285 **Jeffrey T. Neugebauer\*** ([jeffrey.neugebauer@eku.edu](mailto:jeffrey.neugebauer@eku.edu)) and **Muhammad N. Islam**. *Asymptotically periodic solutions of a  $q$ -integral equation.*

Using the definition of periodicity on time scales that are not necessarily additively periodic, given by Adivar, we define the notion of asymptotically periodic functions on the quantum time scale  $q^{\mathbb{N}_0}$ . We study the existence of an asymptotically periodic solution of a Volterra integral equation on  $q^{\mathbb{N}_0}$ . In the process, we study the existence of periodic solutions of an associated equation on the time scale  $q^{\mathbb{Z}}$ . Schauder's fixed point theorem is employed in the analysis. (Received August 18, 2017)

1135-39-388 **James Walsh\*** ([jawalsh@oberlin.edu](mailto:jawalsh@oberlin.edu)). *Nonsmooth invariant manifolds in a conceptual climate model.*

There is widespread agreement that ice sheets flowed into the ocean at sea level in tropical latitudes during the Earth's past. Whether these extreme ice ages were snowball Earth events, with the entire surface covered in ice, or whether ocean water remained ice free in regions about the equator, continues to be controversial. For the latter situation to occur, the effect of positive ice albedo feedback would have to be damped to stabilize an advancing ice sheet shy of the equator. We analyze a system of difference equations modeling the cold world of these great glacial episodes and derived from the coupling of zonally averaged surface temperature to a dynamic ice line. The analysis leads to a nonsmooth singular perturbation problem, for which we prove the persistence of an invariant manifold, thereby gaining insight into model behavior. In particular, a stable climate state with the ice line resting in tropical latitudes, but with open water about the equator, is shown to exist. We also present local smooth and nonsmooth bifurcations as the efficiency of meridional heat transport is varied. (Received August 30, 2017)

1135-39-453 **David M. Chan** and **Candace M. Kent\*** ([cmkent@vcu.edu](mailto:cmkent@vcu.edu)), Dept. of Mathematics & Applied Mathematics, Virginia Commonwealth University, Harris Hall, 4th Floor, 1015 Floyd Avenue, Richmond, VA 23284-2014, and **Vlajko L. Kocic** and **Stevo Stevic**. *A Proposal for an Application of a Max-Type Difference Equation to Epilepsy.*

We propose, *for the sake of dialogue*, that the nonautonomous reciprocal max-type difference equation,

$$x_{n+1} = \max \left\{ \frac{A_n^{(0)}}{x_n}, \frac{A_n^{(1)}}{x_{n-1}}, \dots, \frac{A_n^{(k)}}{x_{n-k}} \right\},$$

where the parameters are positive periodic sequences and the initial conditions are positive, when  $k = 1$  may serve as a *phenomenological model* of seizure activity as occurs in *mesial (or middle) temporal lobe epilepsy*. (Received September 04, 2017)



1135-39-455 **Candace M. Kent\*** ([cmkent@vcu.edu](mailto:cmkent@vcu.edu)), Dept. of Mathematics & Applied Mathematics, Virginia Commonwealth University, Harris Hall, 4th Floor, 1015 Floyd Avenue, Richmond, VA 23284-2014. *Piecewise-Defined Difference Equations with Every Solution Eventually Periodic: Open Problem.*

We consider typical examples of *piecewise-defined difference equations*, autonomous and nonautonomous, whose behavior of solutions is such that *every solution is eventually periodic*, and ask what it is be about this class of equations that can lead to eventual periodicity. (Received September 04, 2017)

1135-39-459 **Carol B Overdeep\*** ([coverdeep@stmartin.edu](mailto:coverdeep@stmartin.edu)), Saint Martin's University, 5000 Abbey Way SE, Lacey, WA 98503, and **Sally Fletcher.** *A Period-Four Solution with Period-Two Parameters Applied to a Rational Difference Equation.*

When applying period-two parameters to a second-order rational difference equation, a period-four solution arises when one of the parameters is set to zero. Further, given the two initial conditions, we can find explicit equations for each term in the period-four solution. (Received September 05, 2017)

1135-39-485 **Robert J Sacker\***, [rsacker@usc.edu](mailto:rsacker@usc.edu), and **George R Sell.** *Almost Periodicity, Ricker Map, Beverton-Holt Map and Others, a General Method.*

It is known that under the certain conditions on the coefficient the Ricker difference equation (or map) has a fixed point that is globally asymptotically stable with respect to the positive reals. We show here that under the same conditions, the Ricker equation with almost periodic coefficient has a globally asymptotically stable almost periodic solution with the same frequency module as the coefficient. This is accomplished by showing that the omega limit set  $\Omega$  of an asymptotically stable solution is a covering space of the omega limit set of the coefficients and the flow on  $\Omega$  is uniquely reversible. We provide a unified framework that allows us to conclude that any system of maps in finite dimensions that has an orbit that is bounded and whose omega limit set is asymptotically stable, also has the property that certain attributes of the coefficients (periodicity, almost periodicity) can be carried over, or lifted to the solution. In particular if the successive compositions are bounded and have Fréchet derivatives with spectrum inside the unit circle in the complex plane then the above conclusions apply. (Received September 05, 2017)

1135-39-506 **Vlajko L. Kocic\*** ([vkocic@xula.edu](mailto:vkocic@xula.edu)), Xavier University of Louisiana, Mathematics Department, New Orleans, LA 70125. *Global asymptotic behavior of certain class of nonlinear nonautonomous difference equations.*

In this paper we study the global asymptotic behavior of positive solutions of the nonlinear nonautonomous difference equation of the form

$$x_{n+1} = a_n x_n f(x_{n-k}), n = 0, 1, \dots$$

with positive initial conditions where the sequence  $a_n$  is positive and periodic with period  $p$ , and the function  $f$  satisfies some additional conditions. Results are applied to some periodically forced classical population models. (Received September 06, 2017)

1135-39-536 **Michael Alexander Radin\*** ([michael.radin@rit.edu](mailto:michael.radin@rit.edu)), Rochester Institute of Technology, College of Science, School of Mathematical Sciences, Rochester, NY 14623. *Rulkov & Izhikevich Neural Networking Models; Master neurons and slave neurons with applications in modeling Alzheimer's Disease. Delays in the signal and eventually periodic solutions.*

The objective of the presentation is to compare the similarities and differences between the Rulkov and the Izhikevich Neural Networking Models. Furthermore, we will compare the similarities and differences in the periodic cycles, the delays of the periodic cycles, the stability of the periodic cycles and the spike patterns of the the periodic cycles too. (Received September 07, 2017)

1135-39-588 **Ziyad AlSharawi\*** ([zsharawi@aus.edu](mailto:zsharawi@aus.edu)), Department of Mathematics and Statistics, American University of Sharjah, PO Box 26666, Sharjah, United Arab Emirates. *On the embedding of a discrete dynamical system into a dynamical system of higher dimension.* Preliminary report.

We consider second order difference equations of the form  $x_{n+1} = F(x_n, x_{n-1})$ , in which  $F$  is non-decreasing in one component and non-increasing in the other. Such equations are knows as difference equations of mixed monotonicity. The embedding technique is known in studying the dynamics of this type of equations. When the obtained dynamical system assumes an invariant box, which could be the positive quadrant, the system can be embedded into a monotonic system of higher dimension, and the new system can be used to investigate the dynamics of the original system. In this talk, we focus on systems of mixed monotonicity that have compact

invariant sets that are not necessarily boxes, then develop the embedding technique to address the issue of asymptotic behavior. (Received September 09, 2017)

1135-39-631 **Jim M Cushing\*** ([cushing@math.arizona.edu](mailto:cushing@math.arizona.edu)), Department of Mathematics, 617 N Santa Rita, University of Arizona, Tucson, AZ 85719, and **Shandelle Henson** and **James Hayward**. *Periodic matrix models for seasonal dynamics of stage structured populations I: A general class of models.*

We formulate and analyze a general class of discrete-time matrix models (systems of difference equations) which arise in the study of stage structured population dynamics. Specifically, the models are designed to account for changes in behavioral tactics within a breeding season and for their dynamic consequences at the population level across breeding seasons. Using bifurcation theoretic techniques, we study the nature of non-extinction, seasonal cycles as a function of model parameters as they are created upon destabilization of the extinction state. Of particular interest are backward bifurcations in that they typically create strong Allee effects in population models which, in turn, lead to the benefit of possible (initial condition dependent) survival in adverse environments when a forward bifurcation would lead to extinction. The models and their analysis are motivated by recent field observations of changed behavioral and life history strategies in seabird populations (of glaucous winged gulls) that are correlated with climate change (mean sea surface temperature rise). (Received September 11, 2017)

1135-39-643 **Horst R Thieme\*** ([hthieme@asu.edu](mailto:hthieme@asu.edu)), School of Mathematical and Stat. Sciences, Arizona State University, Tempe, AZ 85287-1804. *The spectral radius of homogeneous operators as threshold between persistence and extinction in discrete-time population models.* Preliminary report.

If discrete-time population models with infinite dimensional structure incorporate two sexes or are modeled in the space of measures with the flat norm, the first order approximation at 0 may not longer be a linear positive bounded operator on an ordered normed vector space but just a homogeneous order-preserving operator on its cone. The concept of a spectral radius can be extended to such operators so that it still acts as threshold between population persistence and extinction. (Received September 11, 2017)

1135-39-778 **John R. Graef\*** ([john-graef@utc.edu](mailto:john-graef@utc.edu)), Department of Mathematics, University of Tennessee at Chattanooga, Chattanooga, TN, and **S. Heidarkhani**, **L. Kong** and **M. Wang**. *Existence of solutions to a discrete fourth order boundary value problem.*

Criteria are established for the existence of at least two nontrivial solutions to the discrete fourth order boundary value problem

$$\begin{cases} \Delta^4 u(t-2) - \alpha \Delta^2 u(t-1) + \beta u(t) = f(t, u(t)), & t \in [1, N]_{\mathbb{Z}}, \\ u(-1) = \Delta u(-1) = 0, & u(N+1) = \Delta^2 u(N) = 0, \end{cases}$$

where  $N \geq 1$  is an integer,  $\alpha, \beta \geq 0$ , and  $f : [1, N]_{\mathbb{Z}} \times \mathbb{R} \rightarrow \mathbb{R}$  is continuous in the second argument. Applications of the results to a related eigenvalue problem are also presented. The proofs are mainly based on the variational method and the classic mountain pass lemma of Ambrosetti and Rabinowitz. Examples are included to illustrate the applicability of the results. (Received September 14, 2017)

1135-39-850 **Tiffany Jones\*** ([tiffany\\_jones1@baylor.edu](mailto:tiffany_jones1@baylor.edu)), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76798-7328, and **Qin Sheng** ([qin\\_sheng@baylor.edu](mailto:qin_sheng@baylor.edu)), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76798-7328. *On Stabilities of a Decomposed Compact Method for Solving Highly Oscillatory Wave Equations.*

The study of numerical partial differential equations has been an ever-expanding field of computational mathematics. This growth is inspired by the challenges in the theory and methods, but also the significance of the equations involved. And yet, even with this motivation compelling the advances of recent years, the research of numerical partial differential equations is far from completion.

A key component in aforementioned explorations is the stability analysis. In this talk, we will concentrate on two types of the numerical stability for a decomposed compact method, which is highly effective and efficient for solving highly oscillatory Helmholtz equations. A radially symmetric transverse field and polar coordinates are considered.

To remove the singularity due to coordinate transformations, decomposed transverse domains are incorporated. A compact algorithmic structure is thus introduced to raise the accuracy of the finite difference scheme in the transverse direction.

Though the decomposed compact scheme implemented is shown to shy away from conventional stability in the von Neumann sense, it is unconditionally stable with index one in an asymptotical sense. Computer experiments and beam propagation simulations further verify our conclusions. (Received September 15, 2017)

1135-39-937 **Billur KAYMAKÇALAN\*** (billur.kaymakcalan@gmail.com) and **Neslihan Nesliye PELEN**. *Bennet and Leindler type inequalities for Nabla Time Scale Calculus*. Preliminary report.

In this talk, we will present some new dynamic inequalities on a time scale  $\mathbb{T}$  using nabla time scale calculus. When  $\mathbb{T} = \mathbb{N}$  these inequalities contain the discrete inequalities due to Bennett and Leindler which are converses of Copson's inequalities. (Received September 17, 2017)

1135-39-1104 **Allan Peterson\*** (apeterson1@math.unl.edu), Lincoln, NE, and **Baoguo Jia, Feifei Du** and **Lynn Erbe**. *Asymptotic behaviour of nabla fractional  $h$ -difference equations*.

We consider the nabla fractional initial value problem

$$\rho_{(a)} \nabla_h^\nu x(t) = cx(t), x(a) = A > 0, t \in (h\mathbb{N})_{a+h}. \quad (1)$$

where  $\rho_{(a)} \nabla_h^\nu x(t)$  denotes Riemann-Liouville nabla  $h$ -difference of  $x(t)$  on sets  $(h\mathbb{N})_a$ . In this paper, we will discuss the asymptotic behaviour of the solutions of (1) (Received September 19, 2017)

1135-39-1200 **Youssef Naim Raffoul\*** (yraffoul1@udayton.edu), 300 College Park, Dayton, OH 45469-2316. *Nonstandarized Discretization Scheme In Volterra Integro-differential Equations that Preserves Uniform Asymptotic Stability*.

We apply a nonstandarized discretization scheme to continuous Volterra integro-differential equations and show that under our discretization, the necessary and sufficient conditions for uniform asymptotic stability are preserved for continuous dynamical system. Our analysis will be based on the notion of resolvent. An example will be provided as an application to our theory.

(Received September 20, 2017)

1135-39-1226 **Allan Peterson\*** (apeterson1@math.unl.edu), Lincoln, NE 68588, and **Feifei Du, Wei Hu** and **Lynn Erbe**. *Some new integral inequalities on time scales*.

In this paper, we establish some generalizations of inequalities on time scales, which have appeared in different articles. Even the inequalities that we will derive from our results when  $g(t) = t$  are essentially new and of great interest. (Received September 20, 2017)

1135-39-1247 **Scott C. Gensler\*** (scott.gensler@huskers.unl.edu). *The General Operator  $\nabla^\gamma$  and Using It to Extend Results Involving  $\nabla_a^\gamma$  and  $\nabla_{a*}^\gamma$* . Preliminary report.

In this talk we define and explore, for any real number  $\gamma$ , a general fractional operator  $\nabla^\gamma$ . This is a generalization of the familiar, iterated difference operator  $\nabla^N$ . Indeed, when  $\gamma$  is a natural number, the two coincide. These general fractional operators have very nice properties. For example, they commute with each other, compose nicely, and are invertible. While in most cases the general operator,  $\nabla^\nu$ , differs slightly from the Riemann-Liouville operator,  $\nabla_a^\nu$ , and the Caputo operator,  $\nabla_{a*}^\nu$ , it is closely related to both and interacts nicely with them. As such, one of the things it can be used for is to simplify and extend many of the prior results involving the  $\nabla_a^\nu$  and  $\nabla_{a*}^\nu$  operators. (Received September 20, 2017)

1135-39-1270 **Christopher S. Goodrich\*** (cgood@prep.creighton.edu), 7400 Western Ave., Omaha, NE 68114, and **Rajendra Dahal** (rdahal@coastal.edu), Department of Mathematics and Statistics, Wall Building 101 G, Conway, SC 29526. *An Analysis of Nonlocal Elements in Discrete Fractional Calculus*.

In this talk we will describe the effect of nonlocal elements in the discrete fractional calculus. More specifically, we will demonstrate the connections between the sign of a discrete fractional operator acting on a map  $f$  and the associated monotonicity or convexity of  $f$ . Some representative results in the area will be discussed, and, especially, the sharpness of the results will be mentioned. Finally, we will compare and contrast these results with those known for non-fractional difference operators, and it will be shown that in the fractional case there is considerably greater richness and underlying mathematical complexity. (Received September 20, 2017)

1135-39-1389 **Areeba Ikram\*** (areeba@huskers.unl.edu), 203 Avery Hall, PO Box 880130, Lincoln, NE 68521. *Lyapunov Inequalities for Boundary Value Problems involving Fractional Difference Equations*. Preliminary report.

We will present background for the nabla fractional calculus. We will then discuss Green's functions for several boundary value problems involving a Caputo fractional difference equation. Finally, we will show Lyapunov inequalities for these boundary value problems. (Received September 21, 2017)

1135-39-1463 **Jim M Cushing\*** ([cushing@math.arizona.edu](mailto:cushing@math.arizona.edu)), Department of Mathematics, 617 N Santa Rita, University of Arizona, Tucson, AZ 85721. *Equilibrium bifurcations in difference equation models from structured population dynamics: extinction, survival, and strong Allee effects.*

Systems of difference equations, expressed as matrix models of discrete time dynamical systems, are widely used in structured population dynamics. A fundamental bifurcation occurs when the extinction state is destabilized, which results in a local transcritical bifurcation of a continuum of non-extinction equilibria whose stability depends on the direction of bifurcation. I will discuss some global properties of this continuum, the stability and instability of its equilibria outside of the neighborhood of the bifurcation point, and the role that a backward bifurcation and saddle-node bifurcations play in the occurrence of strong Allee effects. Strong Allee effects play a role in studies of population survival in adverse environments (climate change) and the presence of a tipping point at which a sudden collapse to extinction occurs. (Received September 22, 2017)

1135-39-1616 **Yang Li\*** ([yang.li@louisiana.edu](mailto:yang.li@louisiana.edu)). *Modeling releases of sterile mosquitoes with Beverton-Holt survival functions.*

To prevent transmissions of malaria, dengue fever, or other mosquito-borne diseases, one effective weapon is the sterile insect technique in which sterile mosquitoes are released to reduce or eradicate the wild mosquito population. Based on difference equations, we use the Beverton-Holt survival function, instead of the classical Ricker-type of nonlinearity, to formulate several discrete models for the interactive dynamics of the wild and sterile mosquitoes, incorporating different strategies in releasing sterile mosquitoes. Basic analysis, including the existence of fixed points and their stability is given. Numerical examples to demonstrate our findings and brief discussions are also provided. (Received September 23, 2017)

1135-39-1892 **ARZU BILGIN\*** ([bilgin\\_a@my.uri.edu](mailto:bilgin_a@my.uri.edu)). *Basins of Attraction of Period- Two Solutions of Monotone Difference Equations.*

I investigate the global character of the difference equation of the form

$$x_{n+1} = f(x_n, x_{n-1}), \quad n = 0, 1, \dots$$

with several period-two solutions, where  $f$  is increasing in all its variables. I show that the boundaries of the basins of attractions of different locally asymptotically stable equilibrium solutions or period-two solutions are in fact the global stable manifolds of neighboring saddle or non-hyperbolic equilibrium solutions or period-two solutions. An application of our results give global dynamics of three feasible models in population dynamics which includes the nonlinearity of Beverton-Holt and sigmoid Beverton-Holt types. (Received September 25, 2017)

1135-39-2138 **Yun Kang\*** ([yun.kang@asu.edu](mailto:yun.kang@asu.edu)), 7001 E. Williams Field Rd., Mesa, AZ, and **O. Aydogmus, M. E. Kavgaci and H Berketoglu.** *Dynamical effects of nonlocal interactions in discrete-time growth-dispersal models with logistic-type nonlinearities.*

This talk is devoted to the study of discrete time and continuous space models with nonlocal resource competition and periodic boundary conditions. We consider generalizations of logistic and Ricker's equations as intraspecific resource competition models with symmetric nonlocal dispersal and interaction terms. Both interaction and dispersal are modeled using convolution integrals, each of which has a parameter describing the range of nonlocality. It is shown that the spatially homogeneous equilibrium of these models becomes unstable for some kernel functions and parameter values by performing a linear stability analysis. To be able to further analyze the behavior of solutions to the models near the stability boundary, weakly nonlinear analysis, a well-known method for continuous time systems, is employed. We obtain Stuart-Landau type equations and give their parameters in terms of Fourier transforms of the kernels. This analysis allows us to study the change in amplitudes of the solutions with respect to ranges of nonlocalities of two symmetric kernel functions. Our calculations indicate that supercritical bifurcations occur near stability boundary for uniform kernel functions. We also verify these results numerically for both models. (Received September 25, 2017)

1135-39-2164 **Guihong Fan\***, [fan\\_guihong@columbusstate.edu](mailto:fan_guihong@columbusstate.edu), and **Horst Thieme and Huaiping Zhu.** *Global Hopf bifurcation in a delayed model for ticks.*

Lyme disease is an emerging infectious disease which can cause severe health problems in human body. In this paper, we study a three staged tick population model with three different developmental delays. We investigate the impact of delays on the prevalence of the tick population. By employing the Global Hopf bifurcation theorem, we establish the global existence of the nontrivial periodic solutions. This shows that the tick population can persist in a region in a oscillatory manner if the sum of three developmental delays fall into the critical time

interval. To illustrate our theoretical results on global Hopf bifurcation, we use some software to obtain the bifurcation diagram using delay as a bifurcation parameter. (Received September 25, 2017)

1135-39-2180 **Elliott J. Bertrand\*** (elliott.bertrand@rhodeisland.edu), Department of Mathematics, University of Rhode Island, 5 Lippitt Road, Kingston, RI 02881, and **Mustafa R.S. Kulenovic**. *Global Dynamic Scenarios for Competitive Maps in the Plane*.

In this presentation we will discuss some possible global dynamic scenarios for general competitive maps in the plane. We apply these results to the class of second-order autonomous difference equations whose transition functions are decreasing in the variable  $x_n$  and increasing in the variable  $x_{n-1}$ . We illustrate our results with the application to the difference equation

$$x_{n+1} = \frac{Cx_{n-1}^2 + Ex_{n-1}}{ax_n^2 + dx_n + f}, \quad n = 0, 1, 2, \dots,$$

where the initial conditions  $x_{-1}$  and  $x_0$  are arbitrary nonnegative numbers such that the solution is well-defined and the parameters satisfy  $C, E, a, d, f \geq 0$ ,  $C + E > 0$ ,  $a + C > 0$ , and  $a + d > 0$ . We characterize the global dynamics of this equation with the basins of attraction of its equilibria and periodic solutions. (Received September 25, 2017)

1135-39-2352 **Joshua Siktar\*** (jsiktar@andrew.cmu.edu) and **Steven J Miller**. *Generalizations of Zeckendorf's Theorem to Two-Dimensional Sequences*. Preliminary report.

Zeckendorf proved that every positive integer can be written uniquely as a sum of non-adjacent Fibonacci numbers. This is called the Zeckendorf decomposition, and has been generalized to many sequences, especially those arising from recurrence relations of fixed depth and constant non-negative integer coefficients. We generalize results in the subject to certain two-dimensional sequences, in particular proving Gaussianity for the distribution of the number of summands. (Received September 26, 2017)

1135-39-2360 **Steven J Miller\*** (sjm1@williams.edu), **Yujin Kim**, **Shannon Sweitzer**, **Eric Winsor** and **Jianing Yang**. *Distributions in Generalized Zeckendorf Decompositions*.

Zeckendorf's Theorem states that every positive integer has a unique decomposition as a sum of nonadjacent Fibonacci numbers. This property can be used to define the Fibonacci numbers, and thus it's natural to study extensions to other recurrence sequences and decomposition laws. Previous work generalized to legal decomposition where we have infinitely many bins, all of length  $b$ , and one chooses at most 1 element from a bin and never elements from 2 consecutive bins (the Fibonacci case is  $b = 1$ ). In addition to unique decomposition, these sequences led to Gaussian behavior in the distribution of number of summands in decompositions, and geometric decay in the probability of gap lengths. We extend these arguments to allow the bin sizes to vary. Previous combinatorial techniques are no longer applicable, and we use the Lyapunov Central Limit Theorem to prove Gaussian behavior for the number of summands in a decomposition. Gaussian behavior is also observed when we generalize Zeckendorf decompositions by extending the legal number of summands chosen from the  $n$ -th bin to a set  $A_n$ . We further generalize by examining cases where we place adjacency conditions on the bins, where we rely on a version of the CLT for dependent random variables to study the limiting behavior. (Received September 26, 2017)

1135-39-2423 **Toufik Khyat\*** (toufik.khyat@trincoll.edu). *A Difference Equation with Neimark-Sacker Bifurcation*.

We compute the direction of the Neimark-Sacker bifurcation for the difference equation  $x_{n+1} = \frac{x_n}{Cx_{n-1}^2 + Dx_n + F}$  where  $C, D$  and  $F$  are positive numbers and the initial conditions  $x_{-1}$  and  $x_0$  are positive numbers. Moreover, we give the asymptotic approximation of the invariant curve. (Received September 26, 2017)

1135-39-2438 **Oluwaseye Adekanye\*** (seyadekanye@gmail.com) and **Talitha Washington**. *Constructing nonstandard finite difference (NSFD) schemes for dynamical systems*. Preliminary report.

Many real world phenomena can be modeled by dynamical systems that describe the evolution of phenomena over time. For example, the growth and decay equation models how a quantity changes over time. The transport equation with a flux term models the flow of a particle through a given medium. The Airy equation models the diffraction of light. The Tacoma Narrows Bridge models describe the torsional and vertical oscillations on the day of its collapse. Using solutions to subequations of a system, we can develop the foundation for an exact nonstandard finite difference scheme (NSFD) which may preserve properties of the dynamical system into its discretization which leads to dynamical consistency. Some systems may require the NSFD scheme to adhere

to time and space step size constraints. In this talk, we will show how to construct NSFD schemes for various models that may outperform the traditional standard finite difference schemes. (Received September 26, 2017)

1135-39-2674 **Saber Elaydi\*** ([selaydi@trinity.edu](mailto:selaydi@trinity.edu)), One Trinity Place, San Antonio, TX 78212. *A Discrete Mathematical Model for the Aggregation of  $\beta$ -Amyloid*. Preliminary report.

Alzheimer's disease (AD) is an age-related, progressive degenerative disorder characterized by the loss of synapses and neurons from the brain. Monomers of  $\beta$ -Amyloid aggregates to form Oligomers and oligomers aggregate to form fibrils. Our study is based on the assumption that soluble  $\beta$ A oligomers are the causative agents of AD, due to their toxicity to neuron cells in the brain.

We develop a five-dimensional discrete mathematical model for the aggregation of monomers into oligomers. The model establishes a theoretical mechanism to reduce the production of oligomers. We provide conditions for the stability of the aggregation of  $\beta$ -Amyloid. A formula for the number of monomers that is required for the producing oligomers has been established. A mechanism to prevent monomers from aggregating to oligomers is proposed for practitioners in the field of Alzheimer Disease. This provides health providers a method for the prevention of Alzheimer Disease. (Received September 26, 2017)

1135-39-2696 **Saber Elaydi\***, One Trinity Place, San Antonio, TX 78212. *On the global stability of a class of discrete dynamical systems*. Preliminary report.

The question that we are investigating is when does local stability of a fixed point implies global stability. It is known in the literature that certain maps, both multi-dimensional and planar, behave in this way. These maps include triangular maps in higher dimension, the planar Ricker map, the multi-species Leslie-Gower model, and monotone maps in any dimension. The question that we are going to address is what class of maps, that are not monotone or triangular, for which local stability of a fixed point implies its global stability. Open problems and conjectures will be presented. (Received September 26, 2017)

1135-39-2916 **Imelda Trejo\*** ([imelda.trejo@mavs.uta.edu](mailto:imelda.trejo@mavs.uta.edu)), 425 S Oak St, Apt 106, Arlington, TX 76010, and **Hristo Kojouharov** and **Benito Chen-Charpentier**. *Modeling the Effects of Inflammation in Bone Fracture Healing*.

A new mathematical model is presented to study the early inflammatory effects in bone healing. It consists of a system of nonlinear ordinary differential equations that represents the interactions among macrophages, mesenchymal stem cells, and osteoblasts. A qualitative analysis of the model is performed to determine the equilibria and their corresponding stability properties. There are three equilibria which represent the successful healing, nonunion, and dead tissue. A set of numerical simulations is presented to support the theoretical results. The model is also used to numerically monitor the evolution of a broken bone for different types of fractures and to explore possible treatments to accelerate bone healing by administrating anti-inflammatory drugs. (Received September 26, 2017)

## 40 ► Sequences, series, summability

1135-40-3098 **Michael G Augspurger** ([augspurgermg1@gcc.edu](mailto:augspurgermg1@gcc.edu)), 200 Campus Dr. Box #1577, Grove City, PA 16127, and **Rachel E Falk** ([falkre1@gcc.edu](mailto:falkre1@gcc.edu)), **Joseph E Swanson\*** ([swansonje1@gcc.edu](mailto:swansonje1@gcc.edu)) and **Michael A Jackson** ([majackson@gcc.edu](mailto:majackson@gcc.edu)). *Number Sequences of Truncated Simplices*. Preliminary report.

Polytope numbers are sequences of nonnegative integers constructed geometrically from polytopes. Our research primarily investigates truncation of the simplest polytope, the simplex, which is a generalization of a tetrahedron to higher dimensions. Truncation is a geometric process removing of vertices by cutting one-third of each edge off, r-truncation makes cuts beyond the midpoint of each edge for shapes in dimensions higher than 3. Our research combines patterns found in low dimensions with the geometric construction of the r-truncated simplex to create a formula which works for any given dimension. By synthesizing H. K. Kim's method with previous research, we have created a formula which generates the polytope numbers for an r-truncated d-dimensional simplex. (Received September 26, 2017)

## 41 ► *Approximations and expansions*

1135-41-17 **George A Anastassiou\*** ([ganastss@memphis.edu](mailto:ganastss@memphis.edu)), Dept. Mathematical Sciences, 3725 Norriswood Avenue, Memphis, TN 38152. *Multivariate and abstract Approximation theory for Banach space valued functions*. Preliminary report.

Here we study quantitatively the high degree of approximation of sequences of linear operators acting on Banach space valued Fréchet differentiable functions to the unit operator, as well as other basic approximations including ones under convexity. These operators are bounded by real positive linear companion operators. The Banach spaces considered here are general and no positivity assumption is made on the initial linear operators whose we study their approximation properties. We derive pointwise and uniform estimates which imply the approximation of these operators to the unit assuming Fréchet differentiability of functions then we continue with basic approximations. At the end we study the special case where the approximated function fulfills a convexity condition resulting into sharp estimates. We give applications to Bernstein operators. (Received June 14, 2017)

1135-41-19 **George A Anastassiou\*** ([ganastss@memphis.edu](mailto:ganastss@memphis.edu)), Dept. Mathematical Sciences, 3725 Norriswood Avenue, Memphis, TN 38016. *Arctangent function based Banach space valued neural network approximation*. Preliminary report.

Here we study the univariate quantitative approximation of Banach space valued continuous functions on a compact interval or all the real line by quasi-interpolation Banach space valued neural network operators. We perform also the related Banach space valued fractional approximation. These approximations are derived by establishing Jackson type inequalities involving the modulus of continuity of the engaged function or its Banach space valued high order derivative or fractional derivatives. Our operators are defined by using a density function induced by the arctangent function. The approximations are pointwise and with respect to the uniform norm. The related Banach space valued feed-forward neural networks are with one hidden layer. (Received June 14, 2017)

1135-41-232 **George A Anastassiou\*** ([ganastss@memphis.edu](mailto:ganastss@memphis.edu)), Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152. *Vectorial fractional approximation by linear operators*.

In this article we study quantitatively with rates the convergence of sequences of linear operators applied on Banach space valued functions. The results are pointwise and uniform estimates. To prove our main results we use an elegant boundedness property of our linear operators by their companion positive linear operators. Our inequalities are fractional involving the right and left vector Caputo type fractional derivatives, built in vector moduli of continuity. We treat very general classes of Banach space valued functions. We give applications to vectorial Bernstein operators. (Received August 14, 2017)

1135-41-252 **Vira Babenko\*** ([vbabenko@ithaca.edu](mailto:vbabenko@ithaca.edu)), **Vladyslav Babenko** and **Mariya Polischuk**. *Optimal recovery of operators on the classes of functions with values in L-spaces*.

In this talk, we will present a solution to the problem of optimal recovery of operators on the classes of functions with values in L-spaces that have a given majorant of moduli of continuity (we use information given with error). The consideration of classes of functions with values in L-spaces allows us to consider various problems of optimal recovery (for operators in spaces of set-valued functions, fuzzy-valued functions, and others) within one general approach. (Received August 16, 2017)

1135-41-556 **Jean-Paul Berrut\*** ([jean-paul.berrut@unifr.ch](mailto:jean-paul.berrut@unifr.ch)), University of Fribourg, Departement de Mathematiques, Chemin du Musee 23, CH 1700 Fribourg, Switzerland. *Linear barycentric rational interpolation with guaranteed degree of exactness*.

In recent years, linear barycentric rational interpolants, introduced in 1988 and improved in 2007 by Floater and Hormann, have turned out to be among the most efficient infinitely smooth interpolants, in particular with equispaced points. In the present contribution, we introduce a new way of obtaining linear barycentric rational interpolants with relatively high orders of convergence. The basic idea is to modify the interpolant with equal weights of 1988 to force it to interpolate exactly the monomials up to a certain degree. This is obtained by modifying a few weights at each extremity of the interval of interpolation. Numerical experience demonstrates that the method is indeed able to interpolate with much higher orders than the original 1988 interpolant, and in a very stable way. (Received September 21, 2017)

1135-41-1268 **Brian Simanek\*** ([brian\\_simanek@baylor.edu](mailto:brian_simanek@baylor.edu)), Baylor Math Department, One Bear Place #97328, Waco, TX 76798. *Geronimus Polynomials on the Unit Circle*.

The Geronimus polynomials are a family of orthogonal polynomials on the unit circle whose corresponding sequence of Verblunsky coefficients is constant. We will present a new formula for these polynomials in terms of Chebyshev polynomials of the second kind and then consider some applications of this formula. The most substantial application we will discuss is a universality result at the endpoint of an arc of the unit circle that supports the measure of orthogonality. (Received September 20, 2017)

1135-41-1330 **Palle Jorgensen\*** ([palle-jorgensen@uiowa.edu](mailto:palle-jorgensen@uiowa.edu)), Dept Mathematics MLH, University of Iowa, Iowa City, IA. *Transfer operators and their applications*.

By “transfer operator” I mean a family of operators which arise in a host of areas of applications; in dynamics, ranging from signal/image representations to kernel learning, from multi-resolution wavelet theory, to measurable dynamics, from fractals to signal analysis, and from Markov operators and potential theory, both in the discrete as well as continuous settings. In many problems, an initial function space is often not given as a feature space, – its realization as a Hilbert space only comes much later; i.e., the appropriate transfer operators often arise in instances where a Hilbert space is not given directly, and certainly not at the outset. In any case, the spectral theory we consider here differs from that of more traditional settings where transfer operators are also used, as will be outlined in the talk. (Received September 21, 2017)

1135-41-3205 **Xin Li\*** ([xin.li@ucf.edu](mailto:xin.li@ucf.edu)), Department of Mathematics, 4000 Central Florida Blvd, Mathematical Science Building 207, Orlando, FL 32816. *A weighted max-min-max problem on the unit circle*. Preliminary report.

Let  $w$  be a polynomial of degree  $n$  with all zeros contained in  $|z| > 1$  in the complex plane. Consider

$$m_{n,w} = \max_{0 \leq t_1 < t_2 < \dots < t_n < 2\pi} \min_{1 \leq j \leq n} \left( \max_{t_j \leq t \leq t_{j+1}} \left| \frac{\prod_{k=1}^n (e^{it} - e^{it_k})}{w(e^{it})} \right| \right), \quad (t_{n+1} := t_1)$$

When  $w(z) \equiv 1$ , Khrushchev established that  $m_{n,1} = 2$ . Erdelyi, Hardin, and Saff demonstrated this as a consequence of an *inverse* Bernstein inequality for polynomials on the unit circle. In this talk, we will show that the above weighted max-min-max problem is related to an inverse Bernstein inequality for rational functions on the unit circle. (Received September 27, 2017)

## 42 ► *Fourier analysis*

1135-42-113 **Lily Wittle\*** ([lily.wittle@gmail.com](mailto:lily.wittle@gmail.com)) and **Theresa A Thimons** ([theresa.thimons@gmail.com](mailto:theresa.thimons@gmail.com)). *Investigating the Gerchberg-Saxton Phase Retrieval Algorithm*. Preliminary report.

This presentation discusses the investigation of Gerchberg and Saxton’s phase retrieval algorithm. We observed the algorithm’s performance by experimentation using a numerical implementation of the algorithm that we wrote in MATLAB. We found that functions of the form  $f \times g$ , where  $g$  is a Gaussian function, have better success than those of the corresponding  $f$ . We also found that using a constant initial phase estimate produces more consistent and efficient results for non-centrosymmetric input than the random initial phase estimate used in the original algorithm. Our presentation includes a proof of error convergence and a description of the implementation of our modifications. (Received July 28, 2017)

1135-42-1123 **Shiwen Zhang\*** ([zhangshiwen@math.msu.edu](mailto:zhangshiwen@math.msu.edu)). *The Projection of some Random Cantor sets and the Decay Rate of the Favard length*.

The Favard length of a set  $E$  has a probabilistic interpretation: up to a constant factor, it is the probability that the “Buffon’s needle,” a long line segment dropped at random, hits  $E$ . In this talk, we study the Favard length of some random Cantor sets of dimension 1. Replace the unit disc by 4 disjoint sub-discs of radius  $1/4$  inside. By repeating this operation in a self-similar manner and adding a random rotation in each step, we can generate a random Cantor set  $D(\omega)$ . Let  $D_n$  be the  $n$ -th generation in the construction, which is comparable to the  $4^{-n}$ -neighborhood of  $D$ . We are interested in the decay rate of the Favard length of these sets  $D_n$  as  $n \rightarrow \infty$ , which is the likelihood (up to a constant) that the “Buffon’s needle” will fall into the  $4^{-n}$ -neighborhood of  $D$ . It is well known that the lower bound for such 1-dimensional set is constant multiple of  $n^{-1}$ . We show that the upper bound of the Favard length of  $D_n$  is also constant multiple of  $n^{-1}$  in the average sense. (Received September 19, 2017)



1135-42-2061 **John E. Herr\*** (jeherr@butler.edu), **Eric S. Weber** and **Palle E. T. Jorgensen**. *A Matrix Characterization of Boundary Representations of Positive Matrices in the Hardy Space.*

Given a positive matrix  $K_C$  in the Hardy space induced by a bi-infinite matrix  $C$  forming a bounded operator on  $\ell^2$ , we characterize those Borel measures on  $\mathbb{T}$  with respect to which  $K_C$  has a boundary representation. This characterization is in terms of a matrix identity obtained by introducing a new matrix product called the Abel product. (Received September 25, 2017)

### 43 ► *Abstract harmonic analysis*

1135-43-858 **Ryan W Matzke\*** (matzk053@umn.edu), 1920 S 1st St, Apt 506, Minneapolis, MN 55454. *Stolarsky Principle and Energy Optimization on the Sphere.*

The classic Stolarsky Invariance Principle connects the  $L^2$  discrepancy of a finite point set on the sphere to the pairwise sum of Euclidean distances between the points. By extending this result to arbitrary measures and arbitrary notions of discrepancy, we can approach an array of problems of energy optimization from the stand point of discrepancy theory. In particular, we can determine the maximum sum of geodesic distances between points on the sphere, and the arrangements that produce it. (Received September 15, 2017)

### 44 ► *Integral transforms, operational calculus*

1135-44-2099 **Fatma Terzioglu\*** (fatma@math.tamu.edu), **Peter Kuchment** and **Leonid Kunyansky** (leonk@math.arizona.edu). *Mathematics of Compton camera imaging.*

Compton cameras are used for medical (emission) imaging, as well as for astronomy and homeland security applications. Mathematically (although not in terms of the underlying physics) similar neutron detectors are also being developed for homeland security. The talk will survey the mathematics related to such applications. Namely, the so called cone transform, its properties and inversion. (Received September 26, 2017)

1135-44-2168 **Reza R Ahangar\*** (reza.ahangar@tamuk.edu), 700 University BLVD, MSC 172, Mathematics Department, TAMUK, Kingsville, TX 78363. *Variation of Parameters for Causal Operator Differential Equations.*

The operator  $T$  from a domain  $D$  into the space of measurable functions is called a nonanticipating (causal) operator if the past informations is independent from the future outputs. We will investigate the solution to a nonlinear operator differential equation  $y'(t)=A(t)y(t)+f(t,y(t),T(y)(t))$ . We will also study the nonlinear variation of parameters (NVP) for this type of causal operator differential equations and develop Alekseev type of Nonlinear Variation of Parameters formula. (Received September 25, 2017)

### 45 ► *Integral equations*

1135-45-360 **Muhammad Islam\*** (mislam1@udayton.edu), 300 College Park, Dayton, OH 45469, Dayton, OH 45469. *Asymptotically stable and  $L^1$  Solutions of Caputo Fractional Differential Equations.*

The existence of bounded, asymptotically stable, and  $L^1$  solutions of a Caputo fractional differential equation has been studied in this paper. The results are obtained from an equivalent Volterra integral equation which is derived by inverting the fractional differential equation. The kernel function of this integral equation is weakly singular and hence the standard techniques that are normally applied on Volterra integral equations do not apply here. This hurdle is overcome using a resolvent equation and then applying some known properties of the resolvent. In the analysis Schauder's fixed point theorem and Liapunov's method have been employed. The existence of bounded solutions are obtained employing Schauder's theorem, and then it is shown that these solutions are asymptotically stable. Finally, the  $L^1$  properties of solutions are obtained using Liapunov's method. (Received August 28, 2017)

1135-45-821 **Yuzhen Liu\*** (yz10204@auburn.edu), **Lixin Shen**, **Yuesheng Xu** and **Hongqi Yang**. *A Collocation Method Solving Integral Equation Models for Image Restoration.*

Discrete models are usually used in image processing due to their convenience in implementation and their consistence with the usual sampling method. However, since discrete models result from piecewise constant approximation of the integral equation which describes the processing, it imposes a bottleneck model error which

cannot be compensated by any image processing method. To overcome the shortcoming of discrete models in image restoration, the continuous model can be used instead.

In this talk, a continuous model we propose for image restoration from out-of-focus images as well as the multiscale collocation method we used for solving the integral equations that arise will be presented. (Received September 14, 2017)

## 46 ► *Functional analysis*

1135-46-59 **Sheldon Axler\*** (axler@sfsu.edu), Mathematics Department, San Francisco State University, San Francisco, CA 94132. *The Sarason Space  $H^\infty + C$ .*

Let  $\mathbf{T}$  denote the unit circle in the complex plane; let  $L^\infty$  denote the usual space of bounded measurable functions on  $\mathbf{T}$  with respect to arc length measure; let  $H^\infty$  denote the subalgebra of  $L^\infty$  consisting of those  $L^\infty$  functions whose Fourier coefficients corresponding to the negative integers all vanish; and let  $C$  denote the set of continuous complex-valued functions on  $\mathbf{T}$ . The Sarason space  $H^\infty + C$  is the linear span of  $H^\infty$  and  $C$ . Surprisingly,  $H^\infty + C$  is a closed subalgebra of  $L^\infty$ . This expository talk will highlight the many remarkable appearances of  $H^\infty + C$  in various aspects of operator theory and function theory. (Received July 14, 2017)

1135-46-318 **Dejenie A Lakew\*** (dlakew@stratford.edu), Stratford University, Newport News, VA 23601. *On orthogonal decomposition of a Sobolev space.*

The theme of this research is to investigate an orthogonal decomposition of the Sobolev space  $W^{1,2}(\Omega)$  as  $W^{1,2}(\Omega) = A^{2,2}(\Omega) \oplus D^2(W_0^{3,2}(\Omega))$  and look at properties of the inner product therein and the distance defined from the inner product. We also see the structure of the orthogonal difference space  $W^{1,2}(\Omega) \ominus (W_0^{1,2}(\Omega))^\perp$  and the expansion of Sobolev spaces as their regularity increases (Received August 23, 2017)

1135-46-397 **William T Ross\*** (wross@richmond.edu), Department of Mathematics and Computer Science, University of Richmond, Richmond, VA 23173. *Pythagoras Lives!*

Pythagoras of Samos (c. 570–495 BCE) was a mathematician, philosopher, and teacher whose influence is still being felt millennia after he lived. He is known to all school children by this Pythagorean theorem which, in modern notation, gives us a notion of orthogonality and parallelogram laws in Hilbert spaces. In this joint work with Raymond Cheng and Javad Mashreghi, we explore some notions of weak parallelogram laws and how far certain Banach spaces deviate from Hilbert spaces. This talk also explores a more general notion of orthogonality that has applications to problems in complex analysis and prediction theory. (Received August 31, 2017)

1135-46-491 **Torrey Gallagher, Chris Lennard and Roxana Popescu\*** (rop42@pitt.edu). *Weak compactness is not equivalent to the fixed point property in  $c$ .*

We show that there exists a non-weakly compact, closed, bounded, convex subset  $W$  of the Banach space of convergent sequences  $(c, \|\cdot\|_\infty)$  such that every nonexpansive map  $T : W \rightarrow W$  has a fixed point. This answers a question left open in the 2003 and 2004 papers of Dowling, Lennard and Turett. It is also the first example of a non-weakly compact, closed, bounded, convex subset  $W$  of a Banach space  $X$  isomorphic to  $c_0$ , for which  $W$  has the fixed point property for nonexpansive mappings. (Received September 06, 2017)

1135-46-508 **Valmir Bucaj\*** (vb11@rice.edu), 7901 Cambridge st, apt 85, Houston, TX 77054. *Lyapunov exponents for the discrete one-dimensional generalized Anderson model.*

We study a particular class of discrete one-dimensional random Schrodinger operators. Specifically, the ones where a single i.i.d random variable determines the potential on a block of arbitrary, but fixed, size  $\alpha$ , which in literature is also known as the discrete generalized Anderson model. For this model we prove uniform positivity of Lyapunov exponents at all energies. Concretely, we give a lower bound, in terms of the length of the block  $\alpha$ , for the size of the support of the distribution for which one has uniform positivity of Lyapunov exponents at all energies. Moreover, we show that it is not possible to chose the size of the support of the distribution uniformly and still have positive Lyapunov exponents at all energies. (Received September 06, 2017)

1135-46-560 **Jeremy J Becnel\*** (becneljj@sfasu.edu), Department of Mathematics, Stephen F. Austin State University, Nacogdoches, TX 75965. *The Gauss Radon Transform in Infinite Dimensions.*

We describe the Gauss Radon transform in infinite dimensions. We provide a support theorem in this setting and then develop several means of recovering a function from the function's Gauss Radon transform. (Received September 08, 2017)

1135-46-758 **J. William Helton\***, Prof. Bill Helton, Math Dept, UCSD, La Jolla, CA 92093.  
*Bianalytic maps in "matrix" variables.*

The talk concerns inequalities for functions having matrix variables. The functions are typically (noncommutative) polynomials or rational functions. A focus of much attention is the inequalities corresponding to convexity which in turn is bound closely to Linear Matrix Inequalities, LMIs.

Engineering systems problems seldom produce an LMI directly and depend on a change of variables to produce convexity. This talk concerns analytic changes of noncommutative variables to convert one convex set to another. This produces a wide range of subsidiary problems which need to be solved. Most of the work is done jointly by Meric Augat, J. William Helton, Igor Klep and Scott A. McCullough. (Received September 14, 2017)

1135-46-791 **Shmuel Friedland** (friedlan@uic.edu) and **Todd Kemp\*** (tkemp@math.ucsd.edu). *Most Quantum States are Almost Maximally Entangled.*

In 2009, Gross, Flammia, and Eisert showed that the geometric measure of entanglement of a random quantum state is very close to the theoretical maximum, with very high probability; this has some surprisingly negative consequences for quantum computation.

In this talk, I will discuss recent work proving the comparable concentration result for symmetric (Boson) quantum states; with a glimmer of hope for quantum computation.

Our techniques involve concentration methods well known in free probability, and representation theory of unitary groups. (Received September 14, 2017)

1135-46-856 **Andrea Arauza Rivera\*** (arauza@math.ucr.edu), 900 University Ave., Surge 259, Riverside, CA 92521. *Spectral triples and fractal geometry.*

There is a long tradition in mathematics of using algebraic tools to study the geometry of a space. The duality between the category of compact Hausdorff spaces and the category of commutative unital  $C^*$ -algebras means that one can study a topological space  $X$  by studying the algebra of continuous functions on  $X$ . Dropping the commutativity requirement leads to the study of noncommutative  $C^*$ -algebras and hence noncommutative spaces. The field of noncommutative fractal geometry uses operator algebraic tools to study geometry and analysis on fractal sets. We will see how one can use the spectral triples of noncommutative geometry to formulate notions of dimension, metric, and measure on fractal spaces. (Received September 15, 2017)

1135-46-981 **Ioana Ghenciu\*** (ioana.ghenciu@uwrf.edu), 410 S. 3rd Street, River Falls, WI 54022.  
 *$L$ -sets and property  $(SR^*)$  in spaces of compact operators.*

We give a characterization of  $L$ -sets using weakly precompact operators. A Banach space  $X$  has property  $(SR^*)$  (resp.  $(wSR^*)$ ) if every *Right\** subset of  $X$  is relatively weakly compact (resp. weakly precompact). Let  $K_{w^*}(E^*, F)$  denote the set of all  $w^*$ - $w$  continuous compact operators from  $E^*$  to  $F$ . We investigate whether the space  $K_{w^*}(E^*, F)$  has property  $(SR^*)$  (resp.  $(wSR^*)$ ) when  $E$  and  $F$  have the respective property. (Received September 18, 2017)

1135-46-1094 **Adam H. Fuller, Michael Hartz\*** (mphartz@wustl.edu) and **Martino Lupini**.  
*Rectangular boundary representations and rectangular matrix convexity.*

Boundary representations in the sense of Arveson are the noncommutative analogue of the Choquet boundary of a uniform algebra. They play an important role in the theory of operator systems and operator algebras. I will talk about boundary representations for operator spaces, which can be seen as a rectangular version of operator systems. In particular, there are enough boundary representations to generate the triple envelope, which is an operator space analogue of a theorem of Arveson and Davidson and Kennedy. Moreover, I will discuss rectangular matrix convex sets. (Received September 19, 2017)

1135-46-1120 **Donghai Ji, Khazhak Varazdat Navoyan\*** (knavoyan@go.olemiss.edu) and **Qingying Bu**. *Bases in the space of regular multilinear operators on Banach lattices.*

For Banach lattices  $E_1, \dots, E_m$  and  $F$  with 1-unconditional bases, we show that the monomial sequence forms a 1-unconditional basis of  $\mathcal{L}^r(E_1, \dots, E_m; F)$ , the Banach lattice of all regular  $m$ -linear operators from  $E_1 \times \dots \times E_m$  to  $F$ , if and only if each basis of  $E_1, \dots, E_m$  is shrinking and every positive  $m$ -linear operator from  $E_1 \times \dots \times E_m$  to  $F$  is weakly sequentially continuous. As a consequence, we obtain necessary and sufficient conditions for which the  $m$ -fold Fremlin projective tensor product  $E_1 \hat{\otimes}_{|\pi|} \dots \hat{\otimes}_{|\pi|} E_m$  (resp. the  $m$ -fold positive injective tensor product  $E_1 \hat{\otimes}_{|\epsilon|} \dots \hat{\otimes}_{|\epsilon|} E_m$ ) has a shrinking basis or a boundedly complete basis. (Received September 19, 2017)

1135-46-1316 **Keshab Chandra Bakshi, Sayan Das and Zhengwei Liu\*** (zhengweiliu@fas.harvard.edu), 17 Oxford Street, Cambridge, MA , and **Yunxiang Ren.** *An angle between intermediate subfactors and its rigidity.*

We introduce a new angle between intermediate subfactors and prove a uniform 60 to 90 degree bound for the angle between minimal intermediate subfactors of a finite index irreducible subfactor. From this rigidity we can bound the number of minimal (or maximal) intermediate subfactors by the kissing number in geometry. As a consequence, the number intermediate subfactors of an irreducible subfactor has at most exponential growth with respect to the Jones index. This answers a question of Longo published in 2003. (Received September 21, 2017)

1135-46-1424 **James Eldred Pascoe\*** (pascoej@math.wustl.edu). *Invariant theory for non-commutative functions and applications.*

Given a group  $G$  which acts on  $d$ -dimensional space, it is natural to study the ring of polynomials which are invariant under the action of  $G$ , the so-called *ring of invariants*. Two key classical results in invariant theory are the fact that the ring of invariants is finitely generated, and the Chevalley-Shepard-Todd theorem which characterizes their structure. Similarly, one can study the noncommutative ring of invariant free polynomials and other non-commutative functions, which in turn have their own rich theory, starting in 1936, when Margarete C. Wolf showed that the ring of symmetric free polynomials in two or more variables is isomorphic to the ring of free polynomials in infinitely many variables, which corresponds to a free Chevalley-Shepard-Todd type theorem. We will discuss some recent advances in non-commutative invariant theory with some surprising applications to change of variables theory. (Received September 22, 2017)

1135-46-1494 **Michael Brannan\***, Department of Mathematics, Texas A&M University, College Station, TX 77840, and **Roland Vergnioux.** *Orthogonal free quantum group factors are strongly 1-bounded.*

In this talk I will survey some recent results on the structural theory of a class of  $\text{II}_1$ -factors arising from a family of discrete quantum groups, called the orthogonal free quantum groups  $\mathbb{F}O_n$ . A question that has been around for some time is whether or not an orthogonal free quantum group factor  $\mathcal{L}(\mathbb{F}O_n)$  can be isomorphic to a free group factor  $\mathcal{L}(F_k)$ . We answer this question in the negative by proving that  $\mathcal{L}(\mathbb{F}O_n)$  is a strongly 1-bounded von Neumann algebra in the sense of Kenley Jung. We obtain this result by proving a certain spectral regularity result for the edge reversing operator on the quantum Cayley tree of  $\mathbb{F}O_n$  and connect this result to a recent free entropy dimension result of Jung and Shlyakhtenko. This is joint work with Roland Vergnioux. (Received September 22, 2017)

1135-46-1513 **Eric Michael Evert\*** (eric.m.evert@gmail.com). *Matrix Convex Sets Without Absolute Extreme Points.* Preliminary report.

This talk will give an example of a class of closed bounded matrix convex sets which do not have absolute extreme points. The sets considered are noncommutative sets,  $K_X$ , formed by taking matrix convex combinations of a single tuple  $X$ . In the case that  $X$  is a tuple of compact operators with no nontrivial finite dimensional reducing subspaces,  $K_X$  is a closed bounded matrix convex set with no absolute extreme points.

A central goal in the theory of matrix convexity is to find a natural notion of an extreme point in the dimension free setting which is minimal with respect to spanning. Matrix extreme points are the strongest type of extreme point known to span matrix convex sets; however, they are not necessarily the smallest set which does so. Absolute extreme points, a more restricted type of extreme points that are closely related to Arveson's boundary, enjoy a strong notion of minimality should they span. This result shows that matrix convex sets may fail to be spanned by their absolute extreme points. (Received September 22, 2017)

1135-46-1573 **Guillaume Cebon and Ching Wei Ho\***, 9500 Gilman Dr, La Jolla, CA 92093. *The Large- $N$  Limit of the  $q$ -Segal-Bargmann Transform.*

Sniady constructed a random matrix model which has a limiting noncommutative distribution of the  $q$ -Gaussian distribution. We prove that the Segal-Bargmann transform on the Sniady random matrix model converges to the  $q$ -Segal-Bargmann transform in  $L^2$  sense. (Received September 23, 2017)

1135-46-1608 **Lauren C. Ruth\*** (ruth@math.ucr.edu). *Two new settings for examples of von Neumann dimension.*

Let  $G = PSL(2, \mathbb{R})$ , let  $\Gamma$  be a lattice in  $G$ , and let  $\mathcal{H}$  be an irreducible unitary representation of  $G$  with square-integrable matrix coefficients. A theorem in Goodman-de la Harpe-Jones (1989) states that the von Neumann dimension of  $\mathcal{H}$  as a  $W^*(\Gamma)$ -module is equal to the formal dimension of the discrete series representation  $\mathcal{H}$  times the covolume of  $\Gamma$ , calculated with respect to the same Haar measure. We will present two results inspired by

this theorem. First, we show there is a representation of  $W^*(\Gamma)$  on a subspace of cuspidal automorphic functions in  $L^2(\Lambda \backslash G)$ , where  $\Lambda$  is any other lattice in  $G$ , and  $W^*(\Gamma)$  acts on the right; and this representation is unitarily equivalent to one of the representations in [GHJ]. Next, we explain how their proof carries over to a wider class of groups, and we calculate von Neumann dimensions when  $G$  is  $PGL(2, F)$ , for  $F$  a local non-archimedean field of characteristic 0;  $\Gamma$  is a torsion-free lattice in  $PGL(2, F)$ , which, by a theorem of Ihara, is a free group; and  $\mathcal{H}$  is the Steinberg representation, or a depth-zero supercuspidal representation, each yielding a different dimension. (Received September 23, 2017)

1135-46-1630 **Sofya S Masharipova\*** ([ssm57@pitt.edu](mailto:ssm57@pitt.edu)), Department of Mathematics, University of Pittsburgh at Johnstown, 450 Schoolhouse Road, Johnstown, PA 15904. *Positivity and geometry of unit balls in operator algebras on Pontryagin space* $\Pi_1$ . Preliminary report.

In the work we study positivity for operators and geometry of unit balls on symmetric algebras of bounded linear operators on Pontryagin space  $\Pi_1$ . Consideration of all possible cases is based on structure of cones of positive linear operators. Geometry of unit balls of symmetric algebras of operators on Pontryagin space  $Pi_1$  is studied. Connection with V. S. Shulman's types of such algebras is described (see Math. Sbornik, 1972, No 2). (Received September 23, 2017)

1135-46-1632 **Shukhrat M Usmanov\*** ([shukhrat.usmanov@waldorf.edu](mailto:shukhrat.usmanov@waldorf.edu)), Waldorf University, 106 South 6th Street, Forest City, IA 50436. *Periodic automorphisms of special flows on Lebesgue space and factors of type*  $III_0$ . Preliminary report.

In the work we consider a contraction of the periodic automorphism  $\alpha$  on the Lebesgue space  $\{X, \mu\}$  to a periodic automorphism  $\alpha_0$  of the Lebesgue space  $\{Y, m\}$ , where the flow  $\{S_t\}$  on  $\{X, \mu\}$  is a special flow, constructed from the ergodic automorphism  $T$  on Lebesgue space  $\{Y, m\}$ . In the assumption that  $\alpha$  and  $\{S_t\}$  are commuting for any  $t$ , we prove that  $\alpha_0$  and  $T$  commute, too. We will use the construction of a contraction  $\alpha_0$  for building of the discrete decomposition of real factors of type  $III_0$ . (Received September 23, 2017)

1135-46-1682 **Terje Høim\*** ([thoim@fau.edu](mailto:thoim@fau.edu)), Wilkes Honors College, Florida Atlantic University, Jupiter, FL 33458, and **D A Robbins** ([david.robbins@trincoll.edu](mailto:david.robbins@trincoll.edu)), Trinity College, Hartford, CT 06106. *Cover topologies, subspaces, and quotients for some spaces of vector-valued functions.*

Let  $X$  be a completely regular Hausdorff space, and  $\mathcal{D}$  a cover of  $X$  by  $C_b$ -embedded sets. Mati Abel, J. Arhipainen, and J. Kauppi in [Mediterr. J. Math. 7 (2010) 271-282] and [Cent. Eur. J. Math. 10 (2012), 1060-1066] describe the cover topology on  $C_b(X, \mathcal{D})$ , the space of continuous scalar-valued functions on  $X$  which are bounded on each  $D \in \mathcal{D}$ , and investigate the ideal and quotient structures of  $C_b(X, \mathcal{D})$ . We use ideas from the theory of bundles of topological vector spaces (in particular, bundles of Banach algebras), as found in e.g. G. Gierz [Lect. Notes Math. 955 (Springer-Verlag, 1982)] and the present authors' papers [Acta. Comment. Univ. Tartuensis Math. 14 (2010), 75-90] and [Quaest. Math. 34 (2011), 361-376] to define analogous topologies on section spaces of such bundles, and to discuss density properties, ideals, and quotients of such spaces. This work is also related to the authors' paper *Cover-strict topologies, ideals, and quotients for some spaces of vector-valued functions*, Banach J. Math. Anal. 10 (2016), 783-799. (Received September 24, 2017)

1135-46-1757 **Jim Agler\*** ([jagler@ucsd.edu](mailto:jagler@ucsd.edu)), **John E. McCarthy** and **Nicholas Young**. *nc-Manifolds and their Application to the Analysis of Free Holomorphic Functions*. Preliminary report.

The richly developed theory of complex manifolds plays important roles in our understanding of holomorphic functions in several complex variables. It is natural to consider manifolds that will play similar roles in the theory of holomorphic functions in several non-commuting variables. We introduce the class of *nc-manifolds*, the mathematical objects that at each point possess a neighborhood that has the structure of an  $nc$ -domain in the  $d$ -dimensional  $nc$ -universe  $\mathbb{M}^d$ . We then describe applications of  $nc$ -manifolds to the free square root function, free homogenous functions, and free symmetric functions. (Received September 24, 2017)

1135-46-1859 **Sandeepan Parekh\*** ([sandeepan.parekh@vanderbilt.edu](mailto:sandeepan.parekh@vanderbilt.edu)), **Koichi Shimada** and **Chenxu Wen**. *Maximal amenable subalgebras in  $q$ -Gaussian factors*.

For  $1 < q < \infty$ , Bozejko and Speicher's  $q$ -Gaussian factors can be thought of as  $q$ -deformed versions of the free group factor. Indeed they are known to share several properties in common with the free group factors like being non-injective, strongly solid, isomorphic to  $LF_n$  (for  $|q|$  small enough). Continuing this line of investigation, in a joint work with K. Shimada and C. Wen, we show the generator masa in these factors are maximal amenable by constructing a Riesz basis in the spirit of Radulescu. (Received September 25, 2017)

1135-46-1878 **Christopher Michael Schwanke\*** (cmschwanke26@gmail.com), Private Bag X6001, Potchefstroom Campus, North-West University, Potchefstroom, North-West 2520, South Africa. *Turning the Cauchy-Schwarz inequality into an equality.*

It will be illustrated in this talk how the theory of Archimedean vector lattices is a convenient apparatus for shedding new light on classical results. In particular, we provide an identity involving semi-inner product-like maps from the Cartesian square of a vector space into an Archimedean vector lattice from which a generalization of the classical Cauchy-Schwarz inequality immediately follows. (Received September 25, 2017)

1135-46-1879 **Thomas Sinclair\***, Department of Mathematics, 150 N University St, West Lafayette, IN 47907. *On the classification of group von Neumann algebras.*

I will discuss recent progress on structural and classification results for  $\text{II}_1$  factors associated to countable, discrete groups. (Received September 25, 2017)

1135-46-1884 **Jeremiah Hocutt\*** (jhocutt@ufl.edu), Department of Mathematics, PO Box 118105, University of Florida, Gainesville, FL 32611. *'Twisted Duality' in the Clifford Hilbert Space.*

Let  $C(V)$  be the complex Clifford algebra associated to the real inner product space  $V$ . It is a familiar fact that the graded (or super) commutant of the subalgebra generated by a subspace  $Z$  of  $V$  coincides with the subalgebra generated by the orthogonal space  $Z^\perp$ . We show that a corresponding statement is true for the Hilbert space completion  $H(V)$  of  $C(V)$  relative to its tracial inner product; our proof exploits the freedom offered by working within the full antidual  $C'(V)$  comprising all (bounded or unbounded) antilinear functionals on  $C(V)$ . (Received September 25, 2017)

1135-46-1927 **Igor Nikolaev\*** (igor.v.nikolaev@gmail.com), 8000 Utopia Parkway, New York, NY 11439. *Remark on arithmetic topology.*

We formalize the arithmetic topology, i.e. a relationship between knots and primes. Namely, using the notion of a cluster  $C^*$ -algebra we construct a functor from the category of 3-dimensional manifolds  $M$  to a category of algebraic number fields  $K$ , such that the prime ideals (ideals, resp.) in the ring of integers of  $K$  correspond to knots (links, resp.) in  $M$ . It is proved that the functor realizes all axioms of the arithmetic topology conjectured in the 1960's by Manin, Mazur and Mumford. Reference: arXiv:1706.06398 (Received September 25, 2017)

1135-46-1930 **Don Hadwin, Qihui Li, Weihua Li\*** (wli@colum.edu) and **Junhao Shen**. *Topological Orbit Dimension of MF  $C^*$ -algebras.*

This paper is a continuation of our work on D. Voiculescu's topological free entropy dimension in unital  $C^*$ -algebras. In this paper we first prove the topological free entropy dimension of a MF-nuclear and inner QD algebra is irrelevant to its generating family. Then we give the relation between the topological orbit dimension  $K_2^{top}$  and the modified free orbit dimension  $K_2^2$  by using MF-traces. We also introduce a new invariant  $K_3^{top}$  which is a modification of the topological orbit dimension  $K_2^{top}$  when  $K_2^{top}$  is defined. As applications, we prove that  $K_3^{top}(A) = 0$  if  $A$  has property  $C^* - \Gamma$  and has no finite-dimensional representations. We also give the definition of property MF- $C^* - \Gamma$ . We then conclude that, for the unital MF  $C^*$ -algebra with no finite-dimensional representations, if  $A$  has property MF- $C^* - \Gamma$ , then  $K_3^{top}(A) = 0$ . (Received September 25, 2017)

1135-46-1999 **S E Scott\*** (sscott1008@gmail.com), 2973 N 55th Street, Milwaukee, WI 53210. *Ergodicity defect tortuosity metric.*

The detection and measurement of the abnormal tortuosity – i.e., the abnormal bending and winding - of vessels is recognized as an important diagnostic indicator for many diseases, but although several metrics for tortuosity have been proposed, no single one measure is able to capture all types of tortuosity. This talk presents the ergodicity defect tortuosity metric (ED) which is based on the ergodicity defect, a technique which measures the complexity of a system's trajectories in terms of how they cover the space; hence, a vessel that has excessive bending and winding can be considered more complex than nontortuous vessels. The technique is considered with vessels from retinal images (fundus photos) and MRA tumor images. (Received September 26, 2017)

1135-46-2105 **Alvaro Arias\***, Department of Mathematics, University of Denver, 2390 S. York St., Denver, CO 80210, and **Natasha Dobrinen, Gabriel Giron** and **Jose Mijares**. *Banach Spaces from Barriers in High Dimensional Ellentuck Spaces.*

A new hierarchy of Banach spaces  $T_k(d, \theta)$ ,  $k$  any positive integer, is constructed using barriers in high dimensional Ellentuck spaces (introduced by Dobrinen) following the classical framework under which a Tsirelson type norm is defined from a barrier in the Ellentuck space.

It is shown that these spaces contain arbitrarily large copies of  $\ell_\infty^n$ , with the bound constant for all  $n$ . These spaces are  $\ell_p$ -saturated, in fact forming natural extensions of the  $\ell_p$  spaces. They are not isomorphic to each other, but form a strict hierarchy: For fixed  $d$  and  $\theta$ , for any  $j < k$ , the space  $T_j(d, \theta)$  embeds isometrically into  $T_k(d, \theta)$ . (Received September 25, 2017)

1135-46-2179 **Michael Hartglass\*** ([mhartglass@scu.edu](mailto:mhartglass@scu.edu)) and **Brent Nelson**. *Free transport for interpolated free group factors.*

A few years ago in a landmark paper, Guionnet and Shlyakhtenko proved the existence of free transport maps from the free group factors to von Neumann algebras generated by elements which have a joint law “close” to that of the free semicircular law. In this talk, I will discuss how to modify their idea to obtain similar results for interpolated free group factors using an operator-valued framework. This is joint work with Brent Nelson. (Received September 25, 2017)

1135-46-2186 **Michael Hartglass\*** ([mhartglass@scu.edu](mailto:mhartglass@scu.edu)). *Weighted graphs as compact quantum metric spaces.*

I will present a canonical noncommutative  $C^*$ -algebra that one can associate to any graph with a weighting on the vertices. I will show that this forms a compact quantum metric spaces in the sense of Rieffel, and describe partial convergence results in the area of quantum Gromov-Hausdorff distance. This is joint work in progress with Dave Penneys. (Received September 25, 2017)

1135-46-2212 **Valeriano Aiello\*** ([valeriano.aiello@vanderbilt.edu](mailto:valeriano.aiello@vanderbilt.edu)), Vanderbilt University, 1326 Stevenson Center, Nashville, TN 37240. *The inner structure of the 2-adic ring  $C^*$ -algebra and its acquaintances.*

The 2-adic ring  $C^*$ -algebra is the universal  $C^*$ -algebra  $\mathcal{Q}_2$  generated by an isometry  $s_2$  and a unitary  $u$  such that  $s_2u = u^2s_2$  and  $s_2s_2 + us_2s_2u^* = 1$ . Inside  $\mathcal{Q}_2$ , the isometries  $s_2$  and  $us_2$  generate a natural copy of the Cuntz algebra  $\mathcal{O}_2$ . This inclusion turns out to be rigid in the sense that endomorphisms of  $\mathcal{Q}_2$  that restrict to the identity on  $\mathcal{O}_2$  are actually the identity on the whole  $\mathcal{Q}_2$ . Inverting the perspective, I will also discuss the problem of extending Bogolubov automorphisms from  $\mathcal{O}_2$  to  $\mathcal{Q}_2$ . In addition, I will present structure results on endomorphisms and automorphisms that fix the diagonal subalgebra of  $\mathcal{Q}_2$  or the copy of  $C^*(\mathbb{Z})$  generated by  $u$ . Time permitting, I will indicate how these results generalize to certain boundary quotients of right LCM semigroups. This talk is based on a joint work with Roberto Conti (Sapienza Università di Roma), Stefano Rossi (Università di Roma Tor Vergata), and Nicolai Stammeier (University of Oslo). (Received September 25, 2017)

1135-46-2278 **Konrad Aguilar** and **Samantha Brooker\*** ([samijobrooker@gmail.com](mailto:samijobrooker@gmail.com)). *Finite-Dimensional Quantum Metric Spaces.* Preliminary report.

According to K. Aguilar and F. Latrémolière, approximately finite-dimensional  $C^*$ -algebras (AF algebras) can be endowed with a quantum metric, which then allows one to prove that AF algebras are limits of finite-dimensional quantum metric spaces for a noncommutative version of the Gromov-Hausdorff distance, named the quantum Gromov-Hausdorff propinquity. Our research concerns the geometry, for this propinquity, of the class of these finite-dimensional quantum metric spaces. We have established thus far that several of these spaces are not isometric quantum metric spaces. The examples that we studied are described via full matrix algebras endowed with various quantum metrics, and we prove that no automorphism of the full matrix algebra can carry one quantum metric to another. We hope that our work is a first step in establishing lower bounds on the propinquity between these finite-dimensional quantum metric spaces, which in general are difficult to establish. (Received September 25, 2017)

1135-46-2292 **Edward S. Sichel\*** ([edsichel@gmail.com](mailto:edsichel@gmail.com)), Department of Mathematics, 5245 North Backer Avenue, M/S PB108, Fresno, CA 93740. *A Direct Approach to Nonhypercyclicity in the Finite Dimensional Setting.*

A direct eigenvalue approach is found to showing the non-hypercyclicity of a linear operator and a continuous semigroup of linear operators on a complex finite-dimensional Banach space through the use of Jordan normal form. (Received September 25, 2017)

1135-46-2634 **Chase T Worley\*** ([chase.worley@maryvillecollege.edu](mailto:chase.worley@maryvillecollege.edu)), 912 Enterprise Way, Maryville, TN 37801. *Commuting Squares of Bismash Product Hopf Algebras.* Preliminary report.

We construct a new class of commuting squares which we call bismash commuting squares. They are obtained from bismash product Hopf algebras based on exact factorizations of finite groups,  $L$ . We recall Nicoara's definition of the defect of a commuting square, and then investigate the defect of a bismash commuting

square which leads us to the conjecture that the defect of the commuting square is equal to the defect of the group  $L$ . We are able to calculate the defect for easy examples coming from group theory. We prove this conjecture when  $L$  is the direct or semidirect product of two proper subgroups. This is joint work with Remus Nicoara and Ian Francis. (Received September 26, 2017)

1135-46-2829 **Oleg Friedman\*** ([friedman001@yahoo.com](mailto:friedman001@yahoo.com)), Lander College for Men / Touro College, Department of Mathematics, 75-31 150th Street, Flushing, NY 11367, and **Alexander A. Katz** ([katza@stjohns.edu](mailto:katza@stjohns.edu)), St. John's University, SJC, Dep. of Mathematics & CS, 8000 Utopia Parkway, SJH-334-G, Queens, NY 11439. *Some properties and representation theory for real locally  $C^*$ - and locally JB-algebras.*

We discuss some properties and representation theory for locally  $C^*$ -algebras and locally JB-algebras, and their connections with the theory of complex locally  $C^*$ -algebras. (Received September 26, 2017)

1135-46-3093 **Alexander A. Katz\*** ([katza@stjohns.edu](mailto:katza@stjohns.edu)), St. John's University, SJC of LAS, Department of Mathematics & CS, 8000 Utopia Parkway, SJH-334-G, Queens, NY 11439. *On the notion of Stonean  $*$ -algebras.*

We introduce and study Stonean  $*$ -algebras ( $S^*$ -algebras) which are essentially normed (non-commutative) complex algebras with involution  $**$ . Its norm satisfies  $C^*$ -algebra like conditions, but does not have to take just finite values. In addition, its self-adjoint part admits a partial ordering that defines a topology, generally coarser than the topology induced by the norm. In the sequel we study basic properties of these algebras and show how they are related to von Neumann algebras,  $AW^*$ -algebras and  $O^*$ -algebras. (Received September 26, 2017)

1135-46-3108 **Ehssan Khanmohammadi\*** ([ehssan@fandm.edu](mailto:ehssan@fandm.edu)). *A structured inverse spectrum problem for unbounded operators.*

We present our extensions of some recent results on inverse eigenvalue problems of finite graphs to the infinite setting by means of functional analytic methods. Our work is inspired and it involves a discrete version of the potential energy operator in quantum mechanics.

Given an infinite graph  $G$  on countably many vertices and a closed, infinite set  $\Lambda$  of real numbers, we show, among other things, the existence of an unbounded operator whose graph is  $G$  and whose spectrum is  $\Lambda$ . (Received September 26, 2017)

1135-46-3216 **S. K. Upadhyay\*** ([sk\\_upadhyay2001@yahoo.com](mailto:sk_upadhyay2001@yahoo.com)), Department of Mathematics, Indian Institute of Technology Kharagpur, Banaras Hindu University, Varanasi, UP 221005, India. *Extremal bounds of Localization Operators involving Watson Transform Associated with Regular Representations and Wavelet Multipliers.*

In this paper we study the boundedness of localization operators associated with left regular representations of a locally compact group. We use the theory of Watson transform to find relation between wavelet multipliers and localization operators. Search for optimal bounds to find extremal inequalities is under investigation (Received September 27, 2017)

## 47 ► Operator theory

1135-47-79 **Arnab Patra\*** ([arnabpatra@maths.iitkgp.ernet.in](mailto:arnabpatra@maths.iitkgp.ernet.in)), Department of Mathematics, Indian Institute of Technology Kharagpur, State- West Bengal, West Midnapore, 721302, India, and **Riddhick Birbonshi** and **Parmeshwary Dayal Srivastava**. *On some spectral properties of triangular band matrices.* Preliminary report.

The purpose of this work is to investigate the spectrum and fine spectrum of triangular band matrices where the bands are taken as convergent sequences. This kind of matrices can be expressed as a sum of a triangular Toeplitz operator and a compact operator. Our method employs some results of linear difference equations and compact perturbation theory. In this context, a result regarding the location of the roots of a polynomial with respect to the unit circle is derived which is useful to identify the point spectrum of the operator. As a consequence of our result, a class of compact operator is also obtained such that, not only the spectrum but also the fine spectrum of a triangular Toeplitz operator remains invariant under the perturbation given from this class. (Received July 24, 2017)



1135-47-117 **Benjamin Passer\***, benjaminpas@technion.ac.il, and **Orr Moshe Shalit and Baruch Solel**. *Scaling and Inclusion of Matrix Convex Sets.*

For a convex body  $K \subset \mathbb{R}^d$ , there may be many matrix convex subsets of  $\bigcup_{n=1}^{\infty} M_n(\mathbb{C})_{sa}^d$  which have  $K$  as the first level. In particular, one can find a maximal matrix convex set  $\mathcal{W}^{\max}(K)$  and a minimal matrix convex set  $\mathcal{W}^{\min}(K)$  with this property, and these sets may be described in terms of linear inequalities and existence of dilations, respectively. We seek inclusion results, such as  $\mathcal{W}^{\max}(K) \subseteq \mathcal{W}^{\min}(L)$ , especially when  $L$  is a scalar multiple of  $K$ . For example, if  $\overline{\mathbb{B}}_{p,d}$  is the closed unit ball of  $\mathbb{R}^d$  with the  $\ell^p$  norm, then

$$\mathcal{W}^{\max}(\overline{\mathbb{B}}_{p,d}) \subseteq d^{1-|1/p-1/2|} \cdot \mathcal{W}^{\min}(\overline{\mathbb{B}}_{p,d}),$$

and the scale  $d^{1-|1/p-1/2|}$  is optimal. Moreover, the minimal and maximal matrix convex sets over  $K$  are equal if and only if  $K$  is a simplex, and this result may be strengthened to include some dimension control. In this talk, I will describe some explicit dilations related to these results and some of the convex geometry guiding our minimality arguments.

This is joint work with Orr Shalit and Baruch Solel. (Received July 29, 2017)

1135-47-225 **Berhanu T Kidane\*** (berhanu.kidane@ung.edu), University of North Georgia, 1201 Bishop Farms Pkwy., Watkinsville, GA 30677. *The Corona Problem for the Weight Dirichlet Space of Weight 1/2.* Preliminary report.

We prove a corona theorem for infinitely many functions from the multiplier algebras on weighted Dirichlet spaces of weight 1/2 (or 0.5) on the unit disk. In addition, explicit estimate on solution is given. The estimate is given on the harmonic weighted Dirichlet space, which is an extension of our multipliers of the weighted Dirichlet space. (Received August 13, 2017)

1135-47-271 **Milivoje Lukic\*** (milivoje.lukic@rice.edu). *Spectral edge behavior for eventually monotone Jacobi and Verblunsky coefficients.*

This talk is about Jacobi matrices with eventually increasing sequences of diagonal and off-diagonal Jacobi parameters. We describe the asymptotic behavior of the subordinate solution at the top of the essential spectrum, and the asymptotic behavior of the spectral density at the top of the essential spectrum.

In particular, allowing on both diagonal and off-diagonal Jacobi parameters perturbations of the free case of the form  $-\sum_{j=1}^J c_j n^{-\tau_j} + o(n^{-\tau_1-1})$  with  $0 < \tau_1 < \tau_2 < \dots < \tau_J$  and  $c_1 > 0$ , we find the asymptotic behavior of the log of spectral density to order  $O(\log(2-x))$  as  $x$  approaches 2.

Apart from its intrinsic interest, the above results also allow a description of the asymptotics of the spectral density for orthogonal polynomials on the unit circle with real-valued Verblunsky coefficients of the same form. (Received August 17, 2017)

1135-47-305 **Xiao-Xiong Gan\*** (xiao-xiong.gan@morgan.edu), Department of Mathematics, Morgan State University, 1700 E. Cold Spring Lane, Baltimore, MD 21251. *Canonical operator on the space of formal series.*

Let  $\xi_I$  be the characteristic function of the interval  $I \subset \mathbb{R}$  and we define

$$\mathfrak{M}_0 = \left\{ \sum_{n \in \mathbb{Z}} a_n \xi_{(n, n+1]} : (a_n)_{n \in \mathbb{Z}} \subset \mathbb{C} \right\},$$

the subset of extended  $\mathbb{C}$ -valued step functions on  $\mathbb{R}$ . Let  $\mathbb{L}$  be the space of formal Laurent series over  $\mathbb{C}$ , we define the mapping  $A : \mathbb{L} \rightarrow \mathfrak{M}_0$  by

$$A(f) = \sum_{n \in \mathbb{Z}} a_n \xi_{(n, n+1]} \quad \text{for } f(z) = \sum_{n \in \mathbb{Z}} a_n z^n \in \mathbb{L}.$$

The mapping  $A$  is said to be the canonical mapping.

This canonical mapping provides a simpler and interesting approach for us to investigate the rather complicated multiplication, or Cauchy product, for formal Laurent series by connecting the Cauchy product in  $\mathbb{L}$  to the convolution in  $\mathfrak{M}_0$ . Some metric and topology of  $\mathbb{L}$  is also introduced under which the canonical mapping  $A$  is isometric.

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(Received August 22, 2017)

1135-47-308 **Yunied Puig de Dios\*** ([puigdedios@gmail.com](mailto:puigdedios@gmail.com)), University of California Riverside, Department of Mathematics, 900 University Ave, Riverside, CA 92521. *Piecewise syndetic recurrence sets in hypercyclicity.*

We will be concerned with dynamics of linear operators. On one hand, we address the problem of the regularity of frequently hypercyclic vectors. This goes back to Shkarin in 2009 and consists to determine whether a frequently hypercyclic operator admits a frequently hypercyclic vector with some recurrence set having positive and different lower and upper density. As far as we know this problem is still open. We give a positive partial answer replacing upper density by upper Banach density. On the other hand, it is known that reiteratively hypercyclic operators are syndetic transitive. However the converse is not true, and the example is a weighted shift on  $l^p$ . Still, syndetic transitive weighted backward shifts remain very close to reiteratively hypercyclic ones. This is the content of our second result. (Received August 22, 2017)

1135-47-349 **Asuman Güven Aksoy\*** ([aaksoy@cmc.edu](mailto:aaksoy@cmc.edu)), Dept of Mathematics, 850 Columbia Avenue, Claremont, CA 91711. *Deviation of elements of a Fréchet space from a system of subspaces.*

One of the notable theorems used in the constructive theory of functions is Bernstein Lethargy Theorem (BLT). In this talk, we consider Bernstein Lethargy Theorem (BLT) in the context of Fréchet spaces. Let  $X$  be an infinite-dimensional Fréchet space and let  $\mathcal{V} = \{V_n\}$  be a nested sequence of subspaces of  $X$  such that  $\overline{V_n} \subseteq V_{n+1}$  for any  $n \in \mathbb{N}$ . Let  $e_n$  be a decreasing sequence of positive numbers tending to 0. Under an additional natural condition on  $\sup\{\text{dist}(x, V_n)\}$ , we prove that there exists  $x \in X$  and  $n_o \in \mathbb{N}$  such that

$$\frac{e_n}{3} \leq \text{dist}(x, V_n) \leq 3e_n,$$

for any  $n \geq n_o$ . By using the above theorem, we prove both Shapiro's and Tyuremskikh's theorems for Fréchet spaces. We also identify some classical Banach spaces that form a Bernstein pair. (Received August 27, 2017)

1135-47-384 **Ronald G Douglas\*** ([rgd@tamu.edu](mailto:rgd@tamu.edu)), Texas A&M University Mathematics, MS 3368, College Station, TX 778433368. *A New Index Theorem for Monomial Ideals by Resolutions.*

Along with framing the conjecture by Arveson that the closure of a homogeneous polynomial ideal in a Drury-Arveson space is essentially normal, he established it for ideals generated by monomials. Later, the first author showed this conjecture is also valid for the closure of arbitrary ideals generated by monomials in the Bergman space.

In this paper the authors establish this result using a new approach which also yields an index formula. In particular, such ideals are shown to have resolutions by Bergman-like modules.

In this talk we examine some simple cases of this result attempting to clarify the relation between the combinatorial and geometrical pictures. We will conclude by discussing some possible generalizations of these results. (Received August 30, 2017)

1135-47-398 **George R. Exner\*** ([exner@bucknell.edu](mailto:exner@bucknell.edu)). *A mixed hyponormality-contractivity condition.* Preliminary report.

For  $T$  a linear operator on Hilbert space of norm one, and  $n$  a positive integer, define the condition  $C(n)$  on  $T$  by  $\sum_{i=0}^n (-1)^i \binom{n}{i} T^{*i} (T^*T - TT^*) T^i \geq 0$ . (This mixes the familiar positivity condition for hyponormality with the Agler positivity condition for  $n$ -contractivity.) For  $T$  to have  $C(n)$  for all  $n$  it is not sufficient that  $T$  be subnormal; for weighted shifts  $W$ , we exhibit a sufficient condition, related to infinite divisibility of  $W$ , for  $W$  to have  $C(n)$  for all  $n$ , give some further results, and discuss some examples. (Received August 31, 2017)

1135-47-411 **Anthony Vasaturo\*** ([avasatu@rockets.utoledo.edu](mailto:avasatu@rockets.utoledo.edu)), 2801 W Bancroft Street, Toledo, OH 43606, and **Zeljko Cuckovic**. *Carleson measures and Douglas' question on the Bergman space.*

Motivated by Douglas' question about the invertibility of Toeplitz operators on the Hardy Space, we study a related question concerning the Berezin transform and averaging function of a Carleson measure for the weighted Bergman space of the disc. As a consequence, we obtain a necessary and sufficient condition for the invertibility of Toeplitz operators whose symbols are averaging functions of these Carleson measures. (Received August 31, 2017)

1135-47-449 **Javad Mashreghi\*** (javad.mashreghi@mat.ulaval.ca), 1070 Pav. Vachon, Université Laval, Quebec, QC G1S 1M8, Canada, and **Thomas Ransford**, same as above. *Linear maps on function spaces preserving inner functions.*

We show that, for many holomorphic function spaces on the open unit disk, e.g., Hardy, Bergman and Dirichlet spaces, a continuous endomorphism that sends inner functions to inner functions is necessarily a weighted composition operator. (Received September 04, 2017)

1135-47-532 **Kelly A. Bickel, Michael Hartz** and **John E. McCarthy\***, Dept. of Mathematics, Washington University, 1, Brookings Drive, St. Louis, MO 63130. *Multiplier algebra functional calculus.*

A completely non-unitary contraction has not only a disk algebra functional calculus (which follows from von Neumann's inequality) but also an  $H^\infty$  functional calculus. This result was extended to the unit ball by J. Eschmeier (for operators that satisfy von Neumann's inequality for the ball), and by R. Clouâtre and K. Davidson (for row contractions). We extend this result to operators on regular unitarily invariant spaces on the ball, which means spaces with a kernel  $k(z, w) = \sum_{n=0}^{\infty} a_n \langle z, w \rangle^n$  for which  $\lim_{n \rightarrow \infty} a_n/a_{n+1} = 1$ . (Received September 07, 2017)

1135-47-687 **Stephan Ramon Garcia\*** (stephan.garcia@pomona.edu), Department of Mathematics, Pomona College, 610 N College Ave, Claremont, CA 91711. *Toeplitz Operators and Lattices.*

How do Toeplitz operators and frames relate to properties of lattices? We discuss recent results about the interplay between operator theory and lattice theory. This is joint work with A. Böttcher, S. Eisenbarth, L. Fukshansky, H. Maharaj, and D. Needell. (Received September 13, 2017)

1135-47-922 **Ilya M Spitkovsky\***, ims2@nyu.edu, imspitkovsky@gmail.com. *On operators from the algebra generated by two orthogonal projections.*

A canonical representation of a pair of orthogonal projections was obtained by Halmos in his "Two subspaces" paper as early as 1969. I will discuss several new results which either were obtained by using this representation or can be sharpened with its help. (Received September 17, 2017)

1135-47-1044 **Wu Jing\*** (wjing@uncfsu.edu). *Nonlinear \*-Lie Derivations of Standard Operator Algebras.*

Let  $\mathcal{H}$  be an infinite dimensional complex Hilbert space and  $\mathcal{A}$  be a standard operator algebra on  $\mathcal{H}$  which is closed under the adjoint operation. We prove that every nonlinear \*-Lie derivation  $\delta$  of  $\mathcal{A}$  is automatically linear. Moreover,  $\delta$  is an inner \*-derivation. (Received September 18, 2017)

1135-47-1058 **Guy Salomon**, Haifa, Israel, **Orr Moshe Shalit\*** (oshalit@tx.technion.ac.il), Haifa, Israel, and **Eli Shamovich**, Waterloo, Canada. *Algebras of Bounded Analytic Noncommutative Functions on Noncommutative Varieties.*

We study algebras of bounded analytic noncommutative (nc) functions on subvarieties of the nc unit ball. In recent work, it was shown that two such algebras are completely isometrically isomorphic, if and only if the underlying nc varieties are conformally equivalent. Moreover, every completely isometric isomorphism arises as composition with a conformal automorphism of the ball, and vice versa. Thus, the conformal structure of the underlying variety is a complete invariant of the (completely isometric) operator algebraic structure of such an algebra, and one may consider the classification of such algebras up to completely isometric isomorphism as settled.

In this talk I will review the existing results, and then report on our progress in answering the following two questions:

- (1) **The algebraic isomorphism question:** What is the appropriate complete invariant that "classifies" algebras of bounded analytic functions on subvarieties of the nc unit ball up to algebraic (or completely bounded) isomorphism?
- (2) **Going beyond the nc unit ball:** Can algebras of bounded analytic functions on subvarieties of other nc bounded domains be classified along the same lines?

(Received September 19, 2017)

1135-47-1070 **Scott M. LaLonde** and **David Milan\***, dmilan@uttyler.edu, and **Jamie Scott**. *Condition (K) for Inverse Semigroups.* Preliminary report.

A directed graph is said to satisfy Condition (L) if every cycle in the graph has an entrance and Condition (K) if every vertex that is the base of a cycle has at least two distinct return paths. We define Condition (L) and

Condition (K) for inverse semigroups in such a way that a graph inverse semigroup satisfies each condition if and only if the directed graph does. We characterize the ideals in the  $C^*$ -algebras of a class of inverse of inverse semigroups satisfying Condition (K) in terms of ideals in the semigroup, generalizing the description of ideals for graph  $C^*$ -algebras. An application to the inverse semigroups of self-similar graph actions is given. (Received September 19, 2017)

1135-47-1179 **Soumyadip Acharyya\*** (acharyys@erau.edu) and **Timothy Ferguson**. *Algebraic Combinations of Composition and Differentiation Operators on Analytic Function Spaces*. Let  $\varphi$  be an analytic self-map of the open unit disk  $\mathbb{D}$  and  $u$  be an analytic function on  $\mathbb{D}$ . The linear map  $uC_\varphi$  on  $H(\mathbb{D})$  defined by

$$(uC_\varphi)(f)(z) = u(z)(f \circ \varphi)(z), \quad f \in H(\mathbb{D}), z \in \mathbb{D},$$

is called the weighted composition operator with weight  $u$  and symbol  $\varphi$ . For a positive integer  $n$ ,  $D^n$  stands for the iterated differentiation operator defined by  $D^n(f) = f^{(n)}$ .

The talk will provide characterizations of order - boundedness and compactness of sums of products of weighted composition and differentiation operators. We will discuss the testing functions used in the proof. These functions turn out to be solutions to an interpolation problem. (Received September 20, 2017)

1135-47-1202 **Matthew Fleeman** (matthew\_fleeman@baylor.edu), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76798-7328, **Dale Frymark\*** (dale\_frymark@baylor.edu), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76798-7328, and **Constanze Liaw** (liaw@udel.edu), Department of Mathematical Sciences, University of Delaware, 501 Ewing Hall, Newark, DE 19716. *Spectral Properties of Sturm-Liouville Operators via Perturbation Theory*. Preliminary report.

The boundary conditions given to symmetric Sturm-Liouville differential operators that make them self-adjoint can be written as rank-one perturbations. Perturbation theory allows a completely description of the spectrum of these self-adjoint operators as the boundary conditions range over all possible combinations. This description is much more efficient than existing strategies in Sturm-Liouville theory. In particular, explicit forms for eigenfunctions are given for each such set of boundary conditions. (Received September 20, 2017)

1135-47-1217 **Mohammad Ansari\*** (ansari.moh@gmail.com), Azad University of Gachsaran, Gachsaran, Iran. *Strong topological transitivity*.

Let  $X$  be a topological vector space. An operator  $T \in L(X)$  is called strongly topologically transitive if  $X \setminus \{0\} \subset \bigcup_{n=0}^{\infty} T^n(U)$  for any nonempty open set  $U \subset X$ . In this extended abstract, we deal with the strong topological transitivity of some well-known topologically transitive operators. It is proved that, on  $H(\mathbb{C})$ , the derivative operator is strongly topologically transitive but translation operators are not. We present a sufficient condition and a necessary condition for weighted backward shifts on  $C_0$  and  $\ell^p$  to be strongly topologically transitive. We prove that the adjoint of any invertible multiplication operator on  $H^2$  is not strongly topologically transitive but there are non-invertible multiplication operators whose adjoints are strongly topologically transitive. We show that no composition operator on a Banach space  $X$  of analytic functions on the disk is strongly topologically transitive. Finally, it is proved that on every second countable Baire locally convex space  $X$ , the set of all strongly topologically transitive operators is either empty or SOT-dense in  $L(X)$ . (Received September 21, 2017)

1135-47-1235 **Joseph A Ball\*** (joball@math.vt.edu), Department of Mathematics, Virginia Tech, 225 Stanger Street, Blacksburg, VA 24061. *A Positivstellensatz for free noncommutative kernels*.

A **free nc kernel** is a function  $K$  of two matrix  $d$ -tuple arguments  $Z = (Z_1, \dots, Z_d)$  and  $W = (W_1, \dots, W_d)$  (with each  $Z_i$  say of size  $n \times n$  and each  $W_i$  of size  $m \times m$ ) coming from some domain  $\Omega$  with values in the space  $\mathcal{L}(\mathcal{A}^{n \times m}, \mathcal{B}^{n \times m})$  with  $\mathcal{A}$  and  $\mathcal{B}$  two given  $C^*$ -algebras (here  $d$  is fixed while  $n$  and  $m$  are arbitrary), subject to some additional compatibility conditions. Such a kernel  $K$  is said to be **completely positive (cp)** if the  $K(Z, Z)$  is a completely positive map for each  $Z \in \Omega$ . A **Positivstellensatz for free nc kernels** for a given pair of such kernels  $\mathfrak{Q}: \Omega \times \Omega \rightarrow \mathcal{L}(\mathcal{A}, \mathcal{L}(S))_{\text{nc}}$  and  $\mathfrak{S}: \Omega \times \Omega \rightarrow \mathcal{L}(\mathcal{A}, \mathcal{L}(Y))_{\text{nc}}$  is to obtain a **certificate** for when it is the case that  $\mathfrak{Q}(Z, Z)(I) \succ 0 \Rightarrow \mathfrak{S}(Z, Z)(I) \succeq 0$ . The talk will discuss the present status of this problem as well as its connections with free nc Nevanlinna-Pick interpolation and with other Positivstellensätze. (Received September 20, 2017)

1135-47-1450 **Linda J Patton\*** (lpatton@calpoly.edu), Mathematics Department, Cal Poly, San Luis Obispo, CA 93407. *Nevanlinna-Pick interpolation on the boundary of the bidisk.* Preliminary report.

When the Nevanlinna-Pick interpolation theorem is extended to the boundary of the unit disk with angular derivative values included, calculations similar to those in the Carathéodory-Julia theorem arise naturally. Over the last few years, Agler, McCarthy, Young, Pascoe, and Tully-Doyle and others have described the extent to which Carathéodory-Julia results generalize to the bidisk. These ideas will be used to discuss Pick interpolation with analogs of the angular derivative on the boundary of the bidisk. (Received September 22, 2017)

1135-47-1455 **Kristin Courtney\*** (kc2ea@virginia.edu). *A von Neumann-type inequality with universal  $C^*$ -algebras.*

A famous inequality of von Neumann states that, given any polynomial  $p$  in one variable, the maximal norm of the operator  $p(T)$ , as  $T$  ranges over all contractive Hilbert space operators, can be determined by considering only contractive operators on a one dimensional Hilbert space, i.e. elements of the unit disk in the complex plane.

With a universal  $C^*$ -algebra, one can readily establish a von Neumann-type inequality for any non-commutative  $*$ -polynomial  $q$ , which says that the maximal norm of the operator  $q(T)$ , as  $T$  ranges over contractive Hilbert space operators, can be determined by considering only contractive matrices. In this talk, we show why it actually suffices to consider only nilpotent contractive matrices.

Moving to polynomials in two variables, von Neumann's inequality notably extends for pairs of commuting contractive Hilbert space operators. Can we again look to matrices for an analogous extension in the case of noncommutative  $*$ -polynomials in two variables? Surely this question is not too difficult, is it? (Received September 22, 2017)

1135-47-1466 **Adrian Ioana\*** (aioana@ucsd.edu), Mathematics Department, UCSD, La Jolla, CA 92093, and **Ionut Chifan**. *Rigidity for von Neumann algebras of amalgamated free product groups.*

I will explain recent work with Ionut Chifan in which we provide a large family of amalgamated free product groups whose amalgam structure can be completely recognized from their von Neumann algebras. Our result significantly strengthens some of the previous Bass-Serre rigidity results for von Neumann algebras. As a consequence, we obtain the first examples of amalgamated free product groups which are  $W^*$ -superrigid, and the first examples of non-amenable groups which are  $C^*$ -superrigid. (Received September 22, 2017)

1135-47-1469 **Daniel Drimbe, Daniel Hoff\*** (hoff@math.ucla.edu) and **Adrian Ioana**. *Tensorial Relative Solidity.*

A well known and remarkable result of Popa and Vaes from 2012 says that icc hyperbolic groups are relatively strongly solid. As a consequence, if a countable icc group  $\Lambda$  is a product of  $k$  non-amenable hyperbolic groups, then  $L(\Lambda)$  has the following property: For any tracial von Neumann algebra  $M_0$ , and any  $k + 1$  commuting subalgebras of  $M_0 \overline{\otimes} L(\Lambda)$ , the first  $k$  of which are strongly non-amenable relative to  $M_0$ , the  $(k + 1)$ st must strongly intertwine into  $M_0$  in the sense of Popa. In joint work with Daniel Drimbe and Adrian Ioana, we show that this property, although a substantial weakening of relative strong solidity, has the quality that it passes from  $\Lambda$  through measure equivalence; that is,  $L(\Gamma)$  has this property for any icc group  $\Gamma$  which is merely measure equivalent to a product of  $k$  non-amenable hyperbolic groups. In fact, even more is true of  $L(\Gamma)$ , the consequences of which will be the focus of this talk. (Received September 22, 2017)

1135-47-1547 **Tom-Lukas Kriel\*** (tom-lukas.kriel@uni-konstanz.de). *Absolute extreme points and generators of matrix convex sets.*

We consider a closed matrix convex set  $K$  in a finite number of variables. It be written as the matrix convex hull of a single operator tuple  $L$  on a separable Hilbert space. We explain how the absolute extreme points of  $K$  are encoded in  $L$ . In the case that the closed matrix convex hull of the absolute extreme points equals  $K$ , there is a "minimal" choice of the generator  $L$ . (Received September 23, 2017)

1135-47-1588 **Kathryn McCormick\*** (kathryn-mccormick@uiowa.edu), University of Iowa, 14 MacLean Hall, Iowa City, IA 52246. *Holomorphic Bundles Over Riemann Surfaces and Subalgebras of  $n$ -homogeneous  $C^*$ -algebras.*

Let  $\overline{R}$  denote a finite bordered Riemann surface, and let  $\Gamma_c(\overline{R}, \mathfrak{E}(\overline{R}))$  denote the continuous sections of a flat, matrix  $PU_n(\mathbb{C})$ -bundle over  $\overline{R}$ ,  $\mathfrak{E}(\overline{R})$ . We write  $\Gamma_h(\overline{R}, \mathfrak{E}(\overline{R}))$  for the collection of continuous sections of  $\mathfrak{E}(\overline{R})$  that are holomorphic on the interior of  $\overline{R}$ . This is a subalgebra of the  $C^*$ -algebra of continuous sections,  $\Gamma_c(\overline{R}, \mathfrak{E}(\overline{R}))$ . Our first goal is to calculate the boundary representations of  $\Gamma_c(\overline{R}, \mathfrak{E}(\overline{R}))$  for  $\Gamma_h(\overline{R}, \mathfrak{E}(\overline{R}))$ , in the

sense of Arveson, showing that they are point evaluations on the boundary of  $\overline{R}$ . Our second goal is to show  $\Gamma_h(\overline{R}, \mathfrak{E}(\overline{R}))$  is an Azumaya algebra over its center. Together, these two theorems indicate a sense in which our algebra  $\Gamma_h(\overline{R}, \mathfrak{E}(\overline{R}))$  is a nonselfadjoint generalization of an  $n$ -homogeneous  $C^*$ -algebra. (Received September 23, 2017)

1135-47-1589 **Evans M. Harrell\*** ([harrell@mat.gatech.edu](mailto:harrell@mat.gatech.edu)), School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332-0160, and **Anna V. Maltsev** ([annavmaltsev@gmail.com](mailto:annavmaltsev@gmail.com)), School of Mathematical Sciences, Queen Mary University of London, Mile End Rd, London, E1 4NS, United Kingdom. *Localization of eigenfunctions on quantum graphs.*

I'll discuss ways to construct realistic landscape functions for eigenfunctions  $\psi$  of quantum graphs, i.e., metric graphs where a Schrödinger equation operates on the vertices. The term "landscape functions" refers to functions that are easier to calculate than exact eigenfunctions, but which dominate  $|\psi|$  in a non-uniform pointwise fashion constraining how  $\psi$  can be localized. Our techniques include Sturm-Liouville analysis, a maximum principle, and Agmon's method, and we make connections both to the potential energy function and the topology of the graph.

This is joint work with Anna Maltsev of Queen Mary University. (Received September 23, 2017)

1135-47-1593 **Marat V. Markin\*** ([mmarkin@csufresno.edu](mailto:mmarkin@csufresno.edu)), Department of Mathematics, California State University, Fresno, 5245 North Backer Avenue, M/S PB108, Fresno, CA 93740-8001. *On the Gevrey ultradifferentiability of weak solutions of an abstract evolution equation with a scalar type spectral operator.* Preliminary report.

Found are conditions on a scalar type spectral operator  $A$  in a complex Banach space necessary and sufficient for all weak solutions of the evolution equation

$$y'(t) = Ay(t), \quad t \geq 0,$$

to be strongly Gevrey ultradifferentiable of order  $\beta \geq 1$ , in particular analytic or entire, on  $[0, \infty)$  or  $(0, \infty)$ . Certain inherent smoothness improvement effects are analyzed. It is shown that, if all weak solutions are Gevrey ultradifferentiable of orders  $0 \leq \beta < 1$ , then the operator  $A$  is necessarily bounded. (Received September 23, 2017)

1135-47-1690 **Simon Becker** and **Rui Han\*** ([rhan@ias.edu](mailto:rhan@ias.edu)), 027 Simonyi Hall, School of Math, IAS, Princeton, NJ 08540, and **Svetlana Jitomirskaya**. *Cantor spectrum of graphene in magnetic fields.*

In this talk, we present a full analysis of the spectrum of graphene in magnetic fields with constant flux through every hexagonal comb of the graphene structure. In particular, we provide a rigorous foundation for self-similarity by showing that for irrational flux quanta, the electron spectrum of graphene is a Cantor set. (Received September 24, 2017)

1135-47-1692 **Svetlana Jitomirskaya** and **Fan Yang\*** ([yangf@ias.edu](mailto:yangf@ias.edu)), 211 Fuld Hall, School of Math, IAS, Princeton, NJ 08540. *Spectral transition line in phase for the almost Mathieu operator.*

In this presentation, we will talk about the spectral transition line in phase for the almost Mathieu operator in the positive Lyapunov exponent regime. We show both pure point spectrum and purely singular continuous spectrum occur for dense subsets of phases on this transition line. (Received September 24, 2017)

1135-47-1829 **Meric L. Augat\*** ([mلاغat@ufl.edu](mailto:mلاغat@ufl.edu)). *A noncommutative Ax-Grothendieck theorem.* Preliminary report.

The Ax-Grothendieck theorem tells us that a polynomial  $p : \mathbb{C}^g \rightarrow \mathbb{C}^g$  that is injective, is bijective and has a polynomial inverse.

A free map is a noncommutative function acting on tuples of matrices that respects direct sums and joint similarity. James Pascoe proved that if  $p : M(\mathbb{C})^g \rightarrow M(\mathbb{C})^g$  is an injective free polynomial, then  $p$  is bijective, has a free inverse  $q$  and, for each  $n$  there is a free polynomial  $r_n$  such that  $q(X) = r_n(X)$  for each  $X \in M_n(\mathbb{C})^g$ .

Our goal is to prove a strengthening of the free Ax-Grothendieck theorem; a free polynomial is injective if and only if it has a free polynomial inverse. Polynomial identities and degree bounds on polynomial inverses make the task more difficult than initially expected. We present results and techniques associated with proving this stronger Ax-Grothendieck theorem. (Received September 25, 2017)

1135-47-1869 **M. T. Jury** and **R. T.W. Martin\***, Robert.Martin@uct.ac.za. *Non-commutative Clark measures for the free and abelian Toeplitz algebras.*

In classical Hardy space theory, there is a natural bijection between the Schur class of contractive analytic functions in the complex unit disk and Aleksandrov-Clark measures on the unit circle. A canonical several-variable analogue of Hardy space is the Drury-Arveson space of analytic functions in the unit ball of  $d$ -dimensional complex space. Drury-Arveson space can be naturally identified with symmetric Fock space, and under this identification, the canonical non-commutative or free analogue of several-variable Hardy space is the full Fock space over  $d$ -dimensional complex space.

We will extend the concept of Aleksandrov-Clark (AC) measure, the bijection between the Schur class and AC measures, Clark's unitary perturbations and several associated results to the several-variable settings of the analytic Toeplitz algebras for Drury-Arveson and Fock space (the unital (WOT)-closed operator algebras of the Arveson  $d$ -shift and the left free shift of left creation operators). (Received September 25, 2017)

1135-47-1888 **Victor Vinnikov\*** (vinnikov@math.bgu.ac.il), Department of Mathematics, Ben Gurion University of the Negev, Beer Sheva, Israel. *Noncommutative hyperbolic metrics.* Preliminary report.

We define a pseudometric on noncommutative domains that possesses a noncommutative Schwarz–Pick property: every noncommutative function is a contraction. The pseudometric is defined in purely geometric terms and can be calculated analytically for domains defined by a noncommutative hermitian kernel, in particular for “generalized balls” that appear naturally in the study of interpolation problems and that include all matrix convex sets. We show that under natural conditions (the noncommutative hyperbolic metric is nondegenerate and blows up as we approach the boundary), two noncommutative domains admit a noncommutative bijection iff they are isometric. This talk is based on a joint work (in progress) with Serban Belinschi. (Received September 25, 2017)

1135-47-1995 **Ian Charlesworth** and **Ken Dykema\***, Department of Mathematics, Texas A&M University, College Station, TX 77845, and **Fedor Sukochev** and **Dmitriy Zanin**. *Joint spectral distributions in finite von Neumann algebras.*

We find joint spectral distribution measures for families of commuting elements of a finite von Neumann algebra. These generalize the Brown measure for single operators. In the case of a finite tuple of operators, the support of this joint spectral distribution measure is a subset of the Taylor joint spectrum of the tuple. Furthermore, we find a lattice (based on Borel sets) consisting of hyperinvariant projections that decompose the spectral distribution measure. This leads to simultaneous upper triangularization results for commuting operators. Both of these constructions behave well with respect to multivariate holomorphic functional calculus. (Received September 25, 2017)

1135-47-2001 **Sujan Pant\***, 621 Elmer circle, Reading, PA 19605, and **Rolando De Santiago**. *Primeness results for group von Neumann Algebra.*

We provide a new example of prime von Neumann algebra. We expand upon methods used in earlier work by the first author, and demonstrate that the group von Neumann algebra  $L(\Gamma)$  of a poly-hyperbolic group is a tensor product of  $k$  diffuse  $\text{II}_1$  factors precisely when  $\Gamma$  is commensurable to a direct product of  $k$  poly-hyperbolic groups. We improve on the second author's previous work to provide group-level criteria for determining whether a group von Neumann algebra is prime, i.e.,  $L(\Gamma)$  is prime precisely when the group is (virtually) indecomposable as a direct product of non-amenable groups. (Received September 25, 2017)

1135-47-2194 **Kevser Erdem\***, Department of Mathematics, University of Arkansas, Fayetteville, AR , and **Maria Tjani**. *Closed range composition operator on BMOA.*

Let  $\varphi$  be a holomorphic self-map of the unit disk  $\mathbb{D}$ . We explore conditions under which  $\varphi$  induces a composition operator with closed range on the space  $BMOA$ . The Reverse Carleson condition due to Luecking, and also sampling are important tools. (Received September 26, 2017)

1135-47-2196 **Wonhee Na\*** (wonhee@math.tamu.edu). *Basic examples of bi-free product.*

We study the bi-free product of  $C^*$ -algebras generated by pairs of commuting projections. In particular, we describe this  $C^*$ -algebra and the bi-free product state. We employ and prove some general results analogous to Voiculescu's partial S-transform in the bi-free setting.

This is joint work with Ken Dykema. (Received September 25, 2017)

1135-47-2300 **Casey Lynn Johnson\*** (calyjo99@gmail.com). *Sturm-Liouville problems with eigenvalue-dependent boundary conditions*. Preliminary report.

Sturm-Liouville problems with eigenvalue-dependent boundary conditions originate from self-adjoint expansions of non-densely defined symmetric operators. These self-adjoint expansions can be defined in terms of coupling of the boundary spaces of two symmetric operators. Properties of Nevanlinna-Pick functions are applied to study distribution of eigenvalues. Green's functions are constructed for various examples of the operators coupling. (Received September 25, 2017)

1135-47-2507 **Yuwen Wang\*** (wangyuwen1950@aliyun.com). *Metric Generalized Inverses in Banach Spaces and Their Applications in Bifurcation Theory*.

In this talk, we first introduce the development of the concept of the metric generalized inverse of a linear operator in Banach space. Though the concept of the metric generalized inverse in Banach space mainly comes from the solution of a ill-posed linear operator equation in Banach space, but it also has important applications in the bifurcation theory in nonlinear analysis. By the generalized metric generalized inverses in Banach space, two bifurcation theorems from multiple eigenvalues are given in this talk, and a unified perturbation theory of the generalized metric generalized inverse is also given, which include the classical perturbation theorems of the metric generalized inverse or the bounded linear generalized inverse. (Received September 26, 2017)

1135-47-2536 **Lara Ismert\***, lara.ismert@huskers.unl.edu. *A weakly-defined derivation  $\delta_w^D$  and kernels of  $(\delta_w^D)^n$* .

Given a (possibly unbounded) self-adjoint operator  $D$  acting on a Hilbert space  $H$ , an operator  $x \in B(H)$  is said to be  $n$ -times weakly  $D$ -differentiable if for every  $\xi, \eta \in H$ , the function  $t \mapsto \langle e^{itD} x e^{-itD} \xi, \eta \rangle$  is  $n$ -times continuously differentiable. Christensen proved that this definition is equivalent to requiring that for all  $k = 1, \dots, n$ , we have  $x(\text{dom } D^k) \subseteq \text{dom } D^k$  and the  $k$ -times nested commutator  $[D, \dots, [D, x]]$  is well-defined and bounded on  $\text{dom } D^k$ . If  $x$  satisfies this condition, we denote the unique extension of the  $n^{\text{th}}$  nested commutator  $[D, \dots, [D, x]]$  to all of  $H$  by  $(\delta_w^D)^n(x)$ . Nested commutators of this form play a significant role in quantum mechanics. In particular, perturbational computations can be simplified if any number of these nested commutators vanish. In this talk, we will examine how the multiplicity of  $D$  affects the kernels of powers of the derivation  $\delta_w^D$ . (Received September 26, 2017)

1135-47-2592 **scott mccullough\*** (sam@ufl.edu). *Partial Matrix Convexity*. Preliminary report.

Let  $(s, x)$  denote a  $g + h$  tuple of freely non-commuting variables with  $g$   $x$ -variables and  $h$   $s$ -variables. A polynomial  $p = p(x, s)$ , or more generally a matrix of polynomials, in  $(s, x)$  is a free polynomial. A free polynomial  $p(s, x)$  is naturally evaluated on a  $g + h$  tuple  $(S, X)$  of symmetric matrices of the same size and  $p$  is symmetric if the  $p(S, X)$  is a symmetric matrix for all such  $(S, X)$ . This talk will focus on partial convexity conditions on  $p$  and of the set  $D_p = \{(S, X) : I \succ p(S, X)\}$ . This ongoing work is joint with a number of coauthors, including Sriram Balasubramanian, Harry Dym, Adrian Lim, Damon Hay, Bill Helton, Michael Jury, Igor Klep, and James Pascoe. (Received September 26, 2017)

1135-47-2607 **Dhruba R Adhikari\*** (dadhikar@kennesaw.edu), 1100 S. Marietta Pkwy, MD 9085, Marieta, GA 30060. *Existence Results for Multivalued Operators of Monotone Type in Reflexive Banach Spaces*.

Let  $X$  be a real reflexive Banach space and  $X^*$  its dual space. Let  $T : X \supset D(T) \rightarrow 2^{X^*}$  be an operator of class  $\mathcal{A}_G(S_+)$ , where  $G \subset X$ . A result concerning the existence of pathwise connected sets in the range of  $T$  is established, and as a consequence, an open mapping theorem is proved. In addition, for certain operators  $T$  of class  $\mathcal{B}_G(S_+)$ , the existence of nonzero solutions of  $0 \in Tx$  in  $G_1 \setminus G_2$ , where  $G_1, G_2 \subset X$  satisfy  $0 \in G_2$  and  $\overline{G_2} \subset G_1$ , is established. The Skrypnik's topological degree theory is used, utilizing approximating schemes for operators of classes  $\mathcal{A}_G(S_+)$  and  $\mathcal{B}_G(S_+)$ , along with the methodology of a recent invariance of domain result by Kartsatos and the author. (Received September 26, 2017)

1135-47-2770 **Waleed K. Al-Rawashdeh\***, Montana Tech, 1300 West Park Street, Butte, MT 59701. *Generalized composition operators on Weighted Hilbert Spaces of Analytic Functions*.

Let  $\varphi$  be an analytic self-map of the open unit disk  $\mathbb{D}$  and  $g$  be an analytic function on  $\mathbb{D}$ . The generalized composition operator induced by the maps  $g$  and  $\varphi$  is defined by the integral operator

$$I_{(g, \varphi)} f(z) = \int_0^z f'(\varphi(\zeta)) g(\zeta) d\zeta.$$

Given an admissible weight  $\omega$ , the weighted Hilbert space  $\mathcal{H}_\omega$  consists of all analytic functions  $f$  such that  $\|f\|_{\mathcal{H}_\omega}^2 = |f(0)|^2 + \int_{\mathbb{D}} |f'(z)|^2 \omega(z) dA(z)$  is finite. In this talk, we characterize the boundedness and compactness



of the generalized composition operators on the space  $\mathcal{H}_\omega$  using the  $\omega$ -Carleson measures. Moreover, we give a lower bound for the essential norm of these operators. (Received September 26, 2017)

1135-47-2948 **Rachael M. Norton\*** ([rachael.norton@northwestern.edu](mailto:rachael.norton@northwestern.edu)). *Investigating Pick interpolation in the setting of free noncommutative function theory.*

In a paper from 2016, Ball, Marx, and Vinnikov discuss Pick interpolation theorems in the setting of free noncommutative function theory. A consequence of working in this setting is that the target data in the interpolation problem are forced to belong to the double commutant of the initial data. In this talk, we focus on the appearance (or lack thereof) of this condition in Pick interpolation theorems in the  $W^*$ -correspondence setting. (Received September 26, 2017)

1135-47-3016 **Daniel Drimbe\*** ([ddrimbe@ucsd.edu](mailto:ddrimbe@ucsd.edu)), San Diego, CA.  *$W^*$ -superrigidity for coinduced actions.*

We prove  $W^*$ -superrigidity for a large class of coinduced actions. In particular, we prove that if  $\Sigma$  is an amenable almost-malnormal subgroup of an icc (infinite class conjugacy) property (T) countable group  $\Gamma$ , the coinduced action  $\Gamma \curvearrowright X$  from an arbitrary free action  $\Sigma \curvearrowright X_0$  is  $W^*$ -superrigid. This generalizes the  $W^*$ -superrigidity obtained by Ioana for Bernoulli actions of icc property (T) groups. (Received September 26, 2017)

## 49 ► *Calculus of variations and optimal control; optimization*

1135-49-95 **Ugur G Abdulla, Evan Cosgrove\*** ([ecosgrove2011@my.fit.edu](mailto:ecosgrove2011@my.fit.edu)) and **Jonathan Goldfarb**. *On the Frechet differentiability in optimal control of coefficients in parabolic free boundary problems.*

We consider the inverse Stefan type free boundary problem, where the coefficients, boundary heat flux, and density of the sources are missing and must be found along with the temperature and the free boundary. We pursue an optimal control framework where boundary heat flux, density of sources, and free boundary are components of the control vector. The optimality criteria consists of the minimization of the  $L_2$ -norm deviations of the temperature measurements at the final moment, phase transition temperature, and final position of the free boundary. We prove the Frechet differentiability in Besov-Hölder spaces, and derive the formula for the Frechet differential under minimal regularity assumptions on the data. Necessary condition for optimality opens the way to the application of projective gradient methods in Besov-Hölder spaces for the numerical solution of the inverse Stefan problem. The results are published in *Evolution Equations and Control Theory*, **6**, 3(2017). (Received July 26, 2017)

1135-49-230 **Aden Omar Ahmed\*** ([aden.ahmed@tamuk.edu](mailto:aden.ahmed@tamuk.edu)), 700 University BLVD, MSC 172, Kingsville, TX 78363-8202. *Quantization of Two and Three player Games*. Preliminary report.

We present an effect on classical games that is obtained by replacing the notion of probability distribution with the notions of quantum superposition and measurement. Our particular focus will be on two and three player games where each player has precisely two pure strategic choices. (Received August 14, 2017)

1135-49-341 **Thomas Wunderli\*** ([twunderli@aus.edu](mailto:twunderli@aus.edu)), PO Box 26666, The American University of Sharjah, Sharjah, Sharjah POBox26666, United Arab Emirates. *Further Results for Functionals on BV Space with Carathéodory Integrands*. Preliminary report.

In this paper we provide conditions on  $\varphi(x, p)$  for which the formula

$$\begin{aligned} \int_{\Omega} \varphi(x, Du) &:= \sup_{\phi \in \mathcal{V}} \left\{ - \int_{\Omega} u \div \phi + \varphi^*(x, \phi(x)) dx \right\} \\ &= \int_{\Omega} \varphi(x, \nabla u) dx + \int_{\Omega} \psi(x) \|D^s u\| \end{aligned}$$

holds for each  $u \in BV(\Omega)$ . Here  $\varphi : \Omega \times \mathbf{R}^N \rightarrow \mathbf{R}$ ,  $\Omega \subset \mathbf{R}^N$  open and bounded,  $\varphi$  convex in  $p$  for a.e.  $x$ ,  $\varphi$  satisfies the linear growth assumption  $\varphi(x, p) \leq \psi(x)|p| + c$  for each  $|p| \geq \beta$ , for a.e.  $x$ ,  $\mathcal{V}$  is defined by

$$\mathcal{V} = \left\{ \phi \in C_0^1(\Omega, \mathbf{R}^N) : \phi(x) \leq \psi(x) \text{ for all } x \in \Omega \right\},$$

$\varphi^*$  is the convex dual of  $\varphi$ , and  $\|D^s u\|$  is the singular part of  $Du$ . Importantly, continuity in  $x$  is not assumed. Existence results to certain minimization problems involving  $\int_{\Omega} \varphi(x, Du)$  over BV space then easily follow. (Received September 14, 2017)

1135-49-370 **Carol Ann Downes\*** ([downes@hendrix.edu](mailto:downes@hendrix.edu)), 1600 Washington Avenue, Conway, AR 72032. *A Mass Reducing Flow for Transport Networks.*

An oriented transport network can be modeled by a 1-dimensional chain whose boundary is the difference between the supply and demand distributions, represented by weighted sums of point masses. To accommodate efficiencies of scale into the model, one uses a suitable  $M^\alpha$  norm for transportation cost. One then finds that the minimal cost network has a branching structure since the norm favors higher multiplicity edges, representing shared transport. In this talk, we construct a continuous flow that evolves some initial such network to reduce transport cost without altering its supply and demand distributions. Instead of limiting our scope to transport networks, we construct this  $M^\alpha$  mass reducing flow for real-valued flat chains by finding a real current of locally finite mass with the property that its restrictions are flat chains; the slices of such a restriction dictate the flow. Keeping the boundary fixed, this flow reduces the  $M^\alpha$  mass of the initial chain and is Lipschitz continuous under the flat- $\alpha$  norm. To complete the talk, we apply this flow to transportation networks, showing that the flow indeed evolves branching transport networks to be more cost efficient. (Received August 29, 2017)

1135-49-379 **Ellina Grigorieva\*** ([egrigorieva@twu.edu](mailto:egrigorieva@twu.edu)), PO BOX 425262, Denton, TX 76204, and **Evgenii Khailov**. *Reduction of the operation cost via optimal control of an industrial wastewater biotreatment.*

A model of an industrial wastewater bio-treatment by means of autothermal thermophilic aerobic digestion (ATAD) is created and investigated. The model is described by a nonlinear system of three differential equations with one bounded control, the aeration rate. An optimal control problem of minimizing energy consumption on the given time interval is stated and solved analytically with the use of Pontryagin Maximum Principle. Dependence of the optimal solution on the initial conditions and model parameters are established. Results of this study can be immediately applied to practical ATAD reaction design (Received August 29, 2017)

1135-49-434 **Katy Craig\*** ([kcraig@math.ucscb.edu](mailto:kcraig@math.ucscb.edu)). *Gradient flow in the Wasserstein metric.*

For a range of partial differential equations—including the porous medium equation, the Fokker-Planck equation, and the Keller-Segel equation—solutions of the equations can be characterized as gradient flows with respect to the Wasserstein metric on the space of probability measures. This gradient flow structure lies at the heart of many recent analytic and numerical results, particularly regarding questions of stability, uniqueness, and singular limits.

Gradient flows with respect to Hilbert space norms have a long tradition in partial differential equations, but the geometry of the Wasserstein metric presents new challenges. First, even for probability measures on Euclidean space, the Wasserstein metric it is positively curved in dimensions higher than one. Second, the metric lacks a rigorous Riemannian structure, which one would normally use to make sense of the "gradient" in a "gradient flow". In this talk, I will introduce a time discretization of the gradient flow problem, due to Jordan, Kinderlehrer, and Otto, by which these problems can be overcome and present new results which extend the convergence of the time discrete scheme to a new class of partial differential equations of applied interest. (Received September 02, 2017)

1135-49-1209 **Ana-Maria Croicu\*** ([acroicu@kennesaw.edu](mailto:acroicu@kennesaw.edu)), 1000 Chastain Rd, MD# 1102, Kennesaw, GA 30144. *Outbreak or bio-terrorism? An optimal control model to reduce and eradicate Anthrax disease.*

Anthrax is a fatal infectious disease which can affect animals and humans alike. Anthrax outbreaks occur periodically in animals and they are of particular concern in herbivores, due to substantial economic consequences associated with animal death. The purpose of this study is to develop optimal control interventions that focus on vaccinating susceptible animals and/or removing infected carcasses. Our mathematical goal is to minimize the infectious animal population while reducing the cost of interventions. Optimal control interventions are derived theoretically and numerical results with conclusions are presented. (Received September 20, 2017)

1135-49-1305 **Otmar Scherzer\*** ([otmar.scherzer@univie.ac.at](mailto:otmar.scherzer@univie.ac.at)), Computational Science Center, University of Vienna, 1090 Vienna, Austria, and **Jose Iglesias** and **Gwenael Mercier**. *A note on convergence of solutions of total variation regularized linear inverse problems.* Preliminary report.

The recent paper by Chambolle, A. and Duval, V. and Peyre, G. and Poon, C. , 2017, essentially shows that under a standard smoothness condition, first formulated in Burger and Osher ,2004, the level sets of total variation denoised solutions are converging to the level sets of the true solution. In this talk we show how we are extending this work to general linear inverse problems, such as deblurring.

This is joint work with J. Iglesias and G. Mercier (Received September 21, 2017)

1135-49-1414 **Mila Nikolova\*** (nikolova@cmla.ens-cachan.fr), ENS Cachan, 61 av. du pres. Wilson, 94235 Cachan, France, and **Paumine Tan** (pauline.tan@cmla.ens-cachan.fr), ENS Cachan, 61 av. du pres. Wilson, 94235 Cachan, France. *Alternating structure-adapted proximal gradient for nonconvex regularised problems with multiconvex coupling terms. Applications in imaging inverse problems.* Preliminary report.

There has been an increasing interest in constrained nonconvex regularized block multiconvex optimization problems. We introduce an approach that effectively exploits the multiconvex structure of the coupling term and enables complex application-dependent regularization terms to be used. The proposed Alternating Structure Adapted Proximal gradient descent algorithm enjoys simple well defined updates. Global convergence of the algorithm to a critical point is proved using the so called Kurdyka-Lojasiewicz property. Moreover, we prove that a large class of useful objective functions obeying our assumptions are subanalytic and thus satisfy the Kurdyka-Lojasiewicz property.

Two applications of the algorithm to image sequences acquired with an infrared spectro-imaging device are presented: multiplicative fringe separation and nonuniformity correction with fringe preservation. (Received September 22, 2017)

1135-49-1550 **Alexander J Zaslavski\*** (ajzasl@technion.ac.il), Department of Mathematics, The Technion - Israel institute of Technology, 32000 Haifa, Israel. *Subgradient Algorithms in Convex Minimization with Computational Errors.*

We study the subgradient projection algorithm for minimization of convex and nonsmooth functions, under the presence of computational errors. We show that our algorithms generate a good approximate solution, if computational errors are bounded from above by a small positive constant. Moreover, for a known computational error, we find out what an approximate solution can be obtained and how many iterates one needs for this. (Received September 23, 2017)

1135-49-1564 **Brittany Stephenson\***, bstephenson@math.utk.edu, and **Cristina Lanzas, Suzanne Lenhart and Judy Day.** *Optimal Control of Vaccination in an Epidemiological Model of Clostridium difficile Transmission.*

The spore-forming, gram-negative bacteria *Clostridium difficile* can cause severe intestinal illness. A striking increase in the number of cases of *C. difficile* infection (CDI) among hospitals has highlighted the need to better understand how to prevent the spread of CDI. In our work, we first modify and update a compartmental model of nosocomial *C. difficile* transmission to include vaccination. We then apply optimal control theory to determine the time-varying optimal vaccination rate that minimizes a combination of disease prevalence and spread in the hospital population as well as cost, in terms of time and money, associated with vaccination. Various hospital scenarios are considered, such as times of increased antibiotic prescription rate and times of outbreak, to see how such scenarios modify the optimal vaccination rate. By comparing the values of the objective functional with constant vaccination rates to those with time-varying optimal vaccination rates, we illustrate the benefits of time-varying controls. (Received September 23, 2017)

1135-49-1644 **Kathleen Craig\*** (kcraig@math.ucsb.edu), Santa Barbara, CA 93111. *From slow diffusion to a hard height constraint: characterizing congested aggregation.*

For a range of physical and biological processes—from dynamics of granular media to biological swarming—the evolution of a large number of interacting agents is modeled according to the competing effects of pairwise attraction and (possibly degenerate) diffusion. In the slow diffusion limit, the degenerate diffusion formally becomes a hard height constraint on the density of the population, as arises in models of pedestrian crowd motion. Motivated by these applications, we bring together new results on the Wasserstein gradient flow of nonconvex energies with the theory of free boundaries to study a model of Coulomb interaction with a hard height constraint. Our analysis demonstrates the utility of Wasserstein gradient flow as a tool to construct and approximate solutions, alongside the strength of viscosity solution theory in examining their precise dynamics. By combining these two perspectives, we are able to prove quantitative estimates on convergence to equilibrium, which relates to recent work on asymptotic behavior of the Keller-Segel equation. This is joint work with Inwon Kim and Yao Yao. (Received September 24, 2017)

1135-49-1728 **Cory Wright\*** (cwright11@unl.edu), **Petronela Radu** and **Mikil Foss.** *Existence and regularity of minimizers for nonlocal energy functionals.* Preliminary report.

Over the past two decades there has been a surge of mathematical and engineering interest in problems that model phenomena with potentially discontinuous behavior. In 2000 Silling introduced peridynamics, a nonlocal unified framework, which has successfully captured deformations, the structure of fractures, and propagation of

cracks in solid materials. Motivated by this theory, we consider energy functionals that involve integral operators with weakly singular kernels. I will present results with sufficient conditions associated with the existence of minimizers, the necessity of Euler-Lagrange equations along with demonstrating connections with the classical results. We also investigate regularity of solutions to nonlocal equations. (Received September 24, 2017)

1135-49-1929 **Clemens Kirisits\*** (clemens.kirisits@ricam.oeaw.ac.at) and **Otmar Scherzer**. *Polyconvex regularization for inverse problems.*

Polyconvex integrands give rise to a large class of regularization functionals, which are of particular relevance for inverse problems with deformations as unknowns. However, due to their lack of convexity such functionals are generally not covered by existing regularization theory. In this talk we focus on the question of how to translate convergence rates results from the convex to the polyconvex setting. (Received September 25, 2017)

1135-49-1933 **Hang Nguyen\*** (hangnguyen@wayne.edu), **Boris S. Mordukhovich** and **Ebrahim Sarabi**. *Second-Order Variational Analysis in Second-Order Cone Programming.*

This talk conducts a second-order variational analysis for an important class of nonpolyhedral conic programs generated by the so-called second-order/Lorentz/ice-cream cone  $\mathcal{Q}$ . We first present a precise calculation of the graphical derivative of the normal cone mapping to  $\mathcal{Q}$  under the weakest metric subregularity constraint qualification and. The obtained results seem to be the first in the literature in these directions for nonpolyhedral problems without imposing any nondegeneracy assumptions. We then derive a complete characterization of isolated calmness for perturbed variational systems associated with second-order cone programs as an application of the obtained result.

The talk is based on joint work with Boris S. Mordukhovich (Wayne State University, Detroit, MI) and M. Ebrahim Sarabi (Miami University, Oxford, OH). (Received September 25, 2017)

1135-49-2334 **Isabelle Kemajou-Brown\*** (elisabeth.brown@morgan.edu), 1700 E Cold Spring Lane, Actuarial Science Program, Department of Mathematics, Baltimore, MD 21251, **Olivier Menoukeu-Pamen** (o.menoukeu-pamen@liverpool.ac.uk), Institute for Financial and Actuarial Maths, Department of Mathematics, University of Liverpool, L69 3BX, Liverpool, United Kingdom, and **Zhongyang Sun**, School of Mathematical Sciences, Nankai University, Tianjin, 300071, Peoples Rep of China. *Application of Stochastic Maximum Principle Risk-Sensitive Regime Switching in Asset Management.*

We solve a risk-sensitive benchmarked asset management problem of a firm when the stock is modeled by both a Markov regime-switching diffusion process and an external factor. It is also assumed that the benchmark depends on the economic factor. We use the general maximum principle to find the portfolio strategy that minimizes the risk sensitivity of an investor in such environment. This portfolio is given in a feedback form and depends on the solution to a regime-switching Ricatti equation. (Received September 25, 2017)

1135-49-2347 **Vikash Chaurasia\*** (vikash.chaurasia@oist.jp), Vikash Chaurasia, Mathematics, Mechanics and Materials Unit, 1991-1 Tancha, Onna-son, Onna-Village, Japan, and **Eliot Fried** and **Yi Chao Chen**. *Interacting elastic curves on a sphere.*

Confinement of flexible curves endowed with finite bending stiffness to a surface is pervasive in nature. In the case of multiple curves on the surface, the equilibrium configuration of the curves is governed by short and long range interaction between them. We have derived a generalized, coordinate free variational framework for studying the interaction between two curves that are restricted to a sphere. The factors determining the equilibrium configuration of such system are size of the sphere on which the curves are restricted, bending stiffness of the curves and the strength of their interaction potential. Using the framework, we solve an example in which two curves are uniformly charged. We find the stability of the trivial solution analytically. We also solve the discretized version of the equilibrium equation numerically to obtain the non-trivial configurations. We found 2-fold, 3-fold and higher n-fold symmetric solutions for given length of the curve. The stability of the non-trivial solutions are computed using eigenvalues of the hessian obtained by discretizing the second variation expression at the critical points. Results demonstrate that the self energy of the rods stabilize the trivial solution and has a straightening effect on the shape of non trivial solutions. (Received September 26, 2017)

1135-49-2427 **Emily Chang\*** (echang@smith.edu), **Xu Ding**, **Qiaomei Li**, **Chujun He** and **Yijia Cui**. *Some results on optimizing network flows.*

Many real-world problems can be formulated as network flow optimization problems. One classic example is the transportation problem. We can visualize this problem as a graph with edge weights and a flow of products from supply nodes to demand nodes, where we seek to minimize the transportation costs. We discuss results about

a variant of the transportation problem, where we seek to minimize the sum of the transportation cost and the cost of building the edges themselves. (Received September 26, 2017)

1135-49-2539 **Hyun Jin Son\*** (hyunson@auburn.edu), Auburn University, Department of Mathematics and Statistics, 221 Parker Hall, Auburn, AL 36849. *Optimal Control of vertical transmitted vector-borne disease: Treatment and Prevention*. Preliminary report.

The paper considers a deterministic model for a vertical transmitted vector-borne disease. We look at time dependent prevention and treatment efforts where optimal theory is applied. Using analytical and numerical techniques, we can see that there are cost effective control efforts for treatment for hosts and prevention of host-vector contacts. (Received September 26, 2017)

1135-49-2701 **Nicholas H. Nelsen\*** (nnelsen@okstate.edu), Dept. of Mathematics, Dept. of Mechanical & Aerospace Engineering, Oklahoma State University, Stillwater, OK 74074, **Weiwei Hu**, Dept. of Mathematics, Oklahoma State University, and **Omer San**, Dept. of Mechanical & Aerospace Engineering, Oklahoma State University. *PDE-constrained optimal flow control in cooling applications*. Preliminary report.

We present the optimal control problem of convection-cooling between a hot fluid and cold fluid in two bounded domains separated by a common interface. The aim is to devise an optimal boundary control input that stirs the cold fluid at this interface to minimize the average temperature of the hot fluid. This is of relevance for effective heat transfer in systems with rotating walls or applications involving mixing. We control the cold fluid flow velocity to optimally cool the hot fluid in such a way that maintains a more even distribution of cold fluid temperature for improving circulation. The hot fluid is represented by a low Reynolds number Stokes flow heated on the boundary of its domain and the cold fluid is subjected to Navier slip boundary conditions. We use a gradient descent based iterative scheme to implement our control design. (Received September 26, 2017)

1135-49-3031 **Prem Talwai\*** (pmt55@cornell.edu). *Stochastic Proximal Gradient Methods for Metric Learning*. Preliminary report.

In the past decade, nuclear norm regularization has been frequently applied in multi-task learning to obtain low-rank linear predictors. However, a common challenge involved in these applications is the formulation of a computationally efficient optimization procedure for solving the resulting non-smooth, semidefinite convex programming problem. This paper develops a novel stochastic proximal gradient algorithm that applies a symmetric, low-rank stochastic gradient at each gradient descent step, and thereby eliminates the necessity of an expensive subsequent semidefinite projection. The developed algorithm was applied to two regularized “triplet loss” objective functions to learn distance metrics for measuring facial similarity. The resulting learned metrics were evaluated on the popular Labeled Faces in the Wild (LFW) face recognition database, and demonstrated over 90% accuracy in correctly matching face images of the same identity. (Received September 26, 2017)

1135-49-3179 **Jonathan Goldfarb\*** (jgoldfar@fit.edu) and **Ugur G Abdulla**. *Optimal Control of Coefficients in Parabolic Free Boundary Problems Modeling Laser Ablation*.

We develop a new variational formulation of the inverse Stefan problem, where information on the coefficients, heat flux on the fixed boundary, and density of heat sources are missing and must be found along with the temperature and free boundary. We employ an optimal control framework, where the missing data as well as the free boundary are components of the control vector, and optimality criteria consist of the minimization of the sum of  $L_2$ -norm deviations from the available measurement of the temperature flux on the fixed boundary and available information on the phase transition temperature on the free boundary. This approach allows one to tackle situations when the phase transition temperature is not known explicitly, and is available through measurement with possible error. It also allows for the development of iterative numerical methods of less computational cost due to the fact that for every given control vector, the parabolic PDE is solved in a fixed region instead of full free boundary problem. Discretization by finite differences is pursued, and convergence of the discrete optimal control problems to the original problem both with respect to cost functional and control is proven. (Received September 26, 2017)

## 51 ► Geometry

1135-51-6 **Thomas Brooks\*** (tgbrooks@gmail.com). *3-Manifolds with Constant Ricci Eigenvalues  $\lambda, \lambda, 0$* .

We present a classification of all simply connected, complete Riemannian 3-manifolds whose Ricci eigenvalues are  $\lambda, \lambda, 0$  for a constant  $\lambda$ . This curvature condition is equivalent to assuming that the manifold is curvature

homogeneous and has constant vector curvature 0. By a result of Szabó, this gives a classification of curvature homogeneous semi-symmetric 3-manifolds. (Received May 29, 2017)

1135-51-78 **Sandipan Dutta\*** (d.sandipan@maths.iitkgp.ernet.in), Department of Mathematics, IIT Kharagpur, West medinipur, 721302, India, and **Debapriya Biswas** (priya@maths.iitkgp.ernet.in), Department of Mathematics, IIT Kharagpur, West Medinipur, 721302, India. *Constructing new geometries in the light of the Erlangen program*. Preliminary report.

In this paper, we have considered all the possible subgroups of  $SL(2; \mathbb{R})$  from dimension zero to three especially concentrating upon the one-dimensional subgroups  $A$ ,  $N$  and  $K$ . From those, we have made homogeneous spaces of various dimensions and finally, we define an action of  $SL(2; \mathbb{R})$  on those spaces.

This action made new non-Euclidean geometries with the transformation group  $SL(2; \mathbb{R})$ . We have followed the path of Felix Klein's *Erlangen Program* to define these new types of geometries. (Received July 24, 2017)

1135-51-201 **J Mealy\*** (jmealy@austincollege.edu), Austin College, ste 61560, 900 North Grand Avenue, Sherman, TX 75090, and **Aimi Hardy** (ahardy15@austincollege.edu), Austin College, ste 60727, 900 North Grand Avenue, Sherman, TX 75090. *Asymptotic time-like polygons in staircase metric space-times*.

After a brief discussion of the generality of both the category, staircase metric geometry, and its accompanying methodology, we utilize this framework to construct and investigate a variety of general (1,1) space-time systems. We establish examples of systems featuring asymptotic time-like triangles with varied combinations of asymptotic properties – for example, with future complete sides, but with finite space-time area. ‘Gravitational’ aspects of such systems are also analyzed. (Received August 08, 2017)

1135-51-214 **Eliot Bongiovanni\*** (eliotbonge@gmail.com), **Alejandro Diaz** (diaz5252@gmail.com), **Arjun Kakkar** (ak23@williams.edu) and **Nat Sothanaphan** (natsothanaphan@gmail.com). *The Least-Perimeter Tetrahedral Tile of Space*.

We seek least-perimeter tiles of Euclidean space of unit volume. Despite what Aristotle said, the regular tetrahedron does not tile space. We prove the optimal tetrahedral tile. (Received August 11, 2017)

1135-51-322 **Sergiy Koshkin\*** (koshkins@uhd.edu), 1 Main Street, University of Houston-Downtown, Department of Mathematics and Statistics, Houston, TX 77002. *Imaginary Triangles, Pythagorean Theorems and Algebraic Geometry*. Preliminary report.

We study families of triangles with two commensurable angles. Just like the family of right triangles, which is parametrized by a plane algebraic curve whose implicit equation is given by the Pythagorean theorem, they admit “Pythagorean theorems” of their own. We extend the notion of triangle to “imaginary triangles” with complex valued sides and angles, and apply the theory of plane Cremona transformations to find them recursively. (Received August 23, 2017)

1135-51-452 **Derege H Mussa\*** (dxm146130@utdallas.edu), Derege H.Mussa, Department of Mathematical Science, University of Texas at Dallas, Richardson, TX 75080. *Dual Tetrahedra and Their Relation*. Preliminary report.

Tetrahedron (plural Tetrahedra) is a three dimensional solid having four vertices, four triangular faces and six edges which don't lie in a single plane. If the tetrahedron  $T$  with a six tuple  $S = (a, b, c, d, e, f)$  exists if and only if the tetrahedron is facial and the McCrea determinant is positive. If  $S$  is a six tuple for tetrahedron  $T$ ,  $S = (a, b, c, d, e, f)$  then the faces  $a, b, c$ ;  $a, e, f$ ;  $b, d, f$  and  $c, d, e$  and the edges at the vertices has the pattern  $a, b, f$ ;  $a, c, e$ ;  $b, c, f$  and  $d, e, f$ . If the pattern of faces and vertices of a tetrahedron is interchanged then  $T$  is called the Dual of Tetrahedron  $T$  however these two tetrahedron are not congruent. Nets which are obtained by cutting three edges of the tetrahedron at a vertex of the tetrahedron or along a sequence of three edges that visit each vertex exactly once. The paper discusses: new findings about the Dual of Tetrahedra & their Nets and Self Dual Tetrahedron. (Received September 04, 2017)

1135-51-590 **Johannes C. Familton\*** (jfamilton@bmcc.cuny.edu), 199 Chambers Street, New York,, NY 10007. *A Brief introduction to Clifford Algebras*.

Those who are contributing papers to the special session on quaternions are familiar with quaternions, and/or the applications of quaternions to their work. A few are also familiar with Clifford Algebra. Last year I did a presentation “The Joining of Quaternions with Grassmann algebras: William Kingdon Clifford”. This year I will give a very basic mathematical introduction to the subject of Clifford Algebra (Geometric Algebra as Clifford called it). It is hoped that this talk will aid in filling in a few gaps for those who are not familiar with the subject

and be a bridge between quaternions and Dr. Hestenes work in Geometric Algebra and Calculus. (Received September 09, 2017)

1135-51-610 **Nat Sothanaphan\*** ([natsothanaphan@gmail.com](mailto:natsothanaphan@gmail.com)), 500 Memorial Drive, Cambridge, MA 02139, and **Eilot Bongiovanni, Arjun Kakkar** and **Alejandro Diaz**. *Isoperimetry in Surfaces of Revolution*.

The isoperimetric problem with a density or weighting seeks to enclose prescribed weighted volume with minimum weighted perimeter. According to Chambers' recent proof of the log-convex density conjecture, for many densities on  $\mathbb{R}^n$  the answer is a sphere about the origin. We seek to generalize his results to some other spaces of revolution or to two different densities for volume and perimeter. We provide a new approach to proving circles about the origin isoperimetric. (Received September 10, 2017)

1135-51-780 **Peter Buser, Eran Makover, Bjoern Muetzel\*** ([bjorn.muetzel@gmail.com](mailto:bjorn.muetzel@gmail.com)) and **Robert Silhol**. *The Jacobian variety of Riemann surfaces with short simple closed geodesics*.

To any compact Riemann surface of genus  $g \geq 2$  one may assign a principally polarized abelian variety of dimension  $g$ , the *Jacobian* of the Riemann surface. The Jacobian is a complex torus and we call a Gram matrix of the lattice of a Jacobian a *period Gram matrix*. We give explicit estimates for the entries of the period Gram matrix with respect to a suitable homology basis, if the Riemann surface contains a short simple closed geodesic  $\gamma$  and study this matrix, if the geodesic is pinched. If  $\gamma$  is separating, then the limit surface can be split into two surfaces with a cusp. If  $\gamma$  is non-separating, then the limit surface has two cusps. We furthermore show how certain harmonic forms of these limit surfaces extend to harmonic forms on the compact surfaces which we obtain by adding charts at the cusp points. As a consequence we obtain that certain sub-matrices of the period Gram matrices of the pinched surfaces converge to period Gram matrices of the compactified surfaces. (Received September 14, 2017)

1135-51-835 **Gabriel Merced Casiano\*** ([gabriel.merced@upr.edu](mailto:gabriel.merced@upr.edu)). *One of the easiest proof of the Pythagoras theorem*.

In this talk we provide an original and witty proof of the Pythagoras theorem. We came up to this proof at the very end of an ordinary Euclidean Geometry class, combining properties of cords in circles. Classical results of congruence and similarity of triangles are the only needed. At the moment we could not find this proof. (Received September 15, 2017)

1135-51-1008 **Otis Chodosh\*** ([ochodosh@princeton.edu](mailto:ochodosh@princeton.edu)). *Global uniqueness of large stable CMC and isoperimetric regions in asymptotically flat 3-manifolds*.

I will report on recent work resolving the global uniqueness question for large stable CMC (joint with M. Eichmair) and isoperimetric (joint with M. Eichmair, Y. Shi, and H. Yu) regions in asymptotically flat 3-manifolds. (Received September 18, 2017)

1135-51-1306 **David Dumas, Anna Lenzhen, Kasra Rafi** and **Jing Tao\*** ([jing@ou.edu](mailto:jing@ou.edu)). *Geometry of the Thurston Metric on Teichmüller Space*.

The Thurston metric is an asymmetric metric on Teichmüller Space defined using Lipschitz constants of maps between hyperbolic surfaces. This metric was introduced by Thurston in the late 80's, who showed this metric is geodesic, though geodesics are not necessarily unique, and induced by an asymmetric Finsler norm on tangent space. In this talk, I will survey some recent advances in this field, particular on the coarse geometry of the geodesics in the Thurston metric, and some finer properties in the case of the punctured torus. This talk is based on joint work with David Dumas, Anna Lenzhen, and Kasra Rafi. (Received September 21, 2017)

1135-51-1368 **Osman B Okutan\*** ([okutan.1@osu.edu](mailto:okutan.1@osu.edu)), 100 Math Tower, 231 West 18th Avenue, Columbus, OH 43210, and **Facundo Memoli** ([memoli@math.osu.edu](mailto:memoli@math.osu.edu)). *Approximating metric spaces with Reeb type graphs*.

Every length space can be approximated by graphs under the Gromov-Hausdorff distance. However, as the approximation gets better and better, the graphs can get quite complicated, more precisely their genus (i.e. the first Betti number) can go to infinity. We study how well can we approximate the original space (in the Gromov-Hausdorff sense) if we put restrictions on the genus of the graph. Reeb type constructions produce graphs with bounded genus or even sometimes trees. Here we consider how to approximate length spaces with Reeb type graphs and trees. We also consider how similar type of ideas can be adapted to the finite setting.

In particular, we prove that the Gromov-Hausdorff distance between a compact Riemannian manifold and a Reeb graph of certain functions defined on the manifold can be bounded in terms of the volume, the first Betti

number and a novel invariant that we call *thickness*. We also obtained a tree approximation result for metric graphs where the upper bound can be written in terms of the genus and the hyperbolicity of the original graph. Here we observed a connection to the finite setting through poset theoretic ideas. (Received September 21, 2017)

1135-51-1386 **C. Connell, X. Dai, R. Perales, J. Núñez-Zimbrón, P. Suárez-Serrato and G. Wei\*** ([wei@math.ucsb.edu](mailto:wei@math.ucsb.edu)), Department of Math, UCSB, Santa Barbara, CA 93106.  
*Volume Entropy Rigidity For RCD Spaces.*

Volume entropy measures the exponential growth rate of the volume of balls in the universal cover. It's related to the topological entropy, minimal volume, bottom spectrum of the Laplacian of the universal cover, among others. For Riemannian manifolds  $M^n$  with Ricci curvature  $\geq -(n-1)$ , the volume entropy is  $\leq n-1$ . Ledrappier-Wang showed the rigidity that the equality holds iff  $M^n$  is hyperbolic. We extend this to  $RCD(-(N-1), N)$  spaces. While the upper bound follows quickly, the rigidity case is quite involved due to lack of smooth structure in RCD spaces. (Received September 22, 2017)

1135-51-1426 **Nick Edelen\*** ([nedelen@mit.edu](mailto:nedelen@mit.edu)), **Luca Spolaor** and **Maria Colombo**. *The singular set of minimal surfaces near polyhedral cones.*

We adapt the method of Simon to prove a  $C^{1,\alpha}$ -regularity theorem for minimal varifolds which resemble a cone  $C_0^2$  over an equiangular geodesic net. For varifold classes admitting a “no-hole” condition on the singular set, we additionally establish  $C^{1,\alpha}$ -regularity near the cone  $C_0^2 \times R^m$ . Combined with work of Allard, Simon, Taylor, and Naber-Valtorta, our result implies a  $C^{1,\alpha}$ -structure for the top three strata of minimizing clusters and size-minimizing currents, and a Lipschitz structure on the  $(n-3)$ -stratum. This is joint work with Maria Colombo and Luca Spolaor. (Received September 22, 2017)

1135-51-1872 **Alessandro Sisto** and **Samuel Taylor\*** ([samuel.taylor@temple.edu](mailto:samuel.taylor@temple.edu)). *Largest projections for random walks and shortest geodesics in random mapping tori.*

We show that the largest subsurface projection distance between a marking and its image under the  $n$ th step of a random walk in the mapping class group grows logarithmically in  $n$ , with probability approaching 1 as  $n$  goes to infinity. As an application, we confirm a conjecture of Rivin about the asymptotic behavior of the systole of random mapping tori. (Received September 25, 2017)

1135-51-1907 **Sara Venkatesh\*** ([venkatesh@math.columbia.edu](mailto:venkatesh@math.columbia.edu)). *Rabinowitz Floer homology and mirror symmetry.*

Rabinowitz Floer homology is a useful Floer theory for studying contact hypersurfaces in symplectic manifolds. Using intuition from mirror symmetry, I will illustrate how the Rabinowitz Floer homology of a bounding hypersurface detects elements of the Fukaya category living in the hypersurface. This result can be generalized to symplectic cohomology theories for cobordisms. (Received September 25, 2017)

1135-51-2020 **Catherine Kendall Asaro Cannizzo\*** ([cannizzo@math.berkeley.edu](mailto:cannizzo@math.berkeley.edu)). *Homological mirror symmetry for the genus 2 curve in an abelian variety and its SYZ mirror.*  
Preliminary report.

Motivated by observations in physics, mirror symmetry is the concept that certain manifolds come in pairs  $X$  and  $Y$  such that the complex geometry on  $X$  mirrors the symplectic geometry on  $Y$ . It allows one to deduce information about  $Y$  from known properties of  $X$ . Strominger-Yau-Zaslow (1996) described how such pairs arise geometrically as torus fibrations with the same base and related fibers, known as SYZ mirror symmetry. Kontsevich (1994) conjectured that a complex invariant on  $X$ , the bounded derived category of coherent sheaves, should be equivalent to a symplectic invariant of  $Y$ , the Fukaya category. This is known as homological mirror symmetry. In this project, we first use the construction of SYZ mirrors for hypersurfaces in abelian varieties following Abouzaid-Auroux-Katzarkov (2015), in order to obtain the manifolds  $X$  and  $Y$ . The complex manifold comes from the genus 2 curve as a hypersurface in its Jacobian torus, and we equip the SYZ mirror manifold with a symplectic form. We then describe progress made towards an embedding of the category on the complex side into the Fukaya category. (Received September 25, 2017)

1135-51-2160 **Mehmet Kilic\*** ([kompaktuzay@gmail.com](mailto:kompaktuzay@gmail.com)), 850 Columbia Avenue, Claremont, CA 91711.  
*Tight Span of Subsets of The Plane With The Maximum Metric.*

We prove that a nonempty closed and geodesically convex subset of the  $l_\infty$  plane  $\mathbb{R}_\infty^2$  is hyperconvex and we characterize the tight spans of arbitrary subsets of  $\mathbb{R}_\infty^2$  via this property: Given any nonempty  $X \subseteq \mathbb{R}_\infty^2$ , a closed, geodesically convex and minimal subset  $Y \subseteq \mathbb{R}_\infty^2$  containing  $X$  is isometric to the tight span  $T(X)$  of  $X$ . (Received September 25, 2017)



1135-51-2316 **Jane Wang\*** ([janeyw@mit.edu](mailto:janeyw@mit.edu)). *From Mathematical Billiards to Dilation Surfaces.*

Mathematical billiards is the study of the possible trajectories of a particle that bounces around some domain (e.g. a polygon or an ellipse) with elastic reflections. Many recent discoveries about the dynamics of billiards in polygons have utilized the connection between these billiard tables and geometric structures called translation surfaces, which can be thought of as collections of polygons with sides glued in parallel pairs or as Riemann surfaces with some extra structure. If we relax the structure on these translation surfaces just a bit, we get dilation surfaces. In this talk, we will survey the trajectory that we take to get from mathematical billiards to dilation surfaces, and say a bit about what is known and still unknown about the geometry and dynamics of dilation surfaces. (Received September 25, 2017)

1135-51-2359 **Laney Bowden, Andrea Haynes, Clayton Shonkwiler and Aaron Shukert\*** ([aaron.shukert@colostate.edu](mailto:aaron.shukert@colostate.edu)). *Spherical Geometry and the Least Symmetric Triangle.*

"What is the least symmetric triangle?" This question isn't well-posed, but given a quantitative measure of the asymmetry of a triangle, a reasonable answer is: the triangle which maximizes this measure. In turn, if we have a notion of distance on triangle space, the minimum distance to a symmetric triangle (isosceles or degenerate) provides such a measure of asymmetry, and the triangle with the largest such distance can reasonably be called the least symmetric triangle. We use a construction originally developed to study ring polymers like bacterial DNA to define a distance on the space of triangles. Specifically, this will allow us to identify triangles with points on the unit sphere and then use spherical geometry to explicitly find the point on the sphere which is furthest from the isosceles and degenerate triangles: this point then gives us the least symmetric triangle. (Received September 26, 2017)

1135-51-2586 **Elizabeth Theta Brown, Emily Castner\*** ([castn22e@mtholyoke.edu](mailto:castn22e@mtholyoke.edu)), **Stephen Davis, Edwin O'Shea, Edouard Seryozhenkov and Alfredo Vargas.** *Geometries in the Absence of the Side-Angle-Side Axiom.* Preliminary report.

The story of the parallel postulate and non-Euclidean geometry has been oft-told. Less studied as an axiom of geometry is the Side-Angle-Side rule for triangle congruence. Although SAS appears as a theorem in Euclid's Elements, its proof hinges on an implicit assumption of superimposition. We survey non-SAS geometries and axiom equivalences in the context of Hilbert's modern axiomatization of Euclidean geometry. Our perspective is informed by mathematical logic. Our main result is a novel proof that Euclid's parallel postulate is not equivalent to Playfair's axiom in the non-SAS context. (Received September 26, 2017)

1135-51-2785 **Thomas Crawford\*** ([crawfoth@bc.edu](mailto:crawfoth@bc.edu)). *Free Bicuspid Groups in Small-Cusped Hyperbolic 3-Manifolds.* Preliminary report.

One approach to the study of cusped hyperbolic 3-manifolds is to look at the shape of the cusp itself. Cusps lift to collections of horospheres in  $\mathbb{H}^3$ . By specifying parameters which govern aspects such as the size, shape, and twist of the isometries of a manifold, we can determine the relative size and positioning (known as the packing) of these horospheres. Typically hyperbolic 3-manifolds with low volume exhibit cycles of tangent horospheres known as bracelets (or necklaces). We consider manifolds with cusps that have no such bracelets. Put another way, we consider manifolds  $M$  with the property that  $\pi_1(M)$  has a subgroup  $B \cong \mathbb{Z}^2 * \mathbb{Z}$ , known as a free bicuspid group. Specifically we attempt to determine how small a cusp can be if it has no bracelets. (Received September 26, 2017)

1135-51-3183 **Robert D Knight\*** ([puzzled40@yahoo.com](mailto:puzzled40@yahoo.com)), 101 University Dr, Chillicothe, OH 45601. *"Duality" in Laguerre Planes.* Preliminary report.

Laguerre geometry originated in work by Edmond Laguerre in the late 1800s. Van der Waerden and Smid generalized this geometry by introducing a basic axiomatic system in 1935. Their system is intuitively connected to the representation of a Laguerre plane as an incidence geometry of "circles" and points in a plane. Much of the research into Laguerre planes since has utilized idiosyncrasies of this representation. This approach has been fruitful, but there is a tendency to neglect studying objects not easily visualized within this representation. For example, Laguerre planes contain at least two types of pencils of circles. A parabolic pencil of circles is a maximal set of mutually tangent circles and an elliptic pencil of circles is the set of all circles through two given points. Structures involving several such pencils of circles appear quite complex. We describe a different representation of Laguerre planes, based on one first presented by W. Blaschke circa 1929 in which the principal objects are affine planes and the points and lines of these planes. Parabolic and elliptic pencils of circles from the old system become lines, making configurations of these objects easier to study. (Received September 27, 2017)

## 52 ► *Convex and discrete geometry*

1135-52-321 **Kristin DeSplinter, Satyan L. Devadoss\*** (devadoss@sandiego.edu), **Jordan Readyhough** and **Bryce Wimberly**. *Unfoldings of cubes never overlap*. Preliminary report.

The open problem of constructing a *net* (a connected edge-unfolding without overlap) for every convex polyhedron can be traced back 500 years to Albrecht Durer. We explore nets for higher dimensional polytopes, with an emphasis on cubes. A visual algorithm is developed which outputs geometric unfoldings given a spanning tree. This machinery is used to show that *any* unfolding of an  $n$ -cube is without overlap. We close with a look at unfoldings into chains, with an elegant relationship to integer partitions. (Received August 23, 2017)

1135-52-587 **Alan Arroyo** and **Dan McQuillan\*** (dmcquill@norwich.edu), Norwich University, Department of Mathematics, 158 Harmon Drive, Northfield, VT 05663-1035, and **R. Bruce Richter** and **Gelasio Salazar**. *Convex Drawings of the Complete Graph: Topology meets Geometry*. Preliminary report.

We introduce a notion of convex drawings of complete graphs, naturally leading to a hierarchy of drawings, from restrictive to general: rectilinear, face-convex, hereditarily-convex, convex and general topological. We present general results that shed light on Aichholzer's computer proof (personal communication) showing that all optimal drawings of  $K_n$  with  $n \leq 12$  are also convex drawings. We have distinguishing examples, characterizing theorems and many new and exciting open questions. (Received September 09, 2017)

1135-52-870 **J De Loera\*** (deloera@math.ucdavis.edu), Dept of Mathematics, University of California, Davis, CA 95616, and **Thomas Hogan, Frederic Meunier** and **Nabil Mustafa**. *Radon and Tverberg's theorem with concrete coordinates*. Preliminary report.

Let  $a_1, \dots, a_n$  be points in  $R^d$ . If the number of points  $n$  satisfies  $n > (d+1)(m-1)$ , then they can be always be partitioned into  $m$  disjoint parts  $A_1, \dots, A_m$  in such a way that the  $m$  convex hulls  $\text{conv } A_1, \dots, \text{conv } A_m$  have a point in common. This is Tverberg's theorem. My talk will discuss a fascinating way to interpret Tverberg's theorem, now with a view toward number theory, lattices, integer programming, all things discrete not continuous nor topological:

Given a discrete set  $S$  of  $R^d$  (e.g., a lattice, or the Cartesian product of the prime numbers), we study the minimum number of points of  $S$  needed to guarantee the existence of an  $m$ -partition of the points  $A_1, \dots, A_m$  such that the intersection of the  $m$  convex hulls of the parts contains at least  $k$  points of  $S$ . This is joint work with T. Hogan, F. Meunier, N. Mustafa. (Received September 15, 2017)

1135-52-1102 **Federico Castillo** and **Fu Liu\*** (fuliu@math.ucdavis.edu). *Uniqueness of Berline-Vergne's valuation*.

Berline-Vergne constructs a valuation that assigns values to faces of polytope, and it satisfies what we call "McMullen's formula". There are different solutions to the McMullen's formula. Any solution provides a way to write the coefficients of the Ehrhart polynomial of a polytope as positive sums of these values.

We study the Berline-Vergne's valuation on generic permutohedra, and show that their construction is the unique solution to the McMullen's formula that is symmetric about the coordinates. This is joint work with Federico Castillo. (Received September 19, 2017)

1135-52-1294 **Jesus De Loera, Elisha Peterson** and **Francis Edward Su\*** (su@math.hmc.edu). *A polytopal generalization of Sperner's lemma, and applications*.

Sperner's lemma is a statement about labelled triangulations of simplices whose applications include: finding fixed points of highly nonlinear functions, and computing Nash equilibria in game theory. We prove a generalization of Sperner's lemma for polytopes and demonstrate applications to cutting multiple cakes with linked preferences, triangulations of polytopes, and the game of Hex. A constructive proof of the polytopal Sperner lemma gives methods for computing solutions in applications. (Received September 21, 2017)

1135-52-1552 **Gabriele Balletti\*** (balletti@math.su.se). *Classifications of lattice polytopes*.

In this talk I describe how explicit classifications can be used to understand the behaviour of lattice polytopes in low dimensions and approach open problems in Ehrhart Theory. (Received September 23, 2017)

1135-52-1641 **Adam Jaffe\*** (ajjaffe@stanford.edu), 531 Lasuen Mall, P.O. Box 15878, Stanford, CA 94305, and **Henry Adams** and **Samir Chowdhury**. *Vietoris–Rips Complexes of Regular Polygons*.

Persistent homology has emerged as a novel tool for data analysis in the past two decades. However, there are still very few non-convex shapes or manifolds whose persistent homology barcodes (say of the Vietoris–Rips complex) are fully known. Towards this direction, we provide a near-complete characterization of the homotopy types of Vietoris–Rips complexes of the boundary of any regular polygon in the plane. Indeed, for  $P_n$  the boundary of a regular polygon with  $n$  sides, we describe the homotopy types and persistent homology of the Vietoris–Rips complexes of  $P_n$  up to scale  $r_n$ , where  $r_n$  approaches the diameter of  $P_n$  as  $n \rightarrow \infty$ . Surprisingly, these homotopy types include spheres of all dimensions (as  $n \rightarrow \infty$ ) and wedge-sums thereof. Roughly speaking, the number of  $2\ell$ -dimensional spheres in such a wedge sum is linked to the number of equilateral (but not necessarily equiangular) stars with  $2\ell + 1$  vertices that can be inscribed in  $P_n$ . We furthermore show that the Vietoris–Rips complex of an arbitrarily dense subset of  $P_n$  need not be homotopy equivalent to the Vietoris–Rips complex of  $P_n$  itself. As our main tool, we employ the recently-developed theory of cyclic graphs and winding fractions. (Received September 24, 2017)

1135-52-1984 **Maryam Farahmand\***, Mfarahmand@math.berkeley.edu. *Partially Magic Labelings and the Antimagic Graph Conjecture*.

The *Antimagic Graph Conjecture* asserts that every connected graph  $G = (V, E)$  except  $K_2$  admits an edge labeling such that each label  $1, 2, \dots, |E|$  is used exactly once and the sums of the labels of the edges incident to each vertex are distinct. On the other extreme, an edge labeling is *magic* if the sums of the labels on all edges incident to each vertex are the same. In this paper we approach antimagic labelings by introducing *partially magic labelings*, where “magic occurs” just in a subset of  $V$ . We generalize Stanley’s theorem about the magic graph labeling counting function to the associated counting function of partially magic labelings and prove that it is a quasi-polynomial of period at most 2. This allows us to introduce *relaxed antimagic labelings* (for which label repetition is allowed), and we show that every bipartite graph satisfies a weakened version of the Antimagic Graph Conjecture. (Received September 25, 2017)

1135-52-2010 **Moshe Cohen\*** (mcohen@vassar.edu) and **Baian Liu**. *Finding line arrangements with interesting realization spaces*. Preliminary report.

When a line is added in a relatively trivial way to a line arrangement whose realization space has just one connected component, the resulting line arrangement also has a realization space of just one component by a result of Nazir and Yoshinaga. We use this tool and others to search for arrangements with interesting realization spaces. Together with Amram, Teicher, and Ye, the first author found a complete list for arrangements of ten lines. In recent work, we use similar techniques to study arrangements of eleven lines. (Received September 25, 2017)

1135-52-2415 **Anthony W Harrison\*** (aharri60@kent.edu). *Computing the lattice size*.

Let  $P$  be a lattice polygon. The lattice size of  $P$  with respect to the standard 2-simplex  $\Sigma$ , denoted  $ls_\Sigma(P)$ , is the smallest number  $\ell$  such that the image of  $P$  under an affine unimodular transformation is contained within the  $\ell$ -dilate of  $\Sigma$ . This parameter can be used to answer questions about toric varieties. Castryck, Cools, and Schicho showed that there is a recursive algorithm to determine the lattice size by relating  $ls(P)$  to the lattice size of the convex hull of the interior lattice points of  $P$ . We have developed a new algorithm that needs only the vertices of  $P$  and so avoids the computational expense of determining the interior lattice points. We show that if a fixed, finite set of transformations does not yield a “smaller” image of  $P$ , then a translate of  $P$  fits in the smallest possible dilate of  $\Sigma$ . (Received September 26, 2017)

1135-52-2605 **Elizabeth Marie Buchanan\*** (embuchanan@smith.edu), **Corrie Elizabeth Ingall**, **Alex K Perry**, **Rebecca Katherine Rohrlach** and **Ileana Streinu**, Smith College, Northampton, MA 01063. *Rigidity and the Pebble Game algorithm in 3D*. Preliminary report.

The Pebble Game algorithm of Hendrickson and Jacobs completely characterizes rigid frameworks in 2D. We present our investigations of classes of graphs recognized by generalized pebble game algorithms and their connections with 3D rigidity. (Received September 26, 2017)

## 53 ► *Differential geometry*

1135-53-46      **Luis Guijarro** and **Fred Wilhelm\*** ([fred@math.ucr.edu](mailto:fred@math.ucr.edu)), Department of Mathematics, Univ. of Calif., Riverside, CA 92521. *A Focal Radius Finiteness Theorem.*

I will discuss the proof of the following optimal finiteness theorem that is joint work with Luis Guijarro.

**Theorem:** *Let  $M$  be a compact Riemannian manifold. Given  $D, r > 0$  the class  $S$  of closed Riemannian manifolds that can be isometrically embedded into  $M$  with focal radius  $\geq r$  and intrinsic diameter  $\leq D$  is pre-compact in the  $C^{1,\alpha}$ -topology. In particular,  $S$  contains only finitely many diffeomorphism types.* (Received July 02, 2017)

1135-53-51      **Scott Van Thuong\*** ([sthuong@pittstate.edu](mailto:sthuong@pittstate.edu)), Pittsburg State University, Department of Mathematics, 1701 S Broadway, Pittsburg, KS 66762. *Metrics on 4-dimensional unimodular Lie groups.*

We classify left invariant metrics on the 4-dimensional, simply connected, unimodular Lie groups up to automorphism. When the corresponding Lie algebra is of type (R), this is equivalent to classifying the left invariant metrics up to isometry, but in general the classification up to automorphism is finer than that up to isometry. In the abelian case, all left invariant metrics are isometric. In the nilpotent case, the space of metrics can have dimension 1 or 3. In the solvable case, the dimension can be 2, 4, or 5. There are two non-solvable 4-dimensional unimodular groups, and the space of metrics has dimension 6 in both of these cases. (Received July 07, 2017)

1135-53-54      **Lee Kennard\*** ([kennard@ou.edu](mailto:kennard@ou.edu)) and **Burkhard Wilking.** *Positive curvature and torus symmetry.*

It is an open problem whether every simply connected, closed manifold admitting non-negative sectional curvature also admits positive sectional curvature. One conjectured obstruction is due to Hopf: In even dimensions, positive sectional curvature implies positive Euler characteristic. I will discuss joint work with Burkhard Wilking on this problem in the presence of torus symmetry. (Received July 10, 2017)

1135-53-101      **Ryad Ghanam\*** ([raghanam@vcu.edu](mailto:raghanam@vcu.edu)) and **Gerard Thompson.** *Minimal representations of Lie algebras with non-trivial Levi decomposition.*

We obtain minimal dimension matrix representations for each of the Lie algebras of dimensions five, six, seven and eight obtained by Turkowski that have a non-trivial Levi decomposition. We use the invariant subspaces associated to a particular representation of a semi-simple Lie algebra to help in the construction of the radical in the Levi decomposition. (Received July 26, 2017)

1135-53-142      **Brian Allen\*** ([brian.allen@usma.edu](mailto:brian.allen@usma.edu)). *IMCF and the Stability of the PMT and RPI.*

We will discuss the stability, under  $L^2$  metric convergence, of the Positive Mass Theorem (PMT) and the Riemannian Penrose Inequality (RPI) in the case where a region of an asymptotically flat manifold  $M^3$  can be foliated by a smooth solution of Inverse Mean Curvature Flow (IMCF) which is uniformly controlled. (Received August 03, 2017)

1135-53-293      **Ovidiu Munteanu\***, Department of Mathematics, University of Connecticut, 341 Mansfield Road U1009, Storrs, CT 06269. *The Poisson equation on complete manifolds.*

I will present recent results, joint with Chiung-Jue Sung and Jiaping Wang, about existence and estimates of solutions to the Poisson equation on complete manifolds with positive bottom spectrum. In the process, we develop new sharp estimates for the heat kernel and Green's function on such manifolds. (Received August 20, 2017)

1135-53-350      **Garri Davydyan\*** ([garri.davyddyan@gmail.com](mailto:garri.davyddyan@gmail.com)), 213-224 Viewmount Dr., Nepean, Ontario K2E 0B4, Canada. *Carcinogenesis: does indefinite metric of a split-quaternion give a clue?* Preliminary report.

Previously it was assumed that a structure of biologic objects can be described by split-quaternions whose vector part represents basis functional mechanisms of biologic systems (negative feedback, positive feedback and reciprocal links). Split-quaternions with the defined basis have a metric signature  $(- - + +)$  that determines normal systemogenesis. Each hierarchical level of the system is equipped with the indefinite metric inherited from the previous level during maturation. If an individual living cycle does not support the metric structure, it causes a pathologic regulation between hierarchical levels. In the frames of this concept, transformation of the indefinite metric to the positive definite one is a major systemic mechanism for cancer development. (Received August 27, 2017)

1135-53-392 **Justin Sawon\*** ([sawon@email.unc.edu](mailto:sawon@email.unc.edu)), Department of Mathematics, University of North Carolina, Chapel Hill, NC 27510. *Quaternionic manifolds.*

Complex manifolds can be described as open subsets of  $\mathbb{C}^n$ , patched together with biholomorphic maps. But quaternion-analytic maps are too limited to define quaternionic manifolds in a similar way. Instead, one defines an almost quaternionic structure on the tangent bundle, and then imposes one of several possible integrability conditions. This yields “quaternionic manifolds” and “hypercomplex manifolds”. One can also add compatible Riemannian metrics, giving “quaternion-Kähler manifolds” and “hyperkähler manifolds”. In this talk we will describe these different constructions and then survey some of the main facts about the resulting manifolds. (Received August 30, 2017)

1135-53-520 **Patrick B. Eberlein\*** ([pbe@ad.unc.edu](mailto:pbe@ad.unc.edu)), University of North Carolina, Department of Mathematics, Chapel Hill, NC 27599. *Central conjugate locus of 2-step nilpotent Lie groups.*

Our goals are twofold : 1) to compute the conjugate locus of a geodesic that lies in the center of a simply connected, 2-step nilpotent Lie group  $N$  with a left invariant metric 2) compare the isometry types of two such nilpotent Lie groups  $N_1, N_2$  whose conjugate loci for central geodesics are “the same” in a suitable sense. The first goal is achieved. The second goal is elusive, and we obtain a partial result.

Let  $Z$  be an element of the center of a 2-step nilpotent Lie algebra  $\mathfrak{N}$  equipped with an arbitrary inner product  $\langle \cdot, \cdot \rangle$ . Let  $N$  be the simply connected Lie group with the left invariant metric determined by  $\langle \cdot, \cdot \rangle$ . Then  $\gamma_Z(t) = \exp(tZ)$  is a geodesic of  $N$  contained in the center of  $N$ , and all central geodesics of  $N$  starting at the identity arise in this fashion. The conjugate locus of  $\gamma_Z$  is computed explicitly from the eigenvalues of the self adjoint curvature operator  $R_Z : \mathfrak{N} \rightarrow \mathfrak{N}$  given by  $R_Z(X) = R(Z, X)Z$ . Conversely, in “almost every” case the conjugate locus of  $\gamma_Z$  determines the eigenvalues of  $R_Z$ . (Received September 06, 2017)

1135-53-521 **Christine Escher\*** ([tine@math.orst.edu](mailto:tine@math.orst.edu)), Oregon State University, and **Catherine Searle**, Wichita State University. *Non-negative curvature and torus actions.*

The classification of Riemannian manifolds with positive and non-negative sectional curvature is a long-standing problem in Riemannian geometry. In this talk I will summarize recent joint work with Catherine Searle on the classification of closed, simply-connected, non-negatively curved Riemannian manifolds admitting an isometric, effective, maximal torus action. This classification has many applications, in particular the Maximal Symmetry Rank conjecture holds for this class of manifolds. (Received September 06, 2017)

1135-53-544 **Sherry Gong\*** ([sgongli@mit.edu](mailto:sgongli@mit.edu)). *Marked link invariants: Khovanov, instanton, and binary dihedral invariants for marked links.*

We introduce a version of Khovanov homology for alternating links with marking data,  $\omega$ , inspired by instanton theory. We show that the analogue of the spectral sequence from Khovanov homology to singular instanton homology (Kronheimer and Mrowka, *Khovanov homology is an unknot-detector*) collapses on the  $E_2$  page for alternating links. We moreover show that the Khovanov homology we introduce for alternating links does not depend on  $\omega$ ; thus, the instanton homology also does not depend on  $\omega$  for alternating links.

Finally, we study a version of binary dihedral representations for links with markings, and show that for links of non-zero determinant, this also does not depend on  $\omega$ . (Received September 07, 2017)

1135-53-604 **Thomas Murphy\*** ([tmurphy@fullerton.edu](mailto:tmurphy@fullerton.edu)), Dept. Mathematics, CSU Fullerton, McCarthy Hall, 800 N. State College Blvd, Fullerton, CA 92831. *Complex Riemannian foliations of Kaehler manifolds.*

For many natural problems arising in Riemannian geometry, the Kaehler setting is restrictive enough to allow concrete classification results. In this vein I will outline joint work with Paul-Andi Nagy classifying complex Riemannian foliations of any open subset of a Hermitian symmetric space of compact type. This has applications to the study of quaternionic-Kähler metrics with restricted sectional curvatures. General results restricting such foliations on any Kaehler manifold are also derived. (Received September 10, 2017)

1135-53-605 **Thomas Murphy\*** ([tmurphy@fullerton.edu](mailto:tmurphy@fullerton.edu)), CSU Fullerton, Dept. Mathematics, 800 N. State College Blvd, Fullerton, CA 92831. *Bounding the invariant spectrum of toric Kaehler manifolds.*

We generalise a theorem of Engman and Abreu-Freitas which bounds the first invariant eigenvalue of a non-negatively curved  $T^1$ -invariant metric on  $S^2$  to toric Kaehler metrics with non-negative scalar curvature. Bounds for all higher eigenvalues are determined: these can be combinatorially determined from the Delzant Polytope.

Similar results in the extremal case are also derived. This is joint work with Stuart Hall. (Received September 10, 2017)

1135-53-607 **Shengwen Wang\*** ([swang@math.jhu.edu](mailto:swang@math.jhu.edu)). *Hausdorff stability of round spheres under small-entropy perturbation.*

Colding-Minicozzi introduced the entropy functional on the space of all hypersurfaces in the Euclidean space when studying generic singularities of mean curvature flow. It is a measure of complexity of hypersurfaces. Bernstein-Wang proved that round spheres  $S^n$  minimize entropy among all closed hypersurfaces for  $n \leq 6$ , and the result is generalized to all dimensions by Zhu. Bernstein-Wang later also proved that the round 2-sphere is actually Hausdorff stable under small-entropy perturbations. I will present in this talk the generalization of the Hausdorff stability to round hyper-spheres in all dimensions. (Received September 18, 2017)

1135-53-652 **Jinjin Liang\*** ([jliang@math.arizona.edu](mailto:jliang@math.arizona.edu)), 617 N. Santa Rita Ave., Tucson, AZ 85721. *Asymptotic behavior of  $\beta$ -polygon flows.* Preliminary report.

We investigate a family of nonlinear evolutions of polygons in the plane called the  $\beta$ -polygon flow and obtain some results analogous to results for the smooth curve shortening flow: (1) any planar polygon shrinks to a point and (2) a regular polygon with five or more vertices is asymptotically stable in the sense that nearby polygons shrink to points that rescale to a regular polygon. In dimension four we show that the shape of a square is locally stable under perturbations along a hypersurface of all possible perturbations. Furthermore, we are able to show that under a lower bound on angles there exists a rescaled sequence extracted from the evolution that converges to a limiting polygon that is a self-similar solution of the flow. The last result uses a monotonicity formula analogous to Huisken's for the curve shortening flow. (Received September 12, 2017)

1135-53-785 **Theodora Bourni\*** ([tbourni@utk.edu](mailto:tbourni@utk.edu)), **Mat L Langford** ([langford@math.utk.edu](mailto:langford@math.utk.edu)) and **Giuseppe Tinaglia** ([giuseppe.tinaglia@kcl.ac.uk](mailto:giuseppe.tinaglia@kcl.ac.uk)). *Collapsing ancient solutions of mean curvature flow.*

We construct a compact, convex, rotationally symmetric, ancient solution of mean curvature flow that lies in a slab of width  $\pi$  and show that, up to rigid motions, it is the only such solution (which lies in no smaller slab). (Received September 14, 2017)

1135-53-794 **Jonathan J Zhu\*** ([jjzhu@math.harvard.edu](mailto:jjzhu@math.harvard.edu)), 1 Oxford Street, Cambridge, MA 02138. *Min-max theory for constant mean curvature hypersurfaces.*

We describe the construction of closed constant mean curvature (CMC) hypersurfaces using min-max methods. In particular, our theory allows us to show the existence of closed CMC hypersurfaces of any prescribed mean curvature in any closed Riemannian manifold. This work is joint with Xin Zhou. (Received September 14, 2017)

1135-53-886 **Laura Starkston\*** ([l1starkst@stanford.edu](mailto:l1starkst@stanford.edu)). *Arboreal Skeletons.*

We will talk about how to encode the symplectic geometry of a  $2n$ -dimensional Weinstein manifold in an  $n$ -dimensional core subcomplex called the skeleton. The skeleton generally has complicated singularities, but we will discuss how we can work with representative skeleta with only a nice combinatorial and topological class of singularities aligning with Nadler's arboreal singularities. Then we will discuss how to compare Weinstein manifolds like cotangent bundles using arboreal skeleta. (Received September 16, 2017)

1135-53-894 **Lihan Wang\*** ([lihan.wang@uconn.edu](mailto:lihan.wang@uconn.edu)). *Some Laplacians on symplectic manifolds.*

We introduce some Laplacians, different from the usual ones, on symplectic manifolds. These Laplacians includes 2nd order operators and 4th order operators on differential forms. They have close relations with certain types of cohomology. . (Received September 16, 2017)

1135-53-905 **James Isenberg\*** ([isenberg@uoregon.edu](mailto:isenberg@uoregon.edu)), Dept of Math, University of Oregon, Eugene, OR 97403, and **Dan Knopf** ([danknopf@math.utexas.edu](mailto:danknopf@math.utexas.edu)) and **Natasa Sesum** ([natasa.sesum@gmail.com](mailto:natasa.sesum@gmail.com)). *Non-Kaehler Ricci Flows that Converge to Kaehler-Ricci Solitons.*

We consider a family of Riemannian(non-koehler) Ricci flow solutions which develop finite-time (type I) singularities such that parabolic rescalings at the singularities take the form of shrinking Kaehler-Ricci solitons. In particular, the singularity models for these are the "blowdown soliton" studied by Feldman-Ilmanen-Knopf. Our results support the conjecture that the blowdown soliton is stable under Ricci flow. As well, our work provides the first set of rigorous examples of non-koehler Ricci flow solutions which become asymptotically choler in suitable neighborhoods of the developing singularities. (Received September 16, 2017)

1135-53-906 **Shoo Seto\*** ([shoseto@ucsb.edu](mailto:shoseto@ucsb.edu)). *The first eigenvalue of the  $p$ -Laplacian with integral curvature condition.*

The  $p$ -Laplacian is a generalization of the usual Laplacian by minimizing the  $L^p$  energy functional instead of the  $L^2$  energy. While the regularity of solutions to the  $p$ -Laplace eigenvalue equation is different for  $p \neq 2$ , its first nonzero eigenvalue admits generalizations. In this talk, we will discuss various sharp estimates of the first nonzero eigenvalue for the  $p$ -Laplacian with integral curvature condition. In particular, we generalize the Cheng upper bound, the Lichnerowicz-Obata lower bound, and the Faber-Krahn isoperimetric inequality. This is based off joint work with Guofang Wei. (Received September 16, 2017)

1135-53-947 **Rachelle DeCoste\*** ([decoste\\_rachelle@wheatoncollege.edu](mailto:decoste_rachelle@wheatoncollege.edu)), 26 East Main Street, Norton, MA 02766, and **Lisa DeMeyer**, **Meera Mainkar** and **Allie Ray**. *Properties of 2-step nilpotent Lie groups associated with graphs.*

In 2005 Dani and Mainkar introduced 2-step nilpotent Lie algebras associated to simple directed graphs. Ray introduced a construction of 2-step nilpotent Lie groups associated to non-simple directed graphs in 2015. Recent joint work has focused on studying properties of metric Lie algebras associated to graphs, including singularity and the presence of a Euclidean de Rham factor. We explore differences in the two constructions and discuss complications that arise when multigraphs are allowed. (Received September 17, 2017)

1135-53-994 **Anna M Siffert\*** ([siffert@mpim-bonn.mpg-de](mailto:siffert@mpim-bonn.mpg-de)), MPIM Bonn, Vivatsgasse 7, 53111 Bonn, Germany, and **Henrik Matthiesen**. *Existence of metrics maximizing the first eigenvalue on closed surfaces.* Preliminary report.

We prove that for closed surfaces of fixed topological type, orientable or non-orientable, there exists a unit volume metric, smooth away from finitely many conical singularities, that maximizes the first eigenvalue of the Laplace operator among all unit volume metrics. The key ingredient are several monotonicity results, which have partially been conjectured to hold before. This is joint work with Henrik Matthiesen. (Received September 18, 2017)

1135-53-995 **Anna Siffert\*** ([siffert@mpim-bonn.mpg-de](mailto:siffert@mpim-bonn.mpg-de)), Vivatsgasse 7, 53111 Bonn, Germany. *Harmonic maps between manifolds with symmetry.* Preliminary report.

Harmonic maps are critical points of the energy functional. In the last century, Eells and Sampson, started the study of the following very important question:

Does every homotopy class of maps between Riemannian manifolds admit a harmonic representative?

For the special case that the target manifold is compact and all its sectional curvatures are nonnegative, Eells and Sampson answered this question in affirmative. However, for the case that the target manifold also admits positive sectional curvatures the answer to this question is still only known in special cases. In my talk I explain the construction of infinitely many new harmonic maps between Riemannian manifolds with symmetry. (Received September 18, 2017)

1135-53-1003 **Chen-Yun Lin\***, [cylin@math.duke.edu](mailto:cylin@math.duke.edu), and **Hau-Tieng Wu**. *An embedding theorem: differential geometry behind massive data analysis.*

High-dimensional data can be difficult to analyze. Assume data are distributed on a low-dimensional manifold. The Vector Diffusion Mapping (VDM), introduced by Singer-Wu, is a non-linear dimension reduction technique and is shown robust to noise. It has applications in cryo-electron microscopy and image denoising and has potential application in time-frequency analysis.

In this talk, I will present a theoretical analysis of the effectiveness of the VDM. Specifically, I will discuss parametrisation of the manifold and an embedding which is equivalent to the truncated VDM. In the differential geometry language, I use eigen-vector fields of the connection Laplacian operator to construct local coordinate charts that depend only on geometric properties of the manifold. Next, I use the coordinate charts to embed the entire manifold into a finite-dimensional Euclidean space. The proof of the results relies on solving the elliptic system and provide estimates for eigenvector fields and the heat kernel and their gradients. (Received September 18, 2017)

1135-53-1046 **Curtis Pro\*** ([cpro@csustan.edu](mailto:cpro@csustan.edu)) and **Fred Wilhelm** ([fred@math.ucr.edu](mailto:fred@math.ucr.edu)). *Stability, Finiteness, and Dimension Four.*

In 1970, Cheeger showed that the class of  $n$ -dimensional Riemannian manifolds with fixed two-sided sectional curvature bounds, upper diameter bound, and positive lower volume bound contains, at most, finitely many diffeomorphism types. Subsequently, Grove, Petersen, Wu, and, independently, Perelman showed that, if  $n \neq 4$ , the same can be claimed without assuming a uniform upper bound on sectional curvature. Perelman's argument

uses his Stability Theorem, which, in this setting, says: for any Gromov-Hausdorff converging sequence of  $n$ -manifolds all having a uniform lower bound on sectional curvature, upper bound on diameter, and positive lower bound on volume, almost all manifolds in this sequence must be homeomorphic. Combining this with Gromov's compactness theorem and, if  $n \neq 4$ , a topological result by Kirby and Siebenmann, diffeomorphism finiteness in this larger class follows. Perelman's argument begs the question: are almost all manifolds in this sequence also diffeomorphic? In this talk, I will outline the major points in joint work with Fred Wilhelm that show when  $n = 4$ , the answer is yes. In particular, Grove, Petersen, Wu/ Perelman's result also holds when  $n = 4$ , completing the generalization of Cheeger's theorem. (Received September 18, 2017)

1135-53-1049     **Andrew Lee\*** (alee150@ucsc.edu) and **Timothy Perutz**. *An Invariant of Mapping Tori from Moduli of Stable Pairs*.

In this talk we use a construction of a moduli space of stable pairs over a Riemann surface to produce a Floer-theoretic invariant of a mapping torus for a surface diffeomorphism (together with a choice of line bundle on it). We then describe some consequences and how this construction relates to other 3-manifold invariants. This is joint work with Tim Perutz. (Received September 18, 2017)

1135-53-1084     **Hiroshi Tamaru\*** (tamaru@hiroshima-u.ac.jp), Department of Mathematics, Hiroshima University, Higashi-Hiroshima, Hiroshima 739-8526, Japan. *Left-invariant pseudo-Riemannian metrics on some solvable Lie groups*.

For a given Lie group, the set of all left-invariant pseudo-Riemannian metrics with a fixed signature can be identified with some pseudo-Riemannian symmetric space. We are studying left-invariant pseudo-Riemannian metrics, in terms of the action of the automorphism group of the corresponding Lie algebra on this symmetric space. In this talk, we illustrate this framework, and apply it to some particular solvable Lie groups, e.g., the solvable Lie group of the real hyperbolic space. (Received September 19, 2017)

1135-53-1158     **Yongjia Zhang\*** (yoz020@ucsd.edu), Department of Mathematics, University of California, San Diego, San Diego, CA 92093-0112. *On the equivalence between bounded entropy and noncollapsing for ancient solutions to the Ricci flow*.

In Perelman's celebrated paper *the entropy formula for the Ricci flow and its geometric applications* he asserted that for an ancient solution to the Ricci flow with nonnegative curvature operator, bounded entropy is equivalent to  $\kappa$ -noncollapsing on all scales. We prove this assertion with an additional assumption on one time slice of bounded geometry, that is, that the curvature is bounded from above and the volume of unit balls is bounded from below. (Received September 20, 2017)

1135-53-1166     **Jacob A Gross, Daniel R Licata, Max S New, Jennifer Paykin, Mitchell Riley, Michael Shulman and Felix Wellen\*** (felix.wellen@posteo.de). *Differential Cohesive Type Theory*. Preliminary report.

As internal languages of toposes, type theories allow mathematicians to reason *synthetically* about mathematical structures in a concise, natural, and computer-checkable way. The basic system of homotopy type theory provides an internal language for  $(\infty, 1)$ -toposes, and has enabled a significant line of work on synthetic homotopy theory. By introducing modalities into homotopy type theory, a synthetic treatment of geometric and topological aspects of objects of interest in pure mathematics seems to be possible.

We aim at constructing the rules for an extension of homotopy type theory providing all six operations of what are called differential cohesive toposes. Half of the operations of this structure are comonadic and there is no hope of adding those to the type theory as a list of axioms. With a mode theory in adjoint type theory, we give a basis for a solution with nice specialized rules and five different sorts of context. The rules of this differential cohesive type theory manipulate these contexts in subtle ways, which express the relationships between the modalities.

In work in progress, we are extending this type theory to dependent types, which is not straightforward because of the dependency structure of the differential cohesive modalities. (Received September 20, 2017)

1135-53-1167     **Jonas Deré\*** (jonas.dere@kuleuven.be). *Ricci negative solvmanifolds and their nilradicals*.

In the homogeneous case, the only curvature behavior which is far from understood is negative Ricci curvature. In this talk, we will focus on recent progress in the special case of solvable Lie algebras. We will give several examples which indicate that it is a hard job to describe the nilradicals of solvable Lie algebras admitting a Ricci negative metric. Also, we give a characterization of derivations on a nilpotent Lie algebra which give rise to a solvable extension which is Ricci negative. This is joint work with Jorge Lauret (FaMAF - Universidad Nacional de Córdoba). (Received September 20, 2017)



1135-53-1212 **Xiaodong Wang\*** ([xwang@math.msu.edu](mailto:xwang@math.msu.edu)), Well Hall D202, 619 Red Cedar RD, East Lansing, MI 48824. *On the size of compact Riemannian manifolds with nonnegative Ricci curvature and convex boundary.*

Given a compact Riemannian manifold with nonnegative Ricci curvature and convex boundary it is interesting to estimate its size in terms of the volume, the area of its boundary etc. I will discuss some open problems and present some partial results. (Received September 20, 2017)

1135-53-1264 **Christine Escher and Catherine Searle\*** ([searle@math.wichita.edu](mailto:searle@math.wichita.edu)), Dept of Mathematics, Statistics, and Physics, 1845 Fairmount, Wichita State University, Wichita, KS 67260. *Non-negatively curved 6-manifolds with almost maximal symmetry rank.*

We classify closed, simply-connected, non-negatively curved 6-manifolds of almost maximal symmetry rank up to equivariant diffeomorphism. (Received September 20, 2017)

1135-53-1280 **Chao Li\*** ([rchlch@stanford.edu](mailto:rchlch@stanford.edu)), 380 Serra Street, Sloan Hall, Stanford University, Stanford, CA 94305. *Positive scalar curvature in singular spaces.*

Motivated by the study of positive scalar curvature (PSC) on singular Riemannian manifolds, we show that edge singularities along codimension-2 submanifolds do not affect its Yamabe type, as long as the cone angles are all no larger than  $2\pi$ . In three dimensions, we show the same for edge-singularities along 1-skeletons and, in particular, we show that uniformly Euclidean point singularities are “removable” in the study of PSC metrics on 3-manifolds. We also discuss a comparison theorem for manifolds with positive scalar curvature. Part of this talk is based on joint work with Christos Mantoulidis. (Received September 20, 2017)

1135-53-1325 **Ruth Gornet\*** ([rgornet@uta.edu](mailto:rgornet@uta.edu)). *Survey of Spectral and Nilpotent Geometry, with recent results.* Preliminary report.

A survey of results and questions in spectral theory for nilpotent geometry will be discussed, followed by recent results. (Received September 21, 2017)

1135-53-1333 **Anusha Mangala Krishnan\*** ([anushakr@sas.upenn.edu](mailto:anushakr@sas.upenn.edu)), Department of Mathematics, David Rittenhouse Labs., 209 South 33rd Street, Philadelphia, PA 19104. *Ricci flow on cohomogeneity one manifolds.*

A cohomogeneity one manifold is a Riemannian manifold  $M$  admitting an isometric action by a Lie group  $G$ , such that the quotient space  $M/G$  is 1 dimensional. We will explain how in many cases the cohomogeneity one framework has been useful for studying the Ricci flow and Ricci solitons. We will also explain some of the challenges involved in studying cohomogeneity one Ricci flow in general, and mention some newer results. (Received September 21, 2017)

1135-53-1334 **Qiang Guang\*** ([guang@ucsb.edu](mailto:guang@ucsb.edu)), **Martin Man-chun Li** and **Xin Zhou**. *Curvature estimates and compactness for free boundary minimal hypersurfaces.*

Minimal hypersurfaces with free boundary are critical points of the area functional in compact manifolds with boundary. In this talk, we will present curvature estimates for stable free boundary minimal hypersurfaces. In general, a free boundary minimal hypersurface may not be proper, i.e., the interior of the hypersurface may touch the boundary of the ambient manifold. For such improper free boundary minimal hypersurfaces, we will also discuss the curvature estimates and compactness. (Received September 21, 2017)

1135-53-1475 **Chenxu He\*** ([chenxuhe@math.ucr.edu](mailto:chenxuhe@math.ucr.edu)), Department of Mathematics, 202 Surge, University of California at Riverside, Riverside, CA 92521. *Fundamental gap of convex domains in the Riemannian manifold.*

For a bounded convex domain on a Riemannian manifold, the fundamental gap is the difference of the first two non-trivial Dirichlet eigenvalues. In their celebrated work, B. Andrews and J. Clutterbuck proved the fundamental gap conjecture for convex domains in the Euclidean space, showing that the gap is at least as large as the one for a one-dimensional model. They also conjectured that similar results hold for spaces with constant sectional curvature. Very recently, for a convex domain on the round sphere, it is proved that the fundamental gap is greater than the gap of the one dimensional sphere model, in particular,  $\geq 3\pi^2/D^2$ , based on the work of Dai, He, Seto, Wei and Wang. The talk is based on the joint work with Guofang Wei at UC Santa Barbara. (Received September 22, 2017)

- 1135-53-1503 **J. Inoguchi, K. Kajiwara\*** (kaji@imi.kyushu-u.ac.jp), **K. T. Miura, M. Sato, W. K. Schief** and **Y. Shimizu**. *Log-Aesthetic Curves in Industrial Design as a Similarity Geometric Analog of Euler's Elastic Curves*.

We consider a class of plane curves called the log-aesthetic curves and their generalization which is used in the industrial design. We investigate those curves under the similarity geometry and characterize them as stationary integrable flow on plane curves which is governed by the Burgers equation. We propose a variational formulation of those curves whose Euler-Lagrange equation yields the stationary Burgers equation. Our result suggests that the log-aesthetic curves and their generalization can be regarded as the similarity geometric analog of Euler's elastic curves. (Received September 22, 2017)

- 1135-53-1506 **Morgan Weiler\*** (morgan.w@berkeley.edu), Dept. of Mathematics, University of California, Berkeley, 970 Evans Hall #3840, Berkeley, CA 94720-3840. *Mean action of area-preserving diffeomorphisms of the annulus*. Preliminary report.

Given an area-preserving diffeomorphism  $\psi$  of a closed annulus which is a rotation near the boundary, we can define an "action function" from the annulus to the reals which captures the dynamics of  $\psi$ . We study this action function via a filtration on embedded contact homology introduced by Hutchings, which is applied after realizing  $\psi$  as the Poincaré return map of a global surface of section for the Reeb flow on a contact three-manifold. (Received September 22, 2017)

- 1135-53-1519 **Xiaolong Li\*** (xiaolol1@uci.edu), **Lei Ni** and **Kui Wang**. *Four-dimensional Shrinking Ricci Solitons with Nonnegative Isotropic Curvature*.

We show that a four-dimensional complete gradient shrinking Ricci soliton with positive isotropic curvature is either a quotient of

$$S^4$$

or a quotient of

$$S^3 \times R$$

. We also give a classification result on four-dimensional gradient shrinking Ricci solitons with non-negative isotropic curvature. This is joint work with Lei Ni and Kui Wang. (Received September 22, 2017)

- 1135-53-1542 **Zhiqin Lu\***, Department of Mathematics, UC Irvine, Irvine. *Hearing the shape of a trapezoid by its eigenvalues*.

We shall prove that the shape of a trapezoid is determined by its Neumann eigenvalues. The Dirichlet eigenvalue case will also be discussed, with the classification on short families of closed geodesics on triangles and trapezoids. This is the joint work with Hamid Hezari and Julie Rowlett. (Received September 23, 2017)

- 1135-53-1562 **Andres Larrain-Hubach\*** (alarrainhubach1@udayton.edu), Science Center 313, 300 College Park, Dayton, OH 45469. *Decay of Twisted Harmonic Spinors on Taub-NUT Space*.

Given a complex vector bundle over a four-manifold, an Instanton is a unitary connection whose curvature is square-integrable and Anti-Self-Dual. The theory of bow representations, developed by Sergey Cherkis, gives an algebraic method to produce Instantons on Taub-NUT space. He conjectured that every generic Instanton on Taub-NUT corresponds to a bow representation. In a joint project with Sergey Cherkis and Mark Stern, we verified this conjecture. In this talk, I will discuss some results concerning the decay of Twisted Harmonic Spinors on Taub-NUT. These results play a fundamental role in the proof of the correspondence between Instantons and bows. (Received September 23, 2017)

- 1135-53-1597 **Daniel Cristofaro-Gardiner** and **Tara S Holm\*** (tara.holm@cornell.edu), Department of Mathematics, Cornell University, Ithaca, NY 14850, and **Alessia Mandini** and **Ana Rita Pires**. *Symplectic embeddings and toric geometry*. Preliminary report.

I will discuss my ongoing work with Cristofaro-Gardiner, Mandini and Pires. We study symplectic embeddings of four-dimensional ellipsoids into toric symplectic four-manifolds. Embedded contact homology capacities provide complete information about such embeddings. The embedding capacity function admits infinite staircases in several known settings (due to McDuff – Schlenk; Frenkel – Müller; and Cristofaro-Gardiner – Kleinman). We find three more, and conjecture that these are the only infinite staircases among toric surfaces. (Received September 23, 2017)

1135-53-1638 **Raquel Perales\*** (raquel.peralesguilar@gmail.com), Leon 2, Centro, 68000 Oaxaca, Oaxaca, Mexico, and **Anna Siffert, Priyanka Rajan, Maree Jaramillo, Catherine Searle** and **Nan Li**. *Integral Currents on Alexandrov Spaces and IF convergence.*

I will first talk about joint work with Jaramillo, Rajan, Siffert and Searle. We endow each closed, orientable Alexandrov space  $(X, d)$  with an integral current  $T$  of weight equal to 1,  $\partial T = 0$  and  $\text{set}(T) = X$ , in other words, we prove that  $(X, d, T)$  is an integral current space with no boundary.

Finally, I will talk about work with Li. We show that non-collapsing sequences of Alexandrov Spaces with a current structure that satisfies the conditions of the first paragraph and have uniform lower curvature and diameter bounds admit subsequences whose Gromov-Hausdorff and intrinsic flat limits agree. (Received September 24, 2017)

1135-53-1659 **Robert L. Bryant\*** (bryant@math.duke.edu), Duke Mathematics Department, P.O. Box 90320, Durham, NC 27708-0320. *The Concept of Holonomy—Its History and Recent Developments.*

Inspired by the study of ‘rolling constraints’ in mechanics, the concept of *holonomy* was introduced into geometry to describe parallel translation in curved media. In the 1920s, it was applied by É. Cartan and his students to problems such as the classification of real forms of Lie groups. Riemannian manifolds with reduced holonomy made their first appearance in mainstream geometry as Kähler geometry, and, in the 1950s, this motivated M. Berger to classify the possible Riemannian holonomy groups, providing a fruitful taxonomy of geometries. S.-T. Yau’s solution of the Calabi Conjecture fits naturally into this framework and stimulated interest in the other ‘special holonomies’ on Berger’s list. Beginning in the 1980s, the final two ‘exceptional’ cases were shown to exist and to play an essential role in high-energy theoretical physics analogous to the role that Calabi-Yau spaces play in string theory and mirror symmetry. In recent years, many new results have extended our knowledge of these exceptional spaces and their remarkable properties, though much remains mysterious.

In this talk, I will describe this history, develop the basic concepts, and explain some of the recent advances and some of the challenging open problems in the study of holonomy. (Received September 24, 2017)

1135-53-1662 **Evangelia Samiou\***, University of Cyprus, Department of Mathematics and Statistics, P.O. Box 20537, 1678 Nicosia, Cyprus. *The X-ray transform on nilpotent Lie groups.*

The X-ray transform on a Riemannian manifold assigns to a compactly supported continuous function its integrals over geodesics. We prove injectivity and a support theorem for the X-ray transform on 2-step nilpotent Lie groups, and some other classes of nilpotent Lie groups. The results follow from a general principle for manifolds containing a totally geodesic plane through each point. (Received September 24, 2017)

1135-53-1814 **Yuri Nikolayevsky\*** (y.nikolayevsky@latrobe.edu.au), Department of Mathematics and Statistics, La Trobe University, Melbourne, Victoria 3086, Australia. *Geodesic orbit and naturally reductive nilmanifolds associated with graphs.*

We study Riemannian nilmanifolds associated with graphs. We prove that such a nilmanifold is geodesic orbit if and only if it is naturally reductive if and only if its defining graph is the disjoint union of complete graphs and the left-invariant metric is generated by a certain naturally defined inner product. (Received September 25, 2017)

1135-53-1823 **Mainak Poddar\*** (mpoddar@metu.edu.tr), Mathematics Group, Middle East Technical University, Northern Cyprus Campus, Kalakanli, 99738 Guzelyurt, Mersin 10, Turkey, and **Ajay Singh Thakur**. *Complex and SKT structures on the total space of principal bundles.*

We present a new construction of non-Kähler complex manifolds that generalizes the construction of complex structures on compact even dimensional Lie groups by Samelson and Wang, as well as the Calabi-Eckmann construction of complex structures on the product to two odd dimensional spheres. We give sufficient conditions for the existence of SKT structures on a large class of these.

Let  $G$  be a complex linear algebraic group and let  $K$  be a maximal compact Lie subgroup of  $G$ . Let  $E_K \rightarrow M$  be a smooth principal  $K$ -bundle over a complex manifold  $M$ . Assume that  $E_K \rightarrow M$  can be obtained by a smooth reduction of structure group from a holomorphic principal  $G$ -bundle over  $M$ . Then  $E_K$  (respectively,  $E_K \times S^1$ ) admits a family of integrable complex structures if  $K$  has even dimension (respectively, odd dimension).

When  $K$  is an even dimensional unitary, special orthogonal or compact symplectic group and  $M$  is a projective manifold, we can choose complex structures on  $E_K$  so that it admits a family of SKT structures if the characteristic classes of  $E_K \rightarrow M$  satisfy some simple conditions. Formulas for the Picard group and the algebraic dimension of  $E_K$  are obtained when, in addition,  $K$  is simply connected. (Received September 25, 2017)

1135-53-1858 **Armando Jose Cabrera Pacheco**, Auf der Morgenstelle 10, 72076 Tuebingen, Germany, **Carla Cederbaum\***, Auf der Morgenstelle 10, 72076 Tuebingen, Germany, and **Stephen McCormick** and **Pengzi Miao**. *On special Riemannian manifolds related to Bartnik's mass.*

Bartnik data are a Riemannian 2-sphere of positive Gaussian curvature equipped with a non-negative function  $H$  to be thought of as its mean curvature in an ambient Riemannian 3-manifold. Mantoulidis and Schoen suggested a construction of asymptotically flat Riemannian 3-manifolds of non-negative scalar curvature which allow to isometrically embed given Bartnik data of vanishing mean curvature, i.e.  $H=0$ . They use their construction to explore — and disprove — stability of the Riemannian Penrose inequality. We adapt their construction to constant mean curvature (CMC) Bartnik data, i.e.  $H=\text{const.}>0$ . Moreover, we construct asymptotically hyperbolic extensions for minimal and CMC Bartnik data. I will present the constructions as well as the motivation for such constructions which is related to Bartnik's quasi-local mass functional and its minimizing properties. (Received September 25, 2017)

1135-53-1862 **Renato G. Bettiol\*** ([rbettiol@math.upenn.edu](mailto:rbettiol@math.upenn.edu)), University of Pennsylvania, Department of Mathematics, 209 South 33rd Street 4W1, Philadelphia, PA 19104. *Sectional curvature and Weitzenböck formulae.*

We prove an algebraic characterization of any lower or upper sectional curvature bound in terms of the curvature terms in the Weitzenböck formulae for symmetric  $p$ -tensors. By introducing a symmetric analogue of the Kulkarni-Nomizu product, we also derive an explicit formula for such curvature terms, as a function of scalar, traceless Ricci, and Weyl curvatures. This is based on joint work with R. Mendes. (Received September 25, 2017)

1135-53-1891 **Angelynn R. Alvarez\*** ([alvarear@potssdam.edu](mailto:alvarear@potssdam.edu)), Department of Mathematics, SUNY Potsdam, 44 Pierrepoint Avenue, Potsdam, NY 13676. *Holomorphic Sectional Curvature and its Friends on Complex Manifolds.* Preliminary report.

In the world of complex geometry, we are interested in studying how geometric quantities shape the global structure of a manifold. In this talk, we discuss a particular quantity called curvature, as well as the different types of curvature there are. Additionally, we will talk about several results on curvature, namely on the positive holomorphic sectional curvature on projectivized vector bundles over compact complex manifolds. (Received September 25, 2017)

1135-53-1915 **Richard Schoen\*** ([rschoen@uci.edu](mailto:rschoen@uci.edu)), Richard Schoen, Department of Mathematics, University of California, Irvine, Irvine, CA 92697-3875. *Constructing complete manifolds with lots of spectral gaps.*

We will discuss the question of the existence of gaps in the spectrum of the Laplacian for complete manifolds with bounded geometry. In particular we will outline a construction which induces an arbitrarily large finite number of gaps without changing the large scale geometry. This is a joint work with Hung Tran. (Received September 25, 2017)

1135-53-1931 **Ailana Fraser\*** ([afrazer@math.ubc.ca](mailto:afrazer@math.ubc.ca)), University of British Columbia, and **Richard Schoen**, U.C. Irvine. *Extremal questions for Steklov eigenvalues in higher dimensions.* Preliminary report.

In the two-dimensional case we proved that for any compact surface of genus zero with boundary there exists a metric that maximizes the first eigenvalue of the Dirichlet-to-Neumann map, among all metrics of fixed boundary length, and that the maximum of the first eigenvalue is strictly increasing in  $k$  and tends to  $4\pi$  as the number of boundary components  $k$  tends to infinity. In this talk we will discuss similar questions in higher dimensions. (Received September 25, 2017)

1135-53-1932 **Davi Maximo\*** ([dmaxim@math.upenn.edu](mailto:dmaxim@math.upenn.edu)), Department of Mathematics, David Rittenhouse Lab, 209 South 33rd Street, Philadelphia, PA 19104. *On Morse index estimates for minimal surfaces.*

In this talk we will survey some recent estimates involving the Morse index and the topology of minimal surfaces (Received September 25, 2017)

1135-53-1944 **Or Hershkovits\*** ([orher@stanford.edu](mailto:orher@stanford.edu)), Or Hershkovits, Stanford, CA 94305, and **Brian White**, Stanford, CA 94305. *Uniqueness of mean curvature flow through (some) singularities.*

Given a smooth compact hypersurface in Euclidean space, one can show that there exists a unique smooth evolution starting from it, existing for some maximal time. But what happens after the flow becomes singular?

There are several notions through which one can describe weak evolutions past singularities, with various relationship between them. One such notion is that of the level set flow. While the level set flow is unique, it has an undesirable phenomenon called fattening: The “weak evolution” develops an interior in  $\mathbb{R}^{n+1}$ . This fattening is, in many ways, the right notion of non-uniqueness for weak mean curvature flow.

As was alluded to above, fattening can not occur as long as the flow is smooth. Thus it is reasonable to say that the source of fattening is singularities. Permitting singularities, it is very easy to show that fattening does not occur if the initial hypersurface, and thus all the evolved hypersurface, are mean convex. Thus it’s reasonable to conjecture that: ”An evolving surface cannot fatten unless it has a singularity with no spacetime neighborhood in which the surface is mean convex”.

In this talk, we will phrase a concrete formulation of this conjecture, and describe its proof. This is a joint work with Brian White. (Received September 25, 2017)

1135-53-1967 **Mat Langford\*** (mlangfo5@utk.edu) and **Stephen Lynch**. *Optimal curvature pinching in mean curvature flow.*

We will describe how Huisken’s now classical Stampacchia iteration argument can be used, in a simple and direct way, to show that a mean convex solution of mean curvature flow is ‘optimally pinched’ at its first singularity. Time permitting, we will present an application of the result to ancient solutions of the flow and describe how the result may be extended to a large family of fully nonlinear flows. The work on fully nonlinear flows is joint with Stephen Lynch (FU Berlin) (Received September 25, 2017)

1135-53-2018 **Hulya Kadioglu\*** (hkadio@yildiz.edu.tr), Yildiz Technical University, Department of, Mathematics and Science Education, Esenler, 34220 Istanbul, Turkey. *An Induced Isometry on a Vector Bundle.*

Let  $(\mathcal{E}, \pi, \mathfrak{M}, E)$  be a vector bundle. There exists a specific Riemannian metric  $\tilde{g}$  on  $\mathcal{E}$  which is induced from a given Riemannian metric  $g$  on  $\mathfrak{M}$ . In this study, we use this special metric and define an isometry on  $\mathcal{E}$ . (Received September 25, 2017)

1135-53-2072 **David Wiygul\*** (dwiygul@uci.edu). *Some recent gluing constructions of minimal surfaces in the 3-sphere and Euclidean space.*

I will describe new joint work with Nicos Kapouleas constructing some interesting minimal surfaces in the 3-dimensional round sphere and Euclidean space by gluing techniques. (Received September 25, 2017)

1135-53-2074 **Rachelle DeCoste, Lisa DeMeyer, Meera Mainkar and Allie Ray\*** (allie.ray@trincoll.edu). *Nilpotent Metric Lie Algebras Constructed from Schreier Graphs.*

We will consider the properties of a certain two-step nilpotent metric Lie algebra constructed from a Schreier graph. This construction is a generalization of a method used by S.G. Dani and M.G. Mainkar. Results will include necessary and sufficient conditions for extending this two-step nilpotent Lie algebra to a three-step nilpotent Lie algebra. In addition, starting with pairs of Schreier graphs of a Gassmann-Sunada triple, I will consider the geometry (in particular issues of isospectrality and isometry) of the associated metric Lie algebras. Finally, recent results have shown that these Lie algebras are always singular, which contrasts with results using the previous construction method. (Received September 25, 2017)

1135-53-2101 **Michael Benjamin Deutsch\*** (mbddeutsch@gmail.com), Av. Athos da Silveira Ramos 149, Centro de Tecnologia, Bloco C, Cidade Universitaria, Ilha do Fundao, Rio de Janeiro, RJ 21941-909, Brazil. *Flat surfaces in hyperbolic 3-space via twistor reduction.* Preliminary report.

We study a projection map from the moduli space of Bryant surfaces in  $\mathbb{H}^3$  to that of flat surfaces which arises from the reduction process of the twistor space of  $\mathrm{PSL}_2\mathbb{C}$  to its mini-twistor space. Among other applications we will describe, this map provides a complex-geometric interpretation of a special case of the Gálvez-Martínez-Milán deformation for linear Weingarten of Bryant type, a characterization of the local correspondence between flat surfaces in  $\mathbb{H}^3$  and minimal Ribaucour pairs  $\mathbb{R}^3$ , an analogous correspondence for superminimal Ribaucour pairs in  $\mathbb{R}^4$ , and an effective method for constructing examples with prescribed symmetry. (Received September 25, 2017)

1135-53-2110 **Daniel Jacob Ketover\*** (dketover@gmail.com), Fine Hall, Princeton University, Princeton, NJ 08544. *Minimal two-spheres in three-spheres.*

The celebrated theorem of Lusternik-Schnirelman states that for any metric on a two sphere, there are at least three closed embedded geodesics. The corresponding problem for a Riemannian three-sphere asks to find at

least four closed embedded minimal two-spheres. The existence of at least one two-sphere was obtained by Leon Simon and Francis Smith in 1983. I'll explain my joint work with Haslhofer, in which we proved the existence of a second minimal two-sphere. The proof uses many recent developments in min-max theory and mean curvature flow. It also leads to the existence of minimal non-planar two-spheres in ellipsoids in  $\mathbb{R}^4$ , answering a question of Yau. (Received September 25, 2017)

1135-53-2153 **Carolyn Gordon\*** (csgordon@dartmouth.edu), Dept of Mathematics, 6188 Kemeny, Dartmouth College, Hanover, NH 03755, and **Michael Jablonski**, University of Oklahoma. *Symmetry properties of homogeneous Ricci soliton metrics on solvable Lie groups*. Preliminary report.

We say that a left-invariant Riemannian metric on a Lie group is “maximally symmetric” if, up to automorphisms, its isometry group contains that of any other left-invariant metric. We also define a weaker notion of “infinitesimal maximal symmetry”. Homogeneous Einstein metrics of negative Ricci curvature on solvable Lie groups are maximally symmetric. In contrast, we show that expanding Ricci solitons on solvable Lie groups of real type need not be maximally symmetric but are always infinitesimally maximally symmetric. We relate these results to questions of stability of the Ricci flow. This is joint work with Michael Jablonski. (Received September 25, 2017)

1135-53-2197 **Maxwell Stolarski\*** (mstolarski@math.utexas.edu). *Ricci Flow on Doubly-Warped Products*. Preliminary report.

I will discuss aspects of the Ricci flow on multiply-warped products with a focus on doubly-warped products of Einstein manifolds with positive scalar curvature and the dynamics of the flow near finite-time singularities. In this case, previous work on the Ricci flow on singly-warped products and the mean curvature flow of  $SO(p) \times SO(q)$  invariant hypersurfaces have relevance, but the Ricci flow on multiply-warped products presents additional analytic difficulties. The geometry of certain non-compact Ricci-flat spaces asymptotic to Ricci-flat cones and their role in the dynamics will also be discussed. (Received September 25, 2017)

1135-53-2245 **Da Rong Cheng\*** (chengdr@uchicago.edu). *Upper bound for the Green matrix of a class of second order elliptic systems*.

We study a class of second order elliptic systems in divergence form, with Lipschitz leading coefficients and possibly discontinuous lower order terms, and show that the Green matrix, if it exists, grows no faster than  $|x - y|^{2-n}$  near the diagonal. Such systems arise, for instance, when one considers the Hodge Laplacian with respect to a Lipschitz metric. The proof uses perturbation arguments and is based partly on the work of Fuchs, who established a similar upper bound in the absence of lower order terms. (Received September 25, 2017)

1135-53-2266 **Patrick Allmann\*** (pallmann@ucsc.edu), **Longzhi Lin** and **Jingyong Zhu**. *Modified Mean Curvature Flow in Hyperbolic Space*.

In this talk we will sketch a proof of the longtime existence of a solution to modified mean curvature flow in hyperbolic space with initial data an entire locally Lipschitz radial graph, which stays graphic for all time. Time permitting, we will also discuss examples and counterexamples of convergence of the flow as time approaches infinity. This is joint work with Longzhi Lin and Jingyong Zhu. (Received September 25, 2017)

1135-53-2273 **Matias Delgadino**, **Francesco Maggi** and **Cornelia Mihaila\*** (cmihaila@math.utexas.edu), Department of Mathematics, 2515 Speedway Stop C1200, Austin, TX 78712, and **Robin Neumayer**. *Bubbling with  $L^2$  almost constant mean curvature and an Alexandrov type theorem for crystals*.

I will discuss a recent result in which an Alexandrov-type theorem for  $L^2$  almost constant anisotropic mean curvature sets is proven. In addition I will provide a description of critical points/local minimizers for elliptic energies interacting with a confinement potential. Key tools involved are the use of the Finslerian Laplace operator, and the use of a variety of geometric identities. An improvement on previous almost constant mean curvature results is our use of  $L^2$  versus  $C^0$  closeness, since that should have applications in mean curvature flow and is new even in the isotropic case. This talk is based on a joint work with Matias Delgadino, Francesco Maggi, and Robin Neumayer. (Received September 25, 2017)

1135-53-2282 **Kevin Sonnanburg\*** (ksonnanb@vols.utk.edu). *Blow-up Continuity for Mean-Convex, Type-I Mean Curvature Flow*.

Under mean curvature flow, each point of a hypersurface moves with velocity equal to its mean curvature. A closed, embedded hypersurface  $M(t)$  becomes singular in finite time. One of the most basic questions about a PDE that develops singularities is the relationship between the occurrence of its singularities and its initial data.

For certain classes of mean-convex mean curvature flows, we show the first singular time  $T$  and the limit set “ $M(T)$ ” is continuous with respect to the initial hypersurface.

We employ an Angenent-like neckpinching argument to force singularities in nearby flows. However, since we cannot prescribe initial data, we combine Andrews’  $\alpha$ -non-collapsed condition and Colding and Minicozzi’s uniqueness of tangent flows to ensure the development of a neck-like structure and place appropriately sized spheres in the two regions outside the neck. (Received September 25, 2017)

1135-53-2307      **Kevin Sonnanburg\*** (kssonanb@vols.utk.edu). *A Liouville Theorem for Ancient Mean Curvature Flows.*

Under mean curvature flow, each point of a hypersurface moves with velocity equal to its mean curvature vector. An embedded, compact hypersurface must develop a singularity in finite time. Ancient solutions often arise in the study of parabolic blow-ups at singularities.

If we can categorize ancient solutions, we can better understand these blow-up limits. We give a Liouville-type theorem restricting a certain class of ancient, two-dimensional mean curvature flows to just spheres or cylinders. (Received September 25, 2017)

1135-53-2515      **Casey Blacker\*** (cblackerc@math.ucsb.edu), Department of Mathematics, South Hall, Room 6607, University of California, Santa Barbara, CA 93106. *The Moduli Space of Flat Connections over Higher Dimensional Manifolds.* Preliminary report.

While the moduli space  $\mathcal{M}_G(\Sigma)$  of flat connections on a  $G$ -principal bundle over a surface  $\Sigma$  has been extensively studied, the case of a higher dimensional base  $M$  remains largely unexplored. Given a Lefschetz symplectic form on  $M$ , there is an induced symplectic structure on the moduli space  $\mathcal{M}_G(M)$ . We will show that, under the action of the gauge group,  $\mathcal{M}_G(M)$  is a generalized symplectic quotient of the space of all  $G$ -connections over  $M$ , endowed with a natural vector-valued symplectic form. For special cases of  $M$  and  $G$ , we also obtain a description of the topology of  $\mathcal{M}_G(M)$ , as well as an analytic expression for the symplectic volume. (Received September 26, 2017)

1135-53-2547      **Roberta Guadagni\*** (rguadagni@math.utexas.edu). *Symplectic topology of singularities.*

While the study of singularities is a very developed field in many areas from algebraic geometry to PDEs, the symplectic point of view is still very much work in progress. Symplectic forms are naturally defined only on smooth spaces, therefore even giving the right definition of symplectic manifolds with singularities isn’t straightforward. After giving a (very brief) overview of the problem, I will focus on the symplectic geometry (and topology) of algebraic singularities embedded in a Kaehler ambient space. The simplest case in the symplectic Lefschetz singularity. For isolated singularities one can look at symplectic versions of the Milnor fibre (as done by A. Keating). Very little is known about the symplectic geometry of algebraic, non isolated singularities. (Received September 26, 2017)

1135-53-2609      **Jasmine Camero\*** (jasminecamero@csu.fullerton.edu), **Oscar Rocha Rocha** (oscrocha167@csu.fullerton.edu), **Nicholas Brubaker** (nbrubaker@fullerton.edu) and **Bogdan Suceava** (bsuceava@fullerton.edu). *An Explorational of Tangential Curvature.*

In the classical differential geometry of surfaces there are two curvature invariants: the Gaussian curvature (introduced by Gauss in 1827) and the mean curvature (introduced by Sophie Germain in 1831). At the end of the 19th century Casorati investigated another curvature invariant. In the present work we introduce a new curvature invariant, the tangential curvature, and we investigate its fundamental properties. (Received September 26, 2017)

1135-53-2681      **Adam J Moreno\*** (amoreno3@nd.edu). *Point Leaf Maximal Singular Riemannian Foliations in Positive Curvature.* Preliminary report.

Positively curved Riemannian manifolds with “large” symmetry have received special attention since the early 90’s. *Cohomogeneity one* actions, for example, have simple orbit spaces which carry information about the topology of the given manifold. Groups acting *fixed point homogeneously* share an important property with cohomogeneity one manifolds and were classified by Grove and Searle in 1997. Here, we interpret this property in the context of singular Riemannian foliations, where the group action is absent. We find positively curved manifolds that can be equipped with these so-called *point leaf maximal* SRF’s have a nice structure which resembles (cohomologically) that of the compact rank one symmetric spaces. (Received September 26, 2017)

1135-53-2709 **P. Solorzano\*** ([pedro.solorzano@matem.unam.mx](mailto:pedro.solorzano@matem.unam.mx)), **J. Ge** and **L. Guijarro**. *Metric geometric rigidity of Euclidean Space.*

In the many attempts to prove Euclid's Fifth Postulate, several equivalently self-evident assumptions were stated. Perhaps the most famous is Playfair's Axiom: Given a straight line and a point not on it there exists a unique parallel line through the point.

In this report we will analyze the necessity of Euclidean space—even without the isotropy assumption—with the classical Fifth postulate and with Playfair's Axiom. For the latter, we will discuss the seemingly weaker assumption of total curvature.

In particular, we will study the implications at infinity both of the parallel postulate, of volume growth, and of the integrability of the Gaussian Curvature for complete metrics on  $\mathbb{R}^2$ . (Received September 26, 2017)

1135-53-2731 **Michael Jablonski\*** ([mjablonski@math.ou.edu](mailto:mjablonski@math.ou.edu)). *Maximal symmetry in solvmanifolds.*

A Riemannian metric is said to have maximal symmetry if its isometry group contains the isometry group of any other metric on the space (up to conjugacy). We consider this notion in the setting of solvable Lie groups with left-invariant metrics, giving special attention to the cases when there exists either a left-invariant Einstein or Ricci soliton metric. (Received September 26, 2017)

1135-53-2800 **John A Simanyi\***, [simanyi@math.ucr.edu](mailto:simanyi@math.ucr.edu). *A Hodge-type decomposition of holomorphic Poisson cohomology on nilmanifolds.*

The existence of a holomorphic Poisson bivector on a generalized complex manifold allows us to develop cohomology built upon a classic  $\bar{\partial}$ -operator and the adjoint action of the bivector. This, in turn, has an associated spectral sequence, where the first page is the Dolbeault cohomology with coefficients in the sheaf of germs of holomorphic polyvector fields. Considering nilmanifolds that admit abelian complex structures, under certain constraints this sequence degenerates on the first page, and will admit a Hodge type decomposition of the holomorphic Poisson hypercohomology. In this talk, based on a joint publication with Yat Sun Poon, we investigate the construction and constraints, and provide some examples as time permits. (Received September 26, 2017)

1135-53-2845 **Christopher Lopez\*** ([clopez04971@gmail.com](mailto:clopez04971@gmail.com)). *Rigidity of the Ambient Obstruction Flow.*

The ambient obstruction tensor  $\mathcal{O}$  on a Riemannian manifold is a conformally covariant tensor of high order in the Ricci curvature. It is the gradient of the conformally invariant integral of a high order scalar curvature analogue termed  $Q$  curvature. In dimension four,  $\mathcal{O}$  is equal to the Bach tensor. The ambient obstruction flow (AOF) is a parabolic flow by  $\mathcal{O}$  and a conformal group counteracting term involving the scalar curvature. Stationary points of this flow are generalizations of Einstein metrics. We prove rigidity results for AOF that generalize known rigidity results for Bach - flat manifolds. These results permit us to continue to study the behavior of AOF near singularities. (Received September 26, 2017)

1135-53-3026 **Olivier Biquard** and **Heather Macbeth\*** ([hmacbeth@mit.edu](mailto:hmacbeth@mit.edu)). *Kähler-Ricci solitons on crepant resolutions.* Preliminary report.

By a gluing construction, we produce steady Kähler-Ricci solitons on crepant resolutions of  $\mathbb{C}^n/G$ , where  $G$  is a finite subgroup of  $SU(n)$ , generalizing Cao's construction of such a soliton on a resolution of  $\mathbb{C}^n/\mathbb{Z}_n$ . (Received September 26, 2017)

1135-53-3036 **T. H. Wears\*** ([wearsth@longwood.edu](mailto:wearsth@longwood.edu)). *The Geometry of Curves and Surfaces in the Three-dimensional Lie Group  $E(1, 1)$ .* Preliminary report.

We study the geometry of curves and surfaces in the three-dimensional Lie group  $E(1, 1)$  equipped with left invariant Lorentzian metric by utilizing the Fels-Olver moving frame method. In doing so, we present complete sets of differential invariants for curves and surfaces in  $E(1, 1)$  when the dimension of the isometry group is four. We provide a geometric interpretation of the invariants for certain classes of curves and surfaces and provide a brief comparison with differential invariants generated by alternative methods. (Received September 26, 2017)

1135-53-3075 **Joseph Ansel Hoisington\*** ([jhois@math.upenn.edu](mailto:jhois@math.upenn.edu)). *Total Curvature and the Betti Numbers of a Complex Projective Manifold.*

We will show that the sum of the Betti numbers of a complex projective manifold can be estimated in terms of its total curvature, as measured by the second fundamental form of its immersion into projective space, and characterize the manifolds whose total curvature is minimal. These results extend the classic theorems of Chern and Lashof to a new setting. (Received September 26, 2017)



1135-53-3176 **Yucheng Lu\*** (y1u27@ucsc.edu). *Normalized Ricci Flow on Non-Compact Manifolds*. Preliminary report.

We study the behavior of the normalized Ricci flow of initial metrics which are perturbations of an Einstein metric. If the perturbation is small enough, then we show that the normalized Ricci flow exists globally, and converges smoothly and exponentially fast to that Einstein metric as time approaches infinity. (Received September 26, 2017)

1135-53-3217 **Aroldo Kaplan\*** (kaplan@math.umass.edu). *Heisenberg groups and division algebras*.

Associated to every division algebra there are two series of Heisenberg-like algebras. Every non-compact simple Lie algebra other than  $\mathfrak{so}(1,n)$  contains a unique standard parabolic subalgebra whose nilradical is one of those. The corresponding parabolic geometry arises as the infinity of the corresponding Damek-Ricci metric. (Received September 27, 2017)

## 54 ► General topology

1135-54-168 **Gezahagne Mulat Addis\*** (buttu412@yahoo.com), , Ethiopia. *Fuzzy Topological Spaces*. In this paper we give a new definition of fuzzy topological spaces. (Received August 07, 2017)

1135-54-215 **Xiaojun Zheng\*** (zheng2@stolaf.edu), 1520 St. Olaf Ave., Northfield, MN 55057. *Topological Data Analysis on Simple English Wikipedia Articles*. Preliminary report.

Persistent homology is an algebraic method to detect the topological features (such as connected components and holes) of high-dimensional data. In this presentation, we use persistent homology to analyze the structure of a point cloud produced from a semantic analysis of Simple English Wikipedia articles. Specifically, the semantic algorithm converts each article to a 200-dimensional vector. We use the two-parameter persistent homology software RIVET to distinguish the Wikipedia point cloud from a point cloud of similar random vectors. We also compare the topological similarity of Wikipedia articles for major cities using semantic distance and geographic distance between the cities. In addition to analyzing the RIVET plots, we apply statistical tests to the topological differences between the data sets to confirm our conclusions. (Received August 11, 2017)

1135-54-325 **Leo Carlsson, Gunnar Carlsson and Mikael Vejdemo-Johansson\***, 2800 Victory Boulevard, 1S-215, Staten Island, NY 10314. *Fibres of Failure: diagnosing predictive models using Mapper*. Preliminary report.

The Mapper algorithm is able to produce intrinsic topological models of arbitrary data in high dimensions. Through a statistical adaptation of the Nerve lemma, the algorithm can be seen to reproduce the topology and parts of the geometry of the data source under assumptions of dense sampling and good parameter choices.

In this talk, we will describe how by careful choice of the Mapper model parameters, the resulting topological model can be guaranteed to separate input values to the predictive process for prediction error, grouping high-error and low-error regions separately.

This approach produces a diagnostic process where local failure modes can be classified, feeding into either a model development process or a local correction term to improve predictive performance. We have successfully applied this approach to temperature prediction in steel furnaces. (Received August 24, 2017)

1135-54-326 **Logan McKee Higginbotham\*** (lhiggin3@vols.utk.edu) and **Thomas Weighill**. *Coarse quotients by group actions and the preservation of Property A*.

For a discrete metric space (or more generally a large scale space)  $X$  and an action of a group  $G$  on  $X$  by coarse equivalences, we define a type of coarse quotient space  $X_G$ , which agrees up to coarse equivalence with the orbit space  $X/G$  when  $G$  is finite. We then show that if  $X$  has Property A and  $G$  is amenable, then we show that  $X_G$  has Property A. (Received August 24, 2017)

1135-54-407 **Sheldon W Davis\*** (sdavis@uttyler.edu), Department of Mathematics, University of Texas at Tyler, Tyler, TX 75799. *Symmetrizable  $L$ -spaces and Symmetrizable Dowker Spaces*. Preliminary report.

In his famous 1966 survey, A. V. Arhangel'skii said, "So far, everything that has been asserted about symmetric spaces shows that along with ordinary metrics, they refer to the most important of those numerical structures that occur in general topological spaces. Problems connected with them merit special consideration." We discuss the questions of the existence of certain symmetrizable spaces of the types mentioned in the title, and we give some partial results. (Received August 31, 2017)

1135-54-589 **Jila Niknejad\*** (jila@ku.edu). *Normal Images of the Products and Countably Paracompact Images.*

In 1997, Buzjakova proved that for a pseudocompact Tychonoff space  $X$  and  $\kappa = |\beta X|^+$ ,  $X$  condenses onto a compact space if and only if  $X \times (\kappa + 1)$  condenses onto a normal space. This is a condensation form of Tamano's theorem. An interesting problem is to determine how much of Buzjakova's result will hold if "pseudocompact" is removed from the hypothesis.

In this talk, I am going to show for a Tychonoff space  $X$ , there is a cardinal  $\kappa$  such that if  $X \times (\kappa + 1)$  condenses onto a normal space, then  $X$  condenses onto a normal, countably paracompact space. (Received September 09, 2017)

1135-54-664 **Ilesanmi Adeboye, Harrison Bray\*** (brays@umich.edu) and **David Constantine.** *Volume and entropy for Hilbert geometries.*

We prove that for any closed manifold  $M$  admitting a hyperbolic metric, there is a lower bound on the Hilbert volume of convex projective structures on  $M$ . Moreover, the volume growth entropy decreases to 0 if the Hilbert volume of the convex projective structures on  $M$  grows without bound. In dimension three, these results are an application of a volume-entropy rigidity theorem following the classical work of Besson-Courtois-Gallot. This is joint work with Ilesanmi Adeboye and David Constantine. (Received September 12, 2017)

1135-54-992 **William G. Fleissner\*** (fleissn@ku.edu). *Baire, Čech-complete, and Subcompact Spaces.* Preliminary report.

Starting with the Baire category theorem, the talk will continue to topologically complete spaces, Čech-complete spaces, and subcompact spaces. The conclusion is that the following questions are natural and interesting.

Are Čech-complete spaces subcompact? More generally, are  $G_\delta$  subspaces of subcompact spaces subcompact? (Received September 18, 2017)

1135-54-1145 **Gerald Beer and Colin Bloomfield\*** (colinbloomfield1@gmail.com), 405 Lewis st., Los Angeles, CA 90042. *A Closure Operator for Clopen Topologies.*

A topology  $\tau$  on a nonempty set  $X$  is called a clopen topology provided each member of  $\tau$  is both open and closed. Given a function  $f$  from  $X$  to  $Y$ , the operator  $E \mapsto f^{-1}(f(E))$  is a closure operator on the power set of  $X$  whose fixed points are closed subsets corresponding to a clopen topology on  $X$ . Conversely, for each clopen topology  $\tau$  on  $X$ , we produce a function  $f$  with domain  $X$  such that  $\tau = \{E \subseteq X : E = f^{-1}(f(E))\}$ . We characterize the clopen topologies on  $X$  as those that are weak topologies determined by a surjective function with values in some discrete topological space. Paralleling this result, we show that a topology admits a clopen base if and only if it is a weak topology determined by a family of functions with values in discrete spaces. (Received September 19, 2017)

1135-54-1152 **Liljana Babinkostova\*** (liljanababinkostova@boisestate.edu), 1910 University Drive, Boise, ID 83725, and **Marion Scheepers** (mscheepe@boisestate.edu), 1910 University Drive, Boise, ID 83725. *The Selective Strong Screenability Game.*

Selective screenability, introduced in a 1978 paper, and selective strong screenability, coincide in a wide class of topological spaces. The two corresponding games, however, are vastly different from each other. In this talk we describe results that illustrate this remark, and indicate that a deeper investigation of the selective strong screenability game is necessary. (Received September 19, 2017)

1135-54-1171 **Jingyu Zhao\*** (jzhao0105@gmail.com), 70 Adams St, Hoboken, NJ 07030. *A Smith inequality for fixed point Floer cohomology.*

We will describe an analogue of the classical Smith inequality for cyclic group of prime order  $p$  for fixed point Floer cohomology, which compares the ranks of the fixed point Floer cohomology of a symplectomorphism to its  $p$ -th iterations. The proof uses a construction of an equivariant  $p$ -th power map. This work in progress is based on the previous work by P. Seidel in the case of  $p=2$ . (Received September 20, 2017)

1135-54-1181 **Samantha Pezzimenti\*** (spezziment@brynmawr.edu), Bryn Mawr, PA. *Immersed Lagrangian Fillings of Legendrian Submanifolds via Generating Families.*

Given a Legendrian submanifold, we would like to know what geometric restrictions exist for its Lagrangian fillings. When the Legendrian admits a generating family (GF), there is a natural isomorphism between the GF-cohomology groups of the Legendrian and the cohomology groups of any GF-compatible embedded Lagrangian filling. I will show that a similar isomorphism exists for immersed GF-compatible Lagrangian fillings, which imposes restrictions on the minimum number and types of double points for any such filling. I will also give some constructions of immersed GF-compatible fillings. (Received September 20, 2017)

1135-54-1297 **Angelo Bella** and **Santi Spadaro\*** ([santidspadaro@gmail.com](mailto:santidspadaro@gmail.com)), Dipartimento di Matematica e Informatica, Università di Catania, Città Universitaria, viale A. Doria 6, 95125 Catania, Italy. *Arhangel'skii's Theorem and discrete sets.*

Juhász, Tkachuk and Wilson define a space to be *almost discretely Lindelöf* if every discrete set can be covered by a Lindelöf subspace. Any example of an S-space (regular hereditarily separable non-Lindelöf) provides an example of an almost discretely Lindelöf non-Lindelöf space. Inspired by Arhangel'skii's theorem, Juhász, Tkachuk and Wilson ask whether every almost discretely Lindelöf Hausdorff space has cardinality at most continuum. We prove that this is the case under  $2^{<c} = c$  (so, in particular, under Martin's Axiom) and in ZFC within the class of Urysohn spaces. (Received September 21, 2017)

1135-54-1543 **Fortunata Aurora Basile\*** ([basilef@unime.it](mailto:basilef@unime.it)), **Maddalena Bonanzinga** and **Nathan Carlson**. *New cardinality bounds for Urysohn spaces.*

Sapirskii proved that  $|X| \leq \pi\chi(X)^{c(X)\psi(X)}$ , for a regular space  $X$ . We introduce the  $\theta$ -pseudocharacter of a Urysohn space  $X$ , denoted by  $\psi_\theta(X)$ , and prove that if  $X$  is a Urysohn space then  $|X| \leq \pi\chi(X)^{Uc(X)\psi_\theta(X)}$ . The Urysohn cellularity of a space  $X$ , defined by Schröder, satisfies  $Uc(X) \leq c(X)$  and  $\psi(X) \leq \psi_c(X) \leq \psi_\theta(X) \leq \chi(X)$ . Note that if  $X$  is a regular space then  $Uc(X) = c(X)$  and  $\psi(X) = \psi_\theta(X)$ .

We also introduce new cardinal invariants:  $\theta\text{-}aL(X)$ ,  $\theta\text{-}aL'(X)$  and  $t_{\hat{c}}(X)$  in order to prove that if  $X$  is a Urysohn space then  $|X| \leq 2^{\theta\text{-}aL'(X)t_{\hat{c}}(X)\psi_\theta(X)}$  (\*). As  $\theta\text{-}aL(X) \leq aL(X)$  and  $t_{\hat{c}}(X)\psi_\theta(X) \leq \chi(X)$ , this represents an improvement of the Bella-Cammaroto inequality  $|X| \leq 2^{aL(X)\chi(X)}$ . The invariant  $\theta\text{-}aL'(X)$  is constructed by using maximal filters on the family of finite intersections of regular closed sets. Finally, we introduce a new class of topological spaces called *weakly H-closed*, a property related to H-closedness. It follows from (\*) that if  $X$  is a Urysohn, weakly H-closed space then  $|X| \leq 2^{\chi(X)}$ . (Received September 23, 2017)

1135-54-1655 **Jack Porter\*** ([porter@ku.edu](mailto:porter@ku.edu)), University of Kansas, 1460 Jayhawk Blvd, Department of Mathematics, Rm 405, Lawrence, KS 66045. *Extending the Tychonoff process of generating H-closed extensions.* Preliminary report.

In response to a question by Alexandroff and Urysohn in 1924, Tychonoff, in 1930, proved that every Hausdorff space can be embedded in an H-closed space, but he did not know if this embedding was dense. In 1992, the author showed that the Tychonoff embedding was dense. In this talk a number of the results in the 1992 paper about various H-closed extensions and  $\theta$ -covers are extended and analyzed. (Received September 24, 2017)

1135-54-1680 **John S Kulesza\*** ([jkulesza@gmu.edu](mailto:jkulesza@gmu.edu)). *The loss of the Lindelof property in products of Sorgenfrey-like spaces.*

For each subset  $A$  of  $R$ , one can define a topology on  $R$  so that points of  $A$  have usual Euclidean neighborhoods, while elsewhere points have Sorgenfrey neighborhoods. These spaces were introduced by Hattori who along with others, has determined many of the possible properties of  $X_A$ , the space determined by  $A$ . Here we are concerned with how these spaces behave with respect to the Lindelof property in powers of a single  $X_A$ , with these results:

- (1) For each  $n \in N$ , there is  $A \subset R$  for which  $X_A^{2n+1}$  is Lindelof while  $X_A^{2n+2}$  is not.
- (2) If  $X_A^{2n}$  is Lindelof, then  $X_A^{2n+1}$  must also be Lindelof.
- (3) There is an  $A \subset R$  for which, for all  $n \in N$ ,  $X_A^n$  is Lindelof, while  $X_A^N$  is not Lindelof.
- (4)  $X_A^n$  (or  $X_A^N$ ) is not Lindelof if and only if it has a closed and discrete subset of cardinality  $c$  if and only if it is not normal.

(Received September 24, 2017)

1135-54-1776 **Hanbo Shao\*** ([hanbo.shao@coloradocollege.edu](mailto:hanbo.shao@coloradocollege.edu)) and **Lyujiangnan Ye** ([jiangnan.ye@coloradocollege.edu](mailto:jiangnan.ye@coloradocollege.edu)). *On Computing Slice Genus of Non-alternating Prime Knots.*

Knot genus, in both the 3- and 4-dimensional settings, is a well-studied knot invariant. In this project, we calculate the smooth slice genus of the last remaining non-alternating prime knots of twelve or fewer crossings for which this invariant is previously unknown - 11 in all. We do this by performing band moves - additions and deletions - on a knot  $K$  to produce a torus cobordism in 4-ball with another knot  $J$ , and the slice genera of two knots are merely differed by one. Regarding different types of knots, we applied various methods to deduce the slice genus of  $K$  via analysis of  $J$ . (Received September 25, 2017)

1135-54-1817 **Tetsuya Ishiut\*** ([ishiut@miamioh.edu](mailto:ishiut@miamioh.edu)), 123 Bachelor Hall, 301 S. Patterson Ave, Oxford, OH 45056. *Finite products of connected nowhere real linearly ordered sets.*

I will present the result that every continuous injection from the product of finitely many connected nowhere real linearly ordered spaces to another such product is coordinate-wise. (Received September 25, 2017)

1135-54-1898 **Lynne Yengulalp\*** (lyengulalp1@udayton.edu), Lynne Yengulalp, University of Dayton, 300 College Park, Dayton, OH 45469. *Strategies in topological games and completeness.*

We will discuss generalized completeness properties and strategies in topological games. In particular, we will look at statements of the following form:

If  $X$  has completeness property  $P$  then the NONEMPTY player has a winning strategy of type  $S$  in the topological game  $G(X)$ .

We will present some theorems and open questions of that form. (Received September 25, 2017)

1135-54-1996 **Maddalena Bonanzinga\*** (mbonanzinga@unime.it), Messina, Italy, and **Mikhail Matveev.** *Monotone versions of some selection principles.*

A space  $X$  is monotonically Lindelöf if there is an operator  $r$  that assigns to every open cover  $\mathcal{U}$  a countable open cover  $r(\mathcal{U})$  so that  $r(\mathcal{U})$  refines  $\mathcal{U}$ , and  $r(\mathcal{U})$  refines  $r(\mathcal{V})$  whenever  $\mathcal{U}$  refines  $\mathcal{V}$ . We consider the monotone versions of the multcover forms of the Lindelöf property: Menger, Hurewicz, Rothberger.

Coauthor: Mikhail Matveev (Mikhail “Misha” Matveev passed away unexpectedly on May 17, 2011). (Received September 25, 2017)

1135-54-2108 **N. Carlson\*** (ncarlson@callutheran.edu), **J. Porter** and **G.J. Ridderbos.** *On Homogeneity and the H-closed Property.*

We discuss several results of Porter, Ridderbos, and the author concerning the H-closed property and topological homogeneity. A space  $X$  is H-closed if every open cover has a finite subfamily with dense union, and  $X$  is homogeneous if for all  $x, y \in X$  there exists a homeomorphism  $h : X \rightarrow X$  such that  $h(x) = y$ . First, we show that every Hausdorff space can be embedded in a homogeneous space that is the countable union of H-closed spaces. Second, it was shown by Motorov that every compact homogeneous space  $X$  is “1.5-homogeneous”; that is, for every closed set  $A \subseteq X$ ,  $x \in A$  and  $y \notin A$ , there exists a homeomorphism  $h : X \rightarrow X$  such that  $h(y) \in A$  and  $h(x) \notin A$ . We extend this result by showing that the above holds if  $X$  is an H-closed Urysohn homogeneous space and  $A \subseteq X$  is an H-set. Third, we show that the cardinality bound  $2^{t(X)}$ , shown to hold for a compact homogeneous space  $X$  by De La Vega, does not hold in general for H-closed homogeneous spaces. Last, we show the Katětov H-closed extension  $\kappa X$  is never homogeneous if  $X$  is non-H-closed, and the remainder  $\sigma X \setminus X$  in the H-closed Fomin extension  $\sigma X$  is never power homogeneous if  $X$  is locally H-closed. (Received September 25, 2017)

1135-54-2240 **Wade Bloomquist\***, bloomquist@math.ucsb.edu, and **Zhengan Wang.** *Spiders and Asymptotic Faithfulness.*

Quantum representations of mapping class groups arise from (2+1) dimensional TQFTs, and thus modular tensor categories using a construction of Turaev. Given a Lie algebra and a natural number called the level, there exists a modular tensor category and thus a mapping class group representation. Using techniques based on the spiders of Kuperberg for rank 2 Lie algebras we explore properties of these representations as the level tends towards infinity. These spiders exploit the graphical calculus of the modular tensor category in a way that generalizes the Temperley-Lieb algebra in the rank 1 case. (Received September 25, 2017)

1135-54-3111 **Ivan S Gotchev\*** (gotchevi@ccsu.edu), Mathematical Sciences Department, Central Connecticut State University, New Britain, CT 06050. *On cardinal inequalities involving the weak Lindelöf degree.* Preliminary report.

In 1977, Bell, Ginsburg and Woods proved that if  $X$  is a normal  $T_1$ -space, then  $|X| \leq 2^{\chi(X)wL(X)}$  and noticed that the same inequality is not always true for Hausdorff spaces, so they asked if the same inequality is true for every regular  $T_1$ -space. They also showed that not every completely regular space  $X$  satisfies the inequality  $|X| \leq 2^{\psi(X)t(X)wL(X)}$  and asked if the later inequality is true for normal  $T_1$ -spaces.

Trying to answer the above two (still open) questions, for a variety of classes of topological spaces  $X$ , Dow and Porter (in 1982), Gotchev, Tkachenko and Tkachuk (in 2016), and Bella and Carlson (in 2016), among others, proved interesting cardinal inequalities involving the weak Lindelöf degree.

In this talk, among other results, the following cardinal inequalities involving the weak Lindelöf degree will be presented, which shed light on the above-mentioned two open questions and generalize previously-known inequalities.

a) If  $X$  is a Hausdorff space which is locally  $H$ -closed (locally compact), locally Lindelöf or has a regular  $G_\delta$ -diagonal, then  $|X| \leq wL(X)\chi(X)$ ;

b) If  $X$  is a Hausdorff space with a  $\pi$ -base whose elements have compact closures, then  $|X| \leq 2^{\psi_c(X)t(X)wL(X)}$ . (Received September 26, 2017)

## 55 ► Algebraic topology

- 1135-55-112 **Rae Helmreich\*** ([helmreich\\_rae@wheatoncollege.edu](mailto:helmreich_rae@wheatoncollege.edu)), **Anchala Krishnan**, **Nathan Schmitz** and **John Meier**. *Persistent homology and probabilistic models of the Gaussian primes*. Preliminary report.
- Ilan Vardi introduced a probabilistic model for the distribution of Gaussian primes, thought of as a point cloud in  $\mathbb{R}^2$ . We compare patterns in the persistent homology of the Gaussian primes versus Vardi's probabilistic model, as well as two additional models, for the primes out to  $|z| \leq 200$ . The first homology provides statistical evidence that the models miss geometric features of the Gaussian primes. (Received July 28, 2017)
- 1135-55-167 **Sanath Devalapurkar\*** ([sanathd@mit.edu](mailto:sanathd@mit.edu)). *Roots of unity in  $K(n)$ -local stable homotopy theory*.
- We prove that no  $K(n)$ -local  $E_\infty$ -ring can have a primitive  $p^k$ -th root of unity in its homotopy. As a corollary, we find that the Lubin-Tate tower, an important object in arithmetic geometry, does not lift to a tower of derived stacks over Morava  $E$ -theory. (Received August 07, 2017)
- 1135-55-400 **Ivan Dungan\***, [gregory.dungan@usma.edu](mailto:gregory.dungan@usma.edu), and **Ettore Aldrovandi**, [aldrovandi@math.fsu.edu](mailto:aldrovandi@math.fsu.edu). *Algebraic Models of Homotopy  $n$ -Types*. Preliminary report.
- For a while now, we have known that groups model connected homotopy 1-types. In the 1940's, J.H.C. Whitehead introduced crossed modules and showed that there is a correspondence between crossed modules and connected homotopy 2-types. Behrang Noohi (re)proved the folklore that the homotopy category of crossed modules is equivalent to the category of connected homotopy 2-types. However, in forming the homotopy category of crossed modules, one must resort to computing derived morphisms using non-constructive topological methods, but Noohi was able to find an algebraic model of these derived morphisms called butterflies, thus making a completely algebraic model of connected homotopy 2-types. Reduced crossed complexes are a generalization of crossed modules (also introduced by Whitehead) and model a subclass of connected homotopy  $n$ -types. We will present algebraic objects called  $n$ -butterflies which satisfy similar properties to butterflies and begin to generalize the theory of butterflies to model morphisms of connected homotopy  $n$ -types. (Received August 31, 2017)
- 1135-55-528 **Hongwei Wang\*** ([hongwei.wang@tamiu.edu](mailto:hongwei.wang@tamiu.edu)), 302 Bob Bullock Loop, Apt 15301, Laredo, TX 78043. *The Action of Kauffman Bracket Skein Algebra of the Torus on the Skein Module of 3-Twist Knot Complement*.
- Skein modules were introduced by Przytycki to extend the new knot polynomials of the 1980's to knots and links in arbitrary 3-manifolds. They turned out to be very useful for constructing quantum invariants since they were closely related to topological quantum field theory. Let  $R$  be a commutative ring with identity. The skein modules are quotients of free modules of ambient isotopy classes of framed knots and links in orientable 3-manifolds by local skein relations. My research is a continued work of Răzvan Gelca and Fumikazu Nagasoto's work. We consider the particular case of the manifold being  $M = S^3 \setminus N(K)$  where  $N(K)$  is a regular neighborhood of the 3-twist knot  $K$ . We want to understand the action of  $K_t(\mathbb{T}^2 \times [0, 1])$  on  $K_t(S^3 \setminus N(K))$ . We modify the basis of Bullock and Lofaro to use Chebyshev polynomials of second kind  $S_n(x)$ . (Received September 07, 2017)
- 1135-55-678 **Yasuaki Hiraoka\*** ([hiraoka@tohoku.ac.jp](mailto:hiraoka@tohoku.ac.jp)), 2-1-1 Katahira, Aoba-ku, Sendai, Miyagi 980-8577. *Machine learnings on persistence diagrams and materials structural analysis*.
- Persistence diagrams have been widely recognized as a compact descriptor for characterizing multiscale topological features in data. When many datasets are available, statistical features embedded in those persistence diagrams can be extracted by applying machine learnings. In particular, the ability for explicitly analyzing the inverse in the original data space from those statistical features of persistence diagrams is significantly important for practical applications. In this talk, we propose a unified method for the inverse analysis by combining linear machine learning models with persistence images. We study RIDGE and LASSO on persistence diagrams, and introduce a useful concept called "sparse persistence diagram", which explicitly connects learning results to geometric structures. This method is first applied to point clouds and cubical sets, showing the ability of the statistical inverse analysis and its advantages. Then, I explain our recent activity of topological data analysis on materials science based on the this method. (Received September 12, 2017)
- 1135-55-770 **Christina Osborne\*** ([cdo5bv@virginia.edu](mailto:cdo5bv@virginia.edu)). *The classifying diagram of the category of finite sets*. Preliminary report.
- One can classify categories by using the nerve construction. But the nerve cannot determine the difference between certain types of categories. For example, the nerve cannot distinguish the difference between the trivial category and a category with two objects and one nontrivial morphism between the objects. Rezk's classifying

diagram is a generalization of the nerve and can distinguish the difference between these categories. In this talk, we will discuss applying the classifying diagram to the category of finite sets. Time permitting, we will also mention the classification diagram, which is a generalization of the classifying diagram, and our future research goals of using it in functor calculus. (Received September 14, 2017)

1135-55-779 **Killian Meehan, Andrei Pavlichenko and Jan Segert\*** (segertj@missouri.edu), Mathematics Department, 202 Math. Sci. Bldg., University of Missouri, Columbia, MO 65211. *On the Structural Theorem of Persistent Homology.*

We study the categorical framework for the computation of persistent homology, without reliance on a particular computational algorithm. The computation of persistent homology is commonly summarized as a matrix theorem, which we call the Matrix Structural Theorem. Any of the various algorithms for computing persistent homology constitutes a *constructive* proof of the Matrix Structural Theorem. We show that the Matrix Structural Theorem is equivalent to the Krull-Schmidt property of the category of filtered chain complexes. We separately establish the Krull-Schmidt property by abstract categorical methods, yielding a novel *nonconstructive* proof of the Matrix Structural Theorem.

These results provide the foundation for an alternate categorical framework for decomposition in persistent homology, bypassing the usual persistence vector spaces and quiver representations. (Received September 14, 2017)

1135-55-1282 **Nima Rasekh\*** (rasekh2@illinois.edu), Department of Mathematics, Altgeld Hall, 1409 W Green Street, Urbana, IL 61801, and **Ruth Davidson, Rosemary Guzman, Chuan (Sophie) Du, Adarsh Manawa, Christopher Szul and Titan Wibowo.** *Analyzing RGB Images using Topology: How to use discrete Morse theory to study crime data.*

Being able to understand and compare images is as of yet a very computationally demanding task. A group at Australian National University recently developed an open source code that can detect fundamental topological features of a grayscale image in a computationally feasible manner. This is made possible by the fact that a computer stores a grayscale image as a cubical cellular complex, which can be studied using the techniques of discrete Morse theory. We expand the functionality of this code by analyzing images encoded in red, green and blue rather than just grayscale via specifically designed surjective functions that allow us to extract the desired key information via informative persistence diagrams. This system allows us to perform data analysis directly on RGB images that encode meaningful data, such as water scarcity maps or crime data maps. (Received September 20, 2017)

1135-55-1586 **Greg Yang\*** (gregyang@microsoft.com). *A Homological Theory of Functions.*

Classical learning theory studies whether we can learn an unknown function within a known class of functions, given access to example input/output pairs. I will show that, surprisingly, the learnability of a class is controlled by its “homology” — specifically, I will associate to each class a simplicial complex, and I will prove that the VC dimension of the class is bounded above by, roughly speaking, the highest dimension of any nonvanishing homology group of this complex, with equality occurring for many common classes occurring in computer science or algebra. If time permits, I will also mention applications of this homological construction in computational complexity lower bounds and a “homological Farkas lemma” that characterizes, via homology, when a linear subspace intersects the positive cone. (Received September 23, 2017)

1135-55-1736 **Robert Short\*** (rss212@lehigh.edu). *Relative Topological Complexity of a Pair of Spheres.*

In the early 2000s, Michael Farber introduced the notion of Topological Complexity as an invariant addressing the least number of rules needed to perform motion planning for robots in a continuous way. This sparked interest in topological robotics, a field of applied algebraic topology which draws inspiration from problems in robotics to ask interesting topological questions. Much of the research in this field introduces different variants on Topological Complexity inspired by addressing different desirable properties of various robots.

In this talk, we introduce a new variant called the Relative Topological Complexity of a Pair. We draw inspiration from the problem of maneuvering a robot to a specified set of known goal configurations. As spheres arise naturally in both configuration spaces of robots and algebraic topology, we present how to calculate the values for this invariant for spheres as a simple example, and we discuss other possible applications of the result. (Received September 24, 2017)

1135-55-1744 **Henry Adams\*** ([adams@math.colostate.edu](mailto:adams@math.colostate.edu)). *The theory of Vietoris-Rips complexes.*  
 Given a metric space  $X$  and a scale parameter  $r > 0$ , the Vietoris-Rips simplicial complex has as its simplices the finite subsets of  $X$  of diameter less than  $r$ . These complexes have been applied to problems in computational topology and topological data analysis as a way to “thicken” a finite sampling. Indeed, when given a dataset sampled sufficiently densely from a manifold, the Vietoris-Rips complex of the dataset can be used to recover topological information about the manifold. In this talk I will survey known results and open questions about Vietoris-Rips complexes of manifolds, and describe Vietoris-Rips complexes of  $n$ -spheres at the first scale parameter when the homotopy type changes. (Received September 24, 2017)

1135-55-1754 **Killian Meehan** and **David C Meyer\*** ([meyerd@missouri.edu](mailto:meyerd@missouri.edu)). *Generalized persistence modules and taking limits.*  
 Generalized persistence modules are used in topological data analysis to distinguish noise from the legitimate topological features of a finite data set. From an algebraic perspective, generalized persistence modules are finitely-generated modules for a poset algebra. We prove an algebraic analogue of the isometry theorem of Bauer and Lesnick for a large class of posets. This theorem shows that for such posets, the interleaving metric of Bubenik, de Silva and Scott can be realized as a bottleneck metric which incorporates some algebraic information. In addition, when two generalized persistence modules come from data, we associate to them a directed set of algebras over which they can be compared. We recover their classical interleaving distance by taking limits. (Received September 24, 2017)

1135-55-1820 **Leo Carlsson\*** ([leoc@kth.se](mailto:leoc@kth.se)), Maltgatan 12, apartment 1208, 12079 Stockholm, Sweden, and **Mattia De Colle**, **Christoffer Schmidt**, **Mikael Vejdemo-Johansson** and **Pär Jönsson**. *Topology in the Furnace: Using the Mapper Algorithm as a Data Analysis Tool to Evaluate an Electric Arc Furnace Energy Model.*

Finding areas of improvement in metallurgical processes using data analysis is often a complicated task due to the high-dimensionality and complexity of the data. In this study, we explore applications of TDA to analyze a metallurgical process. More specifically, we use the Mapper algorithm as a means to evaluate an energy model that estimates the end-point temperature of steel in an electric arc furnace. A subgroup with high model errors was identified qualitatively by analyzing the output from the Mapper algorithm, which was created by using a high-dimensional and complex data set from the furnace. It was found, through statistical analysis, that the measured energy input in the subgroup was a lot higher than the target energy calculated by the energy model. Two commonly used data analysis methods were used to compare the results from the Mapper algorithm to best practices in data analysis. The identified subgroups were not identifiable as separated subgroups using the other methods. (Received September 25, 2017)

1135-55-1825 **Ananth Kalyanaraman**, **Methun Kamruzzaman** and **Bala Krishnamoorthy\*** ([kbala@wsu.edu](mailto:kbala@wsu.edu)). *Maximal interesting paths in the Mapper.*

The *Mapper* is a highly compact representation of high-dimensional data as a simplicial complex that captures its underlying structure. It has seen increased use recently in various domains ranging from biomedicine to analysis of voting patterns of lawmakers. Researchers have also been working on several algebraic topological aspects of the Mapper, e.g., stability under various choices of parameters. At the same time, the process of navigating the Mapper and extracting testable hypotheses related to the original data has not received much attention. We formulate this problem as one of identifying maximal interesting paths in the Mapper. We study the complexity of this problem. We then present an efficient heuristic based on dynamic programming. We illustrate the effectiveness of our method on a corn phenomics data set. (Received September 25, 2017)

1135-55-1924 **Benjamin Schweinhart\*** ([schweinhart.2@osu.edu](mailto:schweinhart.2@osu.edu)), 231 West 18th Avenue, Mathematics Tower, Columbus, OH 43210. *Persistent Homology and Fractal Dimension.*

We define a notion of a fractal dimension for a subset  $X$  of  $\mathbb{R}^d - \dim_{\text{PH}_i}(X)$  – in terms of the persistent homology of finite point samples of  $X$ . This differs from our previous definition of a persistent homology dimension, which was based on the persistent homology of  $X$  itself and bounds  $\dim_{\text{PH}_i}(X)$  from below. We exhibit hypotheses on  $i$ ,  $d$ , and  $X$  under which  $\dim_{\text{PH}_i}(X) = \dim_{\text{box}}(X)$ , the box-counting dimension of  $X$ . (Received September 25, 2017)

1135-55-1983 **Radmila Sazdanovic** and **Daniel Scofield\***, [dscofie@ncsu.edu](mailto:dscofie@ncsu.edu). *Patterns in Khovanov link and chromatic graph homology.*

Khovanov homology of a link and chromatic graph homology are known to be isomorphic in a range of homological gradings that depend on the girth of a graph. In this talk, we discuss patterns shared by these two homology

theories. In particular, we improve the bounds for the homological span of chromatic homology by Helme-Guizon, Przytycki and Rong. An explicit formula for the rank of the third chromatic homology group on the main diagonal is given and used to compute the corresponding Khovanov homology group and the fourth coefficient of the Jones polynomial for links with certain diagrams. (Received September 25, 2017)

1135-55-2210 **Ayah Almousa, Derek Francour and Megan Maguire\*** ([mmaguire2@math.wisc.edu](mailto:mmaguire2@math.wisc.edu)). *Cohomology algebras of configuration spaces: preliminary report*. Preliminary report.

For a manifold  $X$  with finite-dimensional cohomology, we know that the cohomology algebra of each unordered configuration space of  $X$  is finitely generated, but can we say something stronger about its generators? More precisely, does there exist a  $D$  (depending only on  $X$ ) so that the cohomology algebra of each unordered configuration space of  $X$  can be generated in degree at most  $D$ ? We will answer this question in some cases. (Received September 25, 2017)

1135-55-2255 **Greg Malen\*** ([gmalen@math.duke.edu](mailto:gmalen@math.duke.edu)). *Dense Random Clique Complexes*.

One model for percolation on high-dimensional simplicial complexes is  $X(n, p)$ , the probably space of Vietoris-Rips complexes, or clique complexes, on  $n$  vertices where each edge appears independently with probability  $p$ . This is a natural extension of the Erdős-Rényi random graph model  $G(n, p)$  into higher dimensions. Whereas the random  $d$ -dimensional complexes  $Y_d(n, p)$  provide high-dimensional analogues to the phase transitions observed in sparse Erdős-Rényi random graphs, here we examine the evolution of  $X(n, p)$  as  $p$  increases into the dense and super-dense regimes. In particular, around middle dimension we exhibit non-trivial homology in an interval with length that increases from arbitrary finite lengths when  $p = (\log n)^{-\alpha}$ , to  $O(\log \log n)$  when  $p$  is constant. We also discuss various homology vanishing and collapsibility results, and examine bounds for the threshold in  $p$  at which  $X(n, p)$  becomes contractible with high probability. (Received September 25, 2017)

1135-55-2335 **Greg Dreifus\*** ([gdreifus@mit.edu](mailto:gdreifus@mit.edu)), Department of Mechanical Engineering, 35-214, Massachusetts Institute of Technology, Cambridge, MA 02139, and **Fred Cohen** ([cohf@math.rochester.edu](mailto:cohf@math.rochester.edu)), Department of Mathematics, University of Rochester, Rochester, NY 14625. *A Topological and Algebraic Model for 3D Printing*. Preliminary report.

The purpose of this project is to give a model, which we call  $AM^1(C, L)$ , for 3D printing as follows. Consider the set of ordered pairs  $(\vec{a}, \vec{b})$ , where  $\vec{a}$  is in the graph  $C = (V, E)$  embedded in  $\mathbb{R}^3$ , where  $V$  and  $E$  are the typical vertex and edge sets in graph theory,  $\vec{b}$  is in the x-y plane, and the distance between them is some fixed distance  $L$ . Let  $AM^1(C, L)$  be topologized as a subspace of Euclidean space. We prove that the fundamental group is a product in non-singular cases and that  $\pi_1 AM^1(C, L) = \pi_1(C)$  in singular cases. We examine the connectedness of a subspace of  $AM^1(C, L)$ , which we call  $AM^2(C, L)$ , by restricting the position of the line segment connecting  $\vec{a}$  and  $\vec{b}$  to only intersect with  $C$  at point  $\vec{a}$ , and we examine the connectedness of a subspace of  $AM^2(C, L)$ , denoted  $AM^3(C, L)$ , by imposing a parameterization on  $\vec{a}$ . Analogues with multiple linkages using the configuration space of distinct particles in  $C$  are also developed. (Received September 25, 2017)

1135-55-2387 **Gregory Henselman\*** ([greg.henselman@gmail.com](mailto:greg.henselman@gmail.com)), 2600 South Street, Apt 3F, Philadelphia, PA 19146. *Morse-Witten Theory for Real Operators*. Preliminary report.

Accelerated by applications in mathematical physics from the late 20th century, the interaction of the theories of Morse and Hodge in smooth geometry has propelled remarkable advances in geometric topology over the past four decades. Progress in this domain has considerably outpaced parallel investigations in combinatorial topology, where several of the most basic questions regarding spectral analysis of discrete Morse structures remain outstanding. The present talk introduces a discrete Morse-Witten theory for real-linear operators, a direct extension of the Morse-Witten theory for CW complexes pioneered by Forman in the late 1990's. Time permitting, we will discuss some consequences for spectral analysis of cellular spaces, the surprisingly categorical underpinnings of the Morse-Witten complex, and several future directions. No prior knowledge of Morse-Witten theory will be assumed, smooth or otherwise. (Received September 26, 2017)

1135-55-2454 **Lori Beth Ziegelmeier\*** ([lziegel1@macalester.edu](mailto:lziegel1@macalester.edu)), 1600 Grand Avenue, Saint Paul, MN 55105. *A Complete Characterization of the 1-Dimensional Intrinsic Cech Persistence Diagrams for Metric Graphs*.

Metric graphs are special types of metric spaces used to model and represent simple, ubiquitous, geometric relations in data such as biological networks, social networks, and road networks. We are interested in giving a qualitative description of metric graphs using topological summaries. In particular, we provide a complete characterization of the 1-dimensional intrinsic Cech persistence diagrams for finite metric graphs using persistent homology. Together with complementary results by Adamaszek et al., which imply results on intrinsic



Cech persistence diagrams in all dimensions for a single cycle, our results constitute important steps toward characterizing intrinsic Cech persistence diagrams for arbitrary finite metric graphs across all dimensions. This is joint work with Ellen Gasparovic, Maria Gommel, Emilie Purvine, Radmila Sazdanovic, Bei Wang, and Yusu Wang. (Received September 26, 2017)

1135-55-2465 **Radmila Sazdanovic\*** (rsazdan@ncsu.edu), Department of Mathematics NCSU, Raleigh, NC 27695-8205. *Persistence-Based Summaries for Metric Graphs*.

Metric graphs are omnipresent in data analysis and so are the methods and algorithms for analyzing them. The topic of this talk is analyzing metric graphs using persistent homology with a goal of capturing more intricate graph properties. In particular, we construct qualitative/quantitative summaries of metric graphs, compare their discriminative powers, and describe graph properties detected by these persistence-based summaries. This is joint work with Ellen Gasparovic, Maria Gommel, Emilie Purvine, Bei Wang, Yusu Wang, and Lori Ziegelmeier. (Received September 26, 2017)

1135-55-2505 **Killian F. Meehan\*** (kfmb88@mail.missouri.edu) and **David C. Meyer**. *Persistence and stability for the quiver  $\mathbb{A}_n$* . Preliminary report.

Persistent homology uses generalized persistence modules to discern the topological properties of a finite data set. One typically endows the collection of generalized persistence modules with two different metric structures and proves an algebraic stability theorem showing that the identity function is a contraction or an isometry. When a persistence module comes from data, it admits the structure of a representation of a finite, totally ordered set whose Hasse quiver will be an equioriented  $\mathbb{A}_n$  quiver. Since any orientation of the  $\mathbb{A}_n$  quiver in fact corresponds to the Hasse quiver of some finite poset, it is natural to wonder whether one can prove a stability theorem for these posets. We compare various metrics in this setting. (Received September 26, 2017)

1135-55-2514 **Daniel Cicala, Liron Cohen, Nachiket Karnick, Chandrika Sadanand, Michael Shulman, Amelia Tebbe\*** (antebbe@iu.edu) and **Dmitry Vagner**. *Applications of Cohesive Homotopy Type Theory*. Preliminary report.

In this talk, we apply Shulman's ideas from *Brouwer's Fixed-point Theorem in Real-cohesive Homotopy Type Theory* to other classical results. We consider both a 'sharp' version (i.e. selecting an exact result discontinuously) and an 'approximate' version (i.e. selecting an approximate result continuously) of the following theorems: the fundamental theorem of algebra,  $n$ -dimensional Brouwer Fixed-Point, and  $2n$ -dimensional Hairy Ball. The sharp versions for each are proven, but the only approximate version proven is the fundamental theorem of algebra. However, we do give partial results, which rely on a higher dimensional Van Kampen pushout of a contractible object. Additionally, we give partial results on both sharp and approximate versions of the Borsuk-Ulam theorem via the fact that odd maps on spheres have odd degree. To this end, we investigate getting a 0-type for 2-dimensional real protective space as a homotopy colimit. We conjecture this method can be used to obtain higher dimensional real protective spaces by induction. The sharp version of Borsuk-Ulam follows from this conjecture. (Received September 26, 2017)

1135-55-2518 **Chi-Kwong Fok\*** (chi-kwong.fok@adelaide.edu.au), School of Mathematical Sciences, Ingkarni Wardli Building, the University of Adelaide, Adelaide, SA 5005, Australia, and **Jeffrey D Carlson** (jcarlson@math.toronto.edu), 40 St. George Street, Toronto, Ontario M5S 2E4, Canada. *Equivariant formality of homogeneous spaces*.

Equivariant formality, a notion in equivariant topology introduced by Goresky-Kottwitz-Macpherson, is a desirable property of spaces with group actions. Broad classes of spaces of especial interest are well-known to be equivariantly formal, e.g., compact symplectic manifolds equipped with Hamiltonian compact Lie group actions and projective varieties equipped with linear algebraic torus actions, of which flag varieties are examples. Less is known about compact homogeneous spaces  $G/K$  equipped with the isotropy action of  $K$ , which is not necessarily of maximal rank. In this talk we will review previous attempts of characterizing equivariant formality of  $G/K$ , and present our recent results on this problem using an analogue of equivariant formality in  $K$ -theory. Part of the work presented in this talk is joint with Jeffrey Carlson. (Received September 26, 2017)

1135-55-2595 **Maria Gommel\*** (maria-gommel@uiowa.edu). *Using Topology to Study the Brain: An Analysis of fMRI data using TDA*. Preliminary report.

Topological Data Analysis (TDA) is a relatively new area of study that uses tools from algebraic topology to uncover the underlying shape of a given data set. These methodologies can readily be applied to neuroscience data. My work uses TDA to analyze correlation matrices obtained from resting-state fMRI brain scans of healthy controls, where the entries in the matrix represent correlations between regions of the brain. We compute persistent homology and obtain a persistence diagram for each correlation matrix. We then use several methods

to attempt to group the persistence diagrams by gender and by age, looking for notable differences between groups. We aim to extend this work by comparing the persistence diagrams of data from children who will develop Huntington's disease to those from healthy controls. (Received September 26, 2017)

1135-55-2778 **Alex Sherbetjian\*** ([sherbet@math.ucr.edu](mailto:sherbet@math.ucr.edu)). *Rigidification of algebras over algebraic theories in diagram categories*. Preliminary report.

The notion of algebraic theories, which are able to describe many algebraic structures, has been used extensively since its introduction by Lawvere in 1963. This perspective has been very fruitful for understanding in a wide variety of algebraic structures, including rigidification results for simplicial algebras over algebraic theories by Badzioch and Bergner. In this talk, we will extend the rigidification results to algebras over a larger class of categories, which includes bisimplicial sets. (Received September 26, 2017)

1135-55-2831 **Woojin Kim and Facundo Memoli\*** ([memoli@math.osu.edu](mailto:memoli@math.osu.edu)), 100 Math Tower, Columbus, OH 43210. *Stable signatures for dynamic metric spaces via persistent homology*.

When studying flocking/swarming behaviors in animals one is interested in quantifying and comparing the dynamics of the clustering induced by the coalescence and disbanding of animals in different groups. Motivated by this we study the problem of obtaining persistent homology based summaries of time dependent metric data. In particular, we study the stability of this construction under a suitable variant of the Gromov-Hausdorff distance. (Received September 26, 2017)

1135-55-2841 **Dan Christensen** ([jdc@uwo.ca](mailto:jdc@uwo.ca)), London, Ontario, Canada, **Morgan Opie** ([opie@math.harvard.edu](mailto:opie@math.harvard.edu)), Cambridge, MA, **Egbert Rijke\*** ([erijke@andrew.cmu.edu](mailto:erijke@andrew.cmu.edu)), Pittsburgh, PA, **Luis Scoccola\*** ([lsoccol1@uwo.ca](mailto:lsoccol1@uwo.ca)), London, Ontario, Canada, and **Aliaksandra Yarosh** ([aly21@pitt.edu](mailto:aly21@pitt.edu)), Pittsburgh, PA. *Localization at degree  $p$  maps in Homotopy Type Theory*.

Homotopy type theory (HoTT) is an extension of Martin-Löf's dependent type theory with the univalence axiom and higher inductive types. In HoTT we can interpret types as spaces, and many familiar constructions of homotopy theory can be translated. We use higher inductive types to define the localization at the degree  $p$  maps on the circle, exhibiting degree  $p$ -local types as a reflective subuniverse. Our main result is that the localization of a simply connected pointed type at the degree  $p$  map localizes its homotopy groups away from  $p$ . In doing this we characterize the loop spaces of the  $A$ -nullification of a type, and we give a characterization of the localization at the degree  $p$  map of an  $H$ -space as a sequential colimit. (Received September 26, 2017)

1135-55-2856 **Dominic Klyve\*** ([klyved@cwu.edu](mailto:klyved@cwu.edu)), Dept. of Mathematics, 400 E University Way, Ellensburg, WA 98926, and **Nicholas Scoville** ([nscoville@ursinus.edu](mailto:nscoville@ursinus.edu)), 601 E Main St., Collegeville, PA 19426. *Number Theory meets Graphs in Discrete Morse Theory*.

A **discrete Morse function**  $f$  on a connected graph  $G$  is a function  $f : G \rightarrow \mathbb{R}$  such that for every vertex  $v \in G$ ,  $|\{e : f(v) \geq f(e) \text{ for some edge } e \text{ incident to } v\}| \leq 1$ , and for every edge  $e$ ,  $|\{v : f(v) \geq f(e) \text{ for some vertex } v \text{ incident to } e\}| \leq 1$ . It turns out that can use number-theoretic functions to assign values to the vertices and edges of a graph in such a way that the assignment is a discrete Morse function. This talk will explore some of the work done by students in exploring the relationship between these number theory functions and their emergent topological properties, and suggest new avenues to explore. (Received September 26, 2017)

1135-55-3063 **Santana F. Afton\*** ([csafton@email.wm.edu](mailto:csafton@email.wm.edu)), CSU 0433, 110 Sadler Center, Williamsburg, VA 23185, and **Samuel Freedman, Justin Lanier and Liping Yin**. *Taming the Loch Ness Monster: Symmetries of Infinite Surfaces*. Preliminary report.

The mapping class group of a surface is an algebraic object that encodes the symmetries of the surface. Dehn showed that for many surfaces the mapping class group can be generated by certain infinite-order elements called Dehn twists. However, this result does not apply to "big" surfaces that have an infinite number of holes. Patel and Vlamis recently produced the first generating set for an important subgroup of the mapping class group of a big surface. Their generating set is comprised of Dehn twists as well as symmetries unique to big surfaces called handle shifts. Our work focuses on finding algebraic relations between Dehn twists and handle shifts. As a consequence, we show that big mapping class groups are generated by only a subset of their handle shifts. Additionally, we construct elements of arbitrary order for all surfaces of infinite genus. This is joint work with Justin Lanier and Liping Yin. (Received September 26, 2017)

1135-55-3094 **Matthew Hogancamp\*** (hogancam@usc.edu) and **Eugene Gorsky**. *Link splitting and y-ification of Khovanov-Rozansky homology*.

I will discuss a recently constructed HOMFLY-PT analogue of the Batson-Seed invariant. In this theory, the invariant of a link  $L$  is a triply graded module over a polynomial ring in variables  $x_c, y_c$ , where  $c$  ranges over the components of  $L$ . This invariant has link splitting properties analogous to the Batson-Seed invariant, and conjecturally restores the missing  $q \leftrightarrow tq^{-1}$  symmetry of Khovanov-Rozansky homology for links, and matches several predictions coming from a conjectural connection with the Hilbert scheme of points in the plane. This is joint work with Eugene Gorsky. (Received September 26, 2017)

1135-55-3129 **Maksym Zubkov\*** (mzubkov@uci.edu), 13756 Cantlay St, Van Nuys, CA 91405. *Homotopy Commutative Algebras, Knots and Graphs*. Preliminary report.

Earlier Helme-Guizon and Rong have defined a chromatic graph homology complex for a graph  $G$  and the coefficient commutative DG algebra  $A$ . In a recent joint paper with V. Baranovsky we have extended this definition to the case when  $A$  is an algebra over the brace operad. I plan to explain that when  $A$  is the cochain algebra of a compact oriented manifold  $M$ , this complex computes the homology of a generalized configuration space of  $M$ , constructed from  $G$ . When  $G$  is the complete graph this reduces to the usual configuration space of ordered sets of distinct points in  $M$ . (Received September 26, 2017)

## 57 ► *Manifolds and cell complexes*

1135-57-42 **Vi Hart, Andrea Hawksley, Elisabetta A Matsumoto and Henry Segerman\*** (segerman@math.okstate.edu). *Using quaternions to colour the cells of a tiling of  $\mathbb{H}^3$* .

As part of a recent project to create a virtual reality experience of movement within  $\mathbb{H}^3$ , we needed some form of decoration to put in the space, so that users would be able to see the effects of their movement. We chose to use the  $\{4, 3, 6\}$  tiling of  $\mathbb{H}^3$  by ideal cubes, with six cubes meeting around each edge. This works well, but if all cubes are rendered identically, it is difficult to keep track of movements in which the user moves through multiple cubes. I will describe how we used quaternions to produce an eight-colouring of the  $\{4, 3, 6\}$  tiling, giving users an improved set of landmarks to track their movement. The colouring arises as a kind of branched cover over the  $\{4, 3, 3\}$  tiling of  $S^3$ . (Received June 29, 2017)

1135-57-91 **Erica L Sawyer\*** (ericas1026@gmail.com), **Rita L Post** and **Abigayle L Dirdak**. *Virtual spatial trivalent graphs and braids*. Preliminary report.

A virtual spatial trivalent graph diagram (virtual STG diagram) is a trivalent graph immersed in a plane, which contains finitely many transverse double points, each of which has information of over/under or virtual crossings. We regard virtual STG diagrams as combinatorial objects up to an equivalence relation induced by certain combinatorial moves for virtual STG diagrams. Then a virtual spatial trivalent graph is the equivalence class of a virtual STG diagram.

A virtual trivalent braid is a braid similar to the notion of a classical braid, but may contain trivalent vertices and virtual crossings, in addition to classical crossings. The closure of a virtual trivalent braid with  $n$  endpoints on top and  $n$  endpoints on the bottom is a virtual STG diagram. Therefore, we can study virtual trivalent braids to gain information about virtual spatial trivalent graphs.

In this talk we describe our method for converting any virtual STG diagram into an equivalent diagram in braid form. We also provide conditions for having two virtual trivalent braids whose closures yield equivalent virtual STG diagrams. In other words, we provide Alexander- and Markov-type theorems for virtual spatial trivalent graphs and virtual trivalent braids. (Received July 25, 2017)

1135-57-98 **Gabriel A. Coloma\*** (gabriel.coloma@upr.edu) and **Marguerite R. Davis**. *Trivalent braids and their closures*. Preliminary report.

A spatial trivalent graph is an embedding of a trivalent graph in three-dimensional space. A diagram of a spatial trivalent graph  $G$  is a projection of  $G$  into a plane. In this research, we study these objects by working with trivalent braids and their closures. Similar to a classical braid, a trivalent braid contains classical crossings. In addition, a trivalent braid may contain trivalent vertices. We develop a braiding algorithm for converting a diagram of an oriented spatial trivalent graph into the closure of a trivalent braid. We also use our braiding algorithm to describe trivalent braids whose closures represent the same spatial trivalent graph. (Received July 26, 2017)

1135-57-104 **David Freund\*** ([dfreund@math.dartmouth.edu](mailto:dfreund@math.dartmouth.edu)). *Complexity of virtual multistrings*. Preliminary report.

A virtual  $n$ -string  $\alpha$  is a collection of  $n$  closed curves on an oriented surface  $M$ . Associated to  $\alpha$ , there are two natural measures of complexity: the genus of  $M$  and the number of intersection points. By considering virtual  $n$ -strings up to equivalence by virtual homotopy, i.e., homotopies of the component curves and stabilizations/destabilizations of the surface, a natural question is whether these quantities can be minimized simultaneously. We show that this is possible for non-parallel virtual  $n$ -strings and that, moreover, such a representative can be obtained by monotonically decreasing genus and the number of intersection points from any initial representative. (Received July 28, 2017)

1135-57-148 **Joseph A. Quinn\*** ([josephanthonyquinn@gmail.com](mailto:josephanthonyquinn@gmail.com)) and **Alberto Verjovsky Sola** ([alberto@matcuer.unam.mx](mailto:alberto@matcuer.unam.mx)), Instituto de Matemáticas, Unidad Cuernavaca, Universidad Nacional Autónoma de México, Av. Universidad s/n., Col. Lomas Chamilpa, 62210 Cuernavaca, Morelos, Mexico. *Hilbert-Blumenthal Quaternionic Surfaces*. Preliminary report.

The group  $\mathrm{PSL}_2(\mathcal{O})$ , where  $\mathcal{O}$  is an order in Hamilton's quaternions, acts discretely by Möbius transformations on hyperbolic 4- or 5-space (depending on the definition used for  $\mathrm{PSL}_2$ ), giving rise to hyperbolic 4- or 5-manifolds. We introduce a generalization of this where  $\mathcal{O}$  is instead an order in a definite quaternion algebra over a real quadratic number field, and the action now occurs on a product of two copies of hyperbolic 4- or 5-space via a Galois twist (analogous to the classical Hilbert-Blumenthal surfaces), giving rise to 8- or 10-dimensional manifolds.

We present a fundamental domain for the cusp of such a manifold, which facilitates the study of its topology and dynamics. We also discuss analogous new fundamental domains for classical Hilbert-Blumenthal surfaces that were developed concurrently by the authors with the intention of generalizing to the quaternionic version, and which serve nicely to build intuition for the higher-dimensional setting. (Received August 03, 2017)

1135-57-178 **Jay J. Zimmerman\*** ([jjzimmerman@towson.edu](mailto:jjzimmerman@towson.edu)), **Angel V. Kumchev** and **Coy L. May**. *The Genus Spectrum of Abelian Groups*.

The set  $S$  of positive integers that may appear as the genus of a finite abelian group is called the genus spectrum of abelian groups. We will look at the genus spectrum of abelian groups for the strong symmetric genus  $S_\sigma$  and for the real genus  $S_\rho$ . We obtain a set of (simple) necessary and sufficient conditions for an integer  $g$  to belong to  $S_\sigma$ . We also prove that the set  $S_\sigma$  has an asymptotic density and that density is approximately .3284. The situation for the real genus is considerably more complicated. We obtain a set of necessary conditions for an integer  $g$  to belong to  $S_\rho$ . We also prove that the real genus of an abelian group is not congruent to 3 (modulo 4) and that the real genus of an abelian group of odd order is a multiple of 4. Finally, we obtain upper and lower bounds for the density of the set  $S_\rho$ . (Received August 07, 2017)

1135-57-291 **Eugene Gorsky\***, Department of Mathematics, One Shields Avenue, Davis, CA 95616, and **Andrei Negut** and **Jacob Rasmussen**. *Khovanov-Rozansky homology and Hilbert schemes*.

I will outline several conjectures relating the Khovanov-Rozansky homology of knots and links to algebraic geometry of the Hilbert scheme of points on the plane. For a large class of knots and links, this yields a very concrete combinatorial description of Khovanov-Rozansky homology, which was recently confirmed by Elias, Hogancamp and Mellit. (Received August 19, 2017)

1135-57-338 **Colin C Adams\*** ([cadams@williams.edu](mailto:cadams@williams.edu)), Bascom Hall, 33 Stetson Court, Williamstown, MA 01267. *Open and accessible problems in knot theory*.

Knot theory has pretty pictures and deep mathematics. As such, it is an excellent area in which to involve students in research. We will discuss some generalizations of crossing number - including petal number, when knots are put into projections that resemble daisies - and list a variety of open problems appropriate for undergraduate research. (Received August 25, 2017)

1135-57-419 **Nicolas Petit\*** ([petitnicola@gmail.com](mailto:petitnicola@gmail.com)), 110 Few Circle, Oxford, GA 30054. *Finite-type invariants of virtual knots and tangles*. Preliminary report.

We will discuss some recent discoveries regarding finite-type invariants of order one for virtual knots and virtual tangles (Received September 01, 2017)

1135-57-635 **Bakul Sathaye\*** ([sathaye.2@osu.edu](mailto:sathaye.2@osu.edu)), 231 West 18th Avenue, Columbus, OH 43210.  
*Obstructions to Riemannian smoothings of a locally CAT(0) manifold.*

In this talk I will discuss obstructions to Riemannian smoothings of a locally CAT(0) manifold. I will focus on obstructions in dimension = 4 given by Davis-Januszkiewicz-Lafont and show how their methods can be extended to construct more examples of locally CAT(0) 4-manifolds  $M$  that do not support Riemannian metric with non-positive sectional curvature. Further, the universal cover of such a manifold,  $\tilde{M}$ , satisfies the isolated flats condition and contains a collection of 2-dimensional flats with the property that their boundaries at infinity form non-trivial links in  $\partial^\infty \tilde{M}$ . (Received September 11, 2017)

1135-57-654 **Valentas Kurauskas\*** ([valentas@gmail.com](mailto:valentas@gmail.com)), Institute of Mathematics and Informatics, Vilnius University, Akademijos 4, LT-04812 Vilnius, Lithuania, and **Ugnė Šiurienė**.  
*Symmetric road interchanges.*

A road interchange where  $n$  roads meet and in which drivers are not allowed to change lanes can be naturally modeled as an embedding of a 2-colored (hence bipartite) multigraph  $G$  with equal-sized color classes into an orientable surface such that there is a face bounded by a Hamiltonian cycle. The genus of the underlying surface can be interpreted as the number of bridges in the interchange.

Motivated by this, we study the embeddings of  $K_{n,n}$  where one of the faces is bounded by a Hamiltonian cycle. We determine the minimum genus of such  $n$ -fold rotationally symmetric embeddings (equivalently, the minimum number of bridges in a complete  $n$ -way interchange which is symmetric under the cyclic permutation of its roads). We consider both (a) abstract combinatorial/topological symmetry, and (b) symmetry in the 3-dimensional Euclidean space  $\mathbb{R}^3$ . (Received September 12, 2017)

1135-57-744 **Christian Millichap\*** ([cmillich@linfield.edu](mailto:cmillich@linfield.edu)) and **David Futер**. *Mutations and geometric invariants of hyperbolic links and 3-manifolds.*

A mutation is a modification of a link complement (and more generally, a 3-manifold) obtained by cutting along a hyperelliptic surface, rotating this surface by a hyperelliptic involution, and then regluing along this surface. While mutations often produce a different link or manifold, they are notorious for preserving many classical, quantum, and geometric invariants. In this talk, we will examine conditions under which mutations preserve many geometric invariants of hyperbolic 3-manifolds and discuss some open questions in this area. Geometric invariants of interest will include the volume, the length spectrum, the area spectrum, and the Cheeger constant of a hyperbolic 3-manifold. Part of this project comes from joint work with David Futер. (Received September 13, 2017)

1135-57-772 **Kenneth C Millett\*** ([millett@math.ucsb.edu](mailto:millett@math.ucsb.edu)), Department of Mathematics, UCSB, Santa Barbara, CA 93106. *Knotting and Linking in Proteins.*

Since 2000, it has been widely understood that some proteins contain knots and, as a consequence, the geometrical and topological character of spatial structures has been of interest to mathematicians. Beginning with the ongoing characterization of knotting and the current study of linking, we will consider the underlying mathematics, the variety of knotting and linking that is encountered, and discuss a few of the more enticing current research directions. (Received September 14, 2017)

1135-57-827 **Charles Frohman\*** ([charles-frohman@uiowa.edu](mailto:charles-frohman@uiowa.edu)), **Joanna Kania-Bartoszyńska** ([jkaniaba@nsf.gov](mailto:jkaniaba@nsf.gov)) and **Thang Le** ([1etu@math.gatech.edu](mailto:1etu@math.gatech.edu)). *The structure of the Kauffman bracket skein algebra of a closed surface at roots of unity.*

Let  $F$  be a closed oriented surface and  $\zeta$  an  $n^{\text{th}}$  root of unity. The Kauffman bracket skein algebra  $K_\zeta(F)$  is a prime, affine algebra of finite rank over its center. If  $n \not\equiv 0 \pmod{4}$  the center of  $K_\zeta(F)$  is the ring of  $SL_2\mathbb{C}$ -characters of the fundamental group of  $F$ . If  $n \equiv 0 \pmod{4}$  The center is a twisted version of the same ring. We compute the dimension of the algebra over its center. The localized algebra is Frobenius, and we compute the counit. Finally, we show that the skein algebra as a module over its center is the tensor product of two commutative subalgebras associated with pants decompositions of the surface. (Received September 15, 2017)

1135-57-887 **Laura Starkston\*** ([l1starkst@stanford.edu](mailto:l1starkst@stanford.edu)). *Planar curves symplectified!*

Algebraic curves in the complex projective plane can be explicitly understood as the zero set of a degree  $d$  homogeneous polynomial. They can develop singularities like nodes and cuspidal points, and the moduli space of curves with a particular set of singularities can become very complicated. Complex curves are particular examples of symplectic submanifolds of  $CP^2$ , and we can ask similar questions about the space of all symplectic submanifolds of a particular degree with particular singularities. These questions can be much harder since symplectic submanifolds make up an infinite dimensional space. I will discuss initial explorations of symplectic submanifolds of  $CP^2$  (with prescribed singularities). I will describe some of my results that give similarities

between the symplectic and complex categories, and also some giving differences (parts are joint with Golla). (Received September 16, 2017)

1135-57-1023 **Michael S Willis\*** ([mike.willis@math.ucla.edu](mailto:mike.willis@math.ucla.edu)), UCLA Mathematics Department, 520 Portola Plaza, Los Angeles, CA 90095, and **Michael Abel**. *Colored Khovanov-Rozansky Homology for Infinite Braids*.

We show that the limiting unicolored  $\mathfrak{sl}(N)$  Khovanov-Rozansky chain complex of any infinite positive braid categorifies a highest-weight projector. This result extends an earlier result of Cautis categorifying highest-weight projectors using the limiting complex of infinite torus braids. Additionally, we show that the results hold in the case of colored HOMFLY-PT Khovanov-Rozansky homology as well. (Received September 18, 2017)

1135-57-1077 **Adam R Saltz\*** ([adam.saltz@uga.edu](mailto:adam.saltz@uga.edu)), University of Georgia, Boyd Graduate Studies, Department of Mathematics, Athens, GA 30602. *Strong functoriality for Khovanov-Floer theories*. Preliminary report.

A link homology theory is called functorial if link cobordisms induce maps on homology. The recipe for building a map from a diagrammatic presentation of a cobordism is typically built into the homology theory. The difficulty is in showing that the map does not depend on the presentation.

The notion of a Khovanov-Floer theory was introduced by Baldwin, Hedden, and Lobb to encompass the many link homology theories which admit spectral sequences from Khovanov homology. These include the Heegaard Floer (and monopole, framed instanton, and planar Floer homologies) of the branched double cover, Szabó's 'geometric' spectral sequence, and singular instanton knot homology. They show that all of these theories are weakly functorial: link cobordisms induce well-defined maps on the spectral sequence from Khovanov homology and therefore on the homology. But this map is only part of the natural recipe.

I will discuss a strategy for showing that these theories are functorial in the usual sense by relating these theories to Bar-Natan's formulation of Khovanov homology. (Received September 19, 2017)

1135-57-1081 **Melissa Li Zhang\*** ([melissa.zhang@bc.edu](mailto:melissa.zhang@bc.edu)), Maloney Hall 534, Department of Mathematics, Boston College, Chestnut Hill, MA 02467. *Annular Khovanov homology and 2-periodic links*.

A link in the 3-sphere is periodic if rotation about an unknotted axis preserves the link. By deleting the axis, we may view the periodic link as embedded in a solid torus, or a thickened annulus. For a 2-periodic link, we study the relationship between the periodic link and its quotient link via annular Khovanov homology, a triply-graded variant of Khovanov homology defined by Asaeda, Przytycki, and Sikora for links in thickened annuli. By employing algebraic methods from Smith theory, we obtain a spectral sequence for each pair of quantum and winding number gradings. From this we derive rank inequalities and decategorifications reminiscent of Murasugi's formulas for the Jones and Alexander polynomials. Curiously, the same methods suggest a similar spectral sequence relating the Khovanov homology of a 2-periodic link and the annular Khovanov homology of its quotient link. We discuss partial results on this front. (Received September 19, 2017)

1135-57-1157 **Anar Akhmedov** and **Sumeyra Sakalli\*** ([saka1008@umn.edu](mailto:saka1008@umn.edu)). *Deformation of Singular Fibers of Genus 2 Fibrations and Small Exotic Symplectic 4-Manifolds*.

In 1963, Kodaira classified all singular fibers in pencils of elliptic curves, and showed that in such a pencil, each fiber is either an elliptic curve or a rational curve with a node or a cusp, or a sum of rational curves of self-intersection -2. Later Namikawa and Ueno gave geometrical classification of all singular fibers in pencils of genus two curves. In their constructions they used algebro-geometric techniques. In this talk, I will give topological descriptions of certain singularity types in the Namikawa-Ueno's list by presenting Lefschetz pencils of genus two curves in the Hirzebruch surfaces precisely. I will also discuss 2-nodal spherical deformation of certain singular fibers of genus two fibrations. Then by using them I will provide constructions of exotic, minimal, symplectic 4-manifolds homeomorphic but not diffeomorphic to  $\mathbb{C}P^2 \# 6(-\mathbb{C}P^2)$ ,  $\mathbb{C}P^2 \# 7(-\mathbb{C}P^2)$  and  $3\mathbb{C}P^2 \# k(-\mathbb{C}P^2)$  for  $k = 16, \dots, 19$ . This is a joint work with Anar Akhmedov. (Received September 20, 2017)

1135-57-1349 **Allison N. Miller\*** ([amiller@math.utexas.edu](mailto:amiller@math.utexas.edu)) and **Lisa Piccirillo** ([lpiccirillo@math.utexas.edu](mailto:lpiccirillo@math.utexas.edu)). *Knot traces and concordance*.

A famous conjecture of Akbulut and Kirby from 1978 states that the concordance class of a knot is determined by its 0-surgery. In 2015, Yasui disproved this conjecture by providing pairs of knots which have the same 0-surgeries yet which can be distinguished in (smooth) concordance by an invariant associated to the four-dimensional trace of the 0-surgery. I will discuss joint work with Lisa Piccirillo in which we construct many pairs of knots which have diffeomorphic 0-surgery traces, some of which can be distinguished in smooth concordance by the Heegaard

Floer d-invariants of their double branched covers. I will also discuss the applicability of this work to the existence of interesting invertible satellite maps on the smooth concordance group. (Received September 21, 2017)

1135-57-1391 **Michał Adamaszek, Henry Adams and Florian Frick\*** (ff238@cornell.edu). *Metric reconstruction via optimal transport.*

Given a sufficiently dense point sample in a Riemannian manifold, the manifold can be reconstructed up to homotopy equivalence as the Čech or Vietoris–Rips complex of the sample. I will discuss extensions of this beyond Riemannian manifolds and how optimal transport seems to be the correct point of view for metric reconstruction of metric spaces in addition to recovering the homotopy type. (Received September 21, 2017)

1135-57-1482 **Liam Watson\***, Boul. de l'Université, Sherbrooke, QC J1K 2R1, Canada, and **Jacob Rasmussen and Jonathan Hanselman.** *Heegaard Floer homology for manifolds with torus boundary.*

I'll describe an interpretation of the modules arising in Heegaard Floer homology for manifolds with torus boundary in terms of immersed curves in a punctured torus. (Received September 22, 2017)

1135-57-1531 **David Shea Vela-Vick\*** (shea@lsu.edu) and **Michael Wong.** *A bordered interpretation of Manolescu's unoriented skein exact triangle.*

Manolescu first showed that knot Floer homology satisfied an oriented skein exact triangle. In this talk, we use tools from bordered sutured Floer homology to give a new proof of the existence such this exact triangle. (Received September 22, 2017)

1135-57-1539 **Shijie Gu\*** (shijiegu@uwm.edu). *Compactifications of manifolds with boundary.*

In 1976, Chapman–Siebenmann provided criteria for a Hilbert cube manifold  $X$  to admit a  $\mathcal{Z}$ -compactification. However, the question of (whether) the extension of their characterization can be extended to manifolds still remains open: If  $M^n$  is a finite dimensional manifold and  $M^n \times \mathcal{Q}$  is  $\mathcal{Z}$ -compactifiable, is  $M^n$  itself  $\mathcal{Z}$ -compactifiable? In this talk, the complete characterizations of completable manifolds and pseudo-collarable manifolds will be given, respectively. As two applications, the former one implies a best possible "stabilization theorem":  $M^n \times \mathcal{Q}$  ( $n \geq 4$ ) is  $\mathcal{Z}$ -compactifiable iff  $M^n \times [0, 1]$  is  $\mathcal{Z}$ -compactifiable. Applying the latter characterization together with knot theory and group theory, we prove that there exist  $\mathcal{Z}$ -compactifiable manifolds with boundary which are not pseudo-collarable. This can be viewed as a counterexample of a special case of a question asked by Guilbault–Tinsley [2003]: Can a  $\mathcal{Z}$ -compactifiable open manifold fail to be pseudo-collarable? (Received September 23, 2017)

1135-57-1624 **Radmila Sazdanovic\*** (rsazdanovic@math.ncsu.edu), North Carolina State University Mathematics, Raleigh, NC 27695-8205, and **Vladimir Baranovsky.** *On factorization, Khovanov link, and chromatic graph homology.* Preliminary report.

Factorization homology, introduced by Ayala, Francis, and Tanaka, generalizes Hochschild homology. Chromatic homology, a comultiplication-free Khovanov-type theory for graphs constructed by Helme-Guizon and Rong, approximates Hochschild homology when applied to a circle. We complete this picture by relating factorization and chromatic homology to each other, as well as Khovanov link homology. (Received September 23, 2017)

1135-57-1625 **Karene Chu\*** (chukare@mit.edu). *Long flat virtual knots and its associated graded space.* Preliminary report.

Virtual knot theory, introduced by Kauffman, is a generalization of classical knot theory of interest because its finite-type invariant theory is potentially a topological interpretation of Etingof and Kazhdan's theory of quantization of Lie bi-algebras. Classical knots inject into virtual knots, and flat virtual knots is the quotient of virtual knots which equates the real positive and negative crossings, and in this sense is complementary to classical knot theory within virtual knot theory. We completely classify flat virtual tangles with no closed components (pure tangles). This classification can be used as an invariant on virtual pure tangles and virtual braids. We also describe the target space of any universal finite type invariant on long flat virtual knots. (Received September 23, 2017)

1135-57-1642 **Matthew Hedden** (mhedden@math.msu.edu) and **Miriam Kuzbary\*** (miriam.kuzbary@rice.edu), 6100 Main St, Houston, TX 77005. *A New Concordance Group of Links.* Preliminary report.

The knot concordance group has been the subject of much study since its introduction by Ralph Fox and John Milnor in 1966. One might hope to generalize the notion of a concordance group to links; however, the immediate generalization to the set of links up to concordance does not form a group since connected sum of links is not

well-defined. In 1988, Jean Yves Le Dimet defined the string link concordance group, where a link is based by a disk and represented by embedded arcs in  $D^2 \times I$ . In 2012, Andrew Donald and Brendan Owens defined groups of links up to a notion of concordance based on Euler characteristic. However, both cases expand the set of links modulo concordance to larger sets and each link has many representatives in these larger groups. In this talk, I will present joint work with Matthew Hedden where we define a link concordance group based on the the “knotification” construction of Peter Ozsv’ath and Zoltan Szab’o, giving a definition of a link concordance group where each link has a unique group representative. I will also present invariants for studying this group coming from Heegaard-Floer homology as well as a new group theoretic invariant for studying concordance of knots inside in certain types of 3-manifolds. (Received September 24, 2017)

1135-57-1837     **Liang Chang** and **Shawn X Cui\*** (cuiysh@gmail.com), 382 Via Pueblo Mall, Varian Laboratory of Physics, Stanford, CA 94305. *On Two Quantum Invariants of Three Manifolds from Hopf Algebras.*

We prove a conjecture relating two families of quantum invariants of 3-manifolds, namely the non-involutive Kuperberg invariant and the Hennings-Kauffman-Radford (HKR) invariant, both constructed from certain Hopf algebras. The former can be viewed as a non-semisimple generalization of the Turaev-Viro (TV) invariant and the latter a non-semisimple generalization of the Reshetikhin-Turaev-Witten (RTW) invariant. While in the semisimple case it is a classical result that the TV invariant is equal to the norm square of the RTW invariant, the relation in the non-semisimple case has remained a conjecture. We prove two versions of the conjecture. Let  $M$  be a closed oriented 3-manifold,  $D(M)$  the double of  $M$ ,  $H$  a finite dimensional Hopf algebra, and  $D(H)$  the Drinfeld double of  $H$ . (I) If  $Z(H)$  is ribbon, then the Kuperberg invariant of  $M$  endowed with a framing constructed from  $H$  is equal to the HKR invariant of  $M$  endowed with an appropriate 2-framing constructed from  $D(H)$ ; (II) If  $H$  is ribbon, then the Kuperberg invariant of  $M$  endowed with a framing constructed from  $H$  is equal to the HKR invariant of  $D(M)$  endowed with an appropriate 2-framing constructed from  $H$ . (Received September 25, 2017)

1135-57-1868     **Michelle Chu\*** (mchu@math.utexas.edu), Austin, TX. *Quantifying virtual properties of 3-manifolds.* Preliminary report.

The study of virtually special groups played a key role in the resolutions of some important conjectures in 3-manifold theory. In this talk I will motivate and introduce virtual properties of 3-manifold groups and discuss some new results on quantifying these virtual properties for the class of Bianchi groups. (Received September 25, 2017)

1135-57-1875     **Matt Haulmark\*** (m.haulmark@vanderbilt.edu), Department of Mathematics, 1326 Stevenson Center, Vanderbilt University, Nashville, TN 37240. *Groups with Menger Curve Boundary.*

In this talk I will discuss classification theorems for 1-dimensional boundaries of groups with non-positive curvature and their application to finding non-hyperbolic groups with Menger curve boundary. (Received September 25, 2017)

1135-57-1900     **Caitlin Levenson\***, leverson@math.gatech.edu, and **Dan Rutherford.** *Satellite ruling polynomials and representations of the Chekanov-Eliashberg algebra.* Preliminary report.

Given a pattern braid  $\beta$  in  $J^1(S^1)$ , to any Legendrian knot  $K$  in  $\mathbb{R}^3$  with the standard contact structure, we can associate the Legendrian satellite knot  $S(K, \beta)$ . We will discuss the relationship between augmentations of the Chekanov-Eliashberg differential graded algebra of  $S(K, \beta)$  and certain representations of the Chekanov-Eliashberg differential graded algebra of  $K$ . For certain patterns, we can then relate a specialization of the ruling polynomial of  $S(K, \beta)$  to these representation numbers. (Received September 25, 2017)

1135-57-2066     **Priyam Patel\*** (patel@math.ucsb.edu), **Tarik Aougab** and **Samuel Taylor.** *Lifting curves on surfaces via 3-manifolds and the curve complex.* Preliminary report.

Given two simple closed curves alpha and beta intersecting many times on an orientable surface  $S$ , we are interested in studying the minimal degree of a finite cover of  $S$  such that there is a lift of alpha disjoint from a lift of beta. In joint work with Tarik Aougab and Sam Taylor, we use the geometry of hyperbolic 3-manifolds to obtain lower bounds on this degree in terms of curve complex distance between the curves alpha and beta. After describing some of the techniques we use, I will highlight an interesting application of our work that gives lower bounds on the degrees of special covers for certain cube complexes associated to surfaces. (Received September 25, 2017)



1135-57-2315 **Alisa Leshchenko\*** (alisa.s.l@gmail.com) and **Mikael Vejdemo-Johansson** (mvj@math.csi.cuny.edu). *Adaptive Mapper*. Preliminary report.

We propose an improvement to the highly successful Mapper algorithm in Topological Data Analysis. Mapper creates a simplicial model of the original data by clustering the fibers of a chosen filter function; the goal is to retain useful topological invariants encoded in the data, but in a structure with only trivial local topological complexity. Fidelity of this representation is guaranteed by the Nerve Lemma in algebraic topology, which requires each cluster in each fiber to be acyclic – devoid of higher-dimensional topological structure. If this condition fails, the resultant model will be a lossy projection of the “true” topological structure, and may hide interesting structure present in the data.

Our approach uses persistent homology to dynamically refine the model until the conditions for the Nerve lemma are met. This is achieved by checking each cluster and each intersection of clusters for acyclicity, and using local transformations to split clusters that have non-trivial topology.

In this presentation, we will give a detailed description of our method and demonstrate its performance on some illustrative examples. (Received September 26, 2017)

1135-57-2392 **J. Elisenda Grigsby\*** (grigsbyj@bc.edu), **Anthony M. Licata** and **Stephan M. Wehrl**. *Annular Khovanov-Lee theory, representation theory, braids, and cobordisms*.

I will survey the ways in which key representation-theoretic features of the annular Khovanov-Lee theory of braid closures give information about the surfaces they bound in the 4-ball as well as their dynamics when viewed as mapping classes of the punctured disk. (Received September 26, 2017)

1135-57-2455 **Allison Sullivan\***, 1 College Street, PO Box: 2792, Worcester, MA 01610, and **David Damiano** (ddamiano@holycross.edu), Department of Mathematics & Computer Science, College of the Holy Cross, 1 College St., Worcester, MA. *Topological Modeling of Force Networks in Granular Material*. Preliminary report.

The aggregate properties of granular materials are determined by the network of forces between particles. These networks are prominent in a number of fields, including the pharmaceutical and agricultural industries. Force networks are, however, not well understood. The random organization of these networks makes it difficult to derive properties of a granular material from the local structure of its force network. Here, the method of swatch and cloth is instrumental. Swatch and cloth is a new and promising technique developed by Schweinhart et. al. It can be used to accurately describe a force network by analyzing the statistics of the local structure of the network’s adjacency graph. We used this method to analyze and compare the force networks of planar simulations of granular material. In addition, we analyzed the related community structure of these networks. (Received September 26, 2017)

1135-57-2473 **Shelly Harvey\*** (shelly@rice.edu), **Jung Hwan Park** and **Arunima Ray**. *Pure braids, links bounding Whitney towers, and  $n$ -solvability*.

The  $n$ -solvable filtrations of the knot/link concordance groups were defined as a way of studying the structure of the groups and in particular, the subgroup of algebraically slice knots/links. While the knot concordance group  $C^1$  is known to be an abelian group, when  $m$  is at least 2, the link concordance group  $C^m$  of  $m$ -component (string) links is known to be non-abelian. In particular, it is well known that the pure braid group with  $m$  strings is a subgroup of  $C^m$  and hence when  $m$  is at least 3, this shows that  $C^m$  contains a non-abelian free subgroup. We study the relationship between the derived subgroups of the the pure braid group,  $n$ -solvable filtration of  $C^m$ , links bounding symmetric Whitney towers, and links bounding gropes. (Received September 26, 2017)

1135-57-2519 **Michael Abel\*** (maabel@math.duke.edu). *HOMFLY-PT homology for general link diagrams and braidlike isotopy*.

In the construction of HOMFLY-PT homology, one must start with a link presented as a braid closure. This restriction was expected by Khovanov and Rozansky to be required for the homology to be an isotopy invariant. In this talk we explore the consequences of dropping this requirement and allowing general link diagrams. Finally we will show that the Euler characteristic of this homology theory is a deformed version of the HOMFLY-PT polynomial which detects “braidlike” isotopy of tangles and links. This new polynomial agrees with the HOMFLY-PT polynomial on link diagrams which are presented as closed braid diagrams. (Received September 26, 2017)

1135-57-2597 **Eric Samperton\*** (egsamp@math.ucdavis.edu). *Mapping class group actions on representation sets of surfaces and universality in  $(2+1)$ -D TQFT*.

Fix a finite group  $G$  and let  $S$  be a surface. The mapping class group  $MCG_*(S)$  acts on the set of  $G$ -representations  $\{\pi_1(S) \rightarrow G\}$ . Varying  $S$ , the different actions can be packaged as a  $(2+1)$ -D TQFT, closely

related to the Dijkgraaf-Witten theory associated to  $G$ . More precisely, the actions are those afforded by the  $G$ -crossed modular tensor category of  $G$ -graded vector spaces.

In joint work with Greg Kuperberg, the author proved a precise version of classical computational universality for these actions when  $G$  is a nonabelian simple group. We will discuss these results, and their possible applications to the *quantum* universality of modular tensor categories via the process of gauging. (Received September 26, 2017)

1135-57-2702     **Rosemary K Guzman\*** (rguzma1@illinois.edu) and **Peter B Shalen** (shalen@math.uic.edu). *Quantitative Mostow Rigidity: Relating volume to topology for hyperbolic 3-manifolds.*

A celebrated result of Mostow states that if  $M, N$  are two closed, connected, orientable, hyperbolic  $n$ -manifolds which are homotopy equivalent in dimensions  $n \geq 3$ , then  $M, N$  are equivalent up to isometry.

This unique geometric-topological relationship has been the framework for many important results in the field, including notable results providing lower bounds on the volume of  $M$ , and results relating volume to homology (Culler-Shalen).

In this talk, we will focus on the case where the fundamental group of  $M$  has a property, "k-free," for  $k \geq 5$ , and discuss current work toward an improvement on the volume bound from the current known bound of 3.44 which holds for  $k \geq 4$ . (Received September 26, 2017)

1135-57-2714     **Leona Sparaco\*** (leona.sparaco@scranton.edu). *Character Varieties of a Family of Two-Component Links.* Preliminary report.

Let  $M$  be a finite volume orientable hyperbolic manifold. The  $SL_2(\mathbb{C})$  character variety of  $M$  is essentially the set of all representations  $\rho : \pi_1(M) \rightarrow SL_2(\mathbb{C})$  up to trace equivalence. This algebraic set is related to many geometric properties of the manifold  $M$ . In this talk we will look at the character varieties of a family of two-component links. (Received September 26, 2017)

1135-57-2763     **Justin Lanier, Dan Margalit and Rebecca R Winarski\*** (rebecca.winarski@gmail.com). *The twisted rabbit problem via the arc complex.* Preliminary report.

The twisted rabbit problem is a celebrated problem in complex dynamics. Work of Thurston proves that up to equivalence, there are exactly three branched coverings of the sphere to itself satisfying certain conditions. When one of these branched coverings is modified by a mapping class, a map equivalent to one of the three coverings results. Which one?

After remaining open for 25 years, this problem was solved by Bartholdi—Nekyrashevych using iterated monodromy groups. In joint work with Lanier and Margalit, we formulate the problem topologically and solve the problem using the arc complex. (Received September 26, 2017)

1135-57-2766     **Carmen Caprau\***, Department of Mathematics, 5245 N. Backer Avenue M/S PB 108, Fresno, CA 93740, and **Sarah McGahan**. *On the virtual singular braid monoid.* Preliminary report.

We study the algebraic structures of the virtual singular braid monoid,  $VS\mathcal{B}_n$ , and the virtual singular pure braid monoid,  $VSP_n$ . The monoid  $VS\mathcal{B}_n$  is the splittable extension of  $VSP_n$  by the symmetric group  $S_n$ . We also construct a representation of  $VS\mathcal{B}_n$ . (Received September 26, 2017)

1135-57-2774     **Carl C Hammarsten\*** (hammarsc@lafayette.edu). *Decorated Heegaard Diagrams and Combinatorial Heegaard Floer homology.*

A 3-dimensional closed manifold  $Y$  represented by its branched spine has a canonical Heegaard decomposition. We present this decomposition graphically in the form of a strip diagram. We show that strip diagrams have nice properties which greatly simplify the calculation of Heegaard Floer homology. Motivated by this work, we introduce the idea of a decorated Heegaard diagram. That is, a Heegaard diagram together with a collection of embedded paths satisfying certain criteria. Using this decorated Heegaard diagram, we present a combinatorial definition of a chain complex which is homotopically equivalent to the Heegaard Floer one, yet significantly smaller. Finally, we consider the presentation of a branched spine by its O-graph and show how to reformulate our definition in these terms. (Received September 26, 2017)

1135-57-2818     **Jim Fowler\*** (fowler@math.osu.edu) and **Courtney Thatcher** (cthatcher@pugetsound.edu). *Linear actions of  $\mathbb{Z}/p \times \mathbb{Z}/p$  on  $S^n \times S^n$ .* Preliminary report. In 1925, Hopf first stated the "spherical space form" asking which groups act freely on  $S^n$ . Some fifty years later, Madsen, Thomas, and Wall proved in 1978 that certain necessary conditions—discovered by Smith in 1944

and Milnor in 1957—were in fact sufficient. Easy generalizations of this question, like determining which groups  $G$  can act freely on  $S^n \times S^n$ , are still open.

This talk considers the situation of “linear” actions of  $\mathbb{Z}/p \times \mathbb{Z}/p$  on  $S^n \times S^n$  which can be understood by relating them to the easier case of  $\mathbb{Z}/p$  actions on  $S^n$ , that is, to lens spaces. (Received September 26, 2017)

1135-57-2853 **Mark Hughes\*** ([hughes@mathematics.byu.edu](mailto:hughes@mathematics.byu.edu)) and **Seungwon Kim**. *The immersed cross-cap number of a knot.*

The immersed Seifert genus of a knot  $K$  in  $S^3$  can be defined as the minimal genus of an orientable immersed surface  $F$  with  $\partial F = K$ . By a result of Gabai, this value is always equal to the (embedded) Seifert genus of  $K$ . In this talk I will discuss the embedded and immersed cross-cap numbers of a knot, which are the non-orientable versions of these invariants. Unlike their orientable counterparts these values do not always coincide, and can in fact differ by an arbitrarily large amount. In further contrast to the orientable case, there are families of knots with arbitrarily high embedded 4-ball cross-cap numbers, but which are easily seen to have immersed cross-cap number 1. After describing these examples I will discuss a classification of knots with immersed cross-cap number 1. (Received September 26, 2017)

1135-57-2942 **Steve Boyer** and **Ying Hu\*** ([ying.hu@cirget.ca](mailto:ying.hu@cirget.ca)). *Left-orderable 3-manifold groups, taut foliations and contact structures.*

Let  $M$  be an irreducible  $\mathbb{Q}$ -homology sphere (i.e.  $H_*(M, \mathbb{Q}) \cong H_*(S^3, \mathbb{Q})$ ). The L-space conjecture suggests the following statements are equivalent: a)  $\pi_1(M)$  is left-orderable; b)  $M$  admits a co-orientable taut foliation; c)  $M$  is not a Heegaard Floer L-space. The implication from b) to c) was established by utilizing the contact structure that is close to a given taut foliation.

In this talk, we discuss how contact structures could also play a role in studying the interconnection between the left-orderability of  $\pi_1(M)$  and the existence of co-orientable taut foliations on  $M$ . Following this idea, we confirm the L-space conjecture for cyclic branched covers of families of hyperbolic knots.

An important concept occurs in our argument, called fractional Dehn twist coefficient, which is also related to the Khovanov homology of knots.

This is joint work with Steve Boyer. (Received September 26, 2017)

1135-57-3069 **Mustafa Hajij\*** ([mhajij@usf.edu](mailto:mhajij@usf.edu)), 18008 Allison Park Lane, Tampa, FL 33647. *Twist Regions and Coefficients Stability of the Colored Jones Polynomial.*

We prove that the coefficients of the colored Jones polynomial of alternating links stabilize under increasing the number of twists in the twist regions of the link diagram. This gives us an infinite family of  $q$ -power series derived from the colored Jones polynomial parametrized by the color and the twist regions of the alternating link diagram. (Received September 26, 2017)

1135-57-3159 **Jason Parsley\*** ([parslerj@wfu.edu](mailto:parslerj@wfu.edu)). *Finding accessible open problems in topology & geometry.*

For undergraduate research mentors, one formidable task is finding open problems at appropriate levels for their students. In this talk, we describe several routes for arriving at suitable problems, via two main examples, one from knot theory, one from geometry.

In knot theory, we describe ‘petal knots’ – knots which possess a flower-shaped diagram. Next, we examine the geometry of weighted voting. Each case concludes with some open problems. (Received September 26, 2017)

1135-57-3192 **Jay MJ Leach\*** ([jleach@math.fsu.edu](mailto:jleach@math.fsu.edu)). *Detected slopes of manifolds with symmetries.*

In 3-manifolds which have a single torus boundary component the components of the character variety can detect separate slopes for incompressible surfaces. I will look at what these slopes are for a family of two twist knots, and then state a general result. For Knots with symmetry, if a slope is detected by the canonical component of its character variety then there must exist a surface in the manifold that is preserved by the symmetry. (Received September 27, 2017)

1135-57-3228 **Jonathan Simone\***, University of Virginia. *A new construction of exotic 4-manifolds.*

We will introduce a new cut-and-paste technique similar to the rational blow down. In particular, we construct a class of plumbings whose boundaries also bound rational homology  $S^1 \times D^3$ s and, through a cut-and-paste procedure, construct an exotic  $3CP^2 \# 12\overline{CP}^2$ . This cut and paste procedure relies on a gluing formula for the Ozsváth Szabó 4-manifold invariant, which we also prove. (Received September 27, 2017)

## 58 ► *Global analysis, analysis on manifolds*

1135-58-525 **Jo Nelson\*** ([nelson@math.columbia.edu](mailto:nelson@math.columbia.edu)). *Reeb dynamics and contact invariants.*

Contact geometry is the study of certain geometric structures on odd dimensional smooth manifolds. A contact structure is a hyperplane field specified by a one form which satisfies a maximum nondegeneracy condition called complete non-integrability. The associated one form is called a contact form and uniquely determines a vector field called the Reeb vector field on the manifold. I will explain how to make use of J-holomorphic curves to obtain a Floer theoretic contact invariant whose chain complex is generated by closed Reeb orbits. In particular, I will explain the pitfalls in defining contact homology and discuss my work, in part joint with Hutchings, which gives a rigorous construction of cylindrical contact homology via geometric methods and numerous applications to dynamics. (Received September 07, 2017)

1135-58-759 **Saskia Roos\*** ([saroos@mpim-bonn.mpg.de](mailto:saroos@mpim-bonn.mpg.de)), Max-Planck Institute for Mathematics, Vivatsgasse 7, 53111 Bonn, Germany. *Dirac eigenvalues under codimension one collapse.*

After giving a characterization of a collapse of codimension one we study the behavior of Dirac eigenvalues in that situation. We show that there are converging eigenvalues if and only if there is an induced pin structure on the limit space. In addition, we determine the limit operator which corresponds to the limit spectrum. (Received September 14, 2017)

1135-58-1030 **Katrin Wehrheim\***, Math Department, Evans Hall #3840, Berkeley, CA 94720. *Polyfold Lab Report.*

I will survey various results on applications and extensions of Hofer-Wysocki-Zehnder's polyfold theory, such as fiber products of polyfold Fredholm sections' equivariant transversality - existence and obstructions; equivariant fundamental class; Gromov-Witten axioms; two polyfold proofs of the Arnold conjecture. These are joint with or due to Peter Albers, Ben Filippenko, Joel Fish, Wolfgang Schmaltz, and Zhengyi Zhou. (Received September 18, 2017)

1135-58-1237 **Teresa Arias-Marco, Emily B Dryden, Carolyn S Gordon, Asma Hassannezhad, Allie Ray and Elizabeth Stanhope\*** ([stanhope@clark.edu](mailto:stanhope@clark.edu)). *The Steklov spectrum and the geometry of orbifolds.*

The Steklov spectrum of a compact Riemannian orbifold with boundary is the spectrum of the Dirichlet-to-Neumann operator of the orbifold. In two dimensions we show that the Steklov spectrum detects the number of singular points on the boundary of the orbisurface. In addition, the Steklov spectrum determines the lengths of orbisurface boundary components up to an equivalence relation. We also obtain upper bounds on the Steklov eigenvalues of an orbifold in terms of the isoperimetric ratio and a conformal invariant. (Received September 20, 2017)

1135-58-1601 **Xavier Ramos Olive\*** ([olive@math.ucr.edu](mailto:olive@math.ucr.edu)). *Neumann Li-Yau gradient estimate under integral Ricci curvature bounds.*

We prove a Li-Yau type gradient estimate for positive Neumann heat kernels, in a compact manifold  $\mathbf{M}$  with boundary  $\partial\mathbf{M}$ , with integral Ricci curvature bounds. We do not assume  $\partial\mathbf{M}$  to be convex, but  $\mathbf{M}$  must satisfy the interior rolling  $R$ -ball condition. (Received September 23, 2017)

1135-58-1836 **Tomoyuki Takenawa\*** ([takenawa@kaiyodai.ac.jp](mailto:takenawa@kaiyodai.ac.jp)), 2-1-6 Etchu-jima, Koto-ku, Tokyo, Tokyo 135-8533. *The space of initial conditions for some 4D Painlevé systems.* Preliminary report.

In recent years, research on 4D Painlevé systems have progressed mainly from the viewpoint of isomonodromy deformation of linear equations. In this talk we study the geometric aspects of 4D Painlevé systems by investigating the space of initial conditions in Okamoto-Sakai's sense, which was a powerful tool in the original 2D case. Specifically, starting from known discrete symmetries, we construct the space of initial conditions for some 4D Painlevé systems, and using the Néron-Severi bilattice, clarify the whole group of their discrete symmetries. The examples include the directly coupled 2D Painlevé equations, Noumi-Yamada's  $A_5^{(1)}$  system and the 4D Garnier system. (Received September 25, 2017)

1135-58-2090 **Marco A Guaraco\*** ([guaraco@uchicago.edu](mailto:guaraco@uchicago.edu)). *Geometric Aspects of the Allen-Cahn Equation on closed manifolds.*

In this talk I will discuss both local and global properties of the stationary Allen-Cahn equation in closed manifolds. This equation arising from the theory of phase transitions has a strong connection with the theory

of minimal hypersurfaces. I will summarize recent results regarding the analogy between both theories with focusing on min-max constructions and the multiplicity of transitions layers. (Received September 25, 2017)

1135-58-2136 **Ali Behzadan\*** ([abehzadan@ucsd.edu](mailto:abehzadan@ucsd.edu)), 9500 Gilman Drive, Dept. 0112, La Jolla, CA 92093, and **Michael Holst** ([mholst@ucsd.edu](mailto:mholst@ucsd.edu)), 9500 Gilman Drive, Dept. 0112, La Jolla, CA 92093. *On Approximation of Certain Geometric Operators Between Sobolev Spaces of Sections of Tensor Bundles on Compact Manifolds Equipped With Rough Metrics.*

The study of Einstein constraint equations in general relativity naturally leads to considering Riemannian manifolds equipped with nonsmooth metrics. There are several important differential operators on Riemannian manifolds whose definitions depend on the metric; gradient, divergence, Laplacian, covariant derivative, conformal Killing operator, and vector Laplacian are among those operators. We will examine the possibility of approximation of such operators defined using a rough metric by the corresponding operators defined using a smooth metric. This paves the road to understanding to what extent the nice properties that these operators possess when they are defined by a smooth metric will transfer to the case where the underlying metric is nonsmooth. (Received September 25, 2017)

1135-58-2268 **Melanie Pivarski\*** ([mpivarski@roosevelt.edu](mailto:mpivarski@roosevelt.edu)), Mailstop AUD 402, 430 S Michigan Ave, Chicago, IL 60605. *Gradient bounds of heat kernels in the setting of local Dirichlet spaces as co-compact covers of finitely generated polynomial growth groups.* Preliminary report.

In 2004, Dungey proved spatial gradient bounds on heat kernels of co-compact covering manifolds of finitely generated polynomial growth groups. This result is expanded to the local Dirichlet space setting when the space is complete and satisfies a small number of local geometric assumptions: a local Poincaré inequality, local volume doubling, and boundedness of balls of a fixed radius. The proof combines techniques from Coulhon and Duong with heat kernel bounds from Pivarski and Saloff-Coste. (Received September 25, 2017)

1135-58-2651 **Christine Guenther\*** ([guenther@pacificu.edu](mailto:guenther@pacificu.edu)). *Scaling and Entropy for the RG-2 flow.* The second order renormalization group flow (RG-2 flow) on a Riemannian manifold  $(\mathcal{M}, g)$  is the geometric flow

$$\frac{\partial}{\partial t}g = -\alpha \text{Ric}(g) - \frac{\alpha^2}{2} \mathcal{R}m^2(g),$$

where  $\text{Ric}$  is the Ricci curvature tensor,  $\mathcal{R}m_{ij}^2 := \mathcal{R}_{irmk} \mathcal{R}_j^{mk}$  with  $\mathcal{R}_{irmk}$  the Riemannian curvature tensor, and  $\alpha > 0$  is a parameter. In this talk we develop a (physically motivated) solution to problems resulting from the scaling properties of  $\text{Ric}$  and  $\mathcal{R}m^2$ , through the introduction of a *geometrically defined* coupling constant  $\alpha_g(t)$ .

We further investigate an Entropy for the flow, considering natural extensions of Perelman's entropy for the Ricci flow. This is joint work with Mauro Carfora. (Received September 26, 2017)

1135-58-2747 **Casey Lynn Kelleher\*** ([ckelleher@princeton.edu](mailto:ckelleher@princeton.edu)). *Progress in symplectic curvature flow.* Preliminary report.

Symplectic curvature flow, introduced by Streets and Tian, is a parabolic flow of almost Kahler structures, which provides a natural extension of Kahler Ricci flow to symplectic manifolds. We discuss properties of this flow, recent progress and future directions. (Received September 26, 2017)

1135-58-2756 **Casey Lynn Kelleher\*** ([ckelleher@princeton.edu](mailto:ckelleher@princeton.edu)), **Jeffrey D Streets** ([jstreets@uci.edu](mailto:jstreets@uci.edu)) and **Matthew J Gursky** ([matthew.j.gursky.1@nd.edu](mailto:matthew.j.gursky.1@nd.edu)). *A conformally invariant gap theorem in Yang-Mills theory.*

We show a sharp conformally invariant gap theorem for Yang-Mills connections in dimension 4 by exploiting an associated Yamabe-type problem. (Received September 26, 2017)

1135-58-2944 **John D Ross\*** ([rossjo@southwestern.edu](mailto:rossjo@southwestern.edu)), SU Box 7371, 1001 E. University Avenue, Georgetown, TX 78626. *On the Existence of a Closed, Embedded, Rotational Lambda-Hypersurface.*

In this paper we show the existence of a closed, embedded  $\lambda$ -hypersurface  $\Sigma \subset \mathbb{R}^{2n}$ . The hypersurface is diffeomorphic to  $\mathbb{S}^{n-1} \times \mathbb{S}^{n-1} \times \mathbb{S}^1$  and exhibits  $SO(n) \times SO(n)$  symmetry. Our approach uses a shooting method similar to the approach used by McGrath in constructing a generalized self-shrinking torus solution to mean curvature flow. The result generalizes the  $\lambda$ -torus found by Cheng and Wei. (Received September 26, 2017)

## 60 ► *Probability theory and stochastic processes*

1135-60-1 **Tim Leung\*** (timleung@uw.edu). *Optimal Stopping Problems in Computational Finance and Risk Management.*

This talk provides an overview of a number of optimal multiple stopping problems with applications to computational finance & risk management. Under different stochastic models, we discuss both the analytical and computational methods for solving these problems, and illustrate the practical applications, ranging from options and futures trading to infrastructure investment and commodity storage timing strategies. (Received April 27, 2017)

1135-60-22 **Barbara H Margolius\*** (b.margolius@csuohio.edu), Cleveland State University, Department of Mathematics, Cleveland, OH 44114, and **L Felipe Martins.** *Random Walks in the Quarter Plane with Time-Varying Periodic Transition Rates.* Preliminary report.

Many processes vary periodically with time. Some examples include: the dispersal of wildlife in its habitat, the boundary of snow coverage, the level of water in the Great Lakes, the amount of a pollutant in the air, the volume of calls to a call center, the emergency service demands, and many, many others. In this talk, we consider a random walk which travels on a lattice in the quarter plane. We consider general time-varying transition rates and also those rates which vary periodically with time. We obtain the transient distribution for the location of the walk in the plane at time  $t$ . When rates are periodic and the system is ergodic, we obtain an asymptotic (in time) periodic distribution for the location in the plane. We show that under certain conditions, the distribution is asymptotically geometric (in the distance from the origin) and provide numerical examples. (Received June 19, 2017)

1135-60-36 **Indranil SenGupta\*** (indranil.sengupta@ndsu.edu), Department of Mathematics, North Dakota State University, Minard Hall 408, Fargo, ND 58108-6050, and **William Wilson** and **William Nganje.** *Barndorff-Nielsen and Shephard model- oil commodity hedging with variance swap and option.*

In this presentation we discuss a quadratic hedging procedure for oil price commodities. This is particularly important in recent time when the fluctuating oil price is a major concern. We show that an effective hedging procedure can be derived using a combination of variance swap and option. Numerical results (for the state of North Dakota) are shown to provide the effectiveness of the results. (Received June 26, 2017)

1135-60-49 **Yipeng Yang\*** (yangy@uhcl.edu), 2700 Bay Area Blvd., Houston, TX 77058. *Finite Horizon Optimal Execution with Bounded Rate of Transaction.*

We consider an optimal execution problem with fixed time horizon and bounded transaction rate, which is more natural in practice. We show that, different from traditional stochastic control or singular control problems, this problem is of the stochastic bang-bang control type. Under some parameter settings we show that the optimal control does not involve buy action, and the optimal value function is the viscosity solution to the associated Hamilton-Jacobi-Bellman (HJB) equation. We further show that the optimal policy is unique, and provide a numerical example to explore the form of the optimal control. (Received July 06, 2017)

1135-60-84 **Diana Curtis\*** (jaramillo@cpp.edu), Department of Mathematics and Statistics, 3801 West Temple Ave., Pomona, CA 91768, and **Jennifer Switkes** (jmswitkes@cpp.edu), Department of Mathematics and Statistics, 3801 West Temple Ave., Pomona, CA 91768. *Dynamics of a Predator-Prey Model through Stochastic Methods.*

The classical deterministic Lotka-Volterra predator-prey model famously leads to closed curves in the predator-prey phase plane. A stochastic version of this model has been formulated as a simple birth-death process. The expected value of this process is governed by a system of differential equations that is almost, but not quite, identical in form to the deterministic system. The difference in rate functions is shown to be proportional to the time-dependent covariance of the populations of the two species. We explore the impact of this covariance term on the dynamical behavior of the predator-prey system. In particular, this difference creates expected value trajectories in the phase plane that are no longer closed. (Received July 24, 2017)

1135-60-179 **Sayan Banerjee** (sayan@email.unc.edu), **Maria Gordina** (maria.gordina@uconn.edu) and **Phanuel Mariano\*** (phanuel.mariano@uconn.edu). *Coupling in the Heisenberg group and its applications to gradient estimates.*

We construct a non-Markovian coupling for hypoelliptic diffusions which are Brownian motions in the three-dimensional Heisenberg group. We then derive properties of this coupling such as estimates on the coupling rate, and upper and lower bounds on the total variation distance between the laws of the Brownian motions. Finally

we use these properties to prove gradient estimates for harmonic functions for the hypoelliptic Laplacian which is the generator of Brownian motion in the Heisenberg group. (Received August 07, 2017)

1135-60-205 **Sungwon Ahn\*** ([sahn02@roosevelt.edu](mailto:sahn02@roosevelt.edu)) and **Jonathon Peterson**. *Quenched central limit theorem rates of convergence for one-dimensional random walks in random environments.*

Unlike classical simple random walks, one-dimensional random walks in random environments (RWRE) are known to have a wide array of potential limiting distributions. Under certain assumptions, however, it is known that central limit theorem like limiting distributions hold for the walk under both the quenched and averaged measures. In this talk, we show certain polynomial rates of convergence for the quenched central limit theorems for both the hitting time and position of the RWRE. (Received August 09, 2017)

1135-60-216 **Oleksii Mostovyi\*** ([oleksii.mostovyi@uconn.edu](mailto:oleksii.mostovyi@uconn.edu)), University of Connecticut, Department of Mathematics, U1009, 341 Mansfield Road, Storrs, CT 06269, and **Mihai Sirbu**. *Sensitivity analysis of the utility maximization problem with respect to model perturbations.*

We study the sensitivity of the expected utility maximization problem in a continuous semimartingale market with respect to small changes in the market price of risk. Assuming that the preferences of a rational economic agent are modeled with a general utility function, we obtain a second-order expansion of the value function, a first-order approximation of the terminal wealth, and construct trading strategies that match the indirect utility function up to the second order. If a risk-tolerance wealth process exists, using it as a numeraire and under an appropriate change of measure, we reduce the approximation problem to a Kunita-Watanabe decomposition. This talk is based on the joint work with Mihai Sirbu. (Received August 11, 2017)

1135-60-281 **Aziz Issaka\*** ([aziz.issaka@ndsu.edu](mailto:aziz.issaka@ndsu.edu)), Department of Mathematics, NDSU Dept # 2750, Minard Hall 406A, Fargo, ND 58108-6050, and **Indranil SenGupta**. *Analysis of some variance based instruments for Ornstein-Uhlenbeck type models.*

In this presentation a couple of variance dependent instruments in the financial market are studied. Firstly, a number of aspects of the variance swap in connection to the Barndorff-Nielsen and Shephard model are studied. A partial integro-differential equation that describes the dynamics of the arbitrage-free price of the variance swap is formulated. Under appropriate assumptions for the first four cumulants of the driving subordinator, a Vecer-type theorem is proved. Finally, a price-weighted index modulated by market variance is introduced and empirical data driven numerical examples are provided in support of the proposed price index. (Received August 18, 2017)

1135-60-311 **Jinsu Kim\*** ([jkim522@wisc.edu](mailto:jkim522@wisc.edu)), VanVelck B127, Department of Mathematics, 480 Lincoln Dr., Madison, WI 53706, and **David F Anderson**. *Network conditions for positive recurrence of stochastically modeled reaction networks and mixing times.* Preliminary report.

We consider discrete-space continuous-time Markov models of reaction networks and provide sufficient conditions for positive recurrence. The provided analytical results depend solely on the underlying structure of the reaction network and not on the specific choice of parameters. Our main analytical tool is the "tier structure" previously utilized successfully in the study of deterministic models of reaction networks. We also consider mixing times of associated continuous time Markov chains for stochastic model of reaction networks. (Received August 22, 2017)

1135-60-314 **Semere K Habtemicael\*** ([habtemicaels@wit.edu](mailto:habtemicaels@wit.edu)), 550 Huntington ave, Boston, MA 02115, and **Musie S Ghebremichael** and **Indranil SenGupta**. *Superposition Theorem of Barndorff-Nielson and Shephard Model for Variance and Volatility Swap.* Preliminary report.

We propose superposition BN-S Levy processes to model variance and volatility swap of a financial market. Such a model is analytically flexible and offer the possibilities of capturing the unexpected movement of the stock market than the standard models. Parameter estimate and model performance is assessed on data not used to build the model (i.e., test data). It is shown that the prediction error rate for the models are much lower compared to those from previous related models. Moreover, it is shown that unlike previous related models which are restricted to stable markets, the present approach can be applied to both stable and unstable markets. (Received August 22, 2017)

1135-60-331 **Aurel I Stan\***, 1465 Mount Vernon Avenue, Marion, OH , and **Florin Catrina. A**  
*Hölder inequality for norms of Gamma Wick products.*

The Wick product is a natural product defined on the unital algebra generated by a random variable having finite moments of all orders. Its definition uses the  $L^2$ -orthogonal structure given by the orthogonal monic polynomials, and can be used to derive inequalities about the  $L^2$ -norms of the Wick products. However, to obtain inequalities about the  $L^p$ -norms of the Wick products, with  $p \neq 2$ , an integral representation of the Wick product is needed. We present first an integral representation of the Wick product generated by a Gamma distributed random variable, with mean greater than  $1/2$ . We use this integral representation to prove a Hölder inequality for norms of Gamma Wick products. (Received August 24, 2017)

1135-60-359 **Fubao Xi**, School of Mathematics and Statistics, Beijing Institute of Technology, Beijing 100081, Peoples Rep of China, and **Chao Zhu\*** (zhu@uwm.edu), Department of Mathematical Sciences, University of Wisconsin-Milwaukee, Milwaukee, WI 53092. *Jump Type Stochastic Differential Equations with Non-Lipschitz Coefficients: Non Confluence, Feller and Strong Feller Properties, and Exponential Ergodicity.*

This work considers multidimensional jump type stochastic differential equations with super linear growth and non-Lipschitz coefficients. After establishing a sufficient condition for nonexplosion, this paper presents sufficient non-Lipschitz conditions for pathwise uniqueness. The non confluence property for solutions is investigated. Feller and strong Feller properties under non-Lipschitz conditions are investigated via the coupling method. Sufficient conditions for irreducibility and exponential ergodicity are derived. As applications, this paper also studies multidimensional stochastic differential equations driven by Lévy processes and presents a Feynman-Kac formula for Lévy type operators. (Received August 28, 2017)

1135-60-393 **Ngartelbaye Guerngar\*** (nzg0017@auburn.edu), 560 Perry St, Apt 229, Auburn, AL 36830, and **Erkan Nane** (ezn0001@auburn.edu), Department of Math and Statistics, Auburn Uni, 221 Parker Hall, Auburn, AL 36849. *Some properties of the solution of SPDEs.*

Consider the non-linear space-time-fractional stochastic heat type equation

$$\partial_t^\beta u_t(x) = -\nu(-\Delta)^{\alpha/2} u_t(x) + I_t^{1-\beta} [f(t, x) + \sigma(u_t(x))\dot{W}(t, x)]$$

in  $D \times (0, \infty)$ , where  $D$  is a bounded subset of  $\mathbb{R}^d$ ,  $\nu > 0$ ,  $f$  is a non random source term,  $\sigma$  is a Lipschitz continuous function,  $\beta \in (0, 1)$ ,  $\alpha \in (0, 2]$  and  $d < \min(2, \beta^{-1})\alpha$ .  $\partial_t^\beta$  is the Caputo derivative,  $(-\Delta)^{\alpha/2}$  is the generator of an isotropic stable process,  $I_t^{1-\beta}$  is the fractional integral operator and  $\dot{W}(t, x)$  is a mean zero Gaussian random noise.

Under suitable conditions, the equation above admits a unique (mild) solution. I will discuss some properties of such solution. (Received August 31, 2017)

1135-60-437 **Jonathan Novak\***, University of California San Diego, 9500 Gilman Drive, La Jolla, CA 92093. *A Moment Method for Invariant Ensembles.*

I will present a new approach to the spectral analysis of invariant ensembles of random matrices which is based on the observation that the distribution of any such matrix is completely determined by the joint distribution of its diagonal matrix elements. (Received September 02, 2017)

1135-60-492 **Arash Fahim\***, 1017 Academic Way, 208 LOV, Tallahassee, FL 32306, and **Wan-Yu Tsai**, 1017 Academic Way, 208 LOV, Tallahassee, FL 32306. *A Monte Carlo scheme for a singular control problem.* Preliminary report.

We provide a numerical solution of the nonlinear parabolic double obstacle problem arising from a finite horizon portfolio selection with proportional transaction costs. The problem is mainly governed by a time-dependent Hamilton-Jacobi-Bellman equation with gradient constraints due to the singularity of the control. We propose a numerical method which is composed of Monte Carlo simulation to mitigate the curse of dimensionality and finite difference method to make a fine approximation of the free boundary. Numerical results illustrate behaviors of the optimal trading strategies and also satisfy all qualitative properties from theoretical results. (Received September 06, 2017)

1135-60-494 **Manuela Girotti\*** (manuela.girotti@colostate.edu), Colorado State University, Department of Mathematics, 1874 campus delivery, Fort Collins, CO 80523. *Asymptotics of gap probabilities via Riemann–Hilbert approach.*

Gap probabilities of eigenvalues of notable random matrix ensembles can be effectively studied by the use of Riemann–Hilbert techniques from which important features can be extracted. In particular, using steepest



descent methods, the behaviour of gap probabilities in certain critical regimes can be fully and explicitly analyzed. In the talk we will take into consideration two universal processes: the tacnode and the Meijer-G processes. (Received September 06, 2017)

1135-60-518 **Duy Nguyen\*** ([duy.nguyen2@marist.edu](mailto:duy.nguyen2@marist.edu)), Hancock 3017, Marist College, 3399 North Road, Poughkeepsie, NY 12601. *First hitting time of integral diffusions and applications.*

We study the first hitting time of integral functionals of time-homogeneous diffusions, and characterize their Laplace transforms through a stochastic time change. We obtain explicit expressions of the Laplace transforms for the geometric Brownian motion (GBM) and the mean-reverting GBM process. We also introduce a novel probability identity based on an independent exponential randomization and obtain explicit Laplace transforms of the price of arithmetic Asian options and other derivative prices that non-linearly depend on the integral diffusions. Numerical examples are given to demonstrate the accuracy and efficiency of the proposed method. (Received September 06, 2017)

1135-60-530 **Megan Bernstein\*** ([bernstein@math.gatech.edu](mailto:bernstein@math.gatech.edu)) and **Evita Nestoridi.** *Cutoff for the random-to-random card shuffle.*

We use the eigenvalues of the random to random card shuffle found by Dieker and Saliola to prove a sharp upper bound for the total variation mixing time. Combined with the lower bound due to Subag, we prove that this walk exhibits cutoff at  $\frac{3}{4}n \log n$ , answering a conjecture of Diaconis. (Received September 07, 2017)

1135-60-653 **Dan Cheng\*** ([dan.cheng@ttu.edu](mailto:dan.cheng@ttu.edu)), Department of Mathematics and Statistics, Texas Tech University, 1108 Memorial Circle, Lubbock, TX 79409, and **Armin Schwartzman** ([armins@ucsd.edu](mailto:armins@ucsd.edu)), Division of Biostatistics, University of California San Diego, 9500 Gilman Dr., La Jolla, CA 92093. *Expected number and height distribution of critical points of smooth isotropic Gaussian random fields.*

We obtain formulae for the expected number and height distribution of critical points of smooth isotropic Gaussian random fields parameterized on Euclidean space or spheres of arbitrary dimension. The results hold in general in the sense that there are no restrictions on the covariance function of the field except for smoothness and isotropy. The results are based on a characterization of the distribution of the Hessian of the Gaussian field by means of the family of Gaussian orthogonally invariant (GOI) matrices, of which the Gaussian orthogonal ensemble (GOE) is a special case. The obtained formulae depend on the covariance function only through a single parameter (Euclidean space) or two parameters (spheres), and include the special boundary case of random Laplacian eigenfunctions. (Received September 12, 2017)

1135-60-808 **Hanbaek Lyu\*** ([colourgraph@gmail.com](mailto:colourgraph@gmail.com)), 231 W 18th ave, Columbus, OH 43210, and **David Sivakoff** ([dsivakoff@stat.osu.edu](mailto:dsivakoff@stat.osu.edu)), 231 W 18th ave, Columbus, OH 43210. *Persistence of sums of correlated increments and clustering in cellular automata.*

We consider sums of increments given by a functional of a stationary Markov chain. Letting  $T$  be the first return time of the partial sums process to  $(-\infty, 0]$ , under general assumptions, we determine the asymptotic behavior of the survival probability,  $\mathbb{P}(T \geq t) \sim Ct^{-1/2}$  for an explicit constant  $C$ . Our analysis is based on a connection between the survival probability and the running maximum of the time-reversed process, and relies on a functional central limit theorem for Markov chains. Our result extends the classic theorem of Sparre Anderson on sums of mean zero and independent increments to the case of correlated increments. As applications, we recover known clustering results for the 3-color cyclic cellular automaton and the Greenberg-Hastings model in one dimension, and we prove a new clustering result for the 3-color firefly cellular automaton. (Received September 14, 2017)

1135-60-811 **Hanbaek Lyu\*** ([colourgraph@gmail.com](mailto:colourgraph@gmail.com)), 231 W 18th ave, Columbus, OH 43210, and **Janko Gravner** and **David Sivakoff.** *Limiting behavior of 3-color excitable media on arbitrary graphs.*

Fix a simple graph  $G = (V, E)$  and choose a random initial 3-coloring of vertices drawn from a uniform product measure. The 3-color cycle cellular automaton is a process in which at each discrete time step in parallel, every vertex with color  $i$  advances to the successor color  $(i + 1) \bmod 3$  if in contact with a neighbor with the successor color, and otherwise retains the same color. In the Greenberg-Hastings Model, the same update rule applies only to color 0, while other two colors automatically advance. The limiting behavior of these processes has been studied mainly on the integer lattices. In this paper, we introduce a monotone comparison process defined on the universal covering space of the underlying graph, and characterize the limiting behavior of these processes on arbitrary connected graphs. In particular, we establish a phase transition on the Erdős-Rényi random graph. On infinite trees, we connect the rate of color change to the cloud speed of an associated tree-indexed walk. We

give estimates of the cloud speed by generalizing known results to trees with leaves. (Received September 14, 2017)

1135-60-819 **Zachariah E Tyree\*** (ztyree@fau.edu) and **Andrew Thomack**. *On the expected number of zeros of complex Weyl polynomials.*

The zero sets of harmonic polynomials have been the subject of much recent research. In 2009 Li and Wei studied the zeros of random harmonic polynomials with independent Gaussian coefficients. They derived a formula for the expected number of zeros as well as asymptotics for the case that the polynomials are drawn from the Kostlan ensemble. We will propose a new model of random harmonic polynomials based on the Weyl ensemble, and we present asymptotics for this case. We will also briefly discuss interesting phase transitions exhibited by the so-called "first intensity" (an averaged density of zeros in the complex plane). (Received September 14, 2017)

1135-60-979 **Gabrielle C.W Moss\*** (gmoss2@jhu.edu), 3400 N Charles Street, Whitehead Hall 100, Baltimore, MD 21218. *Investigating bounds for the bond and site percolation thresholds of a 2-uniform lattice.*

A percolation model is an infinite graph, from which edges are deleted independently with probability  $p$ . The percolation threshold of an infinite graph is the critical probability  $p_c$  above which there exists a connected, infinite component. Most research so far has focused on calculating exact values and rigorous bounds for the percolation threshold of one-uniform tilings, known as Archimedean lattices. We will investigate how to calculate the percolation threshold of a two-uniform tiling. In a two-uniform tiling, each vertex of the tiling has one of two sequences of faces surrounding it. We use these vertex configurations to name the tiling. This talk will be focusing on the  $(3,4,3,12)$ ,  $(3,12^2)$  lattice. Each vertex of this graph has one of two face sequences: triangle-square-triangle-dodecagon, or triangle-dodecagon-dodecagon. We will derive bounds for the site and bond percolation thresholds and discuss the methods used for calculating these bounds. (Received September 18, 2017)

1135-60-998 **Stewart N. Ethier\*** (ethier@math.utah.edu) and **Jiyeon Lee**. *The flashing Brownian ratchet and Parrondo's paradox.* Preliminary report.

A Brownian ratchet is a one-dimensional diffusion process that drifts toward a minimum of a periodic asymmetric sawtooth potential. A flashing Brownian ratchet is a process that alternates between a Brownian ratchet and a Brownian motion, producing directed motion. These processes have been studied by physicists and biologists for nearly 25 years. The flashing Brownian ratchet is the process that motivated Parrondo's paradox, in which two fair games of chance, when alternated, produce a winning game. Parrondo's games are relatively simple, being discrete in time and space. The flashing Brownian ratchet is more complicated. We show how one can study the latter process numerically using Markov chains. (Received September 18, 2017)

1135-60-999 **Sheldon Ross\***, smross@usc.edu, and **Yang Cao**, cao573@usc.edu. *A winner plays model.* Two of a total of  $n$  players play a game, with the loser then joining the queue and the winner staying to play a game against another player, chosen according to probabilities depending on how long the players in queue have been waiting. Player  $i$  has value  $v_i$  and when  $i$  plays  $j$ ,  $i$  will win with probability  $v_i/(v_i + v_j)$ . Among other things we determine the proportion of all games played that are won by player  $i$ . (Received September 18, 2017)

1135-60-1011 **Divine Wanduku\*** (dwanduku@georgiasouthern.edu), 65 Georgia Ave., Room 3042, Statesboro, GA 30458. *Exploring the effects of white noise on the dynamics of malaria in a highly random environment.* Preliminary report.

Malaria ranks amongst the world's top killers. According to WHO report released in December 2016, there were 212 million cases of malaria resulting in about 429 thousand deaths. Moreover, more than two thirds of the global malaria related deaths are children under the age of 5. In this study, a class of stochastic dynamic models (Ito-Doob type) for malaria is presented. The class type is determined by the qualitative behavior of the nonlinear incidence rates of the disease. Furthermore, the malaria dynamics is influenced by random perturbations from the disease transmission and natural death rates of humans, which are represented by independent white noise processes. The class of malaria models exhibits three random delays:- two of the delays represent the incubation periods of the malaria plasmodium inside the vector and human hosts, whereas the third delay is the period of effective naturally acquired immunity against the disease. This talk focuses on exploring the impacts of the noises in the system on the eradication of malaria via comparative analyses of the family of stochastic models

under different intensities of the noises in the system. Numerical simulation results are presented. (Received September 18, 2017)

1135-60-1066 **Olusegun Michael Otunuga\*** (otunuga@marshall.edu), Dept. of Mathematics, SH 523, One John Marshall Dr, Huntington, WV 25755. *Global stability for a system of HIV epidemic stochastic model with treatments*. Preliminary report.

In this work, we derive and analyze a  $2n+1$ -dimensional stochastic differential equation modeling the transmission and treatment of HIV (Human Immunodeficiency Virus) disease. A theoretical treatment strategy of regular HIV testing and immediate treatment with Antiretroviral Therapy (ART) is investigated in the presence and absence of noise in the transmission rate. We discuss the asymptotic stability of the infection-free and endemic equilibrium (denoted by  $P_0$  and  $P_1$ , respectively) by first deriving the closed form expression for the deterministic infection-free basic reproduction number,  $R_{0,n}$ , and the endemic elimination threshold parameter,  $R_{t,n}$ . In the presence of noise, we derive closed form expression for the stochastic infection-free basic reproduction number,  $\mathcal{R}_{0,n}$ , (in the case without treatment) and the endemic elimination threshold,  $\mathcal{R}_{t,n}$ , (with treatment) and show that epidemic can grow initially (if  $\mathcal{R}_{0,n} > 1$  and  $R_{0,n} < 1$ ) because of the presence of noise in the transmission rate. A numerical simulation is presented for validation. (Received September 19, 2017)

1135-60-1071 **Steven Noren\*** (srnoren@iastate.edu), 125 Marshall Ave, Unit #205, Ames, IA 50014, and **Arka Ghosh** and **Alex Roitershtein**. *Favorite sites of a persistent random walk on  $\mathbb{Z}$* .

We state and prove a theorem on the bound of the number of favorite (i.e., most visited) sites for the symmetric persistent random walk on  $\mathbb{Z}$ , a discrete-time process typified by the correlation of its directional history. This is a generalization of a result by Tóth used to partially prove a longstanding conjecture by Erdős and Révész. (Received September 19, 2017)

1135-60-1074 **Quan Yuan\*** (qyuan@bsu.edu), Department of Mathematical Science, Ball State University, Muncie, IN 47306, and **George Yin** (gyin@wayne.edu), Department of Mathematics, Detroit, MI 48202. *Infinite dimensional regime-switching stochastic approximation algorithms*.

We analyze a type of stochastic approximation algorithms with regime switching modulated by a discrete Markov chain having countable state spaces and two-time-timescale construction. In the algorithm, the increments of a sequence of occupation measures are updated using constant step size iterates. We show that least squares estimates of the tracking errors can be developed. Under the assumption that the adaptation rates have the same magnitude as that of times-different parameter, we prove the continuous-time interpolation from the iterates converges weakly to a system of ordinary differential equations with regime switching. In addition, we demonstrate that the suitably scaled sequence of the tracking errors converges to a system of switching diffusions. (Received September 19, 2017)

1135-60-1075 **Adina Oprisan\*** (oprisana@canisius.edu), Department of Mathematics and Statistics, Canisius College, 2001 Main Street, Buffalo, NY 14208. *An almost sure central limit theorem for autoregressive processes*.

In this talk we consider additive functionals for a class of autoregressive processes and prove that their empirical measures with logarithmic averaging converge almost surely. We apply the method we developed for additive functionals of ergodic Markov processes to the particular case of autoregressive processes that are uniformly ergodic. We also obtain some equivalent results on almost sure asymptotic behavior. (Received September 19, 2017)

1135-60-1122 **Hui-Hsiung Kuo\*** (kuo@math.lsu.edu), Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803. *An extension of the Ito theory to adapted and instantly independent stochastic processes*.

We develop a new theory to extend the Ito theory of stochastic integration to adapted and instantly independent stochastic processes. The new theory is based on the idea of using the left endpoints and the right endpoints of subintervals to evaluate the Ito part and the counter part, respectively, of an integrand. We extend the Ito formula and the isometry formula to the new stochastic integral. We show that multiple Wiener-Ito integrals can be treated as iterated integrals in the new theory. Moreover, we introduce the concepts of near-martingale property and near-Markov property for the new stochastic integration. (Received September 19, 2017)

1135-60-1267 **Andrey Sarantsev\*** ([sarantsev@pstat.ucsb.edu](mailto:sarantsev@pstat.ucsb.edu)), South Hall 5607A, Department of Statistics & Applied Probability, University of California, Santa Barbara, CA 93106.  
*Brownian particles interacting through their ranks.*

Consider a system (finite or infinite) of Brownian one-dimensional particles with drift and diffusion coefficients depending on current rank relative to other particles. They were originally introduced for financial modeling. We study various aspects: collisions of particles, scaling limits, stationary distributions, long-term convergence. (Received September 20, 2017)

1135-60-1273 **Rajeshwari Majumdar\*** ([rajeshwari.majumdar@uconn.edu](mailto:rajeshwari.majumdar@uconn.edu)), **Phanuel Mariano, Hugo Panzo, Lowen Peng and Anthony Sisti.** *Applications of Multiplicative LLN and CLT for Random Matrices.*

The Lyapunov exponent measures the exponential growth rate of the operator norm of the partial product of an independent and identically distributed sequence of random matrices. It usually cannot be computed from the distribution of the matrices. Furstenberg and Kesten (1960) and Le Page (1982) found analogues to the Law of Large Numbers and Central Limit Theorem, respectively, for the norm of the partial product sequence of such random matrices. We use these analogues to efficiently compute the Lyapunov exponent for a selection of random matrix models and numerically estimate the corresponding variances. For random matrices of order 2, with independent components distributed as  $\xi \text{Bernoulli}(\frac{1}{2})$ , where  $\xi \in \mathbb{R}$ , we obtain analytic estimates for the Lyapunov exponent in terms of a limit involving Fibonacci-like sequences. (Received September 20, 2017)

1135-60-1284 **Nobuhiro Asai\*** ([nasai@aeu.ac.jp](mailto:nasai@aeu.ac.jp)), Department of Mathematics, Aichi University of Education, 1, Hiro-sawa, Igaya, Kariya, Aichi 448-8542, Japan. *Weighted Fock spaces with multiparameter and corresponding probability distributions.* Preliminary report.

In this talk, we shall talk on relationships between weighted full Fock spaces with multiparameter and probability distributions from the point of orthogonal polynomials. (Received September 23, 2017)

1135-60-1286 **Zhixin Yang\*** ([zyang6@bsu.edu](mailto:zyang6@bsu.edu)), Department of Mathematical Science, Muncie, IN 47306, and **Zhuo Jin** ([zhuo.jin@unimelb.edu.au](mailto:zhuo.jin@unimelb.edu.au)), Department of Economics, Melbourne, VIC 3010, Australia. *Mean-Variance Defined Contribution Pension Management under Bounded Rationality.* Preliminary report.

In this work, we considered defined contribution pension management with regime switching. The regime switching is described in terms of a continuous time Markov chain. Due to the bounded rationality and observation cost, we cannot observe the information of Markov chain with infinite precision. Therefore, we examined a hidden Markov chain instead. Our interest is to find out the optimal control under mean-variance criterion. Applying the stochastic dynamic programming techniques, we are able to derive the associated Hamilton-Jacob-Bellman equation. Because closed form solutions are basically impossible to obtain, our main efforts are devoted to designing a convergent numerical algorithm using Markov chain approximation. (Received September 20, 2017)

1135-60-1298 **Kursad Tosun\*** ([kursadtosun@mu.edu.tr](mailto:kursadtosun@mu.edu.tr)), Mugla Sitki Kocman University, Department of Biostatistics, Faculty of Medicine, Mugla, Turkey, and **Cemile Tosun** ([ctosun@siu.edu](mailto:ctosun@siu.edu)), Mugla, Turkey. *An epidemic model obtained by expressing the randomness of contact and recovery rates through Wiener processes.*

We can use stochastic nature of the spread of infectious diseases as an advantage to better understand the dynamics of disease epidemics. In this talk, we will introduce a class of stochastic SIS models for non-constant population. By using simulations, based on Balanced Implicit Method, we will show that the new model is realistic. Next, we will prove the global existence of a unique solution of the given system of nonlinear stochastic differential equations. Finally, we will discuss stochastic asymptotic stability of disease-free and endemic equilibria. (Received September 21, 2017)

1135-60-1318 **Padmanabhan Sundar\*** ([psundar@lsu.edu](mailto:psundar@lsu.edu)), Department of Mathematics, Lockett Hall, Louisiana State University, Baton Rouge, LA 70803. *Stochastic analysis of the Enskog equation.*

A stochastic differential system with jumps that corresponds to the Enskog equation in the kinetic theory of gases is studied under a suitable set of conditions. The existence of a weak solution and the uniqueness of marginal distributions of any solution to the system are established. The existence of a probability density for the time-marginals of the velocity is verified in the case where the initial condition is Gaussian, and is shown to be the density of an invariant measure. This is a joint work with S. Albeverio and B. Ruediger. (Received September 21, 2017)

1135-60-1402 **Anthony Sisti\*** ([anthony.sisti@uconn.edu](mailto:anthony.sisti@uconn.edu)), **Phaniel Mariano, Rajeshwari Majumdar** and **Lowen Peng**. *The Derivation of the Black-Scholes Formula Using a Central Limit Theorem Argument.*

The standard method of deriving the Black-Scholes European call option pricing formula involves stochastic differential equations. In this talk we provide an alternate derivation using the Lindeberg-Feller central limit theorem under some technical assumptions. This method allows us obtain the Black-Scholes formula using undergraduate probability. Theoretical results are supplemented with market simulations. (Received September 21, 2017)

1135-60-1406 **Sevak Mkrtchyan\*** ([sevak.mkrtchyan@rochester.edu](mailto:sevak.mkrtchyan@rochester.edu)), Department of Mathematics 915 Hylan Hall, Box 270138, University of Rochester, Rochester, NY 14627. *Turning point processes in plane partitions with periodic weights of arbitrary period.*

In the thermodynamic limit of the lozenge tiling model the frozen boundary develops special points where the liquid region meets with two different frozen regions. These are called turning points. It was conjectured by Okounkov and Reshetikhin that in the scaling limit of the model the local point process near turning points should converge to the GUE corners process. We will discuss the appearance of frozen regions of arbitrary rational slope when weights in the model are periodic in one direction with arbitrary fixed finite period, and what these new frozen regions mean for the turning process. (Received September 21, 2017)

1135-60-1415 **Nobuaki Obata\*** ([obata@math.is.tohoku.ac.jp](mailto:obata@math.is.tohoku.ac.jp)), Sendai, 980-8579, Japan. *Asymptotic spectral analysis of graphs and orthogonal polynomials in two variables.*

During the last fifteen years the quantum probabilistic methods have been developed for asymptotic spectral analysis of (growing) graphs, see e.g., N. Obata: Spectral analysis of growing graphs - A quantum probability point of view, Springer, 2017. One of the main tools is the so-called *method of quantum decomposition*, which makes it possible for us to study the adjacency matrices of graphs in terms of creation and annihilation operators in interacting Fock spaces. Using these quantum probabilistic structure we may obtain asymptotic spectral distributions as orthogonalizing measures of certain polynomials determined by three-term recurrence relation. Many examples of (growing) graphs have been studied in this line so far, however, there is a crucial restriction caused by the use of orthogonal polynomials in one variable. In this talk, reviewing the old result on limit distributions of growing Hamming graphs, I will discuss a two-variable extension by means of strongly regular graphs, and show an example which yields mixture of Gaussian and Poisson distributions. (Received September 22, 2017)

1135-60-1551 **Alan Krinik\***, [ackrinik@cpp.edu](mailto:ackrinik@cpp.edu), and **Uyen Nguyen, Ali Oudich, Pedram Ostadhassanpanjehali, Luis Cervantes, Chon In Luk, Matthew McDonough, Jeffrey Yeh** and **Lyheng Phey**. *Matrix properties of a class of birth-death chains and processes.*

Consider a birth-death chain on state space  $S = \{0, 1, 2, \dots, H\}$  where  $H \in \mathbb{N}$  with alternating birth probabilities  $c_1, c_2, c_1, c_2, \dots$  and alternating death probabilities  $a_1, a_2, a_1, a_2, \dots$  where  $0 < c_i < 1$ ,  $0 < a_i < 1$  and  $a_i + c_i \leq 1$  for  $i = 1, 2$ . Assume  $a_1 c_2 = a_2 c_1$  or  $a_1 c_1 = a_2 c_2$ . Suppose  $P$  is the one-step transition probability matrix associated with this birth-death chain and let  $P^*$  be the one-step transition probability matrix of the corresponding dual birth-death chain.

Conclusions:

- 1) The set of eigenvalues of  $P$  and  $P^*$  are the same and are determined as a function of  $H$ .
- 2)  $P^n$  and  $(P^*)^n$  can be determined exactly for  $n \in \mathbb{N}$ .
- 3) Birth-death chains that have the same set of eigenvalues are identified.

4) Related results for birth-death processes and non-birth-death chains are discussed. (Received September 23, 2017)

1135-60-1592 **Ryan DeMuse\*** ([ryan.demuse@du.edu](mailto:ryan.demuse@du.edu)), **Danielle Larcomb** and **Mei Yin**. *Phase Transitions in Edge-Weighted Exponential Random Graphs: Near-Degeneracy and Universality.*

Conventionally used exponential random graphs cannot directly model weighted networks as the underlying probability space consists of simple graphs only. Since many substantively important networks are weighted, such as social networks where one wants to consider the closeness or strength of a relationship, this limitation is especially problematic. We extend the existing exponential framework by proposing a generic common distribution for the edge weights. Minimal assumptions are placed on the distribution, that is, it is non-degenerate and

supported on the unit interval. By doing so, we recognize the essential properties associated with near-degeneracy and universality in edge-weighted exponential random graphs. (Received September 23, 2017)

1135-60-1635 **Yuval Peres\*** ([peres@microsoft.com](mailto:peres@microsoft.com)), Microsoft Research, 1 Microsoft Way, Redmond, WA 98052. *Cutoff for a stratified random walk on the hypercube, related to a mysterious random walk on invertible matrices modulo 2.*

Consider the Markov chain on the nonzero vectors in the hypercube, which moves by picking an ordered pair  $(i, j)$  of distinct coordinates uniformly at random and adding the bit at location  $i$  to the bit at location  $j$ , modulo 2. In joint work with Anna Ben Hamou, we showed that this Markov chain has cutoff at time  $(3/2)n \log n$  with window of order  $n$ , solving a question posed by Chung and Graham (1997). The chain records the evolution of a single column in the following random walk on the group of invertible  $n \times n$  matrices over  $\mathbb{F}_2$ : pick an ordered pair  $(i, j)$  of distinct rows uniformly at random and add row  $i$  to row  $j$ , modulo 2. The order of the mixing time for this matrix walk is unknown, but lies between  $n^2/(\log n)$  and  $n^3$ . As suggested by Ron Rivest (motivated by applications in cryptography), if we restrict attention to polynomially-computable distinguishing statistics, the resulting modified mixing time for the matrix walk might be much smaller. (Received September 23, 2017)

1135-60-1660 **Christine Klymko\*** ([klymko1@llnl.gov](mailto:klymko1@llnl.gov)), 7000 East Avenue, Livermore, CA 94550, and **Olivia Simpson** ([oliviamsimpson@gmail.com](mailto:oliviamsimpson@gmail.com)), 345 Park Avenue, San Jose, CA 95110. *Detecting changes in node importance in time evolving networks.* Preliminary report.

One of the most basic questions in network analysis concerns the ranking of nodes in complex networks. Many centrality measures have been for this purpose, however, the majority are meant for use on static (unchanging) networks. In reality most real-world networks are evolving over time. Understanding how a node's importance changes, especially the discovery of large jumps in importance, can illuminate otherwise hidden dynamics. We present a method of detecting changes in node importance relative to a fixed "seed" node in large, time-evolving networks by simulating random walks, factoring in temporal metadata in diverse ways. We show that sampling a sublinear number of random walks in the evolving network is enough to capture a change of specified magnitude that occurs over some duration of time with high probability. (Specifically, we show that for an evolving network on  $n$  nodes with a history of length  $T$ ,  $\mathcal{O}(\frac{T}{\tau\delta} \log n)$  random walkers will capture a change of size  $\delta$  over a period of size  $\tau$  with high probability). The use of random walks allows our method to be adaptable to large and noisy networks as well as allowing for various interpretations of what level of importance change is meaningful. (Received September 24, 2017)

1135-60-1663 **Timothy La Fond** ([lafond1@llnl.gov](mailto:lafond1@llnl.gov)), 7000 East Avenue, Livermore, CA 94550, **Geoffrey Sanders** ([sanders29@llnl.gov](mailto:sanders29@llnl.gov)), 7000 East Avenue, Livermore, CA 94550, **Christine Klymko\*** ([klymko1@llnl.gov](mailto:klymko1@llnl.gov)), 7000 East Avenue, Livermore, CA 94550, and **Van E. Henson** ([henson5@llnl.gov](mailto:henson5@llnl.gov)), 7000 East Avenue, Livermore, CA 94550. *An Ensemble Framework for Detecting Community Changes in Dynamic Networks.*

Dynamic networks, especially those representing social networks, undergo constant evolution of their community structure over time. Nodes can migrate between different communities, communities can split into multiple new communities, communities can merge together, etc. In order to represent dynamic networks with evolving communities it is essential to use a dynamic model rather than a static one. Here we use a dynamic stochastic block model where the underlying block model is different at different times. In order to represent the structural changes expressed by this dynamic model the network will be split into discrete time segments and a clustering algorithm will assign block memberships for each segment. In this paper, we show that using an ensemble of clustering assignments accommodates for the variance in scalable clustering algorithms and produces superior results in terms of pairwise-precision and pairwise-recall. We also demonstrate that the dynamic clustering produced by the ensemble can be visualized as a flowchart which encapsulates the community evolution succinctly. (Received September 24, 2017)

1135-60-1749 **Joseph Jackson\***, [jjackso1@swarthmore.edu](mailto:jjackso1@swarthmore.edu), and **Mike Cinkoske** and **Claire Plunkett**. *On the Speed of an Excited Asymmetric Random Walk.*

An excited random walk is a non-Markovian extension of the simple random walk, in which the walk's behavior at time  $n$  is impacted by the path it has taken up to time  $n$ . The properties of an excited random walk are more difficult to investigate than those of a simple random walk. For example, the limiting speed of an excited random walk is either zero or unknown depending on its initial conditions. While its limiting speed is unknown in most cases, the qualitative behavior of an excited random walk is largely determined by a parameter  $\delta$  which can be computed explicitly. Despite this, it is known that the limiting speed cannot be written as a function of  $\delta$ . We offer a new proof of this fact, and use techniques from this proof to further investigate the relationship

between  $\delta$  and limiting speed. We also generalize the standard excited random walk by introducing a “bias” to the right, and call this generalization an excited asymmetric random walk. Under certain initial conditions we are able to compute an explicit formula for the limiting speed of an excited asymmetric random walk. (Received September 24, 2017)

1135-60-1786 **Robert Buckingham\***, Department of Mathematical Sciences, The University of Cincinnati, P.O. Box 210025, Cincinnati, OH 45221. *Nonintersecting Brownian bridges on the unit circle with drift.*

Nonintersecting Brownian bridges on the unit circle form a determinantal stochastic process exhibiting random matrix statistics for large numbers of walkers. We investigate the effect of adding a drift term to walkers on the circle conditioned to start and end at the same position. We compute the asymptotic distribution of total winding numbers in the scaling regime in which the expected total winding is finite. Furthermore, we show that an appropriate double scaling of the drift and return time leads to a generalization of the tacnode process expressed in terms of generalized Hastings-McLeod functions. Our results follow from Riemann-Hilbert analysis of a family of discrete orthogonal polynomials with a complex weight. This is joint work with Karl Liechty. (Received September 24, 2017)

1135-60-1810 **Ibukun O Amusan\*** (ibukun.amusan@kysu.edu). *Pricing with Markov Chain Monte Carlo and a Coupled Additive-Multiplicative Stochastic Model.*

This paper considers the problem of pricing using Markov Chain Monte Carlo (MCMC). MCMC has been known to be useful for processes whose distributions are complex. The paper will specifically look at MCMC with a coupled additive-multiplicative (CAM) stochastic volatility model, which does not follow a Gaussian distribution. Some of the challenges encountered with pricing under this CAM model will be discussed. Also, some potential applications of this model in actuarial science will be explored. (Received September 24, 2017)

1135-60-2030 **Dylan M Cable\*** (dcable@stanford.edu), 1079 Oak Ridge Dr, Glencoe, IL 60022, and **Antonio Auffinger**. *Pemantle’s min-plus binary tree.*

We consider a stochastic process that describes several particles interacting by either merging or annihilation. When two particles merge, they combine their masses; when annihilation occurs, only the particle of smallest mass survives. Particles start at the bottom of a binary tree of depth  $N$  and move towards the root. Assuming that merging or annihilation happens independently at random, we determine the limit law of the final mass of the system in the large  $N$  limit. We do so by providing precise upper and lower bounds for the difference equation of the cumulative distribution function. If we let  $p$  be the probability of merging, then we observe critical behavior at  $p = 1/2$ . (Received September 25, 2017)

1135-60-2055 **Alisa Knizel\*** (knizel@math.columbia.edu) and **Anton Dzhamay**. *Gap probabilities in  $q$ -Racah tiling model and discrete Painlevé equations.* Preliminary report.

We show that gap probabilities in the  $q$ -Racah tiling model can be computed in terms of the discrete Painlevé equation of type  $E_7^{(1)}$ . We work in the context of isomonodromy deformations and present a symmetric Lax pair for this equation. (Received September 25, 2017)

1135-60-2381 **Giray Okten\*** (okten@math.fsu.edu). *Sensitivity and Robustness of Financial Models.*

Global sensitivity analysis is a widely used tool for modelers in sciences and engineering. I will discuss global sensitivity analysis based on Sobol’ sensitivity indices, and a novel approach to quantify the robustness of a model using randomized sensitivity indices. I will present some applications to interest rate and weather derivative models. (Received September 26, 2017)

1135-60-2398 **Y. M. Dib\*** (youssef.dib@balamand.edu.lb), Koura, North, Lebanon, **H. Greige** (youssef.dib@balamand.edu.lb), Koura, North, Lebanon, **N. Kmeid** (youssef.dib@balamand.edu.lb), Koura, North, Lebanon, and **Y. N. Raffoul** (youssef.raffoul@notes.udayton.edu), Dayton, OH 41432. *Optimization Of Cash Management Fluctuation Through Stochastic Processes.*

We study the optimal level of cash for the firm to hold. We model the cash level with inflows and outflows due to deposits and withdrawals; In between, the cash level is a stochastic process where it signals a time to sell. After modeling the continuous jump, we implemented first step analysis method to find the probability of the event with initial cash and we were able to calculate data driven by set of difference equations. These data are used to determine the length of the period of the investment. Then, we adopt the probabilistic decision model where it goes under mathematical optimization. This model let the investor to maximize the probability of success or to stop on one of the largest fortune using the equation of the principle of optimality. Finally, to solve these optimal

equations, we used the result of positive dynamic programming and we elaborate them by proves. (Received September 26, 2017)

1135-60-2421 **Timothy Chumley\*** ([tchumley@mtholyoke.edu](mailto:tchumley@mtholyoke.edu)) and **Renato Feres** ([feres@wustl.edu](mailto:feres@wustl.edu)).  
*Entropy production in random billiards and the second law of thermodynamics.*

A random dynamical system is said to be time-reversible if the statistical properties of orbits do not change after reversing the arrow of time. The degree of irreversibility of a given system is captured by the notion of entropy production rate. We describe a general formula for entropy production that applies to a class of random billiard systems on Riemannian manifolds with boundary for which it is meaningful to talk about energy exchange between billiard particle and boundary. This formula establishes a relation between the purely mathematical concept of entropy production rate and physics textbook thermodynamic entropy. In particular, it recovers Clausius formulation of the second law of thermodynamics: the system must evolve so as to transfer energy from hot to cold. (Received September 26, 2017)

1135-60-2442 **Kumer Pial Das\*** ([kumer.das@lamar.edu](mailto:kumer.das@lamar.edu)), Kumer Das, PO Box 10119, Lamar University, Beaumont, TX 77710. *Choosing accessible and open problems for undergraduate researchers.*

Students think that most, if not all, problems in mathematical sciences are beyond their ability to research them. However, the growth of REUs, thesis, and research projects in the past two decades show that undergraduates can perform math research, or even benefit from going through a structured and well-administered research experience. A mentor is responsible for preparing young scientists by providing them a balance of research and educational elements. Such balance is necessary for many reasons. Primarily, students appreciate the relevance of their coursework in their research. This study shows how to choose accessible problems for students based on their interest and expertise in courses such as calculus, statistics, and stochastic processes. It focuses on problems related to statistics and big data analytics. One of the effective sources of problems is the work of previously conducted undergraduate research under the guidance of the mentor since there is usually quite a few number of problems left unsolved in every research. The mentor encourages researchers to include a section called “open problems” or “future research” in their final report which may be used by future researchers. An excellent collaboration with industries also provide some open problems each year. (Received September 26, 2017)

1135-60-2463 **Paul H Bezandry\*** ([pbezandry@howard.edu](mailto:pbezandry@howard.edu)), 2441 6th Street, NW, Washington, DC 20059. *On almost periodic Stochastic difference equations with random delay.* Preliminary report.

In this talk we discuss the existence and uniqueness of the almost periodic solution of some system of stochastic semi-linear difference equations with bounded random delay of the form:

$$X(n+1) = A(n)X(n) + f(n, X(n-\tau)), X(n-\tau+1), \dots, X(n-1), X(n)),$$

$n \in \mathbb{Z}_+$ , by means of exponential dichotomy. Some applications will be discussed. (Received September 26, 2017)

1135-60-2496 **Flavia Sancier\*** ([fsancier@antiochcollege.edu](mailto:fsancier@antiochcollege.edu)), 1 Morgan Place, Yellow Springs, OH 45387. *Testing Option Pricing Models with Memory.*

We discuss the derivation and testing of continuous option pricing models that have stochastic volatility with hereditary structure. That is, the stock dynamics follows a nonlinear stochastic functional differential equation. In particular, two models are discussed: one in which the drift and volatility continuously depend only on past stock prices, and one in which the continuous past dependence may include present stock prices. The models are an extension of the well-known Black-Scholes framework. The option pricing formulas are the result of an equivalent (local) martingale measure and therefore are written as conditional expectations that can be simulated via Monte Carlo methods. (Received September 26, 2017)

1135-60-2569 **Adetokunbo I Fadahunsi\*** ([afadahun@mix.wvu.edu](mailto:afadahun@mix.wvu.edu)), 813 Grant Ave Apt 25, Morgantown, WV 26505. *Recovery of ruin probability and Value at Risk from the scaled Laplace transform inversion.*

In this paper, we propose three modified approximations of the the ruin probability and the inverse function of the ruin probability using the inversion of the scaled values of the Laplace transform suggested by Mnatsakanov et al (2015). The problem of numerically evaluating the tail Value at Risk of an insurance portfolio is also discussed briefly. Using several examples, performances of the proposed constructions are demonstrated with graphs and tables. (Received September 26, 2017)



1135-60-2621 **Mark L Huber\*** ([autotomic@gmail.com](mailto:autotomic@gmail.com)), 850 Columbia AV, Claremont, CA 91711. *A new way to view popping algorithms for perfect simulation*. Preliminary report.

One method of sampling exactly from high dimensional distributions is called “popping”, and was first introduced by David Wilson in 1996. In this talk I’ll show how these popping algorithms arise naturally as a special case of a type of acceptance rejection algorithm. In particular, I’ll describe how the algorithm for generating uniformly from the rooted directed spanning trees of a graph, and for generating uniformly from sink free orientations of a graph, both come about from such an analysis. I’ll then talk about how these algorithms can be used to generate perfectly from problems where popping does not apply such as the isolated (aka independent) sets of a graph. For certain parameter values, these algorithms can be shown to run in time linear in the input size of the problem. (Received September 26, 2017)

1135-60-2623 **Mobolaji O Ogunsolu\*** ([ogunsolu@usf.edu](mailto:ogunsolu@usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700, and **Gangaram S Ladde** ([gladde@usf.edu](mailto:gladde@usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700. *Nonlinear Levy-type Dynamic Modeling of Time-to-Event Process*. Preliminary report.

We initiate an innovative interconnected hybrid approach for modeling time-to-event processes under the influence of intervention processes. The model is described by a nonlinear L’evy-type stochastic differential equation. We establish the existence and uniqueness of the solution to our model. Well-known hazard and survival function models are presented to demonstrate usefulness of the approach. Moreover, state and parameter estimation problems are also investigated. A numerical approximation scheme is outlined to illustrate the presented results. (Received September 26, 2017)

1135-60-2635 **Emmanuel A. Appiah\*** ([emmanuel1@mail.usf.edu](mailto:emmanuel1@mail.usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700, and **Gangaram S Ladde** ([gladde@usf.edu](mailto:gladde@usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700. *Stochastic Hybrid Dynamic Modeling of Time-to-Event Processes*. Preliminary report.

In this work, we develop an innovative alternative stochastic hybrid dynamic model for time-to-event processes in a systematic and unified way. The presented approach is motivated by parameter and state estimation of time-to-event processes in biological, chemical, engineering, epidemiological, medical, military, and social dynamic processes under the influence of discrete-time intervention processes. The developed results are applied to time-to-event data sets. (Received September 26, 2017)

1135-60-2671 **Gangaram S. Ladde\*** ([gladde@usf.edu](mailto:gladde@usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700. *Comparison Theorems for Complex Interconnected Large-scale Stochastic Dynamic Systems*. Preliminary report.

In this work, employing Lyapunov like function, we develop comparison results for complex interconnected large-scale stochastic dynamic systems. The developed results provide upper and lower bounds for solution processes. Moreover, the presented study provides a robust and easily verifiable sufficient conditions in terms of system parameters. (Received September 26, 2017)

1135-60-2687 **Aric Hagberg\*** ([hagberg@lanl.gov](mailto:hagberg@lanl.gov)). *Temporal Random Graphs and Reachability in Cybersecurity Networks*.

The complex relational data of networked computer systems naturally leads to modeling and analysis using graphs, or networks. Network science is also beginning to play a role in studying cybersecurity systems where the nodes and edges aren’t just computers and communication links. I will give examples of cybersecurity network problems including the application of temporal random graph models to analyze centralized user authentication systems. (Received September 26, 2017)

1135-60-2786 **Andrea W. Richa\*** ([aricha@asu.edu](mailto:aricha@asu.edu)), Computer Science and Engineering, CIDSE, Arizona State University, PO Box 878809, Tempe, AZ 85287-8809, and **Sarah Cannon, Joshua J. Daymude, Dana Randall and Marta Andres Arroyo**. *Local Stochastic Algorithms for Compression and Shortcut Bridging in Programmable Matter*.

Many programmable matter systems have been developed, including modular and swarm robotics, synthetic biology, DNA tiling, and smart materials. We describe programmable matter as an abstract collection of simple computational elements (particles) with limited memory that each execute fully distributed, local, asynchronous

algorithms to solve system-wide problems such as movement, configuration, and coordination. For the compression problem, in which a particle system is tasked with gathering as tightly as possible, we give a Markov chain based solution that minimizes the overall perimeter of the system via individual particles making decisions based only on information about their local neighborhoods. Variants of this algorithm produce a variety of other useful behaviors, including expansion over as wide an area as possible. Subsequently we present a distributed stochastic algorithm for particle systems forming shortcut bridges, a behavior also observed in army ants, where balancing between two competing global objectives is observed. For all of these problems, tools from Markov chain analysis and distributed algorithms allow us to relate local and globally optimal behavior, and to produce local algorithms that are robust, nearly-oblivious, and truly decentralized. (Received September 26, 2017)

1135-60-2962 **Ryan T. White\*** ([rwhite2009@fit.edu](mailto:rwhite2009@fit.edu)), Department of Mathematical Sciences, 150 W. University Blvd., Melbourne, FL 32940. *Exits of Oscillating Random Walks Under Delayed Observation*.

We study the behavior of the random walk  $A(t)$  of a particle on a random lattice enclosed by an open rectangle. Since the lattice is randomly generated as the particle moves, determining the first exit time and position of the particle upon exit is nontrivial. To further complicate the situation,  $A(t)$  is observed only upon a third-party independent point process  $\{\tau_n\}$ . The observed time series  $\{A(\tau_n)\}$  presents crude, delayed data. This sequence is analyzed to recover probabilistic data about the true process. The characteristic functions of the first observed exit time and position of the process upon observed exit are derived and reduced to explicit expressions for special cases. The process has two intended applications: (1) modeling networks under attacks disabling valuable nodes, where the results can predict their crashes and offer remedies, and (2) modeling the short-term accumulation of funds, which is useful in tracking suspicious financial transactions. (Received September 26, 2017)

1135-60-3061 **Hongwei Mei\*** ([hongwei.mei@ku.edu](mailto:hongwei.mei@ku.edu)), 405 Snow Hall, 1460 Jayhawk Blvd, Lawrence, KS 66045, and **Jiongmin Yong**. *Ergodic Control of Linear-Quadratic Stochastic Systems*.

This paper is devoted to investigate the relationship between an  $L^2$  open-loop control problem on finite horizon and the ergodic control problem for linear-quadratic systems. In our paper it is not required that the diffusion term is degenerate or independent of the control process. We prove that the feedback control on finite horizon will converge to an optimal ergodic control as time horizon tends to infinity. As a byproduct, we also can find the explicit forms of the optimal ergodic cost and one optimal control under what the optimal ergodic cost is achieved. (Received September 26, 2017)

1135-60-3073 **Sunday Amaechi Asogwa\*** ([saa0020@auburn.edu](mailto:saa0020@auburn.edu)), 221 Parker Hall, Auburn University, Auburn, AL 36849. *Intermittency fronts and some non-existence results for the space-time fractional diffusion*.

We consider the following time fractional stochastic heat type equation

$$\partial_t^\beta u_t(x) = -\nu(-\Delta)^{\alpha/2} u_t(x) + I_t^{1-\beta} [b(u) + \sigma(u) \dot{W}(t, x)]$$

in  $(d+1)$  dimensions, where  $\nu > 0, \beta \in (0, 1), \alpha \in (0, 2]$ . The operator  $\partial_t^\beta$  is the Caputo fractional derivative while  $-(-\Delta)^{\alpha/2}$  is the generator of an isotropic  $\alpha$ -stable Lévy process and  $I_t^{1-\beta}$  is the Riesz fractional integral operator. The forcing noise denoted by  $\dot{W}(t, x)$  is a space-time white noise. When  $b = 0$ , we show that : (i) absolute moments of the solutions of this equation grows exponentially; and (ii) the distances to the origin of the farthest high peaks of those moments grow exactly linearly with time.

When  $b(u) = u^{1+\eta}$ , we establish some non-existence of global random field solutions, under some additional conditions, most notably on  $\eta, \sigma$  and the initial condition. Our results complement those of P. Chow “P.-L. Chow. Explosive solutions of stochastic reaction-diffusion equations in mean  $l_p$ -norm. *J. Differential Equations*, 250(5) (2011), 2567–2580”, and Foondun et al. in “M. Foondun, W. Liu, and E. Nane. (Received September 26, 2017)

1135-60-3197 **Ted Theodosopoulos\*** ([ttheodosopoulos@nuevaschool.org](mailto:ttheodosopoulos@nuevaschool.org)), 131 E. 28th Ave., San Mateo, CA 94403. *Persistent homology measures of stochastic network models*. Preliminary report.

We present a class of spin processes on dynamic networks, where the evolution of the network is coupled to the stochastic spin dynamics. Applications of these models are discussed in economics and biology. We investigate the generation of random variables that measure the persistent homology of these evolving networks. The resulting distributions can be used as a data filter to infer properties of the underlying networks. (Received September 27, 2017)

1135-60-3203 **Jebessa B Mijena\*** (jebessa.mijena@gcsu.edu), 2317 Sherry Cir. Apt. 11, Milledgeville, GA 31061, and **Erkan Nane**. *Analysis of space-time fractional stochastic partial differential equations.*

We consider non-linear time-fractional stochastic heat type equation

$$\partial_t^\beta u_t(x) = -\nu(-\Delta)^{\alpha/2} u_t(x) + I_t^{1-\beta}[\sigma(u) \dot{W}(t, x)]$$

in  $(d + 1)$  dimensions, where  $\nu > 0, \beta \in (0, 1), \alpha \in (0, 2]$  and  $d < \min\{2, \beta^{-1}\}\alpha$ ,  $\partial_t^\beta$  is the Caputo fractional derivative,  $-(-\Delta)^{\alpha/2}$  is the generator of an isotropic stable process,  $I_t^{1-\beta}$  is the fractional integral operator,  $\dot{W}(t, x)$  is space-time white noise, and  $\sigma : \mathbb{R} \rightarrow \mathbb{R}$  is Lipschitz continuous. Time fractional stochastic heat type equations might be used to model phenomenon with random effects with thermal memory. We prove existence and uniqueness of mild solutions to this equation and establish conditions under which the solution is continuous. In sharp contrast to the stochastic partial differential equations studied earlier by Foondun and Khoshnevisan and by Walsh, in some cases our results give existence of random field solutions in spatial dimensions  $d = 1, 2, 3$ . Under faster than linear growth of  $\sigma$ , we show that time fractional stochastic partial differential equation has no finite energy solution. (Received September 27, 2017)

1135-60-3207 **Jewgeni H. Dshalalow** and **Kizza M. Nandyose\*** (knandyose2012@my.fit.edu). *Continuous Time Interpolation of Marked Random Measures and their Applications.*

We study a class of monotone and non-monotone delayed marked point processes that model stochastic networks (under attacks), optional trading, status of queuing systems during vacation modes, responses to cancer treatments (such as chemotherapy and radiation), hostile ambushes in economics and warfare. We are interested in the behavior of such a process about a fixed threshold. It presents an analytic challenge, because of the arbitrary nature of random marks. We target the first passage time, pre-first passage time, the status of the associated continuous time parameter process between these two epochs, and the status of the process upon these two epochs. A joint functional of these stochastic quantities is investigated in the transient mode. (Received September 27, 2017)

## 62 ► Statistics

1135-62-339 **Joey L Hart\*** (jlhart3@ncsu.edu), **Julie Bessac** and **Emil Constantinescu**. *Global sensitivity analysis for statistical model parameters.*

In this talk a new method is proposed to perform global sensitivity analysis (GSA) on the parameters of a statistical model. Such analysis is useful for improving model accuracy, encouraging model parsimony, and reducing computational complexity. Statistical models pose challenges to traditional GSA tools because of parameter correlations, multivariate model outputs, and model stochasticity. The proposed method utilizes problem structure to address these challenges in a computationally efficient manner. The motivating wind energy application will be discussed and numerical results given. (Received August 25, 2017)

1135-62-364 **Brice Merlin Nguelifack\*** (nguelifa@usna.edu), 572 Holloway Rd., Ste 300, Stop C, United States Naval Academy, Annapolis, MD 21403. *Generalized signed-rank estimation for regression models with non-ignorable missing responses.*

In this paper, we study the generalized signed-rank estimator of the regression coefficients with non-ignorable missing responses. The generalized signed-rank objective function covers a large class of existing objective functions such as the signed-rank, the least absolute deviation among others. We establish the consistency and the limiting distribution of the proposed estimator of the underlying parameter in the regression model. Finite-samples simulation studies are carried out to evaluate the performance of the proposed estimation method, and a practical application is also given to illustrate our method. Results of these studies show that the proposed approach results in a robust and more efficient estimator compared with the least squares and least absolute deviation approaches, mainly when dealing with heavy-tailed, contaminated model error distributions and/or when data contain gross outliers in the response space. (Received August 28, 2017)

1135-62-549 **Alexander Cloninger\*** (acloninger@ucsd.edu), Department of Mathematics, University of California, San Diego, 9500 Gillman Drive, San Diego, CA 92093. *Prediction Models for Graph-Linked Data with Localized Regression.*

Regression problems typically assume the training data are independent samples, but in many modern applications samples come from individuals connected by a network or some other form of baseline information. In the case that the population is divided into discrete and disjoint classes with no notion of inter-class connection, hierarchical linear modelling can address such issues. We propose a series of optimization schemes that create

generalized linear models, which incorporate both the local neighborhoods and global structure of the population network Laplacian to define localized regression coefficients that smoothly vary with respect to the network. We also provide guarantees of bias and variance of the coefficients. Depending on the context of the data and function, this method can be used to learn local function gradients on a manifold, model and denoise time varying processes with drift terms determined by some initial condition on a network, and perform co-clustering of a matrix or tensor. We also discuss incorporating the local regression coefficient estimates to redefine the network geometry and build a function adapted diffusion process. (Received September 07, 2017)

1135-62-553 **Edmund E Ameyaw\*** (edmund.ameyaw@bison.howard.edu), 5800 Timber Creek Ter, Apt 202, Hyattsville, MD 20782, and **Paul Bezandry, Victor Apprey and John Kwagyan.** *Evaluation of Integrals of the Form  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(s, t) \exp \left\{ -\frac{1}{2(1-\rho^2)} [s^2 + t^2 - 2\rho st] \right\} ds dt$  .*

The Logistic-Gaussian distribution is used in statistical applications to account for clustering among binary outcomes. However, its extension and applicability to bivariate outcomes are limited. We developed a model for correlated bivariate binary data that incorporated the Logistic-Gaussian distribution. Bivariate response probabilities in terms of random effects models are formulated, and maximum marginal likelihood estimation procedures based on Gauss-Hermite quadrature. Application to the analysis of vision loss in diabetic retinopathy are is discussed. keywords: Correlated Data; Logistic -Gaussian Distribution; Maximum Marginal Likelihood; Bivariate Binary Outcomes; Gauss-Hermite quadrature; Simulation. (Received September 08, 2017)

1135-62-1141 **Adam Gustafson, Matthew Hirn, Kitty Mohammed, Hariharan Narayanan and Jason Xu\*** (jqxu@ucla.edu). *A Probably-Approximately-Correct Algorithm for Learning a  $C^{1,1}(\mathbb{R}^d)$  Function from Noisy Samples.*

One means of fitting functions to high-dimensional data is by providing smoothness constraints. Recently, an efficient algorithm is proposed by Herbert-Voss, Hirn and McCollum that constructs  $\hat{f}$  and yields  $\text{Lip}(\nabla \hat{f})$  solving the following function approximation problem: given a finite set  $E \subset \mathbb{R}^d$  and a function  $f : E \rightarrow \mathbb{R}$ , interpolate the given information with a function  $\hat{f} \in C^{1,1}(\mathbb{R}^d)$ , the class of first-order differentiable functions with Lipschitz gradients, such that  $\hat{f}(a) = f(a)$  for all  $a \in E$ , and the value of  $\text{Lip}(\nabla \hat{f})$  is minimal.

We address statistical aspects when reconstructing  $\hat{f}$  from noisy data: we observe samples  $y(a) = f(a) + e_a$  for  $a \in E$ , where  $e_a$  is an IID noise term. We obtain uniform bounds relating the empirical risk and true risk over the class  $\mathcal{F} = \{f \in C^{1,1}(\mathbb{R}^d) \mid \text{Lip}(\nabla f) \leq \bar{M}\}$ , where the quantity  $\bar{M}$  grows with the number of samples at a rate governed by the metric entropy of the class  $C^{1,1}(\mathbb{R}^d)$ . We provide a solution method via Vaidya's algorithm, supporting our results via numerical experiments on simulated data. (Received September 20, 2017)

1135-62-1451 **Emily Nystrom\*** (emily.nystrom@navy.mil). *Applications of Random Forests for Modeling Obsolescence.* Preliminary report.

Machine learning offers a flexible, data-driven framework that can be tailored to study time-to-event data. Compared to traditional methods for modeling temporal processes, machine learning requires fewer prior assumptions or input parameters. The inconvenience of system disruption (e.g. due to component unavailability) may be addressed with the development of analytical tools that can be used to proactively monitor and mitigate risks associated with future obsolescence. We consider modeling obsolescence using machine learning algorithms, leveraging the application-methods pairing highlighted by Jennings, Wu, and Terpenney in 2016. We discuss the selection and evaluation of machine learning techniques for applications in obsolescence and outline characteristics of candidate datasets that may provide insight into modeling and predicting obsolescence. (Received September 22, 2017)

1135-62-1500 **José M Ponciano\*** (josemi@ufl.edu), Biology Department, PO Box 118525, Gainesville, FL 32511, and **Mark L Taper.** *Model projections in model space: Multimodel inference beyond model averaging.* Preliminary report.

Information criteria have had a profound impact on modern science. They allow researchers to estimate which of a set of model is closest to the generating process. Unfortunately, information criterion comparison does not tell how good the best model is. Nor does practitioners fully test the reliability (e.g. error rates) of model selection using information criteria. In this work, I show that these two shortcomings can be resolved with a key observation: in a standard analysis it is ignored that there is an estimable divergence relationship amongst all of the models, as well as divergences from each model to the generating process. I then show that using both sets of divergences, a model space can be constructed including an estimated location for the generating process. Thus, not only an analyst can determine which model is closest to the generating process, she/he can also determine how close to the generating process the best model is. Properties of the generating process estimated from these projections are more accurate than those estimated by model averaging. (Received September 22, 2017)

1135-62-1769 **Eddie Aamari\*** (eaamari@ucsd.edu), Department of Mathematics, University of California, San Diego (UCSD), 9500 Gilman Drive # 0112, La Jolla, CA 92093, and **Jisu Kim, Frédéric Chazal, Bertrand Michel, Alessandro Rinaldo** and **Larry Wasserman**. *Estimating the Reach of a Manifold*.

Various problems within computational geometry and manifold learning encode geometric regularity through the so-called reach, a generalized convexity parameter. The reach  $\tau_M$  of a submanifold  $M \subset \mathbb{R}^D$  is the maximal offset radius on which the projection onto  $M$  is well defined. The parameter  $\tau_M$  renders a certain minimal scale of  $M$ , giving bounds on both maximum curvature and possible bottleneck structures. In this talk, we will study the geometry of the reach through an approximation perspective. We will present new geometric results on the reach of submanifolds without boundary. An estimator  $\hat{\tau}_n$  of  $\tau_M$  will be described, in an idealized i.i.d. framework where tangent spaces are known. The estimator  $\hat{\tau}_n$  is showed to achieve uniform expected loss bounds over a  $\mathcal{C}^3$ -like model. Minimax upper and lower bounds are derived. We will conclude with an extension to a model in which tangent spaces are unknown. (Received September 24, 2017)

1135-62-1940 **Juhee Cho, Donggyu Kim, Yilin Zhang, Song Wang, Thu Le** and **Karl Rohe\*** (karlrohe@stat.wisc.edu). *Spectral clustering is dead! Long live spectral clustering!*

In many empirical settings, spectral clustering finds small and uninteresting clusters, e.g. 10 node clusters in 100k node graphs. Using a very simple model, we (i) explain this phenomenon and (ii) show why perturbed and regularized spectral clustering correct this issue. (Received September 25, 2017)

1135-62-2006 **Ziwei, ZM Ma\*** (ziweima@nmsu.edu) and **Tonghui, TW Wang** (twang@nmsu.edu). *Inferences on the location parameter under multivariate skew normal settings*.

For the multivariate skew normal random matrix  $Y$  with location parameter  $\mathbf{1}_n \otimes \boldsymbol{\mu}'$ , scale parameter  $V \otimes \Sigma$ , and skewness parameter  $\mathbf{1}_n \otimes \boldsymbol{\alpha}'$ ,  $Y \sim SN_{n \times p}(\mathbf{1}_n \otimes \boldsymbol{\mu}', V \otimes \Sigma, \mathbf{1}_n \otimes \boldsymbol{\alpha}')$ . The distribution of sample covariance matrix is derived, and the generalized Hotelling and its properties  $T^2$  are discussed. Based on these results, the inferences on location parameter  $\boldsymbol{\mu}$  with unknown  $\Sigma$  are studied. Examples on real data set and simulated data sets are given for illustration of our results. (Received September 25, 2017)

1135-62-2067 **Emmanuel Abbe, Jianqing Fan, Kaizheng Wang** and **Yiqiao Zhong\*** (yiqiaoz@princeton.edu). *Spectral algorithm without trimming or cleaning works for exact recovery in SBM*. Preliminary report.

The stochastic block model (SBM) is a popular random model in the study of graph clustering. The goal is to recover block membership of vertices from a random graph generated from SBM. In this work, we study the vanilla spectral algorithm for SBM with two equal-sized blocks. The core of our results is an entrywise analysis of eigenvectors: denoting by  $\{u_k\}$ , respectively  $\{u_k^*\}$ , the eigenvectors of the adjacency matrix  $A$ , respectively  $\mathbb{E}A$ , we characterize conditions for which

$$u_k \approx \frac{Au_k^*}{\lambda_k^*}$$

serves as a first-order approximation under the  $\ell_\infty$  norm. The fact that the approximation is both tight and linear in the random matrix  $A$  allows for sharp comparisons of  $u_k$  and  $u_k^*$ . In particular, it allows to compare the signs of  $u_k$  and  $u_k^*$  even when  $\|u_k - u_k^*\|_\infty$  is large, which in turn allows to settle a conjecture that the spectral algorithm without any trimming or cleaning steps achieves exact recovery in SBM. Moreover, it attains minimax misclassification rates below the exact recovery threshold. (Received September 25, 2017)

1135-62-2156 **Steven L Craighead\*** (steven.craighead@pacificlife.com), 4 San Gabriel, Rancho Santa Margari, CA 92688. *Enterprise Risk Management Stochastic Analysis Tools: Risk Drivers Revealed*.

In this talk we will discuss modeling within enterprise risk management (ERM), with special emphasis on the sensitivity of value at risk (VaR) and conditional value at risk (CVaR) to various risk drivers. This sensitivity analysis is conducted by quantile regression (QR) and a linear regression upon those QR residuals. (Received September 25, 2017)

1135-62-2241 **Mingwei Sun\*** (msun5@crimson.ua.edu), The University of Alabama, Department of Mathematics, Box 870350, Tuscaloosa, AL 35487, and **Patrick Wang**. *Regression Models with a Universal Penalized Function*.

Regression problems with many potential candidate predictor variables have brought the intense attention of researchers from many disciplines including statistics, biology, medical care, economics and even arts and humanities among which large volume, complex and growing data is generated. Finding an optimal model via statistical model selection to reduce the number of predictors while providing good predictive performance is

demanded. Some classic techniques include stepwise deletion and subset selection. However, these procedures ignore stochastic errors inherited in the stages of variable selections and the resulting subset suffers from lack of stability and low prediction accuracy. Penalized least squares provide new approaches to the variable selection problems. In this research, we develop penalized regressions with a universal penalty function and prove that the LASSO and elastic net are special cases of our function. The structure and properties of our penalty are studied and the corresponding algorithms are developed. Simulation studies and real-data support the advantageous performance of the proposed methods. (Received September 25, 2017)

1135-62-2350 **Alyssa Hope Columbus\*** (acolumbu@uci.edu). *Probabilistic Models, Machine Learning, and the Future of Breast Cancer Risk Assessment.*

With 1 in 8 women in the United States at risk of being diagnosed with breast cancer at some point in her lifetime, the development and accuracy of breast cancer prediction models is pertinent to reducing the morbidity and mortality rates associated with the disease. Three such prediction models are the Gail, BCSC (Breast Cancer Surveillance Consortium), and Tyrer-Cuzick models, each of which determine a woman's risk of breast cancer from risk factors including family history of cancer and mammographic density. However, these models have been shown to vary in accuracy among women with different ethnic backgrounds, which is why the Athena Breast Health Network, an extensive program at UCI that integrates clinical care and research to drive innovation in the prevention of breast cancer, is building and testing new breast cancer risk assessment models using machine learning. Experimenting with algorithms such as k-nearest neighbors (KNN), support vector machines (SVM), and decision trees (DT), Athena hopes to evaluate new breast cancer risk models and their predictive accuracies. (Received September 26, 2017)

1135-62-2460 **Andre K Waschka\*** (akwaschka@berkeley.edu). *A cross-validated tuning parameter estimator for decision trees.* Preliminary report.

The augmented tree-based method presented here is a procedure that uses cross-validated, variance-bias trade-off to choose the most refined level of stratification in order to minimize misclassification rates, by incorporating differential impacts for a false positive (FP) versus false negative (FN) rates. The new tree-based estimator is characterized by a tuning parameter, which is a loss matrix composed of user-supplied weights (FP; FN). Our optimized CV method directly optimizes the weighted FP to FN ratio while capitalizing on the properties of cross-validation to limit the risk of overfitting. This yields an estimator that minimizes the cross-validated risk estimates.

Clinical applications of this approach suggest this method has great promise as a statistical tool for precision medicine. (Received September 26, 2017)

1135-62-2531 **Eric Ruggieri** (eruggier@holycross.edu) and **Michelle Yu\*** (myu18@g.holycross.edu). *Detecting Change Points in Climate Records.*

Climate change is the result of complex interactions between a wide array of climatic variables. Over a long period of time, climatic patterns can exhibit abrupt shifts. These abrupt shifts occurring over relatively short periods of time are known as change points. During these intervals, different climatic variables may undergo dramatic shifts posing serious consequences for many biological and physical systems. In this talk, we discuss a Bayesian algorithm for detecting the exact timing of change points in time series data. (Received September 26, 2017)

1135-62-2557 **Vindya Kumari Pathirana\*** (vindya.pathirana@uconn.edu), Department of Mathematics, University of Connecticut, 99 E. Main Street, Waterbury, CT 06702. *Mahalanobis Based k-Nearest Neighbor with Change Point Detection for Foreign Exchange Data.* Preliminary report.

Discovering the time points where changes occur, called change point detection is highly important in Foreign Exchange (FX) trading. Researchers have adopted various change point detection methods for identifying the abrupt changes in time series data. We incorporate the change point detection technique with Mahalanobis distance based k-nearest neighbor forecasting procedure for daily exchange rate data. Our methods are based on the directional change in daily currency rates. We perform a comparative study on FX rate forecasting and decision making with change point detection for various currency data sets. (Received September 26, 2017)

1135-62-2590 **William D Shannon\*** (wshannon@linfield.edu), Unit Box 3500, Linfield College, 190 SW Brumback St., McMinnville, OR 97128-6899, and **Jennifer Moranchel.** *Big Data and the Stock Market.*

In our modern competitive market, businesses are seeking efficient and innovative platforms to remain profitable and prepared, especially in the uncertain world of the financial stock market. One possible avenue for improving

stock prices that companies can turn to is harnessing a substantial volume of information, known as big data. However, because of the nature of big data, distilling and analyzing the vast amount of information can require complex analytical methods. Using a keyword selection process based on word frequency, we were able to filter out the data amongst the noise and derive a sector specific keyword list. This list used in combination with a previously created trading method along with the implementation of a thresholding technique, allowed us to develop a more specific trading strategy focused on different market sectors. Our results show that the use of thresholding techniques in addition to the Google Trends strategy may improve returns in the stock market. (Received September 26, 2017)

1135-62-2628 **Joshua M Whitlinger\*** ([whitlingerjm@vcu.edu](mailto:whitlingerjm@vcu.edu)), Virginia Commonwealth University, PO BOX 843083, Richmond, VA 23284, and **Edward L Boone** ([elboone@vcu.edu](mailto:elboone@vcu.edu)) and **Ryad Ghanam** ([raghanam@vcu.edu](mailto:raghanam@vcu.edu)). *A Bayesian Estimation for the Fractional Order of the Differential Equation that Models Transport in Unconventional Hydrocarbon Reservoirs.*

The extraction of natural gas from the earth has been shown to be governed by differential equations concerning flow through a porous material. Recently, models such as fractional differential equations have been developed to model this phenomenon. One key issue with these models is estimating the fraction of the differential equation. Traditional methods such as maximum likelihood, least squares, and even method of moments are not available to estimate this parameter as traditional calculus methods do not apply. We develop a Bayesian approach to estimate the fraction of the order of the differential equation that models transport in unconventional hydrocarbon reservoirs. In this paper, we use this approach to adequately quantify the uncertainties associated with the error and predictions. A simulation study is presented as well to assess the utility of the modeling approach. (Received September 26, 2017)

1135-62-2746 **Benjamin S Lieberman\*** ([b1250604@muhlenberg.edu](mailto:b1250604@muhlenberg.edu)), 7711 Mary Cassatt Dr, Potomac, MD 20854, and **James Russell**. *A Monte Carlo measurement of Gerrymandering in Pennsylvania through random tessellation.*

We examine the prevalence of gerrymandering in Pennsylvania through Monte Carlo simulation. To generate the simulations, the area of Pennsylvania is divided using Centroidal Voronoi tessellation with centroids sampled from inhomogeneous Poisson point processes. To ensure that each of the districts constitutes an approximately equal share of the population, a large number of tiles are generated and assigned to a particular district using an acceptance-rejection sampling algorithm. The vote of each individual is simulated using voter registration information and the results from the simulated election are compared to the actual distribution of congressional seats. (Received September 26, 2017)

1135-62-2794 **Alona Kryshchenko\*** ([alona.kryshchenko@csuci.edu](mailto:alona.kryshchenko@csuci.edu)), One University Dr, Camarillo, CA 93012, **Alan Schumitzky** ([schumitzky@aol.com](mailto:schumitzky@aol.com)), 3220 S Vermont Ave, Los Angeles, CA 90089, **Mike van Guilder** ([uphill@cox.net](mailto:uphill@cox.net)), 4650 Sunset Blvd, Los Angeles, CA 90027, and **Michael Neely** ([mneely@gmail.com](mailto:mneely@gmail.com)), 4650 Sunset Blvd, Los Angeles, CA 90027. *Nonparametric estimation of a mixing distribution for a family of linear stochastic dynamical systems.*

In this paper we develop a nonparametric maximum likelihood estimate of the mixing distribution of the parameters of a linear stochastic dynamical system. This includes, for example, pharmacokinetic population models with process and measurement noise that are linear in the state vector, input vector and the process and measurement noise vectors. Most research in mixing distributions only considers measurement noise. The advantages of the models with process noise are that, in addition to the measurements errors, the uncertainties in the model itself are taken into the account. For example, for deterministic pharmacokinetic models, errors in dose amounts, administration times, and timing of blood samples are typically not included. For linear stochastic models, we use linear Kalman-Bucy filtering to calculate the likelihood of the observations and then employ a nonparametric adaptive grid algorithm to find the nonparametric maximum likelihood estimate of the mixing distribution. We then use the directional derivatives of the estimated mixing distribution to show that the result found attains a global maximum. (Received September 26, 2017)

1135-62-2970 **Mavis Pararai\***, 210 S 10 Street, Stright Hall, Rom 233, Indiana, PA 15705, and **Gayan Warahena-Liyanage** ([warah1g@cmich.edu](mailto:warah1g@cmich.edu)), Pearce Hall, Mt Pleasant, MI 48859. *The log-logistic exponential-Poisson distribution.*

The log-logistic exponential Poisson distribution will be explored. Its properties and applications will be explored. The maximum likelihood method will be used to estimate the parameters. (Received September 26, 2017)

1135-62-3034 **Gokarna R Aryal** (aryalg@pnw.edu), Hammond, IN 46323, **Sher B Chhetri\*** (sbchhetri.math@gmail.com), Boca Raton, FL 33431, **Hongwei Long** (hlong@fau.edu), Boca Raton, FL 33431, and **Alfred A. Akinsete** (akinsete@marshall.edu), Huntington, WV 25755. *Generalizations Using the Composition of Beta Distribution and Truncated Poisson Distribution*. Preliminary report.

In this article, we propose and study a new family of distribution which is defined by using the genesis of the truncated Poisson distribution and the beta distribution. Some mathematical properties of the new family including ordinary and incomplete moments, quantile and generating functions, mean deviations, order statistics and their moments, reliability analysis are discussed. We also discuss the parameter estimation issues and potential application of such generalized family of distributions. (Received September 26, 2017)

1135-62-3160 **Mauricio M Nunez-Regueiro\***, mnureg@yahoo.com, and **Josh Hiller**. *Multinomial logit regression models help describe novel spatiotemporal dimensions in environmental policy enrollment patterns*.

Over the last 50 years, payment for ecosystem services schemes (PES) have been lauded as a market-based solution to simultaneously curtail deforestation while lifting local populations out of poverty. Nowhere in the world are these two goals more evident than in the Argentine Forest Law. This law compensates landowners for enrollment of their land in conservation-type projects on a scale seldom seen in the rest of the world. However, PES programs often fail to conserve sites under strong deforestation pressures and allocate limited financial resources to protecting sites that would likely be conserved in the absence of PES - a problem called adverse selection. Here, we analyze adverse selection patterns of enrollment in this program. We do this by utilizing multinomial logistic regression models in a novel way and examine several socio-economic variables. Among the most striking conclusions is that all large parcels of enrolled land show evidence of spatial or temporal adverse selection. Furthermore, conservation-type projects are less effective in avoiding adverse selection than more profitable approaches such as non-timber forest product production and silviculture. (Received September 26, 2017)

1135-62-3229 **Talithia Williams\*** (twilliams@hmc.edu). *Mathematics for the Masses*.

In recent months, we've witnessed Americans grapple with the significance of Science, Technology, Engineering and Mathematics (STEM) through events ranging from the Paris Agreement to the nationwide March for Science, where people marched to defend the role of science in society. In the wake of a renewed excitement for STEM, I'm thrilled to be joining the Public Broadcasting System (PBS) family as one of the hosts of a new NOVA series called NOVA Wonders premiering this spring. NOVA is the most-watched primetime science series on television, reaching an average of five million viewers weekly. NOVA Wonders is a six-part series that will journey to the frontiers of science, where researchers are tackling some of the most intriguing questions about life and the cosmos. My goal in hosting the show is to open individuals to the power of mathematics and data to pursue answers to questions in a clear and purposeful way. During this talk, I plan to share with you an early clip from NOVA Wonders and discuss ways that we can take our mathematics to the masses and share techniques that have been successful in my mathematical environment. We all have a responsibility to inspire a new generation in STEM and nurture the dreams of future mathematical leaders. (Received September 27, 2017)

## 65 ► Numerical analysis

1135-65-94 **Ugur Abdulla** (abdulla@fit.edu), Mathematical Sciences, Florida Institute of Technology, 150 W. University Blvd, Melbourne, FL 32903, **Vladislav Bukshytynov** (vbukshytynov@fit.edu), Mathematical Sciences, Florida Institute of Technology, 150 W. University Blvd, Melbourne, FL 32901, and **Ali Hagverdiyev\*** (ahaqverdiyev2011@my.fit.edu), Melbourne, FL 32901. *Gradient method in Besov spaces for the optimal control of parabolic free boundary problems*.

We pursue numerical analysis of the optimal control problem introduced recently as a variational formulation of the inverse Stefan problem in *U.G.Abdulla, Inverse Problems and Imaging, 7, 2(2013), 307-340* & *10, 4(2016), 869-898*. By employing Frechet differentiability result of the recent paper by *Abdulla et al., Evolution Equations and Control Theory, 6, 3(2017), 319-344*, iterative numerical algorithm based on the projective gradient method in Besov spaces is implemented. We pursue sensitivity analysis with respect to initial guess, and comparison of alternative approaches of simultaneous reconstruction vs. nested optimization of the control vector components. Numerical results are demonstrated for model examples with various levels of complexity. (Received July 26, 2017)



1135-65-124 **Alejandro L. Garcia\*** ([alejandrogarcia@sjsu.edu](mailto:alejandrogarcia@sjsu.edu)). *Algorithm Refinement - A 20 year Retrospective.*

When a large range of scales must be spanned, computational fluid dynamics (CFD) calculations often employ Mesh Refinement so that a fine grid is used only in those regions that require high resolution. However, hydrodynamic formulations break down as the grid spacing approaches the molecular scale, for example, the mean free path between collisions of molecules in a gas. At these microscopic scales additional physics, such as random molecular motion, can be important yet it is not accurately represented by conventional hydrodynamic models.

Algorithm Refinement is an effective approach for such multiscale problems that span macroscopic to microscopic scales. Algorithm Refinement typically couples two structurally (physically and algorithmically) different computational models, which are used in different regions of the problem. For example in a hypersonic flow simulation a particle-based scheme could be used to resolve shock wave structure and a hydrodynamic solver used to model the rest of the flow.

The talk will present a review of Algorithm Refinement, describing both its strengths and weaknesses. The current state-of-the-art for these hybrid schemes and the promising future directions for Algorithm Refinement will be outlined. (Received July 29, 2017)

1135-65-180 **Zicong Zhou\*** ([zicong.zhou@uta.edu](mailto:zicong.zhou@uta.edu)), 411 S. Nedderman Drive, 478 Pickard Hall, Arlington, TX 76019-0408, and **Guojun Liao**. *Medical Images Atlas by averaging Diffeomorphisms through Mesh Generation.*

As presented in [X.Chen and Guojun Liao, New Variational Method of Grid Generation with prescribed Jacobian determinant and prescribed curl] and [X.Chen and Guojun Liao, New method of averaging diffeomorphisms based on Jacobian determinant and curl vector], the new approach of averaging diffeomorphisms is used in this work to find the image atlas of a group of medical images. The proposed average of diffeomorphisms is found by two steps. Firstly, averaging the Jacobian determinants and the curl vectors of all the given diffeomorphisms which can be determined from the given group of images; then, reconstruct a new transformation which is also a diffeomorphism that will be used in resampling an image. The resampled image is what we define to be the desired image atlas of the given group of medical images. In this work, the theoretical Mathematics will be demonstrated and several numerical results will be showed and discussed. (Received August 07, 2017)

1135-65-257 **Jinyan Fan and Jiawang Nie\*** ([njw@math.ucsd.edu](mailto:njw@math.ucsd.edu)), San Diego, CA 92093, and **Anwa Zhou**. *Tensor Eigenvalue Complementarity Problems.*

This talk discusses tensor eigenvalue complementarity problems. Basic properties of standard and complementarity tensor eigenvalues are discussed. We formulate tensor eigenvalue complementarity problems as constrained polynomial optimization. When one tensor is strictly copositive, the complementarity eigenvalues can be computed by solving polynomial optimization with normalization by strict copositivity. When no tensor is strictly copositive, we formulate the tensor eigenvalue complementarity problem equivalently as polynomial optimization by a randomization process. The complementarity eigenvalues can be computed sequentially. The formulated polynomial optimization can be solved by Lasserre's hierarchy of semidefinite relaxations. We show that it has finite convergence for general tensors. (Received August 16, 2017)

1135-65-259 **Nirjal Shrestha\*** ([nirjal.shrestha@cameron.edu](mailto:nirjal.shrestha@cameron.edu)), 2800 W Gore Blvd, Lawton, OK 73505, and **Ioannis K Argyros** ([iargyros@cameron.edu](mailto:iargyros@cameron.edu)), 2800 W Gore Blvd, Lawton, OK 73505. *Comparing the Local Convergence Analysis of Some Two-Step Newtons Method for Solving Equations.* Preliminary report.

We compare the local convergence analysis of some two step Newton methods used to approximate a locally unique solution of a non-linear equation in Banach Space setting. In particular we compute the radii of convergence of the popular two-step Newton method and the two-step midpoint method. Moreover, we find the error bounds on the distances involved using Lipschitz constants. The theoretical results are applied on concrete numerical examples. (Received August 16, 2017)

1135-65-264 **Harbir Antil** and **Thomas S Brown\*** ([tsbrown@udel.edu](mailto:tsbrown@udel.edu)), Department of Mathematical Sciences, 112 Ewing Hall, Newark, DE 19716, and **Francisco -Javier Sayas**. *Transient Waves in Piezoelectric Media: Analysis, Simulation, and Control.*

We study an initial-boundary value problem involving elastic waves propagating in piezoelectric materials, such as can be found in cigarette lighters, humidifiers, speakers, and many other everyday objects. Using a Finite Element scheme to discretize the problem in space, we use the tools of semigroup theory to provide stability analysis and estimates on the error due to semidiscretization. This requires using a non-standard first order form of the problem where the unknowns are the elastic displacement, accumulated stress, and accumulated electric

field. Additionally, we use the problem as a state equation in the setting of optimal control to study control of the elastic displacement using a boundary condition on the electric flux. (Received August 16, 2017)

1135-65-265 **Abram Kay Rodgers\*** ([aks.rodgers@gmail.com](mailto:aks.rodgers@gmail.com)), 335 Walk Circle, Santa Cruz, CA 95060. *High Speed Numerical Analysis of Experimentally Obtained Data via Low Power Computing Devices*. Preliminary report.

Today in Numerical Analysis, it is extremely common to rely on high-level programming environments (such as Octave, MATLAB, and R) in order to make the process of analyzing large data sets easier. However, this introduces a bottleneck effect which raises the time required to compute. In exchange, we allow ourselves to have easier to read and use code. In this presentation, we challenge the norm of “easy coding needs to be slow computing.” We present a derivation of a commonly used curve fitting method known as “Linear Least Square Regression.” We then provide an implementation of this regression algorithm in the C programming language. As proof of concept, we provide a visual of this algorithm using an old low power micro-controller called the “Nintendo GameBoy Advance,” (a.k.a. “GBA,” a computer over 50 times slower (in clock speed) than a Raspberry Pi 2, which in turn is much slower than a common laptop). All code is open source and hosted with video examples at: <https://github.com/akrodger/brams-math-methods/tree/regression-gba-demo> (Received August 16, 2017)

1135-65-267 **Xinyun Zhu\*** ([zhu\\_x@utpb.edu](mailto:zhu_x@utpb.edu)), Xinyun Zhu, Department of Mathematics, UTPB, Odessa, TX 79762, and **Hongtao Fan** and **Bing Zheng**. *The generalized double shift-splitting preconditioner for nonsymmetric generalized saddle point problems from the steady Navier-Stokes equations*. Preliminary report.

In this paper, a generalized double shift-splitting (GDSS) preconditioner induced by a new matrix splitting method is proposed and implemented for nonsymmetric generalized saddle point problems having a nonsymmetric positive definite (1,1)-block and a positive definite (2,2)-block. Detailed theoretical analysis of the iteration matrix is provided to show the GDSS method, which corresponds to the GDSS preconditioner, is unconditionally convergent. Additionally, a deteriorated GDSS (DGDSS) method is proposed. It is shown that, with suitable choice of parameter matrix, the DGDSS preconditioned matrix has an eigenvalue at 1 with multiplicity  $n$ , and the other  $m$  eigenvalues are of the form  $1 - \lambda$  with  $|\lambda| < 1$ , independently of the Schur complement matrix related. Finally, numerical experiments arising from a model Navier-Stokes problem are provided to validate and illustrate the effectiveness of the proposed preconditioner, with which a faster convergence for Krylov subspace iteration methods can be achieved. (Received August 16, 2017)

1135-65-288 **Jacob R Price\*** ([jrp14@uw.edu](mailto:jrp14@uw.edu)) and **Panos Stinis**. *Renormalized Reduced Order Models with Memory for Long Time Prediction*.

We examine the challenging problem of constructing reduced models for the long time prediction of systems where there is no timescale separation between the resolved and unresolved variables. In previous work we focused on the case where there was only transfer of activity (e.g. energy, mass) from the resolved to the unresolved variables. Here we investigate the much more difficult case where there is two-way transfer of activity between the resolved and unresolved variables. As in the previous cases, even if one starts with an exact formalism, like the Mori-Zwanzig (MZ) formalism, the constructed approximate reduced models can become unstable. We derive a new method of approximating the MZ formalism, and we show how to remedy the resultant stability problems by using dynamic information from the full system to renormalize the MZ reduced models. In addition to being stabilized, the renormalized models can be accurate for very long times. We use the Korteweg-de Vries equation to illustrate the approach. The coefficients of the renormalized models exhibit rich structure, including algebraic time dependence and incomplete similarity. (Received August 18, 2017)

1135-65-476 **Jesse Chan\*** ([jesse.chan@rice.edu](mailto:jesse.chan@rice.edu)). *Discretely entropy stable discontinuous Galerkin methods*.

High order methods offer several advantages in the approximation of solutions to hyperbolic equations, such as improved accuracy and low numerical dispersion and dissipation. However, high order methods also tend to suffer instabilities when applied to nonlinear hyperbolic equations, requiring filtering, limiting, or artificial dissipation to ensure that the solution does not grow unboundedly. At the root of these problems is the fact that the stability of the continuous problem does not imply stability at the discrete level. This talk will review the development of high order collocation schemes based on summation-by-parts operators which recover a discrete statement of entropy stability, and will discuss the extension of such methods to a more general class of discontinuous Galerkin methods. Numerical results for the compressible Euler equations in one and two dimensions support the presented theoretical results. (Received September 05, 2017)

1135-65-495 **Leo G. Rebholz, Alex Viguerie and Mengying Xiao\*** ([mengyix@clemons.edu](mailto:mengyix@clemons.edu)), Clemson University, Clemson, SC 29634. *Efficient nonlinear iteration schemes based on algebraic splitting for the incompressible Navier-Stokes equations.*

We present new, efficient, nonlinear iteration methods for the incompressible Navier-Stokes equations. The methods are constructed by applying Yosida-type algebraic splitting to the linear systems that arise from grad-div stabilized finite element implementations of incremental Picard and Newton iterations. They are efficient because at each nonlinear iteration, the same symmetric positive definite Schur complement needs solved, which allows for CG to be used for inner and outer solvers, simple preconditioning, and reusing of preconditioners. For the incremental Picard-Yosida and Newton-Yosida iterations, we prove under small data conditions that the methods converge to the solution of the discrete nonlinear problem. Numerical tests are performed which verify excellent convergence properties of the methods on a variety of test problem. (Received September 06, 2017)

1135-65-542 **Raju Prasad Bhusal\*** ([rbhusal@bgsu.edu](mailto:rbhusal@bgsu.edu)), [rbhusal@bgsu.edu](mailto:rbhusal@bgsu.edu), and **Tong Sun.** *The Error Analysis for the Cubic Front Tracking and RKDG method Solving Scalar Conservation Laws.* Preliminary report.

The error analysis for the Runge-Kutta discontinuous Galerkin (RKDG) method for solving the scalar nonlinear conservation laws for the case of smooth solution has been given in [1]. For the case of having a fully developed shock, a Front Tracking method coupled with RKDG has been implemented and analyzed in [2]. We use a new Cubic Front Tracking method coupled with RKDG to obtain the solution in-between the cases in [1] and [2], namely, from the time a shock is starting to form until it is fully developed. Thus we have filled the gap. The numerical smoothness based error analysis of [1] and [2] remains to be the main approach in our error analysis.

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(Received September 07, 2017)

1135-65-546 **Eric Joseph Hall\*** ([hall@math.umass.edu](mailto:hall@math.umass.edu)), Department of Mathematics and Statistics, 710 N. Pleasant Street, Amherst, MA 01003, and **Håkon Hoel, Mattias Sandberg, Anders Szepešsy and Raul Tempone.** *Computable error estimates for finite element approximations of elliptic partial differential equations with rough stochastic data.*

The Monte Carlo (and Multi-level Monte Carlo) finite element method can be used to approximate observables of solutions to diffusion equations with lognormal distributed diffusion coefficients, e.g. modeling steady-state groundwater flow. Typical models use lognormal diffusion coefficients with Hölder regularity of order up to  $1/2$  almost surely. This low regularity implies that the high frequency finite element approximation error (i.e. the error from frequencies larger than the mesh frequency) is not negligible and can be larger than the computable low frequency error. We address how the total error can be estimated by the computable error and propose goal-oriented estimates for the pathwise Galerkin and expected quadrature errors that are derived using easily validated assumptions. (Received September 08, 2017)

1135-65-650 **Qing Xia\*** ([xia@math.utah.edu](mailto:xia@math.utah.edu)), University of Utah, Department of Mathematics, 155 S 1400 E, ROOM 233, Salt Lake City, UT 84112. *Adaptive Difference Potentials Method for the Chemotaxis Model in 3D.* Preliminary report.

The Patlak-Keller-Segel chemotaxis system models the directed motion of cells or bacteria in response to chemical stimulus. The model is described by coupled advection-diffusion equation for the density of cells and reaction-diffusion equation for the concentration of chemicals. Chemotaxis system poses numerical challenges in capturing the sharp gradients of solutions observed in the blow-up phenomenon. In this talk, we will discuss an adaptive algorithm based on difference potentials method for such model. Numerical results to illustrate accuracy and efficiency of the proposed algorithm will be presented. Current and future research work will also be discussed. This talk is based on joint work with Y. Epshteyn. (Received September 11, 2017)

1135-65-701 **Andreas Frommer, Kathryn Lund\*** ([katlund@temple.edu](mailto:katlund@temple.edu)) and **Daniel B. Szyld.** *A new framework for understanding block Krylov methods applied to the computation of functions of matrices.*

Since Dianne O’Leary’s seminal paper on block conjugate gradients (CG) in 1980, block Krylov subspace methods have been used widely to solve linear systems with multiple right-hand sides. Recently, these methods have been used to compute  $f(A)B$ , where  $f$  is a scalar function defined on the matrix  $A \in \mathbb{C}^{n \times n}$  and  $B \in \mathbb{C}^{n \times s}$ . Hinging on a generalized framework for block Krylov subspaces that encompasses established results not only

for the “classical” block methods (as in O’Leary’s work), but also for global methods and the newer “loop-interchange” methods, we define a block full orthogonalization method for functions of matrices (B(FOM)<sup>2</sup>) whose approximations lie in a generalized block Krylov subspace. This method is shown to converge for Stieltjes functions and Hermitian positive definite matrices with CG-like bounds, even with restarts. We demonstrate the performance and versatility of B(FOM)<sup>2</sup> in a variety of numerical experiments, even for non-Hermitian matrices and non-Stieltjes functions. Important applications include the sign function in quantum lattice chromodynamics and the matrix exponential in differential equations. (Received September 13, 2017)

1135-65-710 **Xiu Ye\***, 2801 S. University ave, little rock, AR 72223, and **Lin Mu**. *A posteriori error analysis on polytopal meshes and simple methods for the problems with non-divergence forms.*

The goal of this talk is twofold:

First, a posteriori error estimators have been developed for both the weak Galerkin and the discontinuous Galerkin finite element methods. The most existing a posteriori error analysis only work on simplicial elements even for the polygonal and polyhedral finite element methods. Our new residual type estimators can be applied to general meshes such as hybrid mesh, polytopal mesh and mesh with hanging node. In addition, these estimators consist fewer terms and are easy to compute.

Second, we present simple finite element methods for solving some non classic problems such as second order elliptic equations in non-divergence form, Cauchy Problem and simple hyperbolic problem. Error analysis have been provided and extensive numerical examples have been tested. (Received September 13, 2017)

1135-65-760 **Aaron Melman\*** ([amelman@scu.edu](mailto:amelman@scu.edu)). *From scalar polynomial to matrix polynomial bounds.*

Bounds on polynomial eigenvalues are useful, e.g., for their computation by iterative methods, when computing pseudospectra, or, especially, in the analysis of engineering problems. We propose to refine and then generalize apparently little known - but useful - results on bounds for the zeros of scalar polynomials, i.e., polynomials with complex coefficients, to the matrix polynomials appearing in polynomial eigenvalue problems. These eigenvalues are much more difficult to compute than polynomial zeros, making bounds on such eigenvalues more valuable. Contrary to almost all existing bounds, our eigenvalue localization results can be further improved iteratively to achieve significant improvements over existing bounds. Furthermore, we obtain results for a large class of polynomial bases that includes - but is not limited to - all classical orthogonal bases: Hermite, Legendre, Chebyshev, etc. We present several applications from the engineering literature. (Received September 14, 2017)

1135-65-774 **Amanda E. Diegel\*** ([adiegel@lsu.edu](mailto:adiegel@lsu.edu)) and **Shawn W. Walker**. *A phase field model for nematic liquid crystal droplets.*

We develop a novel finite element method for a phase field model of nematic liquid crystal droplets. The continuous model considers a free energy comprised of three components: the Ericksen’s energy for liquid crystals, the Cahn-Hilliard energy representing the interfacial energy of the droplet, and a weak anchoring energy representing the interaction of the liquid crystal molecules with the surface tension on the interface (i.e. anisotropic surface tension). Applications of the model are for finding minimizers of the free energy and exploring gradient flow dynamics. We present a finite element method that utilizes a special discretization of the liquid crystal elastic energy, as well as mass-lumping to discretize the coupling terms for the anisotropic surface tension part. We present a discrete gradient flow method and show that it is monotone energy decreasing. Global discrete energy minimizers converge to global minimizers of the continuous energy in the sense of  $\Gamma$ -convergence. We will show many numerical experiments illustrating different gradient flow dynamics, including droplet coalescence and break-up. (Received September 14, 2017)

1135-65-776 **Jacob Jacavage\*** ([jjacav@udel.edu](mailto:jjacav@udel.edu)) and **Constantin Bacuta** ([bacuta@udel.edu](mailto:bacuta@udel.edu)). *A Nonconforming Saddle Point Least Squares Approach for Elliptic Interface Problems.*

Numerical methods for elliptic interface problems have been widely studied. One difficult challenge when approximating such problems is related to discontinuities of the coefficients at the interface. Body-fitted mesh methods involve aligning the mesh with the interface to capture discontinuities. In this category, we propose a new least squares method to approximate the solution of such problems. The method is based on a general Saddle Point Least Squares (SPLS) method for discretizing mixed variational formulations of boundary value problems and involves finite element discretization with piecewise polynomial spaces. The SPLS method has the advantage that a discrete inf-sup condition is automatically satisfied for standard choices of test and trial spaces. The proposed iterative solver is easy to implement and benefits from the way the discrete trial spaces are

chosen. For interface problems, we will focus on using projection trial spaces. Local projections are used if the gradient of the solution is discontinuous along the interface. This idea, combined with classical gradient recovery techniques, leads to a better approximation of the (global) flux as shown by the numerical results. (Received September 14, 2017)

1135-65-784 **Ulises Fidalgo\*** ([uxf@case.edu](mailto:uxf@case.edu)), Yost Hall 323, 2049 Martin Luther King Jr. Drive, Cleveland, OH 44106-7058. *Interpolatory Quadrature Rules and Orthogonal Polynomials of Varying Rational Weights.*

We give sufficient conditions of convergence for an interpolatory integration rule whose evaluation nodes (scheme of nodes) are zeros of polynomials which are orthogonal with respect to a varying rational weight. This work has a precedent in a paper by T. Bloom, D. S. Lubinsky, and H. Stahl where they stated a necessary condition that the scheme of nodes must satisfy when we have convergent interpolatory quadrature formulae. They also prove this necessary condition holds in the case of a scheme of nodes whose elements are zeros of orthogonal polynomials with respect to some sequences of exponential weights where their exponents are logarithmic potentials. Our convergent interpolatory quadrature rules are contained in the class of candidates given by T. Bloom, D. S. Lubinsky, and H. Stahl. (Received September 14, 2017)

1135-65-910 **Weidong Chen\*** ([weidong.chen@mnsu.edu](mailto:weidong.chen@mnsu.edu)), Department of Mathematics and Statistics, 273 Wissink Hall, Minnesota State University, Mankato, MN 56001. *A Regularized Two-Dimensional Sampling Algorithm.*

In this talk, a regularized sampling algorithm for band-limited signals is presented in the two-dimensional case. The convergence of the regularized sampling algorithm is studied and compared with Shannon's sampling theorem and the Tikhonov Regularization Method by some examples. (Received September 16, 2017)

1135-65-913 **Morteza Shafii-Mousavi\*** ([mshafii@iusb.edu](mailto:mshafii@iusb.edu)), 1700 Mishawaka Ave, P.O. Box 7111, South Bend, IN 46634. *Calculation of Fundamental Matrix for an Absorbing Markov Chain and Applications.* Preliminary report.

In their book Mathematical Modeling, D. Maki and M. Thompson treated Fundamental Matrices and Applications. In this talk, the calculations of the Fundamental Matrices and applications by Excel Spreadsheet is demonstrated by which the absorption time from a starting state to an absorbing state is calculated. (Received September 16, 2017)

1135-65-1000 **Andrew Gillette\*** ([agillette@math.arizona.edu](mailto:agillette@math.arizona.edu)), University of Arizona, Department of Mathematics, Tucson, AZ, and **Tyler Kloefkorn**, AAAS STP Fellow, hosted by NSF, Washington, DC. *An Introduction to Trimmed Serendipity Finite Element Spaces.*

Finite element methods – one of the most widely used techniques for numerical approximation of solutions to PDEs – are partially classified by the Periodic Table of the Finite Elements (see <https://femtable.org/>). In recent work, we described a new family of methods called “trimmed serendipity elements” that fit within the same framework described by the table. The computational effort required to employ a trimmed serendipity element method is significantly less than what is required for comparable alternatives from the table, thereby presenting a host of potential benefits to the speed and accuracy of finite element methods in practice. All these ideas will be described in detail, followed by a discussion of the future directions and applications for these intriguing new methods. (Received September 19, 2017)

1135-65-1033 **Matthew A Beauregard\*** ([beauregama@sfasu.edu](mailto:beauregama@sfasu.edu)), **Rana Parshad** and **Joshua Padgett**. *A variable nonlinear splitting algorithm for reaction diffusion systems with self and cross-diffusion.*

Self and cross-diffusion are important nonlinear spatial derivative terms that are included into biological models of predator-prey interactions. Self-diffusion models overcrowding effects, while cross-diffusion incorporates the response of one species in light of the concentration of another. In this talk, a new nonlinear operator splitting method is presented that directly incorporates both self and cross-diffusion into a computational efficient design. The numerical analysis guarantees the accuracy and criteria for stability. Numerical experiments display its efficiency in simulating a generalized Shigesada-Kawasaki-Teramoto (SKT) model. (Received September 18, 2017)

1135-65-1083 **M. Aggul, J. Connors, D. Erkmén** and **A. Labovsky\*** ([aelabovs@mtu.edu](mailto:aelabovs@mtu.edu)). *High Accuracy Partitioned Method for Fluid-Fluid Interaction.*

A method is proposed to improve two aspects of numerical simulations for a model of two fluids coupled across a flat interface. This problem is motivated by atmosphere-ocean interaction. Currently, there is a need for a higher

accuracy method that would allow for the usage of pre-existing atmosphere and ocean solvers. We propose to lift the numerical order of accuracy formally from first order (very common in applications) to second order, by using the deferred correction approach. The method is based on a stable partitioned scheme. Also, the defect correction is added, to represent artificial diffusion used in the fluid solvers, which is often included to control numerical noise or to model subscale mixing processes. The addition of the defect correction adds only marginally to the expense, but in exchange may provide a significant reduction of overdiffusive effects. As a result, we obtain a stable, second order accurate, partitioned method, which allows to use the legacy atmosphere and ocean codes as-is. A computational example using a known (manufactured) solution illustrates the theoretical predictions of the improved convergence rates. We observe a computational benefit in this example even for coarse time steps and over a wide range of artificial viscosity values. (Received September 19, 2017)

1135-65-1203 **Monika Neda\*** ([monika.neda@unlv.edu](mailto:monika.neda@unlv.edu)), Dept. of Mathematical Sciences, Univ. of Nevada Las Vegas (UNLV), 4505 Maryland PKWY, Box 454020, Las Vegas, NV 89154, and **Sean Breckling**. *Numerical Studies of Navier-Stokes-alpha model*. Preliminary report.

This talk will present the numerical finite element studies of the Navier-Stokes-alpha model, stability and convergence results of the finite element solution using a specific div-free finite elements. Then, sensitivity analysis based on the filter parameter alpha will be presented as well. The sensitivity analysis is based on the sensitivity equation method discretized with finite element in space. Lastly, the joint energy-helicity cascade exhibited by the model's equations will be discussed. (Received September 20, 2017)

1135-65-1278 **David F Gleich\*** ([dgleich@purdue.edu](mailto:dgleich@purdue.edu)) and **Austin Benson**. *Dynamical systems for tensor eigenvectors*. Preliminary report.

We describe a new family of algorithms that compute eigenvectors of a tensor by tracking the trajectory of a dynamical system. The algorithm arises from work that relates tensor eigenvectors to the stationary distributions of stochastic processes such as vertex reinforced random walks. This algorithm has good theory for probability tensors, where it can be shown to converge under some strong regularity conditions. In our empirical investigations, a generalization of this algorithm is able to find a wider range of eigenvectors of symmetric tensors compared with previous methods. (Received September 20, 2017)

1135-65-1281 **Tamara G. Kolda\*** ([tgkolda@sandia.gov](mailto:tgkolda@sandia.gov)), Sandia National Laboratories, Livermore, CA. *Tensor Decomposition: A Mathematical Tool for Data Analysis*.

Tensors are multiway arrays, and tensor decompositions are powerful tools for data analysis. In this talk, we demonstrate the wide-ranging utility of the canonical polyadic (CP) tensor decomposition with examples in neuroscience and chemical detection. The CP model is extremely useful for interpretation, as we show with an example in neuroscience. However, it can be difficult to fit to real data for a variety of reasons. We present a novel randomized method for fitting the CP decomposition to dense data that is more scalable and robust than the standard techniques. We further consider the modeling assumptions for fitting tensor decompositions to data and explain alternative strategies for different statistical scenarios, resulting in a *generalized* CP tensor decomposition. (Received September 20, 2017)

1135-65-1291 **Hengguang Li\*** ([li@wayne.edu](mailto:li@wayne.edu)), Detroit, MI 48202. *Finite Element Approximations of Singular Solutions in  $W_p^1$* .

We discuss recent advances in the development of effective finite element algorithms approximating a class of singular solutions, including corner singularities with different boundary conditions and singularities from the non-smooth points on the interface in transmission problems. Based on a-priori estimates in weighted function spaces, we propose and investigate effective finite element methods approximating singular solutions in the energy and relevant Banach spaces. (Received September 21, 2017)

1135-65-1292 **Hengguang Li\*** ([li@wayne.edu](mailto:li@wayne.edu)), Detroit, MI 48202. *New Finite Element Methods for 3D Anisotropic Singular Solutions*.

We discuss a new construction of 3D anisotropic meshes to improve the finite element approximation of elliptic boundary value problems with singular solutions from the non-smoothness of the domain. These meshes can violate the maximum angle condition. We derive optimal error estimates for the proposed method. (Received September 21, 2017)

- 1135-65-1296 **Bernd Hofmann\*** ([hofmannb@mathematik.tu-chemnitz.de](mailto:hofmannb@mathematik.tu-chemnitz.de)), TU Chemnitz, Fakultät für Mathematik, 09107 Chemnitz, Germany. *Tikhonov regularization with oversmoothing penalty for non-linear ill-posed problems in Hilbert scales.*

We study Tikhonov regularization for ill-posed non-linear operator equations in Hilbert scales. Our focus is on the interplay between the smoothness-promoting properties of the penalty and the smoothness inherent in the solution. The objective is to study the situation when the unknown solution fails to have a finite penalty value, hence when the penalty is oversmoothing. By now this case was only studied for linear operator equations in Hilbert scales. We extend those results to certain classes of non-linear problems. The main result asserts that, under appropriate assumptions, order optimal reconstruction is still possible. Finally, we highlight that the non-linearity assumption underlying the present analysis is met for specific applications.

This talk presents joint work with Peter Mathé (Berlin). Research is partially supported by the Deutsche Forschungsgemeinschaft (DFG) under grant HO 1454/10-1. (Received September 21, 2017)

- 1135-65-1307 **Thomas L Lewis\*** ([t1lewis3@uncg.edu](mailto:t1lewis3@uncg.edu)). *Convergent finite difference methods with second order local truncation error for Hamilton-Jacobi equations.*

A new finite difference (FD) approximation method is proposed for stationary Hamilton-Jacobi problems with Dirichlet boundary condition. The FD methods have second order local truncation error and converge to the unique viscosity solution of the first order fully nonlinear partial differential equation. The new methods are not monotone. Instead, a stabilization term called a numerical moment is used to ensure the proposed schemes are admissible, stable, and convergent. Numerical tests are provided that compare the accuracy of the proposed scheme to that of the Lax-Friedrich's method. (Received September 21, 2017)

- 1135-65-1329 **Prince Chidyagwai\*** ([pchidyagwai@loyola.edu](mailto:pchidyagwai@loyola.edu)), Loyola University Maryland, Department of Mathematics and Statistics, 4501 N. Charles Street, Baltimore, MD 21210. *A multirate decoupling scheme for transient coupled surface-subsurface flows.*

We present a multi-rate decoupling scheme for solving the time dependent coupled free flow with porous media flow problem. The flow is modelled by the coupled (Stokes/Navier–Stokes) – Darcy system with appropriate interface conditions. We discretize in space using a multi-numeric scheme combining the continuous finite element method in the free flow region and the discontinuous Galerkin method in the porous medium. This choice of finite element spaces is due to the discontinuities that are typical in the permeability of the porous medium. The fully discrete problem yields large fully coupled problems that must be solved at each time step. This can be computationally expensive, therefore we propose a decoupling technique that takes advantage of the slow moving flow in the porous medium relative to the free flow. This allows the use of larger time steps in the porous domain compared to the free flow domain. The decoupling is attained by time-lagging unknowns on the interface. We present stability and convergence analysis of the decoupling scheme. We compare the fully coupled scheme to the multi-rate decoupling scheme in terms of CPU time and accuracy and demonstrate the stability and robustness of the scheme with respect to realistic physical parameters. (Received September 21, 2017)

- 1135-65-1388 **José C Garay, Frédéric Magoulés and Daniel B Szyld\*** ([szyld@temple.edu](mailto:szyld@temple.edu)), Department of Mathematics, 1805 N Broad St, Philadelphia, PA 19122. *Asynchronous Optimized Schwarz for the solution of PDEs.*

Asynchronous methods refer to parallel iterative procedures where each process performs its task without waiting for other processes to be completed, i.e., with whatever information it has locally available and with no synchronizations with other processes. In this talk, an asynchronous version of the optimized Schwarz method is presented for the solution of differential equations on a parallel computational environment. Convergence is proved under very mild conditions on the size of the subdomains, when approximate (non-optimal) interface conditions are utilized for Poisson's equation on the plane. A different result for bounded rectangular domains is also shown. Numerical results are presented on large three-dimensional problems illustrating the efficiency of the proposed asynchronous parallel implementation of the method. (Received September 22, 2017)

- 1135-65-1442 **Eric Joseph Hall\*** ([hall@math.umass.edu](mailto:hall@math.umass.edu)) and **Markos A Katsoulakis**, University of Massachusetts Amherst, Department of Mathematics and Statistics, Amherst, MA 01003. *Robust information divergences for model-form uncertainty arising from sparse data in random PDE.*

Random partial differential equations of elliptic type arise in many applications of steady-state subsurface flow, including hydrology, carbon sequestration, and petroleum engineering. A key challenge in this context is the analysis of the propagation of uncertainty from data and model inputs to quantities of interest and model outputs. In this talk we examine a novel application of hybrid information divergences to quantify the propagation of

model-form or epistemic uncertainty with a view toward robust prediction and risk management. In particular, we highlight the use of these hybrid divergences for two significant uncertainty quantification tasks: sensitivity analysis and gauging model misspecification due to sparse data. (Received September 22, 2017)

1135-65-1556 **Eliza Woolworth Matt\*** ([ewm1@williams.edu](mailto:ewm1@williams.edu)), 39 Chapin Hall Drive, Williamstown, MA 01267, and **Julia Vasile** ([julia.vasile@stonybrook.edu](mailto:julia.vasile@stonybrook.edu)) and **Philip de Castro** ([pdecastr@poets.whittier.edu](mailto:pdecastr@poets.whittier.edu)). *Optimizing Sparse Representations of Kinetic Distributions.*

The United States Air Force Research Laboratory uses kinetic simulations to reduce costs in their various research projects, including plasma simulations. When performing these simulations, probabilistic methods are employed to reduce the computational expense of estimating the physical entropy of the system. These techniques introduce an error term in the estimation, which we seek to reduce by developing a more efficient algorithm. We discuss the nature of kinetic simulations, relevant mathematical background, and methods for error analysis. We then present multiple algorithms to estimate the physical entropy from common sampling distributions. Some techniques explore the use of Binary Trees and the roots of Legendre polynomials, as well as a combination of the two. Finally, we discuss the performance of these algorithms and provide suggestions for further research. (Received September 23, 2017)

1135-65-1574 **Chao Yang\***, 1 Cyclotron Rd, Berkeley, CA 94720. *Solving Large-scale Eigenvalue Problems in Electronic Structure Calculations.*

One of the most time consuming tasks in Kohn-Sham density functional based electronic structure calculations is to compute eigenpairs of a Hermitian matrix. The number of eigenpairs to be computed is proportional to the number of atoms in the poly-atomic systems being studied. For large molecules and nanoclusters, this number can be quite large even though it is still a small fraction of the dimension of the matrix. For these problems, many standard iterative methods that use the Rayleigh-Ritz procedure to extract approximate eigenpairs from a single subspace becomes very costly. We examine a number of techniques to overcome this difficulty and improve the efficiency and scalability of the eigenvalue calculation on large-scale high performance parallel computers. (Received September 23, 2017)

1135-65-1628 **Chong Sun\*** ([chong\\_sun@baylor.edu](mailto:chong_sun@baylor.edu)), 1806 south 8th street, apt 148, Waco, TX 76706, and **Qin Sheng**. *Linearly Splitting Methods for a Heston Stochastic Volatility Model.*

Black-Scholes type of partial differential equations are frequently utilized for pricing American/European style options and similar financial derivatives. This is because the price of the underlying asset often follows a Brownian motion with a positive drift term. Black-Scholes equations are anti-diffusive. They are derived from an application of Feymann-Kac formula for corresponding stochastic differential equations.

Our primary concern is the numerical stability of split schemes applied to the aforementioned differential equations. We are particularly interested in the Heston stochastic volatility model possessing cross derivative terms. Those terms have been posing difficulties in proving the anticipated stability in computations. Though traditional analysis are based on von Neumann method, the limitation of such an analysis is obvious due to its boundary condition restrictions.

A novel new discretization strategy is proposed in our study. We further prove the stability of such schemes via spectrum analysis without a restriction of types of boundary conditions considered. Computational experiments are given to illustrate our conclusions. (Received September 23, 2017)

1135-65-1654 **Md Shafiqul Islam\*** ([mdshafiqul@du.ac.bd](mailto:mdshafiqul@du.ac.bd)), Department of Applied Mathematics, Dhaka University, Dhaka, 1000, Bangladesh. *Bezier Polynomials With Applications.*

In this paper, we develop a matrix formulation to solve higher order linear and nonlinear BVPs, arising in various fields, by the well known Galerkin method. Bezier polynomials are exploited as basis functions in the technique. To use the Bezier polynomials we need to satisfy the corresponding homogeneous form of the boundary conditions and modification is thus needed. We verify the proposed formulation by solving a numerous examples. The approximate solutions are found with great accuracy, and are compared to the exact solutions as well as approximate solutions available in the literature. All computations are performed using the software MATHEMATICA. (Received September 24, 2017)



1135-65-1665 **Noah J. DeMoes\*** (noah.demoes@usma.edu), United States Military Academy, Department of Mathematical Sciences, PO Box 0872, West Point, NY 10996, **Gabriel T. Bann** (gabriel.bann@usma.edu), United States Military Academy, Department of Mathematical Sciences, PO Box 0150, West Point, NY 10996, and **Randy Boucher** (randy.boucher@usma.edu), United States Military Academy, Department of Mathematical Sciences, West Point, NY 10996. *A Complex Variable Boundary Element Method for Modeling Fluid Flow in High Aspect Ratio Domains.*

The Complex Variable Boundary Element Method (CVBEM) numerically solves potential problems found in several fields of science, technology, engineering, and mathematics. Approximate solutions to steady-state problems using the CVBEM satisfy the Laplace equation and are continuous on the problem domain, contrary to many conventional discrete methods found in industry such as the Finite Element Method (FEM). Typically, applications of the CVBEM have been limited to low aspect ratio domains. In this work, the CVBEM is applied to modeling applications in high aspect ratio domains and compared to the FEM to determine which method approximates solutions with least squares error.

The problem considered is fluid flow in a 90-degree bend. Comparing the CVBEM to the FEM, 3 different domain ratios, 1:10, 1:25, and 1:50 are used in conjunction with the boundary conditions. The CVBEM will use four different bases function families to determine the best basis function family used in the CVBEM to approximate the solution. The approximations will be compared against the FEM to determine the method to model problems with high aspect ratio domains.

The methods and solutions of the CVBEM in two-dimensional high aspect ratio domains and application to 3 dimensions are discussed. (Received September 26, 2017)

1135-65-1730 **Hessah Faihan Alqahtani\*** (hfq1981@gmail.com), 169 Deerfield lane, Aurora, OH 44202, and **Lothar Reichel** (reichel@math.kent.edu), Kent, OH 44242. *Generalized Block Anti-Gauss Quadrature Rules.* Preliminary report.

Golub and Meurant describe how pairs of Gauss and Gauss-Radau quadrature rules can be applied to determine inexpensively computable upper and lower bounds for certain real-valued matrix functionals defined by a symmetric matrix. However, there are many matrix functionals for which their technique is not guaranteed to furnish upper and lower bounds. In this situation, it may be possible to determine upper and lower bounds by evaluating pairs of Gauss and anti-Gauss rules. Unfortunately, it is difficult to ascertain whether the values determined by Gauss and antiGauss rules bracket the value of the given real-valued matrix functional. Therefore, generalizations of anti-Gauss rules have recently been described, such that pairs of Gauss and generalized anti-Gauss rules may determine upper and lower bounds for real-valued matrix functionals also when pairs of Gauss and anti-Gauss rules do not. The available generalization requires the matrix that defines the functional to be real and symmetric. We extends generalized anti-Gauss rules in several ways: The real-valued matrix functional may be defined by a nonsymmetric matrix. Moreover, extensions that can be applied to matrix-valued functions are presented. Estimates of element-wise both bounds then are determined. (Received September 24, 2017)

1135-65-1741 **Arvind Saibaba\*** (asaibab@ncsu.edu), **Misha Kilmer**, **Eric de Sturler** and **Eric Miller**. *RandMCMC: Randomization based MCMC samplers for Bayesian Inverse Problems.* Preliminary report.

Markov Chain Monte Carlo (MCMC) is the prevalent approach for sampling posterior distributions in Bayesian inverse problems. For inverse problems with many measurements, MCMC methods are prohibitively expensive since overall  $10^6 - 10^8$  large-scale linear systems need to be solved to evaluate the likelihood and explore the parameter space. We propose several different methods of reducing the cost of the likelihood evaluations using techniques from randomized matrix methods. Tradeoffs between computational costs and statistical efficiency will be discussed. The performance of the proposed samplers will be demonstrated on 2D model problems from Diffuse Optical Tomography. (Received September 24, 2017)

1135-65-1778 **Cu Hauw Hung\*** (cuhaaw.hung@biola.edu), 13800 Biola Ave, La Mirada, CA 90639, and **Yue Yi Li** (yli5@macalester.edu), 1600 Grand Ave, St. Paul, MN 55105. *Improving the efficiency of Adaptive (Quasi-) Monte Carlo Methods using (Internal) Control Variates.* Preliminary report.

(Quasi-)Monte Carlo ((Q)MC) simulations are used to compute the means of random variables whose distributions are too complex to admit analytic formulae. This work builds upon the automatic stopping criteria developed for (Q)MC simulations that use theoretically justified, data-based error bounds to determine when the error tolerance has been met. This research extends these automatic (Q)MC algorithms to include control variates and internal control variates. Control variates reduce variance and thus improve computational

efficiency. Internal control variates could further deal with situations where multiples random variables share the same expected value. The stopping criteria must be modified, and the choice of control variate coefficients for QMC must be made differently than for simple MC. Numerical tests of our new algorithms and stopping criteria demonstrate that the error tolerance continues to be and that the computational effort required can be dramatically reduced. (Received September 24, 2017)

1135-65-1783 **Vani Cheruvu\*** ([vani.cheruvu@utoledo.edu](mailto:vani.cheruvu@utoledo.edu)), Department of Mathematics and Statistics, The University of Toledo, Toledo, OH 43606, and **Shravan K Veerapaneni**. *Spherical harmonics based solutions for modified Laplace equation on a sphere.*

We consider a modified Laplace equation on a unit sphere. Spherical harmonics are used for the expansion of the unknown function. We show that on the unit sphere, both modified Laplace single and double layer operators diagonalize in spherical harmonic basis. The analytic expressions for evaluating the operators away from the boundary are also derived. Currently, we are working on the numerical aspects. In this talk, we present both the analytical and numerical results of our work. (Received September 26, 2017)

1135-65-1794 **Willi Freeden\*** ([freeden@rhrkuni-kl.de](mailto:freeden@rhrkuni-kl.de)), MPI Building 26, 67663 kaiserslautern, Rhineland, Germany, and **M. Zuhair Nashed** ([m.nashed@ucf.edu](mailto:m.nashed@ucf.edu)), 347 Hartlepool Court, Oviedo, GU 32765. *Inverse Gravimetry in Reproducing Kernel Spaces.*

This lecture deals with the ill-posed and inverse features of transferring input gravitational information in the form of Newtonian volume integral values to geological output characteristics of the density contrast function. Some properties of the Newton volume integral are recapitulated. Reproducing kernel Hilbert space regularization techniques are studied (together with their transition to mollified variants) to provide geological contrast density distributions by downward continuation from terrestrial and/or spaceborne data. (Received September 24, 2017)

1135-65-1796 **Vrushali A Bokil\*** ([bokilv@math.oregonstate.edu](mailto:bokilv@math.oregonstate.edu)), **Linda J. S. Allen, Michael Jeger** and **Suzanne Lenhart**. *Optimal Control of Vector Transmitted Viral Disease of Crops with Different Replanting Strategies.* Preliminary report.

Vector-transmitted diseases of plants have had devastating effects on agricultural production worldwide, resulting in drastic reductions in yield for crops such as cotton, soybean, tomato and cassava. In this investigation, we formulate a new plant-vector-virus model with continuous replanting from density-dependent replanting of healthy and some infected plants. The new model is an extension of a model formulated by Holt et al., An epidemiological model incorporating vector population dynamics applied to African cassava mosaic virus disease, *Journal of Applied Ecology*, pages 793-806, 1997. Both models are analyzed and thresholds for disease elimination are defined in terms of the model parameters. Parameter values for cassava, whiteflies, and the virus, in African cassava mosaic virus serve as a case study. A numerical investigation illustrates how the equilibrium densities of healthy and infected plants for both models vary with changes in parameter values. Applications of insecticide and roguing to reduce plant disease and to increase the number of plants harvested are studied using optimal control theory. (Received September 24, 2017)

1135-65-1807 **Mingchao Cai\*** ([mingchao.cai@morgan.edu](mailto:mingchao.cai@morgan.edu)), 1700 E Cold Spring Ln, Baltimore, MD 21251, and **Guoping Zhang** ([guoping.zhang@morgan.edu](mailto:guoping.zhang@morgan.edu)), 1700 E Cold Spring Ln, Baltimore, MD 21251. *Some Iterative Algorithms for Biot Equations and Applications in Biomechanics.*

Biot equations have been widely used in Biomechanics. For example, modeling brain edema and cancellous bones. We aim at solving the Biot model under stabilized finite element discretizations. To solve the resulting generalized saddle point linear systems, some iterative methods are proposed and compared. In the first method, we apply the GMRES algorithm as the outer iteration. In the second method, the Uzawa method with variable relaxation parameters is employed as the outer iteration method. In the third approach, Uzawa method is treated as a fixed-point iteration, the outer solver is the so-called Anderson acceleration. In all these methods, the inner solvers are preconditioners for the generalized saddle point problem. In the preconditioners, the Schur complement approximation is derived by using Fourier analysis approach. These preconditioners are implemented exactly or inexactly. Extensive experiments are given to justify the performance of the proposed preconditioners and to compare all the algorithms. (Received September 24, 2017)

1135-65-1813 **Lin Lin\*** ([linlin@berkeley.edu](mailto:linlin@berkeley.edu)), **Wei Hu** and **Chao Yang**. *Projected Commutator DIIS method for linear and nonlinear eigenvalue problems.*

The commutator direct inversion of the iterative subspace (commutator DIIS or C-DIIS) method developed by Pulay in the 1980s is an efficient and the most widely used scheme for solving nonlinear eigenvalue problems

in quantum chemistry. However, for large matrices this method is not practical due to the need of explicit construction of the projector and the commutator. I will discuss a newly developed Projected Commutator DIIS method (PC-DIIS) method to accelerate nonlinear eigenvalue problems and even certain linear eigenvalue problems with much reduced cost. The PC-DIIS method has been successfully applied to accelerate hybrid functional density functional theory calculations in quantum chemistry. (Joint work with Wei Hu and Chao Yang) (Received September 25, 2017)

1135-65-1852     **Samet Y Kadioglu\*** (kadioglu@yildiz.edu.tr), Yildiz Technical University, Department of Mathematical Engineering, Davutpasa-Esenler, Istanbul, Turkey. *A High Resolution Numerical Algorithm for Fluid Flows*. Preliminary report.

We present a high resolution numerical algorithm based on the Spectral Deferred Corrections (SDC) time integration technique and the Essentially Non-Oscillatory (ENO) finite volume method for the compressible fluid flow applications. The SDC technique is used to advance the solutions in time with high-order of accuracy. The ENO method is used to calculate numerical fluxes with non-oscillatory fashion. We have tested the new SDC-ENO technique by solving several test problems involving moderate to strong shock waves and smooth/complex flow structures [3]. Our numerical results show that we have numerically achieved the formally fourth-order convergence of the new method for smooth problems. Our numerical results also indicate that the newly proposed technique performs very well providing highly resolved shock discontinuities and fairly good contact solutions. More importantly, the discontinuities in the flow test problems are captured with essentially no-oscillations. We have numerically compared the fourth-order SDC-ENO scheme to the fourth-order SDC-PPM method for the same test problems. The results are similar for most of the tests except in some cases the SDC-PPM method suffers from minor oscillations compared to SDC-ENO scheme being completely oscillation free. (Received September 25, 2017)

1135-65-1870     **Ivan Mitevski\*** (im85@njit.edu), 6 Carlson Place, Lake Hiawatha, 07034, **Matthew Illingworth** (mri8@njit.edu), 12 Vincent Place, Montclair, NJ 07042, **David Youssif** (dy76@njit.edu), 19 Lakewood Ave, Hohokus, NJ 07423, and **Brittany Froese**. *Numerical methods for solving Monge-Ampere equation*. Preliminary report.

The Monge-Ampere (MA) equation is a fully nonlinear degenerate elliptic partial differential equation that arises in optimal mass transportation, beam shaping, image registration, seismology, etc. In the classical form this equation is given by  $\det(D^2\phi(x)) = f(x)$  where  $\phi$  is constrained to be convex. Previous work has produced solvers that are fast but can fail on realistic (non-smooth) data or robust but relatively slow. The purpose of this work is to build a more robust and time-efficient scheme for solving the MA equation. We express the MA operator as the product of the eigenvalues of the Hessian matrix. This allows for a globally elliptic discretization that is provably convergent. The method combines a nonlinear Gauss-Seidel iterative method with a centered difference discretization on a variety of different coordinate systems, which is stable because the underlying scheme preserves monotonicity. In order to solve these systems efficiently, the V-cycle full approximation scheme multigrid method is exploited with error correction within the recursive algorithm; this scheme is used to leverage the low cost of computation on the coarse grids to build up the finer grids. This work shows computational results that demonstrate the speed and robustness of the algorithm. (Received September 25, 2017)

1135-65-1943     **Manuel Aurelio Aurelio Diloné\***, dilonespinal@hotmail.es. *Semilocal convergence Newton method applied to Kepler equation: New results*.

The aim of this paper is to consider as a starting point the value  $E_0 = \pi$  in the theorems of semilocal convergence of Kantorovich, Gutiérrez,  $\alpha$ -theory of Smale and the  $\alpha$ -theory of Wang-Zhao, to compare the convergence conditions obtained. Once set  $E_0$ , one should calculate the parameters listed in the statement of these theorem. So, we will generalize the study of Diloné-Gutiérrez for the case  $E_0 = M$ . Numeric and graphic calculations were obtained by applying Mathematica V10.

Keywords : Newton method; semilocal convergence theorem; Kepler equation. (Received September 25, 2017)

1135-65-1970     **Jodi Mead\*** (jmead@boisestate.edu), **Diego Domenzain** and **John Bradford**. *Uncertainty in Joint inversion*.

Simultaneous joint inversion involves optimizing a single objective function with information from multiple types of data. The objective function contains multiple terms so that the data types can work cooperatively, while also honoring the physics of the problem. Each term requires an uncertainty estimate, and the estimates will significantly affect the results. We propose to use data error as an initial uncertainty estimate and add parameters, similar to regularization parameters, to account not only for uncertain data but also uncertain forward models.

The additional parameters will be computed by ensuring the minimum value of the objective function satisfies appropriate statistical conditions. We will illustrate the approach by combining observations of energy transfer by electromagnetic fields injected into the earth from a large range of frequencies. (Received September 25, 2017)

1135-65-1977 **Noemi Petra\*** (npetra@ucmerced.edu), **Alen Alexanderian**, **Omar Ghattas** and **Georg Stadler**. *Optimal control of systems governed by PDEs with random parameter fields using quadratic approximations.*

We present a method for optimal control of systems governed by partial differential equations (PDEs) with uncertain parameter fields formulated as a risk-averse optimal control problem. Conventional numerical methods for optimization under uncertainty are prohibitive when applied to this problem. To make the optimal control problem tractable we invoke a quadratic Taylor approximation of the control objective with respect to the uncertain parameter field. This enables deriving explicit expressions for the mean and variance of the control objective in terms of its gradients and Hessians with respect to the uncertain parameter. We apply the proposed method to a control problem governed by an elliptic PDE with an uncertain coefficient field. We derive adjoint-based expressions for efficient computation of the gradient of the risk-averse objective with respect to the controls and use trace estimation to compute the trace of the Hessian (present in the control objective under the quadratic approximation). We show numerical studies for various aspects of the risk-averse measure and of the efficiency of the proposed method. The results show the effectiveness of our approach in computing risk-averse optimal controls in a problem with a 3,000-dimensional discretized parameter space. (Received September 25, 2017)

1135-65-2016 **Chao-Ping Lin\*** (cplin@ucdavis.edu), Department of Mathematics, University of California, Davis, CA 95616, and **Huiqing Xie** and **Zhaojun Bai**. *On the Semi-definite B-Lanczos algorithm for sparse symmetric generalized eigenproblems.*

The shift-invert B-Lanczos algorithm is a variant of the symmetric Lanczos method for extracting eigenpairs of large eigenproblem  $Ax = \lambda Bx$  around a shift, where  $A$  and  $B$  are symmetric and  $B$  is positive definite. A number of “industrial strength” black-box solvers are based on the algorithm. When  $B$  is semi-definite, since the algorithm does not require the inverse of  $B$ , it can proceed formally. It has been a common practice since the inception of the algorithm. However, it has been observed that the components of Lanczos vectors lying in the null space of  $B$  can grow rapidly and cause the failure of the algorithm. The issue has been studied where methods for purifying the null-space component have been proposed. Stewart examined the source and consequence of the growth, and concluded that the purifying will not restore the lost information. In this talk, we present an approach by introducing regularization to transform a semi-definite pencil into a definite one, assuming that a basis for the null space is available. The regularization scheme is embedded into the shift-invert spectral transformation, and existing solvers are immediately applicable. The efficacy of the proposed approach will be demonstrated by Stewart’s example and real-life applications. (Received September 25, 2017)

1135-65-2049 **Selin Sariaydin\*** (selin@vt.edu), Virginia Tech, Blacksburg, VA, and **Eric de Sturler**, **Serkan Gugercin** and **Misha Kilmer**. *Randomization for Efficient Reduced Order Models*. Preliminary report.

Diffuse Optical Tomography (DOT) in medical image reconstruction presents huge computational challenges since it requires at least one forward and adjoint PDE solve for each source and detector at each optimization step. One can use reduced order models (ROM) to reduce the size of the linear systems, but computing the reduced order model still requires solving many systems. In this talk, we propose to reduce the number of large linear solves for constructing a ROM global basis by randomization. (Received September 25, 2017)

1135-65-2098 **Gerry Baygents\***, Department of Mathematics and Statistics, UMKC, 5100 Rockhill Road, Kansas City, MO 64110. *Modeling migratory effects of white-tailed deer on dynamics of hemorrhagic disease: reducing risk of outbreaks via isolation.*

Hemorrhagic disease (HD) is a vector-borne disease that affects deer and other ruminants in the United States. A delay differential equation for endemic HD involving variable deer and midge (vector) populations is analyzed. A threshold parameter  $R_0$  exists and the disease persists if and only if  $R_0 > 1$ . We examine the value as a function of influx rates, interaction rates, and probability of death while migrating and provide numerical simulation of the changes in  $R_0$ . (Received September 25, 2017)

1135-65-2111 **Mikheil Tutberidze\*** (mtutberidze@gmail.com), 3/5 Kakutsa Cholokashvili ave., 0162 Tbilisi, Rep of Georgia. *Investigation and Numerical Solution of Difference Analogous to Initial-Boundary Value Problem for One Nonlinear Partial Differential Equation.*

In the present work the difference scheme for initial-boundary value problem to following nonlinear parabolic equation

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x} \left( \left( x, t, \frac{\partial U}{\partial x} \right) \frac{\partial U}{\partial x} \right) + f(x, t, U)$$

is considered. For the mentioned difference scheme the convergence of its solution to the solution of the source problem is proved when certain conditions hold. For the same difference scheme the comparison theorems and the existence and uniqueness of its solution is proved for the same conditions. The iteration process for finding difference scheme solution is constructed and its convergence is proved. (Received September 25, 2017)

1135-65-2145 **Chung-Nan Tzou\*** (ctzou@wisc.edu) and **Samuel Stechmann** (stechmann@wisc.edu). *Simple Second-Order Finite Differences for Elliptic PDEs with Discontinuous Coefficients and Interfaces.*

Many multiphase flow problems require the solution of Poisson's equation with discontinuous coefficients due to different fluid properties, such as density, in the different phases of the fluid. Here we present a second-order-accurate numerical method for this problem, where the method is based on simple finite difference formulas. The derivation is performed on a Cartesian grid and leads to a symmetric operator, even across the interface, with suitable adjustments of the right-hand side arising in the derivation and accounting for the interface. The right-hand side is then determined using an iterative method. Comparisons with other methods, such as the first-order ghost fluid method and the second-order immersed interface method, will be discussed; for instance, the present method does not require derivatives of jump conditions. This numerical method is mathematically proven to be second-order accurate in one dimension, in which case iterations are not needed. Second-order accuracy is demonstrated via numerical trials in both two and three dimensions. (Received September 25, 2017)

1135-65-2184 **Xiaobing Feng, Zhihao Ge and Yukun Li\*** (li.7907@osu.edu), Columbus, OH 43220. *Analysis of a Multiphysics Finite Element Method for a Poroelasticity Model.*

This talk concerns with finite element approximations of a quasi-static poroelasticity model in displacement-pressure formulation which describes the dynamics of poro-elastic materials under an applied mechanical force on the boundary. To better describe the multiphysics process of deformation and diffusion for poro-elastic materials, we first present a reformulation of the original model by introducing two pseudo-pressures. We then propose a time-stepping algorithm which decouples the reformulated PDE problem at each time step into two sub-problems: one of which is a generalized Stokes problem for the displacement vector field (of the solid network of the poro-elastic material) along with one pseudo-pressure field and the other is a diffusion problem for the other pseudo-pressure field (of the solvent of the material). It is also proved that the solutions of the fully discrete finite element methods fulfill a discrete energy law which mimics the differential energy law satisfied by the PDE solution and converges optimally in the energy norm. Moreover, it is showed that the proposed formulation also has a built-in mechanism to overcome so-called "locking phenomenon" associated with the numerical approximations of the poroelasticity model. (Received September 25, 2017)

1135-65-2261 **Matthew Dunlop\*** (mdunlop@caltech.edu), **Dejan Slepčev, Andrew Stuart and Matthew Thorpe.** *Graph-based learning as a Bayesian inverse problem.*

A common task in machine learning is to divide a set of data points, such as images, into a number of classes. The data points may be viewed as vertices of a graph, with edges weighted according to some similarity measure between points, from which a graph Laplacian  $L$  can be constructed. The spectral properties of  $L$  can contain a significant amount of clustering information. If the classes for a small number of points are known, the problem is to propagate these class labels to all of the data points in an appropriate manner; combining the known labels with  $L$  is a natural approach to this. In the Bayesian approach, a prior distribution is constructed from  $L$ , and Bayes' theorem is used to condition it upon the labeled data. Classification, and quantification of uncertainty associated with it, can then be found via integration with respect to this conditioned distribution. We study large data limits of this problem. Subject to appropriate scaling, the graph Laplacian converges to a certain differential operator; properties of this operator then provide heuristics for selection of parameters underlying the prior when the number of data points is large but finite. We also consider hierarchical Bayesian approaches, in which these parameters are learned from the labeled data. (Received September 25, 2017)

1135-65-2353 **Eugene Vecharynski\*** (evecharynski@lbl.gov), **Fei Xue** and **Chao Yang**. *Generalized Preconditioned Locally Harmonic Residual Method for large non-Hermitian eigenproblems.*

We introduce the generalized preconditioned locally harmonic residual (GPLHR) method for solving standard and generalized non-Hermitian eigenproblems. The method is particularly useful for computing a subset of eigenvalues, and their eigen- or Schur vectors, closest to a given shift. The proposed method is based on block iterations and can take advantage of a preconditioner if it is available. It does not need to perform exact shift-and-invert transformation. Standard and generalized eigenproblems are handled in a unified framework. Our numerical experiments demonstrate that GPLHR is generally more robust and efficient than existing methods, especially if the available memory is limited. (Received September 26, 2017)

1135-65-2371 **R H Hoppe, Rahul Kumar** and **Natasha S Sharma\*** (nssharma@utep.edu), Department of Mathematical Sciences, University of Texas at El Paso, El Paso, 79968, and **James Winkle**. *A Phase Field Model for Polycrystallization Processes in Binary Mixtures.*

In this talk, we consider the phase field model for polycrystallization in the solidification of binary mixtures in the domain  $\Omega \subset \mathbb{R}^2$ . This model is based on a free energy functional in terms of three order parameters: the local crystallinity  $\phi$ , the concentration  $c$  of one of the components of the binary mixture, and the local orientation  $\Theta$  of the crystals. The equations of motion are given by an initial-boundary value problem for a coupled system of partial differential equations consisting of two quasilinear second order parabolic equations (in  $\phi$  and  $\Theta$ ) and one quasilinear fourth order parabolic equation of Cahn-Hilliard type equation in  $c$ . We prove the existence of a weak solution by performing an implicit discretization in time and splitting of the equations. Using regularity results for quasilinear parabolic equations, it is shown that a solution of the time-discrete system converges to a weak solution of the original system. (Received September 26, 2017)

1135-65-2378 **Jerome Gilles, Melissa Tong** and **Luminita Vese\***, lvese@math.ucla.edu. *Restoration of images corrupted by atmospheric turbulence.* Preliminary report.

We study the problem of restoring images distorted by atmospheric turbulence. Geometric distortions and blur are the two main components of degradations due to atmospheric turbulence, and prior work has been done to address these components separately. We propose a joint variational deblurring and geometric distortion correction model and present numerical results on synthetic and real data. (Received September 26, 2017)

1135-65-2521 **Constantin Bacuta\*** (bacuta@udel.edu), 501 Ewing Hall, University of Delaware, Newark, DE 19716, and **Jacob Jacavage** (jjacav@udel.edu), 501 Ewing Hall, Newark, DE 19716. *Saddle point least squares methods for first order systems.*

We introduce a Saddle Point Least Squares method for discretizing first and second order boundary value problems written as primal mixed variational formulations. For the mixed formulation we assume a stability LBB condition and a data compatibility condition at the continuous level. For the proposed discretization method a discrete inf – sup condition is automatically satisfied by the natural choices of test spaces (first) and the corresponding trial spaces (second). The discretization and the iterative approach does not require nodal bases for the trial space and an SPD preconditioner acting on the discrete test space can be adopted to speed up the approximation process. A stopping criterion based on matching the order of the the iteration error with the the order of the expected discretization error can be considered. Applications of the method include discretizations of first order systems of PDEs, such as *div – curl* systems and time-harmonic Maxwell equations. (Received September 26, 2017)

1135-65-2526 **Todd Romutis\*** (toddr@bgsu.edu) and **Tong Sun** (tsun@bgsu.edu). *Numerical smoothness and error analysis for parabolic equations.*

Numerical smoothness is an innovative approach to error analysis for numerical solutions of time-dependent partial differential equations. This talk will focus on the concept of numerical smoothness, its relationship to numerical stability, and the general framework with which the method can be applied to time-dependent PDEs. Past research results and current projects will be presented. (Received September 26, 2017)

1135-65-2582 **Cleve Ashcraft, Eric Darve, Roger G Grimes, Yun Huang, Pierre L'Eplattenier, Robert F Lucas, Francois-Henry Rouet\*** (fhrouet@lstc.com), **Clément Weisbecker** and **Zixi Xu**. *Using Block Low-Rank techniques for industrial problems.*

Matrices that appear in the boundary element methods and finite element methods are often structured (or low-rank, or data-sparse). This means that they exhibit rank-deficient blocks, typically the blocks corresponding to far-range interactions in the physical space. Identifying and compressing these low-rank blocks, e.g., using SVD or a rank-revealing factorization, is the key to reducing the storage and computational requirements of

many matrix operations, such as solving linear systems. In this talk, we discuss the use of Block Low-Rank techniques for dense and sparse linear systems arising from multiphysics simulations.

For dense problems, we discuss a fast matrix assembly scheme based on Skeletonized Interpolation, and we compare factorization techniques based on Block-Low Rank and Hierarchically Semi-Separable representations. Our test matrices come from electromagnetics and acoustics. For sparse problems, we present results using Block-Low Rank techniques embedded within our multifrontal solver. Our test matrices come from implicit mechanics. All the experiments are performed using the multiphysics code LS-Dyna. (Received September 26, 2017)

1135-65-2669 **A. Bass Bagayogo\*** ([abagayogo@ustboniface.ca](mailto:abagayogo@ustboniface.ca)), 200 Avenue de la Cathedrale, Winnipeg, MB R2H 0H7, Canada. *Sine Cardinal Function in Modern Computational Mathematics.*

The sine cardinal function or sinc function is an important function that arises frequently in areas of science and engineering like signal processing, approximate solutions of differential and integral equations, numerical analysis, etc... After a theoretical and practical introduction to the problem with remarkable sinc integrals and sums, I will present results from experimental mathematics techniques by using a modern computer algebra packages in order to get the numerical results with high accuracy and less computational efforts. (Received September 26, 2017)

1135-65-2781 **Martin Licht\***, Department of Mathematics, The University of California, San Diego, 9500 Gilman Dr, # 0112, La Jolla, CA 92093, and **Michael Holst**, Department of Mathematics, The University of California, San Diego, 9500 Gilman Dr, # 0112, La Jolla, CA 92093. *Towards Finite Element Methods over Manifolds via Coordinate Charts.*

We present some results concerning finite element methods for partial differential equations over manifolds. Our approach transforms partial differential equations of tensor fields from a physical manifold to parametric coordinate charts. The parametric problems involve smooth coefficients, which lead to a variational crime in practical finite element methods. Only recent results in approximation theory rigorously prove optimal error estimates. In this talk we use the case of Euclidean domains as a demonstrative example and relate our approach to computational practices in engineering and physics. (Received September 26, 2017)

1135-65-2822 **Adel Faridani\*** ([faridani@oregonstate.edu](mailto:faridani@oregonstate.edu)). *Signal Analysis and Reconstruction Algorithms in 2D Computed Tomography.*

Computed tomography produces images from the interior of opaque objects. A variety of data acquisition geometries and reconstruction algorithms are available for practical use. In this talk we will use Signal Analysis in the sense of applying the Shannon Sampling Theorem and its generalizations in order to identify potentially efficient sampling schemes that would allow reconstruction with a minimal amount of data. For various data acquisition schemes we will then identify and analyze suitable numerical reconstruction algorithms that achieve the desired performance. (Received September 26, 2017)

1135-65-2877 **Yekaterina Epshteyn, Kyle R. Steffen\*** ([steffen@math.utah.edu](mailto:steffen@math.utah.edu)) and **Qing Xia**. *Towards a Difference Potentials Method for the Mullins-Sekerka model.*

A classical problem in mathematical physics is the study of solidification or liquidation of materials. The Mullins-Sekerka model arises in the limit when interfacial motion is slow, relative to heat conduction. It is an elliptic free-boundary problem, with Laplace's equation, a Dirichlet boundary condition, and an interface evolution equation. Instabilities and dendritic growth are interesting phenomena which can occur for an interface between a bounded domain and its complement in all of  $\mathbb{R}^3$ .

The Difference Potentials Method (DPM) is a framework for the high-order accurate and efficient numerical solution of partial differential equations (PDE). It naturally permits the use of uniform grids, which need not conform with boundaries or interfaces. With the DPM, one can consider general boundary or interface conditions, with no change to the discretization of the PDE. Moreover, one can achieve spectral accuracy in the approximation of the solution near boundaries or interfaces.

In this talk (based on joint work with Y. Epshteyn and Q. Xia), I will discuss recent work towards a DPM for the Mullins-Sekerka model, including accurate treatment of the unbounded domain (replaced by a finite domain and an exact artificial boundary condition) and the moving interface. (Received September 26, 2017)

1135-65-2920 **Ivo Babuska** and **Ana Maria Soane\*** ([soane@usna.edu](mailto:soane@usna.edu)), 121 Blake Road, Annapolis, MD 21402, and **Manil Suri**. *Computational modeling of problems on domains with small holes.*

The modeling challenges arising when the problem domain has small supported holes in it are considered through a representative membrane problem. Such problems are sometimes modeled intuitively in engineering practice by

taking the limiting case of holes with zero radius. This intuitive model is incorrect, since it has no mathematical solution. It is demonstrated, however, that finite element approximations based on it can still satisfy verification tests and appear to converge, leading to erroneous recovery of quantities of interest. This points to the need for an alternate approach where the holes of finite radius are properly incorporated in the modeling, and robustness with respect to the radius is maintained. To this end, a computational method is presented which combines analytic knowledge of the solution singularities with finite element approximation of its smooth components. Theoretical and numerical results are provided, establishing the efficacy and robustness of the method in extracting quantities of interest. (Received September 26, 2017)

1135-65-2925 **Douglas N. Arnold\*** ([arnold@umn.edu](mailto:arnold@umn.edu)), School of Mathematics, University of Minnesota, Minneapolis, MN 55442, and **Lizao Li**. *Generalized Regge elements and Regge calculus*.

Regge calculus, proposed by Tulio Regge in 1961, elegantly discretizes general relativity (GR). It may be viewed as a finite element method for the Einstein equations, using piecewise constant elements. It is a consistent method, as shown by Cheeger et al in 1984, and nearly a conforming method, as shown by Christiansen in 2011. It is very much in the spirit of structure-preserving discretization methods, which are currently of high interest. We will describe a recent new family of finite elements, the generalized Regge elements, one for each polynomial order and valid in any number of dimensions, with the lowest order case recovering Regge. These are structure-preserving finite elements for second-order covariant tensors, such as metrics. We will discuss their implementation and performance, first for problems simpler than GR, such as the computation of geodesics using the elements to approximate the metric. Based on work of Sorkin from 1975, a 4D Regge calculus discretization of GR may be implemented with reasonable efficiency. However, as is becoming clear, this approach to numerical relativity, though consistent, is unstable, and often fails. We will explain the modes of failure and propose a 3+1 approach, using 3D generalized Regge elements, as an alternative. (Received September 26, 2017)

1135-65-2993 **Nicole A Fider\*** ([nfider@uci.edu](mailto:nfider@uci.edu)). *A numerical study of color category boundaries and category evolution*. Preliminary report.

Color categorization in humans is a topic in psychology and linguistics which can shed light on human thought and perception in general. Although individuals can divide the color space in different ways, it is accepted that in a linguistically unified society there exists a specific set of basic color categories which speakers use when categorizing the color space. These categories give members of the population the ability to communicate color information with each other, and can evolve over time as the culture and language evolve. We believe that dynamic changes are less likely to occur within categories and more likely to occur on or around category boundaries. We present a mathematical method of identifying a language's set of color boundaries based on color-naming data provided by the World Color Survey Data Archives and discuss the possible dynamics of category evolution and how they can be related to the numerical data. (Received September 26, 2017)

1135-65-2994 **Eric de Sturler\*** ([sturler@vt.edu](mailto:sturler@vt.edu)), Department of Mathematics, 460 McBryde Hall, Blacksburg, VA 24061-0123, and **Misha E. Kilmer**, **Eric L. Miller** and **Arvind K. Saibaba**. *Efficient Bayesian Inversion in High-Dimensional Inverse Problems*. Preliminary report.

We consider randomization and model reduction strategies for Bayesian inversion solving parameterized inverse problems in diffuse optical tomography. We analyze how the trade-off between the accuracy of samples and the number of required samples for convergence influence the total cost of the MCMC process. (Received September 26, 2017)

1135-65-3235 **Adam E Telatovich\*** ([adam.telatovich@gmail.com](mailto:adam.telatovich@gmail.com)), Department of Mathematics, 430 McAllister Building, Pennsylvania State University, University Park, PA 16802. *Operator splitting methods for solving the Langevin equation: new methods and analysis of their convergence*. Preliminary report.

In this talk, I will present new operator splitting methods for solving the Langevin stochastic differential equation, which is a joint project with my thesis advisor. Operator splitting methods for ordinary differential equations have been extensively studied by Gilbert Strang and others. We extend the methods to the Langevin equation. To analyze the convergence orders of the approximations (in the strong and weak sense), we compare them with the so-called Ito Taylor expansion of the exact solution, which is a stochastic analogue of the usual Taylor expansion. The methods are easy to implement. (Received September 27, 2017)



## 68 ► Computer science

1135-68-128 **Johan Ugander\*** ([jugander@stanford.edu](mailto:jugander@stanford.edu)). *A random walk around the block: personalized PageRank and stochastic block models.*

Diffusion-based methods for graph ranking are popular and powerful, with two leading examples being personalized PageRank and heat kernel ranking. These two specific methods can be viewed as different linear discriminant functions in the space of landing probabilities of a uniform random walk – two different ways to linearly weight landing probabilities of different walk lengths. In this talk we investigate optimal discriminant functions in the space of such landing probabilities for the popular model of separable graph structure known as the Stochastic Block Model. Surprisingly, the optimal weights for separating two classes of a Stochastic Block Model have the form of personalized PageRank, for a specific value of the personalized PageRank parameter  $\alpha$  that depends on the parameters of the Stochastic Block Model. This connection provides a novel formal motivation for the success of personalized PageRank in graph ranking. We use this connection to also propose more advanced linear and quadratic discriminant functions that account for higher moments of landing probabilities; we show that this strengthening yields much better results for stochastic block models. This is joint work with Isabel Kloumann and Jon Kleinberg. (Received July 31, 2017)

1135-68-467 **Steven T. Morse\*** ([steven.morse@usma.edu](mailto:steven.morse@usma.edu)), **Marta C. Gonzalez** and **Natasha Markuzon**. *Modeling the Influence Structure of a Network with Hawkes Processes.*

Inferring the influence structure of actors in a large-scale network is an important general problem applicable to finance (stock price fluctuations), neuroscience (neuron firing), seismology (earthquake activity), and social networks (social influence). The Hawkes process is a multidimensional stochastic model that can capture the network structure, mutual influence, and clustered event timing inherent in these applications. However, current methods for estimating the parameters of this model typically require highly sophisticated optimization machinery to prevent overfitting. We propose a simpler approach using a maximum a posteriori (MAP) Expectation-Maximization (EM) scheme, with priors on the network structure to achieve the necessary regularization, that has the added benefit of revealing the hidden branching structure of the network activity. To our knowledge this method is novel. We show the method produces interpretable results using a large mobile phone dataset and email records. (Received September 05, 2017)

1135-68-619 **Nawa Raj Pokhrel\*** ([nawaraj@mail.usf.edu](mailto:nawaraj@mail.usf.edu)), 4501 Montego Bay Court Apt 4, Tampa, FL 33613, and **Chris Tsokos** and **Hansapani Rodrigo**. *Cybersecurity: Time Series Predictive Modeling of Vulnerabilities of Desktop Operating System Using Linear and Non-linear Approach.*

Vulnerability forecasting models help us to predict the number of vulnerabilities that may occur in the future for a given Operating System (OS). There exist few models that focus on quantifying future vulnerabilities without consideration of trend, level, seasonality and non-linear components of vulnerabilities. Unlike traditional ones, we propose a vulnerability analytic prediction model based on linear and non-linear approaches via time series analysis in Artificial Neural Network (ANN) and Support Vector Machine (SVM) setting. Utilizing time series approach, this study has developed a predictive analytic model for three popular Desktop Operating Systems, namely, Windows 7, Mac OS X, and Linux Kernel by using their reported vulnerabilities on the National Vulnerability Database (NVD). Based on these reported vulnerabilities, we predict ahead their behavior so that the OS companies can make strategic and operational decisions like secure deployment of OS, facilitate backup provisioning, disaster recovery, diversity planning, maintenance scheduling, etc. Similarly, it also helps in assessing current security risks along with estimation of resources needed for handling potential security breaches and to foresee the future releases of security patches. (Received September 11, 2017)

1135-68-662 **Lorena Mejia Domenzain\*** ([lorenamejiadomenzain@outlook.com](mailto:lorenamejiadomenzain@outlook.com)), Porfirio Diaz #163 casa 6, 10200 Ciudad de Mexico, Mexico, **Natalia Dibbern** ([ndibbern@lion.lmu.edu](mailto:ndibbern@lion.lmu.edu)), 1 LMU Drive MSB, Los Angeles, CA 7559, and **Harjasleen Malvai** ([jasleenmalvai@gmail.com](mailto:jasleenmalvai@gmail.com)). *Taming Information Leaks in Machine Learning.*

A supervised machine learning algorithm takes a dataset along with known labels as input and outputs a learned function which is used to make predictions about new data points. However, using machine learning could compromise privacy at various stages: private information about the training data could be inferred from the output of the algorithm, or the learned function applied to a new data point could reveal private information about the data point (which we call feature vector). Preserving privacy while providing utility poses a set of interesting problems, the first of which is finding precise, achievable definitions for the concepts of privacy and utility in the context of machine learning algorithms. Semantic security, which implies full privacy, is impossible

to achieve in general. We present a weaker definition of privacy, differential privacy, which quantifies the level of privacy attained as a measure of the probability of having similar outcomes from close inputs. We extend differential privacy to quantify feature vector privacy, and show methods of achieving privacy for training data and feature vectors under these definitions. We also pick measures of utility and then present trade-offs between utility and privacy for various machine learning algorithms. (Received September 12, 2017)

1135-68-792 **Althea I. Jensen\*** (ajensen5@gmu.edu). *Stochastic Enumeration with Importance Sampling.*

A great many varied problems in the computational sciences can be solved by summing a cost function over the nodes of a decision tree, and many approximation algorithms exist to estimate such sums. One of the most recent of these algorithms is Stochastic Enumeration (SE), introduced in 2013 by Rubinstein. In 2015, Vaisman and Kroese provided a rigorous analysis of the variance of SE, and also showed that SE can be extended to a fully polynomial randomized approximation scheme for certain cost functions on random trees. However, no one has yet incorporated an importance function into SE to allow non-uniform selection of tree nodes. This talk will present an algorithm that incorporates an importance function into SE, and will provide theoretical analysis including proof of correctness and bounds on the variance of estimates produced by the algorithm. The talk will also present the results of numerical experiments to measure the variance of applications of the algorithm to practical problems, such as counting linear extensions of a poset. (Received September 14, 2017)

1135-68-804 **Howard S. Cohl\*** (hcohl@nist.gov). *Semantic Preserving Bijective Mappings of Mathematical Expressions between LaTeX and Computer Algebra Systems.*

Document preparation systems like L<sup>A</sup>T<sub>E</sub>X offer the ability to render mathematical expressions as one would write these on paper. Using L<sup>A</sup>T<sub>E</sub>X, LaTeXML, and tools generated at the National Institute of Standards (NIST) Digital Library of Mathematical Functions, semantically enhanced mathematical L<sup>A</sup>T<sub>E</sub>X markup (semantic L<sup>A</sup>T<sub>E</sub>X) is achieved by using a semantic macro set. Computer algebra systems (CAS) such as Maple and Mathematica use alternative markup to represent mathematical expressions. By taking advantage of Youssef's Part-of-Math tagger and CAS internal representations, we develop algorithms to translate mathematical expressions represented in semantic L<sup>A</sup>T<sub>E</sub>X to corresponding CAS representations and vice versa. We have also developed tools for translating the entire Wolfram Encoding Continued Fraction Knowledge and University of Antwerp Continued Fractions for Special Functions datasets, for use in the NIST Digital Repository of Mathematical Formulae. The overall goal of these efforts is to provide semantically enriched standard conforming MathML representations to the public for formulae in digital mathematics libraries. These representations include presentation MathML, content MathML, generic L<sup>A</sup>T<sub>E</sub>X, semantic L<sup>A</sup>T<sub>E</sub>X, and now CAS representations as well. (Received September 14, 2017)

1135-68-826 **Yesom Park\*** (yeisom@naver.com), Science complex A #325, 52, Seodaemun-gu, Seoul, Republic of Korea, Seoul, 03762, South Korea, **Yoonkeong Lee** (lyk905@naver.com), Science complex A #325, 52, Seodaemun-gu, Seoul, Republic of Korea, Seoul, 03762, South Korea, **Chohee Park** (parkchohee5@gmail.com), Science complex A #325, 52, Seodaemun-gu, Seoul, Republic of Korea, Seoul, 30762, South Korea, **Seunghui Park** (96seunghui@gmail.com), Science complex A #325, 52, Seodaemun-gu, Seoul, Republic of Korea, Seoul, 30762, South Korea, and **Chohong Min** (chohongmin@gmail.com), Science complex A #325, 52, Seodaemun-gu, Seoul, Republic of Korea, Seoul, 30762, South Korea. *Three dimensional surface reconstruction from a single portrait based on supervised learning and optimization in Sobolev norm.* Preliminary report.

We introduce a new algorithm that produces a three dimensional surface reconstruction only from a single frontal portrait. Our algorithm is mainly based on supervised learning and radial-basis-function, which seeks an optimal recovery with respect to Sobolev norm. Given prototypical images as a training set, the Adaboost algorithm is employed to automatically detect various facial expressions. Since the human face can be characterized by a few main features such as eyes and lips, our algorithm effectively reconstructs the surface using the main features, the landmarks of a human face. Then, using the classified feature points and a reference surface, the smoothest surface in Sobolev norm is calculated via the polyharmonic radial-basis-function interpolation. (Received September 19, 2017)

1135-68-881 **Yonggi Park\*** (pygabc@gmail.com), 712 Vine street, Kent, OH 44240, and **Lothar Reichel, G Rodriguez** and **X Yu**. *Parameter Determination for Tikhonov Regularization Problems in General Form.* Preliminary report.

A regularization parameter  $\lambda > 0$  balances the influence of a fidelity term, which measures how well the data is approximated, and of a regularization term, which dampens the propagation of the data error into the computed

approximate solution. The value of the regularization parameter is important for the quality of the computed solution: A too large value of  $\lambda > 0$  gives an over-smoothed solution that lacks details that the desired solution may have, while a too small value yields a computed solution that is unnecessarily, and possibly severely, contaminated by propagated error. When a fairly accurate estimate of the norm of the error in the data is known, a suitable value of  $\lambda$  often can be determined with the aid of the discrepancy principle. This paper is concerned with the situation when the discrepancy principle cannot be applied. It then can be quite difficult to determine a suitable value of  $\lambda$ . We consider the situation when the Tikhonov regularization problem is in general form, i.e., when the regularization term is determined by a regularization matrix different from the identity, and describe an extension of the COSE method for determining the regularization parameter  $\lambda$  in this situation. (Received September 16, 2017)

1135-68-1051 **Yunkai Zhang\*** (zhang.yunkai98@gmail.com), **Yu Ma, Zhaoqi Li and Catalina Marie Vajiac.** *Exploration of Numerical Precision in Deep Neural Networks.*

Reduced numerical precision is a common technique to lower computational cost in various Deep Neural Networks (DNNs). While it has been observed that DNNs are resilient to small errors and noise, there exists no general result capable of predicting the sensitivity to reduced precision for a given DNN system architecture. In this project, we emulate arbitrary bit-width using a specified floating-point representation and truncating the remainder after a certain number of bits. This truncation is applied to the neural network after every batch. We show results on two representative networks, MNIST and CIFAR-10. In these, we explore the impact of several model parameters and their impact on the network's training accuracy. We then present a preliminary theoretical investigation of the error scaling in both forward and backward propagations. We end with a discussion of the implications of these results as well as the potential for generalization to other network architectures. (Received September 18, 2017)

1135-68-1312 **Florian T Pokorny\*** (fpokorny@kth.se). *Data-Driven Topological Methods for Reasoning about Motion.*

In this talk, I will review our recent work on algorithms that utilize methods from topological data analysis to enable hierarchical reasoning about sets of trajectories. Applications of our approach include a scalable approach to clustering trajectories by means of persistent homology, both for planar motion and motions in robotic configuration spaces. Further, I will discuss how sampling-based simplicial complex approximations of robotic configuration spaces can be utilized to search for collections of homotopy inequivalent trajectories between two configurations of a robotic system. (Received September 21, 2017)

1135-68-1328 **Anastasiia Varava\***, Teknikringen 14, Stockholm, 11428. *Topological and Geometric Methods in Robotic Manipulation and Path Planning.* Preliminary report.

In this talk, I will present our work on designing algorithms for robotic manipulation and path planning. In robotic manipulation, one says that an object is caged when it cannot escape arbitrarily far from its initial position, or, formally, when it lies in a compact connected component of the collision-free subspace of its configuration space. To derive sufficient conditions for objects to be caged by different types of robotic manipulators, we use two different approaches. The first is based on shape analysis: we consider partially deformable objects with specific shape features such as narrow parts, formalize their deformations as isotopies, abstract the manipulator as a closed curve, and use the concept of linking number to verify if the object is caged by the manipulator. The second approach relies on approximating the collision space of the object by an alpha complex and computing the path-connected components of its complement. This approach is also applicable to proving path non-existence between disjoint collision-free configurations of rigid objects and robots, which is an important problem in path planning. (Received September 21, 2017)

1135-68-1505 **Guillaume Brunerie\*** (guillaume.brunerie@gmail.com), 1 Einstein Drive, Princeton, NJ 08540. *Experiments in cubical type theory.* Preliminary report.

I will present the work of the ‘‘Cubical experiments’’ group during the MRC. The goal of this group was to develop some proofs and programs in cubical type theory (in particular in the `cubicaltt` proof assistant) to investigate the extent to which this improves on working in regular HoTT. We mainly worked on trying to compute the natural number  $n$  such that  $\pi_4(\mathbb{S}^3) = \mathbb{Z}/n\mathbb{Z}$  (we know indirectly that it is equal to 2, but computing it directly is still an open problem). We were not able to finish the computation, but we identified several of the obstacles, which I will describe here. (Received September 22, 2017)

1135-68-1633 **Marinka Zitnik\*** ([marinka@cs.stanford.edu](mailto:marinka@cs.stanford.edu)), **Rok Susic** and **Jure Leskovec**. *Prioritizing network communities.*

Uncovering modular structure in complex networks is fundamental for advancing the understanding of complex systems in biology, physics, engineering, and social sciences. Community detection provides a way to computationally identify candidate modules, which then need to be experimentally validated. However, validation of detected communities requires expensive and time consuming experimental methods, such as mutagenesis in a wet biological laboratory or surveying in sociology. As a consequence only a limited number of communities can be experimentally validated, and it is thus important to determine which communities to select for downstream validation and experimentation. In this talk we present a novel approach for prioritizing network communities and identifying the most promising ones for further experimentation. The proposed approach can be used with any community detection method and scales to large networks. The approach allows for more efficient evaluation of hypotheses brought forward by the analysis of complex networks and thus speeding-up scientific discovery process in experimental network sciences. (Received September 23, 2017)

1135-68-1850 **A. Roxana Pamfil\*** ([roxana.feier@maths.ox.ac.uk](mailto:roxana.feier@maths.ox.ac.uk)), **Sam D. Howison** and **Mason A. Porter**. *Optimal modularity maximization in multilayer networks.*

Identifying clusters or “communities” of densely connected nodes in networks is an active area of research, with relevance to many applications. Recent advances in the field have focused especially on temporal, multiplex, and other kinds of multilayer networks.

One method for detecting communities in multilayer networks is to maximize a generalized version of an objective function known as “modularity”. Writing down multilayer modularity requires the specification of two types of resolution parameters, and choosing appropriate values is crucial for uncovering meaningful community structure. In the simplest case, there are just two parameters:  $\gamma$  and  $\omega$ .  $\gamma$  controls the sizes of detected communities, and  $\omega$  influences how much communities change from layer to layer. By establishing an equivalence between modularity optimization and a multilayer maximum-likelihood approach to community detection, we are able to determine statistically optimal values for these two parameters.

We test our method on several existing multilayer benchmarks, and we find that our optimized approach performs significantly better than using parameter choices guided by heuristics. We also apply the method to supermarket data, revealing changes in consumer behaviour over time. (Received September 25, 2017)

1135-68-1860 **Cynthia Dwork\*** ([dwork@seas.harvard.edu](mailto:dwork@seas.harvard.edu)), Department of Computer Science, Harvard University, 33 Oxford Street, Cambridge, MA 02138. *Privacy in the Land of Plenty.*

Privacy-preserving data analysis has a large literature spanning several academic disciplines over more than half a century. Many early attempts have proved problematic in vivo or in vitro. “Differential privacy,” a notion tailored to situations in which data are plentiful, has provided a theoretically sound and powerful framework, given rise to an explosion of research, and has begun to see deployment on a global scale. We will review the definition of differential privacy, illustrate with some examples, and describe surprising applications to statistical validity under adaptive analysis and fairness in machine learning algorithms, settings in which privacy is not itself a concern. (Received September 25, 2017)

1135-68-1959 **Saket Navlakha\*** ([navlakha@salk.edu](mailto:navlakha@salk.edu)), 10010 North Torrey Pines RD, La Jolla, CA 92037, and **Charles F. Stevens** and **Sanjoy Dasgupta**. *A neural algorithm for similarity search.*

Similarity search, such as identifying similar images in a database or similar documents on the Web, is a fundamental computing problem faced by many large-scale information retrieval systems. We discovered that the fly’s olfactory circuit solves this problem using a novel variant of a traditional computer science algorithm (called locality-sensitive hashing). The fly’s circuit assigns similar neural activity patterns to similar input stimuli (odors), so that behaviors learned from one odor can be applied when a similar odor is experienced. The fly’s algorithm, however, uses three new computational ingredients that depart from traditional approaches. We show that these ingredients can be translated to improve the performance of similarity search compared to traditional algorithms when evaluated on several benchmark datasets. Overall, this perspective helps illuminate the logic supporting an important sensory function (olfaction), and it provides a conceptually new algorithm for solving a fundamental computational problem. (Received September 25, 2017)

1135-68-2032 **Nathan C Carter\*** ([ncarter@bentley.edu](mailto:ncarter@bentley.edu)) and **Kenneth G Monks**. *A Web-Based Toolkit for Mathematical Word Processing Applications with Semantics.*

Lurch is an open-source word processor that can check the steps in students’ mathematical proofs. Users write in a natural language, but mark portions of a document as meaningful, so the software can distinguish content for

human readers from content it should analyze. This talk reports on the Lurch Web Platform, a system of tools the authors have created as part of a project to upgrade Lurch from a desktop application to a web application. That system of tools is available on GitHub for other mathematical software developers to use in their own projects. It includes a web editor with mathematical typesetting, an interface for marking up documents with mathematical (or other structured) meaning, OpenMath support, meaning visualization tools, and document dependence and sharing features, among others. Time permitting, the talk will conclude with design plans for ongoing development of the web version of Lurch that will be built on the Lurch Web Platform. (Received September 25, 2017)

1135-68-2244 **Jamie Pommersheim\*** ([jamie@reed.edu](mailto:jamie@reed.edu)), 3203 SE Woodstock Blvd, Department of Mathematics, Reed College, Portland, OR 97202. *Quantum learning from symmetric oracles.*

Many of the existing algorithms in quantum computation can be fit into the framework of oracle problems, also known as concept learning problems. In this setup, an unknown oracle, represented by a unitary transformation, is chosen from known finite set of oracles. Using queries to this oracle, the learner wishes to determine which oracle was chosen, or at least some information about the chosen oracle. If the problem is symmetric, for example, if the oracles in question form a group, then much can be said about the optimal quantum queries that the learner should make. In this talk, we examine such oracle problems. (Received September 25, 2017)

1135-68-2297 **Erik Winfree\*** ([winfree@caltech.edu](mailto:winfree@caltech.edu)). *Algorithmic Self-Assembly of DNA.*

Self-assembly is a fundamental process in the self-organization of biological as well as non-biological structures. Passive self-assembly of molecular units, being driven just by thermodynamic binding energies and the geometrical structure of the molecules, would seem to be the simplest case to study – but it can be remarkably complicated. In fact, in a model of generalized crystal growth abstracted as the self-assembly of Wang tiles, passive self-assembly can be shown to be Turing universal. This leads to a number of theoretical observations: complex shapes that have concise algorithmic descriptions can be self-assembled from a small number of parts; these self-assembled structures can perform error correction during growth and can self-heal after damage; and as a simple form of self-replication, algorithmic crystals could provide an abiological example of Darwinian evolution. Further, many of these principles can be demonstrated experimentally using molecular Wang tiles constructed from DNA, thus establishing algorithmic self-assembly as part of the molecular programming toolbox for DNA nanotechnology. (Received September 25, 2017)

1135-68-2591 **L. Nate Veldt\***, [lveldt@purdue.edu](mailto:lveldt@purdue.edu), and **David Gleich** and **Anthony Wirth**. *Finding sparse cuts and dense clusters with correlation clustering.* Preliminary report.

We present LambdaCC: a novel framework for graph clustering based on a specially-weighted version of correlation clustering. Our framework unifies and generalizes a number of other well-studied partitioning objectives, including modularity, sparsest cut, and cluster deletion, all of which can be recovered by fixing a key clustering resolution parameter  $\lambda$  in our objective function. By varying  $\lambda$ , our framework effectively interpolates between two separate strategies in graph clustering: identifying sparse cuts and finding dense subgraphs in a network. Based on our theory we develop the first constant factor approximation algorithm for the cluster deletion objective. We also present several other algorithms for approximating the LambdaCC objective that are useful in a number of different application domains. (Received September 26, 2017)

1135-68-2608 **Daniel Copeland\*** ([drcoopela@ucsd.edu](mailto:drcoopela@ucsd.edu)) and **Jamie Pommersheim**. *Quantum query complexity of symmetric problems.* Preliminary report.

Using state preparation and measurement to identify an unknown unitary operation is a crucial step in many quantum algorithms. We measure the difficulty of identifying this operation by the number of queries used by an optimal algorithm. When the unknown unitary is sampled from a finite group of unitaries, one can give a character-theoretic description of the query complexity. We apply this to explain several well-known quantum algorithms (notably the van Dam algorithm, PARITY, and the Bernstein-Vazirani problem) and introduce a new family of tasks which involve identifying an unknown permutation. This provides a large class of nonabelian problems, which may or may not allow quantum advantages. For instance, we provide an example of a  $\Theta(1)$  vs  $\Theta(n)$  speedup in query complexity coming from a finite Heisenberg group, but we also show a quantum computer requires  $\Omega(n)$  queries to identify a random permutation given access to the defining representation of  $S_n$ . (Received September 26, 2017)

1135-68-2662 **David Doty\*** ([doty@ucdavis.edu](mailto:doty@ucdavis.edu)), One Shields Ave, University of California, Davis, CA 95616. *Hardness of computing and approximating predicates and functions with leaderless population protocols.*

Population protocols are a distributed computing model appropriate for describing massive numbers of agents with very limited computational power, such as programmable chemical reaction networks in synthetic biology. A population protocol is said to require a leader if every valid initial configuration contains a single agent in a special "leader" state that helps to coordinate the computation. Although the class of predicates and functions computable with probability 1 is the same whether a leader is required or not, it is not known whether a leader is necessary for fast computation.

We show that a wide class of functions and predicates computable by population protocols are not \*efficiently\* computable, nor are some linear functions even efficiently \*approximable\*. For example, the widely studied parity, majority, and equality predicates cannot be computed in sublinear time. It requires linear time for a population protocol even to approximate division by a constant or subtraction: for sufficiently small  $\gamma > 0$ , the output of a sublinear time protocol can stabilize outside the interval  $f(m) (1 \pm \gamma)$ . We show that it requires linear time to exactly compute many semilinear functions (e.g.,  $f(m) = m$  if  $m$  is even and  $2m$  if  $m$  is odd) and predicates (e.g., parity, equality). (Received September 26, 2017)

1135-68-2775 **Zak Webb\*** ([zakwebb@gmail.com](mailto:zakwebb@gmail.com)). *Quantum advantage via non-local games.*

Provable quantum advantages usually require some assumption on the computational model, either through an explicit computation assumption or else a restricted model of computation. Recently, Bravyi, Gosset, and König gave an explicit problem separating constant depth quantum and classical circuits without these assumptions.

In this work, we extend their results using non-local games and give a problem exactly solvable using a constant depth quantum circuit, but for which any  $k$ -bounded fan-in randomized circuit solving the problem with probability more than  $\exp(-\Theta(n^{1-\gamma}))$  is of depth at least  $\frac{\gamma}{2} \frac{\log n}{\log k}$ , where  $\gamma \in (0, 1)$  is any fixed constant and  $n$  is the problem input size. In other words, we construct a problem that is easily solved by a constant depth quantum circuit, but that no classical circuit can solve with non-negligible probability. Our proof proceeds by showing that any classical circuit solving our problem yields a classical strategy for the parallel-repeated Mermin magic-square game, which previous work has ruled impossible. (Received September 26, 2017)

1135-68-2777 **Daniel B Larremore\*** ([daniel.larremore@colorado.edu](mailto:daniel.larremore@colorado.edu)), **Leto Peel** and **Aaron Clauset**. *The ground truth about metadata and community detection in networks.*

It is common to evaluate the performance of community detection algorithms by their ability to find so-called ground truth communities. This works well in synthetic networks with planted communities because these networks' links are formed explicitly based on those known communities. However, there are no planted communities in real-world networks. Instead, it is standard practice to treat some observed discrete-valued node attributes, or metadata, as ground truth. We show that metadata are not the same as ground truth and that treating them as such induces severe theoretical and practical problems. We prove that no algorithm can uniquely solve community detection, and we prove a general No Free Lunch theorem for community detection, which implies that there can be no algorithm that is optimal for all possible community detection tasks. However, community detection remains a powerful tool and node metadata still have value, so a careful exploration of their relationship with network structure can yield insights of genuine worth. We illustrate this point by introducing two statistical techniques that can quantify the relationship between metadata and community structure for a broad class of models. (Received September 26, 2017)

1135-68-2846 **John R. Johnson\*** ([john.johnson@pnnl.gov](mailto:john.johnson@pnnl.gov)), 902 Battelle Boulevard, P.O. Box 999, MSIN K9, Richland, WA 99352, and **Emilie AH Purvine** ([emilie.purvine@pnnlgov](mailto:emilie.purvine@pnnlgov)), 1100 Dexter Ave N., Suite 400, BSRC, Seattle, WA 98109. *Graph Theory and the Mathematics of Cybersecurity*. Preliminary report.

Cybersecurity has its mathematical roots in formal methods for access control of single systems. However as cyber systems have become ubiquitous over the past few decades, and computers have become massively interconnected, the diversity of mathematical fields that are directly relevant to the study and practice of cybersecurity has rapidly increased. This talk will provide an overview of this mathematical evolution with an emphasis on the growing importance of graph applications (Received September 26, 2017)

1135-68-2855 **Sarah Cannon\*** ([sarah.cannon@gatech.edu](mailto:sarah.cannon@gatech.edu)). *Dyadic Tilings and Phase Transitions.*

We study the weighted edge-flip Markov chain for dyadic tilings, show it exhibits a phase transition, and give results about its behavior at the phase transition. A dyadic tiling of size  $n$  is a tiling of the unit square by  $n$  non-overlapping dyadic rectangles each of area  $1/n$ ; a dyadic rectangle can be written in the form  $[a2^{-s}, (a +$

$1)2^{-s}] \times [b2^{-t}, (b+1)2^{-t}]$  for  $a, b, s, t \in \mathbb{Z}_{\geq 0}$ . The edge-flip Markov chain selects a random edge of the tiling and replaces it with its perpendicular bisector if this yields a valid dyadic tiling. We consider a weighted version of this Markov chain where, given a parameter  $\lambda > 0$ , we generate each dyadic tiling  $\sigma$  with probability proportional to  $\lambda^{|\sigma|}$ , for  $|\sigma|$  the total edge length. We show there's a phase transition: when  $\lambda < 1$ , the edge-flip chain mixes in polynomial time, but if  $\lambda > 1$ , the mixing time is exponential. At the critical point  $\lambda = 1$ , we give the first polynomial mixing time upper bound. We complement this with lower bounds showing the behavior at the critical point is provably different than on either side of the critical point, supporting a broad statistical physics conjecture. Joint work with Sarah Miracle, David Levin, Dana Randall, and Alexandre Stauffer. (Received September 26, 2017)

1135-68-2912     **Amanda Harsy\*** ([harsyram@lewisu.edu](mailto:harsyram@lewisu.edu)) and **Piotr Szczurek** ([szczurpi@lewisu.edu](mailto:szczurpi@lewisu.edu)).  
*Designing and Running a Data Science Program at a Regional Liberal Arts University.*

In just over three years, the Data Science Program at Lewis University has become a popular choice for both graduate and undergraduate math and computer science majors. In this talk, we will share the details of the curriculum design of our graduate data science program, options in which students can participate in our programs (e.g. undergraduate concentrations, minors, fast-tracks, certificates in computational biology and bioinformatics, etc.), and opportunities like DataSail and Datathon which give students a chance to collaborate on research in data science and prepare them to work in this field. Lewis University is a four-year, private Catholic and Lasalian university located in Romeoville, Illinois, 35 miles southwest of Chicago. The enrollment is currently around 6,500 students. The Department of Computer and Mathematical Sciences includes Mathematics, Computer Science, and Computer Engineering and consists of seventeen full-time faculty members serving approximately 275 undergraduate majors and 220 graduate students (110 of which are in data science). (Received September 26, 2017)

1135-68-3158     **Yinshu Wu\*** ([yinshu.wu@aamu.edu](mailto:yinshu.wu@aamu.edu)), Yinshu Wu, madison, AL 35758. *The SIR Modeling for the Biological Flu using Artificial Neural Network.*

The biological flu is a disease spread by contact with infected individuals. Victims recover from the disease and gain further immunity from it. In the paper, the population is separated into different groups using the SIR model. Differential equations involve the susceptible, infected, and recovered groups. The model will use the past ten years biological information to estimate the groups of population for the next year by Artificial Neural Network. Machine learning is the general method for artificial neural networks which is based on the concept of self-adjustment of internal control parameters. (Received September 26, 2017)

1135-68-3224     **Avi Wigderson\*** ([avi@ias.edu](mailto:avi@ias.edu)). *Alternate Minimization and Scaling algorithms: theory, applications and connections across mathematics and computer science.*

This 3-lecture series will revolve around a common heuristic for general optimization problems called *alternate minimization*, and natural *scaling algorithms* which capture many of them. Recent attempts to formally analyze their performance in natural settings have uncovered a surprisingly rich web of connections between diverse areas of mathematics and computer science, all of which contribute and benefit from this interaction.

In this first lecture I will give the general set-up, and examples of problems for which these algorithms are relevant. I will then survey some of the different areas they touch, and how. In mathematics, these include non-commutative algebra, invariant theory, quantum information theory and analysis. In computer science they include optimization, algebraic complexity and pseudorandomness.

In the next two lectures I will survey aspects of two central problems to both math and CS, *Proving algebraic identities* and *Proving analytic inequalities*, influenced by the study above.

All three lectures are designed to be independent of each other. They require no special background knowledge. Lecture notes can be found at <http://www.math.ias.edu/~avi/PUBLICATIONS/CCC-17-tutorial-lecture-notes.pdf> (Received September 27, 2017)

1135-68-3231     **Avi Wigderson\*** ([avi@ias.edu](mailto:avi@ias.edu)). *Proving algebraic identities.*

In numerous mathematical settings, an object typically has several representations. The word (or isomorphism) problem asks: when are two given representations equivalent? Such problems have driven much structural and algorithmic research across mathematics.

We focus on the algebraic setting: our objects are polynomials and rational functions in many variables, represented by arithmetic formulae. Here the word problem is proving algebraic identities. I will describe the history, motivation and the status of this problem in two settings: when the variables commute, and when they do not.

For commuting variables, a probabilistic polynomial time algorithm was known, and a major open problem is to find a deterministic counterpart. To explain this we'll visit the VP versus VNP problem, permanents vs. determinants and more.

For non-commuting variables, I will describe a recent deterministic polynomial time algorithm based on the ideas of the first lecture, appealing to the theory of free skew fields and to degree bounds on invariant rings of linear group actions.

Finally, we'll see how the two settings are related!

This talk is self-contained, and requires no special background. The new material covered is taken mostly from <https://arxiv.org/abs/1511.03730> (Received September 27, 2017)

1135-68-3232 **Avi Wigderson\*** ([avi@ias.edu](mailto:avi@ias.edu)). *Proving analytic inequalities.*

The celebrated Brascamp-Lieb (BL) inequalities, and their reverse form of Barthe, is a powerful framework which unifies and generalizes many important inequalities in analysis, convex geometry and information theory.

I will exemplify BL inequalities, building to the general set-up. I will describe the structural theory that characterizes existence and optimality of these inequalities in terms of their description (called BL-data). But can one efficiently compute existence and optimality from given BL-data?

I will describe a recent polynomial time algorithm for these problems, based on a natural alternate minimization approach and operator scaling analysis discussed in the first lecture. It also supplies alternative proofs to some of the structural results.

This algorithm may be viewed (via the structural theory) in two ways that make it potentially exciting for new applications in optimization. First, it efficiently solves a large natural class of non-convex programs. Second, it efficiently solves a large natural class of linear programs with exponentially many inequalities.

This lecture is self-contained, independent of the previous two. No special background is assumed. Most of this presentation is based on the paper <https://arxiv.org/abs/1607.06711> (Received September 27, 2017)

## 70 ► *Mechanics of particles and systems*

1135-70-116 **Corey Shanbrom\*** ([corey.shanbrom@csus.edu](mailto:corey.shanbrom@csus.edu)), Department of Mathematics and Statistics, Sacramento State University, 6000 J St, Sacramento, CA 95819, and **Victor Dods** ([victor.dods@gmail.com](mailto:victor.dods@gmail.com)). *Numerical methods and closed orbits in the Kepler-Heisenberg problem.* Preliminary report.

The Kepler-Heisenberg problem is that of determining the motion of a planet around a sun in the Heisenberg group, thought of as a three-dimensional sub-Riemannian manifold. The sub-Riemannian Hamiltonian provides the kinetic energy, and the gravitational potential is given by the fundamental solution to the sub-Laplacian. This system is known to admit closed orbits, which all lie within a fundamental integrable subsystem.

Here, we introduce a computer program which finds these closed orbits. Using Monte Carlo optimization with a shooting method, and applying a recently developed symplectic integrator for nonseparable Hamiltonians, we find approximations to periodic orbits whose existence was known. Moreover, we find new orbits with previously unknown symmetry types, and we encode these symmetry types as rational numbers. These results provide explicit visual representations of partially integrable dynamics with non-compact invariant manifolds. Our codebase is free and open-source. (Received July 28, 2017)

1135-70-141 **Lennard F Bakker\*** ([bakker@mathematics.byu.edu](mailto:bakker@mathematics.byu.edu)) and **Mitchell Sailsbery** ([mitchell.sailsbery@gmail.com](mailto:mitchell.sailsbery@gmail.com)). *Topological Existence of Periodic Orbits in a Two-Center Symmetric Pair Problem.* Preliminary report.

The two-center symmetric pair problem is a perturbation of the integrable Euler two-center problem, with the mass of the symmetric pair being the perturbation parameter. The perturbed problem is equivalent to a collinear three-center problem with the mass of the middle center being the parameter. Standard KAM theory shows that many of the quasiperiodic orbits in the Euler two-center problem persist for small values of the mass of the symmetric pair. Numerical evidence suggests that some of these quasiperiodic orbits as well as some periodic orbits persist for all values of the mass of the symmetric pair. We prove the persistence of a so-called planetary type periodic orbit for all choices of the mass of the symmetric pair. We also investigate the non-integrability of the two-center symmetric pair problem. (Received August 02, 2017)



1135-70-861 **Connor Jackman\***, 1156 High Street, Santa Cruz, CA 95064. *The Jacobi-Maupertuis principle in the strong force N-body problem.*

The Jacobi-Maupertuis least action principle reformulates the dynamics of a natural mechanical system as a geodesic problem. In this talk we will consider the curvature properties of this metric associated to the strong force N-body problem and observe some dynamical consequences. (Received September 15, 2017)

1135-70-965 **Richard Montgomery\*** (rmont@ucsc.edu), Math Dept, UCSC, 1156 High Street, Santa Cruz, CA 95064. *Metric Geometry and Marchal's lemma.* Preliminary report.

Excluding collisions from action minimizers is the central problem to be faced in getting variational methods to work for N-body problems. In this regard Marchal's lemma and its generalizations is the most powerful tool. Here we relook at the problem of minimizers approaching collision from the perspective of the Jacobi-Maupertuis [JM] metric. Two salient items arise. (1) If the potential is homogeneous of degree  $-a$ ,  $0 < a < 2$ , then the natural metric completion of the JM Riemannian metric has cone-like singularities at collisions. (2) Geodesics ending in collision are inextendible if and only if Marchal's lemma holds for that potential. These observations allow us to construct potentials which are homogeneous of degree  $-1$  for which Marchal's lemma fails. Our work is related to work of Barutello-Terracini and Hu-Yu on anisotropic Kepler problems and the Morse indices of collision solutions. (Received September 18, 2017)

1135-70-1086 **Wentian Kuang, Tiancheng Ouyang and Zhifu Xie\*** (zhifu.xie@usm.edu), Department of Mathematics, The University of Southern Mississippi, 118 College Drive, #5045, Hattiesburg, MS 39406, and **Duokui Yan.** *The Broucke-Hénon orbit and the Schubart orbit in the planar three-body problem with equal masses.*

Variational Method with Structural Prescribed Boundary Conditions (SPBC) is applied to study periodic solutions in the 3-body problem with equal masses. We show that under an appropriate topological constraint, the action minimizer must be either the Schubart orbit or the Broucke-Hénon orbit. One of the main challenges is to prove that the Schubart orbit coincides with the action minimizer connecting a collinear configuration with a binary collision and an isosceles configuration which must be collinear. A geometric property of the action minimizer is introduced to overcome this challenge. The action minimizer without collisions can be extended to the Broucke-Hénon orbit. (Received September 19, 2017)

1135-70-1111 **Hector Sanchez\*** (hector@matem.unam.mx), **Richard Moeckel** and **Richard Montgomery.** *Free time minimizers for the planar three body problem.*

We prove that any solution to planar Newton's three-body problem which is asymptotic to a Lagrange's parabolic homothetic solution is eventually a free time minimizer. Conversely, we prove that every free time minimizer tends to Lagrange's solution, provided the mass ratios lie in a certain large open set of mass ratios. We exclude being asymptotic to Euler's central configurations by a second variation argument. Central configurations correspond to rest points for the McGehee blown-up dynamics. The large open set of mass ratios are those for which the linearized dynamics at each Euler rest point has a complex eigenvalue. (Received September 19, 2017)

1135-70-1197 **Samuel B Heroy\*** (sam.heroy@gmail.com), Dept of Mathematics, CB 3250, Phillips Hall, UNC Chapel Hill, NC 27599, and **Dane Taylor, F. Bill Shi, M. Gregory Forest** and **Peter J. Mucha.** *Rigidity Percolation in Composite Materials.*

In materials science, a challenge of engineering interest is to capture property enhancements using particle-scale models that both reflect the underlying randomness in particle distribution, and capture the inherent phase transitions in macroscopic behavior. Our particular interest is in using the graph theoretic property of rigidity to model mechanical reinforcement in composites with stiffening particles. We develop an efficient algorithmic approach called rigid graph compression (RGC) to capture rigidity in disordered fiber networks, which form the stiffening phase in many composite systems. To establish RGC on a firm theoretical foundation, we adapt rigidity matroid theory to identify primitive topological motifs as rules for composing interacting rigid particles into larger rigid components. Using numerical experiments, we demonstrate that RGC closely approximates the previously determined rigidity percolation threshold in 2D fiber networks ( $<1\%$  relative error when 3 motifs are used), and predict the rigidity percolation threshold in 3D fiber networks (where no such study has been undertaken). (Received September 20, 2017)

1135-70-1437 **Gareth E. Roberts\*** ([groberts@holycross.edu](mailto:groberts@holycross.edu)), College of the Holy Cross, 1 College Street, Worcester, MA 01610, and **Josep M. Cors** ([cors@epsem.upc.edu](mailto:cors@epsem.upc.edu)) and **Montserrat Corbera** ([montserrat.corbera@uvic.cat](mailto:montserrat.corbera@uvic.cat)). *Classifying Four-Body Convex Central Configurations*. Preliminary report.

A central configuration is a special arrangement of masses in the  $n$ -body problem where the gravitational force on each body points toward the center of mass. Central configurations lead to homothetic and homographic periodic solutions, and play a crucial role in understanding the topological structure of the integral manifolds. Here we strive to classify all four-body convex solutions (i.e., the bodies form a convex quadrilateral), with an eye toward configurations possessing some type of symmetry or special geometric property (e.g., co-circular, trapezoidal, kites, tangential, orthodiagonal, equidiagonal, bisecting diagonals). We find useful coordinates to describe the space and show that the set of all four-body convex central configurations is three-dimensional, a graph over three position variables. (Received September 22, 2017)

1135-70-1512 **Cristina Stoica\*** ([cstoica@wlu.ca](mailto:cstoica@wlu.ca)). *Remarks on the  $n$ -body problem on surfaces of revolution*.

We explore the dynamics of  $n$  mass points constrained to move on a surface of revolution and with mutual interaction given by some binary potential. We discuss symmetries and determine certain invariant manifolds. We show that the equivalent of Saari's conjecture fails. Further, we define homographic motions to be those for which the configuration formed by the bodies is planar, orthogonal to the axis of revolution and remains self-similar in the ambient space. Using the method of discrete reduction, we prove that when the masses are equal, homographic motions form an invariant manifold with dynamics reducible to a one-degree of freedom system. We then find that for generic attractive interactions, regular  $n$ -gon-shaped relative equilibria with trajectories located on geodesic circles experience a pitchfork bifurcation. (Received September 22, 2017)

1135-70-1626 **Martha Alvarez-Ramírez\*** ([mar@xanum.uam.mx](mailto:mar@xanum.uam.mx)), **Esther Barrabés**, **Mario Medina** and **Mercé Ollé**. *Ejection-Collision orbits in the symmetric collinear four-body problem*. Preliminary report.

We consider a collinear symmetric four-body problem, in which four masses  $m_2$ ,  $m_1$ ,  $m_1$  and  $m_2$  are in collinear configuration and ordered from left to right, and move symmetrically about the center of mass. The problem has been studied by different authors. Sweatman (2002) carried out a wide numerical exploration, showing different type of orbits: a Shubart-like periodic orbit, quasiperiodic orbits and scattering orbits. In 2004, Sekiguchi and Tanikawa explored, for the case of equal masses, the global geometry of the phase space via a suitable Poincaré map. Later, Alvarez-Ramírez et al. (2015) studied analytically the quadruple collision, as well as the infinity, by means of the McGeehe's techniques. We will show how the study of the invariant manifolds associated to the equilibrium points inside the collision manifold allow us to complete the previous works. In particular, we will focus on the orbits that tend forwards and backwards in time to the quadruple collision. (Received September 23, 2017)

1135-70-1853 **Holger R. Dullin\*** ([holger.dullin@sydney.edu.au](mailto:holger.dullin@sydney.edu.au)), University of Sydney, School of Mathematics and Statistics, Camperdown Campus, Sydney, NSW 2006, Australia. *The Birkhoff normal form at the Lagrange equilateral solution in the spatial 3-body problem*.

We compute the Birkhoff normal form up to including order 4 at Lagrange's equilateral solution. The coefficients of the normal form are expressed in terms of two symmetric functions of the normalised masses. We show how the normal form of the spatial problem reduces to that of the planar problem and to that of the restricted problem. The normal form is used to determine the co-dimension one curves in normalised mass space for which the iso-energetic non-degeneracy condition is violated, and thus find a few special isolated cases in normalised mass space for which the elliptic equilateral solution of Lagrange can be non-linearly unstable. (Received September 25, 2017)

1135-70-1885 **Ernesto Perez-Chavela\*** ([ernesto.perez@itam.mx](mailto:ernesto.perez@itam.mx)), Rio Hondo 1, Col. Progreso Tizapan, 01080 Mexico, D.F., Mexico. *Euler and Lagrange relative equilibria in the curved 3-body problem*. Preliminary report.

The curved  $N$ -body problem is a generalization of the classical Newtonian  $N$ -body problem to spaces with constant curvature  $K$ , in this talk we will consider the two dimensional case. Using the cotangent potential as a generalization of the Newtonian one, we extend the classical Euler and Lagrange relative equilibria of celestial mechanics to spaces of constant curvature and study the stability of them. (Received September 25, 2017)

1135-70-1887 **Abimael Bengochea\*** ([abimael.bengochea@itam.mx](mailto:abimael.bengochea@itam.mx)), Río Hondo 1, Álvaro Obregón, 04330 México, Mexico. *Exchange orbits in the problem of 1+2n bodies.*

The exchange orbits, aka horseshoe orbits, were studied for the first time by Brown, in the restricted three-body problem. In this talk we present symmetric periodic exchange orbits in the 1+2n body problem. We compute the corresponding families by solving a boundary value problem, and discuss common features of the families for some values of n. (Received September 25, 2017)

1135-70-2364 **Danya Rose\*** ([danya.rose@sydney.edu.au](mailto:danya.rose@sydney.edu.au)) and **Holger R Dullin** ([holger.dullin@sydney.edu.au](mailto:holger.dullin@sydney.edu.au)). *No absolute partial choreographies of the planar 3-body problem with vanishing angular momentum.* Preliminary report.

The geometric phase of a relative periodic orbit of the planar 3-body problem can be computed from changes in its shape over one period. A periodic orbit is called *absolutely periodic* if its geometric phase vanishes, and it is thus periodic in an inertial frame of reference. A partial choreography of the 3-body problem is also known as a 2-1 choreography, in which two of the bodies trace over the same path in one period, while the third body traces over a separate path twice. That is, a partial choreography requires symmetry under the interchange of two equal masses at every half-period, though this symmetry does not guarantee partial choreography.

We prove a theorem establishing a general symmetry condition for the geometric phase of a relative periodic orbit to vanish when the angular momentum is zero, and use this result to show that if the geometric phase vanishes *and* the orbit has partial choreographic symmetry, it must be self-retracing (in the sense that every particle retraces the same path in both directions), and can only appear as a partial choreography in an appropriately chosen rotating frame. (Received September 26, 2017)

1135-70-3005 **Kimberly E. Stubbs\*** ([stubbske@g.cofc.edu](mailto:stubbske@g.cofc.edu)), 196 Curtis Creek Rd, Candler, NC 28715, and **Samuel R. Kaplan** ([skaplan@unca.edu](mailto:skaplan@unca.edu)), 313 Robinson Hall, Asheville, NC 28804. *Dynamics of Co-Orbital Moons Near Collision.* Preliminary report.

This project is about celestial mechanics and dynamical systems. Specifically, the goal is to explore the techniques used in modern celestial mechanics to analyze near-collision dynamics and chaos. The model we're working with is a 3-body co-orbital system. Josep Cors and Glen Hall wrote a paper on 3-body co-orbital systems and determined when the moons will pass each other and/or change orbits. They were only interested in these two occurrences, and so they left out the dynamics of near-collision. We're interested in finding out what happens near collision of the two moons and have done the necessary change of variables to allow analysis of the dynamics and chaos. We'll look into the dynamics and what they mean for the entire system. (Received September 26, 2017)

1135-70-3084 **Fariba Khoshnasib\*** ([khoshnaf@uwec.edu](mailto:khoshnaf@uwec.edu)), 30723 Beechwood St, Apt 46206, Wixom, MI 48393. *Bifurcations of Liouville Tori for Goryachev-Chaplygin system Vs. Poincare system.*

This is a research built upon special, integrable cases of rigid body motion. The differential equations of motion of such systems are Integrable Hamiltonian ones. The Goryachev-Chaplygin case is the focus of our interest. This system is completely integrable and the common level set of the first integrals H and F is a smooth manifold that is invariant under the phase flow of the system. Also, we know every connected component of the common level set (symplectic leaf) is diffeomorphic to a two-dimensional torus. The bifurcation diagram of such completely integrable system is the region of possible motion on the plane of first integrals together with the image of the critical set. In the case of Goryachev-Chaplygin top, one of the connected components on the integral manifold is an ordinary Liouville torus and lacks any critical points. To overcome this issue, using Andoyer variables, the invariant tori are constructed and singular leaves stability behavior are analyzed and compared to the traditional stability analysis using a parameter. Poincare's system models motion of a rigid body filled with viscous fluid and bifurcations of Liouville tori are constructed for this case. This model is a good approximation of motion of the earth. (Received September 26, 2017)

1135-70-3165 **Jacky J. Chong\***, 4176 Campus Drive, College Park, MD 20742-4015. *Dynamics of Large Boson Systems with Attractive Interaction and a Derivation of the Cubic Focusing NLS in  $\mathbb{R}^3$ .*

We consider a  $N$ -particle Boson system with two-body interaction  $N^{3\beta-1}v(N^\beta x)$  where  $v \in C_0^\infty$  for some range of  $\beta$ . We extend the results of Grillakis et al. in *Comm. Math. Phys.*, (2013) and Kuz in arXiv:1511.00487 regarding second-order correction to mean-field evolution of systems with repulsive interaction to the case of attractive interaction for  $0 < \beta < \frac{1}{2}$ . The two key ingredients used to extend to this case of attractive interaction are the proofs of the uniform global well-posedness of solutions to a family of Hartree-type equations and the

corresponding  $L^\infty$ -decay estimates on the solutions. Inspired by the recent works Pickl *J. Stat. Phys.* (2010), Chen et al. in *Theo. and Math. Phys.*, (2013) and Chen et al. in *Arch. Ration. Mech. Anal.* (2016), we also provide both a derivation of the focusing cubic nonlinear Schrödinger equation (NLS) in 3D from the many-body Boson system and its rate of convergence toward mean field. (Received September 26, 2017)

## 74 ► *Mechanics of deformable solids*

1135-74-414 **Eyad Said\*** (esaid1@lsu.edu), Baton Rouge, LA 70803, **Robert Lipton** (lipton@math.lsu.edu), Baton Rouge, LA 70803, and **Prashant Jha** (jha@math.lsu.edu), Baton Rouge, LA 70803. *Free damage propagation with memory.*

We introduce a simple model for free damage propagation based on non-local potentials. Constraints on material compressibility are encoded as well as damage history. We work within the small deformation setting and the model is developed using a state based peridynamic formulation. The resulting evolution is shown to be well posed. At each instant of the evolution we identify the damage set. On this set the local strain has exceeded critical values either for tensile or hydrostatic strain and damage has occurred. For this model the damage set is nondecreasing with time and associated with damage variables defined at each point in the body. We show that energy balance holds for this evolution. Here the space - time volume of the damaging set goes to zero at a rate that is linear with the length scale of non-local interaction. For differentiable displacements away from the damage set we show that the nonlocal model converges to the linear elastic model. (Received September 01, 2017)

1135-74-2216 **Eleni Panagiotou\*** (panagiotou@math.ucsb.edu), Department of Mathematics, University of California, Santa Barbara, Santa Barbara, CA 93106, and **Ken Millett** and **Paul Atzberger**. *Topological Approaches for Characterizing in Polymeric Materials the Local and Global Entanglement of Polymer Chains Relevant to Viscoelastic Mechanical Responses.*

We develop topological methods for investigating the relationship between polymer entanglement and bulk viscoelastic responses in polymeric materials. We demonstrate our approach by performing three dimensional computational simulations for polymer weaves of different topologies and densities. We perform rheological studies by shearing the material over a broad range of frequencies to establish entanglement-mechanics relationships. We consider regimes ranging from loosely entangled to strongly entangled in the collective polymer chain topology. Our results show that our measures of entanglement can classify the systems according to complexity in a way that reflects their mechanical response. (Received September 25, 2017)

1135-74-3103 **Mallikarjunaiah S Muddamallappa\***, 100 Institute Road, Worcester, MA 01545, and **Christopher J Larsen** and **Marcus Sarkis**. *A phase-field description of dynamic brittle fracture model: a single anti-plane shear crack.*

In this talk, we describe an efficient finite element treatment of a variational, time-discrete model for dynamic brittle fracture. We start by providing an overview of an existing dynamic fracture model that stems from Griffith's theory and based on the Ambrosio-Tortorelli crack regularization. We propose an efficient numerical scheme based on the bilinear finite elements. For the temporal discretization of the equations of motion, we use generalized  $\alpha$ -time integration algorithm, which is implicit and unconditionally stable. To accommodate the crack irreversibility, we use a primal-dual active set strategy, which can be identified as a semi-smooth Newton's method. It is well known that to resolve the crack-path accurately, the mesh near the crack needs to be very fine, so it is common to use adaptive meshes. We propose a simple, robust, local mesh-refinement criterion to reduce the computational cost. We show that the phase-field based variational approach and adaptive finite-elements provides an efficient procedure for simulating the complex crack propagation such as crack-branching. (Received September 26, 2017)

## 76 ► *Fluid mechanics*

1135-76-231 **Zhenlu Cui\*** (zcui@uncfsu.edu), 1200 Murchison Rd., Fayetteville, NC 28301. *Confined Flowing Bacterial Suspensions.*

A Leslie-Erickson hydrodynamic model is used to study bacterial suspensions in shear and Poiseuille flows. We focus on the role of the director anchoring boundary conditions at the plates and examine oblique anchoring boundary conditions. We establish the asymptotic formulas of the steady boundary-value problem and establish

remarkable dualities. We find that the director anchoring boundary condition plays a key role in the change of hydrodynamics of and stability of the system. (Received August 14, 2017)

1135-76-251 **Shigeru Masuda\*** ([hj9s-msd@asahi-net.or.jp](mailto:hj9s-msd@asahi-net.or.jp)), Room #305, Heights Esperanza, 100-24, Nishida-Chou, Joudoji, Sakyo-Ku, Kyoto, 606-8417, Japan. *Poisson's hydrodynamics and hydrostatics.*

We discuss Poisson's hydrodynamics and hydrostatics in "A Study of Mathematical Physics." Poisson proposes his academic paradigm of mathematical physics.

Poisson issues previously, the original of the Navier-Stokes equations in the paper 1829, which he doesn't include in "A study ." That is as follows :

$$\begin{cases} \rho(X - \frac{d^2x}{dt^2}) = \frac{d\varpi}{dx} + \beta(\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}), \\ \rho(Y - \frac{d^2y}{dt^2}) = \frac{d\varpi}{dy} + \beta(\frac{d^2v}{dx^2} + \frac{d^2v}{dy^2} + \frac{d^2v}{dz^2}), \\ \rho(Z - \frac{d^2z}{dt^2}) = \frac{d\varpi}{dz} + \beta(\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} + \frac{d^2w}{dz^2}), \end{cases}$$

where,  $\varpi = p + \frac{\alpha}{3}(K + k)(\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz})$ .

We follow two topics.

- Conformation of the original of the Navier-Stokes equations. We discuss the original by Navier, Poisson and Stokes.

- Conjecture of defective proofs on the exact differential. Poisson conjects the defective demonstration of exact differential on the eternity in time-space, which Lagrange 1781's and Cauchy's 1815 propose. After Poisson's death, Stokes proposes his new demonstration in 1849. (Received August 22, 2017)

1135-76-348 **Rab Nawaz\*** ([rabnawaz@comsats.edu.pk](mailto:rabnawaz@comsats.edu.pk)), Department of Mathematics, CIIT, COMSATS IIT, Islamabad, Pakistan, and **Muhammad Afzal** ([mafzalqau@gmail.com](mailto:mafzalqau@gmail.com)), Capital University of Science and Technology, Department of Mathematics, Islamabad, Pakistan. *The wave scattering analysis of flexible trifurcated waveguide using Mode-Matching approach.*

The propagation of waves in a trifurcated waveguide with discrete analogues of hard and flexible outer boundaries is considered. While formulating the related boundary value problem in terms of eigen function expansions in different duct regions, the scattering orthogonal and non-orthogonal duct modes are matched using continuity of pressures and velocities across the interface. In this way, the general expressions for the complex amplitudes in all regions are determined in terms of infinite linear algebraic equations. The obtained solution is justified by providing number of analytical and numerical tests, in particular, the relative powers versus frequency regime are displayed graphically. It is worthwhile to mention that the underlying problem provides a standard procedure to model and solve a range of bifurcated and trifurcated waveguide problems with multiple bounding properties of dynamic types. (Received August 27, 2017)

1135-76-356 **Sergei A Fomin\*** ([sfomin@csuchico.edu](mailto:sfomin@csuchico.edu)), Department of Mathematics and Statistics, California State University, Chico, Chico, CA 95929, **Vladimir A Chugunov**, Moscow City University, Moscow, Russia, **William Noland**, Department of Mathematics, Pennsylvania State University, University Park, PA 16802, and **Marcus Battraw**, Department of Mathematics and Statistics, California State University, Chico, Chico, CA 95929. *Mathematical modeling of the tsunami run-up on a beach with complex seafloor geometry.*

The quasi-linear theory of tsunami run-up on beach profiles of varying complexity is proposed. We begin with the nonlinear shallow-water wave equations, which we consider over a piecewise-linear beach of various geometries. We replace the moving boundary associated with the shoreline motion by a stationary boundary applying a transformation to the spatial variable of our computational domain. A characteristic feature of any tsunami problem is the smallness of the parameter, which represents the ratio of the characteristic amplitude of the wave and the characteristic depth of the ocean. The presence of this small parameter enables us to use the method of perturbations, which leads to an analytical solution via integral transformations. For this solution we introduce the wave no-breaking criterion and determine bounds for the applicability of our theory. Our solution is then shown to predict the shoreline motion analytically with good accuracy. Finally, we can repeat our process to investigate run-up and draw-down for different sea bottom profiles, and test the accuracy of our approximate analytic solution over several beach geometries against exact numerical solutions from a fully nonlinear model. (Received August 27, 2017)

1135-76-487 **Guoyi Ke\*** ([guoyi.ke@ttu.edu](mailto:guoyi.ke@ttu.edu)), Texas Tech University, Mathematics and Statistics Dept., Lubbock, TX 79409-1042, and **Eugenio Aulisa** ([eugenio.aulisa@ttu.edu](mailto:eugenio.aulisa@ttu.edu)), Texas Tech University, Mathematics and Statistics Dept., Lubbock, TX 79409-1042. *New preconditioner techniques for the buoyancy driven flow problems.*

In this work, we study the performances of generalized minimal residual method (GMRES) preconditioned with geometric multigrid (GMG), applied to steady and unsteady buoyancy driven flow problems, discretized with the finite element method. For the unsteady case, the second order Crank-Nicolson method is used for the temporal discretization. At each geometric multigrid level, we use Richardson iterative solvers preconditioned with different combinations of physics-based and domain decomposition preconditioners. Three different preconditioners are considered: incomplete LU decomposition (ILU), overlapping Vanka-type domain decomposition for additive Schwarz method (ASM), and field split (FS) physics-based decomposition. We also analyze the effect on the smoother of how the variables are ordered, and in particular whether the leading variable is the velocity or the temperature, resulting in six classes of preconditioners: ILU.VT, ILU.TV, ASM.VT, ASM.TV, FS.VT and FS.TV. Numerical results show that the pair of FS.VT and FS.TV preconditioners works better than the other two pairs, and that the FS.TV preconditioner always performs the best in terms of the computational time for all the steady and unsteady cases. (Received September 05, 2017)

1135-76-882 **Hangjie Ji\***, Department of Mathematics, University of California, Los Angeles, Los Angeles, CA 90095, and **Thomas P Witelski**. *Instability and dynamics of volatile thin films.*

Volatile viscous fluids on partially-wetting solid substrates can exhibit interesting interfacial instabilities and pattern formation. We study the dynamics of vapor condensation and fluid evaporation governed by a one-sided model in a low Reynolds number lubrication approximation incorporating surface tension, intermolecular effects and evaporative fluxes. Parameter ranges for evaporation-dominated and condensation-dominated regimes and a critical case are identified. Interfacial instabilities driven by the competition between the disjoining pressure and evaporative effects are studied via linear stability analysis. Transient pattern formation in nearly-flat evolving films in the critical case is investigated. In the weak evaporation limit unstable modes of finite amplitude non-uniform steady states lead to rich droplet dynamics, including flattening, symmetry breaking, and droplet merging. Numerical simulations show long time behaviors leading to evaporation or condensation are sensitive to transitions between film-wise and drop-wise dynamics. (Received September 16, 2017)

1135-76-915 **Diane M. Taylor\***, [dtaylor2@scu.edu](mailto:dtaylor2@scu.edu), and **Fotini K. Chow**. *Advection diffusion, atmospheric modeling, and methane transport.*

Landfills are a significant contributor to atmospheric methane, a potent greenhouse gas. An important problem is to quantify landfill methane emissions, but the size and heterogeneous nature of landfills presents difficulties. This talk discusses the use of the tracer dilution method (TDM), as a cost effective method to estimate the emissions. This method involves releasing a tracer gas and then measuring both methane and the tracer gas at a location downwind. This research focuses on the use of atmospheric dispersion modeling to simulate the plumes of methane and tracer gas at a real landfill, and examine how this modeling can be used to assess the TDM's accuracy. (Received September 16, 2017)

1135-76-1275 **Christian Zillinger\***, USC Dornsife Department of Mathematics, 3620 S. Vermont Ave, Los Angeles, CA 90007, and **Michele Coti Zelati**. *On degenerate circular and shear flows: the point vortex and power law circular flows.*

We consider the problem of asymptotic stability and linear inviscid damping for perturbations of a point vortex and similar degenerate circular flows. Here, key challenges include the lack of strict monotonicity and the necessity of working in weighted Sobolev spaces whose weights degenerate as the radius tends to zero or infinity. (Received September 20, 2017)

1135-76-1478 **Dambaru Bhatta\*** ([dambaru.bhatta@utrgv.edu](mailto:dambaru.bhatta@utrgv.edu)) and **Daniel N Riahi**. *Weakly nonlinear convective flow in an aquifer with varying permeability and thermal conductivity in the vertical direction.* Preliminary report.

Here the problem of weakly nonlinear convective flow in a horizontal aquifer layer with horizontal isothermal and rigid boundaries is investigated. Such a layer can be treated as a porous layer, where Darcy's law holds, subjected to the conditions that the porous layer's permeability and the thermal conductivity are variable in the vertical direction. In this layer. In addition, this study is restricted to the case that the subsequent hydraulic resistivity and diffusivity have small rate of change with respect to the vertical variable. Assuming a motionless and vertically variable basic state, weakly nonlinear approach is applied to calculate the solutions

for convective flow quantities such as vertical velocity and the temperature that arise as the Rayleigh number exceeds its critical value. Numerical results for those flow quantities for two-dimensional case are presented. (Received September 22, 2017)

1135-76-1479 **Aseel Farhat\*** (af7py@virginia.edu) and **Zoran Grujić**. *Enstrophy cascade and helical nature of turbulence.*

Direct numerical simulations showed that there is a high probability that velocity and vorticity vectors of the flow are nearly aligned in regions of high vorticity. We will show that that 3D NSE are regular under some appropriate local condition on the helicity in the the regions of intense vorticity. Moreover, we will present a rigorous proof of forward cascade of enstrophy, from large scales to small scales in the flow, under the above mentioned local assumption on helicity. (Received September 24, 2017)

1135-76-1602 **Christopher Curtis, Robert Inley, Eunji Yoo and Hannah Barta\*** (hannah941718@gmail.com), 801 E 6th Ave Apt 18, Ellensburg, WA 98926, and **Phoebe Coy, Thomas Pecha, Thomas Retzloff and Alec Todd**. *Study of Vortex Dynamics With Free Surface*. Preliminary report.

We examine the deformation of varying shapes of collections of point vortices beneath a free surface of an inviscid fluid with a flat bottom boundary. Initial simulations investigate the deformation of vortex sheets in a manner which resembles the Kelvin-Helmholtz instability. With large numbers of point vortices, the simulations we run show that these sheets tend to deform into elliptical patches. Several other shapes of point vortex arrays are simulated, and almost all deform into an elliptical shape. Upon deciding to simulate an ellipse as a starting shape, we observe that an initial elliptical shape deforms the least with time. To increase the realism of these simulations, we introduce a mollified kernel, which slows the speeds of vortices that are close together and yields ellipses that retain their shape with time. We introduce a metric to measure the deformation of these ellipses before and after mollification. (Received September 23, 2017)

1135-76-1610 **Saad Qadeer\*** (saad.qadeer@berkeley.edu) and **Jon Wilkening**. *Simulating Faraday Waves in a Cylinder.*

We numerically demonstrate the Faraday wave phenomenon on a cylinder for an incompressible and irrotational fluid. We solve the free surface evolution equations in time using a high order Spectral Deferred Corrections method. Meanwhile, we adapt the Transformed Field Expansion technique to a cylindrical geometry and couple it with a spectral method to rapidly compute the non-local Dirichlet-to-Neumann operator with high accuracy. The results validate the theoretical results established previously with further avenues open to attack. (Received September 23, 2017)

1135-76-1846 **Nasir Ali\*** (nasir.ali@iiu.edu.pk), Department of Mathematics & Statistics, International Islamic University, Sector H-10, Islamabad, 44000, Pakistan, and **Zeeshan Asghar, Muhammad Sajid and Fazal Abbas**. *A numerical study of bacterial gliding over a non-Newtonian slime.*

This work presents a fluid mechanical model and associated numerical analysis for slime-based gliding motility in a bacterium cell which do not has organelles of motility. The gliding motility is assumed to arise from a traveling waves generated on the surface of the cell, which pushes on the viscous slime and give rise to propulsion. The model explores the effects of the viscoelasticity of the slime on cell speed and power consumed by the cell. The flow patterns of slime during gliding are also shown. (Received September 25, 2017)

1135-76-1908 **Jeffrey K Landgren\*** (jeffrey.landgren@ung.edu) and **Gerhard Strohmmer** (gerhard-strohmmer@uiowa.edu). *An Acoustic Eigenvalue Problem and Its Application to Electrochemistry.*

Recent experiments in the field of Electrochemistry demonstrate that sound waves act as a catalyst for chemical reactions. A model is developed using conservation of momentum and mass, a boundary motion equation, and a surface tension equation. Chemically, it is clear that the catalytic phenomenon is derived from the sound waves and how they are affected by the top boundary. When combining all four equations we arrive at a boundary condition involving the top boundary only. Throughout the problem a self-adjoint invertible operator derived in the top (Neumann) boundary condition is established. Then a discussion ensues regarding regularity and formalizing all other boundary and initial conditions. These conditions will then be applied to the wave equation for later use in the model. The specific chemical reactions where this phenomenon is observed can be found in batteries, capacitors, and solar cells. The reaction takes place at an interface or boundary in each device. Making these devices more efficient can help decrease our negative impact on the environment. (Received September 25, 2017)

1135-76-2045 **Dallas Albritton\***, [alabri050@umn.edu](mailto:alabri050@umn.edu), and **Tobias Barker**. *Global weak solutions of the Navier-Stokes equations with initial values in critical spaces.*

We develop a theory of global weak solutions of the Navier-Stokes equations evolving from initial data in the critical Besov spaces  $\dot{B}_{p,\infty}^{-1+3/p}$  with  $p \in ]3, \infty[$ . Our solutions satisfy certain continuity properties with respect to the weak-\* convergence of initial data. These properties allow us to obtain rather general results concerning blow-up criteria, minimal blow-up initial data, and forward self-similar solutions near the borderline space  $BMO^{-1}$ . (Received September 25, 2017)

1135-76-2188 **William Byrd\*** ([billbyrd320@gmail.com](mailto:billbyrd320@gmail.com)) and **Jose E. Castillo**. *Mimetic Curvilinear Coastal Ocean Modeling.*

Mimetic discretization methods have been applied successfully to different areas of geoscience. We present a curvilinear coastal model constructed using MOLE, Mimetic Operators Library Enhanced, as a building block and PETSc for parallelization. Examples for validation are presented.

(Received September 25, 2017)

1135-76-2195 **Manuel A Valera\*** ([mvalera@mail.sdsu.edu](mailto:mvalera@mail.sdsu.edu)), 5500 Campanile Drive, San Diego, CA 92182-1245, **Mariangel Garcia**, San Diego, CA 92182-1245, and **Paul Choboter, Ryan Walter** and **Jose Castillo**. *Modeling nearshore internal bores and waves in Monterey Bay using the General Curvilinear Coastal Ocean Model (GCCOM).*

Internal waves are ubiquitous features in the coastal ocean. The breaking and eventual transformation into higher-frequency waves and bores, play a significant role in the energy mixing across scales in the coastal environment. The numerical modeling of these waves is especially difficult due to the non-hydrostaticity, nonlinearity and large range of scales involved. Additionally, the interaction of eddies with critically sloped bathymetry is crucial in the simulation of dominant coastal dynamics features. Using finer resolution preserves more structures on the terrain but increases the slope interaction, giving rise to nonlinearity in the form of turbulent flows. In this work a 3D case study for the Monterey Bay is validated to demonstrate the capabilities of the GCCOM model for simulating field-scale experiments including nonhydrostatic, stratified flows, internal bores formations and strongly nonlinear wave processes of generation and propagation, as well as the capabilities of handling abrupt bathymetry through the implementation of a fully 3D curvilinear mesh. An speedup analysis is also presented, based in the recent inclusion of the PETSc libraries to parallelize the model, evaluating the efficiency of the code using up to 32 processors. (Received September 25, 2017)

1135-76-2275 **Alexander O. Korotkevich\*** ([alexkor@math.unm.edu](mailto:alexkor@math.unm.edu)), Department of Mathematics and Statistics, MSC01 1115, 1 University of New Mexico, Albuquerque, NM 87131-0001. *Variation of inverse cascade spectrum for gravity waves due to condensate.*

During most of numerical experiments in wave turbulence of gravity waves we operate on a discrete wavenumbers grid. As a result, if we consider formation of inverse cascade, propagation of action flux to the small wavenumbers is arrested at some scale due to inefficiency of resonant four-waves interactions. It results in formation of strong long wave background, which we call condensate using analogy with Bose-Einstein condensation in Statistical Physics. As it is shown in a long numerical experiment, inverse cascade spectrum in the presence of such a condensate has a different power than predicted by the Theory of Wave Turbulence. We propose some preliminary explanation to this interesting phenomenon. (Received September 25, 2017)

1135-76-2367 **Nicholas D Brubaker\*** ([nbrubaker@fullerton.edu](mailto:nbrubaker@fullerton.edu)), 800 N. State College Blvd, Fullerton, CA 92831. *A Continuation Method for Computing Capillary Surfaces.*

The most common approach for computing the shape of a capillary surface is to first triangulate the given interface and then construct an optimization problem to determine locations of the vertices; however, since the resulting problem is often solved directly, only energetically stable interfaces can be determined. Additionally, these methods suffer in applications since they cannot be easily adapted to find families of surface when parameters, such as hydrostatic pressure difference or prescribed volume, are smoothly varied.

In this talk, we layout a new method for computing capillary surfaces based on solving a quasilinear, elliptic partial differential equation—facilitated by writing the unknown surface as a normal graph—using numerical arc-length continuation. The corresponding algorithm not only naturally produces a continuous family of surfaces, but also identifies both changes in stability and bifurcation points. Test examples will be presented to highlight the efficacy, accuracy and robustness of the proposed approach. (Received September 26, 2017)



1135-76-2477 **Yan Peng\*** (ypeng@odu.edu), 4700 Elkhorn Ave., Norfolk, VA 23693, and **Charles Armstrong**. *Study of Bioparticle Transport Using Lattice Boltzmann Method and Immersed Boundary Method*. Preliminary report.

This talk will present a computational framework to simulate bioparticle transport involving complex fluid-structure interactions. The lattice Boltzmann equation (LBE) is used to model the fluid motion because of its accuracy (low dissipation/low dispersion and better isotropy) and computational advantages including its excellent parallel scalability, absence of the need to solve a time consuming elliptic Poisson-type equation for the pressure field, and ease of representation of complex boundaries on Cartesian grids. The immersed boundary method (IBM) is chosen to track the deformable moving boundaries for its ease of implementation without re-meshing to generate the body-fitted mesh. The full approximation storage (FAS) non-linear multigrid method for steady and unsteady state problems has been proposed to improve the computational efficiency and keep the original characteristics of the LBE. One of the main motivations for this study comes from the Lab-on-a-chip (LoC) application, important for bio-medical, pharmaceutical, and environmental industries. Some numerical results of flow-induced deformation of a red blood cell from shear flow are presented. One or more orders of magnitude increase in solution efficiency using multigrid method is demonstrated. (Received September 26, 2017)

1135-76-2495 **Francesca Bernardi\*** (bernardi@live.unc.edu), **Manuchehr Aminian** (manuchehr.aminian@colostate.edu), **Roberto Camassa** (camassa@amath.unc.edu), **Daniel M. Harris** (daniel\_harris3@brown.edu) and **Richard M. McLaughlin** (rmm@email.unc.edu). *Fine structure symmetry-breaking in decaying passive scalars advected by laminar shear flow*. Preliminary report.

We investigate the dispersion of a passive scalar in laminar shear flow through rectangular and elliptical channels. We show through simulation, analysis and experiments that the cross-sectional aspect ratio sets the sign of the average skewness at long times, which describes the longitudinal asymmetry of the tracer distribution. We then extend the results to study the entire tracer distribution rather than only its longitudinal moments. With an analytical approach, we show that it is possible to describe the behavior of the tracer distribution at long time at any location in the cross-section, in turn highlighting the mechanism by which symmetry is broken. Future directions will be discussed. (Received September 26, 2017)

1135-76-2859 **Zeliha Kilic\*** (zkilic@live.unc.edu), UNC at Chapel Hill, Department of Mathematics, Chapel Hill, NC, and **Richard M McLaughlin** (rmm@email.unc.edu) and **Roberto Camassa** (camassa@amath.unc.edu). *Symmetry Breaking in a Random Passive Scalar*.

We consider the evolution of a decaying passive scalar in the presence of a gaussian white noise fluctuating shear flow. We focus on deterministic initial data and establish the short, intermediate, and long time symmetry properties of the evolving point wise probability measure for the random passive scalar. Analytical results are compared directly to Monte Carlo simulations (Received September 26, 2017)

1135-76-2969 **Andrew Thomack\*** (athomack2013@fau.edu), **Jason Mireles-James** and **Erik Lundberg**. *Validated Numerical Analysis of the Hele-Shaw free boundary problem*.

We consider the classical Hele-Shaw free boundary problem of tracking the shape of a two-dimensional region of viscous fluid driven by a source or sink. When surface tension is neglected, the problem reduces, as a consequence of Richardson's Theorem, to solving an inverse moment problem. The problem is further simplified if the conformal map from the unit disk to the initial domain is assumed to take a simplified form, say, a polynomial. The explicit solution to certain low-degree instances are well-known. For more complicated initial domains, we propose an approach using validated numerical analysis, and we discuss how the integrable structure of the problem makes this possible. (Received September 26, 2017)

1135-76-3076 **Katie L Oliveras\*** (oliveras@seattleu.edu), Mathematics Department, Seattle University, 912 12th Ave, Seattle, WA 98122. *Relationships between pressure, bathymetry, and wave-height*.

A new method is proposed to relate the pressure at the bottom of a fluid, the shape of the bathymetry, and the surface elevation of a wave for steady flow or traveling waves. Given a measurement of any one of these physical quantities (pressure, bathymetry, or surface elevation), a numerical representation of the other two quantities is obtained via a nonlocal nonlinear equation obtained from the Euler formulation of the water-wave problem without approximation. From this new equation, a variety of different asymptotic formulas are derived. The nonlocal equation and the asymptotic formulas are compared with both numerical data and physical experiments. This is joint work with Vishal Vasan and Daniel Ferguson. (Received September 26, 2017)

1135-76-3124 **Stefan G. Llewellyn Smith\*** (sgls@ucsd.edu), MAE, UCSD, 9500 Gilman Drive, La Jolla, CA 92093-0411. *Generalizing point vortices.*

Kirchhoff's equations of motion for point vortices are a paradigm of reduction of an infinite-dimensional dynamical system, namely the incompressible Euler equations, to a finite-dimensional system. Yet the original incompressible Euler equations neglect physical phenomena that may be important, for example compressibility, density differences and other wave fields such as those caused by background vorticity gradients. In addition, one can also examine other generalizations of the point vortex singularity, such as higher singularities or the effect of different desingularizations of the point vortex system. The history of point vortices and a number of these extensions, in particular hollow vortices and Sadvskii vortices, are discussed. Some related mathematical problems are mentioned. (Received September 26, 2017)

1135-76-3134 **Wooyoung Choi\*** (wchoi@njit.edu), Dept. of Mathematical Sciences, University Heights, Newark, NJ 07102. *Spectral formulation for nonlinear ocean waves and reduced models.*

The spectral formulation of Zakharov (1968) for nonlinear ocean waves is revisited to explore its relationship with a pseudo-spectral formulation based on unsteady Stokes expansion. Hamiltonian structures under high-order nonlinear approximations are discussed and further reductions are made to describe the evolution long-crested waves and resonant four-wave interactions. (Received September 26, 2017)

1135-76-3233 **Valeria Barra\*** (vb82@njit.edu), **Shahriar Afkhami** (shahriar@njit.edu) and **Shawn A. Chester** (shawn.a.chester@njit.edu). *Numerical Simulations of Thin Viscoelastic Sheets.*

We present numerical simulations of free-surface flows of thin sheets of nearly incompressible viscoelastic media in three dimensions. To include the viscoelastic stresses we consider linear differential viscoelastic constitutive models. The numerical framework used is an extension of a finite element method for linearly elastic thin shells, in which each triangular element exhibits constant strain and stress. This formulation applied to large deformation theory allows to obtain simulations of different physical phenomena involving thin sheets of viscous and viscoelastic media, such as shearing, stretching, and sagging. (Received September 27, 2017)

## 78 ► *Optics, electromagnetic theory*

1135-78-131 **Alexey Sukhinin\*** (asukhinin@gmail.com). *Collapse events of two-color optical beams.* Preliminary report.

Optical collapse discovered in the 60s has been studied extensively due to a number of important applications. Mathematically, it is modeled by the (2+1)D Nonlinear Schrodinger Equations (NLSE). In this talk, I will describe new types of collapse events of two-color beams both at the resonant and non-resonant regimes. The conditions for such events will also be discussed. (Received September 24, 2017)

1135-78-1838 **Jaemin Shin\*** (jaemin.shin@hanbat.ac.kr). *Dipole approximation for shape magnetic forming.*

Shape magnetic forming is an inverse problem to seek a configuration of magnetic field generators which shapes molten metals to prescribed shapes. This talk introduces a new algorithm for shape magnetic forming based on dipole approximation. Convergence results are presented along with numerical simulations. (Received September 25, 2017)

1135-78-3001 **Sean Horan\*** (sthoran@uci.edu), **Adam Gardner**, **Rolf Saager**, **Anthony Durkin** and **Vasan Venugopalan**. *Use of a novel radiative transport solver to recover optical properties of layered media from Spatial Frequency Domain Imaging data.*

The scattering and absorption spectra of biological tissue carries physiological information relevant to tissue structure and composition. Spatial Frequency Domain Imaging provides a method to probe tissues at various depth, but extraction of layered specific optical properties in systems with characteristic thicknesses comparable to, or smaller than, the transport mean free path of light is difficult using conventional analytic or stochastic methods.

We employ a spherical harmonic decomposition approach to solving the radiative transport equation to predict the SFDI data in conjunction with a multi-stage inversion algorithm to extract optical absorption and scattering properties of a two layered medium.

This method is verified using layered tissue phantoms simulating the absorption and reduced scattering spectra of deoxyhemoglobin of various concentrations, with top layers either 90 and 300 um thick dyed using

naphthol green. We show the ability to extract top and bottom layer absorption and scattering properties with accuracies of +/- 5% for bottom layer scattering, 8% for bottom layer absorption, 15% for top layer scattering and 10% for top layer absorption within 500 to 800 nm. (Received September 26, 2017)

## 81 ► Quantum theory

1135-81-99 **Dionisios Margetis\*** ([dio@math.umd.edu](mailto:dio@math.umd.edu)), 2106 William E. Kirwan Hall, Department of Mathematics, University of Maryland, College Park, MD 20742. *Boson gas at finite temperatures: Effect of pair excitation.*

In this talk, I will discuss the formal derivation from microscopic dynamics as well as predictions of nonlinear evolution equations for a trapped gas of repulsively interacting Bosons at finite temperatures below the phase transition. A particular feature of these equations is that they include the nonlocal effect of "Bogoliubov rotation" or "pair excitation", which is responsible for phonons. (Received July 26, 2017)

1135-81-304 **Radhakrishnan Balu\*** ([radhakrishnan.balu.civ@mail.mil](mailto:radhakrishnan.balu.civ@mail.mil)), [radhakrishnan.balu.civ@mail.mil](mailto:radhakrishnan.balu.civ@mail.mil). *Quantum probability based information processing on growing graphs.* Preliminary report.

Growing graphs that dynamically change their vertices and edges are useful models to investigate information processing in networks such as cellular and social. We use quantum probabilistic approach to develop algorithms to process information and characterize the asymptotic behavior of graphs as they increase in size. Specifically, we construct interacting Fock spaces and define quantum stochastic processes on them to describe their dynamic evolution. Systems of imprimitivity are a more fundamental characterization of dynamical systems when described by a group, from which infinitesimal forms in terms of differential equations (Schrodinger, Heisenberg, and Dirac etc.) and the canonical commutation relations can be derived. The concept of localization, where the position operator is properly defined in a manifold, and covariance in relativistic sense of systems can be completely characterized by systems of imprimitivity. In this context, we construct systems of imprimitivity that live on distance-regular graphs induced by Bose-Mesner algebras and discuss their characteristics. (Received August 22, 2017)

1135-81-408 **Jake Fillman\***, 225 Stanger Street, Blacksburg, VA 24061, and **Darren C. Ong** and **Zhenghe Zhang**. *Spectral Characteristics of the Unitary Almost-Mathieu Operator.*

We will discuss the Unitary Almost-Mathieu Operator, which generates a quantum walk on the integers with quasi-periodically distributed coins. We show that its spectrum is a Cantor subset of the circle of zero Lebesgue measure, that the spectral type is almost surely purely singular continuous, and that every spectral parameter is critical in the sense of Avila's global theory for one-frequency cocycles. The key technical ingredients are self-duality under a version of the Fourier transform and an adaptation of the global theory of analytic one-frequency cocycles to the transfer matrix cocycle, which is meromorphic, but not analytic. (Received August 31, 2017)

1135-81-420 **Radhakrishnan Balu\***, [radhakrishnan.balu.civ@mail.mil](mailto:radhakrishnan.balu.civ@mail.mil), and **Daniel Castillo** and **George Siopsis**. *Topological quantum walks and their experimental implementations.*

We find the time-continuous limit of topological split-step quantum walks, with  $d = 2n$  dimensional walker Hilbert space  $H_w$ , and their physical implementations using  $n + 1$  qubits. The corresponding circuit implementations were then ran experimentally using the IBM-Q 5-qubit quantum computer. Using IBM-Q we were able to experimentally verify the known bound states of the topological split-step quantum walk. (Received September 01, 2017)

1135-81-613 **Alvin S Moon\*** ([asmoon@math.ucdavis.edu](mailto:asmoon@math.ucdavis.edu)), University of California, Davis, 1 Shields Ave, Department of Mathematics, Davis, CA 95616, and **Bruno Nachtergaele**, University of California, Davis, 1 Shields Ave, Department of Mathematics, Davis, CA 95616. *Stability of the spectral gap for open chains of spins or fermions.* Preliminary report.

A stable spectral gap in a quantum spin system guarantees the persistence of many important properties of the quantum system through small perturbations. Using quasi-adiabatic evolution, also known as spectral flow, S. Michalakis and J.P. Zwolak previously established spectral gap stability for a large class of Hamiltonians defined with periodic boundary conditions. In this talk, we will present an extension of their stability result to one dimensional systems of spins or fermions interacting with open boundary conditions, a class of models containing the well known Affleck-Kennedy-Lieb-Tasaki spin chain. (Received September 10, 2017)

1135-81-709 **Vyjayanthi Chari\*** ([chari@math.ucr.edu](mailto:chari@math.ucr.edu)) and **Matheus Brito**. *A Weyl character type formula for cluster variables in type A.*

A prime representation of a Hopf algebra is defined to be one which cannot be written as a tensor product of two non-trivial representations. In 2009, Hernandez and Leclerc showed that a family of representations of the quantized enveloping algebra of an affine Lie algebra were precisely the cluster variables in a cluster algebra of type A.

In recent work, we have proved that these prime representations admit a BGG type resolution and deduce a Weyl character formula for these representations. In this talk we shall discuss the corresponding closed formula for a cluster variable in terms of the initial generators  $x_1, \dots, x_n, x'_1, \dots, x'_n$  of the cluster algebra. (Received September 13, 2017)

1135-81-822 **Zhengan Wang\***, 2237 CNSI Bldg. Microsoft Station Q, Santa Barbara, CA. *Conformal Field Theories as Scaling Limit of Anyonic Chains.*

We provide a mathematical definition of a low energy scaling limit of a sequence of general non-relativistic quantum theories in any dimension, and apply our formalism to anyonic chains. This is a joint work with Modjtaba Shokrian Zini. (Received September 14, 2017)

1135-81-934 **Michael George Dombroski\***, [dombroskistm11@verizon.net](mailto:dombroskistm11@verizon.net). *From Harari, Shupe, and Seiberg; to Kalusa & Einstein, using Transpose(\) with Cispose(/) and Eigen-Matrices, to obtain Fermion and Boson matrices at the origin of the universe.* Preliminary report.

Dr. Don Lincoln, a senior physicist at Fermilab, wrote an article in SA Nov12. about Haim Harari, Michael A. Shupe, and Nathan Seiberg. Here we extend the work of HSS by using two sets of nine real 3x3 Universal Base Matrices (UBS and ubs). The "average" of the elements of one set is (0), the other (+1/3). This is analogous to the electric charges of HSS. The Transpose(\) with the new Cispose(/), generate real, Fermion and Boson Matrices. This gives a new, complete, and extremely high degree of symmetry, to physics. A 9x9 matrix is generated, whose 81 elements are themselves also matrices, but 3x3. They are exclusively only one of 4 types. These "types" of 3x3 integer matrices are called "shapes", depending on the location of the zero elements. They are tentatively named: G, S, W, E, analogous to the 4 forces. One result is an unexpected mathematical symmetry of disorder/order. For successive increased powers, the disorder number's power increases, but the order number's power is hypothetically always constant. Thus we hypothesize a classical, non-statistical, observable, analog of the Big Bang initial conditions. This allows us to "see" the "order" of the origin of the universe, through the fog of seeming "disorder". <http://dombroskiSTM.org/com/net> (Received September 17, 2017)

1135-81-1007 **Jessica R Taylor\*** ([jtaylor50ucmerced.edu](mailto:jtaylor50ucmerced.edu)), 5200 N. Lake Road, Merced, CA 95343, and **Boaz Ilan** and **Kevin A Mitchell**. *Enhanced fractal dynamics of a BEC induced by dipolar interactions.* Preliminary report.

Bose-Einstein condensation (BEC) is arguably one of the most fundamental quantum mechanical structures in condensed matter physics. Of current interest is the ability to stabilize a BEC. Earlier studies show that various configurations of the Nonlinear Schrödinger equation effect the flux / escape rate of a BEC. We extend these studies to the defocusing NLS equation with dipolar interactions. Here, we consider the effects of a attractive dipolar potential on the fractal escape-rates of a BEC with repulsive short-range atom-atom interactions relative to a dipolar ground state initial condition. We conclude that the addition of dipolar effects reduces the dispersion of the system and arrests collapse of representative wavefunction. (Received September 18, 2017)

1135-81-1143 **Rupert L Frank** and **Marius Lemm\*** ([mariuslemm@gmail.com](mailto:mariuslemm@gmail.com)), 80 Meritt Ln, Princeton, NJ 08540, and **Barry Simon**. *Condensation of fermion pairs in a domain.*

We consider a gas of fermions at zero temperature and low density, interacting via a microscopic two body potential which admits a bound state. The particles are confined to a domain with Dirichlet (i.e. zero) boundary conditions. Starting from the microscopic BCS theory, we derive an effective macroscopic Gross-Pitaevskii (GP) theory describing the condensate of fermion pairs. The GP theory also has Dirichlet boundary conditions.

Along the way, we prove that the GP energy, defined with Dirichlet boundary conditions on a bounded Lipschitz domain, is continuous under interior and exterior approximations of that domain. (Received September 19, 2017)

1135-81-1259 **Ambar N Sengupta\*** ([ambar.sengupta@uconn.edu](mailto:ambar.sengupta@uconn.edu)). *Aspects of Large-N Yang-Mills in 2 dimensions.*

We will present an account of the behavior of  $U(N)$  quantum Yang-Mills theory in two dimensions in the large- $N$  limit. We will discuss, with examples, how certain limiting expectation values can be expressed in terms of orthogonal polynomials. (Received September 20, 2017)

1135-81-1295 **Chiara Boccato, Christian Brennecke, Serena Cenatiempo and Benjamin Schlein\*** (benjamin.schlein@math.uzh.ch). *Excitation spectra of Bose gases.*

We establish the validity of Bogoliubov's predictions concerning the ground state energy and the energy of low-lying excitations for systems of  $N$  bosons trapped in a box with volume one in the so-called Gross-Pitaevskii regime, where the scattering length of the interaction is of the order  $1/N$  and  $N$  tends to infinity. (Received September 21, 2017)

1135-81-1454 **Bruno Nachtergaele, Robert Sims and Amanda Young\***, Department of Mathematics, University of Arizona, 617 N. Santa Rita Ave., Tucson, AZ 85721. *On the stability of frustration-free lattice fermion systems.* Preliminary report.

In recent years, there have been several results on the spectral gap stability for frustration-free quantum spin models with topologically ordered ground states. In this talk, we consider frustration-free lattice fermion systems with a non-vanishing spectral gap above one or more (infinite-volume) ground states and describe how the stability results in the quantum spin setting can be extended to these situation. (Received September 22, 2017)

1135-81-1647 **Brandon Hoogstra\*** (bhoogstr@asu.edu). *Models of Quantum Dynamics for a Free Particle.*

The project proposes a time-dependent derivation of the Schrödinger equation which models the behavior of a free particle whose propagation is uninhibited by spatial barriers. Initially, a one-dimensional interpretation of a single particle with a potential energy function that depends solely upon time is considered. The results of the one-dimensional model are then generalized to produce a pragmatic three-dimensional model, as well as a theoretical  $n$ -dimensional model. Physical interpretations of free particles exhibiting time-dependent potential energy functions are provided through examples. The project proposes future research, including the incorporation of complex spacetime and applications of the models to tachyons. (Received September 24, 2017)

1135-81-1679 **Matthew Cha, Pieter Naaijken and Bruno Nachtergaele\*** (bxn@math.ucdavis.edu). *Stability of the superselection sectors of Kitaev's abelian quantum double models.*

Kitaev's quantum double models provide a rich class of examples of two-dimensional lattice models with topological order in the ground states and a spectrum described by anyonic elementary excitations. The infinite volume ground states of the abelian quantum double models come in a number of equivalence classes called superselection sectors. We prove that the superselection structure remains unchanged under uniformly small perturbations of the quantum double Hamiltonians. (Received September 24, 2017)

1135-81-1689 **Houssam Abdul-Rahman\*** (houssam@math.arizona.edu), 617 N. Santa Rita Ave., Tucson, AZ 85721-0089. *Entanglement of a class of non-Gaussian states in disordered harmonic oscillator systems.*

For disordered harmonic oscillator systems over the  $d$ -dimensional lattice, we consider the problem of finding the bipartite entanglement of the uniform ensemble of the energy eigenstates associated with a particular number of modes. Such ensemble define a class of mixed, non-Gaussian entangled states that are labeled, by the energy of the system, in an increasing order. We develop a novel approach to find the exact logarithmic negativity of this class of states. We also prove entanglement bounds and demonstrate that the low energy states follow an area law. (Received September 24, 2017)

1135-81-1845 **Minhae Park\*** (minhae.pak@gmail.com), A323, Science Building A, 52, Ewhayeodae-gil, Seodaemun-gu, Seoul, 03760, South Korea, and **Hyun Kyu Kim** (hyunkyukim@ewha.ac.kr), A323, Science Building A, 52, Ewhayeodae-gil, Seodaemun-gu, Seoul, 03760, South Korea. *Operator Quantization of Non-Regular Flips on Teichmüller Space.* Preliminary report.

The phase space of the 2+1 dimensional pure gravity in physics can be described in terms of Teichmüller space. Quantization of Teichmüller space  $T(S)$  of punctured Riemann surface  $S$  has been established by Kashaev and by Chekhov-Fock, which draws the possibility of solving 2+1 quantum gravity. For this, one chooses a triangulation of  $S$  by edges running between punctures. Each edge yields a nice coordinate function on  $T(S)$ , and the quantization process first deforms the algebra generated by these coordinate functions to a non-commutative algebra, which in turn is realized as an algebra of operators on a Hilbert space. A key point of quantum Teichmüller theory is, per each change of triangulations, to build a unitary map between the Hilbert spaces relating the quantum coordinate operators in a consistent manner. In the literature, such a unitary map is built for each 'flip', which is a change of triangulations that alters only one edge. However, a flip involving triangles with two edges identified has not been fully dealt with, and we construct unitary operator for such 'non-regular' flips, filling a missing gap of quantum Teichmüller theory. (Received September 26, 2017)

1135-81-1909 **Marcel Griesemer\*** ([griesemer@mathematik.uni-stuttgart.de](mailto:griesemer@mathematik.uni-stuttgart.de)), Fachbereich Mathematik, IADM, Universität Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany. *On the dynamics of polarons in the strong-coupling limit.*

The polaron model of H. Fröhlich describes an electron coupled to the quantized longitudinal optical modes of a polar crystal. In the strong-coupling limit one expects that the phonon modes may be treated classically, which leads to a coupled Schrödinger-Poisson system with memory. For the effective dynamics of the electron this amounts to a nonlinear and non-local Schrödinger equation. We use the Dirac-Frenkel variational principle to derive the Schrödinger-Poisson system from the Fröhlich model and we present new results on the accuracy of their solutions for describing the motion of Fröhlich polarons in the strong-coupling limit. Our main result extends to  $N$ -polaron systems. (Received September 25, 2017)

1135-81-2035 **James Tener\***, Department of Mathematics, University of California, Santa Barbara, CA 93103. *The geometry of conformal nets.* Preliminary report.

Conformal nets are a mathematical axiomatization of 1+1 dimensional chiral conformal field theories, expressed in terms of von Neumann algebras of local observables. One advantage of studying topics in mathematical physics is that one may compare different mathematical formulations of the same physical phenomena, and learn new things about mathematics in the process. In this talk, I will discuss the relationship between conformal nets and the more geometric notions of vertex operator algebras and Segal CFT. In doing so I will describe how the von Neumann algebras in question have a geometric description. Part of the talk will be based on joint work with André Henriques. (Received September 25, 2017)

1135-81-2075 **Christoph Fischbacher\*** ([cfischb@uab.edu](mailto:cfischb@uab.edu)) and **Gunter Stolz**. *The Quantum XXZ spin model on general graphs.*

We consider the XXZ spin model on general graphs and show its equivalence to a direct sum of discrete many-particle Schrödinger operators of hard-core bosons with an attractive interaction that can be expressed with the help of symmetric graph products. We discuss the existence of a lowest separated energy band (the droplet band), where we focus on the XXZ model on the strip and on  $\mathbb{Z}^d$ , which are not exactly solvable with the Bethe ansatz. (Received September 25, 2017)

1135-81-2232 **Marius Lemm\*** ([mlemm@math.ias.edu](mailto:mlemm@math.ias.edu)). *Local gap threshold for frustration-free Hamiltonians with open boundary conditions.*

A fundamental question about a quantum many-body system is whether it is "gapped". (We say a system is "gapped" if the distance between the two lowest eigenvalues of the Hamiltonian remains bounded in the thermodynamic limit.)

The question whether a system is gapped is difficult to answer in general (in fact, it is undecidable), and we restrict to the more approachable class of "frustration-free" Hamiltonians. In 1988, Knabe devised a "finite-size criterion" for periodic frustration-free spin chains, which says that if the gap at a fixed system size exceeds a certain threshold, then the gap does not close in the thermodynamic limit. We extend Knabe's finite-size criterion to open boundary conditions and we explore some applications. (Received September 25, 2017)

1135-81-2314 **Grant W Allen\***, [grallen@ucsd.edu](mailto:grallen@ucsd.edu), and **Orest Bucicovschi** and **David A Meyer**. *Semi-Algebraic Laws of Entanglement.*

Polynomials in the wavefunction that remain invariant under local SL operations are non-increasing on average under general local operations and classical communication. Such monotonic behavior is one of the most widely accepted properties that entanglement measures should obey. Algebra-based entanglement measure theory has then revealed a popular set of entanglement measures called the tangles. In this talk we give a complete description of 3-qubit pure state entanglement in terms of the image of the tangles, which is only constrained by a single polynomial inequality. We discuss how this set is related to and in fact stronger than the famous monogamy inequality, and then we discuss the generality of these inequality laws. (Received September 25, 2017)

1135-81-2326 **Iris Cong\*** ([cong@g.harvard.edu](mailto:cong@g.harvard.edu)), **Meng Cheng** and **Zhenghan Wang**. *Defects between gapped boundaries in two-dimensional topological phases of matter.*

Defects between gapped boundaries provide a possible physical realization of projective non-abelian braid statistics. We develop general theories to analyze the topological properties and projective braiding of boundary defects in two-dimensional topological phases of matter. In particular, we model the algebraic structure of boundary defects through multi-fusion categories. We then establish a bulk-edge correspondence between certain boundary defects and bulk symmetry defects, which elucidates the projective braid statistics of boundary

defects. We discuss the example of Majorana/parafermion zero modes, which has important applications to topological quantum computation. (Received September 25, 2017)

1135-81-2405 **Jose M Vega-Guzman\*** ([jose.vegaguzman@lamar.edu](mailto:jose.vegaguzman@lamar.edu)), 200 Lucas Building, PO Box 10047, Beaumont, TX 77710. *Schrodinger Group and the Minimum-Uncertainty Squeezed States for the Non-Relativistic Harmonic Oscillator.*

A multi-parameter family of the minimum-uncertainty squeezed states for the harmonic oscillator in non-relativistic quantum mechanics is explored with the aid of the in terms of the maximum kinematical invariance of the Quantum Harmonic oscillator. It is shown that the product of the variances attains the required minimum value only at the instances that one variance is a minimum and the other is a maximum, when the squeezing of one of the variances occurs. The generalized coherent states are explicitly constructed and their Wigner function is studied. The invariance group of the generalized driven harmonic oscillator is shown to be isomorphic to the corresponding Schrodinger group of the free particle. (Received September 26, 2017)

1135-81-2449 **Peter D. Hislop\*** ([peter.hislop@uky.edu](mailto:peter.hislop@uky.edu)), Mathematics Department, University of Kentucky, 715 Patterson Office Tower, Lexington, KY 40506-0027. *Eigenvalue statistics for random Schrödinger operators with point interactions on  $R^d$ .*

We prove that the local eigenvalue statistics for Schrödinger operators with random point interactions on  $R^d$ , for  $d = 1, 2, 3$ , is given by a Poisson point process in the localization regime. This is the first example of Poisson eigenvalue statistics for multi-dimensional random Schrödinger operators in the continuum. The special structure of the point interactions facilitates the proofs of the Wegner and Minami estimates. This is joint work with M. Krishna and W. Kirsch. (Received September 26, 2017)

1135-81-2579 **Gordon Aiello\*** ([gordon-aiello@uiowa.edu](mailto:gordon-aiello@uiowa.edu)) and **Wayne Polyzou** ([polyzou@uiowa.edu](mailto:polyzou@uiowa.edu)). *The Stieltjes Moment Problem and Euclidean Relativistic Quantum Mechanics - Scattering Asymptotic Conditions.*

One recipe for formulating a relativistic quantum mechanical scattering theory utilizes a two-Hilbert space approach, denoted by  $\mathcal{H}$  and  $\mathcal{H}_0$ , upon each of which a unitary representation of the Poincaré group is given. Physically speaking,  $\mathcal{H}$  models a complicated interacting system of particles one wishes to understand, and  $\mathcal{H}_0$  an associated simpler structure one uses to construct asymptotic boundary conditions on states in  $\mathcal{H}$ .

The above considerations lead to the study of the existence of strong limits of operators of the form  $e^{iHt} J e^{-iH_0 t}$ , where  $H, H_0$  are self-adjoint generators of the time translation subgroup of the unitary representations of the Poincaré group on  $\mathcal{H}, \mathcal{H}_0$ , and  $J$  is a contrived mapping from  $\mathcal{H}_0$  into  $\mathcal{H}$ .

The existence of said limits in Euclidean quantum theories depends on the choice of  $J$  and leads to a connection with the Stieltjes moment problem, which concerns the relationship between numerical sequences  $\{\mu_n\}_{n=0}^\infty$  and the existence/uniqueness of measures  $\alpha(x)$  on the half-line satisfying

$$\mu_n = \int_0^\infty x^n d\alpha(x).$$

(Received September 26, 2017)

1135-81-2789 **Li Chen\*** ([il.chen@mail.utoronto.ca](mailto:il.chen@mail.utoronto.ca)), Department of Mathematics, Bahen Centre, 40 St. George St., Room 6290, Toronto, Ontario M5S 2E4, Canada, and **Israel Michael Sigal** ([im.sigal@utoronto.ca](mailto:im.sigal@utoronto.ca)), Department of Mathematics, Bahen Centre, 40 St. George St., Room 6290, Toronto, Ontario M5S 2E4, Canada. *On the Bogoliubov de Gennes Equations.* Preliminary report.

The Bogoliubov de Gennes (BdG) equations form a microscopic description of superconductivity. These are the static equations of the time dependent version, of which the latter assumes the form of Heisenberg equations with a self-consistent Hamiltonian.

When the temperature  $T$  is lower than a certain critical  $T_c$ , superconducting solutions emerges. Macroscopically, one can capture the behavior of these superconducting states by the celebrated Ginzburg-Landau equations when  $T$  is near  $T_c$ . It turns out that the macroscopic theory is an approximation to the microscopic theory when  $T$  is close to  $T_c$ . In this talk, I will present a sketch of the derivation of the Ginzburg-Landau equations as an effective equation of the BdG equations. The main idea of the derivation is to perform bifurcation at the normal (non superconducting) state and to employ iterated Lyapunov Schmidt maps.

This is a joint work with I. M. Sigal. (Received September 26, 2017)

1135-81-2821 **Heide Gluesing-Luerssen and Tefjol Pllaha\*** (tefjol.pllaha@uky.edu). *Stabilizer Codes over Local Frobenius Rings.*

Let  $R$  be a finite, commutative, local, Frobenius ring. A stabilizer code is a submodule of  $R^{2n}$  which is self-orthogonal with respect to a certain symplectic bilinear form. Stabilizer codes have applications in quantum error-correction and we discuss the motivation. In particular, we focus on the minimum distance and isometries of free stabilizer codes. (Received September 26, 2017)

1135-81-2832 **Derek K Wise\*** (wise@csp.edu), Dept of Mathematics and Computer Science, Concordia University, 1282 Concordia Avenue, St Paul, MN 55014. *Hopf Algebra Gauge Theory on Ribbon Graphs: A Foundation for Kitaev Lattice Models.*

The Kitaev models in quantum computation are built on ingredients suspiciously reminiscent of lattice gauge theory, but where the gauge group is generalized to a Hopf algebra. However, until recently, this was only an imprecise analogy, since no general notion of Hopf algebra gauge theory was available. I will explain our recent construction, with Catherine Meusburger, of Hopf algebra gauge theory on a ribbon graph, and how this theory provides a mathematical foundation for Kitaev models. In particular, while our main goal was a natural framework for Kitaev models, by generalizing gauge theory from groups to Hopf algebras we also recover quantum Chern-Simons theory, thus framing the Kitaev models within a larger body of work. (Received September 26, 2017)

1135-81-2902 **Colleen Delaney\*** (cdelaney@math.ucsb.edu). *Applications of equivariantization/de-equivariantization to topological quantum computing.* Preliminary report.

Equivariantization of fusion categories and its inverse process de-equivariantization provide tools for studying  $G$ -crossed braided fusion categories. When a  $G$ -crossed braided fusion category arises as an extension of a unitary modular tensor category modeling the anyons of a 2+1 dimensional topological phase of matter, it has the interpretation of a mathematical model for the same anyons together with symmetry defects. Like anyons, symmetry defects can in principle be used for topological quantum information processing. We discuss how equivariantization and de-equivariantization can be used to further understand symmetry defect models and their role in topological quantum computing as well as their role in some open problems in fusion category theory. (Received September 26, 2017)

1135-81-2989 **Barbara A. Sanborn\*** (bsanborn@antiochcollege.edu). *Quantum uncertainty,  $J$ -holomorphic curves, and symplectic capacity.*

This talk extends the geometric theory of quantum mechanics as a Hamiltonian dynamical system to include aspects of the symplectic topology of an almost complex quantum phase space by identifying the Robertson-Schrödinger uncertainty relation as the differential version of the energy identity in the theory of  $J$ -holomorphic curves. We consider a family of maps from a Riemann surface into the quantum phase space by using the vector fields generated by two quantum observables, and show that the metric tensor pulls back by such a map to the covariance tensor for the two observables. The uncertainty relation is represented as an equality that compares the map energy differential to the sum of the pull-back of the symplectic form and an anti-holomorphic term. If a  $J$ -holomorphic map of this form can be globally defined on a compact Riemann surface, its image is a minimal surface and the uncertainty product integral is a topological invariant that depends only on the homology class of the curve modulo its boundary. This result generates questions about the relation between the concepts of quantum information capacity and symplectic capacity. (Received September 26, 2017)

1135-81-3215 **Krysta M Svore\*** (ksvore@microsoft.com), 1 Microsoft Way, Redmond, WA 98052. *Mathematics of Topological Quantum Computing and its Implementations.*

In 1981, Richard Feynman proposed a device called a Quantum Computer to take advantage of the laws of quantum physics to achieve computational speed-ups over classical methods. Quantum computing promises to revolutionize how and what we compute. Over the course of three decades, quantum algorithms have been developed that offer fast solutions to problems in a variety of fields including number theory, optimization, chemistry, physics, and materials science. Quantum devices have also significantly advanced such that components of a scalable quantum computer have been demonstrated. In this talk, we will present the mathematics behind topological qubits, as well as the mathematical foundations behind the implementation of large-scale fault-tolerant quantum computers. We will cover the requirement for a quantum computer to execute operations that can be implemented fault-tolerantly, i.e., without a proliferation of errors, which gives rise to problems in unitary approximation. For example, all known fault-tolerant gate sets lead to countable subgroups of  $SU(d)$  generated



by elements with algebraic entries and an interesting family from so-called S-arithmetic groups are conjectured to yield the best possible approximations. (Received September 27, 2017)

## 82 ► *Statistical mechanics, structure of matter*

1135-82-102 **Richard D. James\*** ([james@umn.edu](mailto:james@umn.edu)), Aerospace Engineering and Mechanics, 107 Akerman Hall, University of Minnesota, Minneapolis, MN 55455. *Materials and mathematics*. Preliminary report.

I survey some examples of materials whose recent discovery was based in an essential way on mathematical ideas. The main idea concerns “compatibility”—the fitting together of the phases of a material. Some of the emerging materials have the ability to change heat directly into electricity, without the need of a separate electrical generator. (Received July 27, 2017)

1135-82-472 **Dane Taylor\*** ([danet@buffalo.edu](mailto:danet@buffalo.edu)), **Rajmonda Caceres** and **Peter J Mucha**. *Super-resolution community detection for layer-aggregated multilayer networks*.

Inspired by real-world networks consisting of layers that encode different types of connections, such as a social network at different instances in time, we study community structure in multilayer networks. We study fundamental limitations on the detectability of communities by developing random matrix theory for the dominant eigenvectors of matrices that encode random networks. Specifically, we study modularity matrices that are associated an aggregation of network layers. Layer aggregation can be beneficial when the layers are correlated, and it represents a crucial step for discretizing time-varying networks (whereby time layers are binned into time windows). We explore two methods for layer aggregation: summing the layers’ adjacency matrices and thresholding this summation at some value. We identify layer-aggregation strategies that minimize the detectability limit, indicating good practices (in the context of community detection) for how to aggregate layers, discretize temporal networks, and threshold pairwise-interaction data matrices. This work provides a guide for small-community detection in temporal networks and paves the way for a new class of ‘holistic’ methods that simultaneously address the data-preprocessing and network-analysis steps. (Received September 05, 2017)

1135-82-963 **Ralph M Kaufmann**, 150 N University St, West Lafayette, IN 47907, **Dan Li\***, 150 N University St, West Lafayette, IN 47907, and **Birgit Wehefritz-Kaufmann**, 150 N University St, West Lafayette, IN 47907. *Index theory and K-theory of Topological insulators*.

Topological insulators are new materials observed in nature that can be characterized by a  $\mathbb{Z}/2$ -valued invariant. This topological  $\mathbb{Z}/2$  invariant can be understood as a mod 2 index theorem in KR-theory. I will give some background and talk about the relevant index theory and K-theory. Besides the topological  $\mathbb{Z}/2$  invariant, the bulk-boundary correspondence is also an active research topic in KK-theory. (Received September 17, 2017)

1135-82-1006 **Abel Klein\*** ([aklein@uci.edu](mailto:aklein@uci.edu)), University of California, Irvine, Department of Mathematics, Irvine, CA 92697-3875. *Manifestations of localization in the random XXZ quantum spin chain*. Preliminary report.

We will discuss localization in the random XXZ quantum spin chain and its manifestations. (Received September 18, 2017)

1135-82-1064 **Assane Lo\*** ([assanelo@uowdubai.ac.ae](mailto:assanelo@uowdubai.ac.ae)), University of Wollongong in Dubai Block 15, 204, Knowledge Park PO Box 20183, Dubai, UAE, Dubai, United Arab Emirates. *Witten Laplacian method for Bose Einstein Condensation*.

After six decades of work we still do not have a proof of the Bose Einstein Condensation in any continuum model. Recent experiments on dilute Bose gases of atoms seem to confirm that the Bose Einstein Condensation occurs in weakly interacting systems. We use the Witten Laplacian formalism to derive estimates that are suitable for proving the Bose Einstein Condensation in classical continuum models of Kac type. (Received September 19, 2017)

1135-82-1196 **Thomas Joachim Bothner\*** ([bothner@umich.edu](mailto:bothner@umich.edu)), Department of Mathematics, 530 Church Street, Ann Arbor, MI 48109. *The scaling function constant problem in the two-dimensional Ising model*.

We present a simple derivation of the constant factor in the short-distance asymptotics of the  $\tau$ -function associated with the 2-point function of the two-dimensional Ising model. This factor was first computed by Tracy in [2] via an exponential series expansion of the correlation function. Further simplifications in the

analysis are due to Tracy and Widom [3] using Fredholm determinant representations of the correlation function and Wiener-Hopf approximation results for the resolvent operator. Our method relies on an action integral representation of the  $\tau$ -function and asymptotic results for the underlying Painlevé-III transcendent from [1].

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(Received September 20, 2017)

1135-82-1253 **John Z Imbrie\*** ([imbrie@virginia.edu](mailto:imbrie@virginia.edu)), Department of Mathematics, P. O. Box 400137, Charlottesville, VA 229044137. *Localization and Eigenvalue Statistics for the Lattice Anderson model with Discrete Disorder.*

We prove localization and probabilistic bounds on the minimum level spacing for the Anderson tight-binding model on the lattice in any dimension, with single-site potential having a discrete distribution taking  $N$  values, with  $N$  large. (Received September 20, 2017)

1135-82-1678 **Bruno Nachtergaele\*** ([bxn@math.ucdavis.edu](mailto:bxn@math.ucdavis.edu)), **Robert Sims** and **Amanda Young**. *Quasi-locality in quantum lattice systems.* Preliminary report.

In the past dozen years, Lieb-Robinson bounds have been used to exploit the quasi-locality properties of quantum lattice systems with short-range interactions to derive interesting new results about the ground states, low-lying excitations, adiabatic dynamics, linear response, and stability properties of such systems. We discuss recent extensions of these properties and some new applications. (Received September 24, 2017)

1135-82-2267 **Pavel Bleher**, **Brad Elwood** and **Dražen Petrović\*** ([drpetrov@iupui.edu](mailto:drpetrov@iupui.edu)). *Pfaffian Sign Theorem for the Dimer Model on a Triangular Lattice.*

We prove the Pfaffian Sign Theorem for the dimer model on a triangular lattice embedded in the torus. More specifically, we prove that the Pfaffian of the Kasteleyn periodic-periodic matrix is negative, while the Pfaffians of the Kasteleyn periodic-antiperiodic, antiperiodic-periodic, and antiperiodic-antiperiodic matrices are all positive. The proof is based on the Kasteleyn identities and on small weight expansions. As an application, we obtain an asymptotics of the dimer model partition function with an exponentially small error term. (Received September 25, 2017)

1135-82-2461 **Thomas Chen\*** ([tc@math.utexas.edu](mailto:tc@math.utexas.edu)), **Ryan Denlinger** and **Natasa Pavlovic**. *Dispersive estimates for density matrices in quantum dynamics and kinetic theory.*

In this talk, we survey some recent results on Strichartz estimates for density matrices, and present several examples where they are applied. In particular, those include the analysis of the classical Boltzmann equation in an approach based on the Wigner transform. This is joint work with Ryan Denlinger and Natasa Pavlovic. (Received September 26, 2017)

1135-82-3132 **Reza R. Ahangar\*** ([reza.ahangar@tamuk.edu](mailto:reza.ahangar@tamuk.edu)), 700 University MLVD, MSC 172, Mathematics Department, Texas A & M University-Kingsville, Kingsville, TX 78363. *Unifying Forces in Complex Matter Space (CMS).*

It is assumed that matter has two intrinsic components, i) mass, and ii) charge, that is  $M=m+iq$  for a particle in a complex plane. A different view of matter, energy, and momentum is presented. A consistency of this theory with relativity and quantum mechanics is investigated. The consistency of CMS is demonstrated in Hilbert Space (hyperbolic and Minkowski) Geometry. This approach can be used to study the unifying forces in the universe. (Received September 26, 2017)

## 83 ► *Relativity and gravitational theory*

1135-83-460 **Zhoujian Cao\*** ([zjcao@amt.ac.cn](mailto:zjcao@amt.ac.cn)), Zhongguancun Donglu 55, Beijing, 100190. *Gravitational wave model for an eccentric binary based on EOBNR.*

Binary black hole systems are among the most important sources for gravitational wave detection. And also they are good objects for theoretical research for general relativity. Gravitational waveform template is important to gravitational wave detection data analysis. Effective one body numerical relativity model has played an eccentric role in the LIGO data analysis. For future space-based gravitational wave detection, many binary

systems will admit some orbit eccentricity. At the same time the eccentric binary is also an interesting topic for theoretical study in general relativity. In this paper we construct the first eccentric binary waveform model based on effective one body numerical relativity (EOBNR) framework. Our basic assumption in the model construction is the involved eccentricity is small. We have compared our eccentric EOBNR model to the circular one used in LIGO data analysis. We have also tested our eccentric EOBNR model against to another recently proposed eccentric binary waveform model; against to numerical relativity simulation results; and against to perturbation approximation results for extreme mass ratio binary systems. (Received September 05, 2017)

1135-83-522 **Christina Sormani\*** ([sormanic@gmail.com](mailto:sormanic@gmail.com)) and **Carlos Vega**. *"The Null Distance and Spacetime Intrinsic Flat Convergence"*.

We define the null distance,  $\hat{d}_\tau$ , on a spacetime,  $(M, g)$ , endowed with a time function,  $\tau$ . This distance can be used to convert the spacetime,  $(M, g)$ , with a regular cosmological time function (in the sense of Andersson-Galloway-Howard) into a metric space,  $(M, d_t)$ . Once the spacetime has been converted into a metric space, we describe how one might recover the causal structure of the original spacetime using a formula involving the time function and the distance. The above is joint work with Vega.

We next apply the null distance to define Spacetime Intrinsic Flat (SIF) Convergence. We may present a conjecture (appearing in future joint work with Sakovich) concerning the SIF Almost Rigidity of the Spacetime Positive Mass Theorem. We may also describe possible applications to the SIF convergence of Big Bang Spacetimes (appearing in future joint work with Vega). It is possible some of this work may appear before the meeting but it is not yet on the arxiv now. (Received September 07, 2017)

1135-83-681 **Stefanos Aretakis\*** ([aretakis@math.princeton.edu](mailto:aretakis@math.princeton.edu)). *Conservation laws and asymptotics for linear waves.*

The asymptotic behavior of solutions to the wave equation on curved backgrounds is closely connected with various important open problems in general relativity such as the strong cosmic censorship and the black hole stability problem. In this talk, I will present a new technique that allows us to obtain the precise late-time asymptotics for solutions on such backgrounds and describe their relevance to the aforementioned problems. Our method works for both sub-extremal and extremal black hole backgrounds. This is joint work with Y. Angelopoulos (UCLA) and D. Gajic (Imperial). (Received September 12, 2017)

1135-83-816 **Nishanth Abu Gudapati\*** ([nishanth.gudapati@yale.edu](mailto:nishanth.gudapati@yale.edu)), Department of Mathematics, 10 Hillhouse Avenue, New Haven, CT 06511. *A Positive-Definite Energy Functional for Axially Symmetric Maxwell's Equations on Kerr and Kerr-de Sitter Black Holes.*

An important PDE tool in proving decay in the context of the black hole stability problem is a positive-definite quantity corresponding to the perturbations. However, due to the ergo-region of Kerr (and Kerr de- Sitter) black hole spacetimes, the existence of such a quantity is quite subtle in general. In this talk, we shall discuss the proof of that there exists a positive-definite and conserved energy functional for axially symmetric Maxwell's equations propagating on Kerr (and Kerr-de Sitter) black holes. Joint work in part with V. Moncrief (Yale). (Received September 14, 2017)

1135-83-899 **James Isenberg\*** ([isenberg@uoregon.edu](mailto:isenberg@uoregon.edu)), Dept of Math, Institute for Theoretical Science, University of Oregon, Eugene, OR 97403, **Beverly Berger** ([beverlyberger@me.com](mailto:beverlyberger@me.com)), CA, and **Adam Layne** ([adam.n.layne@gmail.com](mailto:adam.n.layne@gmail.com)), OR. *Asymptotic Boredom and Equipartition in the Expanding Direction of Model Cosmological Solutions of Einstein's Equations.* Preliminary report.

While the nature of cosmological "Big Bang"-type singularities has been extensively studied in model spacetime solutions of Einstein's equations, much less is known about what happens in the expanding direction of such solutions. Using a combination of numerical and analytical tools, we examine the dynamical behavior of the gravitational field in the expanding direction of vacuum solutions with a spatially-acting 2-torus isometry. We find striking dynamical attractor behavior, along with strong indications of asymptotic equipartition of energy between the two gravitational field polarizations. We also find that the behavior in the expanding direction of subclasses of these model spacetimes, such as those which are polarized and the Gowdy spacetimes, is generally unstable. (Received September 16, 2017)

1135-83-1172 **Bruno Premoselli\*** ([bruno.premoselli@ulb.ac.be](mailto:bruno.premoselli@ulb.ac.be)). *Instability of focusing initial data sets in high dimensions.*

In this talk we will investigate blow-up properties for a class of initial data sets for the Einstein equations obtained from the conformal method in a scalar-field theory. In dimensions larger than 6, and when some stability conditions on the physics data are not satisfied, we will show that the conformal method produces

blowing-up families of initial data sets. The proof of this result combines constructive variational methods with a priori asymptotic analysis blow-up techniques. (Received September 20, 2017)

1135-83-1487 **Gregory J Galloway\*** ([galloway@math.miami.edu](mailto:galloway@math.miami.edu)), Mathematics Department, University of Miami, Coral Gables, FL 33124, and **Eric Ling**. *Topology and singularities in cosmological spacetimes obeying the null energy condition*. Preliminary report.

The relationship between the topology of spacetime and the occurrence of singularities (causal geodesic incompleteness) is a topic of long-standing interest. In this talk we focus on the cosmological setting: We consider globally hyperbolic spacetimes with compact Cauchy surfaces under assumptions compatible with the presence of a positive cosmological constant. More specifically, for 3+1 dimensional spacetimes which satisfy the null energy condition and contain a future expanding compact Cauchy surface, we establish a precise connection between the topology of the Cauchy surfaces and the occurrence of past singularities. In addition to (a refinement of) the Penrose singularity theorem, the proof makes use of certain fundamental existence results for minimal surfaces and of some recent advances in the topology of 3-manifolds. (Received September 26, 2017)

1135-83-1561 **David Garfinkle\*** ([garfinkl@oakland.edu](mailto:garfinkl@oakland.edu)), Dept. of Physics, Oakland University, Mathematics and Science Center, Room 190, 146 Library Drive, Rochester, MI 48309. *Gravitational wave memory*.

Gravitational waves are detected by their stretching and squeezing of space. Even after the wave has passed, there is a residual stretch and squeeze, which is called gravitational wave memory. This talk will cover several aspects of memory including memory in the expanding universe, an electromagnetic analog of memory, simple models and simple estimates of memory, and gauge invariant treatment of memory. (Received September 23, 2017)

1135-83-1637 **Paul T. Allen\*** ([ptallen@clark.edu](mailto:ptallen@clark.edu)), **James Isenberg**, **John M. Lee** and **Iva Stavrov Allen**. *Asymptotic gluing of shear-free hyperboloidal initial data*. Preliminary report.

In order that a spacetime development admit a regular conformal boundary at future null infinity, hyperboloidal solutions to the Einstein constraint equations must satisfy satisfy the shear-free condition. Working in the CMC setting, we present a construction for gluing such data along the ideal boundary at infinity. (Received September 24, 2017)

1135-83-1921 **James Dilts\*** ([jdilts@ucsd.edu](mailto:jdilts@ucsd.edu)), 9500 Gilman Drive # 0112, La Jolla, CA 92093-0112, and **Michael Holst** ([mholst@ucsd.edu](mailto:mholst@ucsd.edu)), 9500 Gilman Drive # 0112, La Jolla, CA 92093-0112. *When do spacetimes have constant mean curvature slices?*

Many results in mathematical relativity, both for the initial data problem and for the evolution problem, rely on the existence of a constant mean curvature (CMC) Cauchy surface. However, it is known that not all spacetimes have any CMC Cauchy surfaces. This is a serious problem, especially because it is not even known whether spacetimes with CMC Cauchy surfaces are in any sense generic. In this primarily expository talk, we will discuss known results about the existence (and non-existence) of CMC Cauchy surfaces, and make several conjectures concerning their existence and generality. (Received September 25, 2017)

1135-83-2031 **Henri Petrus Roesch\*** ([hroesch@uci.edu](mailto:hroesch@uci.edu)), 340 Rowland Hall (Bldg.#400), University of California, Irvine, Irvine, CA 92697-3875. *Proof of a Null Penrose Conjecture using a new Quasi-local Mass*.

We define an explicit quasi-local mass functional which is nondecreasing along all null foliations (satisfying a convexity assumption) of null cones. We use this new functional to prove the Null Penrose Conjecture under fairly generic conditions. (Received September 25, 2017)

1135-83-2246 **Candice Davis**, **Scott Geyer\*** ([scott.geyer@usm.edu](mailto:scott.geyer@usm.edu)), **William Johnson** and **Zhifu Xie**. *Inverse Problem of Central Configurations in the Collinear 5-body Problem*.

The n-body problem is as old as physics itself. It was Isaac Newton's fascination with calculating the motion of the Earth, sun, and moon that lead him to his discoveries in both physics and calculus. While Poincare discovered the chaos of the three body problem, mathematicians have found ways to analyze the n-body problem by studying central configurations where the bodies can remain their shape in their orbits. The inverse problem of collinear central configurations has been studied in the collinear 3-body problem by Albouy and Moeckel in 2001 and in the collinear 4-body problem by Ouyang and Xie in 2004. In our research, we examine the collinear 5-body problem by solving for the masses with the positions as given variables. Through this process, we can determine possible positions for central configurations. In the symmetric case, it is surprising that any symmetric configuration

could be a central configuration for some positive masses, which is different from the symmetric case for collinear 4-body central configurations. In general, we identify regions in the configuration space where it is possible to choose positive masses that will make the configuration central. Mathematical software Geogebra, Maple, and SageMath have been used to compute and plot the regions. (Received September 25, 2017)

1135-83-2251 **P. P. Yu\***, 501 Westminster Ave., Fulton, MO 65251. *Lagrange reduction of the variational principle for Einstein's equations*. Preliminary report.

Lagrange reduction of the variational principle is applied to study the infinite-dimensional geometry in the constrained dynamics of Einstein's vacuum gravitational field equations. A degenerate Lagrangian invariant under groups of spatial diffeomorphisms and relativistic time translation is constructed on a principle bundle over the configuration space of Riemannian metrics on a spatial hypersurface in the space-time. With a natural choice of connection and homogenization in the time variable, variations of the reduced Lagrangian are carried out in the associated adjoint bundle. It is shown that vertical Lagrange-Poincare equations arising from variations in the shift vector field and the lapse function yield the diffeomorphism and Hamiltonian constraints as conservation laws whereas horizontal Lagrange-Poincare equations take the form of the reduced Einstein equations free of constraints. The Lagrange equations of evolution for given Cauchy data satisfying the constraints are obtained by the horizontal lift of the reduced Einstein equations. (Received September 25, 2017)

1135-83-2365 **Arthur E Fischer\*** ([aef@ucsc.edu](mailto:aef@ucsc.edu)), Department of Mathematics, University of California, Santa Cruz, CA 95064. *A Simple All-time Model for the Birth, Big Bang, and Death of the Universe*.

We model the standard  $\Lambda$ CDM model of the universe by the spatially-flat Friedmann-Lemaître line element

$$ds_{\Lambda\text{CDM}}^2 = -c^2 dt^2 + \left(\frac{8\pi G \rho_{m,0}}{\Lambda c^2}\right)^{2/3} \left(\sinh\left(\frac{3}{2}\sqrt{\Lambda/3} ct\right)\right)^{4/3} d\sigma_{\text{Euclid}}^2$$

which we *extend* for all time  $t \in (-\infty, \infty)$ . This line element is  $C^\infty$  and *solves Friedmann's equation* for all  $t \neq 0$  and is  $C^1$  at  $t = 0$ . We use this *extended* line element to show that encoded into Friedmann's equation is (1) the prediction that the universe existed *before* the big bang; (2) that the big bang was preceded by a negative time epoch  $(-\infty, 0)$ ; (3) that the universe was asymptotically created out of nothing at  $t = -\infty$  from an *unstable* negative half de Sitter  $ds_{dS_4^-}^2$  initial state; and (4) asymptotically dies at  $t = \infty$  as the *stable* positive half de Sitter  $ds_{dS_4^+}^2$  final state. Since these two de Sitter states are vacuum states, our model shows that the universe was created *de novo* from nothing at  $t = -\infty$  and dies to nothing at  $t = \infty$ , and is thus a variant of the *zero energy universe*, with our extended  $\Lambda$ CDM model interpolating between the initial and final state. (Received September 26, 2017)

1135-83-2377 **Noah Benjamin** and **Iva Stavrov Allen\*** ([istavrov@clark.edu](mailto:istavrov@clark.edu)), 0615 SW Palatine Hill Road, MSC 110, Portland, OR 97219. *The effects of self-interaction on constructing relativistic point particles*.

We introduce a framework for studying the effects of self-interaction on the construction of point particle initial data in General Relativity. Within this framework we rigorously prove the claim of Arnowitt, Deser and Misner that electrically neutral point source modeled by a Dirac delta distribution must have zero ADM mass. We further identify a geometric structure and a scaling parameter that allow one to determine, by controlling the effects of self-interaction, when a sequence of "collapsing" matter distributions yields non-zero mass in the limit. (Received September 26, 2017)

1135-83-2734 **Lee Lindblom\*** ([llindblom@ucsd.edu](mailto:llindblom@ucsd.edu)), CASS UCSD 0424, 9500 Gilman Drive, La Jolla, CA 92093. *Developing Numerical Methods for Solving Geometrical PDEs on Manifolds with Arbitrary Spatial Topologies*.

Recent developments of numerical methods for solving geometrical PDEs on manifolds with arbitrary spatial topologies will be reviewed. In particular the problem of building a coordinate atlas and differential structure suitable for computational work will be discussed. A new symmetric-hyperbolic representation of Einstein's equation for performing numerical evolutions on such manifolds will be presented. And, examples of numerical solutions of Einstein's equation and the Ricci flow equation on manifolds with non-trivial topologies will be displayed. (Received September 26, 2017)

1135-83-2764 **Samuel E. Gralla\*** ([sgralla@email.arizona.edu](mailto:sgralla@email.arizona.edu)), Tucson, AZ. *Gravitational Waves and the Aretakis Instability*.

The field of black hole stability lies at the boundary of mathematics and physics and has enjoyed a productive interaction between the two communities. Recently, the mathematician Aretakis discovered an instability of

extremal black holes that had been overlooked by physicists. We have recovered the instability using physics techniques and generalized to the more realistic setting of nonaxisymmetric perturbations of near-extremal black holes. This forms a foundation for determining the physical, and ultimately observational, consequences of the instability. Special attention will be given to gravitational radiation from rapidly rotating black holes. (Received September 26, 2017)

## 86 ► Geophysics

1135-86-1981 **Mariangel Garcia\***, San Diego State University, 5500 Campanile Dr, San Diego, CA 92182-1245, **Paul Choboter** ([pchobote@calpoly.edu](mailto:pchobote@calpoly.edu)), California Polytechnic State University, San Luis Obispo, CA 93407, **Ryan Walter** ([rkwalter@calpoly.edu](mailto:rkwalter@calpoly.edu)), California Polytechnic State University, San Luis Obispo, CA 93407, and **Jose Castillo** ([jcastillo@mail.sdsu.edu](mailto:jcastillo@mail.sdsu.edu)), San Diego State University, 5500 Campanile Dr, San Diego, CA 92182-1245. *Validation of the Nonhydrostatic General Curvilinear Coastal Ocean Model (GCCOM) for Stratified Flows.*

One of the challenges in the simulation of coastal ocean dynamics is the vast range of length and time scales present. While global- and basin-scale processes and currents can be captured quite well with computationally-inexpensive hydrostatic models, smaller-scale features such as shoaling nonlinear internal waves and bores, coastal fronts, and other convective processes require the use of a nonhydrostatic model to capture dynamics accurately. Here we introduce the nonhydrostatic capabilities of the General Curvilinear Coastal Ocean Model (GCCOM) in a stratified environment. GCCOM is a three-dimensional, nonhydrostatic Large Eddy Simulation (LES) model that can run in a fully three-dimensional general curvilinear coordinate system. While this model was validated for unstratified flows, we present on recent advances of the model to simulate stratified flows. In particular, a suite of test cases widely used as benchmarks for assessing the nonhydrostatic capabilities for gravity-driven flows: the classic lock release and gravity current experiment and internal seiche in a flat bottom tank. These validation experiments demonstrate that GCCOM can resolve complex nonhydrostatic phenomena in stratified flows with numerical accuracy and mass and energy conservation. (Received September 25, 2017)

1135-86-1986 **Yanfei Wang\*** ([yfwang@mail.iggcas.ac.cn](mailto:yfwang@mail.iggcas.ac.cn)), No.19 Beitucheng Xilu, Chaoyang District, Beijing, 100029, Peoples Rep of China. *Sparse optimization methods for reconstruction of the seismic wavefield.* Preliminary report.

In geophysics, nearly all inverse problems are ill-posed because of the limitations of observations and instability during inversion computation. For instance, a direct effect of the limitations of acquisition is the sub-sampled data will generate aliasing in the frequency domain; therefore, it may affect the subsequent processing such as filtering, de-noising, amplitude versus offset analysis, multiple eliminating and migration imaging. In our recent work, we develop some sparse optimization methods for the geophysical data regularization and imaging problems. We consider sparsity-constrained regularization modeling and related solving methodology. Numerical experiments based on theoretical data and field data are performed and interpreted. (Received September 25, 2017)

1135-86-2724 **Kyle Campbell\*** ([kjcampbell@sdsu.edu](mailto:kjcampbell@sdsu.edu)), San Diego State University, 5500 Campanile Drive, San Diego, CA 92182-1326, and **Christopher Paolini** ([paolini@engineering.sdsu.edu](mailto:paolini@engineering.sdsu.edu)) and **Jose Castillo** ([jcastillo@mail.sdsu.edu](mailto:jcastillo@mail.sdsu.edu)). *Modeling Multiphase Buoyancy Driven Plume Migration during Geologic CO<sub>2</sub> Injection.*

A Department of Energy geologic carbon sequestration site selection goal is to ensure, through numerical modeling and simulation that no more than 1% of injected CO<sub>2</sub> escape within 1000 years after injection. To predict long-term retention of CO<sub>2</sub> in a reservoir, the interaction of geochemical and geomechanical effects of injection must be modeled. The transport of gas phase CO<sub>2</sub> through microfractures, at the 100-micrometer scale, in porous sandstone and shale could lead to unwanted release of CO<sub>2</sub> back into the atmosphere. We model the buoyancy driven flow of a two-phase system consisting of a CO<sub>2</sub>-H<sub>2</sub>O vapor mixture phase, and an aqueous phase, composed of formation water, dissolved CO<sub>2</sub>, and charged solutes formed from mineral dissolution. This two-phase system forms a plume of gaseous and aqueous CO<sub>2</sub> that can migrate upward due to differences in density between CO<sub>2</sub>-rich phases and the surrounding formation fluid. We model the gas phase CO<sub>2</sub>-H<sub>2</sub>O composition using a Redlich and Kwong equation of state (EOS) with mixing rules, and the aqueous phase composition using the revised Helgeson Kirkham Flowers model for approximating thermodynamic properties of aqueous electrolytic solutions at high temperatures and pressures. Pitzer equations are used to compute solute activity coefficients. (Received September 26, 2017)

1135-86-2861 **Jonathan Matthews\*** ([jmatthews@mail.sdsu.edu](mailto:jmatthews@mail.sdsu.edu)), San Diego State University, 5500 Campanile Drive, San Diego, CA 92182-1326, and **Christopher Paolini** ([paolini@engineering.sdsu.edu](mailto:paolini@engineering.sdsu.edu)) and **Jose Castillo** ([jcastillo@mail.sdsu.edu](mailto:jcastillo@mail.sdsu.edu)). *Fluid Structure Interaction in Geologic CO<sub>2</sub> sequestration using Coupled Finite Element and Mimetic Methods*.

A Thermal-Hydrologic-Mechanical-Chemical ("THMC") simulator for modeling geologic CO<sub>2</sub> sequestration is presented. The model implements fluid structure interaction ("FSI") by coupling finite element methods for solid mechanics and mimetic methods for chemical transport and reactivity. A mixed finite element method is used for fluid pressure and velocity, while a Galerkin method is used for poroelastic mechanics. The poroelastic and pressure-velocity fields are solved in parallel with MPI using domain decomposition. High-order mimetic operators are used in a mass-transport advection-diffusion model to solve for solute concentration and mineral dissolution and precipitation using the open-source Mimetic Operators Library Enhanced ("MOLE") toolkit. The Distributed Coupling Toolkit ("DCT") facilitates solution data exchange between the finite element and mimetic models. Results of a CO<sub>2</sub> injection simulation are presented and validated against the Frio Test Pilot experiment. (Received September 26, 2017)

1135-86-2910 **Christopher Paolini\*** ([paolini@engineering.sdsu.edu](mailto:paolini@engineering.sdsu.edu)), San Diego State University, 5500 Campanile Drive, San Diego, CA 92182-1326. *Effects of CO<sub>2</sub> Injection on Calcite Saturation during Geologic CO<sub>2</sub> Sequestration*.

Three-dimensional numerical simulations were conducted to investigate the effects of varying aqueous CO<sub>2</sub> concentration on calcite saturation and precipitation during CO<sub>2</sub> injection, under variable injection temperatures and pressures. A nine-mineral kinetic mechanism governing the dissolution of quartz, potassium-feldspar (K-spar), anorthite, albite, calcite, kaolinite, smectite, illite, and halite in a porous medium was used, with the aqueous phase pore water temperature modeled using a transient heat advection-diffusion transport model with non-constant thermal coefficients. Water-rock interaction is coupled with a transient mixed finite element method for fluid pressure and velocity, and a Galerkin method for poroelastic mechanics. Thermal coefficients (specific heat and specific enthalpy) are temperature and pressure dependent and computed using the revised Helgeson-Kirkham-Flowers (HKF) model for approximating the thermodynamic properties of aqueous electrolytic solutions at geologic conditions of high temperature and pressure. The HKF derived heat capacity and enthalpy of charged aqueous species arising from the interaction of CO<sub>2</sub>-rich brine with sandstone are used in the heat transfer model source term for computing aqueous phase volumetric energy generation rate. (Received September 26, 2017)

1135-86-3125 **Jose Navarro Nunez\*** ([jnavarronunez@sdsu.edu](mailto:jnavarronunez@sdsu.edu)), San Diego State University, 5500 Campanile Drive, San Diego, CA 92182-1326, and **Christopher Paolini** ([paolini@engineering.sdsu.edu](mailto:paolini@engineering.sdsu.edu)) and **Jose Castillo** ([jcastillo@mail.sdsu.edu](mailto:jcastillo@mail.sdsu.edu)). *Parallelized Chemical Equilibrium Solver for Geologic CO<sub>2</sub> Sequestration*.

Thermochemical processes occurring during geologic CO<sub>2</sub> sequestration have varying characteristic time scales. While the mineral trapping of CO<sub>2</sub> occurs over many years, there are chemical reactions involved that reach an equilibrium state much sooner. For these reactions, it is safe to ignore the chemical kinetics involved and directly compute the equilibrium composition. We present a chemical equilibrium solver that computes the chemical species composition, pH, alkalinity, and redox potential of a NaCl-H<sub>2</sub>O-CO<sub>2</sub> system. The equilibrium model is based on the minimization of the system Gibbs free energy through the method of augmented Lagrange multipliers. In order to obtain accurate results at the extreme thermophysical conditions encountered in the subsurface, the Pitzer model is used to calculate chemical species activity. The equilibrium solver is parallelized, using OpenMP and MPI, to handle a large number of chemical species and virial terms for the Pitzer equations. The methods implemented are high-level OpenMP and vectorization on Intel's Xeon Phi Knights Landing architecture. (Received September 26, 2017)

## 90 ► Operations research, mathematical programming

1135-90-572 **Jesús De Loera, Jamie Haddock\*** ([jhaddock@math.ucdavis.edu](mailto:jhaddock@math.ucdavis.edu)) and **Luis Rademacher**. *The Minimum Euclidean-Norm Point on a Convex Polytope*. Preliminary report.

We consider the problem of finding the point of minimum Euclidean-norm in a convex polytope; this problem is denoted MNP. In 1974, Philip Wolfe proposed a combinatorial algorithm for MNP but its complexity has remained unknown. Solving MNP via Wolfe's algorithm is currently relevant as it is one of the most practical known algorithms for submodular function minimization, which offers efficient and accurate solutions to problems from

machine learning. We first present additional motivation for considering this problem by showing that linear programming reduces in strongly-polynomial time to MNP over a simplex. Additionally, we present several ‘pivot’ rules for Wolfe’s algorithm and discuss initial results towards complexity of Wolfe’s algorithm for a natural choice of ‘pivot’ rule. (Received September 08, 2017)

1135-90-725 **Ram Verma\*** ([verma99@msn.com](mailto:verma99@msn.com)). *Hybrid fractional integral type programming*. Preliminary report.

A hybrid class of fractional integral type programming problems for minimizing a maximum of several time-dependent ratios involving integral type models based on new generation invexities are investigated toward developing generalized Wolfe type dual models, and generalized Mond-Weir type dual models. Furthermore, some results are established on generalized Wolfe type dual models and generalized Mond-Weir type dual models, which lead to establishing weak, strong, and strict converse duality theorems by applying new generation invexity frameworks. These advanced duality theorems have significant applications to multitime multiobjective variational problems as well as to multiobjective control problems. (Received September 13, 2017)

1135-90-1079 **Jeffrey A Braun\*** ([jbrown8@jhu.edu](mailto:jbrown8@jhu.edu)), 3404 Oakenshaw Pl., Baltimore, MD 21218, and **John C Wierman** ([jwierma1@gmail.com](mailto:jwierma1@gmail.com)), 3400 North Charles Street, Whitehead Hall 101, Baltimore, MD 21218. *Asymmetric and Symmetric Rendezvous on the Unit Cube*. Preliminary report.

A famous open problem in the field of Search Games and Rendezvous is that of the Astronaut Problem. In an effort to develop new strategies and tools to make progress on the Astronaut Problem, we postulated a new, similar scenario that is interesting in its own right. Given two indistinguishable agents randomly placed on the surface of a cube who move at unit speed, their location on the cube defines their vision in the following manner: If they are on a face they can see the entirety of the face (including the edges and vertices that comprise it) and if they are on an edge or a vertex, it is as if they are on the two or three faces simultaneously that join together to make the edge or vertex. Our goal is to either directly derive asymmetric and symmetric rendezvous values associated with this search space or provide good bounds for them. We have defined a strategy space which dominates among a large subset of all possible strategies. Within this strategy space, we closely examine two special cases, which provides us with excellent upper bounds for both rendezvous values. We also derived a common lower bound for both rendezvous values from some ad-hoc reasoning. In the process we also make some remarks on Search Games and Rendezvous problems in general. (Received September 19, 2017)

1135-90-1080 **Gary R Engler\*** ([gengler@stevens.edu](mailto:gengler@stevens.edu)), 1 Castle Point on Hudson, Stevens Institute of Technology, Hoboken, NJ 07030, and **Michael Zabaranin**, 1 Castle Point on Hudson, Stevens Institute of Technology, Hoboken, NJ 07030. *Shortest Path Computations in Stochastic Neural Systems*.

A neuromorphic algorithm capable of solving the shortest path problem on pre-learned graph structures is proposed. The generated network structure implements a network of stochastic spiking neurons where in the stable distribution of network states the shortest path corresponds to the network state with the highest probability. The networks behavior of activity representing potential paths corresponds to recent biological results. Numerical results confirming the theoretical portions are given. (Received September 19, 2017)

1135-90-1322 **Youssef Qranfal\*** ([qranfaly@wit.edu](mailto:qranfaly@wit.edu)), 550 Huntington Avenue, Boston, MA 02115. *Filtering and Positivity Incorporation*.

Often times, the solution of a filtering reconstruction problem lacks the required property of being positive. The projected Kalman method was introduced to solve for filtering and positivity. However, both filtering and positivity are done in two successive steps rather than both done at once. Moreover, this method embeds a major drawback. As a remedy, two filtering and positivity incorporation algorithms, the SMART Filter and the EM Filter, are presented. They are validated numerically by applying them to solve the ill-posed inverse problem of dynamic image reconstruction in medical imaging. (Received September 27, 2017)

1135-90-1974 **Diana Gonzalez\*** ([dee.gonzalez912@gmail.com](mailto:dee.gonzalez912@gmail.com)), Department of Mathematics and Statistics, 1250 Bellflower Blvd, Long Beach, CA 90840-1001, and **Jen-Mei Chang** ([jen-mei.chang@csulb.edu](mailto:jen-mei.chang@csulb.edu)), Department of Mathematics and Statistics, 1250 Bellflower Blvd, Long Beach, CA 90840-1001. *Can decision aid increase college student success?* Preliminary report.

College students across the nation have numerous on-campus resources designed to increase their academic successes; however, students struggle to choose the best one(s) that fit their unique learning styles and needs. This is especially true for incoming freshmen. In this preliminary study, we designed and tested a decision-aid



model to understand students' decision-making process when it comes to choosing academic support services. We built our decision-aid model by studying two existing sets of non-cognitive survey data using Factor Analysis, and framed the matching problem as a constrained least-squares optimization problem where we solve two unknowns simultaneously using a classical eigen-analysis. New survey data collected on roughly 165 incoming students at California State University, Long Beach, were used to train and validate our decision-aid model. This talk reports our preliminary findings and future directions. (Received September 25, 2017)

1135-90-2390 **Kimon Fountoulakis\***, 1947 Center Street, Ste. 600, Berkeley, CA 94704, and **Farbod Roosta-Khorasani, Julian Shun, Xiang Cheng** and **Michael Mahoney**. *Variational Perspective on Local Graph Clustering*.

Modern graph clustering applications require the analysis of large graphs and this can be computationally expensive. In this regard, local spectral graph clustering methods aim to identify well-connected clusters around a given "seed set" of reference nodes without accessing the entire graph. The celebrated Approximate Personalized PageRank (APPR) algorithm in the seminal paper by Andersen et al. is one such method. APPR was introduced and motivated purely from an algorithmic perspective. In other words, there is no a priori notion of objective function/optimality conditions that characterizes the steps taken by APPR. Here, we derive a novel variational formulation which makes explicit the actual optimization problem solved by APPR. We show that an optimization algorithm which apparently requires accessing the entire graph, can be made to behave in a completely local manner by accessing only a small number of nodes. This viewpoint builds a bridge across two seemingly disjoint fields of graph processing and numerical optimization, and it allows one to leverage well-studied, numerically robust, and efficient optimization algorithms for processing today's large graphs. (Received September 26, 2017)

1135-90-2501 **Nandini Rakala\*** ([nrakala2015@my.fit.edu](mailto:nrakala2015@my.fit.edu)), Department of Mathematical Sciences, Florida Institute of Technology, 150 W. University Blvd, Melbourne, FL 32901, **Munevver Mine Subasi** ([msubasi@fit.edu](mailto:msubasi@fit.edu)), Department of Mathematical Sciences, Florida Institute of Technology, 150 W. University Blvd, Melbourne, FL 32901, and **Ersoy Subasi** ([esubasi@fit.edu](mailto:esubasi@fit.edu)), Department of Engineering Systems, Florida Institute of Technology, 150 W. University Blvd, Melbourne, FL 32901. *Multi-Objective Extension to Logical Analysis of Data and Its Applications in Medicine*.

Logical Analysis of Data (LAD) is a classification method based on combinatorics, optimization, and theory of Boolean functions. The recent medical applications of LAD provide excellent solutions to medical problems using clinical, genomic, and proteomics datasets. The key ingredient of LAD is the identification of patterns, distinguishing between disjoint subgroups of observations. We propose an algorithmic approach based on multi-objective optimization to generate Pareto optimal LAD patterns. Given a binary data  $\Omega = \Omega^+ \cup \Omega^-$ , where  $\Omega^+ \cap \Omega^- = \emptyset$ , a pattern  $P$  is a subcube of  $\{0, 1\}^n$ , where  $n$  is the number of features in the dataset:  $P = \bigwedge_{j \in N_P} x_j$ ,

where  $N_P \subseteq \{1, \dots, n\}$  and  $x_j \in \{0, 1\}$ . We define a pattern  $P^*$  to be strong (strict) Pareto optimum for the multi-objective problem iff there is no other pattern  $P$  such that  $Cov(P) \subset Cov(P^*)$ , where  $Cov(P) = \{x \in \Omega : P(x) = 1\}$ .  $P^*$  is weak (non-strict) Pareto optimum iff  $Cov(P) \subseteq Cov(P^*)$  for some pattern  $P$ . The proposed approach identifies the set of strong/weak Pareto optimum patterns to predict slow and rapid progressions of chronic kidney disease patients in African-American Study of Kidney Disease genomic data. (Received September 26, 2017)

1135-90-3074 **Karen T. Hicklin\*** ([khicklin@email.unc.edu](mailto:khicklin@email.unc.edu)), Dept. of Statistics and Operations Research, B24 Hanes Hall, Campus Box 3260, Chapel Hill, NC 27599-3260. *A Bayesian Markov Decision Process to Evaluate Mode of Delivery*.

Optimal mode of delivery decisions lead to better short- and long-term health outcomes for mothers and children. When a woman enters labor, she will delivery in one of two ways: vaginal delivery or cesarean delivery. However, at the onset of labor, the delivery outcome is unknown. Patients who have a slow or stalled labor progression are considered failure-to-progress thus leading to the need for a C-section. We combine Bayesian updating into a Markov decision process to determine under what circumstances it is appropriate to gather more information before making a decision regarding mode of delivery. The goal is to maximize the utility of health outcomes for the mother and child as a function of the belief that the woman will have a safe vaginal delivery as a function of cervical dilation progression. The results of this work can be used in developing a decision aid tool to be used by women and their healthcare providers to explore various options and the risks associated with them. (Received September 26, 2017)

## 91 ► *Game theory, economics, social and behavioral sciences*

1135-91-40 **Indranil SenGupta\*** ([indranil.sengupta@ndsu.edu](mailto:indranil.sengupta@ndsu.edu)), Department of Mathematics, North Dakota State University, NDSU Dept # 2750, Minard Hall 408, Fargo, ND 58108-6050, and **Aziz Issaka**. *Analysis of variance based financial instruments for Ornstein-Uhlenbeck type models.*

In this presentation a couple of variance dependent instruments in the financial market are studied. Firstly, a number of aspects of the variance swap in connection to the Barndorff-Nielsen and Shephard model is studied. A partial integro-differential equation that describes the dynamics of the arbitrage-free price of the variance swap is formulated. Under appropriate assumptions for the first four cumulants of the driving subordinator, a Vecer-type theorem is proved. Finally, a price-weighted index modulated by market variance is introduced. The large-basket limit dynamics of the price index and the “error term” are derived. (Received June 28, 2017)

1135-91-43 **Mason A. Porter\***, [mason@math.ucla.edu](mailto:mason@math.ucla.edu). *Multilayer Networks and their Applications.* In natural and engineered systems, entities interact with each other in complicated patterns that can encompass multiple types of relationships, change in time, and include other types of complications. Such systems can also include multiple subsystems and layers of connectivity, and it is important to take such ‘multilayer’ features into account to try to improve our understanding of complex systems. One way to do this is through so-called “multilayer networks”, a generalization of graphs, whose study has become the most prominent area of network science. In this talk, I’ll give an introduction to multilayer networks, discuss applications (including ecology, multimodal transportation networks, and social networks), and present some ideas for studying concepts such as community structure and centrality. I will include a possibly intriguing application of a multilayer representation of time-dependent networks to examine time-dependent ranking of United States mathematics programs based on a PhD exchange network. (Received June 30, 2017)

1135-91-412 **Micah Pollak** and **Yuanying Guan\*** ([guany@iun.edu](mailto:guany@iun.edu)), 3400 Broadway, Gary, IN 60657. *Historical Capital Buffers, Overlapping Portfolios, and Contagion of U.S. Financial System.*

We investigate the risk and severity of a contagion within a financial network in the presence of several types of interbank linkages. Using a core-periphery model we demonstrate how interbank lending, combined with an overlapping external asset (such as mortgage backed securities) and a partially overlapping internal asset (such as substantial ownership within the banking industry) weakens a financial network and propagates shocks during a financial contagion. The model is calibrated based on historical balance sheet and ownership data for U.S. banks between 2000 and 2015. In addition, we show how the risk and severity of a financial contagion is affected by trends in bank capital buffers, which have been driven in part by changes to the Basel Accords. (Received August 31, 2017)

1135-91-462 **Nishant Malik\*** ([nishant.malik@dartmouth.edu](mailto:nishant.malik@dartmouth.edu)), Department of Mathematics, Dartmouth College, Hanover, NH 03755, **Hsuan-Wei Lee**, Department of Sociology, The University of Nebraska-Lincoln, Lincoln, NE 68588, **Feng Shi**, Odum Institute for Research in Social Science, The University of North Carolina, Chapel Hill, NC 27599, and **Peter J Mucha**, Department of Mathematics, The University of North Carolina, Chapel Hill, NC 27599. *Transitivity Reinforcement in Coevolving Network Models.*

Transitivity is one of the fundamental structural properties of real world networks, yet its role in coevolving network models has remained relatively unexplored. We introduce new modified models for the SIS epidemics and opinion formation on coevolving networks, these new models incorporate innovative rewiring rules which reinforce transitivity. Hence, providing a unique opportunity to study various effects of transitivity on the dynamics of coevolving network models. Using numerical simulations, we identify and examine an extensive set of dynamical features in the new models. Furthermore, we present a derivation of approximate master equations (AME) for the SIS model and show that for some parameter settings, the AME accurately traces the temporal evolution of the system. These methods and results may not only be useful in studying coevolving network models but also in developing ideas for controlling dynamics on networks. (Received September 05, 2017)

1135-91-469 **Alex Bernstein, Tathagata Banerjee and Zachary Feinstein\*** (zfeinstein@ese.wustl.edu). *A time dynamic Eisenberg-Noe financial contagion model.*

In this talk we will consider an extension of the Eisenberg-Noe model of financial contagion to allow for time dynamics in both discrete and continuous time. Derivation and interpretation as a generalization of the Eisenberg-Noe model will be provided. Mathematical results on existence and uniqueness of firm equity and losses under discrete and continuous time models will be given as well as convergence results of the discrete-time setup to a continuous-time differential equation. (Received September 05, 2017)

1135-91-482 **Yuri Yatsenko\*** (yyatsenko@hbu.edu), TX, **Natali Hritonenko** (nahritonenko@pvamu.edu), TX, and **Selkhan Boranbayev**, Kazakhstan. *Modeling of technological renovation under environmental uncertainty and resource scarcity.*

Industrial production consumes enormous amounts of natural resources, which leads to the resource shortage and possible exhaustion. In addition, industrial emissions are among major contributors to environmental contamination. Technological renovation, or replacement of obsolete but still workable equipment with the newer and ecologically friendlier vintages, can greatly reduce the environmental damage. The challenge is that, in a general case, the rational equipment lifetime appears to be different for sequential replacement cycles and depends on future cost dynamics. The talk surveys recent developments in the optimal replacement of productive assets under stochastic evolving costs defined by external technological, economic and environmental factors, including resource scarcity. Two alternative techniques, the infinite-horizon cost minimization and the stopping problem of real options theory, will be compared. Results of numeric experiments and managerial implications of the obtained outcomes are discussed. (Received September 17, 2017)

1135-91-509 **Alexander Munson\*** (munson3141@gmail.com), 329 East King St., 2nd Floor, Chambersburg, PA 17201. *A Rigidity Theorem in Pillage Games.* Preliminary report.

We settle a question posed by MacKenzie, Kerber, and Rowat by proving that a continuous n-person pillage game cannot have infinitely many stable sets under a forward dominance condition. We do so by proving a rigidity theorem on dominance relations. Using this property, we derive a weak convergence theorem on the power set of the allocation space. Combining these results, we prove that pillage games with continuous power functions are essentially determined by its local behavior, leading to the main impossibility theorem. We also discuss the extension of our results in the wider scope where the continuity condition is lifted. (Received September 06, 2017)

1135-91-797 **Timmy Ma\*** (timmym@math.uci.edu), Department of Mathematics, University of California, Irvine, Irvine, CA 92697, and **Natalia L Komarova** (komarova@uci.edu). *Feature-Label-Order Effect in A Noisy Learning Environment.*

As we begin to understand more about symbolic learning, we are inclined to introduce layers of complexity to represent how our minds are able to process information and communicate to the world. We discuss how learners are able to differentiate and predict a label from objects or features and to predict features from a label. The differences between the two processes is known as the Feature-Label-Order effect. We present a stochastic model to study the Feature-Label-Order effect when noise elements are introduced into a learning environment. We report a novel experiment trial with human subjects and confirm with our model that there is a Feature-Label-Order effect when introducing noise. We demonstrate the key differences between the two different type of learners from our model, showing that Feature-Label learners exhibit a frequency boosting (regularization) property and are better equipped to process noise, whereas Label-Feature learners lack this ability. We discuss the implications of our findings to the nature of language learning and its importance to symbolic learning. (Received September 14, 2017)

1135-91-873 **Tucker E Evans\*** (tucker.e.evans.19@dartmouth.edu), Tucker Evans, Dartmouth College, 1314 Hinman, Hanover, NH 03755, and **Feng Fu**. *Why do they not believe?: The network dynamics of opinion.*

We model opinion-holding individuals as nodes in a weighted graph such that the connections between nodes represent the strength of influence that nodes have on the opinions of neighbors. Pulling from cognitive dissonance theory, we allow these connections and the opinions of the nodes to evolve under the influence of a fitness function that penalizes incongruity between the opinions of graph neighbors while rewarding stronger connections and higher magnitudes of opinion. We show that the relative emphases on each of these priorities can be used to characterize the resultant behavior of the network. We apply the model to a case study of the United States House of Representatives and the growing phenomenon of partisanship in American politics. (Received September 23, 2017)

1135-91-1229 **Benjamin M Adenbaum\*** ([adenbaub@lafayette.edu](mailto:adenbaub@lafayette.edu)), 524 Clinton Terrace, Easton, PA 18042, **Robert G Root** ([robroot@lafayette.edu](mailto:robroot@lafayette.edu)), Math Department, Lafayette College, Easton, PA 18042, and **R Carter Tilquist**, 111 Engineering Drive, ECOT 717, 430 UCB, Boulder, CO 80309. *Playing the Iterated Prisoners' Dilemma with Cards*. Preliminary report.

The Prisoners' Dilemma and its iterated form are well studied models in Evolutionary Game Theory. We model the iterated game modified by imposing stochastic constraints on future play with a representation as a card game, the Iterated Prisoners' Dilemma with Cards (IPDwC). In the unconstrained iterated game, the base Nash Equilibrium of always defecting enables the creation of Cooperative Equilibria promised by the Folk Theorem. In IPDwC, away from special cases, the strategy profile of defecting whenever possible ceases to be a Nash equilibrium, and by considering the IPDwC as a Markov process, we are able to identify new Nash equilibria in two distinct regimes. These new equilibria can play the analogous role in a Folk Theorem for the constrained game. Further, we demonstrate that the simplest mixed strategies all result in expected payoffs equivalent to random choice among available strategies. These results together demonstrate that stochastic constraints on future play can lead players to unforced cooperative play, suggesting that such constraints might serve as an evolutionary mechanism in the development of cooperative behavior. (Received September 20, 2017)

1135-91-1257 **Sophia R Mancini\*** ([mancinsr@dukes.jmu.edu](mailto:mancinsr@dukes.jmu.edu)). *Anti-Games on Steiner Triple Systems*. The card game Set can be turned into a two-player tic-tac-toe style game: all cards are laid out on a table, and two players alternate taking one card at a time. The winner is the first to hold a set in their hand. "Anti-Set" is a variation of this game in which the first player to hold a set loses the game. Previous researchers found a first-player winning strategy for Anti-SET and related games. In this talk, I generalize the game of Anti-Set to a larger category of combinatorial objects called Steiner Triple Systems. These well-studied objects share many of the key geometric features that determine the winning strategy for Anti-Set. I establish a winning strategy for these games using geometric and combinatorial properties of Steiner Triple Systems. This research was conducted as part of the 2017 REU program at Grand Valley State University. (Received September 20, 2017)

1135-91-1534 **Ho-Chun Herbert Chang\*** ([herbert.hc.chang@dartmouth.edu](mailto:herbert.hc.chang@dartmouth.edu)), Hinman 3853, Dartmouth College, Hanover, NH 03755. *Selective Hedging Against Regret: The Amnesiac Lookback Option*.

Everyone has had the experience and regret of not buying something at the right time. The Lookback Option, created in the late 20th century for volatile commodities like gold, is a financial instrument that hedges against this regret. The buyer pays a premium for the Lookback Call (Put) Option for the right to buy an underlying asset at its minimum (or sell at maximum) within a specified time period. This allows the holder to effectively buy or sell the underlying at its optimal price. However, as the condition of looking at all the previous prices is so strong, the premium of Lookback Options is very high and prevents the option from being more extensively traded. This talk presents the Amnesiac Lookback Option, which reduces the premium of a discrete Lookback Option by restricting the periods that can be looked back on. A fair price is shown using Monte Carlo simulations and compared with an adjusted binomial valuation. The Amnesiac Lookback's use is then extended to cryptocurrencies, volatile commodities enabled by blockchain technology, which have recently been cleared for options and other financial derivatives trading. We conclude by showing that the region between 0% to 20% of the total periods provide regions for investors to precisely balance their risk and regret. (Received September 22, 2017)

1135-91-1661 **Robert R Reitano\*** ([rreitano@brandeis.edu](mailto:rreitano@brandeis.edu)), Brandeis University, 415 South Street, Mailstop 032, Waltham, MA 02453. *Mathematics of Quantitative Finance and Actuarial Science: A Biased Viewpoint*.

This presentation will be a somewhat biased survey of some of the mathematics used in quantitative finance and actuarial science. It is biased by the background of the author which began with research in mathematical analysis (PDEs), then training as a life insurance actuary, and then a career in investment strategy and risk management. Consequently, mathematical applications in quantitative finance and actuarial science will be largely within the life insurance industry, although most of the models are widely used in the banking and investment management industries with little adjustment. Not surprisingly a wide variety of probability models play dominant roles (elementary and measure-theoretic), but so too does calculus (single/multi-variate), linear algebra (including quadratic forms), and differential equations (mostly SDE/PDE, some ODE). (Received September 24, 2017)

1135-91-1731 **Shiyun Wang\*** ([shiyun.wang@student.csulb.edu](mailto:shiyun.wang@student.csulb.edu)), 5979 E Pacific Coast Hwy Apt 7, Long Beach, CA 90803. *Connection between graphical potential games and Markov Random Fields with an extension to Bayesian Networks.*

Abstract: A probabilistic graphical model is a graphical representation of a joint probability distribution, in which the conditional independencies among random variables are specified via an underlying graph. We review the connection between Markov networks and graphical potential games that was given in (Babichenko and Tamuz, 2016), and show how Bayesian networks can be similarly connected to games on directed graphs. In particular, we study the structure of Bayesian networks, define a class of games on directed graphs, and explore the connections between such games and Bayesian networks. In doing so, we show how to construct such games from Bayesian networks and examine properties of this translation. (Received September 24, 2017)

1135-91-1798 **Rachel Matheson\*** ([ramatheson@vassar.edu](mailto:ramatheson@vassar.edu)), 124 Raymond Avenue, Box 1633, Poughkeepsie, NY 12604-1633, and **Jaysha Camacho, Juliana Noguera, Brandon Summers, Nanda Mallapragada and Baojun Song.** *Transportation Networks Optimized for Various Income Groups and their Impact on the Spread of Airborne Disease.*

With growing reliance on mass transit systems in American cities, questions of access become more important. This study aims to explore the spread of infectious disease across a transportation network created to optimize access to most frequented destinations for distinct socioeconomic groups. First, we develop a theoretical model of a city, based on the Kohl model for urban growth which assumes distinct regions where transit-dependent income groups live and work. In this framework, we maximize “satisfaction,” a measure of how easily the population of a neighborhood can travel to desirable destinations, through placement of bus routes. Within this framework we connect a single-outbreak multi-patch SIR model of Influenza A, incorporating the effects of attraction and travel time into the incidence rate. We track the populations’ interactions through contact within their neighborhoods, within the transit network, and with other transit-connected neighborhoods. We observe how the basic reproductive number is affected by the layout of the optimized transportation network. Results show that use of public transportation largely does not affect the global epidemic but that more equal time spent in transit leads to less disparate patch-specific epidemic outcomes. (Received September 24, 2017)

1135-91-2773 **Daniel B Larremore\*** ([daniel.larremore@colorado.edu](mailto:daniel.larremore@colorado.edu)), **Caterina De Bacco** and **Cristopher Moore.** *A physical model for efficient ranking in networks.*

We present a principled model and algorithm to infer a hierarchical ranking of nodes in directed networks, which we call SpringRank. Unlike other methods such as minimum violation ranking, it assigns real-valued scores to nodes rather than simply ordinal ranks, and it formalizes the assumption that interactions are more likely to occur between individuals with similar ranks. It provides a natural framework for a statistical significance test for distinguishing when the inferred hierarchy is due to the network topology or is instead due to random chance, and it can be used to perform inference tasks such as predicting the existence or direction of edges. The ranking is inferred by solving a linear system of equations, which is sparse if the network is; thus the resulting algorithm is extremely efficient and scalable. We illustrate these findings by analyzing real and synthetic data and show that our method outperforms others, in both speed and accuracy, in recovering the underlying ranks and predicting edge directions. (Received September 26, 2017)

1135-91-2798 **Winfried Just, David Gerberry and Ying Xin\*** ([yx123812@ohio.edu](mailto:yx123812@ohio.edu)). *Should I get a flu shot? How well did this go last year?* Preliminary report.

The problem of how to induce people to make vaccination decisions that would be optimal from the point of view of the population has been widely studied over the last decade. When vaccination carries real or perceived costs, individual rational choices in the so-called vaccination game are known to lead to Nash equilibria that are suboptimal at the population level. In reality, the underlying assumption of perfect rationality is unrealistic; actual decision-making is more likely based on a mixture of rational calculations of (mis)perceived costs, imitation, and individual prior experience. We discuss existing models of these decision-making procedures for flu vaccination and present a model for studying vaccination strategies that are based exclusively on the host’s most recent experience, but are not necessarily rational or deterministic. The model is then used as a tool for elucidating the role of behavior based on individual experience first in isolation and then in combination with decision procedures that incorporate additional information. (Received September 26, 2017)

1135-91-2813 **Oyita Udiani\*** ([oyita.udiani@nimbios.org](mailto:oyita.udiani@nimbios.org)), NIMBioS, University of Tennessee, Knoxville, TN 37966. *Models of social advocacy on networks.*

Members of the public hold opposing positions on a multitude of social issues. For example, although there is scientific consensus that climate change is caused by human activity, a sampling of popular media makes it clear

that opposing discourses are alive and well in the United States. As with many social issues, there are several organizations working at the grassroots to convince the public that climate change is of concern, and should be addressed through changes in policy. Achieving success depends on a number of factors, but none is perhaps more important than understanding how to mobilize and persuade individuals within communities of interest. In this talk, I will introduce a mathematical model to study this question. The model investigates trade-offs between common persuasion strategies (e.g., individual vs. collaborative) relative to their implementation costs and ability to achieve specific network objectives. Examples discussed include campaigns to convert skeptics (maximizing prevalence of beliefs) vs. mobilize converts (maximizing extremism of beliefs). (Received September 26, 2017)

## 92 ► *Biology and other natural sciences*

1135-92-73 **Allison R Torsey\*** (allison.torsey@gmail.com) and **Saziye Bayram** (bayrams@buffalostate.edu). *A Stochastic SIRI Model of Opioid Abuse.*

A opioid epidemic has overtaken the United States in recent years. In 2015, there were 238 opioid overdoses in Erie County, NY alone. Previously, infectious disease models, specifically SIR models, have been modified to apply to drug use. We've developed a stochastic SIRI model to study the behavior of opioid abuse in Erie County, New York. This would be the first SIRI model studying opioids to be applied to a region. We analyzed simulation results, obtained from MATLAB, to predict how the system would progress over the course of a year. This estimation of a stochastic SIRI model might be used to predict the usage and spread of multiple illicit drugs. As well as it might influence legislative decisions on the most effective actions to stop the outbreak in not only Erie County but around the United States. (Received July 19, 2017)

1135-92-81 **John C Lang\*** (jclang@ucla.edu), **Hans De Sterck** and **Daniel M Abrams**. *The statistical mechanics of human weight change.*

Over the past 35 years there has been a near doubling in the worldwide prevalence of obesity. However, the processes that determine the changing shape of Body Mass Index (BMI) distributions in high-income societies are not well understood. By compiling and analyzing the largest data set so far of year-over-year BMI changes, we find that the distribution of human BMIs is intrinsically dynamic and is determined by a balance between deterministic drift towards a natural set point and diffusion resulting from random fluctuations in, e.g., diet and physical activity. We formulate a stochastic mathematical model for BMI dynamics, deriving a theoretical shape for the BMI distribution and offering a mechanism to explain the ongoing right-skewed broadening of BMI distributions over time. The model also provides new quantitative support for the hypothesis that peer-to-peer social influence plays a measurable role in BMI dynamics. Implications for public health interventions are discussed. (Received July 24, 2017)

1135-92-202 **Michelle DeDeo\*** (mdedeo@unf.edu), 1 UNF Dr., Department of Mathematics & Statistics, Jacksonville, FL 32224, and **Maanasi Garg**, 1 UNF Dr., Mathematics & Statistics, Jacksonville, FL 32224. *Seizure Prediction using Spectral Density Analysis on Pediatric EEGs.*

In this paper, the authors analyzed specific frequencies within the gamma band that contribute a relatively high amount of power to the electrical activity during pediatric patients' seizures in order to determine if seizure (pre-ictal) activity was present before a seizure (ictal) event. In order to accomplish this, we evaluated the power spectral density graph not over a range of frequencies, but at specific single frequencies using a variation of log power spectral density. The analysis then compares the log power spectral densities of each patient's ictal and inter-ictal episodes. This allowed identification of frequencies within the gamma band to contribute high amounts of power during seizures relative to the amount of power they contribute normally. In addition, all of the EEG recordings containing each patient's seizures were epoched to extract the "trials" of his study using EEGLAB, a MATLAB add-on, and ERP analysis was performed in an attempt to predict pre-seizure activity. Once the seizure files were epoched, the potentials (amplitudes) of each trial were color-coded and converted to a rectangular color. Proper seizure prediction through a medical device tuned to a particular patient would allow for seizure patients to be given warning before their seizures begin. (Received August 08, 2017)

1135-92-297 **Daniel Maxin**, **Paul Georgescu**, **Laurentiu Segă\*** (lsegă@augusta.edu) and **Ludek Berec**. *Global stability of the coexistence equilibrium for a general class of models of facultative mutualism.*

Many models of mutualism have been proposed and studied individually. In this paper, we develop a general class of models of facultative mutualism that covers many of such published models. Using mild assumptions

on the growth and self-limiting functions, we establish necessary and sufficient conditions on the boundedness of model solutions and prove the global stability of a unique coexistence equilibrium whenever it exists. These results allow for a greater flexibility in the way each mutualist species can be modeled and avoid the need to analyze any single model of mutualism in isolation. Our generalization also allows each of the mutualists to be subject to a weak Allee effect. Moreover, we find that if one of the interacting species is subject to a strong Allee effect, then the mutualism can overcome it and cause a unique coexistence equilibrium to be globally stable.

(Received August 21, 2017)

1135-92-394 **Yijun Lou\*** (yijun.lou@polyu.edu.hk), Department of Applied Mathematics, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, and **Xiao-Qiang Zhao** (zhao@mun.ca), Department of Mathematics and Statistics, Memorial University of Newfoundland, St. Johns, NL, Canada. *A theoretical approach to understanding population dynamics with seasonal developmental durations.*

Understanding the population dynamics subject to seasonally changing weather conditions plays a fundamental role in predicting the trends of population patterns and disease transmission risks under the scenarios of climate change. With the host-macroparasite interaction as a motivating example, this talk proposes a synthesised approach for investigating the population dynamics subject to seasonal environmental variations from theoretical point of view, where the model development, basic reproduction ratio formulation and computation, and rigorous mathematical analysis are involved. The resultant model with periodic delay presents a novel term related to the rate of change of the developmental duration, bringing new challenges to dynamics analysis. The synthesised approach developed here is applicable to broader contexts of investigating biological systems with seasonal developmental durations. (Received August 31, 2017)

1135-92-404 **Jacob P. Duncan\*** (jduncan@saintmarys.edu), **Monica McGrath** and **Teresa M. Aubele-Futch**. *Breaking the Vicious Limit Cycle: Addiction Relapse-Recovery As a Fast-Slow Dynamical System.* Preliminary report.

Symptoms of addictive disorders often manifest as periodic episodes of sudden relapse followed by relatively long periods of recovery. For certain types of addiction, a relapse is precipitated by a state of elevated well-being wherein cravings supersede cessation efforts. A relapse satiates cravings temporarily, but is usually followed by a state of depression, which slowly improves as cravings reintensify. To analyze the underlying mechanisms driving relapse-recovery cycles, we construct a fast-slow dynamical system model of the interaction between an addict's propensity to relapse and their current disposition, i.e., craving and mood. The model captures the dynamics of addiction relapse and recovery phenomenologically by admitting relaxation oscillations, which we prove exist by exploiting timescale separation. We derive predictions of cycle period and amplitude to measure relapse frequency and intensity, respectively. As a parameter identified as being responsive to treatment is varied, the system transitions from a state of periodic relapse-recovery to a relapse-free state through reverse Hopf bifurcation. We calculate the threshold value of the treatment parameter, which corresponds to the equilibrium point passing through the fold of the critical manifold. (Received August 31, 2017)

1135-92-436 **Necibe Tuncer**, **Hayriye Gulbudak**, **Vincent Cannataro** and **Maia Martcheva\*** (maia@ufl.edu), 147 Hudson Cove, Edgewater, NJ 07020. *Structural and Practical Identifiability Issues of Immuno-Epidemiological Vector-Host Models with Application to Rift Valley Fever.*

We discuss the structural and practical identifiability of a nested immuno-epidemiological model of arbovirus diseases. We fit this multi-scale model to multi-scale data. For an immunological model, we use Rift Valley Fever Virus (RVFV) time-series data obtained from livestock under laboratory experiments, and for an epidemiological model we incorporate a human compartment to the nested model and use the number of human RVFV cases reported by the CDC during the 2006-2007 Kenya outbreak. We show that the immunological model is not structurally identifiable for the measurements of time-series viremia concentrations in the host. Thus, we study a scaled version of the immunological model which is structurally globally identifiable. After fixing estimated parameter values for the immunological model, we fit observable RVFV epidemiological data to the nested model for the remaining parameter. Monte Carlo simulations indicate that only three parameters of the epidemiological model are practically identifiable. Alternatively, we fit the multi-scale data to the multi-scale model simultaneously. Monte Carlo simulations for the simultaneous fitting suggest that the parameters of the immunological model and the parameters of the immuno-epidemiological model are practically identifiable.

(Received September 02, 2017)

- 1135-92-442 **Heinz Siedentop\*** (h.s@lmu.de), Mathematisches Institut, Ludwig-Maximilians-Universität München, Theresienstr. 39, 80333 München, Germany, and **Li Chen**. *Blow-Up of Solutions to the Patlak-Keller-Segel Equation in Dimension  $\nu \geq 2$ .*

We prove a blow-up criterion for the solutions to the  $\nu$ -dimensional Patlak-Keller-Segel equation in the whole space. The condition is new in dimension three and higher. In dimension two it is exactly Dolbeault's and Perthame's blow-up condition, i.e., blow-up occurs, if the total mass exceeds  $8\pi$ . (Received September 03, 2017)

- 1135-92-505 **Marissa Renardy\*** (renardy.1@osu.edu), **Tau-Mu Yi**, **Dongbin Xiu** and **Ching-Shan Chou**. *A method for sensitivity analysis and parameter estimation applied to a large reaction-diffusion model of cell polarization*. Preliminary report.

Parameter sensitivity analysis and parameter estimation are challenging in many biological systems due to large parameter counts. Too many parameters can make sampling of the parameter space computationally implausible, especially for partial differential equation models that are expensive to solve. In this work, we propose a method for parameter sensitivity analysis and parameter estimation that can be significantly less costly for large problems. The key component in this method is the construction of a polynomial surrogate model. We demonstrate the method by applying it to a large reaction-diffusion model of yeast cell polarization with 35 parameters. (Received September 06, 2017)

- 1135-92-529 **Ellie Mainou\*** (ellie.mainou@emory.edu), 2310 Ava Place, Decatur, GA 30033, and **Robert Dorit**, **Gwen Spencer** and **Dylan Shepardson**. *Parsing a Crowd of Near Best Fits: Consensus Ranges for Drug-Resistant Tuberculosis Modeling*. Preliminary report.

According to WHO, one third of the world's population is infected with tuberculosis (TB), with drug resistance posing a major challenge to the management of TB. Strains resistant to the most common drugs for treatment (e.g. isoniazid, rifampin) are already widespread. To address the issue, we constructed a mathematical compartmental model described by differential equations depicting the transmission of TB in the US. The model encompasses four strains of different resistance to antibiotics, while taking into consideration immigration, which greatly contributes to the excessive transmission of latent TB. A set of 27 parameters was fitted to recent CDC data on TB morbidity and mortality, using a genetic algorithm to minimize an error function. Producing reliable fits, by generating random parameter values within the ranges obtained from real-world data. Local minima were identified and multiple sensitivity analysis tests were performed to identify which parameters the model is sensitive to. (Received September 07, 2017)

- 1135-92-539 **Ronald E. Mickens\*** (rmickens@cau.edu), Clark Atlanta University, Department of Physics, Atlanta, GA 30314. *Some Methodologies for Constructing Discrete-Time Population Models*.

The lack of fundamental rules for the construction of interacting population models has not prevented their appearance in the research literature. Our goal is to present several methodologies for generating such models and demonstrate their value by explicitly building schemes for both single- and two-interacting populations. This work is an extension of previous results by Bhattacharya [1] and Mickens [2]. The central issue is to derive discrete-time models such that positivity holds for the dynamics of the populations. We also discuss the relevance of these methodologies for the numerical integration of differential equations [3].

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- 1135-92-573 **William F. Fagan\*** (bfagan@umd.edu), Dept. of Biology, University of Maryland, College Park, MD 20742, and **Sharon Bewick**, **Jeffery Demers**, **Folashade Augusto**, **Justin M Calabrese** and **Bingtuan Li**. *How different disease transmission modes (vector-borne and sexual) and different dispersal types (vector and host) affect the spread speed of a Zika virus invasion*.

Increasing human mobility and globalization transport invasive species into new regions. Invasive diseases have particularly dire implications, as demonstrated by the arrival of Zika virus in the Americas. Invasive diseases may spread rapidly from their point of introduction. However, unlike most invasive organisms, the rate of spread of invasive diseases depends less on the dispersal characteristics of the disease itself, and more on the dispersal characteristics of the disease host(s). Because vector-borne diseases involve multiple hosts, they offer particularly



interesting systems for studying disease invasion rates. This is especially true for of Zika, where the different hosts are more/less important, depending on the mode of disease transmission – i.e., vector vs. sexual transmission. Using a reaction-diffusion model, we study how the relative rates of vector and sexual transmission interact with vector and human mobility to determine the overall spread speed of a Zika invasion. Our analysis shows that the balance between transmission mode and host mobility has profound implications for disease invasion rates, even without strongly impacting local dynamics. Ultimately, this offers an explanation for the explosive spread speed of Zika. (Received September 08, 2017)

1135-92-611 **Haiyan Wang\*** ([haiyan.wang@asu.edu](mailto:haiyan.wang@asu.edu)). *Combining networks and PDE models to improve influenza predictions*. Preliminary report.

The rich and big data generated by millions of users on social media can be used for predictions in a rapid and accurate fashion. In recent years many researchers have explored real-time streaming data from Twitter for a broad range of applications such as predicting flu trends. In this talk, we present our design and implementation of a prototype system to collect flu related twitter data. Further we use partial differential equation models describe/predict epidemic outbreaks based on the twitter data. We correlate the results with official statistics from Center for Disease Control and Prevention (CDC). These results demonstrate that the system can be used to real-timely monitor the spread of influenza (Received September 10, 2017)

1135-92-612 **Mihiri De Silva**, Lubbock, TX, and **Sophia Jang\*** ([sophia.jang@ttu.edu](mailto:sophia.jang@ttu.edu)), Lubbock, TX 79409. *Period-doubling and Neimark-Sacker bifurcations in a larch budmoth population model*.

We investigate a discrete consumer-resource system based on a model originally proposed for studying the cyclic dynamics of the larch budmoth population in the Swiss Alps. It is shown that the moth population can persist indefinitely for all of the biologically feasible parameter values. Using intrinsic growth rate of the consumer population as a bifurcation parameter, we prove that the system can either undergo a period-doubling or a Neimark-Sacker bifurcation when the unique interior steady state loses its stability. (Received September 10, 2017)

1135-92-630 **Shandelle M. Henson\*** ([henson@andrews.edu](mailto:henson@andrews.edu)), **J. M. Cushing** ([cushing@math.arizona.edu](mailto:cushing@math.arizona.edu)) and **J. L. Hayward** ([hayward@andrews.edu](mailto:hayward@andrews.edu)). *Periodic matrix models for seasonal dynamics of stage structured populations II: Application to a seabird colony*.

Increased sea surface temperatures (SSTs) are associated with decreased resource levels for surface-feeding seabirds. Recent field observations of colonial seabirds in the Pacific Northwest have shown that increased SSTs also are associated with increased egg cannibalism and every-other-day egg-laying synchrony. Egg cannibalism may be adaptive when resources are low, given that one egg contains about half the daily calories needed for one adult bird. The data also show that eggs laid synchronously have less chance of being cannibalized. We apply the theory in the previous talk, Part I, to a particular class of population models for colonial seabirds that accounts for egg cannibalism and egg-laying synchrony. We track four life-stage categories (first-day eggs, older eggs and juveniles, reproductive adults, and nonreproductive adults) on a daily time scale through the breeding season and then look at the dynamic consequences of cannibalism and synchrony across breeding seasons. The possibility of backward bifurcations and strong Allee effects shows that the adaptive individual behaviors may confer an advantage at the population level, as well. (Received September 11, 2017)

1135-92-645 **Alex P Farrell**, **James P Collins**, **Amy L Greer** and **Horst R Thieme\*** ([hthieme@asu.edu](mailto:hthieme@asu.edu)), School of Mathematical, and Statistical Sciences, Arizona State University, Tempe, AZ. *Can infectious pathogens drive their host populations into extinction?*

Amphibian decline and disappearance have renewed interest in the part infectious diseases have in the extinction of their host species. In simple SI epidemic and endemic models, three classes of incidence functions are identified for their potential to be associated with host extinction: Upper density-dependent incidences are never associated with host extinction. Power incidences that depend on the numbers of infectives and susceptibles by powers strictly between 0 and 1 are associated with initial-constellation-dependent host extinction for all parameter values. Homogeneous incidences, of which frequency-dependent incidence is a very particular case, and power incidences are associated with global host extinction for certain parameter constellations and with host survival for others. This leaves the question undecided that motivated this analysis, namely whether ranavirus epidemics can drive tiger salamander populations into extinction. Laboratory infection experiments with salamander larvae are equally well fitted by power incidences and certain upper density-dependent incidences such as the negative

binomial incidence and do not rule out homogeneous incidences such as an asymmetric frequency-dependent incidence either. (Received September 11, 2017)

1135-92-651 **Azmy S Ackleh\*** (ackleh@louisiana.edu), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504, and **Amy M Veprauskas, John E Banks** and **John D Stark**. *Changes in population outcomes resulting from evolutionary responses to a disturbance*. Preliminary report.

Prolonged exposure to a disturbance such as a toxicant has the potential to result in rapid evolution of toxicant resistance in many short-lived species. This evolution may allow a population to persist at higher levels of the toxicant than is possible without evolution. Here we apply evolutionary game theory to Leslie matrix models to obtain Darwinian equations that couple population and evolutionary dynamics. We consider two cases for which evolution of toxicant resistance may have important dynamic consequences. In the first case, we examine how persistence outcomes for surrogate *Daphnia* species may change when one species is able to persist by evolving toxicant resistance while another is not. In the second case, we consider how evolution of toxicant resistance may impact both predator and prey when a prey species evolves in response to a toxicant but the predator does not due to different time scales. We show that, under certain conditions, predator and prey may have opposite responses to prey evolution. This model is inspired by marine mammals which have significantly longer lifespans relative to their food sources. (Received September 11, 2017)

1135-92-676 **A. N. Laubmeier\*** (anjlaubmeier@gmail.com). *Validating a Trait-Based Model for Predator-Prey Dynamics in a System of Terrestrial Arthropods*.

There is an immediate need for models which can predict the effect of changing ecological communities, either due to species loss or migration, on trophic interactions. However, in order to describe these effects, we rely on generalizable models with foundations in physical traits that can be observed across study systems. We therefore consider the Allometric Trophic Network (ATN) model, a Lotka-Volterra type model parameterized by body mass and motivate a variation on the ATN model which accounts for species habitat use and allows for interspecific predator interference. We estimate model parameters using population data from a greenhouse experiment explicitly designed for the validation of this model. We are particularly interested in the importance of habitat use in determining predators' effects on one another and the implications this might have for the design of field-level experiments.

This is a joint work with collaborators at the Swedish University of Agricultural Sciences in Uppsala and California State University, Monterey Bay. (Received September 12, 2017)

1135-92-708 **Emily Kelting\*** (ekelting@uco.edu), **Brittany Bannish** and **Sean Laverty**. *Dynamics of *Toxoplasma gondii* in feral cats*. Preliminary report.

We discuss the transmission and effects of *Toxoplasma gondii* in a cat population. Due to the severity of *T. gondii* spillover infections in pregnant women and monk seals, understanding its dynamics in cats is key to unlocking preventative measures against this parasite. Taking into account susceptible, acutely infectious, and chronically infectious felines, and the surrounding environment, we build a differential equations model of *T. gondii* transmission in cats. We present our model and the results, identifying conditions under which infection is endemic. We conclude by discussing how this information can be used to minimize the risks to other species. (Received September 13, 2017)

1135-92-721 **Pak-Wing Fok\*** (pakwing@udel.edu), 412 Ewing Hall, Department of Mathematical Sciences, University of Delaware, Newark, DE 19716. *Mucosal Folding and Growth Instabilities in a Finite Element Model of an Atherosclerotic Artery*. Preliminary report.

Atherosclerosis is a disease that involves inflammation and remodeling of blood vessel walls. In this paper, we present a displacement-based finite element model of a growing arterial cross section, based on morphoelasticity theory. The model uses an anisotropic strain energy function to account for the presence of collagen fibers in the media and adventitia, and represents the intima as a growing, isotropic, Neo-Hookean solid.

We show through numerical simulations that intima growth for pseudo-realistic arterial cross sections results in a buckling instability, giving rise to mucosal folding at the lumen-intima interface. This type of growth is consistent with photomicrographs of diseased arteries and makes direct connections with the experimental results of Glagov, *New England Journal of Medicine* (1987). (Received September 25, 2017)

- 1135-92-869 **Hannah Callender Highlander\*** ([highland@up.edu](mailto:highland@up.edu)), 5000 N Willamette Blvd, University of Portland, MSC 60, Portland, OR 97203. *Open Problems for Undergraduates: Vaccination Strategies for the Control of Infectious Disease*.  
 Infectious diseases are a serious threat to our health. Vaccination often can prevent their spread, but typically it is not feasible to vaccinate absolutely everyone. Sometimes it is necessary to carefully target the group of individuals to whom a limited supply of vaccine should be administered in order to achieve the largest amount of overall protection for the whole population. A method for choosing the group to be targeted for maximal effect is called a vaccination strategy. The development of optimal vaccination strategies leads to interesting mathematical problems and requires some knowledge of the contact network of the given population. Here I will introduce several open problems in this area that my coauthor and I recently published in a book dedicated to open problems for undergraduates in mathematics. (Received September 15, 2017)
- 1135-92-939 **Kamuela Yong\*** ([kamuela.yong@hawaii.edu](mailto:kamuela.yong@hawaii.edu)). *Avoidance and perception of risk and their effects on an outbreak*. Preliminary report.  
 During an outbreak of an infectious disease, uninfected individuals often modify their behavior and are either attracted to, or repelled from, locations based on the perceived risks of infection at each location. Such perceived risks do not necessarily depend on the actual transmission rate or number of infected individuals, but rather on public perception of risk. These dynamics were illustrated with the 2014 Ebola outbreak. In this paper we use a two-dimensional reaction-diffusion-advection model to simulate the dynamics of perceived risks. (Received September 17, 2017)
- 1135-92-950 **Elizabeth Gross\*** ([elizabeth.gross@sjsu.edu](mailto:elizabeth.gross@sjsu.edu)), San Jose, CA 95112, and **Colby Long** ([long.1579@mbi.osu.edu](mailto:long.1579@mbi.osu.edu)). *Distinguishing phylogenetic networks*.  
 Phylogenetic networks are increasingly becoming popular in phylogenetics since they have the ability to describe a wider range of evolutionary events than their tree counterparts. In this talk, we discuss Markov models on phylogenetic networks, i.e. directed acyclic graphs, and their associated algebra, geometry, and combinatorics. In particular, assuming the Jukes-Cantor model of evolution and restricting to one reticulation vertex, using tools from computational algebraic geometry and combinatorics, we show that the semi-directed network topology of large-cycle networks is generically identifiable. (Received September 17, 2017)
- 1135-92-968 **Aijun Zhang\*** ([zhang0aijun@hotmail.com](mailto:zhang0aijun@hotmail.com)), Department of Mathematics, Oregon State University, Kidder Hall 368, Corvallis, OR 97331-4605. *Nonlocal Dispersals with Moving Habitats*.  
 The current talk is devoted to the study of nonlocal evolution equations with moving habitats. The reaction of the populations will change with the moving habitat of speed  $c$ . The species will become extinct in the habitat if the moving speed  $c$  goes faster than the spreading speed  $c^*$ , where  $c^*$  is determined by the maximum linearized growth rate function. The persistence of the species will depend on the patch size of the habitat if the moving speed  $c$  goes slower than the spreading speeds  $c^*$ . Due to lack of the techniques as those for reaction diffusion equations like comparison(maximum) principle, Harnack inequality, regularity theory etc, the approaches for reaction diffusion equations can not be applied directly. The tools such as spectral theory, comparison principle etc. of nonlocal equations we developed in the current study possess their own interests in further studying such nonlocal equations. This is a joint work with Drs. Patrick De Leenheer and Wenxian Shen (Received September 18, 2017)
- 1135-92-1014 **Xiang-Sheng Wang\*** ([xswang@louisiana.edu](mailto:xswang@louisiana.edu)), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70503. *Traveling waves in epidemic models: non-monotone diffusive systems with non-monotone incidence rates*. Preliminary report.  
 We study the existence and nonexistence of traveling waves of diffusive epidemic models with general incidence rates. The model systems are non-monotone because of the intrinsic predator-prey interaction between the susceptible and infective compartments in epidemic systems. Moreover, the incidence rate may not be monotone in the infected population because social behaviors and collective activities may change in response to the prevalence of disease. To find positive traveling solutions of the non-monotone system with a non-monotone incidence function, we will construct a suitable convex set in a weighted function space, and then apply Schauder fixed point theorem. It turns out that the basic reproduction number of the corresponding ordinary differential equations plays an important role in the existence theory of traveling waves. Moreover, the critical wave speed can be explicitly obtained in terms of the diffusion coefficient, recovery rate and removal rate for infected group, and partial derivative of incidence function at the disease-free equilibrium. Finally, we prove that the positive

traveling wave solution does not exist if the basic reproduction number is no more than one, or the wave speed is less than the critical value. (Received September 18, 2017)

1135-92-1026 **hal L Smith\*** (halsmith@asu.edu), School of mathematical & Statistical Sciences, Arizona State University, Tempe, AZ 85287. *Tragedy of the Commons in the Chemostat.*

We present a proof of principle for the phenomenon of the tragedy of the commons that is at the center of many theories on the evolution of cooperation. We establish the tragedy in the context of a general chemostat model with two species, the cooperator and the cheater. Both species have the same growth rate function and yield constant, but the cooperator allocates a portion of the nutrient uptake towards the production of a public good -the “Commons” in the Tragedy- which is needed to digest the externally supplied nutrient. The cheater on the other hand does not produce this enzyme, and allocates all nutrient uptake towards its own growth. We prove that when the cheater is present initially, both the cooperator and the cheater will eventually go extinct, hereby confirming the occurrence of the tragedy. We also show that without the cheater, the cooperator can survive indefinitely, provided that at least a low level of public good or processed nutrient is available initially. Our results provide a predictive framework for the analysis of cooperator-cheater dynamics in a powerful model system of experimental evolution. (Received September 18, 2017)

1135-92-1106 **Robert Stephen Cantrell, Chris Cosner\*** (gcc@math.miami.edu) and **Xiao Yu.**  
*Dynamics of populations with individual variation in dispersal on bounded domains.*

Most classical models for the movement of organisms assume that all individuals have the same patterns and rates of movement, but there is empirical evidence that movement rates and patterns may vary among individuals. One way to capture variation in dispersal is to allow individuals to switch between two distinct dispersal modes. We consider models for populations with logistic-type local population dynamics whose members can switch between two different nonzero rates of diffusion. The resulting reaction-diffusion systems can be cooperative at some population densities and competitive at others. We analyze the dynamics of such systems on bounded regions. (Traveling waves and spread rates have been studied by others for similar models in the context of biological invasions.) The analytic methods include ideas and results from reaction-diffusion theory, semi-dynamical systems, and bifurcation/continuation theory. (Received September 19, 2017)

1135-92-1125 **Jimin Zhang and Junping Shi\*** (jxshix@wm.edu), Department of Mathematics, College of William and Mary, Williamsburg, VA 23187-8795, and **Xiaoyuan Chang.** *A mathematical model of interaction of pelagic algae, benthic algae and nutrients in an oligotrophic shallow aquatic ecosystem.*

A coupled system of ordinary differential equations and partial differential equations is proposed to describe the interaction of pelagic algae, benthic algae and one essential nutrient in an oligotrophic shallow aquatic ecosystem with ample supply of light. The existence, uniqueness and stability of non-negative steady states are shown, and these results characterize some threshold conditions for the regime shift. The influence of environmental parameters on algal biomass density is also considered, which is an important indicator of algal blooms. (Received September 19, 2017)

1135-92-1128 **Wenjie Ni and Junping Shi\*** (jxshix@wm.edu), Department of Mathematics, College of William and Mary, Williamsburg, VA 23187-8795, and **Mingxin Wang.** *Global stability and pattern formation in a nonlocal diffusive Lotka-Volterra competition model.*

A diffusive Lotka-Volterra competition model with nonlocal intraspecific and interspecific competition between species is formulated and analyzed. The nonlocal competition strength is assumed to be determined by a diffusion kernel function to model the movement pattern of the biological species. It is shown that when there is no nonlocal intraspecific competition, the dynamics properties of nonlocal diffusive competition problem are similar to those of classical diffusive Lotka-Volterra competition model regardless of the strength of nonlocal interspecific competition. Global stability of non-negative constant equilibria are proved using Lyapunov or upper-lower solution methods. On the other hand, strong nonlocal intraspecific competition increases the system spatiotemporal dynamic complexity. For the weak competition case, the nonlocal diffusive competition model may possess nonconstant positive equilibria for some suitably large nonlocal intraspecific competition coefficients. (Received September 19, 2017)

1135-92-1169 **Suzanne Lenhart\*** (lenhart@math.utk.edu), University of Tennessee, NIMBioS, 1122 Volunteer Blvd. Suite 106, Knoxville, TN 37996-3410. *Modeling the spread of La Crosse virus in Knox County, Tennessee.* Preliminary report.

La Crosse virus (LACv) is an arbovirus found commonly in southern Appalachia, and is of particular public health concern because of its potential for negative health impacts on children. LACv transmission depends

on mosquito population dynamics, which in turn depend on environmental factors such as temperature and precipitation; generally, higher temperatures and increases in accumulated precipitation correspond to greater mosquito population. We use a system of ordinary differential equations to investigate the role of temperature and precipitation on the dynamics of LACv vectors over a single season in Knox County, TN. We use locally-collected mosquito population data (in collaboration with R. Trout Fryxell) to parametrize our model. Some of this work is from an REU project. (Received September 20, 2017)

1135-92-1188 **Ricardo Sanchez\*** (rsanchezh70@hotmail.com) and **Lucia Barrera Arenas**. *Key concepts on mathematics for bioinformatics.*

The mathematical community and the non-mathematical community working on Bioinformatics have the benefit of numerous algorithms available for the analysis of the genomes and associated concepts. Concepts such as the Manhattan Tourist Problem, Euclidean Distance, Hamming Distance use equations such as  $d(P,Q)+d(Q,R) \geq d(P,R)$ , known as the triangle inequality may use the BLAST program on the NIH website. Other Graph Theory algorithms are available for the construction of phylogeny trees inside BLAST and others such as MEGA that includes multiple alignment, EMBOSS for Global and Local Alignments. There are more concepts applied here from Combinatorics, String Theory to analyze insertions and deletions on the sequences  $(v,w)$ ;  $d(v,w)=n+m-2s(v,w)$ , with  $s$ , the length of the string. Many of these algorithms are available for researchers and students. (Received September 20, 2017)

1135-92-1193 **Madel R. Liquido\*** (mliquido15@saintpeters.edu), Saint Peter's University, Department of Mathematics, 2641 Kennedy Blvd, Jersey City, NJ 07306, and **Nickolas Kintos** (nkintos@saintpeters.edu), Saint Peter's University, Department of Mathematics, 2641 Kennedy Blvd, Jersey City, NJ 07306. *A simplified mathematical model to explore the output of a rhythmic neural network.*

The electrical output of rhythmically active neural networks is ubiquitous in the nervous systems of animals. Such networks are driven by a core circuit called a central pattern generator (CPG), and, due to their stereotyped activity, CPGs are tractable for mathematical analysis. We examine the gastric mill rhythm (GMR) of the crab, *Cancer borealis*, whose CPG activity is composed of the half-center oscillation between the lateral gastric (LG) neuron and interneuron 1 (Int1). The GMR is activated by synaptic input from the axon terminals of the projection neuron, modulatory commissural neuron 1 (MCN1). Using the difference in fast-slow time scales that exist within the biological system, we construct a simplified, 2-dimensional ODE model of the GMR. Through phase plane analysis of our singularly perturbed system, we show that the MCN1-LG synaptic interactions drive the GMR oscillations in the model. Also, we show that the addition of a fast, synaptic input onto Int1 strongly influences the GMR frequency in the model. Our results agree with those of experiments, and our simplified model captures the underlying network dynamics of this CPG circuit. (Received September 20, 2017)

1135-92-1241 **Mansoor A Haider\*** (mahaider@ncsu.edu), **Christine Mennicke** and **Troy Ghashghaei**. *Branching random walk models for cell differentiation in developmental neurobiology.*

We model the balance between cell expansion and differentiation in the neurogenesis-to-gliogenesis switch (NGS) regime of developmental neurobiology. Models are developed in the context of Mosaic Analysis with Double Markers (MADM), an advanced imaging technique that tracks cell lineage trees. The stochastic nature of cell differentiation, imaging of only terminal neuron and glial cells, and variability across mice make the inverse problem challenging. We present a branching random walk (BRW) model for the temporal evolution of cell lineage trees based on two fundamental differentiation rules for neural and glial stem cells. Distributions of cell lineage trees are computed via simulations over many realizations and compared to the data. The underlying forward problem is combinatorially complex while the associated inverse problem involves significant uncertainty. Recursive analytical formulas for terminal cell counts were derived for the simpler cases of neuron-only trees and glia-only trees. For the more complex cases of mixed trees, direct numerical simulations were performed using the BRW model. Results are presented to assess how well probabilities in the BRW model can quantify and delineate developmental stages as well effects of deletion of genes implicated in the NGS. (Received September 20, 2017)

1135-92-1337 **Josh Hiller\*** (johiller@adelphi.edu). *Modeling market based deforestation prevention policy: the effect of fluctuating commodity prices and industrial agriculture.*

Over the past several decades, market based deforestation-prevention policies have been lauded as a "free-market" approach to deforestation prevention in some of the most endangered ecosystems of the world. Many Markov models have been proposed to capture different aspects of the decision making process. However, most of these

models have assumed that agricultural utility remains constant throughout time. In some cases this may be appropriate. Recently though, evidence has been presented suggesting that deforestation in some ecosystems is primarily driven by fluctuations in commodity prices and the presence of modern agricultural technologies. In this talk we will discuss a simple Markov model which tries to capture the effect of commodity fluctuations and industrial agriculture on the effectiveness of market based deforestation prevention policies. If time permits, we will also touch on issues of spatial-temporal adverse selection (Received September 21, 2017)

1135-92-1366 **Andrea Young\***, [younga@ripon.edu](mailto:younga@ripon.edu). *Outsourcing Undergraduate Research: Using Community Based Research to Generate Undergraduate Research Problems*. Preliminary report.

Community based research is a collaborative endeavor between students, faculty, and community members that addresses a need identified by the community. From the standpoint of undergraduate research, community based problems can provide a rich mathematical experience for students as well as a broader context into which they can situate their work. I will talk about my experience doing community based research, and I will give some thoughts about how to engage your community in mathematical research. (Received September 21, 2017)

1135-92-1429 **Lake R Ritter\***, Math Building D, 1100 S. Marietta Pkwy MD 9085, Marietta, GA 30060. *A delay differential equation model of activation of endothelial nitric oxide synthase*.

Regulation of nitric oxide (NO) serves several functions in the vasculature related to homeostasis, adaptation, and development. Endothelial nitric oxide synthase (eNOS) is the primary enzyme in the vasculature that synthesizes and regulates NO. Various enzymes, kinases and phosphatases, influence eNOS through phosphorylation and dephosphorylation of its amino acid sites. Evidence of oscillation between inactive and active states of eNOS has been detected experimentally consistent with feedback mechanisms in signal transduction. Here we consider a feedback model of eNOS activation in the form of a system of coupled ordinary differential equations. By the introduction of time delays, we account for the more complex dynamics of a signal cascade (formation of protein complexes, diffusion, interactions of unspecified intermediaries, etc.). Under conditions on the model parameters, varying the time delay may give rise to a Hopf bifurcation. Properties of resulting oscillatory solutions are discussed. (Received September 22, 2017)

1135-92-1443 **Diana Schepens\*** ([diana.schepens@gmail.com](mailto:diana.schepens@gmail.com)), **Ross Carlson, Jeff Heys, Ashley Beck** and **Tomas Gedeon**. *Emergence of Cooperativity in Microbial Consortia*.

Metabolic cross-feeding between microbes is observed in many microbial communities. It has been experimentally observed that cross-feeding synthetic communities have an increased level of fitness and cell growth as compared to wild type cells. There are also numerous examples of cross-feeding communities in nature.

We developed a model to analyze the conditions under which cross-feeding emerges in a microbial community. Specifically, we consider how over-production rates in community specialists affect the emergence of cooperation in microbial consortia. We also analyze the different behavior resulting from models which use passive transport of metabolites compared to models that incorporate active transport. (Received September 22, 2017)

1135-92-1447 **Ben G Fitzpatrick\*** ([bfitzpatrick@lmu.edu](mailto:bfitzpatrick@lmu.edu)), 1 LMU Drive, UH 2700, Loyola Marymount University, Los Angeles, CA 90045. *Systems Approaches to Optimization and Control with Agent-Based Models*.

Agent-based models (ABMs) have become an increasingly important mode of inquiry for the life sciences. They are particularly valuable for systems that are not understood well enough to build an equation-based model. These advantages, however, are counterbalanced by the difficulty of analyzing and using ABMs, due to the lack of the type of mathematical tools available for more traditional models, which leaves simulation as the primary approach. Rather than viewing the ABM as a model, it is to be viewed as a surrogate for the actual system. For a given optimization or control problem (which may change over time), the surrogate system is modeled instead, using data from the ABM and a modeling framework for which ready-made mathematical tools exist, such as differential equations, or for which control strategies can be explored more easily. Once the optimization problem is solved for the model of the surrogate, it is then lifted to the surrogate and tested. The final step is to lift the optimization solution from the surrogate system to the actual system. In this talk, we illustrate this process using two different strategies to optimize a simple agricultural pest control problem built on an ABM with predator-prey structure. (Received September 22, 2017)

1135-92-1460 **Jim M Cushing\*** ([cushing@math.arizona.edu](mailto:cushing@math.arizona.edu)), Department of Mathematics, Program in Applied Mathematics, University of Arizona, Tucson, AZ 85721. *Remarks on the definition of  $R_0$ .*

In population dynamics, the basic reproduction number  $R_0$  is, by definition, the expected life time number of offspring produced by a newborn individual. The formulation of  $R_0$  in a model requires the specification of what events are considered as "reproduction" events and what events are considered as a "transitions from one individual state to another". Thus, an element of choice can creep into the definition and there are multiple possible net reproduction numbers. While the biological meaningfulness of each can be debated, mathematically there is no "correct" definition. Moreover, general theorems guarantee that all serve to determine population growth or decay (depending on the relationship to 1). Similar remarks apply to the definition of  $R_0$  in models of infectious diseases. Some simple model examples will be used to illustrate these points. (Received September 22, 2017)

1135-92-1462 **Atanaska Dobрева\***, 1607 Sauls St., Unit A, Tallahassee, FL 32308, and **Nick G Cogan** and **Ralf Paus**. *Mathematics-driven insights into the hostile immune system behavior toward hair follicles.*

Alopecia areata is a hair loss disease in which the immune system attacks and kills hair-producing cells, and the hairless lesions are characterized by distinct spatial patterns. I will present an ODE model that reflects the disease dynamics over time in a small cluster of homogeneous follicles. The model describes the behavior of key substances and immune cells involved in the development of alopecia areata, and it incorporates interactions between hair follicles and the immune system. I will also discuss our on-going efforts to construct a PDE model which can capture the characteristic hairless patterns caused by the disease. Time permitting, I will address some results obtained by applying sensitivity analysis to further investigate the qualitative behavior of the models. Our findings indicate that the signaling pathway via the pro-inflammatory messenger protein interferon-gamma is crucial, and this could help in the development of more effective treatment strategies. (Received September 22, 2017)

1135-92-1480 **Naveen K. Vaidya\*** ([nvaidya@sdsu.edu](mailto:nvaidya@sdsu.edu)), San Diego State University, 5500 Campanile Drive, San Diego, CA 92182. *Impact of Environmental Temperature on Dengue Epidemics: Mathematical Models.*

Growing spread of dengue, a mosquito-borne disease, has been a major international public health concern. In this talk, I will present mathematical models to describe the impact of environmental temperature on the dynamics of dengue epidemics. To establish the global threshold dynamics of the models, we formulate the basic reproduction number and the infection invasion thresholds. I will also present numerical simulation results to demonstrate how the environmental temperature affects the disease dynamics. (Received September 22, 2017)

1135-92-1489 **Aida Abiad** ([aidaabiad@gmail.com](mailto:aidaabiad@gmail.com)), **Elie Alhajjar** ([eahajja@gmu.edu](mailto:eahajja@gmu.edu)) and **Pavel Avdeyev\*** ([avdeyev@gwu.edu](mailto:avdeyev@gwu.edu)), VA 20147, and **Martin Balko** ([balko@kam.mff.cuni.cz](mailto:balko@kam.mff.cuni.cz)), **Garner Cochran** ([gcochran@math.sc.edu](mailto:gcochran@math.sc.edu)), **Kenan Ince** ([kince@westminstercollege.edu](mailto:kince@westminstercollege.edu)) and **Gelasio Salazar** ([gsalazar@ifisica.uaslp.mx](mailto:gsalazar@ifisica.uaslp.mx)). *Rectilinear Crossing Number*. Preliminary report.

A rectilinear drawing of a graph  $G$  is an embedding of  $G$  into the plane such that the vertices of  $G$  are mapped to distinct points, and an edge connecting two vertices are mapped to the straight segment joining the corresponding points. In a rectilinear drawing, edges are allowed to cross, but an edge may not contain a vertex other than its endpoints. The rectilinear crossing number of  $G$ ,  $\overline{cr}(G)$ , is the smallest possible number of pairwise edge crossings in a rectilinear drawing of  $G$ . We investigate rectilinear crossing number of partite graphs. Namely, we discuss the following two problems.

The first problem is to establish the relationship between rectilinear crossing number and crossing number of 4-partite graphs. We show that there is a 4-partite graph, whose rectilinear crossing number and crossing number differ.

The second problem is Zarankiewicz's Conjecture for the complete bipartite graph  $K_{n,m}$  with  $n$  and  $m$  vertices in its partite vertex sets. We consider a special case of Zarankiewicz's Conjecture under the extra assumption that the partite vertex sets are separated by a straight line. We describe the current progress of proving Zarankiewicz's Conjecture in this case. (Received September 22, 2017)

1135-92-1507 **Alan Hastings\*** ([amhastings@ucdavis.edu](mailto:amhastings@ucdavis.edu)). *Simple approaches to manage natural systems when density dependence can be ignored.*

I will review simple models that describe how to determine optimal approaches for managing natural systems when density dependence can be ignored. In this case, one very useful approach is to approximate the dynamics by

linear discrete time models. Then, ideas based on linear programming can be used. I will consider applications to control of invasive species and when there are multiple objectives. The importance of restricting possible actions to match biological limitations will be emphasized. The approximations used will be shown to be robust by considering more general optimization methods and the role of stochasticity. (Received September 22, 2017)

1135-92-1527 **Jin Wang\*** (jin-wang02@utc.edu). *Analyzing the impact of awareness programs on cholera dynamics.*

We propose mathematical models to investigate the impact of media and awareness programs on the transmission and spread of cholera, a water-borne intestinal disease, based on a differential equation framework. We study the essential dynamical properties of each model, using both analytical and numerical approaches. We find that these models, though closely related, may exhibit significantly different dynamical behaviors ranging from regular threshold dynamics to backward bifurcations. Our results underscore the importance of validating key modeling assumptions in the development and selection of mathematical epidemic models toward practical applications. (Received September 22, 2017)

1135-92-1530 **Jin Wang\*** (jin-wang02@utc.edu). *Modeling and simulating tumor virotherapy.*

We present a mathematical modeling framework, where a free boundary problem is formulated based on a system of nonlinear PDEs, for the study of tumor growth under the treatment of virotherapy. We conduct extensive numerical simulation to examine tumor development under a wide range of biological settings, and discuss possible improvement on the efficacy of the treatment. We particularly emphasize the multiple interactions among tumor cells, oncolytic viruses, and immune systems that shape the complex pattern of tumor growth. (Received September 22, 2017)

1135-92-1569 **Najat Ziyadi\*** (najat.ziyadi@morgan.edu), Morgan State University, 1700 East Cold Spring Lane, Baltimore, MD 21251. *A mathematical model of Nutrients-Phytoplankton-Oysters in a bay ecosystem with oyster filtration based on Moser equation.*

In this talk, we will introduce a parameterized mathematical model with oyster filtration that is based on the Moser equation to describe the interactions of nutrients, phytoplankton and oysters in a bay ecosystem. Using the model, we will derive verifiable conditions for the persistence and extinction of phytoplankton and oysters in the bay system. In addition, we will use sensitivity analysis and simulations to illustrate how human activities such as increased nutrients inflow can generate phytoplankton blooms via Hopf bifurcation with corresponding oscillations in the oyster biomass and nutrients level in the bay ecosystem. (Received September 23, 2017)

1135-92-1576 **Abdul-Aziz Yakubu\*** (ayakubu@howard.edu), Department of Mathematics, Howard University, 2441 6th Street NW, Washington, DC 20059, and **Pauline Van den Driessche**. *Discrete-Time Infectious Disease Models*. Preliminary report.

We will focus on discrete-time infectious disease models in populations that are governed by constant, geometric, Beverton-Holt or Ricker demographic equations, and give a method for computing the basic reproduction number,  $R_0$ . When  $R_0 < 1$  and the demographic population dynamics are asymptotically constant or under geometric growth (non-oscillatory), we prove global asymptotic stability of the disease-free equilibrium of the disease models. Under the same demographic assumption, when  $R_0 > 1$ , we prove uniform persistence of the disease, and the existence of at least one endemic equilibrium (EE). We apply our theoretical results to specific discrete-time epidemic models that are formulated for SEIR infections and cholera in humans. Our simulations show that the unique EE of each of the two specific disease models is asymptotically stable whenever  $R_0 > 1$ . (Received September 23, 2017)

1135-92-1577 **Abdul-Aziz Yakubu\*** (ayakubu@howard.edu), Howard University, Mathematics Department, 2441 6th Street NW, Washington, DC 20059, and **Pauline van den Driessche**. *A discrete-time anthrax epidemic model*. Preliminary report.

A discrete-time mathematical model of anthrax (caused by *Bacillus anthracis*) transmission is formulated that includes live animals, infected carcasses and spores in the environment. The basic reproduction number  $R_0$  is calculated, and existence of at least one endemic equilibrium is established for  $R_0$  above the threshold value 1. In particular, if  $R_0 < 1$  we prove global asymptotic stability of the disease-free equilibrium. If  $R_0 > 1$  we prove the uniform persistence of the anthrax model. (Received September 23, 2017)

1135-92-1591 **Yu Jin\*** (yjin6@unl.edu). *Population persistence in a benthic-drift river environment.*

We consider a river environment where species grow on the benthos, drift in the water column, and transfer between the water column and the benthos. We use reaction-diffusion-advection equations coupled with ordinary



differential equations to describe the dynamics of a single species and of two competitive species. We study the population persistence criteria, based on persistence measures, including the net reproductive rate and eigenvalues of corresponding eigenvalue problems. We then use these measures to numerically investigate the influences of factors, such as the birth rate, various flow regimes, diffusion rates, competition rates, transfer rates, and spatial heterogeneity on population persistence. The theory developed here provides the basis for effective decision-making tools for water managers. (Received September 23, 2017)

1135-92-1603 **Fred Brauer\*** (brauer@math.ubc.ca), Department of Mathematics, University of British Columbia, 1984 Mathematics Road, Vancouver, BC, Canada. *An epidemic model with superspreading.*

It appears that superspreading events may be common in epidemics. We analyze a simple compartmental model for superspreading that may be useful early in a disease outbreak for estimating the final size of the epidemic. The model predicts fewer disease cases than a simple epidemic model with the same reproduction number in almost all cases, and may be better for estimating the effects of control measures. (Received September 23, 2017)

1135-92-1606 **Mary Gockenbach\*** (mary.gockenbach@mavs.uta.edu) and **Tim Barry**. *A Model of Iron Metabolism in the Human Body.*

Iron-related disorders are prevalent throughout the world. Anemia, which has iron deficiency as a major cause, affects nearly one quarter of the world's population. Hereditary hemochromatosis, a disease of iron overload, is the most common inherited disease of gene mutation in Caucasians. Understanding the mechanisms of iron metabolism in the human body will advance individualized treatments strategies for these and other conditions. A mathematical model using ordinary differential equations is developed to simulate the distribution of iron in the major organs of the body. The model is calibrated for a healthy person using experimental time course data obtained from literature. The inclusion of hormones in the model, such as erythropoietin and hepcidin, enable the investigation of common iron disorders and potential treatments. This model provides a foundation for the creation of a personalizable model in which the specifics of an individual's condition form the parameter set so that the outcomes of various treatments can be predicted. (Received September 23, 2017)

1135-92-1612 **Cheryl J Briggs\*** (briggs@lifesci.ucsb.edu), Dept. of Ecology, Evolution & Marine Biology, University of California, Santa Barbara, Santa Barbara, CA 931069620, and **Mark Q Wilber** (mark.wilber@lifesci.ucsb.edu), Dept. of Ecology, Evolution & Marine Biology, University of California, Santa Barbara, Santa Barbara, CA 931069620. *Integral Projection Models for host/fungal-pathogen systems: applications to the amphibian chytrid fungus.*

A number of recently emerging fungal diseases have led to population declines and extinction in wildlife species. Many infectious disease agents, including most fungal pathogens, have attributes of both microparasites and macroparasites. We illustrate how Integral Projection Models (IPMs) provide a novel modeling framework to represent fungal pathogens. We build a simple host-parasite IPM that tracks both the number of susceptible and infected hosts and the distribution of parasite burdens in infected hosts.

We parameterize the IPM using data from experiments on an amphibian species suffering population extinctions from the fungal pathogen *Batrachochytrium dendrobatidis* (Bd). We show that while transmission from an environmental Bd reservoir increased the ability of Bd to invade an amphibian population and the extinction risk of that population, Bd-induced extinction dynamics were far more sensitive to host resistance and tolerance than to Bd transmission. We demonstrate that this is a general result for load-dependent pathogens, where non-linear resistance and tolerance functions can interact such that small changes in these functions lead to drastic changes in extinction dynamics. (Received September 23, 2017)

1135-92-1617 **Thomas G Stojanovic\*** (tgs@uwm.edu), 1428 E Capitol Drive, Apartment 2, Shorewood, WI 53211. *Evolutionary stable strategy for a multispecies phytoplankton competition model.*

Phytoplankton live in a complex environment with two essential resources forming various gradients. Light supplied from above is never homogeneously distributed in a body of water due to refraction and absorption from biomass present in the ecosystem and from other sources. Nutrients in turn are typically supplied from below. In poorly mixed water columns, phytoplankton can be heterogeneously distributed forming various layering patterns. In this talk, a reaction-diffusion-taxis model describing the vertical distribution of two phytoplankton species competing for light and resources is presented. Using simulations, we exhibit evidence of thin layer formation for motile phytoplankton in poorly mixed environments. A game theoretic approximation is considered,

where the depth of the phytoplankton layer is treated as the strategy is then analyzed. The evolutionary stable strategy (ESS) is the depth at which the phytoplankton are equally limited by both resources. We derive the ESS of the game theoretic model and draw connections to the simulations of the original model. (Received September 23, 2017)

1135-92-1634 **Yang Kuang\*** (kuang@asu.edu). *Rich Dynamics of a Food Chain Model with Two Limiting Nutrients.*

Ecological stoichiometry studies the balance of energy and multiple chemical elements in ecological interactions to establish how the nutrient content affect food-web dynamics and nutrient cycling in ecosystems. Stoichiometric population models aim to describe fully the complex dynamics often observed in nature in a simple and sound setting. In this study, we formulate a food chain with two limiting nutrients in the form of a stoichiometric population model. This model naturally extends the model due to Loladze, Kuang and Elser. A comprehensive global analysis of the rich dynamics of the targeted model is explored both analytically and numerically. Chaotic dynamic is observed in this simple stoichiometric food chain model and is compared with traditional model without stoichiometry. Our finding shows that decreasing producer production efficiency may have only a small effect on the consumer growth but a more profound impact on the top predator growth. (Received September 23, 2017)

1135-92-1639 **Tufail M Malik\*** (tufail@asu.edu), Science and Mathematics, Arizona State University, Mesa, AZ 85142. *A Discrete Time West Nile Virus Transmission Model with Optimal Bird- and Vector-Specific Controls.*

A discrete-time model describing the west nile virus transmission among the mosquito, wild bird, and domestic bird populations will be presented. The expressions for the basic reproduction number and the disease-free fixed point of the model will be discussed. Correspondingly the local stability of the disease-free fixed point will be established. Optimal control theory will be used to devise the most effective administration profile of mosquito larvicide, mosquito adulticide and domestic bird-protection in controlling the virus transmission among the mosquito - wild bird - domestic bird community. (Received September 24, 2017)

1135-92-1646 **Zhisheng Shuai\*** (shuai@ucf.edu), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Graph Design and Reduction to Global Lyapunov Functions in Mathematical Biology.* Preliminary report.

The graph-theoretic approach has become a standard method to construct global Lyapunov functions for large-scale differential equation systems. Appropriate graph/network design and reduction is the key in the successful application of the approach. We illustrate these graph/network techniques using various biological models in the literature. (Received September 24, 2017)

1135-92-1649 **Timothy D Comar\*** (tcomar@ben.edu) and **Elizabeth Rodriguez.** *An Agent-Based Model for Integrated Pest Management with Periodic Control Strategies.*

We consider an agent-based model (ABM) for integrated pest management (IPM). The model incorporates stage structure for both the pest species and the predator species. In this model, the two control strategies of augmentation of predator species and application of pesticide and the pest births occur periodically at possibly different frequencies. We determine conditions under which either the pest species is eradicated or both species persist. We also investigate how varying the frequency of the augmentation of the predator species and the application of pesticide with respect to the frequency of the pest births affects the amounts of augmentation and pesticide needed to obtain pest eradication and permanent solutions. We then compare pest eradication and permanent solutions in the ABM to those in an analogous models using impulsive differential equations and difference equations. (Received September 24, 2017)

1135-92-1652 **Florian Klimm\***, f.klimm@gmail.com, and **Gorka Zamora-López, Jonny Wray, Charlotte M. Deane, Jürgen Kurths and Mason A. Porter.** *Promiscuity of Nodes in Multilayer Networks.*

Recent advances in the complexity sciences allow the analysis of *multilayer networks*. The question to what extent nodes are similarly important in all layers arises naturally. We define the *promiscuity* of a node as a measure of the variability of its degree across layers in comparison to a random null model.

We show that empirical networks show different promiscuity distributions. Transportation networks, for example, where the layers represent different modes of transportation tend to have a majority of low promiscuity nodes. A few hub nodes with high promiscuity enable the transit between different modes of transportation. The representation of global trade as a multilayer network reveals that country's imports are often very diverse

whereas the export of some countries depends extremely on a single commodity. Employing the promiscuity on transcription factor interaction in multiple cell types reveals proteins that are potential biomarkers of cell fate.

Despite its simplicity, the presented framework gives novel insights into numerous types of multilayer networks and expands the available toolbox for multilayer network analysis. (Received September 24, 2017)

1135-92-1707 **Gregory Roth, Paul L. Salceanu and Sebastian J. Schreiber\*** (sschreiber@ucdavis.edu), Department of Evolution and Ecology, University of California, Davis, CA 95616. *Robust permanence for ecological maps.*

We consider ecological difference equations of the form  $x_{t+1}^i = x_t^i A_i(x_t)$ , where  $x_t^i$  is a vector of densities corresponding to the subpopulations of species  $i$  (e.g., subpopulations of different ages or living in different patches),  $x_t = (x_t^1, x_t^2, \dots, x_t^m)$  is the state of the entire community, and  $A_i(x_t)$  are matrices determining the update rule for species  $i$ . These equations are permanent if they are dissipative and the extinction set  $\{x : \prod_i \|x^i\| = 0\}$  is repelling. If permanence persists under perturbations of the matrices  $A_i(x)$ , the equations are robustly permanent. Sufficient and necessary conditions for robust permanence in terms of Lyapunov exponents for invariant measures supported by the extinction set will be given. The methods will be illustrated with applications to ecological, evolutionary, and epidemiological models. (Received September 24, 2017)

1135-92-1716 **Jesse M. Kreger\*** (kregerj@uci.edu), **Natalia L. Komarova** and **Dominik Wodarz**. *Mathematical models of virus infections.* Preliminary report.

Motivated by recent experimental data, this talk will investigate mathematical models regarding the evolutionary outcomes of viral infections, specifically human immunodeficiency virus (HIV), in humans. The presentation will analyze how the interplay between multiplicity of infection, synaptic cell-to-cell transmission of the virus, and free virus transfer can affect the dynamics of an infection taking place. We consider models with competition between virus strains, characterized by different mutations, to see how each strain's infection strategy can affect outcome. Finally, we will discuss how recombination between virus strains can change the evolutionary outcomes of infection and influence the course of disease. The overall goal of the project is to better understand the dynamics of viral infections, specifically HIV, and to help design more effective healthcare and vaccination approaches. (Received September 24, 2017)

1135-92-1724 **Amy Veprauskas\*** (aveprauskas@louisiana.edu), **Azmy S. Ackleh**, **Ross A. Chiquet** and **Tingting Tang**. *Assessing the effect of environmental disturbances on population recovery and persistence with application to marine mammals.*

We develop nonautonomous matrix models to examine the possible long-term effects of environmental disturbances, such as oils spills, floods, and fires, on population recovery and persistence. We first examine population recovery following a single disturbance, where recovery is defined to be the return to the pre-disturbance population size. We assume that the disturbance results in reductions in either survival or fecundity for a period of time, after which the vital rates recover to their original values. Using this model formulation, we apply matrix calculus methods to derive explicit formulas for the sensitivity of the recovery time with respect to properties of the population or the disturbance. We then develop a model to consider the effect of repeated disturbances on population persistence. Disturbances occur stochastically according to a two-state Markov chain with their frequency depending on the average length of effect of a disturbance and the average time between disturbances. Motivated by the 2010 Deepwater Horizon oil spill, we apply the results of both models to examine the possible response of a sperm whale population to environmental disturbances. (Received September 24, 2017)

1135-92-1755 **Fabio A Milner\*** (milner@asu.edu), P.O. Box 871804, Tempe, AZ 85287-1804, and **Delphine Picart** and **Denis Thiery**. *Optimal treatment schedule for a vineyard pest.*

A model for the control of the European grapevine moth (*Lobesia botrana*) will be presented, including two control methods: insecticides and mating disruption. It is designed to yield the combination and schedule of application that minimizes the cost and losses due to the pest. A simulation of an experimental situation based on real-life data will be presented. (Received September 24, 2017)

1135-92-1777 **Christina Joy Edholm\*** (cedholm@utk.edu). *Modeling the role of superspreaders in infectious disease outbreaks.* Preliminary report.

In an infectious disease outbreak, individuals who transmit the disease to a large number of susceptible individuals are known as superspreaders. To investigate the role superspreaders play in an outbreak, we construct both deterministic and stochastic models with two classes of individuals, superspreaders and nonsuperspreaders. We analyze these models and then run numerical simulations for the cases of Middle East respiratory syndrome

(MERS) and Ebola. From the analysis and simulations, we gain insight into superspreaders role in the outbreak timeline and severity of the outbreak. (Received September 24, 2017)

1135-92-1804 **Paul L. Salceanu\*** ([salceanu@louisiana.edu](mailto:salceanu@louisiana.edu)), Lafayette, LA 70503, and **Hao Wang**. *Competitive outcomes between zebra and quagga mussels in a discrete-time model with migration among patches*. Preliminary report.

We develop and use a mathematical model to investigate interactions and competitive outcomes between two invasive species of mollusks: zebra mussel (*Dreissena polymorpha*) and quagga mussel (*Dreissena rostriformis bugensis*). The model has both spatial structure (patches) and temporal structure (age: juvenile and adult individuals). We show that, when migration among patches does not take place, in each patch one species eliminates the other, and it settles at a constant population size. Further, we investigate both numerically and analytically how migration among patches might affect this outcome. (Received September 24, 2017)

1135-92-1854 **Erich Baker, Sharon Moore, Ami Radunskaya and Elizabeth Zollinger\*** ([ezollinger@sjcny.edu](mailto:ezollinger@sjcny.edu)), 245 Clinton AVE, Brooklyn, NY 11231. *Modeling non-human primate alcohol consumption: a Markov approach*. Preliminary report.

We develop a probabilistic model to classify the level of drinking of a male rhesus macaque given its initial characteristics such as age at induction, bone mineral density, and electrophysiological data. The Monkey Alcohol and Tissue Research Resource (MATRR) has a wealth of behavior and physiological data which we use to calibrate the model. MATRR has standard classifications for long term drinking behavior of the monkey which are very heavy drinker, heavy drinker, binge drinker and low drinker. Using the data of 6 months of drinking for a given cohort, we model second by second drinking by looking at the rate at which a monkey is drinking at any given time. We set the following four states for drinking: not drinking, low drinking, medium drinking, and high drinking. The Markov model has a function describing the likelihood for leaving each state and a probability distribution for transitioning to another state given the state the monkey is currently in. The parameters for each probability is estimated given a set of “training” monkeys. We then test the model by inputting an alternate cohort of known monkeys to simulate drinking and predict the classification for each monkey to see if the model aligns with the original classification. Funded by grant NIAAA #AA019431 (Received September 25, 2017)

1135-92-1913 **Shelby Stanhope\*** ([stanhope@temple.edu](mailto:stanhope@temple.edu)), Department of Mathematics, Temple University, 1805 N. Broad St, Wachman Hall, Philadelphia, PA 19122, and **Isaac Klapper** ([klapper@temple.edu](mailto:klapper@temple.edu)). *Mathematical Models of the Immune System’s Response to Bacterial Infection on the Surface of an Implant Device*. Preliminary report.

The occurrence of biofilms on the surface of implant devices is a problem of great concern in the medical community. Persistent infection may ultimately require replacement of the device, thus increasing medical expenses and causing pain and suffering for the patient. In this study, we use mathematical modeling, in conjunction with data collected from laboratory studies, to model the immune response to bacterial infection on the surface of implant devices. Focusing on the initiation of infection, we aim to understand which factors determine the elimination of infection and which eventually lead to the development of a biofilm on the device. One of the first responders in the body’s innate immune response are neutrophils, which follow chemical signals to locate and phagocytose bacteria. Using a system of partial differential equations, we model the growth of bacterial populations, their release of chemoattractants (which initiate the immune response), and the chemotactic movement of neutrophils to the sites of infection. By varying the amount of bacteria present in the initial infection and the strength of the body’s immune response, we observe cases where the infection is completely eradicated and others where the infection persists. (Received September 25, 2017)

1135-92-1934 **Bin Xu\*** ([bxu2@nd.edu](mailto:bxu2@nd.edu)), ACMS Department, 153 Hurley Hall, Notre Dame, IN 46556. *Modeling the dynamics of Cdc42 oscillation in fission yeast*.

We present a mathematical model of the core mechanism responsible for the regulation of polarized growth dynamics during the fission yeast cell cycle. The model is based on the competition of growth zones of Cdc42 localized at the cell tips for a common GEF distributed uniformly in the cytosol. We consider several potential ways of implementing negative feedback between Cdc42 and its GEF in this model that would be consistent with the observed oscillations of Cdc42 in fission yeast. We analyze the bifurcations in this model as the cell length increases and the total amount of Cdc42 increases. (Received September 25, 2017)

1135-92-1982 **Heather Z. Brooks\*** ([heather@math.utah.edu](mailto:heather@math.utah.edu)), 155 S 1400 E Rm. 233, Salt Lake City, UT 84112, and **Paul C. Bressloff**. *Pattern formation mechanism for homeostatic control of synapse density during *C. elegans* growth.*

We propose a novel mechanism for Turing pattern formation that provides a possible explanation for the regular spacing of synapses along the ventral cord of *C. elegans* during development. The model consists of two interacting chemical species, where one is passively diffusing and the other is actively trafficked by molecular motors; we identify the former as the kinase CaMKII and the latter as the glutamate receptor GLR-1. We use linear stability analysis to derive conditions on the associated nonlinear interaction functions for which a Turing instability can occur. We find that the dimensionless quantity  $\gamma$ , the ratio of switching rate and diffusion coefficient to motor transport velocity, must be sufficiently small for patterns to emerge. One consequence is that patterns emerge outside the parameter regime of fast switching where the model effectively reduces to a two component reaction-diffusion system. Furthermore, these patterns are also maintained during domain growth. We discuss selection and stability of patterns for this mechanism in both 1- and 2-dimensional domains. (Received September 25, 2017)

1135-92-2008 **Evan Milliken\*** ([evan.milliken@asu.edu](mailto:evan.milliken@asu.edu)) and **Hal L. Smith**. *Embedding a One-to-one Infection Network into a Nested Infection Network.*

Bacteria and bacteriophages have been observed in the wild in complex infection networks. It has been shown that a perfectly nested or one-to-one infection network is permanent, or uniformly persistent, as long as the bacteria and bacteriophages obey certain orderings with respect to their efficiencies. In this talk, a model is proposed for the case in which a one-to-one network is embedded into a nested infected network. Conditions are given for the existence of a positive coexistence equilibrium and for the permanence of the network. The stability of the coexistence equilibrium is analyzed and results are compared to stability results for networks with either one-to-one or nested structure, but not both. (Received September 25, 2017)

1135-92-2037 **Krystin E. Steelman Huff\*** ([krystin.huff@ttu.edu](mailto:krystin.huff@ttu.edu)) and **None**. *Stochastic Models of Within-Host Viral Infection.* Preliminary report.

Viral infections caused by influenza virus, Ebola virus, and hantavirus are of serious public health concern. Each virus replicates within specific target cells. For example, hantavirus replicates within the lung microvascular endothelial cells. A well-known target cell model for the early stage of infection is a system of ODEs which includes healthy target cells, latent cells, infected cells, and free viruses. The ODE model serves as a framework for formulation of new stochastic models, a continuous-time Markov chain (CTMC) and a system of stochastic differential equations (SDEs). The stochastic models account for variability in the transition between states and the transmission process. During the early stages of the infection, estimates of the probability of a successful infection is obtained from a multi-type branching process approximation of the CTMC model. The estimates depend on the initial concentration of virions, latent cells, and infected cells. After the infection is established the SDE model shows the variability in timing of the peak infection. Numerical examples illustrate the results for influenza A virus infection. (Received September 25, 2017)

1135-92-2057 **Erica J Graham\*** ([ejgraham@brynmawr.edu](mailto:ejgraham@brynmawr.edu)). *Sensitivity and bifurcation analysis of disruptive mechanisms in a menstrual cycle model.*

A normally functioning menstrual cycle requires significant crosstalk between hormones originating in ovarian and brain tissues. Reproductive hormone dysregulation may disrupt function and can lead to infertility, as occurs in the common endocrine disorder polycystic ovarian syndrome (PCOS). To examine mechanisms of ovulatory dysfunction, we analyze global parameter sensitivity and bifurcations in a model of the menstrual cycle. In particular, the model highlights androgen-mediated dysfunction and the influence of elevated insulin, both of which are common in PCOS. We discuss conditions under which physiological stable oscillations are lost to pathological outcomes. (Received September 25, 2017)

1135-92-2073 **Trey Ideker\*** ([trey.ideker@gmail.com](mailto:trey.ideker@gmail.com)). *Using Deep Learning to Model the Hierarchical Structure and Function of a Cell.*

Although artificial neural networks capture a variety of human functions, their internal structures are hard to interpret. In the life sciences, extensive knowledge of cell biology provides an opportunity to design ‘visible’ neural networks (VNNs) which couple the model’s inner workings to those of real systems. Here we develop DeepCell, a VNN embedded in the hierarchical structure of 2526 subsystems comprising a eukaryotic cell (<http://deep-cell.ucsd.edu/>).

Trained on 12 million genotypes, DeepCell simulates cellular growth nearly as accurately as laboratory observations. During simulation, genotypes induce patterns of subsystem activities, enabling in-silico investigations of

the molecular mechanisms underlying each genotype-phenotype association. These mechanisms can be validated and many are unexpected; some are governed by Boolean logic. Cumulatively, 80% of the importance for growth prediction is captured by 484 subsystems (21%), reflecting the emergence of a complex phenotype. DeepCell provides a foundation for decoding the genetics of disease, drug resistance, and synthetic life. (Received September 25, 2017)

1135-92-2125 **Arjun Kakkar\*** (ak23@williams.edu), 2849 Paresky Center, Williamstown, MA 01267. *Vegetation Patterns in Semi-Arid Regions.*

The appearance of striped vegetation patterns along topographical gradients has been well documented in a variety of semi-arid regions. The development of these patterns has been accounted for in past literature by using a system of nonlinear PDEs that model plant and water interactions. In these models, the striped patterns originate as bifurcations of the homogeneously vegetated state as the amount of precipitation decreases or as plant mortality increases. Such models, however, might admit solutions which physically correspond to continuous heating or cooling in the region. In lieu of this, we propose a new modeling strategy using a global energy functional which penalizes the system for deviating from energy balance. This functional is studied by numerical minimization over a discretized region. Furthermore, application of a naive gradient descent on this functional generates a structure similar to the traditional dynamical PDE models. (Received September 25, 2017)

1135-92-2150 **Reginald L. McGee\*** (mcgee.278@mbi.osu.edu), **Gregory K. Behbehani** and **Kevin R. Coombes.** *The geometry of leukemic proliferation.*

Complex protein interactions networks complicate the understanding of what most promotes the rate of cancer progression. High dimensional data provides new insights into possible mechanisms for the proliferative nature of aggressive cancers, but these datasets often require fresh techniques and ideas for exploration and analysis. In this talk, we consider expression levels of tens of proteins that were recorded in individual cells from acute myeloid leukemia (AML) patients via mass cytometry. After identifying immune cells subpopulations in this data using an established clustering method, we use topological data analysis to search for subpopulations that are most actively proliferating. To conduct the search within these subpopulations, we build on the differential geometric perspective that led to our recent statistic for testing aggregate differences in protein correlations between patients with different subtypes of AML. (Received September 25, 2017)

1135-92-2199 **Kaia Lindberg\*** (klindberg923@rwu.edu) and **Edward Dougherty.** *Computational Simulation of a Partial Differential Equation Based Model of Electrostatic Forces on Neuronal Electrodynamics.* Preliminary report.

Neurostimulation therapies demonstrate success as a medical intervention for individuals with neurodegenerative diseases. Despite promising results from these treatments, the influence of an electric current on ion concentrations and subsequent transmembrane voltage is unclear. This project focuses on developing a unique cellular-level mathematical model of neurostimulation to better understand its effects on neuronal electrodynamics. The Poisson-Nernst Planck system of PDEs is used to model electric potential, transmembrane voltage, and ion concentrations. This system is decoupled using the Gauss Siedal method and then the equations are solved using the finite element method on a biologically-inspired discretized domain. Using FEniCS we have conducted numerous numerical experiments on several two-dimensional neuronal geometries involving action potential generation and external current application. Preliminary results demonstrate the influence of applied external currents on membrane voltage. Future work will include extending these computational simulations to three-dimensional neuron domains and integrating an ODE based intracellular signaling pathway model. Hopefully this work will ultimately help elucidate the principles by which neurostimulation alleviates disease symptoms. (Received September 25, 2017)

1135-92-2257 **Aleesa Monaco\*** (aleemonaco@gmail.com), **John D. Nagy** (jdnagy@asu.edu) and **Kalle Parvinen** (kalparvi@utu.fi). *Coevolving cancer hallmarks: The angiogenic switch is modulated by clonal selection on proliferation.*

Angiogenesis and dysregulated tissue homeostasis are thought to arise by clonal selection. However, natural selection's role in generating the angiogenic switch is not well understood. Here we show that the angiogenic switch is likely to evolve by early positive selection on angiogenic ability which eventually becomes reversed to negative selection in older tumors. Importantly, this reversal is driven by directional selection on proliferation ability. In our model, competing clones vary their ATP allocation to proliferation, angiogenic signaling, and cell maintenance in a realistic way. Adaptive dynamics analysis of this coevolutionary dynamic predicts that as selection drives proliferation towards its ESS, the once-favored angiogenic clones become susceptible to "free-rider" mutants, which reallocate metabolic energy from angiogenesis production to proliferation. These free-rider

clones have an evolutionary advantage over their angiogenic counterparts. The ultimate result is predicted to be necrosis by vascular hypoplasia. However, an analogous stochastic model shows that these deterministic endpoints are rarely realized. Tumors often reach lethal size before selection on angiogenesis has much impact. (Received September 25, 2017)

1135-92-2301 **Lisette dePillis\*** ([depillis@hmc.edu](mailto:depillis@hmc.edu)), Department of Mathematics, Harvey Mudd College, 301 Platt Blvd, Claremont, CA 91711. *Mathematical Modeling of Immune Dynamics in Disease.*

Immune system dynamics in the context of a number of diseases, including certain cancers and type I diabetes, continues to play an increasingly central role in the development of new treatment strategies. The critical importance of the immune system in fighting such diseases has been verified clinically, as well as through mathematical models. Many open questions remain, however, including what may lead to non-uniform patient responses to treatments, and how to optimize and personalize therapy protocols. In this talk, we will present a sampling of mathematical disease models we have developed that help us to simulate immune system interactions, disease dynamics, and treatment approaches that may slow, or even reverse, disease progression. (Received September 25, 2017)

1135-92-2313 **Yanyu Xiao\***, 4199 French Hall West, 2815 Commons Way Cincinnati, Cincinnati, OH 45221. *Dynamics of an Age-Structured SIR Epidemic Model.*

We investigate an SIR epidemic model with discrete age groups to understand the transmission dynamics of an infectious disease in a host population with an age structure. Basic reproduction number  $R_0$  is derived and shown as a sharp threshold parameter. The global stability of the endemic equilibrium when  $R_0 > 1$  is established under a mild condition. This model is also used to analyze the measles outbreaks in India. (Received September 25, 2017)

1135-92-2325 **Yanyu Xiao\***, 4199 French Hall West, 2815 Commons Way, Cincinnati, OH. *Risk assessments on eradication of aquatic invasive species.*

In the work, we evaluate the performance of machine learning approaches for predicting successful eradication of aquatic invasive species and assess the extent to which eradication of an invasive species depends on the various specific ecological features of the target ecosystem and/or features that characterize the planned intervention. (Received September 25, 2017)

1135-92-2329 **Naveen Vaidya and Libin Rong\*** ([libinrong@ufl.edu](mailto:libinrong@ufl.edu)), Department of Mathematics, University of Florida, Gainesville, FL 32611. *Modeling pharmacodynamics on HIV latent infection.* Preliminary report.

Highly active antiretroviral therapy has successfully controlled HIV replication in many patients. The treatment effectiveness may depend on the pharmacodynamics of antiretroviral drugs. We integrate several drug-related parameters into an HIV infection model to investigate the effects of drug pharmacodynamics on the HIV latent reservoir and viral load dynamics. We show that pharmacodynamic characteristics of drugs and the dosing schedule can significantly affect the outcome of either early or late treatment. This is a joint work with Naveen Vaidya. (Received September 25, 2017)

1135-92-2330 **Jordan J. Pellett\*** ([pellett.jordan@uwlax.edu](mailto:pellett.jordan@uwlax.edu)). *Modeling Chronic Vascular Responses Following a Major Arterial Occlusion.*

Peripheral arterial disease is a serious health concern characterized by a full or partial occlusion of a major artery in the systemic vasculature. Following an occlusion, blood supply to peripheral tissues is significantly reduced, causing patients to experience severe pain and reduced mobility. This study uses mathematical modeling to investigate the role of different vascular segments in restoring blood flow following a major occlusion. Vascular adaptations to collateral arteries and the microcirculation distal to the occlusion have been observed to occur on both acute and chronic time scales. Here two chronic vascular responses, arteriogenesis (increased diameter of existing vessels) and angiogenesis (new vessel formation), are investigated in a single vessel and a complex network. By coupling these chronic responses to acute responses, the model provides a framework for understanding the time frame and significance of vascular responses that help restore flow. Preliminary results suggest the number of collaterals increases following an occlusion while fewer vessels distal to the occlusion are required for optimal flow restoration. Ultimately, the model can be used to identify the most important vessels to target for future therapies. (Received September 26, 2017)

1135-92-2344 **Patrick De Leenheer\*** ([deleenhp@math.oregonstate.edu](mailto:deleenhp@math.oregonstate.edu)), Department of Mathematics, Kidder Hall, Corvallis, OR 97331. *Negative density dependence can cause partial migration.*

We consider the phenomenon of partial migration which is exhibited by populations in which some individuals migrate between habitats during their lifetime, but others do not. Using an adaptive dynamics approach, we show that partial migration can be explained on the basis of negative density dependence in the per capita fertilities alone, provided that this density dependence is attenuated for increasing abundances of the subtypes that make up the population. (Received September 26, 2017)

1135-92-2349 **Patrick De Leenheer\*** ([deleenhp@math.oregonstate.edu](mailto:deleenhp@math.oregonstate.edu)), Department of Mathematics, Kidder Hall, Corvallis, OR 97331. *A mathematical proof of the tragedy of the commons in the chemostat.*

We present a proof of principle for the phenomenon of the tragedy of the commons that is at the center of many theories on the evolution of cooperation. We establish the tragedy in the context of a general chemostat model with two species, the cooperator and the cheater. Both species have the same growth rate function and yield constant, but the cooperator allocates a portion of the nutrient uptake towards the production of a public good -the “Commons” in the Tragedy- which is needed to digest the externally supplied nutrient. The cheater on the other hand does not produce this enzyme, and allocates all nutrient uptake towards its own growth. We prove that when the cheater is present initially, both the cooperator and the cheater will eventually go extinct, hereby confirming the occurrence of the tragedy. We also show that without the cheater, the cooperator can survive indefinitely, provided that at least a low level of public good or processed nutrient is available initially. (Received September 26, 2017)

1135-92-2361 **Natalia L Komarova\***, Department of Mathematics, University of California Irvine, Irvine, CA 92697. *Fixation of neutral mutants in random environments: minority rules.*

Fixation probability, the probability that the frequency of a particular individual in an initially heterogeneous population will eventually reach unity, is a fundamental quantity in ecology and evolution. Here we study the effect of spatial randomness on the chances of mutant fixation in a population of cells of a constant size. Such problems arise in the models of cancer initiation and progression, bacterial dynamics, and drug resistance. It turns out that spatial heterogeneity redefines the notion of neutrality, allowing, e.g., a minority of cells (whose fitness values are drawn from the same distribution as that of the wild type) to behave as if they had a selective advantage. The effect can be very significant (increasing the probability of mutant invasion by orders of magnitude), it increases with the standard deviation of the underlying probability distribution and decreases with the skewness. It is the largest when the fitness values of the mutants and wild types are anti-correlated. We discuss the results for both a spatial ring geometry of cells (such as that of a colonic crypt) and a mass-action (complete graph) arrangement. (Received September 26, 2017)

1135-92-2362 **Marily Barron\*** ([mbarron4@ucmerced.edu](mailto:mbarron4@ucmerced.edu)), 276 Dunbarton Road, Aromas, CA 95004, and **Jaapna Dhillon, Syed A. Asghar, Quintin Kuse, Natalie De La Cruz, Emily Vu, Suzanne S. Sindi and Rudy M. Ortiz.** *Assessing the Impact of 8 Weeks of Almond Consumption on Anthropometric and Clinical Measurements in College Freshmen.* Preliminary report.

Biomedical research has provided support for the dietary benefit of almond consumption. However, most of the studies to date have considered only adult populations (age 40+). In this study, we assess the impact of chronic (8-weeks) almond snacking on college freshmen at UC Merced. We observed 73 UC Merced freshmen (mean age: 18.08, mean BMI: 25.44) for eight weeks. 35 consumed a control snack of graham crackers and 38 consumed an isocaloric amount of almonds (2 ounces, 325 kcal) for 8 weeks. Anthropometric and clinical measurements were collected before the study began and at the fourth and the eighth week. With this data, we have outlined a plan to generate multiple models to apply theoretical analyses to robustly assess the effects of chronic almond snacking on anthropometric and clinical outcomes. To date, we have not detected a significant difference in any of the anthropomorphic measurements. We continue to perform many of the biochemical measurements, but of those completed, we observe an increase in plasma HDL in the almond group at week 8 suggesting that chronic almond snacking has the potential to improve the metabolic profile independent of profound changes in anthropomorphic measures such as body mass or adiposity. (Received September 26, 2017)

1135-92-2366 **Lora D Weiss\*** ([ldweiss@uci.edu](mailto:ldweiss@uci.edu)) and **Natalia L Komarova.** *The stability of cellular networks against mutations.* Preliminary report.

Cellular tissue consists of stem cells and a hierarchy of more differentiated cells. It is constantly in flux, with differentiated cells dying, and stem cells replenishing the removed cells. In order to maintain constant size,



cellular processes must be regulated by control networks of intra-cellular signaling. Here we study the robustness of such control networks against cancerous mutations. Beginning with a stable cellular network, we investigate consequences of different types of mutations, and in particular, which mutations cause a stable network to fail. Relevant to the theory of cancer, these network failures may lead to unlimited growth of mutant cell populations. We use differential equations, linear analysis, and modeling to analyze different networks and how they are affected by different mutations. We find that only specific mutations may cause a stable network to fail. (Received September 26, 2017)

1135-92-2373 **Yun Kang\*** (yun.kang@asu.edu), 7001 E. WilliamsField Rd., Mesa, AZ 85212, and **C. Castillo-Chavez** and **F. Rao**. *Dynamics of a Diffusion Reaction Prey-Predator Model with Delay in Prey: Effects of Delay and Spatial Components*.

We study the complex dynamics of a Monod-Haldane-type predator prey interaction model that incorporates: (1) A constant time delay in the prey growth; and (2) diffusion in both prey and predator. We provide the rigorous results of our system including the asymptotic stability of a positive equilibrium; Hopf bifurcation; and the direction of Hopf bifurcation and the stability of bifurcated periodic solutions. We also perform numerical simulations on the effects of diffusion or/and delay when the corresponding ODE model has either a unique interior equilibrium with two interior attractors or two interior equilibria. Our theoretical and numerical results show that diffusion can either stabilize or destabilize the system; large delay could destabilize the system; and the combination of diffusion and delay could intensify the instability of the system. Moreover, when the corresponding ODE system has two interior equilibria, diffusion or time delay in prey or both could lead to the extinction of predator. Our results may provide us useful biological insights on population managements for prey-predator interaction systems. (Received September 26, 2017)

1135-92-2430 **Khanh P Nguyen\*** (kpnguyen21@yahoo.com), 12218 North Palm Lake Dr., Houston, TX 77034, and **Kresimir Josic** and **Zachary Kilpatrick**. *Intertrial correlations in sequential decision-making tasks*. Preliminary report.

Nearly all organisms accumulate evidence to make decisions, in order to survive in their environment. Understanding the mechanism behind these decisions is a focus of current efforts in experimental and theoretical neuroscience. Drift-diffusion models are commonly used to study decision making because they are mathematically tractable, provide a good description of experimentally observed behavior, as well as the underlying neural activity. Most mathematical studies of decision-making have focused on idealized situations, such as static environments or cases where a subjects' decisions and actions have no impact on the environment. In reality, the environment can change in response to the actions that organisms make. In this paper, we extended the classic modeling framework to include spontaneous and action-induced environmental changes. We have modeled spontaneous changes as stochastic switches in the correct choice from trial to trial. A fixed threshold fixes the correct percentage across trials, and we conjecture that the threshold should be dynamically increased to maximize the reward rate. This more realistic model can help us understand the neural computations underlying decisions, and identify the biophysical mechanisms that make such computations possible. (Received September 26, 2017)

1135-92-2435 **Eric T. Funasaki\*** (eric.funasaki@sulross.edu), Department of Computer Science & Mathematics, Sul Ross State University, P.O. Box C-18, Alpine, TX 79832, and **Shandelle M. Henson** and **James L. Hayward**. *Transient predator-prey cycles in bald eagles and glaucous-winged gulls at Protection Island, Washington*.

Widespread use of DDT in the environment during the 1940s had a catastrophic effect on the survival of bald eagle embryos due to eggshell thinning and cracking. Bald eagle populations began to recover with the banning of DDT in the 1970s. By the 2000s eagle resurgence in the Pacific Northwest was linked with declines in seabird prey such as glaucous-winged gulls. We monitored and mathematically modeled numbers of gull residents as well as bald eagle residents and visitors at Protection Island National Wildlife Refuge during 1993-2005. The transient dynamics are well explained by a Lotka-Volterra-type predator-prey model having logistic growth for each species in the absence of the other. (Received September 26, 2017)

1135-92-2439 **Kevin B. Flores\***, Department of Mathematics, North Carolina State University, Raleigh, NC 27540. *Forecasting and uncertainty quantification using a hybrid of mechanistic and non-mechanistic models*.

Traditionally, either mechanistic or non-mechanistic modeling techniques have been used for prediction, however it is uncommon for the two to be incorporated together. We compare the forecast accuracy of mechanistic modeling, using Bayesian inference, a non-mechanistic modeling approach based on state space reconstruction, and a hybrid composed of the two using simulated and experimental data. The experimental data come from

cannibalistic flour beetle populations, in which it has been observed that the adults preying on the eggs and pupae results in non-equilibrium population dynamics. Uncertainty quantification methods for the hybrid models are outlined and illustrated on these data. We perform an analysis of the results from Bayesian inference for the mechanistic model and hybrid models to suggest reasons why hybrid modeling may enable more accurate forecasts of multivariate systems than traditional approaches. (Received September 26, 2017)

1135-92-2488 **Benjamin Levy\*** (blevy1@fitchburgstate.edu), **Christina Edholm, Suzanne Lenhart, Orou Gaoue, Edward Lungu, Moathodi Kgosimore and Farai Nyabadza.** *Modeling the Role of Public Health Education in Ebola Virus Disease Outbreaks in Sudan.*

The 2014 outbreak of Ebola in West Africa was the most significant and detrimental to date. Preparedness of health care facilities and health care workers as well as individual human behavior driven by cultural practices contributed significantly to new infections. In light of this fact, we derive a model that partitions the population into those who take precautions against contracting the disease and those who do not. We also consider new infections arising in both a hospital setting as well as in the community, and include transmission from dead bodies as well as the environment. We estimate parameters using available data from two Ebola outbreaks in Sudan and consider implications of a new strain with respect to different death rates and recovery rates. Our goal is to illustrate the role of education in limiting potential future Ebola outbreaks in Sudan. (Received September 26, 2017)

1135-92-2511 **Nataša Jonoska\***, 4202 East Fowler Av. CMC 342, Tampa, FL 33620, and **Masahiko Saito.** *Detecting complexity and patterns in a scrambled genome.*

DNA recombination occurs at both evolutionary and developmental levels, and is often studied through model organisms such as ciliate species *Oxytricha* and *Stylonychia*. These species undergo massive genome rearrangements during their development of a somatic macronucleus from a zygotic micronucleus. Gene segments that recombine during DNA rearrangement processes may be organized on the chromosome in a variety of ways. They can overlap, interleave or one may be a subsegment of another. We use colored directed graphs to represent contigs containing rearranged segments where edges represent recombined segment organization. Using graph properties we associate a point in a higher dimensional Euclidean space to each graph such that cluster formations and analysis can be performed with methods from topological data analysis. We find that there are specific star-like graph structures that describe most complex interleaving gene patterns. We also use word patterns to investigate genome-wide scrambled gene architectures that describe the precursor-product relationships. Our studies show that there are two general patterns, reoccurring genome wide, that describe over 90% of the *Oxytricha*'s scrambled genes. (Received September 26, 2017)

1135-92-2517 **Mark C Agrios\*** (magrios@email.wm.edu). *Simplicial Homology and Neural Networks: An analysis of biological neural networks using persistent homology.* Preliminary report.

In this study, we use techniques from algebraic topology to study structure and dynamics in biological neural networks. Neural networks can be represented as simplicial complexes, topological structures from which we can extract algebraic information. Using a technique called persistent homology, we can classify important features of these simplicial complexes while also filtering out the "noise" often found in these complex dynamical systems. In this study we look at two types of persistence we can construct and how they can help us identify different topological and dynamic properties of these networks. (Received September 26, 2017)

1135-92-2552 **Theodore P. Pavlic\*** (tpavlic@asu.edu), ASU - CIDSE, P.O. Box 878809, Room 553, Tempe, AZ 85287-8809. *Distributed Algorithms in Biological Systems: The Diverse Architectures of Natural Computation.*

A popular view of "emergence" in biological systems evokes imagery of murmurations of bird flocks, where macroscopic shapes appear out of a symphony of sufficiently many interacting individuals each following simple rules. However, despite their visual appeal, the macroscopic structures in a cohesive flock of birds or school of fish are only an epiphenomenon; the structures themselves provide no adaptive value to the group. There is no "collective mind" in a school of fish; there are only individuals maximizing local efficiency and minimizing local relative risk. To understand the mechanisms of true distributed computation in nature, it is important to focus on groups for which selection does act at multiple organizational levels. In this talk, several examples from natural systems that do achieve some form of collective computation are given, and the development of novel distributed computing paradigms for engineering systems are discussed. Examples come from socially foraging finches, cooperatively breeding cichlids, and social insects (such as ants) in which computation emerges as an out-of-equilibrium structure that makes use of both regularities in individual behaviors and regularities in the environment to compute solutions to naturally occurring problems. (Received September 26, 2017)

1135-92-2558 **Timothy C Reluga\*** ([treluga@math.psu.edu](mailto:treluga@math.psu.edu)). *Social distancing epidemic games, diagnosis, and incomplete information.*

Within the epidemic game literature, there is a disagreement over whether increases in infection risk can discourage prevention. In one theory, increases in infection pressure motivate prevention, while in another, there is a turning-point beyond which investments drop off. The discrepancies between these theories can be resolved by modeling diagnosis events explicitly. Here, we show a unified theory exhibits a turning-point in the optimal social-distancing response to infection pressure. The turning point is a consequence of decision making under incomplete information. With reliable diagnosis, prevention always offsets infection risk. Without reliable diagnosis, individuals may be best-off assuming they are infected once infection pressures are sufficiently high. This creates a public-health trap with high prevalence that can not be escaped by unilateral action. We estimate conditions for HIV, and conclude that turning-point effects likely contribute to traps for core-groups found in some sub-Saharan countries. (Received September 26, 2017)

1135-92-2561 **Farrah Sadre-Marandi\*** ([sadre.1@mbi.osu.edu](mailto:sadre.1@mbi.osu.edu)), 376 JENNINGS HALL, 1735 NEIL AVENUE, Columbus, OH 43210. *Gender differences in hepatic one-carbon metabolism.* Preliminary report.

It is known that there are gender differences in one-carbon metabolism (OCM). Women in the child-bearing years exhibit lower plasma homocysteine, higher betaine and choline, and lower S-adenosylmethionine. Various enzymes in OCM are up-regulated or down-regulated in women due to estrogen. Insulin and glucose affect some enzymes of OCM and change furthermore during pregnancy. All of these results suggest that a mechanistic understanding of how enzymatic differences in women affect OCM is important for precision medicine.

The reaction diagram for the folate and methionine cycles in OCM is very complicated consisting of loops within loops. Furthermore, many substrates in the network influence, through allosteric binding, the activity level of enzymes at distant locations in the network. A mathematical model of folate and methionine metabolism is used to study the enzymatic changes in women of child-bearing age and the resulting concentrations of metabolites. In each case the results are compared to clinical and experimental studies. The causal mechanisms by which the gene expression or enzyme activity changes in women that lead to the metabolite changes will be discussed. (Received September 26, 2017)

1135-92-2602 **Timothy Patrick Barry\*** ([timbarry@umd.edu](mailto:timbarry@umd.edu)). *A mathematical model of iron metabolism in the human body.*

Iron-related diseases are prevalent throughout the world. Anemia, which has iron deficiency as a major cause, affects nearly one quarter of the world's population. Hereditary hemochromatosis, a disease of iron overload, is the most common inherited disease of gene mutation in Caucasians. Understanding the mechanisms of iron metabolism in the human body can facilitate individualized treatments strategies for these and other conditions. A mathematical model using ordinary differential equations is developed to simulate the distribution of iron in the major organs of the body. The model is calibrated for a healthy person using experimental time course data obtained from literature. The inclusion of hormones in the model, such as erythropoietin and hepcidin, enable the investigation of common iron disorders and potential treatments. This model provides a foundation for the creation of a personalizable model in which the specifics of an individual's condition form the parameter set, so that the outcomes of various treatments can be predicted. (Received September 26, 2017)

1135-92-2616 **Abbas Fazal\*** ([drfabbas@gmail.com](mailto:drfabbas@gmail.com)), School of Mathematical Sciences, 1 Lomb Drive, Rochester, NY 14623, and **Eberl J Hermann**. *Analytical and Numerical Study of Detachment Effects for the Upscaled Porous Medium Biofilm Reactor Model.*

Starting from the traditional mesoscopic one-dimensional biofilm model we derive a macroscopic model of a simple porous medium biofilm reactor in the convection dominated, laminar regime. The mesoscopic processes included in this model are biofilm growth due to substrate consumption and biomass loss due to cell death and biofilm detachment. The upscaling to the macroscale leads to a stiff quasilinear hyperbolic system of balance laws. In numerical simulations we investigate the role of the mesoscopic detachment description for the macroscopic model. To this end we compare four mesoscopic detachment models that are based on different model assumptions and lead to different mathematical expressions. We find that the particular choice plays only a minor role for macroscopic behavior, both from a quantitative and qualitative aspect. Similarly, we find that the overall reactor performance is rather insensitive with respect to the parameters of the detachment rate expressions. (Received September 26, 2017)

1135-92-2650 **Ryan Nikin-Beers, Lauren M Childs, Julie Blackwood and Stanca M Ciupe\*** (stanca@vt.edu). *Unraveling within-host signatures of dengue infection at the population level.*

Dengue virus causes worldwide concern with nearly 100 million infected cases reported annually. The within-host dynamics differ between primary and secondary infections, where secondary infections with a different virus serotype typically last longer, produce higher viral loads, and induce more severe disease.

We build upon the variable within-host virus dynamics during infections resulting in mild dengue fever and severe dengue hemorrhagic fever. We couple these within-host virus dynamics to a population-level model through a system of partial differential equations creating an immuno-epidemiological model. The resulting multiscale model examines the dynamics of between-host infections in the presence of two circulating virus strains that involves feedback from the within-host and between-hosts interactions, encompassing multiple scales. We analytically determine the relationship between the model parameters and the characteristics of the models solutions, and find an analytical threshold under which infections persist in the population. (Received September 26, 2017)

1135-92-2679 **Timothy C Reluga\*** (treluga@psu.edu). *The Importance of being Atomic: Ecological Invasions as Random Walks.*

A number of arguments over the sensitivities of ecological invasion speeds to stochasticity, density-dependence, dimension, and discreteness persist in the literature. The standard mathematical approach to invasions is based on Fisher's analysis of traveling waves. In this talk, I will consider an alternative theory based on the premise that living organisms are essentially discrete and invasions are best interpreted as random walks. Using a density-dependent invasion model in a stationary environment with atomic individuals where reproduction and dispersal are stochastic and independent, I show 4 key properties of invasions: (1) greater spatial dispersal stochasticity quickens invasions, (2) greater demographic stochasticity slows invasions, (3) negative density-dependence slows invasions, and (4) greater temporal dispersal stochasticity quickens invasions. The talk concludes with a complete classification of invasion dynamics based on dispersal kernel tails. (Received September 26, 2017)

1135-92-2712 **Christine Sample\***, Emmanuel College, 400 Fenway, Boston, MA 02215, and **Joanna Bieri, Benjamin Allen, Yulia Dementieva, Alyssa Carson, Shirley Qiu, Connor Higgins and Sadie Piatt.** *A model for quantifying the importance of habitats in a migratory network.* Preliminary report.

Understanding the influence of discrete habitats on the survival of a migratory species is an essential part of making successful conservation and management decisions. Migration is a complicated process, and mathematical models of migratory networks offer a way to understand the importance of different parts of an organism's annual cycle. Representing the system as a graph in which habitats are nodes and migratory paths are edges, the reproduction, survival, and movement of the population are modeled with time- and density-dependent functions. Under this network framework, we assess the importance of each node by calculating two values, the C-metric and K-metric. We generalize an existing C-metric, which estimates the per capita contribution of cohorts using a given node, to account for multiple seasons and classes. We develop the K-metric to estimate the overall growth rate when every individual must pass through a given node. We demonstrate how the proposed metrics can be used to analyze a variety of populations including the monarch butterfly, northern pintail, Yellowstone elk, and a plant species. (Received September 26, 2017)

1135-92-2715 **Cecilia Diniz Behn\*** (cdinizbe@mines.edu), Colorado School of Mines, Department of Applied Math and Statistics, 1500 Illinois Ave., Golden, CO 80401, and **Kelsey Kalmbach and Victoria Booth.** *A map-based approach to understanding sleep/wake dynamics in early childhood.*

A homeostatic need for sleep increases with time awake and decreases during sleep, and the dynamics of this process change across development. In adult humans, interactions between the homeostatic sleep drive and the  $\sim 24$ h circadian drive produce one nighttime sleep episode per day. By contrast, preschool children experience two sleep episodes per day: a daytime nap and nighttime sleep. To investigate the role of the dynamics of the homeostat in the transition between one and two sleep episodes per day, we analyzed bifurcations in a model for human sleep/wake dynamics as the time constants related to the build up and recovery of sleepiness are varied. As the time constants decrease, the system exhibits an incremental increase in the number of sleep episodes per day. Using a one-dimensional map to represent the dynamics of the system, we relate this map to a normal form for a piecewise continuous system which undergoes a border collision bifurcation, and we provide numerical evidence for period-adding behavior. This analysis has implications for understanding the dynamics of the transition from napping to non-napping behavior in early childhood. (Received September 26, 2017)

1135-92-2750 **Nora Youngs\***, nora.youngs@gmail.com, and **Carina Curto**. *Maps between neural codes*. Understanding how the brain stores and processes information is central to mathematical neuroscience. Neural data is often represented as a neural code: a set of binary firing patterns  $\mathcal{C} \subset \{0,1\}^n$ . We have previously introduced the neural ring, an algebraic object which encodes combinatorial information, in order to analyze the structure of neural codes. We now relate maps between neural codes to notions of homomorphism between the corresponding neural rings. Using natural operations on neural codes (permutation, inclusion, deletion) as motivation, we search for a restricted class of homomorphisms which correspond to these natural operations. We choose the framework of linear-monomial homomorphisms, and find that the class of associated code maps neatly captures these three operations, and that the class of isomorphisms includes two others - repetition and adding trivial neurons - which are also meaningful in a neural coding context and correspond to codes with no important differences in structure. (Received September 26, 2017)

1135-92-2758 **Angelica Estrada\*** (aestrada@smith.edu), **Elizabeth Fitzpatrick**, **Oumayma Koulouh**, **Salomea Jankovic** and **Yixuan Zhang**. *Non Fibonacci phyllotaxis*. Preliminary report.

Plant organs are often organized in lattice-like patterns, with two families of helices winding in opposite directions around the stem. The number of helices in these two families are usually successive Fibonacci numbers. This work focuses on the case when they are *not*: in many non Fibonacci cases, the number of helices tend to be close to equal. We provide statistical evidence for this phenomenon of “quasi-symmetry”, which has only been recognized recently, and point to a mechanism that explains both types of patterns. (Received September 26, 2017)

1135-92-2780 **Raina S Robeva\***, robeva@sbc.edu. *Stochastic dynamical models for assessing the effects of radiation on living cells*. Preliminary report.

Using models to determine the effect of radiation on the ability of cells to reproduce and form colonies is essential in radiotherapy and for assessing the risks of radiation. The classical target theory and many of its modern modifications take the administered dose as input, but do not account for the time duration over which the dose is delivered, thus essentially ignoring the ongoing repairs in the cell. We present a mechanistic stochastic framework that describes a dynamic dose-effect response, taking into consideration the underlying mechanisms of cell recovery. (Received September 26, 2017)

1135-92-2788 **Seema Nanda** (seema.nanda@dartmouth.edu), 6188 Hinman, Hanover, NH 03755, and **Yash Vyas\***, Dept of Statistics, Palo Alto, CA. *Parameter Analysis for a model for Tuberculosis Spread in India*.

We use a model for spread of tuberculosis (TB) in a closed community and deduce information about the prevalence of disease in a rural district of India. Parameter estimation of the model demands very specific data which is not easily available. We focus our study on the importance of these parameters in the model to identify information about spread of TB. We examine the case of correlated and uncorrelated parameters values and compare our conclusions in the two cases. (Received September 26, 2017)

1135-92-2811 **Alice C. U. Schwarze\*** (alice.schwarze@maths.ox.ac.uk), Mathematical Institute, University of Oxford, Woodstock Road, Oxford, OX2 7GG, United Kingdom, and **Mason A. Porter** and **Jonny Wray**. *Structural and Functional Redundancy in Biological Networks*.

Several scholars of evolutionary biology have suggested that functional redundancy (also known as biological “degeneracy”) is important for robustness of biological networks. Structural redundancy indicates the existence of structurally similar subsystems that can perform the same function. Functional redundancy indicates the existence of structurally different subsystems that can perform the same function. For networks with Ornstein–Uhlenbeck dynamics, Tononi et al. [Proc. Natl. Acad. Sci. U.S.A. **96**, 3257–3262 (1999)] proposed measures of structural and functional redundancy that are based on mutual information between subnetworks. For a network of  $n$  vertices, an exact computation of these quantities requires  $O(n!)$  time. We derive expansions for these measures that one can compute in  $O(n^3)$  time. We use the expansions to compare the contributions of different types of motifs to a network’s structural and functional redundancy. We compute structural and function redundancy for protein-interaction networks and find that these networks have larger functional redundancy than corresponding realisations of several random-graph models. (Received September 26, 2017)

- 1135-92-2814 **Kamal K Barley\*** ([barleykk@ucmail.uc.edu](mailto:barleykk@ucmail.uc.edu)), 435 Probasco St. Apt. 2, Cincinnati, OH 45220. *Implications from Modeling of Visceral Leishmaniasis Transmission: A Region-dependent Characterization of Risks.*

Approximately 500,000 new annual cases with more than 50,000 deaths occur globally as a result of Visceral Leishmaniasis (VL), a Neglected Tropical Infectious Disease. VL is the second deadliest parasitic killer after malaria, and is endemic in 47 countries, with India and Sudan having the highest burden. The risk factors associated with VL are either unknown in some regions or vary drastically among empirical studies. In this talk, I will present a new dynamical vector-borne epidemic model motivated by alternate field data from India and Sudan, which captures multiple VL risks. The model allows us to identify and quantify the impact of region-dependent risks on VL transmission dynamics. Our analysis shows that VL prevalence of symptomatic and asymptomatic host population depends highly on the progression of disease via development rates of symptoms and rates of treatment. Sensitivity analysis suggests that these two parameters are highly sensitive for both countries. In addition, we find that the mean estimates of transmission probability are significantly different for India and Sudan. I will discuss the implications of our results for VL prevention policy and control programs both in India and Sudan. (Received September 27, 2017)

- 1135-92-2849 **M Banelos\*** ([mbanuelos4@ucmerced.edu](mailto:mbanuelos4@ucmerced.edu)) and **S S Sindi**. *Modeling Transposable Element Dynamics with Fragmentation Equations.*

Transposable elements (TEs), segments of DNA capable of self-replication, are abundant in the genomes of most organisms. Because they are generally unnecessary, the host genome consists of both full-length (actively replicating) and partial length (inactive remnant) copies of TEs. Advances in sequencing efforts have led to TE annotations of many species and has revealed numerous partial length TEs remaining in host genomes.

We derive a novel mathematical formulation of TE dynamics that models the density of full and partial length copies with fragmentation equations and thus leverages the full TE annotation of a genome. We derive both an explicit form of the time-varying TE density and steady-state distribution. We fit our model distributions to present-day annotated collections of TEs from the genomes of species of fruit-flies and birds and uncover quantitative relationships of TE dynamics. More specifically, we determine species-specific transposition relationships within specific families of TEs and use cluster analysis of these rate estimates to assess the evolutionary history of these species. As a result, our work provides support for several hypotheses in the literature for the role of horizontal transfer of TEs between species. (Received September 26, 2017)

- 1135-92-2850 **Jason E McDermott\*** ([jason.mcdermott@pnnl.gov](mailto:jason.mcdermott@pnnl.gov)), **Jennifer Webster**, **Ryan McClure**, **Stephen J Young** and **Mahantesh Halappanavar**. *Biological graph analysis applications connecting molecular measurements with phenotype.* Preliminary report.

Biological graphs are commonly inferred from high-throughput measurements of biological molecules (mRNA, proteins, lipids, metabolites) from cells or tissues. Advances in 'omics technologies have driven the generation of multi-omic datasets, and thus integrated multi-omic graphs. In these graphs nodes represent different molecular species (mRNA, protein, post-translational modifications, lipids, metabolites) and edges represent statistical and/or predictive relationships between molecules based on their abundance patterns over a range of different observations (time points, treatments, patients). Clusters of nodes in these graphs commonly represent important and coherent functional groups whose response to specific conditions lead to phenotypic outcomes of the overall system. In this talk I will describe the development of approaches for inferring multi-omic graphs, use of graph analytic clustering approaches to better characterize functional modules, and how these graphs can be used to formulate hypotheses about biological mechanisms in cancer, infectious disease, and soil ecosystems. (Received September 26, 2017)

- 1135-92-2869 **Zvi H. Rosen\*** ([zhrosen@berkeley.edu](mailto:zhrosen@berkeley.edu)), Berkeley, CA 94703, and **Anand Bhaskar** and **Yun S. Song**. *The Geometry of the Sample Frequency Spectrum.*

The sample frequency spectrum (SFS) is a central tool in population genetics, used to infer historical population size, times to most recent common ancestor, and more. The SFS is the vector of probabilities of a given mutation being shared by  $k$  out of the  $n$  individuals in a sample, where  $n$  is fixed and  $k$  goes from 1 to  $n - 1$ . Using the toolbox of algebraic and convex geometry, we analyze the geometry of the set of all possible expected SFS for sample size  $n$ , under neutral selection. Our main result: the expected SFS for any population size history can be obtained by a piecewise-constant population size history with at most  $2n - 2$  epochs. (Received September 26, 2017)

1135-92-2975 **Jamie Prezioso\*** (jep127@case.edu), Department of Physiology & Biophysics, Case Western Reserve University, 10900 Euclid Ave., MS4970, Cleveland, OH 44106, and **Daneila Calvetti, Erkki Somersalo, Rossana Occhipinti and Walter Boron.** *Computational Model of Electrode-induced Microenvironmental Effects on pH Measurements Near a Cell Membrane.*

When studying gas channels in cells, surface and intracellular pH are determined in an oocyte using electrodes which measure the concentration of hydrogen ions ( $H^+$ ). It has been hypothesized that the presence of the electrode creates microenvironmental effects on pH measurements near the cell membrane. We develop a mathematical reaction-diffusion model of the electrode tip at the cell membrane which is restricted into a subdomain with artificial boundary conditions to keep the computational burden manageable. We consider an oocyte immersed in a liquid in which the pH is modulated by controlling the contents of carbon dioxide,  $CO_2$ , which reacts with water forming carbonic acid,  $H_2CO_3$ , which further dissociates to bicarbonate  $HCO_3^-$ , thus releasing a proton  $H^+$ . The local concentrations change due to diffusion, reaction, and gas transport through the cell membrane. To implement the solution numerically, we first discretize the solution in the spatial direction using a finite element scheme, reducing the problem into a system of ODEs. Through a series of numerical experiments, we show that the electrode does, in fact, create a microenvironment impacting the extracellular pH measurements. (Received September 26, 2017)

1135-92-2982 **Devin Mattoon\*** (dmattoon@missouriwestern.edu), **Altan Tutar, Hannah S Sinks, Autumn Estrada, David Sullivan, Daniel Zweerink, A. Malcolm Campbell, Todd T Eckdahl, Jeffrey L Poet and Laurie J Heyer.** *Modeling directed evolution with coupon-collecting and mixing problems.* Preliminary report.

Our interdisciplinary, interinstitutional team of undergraduate researchers sought to evolve a DNA sequence in a bacterial virus, using natural selection to alter the specificity of an RNA regulatory element called a riboswitch. We modeled random mutations in the virus, accounting for lethality due to damaged genes. We applied a generalized solution of the coupon collector's problem to calculate the expected value and a confidence interval for the number of viral genomes required to observe all possible single, double and triple base substitutions in the DNA sequence, after removing from the population all viruses with lethal mutations. We used our probability model to ensure that the spatial and temporal scale of the wet lab experiment was sufficient to explore the extremely large space of all possible DNA sequences of a given length. Further, we used differential equations to predict the time required for a small molecule to reach each of its target concentrations in a dynamic solution, and for particular virus genomes to dominate the population in each of three parallel compartments of a chemostat. Finally, we built web tools to make generalized versions of our probability and differential equation models available to biologists for the design of future experiments. (Received September 26, 2017)

1135-92-2998 **Dan Hrozencik\*** (dhrozenc@csu.edu), 9501 S. King Dr., HWH 332, Chicago, IL 60628, and **Olcay Akman, Devin Akman and Tim Comar.** *Reconstructing Stochastic Gene Regulatory Networks with Recurrent Neural Network Models Using Particle Swarm Optimization.*

In previous work, the authors developed a model to examine the state transition variability of gene regulatory networks (GRNs) due to stochastic variation of propensity probabilities caused by internal noise at the molecular level. Using this model, the authors now show how to optimize the GRNs from gene expression data. In particular, the authors propose to use recurrent neural networks and particle swarm optimization to reconstruct the GRN from data via self-adapting intelligent optimization. (Received September 26, 2017)

1135-92-3045 **Zhisheng Shuai\*** (shuai@ucf.edu), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Coupled Infectious Disease Models via Asymmetric Movements.* Preliminary report.

Many recent outbreaks and spatial spread of infectious diseases have been influenced by human movement over air, sea and land transport networks, and/or anthropogenic-induced pathogen/vector movement. These spatial movements in heterogeneous environments and networks are often asymmetric (biased). The effects of asymmetric movement versus symmetric movement will be investigated using several epidemiological models from the literature. These investigations provide a better understanding of disease transmission and control in the real life application. (Received September 26, 2017)

- 1135-92-3150 **Stanca Ciupe** and **Jonathan Forde\*** ([forde@hws.edu](mailto:forde@hws.edu)), Dept. of Mathematics and Computer Scienc, 300 Pulteney St., Geneva, NY 14456, and **Christopher Miller**. *Bistable dynamics in a model of SIV infection with antibody response*.

Experiments in rhesus macaques have shown that for simian immunodeficiency virus the size of the viral inoculum and the infection stage of the donor animal alter the likelihood of establishment of infection. In this study, we postulate a role for the host and donor antibodies in explaining the dependence of infectiousness on donor infection stage. The resulting mathematical model exhibits bistable dynamics, with viral clearance and persistence dependent on initial conditions. We fit the model to temporal virus data using a censored data approach for measurements below the limit of detection, and make predictions about the minimal viral load in the inoculum required for persistent infection. (Received September 26, 2017)

- 1135-92-3204 **Olcay Akman\*** ([oakman@ilstu.edu](mailto:oakman@ilstu.edu)), Department of Mathematics, Box 4520, Normal, IL 61790-4520, **Timothy Comar**, Department of Mathematics, Lisle, IL 60532, and **Daniel Hrozencik** ([dhro@att.net](mailto:dhro@att.net)), Department of Mathematics, Chicago, IL 60628. *Environmental effect estimation under hidden heterogeneity*. Preliminary report.

Most epidemic related models that involve lifetime estimation or ecological models used to predict survival rates assume that the members of the population in question are affected from the environment identically. However it is not unreasonable to assume that the environmental effect on certain individuals vary due to their frailty. The concept of frailty provides a suitable way to introduce random effects in the model to account for the unobserved heterogeneity of the population. In its simplest form, a frailty is an unobserved random factor that modifies the lifetime of an individual or a group or cluster of individuals. In this talk we will explore extending commonly used models to accommodate the frailty component due to environmental effects. (Received September 27, 2017)

- 1135-92-3220 **Owen L Lewis\***, 155 S 1400 E, ROOM 233, Salt Lake City, UT 84112, and **James P Keener** and **Aaron L Fogelson**. *Maintenance of the pH gradient in the gastric mucus layer*.

The gastric mucus is a complex gel-like layer of various proteins and solutes coating the epithelial surface of the stomach. This layer is widely recognized to serve a protective function, shielding the epithelium and the rest of the gastric mucosa from the extremely low pH and digestive enzymes present in the stomach lumen. Often described as a “diffusion barrier” the mucus layer is thought to hinder the transport of diffusive species from the lumen, to the stomach wall. However, there is still a lack of consensus on the mechanism by which the mucus layer hinders lumen-to-wall transport while allowing acid and enzymes secreted from the mucosa unimpeded transport to the lumen. Using a model of two-phase fluid motion coupled with electro-diffusion, we test one hypothesis. Furthermore, we explore what regulatory mechanisms are necessary to segregate an acidic stomach lumen from a pH neutral stomach wall. (Received September 27, 2017)

- 1135-92-3234 **Vanessa Danielle Machuca\*** ([vdm02014@mymail.pomona.edu](mailto:vdm02014@mymail.pomona.edu)), 2484 Paloma St., Pasadena, CA 91104. *Oscillations and Macropatterning in Electroless Silver Deposition on Copper*. Preliminary report.

This work explores the  $AgNO_3/Cu$  system as a model system for self-assembly in electroless deposition with potential analogies to disease propagation. Parameters like  $AgNO_3$  concentration, solution viscosity, and copper electrode surface area were varied to explore the dynamics of the  $AgNO_3/Cu$  system. Dynamic regions were defined by the stability and frequency of redox potential oscillations. Macropatterning in this system was also investigated, using image and video analysis. Spatiotemporal plots of silver aggregate diameter were constructed and fractal dimensions calculated for the various growth regimes exhibited in the macropatterns observed. Simulations were also run using a previously developed differential equation model, and connections made to the well-known diffusion-limited aggregation model. (Received September 27, 2017)

## 93 ► *Systems theory; control*

- 1135-93-327 **Martin G. Vieten\*** ([mgvieten@uwm.edu](mailto:mgvieten@uwm.edu)) and **Richard H. Stockbridge**. *Linear Programming Formulations for Singular Stochastic Control Problems with A Focus on Numerical Policy Evaluation*. Preliminary report.

Infinite-dimensional linear programming formulations for occupation measures of singular stochastic processes have been extensively studied by Stockbridge, Kurtz et al. Their equivalence to control problems posed by relaxed martingale problem formulations has been shown and has been instrumental in finding analytic solutions



to a line of singular stochastic control problems.

Recent research focused on establishing a numerical method solving infinite-dimensional linear programs using approximation techniques borrowed from finite element theory. In particular, a certain type of discretization of the relaxed controls was proposed, and its asymptotic optimality was shown using weak convergence arguments. This talk introduces the linear programming approach and outlines the main results for the proof of the asymptotic optimality of the proposed numerical approximation. Then, it presents how the linear programming formulations can be used to evaluate the cost criterion of an arbitrary, not necessarily optimal control. An existence and uniqueness argument is derived from approximate inversions of the infinitesimal generators and classic arguments from measure theory. The performance of the numerical approximation is illustrated with an example of stochastic logistic growth. (Received August 24, 2017)

1135-93-343 **Michael Malisoff\***, Department of Mathematics, Louisiana State University, 303 Lockett Hall, Baton Rouge, LA 70803-4918. *Sequential Predictors for Input Delay Compensation in Control Systems*.

Control systems arise in a plethora of engineering applications in which measurements of the current state of the system may not be available. This has given rise to a large literature on the design of feedback controls that achieve uniform global asymptotic stabilization using only time lagged observations of the state, which produces control systems with input delays. One approach to addressing this challenge is to solve the feedback stabilization problem for the system with the input delay set equal to zero, and then to compute upper bounds on the allowable delays that ensure that the feedback control continues to ensure uniform global asymptotic stabilization when input delays are present. Other approaches include prediction, which can compensate for arbitrarily long input delays, but which often produces feedback controls with distributed terms that can be difficult to compute. This talk will present the speaker's alternative approach to delay compensation based on sequential predictors, which can compensate for arbitrarily long input delays, including in nonlinear control systems, or systems with time-varying delays, without using distributed terms. No prerequisite background in control theory will be needed to understand this talk. The work is joint with Frederic Mazenc. (Received August 26, 2017)

1135-93-501 **Billy Jackson\*** (jacksonbi@geneseo.edu) and **John Davis** (john\_m\_davis@baylor.edu). *A Wholistic Solution to the Problem of Determining the Stability Region on Arbitrary Time Scales*. Preliminary report.

We consider methods for computing solutions to the integral condition

$$\limsup_{T \rightarrow \infty} \frac{1}{T - t_0} \int_{t_0}^T \frac{\log |1 + \mu\lambda|}{\mu} \Delta t < 0$$

given by Pötzche, Siegmund, and Wirth in 2001 for necessity and sufficiency of solutions to  $x^\Delta = \lambda x$  to be asymptotically stable. In particular, we expound upon earlier work by the authors in the purely discrete case to include examination of continuous intervals by looking at the embedded sequence of intervals and discrete points as they appear in the arbitrary time scale  $T$ . In contrast with other work in this direction, our results make no commutativity assumptions arising from the system matrix. (Received September 06, 2017)

1135-93-626 **Qing Zhang** and **Caojin Zhang\*** (czhang@wayne.edu), Wayne St Univ, Department of Mathematics, 656 W Kirby, Detroit, MI 48202, and **George Yin**. *Optimal Stopping of Two-Time Scale Markovian Systems*.

Numerous systems arising in applications are subject to uncertainty and stochastic influence. They are often of large scales and have complex structures. They may also display hybrid behavior represented by regime-switching dynamic systems. Exact optimal control policies for these systems are difficult or virtually impossible to obtain. In this paper, we study a representative of such systems, which is an optimal stopping problem driven by a Markov chain. We consider the case that the chain has a large state space. Then, it is natural to divide the states into a number of groups so that the chain jumps frequently within each group and only occasionally among different groups. We develop a two-time-scale approach to reduce the overall dimensionality and construct near-optimal strategies for the original problem. Examples are provided to illustrate the results. Application examples to equity markets are also considered as well. (Received September 11, 2017)

1135-93-656 **Dylan R Poulsen\*** (dpoulsen2@washcoll.edu), 300 Washington Ave, Chestertown, MD 21620. *Linear Mean-Square Stability Theory for Stochastic Pulse Time Scales*. Preliminary report.

We are motivated by the consensus problem in multi-agent systems, wherein agents try to reach agreement about the value of a certain quantity by communicating with one another. If the line of communication between two

agents is unreliable, then the time set on which the agents can communicate forms a pulse time scale with varying and random gap sizes and interval lengths. Solving the consensus problem on time scales can be related to the asymptotic stabilization problem on time scales. Therefore, we discuss the stability theory for linear systems on such stochastic pulse time scales, where the length of communication uptime and downtime are random variables. (Received September 12, 2017)

1135-93-1523 **Hanbaek Lyu\*** (colourgraph@gmail.com), 231 W 18th ave, Columbus, OH 43210. *Global synchronization of pulse-coupled oscillators on trees.*

Consider a distributed network on a simple graph  $G = (V, E)$  with diameter  $d$  and maximum degree  $\Delta$ , where each node has a phase oscillator revolving on  $S^1 = \mathbb{R}/\mathbb{Z}$  with unit speed. Pulse-coupling is a class of distributed time evolution rule for such networked phase oscillators inspired by biological oscillators, which depends only upon event-triggered local pulse communications. In this paper, we propose a novel inhibitory pulse-coupling and prove that arbitrary phase configuration on  $G$  synchronizes by time  $51d$  if  $G$  is a tree and  $\Delta \leq 3$ . We extend this pulse-coupling by letting each oscillator throttle the input according to an auxiliary state variable. We show that this adaptive pulse-coupling synchronizes arbitrary initial configuration on  $G$  by time  $83d$  if  $G$  is a tree. As an application, we obtain a universal randomized distributed clock synchronization algorithm, using  $O(\log \Delta)$  memory per node with  $O(|V| + (d^5 + \Delta^2) \log |V|)$  expected worst case running time. (Received September 22, 2017)

1135-93-1615 **Guangliang Zhao\*** (dr.gzhao@gmail.com), **Zhixin Yang, Quan Yuan and Leyi Wang.** *Controllability in Term of Mean Reaching Time for Markovian Switching Diffusion System.*

For systems of switching diffusions, a concept of controllability is proposed by using the finite expected value of the first reaching time (or mean reaching time) to an arbitrarily small open set of the terminal point. A necessary and sufficient condition is obtained utilizing positive recurrent. A verification criterion is provided by using Liapunov functions, and a matrix form criterion if the system is linear. Finally, a simulation example is provided to show that when coupled with regime switching, the hybrid system of uncontrollable ODEs could be controllable. (Received September 23, 2017)

1135-93-2338 **Marisa C Eisenberg\*** (marisae@umich.edu). *Model and Parameter Uncertainty in Environmentally Driven Disease Models.*

Environmentally-driven diseases such as cholera vary widely in their structures, in terms of transmission pathways, loss of immunity, and a range of other features. These differences can affect model dynamics, with different models potentially yielding different predictions and parameter estimates from the same data. Given the increasing use of mathematical models to inform public health decision-making, it is important to assess uncertainty both in parameter estimates and in model structure. In this talk, we will examine how alternative model structures can affect forecasting and parameter identifiability, evaluating whether the parameter values, model behavior, and forecasting ability can accurately be inferred from data. We will also examine how parameter estimation of environmentally driven disease models using alternative data sources, such as environmental surveillance. (Received September 26, 2017)

## 94 ► *Information and communication, circuits*

1135-94-767 **Leigh Metcalf\***, Software Engineering Institute, 4500 Fifth Ave, Pittsburgh, PA 15213. *The Internet is a Hall of Mirrors.*

The Internet is a complex network with no standard coherent frame of reference, hindering analysis. In this talk we discuss why this is true and give examples drawn from real data. (Received September 14, 2017)

1135-94-833 **Günter M. Ziegler\*** (ziegler@math.fu-berlin.de), Institute of Mathematics, Freie Universität Berlin, Arnimallee 2, 14195 Berlin, Germany. *Let's make a DEAL: Open Access, Business Models, and Transformation Dynamics.*

Open Access has been a dream for many years – but the Publishing Industry has embraced this mostly as a source of additional revenues, but resisted all attempts and requests for transformations. This is changing in the times of electronic communication, open repositories (like the arXiv) and wide-spread piracy. Why should anyone still want to pay for reading? On the other hand, publishers should be paid for (quality) publishing. Ongoing negotiations might force major publishers into re-thinking their business models. Science has to support change, grab the opportunities, but also stay alert. (Received September 15, 2017)

1135-94-855 **Nathan Albin, Jason Clemens and Nethali Fernando\***, Department of Mathematics, Kansas State University, Manhattan, KS 66506, and **Pietro Poggi-Corradini**. *A new proof that effective resistance is a metric on graphs.*

We explore the implications of blocking duality—pioneered by Fulkerson et al.—in the context of  $p$ -modulus on networks. Fulkerson’s blocking duality is an analogue on networks to the method of conjugate families of curves in the plane. The technique presented here leads to a general framework for studying families of objects on networks; each such family has a corresponding dual family whose  $p$ -modulus is essentially the reciprocal of the original family’s.

As an application, we give a modulus-based proof for the fact that effective resistance is a metric on graphs. This proof immediately generalizes to yield a family of graph metrics, depending on the parameter  $p$ , that continuously interpolates among the shortest-path metric, the effective resistance metric, and the mincut ultrametric. (Received September 15, 2017)

1135-94-985 **Kaitlyn Myers and Hieu D Nguyen\*** ([nguyen@rowan.edu](mailto:nguyen@rowan.edu)), Rowan University, Department of Mathematics, 201 Mullica Hill Rd., Glassboro, NJ 08028. *A new efficient decoding algorithm for correcting multiple insertion or deletion errors in Helberg codes.*

Helberg codes are binary error-correcting codes capable of correcting multiple insertions/deletion errors. A decoding algorithm was recently found by Le and Nguyen to correct multiple deletion errors in these codes. We present a new efficient decoding algorithm for correcting multiple insertion (or deletion) errors. (Received September 18, 2017)

1135-94-1436 **Thierry Bouche\*** ([thierry.bouche@univ-grenoble-alpes.fr](mailto:thierry.bouche@univ-grenoble-alpes.fr)). *Supporting the move to diamond open access: The launch of Centre Mersenne*. Preliminary report.

I will introduce the newly launched Centre Mersenne, a French publishing infrastructure support journals publishing under the Diamond Open Access model (aka Gold without APC). (Received September 22, 2017)

1135-94-1627 **Arthur M Jaffe\*** ([jaffe@g.harvard.edu](mailto:jaffe@g.harvard.edu)). *Charged String Languages in Quantum Information.*

We give an introduction to the two-string pictorial language and the four-string quon language defined by the author in collaborators Zhengwei Liu and Alex Wozniakowski. (Received September 23, 2017)

1135-94-2356 **Andrea L Bertozzi\*** ([bertozzi@math.ucla.edu](mailto:bertozzi@math.ucla.edu)), Department of Mathematics, 520 Portola Plaza, UCLA, Los Angeles, CA 90095. *Geometric graph-based methods for high dimensional data.*

We present new methods for segmentation of large datasets with graph based structure. The method combines ideas from classical nonlinear PDE-based image segmentation with fast and accessible linear algebra methods for computing information about the spectrum of the graph Laplacian. The goal of the algorithms is to solve semi-supervised and unsupervised graph cut optimization problems. The methods make parallels between geometric ideas in Euclidean space such as motion by mean curvature, ported to a graphical framework. These ideas can be made rigorous through total variation minimization, and gamma convergence results, and convergence of time stepping methods in numerical analysis. I will show diverse examples including image processing applications such as image and video labeling and hyperspectral video segmentation, and machine learning and community detection in social networks, including modularity optimization posed as a graph total variation minimization problem. (Received September 26, 2017)

1135-94-3079 **Qiyu Sun\*** ([qiyu.sun@ucf.edu](mailto:qiyu.sun@ucf.edu)), University of Central Florida, Orlando, FL 32816. *Phaseless sampling and reconstruction of signals with finite rate of innovation*. Preliminary report.

A signal is determined by its evaluation on its domain. In this talk, we consider the problem when and how a signal with finite rate of innovation can be reconstructed, up to a global phase, from its magnitudes on its domains or its sampling set. (Received September 26, 2017)

1135-94-3107 **Jessalyn Bolkema\*** ([jessalyn.bolkema@huskers.unl.edu](mailto:jessalyn.bolkema@huskers.unl.edu)). *Results on polar codes via configurations on multitrees.*

While Arikan’s polar codes have been celebrated for their capacity-achieving performance since first presented in 2008, the questions of optimal finite-length design and decoding remain open. In this talk, we explore a graph-theoretic construction of computation multitrees applicable to belief propagation decoding. We describe configurations on these graphs that provide insight into the structure and decoding properties of polar codes. (Received September 26, 2017)

1135-94-3141 **Rachid Ait Maalem Lahcen\*** (rachid@ucf.edu), Department of Mathematics, University of Central Florida, ORLANDO, FL 32816, and **Ram N Mohapatra** (ramm1627@gmail.com), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Extremal Graph Theoretic Approach to Study Network Vulnerability and Cyber Security.*

In this preliminary study we shall mention the importance of the three layers to study cyber security. Kyle Ingols, Richard Lippmann, and Keith Piowowski of MIT Lincoln Laboratory, have studied practical attack graphs for network defense. They have shown that Attack graphs are a valuable tool to network defenders, illustrating paths an attacker can use to gain access to a targeted network. Our objective is to see how these methods can be used to protect the communication networks for UAVs. The research is at the exploratory stage. (Received September 26, 2017)

## 97 ► *Mathematics education*

1135-97-13 **Michael Alexander Radin\*** (michael.a.radin@gmail.com), Rochester Institute of Technology, School of Mathematical Sciences, Rochester, NY 14623. *Communication with students outside the classroom and international education and learning.*

Professors encourage and expect students to learn outside the classroom and demonstrate their competence; in fact, 80% of the learning occurs outside the classroom (Brown, 1958). Now it is of paramount importance to address the following questions: How should professors teach students and interact with students outside the classroom and enhance their competence (Ostman and Wickman, 2014)? How should professors get started? What opportunities and resources are available to professors to enhance the students' competence? How should faculty encourage students to come to office hours and how should faculty increase and retain the attendance during office hours (Radin, 2016)? By how much percent will these new innovative ideas increase the class attendance and enhance the students' academic performance? How to extend our teaching boundaries outside the classroom and internationally (Shields, 2003)? What new challenges will professors experience (Von Glaserfeld, 1989)? How will this change and affect professors' future teaching style(s)? Will these techniques work on the international level outside the U.S.? What adjustments will be necessary to acclimate? (Received June 07, 2017)

1135-97-16 **Steven J Miller\*** (sjm1@williams.edu), 18 Hoxsey St, Williamstown, MA 01267. *YouTube University: Shared upper level math courses across schools.*

For the past several years I've recorded my lectures and uploaded them to YouTube. Once I was fortunate enough to have OIT make professional recordings; the other classes I've used an iPad. This has allowed students at other liberal arts colleges to remotely take my classes (this is very useful, as small institutions cannot always offer the classes our students need at the time they need to take it). Additionally, if my students miss a lecture (illness, sports meet) they can see exactly what they missed. I'll discuss some of the opportunities and challenges in collaborating across institutions. (Received June 09, 2017)

1135-97-39 **Kevin C. Moore\*** (kvcmoore@uga.edu), 105 Aderhold Hall, Dept. of Mathematics & Science Education, University of Georgia, Athens, GA 30602. *Visualization: Constructing what is "out there".*

The use of representations is pervasive to the teaching and study of mathematics, so much so that individuals often treat representations as being self-evident with respect to the ideas they are intended to represent. In this report, I draw on Piagetian ideas to characterize representational activity as a complex process of reflection, abstraction, and internalized mental coordinations that ultimately influences what we perceive as being "out there". Specifically, I focus on evolutions in students' ways of thinking for graphing to illustrate differences between thought dominated by sensorimotor experience and thought dominated by internalized mental operations that, in turn, become objects of thought. By focusing on the aforementioned differences in students' thinking, I argue that the former ways of thinking—those dominated by sensorimotor experience—are less productive for students' learning than the latter ways of thinking—those dominated by internalized mental operations. I also argue, however, that common instructional practices foster ways of thinking that foreground sensorimotor experience, thus not affording students sufficient opportunities to engage in representational activity that is conducive for their constructing generative mathematical ways of thinking. (Received June 28, 2017)

1135-97-97 **Nathan Pennington\*** ([nathanpennington@creighton.edu](mailto:nathanpennington@creighton.edu)), 2500 California Plaza, Omaha, NE 68178. *Modeling in an Integrated Mathematics and Physics course.*

Modeling the Physical World is a year long project-based integrated Mathematics and Physics course aimed at advanced incoming first-year students. While the students receive credit for Calc II/Calc III and General Physics I/II, many of the projects require the use of differential equations and would be suitable for a differential equations class. In this talk we discuss the structure of the course, finding compelling modeling problems, and ideas for incorporating modeling with differential equations into a variety of courses. (Received July 26, 2017)

1135-97-127 **Martin Flashman\*** ([flashman@humboldt.edu](mailto:flashman@humboldt.edu)). *Mapping Diagrams Visualizing Functions and Equation Solutions: From Algebra Basics to Real and Complex Analysis.* Preliminary report.

A mapping diagram is an alternative to a Cartesian graph that visualizes a function using parallel axes. Like a table, it can present finite data, but it also can work continuously and dynamically with technology. Using GeoGebra as the primary technology, this presentation will cover topics from visualizing the algebra in solving simple linear equations to understanding the functions of real analysis for differentiation and integration. The conclusion will focus on complex analysis with new dynamic 3 dimensional mapping diagrams for visualizing complex polynomial functions and solutions to cubic equations. Background and examples can be found at Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(differential) E(uation)s [<http://users.humboldt.edu/flashman/MD/section-1.1VF.html>] and at Mapping Diagrams to Visualize Complex Analysis [<https://www.geogebra.org/m/Ni69jyKs>]. (Received July 31, 2017)

1135-97-276 **Julie Barnes\*** ([jbarnes@email.wcu.edu](mailto:jbarnes@email.wcu.edu)). *Mathematical visualization using Wikki Stix, yarn, or pipe cleaners.* Preliminary report.

In this talk, we discuss using hands-on class activities involving Wikki Stix, yarn, or pipe cleaners to help students more deeply understand topics from a variety of undergraduate mathematics courses. Examples include physically acting out function transformations, using properties of derivatives to graph associated functions, and dynamically demonstrating the epsilon-delta definition of continuity. In all cases we look at logistical considerations and observed benefits from using these activities. (Received August 17, 2017)

1135-97-292 **Alexander Fedorovich Mironychev\*** ([amironit@houstonisd.org](mailto:amironit@houstonisd.org)), 5601 West Loop South Freeway, Challenge Early College, Houston, TX 77081. *SAS and SSA Conditions for Congruent Triangles.*

The traditional set of theorems at the high school and undergraduate college level regarding congruent triangles include the following – SAS, ASA, AAS, SSS, and HL. However, the set of statements and theorems describing congruent triangles can be extended to include one more theorem which is often overlooked by most texts: SSA. For two triangles to be congruent, SAS theorem requires two sides and the included angle of the first triangle to be congruent to the corresponding two sides and included angle of the second triangle. If the congruent angles are not between the corresponding congruent sides, then such triangles could be different. It turns out that it is possible to describe four cases in which triangles are congruent even though congruent angles are not between the corresponding congruent sides. Such a theorem could be named, for example, SSA theorem. Many texts state that two triangles cannot be shown to be congruent if the condition of SSA exists. However, the author describes cases in which such triangles could be proven congruent with the SSA theorem. An immediate consequence of this new understanding is the necessity of revising many problems and answers in high school and college-level texts related to congruent triangles. (Received August 19, 2017)

1135-97-319 **Allison J Dorko\*** ([allison.j.dorko@okstate.edu](mailto:allison.j.dorko@okstate.edu)), 676 Waterville Road, Skowhegan, ME 04976. *Assimilation, Accommodation, and Graphing in R3.*

The ability to construct a graph in R3 is important for student success in many multivariable topics, but many students struggle to create correct R3 graphs. In this talk, I will discuss students' strategies for visualising some basic R3 graphs. I will frame the talk with Piagetian learning theory. Specifically, I will argue that characterising students' graphing strategies in terms of assimilation and accommodation provides an explanatory mechanism for students' difficulty visualising R3 graphs. I will conclude with instructional suggestions for helping students create correct graphs. (Received August 23, 2017)

1135-97-377 **Polina D Sabinin\*** ([psabinin@bridgew.edu](mailto:psabinin@bridgew.edu)), 374 Great Rd, Unit 12, Acton, MA 01720. *Play again! Developing spatial reasoning and visualization through games and puzzles (with emphasis on the mathematical mindset).*

Many students in introductory mathematics courses struggle with insecurities, fear, and anxiety. Anxiety and aversion to risk-taking create a vicious cycle of underperformance in mathematics. To break this cycle, instructors

must create a learning-centered atmosphere with highest value being placed on risk-taking and ‘stretch’-mistakes. Fortunately, there are ample educational games and puzzles which develop students’ spatial reasoning in a non-threatening environment in class. In this workshop, the speaker will present a number of spatial reasoning games which she has used in Second Year Seminars and outreach programs, ways to assess students’ learning through games, as well as provide a short overview of the impact of mathematical mindset on meaningful learning. (Received August 29, 2017)

1135-97-484 **Natali Hritonenko\*** ([nahritonenko@pvamu.edu](mailto:nahritonenko@pvamu.edu)). *Facilitating the understanding of mathematics curriculum in students through problem development exercises.*

Various pedagogical ideas have been suggested to foster students’ understanding of mathematical topics, help them see relations among different techniques and choose the best strategy for solving specific problems. Warm-ups, games, bonus questions, and problem creation are among them. Their descriptions and examples, suitable to different levels of mathematical courses, will be presented. Emphasis will be made on exercises asking students to develop problems with specific requirements. The benefits and challenges of these activities, from both assessment and students’ perspectives, will be discussed. (Received September 05, 2017)

1135-97-598 **William R. Penuel\*** ([william.penuel@colorado.edu](mailto:william.penuel@colorado.edu)), School of Education, UCB 249, Boulder, CO 80305. *Strengthening Infrastructures for Promoting Equity in Mathematics Education through Research-Practice Partnerships.*

Research-Practice Partnerships (RPPs) are long-term collaborations between researchers and practitioners. RPPs are intentionally organized, focused on persistent problems of practice, mutualistic, and long-term. In RPPs, researchers may collaboratively design new programs with educators, support implementation, and study implementation and outcomes. There is strong evidence that interventions designed by RPPs can improve student outcomes and the quality of teaching (Coburn & Penuel, 2016).

RPPs can be an important form of infrastructure for promoting equity. For one, they empower educators to help shape reforms. Second, the involvement of researchers over the long-term allows for programs to be adapted and studied when they fail to achieve equitable implementation and outcomes. Third, the research conducted by RPPs can provide a powerful justification for equity-oriented reforms.

Successful RPPs are not easy to develop or sustain, however. Success depends on mutual regard for the expertise of researchers and practitioners, a commitment to ongoing communication, tools and routines for ensuring mutualism, and intentional efforts to build trust while also addressing mistrust that arises from educational inequity and status differences between researchers and practitioners. (Received September 09, 2017)

1135-97-608 **Dina Yagodich\*** ([dyagodich@frederick.edu](mailto:dyagodich@frederick.edu)), **Brian Winkel** ([brianwinkel@simiode.org](mailto:brianwinkel@simiode.org)), **Corban Harwood** ([rhharwood@georgefox.edu](mailto:rhharwood@georgefox.edu)) and **William Skerbitz** ([william.skerbitz@wayzata.k12.mn.us](mailto:william.skerbitz@wayzata.k12.mn.us)). *Panel Discussion and Audience Participation: Experience with Engaging in Modeling in a Differential Equations Course.*

The panel will share personal experiences with engaging in modeling in a differential equations course and invite the audience to share in their own experiences, ask questions, and enter a conversation. (Received September 10, 2017)

1135-97-671 **Margo Alexander\*** ([malexander@gsu.edu](mailto:malexander@gsu.edu)), 25 Park Place, Suite 1400, Atlanta, GA 30303. *Collaborative Teaching: Math History and Ethno-mathematics.*

There has been a growing recognition among educators of the important role through intercampus collaboration can have in enhancing many aspects of teaching and learning. Collaborative teaching is valuable for promoting integrative thinking but it is costly to commit a couple of faculty members to one course. However, this major university sees the benefits and is willing to go the extra mile to facilitate in the improvement of instruction. The collaboration will be between a mathematics course of the History of Mathematics and a mathematical educator’s course of Ethno-mathematics. The primary purpose of this study is to provide students with an alternative lens to view mathematical knowledge as it transpired across civilizations and within various non-western cultures and to help endow students with the necessary predispositions to look at the world through ethno-mathematical eyes. That is, this course dovetails two components: the exploration of the historical development of mathematics and second, culturally immersing students firsthand in a field experience of an authentic, elusive, and problematic diverse setting. The purpose of this talk is to describe the impact of this collaborative teaching and learning experience as well as student perceptions. (Received September 12, 2017)

1135-97-893 **Andrew D. Jones\*** ([drandrewjones@outlook.com](mailto:drandrewjones@outlook.com)), 16023 Henley Road, East Cleveland, OH 44112. *The Best Way to Teach Math.*

Too often it is asked, “What is the best way to teach mathematics?” This question is equivalent to, “What is the best shoe size?” The answer is clearly, “The size that fits the individuals foot”.

Just as a good shoe store has a variety of lengths and widths for each style of shoe, a good school should have a variety of content and delivery systems mathematics. This can be done at little to no additional cost.

In this presentation, we examine different content and delivery systems that have been employed to teach mathematics. We will give special attention to data from a precalculus course that employed rigorous content and Socratic delivery. (Received September 16, 2017)

1135-97-1124 **Manmohan Kaur\*** ([mkaur@ben.edu](mailto:mkaur@ben.edu)), Department of Mathematics, Benedictine University, 5700 College Road, Lisle, IL 60532. *An Undergraduate Number Theory Course in Disguise.* Preliminary report.

In order to get undergraduates interested in mathematics, it is essential to introduce them to interesting and challenging problems in the field. Cryptology, the science of sending and receiving secret messages, encompasses every aspect of modern life – online financial transactions, digital signatures, cloud computing, and so on. The esoteric nature of cryptology makes it naturally interesting to the undergraduates at our liberal arts institution, whereas the same set of students may not choose a more ‘abstract sounding’ course. In Spring 2004, we started teaching a course entitled ‘Introduction to Cryptology’, which essentially is a modified number theory course, complete with all the rigor of abstract mathematics. The first few iterations of the course were taught from Rosen’s number theory book, while more recently we have moved to other texts. In this presentation, we will discuss this Cryptology course, which has become a popular elective for our both our math majors and math minors. (Received September 19, 2017)

1135-97-1210 **Tim Flood\*** ([tflood@pittstate.edu](mailto:tflood@pittstate.edu)), Mathematics Department, Pittsburg State University, 1701 S Broadway, Pittsburg, KS 66762. *Using GeoGebra to Visualize and Investigate Complex Valued Functions of a Complex Variable.*

Students are very well versed at visualizing real valued functions of a real variable. They are adept at producing graphs and determining a variety of characteristics of functions by analyzing the properties of the graphs of the functions. Since complex valued functions of a complex variable map the plane to the plane, it is much more difficult to visualize these functions and to investigate the characteristics of the function. Properties such as one-to-one, onto, and continuity are difficult, if not impossible, to visualize without the use of some type of technology. A very simple GeoGebra worksheet can be used for students to visualize and explore complex valued functions of a complex variable. The basics of creating the worksheet, along with demonstrations of its use in investigating the properties of complex functions will be presented. (Received September 20, 2017)

1135-97-1484 **Paulina Chin\*** ([pchin@maplesoft.com](mailto:pchin@maplesoft.com)), 615 Kumpf Drive, Waterloo, Ontario N2L6N4, Canada, and **Louise Krmptotic** ([louisek@maplesoft.com](mailto:louisek@maplesoft.com)), 615 Kumpf Drive, Waterloo, Ontario N2L6N4, Canada. *Automated Grading of Sketched Graphs in Introductory Calculus Courses.* Preliminary report.

Automated grading of mathematics is being used widely. However, the assessment of questions involving sketched graphs still poses many challenges for the designers and users of such software. Apart from determining a reasonable numeric grade, we also wish to provide useful interactive visual feedback to the student when the work is being done for practice and self-assessment. The goal is to allow flexibility in the creation of the question and in the student’s input methods.

In this talk, we will discuss our experiences with automated tools for the assessment of sketched graphs in first-year calculus courses. We will show strategies used and pitfalls encountered in our work on automated graph grading in the Maple package, as well as describe the experiences as conveyed to us of educators who have been using these tools. (Received September 22, 2017)

1135-97-1511 **Myron Minn-Thu-Aye** ([myron.minn-thu-aye@uconn.edu](mailto:myron.minn-thu-aye@uconn.edu)), 341 Mansfield Road U1009, Storrs, CT 06269, **Anthony Rizzie** ([anthony.rizzie@uconn.edu](mailto:anthony.rizzie@uconn.edu)), 341 Mansfield Road U1009, Storrs, CT 06269, and **Amit Savkar\*** ([amit.savkar@uconn.edu](mailto:amit.savkar@uconn.edu)), 341 Mansfield Road U1009, Storrs, CT 06269. *On the effect of using Play-Doh as a modeling tool on student’s visual understanding and success in multivariable calculus.* Preliminary report.

Geometric interpretations and questions arise throughout multivariable calculus, with many students identifying their struggle with visual reasoning as a potential roadblock to success in the course. At the University of Connecticut, a series of Play-Doh modeling activities has been developed to give students hands-on experience with various geometric objects and concepts in multivariable calculus. These activities were developed and

implemented in Fall 2016 and are now in their second iteration. We will discuss the development and use of Play-Doh activities across three semesters. We will present preliminary results that compare students' self efficacy on the use of Play-Doh for visual understanding of multivariable calculus concepts to the performance of students in some specific guided assessments. We will also look for differences in the perception and performance of students between classes where Play-Doh was used with guided instruction and where very limited instructions were offered on the use of Play-Doh. (Received September 23, 2017)

1135-97-1544 **Chris McCarthy\*** ([cmccarthy@bmcc.cuny.edu](mailto:cmccarthy@bmcc.cuny.edu)), 199 Chambers Street, New York, NY 10007. *Modeling with Differential Equations: Student Projects and Research Opportunities at a 2 Year College.*

A presentation on teaching mathematics in context via differential equation models. The presentation will focus on collaborative faculty – student research projects carried out at a 2 year college. (Received September 23, 2017)

1135-97-1696 **Jennifer Ann Czocher\***, Texas State University, 601 University Drive, San Marcos, TX 78666. *What outcomes might we anticipate from students engaging concurrently with modeling and differential equations content?*

There is always some debate about whether differential equations content should be taught before modeling or whether the content should be discovered through modeling. In this session, I will first present an overview of mathematical modeling that emphasizes students' mathematical thinking during modeling. I will then present examples from two recent educational studies of students engaging with differential equations content without having learned the content first. The first examines learning outcomes in an undergraduate classroom where the instructor maintained emphasis on the relationship between real world conditions and assumptions and mathematical properties and parameters. The second examines the self-confidence of student teams and their teacher-leaders as they participated in an extracurricular modeling competition. (Received September 24, 2017)

1135-97-1772 **W James Lewis\*** ([jlewis@unl.edu](mailto:jlewis@unl.edu)), Department of Mathematics, University of Nebraska-Lincoln, Lincoln, NE 68502. *The Nebraska Center for Science Mathematics and Computer Education.* Preliminary report.

In 1991, the University of Nebraska-Lincoln (UNL) received an NSF Statewide Systemic Initiative (SSI) award. One product resulting from the SSI grant was the establishment of UNL's Center for Science Mathematics and Computer Education (CSMCE). Over the past 20 years, the CSMCE has provided the foundation for a collaborative infrastructure that has led to partnerships among UNL, other Institutions of Higher Education (IHEs) and K-12 school districts across the state and several major NSF grants. We will discuss the CSMCE's role in increasing capacity for mathematics and mathematics education faculty at UNL to develop strong partnerships with both other IHEs and Nebraska school districts and compete successfully for NSF grants. (Received September 24, 2017)

1135-97-1805 **Lochana Siriwardena\*** ([siriwardenal@uindy.edu](mailto:siriwardenal@uindy.edu)) and **Yasanthi Kottegoda** ([ykottegoda@newhaven.edu](mailto:ykottegoda@newhaven.edu)). *Assessing an Inquiry Based Curriculum using Bayesian Networks.* Preliminary report.

In this paper we discuss an application of Bayesian networks to assess an inquiry based learning curriculum for Calculus II. We model connections between different concepts and tasks as a Bayesian network to infer certain conditional probabilities. Bayesian network allows us to quantify multiple dependencies of different concepts and their impact on final assessment of the students. This approach is important to measure the effectiveness of the proposed curriculum. For example, If the conditional probability of a student passing the final exam given the student passed all the previous inquiry tasks is very low, that shows a discontinuity between supporting inquiry based tasks and the comprehensive knowledge of concepts. It is also possible to analyze student performance such as the probability of a student failing to master a certain concept given that the student failed to pass the final assessment, using this approach. (Received September 24, 2017)

1135-97-1906 **Mariya Boyko\*** ([mariya.boyko@mail.utoronto.ca](mailto:mariya.boyko@mail.utoronto.ca)). *The role of socialist competition in the Soviet mathematics curriculum reform of the 1960's and 1970's.*

The role of socialist competition in the Soviet mathematics curriculum reform of the 1960's and 1970's

In 1958 the Soviet government led by Nikita Khrushchev initiated a major reform of education in order to bridge the gap that then existed between the school curriculum and the practical needs of the state. Prominent mathematicians and educators (including Andrei Kolmogorov) were involved in re-writing the mathematics curriculum. However, the content of the new curriculum proved to be unsuitable for the general audience of students who were not highly interested in mathematics a priori. There are numerous academic factors that influenced



such an outcome, but it is also important to explore the ideological context in which the curriculum reform was taking place. Socialist competition was one of the most prevalent ideological phenomena in the 1950's which influenced social and academic life of the state. In this talk we will focus on the role of socialist competition in the math education reform which often gets overlooked in the literature. We will define the socialist competition on international, inter-state and interpersonal level, and explore specific examples of manifestation of the socialist competition in high school and elementary school setting. (Received September 25, 2017)

1135-97-1919 **Alicia Machuca\***, amachuca@twu.edu, and **Junalyn Navarra-Madsen**, jnavarramadsen@twu.edu. : *An Undergraduate Research Experience with Emphasis on Topics Covered in the GRE Math Test*. Preliminary report.

During the summer of 2017 a bridge program was held at Texas Woman's University with the goal of preparing students for graduate school in two ways; the first is an undergraduate research project, the second is a GRE subject test preparation course. To promote success in graduate school the students' summer research project was chosen to emphasize applications of linear algebra, L<sup>A</sup>T<sub>E</sub>X, and programming in MatLab. In this talk we will discuss how we developed an undergraduate research topic from our own research area and provide ideas to form your own GRE preparation course. (Received September 25, 2017)

1135-97-1948 **Corinne A. Manogue\*** (corinne@physics.oregonstate.edu), Dept of Physics, Oregon State University, Corvallis, OR 97331, and **Tevian Dray** (tevian@math.oregonstate.edu), Dept of Mathematics, Oregon State University, Corvallis, OR 97331. *Kinesthetic Activities*.

One of the best ways for students to interpret geometric concepts is for them to use their own bodies to represent interrelationships. In this participatory talk, we will ask the audience to try out several examples of kinesthetic activities we use in the classroom. Our topics are drawn from multivariable and vector calculus, complex numbers, and a few from the physics of electromagnetism and thermodynamics. (Received September 25, 2017)

1135-97-1994 **Gwen Spencer\*** (gwenspencer@gmail.com), 180 Pleasant Street, APT 306, Easthampton, MA 01027. *Too Big is Better than Too Small: In Pursuit of Just Right*.

Undergraduates often have significant prior practice in small-scale well-defined tasks that can be resolved in minutes or hours. The persistent dilemma in selecting research projects for undergraduates is choosing a problem big enough to provide a meaningful challenge, and small (or curated) enough so that the student can get some traction.

As an applied mathematician, I advocate starting big, involving students in problem formulation, and designing projects where the students act as the primary expert on some part of the project. I'll mention several personal experiences in advising interdisciplinary applied undergrad research projects, and mention strategies I've used to both stretch and focus the scope of such projects in real time. My top goals in research advising are that the student should gain an authentic sense of ownership, view their contribution in the context of a larger body of knowledge, and have personal questions and ideas about next steps by the end of the project. (Received September 25, 2017)

1135-97-2176 **David Uminsky\*** (duminsky@usfca.edu), 101 Howard St. Suit 500, San Francisco, CA 94105. *Teaching data science at the epicenter of data science, a reflection on both graduate and undergraduate programs @University of San Francisco*. Preliminary report.

The University of San Francisco was early to market in the education of data science. The graduate masters program is entering its 6th year and was just one of a half dozen or so programs at launch in this space and the undergraduate BS in Data Science was launched a few years ago as the third BSDS in the country. Since their launch there is now over 150 peer graduate programs and dozens of BS in Data Science programs coming on line.

This talk will address the success and lessons of these two programs and include the key differentiators such as practicum, multidisciplinary and non-traditional faculty and curricular flexibility. Both programs are de-facto housed in the mathematics and statistics department and we will discuss how that position along with being located in San Francisco has influenced how to manage a multidisciplinary faculty group that includes computer scientists, Operations Research and even business faculty teaching in the core programs. (Received September 25, 2017)

1135-97-2177 **Brianna Donaldson\*** ([brianna@aimath.org](mailto:brianna@aimath.org)), American Institute of Mathematics, 600 E. Brokaw Rd., San Jose, CA 95112. *Seeding Collaboration Through Math Teachers' Circles*. Preliminary report.

Math Teachers' Circles (MTCs) are communities of K-12 teachers and mathematicians who meet regularly to do mathematics together. The goals are to support teachers as mathematicians and to connect mathematics professors with K-12 education. With more than 100 MTCs in 39 states, many hosted by mathematics departments, MTCs represent an increasingly widespread, relatively simple infrastructure for fostering ongoing interactions that contribute to developing a sense of shared mathematical community across the education spectrum. We will discuss examples of how MTCs can extend the impact of shorter-term collaborations, as well as prepare fertile ground for the development of larger and more complex collaborative infrastructures. In addition, we will describe the infrastructure provided by the national MTC Network, which has taken an implementation sciences-based approach to scaling, disseminating resources, and connecting local MTCs so that they may learn from each other. (Received September 25, 2017)

1135-97-2227 **Aaron D Wangberg\*** ([awangberg@winona.edu](mailto:awangberg@winona.edu)), **Elizabeth Gire** and **Robyn Wangberg**. *From concrete to abstract: Student discoveries and conjectures using physical surfaces in multivariable calculus and physics*.

By working with physical, dry-erasable surfaces and other representations of multivariable functions, students in the 'Raising Calculus to the Surface' and 'Raising Physics to the Surface' projects (NSF DUE-1246094 and NSF DUE-1611946, resp.) are able to explore new concepts in small groups and contribute their knowledge to the classroom. In this talk, I'll share how the classroom activities are designed to encourage student collaboration and exploration of new (and old) concepts, and how instructors utilize student ideas and conjectures to build the course content upon student ideas. (Received September 25, 2017)

1135-97-2253 **Kirk Walters\*** ([kwalters@air.org](mailto:kwalters@air.org)), 1000 Thomas Jefferson St. NW, Washington, DC 20007, and **Toni S. Smith**, 1000 Thomas Jefferson St. NW, Washington, DC 20007. *Forging Research and Practice with Improvement Science: Early Lessons from the Better Math Teaching Network*. Preliminary report.

The Better Math Teaching Network (BMTN) is a networked improvement community of researchers, teachers, and instructional leaders from New England who are using principles of improvement science to increase the number of students who are deeply and actively engaged in understanding algebra. The BMTN's quick-cycle testing involves refining and sharing student-centered instructional routines across the network. Presenters will discuss challenges and opportunities related to this collaborative model, and share emerging findings. These include: (1) BMTN members have demonstrated strong levels of buy-in and participation in the networked improvement community structure and quick-cycle testing procedures; (2) BMTN teachers have tested and refined a variety of student-centered instructional routines that will be shared throughout the network and beyond in 2017-18; (3) BMTN teachers' choice of tasks and use of scaffolds determined the depth and degree to which students took academic ownership of the learning environment; and (4) A majority of students in BMTN teachers' classrooms reported experiencing learning environments that closely aligned with the goals of the network, with statistically significant increasing levels from fall 2016 to spring 2017. (Received September 25, 2017)

1135-97-2260 **W. Gary Martin\*** ([martiwg@auburn.edu](mailto:martiwg@auburn.edu)), Department of Curriculum and Teaching, 5040 Haley Center, Auburn University, AL 36849, and **Marilyn E. Strutchens** ([strutme@auburn.edu](mailto:strutme@auburn.edu)), Department of Curriculum and Teaching, 5040 Haley Center, Auburn University, AL 36849. *The Mathematics Teacher Education Partnership: A Networked Improvement Community of Universities and School Systems to Transform the Preparation of Secondary Mathematics Teachers*.

The Mathematics Teacher Education Partnership (MTE-Partnership) was formed by the Association of Public and Land-grant Universities in 2012 to address the undersupply of new secondary mathematics teachers who are well prepared to help their students attain the goals of the Common Core State Standards for Mathematics (CCSSM) and other rigorous state mathematics standards. This consortium of over 90 universities and over 100 school systems has a common goal of transforming secondary mathematics teacher preparation using the Networked Improvement Community (NIC) design (Bryk et al., 2015). The Partnership currently has five Research Action Clusters (RACs) consisting of 6-20 universities working collaboratively to address "primary drivers," hypotheses about the main areas of influence necessary to advance the improvement aim of increasing the quality and quantity of students prepared to teach secondary mathematics. In particular, the RACS address experiences in introductory mathematics courses, the development of mathematical knowledge for teaching, clinical

experiences, and recruitment of secondary mathematics teacher candidates, and retention of those candidates in the profession upon program completion. Particular opportunities and challenges of the NIC design will be addressed. (Received September 25, 2017)

1135-97-2293 **Maria Grazia Viola\*** ([mviola@lakeheadu.ca](mailto:mviola@lakeheadu.ca)). *Separable exact  $C^*$ -algebras non-isomorphic to their opposite algebras.*

There are several examples in the literature of factors of type  $II_1$  and type III which are not isomorphic to their opposite algebras. Since a  $C^*$ -algebra isomorphism of von Neumann algebras is necessarily a von Neumann algebra isomorphism, these are therefore also examples of simple  $C^*$ -algebras not isomorphic to their opposite algebras. However, none of these examples is separable or exact in the  $C^*$ -algebra sense. In a joint work with N. C. Philipps we construct uncountably many mutually nonisomorphic simple separable stably finite unital exact  $C^*$ -algebras which are not isomorphic to their opposite algebras. In particular, we prove that there are uncountably many possibilities for the  $K_0$ -group, the  $K_1$ -group, and the tracial state space of such an algebra. We give further information on the algebras we constructed, including showing that the order on projections is determined by traces, computing the Cuntz semigroup, and showing that the algebras have stable rank one and tensorially absorb the Jiang-Su algebra. We also show that these  $C^*$ -algebras satisfy the Universal Coefficient Theorem. (Received September 25, 2017)

1135-97-2339 **Paul E. Seeburger\*** ([pseeburger@monroecc.edu](mailto:pseeburger@monroecc.edu)), 1000 E. Henrietta Rd., Rochester, NY 14623, and **Monica VanDieren** ([vandieren@rmu.edu](mailto:vandieren@rmu.edu)) and **Deborah Moore-Russo** ([dam29@buffalo.edu](mailto:dam29@buffalo.edu)). *Improving Conceptual Understanding of Multivariable Calculus & Differential Equations Through 3D Visualization Using CalcPlot3D*. Preliminary report.

A presentation of CalcPlot3D, an interactive online 3D JavaScript app designed to enhance the teaching and learning of multivariable calculus, differential equations, linear algebra and other related topics. CalcPlot3D brings the concepts of these courses to life, making it easy to visually explore the concepts and relationships between them. Through visual verification and exploration of problems involving surfaces, contour plots, curves, velocity and acceleration, directional derivatives, gradients, vector fields/phase portraits, etc., our project seeks to improve students' geometric intuition so they more fully understand the application of these concepts in other STEM coursework.

In addition to the creation of this app, our project seeks to create a series of new visual concept explorations to improve student understanding of multivariable calculus, differential equations and linear algebra and to use the app to conduct research investigating how student understanding of these concepts changes through the use of visualization and dynamic concept explorations.

CalcPlot3D also supports 3D glasses and facilitates the generation of 3D printed surfaces. See <http://web.monroecc.edu/calcNSF/>. This project is funded by NSF-IUSE 1524968, NSF-IUSE 1523786, and NSF-IUSE 1525216. (Received September 26, 2017)

1135-97-2393 **Bill McCallum\*** ([wmc@math.arizona.edu](mailto:wmc@math.arizona.edu)) and **Kristin Umland** ([kristin@illustrativemathematics.org](mailto:kristin@illustrativemathematics.org)). *Two case studies in K-20 collaboration: The Institute for Mathematics & Education and Illustrative Mathematics.*

The Institute for Mathematics & Education (IM&E) at the University of Arizona was founded in 2006 as a neutral space where mathematicians, education researchers, and teachers could meet to discuss pressing, and sometimes controversial, problems in mathematics education, under strong norms of respect for each others' professional knowledge.

Illustrative Mathematics (IM) started in 2010 as a project of IM&E and in 2013 moved out of the university to become a free-standing non-profit 501(c)3 company with a mission of creating a world where learners know, use, and enjoy mathematics. It has built an energetic community of teachers and mathematicians that embodies the norms expressed above. It recently completed a grades 6-8 mathematics curriculum freely available under a Creative Commons Attribution 2.0 Generic license.

IM&E and IM present two very different examples of infrastructures supporting collaboration between higher education and K-12. We will describe the origins and development of both, look at the day-to-day relationship-building and collaborations involved, and discuss what, if any, general conclusions can be drawn about building sustainable collaborative efforts. (Received September 26, 2017)

1135-97-2466 **Younggon Bae** and **V. Rani Satyam\***, [satyamvi@msu.edu](mailto:satyamvi@msu.edu), and **Mariana Levin**, **Kevin Voogt** and **John P Smith III**. *How do students experience the transition to proof?*

Proof is a foundational activity for upper-level undergraduate mathematics courses. Learning how to prove is a difficult transition, shifting from largely computation to now argument-based work. While we know many of

the difficulties inherent in learning how to prove, we do not know much yet from the students' perspective about how they experience this transition. We interviewed 14 undergraduates about their experience in a semester long introduction to proof course. This work differs from previous work in the focus on experience as the object of interest. We report on how students describe their reasoning while proving, the challenges they face, the adaptations they make to their learning activity, and their sense of success. Results indicate that students are acutely aware of the differences in what they are now being asked to do but continue to re-use or adapt past resources and behaviors, to try to be successful at this new activity. This work has implications for the design of introduction to proof courses and in exploring how students deal with transitions in mathematics. (Received September 26, 2017)

1135-97-2606      **Cameron Sweet\*** ([csweet@math.wsu.edu](mailto:csweet@math.wsu.edu)), Washington State University, Department of Mathematics and Statistics, Pullman, WA 99164-3113. *Symbol System Flexibility and Transfer in Multiplying Polynomials*. Preliminary report.

While there is an extensive amount of research demonstrating that the ability to relate one representation of a function to another is necessary for understanding the concept of function, there are few studies on using multiple representations to help high school algebra students relate multiplication of polynomials to multiplication of integers. A representational dilemma emerges when students are taught the unfamiliar concept of multiplying polynomials using the unfamiliar symbolic representation for the distributive property. The goal of this study is to gain an understanding of whether presenting multiplication of polynomials using the same methods in which integers are multiplied may be beneficial to students' ability to make appropriate representational choices when multiplying polynomials. Study participants are students enrolled in high school algebra classes in which multiplication of polynomials is introduced and the teachers have been observed to encourage students to use multiple methods for problem solving. Assessments and interviews will be conducted to determine student justifications for their choices and how students relate multiplication of polynomials to multiplication of integers. (Received September 26, 2017)

1135-97-2619      **Orlando B. Alonso\*** ([orlando.alonso@lehman.cuny.edu](mailto:orlando.alonso@lehman.cuny.edu)), 3635 Johnson Avenue, Apt. 3A, Bronx, NY 10463. *Deepen student understanding of mathematics using technology: Justifying a hierarchical classification of convex quadrilaterals using dynamical software*.

This presentation show how the dimension added by the capabilities of geometric dynamical software enables pedagogical approaches that enhance student conceptualization techniques and involvement in mathematics generalizations processes promoting students to fully participate in the development of the classroom mathematical discourse. The legitimization of a hierarchical (inclusive) classification of quadrilaterals is attained while distinguishing different representations of quadrilaterals involved from an exclusive conceptual perspective using virtual manipulative tools designed in a dynamical software environment leading the audience to understand how basic properties for families of convex quadrilaterals could be associated with the shape construction of members of quadrilaterals families (isosceles trapezoids, kites, and parallelograms) following a constructivist approach. The discussion focuses in an innovative system of observable features that looks for side length and angle measure patterns as classification criterion. Investigation results are recorded on a semantic feature analysis chart that enables the coexistence of inclusive and exclusive conceptualization paths to classification, which until now have competed for the available geometric terms. (Received September 26, 2017)

1135-97-2643      **Elizabeth Theta Brown\*** ([brownet@jmu.edu](mailto:brownet@jmu.edu)), Dept. of Mathematics and Statistics, MSC 1911, 800 South Main, Harrisonburg, VA 22807. *Calculus for the 21st century*. Preliminary report.

This talk will report on a redesign of the calculus sequence that seeks to erase the success, persistence, and achievement gaps between students from majority and underrepresented groups using methodologies whose effectiveness has been documented in, eg, the Proceedings of the National Academy of Sciences. The reforms implemented involve both pedagogy and content. Strategies were chosen for their robust vetting in the literature and potential for sustainability in a typical calculus sequence, in which the syllabus is already tight and the individual instructor has limited agency to affect course features, like text, topics, etc. Curricular adjustments include readings, applications, examples, and projects that illuminate calculus' relevance to wide variety of human and social concerns, and framing the subject in an accurate historical context, which helps students understand mathematics as a broadly human endeavor. Pedagogical efforts include value affirmation, problem framing as a way to evade stereotype or identity threat, and student-empowering course management software.

The talk will offer quantitative as well as qualitative measures of the project's efficacy. References and resources will be provided. (Received September 26, 2017)

1135-97-2680 **Mara Alagic\*** ([mara.alagic@wichita.edu](mailto:mara.alagic@wichita.edu)). *Nurturing Preservice Elementary Teachers' Creative Mathematical Thinking.*

This abstract provides a snapshot from a longitudinal study focused on developing math related pedagogical content knowledge (Ball, 2000) in the context of visual and creative digital environments. In a classroom culture in which teachers view math most often as the completion of algorithms and usually associate creativity only with arts, they struggle to understand how creativity, art and technology might improve math learning. "A worst-case scenario would involve newly qualified teachers entering the classroom unable to recognise creativity, ultimately discouraging those creative pupils from pursuing mathematics or, at least, from being mathematically creative" (Bolden, Harries, & Newton, 2010, p. 154). Preservice elementary teachers were required to design an artistic, visual and dynamic representation of a math concept of their choice and provide an extensive (guided) explanation of how they might use their work in a math classroom. The resulting research presentation will consist of two parts: (i) sample student's projects (e.g., dynamic GIFs), and (ii) data analysis of teachers' mathematical thinking about their own learning processes and changing (or not) perceptions about ways in which they intend to teach while nurturing creative ways of understanding/doing math. (Received September 26, 2017)

1135-97-2723 **Talia Milgrom-Elcott\*** ([tme@100kin10.org](mailto:tme@100kin10.org)) and **Grace Doramus** ([grace@100kin10.org](mailto:grace@100kin10.org)). *The Grand Challenges: Mapping the System of K-12 STEM Education.* Preliminary report.

Despite billions of dedicated dollars, many of our nation's kids do not have access to quality science, technology, engineering, and math education. This talk introduces a new approach to reverse this story and enable lasting change, grounded in a simple insight: you can't solve a problem you don't understand.

100Kin10's grand challenges is an innovative approach to mapping a social system, grounded in ecology and network science and resulting in an unprecedented representation of the challenges facing quality K-12 STEM education (<https://grandchallenges.100kin10.org/>). Through 2+ years of data collection and analysis with thousands of stakeholders, we see that, like systems in the natural world, each problem in the STEM education system is connected to at least one other problem; but despite this interconnection, the problems are not all equal. There are some that, if solved, have greater influence across the system.

With this map, all kinds of stakeholders - from teachers and policymakers to researchers and district leaders - can access a full picture of the system, gain a deeper understanding about its many components, and see how they can contribute. It also enables the design and development of systems-wide solutions to improve STEM learning across our nation. (Received September 26, 2017)

1135-97-2757 **Girija Sarada Nair-Hart\*** ([nairhaga@uc.edu](mailto:nairhaga@uc.edu)), 3585, Applewood Drive, Amelia, OH 45102. *Evidence for Concept Modification – The limit Concept of Post-secondary Students.*

During the research that investigated innovative teaching techniques that promoted conceptual change in calculus concepts, data was collected through two problem solving interviews and one six-week long teaching episodes that was modeled after the constructivist theory of learning. In the end, students who held inconsistent conceptions of the limit construct was able to understand the concept in a level that was beyond the procedural level. Based on the evidence presented through during the research, I will detail the instructional undertakings that lead to the vital concept modification. Data was collected through detailed field notes, and video taped conversations test were parts of the teaching episodes. The concluding problem-solving interview that lasted 2 hours provided the data that highlighted conceptual change in students. (Received September 26, 2017)

1135-97-2823 **Uri Treisman\*** ([uri@austin.utexas.edu](mailto:uri@austin.utexas.edu)). *Strengthening the Improvement Infrastructure for Mathematical Sciences Education.*

Everyday and everywhere myriad mathematics instructors are working to improve their teaching, strengthen their programs, and respond as educational leaders to mathematics' increasing importance to society and to students' prospects for upward mobility. As a community, we are awash in a sea of educational innovation made turbulent by tectonic plate shifts in our understanding of how people learn, in the public policies shaping the demand for and funding of mathematics instruction, and, of course, in the mathematical sciences themselves. Yet little of this innovation goes to scale or becomes normative practice. All too often, the learning from even the most successful, well-designed experiments and pilot programs dissipates in a fog of collective amnesia.

In the last few years, several high-profile efforts have been launched to build a robust infrastructure for strengthening and modernizing mathematics education. These coexist with large-scale movements often launched from outside the mathematics community to reshape national practice.

This talk describes the design principles behind these recent efforts and their theoretical and organizational bases. It hopes to shed light on their successes and failures with lessons for a discipline in the policy spotlight. (Received September 26, 2017)

1135-97-2987 **Daniel J. Teague\*** ([teague@ncssm.edu](mailto:teague@ncssm.edu)), NCSSM, 1219 Broad Street, Durham, NC 27705. *Comparing Simulations with Solutions for Classical Differential Equation Models.*

Students often have difficulty understanding the role of the parameters in classical interaction models like the SIR infectious disease model. At the North Carolina School of Science and Mathematics, we have found that simulating the process defined by the DE first by using dice and then with agent-based models helps clarify probabilistic assumptions built into the model. By comparing the deterministic results to those generated by the simulations, students can see the standard "solution" to the differential equations as an averaging of the different possible actual paths. (Received September 26, 2017)

1135-97-2996 **Nathan Tintle\*** ([nathan.tintle@dordt.edu](mailto:nathan.tintle@dordt.edu)), Department of Mathematics and Statistics, 498 4th Ave NE, Sioux Center, IA 51250. *A conceptual approach to financial mathematics.*

Financial mathematics courses often get lost in a sea of technical notation, formulas and nuanced financial and mathematical details, causing students to lose sight of overarching mathematical and financial concepts. In this talk, I will describe our recent efforts to develop a radically different approach to teaching financial mathematics. This new approach (1) Develops financial mathematics thinking by re-ordering the typical content to embrace a spiral approach to learning, (2) Uses real financial data, big data contexts and case studies, (3) Stresses conceptual understanding, rather than mere knowledge of procedures, (4) Fosters active learning in the classroom through hands-on simulations and case studies, (5) Uses technology for developing conceptual understanding and, (6) Uses assessments to improve and evaluate student learning. We have implemented this course as part of our actuarial preparation program for students during their freshman year, with other students take the course to meet general quantitative reasoning requirements at the college. We will also describe efforts to develop an assessment tool to use in financial mathematics courses to better capture what students know and don't know, before and after financial mathematics courses. (Received September 26, 2017)

1135-97-3090 **Levi J. Patrick\*** ([levi.patrick@sde.ok.gov](mailto:levi.patrick@sde.ok.gov)), Oklahoma State Department of Education, 2500 N. Lincoln Blvd., Suite 315, Oklahoma City, OK 73105, and **Michael A. Tallman.** *Like Terms Communities: Developing Mutualistic K-20 Partnerships within a Statewide Networked Improvement Community to Advance Teacher Content Knowledge.* Preliminary report.

To accomplish meaningful and effective implementation of mathematics learning standards, teachers need opportunities for continued professional growth in their mathematical knowledge for teaching (CBMS, 2012). Indeed, the mathematical demands of teaching require specialized mathematical knowledge not needed in other settings (Ball et al., 2008).

Based upon the input from the 20 member organizations of the Oklahoma Mathematics Alliance, a root cause analysis indicated that limited access, time, and funding for highly relevant, comprehensive professional learning experiences, along with the pressures of state assessments and accountability measures, often lead to minimal intervention in this critical component of excellent teaching.

The Like Terms Community is Oklahoma's initial solution to this complex issue. We are actively engaging in monthly virtual department meetings, called Like Terms Communities, that incorporate mathematics content knowledge across the grade bands and rely on a collaborative leadership triad structure comprised of representatives from the K-12 mathematics education community, the pure and applied mathematics community, and the pre-service teacher education community. (Received September 26, 2017)

1135-97-3123 **Kristin P Bennett\*** ([bennek@rpi.edu](mailto:bennek@rpi.edu)), IDEA, CII 3129, 110 8th Street, Troy, NY 12180. *Data Analytics Throughout Undergraduate Mathematics.*

We discuss an innovative pipeline through mathematics for creating the next generation of agile data scientists and data users. The program is designed to rapidly meet the unmet need for data scientists by providing novel low barrier "on ramps" into data science. Freshman and sophomore students enter this pipeline by taking an innovative data math course; "Introduction to Data Mathematics" teaches linear algebra and high-dimensional geometry using data analytics methods as driving applications. Student then immediately participate in applied data analytics research on compelling open real-world problems for clients in industry and research as part of the Data INCITE Research Lab. Students go on to pursue existing and enhanced data science related courses

in math, computer science and other disciplines. This early education and research program in data sciences engenders engaged students with data analytics skills who are highly recruited for internships, coops, and jobs. The program increases the diversity and number of students pursuing careers and graduate work in data science by drawing students from outside of computer science and statistics curricula. (Received September 26, 2017)

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## MAA Invited Addresses, Presentations by Teaching Award Winners, and SIGMAA Guest Lecturers

1135-A0-108 **Alissa Crans\***, Loyola Marymount University, Los Angeles, CA. *Quintessential quandle queries*

Motivated by questions arising in starkly different contexts, quandles have been discovered and rediscovered over the past century. The axioms defining a quandle, an analogue of a group, simultaneously encode the three Reidemeister moves from knot theory and capture the essential properties of conjugation in a group. Thus, on the one hand, quandles are a fruitful source of applications to knots and knotted surfaces; in particular, they provide a complete invariant of knots. On the other, they inspire independent interest as algebraic structures; for instance, the set of homomorphisms from one quandle to another admits a natural quandle structure in a large class of cases. We will illustrate the history of this theory through numerous examples and survey recent developments. (Received July 27, 2017)

1135-A0-155 **Maria Klawe\***, Harvey Mudd College, Claremont, CA. *Transforming learning: building confidence and community to engage students with rigor.*

As the first woman and the first mathematician to become president of Harvey Mudd College, I have been delighted to see our departments transform the teaching of rigorous mathematical content in ways that attract and retain female students in mathematics, computer science, engineering and physics. This talk describes the curricular and classroom transformations that have taken place over the last decade and the significant increases in diversity that have occurred as a result. Just in the last three years we have seen graduating classes in computer science, engineering and physics that were more than 50% female. I hope that attendees will leave energized and inspired to experiment in their own departments. (Received August 05, 2017)

1135-A0-156 **Tadashi Tokieda\***, University of Cambridge, Cambridge. *Toy models.*

Would you like to come see some toys?

‘Toy’ here has a special sense: an object from daily life which can be found or made in minutes, yet which, if played with imaginatively, reveal behaviors that intrigue scientists for weeks. We will explore table-top demos of several such toys, and extract a mathematical story. So some of the toys will be classical but revisited, others will be original, and all will be surprising to mathematicians/physicists and amusing to everyone else. (Received August 05, 2017)

1135-A0-157 **William Cook\***, University of Waterloo, Waterloo, ON. *Information, computation, optimization: connecting the dots in the traveling salesman problem.*

Few math models scream impossible as loudly as the traveling salesman problem. Given  $n$  cities, the TSP asks for the shortest route to take you to all of them. Easy to state, but if  $\mathcal{P} \neq \mathcal{NP}$  then no solution method can have good asymptotic performance as  $n$  goes off to infinity. The popular interpretation is that we simply cannot solve realistic examples. But this skips over nearly 70 years of intense mathematical study. Indeed, in 1949 Julia Robinson described the TSP challenge in practical terms: “Since there are only a finite number of paths to consider, the problem consists in finding a method for picking out the optimal path when  $n$  is moderately large, say  $n = 50$ .” She went on to propose a linear programming attack that was adopted by her RAND colleagues Dantzig, Fulkerson, and Johnson several years later.

Following in the footsteps of these giants, we use linear programming to show that a certain tour of 49,603 historic sites in the US is shortest possible, measuring distance with point-to-point walking routes obtained from Google Maps. We highlight aspects of the modern study of polyhedral combinatorics and discrete optimization that make the computation feasible. (Received August 05, 2017)

1135-A0-158 **Jo Boaler\***, Stanford University, Stanford, CA. *Changing mathematical relationships and mindsets: how all students can succeed in mathematics learning.*

This talk and discussion will consider how important new brain science can change students’ ideas and approaches to mathematics, change students’ mathematics pathways dramatically, and promote equity in mathematics classrooms. We will hear about research in neuroscience and education, watch classroom videos and consider mathematics transformations for school and college students. (Received August 05, 2017)

- 1135-A0-159 **James Tanton\***, MAA Mathematician at Large, Washington, D.C.. *How many degrees are in a Martian circle? And other human—and nonhuman—questions one should ask about everyday mathematics.*

Who chose the number 360 for the count of degrees in a circle? Why that number? And why do mathematicians not like that number for mathematics? Why is the preferred direction of motion in mathematics counterclockwise when the rest of world naturally chooses clockwise? Why are fingers and single digit numbers both called digits? Why do we humans like the numbers 10, 12, 20, and 60 particularly so? Why are logarithms so confusing? Why is base  $e$  the "natural" logarithm to use? What happened to the vinculum? (Bring back the vinculum, I say!) Why did human circle-ometry become trigonometry? Let's spend a session together exploring tidbits from the human—and nonhuman—development of mathematics. (Received August 05, 2017)

- 1135-A0-160 **Moon Duchin\***, Tufts University, Boston, MA. *Political Geometry: Voting districts, "compactness," and ideas about fairness.*

The U.S. Constitution calls for a census every ten years, followed by freshly drawn congressional districts to evenly divide up the population of each state. How the lines are drawn has a profound impact on how the elections turn out, especially with increasingly fine-grained voter data available. We call a district gerrymandered if the lines are drawn to rig an outcome, whether to dilute the voting power of minorities, to overrepresent one political party, to create safe seats for incumbents, or anything else. Bizarrely-shaped districts are widely recognized as a red flag for gerrymandering, so a traditional districting principle is that the shapes should be "compact"—since that typically is left undefined, it's hard to enforce or to study. I will discuss "compactness" from the point of view of metric geometry, and I'll overview opportunities for mathematical interventions and constraints in the highly contested process of electoral redistricting. To do this requires a rich mix of law, civil rights, geometry, political science, and supercomputing. (Received August 05, 2017)

- 1135-A0-166 **Robert Gould\***, University of California, Los Angeles, Los Angeles, CA. *We are all data scientists (or we should be)*

In this talk, I make the argument that data literacy—a combination of statistical thinking, computational thinking, and mathematical thinking—is too important to ignore, and yet it has been and continues to be ignored. I'll also discuss several projects at the K–16 level that have attempted to improve data literacy. (Received August 06, 2017)

- 1135-A0-184 **Lázló Babai\***, University of Chicago, Chicago, IL. *Groups, graphs, algorithms: The Graph Isomorphism problem.*

Deciding whether or not two given finite graphs are isomorphic has for decades been known as one of a small number of natural computational problems with unsettled complexity status within the P/NP theory.

Building on a framework introduced in a seminal 1980 paper by Eugene M. Luks, recent algorithmic progress on this problem involves an interplay between finite permutation groups, graphs and more generally, relational structures with low arity, and algorithmic techniques such as the "Divide and Conquer" principle. The talk will attempt to illustrate some of the components of this work. (Received August 08, 2017)

- 1135-A0-245 **Joseph W. Dauben\***, Herbert H. Lehman College, Bronx, NY 10468. *The history of Chinese mathematics: 60th anniversary of the founding of the IHNS (CAS), Beijing.*

This talk will highlight the 60th anniversary of the founding of the Institute for History of Natural Sciences in Beijing, in which historians of mathematics, east and west, also played leading roles in bringing the importance and international significance of mathematics in China to the attention of the world at large. The role that Joseph Needham played in promoting history of science generally, and history of mathematics in particular, will also be discussed, along with the extent to which our understanding of the early history of mathematics in China has changed over the past 60 years thanks in large measure to recent archaeological discoveries of which Needham and the founders of the Institute were unaware in the 1950s. In conclusion, this presentation will also consider the role that mathematics plays-or ought to play-in the history of science globally, and the extent to which proofs may be conveyed with or without diagrams in providing convincing arguments for the validity of results obtained by mathematicians in ancient China. (Received August 15, 2017)

- 1135-A0-250 **Hortensia Soto\*** ([hortensia.soto@unco.edu](mailto:hortensia.soto@unco.edu)), 6151 Spearmint Court, Fort Collins, CO 80528. *Teaching On Purpose and With a Purpose: The Scarecrow, the Lion, and the Tin Woodman.*

In this presentation, I will discuss characteristics of teaching on purpose and with a purpose and how intentional teaching can create a sense of community in the classroom. I have found that such communities can result in

students embracing rich tasks, that they might not otherwise. Most importantly, students contribute more to the classroom and take responsibility for their own learning of mathematics. As part of this presentation we will learn how the Scarecrow, the Lion, and the Tin Woodman can assist us in our teaching endeavors. (Received August 15, 2017)

1135-A0-752 **Rafael Núñez\*** (rnunez@ucsd.edu). *Philosophy of mathematics in the 21st century: why does it need the sciences of the mind?* Preliminary report.

Mathematics is about abstract concepts, precise idealizations, relations, calculations, and notations, all of which are made possible by the amazing (albeit limited) workings of the human mind and the biological apparatus that supports it. Over the past 50 years the scientific study of mental phenomena has made enormous progress in understanding their psychological, linguistic, and neurological underpinnings. Traditional approaches in Philosophy of Mathematics such as Platonism, Formalism, Logicism, and Intuitionism - developed many decades, if not centuries prior to these developments - could not benefit from these findings. I argue that today, in the 21st century, philosophical investigation - e.g. What is mathematics? What is it for? How does it work? - should be informed by, and be compatible with findings in the sciences of the mind. I'll illustrate my arguments with research addressing issues in hyperset theory, infinitesimal calculus, and mathematical induction. (Received September 14, 2017)

1135-A0-2444 **Jim Fowler\*** (fowler@math.osu.edu) and **Bart Snapp** (snapp.14@osu.edu). *Using Ximera to build online interactive math activities.*

Many mathematicians are comfortable authoring handouts in LaTeX, but there is some pain in converting a paper or PDF handout into an interactive webpage with autograded questions. A solution is Ximera, an open-source system for deploying interactive LaTeX documents on the web. For instance, Ximera converts markup like  $1 + 2 = \text{\answer{3}}$  into  $1 + 2 =$  followed by a computer-graded answer blank, and then records whether or not a particular student entered a correct response into the gradebook in the LMS. By making deep use of git, Ximera enables teams of instructors to collaborate on the creation of online activities. The creators of Ximera will demonstrate the system and discuss the results of experiments comparing the performance of students using Ximera to students using commercial systems. (Received September 26, 2017)

1135-A0-3257 **Stephen Hobbs\***, Space and Naval Warfare Systems Center, San Diego, CA. *Using Navy carriers for disaster relief, and the remarkable Hilbert space.*

In this talk I will discuss an optimization problem that arises when a Navy ship is tasked with airlifting supplies to an area that has been struck by a natural disaster. In such a situation hundreds, perhaps thousands, of lives may depend on the speed of delivery. The solution will highlight Hilbert space methods, and in the process of setting up the problem, I will point out some of the numerous uses of Hilbert space methods in modern engineering. Among the most important properties that Hilbert spaces possess for these applications are: projections (for solutions of many types of least squares estimation problems), the Riesz representation theorem (for solutions to many important elliptic boundary value problems), compact operators (for solutions of parabolic and hyperbolic equations), the Fourier transform (for frequency domain processing and analysis of almost all signals in modern communication systems), and reproducing kernels (which are finding a rapidly expanding number of applications in machine learning, automated systems, statistics, and big data). I hope this discussion promotes the inclusion of some of these results and methods, as well as some of their applications, in the graduate curriculum. (Received October 18, 2017)

## *Innovative Mathematical Outreach in Alternative Settings*

1135-A1-480 **Daniel Zaharopol\*** (danz@beammath.org) and **Ruthi Hortsch** (ruthi@beammath.org). *Advanced, underserved students: strategies for program design.*

A wealth of programs exist to help K-12 students study advanced mathematics. From summer programs to math circles to after-school programs, these opportunities give access to abstract mathematics and problem-solving skills. However, most programs have limited outreach and support for reaching underserved students.

When working with students who have not received this invitation into mathematics, what program design features help them make the most progress and feel a part of the community? We will share the experiences of Bridge to Enter Advanced Mathematics (BEAM), a program that has been operating in New York City since 2011 and is opening in Los Angeles beginning 2018. BEAM reaches hundreds of students each year through summer programs, weekend programs, and mentoring from 6th grade through 12th grade. Curriculum includes math ranging from logical reasoning through number theory, combinatorics, and group theory, in addition to

college access work and educational advising. Ultimately, BEAM's goal is simple: give its students access to the same resources as more affluent peers. (Received September 05, 2017)

1135-A1-628 **Caleb C Moxley\*** (ccmoxley@bsc.edu), 900 Arkadelphia Road, Birmingham, AL 35254. *St. Mary's GED: Mathematics Underpinning GED Education.*

While each general education diploma (GED) student faces unique challenges, mathematical competency is commonly a difficult skill for GED students to develop, and the mathematics portion of the GED exam and other mathematical concepts throughout the exam are often obstacles to students obtaining GEDs or to achieving college-ready designations in their GED scores. What is more, students who do obtain GEDs often do not retain their mathematical skills and do not command basic levels of mathematical reasoning. St. Mary's-on-the-Highlands GED program, which exclusively caters to single mothers living in the metro area of Birmingham, Alabama, provides students with large amounts of time to develop their mathematical skills, connects mathematics to other GED curriculum, and encourages students to attach their mathematical development to that of their children through a student-to-teacher model. This talk will discuss these techniques in the context of single-sex, single-parent GED education and their relevance to more general GED-seeking populations. (Received September 11, 2017)

1135-A1-2154 **Catherine Paolucci\*** (paoluccic@gmail.com) and **Helena Wessels.** *Initial outcomes and lessons from the International Mathematics Enrichment Project (IMEP).* Preliminary report.

Mathematical disciplines continue to face challenges with broadening participation. Efforts to increase diversity among those who study mathematics to advanced levels must include strategies for building an early pipeline at the elementary and secondary levels. This often rests upon the shoulders of the teachers who shape these students' early experiences with mathematics. The International Mathematics Enrichment Project (IMEP) was created to both support mathematical learning among children in some of South Africa's high-poverty communities and provide opportunities for teachers to learn strategies for empowering these children and developing their self-efficacy and dispositions toward mathematics.

This session will discuss the initial outcomes from the first IMEP implementation. The project engaged pre-service teachers from New York and South Africa in the collaborative design and delivery of a mathematics enrichment program for South African children. In addition to presenting positive learning outcomes among the children, the session will highlight initial evidence of the potential for this service-learning model to help pre-service teachers gain first-hand experience with important educational issues that can help to break down barriers to the field. (Received September 25, 2017)

1135-A1-2270 **Katie Haymaker\*** (kathryn.haymaker@villanova.edu) and **Beth Malmskog.** *Math Outreach at Graterford State Correctional Institution.*

Graterford State Correctional Institution is a maximum-security state prison near Philadelphia, PA. With help from Villanova University's degree program at Graterford, we developed a new model of Math Circle, made up of incarcerated students with an interest in mathematics. This talk will describe our experience with this Math Circle, as well as in teaching a college-level core mathematics course at the institution. We will discuss the logistics of teaching at a secure facility, the problems and curriculum we found to be successful, and what we learned in the process. (Received September 25, 2017)

1135-A1-2425 **Robert Scott\*** (robscott@cornell.edu), 300 Kennedy Hall, Ithaca, NY 14850. *Teaching Math in Prison: Observations from the Cornell Prison Education Program.*

This paper explores the challenges of teaching college-level mathematics in prison. For this study, the author interviewed several math instructors from a college-in-prison program in upstate New York. It is common for prison educators to hope that education will help incarcerated students think outside of their current circumstances. Instructors interviewed for this study discussed the challenges of using mathematics in prison as a vehicle for empowerment. Several people have observed how prison strictures, the insistence on the abiding of rules, and the drills/repetition experienced in prison recapitulate challenging experiences of math classrooms that the incarcerated students experienced before their incarceration. Still, few of the math instructors interviewed looked outside of the methodology of practice and repetition as a means to success in their math classrooms. In the absence of an extra-mathematical curriculum of social justice, the paper explores the descriptions of successful math students in the prison environment, and considers their prison-specific metrics of success. The author explores what the incarcerated students themselves, and their definitions of success, say about the college campus and its portrait of the successful math student. (Received September 26, 2017)

1135-A1-2622 **Branden Stone\*** ([bstone@adelphi.edu](mailto:bstone@adelphi.edu)), Dept of Math and Computer Science, Adelphi University, Garden City, NY 11530. *Advising undergraduate research in prison, what worked and what didn't work.*

Advising undergraduate research can be difficult, even in a normal campus environment. The difficulty only increases when you are advising incarcerated students without access to the internet, libraries, or even a calculator. In this talk we will discuss my experience advising students in the Bard Prison Initiative. In those two years the students and I were faced with questions such as: How do we type up the results?; How can we find articles related to their research?; What topics are accessible with limited resources? We will address these questions and more, focusing on choosing topics and ways to encourage students to take ownership of their own research. (Received September 26, 2017)

1135-A1-2889 **Brandy S Wieggers\*** ([brandy.wieggers@cwu.edu](mailto:brandy.wieggers@cwu.edu)), 400 E University Way/ 7424, Ellensburg, WA 98926. *Kittitas Valley Math Circle: Circling parents, guardians, and other adults to join in their student's Math Circle experience.*

Gryffin had tried Cub Scouts and soccer, and while he enjoyed these activities there was never a thrill in his eye or jump in his step. That all changed this year when Gryffin joined the Kittitas Valley Math Circle, a community of 2nd-6th-grade students who gather once a week to celebrate the joys of problem-solving.

This Math Circle is unique in providing family members for students like Gryffin an opportunity to engage in their students' problem-solving experience. As students tackled the posed problem of the day, their family worked equally as hard next door in an adult only session that includes the opportunity to discuss educational issues, explore mathematics, and answer questions while sharing ideas. After a thorough exploration, adults were provided additional problems that they could work on at home with their student.

Through this program, the Kittitas Valley Math Circle has been successful at building adults' confidence in engaging with their students in continuing reasoning and problem-solving beyond the Math Circle. We've seen grandparents and older siblings join the effort and all leave feeling more prepared to support students like Gryffin. This talk will present the resources we've developed and share ideas for how to start your own program. (Received September 26, 2017)

1135-A1-3113 **Zeinab Bandpey\*** ([zeinab.bandpey@morgan.edu](mailto:zeinab.bandpey@morgan.edu)), Baltimore, MD 21251. *An Iranian woman studying in a historical black university teaches College Algebra in prison. . . .*

Prisoners are provided college education so that, when they are released, they will adjust easily to society and won't get sent back to prison! I was fascinated by the idea so much that I wanted to be a part of it. As a result, I have been teaching in prison for two consecutive semesters. In this paper, I will explain how the fact that I'm an immigrant from Iran having a single-entry visa helped me to get along with students in prison and also motivated them to rely on themselves, focus on their successes and do better in math. I will talk about the challenges my students and I have gone through and at the end I will come up with some suggestions that I believe would help any prisoner attending math class in prison. (Received September 26, 2017)

1135-A1-3144 **Tatiana Dezbah Shubin\*** ([tatiana.shubin@sjsu.edu](mailto:tatiana.shubin@sjsu.edu)). *The Alliance of Indigenous Math Circles.*

The Alliance of Indigenous Math Circles (<https://aimathcircles.org/>) is an initiative that grows out of the five-year old Navajo Nation Math Circles project. The project launched in 2012 has demonstrated that math circles model can be successfully implemented and used by Native American communities. In this talk we will share the experience gained through the work in the Four Corners region and beyond. (Received September 26, 2017)

1135-A1-3236 **Mark Saul\***, [marksaul@earthlink.net](mailto:marksaul@earthlink.net), and **Nancy Blachman**. *Julia Robinson Mathematics Festivals—An Alternative to Competition.*

A Julia Robinson Mathematics Festival is an after-school mathematical event in which students engage in unusual and deep mathematics in a cooperative environment. Guided by a facilitator, they work in groups of 6-10 on problem sets, activities, or games with mathematical content. The facilitator questions, supports, and guides, but does not teach in the traditional sense. The problems are offered at tables scattered around a large area, such as a gymnasium or cafeteria. Students choose which activities to engage in, and decide for themselves how long they will stay with each problem. The latter, rather than speed or even achievement level, is the measure of success for the program.

Competitions are by now a tradition in American mathematics. But they leave out important communities: minority communities and girls. We are developing a parallel tradition which includes these communities. In the process, we hope to affect the pedagogical styles of teachers involved in our program. (Received September 27, 2017)

## *Philosophy of Mathematics as Actually Practiced*

1135-A5-144      **David M Shane\*** ([dshane@student.methodist.edu](mailto:dshane@student.methodist.edu)), 5400 Ramsey Street, Department of Mathematics, Fayetteville, NC 28311. *The Eroding Foundation of Mathematics*.

The very foundation of the physical sciences is mathematics, which is arguably the most fascinating conversation in the philosophical arena. Paradoxically, the majority of mathematicians in modernity decline this engagement, and instead, set their cross hairs primarily on physical science and technological applications; which is to say that mathematicians are neglecting to develop new theoretical frameworks for their trade. This tendency may be directly observed within even the purest of mathematical branches, number theory, where the tip of the spear points to every direction, yet seldom a philosophical one. I contest that mathematicians need to acknowledge this shortcoming and reunite themselves with the philosophical network. While we may not agree completely with the assertions made by Pythagoras, Fermat, Russell, or Gödel, one cannot deny that their mathematical practices are interwoven in the fabric of their philosophical tapestry. In short, mathematics needs to spawn its own philosophers in order to facilitate the growth of new branches: the community needs avant-garde theories which are “for us, by us” and that pioneer into the wilderness of abstract thought. Keywords: Philosophy of Mathematics, History of Mathematics, Number Theory, Logic. (Received August 03, 2017)

1135-A5-301      **Jae Yong John Park\*** ([parkjohn0109@gmail.com](mailto:parkjohn0109@gmail.com)), 16 Warren Street, Apt. 5, New York, NY 10007. *Fictionalism, Constructive Empiricism, and the Semantics of Mathematical Language*.

In Field’s fictionalism, good mathematical theories do not need to be true, but rather must be consistent and conservative. Likewise, Van Fraassen views science to be nothing more than a study of obtaining truths about the observable phenomena of the world, so good scientific theories need not be true, merely empirically adequate. The takeaway from this comparison is that the concepts of acceptance and empirical adequacy of constructive empiricism can be used to better understand fictionalism. At first glance, constructive empiricism does not seem to help the case of fictionalism because the constructive empiricism is based on the possibility of empirical verification, which is also the basis of indispensability argument. This paper argues, however, that the empirical adequacy of scientific theories is comparable to the conservative nature of mathematical theories. By understanding that a scientific theory need not be entirely true, the falsity of mathematical statements in the fictionalist view becomes more graspable. (Received August 21, 2017)

1135-A5-362      **James Henderson\*** ([jrh66@psu.edu](mailto:jrh66@psu.edu)). *When Physicists Teach Mathematics*.

The teaching of mathematics is one of the field’s most straightforward applications. Because the math lessons 10-year-olds absorb are of a very different sort than those mathematicians-in-training endure while studying functions of a complex variable in graduate school, it is not surprising that the material varies quite a bit in many settings in which math is taught, but the presentation of the material does, too, depending on the professor, the audience, and the purpose of the class. Further, it’s not just mathematicians who teach math. Sometimes, for instance, physicists teach math to physics majors explicitly for use in physics courses. This was most famously done in 1961 and 1962 by Richard Feynman at Caltech. (Math was not all he taught, but algebra did warrant a chapter in his celebrated *The Feynman Lectures on Physics* [1963].) How does the presentation differ when a mathematician teaches math to math students and a physicist teaches math to physics students? What are the primitive terms and rules of inference in each case? How do these differences define the process of teaching? To answer these questions, I will rely on Feynman’s lecture on algebra, input from a physicist friends, and my own experiences as a graduate student in a mathematics program. (Received August 28, 2017)

1135-A5-609      **Daniel C. Sloughter\*** ([dan.sloughter@furman.edu](mailto:dan.sloughter@furman.edu)), Department of Mathematics, Furman University, Greenville, SC 29613. *Hardy, Bishop, and making hay*. Preliminary report.

I find many parts of constructive mathematics appealing. For example, I find Kronecker’s dictum “[d]ie ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk,” an attractive starting point. Philosophically, I am intrigued by Errett Bishop’s addition that “mathematics belongs to man” and “[i]f God has mathematics of his own that needs to be done, let him do it himself.” Yet, when confronted with, say, the version of the Hahn-Banach theorem in Bishop’s *Constructive Analysis*, I find myself turning back to the cleaner statement in classical (“God’s”) mathematics. And I recall G. H. Hardy’s remark that he was willing to accept the Axiom of Choice, in part, because to deny it “seems to make hay of a lot of the most interesting mathematics.” In this talk, I will discuss the context of Hardy’s comment and how the creation of interesting and aesthetically pleasing mathematics influences one’s acceptance of axioms, and hence one’s philosophy of mathematics. (Received September 10, 2017)



1135-A5-814 **Chandra Kethi-Reddy\*** ([chan.dra@knights.ucf.edu](mailto:chan.dra@knights.ucf.edu)). *Gian-Carlo Rota and the Phenomenology of Mathematics.*

The celebrated combinatorialist Gian-Carlo Rota arguably produced a philosophy of mathematics more faithful to the actual practice of mathematics than any other American mathematician or philosopher of his time. While among his peers at MIT or the Los Alamos Research Laboratory, even while he was Vice President of the AMS, Rota had to fight the hegemonic and exclusionary institution of analytic philosophy in order to justify the intelligibility and practicality of his unique phenomenology of mathematics. In this presentation, I will take the audience through Gian-Carlo Rota's "Phenomenology of Mathematical Proof" in order to demonstrate the place of phenomenology in any rigorous philosophy of mathematics. I will also go through a selection of his advice from "Ten Lessons I Wish I Had Been Taught" and "Ten Lessons for the Survival of a Mathematics Department" to show how his philosophy can be lively, humorous, and close to life. I hope that this presentation will reinvigorate research in the phenomenology of mathematics and interest in the life and work of this titan. (Received September 14, 2017)

1135-A5-1883 **sarah-marie belcastro\*** ([smbelcas@toroidalsnark.net](mailto:smbelcas@toroidalsnark.net)). *Does Inclusivity Matter in Mathematical Practice?*

Many in the mathematical community believe that it is important to welcome participation from people with a variety of backgrounds and in particular from members of underrepresented groups. From a philosophical point of view, inclusivity is a broader concept than welcoming a diversity of human experience; for example, it includes welcoming a variety of mathematical perspectives (epistemic diversity).

Are there implications of encouraging inclusivity for the production of mathematical knowledge, and if so, what are they? And are they positive or negative? Conversely, are there implications of our mathematical practices, in terms of producing knowledge (theorems, proofs, etc.), on inclusivity?

We will carefully describe inclusivity as framed in the literature on scientific values, and restrict our discussion to epistemic values, and then to mathematical epistemic values. We will then examine what impacts the epistemic value(s) of inclusivity may have on mathematical practices, and what impacts current mathematical practices may have on inclusivity, and include specific examples. Finally, we will pose changes/actions that individuals or the community might make/take, in accordance with common mathematical values, and evaluate their impact relative to inclusivity. (Received September 25, 2017)

## *Trends in Mathematical and Computational Biology*

1135-AA-1677 **Katie Morrison\*** ([katherine.morrison@unco.edu](mailto:katherine.morrison@unco.edu)), 501 20th St CB 122, Greeley, CO 80639. *Emergent dynamics from neural network connectivity.*

Even in the absence of changing sensory inputs, many networks in the brain exhibit *emergent dynamics* – that is patterns of activity that are shaped by the intrinsic structure of the network rather than modulated by an external input. Such dynamics are believed to underlie central pattern generators (CPGs) for locomotion, oscillatory activity in both hippocampus and cortex, and the complex interplay between sensory-driven responses and ongoing spontaneous activity. To isolate the role of network connectivity alone in shaping these dynamics, we introduce the *Combinatorial Threshold Linear Network* (CTLN) model, a minimal model with binary synapses, simple perceptron-like neurons, and flat external input, whose dynamics are controlled solely by the structure of an underlying directed graph. By varying only the underlying graph, we observe the full variety of nonlinear dynamics: multistability, limit cycles, chaos, and even quasiperiodic behavior. The simplicity of the model makes these dynamics amenable to mathematical analyses, allowing us to connect various properties of the emergent dynamics to the connectivity structure of the underlying directed graph. (Received September 24, 2017)

1135-AA-1729 **Elena S. Dimitrova\*** ([edimit@clemson.edu](mailto:edimit@clemson.edu)), **Reinhard Laubenbacher** and **Linda H. Shapiro**. *A discrete multiscale modeling perspective to the innate immune response to ischemic injury.*

Regardless of the initial insult, optimal healing of damaged tissue relies on the precise balance of pro-inflammatory and pro-healing processes of innate inflammation to the extent that variations in either arm can exacerbate many diseases, from obesity to autoimmunity. In this talk, we will present a discrete multiscale mathematical model that spans the tissue and intracellular scales, and captures the consequences of targeting various regulatory components of injury-induced TLR4 signal transduction on potential pro-inflammatory or pro-healing outcomes. We apply known interactions to simulate how inactivation of specific regulatory nodes affects dynamics in the context of injury and to predict phenotypes of potential therapeutic interventions. We propose logical rules to link model behavior to qualitative estimates of pro-inflammatory signal activation, macrophage infiltration,

production of reactive oxygen species, and resolution. It will be described how the mathematical model can form the basis of a conceptual framework focusing on TLR4-mediated ischemic repair to assess potential molecular targets that can be utilized therapeutically to improve efficacy and safety in treating ischemic/inflammatory injury. (Received September 24, 2017)

1135-AA-2014 **Tim Lewis\*** ([tjlewis@ucdavis.edu](mailto:tjlewis@ucdavis.edu)), Department of Mathematics, University of California, One Shields Avenue, Davis, CA 95616. *Limb Coordination in Crustacean Swimming: Neural Mechanisms and Mechanical Implications.*

Despite the general belief that neural circuits have evolved to optimize behavior, few studies have clearly identified the neural mechanisms underlying optimal behavior. The distinct limb coordination in long-tailed crustacean swimming and the relative simplicity of the neural coordinating circuit have allowed us to show that the interlimb coordination in crustacean swimming is biomechanically optimal and how the structure of underlying neural circuit robustly gives rise to this coordination. Specifically, we use a *computational fluid dynamics* model to demonstrate that the crustacean stroke pattern is the most effective and mechanically efficient paddling rhythm across the full range of biologically relevant Reynolds numbers. We then use *coupled oscillator theory* to show that the organization of the neural circuit underlying swimmeret coordination provides a robust mechanism for generating this stroke pattern. Our result provide a concrete example of how an optimal behavior arises from the anatomical structure of a neural circuit. Furthermore, they suggest that the connectivity of the neural circuit underlying limb coordination during crustacean swimming may be a consequence of natural selection in favor of more effective and efficient swimming. (Received September 25, 2017)

1135-AA-2382 **Katja A Lamia\*** ([klamia@scripps.edu](mailto:klamia@scripps.edu)). *Cryptochromes link circadian clocks with metabolism and cancer.*

Circadian clocks modulate a wide variety of cellular and physiological functions including metabolism and genome protection in a time-of-day dependent manner. Mammalian circadian clocks involve a transcription and translation feedback loop in which the DNA-binding transcriptional activators CLOCK and BMAL1 drive expression of genes encoding PERIOD (PER1-3) and CRYPTOCHROME (CRY1,2) repressors, which inhibit CLOCK:BMAL1, resulting in oscillating transcription. I will discuss several pathways by which CRY1 and CRY2 mediate circadian responses to extracellular stimuli by regulating the activity of transcription factors. Among the transcription factors that are regulated by CRY1/2 are several nuclear hormone receptors, including sensors for stress hormones and dietary lipids, and c-MYC, which is the most commonly amplified oncogene in human cancers. I will describe some of the molecular mechanisms by which CRY1 and CRY2 suppress the activities of diverse transcription networks and the approaches that we are using to further understand interactions between circadian clocks, metabolism, and cancer. (Received September 26, 2017)

1135-AA-2538 **Richard Schugart\*** ([richard.schugart@wku.edu](mailto:richard.schugart@wku.edu)). *Using a Mathematical Model with Individual Patient Data to Quantify Differences Between Patients with Diabetic Foot Ulcers.* Preliminary report.

In this work, we quantify differences in healing responses between type-II diabetic patients with foot ulcers. This work builds off of our previous publication (Krishna et al., B Math Biol, 2015), where we formulate a mathematical model to describe healing responses using averaged time-course data from another study (Muller et al., Diabet Med, 2008). In Muller's work, they collect data from 16 patients with type-II diabetes. In addition to recording wound areas, Muller also measures levels of matrix metalloproteinases and their inhibitors at Weeks 0, 1, 2, 4, 8, and 12, collected from wound fluid. The patients are divided into two groups categorized as "good healers" and "poor healers" dependent upon the healing response at the four-week point. In our previous work, we use the average data to calibrate our mathematical model and quantify differences between the two groups. In our current work, we have calibrated our mathematical model for each individual patient and have quantified differences between these patients. In this presentation, we will discuss how our model has identified differences across patients using a variety of techniques. (Received September 26, 2017)

1135-AA-2885 **Richard Leonard and Shantia Yarahmadian\***, Department of Mathematics and Statistics, Mississippi State University, Mississippi State, MS 39762, and **Seth Oppenheimer**. *A Mathematical Model for the Cholinesterase Inhibitors in the Treatment of Alzheimer's Disease.*

Recent studies reveals that from pathological standpoint, Alzheimer disease (AD) is described by the cerebral deposition of amyloid- peptides in the form of amyloid plaques. We introduces a new mathematical model for the treatment of Alzheimer disease in the presence of inhibitory drugs. Two types of drugs are considered. Anti-inflammatory drugs (NSAIDs), which reacts with amyloid-beta 42 (A 42) protein monomers and the second

type, which reacts with amyloid- beta 42 aggregated polymers and clears them from creating around neurons. (Received September 26, 2017)

## *Teaching for Equity and Broader Participation in the Mathematical Sciences*

1135-AB-1227 **Chris L Rasmussen\*** ([chris.rasmussen@sdsu.edu](mailto:chris.rasmussen@sdsu.edu)). *From Inquiry to Critical Inquiry*.

There is increasing convergence on the following three principles that characterize inquiry-based or inquiry-oriented instruction: deep engagement in mathematics, peer-to-peer interaction, and instructor interest in and use of student thinking. These principles speak to both student activity and instructor activity. Missing from these principles, however, is explicit attention to equity. One way to focus on equity is to embrace a critical stance, which suggests we make a move from inquiry to critical inquiry. Embracing critical inquiry would involve creating a space for students to reflect on past, current, and future experiences. Consistent with this perspective, I present results from an analysis of student responses to an end of the semester portfolio assignment in a differential equations class. Student responses to the assignment, which was intended to focus on mathematical progress, also included unprompted reflection on the nature of mathematics, understanding mathematics, and teaching practice, both in terms of past experiences and future possibilities. Illustrative examples of these themes will be presented and related to classroom experiences that opened up the possibility for students to embrace a critical stance. (Received September 20, 2017)

1135-AB-1514 **Dan Battey\*** ([dan.battey@gse.rutgers.edu](mailto:dan.battey@gse.rutgers.edu)), Graduate School of Education, 10 Seminary Place, 232, New Brunswick, NJ 08901. *Building strong relationships with underrepresented students in undergraduate mathematics: Drawing on students' voices and exemplars from K-12 mathematics teaching*.

Mathematics is known as being a subject that can produce feelings of anxiety, inadequacy, embarrassment, and failure for many students. Adding to this, relationships with African American and Latin@ students have been found to be overly conflictual in mathematics (Jerome, Hamre, & Pianta, 2009). Conflicts can play out in implicit and unconscious ways in terms of missing students' mathematical contributions, discussing intellectual limitations of students, and dismissing incorrect answers as having no value (Battey, 2013). These moments serve as critical points to challenge and transform the way we interact with students. Thus, while important to understand the relational dimensions of instruction, it is important to understand the types of interactions as well as the messages that are being conveyed. The presentation will describe relational ways for instructor to support of historically marginalized students in mathematics classrooms, drawing on work from K-12 schooling (Battey, Neal, Leyva, & Adams-Wiggins, 2016). These forms of interactions include finding the value in students' incorrect answers, clearly noting students' competence, explicitly citing examples that counter stereotypes in mathematics, and providing safe emotional spaces for students to engage mathematically. (Received September 22, 2017)

1135-AB-2139 **Rochelle Gutiérrez\*** ([rg1@illinois.edu](mailto:rg1@illinois.edu)), University of Illinois at Urbana-Champaign. *Rehumanizing mathematics: should that be our goal?*

For far too long, we have embraced an "equity" standpoint that has been poorly defined (Gutierrez, 2002) or constantly shifting (NCTM, 2008). It has been difficult to assess progress beyond closing the achievement gap or recruiting more diverse students into the mathematical sciences. Instead, we should rehumanize mathematics, which considers not just access and achievement, but the politics in teaching and mathematics. This approach highlights: 1) what might be some dehumanizing experiences in mathematics for students and teachers and 2) how students could be provided with windows and mirrors onto the world and ways of relating to each other with dignity. As such, we can begin to think differently about student misconceptions, teachers as identity workers, and why it is not just that diverse people need mathematics but mathematics needs diverse people (Gutierrez, 2002; 2012). I present eight dimensions of a rehumanized mathematics classroom (participation/positioning; cultures/histories; windows/mirrors; living practice; broadening maths; creation; body/emotions; and ownership) as well as how mathematicians and mathematics educators can take risks in ensuring those happens in small and large ways. (Received September 25, 2017)

1135-AB-2742 **Uri Treisman\*** ([uri@austin.utexas.edu](mailto:uri@austin.utexas.edu)). *Experiments in Inclusion: Designing Instruction that Welcomes Students into the Mathematics Community*.

Recent advances in social psychology and behavioral economics suggest new approaches to perennial problems of mathematics instruction. How, for example, can we reliably promote students' productive persistence in the

face of mathematical challenges? How can we help students develop committed interest in mathematics from the short-term situational interest that can arise from well-designed instruction? How can we develop students' ability to use the mathematical tools they acquire in well-taught classrooms to create new mathematics, or at least mathematics that is new to them? How can we, in the context of a content-rich curriculum, help students become more strategic and effective learners?

The talk will describe a set of novel instructional protocols for addressing these and related questions and their implementation in a large lecture section of freshman calculus at The University of Texas at Austin. Preliminary results using a novel set of practical measurement tools will be shared. A special focus of the talk will be on the challenges of designing research-based instructional protocols for culturally and economically diverse student populations. (Received September 26, 2017)

1135-AB-3243 **Danny Bernard Martin\*** (<mailto:dbmartin@uic.edu>), University of Illinois at Chicago, 1040 W. Harrison Street (MC147), Chicago, IL 60607. *Black learners, citizenship, and the desegregation of mathematics.*

How has equity-oriented discourse in mathematics (education) sustained itself despite the failure of multiple efforts to radically respond to Black oppression and dehumanization? One reason is that equity is offered up to dominant white audiences and marginalized Black audiences with similar appeals but different promises and consequences. Nearly all modern mathematics education reforms have anchored themselves in the slogan system of Mathematics for All. This weak call to desegregate resonates with white liberalism but does not radically alter the status quo. White institutional spaces, including mainstream mathematics (education), remain content with metering out incremental, slow changes in Black "achievement" and representation. For Black learners, the lure of inclusion and broader participation in mainstream mathematics often include implied promises of integration and fuller citizenship but in reality require assimilation into the existing culture of mathematics (education). Rarely are the reform rhetorics of inclusion and broader participation interrogated to ask what kind of participants and citizens Black learners are expected or allowed to be within the political project of mainstream mathematics (education). In this talk, I offer brief elaborations of these points. (Received October 06, 2017)

1135-AB-3256 **Rachel Levy\*** ([Levy@HMC.edu](mailto:Levy@HMC.edu)), Harvey Mudd College, 301 Platt Blvd, Claremont, CA 91711. *Mathematical modeling and inclusivity: tales from teacher collaborators and their classrooms.*

Mathematical Modeling as a practice can have many elements that can support inclusivity: modelers can make genuine decisions as they pose and refine meaningful problems in a real world context, choose mathematical tools to apply to that problem and then communicate their solution to a client in a real world context. I will discuss two NSF-funded projects that explore how mathematical modeling can be enacted by students and teachers. I will focus on what mathematical modeling can afford as a practice while considering ways that potentially inclusive practices can go awry. (Received October 13, 2017)

## MAA Instructional Practices Guide

1135-AC-479 **April D. Strom\*** ([april.strom@scottsdalecc.edu](mailto:april.strom@scottsdalecc.edu)), 9000 E Chaparral Rd, Scottsdale, AZ 85256. *MAA IP Guide: A Resource for Implementing Meaningful Mathematical Tasks to Foster Student Engagement.*

The MAA Instructional Practices Guide provides a chapter on Classroom Practices devoted to fostering student engagement. Specifically, we offer examples of selecting mathematical tasks to achieve high levels of engagement in the classroom using ideas of appropriateness, motivation, and communication of ideas (such as critiquing the reasoning of others). Participants will learn about the MAA IP Guide and ideas for promoting active and cognitive student engagement when teaching collegiate mathematics. (Received September 05, 2017)

1135-AC-625 **Benjamin Braun\*** ([benjamin.braun@uky.edu](mailto:benjamin.braun@uky.edu)). *Broadening Assessment Methods in Postsecondary Mathematics.* Preliminary report.

Traditional lecture-based instruction methods have been associated with traditional summative assessment procedures such as timed exams with questions in very specific formats. Recent research in mathematics education, coupled with new technology, has encouraged classroom practices that promote student engagement. The Assessment chapter of the MAA Instructional Practices guide gives examples of formative and summative assessments

that are a better fit for the modern classroom. We highlight some of these examples and the underlying principles that could allow instructors to be creative in designing appropriate and equitable assessments for their own courses. (Received September 11, 2017)

1135-AC-637      **Richard Cleary\*** ([rcleary@babson.edu](mailto:rcleary@babson.edu)). *Assessment for Teaching and Learning*.

The opening statement of the Assessment chapter of the MAA Instructional Practices guide make the following claim: Effective assessment occurs when we clearly state high-quality goals for student learning, give students frequent informal feedback about their progress toward these goals, and evaluate student growth and proficiency based on these goals. We detail some of the ways that effective assessment can be implemented in various types of courses. We consider ways to make assessment consistent with course design and practice to promote effective learning for all students, as well as the value of creating a positive culture of assessment for faculty and departments. (Received September 11, 2017)

1135-AC-1165      **Elizabeth A. Burroughs\***, Department of Mathematical Sciences, Montana State University, PO Box 172400, Bozeman, MT 59717, and **Elizabeth G. Arnold**, Department of Mathematical Sciences, Montana State University. *The MAA IP Guide in the Context of Partner Organizations and Departmental Initiatives*.

The IP Guide provides support for undergraduate mathematics and statistics instructors and exposes them to effective teaching strategies and techniques they can implement to foster student engagement in their classrooms. The guide complements the curricular vision outlined in the MAA's 2015 *CUPM Curriculum Guide*, and supplements the recommendations made in the NCTM's 2014 *Principles to Actions* and the AMTE's 2017 *Standards for Preparing Teachers of Mathematics*. We will provide an example of how departments might make use of the guide, using our department's ongoing year-long graduate teaching assistant (GTA) training program, providing examples of what worked particularly well. We will also share GTAs' responses to workshops influenced by the IP Guide, providing some insight into the overall effectiveness of the guide. (Received September 20, 2017)

1135-AC-2088      **James A. Mendoza Álvarez\*** ([james.alvarez@uta.edu](mailto:james.alvarez@uta.edu)). *MAA IP Guide: Entry Points for Fostering Student Engagement in the Classroom*.

Increasingly, research findings indicate that enhanced learning in the classroom occurs when students are engaged or active in their learning. For college and university faculty, creating a classroom environment that fosters student engagement may be challenging due to insufficient familiarity with suitable techniques appropriate for a wide spectrum of classroom settings. The MAA Instructional Practices Guide provides examples of strategies and techniques that college and university faculty can use to foster student engagement in the classroom. We discuss classroom vignettes that illustrate these approaches in action as well as highlight some of the strategies and techniques that serve as natural entry points or initial steps for faculty aiming to increase student engagement in their classrooms. (Received September 25, 2017)

1135-AC-2793      **Karen Allen Keene\*** ([kakeene@ncsu.edu](mailto:kakeene@ncsu.edu)). *Incorporating the MAA IP Guide Design Practices into Instructional Planning: Principles to Know and Questions to Ask*.

All mathematics instructors know about planning for their classes. Planning involves choosing a textbook, setting a syllabus, and making notes for the class. If we think about what we do to prepare for class more deeply, we can consider Design Practices for instruction in undergraduate mathematics. Designing for Instruction is complex and time consuming; it involves considering students, the learning environment, content, and instructional practice. In this session, I will talk about best practices for design of instruction as considered in the new MAA Instructional Practices guide. Questions for an instructor to consider as she plans her course will be presented and participants will have an opportunity to think about the deep ideas of design. Time will be provided for discussion in this interactive presentation. (Received September 26, 2017)

## Quandle Questions

1135-AD-427      **Sam Nelson\*** ([sam.nelson@cmc.edu](mailto:sam.nelson@cmc.edu)). *Quandle Generalizations and Enhancements*.

In this talk we will take a brief tour of the world of generalizations of quandles, encountering structures such as racks, biquandles, and kei, and we will see a unified strategy for using these objects to define invariants of knotted objects. (Received September 01, 2017)

1135-AD-435     **Seung Yeop Yang\*** ([seungyeop.yang@du.edu](mailto:seungyeop.yang@du.edu)). *An introduction to quandle cocycle invariants of knots.*

There is a close relationship between quandles and knots; quandle theory has its origins in knot theory. We study an application of quandles to show how quandle cohomology theory, introduced by J. S. Carter, D. Jelsovsky, S. Kamada, L. Langford, and M. Saito, can be used to distinguish two knots. This talk is suitable for undergraduates. (Received September 02, 2017)

1135-AD-885     **Jieon Kim\*** ([jieonkim7@gmail.com](mailto:jieonkim7@gmail.com)), 1104-ho, 146, Surim-ro72beon-gil, Geumjeong-gu, Busan, Busan 46242. *Quandle coloring invariants of knots and surface-knots.* Preliminary report.

A knot is an embedding of a circle  $S^1$  in  $\mathbb{R}^3$ . Two knots are equivalent if one can be transformed into the other via a deformation of  $\mathbb{R}^3$  upon itself. A surface-knot is an embedding of a surface in  $BbbR^4$ . Two surface-knots are equivalent if one can be transformed into the other via a deformation of  $\mathbb{R}^4$  upon itself. A quandle is an algebraic structure with a binary operation satisfying certain conditions derived from Reidemeister moves which are local moves of knot diagrams. In this talk, I'll introduce invariants of links and surface-links, called quandle coloring invariants. (Received September 16, 2017)

1135-AD-1056     **Kanako Oshiro\*** ([oshirok@sophia.ac.jp](mailto:oshirok@sophia.ac.jp)). *G-families of quandles, their homology theory and cocycle invariants of handlebody-knots.*

This talk is based on the paper "A. Ishii, M. Iwakiri, Y. Jang and K. Oshiro, A G-family of quandles and handlebody-knots, Illinois J. Math. 57 (2013), no. 3, 817–838". G-families of quandles are used to produce invariants of handlebody-knots. As in the case of quandles, we can construct their (co)homology theory and as an application, we have cocycle invariants for handlebody-knots. I would also like to show the Nosaka's method to construct cocycles of a given G-family of quandles. I remark that any invariant of handlebody-knots is also useful as that of classical-knots. (Received September 19, 2017)

1135-AD-1477     **Jim Hoste\*** ([jhoste@pitzer.edu](mailto:jhoste@pitzer.edu)), Pitzer College, 1050 N Mills Ave, Claremont, CA 91711, and **Patrick D. Shanahan**. *Quotients of the fundamental quandle of a link.*

Associated to every knot and link is its *fundamental quandle*, an algebraic object shown to be a complete knot invariant (up to mirror reversal) by both Joyce and Matveev. The fundamental quandle of a link is almost always infinite—it is finite only for the trivial knot and the 2-component Hopf link, the simplest nontrivial link. Being a complete knot invariant, the fundamental quandle is essentially no easier to study than knots themselves. However, as with most algebraic objects, one can pass to various quotients of the fundamental quandle to obtain more practicable, albeit less sensitive, invariants of knots. In this talk I will discuss certain quotients of the fundamental quandle that are sometimes finite and therefore, more easily investigated than the fundamental quandle. (Received September 22, 2017)

1135-AD-1491     **Patrick D Shanahan\*** ([pshanahan@lmu.edu](mailto:pshanahan@lmu.edu)), Loyola Marymount University, Dept. of Mathematics, 1 LMU Dr., Los Angeles, CA 90045, and **Jim Hoste**. *Enumerating cosets, quandles, and coset-quandles.*

In the 1930's, Todd and Coxeter developed a process to enumerate the cosets of a subgroup of a group. This process was a fundamental development in the emerging field of computational group theory. Some 50 years later, Joyce and Matveev independently introduced quandles as a new type of algebraic knot invariant. Shortly thereafter, a graph based Todd-Coxeter like process to enumerate the elements of an  $n$ -quandle of a knot was produced by Winker. In this talk, we will present an enumeration process for the elements of a rack given by a presentation. Since every quandle is a rack, our process generalizes that of Winker. Unlike Winker's method, our approach is modeled after current table-based implementations of the Todd-Coxeter process. Using similar methods to those in computational group theory, we prove that our process terminates if and only if the order of the rack is finite, in which case the procedure outputs an operation table for the rack. We conclude with an application to  $n$ -quandles of knots and discuss the relationship between the  $n$ -quandle of the knot and the coset-quandle of the peripheral subgroup. This project is joint work with Jim Hoste of Pitzer College and was, in part, motivated by the undergraduate senior thesis of Sarah Yoseph. (Received September 22, 2017)

## *Research in Improving Undergraduate Mathematical Sciences Education: Examples Supported by the National Science Foundation's IUSE: EHR Program*

1135-AE-1352     **Petra Bonfert-Taylor\*** (petra.b.taylor@dartmouth.edu), 14 Engineering Drive, Hanover, NH 03755, and **Sarah Eichhorn** (s.eichhorn@austin.utexas.edu), **Jim Fowler** (fowler@math.osu.edu) and **David Farmer** (farmer@aimath.org). *CuratedCourses in Mathematics.*

CuratedCourses in Mathematics is a project to create, gather, curate, tag, review, organize and make available high quality online open educational mathematics resources. The project aims to coordinate work being done at multiple institutions on similar courses, enabling faculty to share resources they create or curate from other sources. By creating a system for curating and tagging resources our hope is that faculty can more easily find high quality materials to utilize in their classes and more broadly disseminate good resources they create.

We will describe the project itself, describe resources we have created for faculty about how to design and produce online mathematics content, describe our tagging system for content submitted to our site as well as present our future goals for the project. (Received September 21, 2017)

1135-AE-1767     **Victor J. Donnay\*** (vdonnay@brynmawr.edu), Bryn Mawr College, 101 N. Merion Ave, Bryn Mawr, PA 19010, and **Ellie Goldberg.** *The Teaching Experience for Undergraduates (TEU) Summer Program: an immersive experience in mathematics pedagogy for students from liberal arts colleges interested in exploring a career in teaching.* Preliminary report.

The Summer STEM Teaching Experiences for Undergraduates from Liberal Arts Institutions (TEU) program is developing and testing a model program that provides undergraduate STEM majors with an immersive summer experience in secondary mathematics or science education. The model integrates a high-quality discipline specific pedagogy course and teaching practicum with a focus on urban education, and other activities during the academic year. During the five-year grant, 120 undergraduates (60 in mathematics; 60 in science) from a network of 61 liberal arts institutions will take a pedagogy course in secondary mathematics or science integrated with a teaching practicum. The project team will investigate the degree to which the TEU program provides a robust model for STEM teacher recruitment and preparation that could be replicated nationally.

TEUs work in collaborative teams to design and teach lessons and receive daily feedback and support from mentors. TEUs live together in dorms and thus are immersed 24/7 in a community of peer novice teachers. The program also includes a focus on teacher leadership development. We view the TEU program as a teaching analogue of the successful NSF Research Experiences for Undergraduates (REU) program. (Received September 24, 2017)

1135-AE-1873     **Susan L. Ganter\*** (susan.ganter@erau.edu), **Jack Bookman**, **Rhonda Ellis** and **Rebecca Segal.** *A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P).* Preliminary report.

SUMMIT-P is a consortium of eleven institutions that have formed an interdisciplinary collaborative to renew the undergraduate mathematics curriculum based on research about the needs of partner disciplines, as identified in a series of 22 workshops organized by MAA's Committee on Curriculum Renewal Across the First Two Years (CRAFTY). The Curriculum Foundations (CF) reports, written by partner discipline representatives, indicate a need for mathematics that emphasizes problem solving, statistics and quantitative data, problems in context, and appropriate technologies. Mathematicians and partner discipline faculty are investigating how the CF recommendations can be used to improve course content. Impact across the consortium will be measured using the following research questions: 1. What are the beliefs of SUMMIT-P faculty concerning the importance of interdisciplinary collaborations? 2. How and to what extent are institutions implementing and supporting interdisciplinary collaborations? 3. Has the SUMMIT-P work benefited from faculty learning communities (FLCs)? Have these FLCs supported meaningful and authentic collaborations across the disciplines? 4. Have the affected courses and student learning/attitudes improved as a result of the SUMMIT-P work? (Received September 25, 2017)

1135-AE-2259 **Kathy Cousins-Cooper\*** ([cousinsk@ncat.edu](mailto:cousinsk@ncat.edu)), 1601 E. Market St., Mathematics Department, North Carolina A&T State University, Greensboro, NC 27411, and **Dominic P. Clemente** and **Katrina N. Staley**. *Assessing the Impact of the Emporium Model on Student Persistence and Dispositional Learning by Transforming Faculty Culture.*

We present a progress report on the implementation of the Mathematics Emporium Model (MEM) in the Algebra course sequence at North Carolina A&T State University (NC A&T SU). The purpose of the project is to improve performance and pass rates in four College Algebra and Trigonometry (CAT) courses NC A&T SU. At NC A&T SU, these courses fill remedial and general education (GE) requirements or serve as prerequisites for a wide range of higher-level courses in various subjects, and are therefore, gatekeeper courses for a vast range of majors — and in particular those in the high-demand science, technology, engineering, and mathematics (STEM) fields. The following four courses were redesigned to eliminate identified success inhibitors: Math 101, Math 102, Math 103, Math 104, all 3-credit hour College Algebra and Trigonometry courses offered in two sequences, Math 101-102 and Math 103-104. This presentation will discuss the results of the re-design to date, including learning space re-design and student performance comparisons of traditional and the MEM course designs. In addition, this project aims to generate a transformative change in teaching practices. We will report the results of surveys administered to evaluate faculty perceptions of the new learning environment. (Received September 26, 2017)

1135-AE-2965 **Nathan Tintle\*** ([nathan.tintle@dordt.edu](mailto:nathan.tintle@dordt.edu)), 498 4th Ave NE, Department of Mathematics and Statistics, Sioux Center, IA 51250. *Broadening the impact and evaluating the effectiveness of simulation-based curricula for introductory statistics.*

The demands for a statistically literate society are increasing, and the introductory statistics course "Stat 101" remains the primary venue for learning statistics for the majority of high school and undergraduate students. Our group has developed and implemented one of the first cohesive curricula that (a) emphasizes the core logic of inference using simulation-based methods in an intuitive, cyclical, active-learning pedagogy, and (b) emphasizes the overall process of statistical investigations, from asking questions and collecting data through making inferences and drawing conclusions. In this talk, I will highlight two primary efforts taken over the last five years as part of an NSF-funded project. First, we have been working to expand implementations of simulation-based curricula for introductory statistics by (a) conducting a series of professional development workshops involving hundreds of instructors, and (b) developing and supporting an online learning community, which have provided free resources to hundreds more. Second, we are evaluating students' attitudes, conceptual understanding, and learning trajectories in simulation-based curricula through a large scale, multi-site assessment project involving over 10,000 students and 200 instructors. (Received September 26, 2017)

1135-AE-3085 **Jeffrey C Humpherys\*** ([jeffh@math.byu.edu](mailto:jeffh@math.byu.edu)), TMCB 386, Department of Mathematics, Brigham Young University, Provo, UT 84602, and **Tyler J Jarvis** ([jarvis@math.byu.edu](mailto:jarvis@math.byu.edu)), TMCB 275, Department of Mathematics, Brigham Young University, Provo, UT 84602. *Applied and Computational Mathematics: A New Degree for 21st Century Discovery and Innovation.*

Faculty from BYU's Math Department have created a new upper-division curriculum in Applied and Computational Mathematics that can serve as a national model to expand the pipeline of young scholars in the mathematical sciences who will be well equipped to face the challenges of the 21st Century and become leaders in the globally competitive STEM workforce.

The main elements of this program are:

- (1) A new and completely redesigned upper-division lockstep curriculum that provides students with a rigorous foundation in mathematics, statistics, and computation.
- (2) Horizontal integration across multiple quantitative disciplines.
- (3) Socialization, soft skills and leadership training to foster team building among diverse groups of students.
- (4) A capstone experience, through either a research experience or an internship.

The curriculum development has been seeded by an NSF DUE TUES Phase II award, which is currently supporting the creation of 4 textbooks, which are being published by SIAM, and 4 lab manuals that are now freely available on GitHub.

In this talk, we examine several of the program's features, where we've had successes and failures, and how we've made improvements along the way. (Received September 26, 2017)



## *Polyhedra, Commemorating Magnus J. Wenninger*

1135-AF-206 **Thomas Q Sibley\*** ([tsibley@csbsju.edu](mailto:tsibley@csbsju.edu)). *Nurturing Geometrical Intuition.*

For millennia geometry has fostered a fruitful synthesis of intuition and proof. But our current pedagogical emphasis on algebraic thinking has eroded geometric intuition. Fr. Magnus Wenninger's marvelous models provided a welcome balance, showing the value of visual and tactile understanding in geometry. He also shared his passion for the intrinsic beauty of polyhedra. I will discuss some ways to build on his legacy, increasing students' geometrical intuition with physical models. (Received August 09, 2017)

1135-AF-634 **Joseph D Clinton\*** ([jclintonds@gmail.com](mailto:jclintonds@gmail.com)), 334 MacKenzie Drive, West Chester, PA 19380. *A Tribute to Magnus Wenninger: A Visual Adventure in Mathematical Thought.*

Father Magnus Wenninger spent 79 years of his 98 years of life as a Benedictine monk, educator, mathematician, author, student of philosophy and polyhedrons. He made major contributions to help rekindle the interest in polyhedron as an integral part of mathematics education. Through his polyhedron, modeling he demonstrated the importance of visual thinking as an aid to solving complex problems. This presentation will focus on his influence on the advantages of visual thought. Model building as a means of making connections within a family of polyhedron transformations will be discussed. As an example, a short animated film will be shown illustrating a series of polycylinderhedron relationships. (Received September 11, 2017)

1135-AF-1342 **Vladimir Bulatov\*** ([info@bulatov.org](mailto:info@bulatov.org)), Corvallis, OR. *Wythoff Polyhedra Construction and its Generalizations.* Preliminary report.

The Wythoff construction of uniform polyhedra is based on reflections in sides of spherical triangles. The base vertex is placed inside of the triangle and series of reflections in triangle sides generate other equivalent polyhedron vertices and faces. The condition of uniformity restricts possible vertex location to few special points. Two generalization of the construction are considered. We allow the base vertex to be located anywhere on the surface of the sphere and we allow arbitrary spherical polygon to be used as generator for the polyhedron faces. The generalization produces continuous families of kaleidoscopic polyhedra. Numerous illustrations and animations of such polyhedral families will be presented for spherical, plain and hyperbolic geometries. (Received September 21, 2017)

1135-AF-1596 **Thomas Banchoff\*** ([thomas\\_banchoff@brown.edu](mailto:thomas_banchoff@brown.edu)), RI. *Interviewing Magnus Wenninger.* Magnus Wenninger's polyhedral models are striking and seeing him work on new models at conferences on mathematics and art would make anyone wonder about his story. This presentation describes visits with him in the spring of 1995 at St. John's College in Minnesota, and at the Geometry Center in the University of Minnesota, leading to a biographical interview with him published in 2002. (Received September 23, 2017)

1135-AF-2181 **Rona Gurkewitz\*** ([gurkewitzr@wcsu.edu](mailto:gurkewitzr@wcsu.edu)), WCSU, 181 White Street, Danbury, CT 06810, and **Bennett Arnstein.** *From Impossible to Obvious: Exploring Origami Polyhedra, and Participating in Wenninger's Polyhedron Email List.*

I started on the mailing list by getting a reference to one of Magnus' books. Later, the reaction to my claim about some origami polyhedra was "impossible". Yet a year later it was described as "obvious". I will show the origami polyhedra and their special mathematical property. (Received September 25, 2017)

1135-AF-2897 **Vincent J. Matsko\*** ([vince.matsko@gmail.com](mailto:vince.matsko@gmail.com)). *Working with Magnus Wenninger.*

After becoming enchanted by his books, I began corresponding with Magnus Wenninger in the early 1990's. He was a significant influence in the way I looked at polyhedra, and my visits with him were especially wonderful. Much of my work with polyhedra is a direct consequence of knowing and working with Magnus. I will share several examples, including a textbook on polyhedra, dual polyhedra, and stellations of polyhedra. (Received September 26, 2017)

## *Research in Undergraduate Mathematics Education: Highlights from the Annual SIGMAA on RUME Conference*

1135-AG-287 **Morgan Early Sellers\*** ([mearly1@asu.edu](mailto:mearly1@asu.edu)), SoMSS, Arizona State University, P.O. Box 871804, Tempe, AZ 85287-1804, **Kyeong Hah Roh** ([khroh@asu.edu](mailto:khroh@asu.edu)), SoMSS, Arizona State University, P.O. Box 871804, Tempe, AZ 85287-1804, and **Erika Johara David** ([ejdavid@asu.edu](mailto:ejdavid@asu.edu)), SoMSS, Arizona State University, P.O. Box 871804, Tempe, AZ 85287-1804. *A Comparison of Calculus, Transition-to-Proof, and Advanced Calculus Student Quantifications for Complex Mathematical Statements.*

This study investigates Calculus, Transition-to-Proof, and Advanced Calculus students' quantifications for conditional statements involving multiple quantifiers. Three students from each course participated in clinical interviews. During the interviews, we presented the Intermediate Value Theorem (IVT) and three other statements. All four statements have the same hypothesis and predicate, but none of them are logically equivalent. During the first half of the interviews, students were asked to evaluate and interpret these statements. During the second half of the interviews, students used graphs to justify their reasoning for their evaluations. In the data analysis, we focused on student utterances, gestures, and markings on graphs to analyze students' evaluations of and quantifications for the statements. Several different types of student quantifications for the statements emerged from the data analysis. In this presentation, we will discuss how these student quantifications impact student evaluations of the statements. We also address differences in student quantifications across mathematical level and discuss some teaching implications associated with these findings. (Received August 18, 2017)

1135-AG-466 **Nicholas Wasserman\*** ([wasserman@tc.columbia.edu](mailto:wasserman@tc.columbia.edu)), **Keith Weber** and **William McGuffey**. *Leveraging Real Analysis to Foster Pedagogical Practices.*

Real analysis is frequently a required course for prospective secondary mathematics teachers. However, most teachers view real analysis as unnecessary and unrelated to the work of teaching secondary mathematics. In accord with a theoretically-motivated instructional model for improving the teaching of advanced mathematics courses for teachers, we implemented a course that framed real analysis content by 'building up from' and 'stepping down to' teaching practice. In this session, we describe how this model was implemented in a single module – about “attending to scope” – and analyze secondary mathematics teachers' engagement in and reflections on the desired pedagogical aims. In addition, we followed six of these secondary teachers into their classrooms to observe their subsequent teaching. Both in-class data and teaching observation data provide evidence that what they learned in the real analysis module was useful for informing their pedagogical practice. We discuss the design approach and its potential implications for secondary mathematics teacher education. (Received September 05, 2017)

1135-AG-486 **Naneh Apkarian\*** ([naneh.apkarian@gmail.com](mailto:naneh.apkarian@gmail.com)) and **Chris Rasmussen**. *Mathematics Instruction Leadership in Undergraduate Departments.*

Amid the myriad calls for improving post-secondary mathematics instruction, many universities are coordinating their introductory mathematics courses. Robust coordination systems consist of two major elements: uniform course elements (e.g., common text; exams) and regular instructor meetings - the latter of which can engender productive collaboration among faculty. Of particular importance are those who act as leaders (formally and/or informally) within these coordination systems, because they have the potential to influence instructional practice. This study uses social network data to investigate instructional leaders at five diverse institutions, considering both formal and informal coordination phenomena. Our results indicate that, in these successful programs, formal and informal leadership is in alignment and instructors are frequently involved in conversations about instruction. These results speak to the role of social interactions in mathematics departments and the potential of coordination systems to support and sustain change at the department level. (Received September 05, 2017)

1135-AG-1221 **Hortensia Soto-Johnson** ([hortensia.soto@unco.edu](mailto:hortensia.soto@unco.edu)), School of Mathematical Sciences, Ross Hall 2240C, Greeley, CO 80639, **Michael Oehrtman\*** ([michael.oehrtman@okstate.edu](mailto:michael.oehrtman@okstate.edu)), Department of Mathematics, Oklahoma State University, Stillwater, OK 74074, and **Brent Hancock** ([brent.hancock@unco.edu](mailto:brent.hancock@unco.edu)), School of Mathematical Sciences, Ross Hall 2210E, Greeley, CO 80639. *Mathematicians' construction of meaning for derivatives and integrals of complex-valued functions.*

We engaged five research mathematicians in describing their images of differentiation and integration for functions of complex variables. Our data collection and analysis focused on characterizing these experts' movement between domains of thought ranging from real world experience to formal mathematical theory. The mathematicians relied heavily on direct application of concepts from differentiation of real-valued functions and employed rotation and

stretching as a local linear description of the action of the function. They employed reasoning about real-valued line integrals to generalize to complex functions, but several struggled to interpret what was being accumulated in this case. In some cases, the experts' fixation on particular features created barriers to progress developing their geometric reasoning. All of the participants began reasoning about the tasks by invoking symbolic or formal reasoning. Only later did some of the mathematicians invoke corresponding geometric interpretations, often carefully crafted to represent features of their formal reasoning. We use these examples to explore the implications for instruction of attending to the interplay of concrete and formal mathematical reasoning, rather than presuming meaning develops only from the former to the latter. (Received September 20, 2017)

1135-AG-1533     **Aditya P. Adiredja\*** ([adiredja@math.arizona.edu](mailto:adiredja@math.arizona.edu)) and **Michelle Zandieh** ([zandieh@asu.edu](mailto:zandieh@asu.edu)). *Using Intuitive Examples from Women of Color to Reveal Nuances about Basis.*

Research and surveys continue to perpetuate deficit narratives about women of color, particularly regarding their participation in and contribution to mathematics. Following the broader call for more research concerning STEM learning experiences of women of color, this study focuses on the sense making of eight women of color regarding their understanding of basis in linear algebra. We documented diverse ways that these women creatively explained the concept of basis using intuitive ideas from their everyday lives. These examples revealed important nuances and aspects of understanding of basis that are rarely discussed in instruction. These students' ideas can also serve as potentially productive avenues into the topic. Our results also challenge the existing broader narrative about academic underachievement of women of color in mathematics. (Received September 22, 2017)

## *Differential Equations and Their Applications to Neuroscience*

1135-AH-283     **Honghui Zhang\*** ([haozhucy@nwpu.edu.cn](mailto:haozhucy@nwpu.edu.cn)), New Bedford, MA 02740. *Seizure dynamics of coupled-oscillator studied by Epileptor field model.*

The focus of our study is to investigate the mechanisms of seizure recruitment and propagation through a neural field model called Epileptor, which is coupled via signaling through slow permittivity variables. We also investigate the influence of electro-tonic couplings and chemical synaptic coupling in the generation, propagation and termination of spontaneous seizure-like events. Our analysis provide a set of indices conditions that seizures propagate to nonepileptogenic brain regions. These results are hoped to give some useful tips on clinical therapy of partial epilepsies. (Received August 18, 2017)

1135-AH-340     **Sam Behseta\*** ([sbehseta@fullerton.edu](mailto:sbehseta@fullerton.edu)), 800 N. State College Blvd., Department of Mathematics, California State University, Fullerton, Fullerton, CA 92832. *Bayesian Modeling of Neuronal Spike Trains.*

In this talk, I will give an overview of some of the work that I have been involved with in the past few years, including a series of collaborative efforts utilizing Bayesian hierarchical models for the analysis of point processes associated with the neuronal spike train data, I will talk about Bayesian Functional Data Analysis for the comparative analysis of neuronal spiking activities recorded under multiple experimental conditions and Bayesian nonparametric techniques utilizing Dirichlet Processes for the same objective. Additionally, I will explain the advantages of applying a Bayesian framework through Gaussian Process models for decoding information associated with multiple spike trains obtained from simultaneously recorded neurons. Finally, I will offer a few thoughts about some exciting opportunities for future research. (Received August 25, 2017)

1135-AH-872     **Jeffrey Kopsick\*** ([kopsicjd@dukes.jmu.edu](mailto:kopsicjd@dukes.jmu.edu)) and **James Sochacki**. *Investigating How Neurons Communicate Through the Power Series Method (PSM)*. Preliminary report.

Based on the physical conceptions and processes underlying the electrical activity of neurons, Hodgkin and Huxley created the first biophysical model for the giant squid axon. Their Nobel Prize winning work and ideology has helped shape our current understanding of conductance-based models. The canonical numerical methods used to approximate the solutions of these models are Runge-Kutta and variants of it. Stemming from the computational neuroscience results of R.D. Stewart and W. Bair's work and the complexity of these dynamical systems, we have developed an algorithm using the Power Series Method (PSM) to improve the numerical solutions to the differential equations that describe these types of models. This talk will focus on comparative studies of the PSM and Runge-Kutta methods in different conductance-based models, motivated by the work of Hodgkin and Huxley. Applications of our work include improving parameter estimation (e.g.

rate constants), better understanding neuronal response to external current and the transition state parameters effects on the action potential.\*

\*This joint work between James Sochacki and I was made possible by the Department of Mathematics and Statistics at James Madison University. (Received September 15, 2017)

1135-AH-1287 **Jianzhong Su\*** (su@uta.edu), Department of Mathematics, University of Texas at Arlington, Arlington, TX 76019, and **Sat byul Seo** and **Ege Kavalali**. *Discontinuous coefficient diffusion models of neurotransmitter release for independent synaptic currents.*

Synapses play a major role in neuron communications in the brain. The synapses act through a chemical process called synaptic fusion between pre-synaptic and post-synaptic terminals. In the paper, we develop a mathematical model in 3-D to emulate spontaneous and evoked neurotransmissions resulted from glutamate release within a single synapse. We propose numerical methods for solving piecewise continuous heat diffusion equation, estimate and verify its errors of second order accuracy. In order to identify the spatial relation between spontaneous and evoked glutamate releases, we consider quantitative factors, such as the size of synapses, inhomogeneity of diffusion coefficients, the geometry of synaptic cleft, and the release rate of neurotransmitter, that will affect postsynaptic currents. We conclude quantitatively that as a synapse's size is smaller and if the synaptic cleft space is less diffusive in the peripheral area than the central area, then there is high possibility of having crosstalk between two signals from spontaneous and evoked releases. The computed results match well with existing experimental findings and provide a quantitative map of boundaries of physical constraints for having independent synaptic fusion events. (Received September 20, 2017)

1135-AH-1849 **Daqing Guo\*** (dqguo@uestc.edu.cn), **Mingming Chen**, **Yang Xia**, **Yangsong Zhang** and **Dezhong Yao**. *Mesoscopic Neural Field Model of Absence Epilepsy.*

Epilepsy is a general term for conditions with recurring seizures. Absence seizures are one of several kinds of seizures, which are characterized by typical 2–4 Hz spike-and-slow wave discharges (SWDs). Recent data suggested that the basal ganglia (BG) might regulate absence seizures, but the related biological mechanisms are still unclear. Here we establish a mesoscopic neural field model for the basal ganglia-corticothalamic network to investigate the roles of BG in controlling absence seizures. Using this model, we demonstrate that the typical absence seizure activities can be controlled and modulated by the direct GABAergic projections from the substantia nigra pars reticulata to either the thalamic reticular nucleus or the specific relay nuclei of thalamus, through different biophysical mechanisms. Under certain conditions, these two types of seizure control are observed to coexist in the same network. Moreover, we also observe that both increasing the activation of neurons in globus pallidus externa and enhancing the coupling strength of the inhibitory pallido-cortical pathway can suppress the bilaterally synchronous 2–4 Hz SWDs during absence seizures. Overall, these findings highlight the multiple functional roles of BG in regulating absence seizure. (Received September 25, 2017)

1135-AH-3166 **Gabriel A Silva\*** (gsilva@ucsd.edu), Department of Bioengineering, University of California San Diego, 9500 Gilman Drive, MC0412, La Jolla, CA 92093-0412. *Inference of actualized subsets of geometric association graphs based on context and a neural derived dynamic competition model.*

We have developed a generalized neural derived framework capable of mining contextual geometric association graphs in unique ways. ‘Contextual’ and ‘associative’ in that the input data being learned and the connections between data elements reflect semantic and situational relationships organized on a temporal dynamics. ‘Geometric’ in that the adjacency matrices are not binary but weighted by a distance metric that reflects a computed ‘closeness’ between data elements. Our methods allow us to carry out contextual dynamic inference on such graphs. An association graph encodes every possible and realizable connection between different learned data elements, along with a measure of the probability or strength of the interactions between them. Call this the total solution space. However, at any given moment, how such a graph is actually used, how subsets of the total solution space are called upon and actualized in response to a contextual demand external to the system, will vary over time dependent on the context and situation. We show we can efficiently compute the subset of associations that form functional paths using a neural derived competitive refractory model that computes how the timing of signals propagating on the graph compete to ‘activate’ vertices they connect into. (Received September 26, 2017)

1135-AH-3227 **Qinyu Chao** and **Lixia Duan\*** (duanlx@ncut.edu.cn), School of Mathematics, North China University of Technology, Beijing, 101300, Peoples Rep of China. *Dynamics analysis of the pre-Botzinger complex under magnetic flow.*

Breathing is a complex rhythmic movement, normal breathing rhythm is substantially uniform, and pathological conditions will appear a variety of changes in respiratory rhythm. Therefore, the dynamics of neuron firing

activity has important implications for understanding the reason of morbid respiratory rhythm. With the development of electrophysiological and electromagnetism theory, there are complex distribution of electromagnetic field in the neuronal system. According to the Maxwell electromagnetic induction theorem, internal bioelectricity of nervous system plays a key role for electric activity in each neuron. This talk presents the improved Butera neuron model. The model are created using magnetic flow to describe the influence of electromagnetic induction on neuronal activities. By adding memristive current on the membrane variable, we explore the effects of magnetic flux on the membrane potential. The effects of electromagnetic radiation on the discharge activities are studied. Multiple modes of electric activities can be observed with the fixed initial value by changing the external stimulus currents. Further studies on the relationship between the initial value  $k_1$  and the current are studies by two parameter bifurcation analysis. (Received September 27, 2017)

## *Accessible Problems in Modern Number Theory*

1135-AI-382      **Nathan Kaplan\*** (nckaplan@math.uci.edu), Department of Mathematics, 340 Rowland Hall, Irvine, CA 92697. *Counting Numerical Semigroups.*

A numerical semigroup  $S$  is an additive submonoid of  $\mathbb{N}_0 = \{0, 1, 2, \dots\}$  where  $\mathbb{N}_0 \setminus S$  is finite. The size of  $\mathbb{N}_0 \setminus S$  is called the genus of  $S$ . Let  $N(g)$  be the number of numerical semigroups of genus  $g$ . How does this sequence behave?

Bras-Amorós computed the first 50 values of  $N(g)$  and noticed some striking patterns. She conjectured that  $N(g)$  grows approximately as fast as the Fibonacci sequence and that  $N(g-1) + N(g-2) \leq N(g)$ . Zhai proved the conjecture on the Fibonacci-like growth of  $N(g)$ , but even the weaker conjecture that  $N(g-1) \leq N(g)$  remains unsolved. We will give an overview of problems about counting numerical semigroups, highlighting some accessible questions that remain open. (Received August 29, 2017)

1135-AI-1809      **Paul Pollack\*** (pollack@uga.edu). *Squares mod  $p$ .*

Take the collection of perfect squares  $— 1, 4, 9, 16, \dots —$  and reduce modulo a prime number  $p$ . What can be said about the resulting list? I will survey results, old and new, as well as outstanding open problems. (Received September 24, 2017)

1135-AI-2036      **J. Brian Conrey\***, 600 East Brokaw Rd., San Jose, CA 95112, and **Mark Holmstrom.** *Smooth values of quadratic polynomials.* Preliminary report.

A number is  $z$ -smooth if all of its prime factors are at most  $z$ .

It turns out that for any  $a \neq 0$  and any  $z$  the set of  $z$ -smooth numbers of the form  $q^2 + a$  is a finite set!

We have found an algorithm which quickly finds, for any  $z$  and any  $a \neq 0$ , almost all of the set of  $z$ -smooth values of  $q^2 + a$ .

We have compiled the sets resulting from our algorithm for  $z = 500$  and  $1 \leq |a| \leq 25$ . These sets are large, usually around 10 to 20 thousand. There is a lot of data to parse here and inspecting it leads to many questions and conjectures. For example, it appears that for each  $z$  and  $a$ , the set of the logarithms of the numbers in the set are normally distributed. If so, what are the mean and the standard deviation?

There are many open questions that arise about the sizes of these sets, about the maximal number in each set, and about a surprising divisibility property of the numbers in the sets. (Received September 25, 2017)

1135-AI-2215      **Adriana Salerno\***, 3 Andrews Rd, Lewiston, ME 04240, and **Juergen Kritschgau.** *Origami constructions of rings of integers of imaginary quadratic fields.*

In the making of origami, one starts with a piece of paper, and through a series of folds along seed points one constructs complicated three-dimensional shapes. Mathematically, one can think of the complex numbers as representing the piece of paper, and the seed points and folds as a way to generate a subset of the complex numbers. Under certain constraints, this construction can give rise to a ring which we call an origami ring. We will talk about the basic construction of an origami ring due to Buhler, Butler, de Launay, and Graham, and further extensions and implications of these ideas in algebra and number theory. In particular in this talk, we show that it is possible to obtain the ring of integers of an imaginary quadratic field through an origami construction. (Received September 25, 2017)

1135-AI-2548      **David Zureick-Brown\*** (dzb@mathcs.emory.edu), 400 Dowman Drive, Atlanta, GA 30322. *Beyond Fermat's Last Theorem.*

Diophantine geometry is the study of integral solutions to a polynomial equation. For instance, for integers  $a, b, c \geq 2$  satisfying  $1/a + 1/b + 1/c < 1$ , Darmon and Granville proved that the individual generalized Fermat

equation  $x^a + y^b = z^c$  has only finitely many coprime integer solutions. Conjecturally something stronger is true: for  $a, b, c \geq 3$  there are no non-trivial solutions.

I'll discuss various other Diophantine problems, with a focus on the underlying intuition and conjectural framework. I will especially focus on the uniformity conjecture, and will explain new ideas from tropical geometry and our recent partial proof of the uniformity conjecture. (Received September 26, 2017)

1135-AI-3195 **Kristin E Lauter\*** (klauter@microsoft.com). *New number theoretic problems in Cryptography*. Preliminary report.

New problems in number theory are being considered as the basis for post-quantum cryptography. This talk will describe new problems arising from lattice-based cryptography and supersingular isogeny graph cryptography. These problems are accessible, but not necessarily easy. (Received September 27, 2017)

## Mathematics and Sports

1135-B1-41 **Stephen Szydluk\*** (szydliks@uwosh.edu), Mathematics Department, University of Wisconsin Oshkosh, 800 Algoma Blvd., Oshkosh, WI 54901-8631. *You win some and you lose some: scoring a two-team cross-country race*.

Scoring a cross-country running race between two teams is straightforward: essentially, a team's score is determined by finding the sum of the placings of its runners, with the winner being the team with the lower score. This "standard scoring method" is hardly the only way that the winner could be chosen, however. In this presentation, we'll take explore alternative scoring methods, we'll consider some attractive properties that we would want a scoring method to possess, and we'll see connections to the mathematics of social choice. Runners and mathematicians (and especially running mathematicians) of all ages are welcome to attend. Lace up your shoes and join us! (Received June 28, 2017)

1135-B1-88 **Stanley Rothman\*** (stanley.rothman@quinnipiac.edu), Quinnipiac University, Hamden, CT 06518. *New Theorems to Predict Winning Percentages and Compare Parity in the MLB, NBA, and NFL*. Preliminary report.

My talk will introduce a new extension to my prior research on the concept of predicting a team's actual winning percentage ( $Y = W\%$ ) using  $X = RS-RA$  in baseball and  $(PS-PA)$  in the NBA and NFL. The first major result was Bill James' Pythagorean Formula of Baseball which states  $W\% = (RS)^2 / ((RS)^2 + (RA)^2)$ . Other researchers found the exponents 2.37 and 13.91 work for the NFL and NBA. Using linear regression I developed the Linear Formula for Baseball  $W\% = .000673*(RS - RA) + 1/2$ . Wanting to extend my linear formula to the other professional leagues led me to create a new General Linear Theorem. As a corollary to my Linear Theorem, new linear equations for the MLB, NBA, and NFL were established. I showed these three equations were accurate for the MLB since 1901 and for the other leagues after their mergers. Using prediction intervals from these linear equations, at any point in a season, we can identify under-performing and over-performing teams. Finally, my newest theorem shows, that after normalizing both the X and Y above, the slopes of the new linear equations for any league for any year are the correlation coefficients from the original linear equations for X and Y. The results of this new theorem are then used to examine which sports league has the most parity. (Received July 25, 2017)

1135-B1-90 **Diana S Cheng\*** (dcheng@towson.edu), 8000 York Road, Towson, MD 21286, and **Peter Coughlin**. *The Value of Placement: Athletes' Contributions to Figure Skating Teams*.

We demonstrate how the Shapley-Shubik and Banzhaf indices can be used to analyze contributions of athletes to their teams in figure skating team events. We illustrate this approach by analyzing the results from the 2014 Winter Olympic Games figure skating team event. We also discuss additional ways in which the numbers assigned by the equations from power indices can be used in the figure skating context. (Received July 25, 2017)

1135-B1-125 **Tetyana Berezovski\*** (tberezov@sju.edu), Saint Joseph's University, Department of Mathematics, 5600 City Ave, Philadelphia, PA 19131. *Defining SPORTHEMATICS: Characterizing Task-Design for Sports and Mathematics Education*.

SPORTHEMATICS is the study of the relationship between mathematics and sports. It refers to a broader cluster of ideas ranging from mathematics of scheduling tournaments to sports and mathematics education. The goal of SPORTHEMATICS is to contribute both to the understanding of sports and the understanding of mathematics, along with underlining the importance of the connections between the two. In this study, the author generalizes the existing body of research and focuses the discussion on sports and mathematics education. The

presented study is based on the author's six years of engagement in the development of sport based mathematical models, as well as the design of sports-related tasks appropriate to students ranging from the middle school to the undergraduate level. (Received July 29, 2017)

1135-B1-375 **Keri D'Angelo, Quyen Do, Jamie Kunzmann, Joshua Radack\*** (radackj@lafayette.edu) and **Trent Gaugler**. *Predicting Outcomes of College Football Games*. Preliminary report.

It is common to try to predict who will win a football game, and pro-football-reference.com produced a well-known model (called the PFR model) for predicting such outcomes for NFL games. Their model relies heavily on the point spread that sportsbooks use to balance the wagering on the game. Our focus will be on FBS college football games, and we seek to compare this PFR model to new logistic regression models built using data from 2008 to 2016. In addition to asking who will win the game, it is sometimes more interesting to ask if the favored team will win by a certain margin. We also use variants of our models to address these questions. We finally discuss applying these predicted probabilities for use with the Kelly Criterion for simultaneous wagering on outcomes of FBS games. (Received August 29, 2017)

1135-B1-409 **John Mayberry\*** (jmayberry@pacific.edu), 3601 Pacific Ave, Stockton, CA 95211. *What can a jump tell us about a pitcher?*

Force plate testing is commonly used by trainers to monitor athlete strength and health. For example, ground reaction forces during a counter-movement jump (CMJ) test can provide information about movement deficiencies which can in turn be addressed through prescribed exercise plans. In this talk, we will discuss two surprising links between CMJ testing and baseball pitcher wellness. First, we establish a link between CMJ and UCL strains which in worst case scenarios can lead to Tommy John Surgery. In particular, pitchers who rely too much or too little on impulse momentum compared to force production during CMJ are three times as likely to sustain elbow injuries compared to players with more balanced profiles. Second, we provide evidence of a correlation between CMJ measurements and Field Independent Pitching (FIP) in a sample of college pitchers. Combined, these results suggest that CMJ monitoring is a useful tool for tracking pitcher development and ability. (Received August 31, 2017)

1135-B1-562 **Andrew B Perry\*** (aperry@springfieldcollege.edu). *Statistical Evidence of Referee Bias in the NBA and NFL*.

In an effort to be transparent, major sports leagues such as the National Basketball Association (NBA) and the National Football League (NFL) are publishing lists of officials and referees who call their games. In some cases they are even publishing statistics for these referees, for example, the number of fouls called per game. Unfortunately, this data reveals some unpleasant truths. For example, some referees have an apparent bias toward the home team. Home teams win the games that these particular referees work at a highly improbable rate. Other statistical irregularities related to referees in basketball and football will be considered as well. For example, football penalties that are subjective and tricky to call, like "holding", are rarely called at the start or end of games. This irregularity is not in and of itself evidence of impropriety, but it does raise questions. (Received September 08, 2017)

1135-B1-621 **Eric Eager\*** (eeager@uwlax.edu). *Using Machine Learning to Classify Quality and Style of Play at the Quarterback Position*.

When it comes to winning football games, success at the quarterback position is the most highly correlated and predictive variable. As such, finding, evaluating, and sustaining a high-level passing attack is one of the most important tasks in all of pro sports. Using Pro Football Focus data, we determine aspects of a quarterback's throw profile that are the most stable season to season, as well as those that are most predictive of future performance. With a variety of machine learning techniques at our disposal, we use these insights to classify quarterback play, and these groups provide substantial information for both explanatory and predictive purposes for teams moving forward. (Received September 11, 2017)

1135-B1-627 **Tien Chih\*** (tien.chih@msubillings.edu), 1500 University Drive, Billings, MT 59101, and **Demitri J Plessas**. *A Search for Champion Boxers*.

In the world of boxing, fighters are judged by the quality of their opponents. This way of determining value is reminiscent of Google's PageRank algorithm, which assigns values to websites based on the value of sites linked to it. We describe the mathematics behind PageRank and give a scheme to adapt it to boxing. We illustrate how this algorithm would behave in several scenarios. Then 20 top heavyweight boxers from the 1990's are selected

and ranked using this method. We also demonstrate show how Sage code may be easily implemented to execute this algorithm and finally, describe some possible extensions of this method. (Received September 11, 2017)

1135-B1-771 **Adam Childers\*** ([childers@roanoke.edu](mailto:childers@roanoke.edu)), 2521 Robin Hood Rd, Roanoke, VA 24014. *Serving up a Winner: Modeling Tennis Match Win Probability.*

Winning a tennis match depends on a player's ability to serve, return serve, and win points that enter open play. In this presentation, we will present a closed-form tennis match win probability model with parameters allow us to understand how each of these facets of the game interact and how they influence the outcome of a match. The model will allow us to explore distinctive styles of play and investigate the sensitivity of match win probability with respect to service winners and open play. Using player data from the ATP, we will use the model to determine the win probability for matches between prominent players. (Received September 14, 2017)

1135-B1-866 **Reza O. Abbasian\*** ([rabbasian@tlu.edu](mailto:rabbasian@tlu.edu)), TLU , Dept. of Math-CS-IS, 1000 W. Court St., Seguin, TX 78155, and **John T. Sieben** ([jsieben@tlu.edu](mailto:jsieben@tlu.edu)), TLU, dept of Math-CS-IS, 1000 W. Court st., Seguin, TX. *Statistical Modeling of a Mercy Rule in College Football to Reduce Major Injuries: A Second Report.*

In this presentation, we continue our previous work where we utilized twelve years of data from over 10000 college football games to develop statistical models which can be used to approximate the probability of winning as a function of score difference at various stages of the game for the winning team. We have added the ranking difference as a second variable to produce a more accurate model. Using the model we can then determine the stopping time and score difference when there is a near-zero probability of a comeback. The proposed mercy rule will lead to significant shortening of certain games. Approximately 60% of major injuries in college sports are attributed to athletes playing football. The saved minutes and the probability distribution of major injuries in college football will help us to estimate the reduction in the number of catastrophic injuries. (Received September 15, 2017)

1135-B1-1034 **Sheldon H Jacobson\*** ([shj@illinois.edu](mailto:shj@illinois.edu)), 201 n. goodwin avenue MC258, Urbana, IL 61801-2302, and **Douglas M King** ([dmking@illinois.edu](mailto:dmking@illinois.edu)) and **Arash Khtaibi** ([khatibi2@illinois.edu](mailto:khatibi2@illinois.edu)). *Bits and Bytes in March Madness.*

There are 67 games (four in the First Four and 63 in the main bracket) that are played during the NCAA Division I Men's Basketball Championship Tournament (also known as March Madness). The structure of the main bracket allows one to represent the outcome of these games using a 63 bit string. Our research does not attempt to predict the outcome of each such game, but rather, uses this 63 bit representation to generate sets of brackets based on the historical patterns (since 1985) observed in these 63 bit strings. Since this is done without any knowledge of the teams in the tournament, most of the brackets in the sets are not particularly close to the actual outcome. However, numerical experiments suggest that, when the number of brackets in the set is comparable to the number of brackets submitted to the ESPN Bracket Challenge (approximately ten million brackets), these bit string generators often produce "bracket nuggets" that would rank among the ESPN Top 100 brackets, and sometimes produce brackets that would rank first among the ESPN Top 100 brackets. We reports results of this approach for several recent tournaments, and compare these "bracket nuggets" to the best scores reported in the ESPN Bracket Challenge. (Received September 18, 2017)

1135-B1-1085 **Juan Sanchez** and **Noah Weiss\***, 2001 Alford Park Drive, Kenosha, WI 53140. *Stayman and Four-Way Transfers in Contract Bridge.*

Contract bridge is a popular card game, and many interesting applications of probability and statistics are possible from its bidding and play-of-the-hand phase. In the bidding phase of the hand, the sequences of bidding act as a context-dependent language, and each bid can be natural or conventional. In this project, a card-dealing simulator was designed, and statistical tests were used to determine whether the Stayman and four-way transfer conventions usefully replace their natural bids of 2 clubs and 2 notrump respectively, following an opening of one notrump. (Received September 19, 2017)

1135-B1-1139 **Rachael Talbert\*** ([rtalbe1@students.towson.edu](mailto:rtalbe1@students.towson.edu)), **Diana Cheng** and **Tetyana Berezovski**. *Mathematical interpretations of figure skaters' blade tracings.*

We show a series of mathematical modelling problems about figure skaters' blade tracings. These problems were solved by undergraduate pre-service middle school mathematics teachers, who interpreted skating rink diagrams using their knowledge of circumferences of circles, expressions and equations, scaling, and rates. In an open-ended problem, the pre-service teachers were asked to choreograph their own skating step sequences and report on their mathematical properties. We report on some sample student work. (Received September 19, 2017)



1135-B1-1340 **Steve Bacinski\*** ([sbacinski@davenport.edu](mailto:sbacinski@davenport.edu)), Davenport University, 6191 Kraft Ave SE, Grand Rapids, MI 49512. *The Ins and Outs of the Elo Rating System.*

Originally designed for chess players, the Elo rating system has now been extended and adapted to many other sports including basketball, football, soccer, and tennis. This powerful and self-correcting tool allows us to make predictions for upcoming games/matches, and compare teams across eras. We will explore the "ins" (fundamental assumptions and driving formulas to build the model) and the "outs" (interpretations and results) of the Elo rating system for various leagues, and include applications to the classroom. (Received September 21, 2017)

1135-B1-1383 **Megan O Powell\*** ([mpowell@stfrancis.edu](mailto:mpowell@stfrancis.edu)), 510 N. Praire, Joliet, IL 60435, and **Renee Martin** ([reneeemartin1@stfrancis.edu](mailto:reneeemartin1@stfrancis.edu)), 103A West Edson, Poplar Grove, IL 61065. *Analyzing NFL Overtime.* Preliminary report.

In this talk, we consider the National Football League's rules for overtime. We use Markov chain models to represent sudden death, modified sudden death 15-minute overtime, the newly changed modified sudden death 10-minute overtime, a theoretical alternative modified sudden death where each team is required to possess the ball at least once. Through our model analysis, we find the average length of overtime and the probability of the team possessing the ball first during overtime winning the game. Furthermore, we predict how the new 10-minute overtime length may affect game outcomes. (Received September 21, 2017)

1135-B1-1667 **David J Hunter\*** ([dhunter@westmont.edu](mailto:dhunter@westmont.edu)). *Measuring Umpire Consistency.*

The availability of pitch-tracking data has led to increased scrutiny of Major League Baseball umpires. While many studies have attempted to rate umpires based on their conformity to the rule book strike zone, players and managers tend to accept deviations from this zone, provided that umpires establish consistent zones within a game. Using tools from computational geometry, we construct a new metric to assess the consistency of an umpire's ball and strike calls over the course of a game. This metric yields a consistency ranking of MLB umpires using pitch-tracking data on all ball and strike calls made during the 2017 MLB regular season. It can also be used to measure consistency of umpires under various conditions (e.g., for different pitch types, stadiums, and catchers). This talk will address the motivation for this metric, explore some of its applications and generalizations, and discuss its statistical properties. (Received September 24, 2017)

1135-B1-1784 **Douglas VanDerwerken** and **Franklin H. J. Kenter\*** ([kenter@usna.edu](mailto:kenter@usna.edu)). *A Generative Markov Model for Bowling Scores.*

We create a data-driven Markov model for generating 10-pin bowling scores from the Professional Bowlers Association. The model incorporates insights from the hot hand literature and makes use of Bayesian shrinkage. For realistic sample sizes, the proposed approach is superior to modeling via the empirical distribution. Investigation of player-specific model components allows for a richer comparison of players than is possible using raw game scores alone. (Received September 24, 2017)

1135-B1-1856 **Roland Minton\*** ([minton@roanoke.edu](mailto:minton@roanoke.edu)), 620 Beech Road, Salem, VA 24153. *Homers and Roadies in March Madness.* Preliminary report.

It is often stated that teams that play well on the road and play strong defense are the best bets to win championships. The Massey rating system is adapted to rate college basketball teams separately on offensive, defensive, home and away play during the regular season. The component ratings are then used to predict the postseason tournament. Road play and defense are not as important as typically stated, and there are differences between Division I and Division III play. (Received September 25, 2017)

1135-B1-2306 **J. Christopher Tweddle\*** ([ctweddle@govst.edu](mailto:ctweddle@govst.edu)). *Modeling and simulation of a bicycle race.* Preliminary report.

The sport of cycling includes a wide variety of racing situations and disciplines. Track races are held both indoors and out in velodromes on an oval track with banked corners; road races are held on flat, hilly, and mountainous courses on urban and rural paved streets; mountain bike and cyclocross races are held on dirt tracks that may include obstacles and hills. Disciplines include individual and team races, sprint and endurance distances, as well as single-day and multi-stage events. In this presentation, we will discuss the assumptions and features of the proposed model and present preliminary simulation results beginning with track events, such as individual and team sprint and pursuit races. We will then explore how the basic model may be adapted to more complicated racing scenarios, such as the road team time trial that includes varied terrain and wind. We conclude with thoughts on how the model may be extended to mass-start road race that may include nearly 200 riders. (Received September 25, 2017)

1135-B1-2402 **Robert Franzosa\*** ([franzosa@maine.edu](mailto:franzosa@maine.edu)), 220 Neville Hall, University of Maine, Orono, ME 04401. *What is a walk a week worth?*

The Baseball Simulator is a baseball simulation program developed by the author that replays Major League Baseball (MLB) games and seasons using team (not individual player) statistics. We introduce the program, demonstrating its accuracy recreating past MLB seasons. Then we show how—by modifying statistics for teams—we can answer questions like the following: How much better would a team perform if they had one more walk each week during the season? How much better would a team perform if they had one more home run per game? Furthermore, we present results from investigating some of these questions. (Received September 26, 2017)

1135-B1-2516 **Tyler Skorczewski\*** ([skorczewskit@uwstout.edu](mailto:skorczewskit@uwstout.edu)), 311 Jarvis Hall Science Wing, 410 10th Avenue E., Menomonie, WI 54651. *Exploring how students learn in youth archery.* Preliminary report.

The National Archery in the Schools Program provides archery education in physical education classes and competitive archery tournaments to thousands of students across the United States. In an NASP tournament student archers using standardized equipment shoot 30 arrows that each score between 0 and 10 (bullseye) giving a maximum tournament score of 300. Previous investigations of archer data show evidence that students are learning archery skills as measured by improving tournament scores over time. In this study we try to determine how the students are learning by comparing the fits of models with different learning assumptions. One model uses a discrete difference approach and assumes a constant rate of learning. Another model assumes learning happens in a series of ‘ah ha’ moments and takes a time-series approach. Results indicate that the time-series approach fits archer data better for most cases, but that there are benefits and drawbacks for each approach. This project is accessible to undergraduates. (Received September 26, 2017)

1135-B1-2718 **Thomas W Polaski\*** ([polaskit@winthrop.edu](mailto:polaskit@winthrop.edu)), Department of Mathematics, Winthrop University, Rock Hill, SC 29733. *On Bonobos and Baseball: an Application of David’s Score.*

In the 1980’s the statistician H. A. David developed a method for ranking the members of a social group in a dominance hierarchy. This method, called David’s score, takes into account the highly variable number of encounters between the animals competing for dominance and the relative strength of each animal’s opponents. David’s score has been used to model social structure in animals including bonobos, as well as to study the steepness of the dominance hierarchy in bonobo colonies. In this talk, the author will discuss David’s score and its connection to other ranking methods popular in sports analytics. The score will then be applied to Major League Baseball, where the number of encounters between teams is likewise highly variable and relative strength of opponents is included in many ranking systems. (Received September 26, 2017)

1135-B1-2963 **Marissa Koronkiewicz\*** ([marissalkoronkiewi@lewisu.edu](mailto:marissalkoronkiewi@lewisu.edu)), **Carley Maupin** ([carleymaupin@lewisu.edu](mailto:carleymaupin@lewisu.edu)) and **Amanda Harsy** ([harsyram@lewisu.edu](mailto:harsyram@lewisu.edu)). *Predictive Modeling and Analysis of Golf and Softball Teams Using Linear Algebra.* Preliminary report.

Ranking sports teams and predicting post-season results from seasonal games can be challenging. Among the many mathematically inspired sports ranking systems, the Colley and Massey methods are relatively simple and can easily be introduced to undergraduate students who have taken a linear algebra course. At their most basic level, these methods are useful for sports rankings, but unfortunately, they are not particularly strong at predicting future outcomes of games. One way to possibly improve these methods for ranking and predicting future outcomes is by introducing weights to these systems and by using cross-validation to help determine the quality of our models. In this talk, we will discuss the results of two undergraduate student projects which tested the predictive power of weighted Colley Method and Massey Methods using data from softball and golf teams in the Great Lakes Valley Conference. (Received September 26, 2017)

## *The Scholarship of Teaching and Learning in Collegiate Mathematics*

1135-B5-71 **Lisa Bromberg\*** ([lisa.bromberg@usma.edu](mailto:lisa.bromberg@usma.edu)) and **Michael Seminelli** ([michael.seminelli@usma.edu](mailto:michael.seminelli@usma.edu)). *The Death of Paper: Should we use Digital Assessments in Undergraduate Mathematics?*

As technology continues to establish a more prominent role in the classroom, educators are turning to digital assessment tools to enhance the instruction of mathematics. The ease of grading and ability to provide the

student quick feedback makes this style of assessment appealing to the instructor. This study investigates the effect of digital versus paper quizzes on student performance in a first year collegiate mathematics course. A sample of 150 students from nine sections were randomly assigned an assessment format: paper, digital through WebAssign, or the option to select either form. Students also provided qualitative data by responding to surveys at the beginning and end of the semester answering questions on their preference for quiz format. The study shows no difference in student performance based on assessment format; however the qualitative responses are overwhelmingly in favor of paper quizzes. (Received July 19, 2017)

1135-B5-92 **Yevgeniya Rivers\*** ([ydrivers@newhaven.edu](mailto:ydrivers@newhaven.edu)), Department of Math & Physics, 300 Boston Post Rd, West Haven, CT 06515. *How is growth mindset and metacognitive self awareness linked in developmental math students?* Preliminary report.

Introductory level college math students have been studied with regard to metacognition and mindset assessment in separate settings but have only been minimally investigated as necessarily accompanying variables. In particular those investigations have focused primarily on K – 12. In one study in the community college setting, while there seemed to be no effects on pass rates, a mindset intervention in intermediate algebra courses resulted in lower dropout rates (6.8 percent) as compared to a control group (11.6 percent) (Kosovich 2014). Research into performance on international measures of achievement have shown that, “schools in other countries that score significantly lower . . . including England and the United States of America, frequently base their schooling practices upon ideas about ability that have now been shown to be incorrect.” (Boaler, 2013). This study - which was initiated in the Spring 2017 term with 558 students - was designed to investigate how developmental math students perceive their test performance in comparison to their actual performance and if this correlates with their mindset assessment profile. The outcome of the study will determine whether and what kind of a mindset intervention should be implemented. (Received July 25, 2017)

1135-B5-315 **Rachel M Bates\*** ([rachel.bates@redlandsc.edu](mailto:rachel.bates@redlandsc.edu)), 1300 S. Country Club RD, El Reno, OK 73036. *Computers and Constructivist Learning Environments: Pedagogy and Achievement in College-Level Mathematics.* Preliminary report.

To what extent can computers be used to help teachers create a constructivist learning environment in the post-secondary mathematics classroom? In this study, data from 46 post-secondary college algebra classes and 7 teachers were examined to determine the extent to which computers can alter pedagogy and student achievement. This study examined two groups of teachers: Experienced users of the new pedagogy and materials and beginning users of the new pedagogy and materials. Results suggest computers can positively alter both pedagogy and student achievement. Implications for using computers in the classroom are discussed. (Received August 23, 2017)

1135-B5-888 **Lauren Klamerus\*** ([laurenaklamerus@lewisu.edu](mailto:laurenaklamerus@lewisu.edu)), **Christina Carlson** ([christinacarlson@lewisu.edu](mailto:christinacarlson@lewisu.edu)) and **Amanda Harsy** ([harsyram@lewisu.edu](mailto:harsyram@lewisu.edu)). *Analyzing the Impact of Mastery-based Testing in Mathematics Courses.*

As educators, it is important to recognize that our assessment methods affect student attitudes. If we want students to learn from their mistakes and counteract a fixed-mindset of learning, perhaps we should look at what we incentivize in the classroom. One way that professors are attempting to counteract math anxiety, poor STEM retention, and a fixed-mindset of learning is through using and researching a new assessment model called “Mastery-based Testing” (MBT). In MBT, students are given problems in which they can only receive full credit for the problem after they demonstrate mastery of the concept being tested. Each test includes similar questions over the same concepts from previous tests which allows students who have not mastered an idea to retest and reevaluate old concepts. In this talk, we will present the results of qualitative and quantitative data from calculus, linear algebra, and real analysis MBT classes. We will also present the results from a two-year study comparing Mastery-based and traditional assessments in six Calculus II classes. (Received September 26, 2017)

1135-B5-1367 **Jen-Mei Chang\*** ([jen-mei.chang@csulb.edu](mailto:jen-mei.chang@csulb.edu)), Department of Mathematics and Statistics, 1250 Bellflower Blvd, Long Beach, CA 90840-1001, and **Joshua Chesler** ([josh.chesler@csulb.edu](mailto:josh.chesler@csulb.edu)), Department of Mathematics and Statistics, 1250 Bellflower Blvd, Long Beach, CA 90840-1001. *Developing a course in mathematical modeling for pre-service secondary teachers: challenges and opportunities.* Preliminary report.

In Spring 2017, California State University Long Beach offered the first iteration of a course in mathematical modeling for pre-service secondary teachers that was co-developed and co-taught by an applied mathematician and a mathematics educator. The development and teaching of the course was approached as a design experiment;

data included all course artifacts, instructor reflections, and student surveys. We report on preliminary findings and on emergent hypotheses that will guide the teaching and research of the next iteration of the course in Fall 2018. In particular, we will discuss (1) the role that students' quantitative and covariational reasoning played in shaping the course activity, (2) how early direct instruction about the modeling cycle may have hindered students, and (3) insights about developing pre-service teachers' pedagogical content knowledge of mathematical modeling. (Received September 21, 2017)

1135-B5-1392 **Kirthi Premadasa\*** ([kirthi.premadasa@uwc.edu](mailto:kirthi.premadasa@uwc.edu)), **Geetha Samaranyake** ([samarang@uww.edu](mailto:samarang@uww.edu)), **Rajee Amarasinghe** ([ramarasi@csufresno.edu](mailto:ramarasi@csufresno.edu)) and **Khyam Paneru** ([paneruk@uww.edu](mailto:paneruk@uww.edu)). *Does collaboration help create change in college teachers?*

Japanese Lesson Study has become one of the popular ways through which college mathematics professors get introduced to the discipline of SoTL. Lesson study essentially involves teachers collaboratively developing and teaching a lesson meant to bring out one or more broad objectives in students. Through lesson study teachers get an opportunity to have insight into student thinking. In our study, we asked the research question, "Does collaboration among college teachers help create teacher change?" We surveyed 27 college teachers who participated in lesson study projects in WI. The survey measured the change in the beliefs and attitudes of teachers before and after a lesson study as well as the effect of the lesson study experience several years later. Our study conclusively shows that teacher change occurs significantly after a college lesson study experience. (Received September 21, 2017)

1135-B5-1473 **Jenny G Fuselier** ([jfuselie@highpoint.edu](mailto:jfuselie@highpoint.edu)) and **Laurie Zack\*** ([lzack@highpoint.edu](mailto:lzack@highpoint.edu)). *Increasing Growth Mindset in Business Calculus Students*. Preliminary report.

We present preliminary results from a three-semester study of the effectiveness of growth mindset interventions on business calculus students at a liberal arts institution. Some students participated in short activities related to growth mindset on the first day of class. Other students attended evening learning labs during which course concepts and persistence were reinforced by working practice problems separate from the student's graded homework. Students were assessed via a pre and post growth mindset survey as well as by final exam score. (Received September 26, 2017)

1135-B5-1821 **Adam J. Castillo\*** ([adacasti@fiu.edu](mailto:adacasti@fiu.edu)), STEM Transformation Institute, VH 160D, Florida International University, 11200 SW 8th St., Miami, FL 33199. *Understanding How Two-Year College Math Faculty Perceptions of Cooperative Learning Influence Its Use in Math Courses*.

Cooperative learning, where students are actively working in groups, is a well-documented instructional strategy that promotes student learning and positive relationships among students. Yet, despite ample research on cooperative learning in the K-12 setting, there is little research on two-year college mathematics faculty perceptions and use of cooperative learning in mathematics courses. A mixed methods study, involving an online survey, interviews, and classroom observations, was conducted on two-year college mathematics faculty at Texas two-year colleges to understand: how do two-year college mathematics faculty perceptions of cooperative learning influence its use in college mathematics courses? Results will highlight how faculty characterize and implement cooperative learning in developmental and college-level mathematics courses. (Received September 25, 2017)

1135-B5-1918 **William O. Martin\*** ([william.martin@ndsu.edu](mailto:william.martin@ndsu.edu)), NDSU Department of Mathematics, Department 2750, PO Box 6050, Fargo, ND 58108-6050, and **Ben Duncan** ([benton.duncan@ndsu.edu](mailto:benton.duncan@ndsu.edu)), **Josef Dorfmeister** ([josef.dorfmeister@ndsu.edu](mailto:josef.dorfmeister@ndsu.edu)) and **Friedrich Littmann** ([friedrich.littmann@ndsu.edu](mailto:friedrich.littmann@ndsu.edu)). *A Systematic Departmental Effort to Improve Undergraduate Proof Capabilities*. Preliminary report.

How well is our undergraduate program promoting mathematics majors ability to read, understand, use and write proofs by the time they graduate? Each semester our department administers and analyzes student performance on assessments at three points in the program: Before and after our Introduction to Abstract Mathematics course and during the final capstone course for mathematics majors. The assessments are based on a framework proposed by Mejia-Ramos, Fuller, Weber, Rhoads, & Samkoff (2012). The framework allows faculty to monitor the development of student reasoning and proof comprehension during the undergraduate program. Our cyclic, iterative assessment model develops detailed insights about undergraduates' abilities to read, interpret, critique and write proofs. These insights are used to collaboratively develop, implement and revise pedagogy and curricula. Several goals of the project are to: \*Engage mathematicians in systematic reflection on the nature of student proof capabilities. \*Collaborate with mathematicians to devise and implement pedagogical

changes in the transitional undergraduate proof course that lead to measurable improvements in students' proof capabilities. (Received September 25, 2017)

1135-B5-2128 **Tharanga M. K. Wijetunge** ([tharanga.wijetunge@lyon.edu](mailto:tharanga.wijetunge@lyon.edu)) and **Kirthi Premadasa\*** ([kirthi.premadasa@uwc.edu](mailto:kirthi.premadasa@uwc.edu)). *How do students select questions in a math exam?* Preliminary report.

When instructors set math exams for their students, they do not necessarily follow an order in which the questions are presented in the exam. As a result, for example more difficult problems could be presented at the beginning of the exam and easier questions towards the end, sometimes causing students to run out of time when the students simply follows the order of the exam. In this light, it is important to find out whether there are any popular practices among students as to the order in which they typically answer questions in math exams. Do students simply answer the questions in the way they are presented or do they reflect on the questions prior to answering, and answer in the ascending order of difficulty? Is it also possible that students answer questions in a random order? Answers to these questions can be key in providing interventions towards improving student success in math exams. Our study asks the research question "How do students select questions in a math exam?" The study was carried out in several sections ranging from intermediate algebra to calculus at two liberal arts colleges in Wisconsin and Arkansas. We will present the results of this study as well as possible implications on these results on student success. (Received September 25, 2017)

1135-B5-2450 **Jim Fowler\*** ([fowler@math.osu.edu](mailto:fowler@math.osu.edu)) and **Bart Snapp** ([snapp.14@osu.edu](mailto:snapp.14@osu.edu)). *The Calculus Knowledge Assessment: an open-source instrument for measuring learning gains in calculus courses.*

The Calculus Knowledge Assessment (CKA) is a large bank of openly licensed multiple-choice calculus items created at Ohio State. There are now over 600,000 student responses to questions from the CKA bank and for most of these learners, there is matched face-to-face classroom data as part of an IRB-approved study. This talk presents data on how the CKA can be used to measure learning gains (e.g., in August 2016, a student submitted on average 35 correct answers during the pretest while at the end of the course, a student submitted on average 47 correct answers) and how certain CKA items are highly correlated with success on traditional in-class exams. The fact that the CKA is openly licensed and served via the open-source Ximera platform makes it easy for other schools to adopt the CKA. (Received September 26, 2017)

1135-B5-2502 **Deborah Hughes Hallett\*** ([dhh@math.arizona.edu](mailto:dhh@math.arizona.edu)), Department of Mathematics, University of Arizona, Tucson, AZ 85721. *Prerequisites: Past and Future.* Preliminary report.

Many colleges and universities are concerned about student attrition, often reflected in the high failure rates in remedial mathematics courses. One approach that shows promise is co-remediation, or just-in-time teaching that allows students who do not place into a course to take it by studying prerequisite material concurrently. In this talk we will talk about what can makes such an approach successful and what the dangers are. With examples from several courses (calculus, precalculus, algebra, statistics), we will look at principles and questions to consider in designing co-remediation. How should background material be designed? Where and when should it be introduced? What pedagogy is most effective? How much emphasis should be placed on testing? On study skills? We will also talk about how a department can manage and evaluate co-remediation efforts. (Received September 26, 2017)

1135-B5-2513 **Keri Kornelson\*** ([kkornelson@ou.edu](mailto:kkornelson@ou.edu)). *Course Innovation: A large-scale active learning program in pre-calculus and trigonometry.* Preliminary report.

Using internal funds for a major re-design of large enrollment courses, our team in the Math Department at the University of Oklahoma developed a pre-calculus and trigonometry course that is concentrated on peer-learning activities. This course is the immediate prerequisite for Calculus I, and the enrollment reaches over 1000 students per year. Features of our course include a cap of 36 students per section, class time devoted primarily to group work, and undergraduate Learning Assistants (LAs) who assist the instructor. The administration aided our efforts by scheduling our courses in classrooms with furniture and whiteboards that facilitate this student-centered pedagogy. Our new program also involves an intensive pre-term training session for the instructors, who are primarily graduate students.

The team had very little direct experience with active learning methods. Our questions were concentrated on assessment: "Does the OU active-learning pedagogy implementation impact the math attitudes, grades in current and future courses, and mindset of our students?" In this talk, we will discuss our answers based upon

evidence from student grades, pre- and post- surveys, and some interviews. Future studies will also address the impact on the graduate student instructors. (Received September 26, 2017)

1135-B5-2640 **Andrew G. Bennett** and **Ian Manly\*** ([imanly62@ksu.edu](mailto:imanly62@ksu.edu)), Kansas State University, Dept. of Mathematics, Cardwell Hall, Manhattan, KS 66506. *Bailout Pre-Calculus: An Approach to Improving Retention*.

One of the issues facing higher education, especially in STEM fields, is that of retention - what can we do to keep students studying STEM fields, or even just keep them in college? Kansas State University has introduced a "bailout" course for Calculus 1. After the first exam of the semester, students who performed poorly are given the opportunity to have a clean start in a new section of Pre-calculus, for no cost and with no record of withdrawal from Calculus. This has made an impact on retention as measured a year later. In this talk we will discuss the specific assignments and student work, including our analysis of which elements seem most critical to student success. (Received September 26, 2017)

1135-B5-2675 **Susan Ruff\*** ([ruff@mit.edu](mailto:ruff@mit.edu)) and **Lisa Emerson**. *Teaching Writing in Mathematics: What does it mean to communicate as a mathematician?* Preliminary report.

How can we help students learn to communicate as mathematicians? To answer this question we would like to know, more specifically,

- (1) What does it mean to communicate as a mathematician? What are mathematicians' communication purposes? audiences? practices? challenges?
- (2) How did current research mathematicians learn to write mathematics? Do they feel these teaching and learning strategies were effective?
- (3) What are mathematicians' attitudes and beliefs about the role of writing in mathematics? The answers to this question will enable future research to test whether student adoption of expert-like attitudes and beliefs about writing correlates with student success in writing mathematics.

To answer the questions above, we are collecting interview and questionnaire data from established and emerging mathematicians. Interview responses will yield rich literacy narratives that demystify for students what it means to communicate as a mathematician; while analyses of the interviews and responses from the more widely distributed questionnaires will provide complementary quantitative data about mathematicians' attitudes, beliefs, and experiences of writing mathematics. Preliminary results will be presented. (Received September 26, 2017)

1135-B5-2741 **Greg Mayer\*** ([greg.mayer@gatech.edu](mailto:greg.mayer@gatech.edu)) and **Asad Abbas** ([asad.97.abbas@gmail.com](mailto:asad.97.abbas@gmail.com)). *Supporting Teaching Assistants Facilitating Instruction in a Blended Synchronous Learning Environment*. Preliminary report.

This presentation describes the results of an ongoing project aimed at characterizing the teaching practices of TAs who facilitate instruction in a blended learning environment for a linear algebra course. Using an exploratory, mixed methods sequential study design, we collected teaching observation and interview data from TAs over two semesters to gain insight into their teaching methods and perspectives on teaching in blended settings. Observation data was collected using a modified version of the Classroom Observation Protocol for Undergraduate STEM (COPUS) tool. Drawing connections to Self-Determination Theory, our preliminary study findings offer some evidence that providing an adequate level of structure, community, and autonomy for our teaching assistants helped them achieve success in their instructional roles. (Received September 26, 2017)

1135-B5-2751 **Girija Sarada Nair-Hart\*** ([nairhaga@uc.edu](mailto:nairhaga@uc.edu)), 3585, Applewood Drive, Amelia, OH 45102. *Inquiries into Student Understanding and Insights into Student Misunderstanding*.

Students' (mis) understandings can be better explained in the context of mismatch between student 'concept images' and formal concept definition. Until a balance is maintained between these two constructs student understanding of a mathematical concept remains askew. A research was conducted to investigate how non math majors developed their mental models of various calculus concepts. Qualitative research methodology of teaching experiment was employed to collect data during the three-part research that consisted of two pre and post problem solving interviews; and a series of teaching episodes in the middle.

Data was collected based on pre and post problem solving interviews and a series of teaching episodes with small numbers of students. Students' oral and written artifacts were collected during the problem solving interviews. Students explained their thinking process and elaborated on the rationale for the pursuit of their selected problem solving techniques. Problem solution was studied and video-taped conversations and think alouds were explored. During the presentation students' concept images of asymptotes and limits as discovered

during this research and the factors that influenced such concept images will be discussed. (Received September 26, 2017)

1135-B5-2830 **Ben Hartlage, David Higginbotham, Ian Pierce\*** ([ian.pierce@usafa.edu](mailto:ian.pierce@usafa.edu)) and **Beth Schaubroeck**. *Grit and Diligence as Predictors of Success in Mathematics Courses*. Preliminary report.

All students at the United States Air Force Academy (USAFA) must complete a two-course sequence in calculus; for many students this is a significant hurdle that they must clear at the beginning of their course of study. Previous investigations at USAFA examined what factors predict success in our lowest-level mathematics course, Algebra and Trigonometry, and Calculus I. Academic measures such as scores on the mathematics placement test and incoming ACT/SAT scores did not do well at predicting success in either of these courses. Anecdotal evidence indicated that student success was much more determined by non-academic factors. We are currently investigating the contributions of the personality traits of Grit and Diligence (Duckworth et al 2007; Galla et al 2014) to academic success. Are either Grit or Diligence useful predictors for student retention at USAFA (do the students complete their first, second, and fourth semesters at USAFA)? Are Grit and Diligence useful predictors for student success in mathematics courses and other courses (such as Physics) that build upon these mathematics courses? We have now gathered and are analyzing our first year's data with the aim of applying Latent Path Structural Equation Modeling in the hope of answering these questions. (Received September 26, 2017)

1135-B5-2930 **Alison G. Lynch\*** ([alynch@csusb.edu](mailto:alynch@csusb.edu)), Seaside, CA 93955. *Peer Problem Review in Calculus I*.

In large sections of Calculus, it can be challenging to provide frequent and detailed feedback on student work. One possible solution is to have students provide peer feedback on each other's work. However, since the students are themselves just learning the material, the feedback they provide may be incomplete or incorrect. In this study, I investigated the impact on student learning of a structured Peer Problem Review activity in Calculus I, in which each student completed a problem individually, then reviewed a peer's solution using a detailed rubric. After participating in this activity, students scored higher on relevant quiz problems than their peers in other sections. Students also reported that the activity helped them to learn from their own mistakes and gave them a better idea of what was needed for a complete solution. Evidence from student work and student feedback will be presented. (Received September 26, 2017)

1135-B5-2984 **Brett C Smith\*** ([brett.c.smith@yale.edu](mailto:brett.c.smith@yale.edu)), 109 Church St., unit 302, New Haven, CT 06510, and **James S Rolf**. *Improving Preparation for Calculus with Gaming*. Preliminary report.

Success in calculus depends on a strong understanding of both the equations and graphs that represent functions. We created a video game called *Cartes* to strengthen students' intuition for the relationship between these representations.

We will report on *Cartes*' impact on student motivation and learning in an integrated pre-calculus/calculus course. Students in this course engage in single- and multi-player gameplay both in- and outside of class. We will report on data from pre/post concept tests as well as pre/post surveys on student attitudes. In particular, we will look at how immediate feedback and peer instruction involved in gameplay impact motivation and learning. (Received September 26, 2017)

## *Innovative and Effective Ways to Teach Linear Algebra*

1135-C1-306 **Jennifer R. Galovich\*** ([jgalovich@csbsju.edu](mailto:jgalovich@csbsju.edu)). *Di-Eigenals*. Preliminary report.

What happens to the diagonals of the unit square under a linear transformation? This is the story of a conjecture that failed; what happened next involved using all our "eigentools" and proved to be an effective way to summarize the "eigenstuff". Some answers emerged, but there are lots of questions left to explore. (Received August 22, 2017)

1135-C1-503 **Naima Naheed\*** ([naheedn@benedict.edu](mailto:naheedn@benedict.edu)), Naima Naheed, 1600 Harden St., Columbia, SC 29204. *Linear Algebra in Digital World*.

The aim of this presentation is to show the application of Linear algebra in Computer Graphics. Everyone is familiar with the tremendous popularity of animated movies such as Toy Story or Shrek. Similarly 3D computer games has acquired a large place in everyone's mind. It looks like 3D entertainment sounds like more fun than

studying a linear algebra book. As mathematicians, we have to remind our students that because of linear algebra those movies and games are brought to TV or computer Screen. When we see a character move on the screen, it is animated using some equation straight out of Linear Algebra. In this sense, Linear Algebra is a driving force of our new digital world. In my presentation, I want to share with you how I introduce local and global coordinates, their conversion and importance. Then Bezier curves comes next. Bezier curves were first developed by automobile designers to describe the shape of exterior car panels. It is a smooth curve with only few points to reduce the storage requirements. Application of Linear Algebra on these curves makes it more amenable to computerization. (Received September 18, 2017)

1135-C1-557 **Cheyne J Miller\*** ([cmiller5@sjcny.edu](mailto:cmiller5@sjcny.edu)), Dept Of Mathematics and Computer Science, St. Joseph's College, 155 W. Roe Blvd., Patchogue, NY 11772. *From Linear Algebra to Cech Cohomology in one Undergraduate Semester*. Preliminary report.

This paper will discuss how an undergraduate linear algebra syllabus can comfortably fit *Cech Cohomology* calculations and subsequent discussions surrounding modern applied topology. The author's experiences are based off of two semesters of successful implementation at a liberal arts college. We will assume that by the end of the semester, the student knows how to compute the rank of a matrix and understands how to apply the *Rank-Nullity Theorem*. With about three additional hours of lecture time, we will explain how the student can then efficiently and enthusiastically learn how to compute the *Cech Cohomology* of various manifolds. While this computation is a great way for students to connect many different ideas and techniques they've learned throughout the semester, it also has a nice blend of geometry and algebra that was appreciated by the students. Moreover, this foray into applied topology allows the instructor to end the semester with a peek at *Persistent Homology*, a modern data analysis technique which would theoretically use these same calculations. (Received September 08, 2017)

1135-C1-1020 **David M McClendon\*** ([mcclemd2@ferris.edu](mailto:mcclemd2@ferris.edu)), Department of Mathematics, ASC 2021, Big Rapids, MI 49307. *A thematic linear algebra course focused on four problems of the form  $T(x) = b$* . Preliminary report.

Past assessment from our linear algebra course at Ferris State University reveals that our students were very competent at performing linear algebra computations (row reductions, matrix multiplication, etc.) but weak at demonstrating understanding of linear algebra terminology and theory.

To address this issue, I developed a linear algebra course centered around four "key" problems: a traditional system of linear equations, an indefinite integral, a linear differential equation, a problem asking for an equation of a curve which best fits some data. Throughout the semester, the terminology and theory of linear algebra is developed as a mechanism to observe similarities between these problems. In this talk, I'll discuss more about the structure of this course, and reveal some of its successes. (Received September 18, 2017)

1135-C1-1445 **Gilbert Adam** ([a.gilbert1@snhu.edu](mailto:a.gilbert1@snhu.edu)), 2500 N River Rd., Manchester, NH 03106, **William Jamieson\*** ([w.jamieson@snhu.edu](mailto:w.jamieson@snhu.edu)), 2500 N River Rd, Manchester, NH 03106, **Megan Sawyer** ([megan.sawyer@snhu.edu](mailto:megan.sawyer@snhu.edu)), 2500 N River Rd, Manchester, NH 03106, and **Christina Starkey** ([c.starkey@snhu.edu](mailto:c.starkey@snhu.edu)), 2500 N River Rd, Manchester, NH 03106. *Exploring Linear Algebra through SageMath Labs*. Preliminary report.

In this talk we will discuss the creation and implementation of weekly computer-based labs for our linear algebra students using SageMathCloud, a free cloud-based mathematical computing environment. Some of the lab topics were the electronic game Lights Out, Hamming codes, image manipulation, and Google PageRank. The goals of this lab were to provide an opportunity for our students to become familiar with coding, and to explore interesting applications of linear algebra topics from class. The talk will include how we introduce our students to coding using SageMath, survey feedback from students about their interpretations of the labs, examples of the labs, and how we assess students' lab performance. (Received September 22, 2017)

1135-C1-1545 **Karsten Schmidt\***, Faculty of Business and Economics, Schmalkalden University of Applied Sciences, Blechhammer, 98574 Schmalkalden, Germany. *Teaching Matrix Algebra Using Technology – Do the Students' Attitudes Change with Time?*

In the Faculty of Business and Economics at Schmalkalden University, the matrix algebra course of the bachelor program has been taught in the PC lab for many years. A Computer Algebra System (CAS) is used throughout the course. Students can install the CAS on their private PCs, and have access to it during the final exam in the PC lab. Other courses, like Introduction to Mathematics, and Introduction to Statistics, are still taught in a traditional classroom setting. At the beginning of the 2010/11 winter semester, a survey was carried out to investigate whether the students preferred traditional or technology-based courses in mathematics, and how well



they coped with the technology. During the 2015/16 winter semester, a similar survey was carried out to check whether students' attitudes towards the use of technology in the teaching of mathematics have changed over time. In this presentation we will look at the key questions of the questionnaire, display descriptive statistics and charts of the variables generated from the gathered data, and analyze the effect that certain characteristics of the students (e.g. male vs. female, or students who like math vs. those who do not) have on their answers. The new results will be also compared to those found five years ago. (Received September 23, 2017)

1135-C1-1636 **Daniel E. Otero\*** ([otero@xavier.edu](mailto:otero@xavier.edu)). *Determining the Determinant: learning in the footsteps of Cramer and Cauchy.*

This talk will report on the development of a Primary Source Project by the author (under the aegis of the NSF-supported TRIUMPHS grant) for use in a standard linear algebra course to assist students in their understanding of the determinant, its computation and its fundamental properties. The design of the PSP has students read and work through works by Gabriel Cramer — in which he proposes his eponymous Rule — and by Augustin-Louis Cauchy, who, in his early twenties, laid out a fully developed theory of alternating symmetric polynomial functions, including a framework for the analysis of permutations on  $n$  objects and their signs, together with an early form of matrix multiplication, from which the fundamental properties of the determinant fall like ripe fruit from a tree. Time will be devoted to a discussion of the pedagogical effects of having undergraduate students “read the masters” as part of their initial introduction to mathematical ideas. (Received September 24, 2017)

1135-C1-1698 **Robert A Beezer\*** ([beezer@ups.edu](mailto:beezer@ups.edu)). *Teaching introductory linear algebra with open software and textbooks.*

Sage is open source software for mathematics and has research-level capabilities for both exact and numerical linear algebra, over a wide variety of fields and rings. It is also intuitive enough that a beginning student of linear algebra can use it profitably. When integrated into an openly licensed online textbook, it becomes a powerful tool for exploring basic concepts, while also gaining competency with a useful tool for computational approaches to many different areas of mathematics.

We will describe the use of Sage in all aspects of an introductory linear algebra class: integration with the online textbook, classroom presentations, student work, and examinations. This approach has been developed through ten iterations of an introductory course. (Received September 24, 2017)

1135-C1-1896 **Sarah E. Eichhorn\*** ([s.eichhorn@austin.utexas.edu](mailto:s.eichhorn@austin.utexas.edu)), **Petra Bonfert-Taylor**, **David Farmer** and **Jim Fowler**. *Linear Algebra using Curated Courses Open Educational Resources.* Preliminary report.

Curated Courses in Mathematics was developed under NSF-IUSE funding in order to provide high-quality instructional resources to instructors and students. These resources are reviewed and tagged to facilitate discovery and use. We will discuss how the Curated Courses materials can be used to implement a variety of evidence-based teaching practices in a Linear Algebra course. (Received September 25, 2017)

1135-C1-2086 **David Strong\*** ([david.strong@pepperdine.edu](mailto:david.strong@pepperdine.edu)), Pepperdine University, Natural Science Division, 24255 Pacific Coast Highway, Malibu, CA 91214. *Meaning and context in teaching linear algebra.* Preliminary report.

In a linear algebra course we often present and develop a new concept without much motivation, real-life meaning or context. Textbooks typically do a good job with the “what” and the “how,” but not as well with the “why.” Often it isn’t until subsequent sections that students begin to understand the importance and use of the ideas learned in the previous sections. While this is sometimes the inherent nature of mathematics, it doesn’t usually have to be this way. Instead, we (textbook authors and course instructors) have a golden opportunity to simultaneously motivate the need for the ideas and motivate the students to want to learn about those ideas.

I will discuss how we can better address the “why” through relevant and thought-provoking examples to better motivate the need for the ideas taught in the course and to simultaneously pique the interest of the student. I will also talk about how we can give better context and meaning to ideas to enrich student learning. (Received September 25, 2017)

1135-C1-2120 **James D. Factor\*** ([james.factor@alverno.edu](mailto:james.factor@alverno.edu)), Alverno College, Milwaukee, WI 53234. *Visualization of each Step and the Solution of Gauss-Jordan Elimination using GeoGebra.* Preliminary report.

This presentation will use an interactive GeoGebra applet to show the visual geometric changes for each step of the Gaussian–Jordan elimination process as the solution unfolds in achieving reduced row echelon form. The cases of one solution, no solution, and infinite solutions (linear and planar) will be illustrated dynamically.

This applet, along with others, is freely available for instructor demonstration and student discovery of the meaning of linear algebra concepts. Associated activities have been designed to guide students in using the interactive applets to enhance learning. This work is part of the NSF project entitled Transforming Linear Algebra Education with GeoGebra Applets (NSF TUES Grant DUE-1141045). Additional information about the project developed applets is available at the MAA/NSF Poster Session. (Received September 25, 2017)

1135-C1-2312 **Mary Flagg\*** ([flaggm@stthom.edu](mailto:flaggm@stthom.edu)), University of St. Thomas, 3800 Montrose, Houston, TX 77006. *Solving a system of linear equations using ancient Chinese methods.*

Gaussian elimination was invented by Carl Friedrich Gauss in the nineteenth century, or was it? The 2000 year old classic Chinese textbook, *The Nine Chapters on the Mathematical Art*, contains a whole chapter on solving systems of linear equations using the Fangcheng Rule. The Fangcheng Rule is equivalent to Gaussian elimination! I created a curriculum module (a Primary Source Project or PSP) designed to be used in an active learning environment that introduces elimination by guiding students through the English translation of the original Chinese text then introduces the modern method. In this talk I will introduce my PSP, describe the results of testing my PSP in linear algebra classrooms in the fall of 2017, and explain how participants may also try my PSP. My project is part of a larger collaborative effort partially supported by the NSF, Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS). (Received September 25, 2017)

1135-C1-2432 **Mile Krajcevski\*** ([mile@mail.usf.edu](mailto:mile@mail.usf.edu)), Department of Mathematics & Statistics, University of South Florida, CMC342, 4202 East Fowler Av., Tampa, FL 33620-5700. *Investigating drawing as a cognitive strategy in undergraduate linear algebra course.* Preliminary report.

Gradual changes in the undergraduate curriculum in the last decade have brought linear algebra courses to the forefront of undergraduate curriculum. For the first time in their undergraduate mathematical education, students encounter a notion of an algebraic structure on a set of objects, mappings that preserve this structure, and multiple ways of representing the elements of this structure. Each of the approaches in the teaching of linear algebra carries its own challenges, and different pedagogical models have been developed to support students' learning processes. We propose a model that will continue with the practice of using manipulatives in learning of mathematical concepts, utilizing drawing as activity that will produce objects that can be manipulated. Illustrations of mathematical objects help students concretize variety of concepts like vectors, eigenvectors, or linear transformations. Students will have an opportunity to draw vectors, subspaces or represent linear transformations in a synthetic environment, that is, without a preferred coordinate system. We will present some of the pedagogical materials we've used with this approach, together with the results and respond from students. (Received September 26, 2017)

1135-C1-2489 **Sepideh Stewart\*** ([sepidehstewart@ou.edu](mailto:sepidehstewart@ou.edu)), 601 Elm Ave, Norman, OK 73019. *Moving between the Three Worlds of Mathematical Thinking in Linear Algebra.* Preliminary report.

Linear algebra consists of many languages and representations. Instructors often move between these languages and modes fluently and expect students to follow along. In reality, many students do not have the cognitive framework to perform the move that is available to the experts. In this talk, employing Tall's three-world model, I will present a set of linear algebra tasks that are designed to encourage students to move between the embodied, symbolic and formal worlds of mathematical thinking. We anticipate that creating opportunities to move between the worlds of mathematical thinking will encourage students to think in multiple modes and in the long term broaden their mathematical knowledge. Some preliminary data illustrating students' abilities in moving between the worlds will be presented. (Received September 26, 2017)

1135-C1-2838 **Michelle L Ghrist\*** ([ghrist@gonzaga.edu](mailto:ghrist@gonzaga.edu)), Department of Mathematics, 502 E. Boone Avenue MSC 2615, Spokane, WA 99258. *Exploring Subspaces and Bases through Magic Squares.*

Here, I present more details on a project in which students explore the concepts of subspaces and bases through the lens of magic squares, expanding on an idea first mentioned in my talk at the 2015 Joint Mathematics Meetings (which was originally inspired by a problem from Strang's book *Linear Algebra and Its Applications*). In addition, I also discuss how some these ideas generalize to larger magic squares. One key new question: Can one find a relationship between the dimension of the subspace and the size of the magic square? (Received September 26, 2017)

1135-C1-2898 **Chon In Luk and Jeffrey Yeh\***, jeffreyyeh@cpp.edu, and **Lyheng Phey, Luis Cervantes, John Kath and Tanner Thomas**. *Powers of Matrices and Exponential Matrices*. Preliminary report.

We are interested in finding computationally efficient and accurate ways to find powers of an  $n \times n$  matrix,  $M$ , and exponential matrices,  $\exp(M)$ , under the assumptions that  $M$  has  $n$ -distinct known real eigenvalues and when  $n > 10$ . We compare the traditional method of diagonalization to a Cayley-Hamilton, Vandermode matrix approach. As applications, we consider powers of one-step transition probability matrices,  $P$ , representing certain Markov chains and the matrix of transition probability functions,  $\exp(Q_t)$ , corresponding to Markov processes where  $Q$  is an  $n \times n$  constant rate matrix and  $t$  is time. (Received September 26, 2017)

1135-C1-3072 **Muhammad Qadeer Haider\*** (mqh14@my.fsu.edu), 1114 W Call St., Stone Building, School of Teacher Education, Tallahassee, FL 32306. *Development and Validation of an Assessment for Introductory Linear Algebra Courses*.

The central goal of the study is to elaborate the development and validation process of an assessment instrument which is sensitive to students' ways of reasoning and understanding of introductory linear algebra concepts. This study is a part of a broader NSF project which was designed to support college instructors for inquiry-oriented teaching. We developed a test to gauge students' conceptual understanding of four focal topics: span and linear independence, systems of linear equations, linear transformations, and eigenvalues and eigenvectors. Instructors mostly cover the four topics in introductory linear algebra courses. The linear algebra assessment is a paper-pencil based test and was given as a post-test to 500 students in 18 linear algebra classes at 15 institutions. The assessment carries nine questions which are a combination of multiple-choice and open-ended questions. The content validity of the test items was established through expert validation, and preliminary analysis of a subset of the data showed that the entire test and the individual test items are reliable. Linear algebra instructors can use the assessment to measure their students' conceptual understanding of linear algebra concepts and can identify the concepts which are vexatious for their students. (Received September 26, 2017)

## *The Teaching and Learning of Undergraduate Ordinary Differential Equations*

1135-C5-153 **Karoline Hood\*** (karoline.hood@usma.edu) and **Paul Goethals**. *Inspiring Engineers to Study Ordinary Differential Equations with Open-Ended Modeling Problems*.

Mathematics is the language of engineering. In order to solve engineering problems, students must learn this language in sufficient depth and breadth. Almost all engineers are required to take an engineering math course where they learn this breadth of knowledge to include a heavy focus on ordinary differential equations (ODEs). We examined the implementation of engineering projects in an undergraduate mathematics course, specifically designed to present open-ended problems related to a student's major. The problems, which utilize second-order ODEs in their formulation, include modeling beams oscillating due to earthquakes, the development of shock absorbers used by trucking companies, and measuring the electrical vibrations of a radio. Students were required to conduct their own research, use facts and assumptions to make their model as realistic as possible, and present their research to the instructor at the end of the course. One major focus of this project is the assessment of the student's ability to take his or her knowledge on ODEs, apply it to a model, and effectively communicate the model through visualization tools. Our talk will discuss the different projects, student output and performance, as well as the effect of open-ended applications-based work in the classroom. (Received August 04, 2017)

1135-C5-280 **Stephen H Sapertone\*** (sap@gmu.edu), Department of Mathematical Sciences, George Mason University, Fairfax, VA 22030. *Web based lectures for ODEs with Interactivity*. Preliminary report.

ODEweb (for Windows and OS X) is a browser-based interactive digital textbook for undergraduates in ordinary differential equations in mathematics, science, engineering, and economics. The material consists of 27 lectures plus corresponding exercises. Applications are interspersed and integrated throughout the lectures. A unique feature of ODEweb is a dynamic component not found in printed books. This feature is implemented in two ways: (1) a nonlinear layout and (2) interacts.

(1) Nonlinear layout - composed of hyperlinks and notelinks. Hyperlinks take the reader back (or forward) to referenced material. Notelinks ("pop-ups") contain material such as proofs and calculations that can be skipped on first reading so as not to interrupt the flow of text.

(2) Interacts - based on Mathematica CDF files embedded in the textual material. They demonstrate a variety of dynamically controlled “what if’s”, variable slope and direction fields, and parameter dependence of solutions to ODEs. Visualization tools provide a 3d framework for interpretation of implicit solutions to first-order ODEs and a 3d representation of solutions to 2nd order (or 2d-systems) with their phase plane projections. (Received August 18, 2017)

1135-C5-425 **James Walsh\*** ([jawalsh@oberlin.edu](mailto:jawalsh@oberlin.edu)). *What happens when you periodically force a nonlinear oscillator?*

Periodic forcing of a harmonic oscillator is a fundamental topic in the sophomore-level ODEs course. Interesting and important behaviors such as beats and resonance are known to occur in this setting. Due to the linear nature of the spring’s restoring force, however, more intricate behaviors such as chaotic trajectories cannot occur. What if the oscillator is nonlinear? I will present a simple periodically force nonlinear oscillator model designed by an undergraduate at Oberlin. As an analysis of the model in the presence of damping appeared previously in the *CODEE Journal*, I will focus on the undamped case. The use of *Mathematica* yields great insight into model behavior, illustrating the coexistence of stable and chaotic motions for certain parameter regimes, as well as the persistence of invariant curves and quasiperiodic motion for the associated Poincaré map for small amplitude forcing. The model would fit nicely in an applied mathematics or modeling course in which the study of ODEs incorporates a dynamical systems perspective. (Received September 01, 2017)

1135-C5-903 **Jakob Kotas\*** ([kotas@up.edu](mailto:kotas@up.edu)), University of Portland, MSC60, 5000 N Willamette Blvd, Portland, OR 97203. *Standards-based grading for ordinary differential equations.*

Standards-based grading, in which students are given several opportunities to demonstrate mastery of key concepts over the course of the academic term, was attempted in two 25-student sections of an introductory-level ordinary differential equations class. This talk gives a retrospective on the implementation and results. A comparison with a traditional grading approach from the previous year is presented. (Received September 16, 2017)

1135-C5-1438 **Michael A. Karls\*** ([mkarls@bsu.edu](mailto:mkarls@bsu.edu)), Department of Mathematical Sciences, Ball State University, Muncie, IN 47306. *Verifying One-Dimensional Groundwater Flow with Incomplete Data.*

In 2009 I began a series of student research projects\* aimed at validating classic groundwater flow models that involve the heat equation. We will look at the following problem, based on the data collected for one of these projects: *Suppose you have collected head level data measured at three water wells in a row, but are unsure of the fixed head levels at boundaries to the left and right of these wells. Find a model for this data.* We offer two ways to approach this problem which lead to models that produce an excellent match to the data.

\*Portions of this work were supported by an Indiana Space Grant Consortium Grant. (Received September 22, 2017)

1135-C5-1504 **Jean Marie Linhart\*** ([jmlinhart@cwu.edu](mailto:jmlinhart@cwu.edu)), Department of Mathematics, Central Washington University, 400 E. University Way, Ellensburg, WA 98926. *Differential Equations and the United States Census Data.*

The United States Census data is a compelling data set for modeling with differential equations. If you introduce this data set at the beginning of the term, many students will suggest modeling it with the exponential function, from which the instructor can introduce the exponential differential equation and its assumption that the per-unit population growth is constant. The exponential model is easily criticized for growing without bound, and it is an easy conceptual leap to suggest a model that levels off after some time, leading to a steady-state population. Now the students can derive the logistic differential equation from the exponential differential equation; it has per-unit population growth that go to zero as the population approaches its steady-state. If this is done at the beginning of the term, this is now a good jumping off point for introducing the method of separation of variables. This topic can also introduce slope fields and equilibria. (Received September 22, 2017)

1135-C5-1579 **Timothy A Lucas\***, [timothy.lucas@pepperdine.edu](mailto:timothy.lucas@pepperdine.edu). *Slopes: An Interactive App for Exploring Differential Equations.*

A proper study of differential equations requires that students visualize solutions and analyze slope fields and phase planes. Although there is software that performs these functions (Maple, MATLAB, or Mathematica), these programs are expensive and students must invest a significant amount of time to learn the functions and proper syntax. *Slopes* is an interactive mobile app developed by faculty and students at Pepperdine University that allows users to explore numerical methods and graphical solutions to differential equations. The name of the app originates from interpreting the derivative as slope and most activities revolve around plotting slopes. One

advantage of using the app is that iPhones and iPads are highly portable and feature larger touch screens that allow students to view and manipulate content easily. Research based on observations of mathematics courses at Pepperdine University has shown that students are more willing to collaborate and share their results when using tablets such as the iPad (Fisher, Lucas et al. 2013). The intuitive interface of *Slopes* invites students to fully immerse themselves in the world of differential equations so that they can understand the concepts from not only algebraic, but also graphical and numerical perspectives. (Received September 23, 2017)

1135-C5-1668 **Justin Dunmyre\*** (jrdunmyre@frostburg.edu), **Nicholas Fortune**, **Karen Keene** and **Chris Rasmussen**. *An algorithm founded in intuition: Guiding students to reinvent Euler's Method.*

We present an inquiry-oriented sequence of tasks that elicit student intuitions about slope fields; the instructor can leverage those intuitions into class discussion. The students then solidify the geometric interpretation of the technique via a GeoGebra applet that requires them to manually move and lock vectors into place to generate the numerical solution. Finally, the students develop a tabular method of finding the numerical solution, and formalize their method as a reinvention of Euler's method. We leverage student thinking that the initial value of  $\frac{dy}{dt}$  gives the change in the  $y$ -variable per unit change in the  $t$  variable as a foundational stepping stone to the algorithm. This leads to a natural understanding of the use of smaller step sizes. The end result is that, for our students, Euler's method is no longer a dry algorithm, yet, it is a fundamental and intuitive tool for understanding differential equations. (Received September 24, 2017)

1135-C5-1812 **Alexander M. Gofen\*** (gallex@ski.org), 333 Fell St. #218, San Francisco, CA 94102. *Teaching ODEs with dynamics.* Preliminary report.

This is about a teaching potential of unique software for integration, plotting, and exploring ODEs. an all-in-one graphics package for Windows called the Taylor Center [1]. It's a powerful tool with unique features both for experiments in research and for teaching various aspects of ODEs. Dynamics are crucial while teaching applied ODEs.

This software plots solutions in a real time animation, giving the viewer not only the final shape of the trajectory as a still image, but also the dynamical evolution of the curve in real time, as a trajectory of a bullet (available both for planar trajectories in 2D and also in 3D viewed via a pair of cheap anaglyph (red/blue) glasses). Moreover, a versatile Phase Portrait Designer visualizes the image of the general solution of ODEs (see pictures in [1]).

Besides this straightforward purely graphical assistance in teaching, there are many examples how this package can be used for demonstration of more sophisticated properties of ODEs [2]. This package is preloaded with various illustrative examples in classical ODEs. However it is possible to develop sets of samples supporting nearly every textbook on ODEs, making such textbooks more attractive and motivating.

1. [taylorcenter.org/Gofen/TaylorMethod.htm](http://taylorcenter.org/Gofen/TaylorMethod.htm)
2. [taylorcenter.org/Gofen/Teaching](http://taylorcenter.org/Gofen/Teaching) (Received September 25, 2017)

1135-C5-1839 **Chris Oehrlein\*** (cdoehrlein@gmail.com) and **Jessica Oehrlein**. *First and Second-Order Models of Vertical Motion of Dry Air Parcels.*

Using basic thermodynamic principles as the foundation, Atmospheric Physics contains many relationships that Introductory Differential Equations students can model. Assuming no exchange of heat as a dry air parcel rises leads to a separable equation relating the parcel's temperature to its pressure. Comparing the atmosphere's natural change in temperature due to altitude to a dry air parcel's same rate of change leads to a second order linear equation modeling the change in vertical position of the air parcel. Collaborating with a graduate student in Applied Math and Atmospheric Science, a differential equations instructor now has a couple of homework problems that are very unique when compared with applications in standard Differential Equations textbooks. (Received September 25, 2017)

1135-C5-2146 **P. P. Yu\***, 501 Westminster Ave., Fulton, MO 65251. *What happens when a physicist teaches Ordinary Differential Equations?* Preliminary report.

As a mathematical relativist with formal training in physics and engineering, I will discuss in this talk some pedagogical approaches I have implemented in several distinct undergraduate courses on Ordinary Differential Equations (ODEs). Specific topics include (1) Use planar systems as a vehicle to teach ODEs simultaneously with linear algebra inherently embedded therein. I will lay out a pathway through the text by Hirsch, Smale, and Devaney I experimented when teaching ODEs without linear algebra as a prerequisite. In particular, I will underscore how concrete examples from classical and quantum mechanics as well as feedback control design can be naturally incorporated in a reverse-engineering strategy to help reduce the barrier of the eigenvalue problem.

(2) Emphasize the role symmetry plays in ODEs through a geometric (re)interpretation of systems of ODEs using low-dimensional Lie algebras of vector fields. The cost of having to deal with partial derivatives proved to be minimal; however, the real challenge seemed to consist in establishing a tangible feel in students about flows and brackets. I will share my efforts of introducing the most quintessential components of the Lie theory in a first course on ODEs to math majors with knowledge of Calculus III and Linear Algebra. (Received September 25, 2017)

1135-C5-2163 **Paul D. Olson\*** ([pdo2@psu.edu](mailto:pdo2@psu.edu)), Penn State Erie, the Behrend College, School of Science, 4205 College Drive, Erie, PA 16563-0203. *Laplace Transforms vs. The Method of Undetermined Coefficients.*

In undergraduate applied differential equations courses (the first course in ODE's for some engineering majors), we consider several methods of solving second order, linear, nonhomogeneous ODE's. After several years of teaching these courses, the results are in: most engineering majors prefer using Laplace Transforms over using the Method of Undetermined Coefficients. In this talk, we explore reasons why the engineering students prefer using Laplace Transforms. (Received September 25, 2017)

1135-C5-2281 **William Skerbitz\*** ([william.skerbitz@wayzata.k12.mn.us](mailto:william.skerbitz@wayzata.k12.mn.us)), Wayzata High School, 4955 Peony Lane N, Plymouth, MN 55441. *Strategic use of technology and modeling to motivate, investigate, and illuminate.*

The continual development of open-sourced and freely available applets, mathlets, mobile device apps, software, and internet-based platforms has greatly expanded our technology toolboxes for exploring and teaching differential equations. At the same time, there is continually growing interest in a modeling-first approach to teaching ODE. To successfully incorporate and interweave both modeling and technology into our courses in a manner that is motivating, illuminating, and engaging for students might require us to make difficult choices about topic coverage and to come clean about what it is we truly value in our typical ODE curricula. (Received September 26, 2017)

1135-C5-2357 **Paul E. Seeburger\*** ([pseeburger@monroecc.edu](mailto:pseeburger@monroecc.edu)), 1000 E. Henrietta Rd., Rochester, NY 14623. *Visualizing Topics from Differential Equations Using CalcPlot3D.* Preliminary report.

A demonstration of how to use CalcPlot3D to visually verify and explore various topics in differential equations. Visually verify solutions of first order differential equations by varying the integration parameter in the general solution to show that it always fits the corresponding direction field. Explore the solutions of systems of two or three differential equations in their corresponding 2D and 3D phase plots. Consider the effect of a time parameter in non-autonomous systems.

CalcPlot3D is a freely available interactive online JavaScript app designed to enhance the teaching and learning of multivariable calculus, differential equations, and linear algebra.

In addition to the creation of CalcPlot3D, our project seeks to create a series of guided visual concept explorations to improve student understanding of multivariable calculus, differential equations and linear algebra and to use CalcPlot3D and these explorations to conduct research investigating how student understanding of these concepts changes through the use of visualization and dynamic concept explorations.

3D glasses will be provided for viewing 3D vector field phase portraits.

See <http://web.monroecc.edu/calcNSF/>. This project is funded by NSF-IUSE 1524968, NSF-IUSE 1523786, and NSF-IUSE 1525216. (Received September 26, 2017)

1135-C5-2815 **Andrew G. Bennett\*** ([bennett@ksu.edu](mailto:bennett@ksu.edu)), Kansas State University, Department of Mathematics, Cardwell Hall, Manhattan, KS 66506. *Deriving Kepler's Laws in a Differential Equations class.*

In a large differential equations class, students were asked to choose between several possible extension topics. The result was a landslide in favor of deriving Kepler's laws from Newton's laws. Many students expected to learn how Newton's laws lead to Kepler's laws in Calculus, but this topic is often skipped. That expectation not having been met in Calculus only increases students interest in this topic. While usually included in multivariable calculus books, this derivation is more comprehensible when approached using Differential Equations (with some material from Calculus 2 as well of course). (Received September 26, 2017)

1135-C5-3010 **Douglas B. Meade\*** ([meade@math.sc.edu](mailto:meade@math.sc.edu)), Department of Mathematics, University of South Carolina, Columbia, SC 29208. *Using Dynamic Visualization to Better Understand the Tractrix and Other "Pulling" Curves.*

The tractrix is a classic problem that is well understood from many perspectives. In this talk we revisit the classical tractrix problem as an initial value problem in phase space, and note that this perspective does not provide any information about how the curve would be traversed in time. We present different parameterizations, and discuss their physical interpretations (or lack thereof). We also present a vector-based model for the tractrix from which we can pursue related problems. A common theme found in many of these formulations is the mathematical benefits of parameterizations with respect to arclength (in one form or another). These examples also exhibit benefits of dynamic visualization in modern investigations of classical problems. (Received September 26, 2017)

1135-C5-3181 **William W Hackborn\*** ([hackborn@ualberta.ca](mailto:hackborn@ualberta.ca)), University of Alberta, Augustana Campus, Camrose, Alberta T4V2R3, Canada. *Resisted Projectile Motion: a Trove of ODE Applications/Projects.*

Problems involving a body moving vertically up or down, subject to air resistance of various kinds, are standard in ODE courses. Even so, such problems are still a source of novel results, worthy of student research projects. A source of more challenging applications/projects involves a body launched at an oblique angle: if the resistance on the body is proportional to its speed, the ODEs for the body's motion can be solved quite easily, but interesting questions can still be asked and answered. This talk, however, will focus on a body projected obliquely and subject to resistance varying (more realistically) as the square of its speed. What makes this case a trove of engaging projects is the fact that the solution can (it seems) be expressed only using intractable integrals, and so one must resort to qualitative approaches or approximations based on simplifying assumptions (such as a small launch angle, or weak resistance, for which a regular perturbation approach is fruitful) to analyze the solution. Leonhard Euler himself tackled this ballistics problem while in the service of Frederick the Great and described the solution qualitatively in addition to devising a numerical technique used on the battlefield as recently as World War II. (Received September 27, 2017)

## *Trends in Undergraduate Mathematical Biology Education*

1135-D1-629 **Carrie Diaz Eaton\***, Unity College, 90 Quaker Hill Rd, Unity, ME 04988, and **Emma Perry**, Unity College, 90 Quaker Hill Rd, Unity, ME 04988. *Translating Marine to Math.* Preliminary report.

SUMMIT-P is an NSF IUUSE grant which brings together interdisciplinary collaborations at ten institutions. At Unity College, this work focuses on collaboration between calculus and marine biology with an emphasis on sustainability science and studies. In this talk, we present a framework for models and modeling that works to unite mathematical and biological sciences. After a year of marine and math classroom observations and interdisciplinary coaching, we discuss language shift opportunities in the calculus classroom and modeling opportunities and language in the marine classroom. This is accompanied by a thematic analysis and preliminary bipartite network language analysis. Lastly, we explore opportunities to extend the models and modeling framework to engage multiple disciplines beyond marine biology. (Received September 11, 2017)

1135-D1-1242 **Elizabeth Stanhope\*** ([stanhope@clark.edu](mailto:stanhope@clark.edu)). *Development of a Biological Science Quantitative Reasoning Exam (BioSquare).*

National reports such as Vision and Change, Preparing Future Physicians, PCAST and BIO2010 assert that a student's quantitative preparation correlates with persistence and success in the life sciences. Unfortunately, among the students who take the ACT exam, only 43% achieve a score that indicates they have a 50% chance of earning a B or above in their first college-level math class. More disconcerting is that, of 12th grade students with high STEM interest, fully 45% have weak mathematical preparation as measured by the ACT. Here we describe the efforts of an HHMI funded Consortium to develop, pilot, and refine a 22-item instrument called the Biology Science Quantitative Reasoning Exam (Biosquare) that probes student skills with the quantitative topics deemed essential for undergraduate biology. We envision that the Biosquare will communicate to entering life science students areas of quantitative strength and weakness, provide data for faculty on what students know, and, at the programmatic level, highlight the quantitative skills biology instructors consider to be important. The Biosquare will offer empirical insight into the quantitative skills gap, informing efforts to bolster life science student success through strengthened quantitative preparation. (Received September 20, 2017)

1135-D1-1650 **Timothy D Comar\*** (tcomar@ben.edu). *The Dynamics of Pulse Vaccination Models for the Spread of Disease.*

This talk focuses on preparing students for research in developing and analyzing pulse vaccination epidemic models using impulsive differential equations. We discuss how students can be prepared for such studies either through course work or independent mentoring. Skills students need to develop include reading journal articles, programming in MATLAB, working with data, model development, and mathematical skills. We discuss how to manage a group of several undergraduate research students of varying levels of experience. We conclude by highlighting some research projects, including the development and analysis of an age-structured pulse vaccination model for HPV. In particular, we draw conclusions the impacts on the spread of HPV in the the population if some individuals receive less than the typical, full three-dose vaccination regimen (Received September 24, 2017)

1135-D1-1951 **Rebecca Sanft\*** (bsanft@unca.edu) and **Anne Walter.** *Exploring Mathematical Modeling in Biology Through Case Studies and Experimental Activities.* Preliminary report.

We describe a book project that is a stand-alone compendium of exercises, cases and wet labs designed to help mathematics and life-sciences college students integrate mathematical, computational and research approaches to addressing real problems. The book consists of four units plus supplementary materials for projects and technical notes for laboratory activities, and it is written from the perspectives of a biologist and a mathematician. Each unit begins with a biological background and motivation for the topic, and a mathematical and computational background with embedded R code examples, followed by three case studies across multiple scales and areas of biological inquiry, and a wet lab with opportunities for students to generate their own hypotheses and test them. Each case study and lab is motivated through a biological question, and then guides the students through the steps of model formulation using discrete-time models or differential equation models, parameter estimation, model validation, and analysis. Students see the utility of models for identifying further questions, predicting outcomes, and understanding complex systems. (Received September 25, 2017)

1135-D1-2235 **Hannah Callender Highlander\*** (highland@up.edu), 5000 N Willamette Blvd, MSC 60, Portland, OR 97203, and **Carrie Eaton.** *A “Rule-of-Five” Framework for Models and Modeling to Unify Mathematicians and Biologists and Improve Student Learning.*

Despite widespread calls for the incorporation of mathematical modeling into the undergraduate biology curriculum, there is lack of a common understanding around the definition of modeling, which inhibits progress. Here we extend the “Rule of Four,” initially used in calculus reform efforts, to a framework for models and modeling that is inclusive of varying disciplinary definitions of each. This unifying framework allows us to both build on strengths that each discipline and its students bring, but also identify gaps in modeling activities practiced by each discipline. We will also discuss benefits to student learning and interdisciplinary collaboration. (Received September 25, 2017)

1135-D1-2567 **Marcus Roper\*** (mroper@math.ucla.edu) and **Heather Dallas** (dallas@math.ucla.edu). *Modeling and simulation in the calculus classroom.*

Calculus teachers are under increasing pressure to teach modeling and simulation skills in introductory calculus courses. These skills can enrich the study of calculus, by connecting it to real world applications that go far beyond ‘word problems’. They are also highly useful for students taking calculus into follow on courses in the life sciences. But teaching these skills in a way that doesn’t overburden students or teachers is huge challenge. Here we will describe some methods for introducing modeling and simulation into calculus courses, while maintaining a focus on teaching the rudiments of calculus, and allowing the teacher to introduce as little or as much computer programming or specialized software as they want to. (Received September 26, 2017)

1135-D1-2706 **Eric J Kostelich\*** (kostelich@asu.edu), School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287-1804. *REU projects on mathematical biology at Arizona State University.*

This talk will survey some projects that undergraduates have attempted over the past 10 years under the auspices of the NSF Computational Science Training for Undergraduates in the Mathematical Sciences and Mentoring through Critical Transition Points initiatives. These programs supported 8-week, full-time research experiences for students with two to three years of previous undergraduate work. The projects involve simulations of social insect networks (using data from ant colonies); dynamics of hormone therapy for prostate cancer; simulations of brain tumor growth using data from a laboratory experiment involving murine glioma; and a model of tumor vasculature growth and collapse. Some of these examples have been incorporated into an undergraduate topics course called Mathematics and Cancer. (Received September 26, 2017)



1135-D1-2929 **Alan Garfinkel\*** ([agarfinkel@mednet.ucla.edu](mailto:agarfinkel@mednet.ucla.edu)). *Teaching Modeling and Dynamics to Freshman Biology Students.*

We designed a class to teach modeling, using ordinary differential equations, to freshmen biology students. We designed a 2-quarter class that had

- multi-variable nonlinear differential equations, as models for biological and physiological processes
- numerical integration via Euler’s method
- an introduction to bifurcation theory
- linear algebra including eigenvectors and eigenvalues

The key step is the 20th century concept of a **vector field**. A vector field is a rigorously defined notion that replaces the un-rigorous and unhelpful concept of a differential equation. A vector field is formally a cross-section of the tangent bundle of a differentiable manifold. Intuitively, a vector field is a function that assigns “change vectors” (elements of the tangent bundle) to points in state space. The state point, starting from an initial condition, everywhere follows the change vectors, a trajectory that is guaranteed to exist by theorem. We can approximate that trajectory to any desired degree, using Euler’s method. Our experience at UCLA is that **these 20th century geometric concepts also make excellent pedagogy, even at the freshman level, far superior to traditional calculus.** (Received September 26, 2017)

## *Inquiry-Based Teaching and Learning*

1135-D5-623 **Mitchel T. Keller\*** ([kellermt@wlu.edu](mailto:kellermt@wlu.edu)), Department of Mathematics, 204 W Washington St, Lexington, VA 24450, and **Oscar Levin** and **Kent E. Morrison**. *Open Educational Resources and Inquiry-Based Learning: Bogart’s Combinatorics through Guided Discovery Gets New Life.*

Prior to his death, Ken Bogart was developing an IBL text (*Combinatorics through Guided Discovery*) to be used in an upper-division combinatorics course. Since he passed before the book could be published, his family released it under an open source license. Unfortunately, his L<sup>A</sup>T<sub>E</sub>X and PDF differed in a number of places, which limited the usability for those who wanted to make corrections or adaptations. Despite this, the book has been used and appears on the AIM list of endorsed open math books.

This year, we converted Bogart’s L<sup>A</sup>T<sub>E</sub>X code to PreTeXt, which allows simple conversion to (very nice) HTML and L<sup>A</sup>T<sub>E</sub>X. We updated the source to match Bogart’s PDF and prepared a print version. The PreTeXt source is on GitHub. PreTeXt HTML books offer features not possible in print that will appeal to IBL practitioners, and we plan to use those more heavily in future releases with community support.

This talk will briefly discuss some open source licenses (and why one should use them). We will also demonstrate some features of the HTML version of *CtGD*. The talk will be of interest to those who seek to share their IBL materials, use materials developed by others, easily develop interactive HTML-based IBL materials, or collaboratively develop materials. (Received September 11, 2017)

1135-D5-667 **Keith M Jones\*** ([keith.jones@oneonta.edu](mailto:keith.jones@oneonta.edu)) and **Toke Lindegaard Knudsen**. *Engaging students with the Pell equation through inquiry with primary historical sources.* Preliminary report.

TRIUMPHS (Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources) is an NSF-funded project to develop inquiry-based modules for student engagement in mathematical topics by working with and interpreting primary historical sources. As a part of this project, we have created a module, “The Pell Equation in India”, which guides students through source translations of two of medieval India’s most important mathematicians: Brahmagupta and Bhāskara II. This module begins with an exploration of some basic properties of the Pell equation,  $Nx^2 + 1 = y^2$ , moves through crucial tools developed by Brahmagupta for understanding this equation, and concludes with Bhāskara’s “Cyclic Method” for solving the Pell equation. The student discovery process is guided by tasks which ask students to explore calculations, engage with the primary source, and make, test, and prove conjectures. (Received September 12, 2017)

1135-D5-731 **Diana Davis\*** ([ddavis3@swarthmore.edu](mailto:ddavis3@swarthmore.edu)), 500 College Avenue, Swarthmore, PA 19081. *A “Harkness” discussion-based, problem-centered first-year honors calculus course.* Preliminary report.

We describe a case study of a problem-solving section, using the “Harkness” discussion method, of an honors multivariable calculus course. Students in the problem-solving section had equivalent outcomes on exams, reported higher ratings in self-assessments of skills, and took more math classes in the following year, compared

to students in the lecture-based sections. This talk will include examples, graphs, and lots of pictures. (Received September 13, 2017)

1135-D5-904 **Christine von Renesse\*** ([cvonrenesse@westfield.ma.edu](mailto:cvonrenesse@westfield.ma.edu)) and **Volker Ecke** ([vecke@westfield.ma.edu](mailto:vecke@westfield.ma.edu)). *Engaging Faculty using Inquiry.*

How can we best support faculty in learning how to incorporate new teaching techniques? If people learn best by “actively doing” then what should learning opportunities for faculty look like?

During this talk faculty will have the opportunity to explore their own learning edge in teaching mathematics. We will use our reflections to think about the implications for facilitating workshops for faculty. We claim that workshops can engage participants in “actively doing teaching in community” as a way to empower faculty to try new methods in their classrooms. (Received September 16, 2017)

1135-D5-1019 **Gabriella Harris\***, Towson University, Department of Mathematics, 8000 York Rd., Towson, MD 21252, and **Kimberly Corum**. *Using Multivariable Mathematical Modeling Activities to Facilitate Inquiry-Based Teaching and Learning: The Derivation of Ampere’s Law.*

Mathematical modeling is the process of using mathematics as a way to understand and solve real-world problems. Modeling activities can be used to facilitate inquiry-based learning (IBL), as they provide an environment for students to engage in authentic mathematical practices. One such modeling activity is the derivation of Ampere’s Law. Ampere’s Law ( $B = \mu \frac{N}{L} I$ ) relates the strength of the magnetic field generated by a solenoid ( $B$ ) to the number of coils of wire ( $N$ ), the length of the solenoid ( $L$ ), and electric current ( $I$ ). A solenoid is a coil of conductive wire; when electric current flows through the wire, the coil generates a magnetic field. Solenoids are an integral component in a number of historic and modern-day technologies. While Ampere’s Law is a multivariable mathematical model, it can be derived experimentally by systematically varying the different parameters of a solenoid. By sharing our findings from several successful implementations of the Deriving Ampere’s Law activity with students, ranging from rising eighth grade students to undergraduate mathematics and science majors, we hope that others will be inspired to develop similar activities to use in their own teaching. (Received September 19, 2017)

1135-D5-1093 **Angie Hodge\*** ([angie.hodge@nau.edu](mailto:angie.hodge@nau.edu)), **Xiao Xiao**, **Volker Ecke**, **Chuck Hayward**, **Philip Hotchkiss**, **Sandra Laursen**, **Chrissi von Renesse** and **Stan Yoshinobu**. *Inquiry-based learning workshops: Short workshops leading to longer workshops.*

As part of a five-year project to expand the professional development capacity in undergraduate mathematics, workshops on inquiry-based learning (deep engagement of students in rich mathematical tasks) are being offered to postsecondary mathematics instructors. This talk will describe the model being used in the four day intensive workshops as well as offerings in short workshops. In particular, this talk will focus on discussing the connection between a short workshop, which you could offer at a section meeting or in your department, and the four day intensive workshops. (Received September 19, 2017)

1135-D5-1182 **W. Ted Mahavier\*** ([ted.mahavier@lamar.edu](mailto:ted.mahavier@lamar.edu)). *JIBLM – yes, the IBL material you developed does count toward tenure!*

The {Journal of Inquiry-Based Learning in Mathematics} now publishes three distinct types of materials: {Modules}, {Course Notes} and {User Reviews}. We will address three topics: (1) how to prepare a submission to increase the odds of a smooth path to publication in JIBLM, (2) our personal experience with the refereeing process, and (3) the career benefits of publishing one’s IBL activities, course notes and reviews of existing JIBLM notes. (Received September 20, 2017)

1135-D5-1324 **Mairead K Greene\*** ([mairead.greene@rockhurst.edu](mailto:mairead.greene@rockhurst.edu)). *Creating an Inquiry-Based Experience Online.*

Over the past number of years I have given a great deal of thought to how to create a truly inquiry based experience in an online class. I have taught both Precalculus and Calculus I online using an inquiry-based approach and in this talk will discuss the features of these classes that I believe helped to create an inquiry-based experience for my students. I will provide examples of how I incorporated these features into my classes and discuss the tools that made this possible. Finally I will point to additional elements that I believe would improve the learning for my students and how I hope to incorporate these new elements going forward. (Received September 21, 2017)

1135-D5-1332 **Mark L. Daniels\*** ([mdaniels@math.utexas.edu](mailto:mdaniels@math.utexas.edu)), Professor Mark Daniels, 2515 Speedway - Stop C1200, Austin, TX 78717. *An IBL Freshman Research Initiative Course: IBL FRI - Symmetry*. Preliminary report.

We will discuss the creation and implementation of a special IBL discrete/intro to proof course for 2nd semester freshman. Students in this IBL course learn many of the concepts of a discrete mathematics using the topic of symmetry as a theme. In addition, students are expected to choose a research topic and learn the skills of researching that topic in mathematics journals by writing a summary report of findings in a professional manner. (Received September 21, 2017)

1135-D5-1605 **Michael G. Dabkowski\*** ([mdabkowsk@1tu.edu](mailto:mdabkowsk@1tu.edu)), 11403 Waverly, Plymouth, MI 48170. *Course Based Research Experience at the Sophomore Level*. Preliminary report.

Through a college wide effort to bring research into the undergraduate curriculum, the math department at Lawrence Tech is transitioning sophomore level mathematics courses into research-based experiences. Working in the genesis of this process to develop relevant projects germane to subject matter and respecting the course learning objects has proved challenging. In its current form, students are grouped into teams of four and are given three pages of basic theory of a more advanced topic along a "further direction" topic which they will present to the entire class. We will discuss the particular projects used in these courses and show clips of student presentations. (Received September 23, 2017)

1135-D5-1703 **Matthew Fischler\*** ([matthew.fischler@gmail.com](mailto:matthew.fischler@gmail.com)). *Question Formulation Technique: Fostering engagement through student-driven questioning*.

Mathematics educators wish to instill an enduring love and appreciation for the mathematics they teach. However, the combination of fragmented curricula and an over-emphasis on skills-based instruction has been stifling the curiosity required for students to make deep, long-lasting connections with mathematics.

After many years of experimenting with strategies to bolster student engagement, I have applied and developed a simple, effective method to help students have meaningful experiences with rich mathematical concepts. This strategy, known as Question Formulation Technique, or QFT, skirts much of the problem solving process and instead focuses more on problem posing.

Created by The Right Question Institute, QFT follows an intuitive process to problem creation, regardless of students' skill levels. After using QFT with several classes across different levels of mathematics, quantitative metrics have shown an improvement in students' engagement and perception of mathematics. (Received September 24, 2017)

1135-D5-1714 **Monika Keindl\*** ([monika.keindl@nau.edu](mailto:monika.keindl@nau.edu)). *Building problem solving skills from existing mathematical knowledge*.

There is a common misconception about mathematics among prospective math majors and minors. Many of them equate procedural competency with general mathematical skills. A problem solving course was developed to remedy this misconception. This new course is designed as a possible prerequisite for a first course in proof writing. The focus is on mathematical reasoning and communication through exposure to multi-step problems that only require elementary mathematical knowledge not much beyond precalculus. This talk will explore experiences and challenges in teaching this course using IBL approaches. (Received September 24, 2017)

1135-D5-1746 **Ju Zhou\*** ([zhou@kutztown.edu](mailto:zhou@kutztown.edu)). *Application of Inquiry Based Learning in Mathematics of Finance*.

Mathematics of Finance is a course designed for students who are interested in applying Mathematics in Finance field. In particular, this course will prepare students who are interest in pursuing an actuarial career for the Professional exam FM/2. The author will share her experience in applying Inquiry Based Learning in teaching this course to strengthen students' understanding of the basic financial concepts. (Received September 26, 2017)

1135-D5-1785 **Mona Mocanasu\*** ([mmocanas@msudenver.edu](mailto:mmocanas@msudenver.edu)). *Is it still called a textbook if it is for an IBL course?*

I have been using IBL methods in my Abstract Algebra courses for several years, but my students always ask for a course textbook. These two statements are logically contradictory – as a traditional undergraduate textbook, even if used just as a reference text, defies the IBL teaching method. The solution seems straightforward – simply write a textbook for my students and my own teaching style. This talk presents the journey of transforming daily class handouts and many-times revised sets of problems into a complete course textbook. In particular, I document the hard choice of when and how to include standard algebraic definitions, notations and examples

into what is, basically, a list of problems. This presentation concludes with my students' reactions to using this new document instead of a traditional textbook. (Received September 24, 2017)

1135-D5-1808     **Jaewoo Lee\*** ([jaelee@bmcc.cuny.edu](mailto:jaelee@bmcc.cuny.edu)), Department of Mathematics, Borough of Manhattan Community College, 199 Chambers Street, New York, NY 10007. *What to ask next, as students explore Egyptian Fractions.*

Egyptian Fractions can be used as an introductory subject for undergraduate students to the research in number theory. Students often find it surprising to see a simple idea can lead to so many different meaningful (and sometimes hard) questions. This talk will share how to lead students to think about what to ask next, using the results they obtain at the early stages and studying certain patterns. (Received September 24, 2017)

1135-D5-1866     **David M Clark\*** ([cclarkd@newpaltz.edu](mailto:cclarkd@newpaltz.edu)) and **Samrat S Pathania**. *Advanced Euclidean Geometry via IBL*. Preliminary report.

The speaker's text, *Euclidean Geometry: A Guided Inquiry Approach*, is written for a one semester undergraduate IBL course particularly designed to give preservice teachers a deeper understanding of the content they will teach. It gives an axiomatic development of the standard topics of synthetic plane geometry: congruence and similarity through transformations, area measure, angle measure, trigonometry and circles. Obviously it is not possible to do all of this in full Hilbert style detail in one semester. To accommodate this fact the development includes logical gaps and unstated assumptions that are carefully chosen to be transparent to undergraduate students.

This talk will report on a follow up IBL course that will fill in all of those gaps without any unstated assumptions. It is intended for advanced students who have completed the previous text. It is also intended for college and university instructors who would themselves like to have a deeper understanding of the content that they will teach. (Received September 25, 2017)

1135-D5-1867     **Phong Le\*** ([phong.le@goucher.edu](mailto:phong.le@goucher.edu)), Goucher College, Data Math and CS, JR 137, 1021 Dulaney Valley Road, Baltimore, MD 21204. *Data, Statistics and Inquiry*. Preliminary report.

Where does inquiry naturally live in a modern introductory level data and statistics course? Within the IBL community, there are many examples of proofs, calculus and geometry. Far fewer resources exist for data and statistics. Intro stats is quite different from intro calculus or linear algebra, both in student population and often, ideologically.

One place of inquiry in statistics lies within data itself. The inferential mindset in statistics lends itself to inquiry. Students given the opportunity to generate their own critiques and insights into quantitative studies and datasets form a lasting appreciation and understanding of the discipline. An example of an in-class inquiry exercise will be presented along with student examples and feedback. The author will also provide reflections on the state of inquiry and statistics. (Received September 25, 2017)

1135-D5-1956     **Ekaterina Lioutikova\*** ([elioutikova@usj.edu](mailto:elioutikova@usj.edu)), University of Saint Joseph, 1678 Asylum Ave., West Hartford, CT 06117. *Fostering engagement and inquiry in a linear algebra class*. Preliminary report.

In this talk, I will share my experience teaching a "hybrid IBL" linear algebra class, where a course previously taught in a more traditional way was infused with a variety of inquiry-based, active-learning activities. Students spent a significant portion of each class period collaborating on conceptually focused tasks and sharing their reasoning with peers. I will discuss the benefits and challenges of the inquiry-based approach from both the instructor's and students' perspectives. (Received September 25, 2017)

1135-D5-2005     **Stuart Boersma\*** ([stuart.boersma@cwu.edu](mailto:stuart.boersma@cwu.edu)) and **Frank Savina**. *Inquiry-based pathway to Calculus*. Preliminary report.

Reasoning with Functions is a new and innovative pathway to Calculus for STEM intending majors. Student engagement, inquiry, discovery, and exploration are at the heart of this innovative curriculum. Based in best-practices, Reasoning with Functions relies on the growing body of published evidence which supports an inquiry-based approach to student learning. Active learning, coupled with the intentional development of functions as processes, are integrated into daily lessons for students and supporting materials for instructors. The authors will discuss the design and the components of these lessons and share specific examples from the curriculum. (Received September 25, 2017)

1135-D5-2024 **Benjamin Wilson\*** ([bwilson4@stevenson.edu](mailto:bwilson4@stevenson.edu)), Stevenson University, 11200 Ted Herget Way, Owings Mills, MD 21117. *Active First Semester Calculus*.

We will examine inquiry-based learning methods used in teaching Calculus at Stevenson University. In this presentation we will focus on first semester Calculus. Specifically, we will discuss the resources we use, the overall framework of the course, the daily structure of class meetings, some of the challenges faced, and some of the benefits seen since beginning this approach in Fall 2016. We will also explore some of the changes that have been made to improve the course over the past three semesters and some ideas for further improvement. (Received September 25, 2017)

1135-D5-2173 **Janelle Krause Lorenzen\*** ([jlorenzen@southeastern.edu](mailto:jlorenzen@southeastern.edu)), SLU 10749, Hammond, LA 70402. *The Effect of Instructional Strategies on Preservice Elementary Teachers Math Anxiety and Achievement*.

This study addressed how different instructional strategies affected preservice elementary teachers' levels of math anxiety and their achievement in a math content course. The instructional strategies used were traditional teaching methods and inquiry-based learning (IBL). Participants completed the Mathematics Anxiety Rating Scale–Short Version (MARS-S) and completed a 20-item content knowledge assessment pre- and post-intervention. Participants' journal entries throughout the semester contained self-reported measures of math anxiety and understanding of course content as well as descriptions of their experiences in the course. Significant results showed that as the semester progressed, the math anxiety of IBL participants decreased, whereas the math anxiety of traditional participants increased. Differences between the groups in terms of their level of achievement were not significant even though within both groups, participants experienced significant learning gains. Statistical tests revealed that the IBL participants had significantly more positive opinions on their classroom experiences and preferences in mathematics classrooms. (Received September 25, 2017)

1135-D5-2200 **Gary A. Olson\*** ([gary.olson@ucdenver.edu](mailto:gary.olson@ucdenver.edu)). *College Algebra Reform and Redesign Utilizing Active Learning and a TA Coach*. Preliminary report.

In this session, I will share the key components and results of recent College Algebra reform efforts at the University of Colorado Denver. The reform and redesign centered on the use of a one credit hour Active Learning mathematics recitation to support student success. In addition, a TA Coach was assigned to help support and mentor new instructors in their transition to an active learning classroom. The impact of this reform effort on DFW rates and final exam performance will also be discussed. This work is sponsored by an NSF DUE grant #1539602. (Received September 25, 2017)

1135-D5-2321 **Elizabeth Thoren\*** ([ethoren@pepperdine.edu](mailto:ethoren@pepperdine.edu)). *Trust Your Liberal Arts Students with Proof*.

A key component of meaningful mathematical inquiry requires determining, as a community, what constitutes a convincing mathematical argument. In this talk I will discuss some challenges and strategies for supporting this work in a Mathematics for Liberal Arts course. (Received September 25, 2017)

1135-D5-2336 **Germaine Kamleu-Ndouma\*** ([3008010@myuwc.ac.za](mailto:3008010@myuwc.ac.za)), 3872 Saybrook court, Jarrettsville, MD 21084, **Lorna Holtman** ([lholtman@uwc.ac.za](mailto:lholtman@uwc.ac.za)), Robert Sobukwe Rd, Bellville, Cape Town, 7535, South Africa, Cape Town, 7535, South Africa, and **Bingwen Yan** ([yanb@cput.ac.za](mailto:yanb@cput.ac.za)), Symphony Way, Bellville, Cape Town, 7535, South Africa, Cape Town, South Africa. *Ordinal Regression to Analyze Postgraduate Students' Attitudes toward the Application of Statistical Procedures in the Western Cape Institutions, South Africa*.

The ordinal regression method was used to model the relationship between the different levels of students' ability regarding the overall learning experience in the application of statistical procedures in both University of Cape Town (UCT) and University of the Western Cape (UWC), and the explanatory variables concerning demographics, emotions, students' attitude and students learning environment in these institutions. The outcome variable for students' attitudes was measured on an ordered, categorical six-point Likert scale. The major decisions involved in the model building for ordinal regression were deciding which explanatory variables should be included in the model and choosing the link function (e.g. logit link, probit link, cauchit link, negative log-log link and complementary log-log link) that demonstrated the model appropriateness. In addition, the model fitting statistics, the accuracy of the classification results and the validity of the model assumptions, e.g., parallel lines, were essentially assessed for selecting the best model. One of our main finding is that suitable environment for development of student capabilities to learn the skills of solving real life problems are highly significant with the application of statistical procedures. (Received September 25, 2017)

1135-D5-2441 **Daniel Dobbs\*** ([dobbsd@trine.edu](mailto:dobbsd@trine.edu)), One University Avenue, Angola, IN 46703.  
*Experiment, conjecture and ...*

After spending the last few semesters dipping my toes in the IBL waters, I was finally brave enough to get my hair wet only to find that my audience was content tanning on the beach. In this talk I will present my recent attempt at implementing an inquiry-based learning curriculum in a linear algebra course with twenty eight engineering students and zero math majors. I will share successes, challenges and reflections on what could be done differently in a future iteration of the course, including how to replace the ellipsis in the title. (Received September 26, 2017)

1135-D5-2503 **Susan B Crook\*** ([susan.crook@loras.edu](mailto:susan.crook@loras.edu)). *Austism Spectrum Students in IBL Classrooms*. Preliminary report.

As the push to use inquiry-based learning pedagogy in college and K-12 classrooms gains momentum, our community must consider and address how this pedagogy affects students with learning differences. While there has been discussion and research about minority students and IBL, there has been less widespread discussion of students with learning disabilities. In my classes, I commonly have students with Autism Spectrum Disorders and would like to share my observations helping this subset of students be successful in a full or partial IBL college classroom. These are preliminary and anecdotal thoughts, but mathematics education resources will be cited when possible. (Received September 26, 2017)

1135-D5-2560 **Michael Starbird\*** ([starbird@mail.utexas.edu](mailto:starbird@mail.utexas.edu)), Department of Mathematics, RLM 8.100, The University of Texas at Austin, 2515 Speedway, Stop C1200, Austin, TX 78712.  
*Helping Underprepared Calculus Students to Learn to Think*. Preliminary report.

Students' specific mathematical deficits in their preparation for calculus might be viewed as symptoms of deeper thinking issues rather than being the core problems. This talk will report on a project at UT Austin that is attempting to systematically help students learn effective practices of mind while they learn both pre-calculus and calculus meaningfully. Inquiry methods are central to this effort. This talk will describe instructional methods and assignments that were used and their successes and failures. (Received September 26, 2017)

1135-D5-2566 **Ryan Gantner\***, St. John Fisher College, 3690 East Ave., Rochester, NY 14618, and **Erica Johnson**, St. John Fisher College, 3690 East Ave., Rochester, NY 14618. *It's the little differences*.

Two professors. One set of materials. And several differences. We made the decision to use presentation-based IBL and make our sections look fairly similar. As is the natural order of the universe, the classes had their differences and it was precisely those differences that have led to the richest discussions during and after the course. Here we present a brief tour of two professors' experiences using the same IBL materials (for the first time) in a first semester Calculus course. (Received September 26, 2017)

1135-D5-2596 **Mary A Nelson\*** ([mnelso15@gmu.edu](mailto:mnelso15@gmu.edu)), 4400 University Drive, Fairfax, VA 22030, and **Maddie Trieber**. *Oral Reviews: Leveling the Playing Field*.

Failure rates in introductory mathematics courses often result in students becoming discouraged and even the dropping their STEM major. The gatekeeper of most STEM majors is Calculus I, where the national failure rates have been about 40%. More troubling is the fact that many rural and inner city high schools are unable to provide high level mathematics courses, typically available to students in the suburbs. Hence some students begin STEM majors at a distinct disadvantage. Our study showed strong correlations between the use of orals and a leveling of the playing field for mathematically under-prepared students. Orals are voluntary, ungraded discussion-based review sessions that take place before written exams. A group of 5-6 students discuss important concepts with each other and a knowledgeable facilitator. They discuss concepts to be covered on upcoming written exams, and draw graphs and diagrams to clarify important ideas. Students gain a more conceptual understanding of course materials through the interactive feedback. The use of orals has been shown to be highly correlated to improved exam scores and to a decrease in DFW rates. At one university, for example, a ten year average failure rate for Calculus I of 31% was dropped to a five year average of 22%. (Received September 26, 2017)

1135-D5-2627 **Ali S Shaqlaih\*** ([ali.shaqlaih@untDallas.edu](mailto:ali.shaqlaih@untDallas.edu)), Dr. Ali Shaqlaih, University of North Texas at Dallas, 7400 University Hills BLVD, Dallas, TX 75241. *Inquiry Based Learning Integrated with Technology in a Geometry for Teachers Course*.

Inquiry based learning (IBL) is an efficient approach in teaching mathematics especially for pre-service teachers. Integrating IBL with technology makes the approach more applicable; with IBL integrated with technology, not only teachers will have a better conceptual understanding of the math content but they will more likely to use

it in their future schools. In this talk, a practical realistic implementation of the IBL integrated with technology in classroom settings will be presented. Students' engagement, class dynamics and evaluation methods will be discussed as well. (Received September 26, 2017)

1135-D5-2673 **Robert L. Sachs\*** (rsachs@gmu.edu). *The i road to upper-level mathematics.* Centering a "transitions" course around the complex number system, complex polynomials, and the basic complex transcendental functions has multiple advantages, particularly for student inquiry. Historically the complex numbers were both useful and in need of justification. Familiar properties of functions may or may not persist in the complex domain. Most topics in upper-level undergraduate mathematics are connected in part to this material. Several examples of student inquiry tasks will be given, but student work cannot be described yet as the author will be giving this course in the spring term. (Received September 26, 2017)

1135-D5-2795 **Judith Lynn Gieger\*** (lgieger@oglethorpe.edu), 4484 Peachtree Rd. NE, Atlanta, GA 30319. *What if there was no 2?* This presentation focuses on a classroom activity in which students are asked to explore properties of prime numbers in a natural number system which does not include the number "2". The setting is a liberal arts mathematics course, heavily (but not exclusively) populated by students who are either anxious or ambivalent about mathematics. Even those who have had prior success in their mathematical coursework are often uncertain about what it means to become a producer and not just a consumer of mathematics. Therefore, an activity that focuses on a relatively familiar and deceptively straightforward numerical concept provides a productive springboard for re-examining and questioning a core mathematical definition with a group of students of diverse mathematical abilities and inclinations. The presentation includes both a detailed description of the activity itself as well as student artifacts in the form of their proposal and justified results. (Received September 26, 2017)

1135-D5-2911 **Tara T. Craig\*** (tcraig@coastal.edu). *Transforming Future Math Teachers into Mathematicians.* Since the turn of the century, engagement in authentic learning experiences has been a major initiative for reforming school classrooms, but our future math teachers are often taught mathematics in teacher-centered, lecture halls. This lack of personal experience with student-centered instruction makes it difficult for a future educator to see the value of these best practices. In a course restricted to math majors interested in teaching, students' engaged in inquiry-based learning (IBL) explorations the duration of the semester with the goal of deepening and broadening their understanding of topics focused on in secondary mathematics. In this session, I will highlight some of the most impactful explorations from the course and share the effect IBL had on students' understanding of foundational mathematics topics, their ability to think more effectively, their general perceptions of math, and further their teaching philosophy. (Received September 26, 2017)

1135-D5-2914 **J D. Berg\*** (jberg5@fitchburgstate.edu). *Specifications grading in an IBL-ish Abstract Algebra course.* Preliminary report. Inquiry based learning instructional methods develop a sense of student ownership of learning. Specifications grading is an assessment system that relies on student exhibition of clearly stated learning objectives. This talk will discuss how the author merged these two methods into a standard fall term senior year abstract algebra course. The talk will include student responses from mid-term and end of term surveys. (Received September 26, 2017)

1135-D5-2995 **Rebekah B Johnson Yates\*** (rebekah.yates@houghton.edu). *Knowing What You Know.* One of the goals of using inquiry-based learning is getting students to take ownership of their own learning, but even in an IBL course, students do not always recognize what they know and what they don't know. In this talk, I'll share assignments I've used to help students in my inquiry-based courses reflect on their own learning. I'll also share some student responses to such assignments, which I have given in a math for liberal arts course, Calculus II, and Real Analysis I. (Received September 26, 2017)

1135-D5-3004 **Katherine Vance\*** (katherine.vance@simpson.edu). *Supervised in-class proving.* Many math departments offer an introduction to proofs course to prepare students for proof-based upper-level mathematics courses. Even after taking such a course, many students struggle to find their sea legs writing proofs in a new course. In this talk, I will discuss how I have used collaborative in-class proof-writing tasks in both a bridge to proofs course and upper-level pure math courses to help my students gain proficiency and confidence in their proof-writing skills. (Received September 26, 2017)

- 1135-D5-3029 **Katrina Morgan\*** (katri@live.unc.edu) and **Francesca Bernardi** (bernardi@live.unc.edu). *Inquiry-Based Learning Problem Sets in an Outreach Program for High School Girls: Increasing Confidence and Strengthening Interest Among Underrepresented Groups*. Preliminary report.

Inquiry-based learning is implemented in Girls Talk Math, a free summer day camp for high schoolers who identify as girls and are interested in math held at the University of North Carolina at Chapel Hill. IBL helps accomplish two program goals: increasing confidence and strengthening interest in taking college math courses. In IBL students feel ownership of their work, which increases confidence. IBL's emphasis on creative problem solving tends to be more engaging and interesting than the algorithmic approach often taken in high school courses. Increased engagement and interest is expected to improve the likelihood of participants pursuing further math. During camp, participants break into 8 groups of 4-5 campers to complete a lengthy problem set on a math topic usually not encountered until college or graduate school. Problem set topics include: Elliptic Curve Cryptography, Special Relativity, Scientific Computing, Fluid Dynamics, and Knot Theory. Team Leaders provide guidance, but campers decide how to spend their time and are responsible for their learning. Each group writes a blog post about the math they learn, allowing participants to assimilate new knowledge through reflection and communication. Analysis of pre- and post- survey data will be presented. (Received September 26, 2017)

- 1135-D5-3070 **Milos Savic\*** (savic@ou.edu), **Houssein El Turkey**, **Emily Cilli-Turner**, **Gail Tang**, **Gulden Karakok** and **Mohamed Omar**. *Exploring the Intersection of Fostering Mathematical Creativity and Inquiry Teaching*. Preliminary report.

Few mathematics education studies on creativity have been undertaken at the undergraduate level. In particular, investigations on pedagogical actions that foster creativity in undergraduate mathematics courses are rare. Coincidentally, there is a dearth of research on pedagogical actions that define inquiry-based learning. Our aim is to add to both fields by illuminating the pedagogical actions in which inquiry teaching could help foster creativity. We collected and transcribed Livescribe pen audio of five class sessions of an introduction-to-proofs course, interviewed seven students, and obtained instructor journals of preparation for and reflection on the course. A preliminary analysis utilizing six principles to maximize creativity (Sriraman, 2005; Moore-Russo & Demler, in press) along with four components of inquiry-oriented instruction (Kuster, Johnson, Keene, & Andrews-Larson, 2017), showed two of the six principles had a clear alignment with three of the components of inquiry-oriented instruction: generating reasoning, building on student thinking, and developing a shared understanding. We detail those two principles, and discuss the reasons for which the other four principles did not purely align. (Received September 26, 2017)

- 1135-D5-3088 **Jane H Long\*** (longjh@sfasu.edu), Department of Mathematics & Statistics, Stephen F. Austin State University, P.O. Box 13040, SFA Station, Nacogdoches, TX 75962-3040. *One Model for a Successful Capstone Course in Problem Solving*. Preliminary report.

The MAA's Committee on the Undergraduate Program in Mathematics recommends that majors in the mathematical sciences should work on "a substantial mathematical project that involves techniques and concepts beyond the typical content of a single course." Constraints including number of required course hours for students and advising time for faculty can make this important recommendation difficult to implement. We present one model for a successful one-hour capstone course that prompts students to synthesize the mathematical knowledge they have gathered through the exploration of simply stated but non-routine problems, with both quick and extended investigation. Examples of problems and administrative details of the course will be presented. (Received September 26, 2017)

- 1135-D5-3097 **Aaron Wangberg\*** (awangberg@winona.edu). *Inquiry in Multivariable Calculus using tangible surfaces*.

Raising Calculus to the Surface (NSF DUE 1246094) utilizes dry erasable tangible surfaces, measurement tools, and small group activities to let students explore and discover multivariable calculus concepts prior to formal lecture. By providing access to various representations of multivariable functions, students are able to seek answers to important questions, propose and discuss possible conjectures, and contribute their knowledge to the course even before knowing formal definitions. In this talk, we'll share the experiences of several instructors who used the materials and highlight how the project has helped them incorporate inquiry and student contributions into their classrooms. (Received September 26, 2017)



1135-D5-3138 **Kathleen M Clark\***, Florida State University, School of Teacher Education, 1114 West Call Street, Tallahassee, FL 32306-4459, and **Cihan Can** (cc15m@my.fsu.edu), Florida State University, School of Teacher Education, 1114 West Call Street, Tallahassee, FL 32306-4459. *The Possibility of Inquiry and its Role in the Pursuit of Personally Relevant Mathematics*. Preliminary report.

In Fall 2017 the course, “Personally Relevant Mathematics,” was offered at Florida State University for the first time. The course was designed to fulfill a Liberal Studies quantitative and logical thinking course requirement and in its first offering, included 40 students (representing some 20 different majors). During the course, students were asked to investigate a mathematical idea that is of interest to them (e.g., “personally relevant”) as a culminating Inquiry Project, while investigating other mathematical concepts and topics during class sessions. To model how to engage in inquiry – particularly in the pursuit of a problem, question, or topic for which an answer or solution is not readily obvious – we used the aspects of the “What-If-Not” strategy (Brown & Walter, 2005) in whole-class and small group sessions while working on mathematical explorations that addressed a broad range of mathematical topics. In this talk we present several examples of tasks used as class explorations, describe the various assignments designed to support students in their pursuit of their Inquiry Project, and share examples of students’ Inquiry Projects, as well as their perceptions about the course experience in particular and their view of mathematics in general. (Received September 26, 2017)

1135-D5-3170 **Karen Heavin\*** (karen.heavin@kysu.edu), Kentucky State University, 400 East Main, Frankfort, KY 40601, and **Fariba Bigdeli-Jahed** (fariba.bigdeli-jahed@kysu.edu), Kentucky State University, 400 East Main, Frankfort, KY 40601. *Inquiry Based Learning (IBL) and Culturally Responsive Teaching (CRT): Increasing Equity, Access, and Success in underrepresented populations through a combination of IBL and CRT*.

Cooperation, collaboration, and community are prominent and successful practices in minority student education (Gaye, 2000). Learning math involves a process of making sense of and connections to the content through personal understanding and meaningful cultural experiences. Math faculty at KSU developed a unique CRT/IBL math curriculum specifically designed to serve students attending the Historically Black University (HBCU) in Frankfort, Kentucky. Faculty, local secondary teachers, evaluators and a CRT specialist participated in a week long professional development workshop, July 2016. The resulting collaborative math curriculum/pedagogy was implemented during the 2016-2017 academic year. The relationship based focus of the CRT/IBL freshman level courses emphasized conceptual understanding as well as procedural skill development, critical thinking, persistence and productive struggle. Student passing rates, grades, ACT math subscores, fall to spring longitudinal success and affective measures were evaluated. The results of the study showed significant correlations between student success, self-determination and perceptions of the teacher and classroom environment. This session will share the successes and failures experienced while transitioning from training to practice. (Received September 26, 2017)

1135-D5-3185 **Shari Samuels\*** (ssamuels@lowercolumbia.edu). *The Evolution of Prospective Elementary Teachers’ Competencies: Procedural Knowledge, Mathematical Knowledge for Teaching, Attitudes, and Enactment of Mathematical Practices*.

The purpose of this research was to explore the evolution of prospective elementary teachers’ competencies. This was conducted as a case study of the first two of three inquiry-based mathematical content courses for elementary teachers. Both qualitative and quantitative data was collected from a cohort of students. Results showed an increase in prospective elementary teachers’ mathematical knowledge for teaching scores over time, but no change in their procedural knowledge or attitude scores. Overall, students grew in their ability to problem solve and construct viable arguments in mathematics while moving through the curriculum, with a few exceptions. Three factors contributed to students’ learning in the curriculum: the amount of effort made by the student, the atmosphere and attitudes of students in the class, and the nature of the content and questions asked in the curriculum. Another important consideration which arose from the data analysis was the opportunities the curriculum allowed for the practice of written versus verbal explanations, and what was formally assessed. Designers of teacher education programs using a similar curriculum should evaluate the importance of written versus verbal explanations in the goals of the course, and appropriately assess the students. (Received September 27, 2017)

1135-D5-3187 **Michael Sebastian Gagliardo\*** ([mgagliar@callutheran.edu](mailto:mgagliar@callutheran.edu)). *3D Printed Surfaces in an IBL Multivariable Calculus Course.*

In Fall of 2016, I adapted my IBL multivariable calculus course by including 3D printed manipulatives to strengthen student understanding of contour plots and quadratic surfaces. In this talk, I will discuss my motivation for using physical 3D printed surfaces and the activities students worked through. (Received September 27, 2017)

1135-D5-3198 **Ted Theodosopoulos\*** ([ttheodosopoulos@nuevaschool.org](mailto:ttheodosopoulos@nuevaschool.org)), 131 E. 28th Ave., San Mateo, CA 94403. *Exploring polygonal centers.*

We present a module for an inquiry-based applied geometry course for high school students. This module is inspired by the multiple non-equivalent notions of center for a triangle. Students explore the utility of different competing centers and seek to generalize them to quadrilaterals and beyond. The presentation includes several examples of student work and the writing that accompanied it. (Received September 27, 2017)

1135-D5-3202 **Gabriel Sosa\*** ([gsosa@amherst.edu](mailto:gsosa@amherst.edu)), Amherst College, Amherst, MA 01002. *Collaborative Activities in an undergraduate Abstract Algebra Course.*

Many students struggle with specific topics on Abstract Algebra. In some cases this is due to the student facing definitions that are too technical when compared to their mathematical maturity and, in others, to a lack of opportunity to experiment with the objects that are being studied. We will discuss some activities, along with their implementation and outcomes, related to modular arithmetic, the dihedral group and normal subgroups. (Received September 27, 2017)

## Teaching Abstract Algebra: Topics and Techniques

1135-E1-802 **Joe Stickles\*** ([jstickles@millikin.edu](mailto:jstickles@millikin.edu)). *Graphs and Zero-Divisors.*

There has been an explosion of research in graphical structures associated with algebraic objects, by professional mathematicians and undergraduates. The objective is to use these graphs to elucidate algebraic information. One particularly appealing graph is the zero-divisor graph of a commutative ring. This topic is approachable by anyone with one or two semesters of abstract algebra. We give the basic definitions, sample results proven by undergraduates, and provide a list of possible projects that an interested undergraduate can investigate. (Received September 14, 2017)

1135-E1-1073 **Elizabeth Wilcox\*** ([elizabeth.wilcox@oswego.edu](mailto:elizabeth.wilcox@oswego.edu)). *Learning to Read Math Papers: Two activities for an abstract algebra course.*

Reading mathematics is not an easy task – not for undergraduates, graduates, or even research mathematicians. Of course, a key difference between these groups is that undergraduates often expect reading math to come naturally and are deeply discouraged to realize the challenge of the “simple” act of reading (and understanding). For this reason, I’ve developed several activities for varying versions of Abstract Algebra courses (and other upper level courses) that engage students in the practice of reading and writing mathematics. I’ll discuss the benefits and costs of two activities that I’ve used and share my resources for these activities. (Received September 19, 2017)

1135-E1-1580 **Kristofer D Jorgenson\*** ([kjorgenson@sulross.edu](mailto:kjorgenson@sulross.edu)), Box C-18, Sul Ross State University, Alpine, TX 79832. *Teaching Modern Algebra Through Applications.*

I would like to discuss methods I’ve used and continue to modify and perfect for teaching an introductory course in rings, fields, and groups that ties these abstract concepts to real-life applications such as: (a) showing how every field axiom is used in solving a first degree equation in the real numbers. (b) showing how modular arithmetic in  $\mathbb{Z}_n$ , vector spaces, or basic group properties are used to solve problems in linear codes, bar codes and error-correcting codes. (c) showing how field extensions are used to solve Ancient Greek compass-and-straightedge geometric construction problems. (d) showing how quotient rings are used in the mathematics of general finite fields and error-correcting codes. (Received September 26, 2017)

1135-E1-1666 **Jeffrey W. Clark\*** ([clarkj@elon.edu](mailto:clarkj@elon.edu)). *Using Galois Theory as a Motivation for Learning About Permutation Groups.*

A number of our students have been conducting undergraduate research with one of my department members into Galois Theory. As a result an even larger number of students in our department are familiar with the notion of Galois groups permuting the roots of an irreducible polynomial. This talk will address my experiences in

using Galois Theory as a motivation and source of examples for students learning about permutation groups in an introductory class on abstract algebra. (Received September 24, 2017)

1135-E1-1775 **Joshua K. Lambert\*** ([joshua.lambert@armstrong.edu](mailto:joshua.lambert@armstrong.edu)), 11935 Abercorn Street, Savannah, GA 31419, Savannah, GA 31419. *Gamifying Abstract Algebra*.

From an early time in our development, we build an affinity for games. Whether our introduction began with a game of peekaboo or started with a round of poker, we have seen games in a wide variety of forms. Our fondness for these games led to instructors gamifying their own classrooms in an attempt to motivate participation and engagement with the material. This talk discusses the adventures and misadventures of gamifying an Abstract Algebra class. From the design of the course to the students' ability to meet the learning outcomes set for the semester, we explore the ups, downs, and lessons learned from taking this approach to such a course. (Received September 24, 2017)

1135-E1-1865 **Sarah Wolff\*** ([wolffs@denison.edu](mailto:wolffs@denison.edu)). *Teaching Abstract Algebra with Pre and Post Quizzes*.

When teaching abstract algebra, I added daily reading pre-quizzes and weekly post-quizzes in order to have students engage more deeply with the material. The reading quizzes gave students a chance to test their understanding in a low-stakes environment, while the post quizzes gave them an opportunity to see harder problems under a time constraint, but again in a low-stakes environment. In this talk I will discuss motivation and implementation of these activities, as well as lessons learned. (Received September 25, 2017)

1135-E1-2429 **Jessie Lenarz\*** ([jklenarz@stkate.edu](mailto:jklenarz@stkate.edu)). *Oral Presentations as Assessment in Abstract Algebra*.

We will discuss the use of oral presentations in an undergraduate abstract algebra course. Oral presentations can be used as both formative and summative assessments. Procedures for both will be discussed as well as anecdotal evidence of the degree of success of the assessments. (Received September 26, 2017)

1135-E1-2699 **Silas Johnson\*** ([sjohnson@math.northwestern.edu](mailto:sjohnson@math.northwestern.edu)). *An Abstract Algebra Course Capstone*.

I'll discuss details and results of a "mini-capstone" project at the end of a full-year abstract algebra course for undergraduates. The project is intended to give students an exploratory mathematical experience, look back on course material, and potentially spark interest in research. (Received September 26, 2017)

1135-E1-2752 **Lauren Keough\*** ([keoulaur@gvsu.edu](mailto:keoulaur@gvsu.edu)). *An Alternative Assessment Technique in Abstract Algebra Lowers Stress for All*.

Mastery based or specifications based grading has been an up and coming method for assessment in many math courses. There are many ways to implement this, but the core principles are that there is a set of standards which is openly shared with the students and the students get multiple attempts. The claim is that this is great for students, but implementing mastery based grading can feel overwhelming. In this talk I will discuss two mastery based assessment techniques I implemented - mastery based quizzing and mastery based proofs. The strategies were easy and low stress for me to implement, made grading more fun, and improved both class performance and attitude. (Received September 26, 2017)

1135-E1-2834 **Yevgeniy V. Galperin\*** ([egalperin@po-box.esu.edu](mailto:egalperin@po-box.esu.edu)), 200 Prospect St, East Stroudsburg, PA 18301. *Abstract Algebra based on Coding and Cryptography*.

Our course is designed based on examples taken from theory and practice of coding and cryptography. This approach helps the audience stay motivated through all the abstract topics and also allows us to keep the material fairly elementary and readily accessible to the students. (Received September 26, 2017)

1135-E1-3157 **Ethan Berkove\*** ([berkovee@lafayette.edu](mailto:berkovee@lafayette.edu)), Department of Mathematics, Lafayette College, Easton, PA 18042. *Hands-on group theory: worksheets and beyond*.

How do you get students to engage in real-time with the material in abstract algebra? We'll discuss how we used in-class worksheets and group work to give students hands-on experience to build their intuition about groups and rings as well as notions like order, symmetry, isomorphism and homomorphism. In addition, we'll describe how we extended some of the worksheet problems to more complicated homework examples involving symmetry groups of polyhedra and the Rubik's Cube. In the latter case, one student's work led her to an open problem in mathematics and her first experience reading a mathematical paper. (Received September 26, 2017)

## Meaningful Modeling in the First Two Years of College

1135-E5-145      **Blain Anthony Patterson\*** (bapatte3@ncsu.edu) and **Sarah Elizabeth Ritchey.**  
*From Startups to World Hunger: A Mathematical Perspective.*

Where is the best place to raise a child? Should the penny be discontinued? How long will it take the United States to completely switch to renewable energy? These are just some of the questions that can be answered using elementary mathematical modeling techniques such as regression, ranking, matrix models, and differential equations. In this presentation, the authors will discuss their experience teaching a two-week mathematical modeling course to academically gifted high school students. This project based course enabled students to explore mathematical careers in industry, learn modeling techniques, and solve meaningful open-ended problems. (Received August 03, 2017)

1135-E5-534      **Filippo Posta\*** (filippo.posta@gcu.edu), 3300 W Camelback Rd Rm 16-323, Phoenix, AZ 85255. *Balancing Chemical Reactions: a Modeling-based Exploration of Solutions of Linear Systems.*

Mathematics has a pivotal role during the first two years of college: to provide critical thinking and quantitative reasoning skills. Mathematical modeling provides the perfect environment for this purpose by encouraging the learner to connect abstract mathematical procedures to meaningful workplace problems. In this paper, we use the problem of balancing chemical equations to explore how to build a linear system and interpret its solutions. This modeling exercise provides a deeper understanding of the difference between coefficients and parameters as well as the meaning and handling of infinite solutions. We also present how the project is incorporated in a College Algebra course and its learning outcomes. (Received September 07, 2017)

1135-E5-1114      **Sara L Malec\*** (malec@hood.edu), 401 Rosemont Ave, Frederick, MD 21701. *Optimizing Idling: the Mathematics of Incremental Games.*

Incremental games, also called idle games, are simple video games where the player usually clicks a mouse repeatedly to earn some kind of currency. That currency can then be exchanged for other objects to help earn that currency more quickly. The gameplay is simple, but playing optimally involves a variety of interesting mathematical topics from modeling to calculus to statistics. We'll explore several areas of these games and the mathematics that underlie them, and see how they can be used as a teaching tool in a variety of mathematics classes. (Received September 19, 2017)

1135-E5-1163      **Khairul Islam\*** (kislam@emich.edu), Department of Mathematics and Statistics, Eastern Michigan University, Ypsilanti, MI 48109. *On Assessing Adequacy of Fitted Models.*

Incorporate modeling in undergraduate mathematics curricula is an important step. It allows significant experiences of modeling to students at an early stage. Alongside, the practice of assessing an incorporated model is equally important. This leads answers to questions such as: Which model is the best model? Or, how is a given model best? It makes students competitive and confident than others without modeling experiences. This presentation is targeting towards assessing adequacy of a given fitted model. Particularly, linear and non-linear models will be assessed for adequacy using real-life examples and applications. (Received September 20, 2017)

1135-E5-1761      **Annella R Kelly\*** (annela.kelly@regiscollege.edu). *Data Analysis in Precalculus.* Preliminary report.

The presentation will showcase the mathematical modelling project in precalculus class. The students use technology to obtain regression polynomials that model data and analyze it at precalculus level. The project emphasizes the properties of polynomial functions and their connection to the real world data. The effectiveness of this project will be assessed through survey in the end of the semester. (Received September 24, 2017)

1135-E5-1992      **Larissa B. Schroeder\*** (schroeder@hartford.edu). *The Eiffel Tower and Lake Mead: Collaborative Projects in Calculus II.* Preliminary report.

Research has found that solving routine problems in Calculus does not necessarily translate into the ability to solve non-routine problems. At the University of Hartford, we are developing a set of ill-structured, non-routine problems to be used as collaborative projects in Calculus II. The goal is to have students answer accessible but substantive questions with no clear solution path and to present their results. This talk describes our pilot work with two such questions and our strategy for organizing student presentations. We will also share student work samples illustrating the range of solution strategies, video clips of student presentations, and discuss unanticipated challenges. (Received September 25, 2017)

1135-E5-2042 **Frank Savina\*** ([fsavina@austin.utexas.edu](mailto:fsavina@austin.utexas.edu)). *Mathematical Modeling for STEM Intending Students.*

The Charles A. Dana Center at the University of Texas at Austin has developed a new and innovative pathway to Calculus for STEM intending students. This curriculum provides students a meaningful pre-calculus experience that values and honors their STEM aspirations by providing extensive opportunities to explore mathematical modeling within authentic STEM contexts. In order to provide meaningful modeling opportunities while developing the skills necessary for success in Calculus, many student activities were designed in collaboration with faculty members from other disciplines. This paper will describe these collaborations and provide specific examples of mathematical modeling within STEM disciplines. (Received September 25, 2017)

1135-E5-2543 **Erika L. Ward\*** ([eward10@ju.edu](mailto:eward10@ju.edu)). *Are There Enough Sidewalks? A Graph Theory Project for Non-Majors.*

At Jacksonville University, our mathematics for non-majors course asks students to explore mathematical topics that they haven't seen before, with the goal of gaining systematic problem solving skills and understanding ways that mathematics describes the real world. As the capstone to a section about graph theory, students create a graph that represents a portion of campus and the sidewalks that connect the features they've chosen to map. They then use the vocabulary, concepts, and skills they have learned to evaluate whether the sidewalks on campus are sufficient and useful. This modeling project not only helps them see how the mathematics they are learning applies to the world around them, but also helps them to clarify the concepts that they are trying to understand. It also encourages students to incorporate skills and interests from other areas to create maps that are relevant and even beautiful. (Received September 26, 2017)

1135-E5-3035 **Mary R Parker\*** ([mparker@austinctc.edu](mailto:mparker@austinctc.edu)). *Spreadsheets in a Math for Liberal Arts Course.* Preliminary report.

Students in our course learn to use spreadsheets to address more complex problems than they would do with formulas. They create many spreadsheets and use some more complicated spreadsheets (created by the teachers and from various financial planning websites) to address multi-step problems in personal finance. They fit linear, quadratic, and exponential models to data, interpret the parameters, and answer various questions using those models. In solving many other types of problems relevant to topics in a math for liberal arts course, they use spreadsheets as "smart scratch paper." (Received September 26, 2017)

## Arts and Mathematics: The Interface

1135-F1-7 **Sandra Fital-Akelbek\*** ([sfitalakelbek@weber.edu](mailto:sfitalakelbek@weber.edu)), Weber State University, 1415 Edvalson St DEPT 2517, Ogden, UT 84403. *Influence of Art on the Development of Projective Geometry.*

During a summer project, an undergraduate student learned how projective geometry was influenced by art. The demand and development of new techniques used in art were the greatest discovery of the fifteenth century Renaissance. In this talk, we present what the student and his mentor learned while working of this project and we discuss the math behind the perspective projection and the growing achievements in art. (Received May 29, 2017)

1135-F1-235 **Gary R Greenfield\*** ([ggreenfi@richmond.edu](mailto:ggreenfi@richmond.edu)), Department of Math & Comp. Sci., University of Richmond, Richmond, VA 23173. *Harnessing chaos for generative art and an unusual packing problem.* Preliminary report.

Label the edges of an  $m \times n$  grid with zeros and ones. Treat each row of edges and each column of edges as inputs to some chaotic one-dimensional cellular automaton. After a large number of iterations evaluate, starting counterclockwise from the top, the edges of each grid cell to assign cells hexadecimal digits. Assign colors to digits and generate artworks by invoking a hill-climbing algorithm to maximize the number of occurrences of two distinguished digits. This gives rise to packing problems such as: How many times can you successively place a Y and a B in an  $m \times n$  grid such that the cells to the left and above each Y are always blank? (Received August 14, 2017)

1135-F1-477 **Natasha Rozhkovskaya\*** ([rozhkovs@math.ksu.edu](mailto:rozhkovs@math.ksu.edu)), Department of Mathematics, Kansas State University, 138 Cardwell hall, Manhattan, KS 66506. *Teaching Mathematics in a Fine Arts Museum.*

We will describe our experience of teaching mathematics concepts to school students, undergraduate students and educators through the discussion of artworks in a gallery of a fine arts museum. We will share the examples

of math topics that could be covered at such workshops, discuss the usage of digital collections of prominent world art museums and other resources in fine arts and mathematics. The talk is based on materials accumulated over eight years of collaboration between Math Circle Seminar at Kansas State University and the Marianna Kistler Beach Museum of Art (Manhattan, Kansas). (Received September 05, 2017)

1135-F1-483 **Rachel Schmitz\*** ([rschmi9@students.towson.edu](mailto:rschmi9@students.towson.edu)), 8000 York Road, Towson, MD 21252.  
*Vanishing points in paintings: An algebra educational activity.*

In a two-dimensional representation of lines that are parallel in 3-dimensional reality, lines converge to a point called the vanishing point. Several famous paintings make use of the vanishing point in order to show perspective within their paintings. This presentation shows a middle school algebra classroom activity that addresses a Common Core State Standard in Expressions and Equations, and discusses how paintings can be used as a context to find the point of intersection of two lines. (Received September 05, 2017)

1135-F1-741 **Robert W Fathauer\*** ([tessellations@cox.net](mailto:tessellations@cox.net)). *Mathematical Art and Recreation*  
*Based on Kite Tiling Rosettes.*

A kite rosette is a tiling with a single kite-shaped prototile and a singular point about which the tiling has rotational symmetry. A tiling with  $n$ -fold symmetry has  $n$  kites of the same size arranged in a ring, with kite size increasing with distance away from the singular point. The prototile can be either convex or concave (also known as a dart). Such tilings can be constructed over a wide range of kite shapes for all  $n > 2$ , and for convex kites for  $n = 2$ . They are two colorable and possess mirror symmetry about lines passing through the singular point. A finite patch of adjacent rings of tiles can serve as a scaffolding for constructing knots and links, with strands lying along tile edges. The number of strands can be calculated from the rotational symmetry and number of rings of tiles. Such patches serve as convenient templates for attractive graphic designs, and original examples will be presented in addition to prints by M.C. Escher. In addition, these patches can be used as grids for a variety of puzzles and games. Three-dimensional structures created by giving the tiles thickness are also explored. (Received September 13, 2017)

1135-F1-846 **Jordan Schettler**, [jordan.schettler@sjsu.edu](mailto:jordan.schettler@sjsu.edu), and **Kecia Sako\*** ([kecia.sako@sjsu.edu](mailto:kecia.sako@sjsu.edu)).  
*Recursively Constructed Keyboards for Non-Standard Musical Scales.*

We will discuss recursive arrangements of black and white keys for macro/microtonal scales (like Bohlen-Pierce and the gamma scale of Wendy Carlos) which generalize the familiar layout on a standard piano keyboard. The number of notes in certain scales with roughly equal spacing (e.g., the 12 notes in an octave on a standard keyboard) is well-known to correspond with the denominator of a convergent in the continued fraction expansion of  $\log_b(a)$  where  $a$  and  $b$  are rational numbers. We take this a step further to describe a procedure for constructing keyboard layouts themselves using only the coefficients in the continued fraction. This framework is used to examine 41 equal temperament and help quantify why it is so under-used in music. The investigation was part of a URG (undergraduate research group) funded by San Jose State University. (Received September 15, 2017)

1135-F1-880 **Vi Hart, Andrea Hawksley, Elisabetta Matsumoto and Henry Segerman\***  
([segerman@math.okstate.edu](mailto:segerman@math.okstate.edu)). *Non-euclidean virtual reality.*

Non-euclidean spaces are often thought of as unintuitive and exotic, but with direct immersive experiences we can get a better intuitive feel for them. The latest wave of virtual reality hardware, in particular the HTC Vive, tracks both the orientation and the position of the headset within a room-sized volume, allowing for such an experience. We use this technology to explore two of the three-dimensional geometries of the Thurston/Perelman geometrization theorem:  $\mathbb{H}^3$  and  $\mathbb{H}^2 \times \mathbb{E}$ . (Received September 16, 2017)

1135-F1-897 **Leo S Bleicher\*** ([l\\_bleicher@pacbell.net](mailto:l_bleicher@pacbell.net)), 555 Rushville St., San Diego, CA 92037.  
*Evolving Paintings with Sequences of Coordinate Transforms in 2D and 3D.*

Complex shapes can be generated through the application of sequences of symmetry breaking coordinate transformations. For example, a square or circle, through a series of polar coordinate transforms and rotations can become something much more interesting.

A specific sequence of transformations is a code describing a method for generating one member of a family of related shapes. Such sequences can be recombined, allowing the resultant images to 'evolve' with the application of suitable fitness function. Software has been developed to initialize sequences, generate renderings, evaluate output images and recombine my favorites. Over generations the sequence length, complexity and diversity of images generated in this way can tend to increase.

In 3D still more complexity and diversity can be discovered using additional coordinate systems. In this case Pipeline Pilot is used to generate renderable scene descriptions and launch the rendering engine. Sequences

having transformations with continuous domains (e.g. rotations) can be rendered at many points along a vector through parameter space to yield movies. (Received September 16, 2017)

1135-F1-909 **Samaneh G. Hamidi\***, Department of Mathematics, Brigham Young University, Provo, UT 84602. *Mathematics in Persian Art II*. Preliminary report.

Persia has left numerous marks on the civilizations and cultures of mankind dating back to the early 5th millennia BC. Persian architecture has been a comprehensive embodiment of Iranian psychology and characteristics in different historical periods from the first scientific notions of astronomy with the measurement of the celestial sphere and determination of the new year according to the vernal equinox to the mathematics of shapes, solids, and numbers. Persians have combined skills in theoretical mathematics and practical techniques such as ceramics, along with artistic ideas from their own history and around the world. Putting these together, they have expressed several appealing forms that bridge mathematics and the arts. The purpose of this talk is to get a glimpse of mathematics in Persian arts in different eras and also to present a more detailed visual analysis of a ceramic design. (Received September 16, 2017)

1135-F1-1087 **Josh Hallam\*** ([hallamjw@wfu.edu](mailto:hallamjw@wfu.edu)), Wake Forest University, Department of Mathematics and Statistics, Winston-Salem, NC. *Creative Writing Projects in Mathematics Courses*.

During the last few years I have started to implement a creative writing component in my classes. For example, in discrete mathematics courses I have students create short science fiction stories which are based on Isaac Asimov's Three Laws of Robotics. Additionally, in calculus courses I have students create short stories where calculus plays a major role in the plot. Students have used several mediums to tell their story including written short stories, play scripts, short videos, and computer games. I will discuss some of these projects including some that are clever and humorous and others that generated interesting philosophical discussions. (Received September 19, 2017)

1135-F1-1144 **Sara Jensen\*** ([sjensen1@carthage.edu](mailto:sjensen1@carthage.edu)), 2001 Alford Park Dr., Kenosha, WI 53140. *How Many Different Patterned Stockings Can You Knit?* Preliminary report.

Fair Isle knitting is a type of knitting using two colors at a time to create patterns. When knitting a Fair Isle Christmas stocking, I began wondering just how many different Fair Isle patterns were possible given  $n$  stitches. Although an innocent enough question to ask, the solution to this problem was anything but straightforward. The journey to a solution involved programming, some Abstract Algebra, and a surprise appearance by the partition function. (Received September 19, 2017)

1135-F1-1289 **M. Carol Williams\*** ([carol.williams@ttu.edu](mailto:carol.williams@ttu.edu)), Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409-1042. *Quilt Designs Inspired by Ruler and Compass Constructions*.

We will describe how our work in the quilting arts has been inspired by classical Euclidean ruler and compass constructions. Bringing this work into the classroom, we have greatly increased our effectiveness in motivating our students, mostly future teachers, in appreciating the beauty of constructions by sharing the beauty of the quilts. (Received September 21, 2017)

1135-F1-1335 **Emily Gullerud\*** ([gullerej@uwec.edu](mailto:gullerej@uwec.edu)) and **James S. Walker** ([walkerjs@uwec.edu](mailto:walkerjs@uwec.edu)). *Creating Hyperbolic Wallpapers and Animations*. Preliminary report.

We extend the work of Dr. Frank Farris by developing a computationally efficient way of generating hyperbolic wallpaper designs. Using Matlab, we compute an array of linear transformations through the use of a trinary tree of Pythagorean pairs and their corresponding Bézout coefficients. Entries in this array are then used to generate the designs, bypassing the need for the greatest common divisor function in Matlab. Further, we create animations of these wallpapers through various techniques. (Received September 21, 2017)

1135-F1-1363 **Vladimir Bulatov\*** ([info@bulatov.org](mailto:info@bulatov.org)). *3D Hyperbolic Tilings and Horosphere Cross Sections*. Preliminary report.

The images of two dimensional hyperbolic tilings are familiar and were popularized by M.C.Escher Circle Limit woodcuts. Tilings of 3 dimensional hyperbolic space are much harder to visualize because viewer has to be inside of the hyperbolic space to be able to see the tiling completely. We try to display 3D tilings as a two dimensional cross sections with horospheres. The intrinsic geometry of horosphere is euclidean plane therefore horosphere can naturally be displayed in two dimensions without distortions. Such a horosphere cross sections display interesting properties. It is infinite and is somewhat similar everywhere but non repeating. The apparent size of individual tiles sections remains constant in contrast to 2d tiling where tiles size rapidly tends to zero

when tiles approach the hyperbolic plane horizon. We will provide few examples of such a tilings and several animations.

Tiling <https://vladimir-bulatov.deviantart.com/art/Horosphere-Tiling-III-280995906>

Animation <https://youtu.be/KrAqh48-imA>

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1135-F1-1526 **Alice E Petillo\*** ([alice.petillo@marymount.edu](mailto:alice.petillo@marymount.edu)), 2807 N Glebe Road, Arlington, VA 22207. *An Exploratory Approach to Polyhedra Using the Open-Access Software, Archimedean.*

Polyhedra have appeared in the intersection of mathematics and art on an ongoing basis across time and cultures, including today's digital culture. As we remember the contributions of Father Magnus J. Wenninger, many of us hope to inspire future generations of students to see and experience the beauty of polyhedra in the world around them. Archimedean, an open-access 3D construction and rendering tool, has an exploratory approach that allows users to virtually construct, transform and analyze polyhedra. Dr. Alice Petillo will share how the Archimedean program has been used in Liberal Arts Core mathematics content classes to introduce undergraduate students to known polyhedra along with new objects that are the result of allowing faces to traverse the surface more than once before joining edges. (Received September 22, 2017)

1135-F1-1710 **Donald J Plante\*** ([donald.plante@unh.edu](mailto:donald.plante@unh.edu)), 88 Commercial Street, Manchester, NH 03101. *Automating String Art through the use of 3D Printers.*

String art, first popularized by Mary Everest Boole, beautifully demonstrates geometric patterns that are formed when arranging thread by stringing it between multiple points. This mathematical art is both pleasing to observe as well as time consuming to create. By taking advantage of 3D printers natural tendency to form strings of filament during the printing process, we have automated this labor-intensive art form. This is accomplished by programming the movements of 3D printers in G-code so that these models can be quickly reproduced and enjoyed by a wider audience. (Received September 24, 2017)

1135-F1-1732 **Robert Bosch\*** ([rbosch@oberlin.edu](mailto:rbosch@oberlin.edu)). *3D printed tours.* Preliminary report.

TSP Art involves converting a grayscale target image into a collection of points, treating the points as the cities of a Traveling Salesman Problem (TSP), and then finding a high quality tour (Hamiltonian cycle) that passes through them. If all goes well, the salesman's tour will be a continuous line drawing that resembles the target image. In this talk, I will discuss my ongoing experiments in incorporating 3D printing into TSP Art and other tour-based artwork. My goal is to use the technology to create sculptures that pull the viewer into the artwork, make discoveries about it, or see something in a new light. (Received September 24, 2017)

1135-F1-1758 **Margaret Kepner\*** ([renpek1010@gmail.com](mailto:renpek1010@gmail.com)). *Crowdsourcing the Magic.* Preliminary report.

Magic squares have been around for thousands of years. But there is always more magic to be found. Computers and the Internet have facilitated new ways of exploring the properties of magic squares. I will discuss a distributed computing project that put to rest a long-standing problem concerning magic squares containing knight's tours, as well as an online contest involving magic squares that 'hold water.' These efforts have added additional layers to the subject of magic squares. I will describe several examples of interesting magic squares, and illustrate how I have employed them in developing my artwork. (Received September 24, 2017)

1135-F1-1827 **Vincent J. Matsko\*** ([vince.matsko@gmail.com](mailto:vince.matsko@gmail.com)). *The Combinatorics of Binary Trees.*

Binary trees as considered in the literature are usually symmetric binary trees, where left and right branchings are simple scaled rotations. In these cases, the number of new nodes typically doubles with each iteration. When more general affine transformations are allowed for branchings, it is possible that different branching sequences produce the same node, so that the number of nodes at a given iteration is consistently less than twice that of the previous iteration. This talk will focus on how to force different branching sequences to produce the same node, and in such cases, how to count the number of nodes at a given depth. (Received September 25, 2017)

1135-F1-1831 **Chamberlain Fong\*** ([chamberlain@alum.berkeley.edu](mailto:chamberlain@alum.berkeley.edu)), San Francisco, CA 94110. *Squirclular Calculations.*

The Fernandez-Guasti squircle is a plane algebraic curve that is an intermediate shape between the circle and the square. It has characteristics that are very similar to the more famous Lamé curve. However, unlike the Lamé curve which has unbounded polynomial exponents, the Fernandez-Guasti squircle is a low degree quartic curve. This makes it more amenable to algebraic manipulation and simplification. For instance, we have used this curve to formulate invertible mappings between the circular disc and the square.



In this paper, we shall motivate the use of the squircle for artistic purposes. For example, it can be used to convert M.C. Escher's circle limit lithographs into squares. Also, it can be used to stylize rectangular paintings into oval regions. Another artistic use involves the visualization of 3D surfaces derived from the squircle. See <https://squircular.blogspot.com> for sample results. (Received September 26, 2017)

1135-F1-1851     **Debra K Borkovitz\*** ([dborkovitz@wheelock.edu](mailto:dborkovitz@wheelock.edu)), [dborkovitz@wheelock.edu](mailto:dborkovitz@wheelock.edu), and **Karl Schaffer**, [karl\\_schaffer@yahoo.com](mailto:karl_schaffer@yahoo.com). *A truncated octahedron in dance, art, music, and beyond*.

A graph where the vertices are permutations on four elements and the edges connect permutations that switch adjacent elements forms a truncated octahedron, isomorphic to a graph known as the permutahedron. This construction lends itself to artistic interpretation in a variety of media – one of us started with dance and the other with temari (a Japanese/Chinese art of embroidery on a sphere), as well as to interesting classroom activities. For this talk, we will survey connections between this truncated octahedron and dance, art, music, and computer science; we will also discuss classroom activities. (Received September 25, 2017)

1135-F1-1864     **Susan Schmoyer\*** ([schmoyes@tcnj.edu](mailto:schmoyes@tcnj.edu)), Ewing Township, NJ. *A Mathematician Plays with a Spirograph*. Preliminary report.

If you've ever played with a Spirograph toy, you know how hard it is to (1) draw a perfect curve without the pen or gear slipping, and (2) draw curves as amazing as the samples that come with the toy. Using a computer algebra package like CoCalc (formerly SAGE), parametric equations, and a little linear algebra, ANYONE can make the most amazing Spirograph designs. This paper presents the math and code needed to start making your own beautiful hypotrochoid and epitrochoid designs. (Received September 25, 2017)

1135-F1-1910     **Eve Torrence\***, [etorrenc@rmc.edu](mailto:etorrenc@rmc.edu). *Hypar Zonohedra*.

A pleated hypar is made by folding paper to form an origami model that approximates a hyperbolic paraboloid. The term hyparhedra was coined by Erik Demaine, Martin Demaine, and Anna Lubiw in their 1999 Bridges Conference paper in which they give an algorithm for making paper sculptures based on the Platonic solids from hypars folded from square paper. By experimenting with rhombic paper and zonohedra I have created more symmetric versions of some of the Demaine et al models. Several of these sculptures have been exhibited in Bridges Conference Art Exhibitions.

In this talk I will explore the possibility of building more complex hypar zonohedra by using different shapes of rhombic paper to model a variety of polyhedra with rhombic faces. (Received September 25, 2017)

1135-F1-1976     **Maria Antonieta Emparan\*** ([antonieta.emparan@gmail.com](mailto:antonieta.emparan@gmail.com)), Departamento de Matematica y CC of. 426, USACH, Las Sophoras 175, Estacion Central, 9170020 Santiago, Chile. *Curatorial project "Des linkages between art and mathematics"; work in progress*. Preliminary report.

This paper addresses to the curatorial project presented to FONDART (funds from the State of Chile to finance artistic projects) for an exhibition linking visual arts and mathematics. In Chile, both at school and university levels, there is a disconnection between visual arts and mathematics. This curatorial project opens a reflective space around our teaching methods current state and the possibilities of re-linking both disciplines. The group of artists is formed by a mathematician dedicated to kinetic sculpture on paper; a visual artist who will perform an installation reviewing the discussions of the Ministry of Education about the hours of Geometry in middle education and the current curricula of Visual Arts, Design, Bachelor in Mathematics and Engineering in Mathematics; A photographer who will perform a fieldwork with the New Age practices associated with the use of "sacred geometry". And, on the other hand, a public appeal will be made to young artists to integrate the work team with a proposal according to the curatorial spirit. This group of artists, besides receiving advice from the history of art, will also receive mathematical advice from the current president of the Society of Mathematics of Chile, who forms the team with the three artists and the curator. (Received September 25, 2017)

1135-F1-1990     **Teresa L Downard\*** ([teresa.downard@wwu.edu](mailto:teresa.downard@wwu.edu)), 516 High Street, Mathematics Department MS 9063, Bellingham, WA 98225. *Problem solving and creativity*. Preliminary report.

Sharing math/art problems and student work from in a new Math and Art general education course at Western Washington University. This is a follow-up to the talk "Folding, imagining, and constructing a math and art class" with Dina Buric at JMM 2015. (Received September 25, 2017)

1135-F1-2025 **Anil Venkatesh\*** ([anilvenkatesh@ferris.edu](mailto:anilvenkatesh@ferris.edu)). *Symmetric Substructures in Musical 12-Tone Rows*. Preliminary report.

In music, a 12-tone row is any of the  $12!$  possible orderings of notes in the Western chromatic scale. The musical notes of a 12-tone composition must always arise in the same order, cycling repeatedly through a predetermined “row” of twelve notes. The repetitive structure of 12-tone music lends itself to mathematical study. In 2003, Hunter and von Hippel investigated symmetry in 12-tone rows, using group theory to enumerate equivalence classes of rows under a group of music-theoretic symmetries. They found that highly symmetric rows constitute just 0.13% of the  $12!$  possibilities, and yet these rows arise in 20% of actual compositions. While this result clearly indicates that composers prefer symmetric rows, a mathematical motivation for the remaining 80% of 12-tone compositions has not yet been found. While Hunter and von Hippel worked on the level of entire 12-note sequences, the human ear is also sensitive to shorter repetitions and symmetries. In this talk, we argue that an investigation of these symmetric substructures of 12-tone rows may lead to a more complete account of the aesthetics of composition. (Received September 25, 2017)

1135-F1-2149 **Maria C Mannone\*** ([manno012@umn.edu](mailto:manno012@umn.edu)), Minneapolis, MN 55455. *Categories and Arts: Some cARTegory Theory of Music and Visual Arts*.

Can we “say (something about math) with flowers?” The formalism of category theory can be successfully applied to classify and investigate transformational processes of shapes of plants and flowers, as well as other images from nature. Through the same formalism, we can create a model of objects and processes that are relevant in music. In daily practice, musicians deal with objects, transformations, and transformations of transformations: for example, a crescendo connects different levels of loudness; an accelerando transforms a slow crescendo into a fast crescendo. Moreover, sonification strategies (translational processes of some data into sounds) may be enhanced through a categorical description: we are not only able to hear “the sound of a drawing” but we can also convert a transformational process in visual arts into a transformational process in musical art. These techniques themselves might unfold new strategies to creatively explore the fundamentals of category theory. Artworks of different media can be compared, highlighting eventual similarities between the technique to make drawings and the technique to make music. Some working examples deal with the musical rendition of flowers, animals’ armors, and cathedrals’ profiles. (Received September 25, 2017)

1135-F1-2157 **Frank A. Farris\*** ([ffarris@scu.edu](mailto:ffarris@scu.edu)), 500 El Camino Real, 314 O’Connor Hall, Santa Clara University, Santa Clara, CA 95053. *Fibonacci Wallpaper Spirals: Painting and Animation*.

Inspired by John Edmark’s “Blooms,” I show how to take any wallpaper pattern and wind it around the plane into a family of Fibonacci spirals. The domain-coloring algorithm allows us to paint spiral patterns with the colors and textures of any photograph. When we view rotations of spirals like these in the correct time sequence, they produce a remarkable blooming effect. I will show animations of flat spiral images, as well as virtual 3D models. (Received September 25, 2017)

1135-F1-2182 **Mary D Shepherd\*** ([msheprd@nwmissouri.edu](mailto:msheprd@nwmissouri.edu)). *Two-color wallpaper groups in quarter-circle quilting patterns*.

The quilting tile commonly called Drunkard’s Path quilt block is a simple square with a quarter circle in a contrasting color in one corner. The radius of the quarter circle is usually slightly larger than half the side length. The quarter circle can appear in any of the 4 corners, and the contrasting colors can be reversed. This gives 8 possible quilt blocks. These can be viewed as representing the group of order  $8 \mathbb{Z}_4 \times \mathbb{Z}_2$ . Frequently 4 quilt blocks are arranged in a 2 by 2 square and then four of these 2 by 2 squares are put together with some type of symmetry, rotational, reflection or color balance, to create a 4 by 4 block that is used to tile the quilt top. The complexity of the two-color wall-paper patterns that emerge create some beautiful patterns, not all of which appear to have been created into actual quilt tops. This presentation will describe many of the amazing patterns that emerge from this bottom-up construction. (Received September 25, 2017)

1135-F1-2368 **Kerry Mitchell\*** ([1kmitch@gmail.com](mailto:1kmitch@gmail.com)), 3833 N. 30th Street, #247, Phoenix, AZ 85016. *Popcorn and Photographs: Manipulating Images with Iterated Functions*.

In this work, I explore techniques for generating two-dimension images using iterated discrete versions of non-linear differential equations. Of particular interest is Pickover’s Popcorn formula, which involves compositions of transcendental functions of complex variables. Generalizations and variations of the formula are introduced, as well as a variety of mechanisms for creating art using the formulas. One favorite technique is to use the formula as a warping of the complex plane, onto which photographs or other images are mapped. The result

can be combined with others, by layering, resulting in transformative artwork that may or may not resemble the original piece. (Received September 26, 2017)

1135-F1-2431 **Elijah m Allen\*** ([pupilofyah@gmail.com](mailto:pupilofyah@gmail.com)), 159 N Streeper, Baltimore, MD 21224. *An Edgematching Puzzle with a Twist*. Preliminary report.

I present the results of an exploration of color distribution and topological objects represented on squares with a set of 9 unique edge-colored square tiles using 4 colors (designed by Kate Jones), reminiscent of Major Percy MacMahon's Three-Color Squares as described in his *New Mathematical Pastimes*, 1921. The delight of such a puzzle is the artistic surprise that each new solution brings. (Received September 26, 2017)

1135-F1-2529 **John Shier** and **Douglas Dunham\*** ([ddunham@d.umn.edu](mailto:ddunham@d.umn.edu)), Department of Computer Science 320 HH, 1114 Kirby Drive, Duluth, MN 55812-3036. *New Metamorphosis Patterns*. Preliminary report.

M.C. Escher utilized transformations in a number of his prints, notably in his *Metamorphosis I, II, and III*. We show some new Escher-like transition patterns and some new transition patterns in different directions than Escher explored.

Among the Escher-like patterns, we show simple a simple squares-to-hexagons transition, a transition from a simple black-white tessellation to a pattern of colored butterflies, and "Night into Day", our version of Escher's "Day and Night" with black-white symmetry. We also show a transition from the center outward starting with an equilateral triangle tessellation to one of interlocked birds.

Most of our non-Escher patterns start with a simple regular tessellation on the left which is gradually transformed to a non-regular or even random pattern on the right.

All of our transitions are accomplished by using continuous random transition functions that are applied to the motifs to make changes in position, shape, color, and interior decoration. These functions start at "identity" values which are then morphed to non-regular final values. (Received September 26, 2017)

1135-F1-2541 **Tom Bates\*** ([tombagoo@gmail.com](mailto:tombagoo@gmail.com)). *A Generalization of the Chaos Game*.

The well known "chaos game" algorithm that generates the Sierpinski gasket stochastically is generalized beyond the equilateral triangle to any order of regular polygon. A simple formula is derived that gives a value, referred to as the "kissing number," to produce a crisp analog to the Sierpinski gasket for any order of regular polygon and yields an easily understood geometric reason for the value of 0.5 in the equilateral case and the correct value for all other cases. In addition a secondary kissing number is revealed that produces a second fractal figure for each polygon.

Finally, some surprising aspects of what the algorithm is actually doing are revealed, including that the figures created by it contain a deep history of the order of vertices targeted by the algorithm, which is illustrated by assigning a color to each vertex.

This work was performed both to generalize the usual algorithm and to use the understanding gained thereby towards artistic ends. (Received September 26, 2017)

1135-F1-2580 **Susan Happersett\*** ([fibonaccisusan@yahoo.com](mailto:fibonaccisusan@yahoo.com)), 2 East Main Street, Roundhouse Unit #7, Beacon, NY 12508. *Altering Symmetries: Expanding on Cartesian Lace Drawings*.

I have created a new series of mathematically inspired drawings that I call "Cartesian Lace". For these drawings I have developed a drawing process based on Set Theory using bijective (one point to one point) and non-bijective (one point to many points) mapping patterns. I used a set of 4 axes, intersecting at a central point, creating 90 degree and 45 degree angles. These figures possessed order-4 or order-8 rotational symmetry. To build more complex patterns, I changed the number of axes to 3 or 6, but continued to use the same mapping procedures. The symmetries changed to order 3, 6 or 12 rotational symmetries. I repeat these new forms as building blocks within a grid formation. These new tilings give me the opportunity to further explore wall paper patterns. As I continue to experiment with scale and grid structures, I have been able to present more complicated geometric networks. This new work began as a vehicle to explore the visualization of networks such as neural connections and the diagrams for technology systems, as well as lace making. Expanding my artistic practice by hand drawing these tessellations, I have broadened the scope of my work. (Received September 26, 2017)

1135-F1-2611 **Joshua Brandon Holden\*** ([holden@rose-hulman.edu](mailto:holden@rose-hulman.edu)), Department of Mathematics, Rose-Hulman Institute of Technology, 5500 Wabash Ave., Terre Haute, IN 47803. *Rock Me Fibonacci: Using Recurrence Relations to Count Rock Drum Fill Patterns*. Preliminary report.

Drum fills are a way of "filling time" during a short break in a rock or pop song, usually using a rapid sequence of notes played in succession across multiple drums in a drum kit. Considering the common configuration known as

a five-piece kit, we see that moving from one drum to another is considerably easier in some combinations than others. We construct a set of rules to model patterns which avoid these difficult transitions, and then construct and solve recurrence relations to count the number of  $n$ -beat patterns which fit the rules. Variations on the theme are provided by varying the durations of notes and the numbers and types of drum allowed. Warning: Most of the integer sequences will actually be generalizations of the Fibonacci numbers. Expect less “two, three, five, eight” and more “one, two, tres, quatro”. (Received September 26, 2017)

1135-F1-2648      **Susan Goldstone\*** ([sgoldstine@smcm.edu](mailto:sgoldstine@smcm.edu)), Dept of Mathematics and Computer Science, St. Mary’s College of Maryland, 47645 College Drive, St. Mary’s City, MD 20619. *Knitting Symmetries: Yarn, Stitch, and Fabric*.

Symmetry has long been a key element of the decorative arts. As more artists and mathematicians turn their attention to the interface between mathematics and the arts, we have paid closer and closer attention to which symmetry types are possible in different art forms. Here, we take a deep dive into the frieze and wallpaper symmetries in different styles of knitting and find that some symmetries are more deeply embedded in the fiber of knitting than others. (Received September 26, 2017)

1135-F1-2711      **Andrew Diener\*** ([adiener@cbu.edu](mailto:adiener@cbu.edu)), 36 South Merton, Memphis, TN 38112, and **Cathy Grilli**. *Using Music to Increase Understanding and Performance in Trigonometry*. Preliminary report.

Music is an art that is particularly appropriate to trigonometry classroom. Aspects of it are an application of trig, and it can also be used as a device to remember basic rules. In a Student MAA meeting we used graphing calculators and tuning forks to see that sinusoids really do occur outside of the classroom. We wanted to connect the student experience in this activity to actual course material. A consideration in activities is the time restriction in a class so we invited students to an out of class session. There they produced graphs of several notes using a tuning fork app. As a group activity, they determined equations for the functions describing those graphs. The period was compared with the frequency of each note. There was some investigation of chords. We next appealed to their creative side. A student shared a jingle learned in high school as a memory device for trig identities. We challenged each group to come up with either a musical or graphic representation that would help other students remember some identity or concept from trig. This activity was a joint project from different pathways we offer to prepare students for Calculus I. In this paper we will share our experiences. (Received September 26, 2017)

1135-F1-2805      **Zdenka Guadarrama\***, 1100 Rockhurst Road, Kansas City, MO 64110. *Examples of connections to mathematics through the lens of art*.

In this talk, I will engage the audience in a short activity to illustrate the idea that reaching into mathematics through the lens of art provides very rich platform for hands on interactions with mathematics in the classroom. I will then showcase various examples of art projects and objects which my students and I have created for the exploration, analysis and synthesis of mathematical concepts. Some of these examples will come from outreach programs to elementary schools and high schools, other examples will come from work in different areas of the undergraduate curriculum, including the calculus sequence, and my experience using art in undergraduate research. (Received September 26, 2017)

1135-F1-2886      **Laura Taalman\*** ([laurataalman@gmail.com](mailto:laurataalman@gmail.com)). *Math as Design Engine: Leveraging mathematics to create 3D printed art*.

Mathematicians have a powerful secret universe to draw from when creating 3D-printable designs. In this talk we’ll discuss how software like Grasshopper, OpenSCAD, Structure Synth, and TopMod can be used to turn mathematical knots, curves, polyhedral wireframes, and procedurally generated forms into 3D printed jewelry, housewares, and art. We will also display printed pieces from the collection at [mathgrrl.com/hacktastic/designs](http://mathgrrl.com/hacktastic/designs). (Received September 26, 2017)

1135-F1-2937      **Phil Webster\*** ([phil@philwebsterdesign.com](mailto:phil@philwebsterdesign.com)), 950 Bethany Drive, Scotts Valley, CA 95066. *A Methodology for Creating Fractal Islamic Patterns*. Preliminary report.

Historically, a small proportion of Islamic geometric patterns were created that had various sorts of self-similarity, but only at two different levels at most. This paper presents a methodology for systematically creating truly fractal Islamic patterns with self-similarity at infinite levels. Centers of local symmetry are established using fractal trees, and patterns are then constructed using the traditional polygon technique. The adaptations required to accommodate the fractal arrangement of the polygons, as well as decoration options specific to fractal Islamic patterns, will also be discussed. (Received September 26, 2017)

- 1135-F1-3066 **Cindy Marie Blois\*** (cblois@usc.edu), 3620 S. Vermont Ave, KAP 104, Los Angeles, CA 90089. *Developing Mathematical Metaphors Through Open-Ended Art Projects*. Preliminary report.

When you think about a given theorem, such as the Bolzano-Weierstrass Theorem, what images or metaphors does your brain evoke? According to cognitive science (Lakoff and Nuñez, 2000), our understanding and practice of mathematics is underpinned by our conceptual metaphors for abstract mathematical ideas. In this talk, we'll discuss the use of open-ended art projects as a means for allowing students to develop and share their own conceptual metaphors in the classroom. Specific examples from undergraduate math courses will be presented. (Received September 26, 2017)

- 1135-F1-3101 **David A Reimann\*** (dreimann@albion.edu), Math/CS Department, Albion College, 611 E. Porter St., Albion, MI 49224, and **Antoni Fodor**. *Panoramic Photographic Polyhedral Pavilions*.

Conventional cameras view a small solid angle, limiting the both the field of view and projective distortion. However, multiple individual pictures are need to have full spherical coverage. Cameras that can directly take spherical panoramic photos, such as the Ricoh Theta S, have become available as relatively inexpensive consumer products. Unlike a traditional camera, this camera has two hemispherical lenses, allowing it to see simultaneously in every direction around the camera. These cameras produce an equirectangular projection, where each latitude row has the same number of pixels, which has severe distortion at the poles. One approach to hardcopy display is to map the image onto the surface of a small polyhedron, such as a Platonic or Archimedean solid, which reduces the distortion. Using such polyhedra resembles the process artist Dick Termes uses for painting on a sphere, which he calls a Termesphere. These techniques force the viewer to see the world inside-out. Our work maps the spherical photo to the inside of a large polyhedra to create a miniature pavilion which can be entered for a personal panoramic experience. (Received September 26, 2017)

- 1135-F1-3119 **Karl M Kattchee\*** (kkattchee@uwlax.edu). *Art of de Bruijn Sequences*.

A binary *de Bruijn sequence* is a circular bit string of length  $2^n$  with the property that every bit string of length  $n$  occurs once as a contiguous substrings. The existence of de Bruijn sequences is somewhat surprising, and the concept provides fertile ground for visual arts. We give a sketch of the existence proof and examples of artwork inspired by the idea of de Bruijn sequences. (Received September 26, 2017)

- 1135-F1-3149 **Karl H. Schaffer\*** (karl\_schaffer@yahoo.com), 325 Lucinda St., Scotts Valley, CA 95066, and **Joseph Thie** (jathie@utk.edu) and **Kasia Williams** (kwilliams@hw.com). *Quantifying the Center of Attention (CA) for Describing Dance Choreography*.

A center of attention (CA) was defined by Kasia Williams in a paper at the international 2012 Bridges conference as a means for mathematically studying how choreographers and dancers tend to create and manipulate the CA in dance works. In the arts, qualitative usage of the CA has long been successfully employed by painters. For dance it is quantitatively calculated by an interpreter of video frames assigning weights to the dancers based on the choreographer's intent as applied to dancer configurations and locations on stage as the performance proceeds in time. The presenters, using Excel and Mathematica, have been exploring methods of calculating and displaying the CA as a trajectory along with statistical and other quantifiers from analyses of a variety of dance performances. The hope is that the center of attention might emerge as a new useful tool in matters such as understandings, classifications, and improving audience enjoyment. We will present our latest results and also suggest ways how these ideas might be adopted beyond dance performances. These include classroom activities and perhaps related attention-producing technologies dominated by configurations changing in space and time. (Received September 26, 2017)

- 1135-F1-3156 **Darrah P. Chavey\*** (chavey@beloit.edu), Beloit College, 700 College St., Beloit, WI 53511. *Symmetries in the Woven Tunics of Oaxaca, Mexico*. Preliminary report.

The clothing weavers of the state of Oaxaca in Mexico create highly decorated "huipil" tunics, which generally consist of multiple geometric patterns woven together. As with many cultures that enjoy such designs, all 7 strip symmetry groups appear in the collection of huipils we investigated, but some strip patterns were substantially rarer than others. There appear to be weaving reasons for some of the difference in frequencies. One strip pattern, pmm2, essentially only appears on the shoulder bands of these tunics, and may be related to a desire for a 3-dimensional "front-to-back" symmetry imposed upon the symmetries that appear in a more "standard" Oaxaca design. A surprisingly large number of variations of strip symmetries appear in these huipil, including 3-color and 4-color symmetry; combinations of adjacent strips with varying symmetries; and deliberate symmetry breaking. One huipil gives the impression of a weaver consciously playing with, or experimenting with, glide

reflections. We will look at this collection of huipil tunics as an example of the interaction between geometric art and the cultural standards of the Oaxaca weavers. (Received September 26, 2017)

1135-F1-3199 **Gregg Helt\*** ([gregghelt@gmail.com](mailto:gregghelt@gmail.com)). *Mandelboxen: mathematical extensions to the artistic toolkit for 2D and 3D Mandelbox fractals.*

The Mandelbox is a recently discovered class of escape-time fractals that use a conditional combination of reflection, spherical inversion, scaling, and translation to transform a point under iteration. Although most artistic explorations of the Mandelbox have focused on 3D versions, it can be generated in any number of dimensions. In this presentation we first review existing Mandelbox variants, then introduce new data visualization techniques to help explore versions of the 2D Mandelbox. We then propose several new extensions to the 2D Mandelbox. Perhaps most intriguing is the introduction of shape inversion (or pseudoinversion) as a substitute for spherical inversion. Shape inversion generalizes standard spherical inversion to other shapes, and although shape inversions are not conformal transformations, they preserve many other properties of spherical inversion. We further explore applying shape inversion and other new extensions to the 3D Mandelbox. Source code for the presented work is freely available on GitHub. (Received September 27, 2017)

## *Mathematical Knowledge for Teaching Grades 6–12 Mathematics*

1135-G1-679 **Alyson E. Lischka\*** ([alyson.lischka@mtsu.edu](mailto:alyson.lischka@mtsu.edu)), 1301 E. Main Street, Box 34, Murfreesboro, TN 37132, and **Candice M. Quinn**. *MODULE(S2) Project: Developing Prospective Secondary Mathematics Teachers' Mathematical Knowledge for Teaching in College Geometry.*

The MODULE(S2) Project (NSF-IUSE Award #1726723) is focused on developing mathematical knowledge for teaching (MKT) in College Geometry, Statistics, Abstract Algebra, and Modeling through simulations of teaching practice incorporated in materials for these courses. By engaging prospective teachers in simulations of teaching practice as they develop content knowledge, we strive to also develop the knowledge they need to leverage their understanding of upper level mathematical concepts while doing the work of teaching. We will share data and experiences from the pilot study of the Geometry materials which indicate that the materials allow for an increase in MKT along with growth in understanding of the content. Both quantitative and qualitative data in the study point to an increase of MKT among students engaged with the materials, specifically in understanding student thinking. Prospective teachers generated more specific descriptions of the mathematical concepts and produced questions that were more specific to the mathematical content after engaging with the materials than they did at the onset of the course. Future work includes further development of these modules and those for Statistics, Algebra, and Modeling along with assessment of the effectiveness of implementation. (Received September 12, 2017)

1135-G1-733 **Cynthia Oropesa Anhalt\*** ([canhalt@math.arizona.edu](mailto:canhalt@math.arizona.edu)) and **Ricardo Cortez** ([rcortez@tulane.edu](mailto:rcortez@tulane.edu)). *A Report on Prospective Teachers' Development of Content Knowledge in Mathematical Modeling and the Use of Reflections to Promote the Emergence of Mathematical Knowledge for Teaching.*

Recent infusion of mathematical modeling in k-12 has generated research and teacher preparation activity to understand the modeling process as content and to develop mathematical knowledge for teaching (MKT) modeling. For mathematics majors in teacher preparation programs, this new activity is aligned with CUPM (2015) cognitive and content modeling recommendations. Given that most prospective teachers (PTs) are not familiar with mathematical modeling, there is an urgent need to design ways for them to simultaneously develop modeling content knowledge and MKT. This presentation focuses on PTs' development of specific modeling competencies that indicate both the possession of content knowledge and the practice of "modeling thinking" as a cognitive goal for PTs to develop MKT. We report on the implementation of a mathematical modeling module in secondary teacher preparation courses that emphasizes multiple solution paths to open-ended problems and assumption making to develop modeling competencies combined with reflections on teaching. We will present results from studies with PTs using theoretical frameworks for MKT (Ball, Thames, and Phelps 2008, *Journal of Teacher Education*, 59(5), 389-407) and competencies (Maaß 2006, *ZDM*, 38(2), 113-142). (Received September 13, 2017)

- 1135-G1-839 **Nicholas Wasserman\*** ([wasserman@tc.columbia.edu](mailto:wasserman@tc.columbia.edu)), **Keith Weber**, **Juan Pablo Mejia-Ramos** and **Tim Fukawa-Connelly**. *Designing Real Analysis courses for Secondary Mathematics Teachers*.

Real analysis is frequently a required course for prospective secondary mathematics teachers. However, most teachers view real analysis as unnecessary and unrelated to the work of teaching secondary mathematics. In accord with an instructional model for improving the teaching of advanced mathematics courses for teachers, we implemented a real analysis course that framed content by ‘building up from’ and ‘stepping down to’ teaching practice. In this session, we exemplify our approach with materials from two sample modules. These modules cover proofs of the algebraic limit theorems for sequences and of various derivative rules (e.g., product rule), and elaborate on connections made to secondary mathematics teaching. In addition, since we consider teachers’ mathematical knowledge primarily in relation to their own teaching practices, we report on data from subsequent classroom observations for six secondary teachers. The observation data provide evidence that what they learned from the real analysis modules was useful for informing their pedagogical practice. We discuss the instructional approach in this real analysis course and its potential implications for secondary mathematics teacher education. (Received September 15, 2017)

- 1135-G1-854 **Marshall Gordon\*** ([mgordon417@aol.com](mailto:mgordon417@aol.com)), 5611 Mirrorlight Place, Columbia, MD 21045. *Mathematics Education of Teachers - Developing a Language for Engaging Mathematics Problems*.

MET II [CBMS, 2012] locates a most pressing concern regarding the education of mathematics teachers in grades 6-12. It is that “A primary goal of a mathematics major program is the development of mathematical reasoning skills. This may seem like a truism to higher education mathematics faculty to whom reasoning is second nature. But precisely because it is second nature, it is often not made explicit in undergraduate mathematics courses [p. 55].” In the absence of mathematics faculty making explicit means to engage mathematics problems, students are left to their own devices, as texts provide the most elegant/efficient formal expression, which means that whatever is learned that is helpful in mathematics problem solving, it will be of an informal nature. The proposed presentation will focus on solving mathematics problems by highlighting mathematical habits of mind that enable students to engage productively problems that lack an explicit procedure for solution. The argument will be made is that mathematical habits of mind need to be considered as content for a productive education of mathematics teachers. (Received September 15, 2017)

- 1135-G1-1027 **Jennifer Earles Szydlik\*** ([szydlik@uwosh.edu](mailto:szydlik@uwosh.edu)), Math Department, UW Oshkosh, 800 Algoma Blvd, Oshkosh, WI 54901. *Number Talks: Building Numeracy and Conceptual Understanding Ten Minutes at a Time*.

A Number Talk is a short class discussion around a particular problem that is designed to elicit invented algorithms, improve mathematical communication and develop numeracy and mathematical creativity (see Humphreys and Parker 2015). This instructional tool has become an important part of many school math classrooms. In this talk, we will engage in a Number Talk, and we will discuss how to use this tool in courses for pre-service middle and high school teachers to strengthen their mathematical knowledge for teaching. (Received September 18, 2017)

- 1135-G1-1101 **Theresa A. Jorgensen\*** ([jorgensen@uta.edu](mailto:jorgensen@uta.edu)), **Kathryn Rhoads** and **James Mendoza Alvarez**. *Preservice Secondary Mathematics Teachers’ Conceptions of Functions and Equations*.

Research supports that future secondary mathematics teachers must learn specialized knowledge for teaching mathematics (MKT), connect advanced mathematics topics to school mathematics, and engage in inquiry-based learning. During an early undergraduate mathematics course experience with an eight-week unit on functions from an advanced perspective, participants (n=24 mathematics majors seeking secondary mathematics teaching certification) completed a ten-item pre- and post-assessment on functions and equations. We highlight a research-based exploration that illustrates an inquiry approach to addressing MKT and connections between advanced mathematics and school mathematics, focusing on distinctions between the concepts of function and equation. In addition, we discuss six think-aloud, hour-long interviews on functions and equations. Open coding techniques were used to identify emerging categories that describe participants’ distinctions between the concepts. Preservice teachers struggled to explain connections and differences between the two concepts, which has implications for curriculum revisions that address this MKT. (Received September 19, 2017)

1135-G1-1464 **Robert A Powers\*** ([robert.powers@unco.edu](mailto:robert.powers@unco.edu)), School of Mathematical Sciences, 501 20th Street, Campus Box 122, Greeley, CO 80639-0098. *Implications of Expert Teachers' MKT of Analyzing Exponential Functions Tasks on the Preparation of Future Secondary Mathematics Teachers*. Preliminary report.

Our current work endeavors “to understand expert secondary mathematics teachers’ knowledge for teaching exponential functions” (Oehrtman, Novak, Parker, & Powers, 2015, p. 1). We focus on exponential functions because of the increased attention in the Common Core State Standards (National Governors Association Center for Best Practices, 2010), especially during the first year of algebra. As a result, deep content knowledge for teaching exponential functions is necessary for all preservice secondary mathematics teachers, including those who plan to teach functions in grade 8 (CBMS, 2012). Part of our work investigates the decisions expert teachers make analyzing an exponential functions task (Mathalicious, 2017) and indicates these teachers draw on their knowledge gained while teaching (Parker, Troutd, & Powers, in progress). The goal of this session is to present a critical reflection of mathematical knowledge needed in the preparation of secondary teachers based on the evidence from expert secondary mathematics teachers. The session will conclude with recommendations for improved development of the mathematical knowledge for teaching of future secondary mathematics teachers. (Received September 22, 2017)

1135-G1-1826 **Christina Eubanks-Turner\*** ([ceturner@lmu.edu](mailto:ceturner@lmu.edu)), Loyola Marymount University, Department of Mathematics, 1 LMU Drive, University Hall, Suite 2714, Los Angeles, CA 90045. *Connecting the Common Core Math Practices to Mathematical Proof in a course for Middle and Secondary Teachers*. Preliminary report.

In this talk I will present findings related to work done with teachers who took a graduate-level Introduction to Mathematical Proofs course for Middle and Secondary Teachers. The talk will focus on teacher work from a culminating project that asked teachers to create a classroom lesson focused on one of the mathematical practices and also tell how that particular practice connects to the work done in a graduate-level introduction to mathematical proofs course. I will show examples of student work and discuss implications of this work to Math Teacher Educators. (Received September 25, 2017)

1135-G1-1920 **James Álvarez, Elizabeth G. Arnold, Elizabeth A. Burroughs\*** ([burroughs@montana.edu](mailto:burroughs@montana.edu)) and **Elizabeth Fulton**. *The Mathematical Education of Teachers as an Application of Undergraduate Mathematics*.

The Mathematical Education of Teachers as an Application of Undergraduate Mathematics (META Math) is a project to create, pilot, and field-test materials for use in undergraduate mathematics and statistics courses taken by pre-service teachers. Materials in calculus, discrete mathematics, algebra, and statistics showcase vital connections between college mathematics and the mathematics taught in high school. Drawing on recommendations in the *Mathematical Education of Teachers II* (CBMS, 2012) and the *Statistical Education of Teachers* (ASA, 2014), the project puts attending to the needs of pre-service teachers on par with attending to the needs of other undergraduate students by focusing on applications related to high school mathematics teaching. These applications will benefit all undergraduates in these courses, not just those preparing to be teachers. (Received September 25, 2017)

1135-G1-1962 **Priya V Prasad\*** ([priya.prasad@utsa.edu](mailto:priya.prasad@utsa.edu)), Department of Mathematics, The University of Texas at San Antonio, One UTSA Circle, San Antonio, TX 78249, and **Steven Boyce** ([sboyce@pdx.edu](mailto:sboyce@pdx.edu)), Neuberger Hall, Room 334, 724 SW Harrison Street, Portland, OR 97207-0751. *Supporting prospective teachers' understandings of triangle congruence*.

We will discuss our collaborative efforts to design instruction to support prospective secondary mathematics teachers’ (PTs’) construction of mathematical content knowledge for teaching a “transformations-first” approach to triangle congruence criteria. We conducted a design experiment to support our students’ understanding of the CCSSM standard: “Explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motions.” Our aim was to perturb our students’ existing understandings of triangle congruence criteria so that they might construct transformational proof schemes for them (Harel, 2013). We conjectured that after learning that some Euclidean isometries are not isometries in the taxicab metric, our PTs would understand triangle congruence criteria in Euclidean geometry as a result of the properties of isometries and thus be better-prepared to support their future students in constructing that way of thinking about triangle congruence criteria. We will discuss how PTs went from interpreting congruence criteria as axiomatic to referring to the properties of Euclidean isometries in their justifications for the congruence criteria. (Received September 25, 2017)



- 1135-G1-2065 **Andrew M. Ross\*** (aross15@emich.edu), **Stephanie Casey**, **Randall Groth**, **Rita Zejnullahi** and **James Albert**. *Prioritizing Statistical Knowledge for Teaching: Designing and Testing a Curriculum Module on Categorical Association.*

We will share our work in designing, implementing, and evaluating a curriculum module for pre-service teachers that addresses statistical knowledge for teaching categorical association (e.g., contingency tables, graphical representations of data, and chi-squared testing) in introductory statistics classes. Categorical association is an important topic taught at both the middle and high school levels, as required by the Common Core State Standards in Mathematics. As our module introduces pre-service teachers to the required content, it also has them consider aspects of how to teach it to their future students, addressing all six domains in the Mathematical Knowledge for Teaching (MKT) framework. We will present portions of the curriculum module and data from pilot studies showing how pre-service teachers performed both before and after using the materials, particularly on creating appropriate graphs, interpreting and responding to student-created graphs, and identifying potential student misconceptions. This curriculum will become part of a larger NSF-funded project, MODULE(S2), that is creating secondary teacher education materials for college classes in statistics, geometry, modeling, and abstract algebra. (Received September 25, 2017)

- 1135-G1-2085 **Ashlee LeGear\*** (ashlee.legear@uwrf.edu), 214C North Hall, River Falls, WI 54022, and **Erick Hofacker** and **Sherrie Serros**. *Changing Classroom Expectations and Culture through Mathematical Modeling.*

When the Common Core State Standards of Mathematics were adopted by many states in the U.S., it provided a new spotlight on the importance for implementing mathematical modeling in the secondary curriculum. Through the infusion of modeling into the classroom, it provides students a new lens for looking at and solving meaningful problems. By introducing students to the mathematical modeling cycle, it suggests to them that mathematics problems should involve choice and be completed throughout multiple iterations.

Work being completed within a professional development project involving 20 secondary mathematics teachers will be shared. We will showcase how teachers can transfer a word problem to a modeling-based problem. Rich tasks will be presented to show the types of opportunities that teachers need to engage in so that they see the importance and need to bring these types of problems into their classroom. (Received September 25, 2017)

- 1135-G1-2134 **Melissa L Troutd\*** (melissa.troutd@unco.edu) and **Jodie Novak**. *Developing prospective secondary mathematics teachers' knowledge of exponential functions through engaging in multiplicative reasoning and the work of teaching.* Preliminary report.

To better understand what it takes to unpack prospective secondary mathematics teachers' understandings of secondary mathematical content and how it can be leveraged into mathematical knowledge for teaching, we collected data from a course intended to support prospective secondary teachers' (PSTs) understanding of content and ways of reasoning relevant to grades 7-12 mathematics. One emphasis of the course was using multiplicative reasoning to make sense of situations or tasks involving exponential growth and decay. The PSTs also engaged in the work of teaching by analyzing tasks for potential learning and articulating mathematical ideas relevant in solving tasks involving exponential growth. We are investigating the hypothesis that the focus on multiplicative reasoning and the engagement in the work of teaching can support the PSTs in making meaning of exponential functions ideas and in their ability to recognize what mathematical ideas are relevant to a mathematical task. We have evidence the PSTs entered the course with little proclivity to solving or discussing the thinking involved with solving exponential functions tasks without the use of algebraic formulas. In this session, we plan to discuss the results of further analysis and the implications of this work. (Received September 25, 2017)

- 1135-G1-2263 **Steven R LeMay\*** (slemay@fairfield.edu) and **Fabiana Cardetti** (fabiana.cardetti@uconn.edu). *Argumentation as a Habit of Mind in the Preparation and Professional Development of Teachers.*

Mathematical argumentation has recently received more prominent attention in K-12 classrooms which has immediate consequences in the preparation and professional development of teachers, including the critical intersection with representing mathematical concepts. It is important to understand teachers' perceptions of this intersection at all levels as they have a significant impact not only on the skills but also on the habits of mind that their students develop with respect to mathematical argumentation. This talk discusses results from a qualitative study that investigated (1) how educators conceptualized argumentation, (2) the role(s) and purpose(s) they attribute to their representations within argumentation, and (3) the criteria for representations they use/offer when arguing claims of generality. Moreover, this talk discusses the results' implications on the preparation and professional development of middle and high school teachers with respect to mathematical argumentation and proof. (Received September 25, 2017)

1135-G1-2467 **Yvonne Lai\*** ([yvonnexlai@unl.edu](mailto:yvonnexlai@unl.edu)). *Understandings that Prospective Secondary Teachers bring to Geometry from a Transformation Viewpoint*. Preliminary report.

With the Common Core's demand to change approach to school geometry from traditional to transformational, there is a gap between how geometry teaching and learning has most often been considered and what is needed to prepare future geometry teachers. In this talk, I will discuss key issues in congruence and similarity proofs from a transformation viewpoint, and how these issues are “new” in the sense that they differ from the knowledge, skills, and dispositions needed for traditional congruence and similarity proofs. The analysis that I present is based on examining the understandings and challenges of 15 pre-service teachers over 13 weeks of learning geometry from a transformation approach after learning geometry from a traditional approach the prior semester. I will additionally provide an overview of mathematical and educational literature on geometry from a transformation viewpoint, tying this together with recent results on designing tasks to elicit teachers' mathematical knowledge for teaching, and how these results informed the tasks used in the course. I will share tasks that “worked” as well as tasks that did not and why, lessons learned, and implications for structuring a course for teachers on geometry from a transformation approach. (Received September 26, 2017)

1135-G1-2484 **Cody L Patterson\*** ([cody.patterson@utsa.edu](mailto:cody.patterson@utsa.edu)), Department of Mathematics, One UTSA Circle, San Antonio, TX 78249, and **Rebecca McGraw** ([rmcgraw@math.arizona.edu](mailto:rmcgraw@math.arizona.edu)), 617 N. Santa Rita Ave., PO Box 210089, Tucson, AZ 85721. *Making sense of students' thinking about graphing, covariation, and linearity*.

Current standards for high school mathematics call for students to recognize a linear function as one in which uniform changes in the input variable correspond to uniform changes in the output variable (e.g., CCSSM.HSF-LE.A.1). We investigated teachers' and students' understanding of this notion of linearity by recording their work on a task that calls for a graph of the relationship between two quantities that vary nonlinearly with time, but that are linearly related to each other. Video interviews with students working on this task revealed ways of thinking about graphing and covariation that, by turns, helped and hindered them in discovering the invariant linear relationship between the two quantities. We argue that explicit attention to these ways of thinking in the preparation of secondary mathematics teachers may better equip them to help students learn to analyze quantitative relationships and recognize linear functions in context. (Received September 26, 2017)

1135-G1-2864 **Carren Senn Walker\*** ([carren@me.com](mailto:carren@me.com)), CRMSE, 6505 Alvarado Rd. #201, San Diego, CA 92129, **William Zahner**, CRMSE, 6505 Alvarado Rd. #201, San Diego, CA 92120, and **Lynda Wynn**, CRMSE, 6505 Alvarado Rd. #201, San Diego, CA 92120. *Examining the Development of the Concept of Slope in one CCSSM-aligned Secondary Mathematics Text with a Focus on Enhancing Access for Linguistically Diverse Students*. Preliminary report.

We present the results of our conceptual analysis of linear slopes as represented in one Common Core-Aligned Secondary Mathematics Text (Cuoco & Kerins, 2013) as part of a project to redesign secondary mathematics content for greater accessibility for linguistically diverse learners. To experts, the slope of a line might appear as a single, unified concept. Yet, our analysis revealed three distinct conceptions of slope as developed in the text. Each of these three conceptions induces different forms of mathematical and linguistic complexity. One conception is slope as a measure of steepness. This conception quantifies everyday terms like “steeper” and “flat.” A second conception is slope represents a rate of change in a quantitative relationship. This conception is realized in graphs of real-world contexts, and connects to calculus concepts. Third, the slope can be conceptualized as an invariant property of collinear points. This conception emphasizes the idea of constant slope and collinear points. In our analysis, we explore the affordances and constraints of these three conceptions and, in particular, we consider the conceptual and linguistic demands for each conception of the slope of a line. (Received September 26, 2017)

1135-G1-2887 **Stacy Musgrave\*** ([smmusgrave@cpp.edu](mailto:smmusgrave@cpp.edu)). *Teachers' Structural Reasoning with Algebraic Expressions and Equations*.

One of the mathematical practices being promoted in today's mathematics classrooms is reasoning structurally. In this presentation, I highlight what it means to reason structurally about expressions and equations. I draw upon data from Project Aspire (PI, P. Thompson), a research program that developed an diagnostic tool, the Mathematical Meanings for Teaching secondary mathematics (MMTsm), to characterize secondary teacher's mathematical meanings for foundational ideas in the high school curriculum. (Received September 26, 2017)

## *Innovative Curricular Strategies for Increasing Mathematics Majors*

1135-G5-401 **Ivan Dungan\*** ([gregory.dungan@usma.edu](mailto:gregory.dungan@usma.edu)) and **Randy Boucher** ([randy.boucher@usma.edu](mailto:randy.boucher@usma.edu)). *An Alternative Calculus Sequencing for an Undergraduate Core Mathematics Program.*

We will discuss a calculus sequencing where an introductory ordinary differential equations course is introduced immediately after a single variable calculus course. This is a slight variation of the usual sequencing of calculus courses, but we propose that the benefits are great especially when modeling real-world phenomena is integrated into the course. We will highlight these benefits which give students an early understanding of the power of mathematics in the academic and professional world with hope to create more math majors and in general, more STEM graduates. (Received August 31, 2017)

1135-G5-1108 **Kimberly J Presser\*** ([kjpres@ship.edu](mailto:kjpres@ship.edu)), 23 Glenwood Drive, Biglerville, PA 17307, and **Luis Melara** ([lamelara@ship.edu](mailto:lamelara@ship.edu)) and **Douglas Ensley** ([deensley@ship.edu](mailto:deensley@ship.edu)). *Entry Year Experience for New Mathematics Majors: Creating and refining a course tailored to your mathematics program.* Preliminary report.

Entry Year Experience (EYE) classes come in a wide variety of shapes and forms. While many schools have EYE programs which focus on broad topics and acclimating to college in general, the focus for this talk is on the development of an EYE program designed to focus specifically on acclimation to the mathematics major. At Shippensburg University, we are currently in the third iteration of our EYE course. The class was created to give students a broader view of mathematics at the start of their program of study. To address the difficulties of both internal and external transfers, this EYE is required of all students who are new to the major – whether freshmen, transfers or students who have changed majors. In addition to perspective, the course focuses on the development of skills which are needed, but not central to the coursework in their mathematics program of study. This presentation will focus on our motivations for creation of the course, goals and objectives for the course, outcomes, obstacles, and future plans. (Received September 20, 2017)

## *Flipped Classes: Implementation and Evaluation*

1135-H1-172 **John A Kerrigan\*** ([johnkerr@rci.rutgers.edu](mailto:johnkerr@rci.rutgers.edu)), 110 Frelinghuysen Road, Piscataway, NJ 08854. *Productive Failure in the Flipped Mathematics Classroom.* Preliminary report.

Over the past few years, the flipped classroom approach has been gaining popularity in higher education (Abeysekera & Dawson, 2015), particularly in mathematics (Muir & Geiger, 2015). While many studies have addressed differences between the flipped classroom and traditional methods of instruction, few have closely examined how to design activities in a flipped classroom that develop students' higher-order thinking skills (O'Flaherty & Phillips, 2015; Song & Kapur, 2017). Kapur's (2008) theory of productive failure states when students have an opportunity to generate and explore solutions to a challenging task prior to being instructed on it, they are better positioned to consolidate their knowledge during and after instruction. This mixed methods study involves two undergraduate flipped math courses at a large state University, one of which was taught using the productive failure model and the other the traditional flipped classroom model. Quantitative data from a survey and from course assessments will be used to help explain students' performance in both treatment conditions, while qualitative data from a focus group interview and video footage of in-class problem solving will be used to better understand the learning and problem-solving processes in both treatment conditions. (Received August 07, 2017)

1135-H1-329 **Benjamin V.C. Collins\*** ([collinbe@uwplatt.edu](mailto:collinbe@uwplatt.edu)). *Flipping the Precalculus Classroom: A Quasi-Experimental Study.* Preliminary report.

In this quasi-experimental study, I taught parallel sections of Precalculus, one with a flipped classroom and one with traditional lectures, holding all other variables constant (as much as possible). I compared attendance and completion of assignments, performance on exams, and overall grades. I also distributed a survey to compare student attitudes towards the course. This is a follow-up to my study of the flipped classroom in Precalculus, reported at MathFest 2017. (Received August 24, 2017)

1135-H1-448 **Mike Janssen\*** ([mike.janssen@dordt.edu](mailto:mike.janssen@dordt.edu)), 498 4th Ave NE, Sioux Center, IA 51250. *A first experience in a flipped Calculus II course.*

In this talk, we will discuss a first experience of flipping a standard second semester course in calculus. We will describe the day-to-day rhythms of the course and the tools and resources used to create the pre-class activities, as well as the course assessment structure. Mid- and post-semester student responses will also be shared. (Received September 04, 2017)

1135-H1-474 **Barbara J Wilkins\*** ([bwilkins@mst.edu](mailto:bwilkins@mst.edu)) and **Paul N Runnion** ([prunnion@mst.edu](mailto:prunnion@mst.edu)). *Digging Out of the Hole – One Solution for Struggling Calculus Students.*

Struggling students are frequently the least likely to seek out assistance or work through extra problems, and often find themselves in an insurmountable hole by midterm. These students generally learn best by doing rather than watching, and to assist these students, we offer a half-semester calculus recovery course using a flipped and blended format where video content is included but not emphasized. Outside of the classroom, students are guided through a variety of research-driven activities which help develop their ability to read mathematics and introduce a variety of student success skills. In the classroom, students are guided through small-group problem solving activities. It is our goal that this course will equip students for their collegiate careers rather than just future calculus coursework. This course has now been offered for two full years, and we will discuss how the structure has evolved and its impact on our students. (Received September 05, 2017)

1135-H1-948 **Sheeva Doshireh\*** ([sdoshire@gmu.edu](mailto:sdoshire@gmu.edu)). *Learning Assistants' Roles in Flipping Large Classrooms.*

Over the past few semesters, learning assistants have worked with professors to create flipped classrooms with large class sizes averaging around ninety students per class. Flipped classrooms have a unique structure, generally consisting of the lecture portion of the class being completed outside of the classroom through videos or notes, while in-class time is spent working with students through group work and practice problems. The presentation will discuss the in-class procedures and outside-of-class preparations, examine the professors' and students' views of flipped classrooms, and how the learning assistant program is key to flipping a large classroom. (Received September 17, 2017)

1135-H1-1126 **Vinod Arya\*** ([vinod.arya@untDallas.edu](mailto:vinod.arya@untDallas.edu)), University of North Texas at Dallas, 7400 University Hills Blvd., Dalla, TX 75077. *Developing and Implementing a Modularized Flipping-The-Class Model.*

A large number of institutions are struggling with low success and retention rates in Mathematics courses. To deal with this challenging problem, several educational strategies such as Emporium model, have been developed and successfully employed by a large number of educational institutions. A model showing considerable promise and success is the flipping-the-class model. This paper presents a self-paced, modularized flipping-the-class (Emporium based) model of instruction developed by the author and implemented for enhancing the student retention and success rate at UNTD. The model has been modified to specifically suit the needs of students who struggle in Gatekeeper Mathematics courses such as College Algebra and Calculus. Consequently, this flipping-the-class model adopts a modularized structure where the entire course content is divided into an appropriate number of modules. As a pilot program, this modularized flipping-the-class model was developed and adopted for a Gatekeeper - College Algebra course. The benefits of employing the model, in terms of enhanced student success rates and performances, have been assessed and the relevant data exhibiting the success of the model in achieving its objectives is presented. (Received September 19, 2017)

1135-H1-1347 **Rebecca Swanson\*** ([swanson@mines.edu](mailto:swanson@mines.edu)), 1015 14th St., Applied Mathematics and Statistics - CSM, Golden, CO 80401, and **Deb Carney**. *Flipping Calculus II - Creating Materials that Others Can Use.* Preliminary report.

Many of us teach in multi-section settings where we may want to flip our own classroom, but we either do not want to or cannot require the same of others teaching the same course. The authors developed a flipped model for Calculus II that involves pre-class videos, pre-class questions, in-class activities, and out-of-class homework/exams. This model was implemented in a multi-section setting beginning in fall 2016, and materials were designed with the intent that although flipping is not required of other instructors, the materials would be easy for other faculty to choose to use, while at the same time maintaining the coordinated common homework and exam schedule with instructors who chose not to flip their classes. In this talk, we will discuss our model design, how we developed daily activities, and some of our preliminary data, including information regarding both student attitudes and the extent to which students attained course learning outcomes. (Received September 21, 2017)

1135-H1-1379 **James T Sandefur\*** ([sandefur@georgetown.edu](mailto:sandefur@georgetown.edu)). *Flipping a Proof Class using Faculty and Student Videos.*

We discuss a Proof class taught using a combination of short video lectures and short videos of similar students attempting to prove mathematical statements. The class is usually assigned reading and watching short video-lectures, so class time can be primarily spent working on developing proofs in groups. A unique feature of this course is the use of the videos of students attempting to write proofs. By reflecting on the thinking seen in these videos, both through classroom discussion and written assignments, students learn to reflect on their own thinking.

In one early lesson, the class is assigned to watch a video in which a pair of students develop a backwards proof of a statement. Each student then converts this informal proof into an acceptable written proof, forcing the class to focus on the actual writing process. In another lesson, students watch two pairs of students working on a set inclusion problem. Each student then writes a paper comparing the ways each pair of students worked together, how they approached the problem, and their final proof. This has resulted in students learning to be better group members and developing better problem solving skills. The IRB approved videos are available online. (Received September 21, 2017)

1135-H1-1381 **Casey Mann\*** ([cemann@uw.edu](mailto:cemann@uw.edu)), School of STEM, 18115 Campus Way N.E., Box 358538, Bothell, WA 98011. *Two Flipped Introductory Real Analysis Courses.* Preliminary report.

In winter 2017 we designed two flipped courses on introductory real analysis. One course was 100% flipped and was offered in a 50% hybrid online format, while the other course was 50% flipped and 50% lecture. Both courses used in-class time to help students collaboratively develop proof writing skills and to get started on what we called “mastery” homework problems (proofs in analysis) through group work, while pre-class time was spent on reading comprehension and basic computational and definitional learning. We will present the structure of these courses and how we used technology to facilitate the approach. We also provide data comparing final exam scores, homework averages, and GPAs of the students in the flipped courses with previous students taught in a traditional lecture format. (Received September 21, 2017)

1135-H1-1472 **Theresa Laurent\*** ([tlaurant@stlcop.edu](mailto:tlaurant@stlcop.edu)) and **Jamie Pace.** *Designing Pre-Class Activities for a Flipped Calculus Course Based on Learning Theory Principles.*

This presentation will review Lev Vygotsky’s Zone of Proximal Development (ZPD) and how it was used as a basis for the development of pre-class assignments in an applied calculus course. The authors will discuss the five-year process of refining these assignments to meet the needs of the students and make class time more productive. The content, structure and evaluation of these assignments evolved as a result of analyzing students’ participation and class preparedness. The authors will discuss how decisions were made on the content of videos and how they adapted that content to meet the needs of students with diverse backgrounds. Methods for assessing student understanding of pre-class content and methods for motivating students to complete these activities will also be discussed. (Received September 22, 2017)

1135-H1-1567 **Mike Weimerskirch\*** ([weim0024@umn.edu](mailto:weim0024@umn.edu)), 206 Church St. SE, Minneapolis, MN 55441. *Video Textbooks in the Active Learning Classroom.*

As our computational power increases with each new technological advance, our need for proficiency in higher-order thinking skills grows. That is the thought behind the flipped classroom, which enables instructors to focus on deeper thought processes during class time, while relegating routine tasks to readings, videos and practice that students do outside of the classroom. The standard approach is to record video lectures based on a printed text. A better approach is to begin the instruction with the video as the primary source of imparting information to students. Video offers a greater flexibility in presenting concepts and connecting ideas. The University of Minnesota now uses open source video textbooks to teach its pre-calculus courses in active learning classrooms. Data will be presented on the effectiveness of this initiative, which is now in its fifth year. (Received September 23, 2017)

1135-H1-1914 **Meghan M De Witt\*** ([mde Witt@stac.edu](mailto:mde Witt@stac.edu)), Sparkill, NY. *Course notes to augment a flipped classroom.*

The author presents the results of the past three years of running a flipped classroom in a pre-calculus class. This includes data on student performance as different techniques were implemented—including, but not limited to, the exact method of working problems in the classroom and the use of a course pack providing a structure for student note taking. Students are required to make use of the course pack, and it is purchased in place of a standard textbook. The course pack follows the material of the videos exactly ensuring notes are taken in a correct and efficient manner. (Received September 25, 2017)

1135-H1-2043 **Violeta Vasilevska\*** ([violeta.vasilevska@uvu.edu](mailto:violeta.vasilevska@uvu.edu)). *Enhancing the Hybrid College Algebra Course with a Flipped Classroom*. Preliminary report.

In the last few years, the presenter has been teaching a hybrid College Algebra class in a flipped classroom. For the on-line learning activities, students use MyMathLab. Students are asked to view the video lectures on-line and work on the on-line homework before they come to class. The in-class activities are divided in two parts. The first part is a question/answer segment. During this part, the instructor (presenter) does not lecture, but rather just helps answer questions about concepts/topics that caused difficulty to students at home. The second part is a problem-solving segment. Students are given a group quiz, which is a unique way to involve students in collaborative problem-solving active learning. This structure allows students to go over/review the new concepts several times, to encourage them to ask questions, to lead them to answer their own questions, and to spark communication between them. The presentation will demonstrate how the question/answer part of the class is conducted. In addition, results from student surveys about the structure and effectiveness of this course will be shared. Moreover, it will be discussed how student feedback has been incorporated in the structure of the class, and how those changes have impacted student learning. (Received September 25, 2017)

1135-H1-2193 **Ranthyony A.C. Edmonds\***, [Ranthyony-Edmonds@uiowa.edu](mailto:Ranthyony-Edmonds@uiowa.edu). *A Case for a Partial Flip: A Blended Model for a College Trigonometry Course*.

This talk describes a partially flipped model developed for a college trigonometry course in the Spring of 2017 at the University of Iowa. Motivation and implementation of the design will be described in detail. The model allowed for one day of flipped instruction, and two days of traditional lecture in a class that met Monday, Wednesday, and Friday. The main features of this model included instructional videos, created with Doceri for iPad, which were viewed outside of class once a week by students, coupled with a short assessment based on that instruction. The following 'flipped' period involved individual and/or group activities expanding upon concepts introduced in the videos. There were a wide range of activities, including worksheets, games, and team focused inquiry based learning activities. The most popular activities amongst students will be described in detail. Canvas by Instructure was used heavily throughout the course. Quantitative data with regards to assessments will be shared as well as the results of a qualitative survey given to students about their experience in the course. Possible extensions of this model for singular activities and/or other introductory courses will also be discussed, along with reflections on the successes and shortcomings of the model. (Received September 25, 2017)

1135-H1-2203 **Jennifer Elyse Clinkenbeard\*** ([jennifer.clinkenbeard@csuci.edu](mailto:jennifer.clinkenbeard@csuci.edu)). *Evaluating A Promising Practice: A Multi-Year Study of Student Outcomes in Flipped and Traditional College Algebra and Pre-Calculus*.

This study investigated the impact of flipping college algebra and pre-calculus classes with respect to key student outcomes. Both college algebra and pre-calculus are considered 'bottleneck' courses, with high enrollment demand but low success rates. This study took place at an urban public university comprised of about 40,000 students. The study included about 20,000 student records from 2010-2015. About 1300 of these students participated in a flipped section.

The flipped pedagogy for this study included three components: modular, interactive video lectures; a pre-assignment; and problem solving during class (Ichinose & Clinkenbeard, 2016). Identical assessments were used in both the traditional and flipped classes, and students did not self-select. There were no significant differences between flipped and traditional students on background measures, including high school GPA and math SAT score.

Significant differences between the flipped and traditional groups were observed in pass rates, course grades, and common exam scores, favoring the flipped groups in both college algebra and pre-calculus. Analysis with respect to gender and ethnicity showed that gaps in achievement in a traditional environment were smaller or nonexistent in the flipped environment. (Received September 25, 2017)

1135-H1-2413 **Sarah Dumnich\*** ([scdumnich@frostburg.edu](mailto:scdumnich@frostburg.edu)), 101 Braddock Rd, Frostburg, MD 21532. *Flipping Calculus II*.

Research shows that students in flipped classrooms outperform those in traditional lecture classrooms. These students also report higher levels of satisfaction with the course overall. With this in mind, I flipped my Calculus II course. I coupled this course structure with an emphasis on communication and leadership. Flipping the class gave me the time and opportunity to give formative feedback and highlight the importance of communication in mathematics. Through their leadership roles, students took ownership over their work. In this talk, I will discuss the structure of the class, the technology and text that were used, and include student feedback. (Received September 26, 2017)

1135-H1-2559 **Melissa Lindsey\*** ([melissa.indsey@dordt.edu](mailto:melissa.indsey@dordt.edu)), Dordt College, 498 4th Ave NE, Sioux Center, IA 51250. *Designing and assessing out-of-class activities in Technical Mathematics*. Dordt College launched a professional and technical program (Pro-Tech) in the Fall of 2017 offering two-year programs in Manufacturing Technology and Farm Operations Management. The mathematics course for these programs has two main goals: to ensure that all students have a foundation of basic mathematics skills and to introduce students to the ways in which they can apply those mathematical skills to their chosen vocations. In order to accommodate a wide variety of mathematical backgrounds and to ensure optimum time for exploring applications, a flipped learning strategy was implemented in this course. An overview of how the course was structured will be discussed with an emphasis on how the out-of-class assignments were designed, created, and assessed. In particular, the talk will explain how the out-of-class activities provided students with the ability to learn the basic mathematics skills needed at their own pace and prepared them to explore applications of those skills during class time. Though conceived with a specific demographic of student in mind, the techniques mentioned are applicable to most lower-level mathematics courses. (Received September 26, 2017)

1135-H1-2837 **Selin Kalaycioglu\*** ([sk168@nyu.edu](mailto:sk168@nyu.edu)), 251 Mercer St, New York, NY 10012, and **Matthew Leingang** ([leingang@nyu.edu](mailto:leingang@nyu.edu)), 251 Mercer St., New York, NY 10012. *A flipped classroom journey from all angles: Planning, creating, launching, evaluating, and improving*.

The flipped classroom model applied to Calculus I is one of the major technology-enabled education efforts at the NYU Courant Institute of Mathematical Sciences. We will share our journey with flipped Calculus I that started in Spring 2014. Included in the discussion is the technology used in the design of the flipped course content for both outside and inside the classroom, and various changes in implementation over time, supported by the data collected and several studies done to improve both the learning experience and outcome. (Received September 26, 2017)

1135-H1-2988 **James S Rolf\*** ([jim.rolf@yale.edu](mailto:jim.rolf@yale.edu)), **Brett Smith** and **Sarah Vigliotta**. *Flipping an Integral Calculus Course with Multiple Instructors*. Preliminary report.

Yale begin flipping Integral Calculus in the fall of 2013. Currently, all instructors of this course utilize the same pre-class materials in a coordinated course format. We report on the structure of our implementation, the challenges and successes of involving multiple instructors (including new instructors with little teaching experience) into a coordinated course environment using the same pre-class video and quizzing content to flip the course. We also report on student performance in the course, attitudes from student surveys and focus groups, and analysis of the downstream impact of this experience on students as they take other mathematics courses. (Received September 26, 2017)

1135-H1-2999 **Anne M Fernando\*** ([amfernando@nsu.edu](mailto:amfernando@nsu.edu)) and **Rhonda D Fitzgerald** ([rdfitzgerald@nsu.edu](mailto:rdfitzgerald@nsu.edu)). *Results from applying a Flipped model in Pre-calculus to Engage Students for Higher Retention and Building Stronger Foundations: Results after One Semester*. Preliminary report.

The overall goal of teaching pre-calculus using a flipped model is to increase the pass rate and to provide a strong foundation in the course to increase retention in STEM majors. Weaving major specific mathematics applications is easier with this approach. Pre-Class Implementation—In the days prior to class, students are provided with two ways to access links featuring mini-lectures introducing new topics. Each concept may be explained via several pre-recorded videos not to exceed 20 minutes in total. In-Class Implementation—The first focus is to assure that everyone has the basic prerequisite concepts for this class meeting (readiness assurance). The second will be to apply the knowledge acquired in the pre class assignment. The most vulnerable part of the learning process takes place in class. Post-Class Implementation—Online labs will be assigned per section via Cengage WebAssign. Out-of-class sessions with student instructors will focus on student driven problem solving and model successful learning strategies. Following an exam, students will answer a post-exam survey to help them reflect and focus on effective exam preparation strategies. In this Poster or talk we discuss some results after implementing this flipped approach for one semester in 2 pre-calculus sections. (Received September 26, 2017)

1135-H1-3038 **Rachelle Bouchat\*** ([rbouchat@iup.edu](mailto:rbouchat@iup.edu)), Indiana University of Pennsylvania, 210 South Tenth Street, Stright Hall, Room 233, Indiana, PA 15701. *Linear Algebra: A Flipping Success*. Preliminary report.

The course that will be discussed is a computational linear algebra course that is taken before an introduction to proofs class by freshman and sophomore level STEM majors. Historically, this class has been a gateway course for mathematics majors, causing many of them to change majors while enrolled in this course. I previously taught

the course using a traditional lecture style in 3 separate class sections over 2 different semesters (Spring 2015 and Fall 2016). In the Spring 2017 semester, I taught the course to 2 separate classes using a flipped classroom model. During this talk, I will share the structure and implementation of the flipped classroom environment within the course, including both expectations and evaluation methods. Additionally, I will present data on the increase in grades when the flipped classroom environment is compared with the traditional lecture classroom environment, as well as share comparisons of student feedback in the open response section of the student evaluations for these different classroom experiences. (Received September 26, 2017)

1135-H1-3051 **Darci L. Kracht\*** ([darci@math.kent.edu](mailto:darci@math.kent.edu)). *Using Collaborative Annotation to Flip a Trigonometry Course*. Preliminary report.

For many years I have grappled with the problem of getting students to engage with course material before coming to class. Students are generally poorly prepared to read mathematics and feel discouraged when asked to learn on their own. However, these are important skills to develop. Many of the methods I have tried, such as reading quizzes, worksheets, and video lectures, are labor intensive for the instructor. This semester I am testing a free collaborate annotation tool called Perusall, along with an open-source textbook in a section of Trigonometry. Students are given reading assignments to prepare for each class. They are required to annotate the electronic text with comments and questions. Students can see and respond to their classmates' annotations. The software automatically scores each student's annotations and creates a "Confusion Report" for the instructor, highlighting three or four of the most difficult topics. This opens up class time for group activities targeting the problem areas. I will report on students' and instructor's perceptions of the successes and failures of this approach. (Received September 26, 2017)

## *Humanistic Mathematics*

1135-H5-83 **Linda McGuire\*** ([lindamcguire@muhlenberg.edu](mailto:lindamcguire@muhlenberg.edu)), Department of Mathematics & Computer Science, Muhlenberg College, Allentown, PA 18104. *Understanding Mathematics using Historical, Cultural, and Sociological Perspectives from Feminist Theory*. Preliminary report.

This presentation will detail a general-education course designed to analyze and discuss women's contributions to mathematical and scientific knowledge, history, society and culture, both in American society and globally. How have women served both as subject to and producers of mathematical and scientific research? What contributions have they made to such fields? What role has gender played in the history of mathematics and science? What has the influence of feminism been on the ways we analyze and think about the practice, theories, and history of mathematics and science? This seminar class explored answers to these questions through an examination of social, political and mathematical/scientific influences as suggested by a series of thematic readings grouped into categories: the history of women in math and science, the scientific construction of gender, and the gendered construction of science (and scientists). The presentation will highlight the construction of the course, its relationship to a theatre course it was "linked" to, student assignments and assessment, and, most importantly, how students successfully integrated the study of mathematics with the study of feminist theory. (Received July 24, 2017)

1135-H5-135 **Brittany Anne Carlson\*** ([bcar1005@ucr.edu](mailto:bcar1005@ucr.edu)). *Victorian Puzzle Addiction: "The Final Problem" as a Mathematical Puzzle*.

"Victorian Puzzle Addiction: 'The Final Problem' as a Mathematical Puzzle" examines the social conditions leading to the popularity of the Sherlock Holmes canon and the Victorian fascination with puzzles found in both detective fiction and recreational mathematics. This paper argues that Sir Arthur Conan Doyle's "The Final Problem," uniquely functions as both detective fiction and a mathematical puzzle, forcing its audience to think beyond the text to derive a solution to what game theoretical scholars term the "Holmes-Moriarty Paradox." In "The Final Problem," Holmes and Moriarty allegedly arrive at their untimely deaths, with no witnesses, at Reichenbach Falls. The "Holmes-Moriarty Paradox" arises out of the tensions leading to their deaths at Reichenbach Falls when the audience is confronted with the question of who will prevail and how: Professor Moriarty, who is an unstoppable evil genius, or Sherlock Holmes and his untouchable facilities of logic. This paper asserts that although Conan Doyle attempts to transcend the bounds of the short story genre with a witty paradoxical puzzle to distract his fans from the loss of Holmes, it is a failure, forcing Conan Doyle to revive him in "The Empty House." (Received August 01, 2017)



1135-H5-374 **Izabel Aguiar\***, [izabel.p.aguiar@gmail.com](mailto:izabel.p.aguiar@gmail.com), and **Jessica Deters and Jacquie Feuerborn**. *The Mathematics of Gossip*.

In this project we develop a numerical model to investigate the spread of rumors, lies, and gossip throughout a community using an adapted susceptible-infected-recovered (SIR) dynamical system. We explore parameter dependence of the model in forms of various 'rumor spreaders'. We highlight how mathematics can be applied in creative ways to illuminate and explore new quantitative aspects of sociological systems. (Received August 29, 2017)

1135-H5-471 **Melanie Butler\*** ([mbutler@msmary.edu](mailto:mbutler@msmary.edu)), 16300 Old Emmitsburg Road, Emmitsburg, MD 21727. *Learning from the humanities: mathematics reading comprehension*.

This talk will build on connections between mathematics and the humanities to find takeaways for teaching students to read mathematics. Common themes from research will be used to motivate the need for reading comprehension instruction in college mathematics. Associations will be drawn between different research studies that apply to college mathematics instruction. In addition, the talk will give results of a research study involving interviews about reading comprehension strategies and instruction with interdisciplinary faculty from several institutions. (Received September 05, 2017)

1135-H5-670 **Steven M Deckelman\*** ([deckelmans@uwstout.edu](mailto:deckelmans@uwstout.edu)), 305 Jarvis Hall Science Wing, 410 10th Avenue East, Menomonie, WI 54751. *Incorporating Philosophy, Theology and the History of Mathematics in an Introduction to Proof Course*.

Kurt Gödel (1906-1978) is well known for this celebrated incompleteness theorem. Perhaps less well known is that he also composed at least two versions of an ontological argument for the existence of God using modal logic. His first formulation occurred around 1941 when he was still in his thirties although he did not make this public until 1970 when he discussed it with Dana Scott and Oskar Morgenstern at a time he thought he might be dying. Gödel never published it during his life time, though it now can be found in his collected works and Dana Scott presented it in a seminar at Princeton in 1970. We describe a project activity for an undergraduate proof course aimed at lower division mathematics and computer science majors that expands upon the usual propositional and predicate logic present in such courses by including modal logic. In this way connections are made with philosophy, theology and the history of mathematics through introducing undergraduate students to Kurt Gödel's proof of the existence of God. Moreover, as a pedagogy, the project involves a broader range of critical thinking skills than is usual in such courses and affords opportunities to expose students to L<sup>A</sup>T<sub>E</sub>X and Writing Across the Curriculum efforts. (Received September 18, 2017)

1135-H5-787 **Mark Branson\*** ([mbranson@stevenson.edu](mailto:mbranson@stevenson.edu)), Department of Mathematics, Stevenson University, 11200 Gundry Ln, Owings Mills, MD 21117. *Fighting Alternative Facts: Teaching Quantitative Reasoning with Social Issues*.

Mathematics has a unique and powerful role to play in the teaching of social justice issues. There is substantial quantitative evidence for social injustice, but many citizens lack the quantitative skills to understand that evidence. A course in quantitative reasoning is a unique opportunity to provide this quantitative understanding to a wide range of students in a general education context. Quantitative reasoning skills provide citizens with the tools they need to critically analyze misinformation and make good decisions about civic issues. This session will provide faculty with examples of ways to infuse social justice into their quantitative reasoning courses. (Received September 14, 2017)

1135-H5-911 **Satish C. Bhatnagar\*** ([bhatnaga@unlv.nevada.edu](mailto:bhatnaga@unlv.nevada.edu)), 308 CAVALLA STREET, HENDERSON, NV 89074-5976. *Life Values and Mathematics*.

The humanistic side of mathematics blooms only when mathematics is consciously practiced – whether through teaching, research, or service. However, it does take a couple of decades before mathematics and its humanistic side integrates in one's everyday life. Every concept and definition in mathematics can take on a humanistic ride – be it that of point, line, limit, continuity and differentiability, integration, linearly independence and dependence, groups, bases, and so on. In teaching at lower division level, the first challenge is to be able distill the humanistic side of a mathematical idea. A bigger challenge is about imparting it, as the students in these course are under stress of tight syllabi, homework, quizzes, and tests. They find any direct reference to some philosophy and/or history of mathematics unacceptable. Humanistic mathematics borders on psychology. In this paper, the example of Proper Choices of Axes, as introduced in analytic geometry and pre-calculus, is taken for the pedagogy of its humanistic values. It has to be done in 60-90 seconds! One must seize upon that humanistic moment and use one's communication skills to make a connection and then smoothly get back on the track – leaving some students wondering. (Received September 16, 2017)

1135-H5-1076 **Randall E. Cone\*** ([recone@salisbury.edu](mailto:recone@salisbury.edu)), 128 Henson Hall, Salisbury, MD 21801.  
*Visualizing the Mathematics of Hate.*

Hate is an ugly word, yet it is part of the Human Experience. In this presentation, we examine the use of computational mathematics in detecting hate speech within a variety of linguistic sources. Digital visualizations of such mathematics are then discussed, with the goal of finding early opportunities to counter hate speech through easy detection of defined, idiosyncratic markers. Philosophically, the point of this work is to foster early, even pre-emptive, positive communication between opposing groups. (Received September 19, 2017)

1135-H5-1099 **James M. Henle\*** ([jhenle@smith.edu](mailto:jhenle@smith.edu)), 105 Prospect St, Northampton, MA 01060. A  
*Studio Course in the Mathematical Art.* Preliminary report.

The objects of the mathematical art are mathematical structures. A mathematical structure succeeds as art if it attracts interest, if members of the mathematical public are drawn to it and want to explore it, if they find intriguing or elegant aspects of it. The class I taught last Spring was for students with no special mathematical background. I will present some of the most successful structures the students created. (Received September 19, 2017)

1135-H5-1252 **Manmohan Kaur\*** ([mkaur@ben.edu](mailto:mkaur@ben.edu)), Department of Mathematics, Benedictine University, 5700 College Road, Lisle, IL 60532. *An Inquiry General Education Mathematics Course for Students in the Humanities, Fine Arts, and the Social Sciences.* Preliminary report.

Over the centuries, mathematics has played an important role in almost every aspect of human development. Mathematics is more than crunching numbers and following rules. It is about finding patterns, solving problems, and thinking logically and critically. It is about interpolating, extrapolating, building models, and solving puzzles. In this presentation, we will discuss a course that presents the beauty and elegance of mathematics to liberal arts students in their first two years of college, by delving into some intriguing and ground breaking mathematical concepts. This survey course is designed to provide the students with a broader understanding and appreciation of mathematics. It has less focus on algebra, and more on discovery. The course satisfies the ‘quantitative and computational’ mode of inquiry requirement of the liberal arts curriculum at our institution, and is designed for students who do not require a mathematics course for their intended major. (Received September 20, 2017)

1135-H5-1657 **Jennifer Wilson\*** ([wilsonj@newschool.edu](mailto:wilsonj@newschool.edu)). *Data, Design and Social Justice.*

Writing On It All is a participatory writing project in which members of the public are invited to write on interior walls of out-of-use homes, or more recently, a building on my campus, scheduled for construction. The on campus event was facilitated by faculty and students in a writing class, who developed prompts by which participants could respond to recent political events on the themes of “the wall” and “sanctuary.” In this talk, I discuss the involvement of students in a “Data, Design and Social Justices” class who developed additional activities for participants of the Writing On It All project allowing them to think quantitatively and graphically about the impact of proposed legislative changes to immigration policy. Students created a number of interactive experiences including ways to visualize the cost of a projected wall along the U.S.’s southern border, and maps to visualize the journeys of different migrant groups around the globe. The project allowed Data, Design and Social Justice students to see how data visualization could be part of a larger discourse involving language, journalism, and political activism. (Received September 24, 2017)

1135-H5-1972 **Emelie A Kenney\*** ([kenney@siena.edu](mailto:kenney@siena.edu)), Department of Mathematics, 515 Loudon Road, Loudonville, NY 12211. *Clandestine Mathematics in Poland: The World War II Years.* Preliminary report.

Government interference in educational opportunities for citizens is particularly evident in dictatorial regimes. Because Poland was partitioned over a 123-year period among the Prussian, Russian, and Austro-Hungarian empires, education and knowledge of Polish literature, geography, culture, and history had to be conducted underground in what was known as the Flying (or Floating) University. World War II and, later, Soviet domination of Poland prevented a full educational experience for Poles, so clandestine education was practiced in those times, as well, even though secret scholars could be murdered if caught. A large number of Polish mathematicians, mathematics educators, and students of mathematics were involved in such activities in WWII. Here, we hope to illuminate the lives and works of some of the lesser known women participants in underground mathematics teaching and learning during the Nazi occupation. (Received September 25, 2017)

1135-H5-2909 **Felicia Yeung Tabing\*** (tabing@usc.edu), USC Department of Mathematics, 3620 S. Vermont Ave., KAP 104, Los Angeles, CA 90089. *Ikebana: A mathematical experience.*

Ikebana, the art of Japanese flower arranging, is an art that I have been interested in learning for years. This stemmed from my curiosity of the rules of Ikebana, as well as my interest in Japanese aesthetics and mathematics. After moving to Los Angeles for my new job, I finally have the opportunity to learn Ikebana, given my proximity to a variety of schools that teach this art form. As a mathematics educator, I try to take opportunities to put myself back in the experience of being a student, as a reminder of the student experience. Taking an Ikebana class has been a fruitful experience for me, not only as a relaxing and creative experience, but also mathematically. From the lens of someone trained in mathematics, I made connections with aspects of Ikebana to mathematics. I will describe the connections I made between the art of Ikebana, mathematics, and how the class gave me space to reflect on mathematics and teaching. (Received September 26, 2017)

1135-H5-3089 **Milos Savic\*** (savic@ou.edu), **Devon Gunter**, **Emily Curtis** and **Ariana Paz Pirela**. *Productive Failures: From Class Requirement to Fostering a Peer-led Support Group.*

Mistakes occur frequently in mathematics. In two classes (Abstract Algebra and Calculus II), mistakes were brought to the forefront in the form of a “productive failure.” Through five interviews with students, we initially looked for affectual responses to the pedagogical allowance and student-led demonstration. Many of the responses, both benefits and drawbacks of the productive failure, were interpreted by the research group to resemble peer-led support groups such as Alcoholics Anonymous. Descriptions of both productive failure and support groups, as well as quotes from the students, aim to shed light on psychological benefits of valuing mistakes. (Received September 26, 2017)

1135-H5-3191 **Brian P Katz\*** (briankatz@augustana.edu). *(De)Constructing Mathematical Authority.* Mathematics is a human endeavor, so it involves power and privilege. I think that this exclusionary system is sustained, in part, by our discipline’s curious views on authority and the ways that authority is constructed and recreated in mathematics classrooms. This talk will focus on the interconnected issues I have faced while trying to build a classroom community that critiques and resists this system. (Received September 27, 2017)

## *Mathematical Experiences and Projects in Business, Industry, and Government (BIG)*

1135-I1-316 **Paul E. Fishback\*** (fishbac@mail.gvsu.edu), Dept. of Mathematics, Grand Valley State University, 1 Campus Drive, Allendale, MI 49401. *A Mathematician, Engineer, and Brain Surgeon Walk into a Bar: Collaborating with Biomedical Engineers and Neuroscientists in the Study of Epilepsy.* Preliminary report.

Evaluation of an epilepsy patient’s candidacy for surgical intervention requires determining the portion of cerebral tissue that constitutes the seizure onset zone (SOZ). Recent research indicates that biomarkers of the SOZ include sources of high frequency oscillations and the presence of network community structures, all of which can be obtained from analyzing long-term electroencephalography (EEG) records. Mathematical tools used in identifying these biomarkers include wavelets and other integral transforms, time-frequency analysis, machine learning, and network science measures. This talk will provide a brief overview of how some of these tools are used as well as a reflection by the speaker on his experiences collaborating with individuals from the biomedical engineering and neurology disciplines. (Received August 23, 2017)

1135-I1-366 **William P. Fox\*** (wpfox@nps.edu), **Robert Burks** (reburks@nps.edu), **Bradley Greaver** (bradley.greaver@socom.mil), **Leo Raabe** (leopele.raabe@gmail.com), **Reed Kitchen** (rakiche@nps.edu), **Steven Powell** (sepowell11@nps.edu) and **Gregory Spence** (grspence@nps.edu). *CARVER 2.1: Weighting Methods.*

This talk compares the AHP weighting scheme to other weighting schemes for use in multi-attribute decision making processes to improve the CARVER center of gravity analysis that is currently used by Special Operations Forces. We employ the rank order centroid method, ratio method, and entropy methods for obtaining weights to compare to the pairwise comparison and equal weights methods. First, we show the standard current CARVER method as outlined in FM 34-36. Next, we apply several MADM methods using our suggested various weighting schemes to obtain the rankings of the alternatives. We compare the results and provide sensitivity analysis to examine the robustness of each MADM analysis. We conclude that any decision methodology for CARVER that includes a weighting scheme by each decision maker is better than not using any weighting scheme. (Received August 28, 2017)

1135-II-538 **James H Fife\*** ([jfife@ets.org](mailto:jfife@ets.org)), 660 Rosedale Road, Princeton, NJ 08541. *Capturing Equations for Automated Scoring—Typed or Handwritten.*

In this talk I will describe a project with two research questions: (1) Do high school students who enter responses on a computer, using an equation editor, perform as well as if they had handwritten the responses, and (2) Is handwriting recognition software sufficiently reliable to capture handwritten responses on an iPad for accurate automated scoring? Two parallel forms of an assessment were created; students were asked to copy a series of successively more complicated equations, ranging from simple arithmetic equations to the quadratic formula. Approximately 475 students were recruited from four high schools in different parts of the US; each student took each assessment, one by entering responses on a computer, using an equation editor, and one by handwriting responses on an iPad. The computer responses were captured as MathML, while the handwritten responses were converted by handwriting recognition software into MathML. The MathML from both forms of responses was then evaluated for correctness. In this talk we will discuss our findings. (Received September 07, 2017)

1135-II-786 **Robert E Burks\*** ([reburks@np.edu](mailto:reburks@np.edu)), 1 University Circle, Defense Analysis Department, Monterey, CA 93940, and **Peter Wilcox** and **Jeff Applegate**. *Modeling the Human Terrain*. Preliminary report.

Decisions about policies involving the defeat of the Islamic State is a tug-and-pull between stakeholders' competing interests, whether it's the Iraqi government and its various federations, the international community, or special interest groups where each stakeholder's preference varies in scope and intensity. This complex tug-and-pull scenario is just one of many examples of the expanding complexity of sub-national and cross-national threats to national security that strains the analytical capabilities of the Department of Defense (DOD), highlighting the need for methods and tools that can make this emergent complexity manageable. Today's computational power allows for sophisticated agent based simulations, with thousands of agents operating in a simulated environment to potentially address DOD shortcomings. This research effort looks at the potential use off the shelf technology to simulate the iterative political decision making calculus among stakeholders with different interests in and varying influence on the political process. This talk will review the current progress and the future potential to streamline and automate the necessary data collection for these models. (Received September 14, 2017)

1135-II-1137 **Aihua Li\*** ([lia@mail.montclair.edu](mailto:lia@mail.montclair.edu)), 1 Normal Avenue, Montclair, NJ 07043. *Emerging Data Science and Mathematics in Classroom – A PIC Math Approach.*

In my recent PIC Math class (Preparation for Industrial Careers in Mathematics), I organized 4 student groups each of them working on an industry problem given by our industry conduct. All projects dealt with data analysis and 3 of the 4 worked on large data. It is evident that data science techniques played an important role in solving these problems, which can be seen from the project titles: 1. Search for Nearest Correlation Matrices to Guide Risk Management of Stock Activities 2. Mathematical Modeling of Upper Tolerance Limits for Metal Concentrations in Certain Regions 3. Mathematical Modeling of Likelihood of Customers' Buying at Staples 4. Group Marketing Analysis of the Influence of Email Responses on DeBoer's Auto Sales and Services

When working on the projects, students must lean a software requested by the industry representatives and advanced methods to analyze large data. Python, R, and Matlab, were heavily used throughout. I will introduce the organization of the class, benefits students obtained and challenges students faced when working on the projects. Summary of results of the projects will be discussed. (Received September 19, 2017)

1135-II-1140 **Shirley L Yap\*** ([shirley.yap@csueastbay.edu](mailto:shirley.yap@csueastbay.edu)). *Vector Space Embeddings of Words*. Preliminary report.

As part of the PIC Math program, I worked with a team of students and the natural language processing company First Rain, to use artificial neural networks to help them model their documents. Artificial neural networks are machine learning models mimicking the behavior of axons in a biological brain. Neurons are organized in layers, with weights between each layer that control how connected one neuron is to another. Neural networks are a supervised learning system, where inputs are fed into a system with known outputs. When inputs are entered into the network, the outputs will result in an error function, which is minimized via gradient descent. Neural networks are powerful modeling systems so that even a two layers network can represent any bounded continuous function within an arbitrary degree of accuracy. In this project, we applied a neural network to a natural language processing problem. Specifically, we fed hundreds of thousands of text documents into a neural network in order to find synonyms for given glossaries. Our particular focus was to find the optimal embedding space dimension for the embedding space of the words (Received September 19, 2017)

1135-II-1497 **Joseph A Eichholz\*** (eichholz@rose-hulman.edu), 5500 Wabash Ave, Terre Haute, IN 47803. *Preparation for Industrial Careers in Mathematics at Rose-Hulman Institute of Technology*. Preliminary report.

In AY2015-2016 and AY2016-17 Rose-Hulman Institute of Technology offered a Preparation for Industrial Careers in Mathematics (PICMath) course to undergraduate students. 16 students enrolled across the two academic years and successfully completed 4 different projects. In this talk we detail our experience soliciting sponsors for project, coordinating effort with the sponsors, and shepherding students through the research experience. We hope to offer a perspective that is somewhat different from that of other schools, as many of our students are primarily engineering or computer science majors with a second major or minor in mathematics. In particular, we describe how our motivational and assessment techniques changed from year to year as we gained experience. (Received September 22, 2017)

1135-II-1651 **Uma Ravat\***, ravat1@illinois.edu. *Strategies for finding and incorporating projects from BIG in an undergraduate classroom*.

This talk will draw from the authors' experiences being a faculty participant in the PICMath (Preparation for Industrial Careers in Mathematical Sciences) program over the past two years. As part of the PIC Math program, we developed an industry collaboration course for undergraduates. In this course students worked in small groups to model and solve a problem of current interest to the industry sponsors. In this talk, we will discuss strategies for finding projects in BIG, tips on choosing projects and adapting them for incorporating in an undergraduate statistics classroom. We will also give a brief overview of projects undertaken and project outcomes, student reflections on completing the course and faculty and department reflections on the course and industry sponsor reflections. (Received September 24, 2017)

1135-II-2744 **Patrick Kuiper\*** (patrick.kuiper@usma.edu), MADN-MATH, United States Military Academy, 646 Swift Road, West Point, NY 10928, and **Karoline Hood**. *Improving Student Surveys With Natural Language Processing (NLP)*.

Stakeholders from academic institutions across the world employ surveys to assess the quality of their work. With surveys these stakeholders attempt to obtain quantified, structured, and directed data in order to make decisions. Often these stakeholders employ long, directed Likert scaled surveys to gain this information. We propose an alternate construction for academic surveys, where stakeholders provide 1-3 open ended "free text" questions, allowing students to lead the discussion. We call this survey methodology "Student Directed Discussion Surveys" (SDDS). SDDS retain the ability to provide quantified, structured, and directed results by employing Natural Language Processing (NLP). We confirm the accuracy of SDDS in relation to traditional Likert scaled surveys with a permutation test confirming a negligible statistical difference between SDDS and Likert surveys using real data. We then show the utility of SDDS by employing word frequency and sentiment analysis, providing important unbiased decision making information, which is limited when traditional Likert scaled surveys are administered. (Received September 26, 2017)

1135-II-3226 **Tracy A Bibelnieks\*** (tbibelni@d.umn.edu), 140 Solon Campus Center, 1117 University Drive, Duluth, MN 55812. *Data Analytics Competitions: The New PIC Math Classroom for Teaching Data Science*.

For the past three years I have mentored teams of undergraduate students in data analytics competitions as part of my participation as a faculty member in PIC Math (an MAA program). The competitions are month long student-focused analytics events inviting participants to explore real-world data while enhancing their skills. Students go through the process of cleaning data, visualizing and analyzing data, and then presenting results as a data story at a culminating competition. The beauty of these competitions is that the student teams own the problems and their data solutions. They are invested in acquiring technology skills that will allow them to chase their creative brainstorming to arrive at a solution to the problem. As the faculty mentor, I provide assistance in helping them learn the technology or mathematics needed but I don't prescribe the solution approach. The approach comes from student experiences in math/stats courses and external resources and ideas. This talk will share my experience in creating a novel type of a PIC classroom where, "I've learned more in this experience than I did in all of my math/stats coursework," shows up on the evaluation of the experience. (Received September 27, 2017)

## 20th Anniversary—The EDGE (Enhancing Diversity in Graduate Education) Program: Pure and Applied Talks by Women

1135-I5-569 **Nida Obatake\***, 4503 College Main St Apt B, Bryan, TX 77801, and **Elizabeth Gross** and **Nora Youngs**. *Place field diagrams of neural codes.*

A rat has special neurons that encode its geographic location. These neurons are called place cells and each place cell corresponds to a region in the space, called a place field. Neural codes are collections of the firing patterns of place cells. In this talk, we investigate how to algorithmically draw a place field diagram of a convex neural code, building on existing work studying neural codes, ideas developed in the field of information visualization, the toric ideal of a neural code, and the neural ideal and general relationship graph of a neural code. This talk is based on joint work with Elizabeth Gross and Nora Youngs. (Received September 08, 2017)

1135-I5-570 **Ryan R. Martin** and **Shanise Walker\*** ([shanise1@iastate.edu](mailto:shanise1@iastate.edu)). *A lower bound for a vertex-identifying code in  $(p, \beta)$ -jumbled graphs.*

Let  $N[v]$  denote the closed neighborhood of a vertex  $v$ . For a finite graph  $G$ , a vertex-identifying code in  $G$  is a subset  $C \subset V(G)$ , with the property that  $N[u] \cap C \neq N[v] \cap C$ , for all distinct  $u, v \in V(G)$  and  $N[v] \cap C \neq \emptyset$ , for all  $v \in V(G)$ . A graph  $G$  on a vertex set  $V$  is  $(p, \beta)$ -jumbled if, for all vertex subsets  $X, Y \subseteq V(G)$ ,  $|e(X, Y) - p|X||Y|| \leq \beta\sqrt{|X||Y|}$ , where  $e(X, Y)$  is the number of edges between  $X$  and  $Y$ . Let  $n$  be an integer,  $0 < p < 1$  where  $p$  is fixed, and let  $\beta = o(\sqrt{n \log_2 n})$ . We prove there exists an  $\varepsilon = o(1)$  such that if  $G$  is a  $(p, \beta)$ -jumbled graph on  $n$  vertices, then every vertex-identifying code in  $G$  has cardinality at least  $\frac{(1-\varepsilon) \log_2 n}{H_2(p)}$ . (Received September 08, 2017)

1135-I5-614 **Angelica Gonzalez\*** ([agonzalez@math.arizona.edu](mailto:agonzalez@math.arizona.edu)), Dept. of Mathematics, University of Arizona, 617 N. Santa Rita Ave., P.O. Box 210089, Tucson, AZ 85721-0089. *A Random Graph Model Related to One Face Maps*. Preliminary report.

Expander graphs, which are simultaneously sparse yet highly connected and robust, have many mathematical, computational, and physical applications. It has been shown that random  $d$ -regular graphs are likely to be expander graphs. In this talk we will consider a random class of graphs that is directly related to one-face maps. We will discuss how this class embodies many of the aspects of regular graphs that are optimal from the perspective of expansion. (Received September 10, 2017)

1135-I5-844 **Rachelle DeCoste\*** ([decoste\\_rachelle@wheatoncollege.edu](mailto:decoste_rachelle@wheatoncollege.edu)), 48 Branch St, Mansfield, MA 02048. *20 years of Density of Closed Geodesics on 2-step Nilmanifolds.*

In the study of geometric properties of nilmanifolds constructed from 2-step nilpotent Lie groups, the distribution of closed geodesics has been well considered. Eberlein, Lee-Park and Mast produced some early results that completely answered the question of whether a nilmanifold has a dense set of closed geodesics when it arises from a nonsingular 2-step nilpotent Lie algebra. In the years since, progress has been made on answering the question for certain classes in the singular case. We will discuss important findings, including the Heisenberg-like case, and give recent results on nilmanifolds arising from a graph construction. (Received September 15, 2017)

1135-I5-1154 **Kendra E Pleasant\*** ([kendra.pleasant@morgan.edu](mailto:kendra.pleasant@morgan.edu)). *Building Partition Regularity.*

Ramsey Theory is a mathematical study of combinatorial objects in which a certain degree of order must occur as the scale of the object becomes large. A standard problem in Ramsey Theory starts with some mathematical object and breaks it into several pieces. How big must the original object be for the pieces to have a certain property? This is described as *partition regularity*. Let  $u, v, n \in \mathbb{N}$  and let  $A$  be a  $u \times v$  matrix of rank  $n$  with integer entries. We show that there is a  $u \times n$  matrix  $B$  with integer entries such that

$$\{A\vec{k} : \vec{k} \in \mathbb{Z}^v\} \cap \mathbb{N}^u = \{B\vec{x} : \vec{x} \in \mathbb{N}^n\} \cap \mathbb{N}^u.$$

We also consider similar results dealing with an arbitrary commutative cancellative semigroup  $(S, +)$  and its group of differences,  $G$ . (Received September 19, 2017)

1135-I5-1175 **Jill E Jordan\*** ([jill.jordan@houghton.edu](mailto:jill.jordan@houghton.edu)), 1 Willard Ave, Houghton College, Houghton, NY 14744. *From Apprehension to Enthusiasm: Getting Students on Board with Inquiry-Based Learning.*

While many mathematics teachers are convinced that inquiry-based teaching methods result in better, deeper learning, it can take time and effort to get students on board. In this talk I will discuss challenging student attitudes that instructors may face when implementing inquiry-based methods, and I will share some techniques

for increasing student buy-in and confidence in inquiry-based mathematics classes. (Received September 20, 2017)

1135-I5-1189 **Susan D'Agostino\*** ([s.dagostino@snhu.edu](mailto:s.dagostino@snhu.edu)). *The EDGE Program Turns 20: What Have We Learned?*

The EDGE Program (Enhancing Diversity in Graduate Education) began twenty years ago to provide support for women entering doctoral programs in the mathematical sciences. For the first four years, the program alternated between Bryn Mawr and Spelman Colleges. In the intervening years, EDGE has been hosted on campuses around the nation and expanded to offer support for women throughout their graduate school and professional careers. With its steadfast commitment to diversity among participants, faculty, and staff, what impact has EDGE had on the national mathematics community? What impact has EDGE had on its alumnae? Has EDGE succeeded in producing a diverse new generation of leaders in the mathematics community? Lessons learned that may benefit others involved in related efforts will be shared. This talk is based on a paper coauthored with EDGE founders Sylvia Bozeman and Rhonda Hughes, to be published in Springer's 2018 *Women in Mathematics: Celebrating the Centennial of the Mathematical Association of America*. (Received September 20, 2017)

1135-I5-1225 **Carolyn Reinhart\*** ([reinh196@iastate.edu](mailto:reinh196@iastate.edu)). *Applications of SMP to the determination of the minimum number of distinct eigenvalues.*

The minimum number of distinct eigenvalues for a graph  $G$ ,  $q(G)$ , is the minimum number of distinct eigenvalues over all real symmetric matrices whose off-diagonal entries correspond to adjacencies in  $G$ , denoted  $S(G)$ . This relatively new parameter is of interest due to its relationship to the inverse eigenvalue problem which tries to determine all possible spectra for matrices in  $S(G)$ . The Strong Multiplicity property (or SMP) is a strong matrix property which will be applied to determine possible spectra of supergraphs and their associated multiplicities. New results to be presented include applications of SMP to find bounds on  $q(G)$  for graph products as well as the determination of  $q(G)$  for all connected graphs on 6 vertices.

This talk is based on joint work with Beth Bjorkman, Leslie Hogben, Scarlitte Ponce, and Theodore Tranel. (Received September 20, 2017)

1135-I5-1326 **Amy Buchmann\*** ([abuchman@tulane.edu](mailto:abuchman@tulane.edu)), **Lisa J Fauci**, **Karin Leiderman**, **Eva Strawbridge** and **Longhua Zhao**. *Mixing and pumping by pairs of helices in a viscous fluid.*

It is difficult to mix and pump fluid in microfluidics devices because the traditional methods of mixing and pumping at large length scales don't work at small length scales. Experimental work has suggested that rotating helical flagella may be used to effectively mix and pump fluid in microfluidics devices. To further explore this idea and to characterize the flow features around rotating helices, we study the hydrodynamic interactions between two rigid helices rotating at a constant velocity. Helices are coupled to a viscous fluid using a numerical method based upon a centerline distribution of regularized Stokeslets, and we analyze the effects of spacing and phase shift on mixing and pumping. (Received September 21, 2017)

1135-I5-2062 **Raegan Higgins\*** ([raegan.higgins@ttu.edu](mailto:raegan.higgins@ttu.edu)) and **Kristen Weasenforth**. *An SIR Model on Time Scales*. Preliminary report.

The goal is to create a time-scale version of the classic SIR epidemic model. One issue with transforming the SIR model into another is that it cannot be solved in closed form. We will use the idea of expressing  $S(t)$  as an exponential function on a general time scale  $T$  and solve for  $I(t)$  and  $R(t)$  to obtain the SIR system on  $T$ . (Received September 25, 2017)

1135-I5-2096 **Nakeya D Williams\*** ([nakeya.williams@usma.edu](mailto:nakeya.williams@usma.edu)), West Point, NY 10996. *Mathematical Modeling of Cardiovascular Dynamics during Orthostatic Stress*. Preliminary report.

This study develops a non-pulsatile model, from the integration of a pulsatile model, for the prediction of blood flow and pressure during orthostatic stress. Patients suffering from orthostatic intolerance, are diagnosed via a head-up tilt (HUT) test. This test is used to diagnose potential pathologies within the autonomic control system, which is complex and difficult to study in vivo. Here we show that mathematical modeling can be used to predict changes in cardiac contractility, vascular resistance, and arterial compliance, quantities that cannot be measured, but are useful to assess the system's state. The cardiovascular system is pulsatile, but predicting the control in response to HUT (over 10-40 min) is computationally challenging, and limits the applicability of the model. To overcome this complexity, we develop a simple non-pulsatile model that can be interchanged with the pulsatile model, yet both models are able to predict internal variables. Having models with various levels

of complexity formulated with a common set of parameters, allows us to combine long-term average simulations with pulsatile simulations on a shorter time-scale. (Received September 25, 2017)

1135-I5-2298 **Carla D Cotwright-Williams\*** ([carla.cotwright@gmail.com](mailto:carla.cotwright@gmail.com)), Baltimore, MD.  
*Mathematics in Public Service.*

A brief exploration of the use of mathematics to shape the policies which protect and support the lives of citizens and residents across the U.S. Several real world examples and their potential policy implications will be shown. This overview will be followed by a discussion on ways mathematicians and scientists can influence policy makers (e.g. U.S. Congress, federal, state and local officials) to improve (or change) the policies which shape society. (Received September 25, 2017)

1135-I5-2351 **Carmen Wright\*** ([carmen.m.wright@jsums.edu](mailto:carmen.m.wright@jsums.edu)), **Jasleen Kaur**, **Abigail Newsome** and **Charles Bland**. *Using Classification Algorithms to Predict Promoter Regions in E. Coli Based on DNA Structural Properties.*

One of the major challenges in biology is the correct identification of promoter regions. Computational methods based on motif searching have been the traditional approach taken. Studies have shown that DNA structural properties related to the bendability, curvature, and stability under certain stresses of the DNA structure itself are useful in promoter classification, as well. In particular, we compare the three structural properties of free energy, curvature, and stress-induced duplex destabilization (SIDDD) for their effectiveness in correctly identifying promoters with several classification algorithms, including Naive Bayes, K-nearest neighbors, and the decision tree. (Received September 26, 2017)

1135-I5-2372 **Emelie Jeanine Curl\*** ([ecurl@iastate.edu](mailto:ecurl@iastate.edu)), 205 South 5th Street, APT # 705, Ames, IA 50010. *Generalized Petersen Graphs with Maximum Nullity Equal to Zero Forcing Number.*

The maximum nullity of a simple graph  $G$ , denoted  $M(G)$ , is defined to be the largest possible nullity over all symmetric real matrices whose  $ij$ th entry is nonzero exactly when  $\{i, j\}$  is an edge in  $G$  for  $i \neq j$ , and the  $ii$ th entry is any real number. The zero forcing number of a simple graph  $G$ , denoted  $Z(G)$ , is the minimum number of blue vertices needed to force all vertices of the graph blue by applying the color change rule. The motivation for this research is the longstanding question of characterizing graphs  $G$  for which  $M(G) = Z(G)$ . The following conjecture was proposed at the 2017 AIM workshop *Zero forcing and its applications*: If  $G$  is a bipartite 3-semiregular graph, then  $M(G) = Z(G)$ . A counterexample was found, but questions remained as to which bipartite 3-semiregular graphs have  $M(G) = Z(G)$ . This talk concentrates on one family of graphs known as the Generalized Petersen graphs. These graphs are 3-regular and are only bipartite in specific cases. We were able to establish  $M(G) = Z(G)$  for certain Generalized Petersen graphs. (Received September 26, 2017)

1135-I5-2480 **Wanda Strychalski**, **Laura Miller**, **Sarah Bryant\*** ([sarahbryant456@gmail.com](mailto:sarahbryant456@gmail.com)), **Baasansuren Jadamba**, **Eirini Kilikian**, **Xiulan Lai**, **Leili Shahriyari**, **Rebecca Segal** and **Ning Wei**. *Why So Fast? Investigation of the Superfast Discharge of Nematocysts.* Preliminary report.

In April 2017, the Mathematical Biosciences Institute hosted the AWM Women Advancing Math Bio workshop. This workshop drew together nearly 50 women to tackle six complex math bio projects. In this talk, I will speak about my participation in this program and share preliminary results from my group project. We are investigating the discharge mechanics of nematocysts, a type of specialized organelle that shoots a harpoon-like projection into prey. This discharge is extremely fast and therefore understudied. Recent advances in electronic microscopy and high frame rate recording have allowed for improved measurements of nematocysts and their rate of discharge. Using IBAMR (Immersed Boundary Adaptive Mesh Refinement), an open-source library with an implementation of the immersed boundary with Cartesian grid adaptive mesh refinement, we were able to run simulations to better understand the discharge mechanics. Preliminary results point to a need for extremely fast discharge in order for the nematocysts to overcome the dampening of movement in water seen at small scales and actually allow the harpoon-like nematocysts to reach the prey rather than merely push it away. (Received September 26, 2017)

1135-I5-2927 **Karoline P Pershell\*** ([karoline@srtlabs.com](mailto:karoline@srtlabs.com)) and **Jamie Haddock**. *How do robots find their way home?: Optimizing Bluetooth beacon placement for robot localization and navigation in indoor spaces.* Preliminary report.

While map apps on mobile devices are excellent for navigating around town, they are not precise enough for use within buildings. Service Robotics & Technologies is currently working on deploying service robots (vacuuming, security, mail delivery) throughout a facility, and the robotic systems must navigate the space based on a pre-made facility map and built-in obstacle avoidance technology. However, a robot still needs to localize itself



within the map at regular intervals. Using Bluetooth Low Energy (BLE) beaconing technology for triangulating positions is a promising option for localization. Since signal strength is variable, even at a fixed distance, due to the physics of the signal, reflections based on room dynamics, and inherent inaccuracies in the hardware, we also discuss our choices for filtering noise. Given a map and signal readings from multiple beacons along a path, we extrapolate the BLE signal strength to any point in the map, using signal triangulation to allow the robot to localize. We discuss the process for data simulation and testing, and intended future work in localization and navigation. This work was made possible by the Institute for Mathematics and its Applications 2017 Math-to-Industry Bootcamp. (Received September 26, 2017)

## *Using Mathematics to Study Problems from the Social Sciences*

1135-J1-68      **Sirui Wang\*** ([siruiw@wharton.upenn.edu](mailto:siruiw@wharton.upenn.edu)). *Identifying Communities of Specialized Knowledge in a Tech Economy*. Preliminary report.

In the past decade, New York City has seen tremendous growth in its technology sector, making it a prominent player in the world of technology today. The division of labor in such a tech economy can be attributed to specialization of knowledge and skills within the community. We study this organization of knowledge by building dynamic social networks with data from a large online group focused on tech-based startups in the New York area. With a given network, the problem of community detection can be cast as a constrained integer linear program that optimizes network modularity. Using the Louvain method to approximate sub-communities in the overall tech economy, we find that the technology sector in New York is not only growing in size, but also in knowledge diversity, with the period of greatest change at around the end of the 2008 financial crisis. (Received September 16, 2017)

1135-J1-89      **John C Lang\*** ([jclang@ucla.edu](mailto:jclang@ucla.edu)) and **PJ Lamberson**. *On the Shoulders of Giants: When is it better to work in sequence versus in parallel?*

In the past decade, there has been an explosion of research into the ability of teams of individuals to find solutions to complex problems that no single individual would be able to solve on their own. This broad field of study, which includes the study of collective intelligence and the wisdom of crowds, becomes ever more relevant as developed countries become increasingly reliant on the knowledge economy as a driver of productivity and economic growth. Here we study a seemingly simple and fundamentally important question in the context of teams of problem solvers: When is it better to build on the work of your predecessors (i.e. to work in sequence), and when is it better to start from scratch (i.e. to work in parallel)? This dilemma appears in one form or another in nearly every profession and research domain, including fields as diverse as economics, medicine, engineering, etc. Using elementary techniques from network theory, probability theory, and calculus, we develop a simple mathematical model that we analyze to show that for difficult problems it is always better for problem solvers to work in parallel rather than in sequence. Finally, we provide counter-examples showing cases where, for simple problems, it can be better for problem solvers to work in sequence. (Received July 25, 2017)

1135-J1-199      **William J. Britt** and **William E. Gryc\*** ([wgryc@muhlenberg.edu](mailto:wgryc@muhlenberg.edu)), Department of Mathematics & Computer Science, Muhlenberg College, 2400 Chew St, Allentown, PA 18104, and **Jamie A. Oliva**, **Brittney N. Tuff** and **Charli E. White**. *Strategies for “Buy-It-Now or Make Offer” Auctions on eBay*.

eBay is the top auction website in North America. Some of the auctions on eBay have a fixed “Buy-It-Now” price for which the item can be instantly purchased with an opportunity for interested bidders to instead “Make (an) Offer” that the seller can accept or reject. This format presents some basic questions for the bidder. Should the bidder risk making an offer or just buy the item? If making an offer, how much should the bidder offer? And on the seller side, how high should the seller set the “Buy-It-Now” price? In this talk we present research on a model for this auction format and give mathematical answers to all the preceding questions. (Received August 08, 2017)

1135-J1-369 **Carol Ann Downes\*** ([downes@hendrix.edu](mailto:downes@hendrix.edu)), 1600 Washington Avenue, Conway, AR 72032, and **Philip D Waggoner** ([philip.waggoner@gmail.com](mailto:philip.waggoner@gmail.com)), University of Houston, Department of Political Science, 3551 Cullen Blvd., Room 447, Houston, TX 72204.  
*Explaining the Impact of Cosponsorship in Legislative Processes: An Application of Optimal Transport Theory.*

While cosponsorship is a relatively low cost form of legislative behavior, its impact on a bill is consequential. Adapting optimal transport theory for the American legislative context, we offer a model of the legislative process where cosponsors add their ideological and political pull to a bill as it makes its way through the chamber. The final form of a bill, as well as its position in ideological space, may be considered as a function of the ideological and political pull of all collaborators (i.e., the initial sponsor and all cosponsors). Being robust and internally consistent, our model shows cosponsorship is impactful on the legislative process despite the low cost to collaborators. (Received August 29, 2017)

1135-J1-594 **Sara M Clifton\*** ([smc567@illinois.edu](mailto:smc567@illinois.edu)), **Eileen Herbers**, **Jack Chen** and **Daniel M Abrams**. *The tipping point: a mathematical model for the profit-driven abandonment of restaurant tipping.*

The custom of voluntarily tipping for services rendered has gone in and out of fashion in America since its introduction in the 19th century. Restaurant owners that ban tipping in their establishments often claim that social justice drives their decisions, but we show that rational profit-maximization may also justify the decisions. Here, we propose a conceptual model of restaurant competition for staff and customers, and we show that there exists a critical conventional tip rate at which restaurant owners should eliminate tipping to maximize profit. Because the conventional tip rate has been increasing steadily for the last several decades, our model suggests that restaurant owners may abandon tipping en masse when that critical tip rate is reached. (Received September 09, 2017)

1135-J1-1515 **Steven A Reed** ([sareed5@asu.edu](mailto:sareed5@asu.edu)), 45 N. Phyllis #210, Mesa, AZ 85201, and **James PR Larrison\*** ([jlarrison@asu.edu](mailto:jlarrison@asu.edu)), 2614 N 73rd Dr, Phoenix, AZ 85035. .

We investigate means of decreasing variance of wealth distributions utilizing different tax structures and a novel proximity-based exchange system where agents are given positions and the probability of exchange with a given agent is distance-dependent. The results of our numerical simulations show that economic systems with wealth-biased taxation and poverty-biased distribution decrease variance of the wealth distribution. Additionally, we show that variance can be decreased through a proximity-based exchange system by utilizing the inherent wealth-inequality.

(Received September 22, 2017)

1135-J1-1709 **M. Leigh Lunsford\*** ([lunsfordml@longwood.edu](mailto:lunsfordml@longwood.edu)), **JoEllen G. Pederson** and **Marcus H. Pendergrass**. *Body Image in Popular Culture: A Comparison of Body Mass Index (BMI) among Celebrities, Students, and Superheroes.* Preliminary report.

Sociologists and psychologists have studied the effects of unrealistic media representations of body images on appearance and self-satisfaction with appearance. Their research argues that women feel their bodies do not meet the standards of physical attractiveness, athleticism, and body slenderness highlighted in popular culture. Is there statistical evidence that women are portrayed more unrealistically than men in popular culture? How does this portrayal compare to the body types of ordinary men and women? Interestingly, there is little existing research using BMI to make these comparisons. We will discuss our methodology using BMI, including its strengths and weaknesses, and the robustness of our results, which suggest there is indeed an unrealistic portrayal of women's bodies in popular culture. This research has also resulted in a project we now give our second semester statistics students. They compare their BMI to that of celebrities and superheroes using several basic statistical methods, including the two-sample t-test and one and two-way ANOVA. We have found that while students are generally aware of the pressure to conform to body images as portrayed in popular culture, they have not had the evidence to support their feelings or understand this issue statistically. (Received September 24, 2017)

1135-J1-2019 **Christopher S Shaw\*** ([cshaw@colum.edu](mailto:cshaw@colum.edu)), Science and Mathematics Department, 600 S Michigan Ave, Chicago, IL 60640. *Understanding Legal Terminology through Symbolic Logic.* Preliminary report.

The language employed in legal proceedings may often be filled with complex rhetorical devices that are rarely used in colloquial speech and writing; as a result, it is not uncommon for juries composed of people with a variety of educational backgrounds to make mistakes in interpreting the language of the cases before them. In this work, we examine some examples of complex legal language that are commonly used in the course of legal proceedings, and discuss how basic symbolic propositional and predicate logic may provide a framework for

simplifying this language. As part of this discussion, we present some preliminary data collected from student surveys. (Received September 25, 2017)

1135-J1-2054 **Linn E. Carothers\*** (lcarothers@calbaptist.edu), 8432 Magnolia Ave, Riverside, CA 92504. *Youth @ Risk: Data Mining A Longitudinal Cohort to Predict Patterns of Family Instability and Crime.*

Data mining a 18 year follow-up study of a large Danish Prospective Perinatal cohort identified key correlates of family instability that contribute to adolescent and young adult criminality. Comparative log-linear analyses of paternal crime, descriptions of the families' patterns of stability, and socioeconomic status changes over the life of the offspring revealed predictive gender, family constellation and SES combinations at risk for criminal behavior. These techniques, visualizations and modeling provide socially significant examples of using applied mathematical techniques to suggest crime prevention interventions. (Received September 25, 2017)

1135-J1-2345 **Zakariya Mohamed Kmir\*** (zkmir1@umbc.edu), Ellicott City, MD 21043. *Measuring Health Outcomes of Uncovered Employment: A study of income, social mobility, equality, and health indicators in an overlooked segment of the labor force.*

Economists have strongly supported that unemployment causes many undesirable health outcomes. However, how does belonging to a different sector of employment tied closely to changes in minimum wage and inflation relate to overall health. To properly understand the numerical significance of health disparities in the uncovered sector of employment, this research is targeted at quantifying the relationship between the insured and non-insured within the uncovered sector. By substantiating the existence of severe health disparities as a function of the labor force dynamic this research subsequently estimates the amount of inefficiency in the US economy that can be attributed to minimum wage hikes, the single cause for uncovered employment. Several single regressions are conducted to understand the statistical significance of health spending and life expectancy, and youth unemployment and overall unemployment. Youth unemployment is a function of employment, which this research analyzes to see as a significant indicator for social mobility and policy demand for rises in the minimum wage. The data in this research comes mostly from the World Bank, Organization for Economic Cooperation and Development, and individual US state censuses. (Received September 26, 2017)

1135-J1-2418 **Abdur-Rahman Munajj\*** (rmunajj@gmail.com), 112-15 209 Street, Queens Village, NY 11429, **Shaquille Winston** (shaquille.winston42@myhunter.cuny.edu), 51-45 Alameda Avenue, Far Rockaway, NY 11691, and **Brett Sims** (bsims@bmcc.cuny.edu), 199 Chambers Street, Department of Mathematics, New York, NY 10007. *Topologies on Cognitive Substructures.* Preliminary report.

Kurt Lewin, in his book Principles of Topological Psychology (1936), wrote that topology is a suitable mathematical theory to describe properties and dynamics in psychological spaces. He presented a detailed analysis of psychological space and dynamics in topological terms of interior and exterior regions, boundary zones, paths, and connectivity. In 2011, Falmagne and Doignon defined cognitive substructures as Knowledge Spaces composed of subsets of states of knowledge that a learner can "move" through, during problem solving. In our work, we consider cognitive substructures of the mind to be sets of many-sorted formulas. These formulas, as defined by Eysenck and Keane (2000), are mental representations of perceived aspects of environments (subsets) of the real-world, or mental representations of things imagined in the mind itself. We discuss Hausdorff and non-Hausdorff topologies on cognitive substructures of a space of mental representations. We explore the degree of "Hausdorff-ness" of a cognitive structure with respect to the emotional magnitude imposed on that structure. Bases are defined for topologies on subsets of the real-world and cognitive mental substructures, and we construct a type of mental covering space and map for selected subsets of the real-world. (Received September 26, 2017)

1135-J1-2424 **Abdur-Waajid Munajj\*** (munajjwaajid@yahoo.com), 112-15 209 Street, Queens Village, NY 11429, and **Brett Sims** (bsims@bmcc.cuny.edu), 199 Chambers Street, Department of Mathematics, New York, NY 11429. *Induced Mental-Endomorphisms.* Preliminary report.

This research in mathematical psychology is motivated by studies of cognitive dynamics brought on by the person "stimulating" their own mind, while a psycho-social study is motivated by cognitive dynamics stimulated by person to person interaction- between the minds of individuals through any form of communication. Carl Jung (1921), a Swiss psychiatrist defined the self (person) as the main entity governing its individuation out of conscious and unconscious states; however, it is not determined whether the self (person) is disjoint from the mind or not. In this research, the person (individual) is taken to be a separate entity, disjoint from the mind, but "owning" a mind. The action of the person on the mind is discussed in terms of Actions on Sets, where the person, as the action domain, is a magma of potentials, with the sentient properties of awareness, perception,

and the ability to cause. The structures acted on in the mind are cognitive substructures defined by abstract simplicial complexes, whose vertices can be many-sorted formulas. We model mental actions, implicitly, by induced mental-endomorphisms on complexes. We also construct a scenario for induced mental-endomorphisms stimulated by a semantic preserving “homomorphic” interpersonal communication map. (Received September 26, 2017)

1135-J1-2483     **Jonathan C. Davis** and **Jeffrey S. Powell\*** ([jspowe11@samford.edu](mailto:jspowe11@samford.edu)), Dept. of Math & CS–Samford University, 800 Lakeshore Dr, Birmingham, AL 35229. *Gatekeeping and the Professional Network of Therapist Supervisors*. Preliminary report.

As marriage and family therapist (MFT) supervisors play a critical role in the licensing of new marriage and family therapists, the underlying professional network of these supervisors is particularly important in light of their gatekeeping function (i.e. ensuring that “bad candidates” do not get licensed). To study the network of one state’s supervisors, Dr. Davis performed a census which asked each MFT supervisor to assess their relationships to all other MFT supervisors in state. This information was used to construct an approximation of the underlying professional network. In this talk, we will examine the structure of this network and discuss several relevant measures and properties. We will present both the positive and negative implications of the network structure from the perspective of the supervisors’ roles as gatekeepers. The research in this talk represents a collaboration between the Department of Mathematics and Computer Science and the Department of Human Development and Family Life Education (Received September 26, 2017)

1135-J1-2509     **Steven D. Leonhardi\*** ([sleonhardi@winona.edu](mailto:sleonhardi@winona.edu)), Winona State University, Winona, MN 55987. *Modeling and Experiencing the Tragedy of the Commons*. Preliminary report.

The “Tragedy of the Commons” describes a classical problem arising from shared usage of any natural resource such as freshwater, forests, and fish populations: individual parties each trying to maximize their own private gain can lead to loss of the resource value, and thus damage the common good of the users when considered as a collective group. This talk will discuss attempts to model this situation using Mathematical Game Theory concepts and techniques. We will present specific classroom activities and educational materials designed to help students experience first-hand the nature of this problem, and to explore possible solutions. No definitive solution has been found to this problem, nor is an all-purpose solution likely to be found. Nevertheless, the Tragedy of the Commons serves as an excellent context for the application of game theory which also draws upon concepts from economics, political science, and even psychology. (Received September 26, 2017)

1135-J1-2564     **McKenzie Lamb\*** ([1lamb@ripon.edu](mailto:1lamb@ripon.edu)), Ripon College, 300 W Seward St, Ripon, WI 54971. *Using the Status Quo to Define Fair District Plans*. Preliminary report.

Samuel Wang has proposed a test for gerrymandering in federal congressional district plans that compares the number of democratic seats won in a given state to the number of democratic seats won in other, similar, sets of districts drawn from around the rest of the country. In essence, this test defines “fairness” in terms of the current state of the country. We use computer simulations to highlight some of the drawbacks of this kind of test. (Received September 26, 2017)

1135-J1-2854     **Marcus Pendergrass\*** ([mpendergrass@hsc.edu](mailto:mpendergrass@hsc.edu)), P.O. Box 174, Hampden-Sydney, VA 23943. *How To Assign Win Probabilities In An Election Based On Polling Results*. Preliminary report.

Suppose you want to assign a candidate’s probability of winning an election based on polling results. Is there a “right” way to do this? I explore this question mathematically in the context of a two-candidate race. The approach is game-theoretical. I posit a game in which the “house” publishes probabilities of winning for each candidate in an election. The house then accepts bets on the outcome of the election. If a player bets on a candidate, and that candidate loses the election, the player pays the house one dollar. But if the candidate wins, the house pays the player an amount that depends on the candidate’s chances of winning: the lower the candidate’s chances of winning, the higher the payoff. The exact amount of the payoff is calculated according to *fair house rules*: if the published house probability were the actual probability that the candidate wins, then the house’s net gain, averaged over many bets, would be zero. The house’s strategy in this game is the choice of how to assign win probabilities based on poll results. The player’s strategy is the choice of which candidate to bet on. Thus, different ways for the house to assign win probabilities can be compared, by seeing how they fare against an optimal player in this game. (Received September 26, 2017)

1135-J1-3128 **Alex John Quijano\*** (aqui\_jano4@ucmerced.edu), Merced, CA 95343. *An Unsupervised Machine Learning Approach for Detecting the Language of Prejudice Including Micro-aggressions on Social Media.*

This study focuses on integrating semantic and sentiment analysis on the detection of the language of prejudice on Twitter. We explored NLP methods and historical word usage to develop a machine learning algorithm that detect the language of prejudice. The use of prejudicial language is often prevalent in social media during significant events such as presidential elections, holidays, and tragedies. It is important to note that there are two main ways on how users express prejudice, (1) by directly using active words, or (2) by forming a sentence in a form of micro-aggression, which is normally harder to detect. These type of prejudicial language are learned based on culture and it undergoes accumulation of random changes in time. Take the word “gay” for instance. It was originally defined as “cheerful” or “happy”. As recent as the 20th century, the definition of “gay” has become polysemous where it is defined as “homosexual” and it is used as either a description or a pejorative. When using the sentence “That’s so gay!” today would be insidious. In this talk, we present a classification problem that detects such language based on historical and sociocultural accounts to understand why and how prejudicial language change in time and across cultures. (Received September 26, 2017)

## *Research in Undergraduate Mathematics Education (RUME)*

1135-J5-111 **Mary E. Pilgrim\*** (mpilgrim@rams.colostate.edu), 1874 Campus Delivery, Fort Collins, CO 80523-1874, and **Daniel Reinholz** and **Joel Corbo**. *Improving STEM Education Through Departmental Action Teams.*

To improve undergraduate STEM education, we have been facilitating and studying Departmental Action Teams (DATs). A DAT is a departmentally-based team of faculty, students, and staff working together to address a broad-scale educational issue. Rather than making “quick fixes” to try to address a “problem” in the short term, DATs aim to build structures to create sustained and ongoing improvements. Given that quick fixes have yet to improve education at scale (e.g., resulting in the widespread use of innovative teaching and learning techniques), we argue that new approaches, like DATs, are necessary. DATs are supported by external facilitators who provide DAT participants with expertise in education and institutional change research, logistical support, campus connections, and a functional process for achieving their goals. In this presentation we will share the DAT model, successful pilot DATs, and the expansion of the DAT model to two universities. Further, we will discuss lessons learned with regard to departmental and institutional contexts, as well as our process for developing robust measures of departmental culture around education. (Received July 28, 2017)

1135-J5-150 **Samer Habre\*** (shabre@lau.edu.lb), Beirut, Lebanon, **Iman Osta**, Beirut, Lebanon, and **Hana Shatila**, Beirut, Lebanon. *Students’ Conceptual Understanding of Derivatives in Freshmen Calculus.*

This study examines students’ conceptual understanding of derivatives in a calculus I course offered at a private Lebanese university. Two groups are considered: a control group learning derivatives with an emphasis on the symbolic approach of the concept, and an experimental group where the emphasis is on multiple- representations. In the experimental group, cooperative learning, technology, and a series of activities incorporating the APOS (action- process, object- schema) levels were integrated in the teaching and learning of derivatives. Data were collected using qualitative and quantitative methods. Results show that students in the experimental group have an object conception and an almost comprehensive understanding of the derivative, particularly concerning the slope of a tangent line at a point, the instantaneous rate of change, and the relation between a function and its derivative. Students in the control group however have deficiencies in their understanding, showing only action and process conceptions of the derivative. Quantitatively, results show significant statistical differences in the mean scores between the two groups in favor of the experimental group. (Received August 04, 2017)

1135-J5-299 **Brent Hancock\*** (brent.hancock@unco.edu). *Collective Argumentation Regarding Integration of Complex Functions within Three Worlds of Mathematics.*

Although undergraduate complex variables courses often do not emphasize formal proofs, many widely-used integration theorems contain nuanced hypotheses. Accordingly, students invoking such theorems must verify and attend to these hypotheses via a blend of symbolic, embodied, and formal reasoning. This report explicates a study exploring student pairs’ collective argumentation about integration of complex functions, with emphasis placed on students’ attention to hypotheses of integration theorems. Data consisted of task-based, semistructured interviews with pairs of undergraduates, as well as classroom observations. Findings indicate that participants’ explicit qualifiers and challenges to each other’s assertions catalyzed new arguments allowing

students to reach consensus or verify conjectures. Although participants occasionally conflated certain formal hypotheses, their arguments married traditional integral symbolism with dynamic gestures and clever embodied diagrams. Participants also took care to avoid invoking attributes of real numbers that no longer apply to the complex setting. Teaching and research implications are discussed as well. (Received August 21, 2017)

1135-J5-312 **Daniel L Reinholz\*** ([daniel.reinholz@sdsu.edu](mailto:daniel.reinholz@sdsu.edu)), Department of Mathematics & Statistics, Department of Mathematics & Statistics, San Diego, CA 92115. *Support mathematical proofs through peer annotations.*

This talk focuses on the application of peer assessment to the teaching and learning of mathematical proof. Students in an analysis course annotated the work of their peers and revised their work based on feedback received. This structured review process was based on Peer-Assisted Reflection (PAR), a process previously used for mathematical problem solving. Students used a modified version of two-column proofs for the PAR process, and also applied this structure to their other proofs in the class. Analyses showed that this structure afforded students opportunities to engage in a variety of practices related to proof comprehension. Student conversations further indicated that students were still grappling with a number of proof-related ideas that one would normally take for granted in advanced courses. Finally, this process afforded students with agency, helping to make the proof process feel more personal. This contrasted their prior experiences of feeling alienated by proof-based courses. (Received August 22, 2017)

1135-J5-333 **Eyob S Demeke\*** ([edemeke@calstatela.edu](mailto:edemeke@calstatela.edu)), Department of Mathematics, 5151 State University Dr, Att: Dr. Eyob Demeke, Los Angeles, CA 90032-8204. *Mathematics PhD students' interpretation of explanatory proofs.*

Proofs are crucial in conveying mathematical knowledge. Both mathematicians and mathematics educators have argued that a proof is more valuable to students when it explains why a theorem is true. In this contributed report, I discuss attributes of explanatory proofs that eleven doctoral students in mathematics described. Doctoral students in this study interpreted the nature of mathematical explanation in the context of a proof in a wide range of ways. In particular, these participants expressed that they are more likely to consider a proof more explanatory when it succeeds in providing (a) insight into the derivation of certain formulas, (b) intuition as to why the theorem is true, or (c) insight into how the author or the reader could have discovered the proof in practice. (Received August 24, 2017)

1135-J5-345 **Benjamin David Sencindiver\*** ([sencindi@math.colostate.edu](mailto:sencindi@math.colostate.edu)), **Mary E. Pilgrim** ([mpilgrim@rams.colostate.edu](mailto:mpilgrim@rams.colostate.edu)) and **James E. Folkestad**. *Measuring Self-Regulated Learning: A Tool for Understanding Disengagement in Calculus I*. Preliminary report.

Calculus 1 has been and continues to be a key gateway course to STEM majors, which contributes to a loss of students in the STEM pipeline. While many factors play a role in the student experience, student-learning behaviors are particularly important in learning. By analyzing early online homework activity and help seeking, rich descriptions of student can be used for early prediction for at-risk students, but can be misrepresentative for students who have not yet engaged with these resources. This preliminary report presents self-regulated learning (SRL) theory as a way to understand student behaviors. Using this framework, online tools were designed to collect behavioral data which was used to create a SRL score based on in-course student activity. This preliminary report presents findings on the relationship between student behaviors in Calculus I, a behavioral SRL score, and failure rates, particularly with students disengaged with Calculus I course content. (Received August 26, 2017)

1135-J5-347 **Erica R Miller\*** ([erica.miller@huskers.unl.edu](mailto:erica.miller@huskers.unl.edu)). *Mathematical Knowledge for Teaching Examples in Pre-Calculus: A Collective Case Study*. Preliminary report.

The purpose of this collective case study is to examine mathematical knowledge for teaching examples in pre-calculus. The instructors involved in the study were experienced graduate teaching assistants who were teaching their course for the third time and were identified as good teachers. Utilizing a social constructivist and cognitive theory approach, I analyzed video recordings of enacted examples. The central question that guided this analysis was: What is the mathematical knowledge for teaching examples in pre-calculus? The goal of this study is to examine undergraduate mathematical knowledge for teaching from the perspective of practice, instead of relying on existing frameworks. As a result of this study, the author developed a model of mathematical knowledge for teaching examples in pre-calculus that includes knowledge of representations, students, instruction, specialized content, and connections when enacting high cognitive demand examples. (Received August 26, 2017)

- 1135-J5-403      **Gabriela Maria Hernandez\***, 1874 Campus Delivery, Fort Collins, CO 80523-1874, and **Mary E Pilgrim**, 1874 Campus Delivery, Fort Collins, CO 80523-1874. *GTA Professional Development: Lessons Learned from a Non-STEM Department.*

It is not uncommon for Graduate Teaching Assistants (GTAs) to be utilized as instructors for first and second year STEM courses. Given that the first two-year experience plays a key role in retaining STEM majors (Seymour & Hewitt, 1997), the pedagogical preparation of GTAs is important. As part of a larger project, a non-STEM GTA pedagogy course was observed in order to identify successful practices that could be translated to a Mathematics department to enhance an existing GTA professional development program. Qualitative data gathered through participant observations and interviews were thematically analyzed. In this report we present findings that highlight how apprenticeship of observation can be leveraged to foster empathy and compassion in GTA professional development. (Received August 31, 2017)

- 1135-J5-514      **Hyunkyong Yoon\*** (hyunkyong.yoon@asu.edu), 1720 E Broadway Rd Apt 1165, Tempe, AZ 85282. *Relationships Between What a Teacher Knows, What a Teacher Does in Classroom, and What His or Her Students Learn.*

This study is designed to investigate the relationships between teachers' meanings for the ideas they teach, the ways they express these meanings in instruction, and how students' meanings are affected by their attempts to understand what the teacher intends. I observed three teachers' lessons about slope, and interviewed them and six students. My analysis shows that teachers' meanings and their assumptions about what students already understood influenced the ways they expressed their meanings in instruction. Then, students developed their meanings in trying to understand what the teacher said and did. The results demonstrated that conveyed meanings from a teacher to students were not always the same as either teachers' meanings or what the teacher intends when the teacher had no image of how students might understand his or her statements or actions. This study could inform instructional design for pre-service teacher preparation. (Received September 06, 2017)

- 1135-J5-618      **Emilie R Hancock\*** (emilie.hancock@unco.edu). *An Activity Theory Approach to Mediating the Development of Metacognitive Norms During Problem Solving.*

Metacognition has long been identified as an essential component of the problem-solving process. Research on metacognition and metacognitive training has historically adopted an acquisitionist view. This study takes a participationist lens by considering metacognition as a habit of mind or dispositional tendency. Problem-solving habits of mind can be viewed as normative ways of thinking to which students become attuned by participating in authentic problem-solving situations. This study explored one such situation, in which portfolio problem-solving sessions and write-ups were used to mediate metacognitive thinking in a first-year mathematics content course for pre-service elementary teachers. Periodically, students worked together on non-routine problems and submitted individual write-ups documenting their judgement and decision-making processes. Analysis utilized Activity Theory, which operationalizes the participation structure of a classroom, to document the nonlinear development of classroom metacognitive norms during problem solving. Micro-analysis revealed a shift from product- to process-oriented metacognitive norms. Macro-analysis situated these results, highlighting social mediators of activity and contradictions as catalysts for change. (Received September 11, 2017)

- 1135-J5-682      **Thorsten Scheiner\*** (t.scheiner@auckland.ac.nz). *On the dialectic of extracting meaning and ascribing meaning.*

Previous research in mathematics education has distinguished between various forms of mathematics cognition, often contrasting between processes of extracting meaning from mathematical objects and processes of ascribing meaning to mathematical objects. The purpose of this presentation is to argue that such dichotomist views cannot adequately address the complex emergence of evolving forms of meaning. A new theoretical framing is discussed that acknowledges mathematics cognition as entangled processes of extracting meaning and ascribing meaning. (Received September 12, 2017)

- 1135-J5-1097      **Constantin Dorin Dumitrascu\*** (ddumitrascu@adrian.edu). *Assessment of Flipping the Introductory Statistics Course.*

We discuss an assessment of the implementation of the flipped pedagogy in the Introductory Statistics course offered by a mathematics department. The study is based on the comparison of five flipped sections (n=122, over three semesters) and five traditional sections (n=130, over three semesters). We analyze the acquisition of statistical knowledge and the evolution of students' attitudes toward statistics between the two groups. In a preliminary analysis of the data, we found that the flipped pedagogy did not generate statistically significantly different scores in the instruments used to assess statistical knowledge (basic statistics knowledge questionnaire,

final exams scores, WebAssign homework scores). We also found that the flipped pedagogy did generate statistically significantly better scores in the affect and cognitive competence of the students, as measured by the pre-test and post-test answers to the Survey of Attitudes Toward Statistics (SATS-36) of Professor Schau. We also provide details about the instructional methods used to implement the flipped pedagogy during the semester. The presentation is based on joint work with Elizabeth Lamprecht (Adrian College) and Roy St. Laurent (Northern Arizona University). (Received September 19, 2017)

1135-J5-1148 **Halil I. Tasova\*** ([halil.tasova25@uga.edu](mailto:halil.tasova25@uga.edu)), 105 Aderhold Hall, 110 Carlton Street, Athens, GA 30602, and **Kevin C. Moore** ([kvcmoore@uga.edu](mailto:kvcmoore@uga.edu)). *Justification of an invariant relationship between two quantities: Coordinating quantities vs. steepness of tangent lines.*

Several researchers have illustrated that teachers' covariational reasoning is critical to their supporting students in understanding major precalculus and calculus ideas. In this presentation, we characterize pre-service teachers' (PSTs') thinking during a teaching experiment as they reasoned about rate of change between the distance a rider has traveled (i.e., arc length) around a Ferris wheel and the rider's distance from the horizontal diameter (i.e., height, and together the sine relationship). Both PSTs described "the height is increasing less and less as the arc length increases"; however, their reasoning entailed marked differences. Our analyses illustrate the PST who constructed a structure of the related quantities (i.e., equally partitioning the arc length and comparing the corresponding variations in height) was able to provide productive justifications regarding rate of change and the curvature of her constructed graphs, including in related but different situations than the Ferris wheel. In contrast, the PST who reasoned about perceptual features of graphs and situations (e.g. steepness of tangent lines) was not able to provide justifications regarding the rate of change and curvature of their graphs that were generalizable to other situations and graphs. (Received September 19, 2017)

1135-J5-1185 **Aubrey Kemp\*** ([akemp9@student.gsu.edu](mailto:akemp9@student.gsu.edu)) and **Draga Vidakovic**. *Ways secondary mathematics teachers transfer and apply definitions from Euclidean to Taxicab context: An example of a real-life situation.* Preliminary report.

It has been known that definitions in mathematics are an integral part of understanding concepts, and are often not used correctly by students in mathematical proofs and problem-solving situations. In addition, research shows that by observing properties and making conjectures in non-Euclidean geometry, students can better develop their understanding of concepts in Euclidean geometry. To further investigate this, APOS Theory is used as the framework in this preliminary data analysis of responses to a real-life situation from eleven secondary mathematics teachers enrolled in a College Geometry course at a university in Fall 2016. Within the context of APOS Theory, this preliminary analysis provides illustrations of the conceptual understandings found among these participating teachers in relation to this real-life situation. By adapting and transferring their knowledge of definitions from Euclidean geometry to Taxicab geometry, these participating teachers demonstrated a variety of responses, providing detailed explanations of their thought processes. (Received September 20, 2017)

1135-J5-1265 **Anna Marie Bergman\*** ([a.bergman@pdx.edu](mailto:a.bergman@pdx.edu)). *Students' Emerging Ideas of the Symmetries of Molecular Structures.* Preliminary report.

The aim of the work described here is to share students' initial ideas related to symmetry in the context of molecular structures. During an ongoing design experiment intended to develop a local instructional theory (Gravemeijer, 1998) for students reinvention of a classification of chemically important point groups, students were asked how they could identify all the symmetries of a molecule. This report shares how pairs of group theory students in a series of teaching experiments (Steffe, 1991), begin interacting within the chemistry context to identify the symmetries of various molecules. Initial analysis shows that students benefit greatly from the use of 3-d models and often attend to symmetry elements (geometric objects such as rotational axes) over symmetry operations (rigid motions about geometric objects such as a 120 rotation). (Received September 20, 2017)

1135-J5-1490 **Rebecca Hartzler\*** ([rebeccahartzler@austin.utexas.edu](mailto:rebeccahartzler@austin.utexas.edu)), Charles A Dana Center, UT Austin, 1616 Guadalupe, Suite 3.206, Austin, TX 78701, and **Jeremy Martin** ([jeremyryanmartin@austin.utexas.edu](mailto:jeremyryanmartin@austin.utexas.edu)), Charles A. Dana Center, 1616 Guadalupe, Suite 3.206, Austin, TX 78701. *Dana Center Mathematics Pathways: Early findings from a randomized study of pathways curriculum implementation in four colleges.* Preliminary report.

The Center for the Analysis of Postsecondary Readiness, led by MDRC and Community College Research Center (CCRC), just released a research brief containing early findings from their randomized study of Dana Center Mathematics Pathways. The findings are encouraging: Students enrolled in Dana Center curriculum based on the DCMP model "are having qualitatively different classroom experiences from those of students in traditional



developmental math courses and are enrolling in and passing these courses at higher rates.” Students responded that participating in the Foundations for Mathematical Reasoning course made them more confident in their math ability and more interested in math in general, while teaching them how to struggle through problems.

These early findings show the contrast in student experience from just one semester of using the Dana Center curriculum. In this session we will review these early findings and discuss the upcoming findings from the final report. (Received September 22, 2017)

1135-J5-1510 **Jessica Ellis Hagman\*** ([jess.ellis@colostate.edu](mailto:jess.ellis@colostate.edu)), Fort Collins, CO 80524, and **Matt Voigt** ([mkvoigt@gmail.com](mailto:mkvoigt@gmail.com)), **Nancy Kress** ([nancy.kress@colorado.edu](mailto:nancy.kress@colorado.edu)) and **Jessica Gehrtz** ([jrgehrtz@gmail.com](mailto:jrgehrtz@gmail.com)). *Exploring the inequitable experiences of students in Calculus II.*

While active learning has been shown to support students from some marginalized populations, little research has been done to actually understand the experiences of students from marginalized populations in active learning undergraduate mathematics classes. In this presentation, we will investigate the experiences of students in Calculus II at one highly selective and technical campus. Half of the calculus II courses were taught using a traditional, lecture approach while the other half were taught using a more active approach. We will compare the experiences of students from the two approaches, and will focus on the experiences related to equitable engagement among students from historically marginalized populations in mathematics, including racial and ethnic minorities, women and gender diverse students, first-generation students, low-income students, and queer students. Specifically, we will analyze survey responses from 40 students who reported they were given less opportunity to learn in class compared to other students. We will use thematic analysis to identify commonalities and patterns among students’ experience in Calculus II, attending to patterns related to course approach (traditional or active) and student identities. (Received September 22, 2017)

1135-J5-1529 **Chris Watkins\*** ([watki115@mail.chapman.edu](mailto:watki115@mail.chapman.edu)), Chapman University, One University Drive, Orange, CA 92866, and **Mary Beisiegel** ([mary.beisiegel@oregonstate.edu](mailto:mary.beisiegel@oregonstate.edu)), OSU Department of Mathematics, 368 Kidder Hall, Corvallis, OR 97330. *The Impact of Mandatory Participation in Math Excel Labs Associated with Calculus Courses.*

About twenty years ago, the mathematics department of a university in the pacific northwest of the United States established the Math Excel program based on Uri Treisman’s Emerging Scholars Program. During the 2013-2014 academic year, one section each of differential calculus, integral calculus, and vector calculus was dedicated to the Math Excel Program. In addition to three hours of lecture, students in Math Excel sections were required to attend four hours of calculus lab during which they worked on challenging problems. We investigated whether pass rates, grades, and future grades of Math Excel students were higher than students in the traditional version of the course. The goal of this study was to determine how the program affects student success in the Math Excel course and other mathematics courses they take in the future. We found that there is not significant evidence that the Math Excel program has a greater effect on student pass rates, grades, or future grades in calculus courses than traditional versions of the calculus courses. Based on these and prior results, we recommend that students not be required to enroll in additional lab hours, but instead be allowed to choose that option. (Received September 22, 2017)

1135-J5-1538 **Soofia Malik\*** ([smalik@unoh.edu](mailto:smalik@unoh.edu)), 1441 N Cable Rd, Lima, OH 45805. *Development of Instrument to Assess Undergraduates’ Attitudes toward Mathematics.*

The present study examined the factor structure of newly-developed attitude toward mathematics instrument (MAT). The 25-item MAT was so developed that it consisted of five subscales: anxiety, enjoyment, self-confidence, value, and technology. These subscales were found as a result of thorough literature review. The sample consisted of 163 ( $M = 82$ ,  $F = 81$ ) undergraduates enrolled in Algebra course in the Spring 2017 quarter at a university located in the midwestern region of the United States. The scores obtained from the survey were subjected to Exploratory Factor Analysis. The survey data was also subjected to independent samples t-tests and one-way ANOVA to investigate the differences between undergraduates’ attitudes toward mathematics based on their gender, college-year, and major field of study. This presentation will expand upon the development of the MAT instrument, the results, and the future directions for this study. (Received September 23, 2017)

1135-J5-1669 **Caroline J. Merighi\*** ([caroline.merighi@tufts.edu](mailto:caroline.merighi@tufts.edu)). *Introductory calculus students’ approaches to conceptual problems and what it reveals about their understandings of core calculus concepts.*

Prior studies have found that even students who are successful in calculus courses can hold weak understandings of concepts that underlie the algorithms they routinely use. In this study, we investigate these ideas further

by examining the written work of first-semester calculus students on problems that address the foundational concepts of rate of change and accumulation without providing explicit equations to manipulate. We also analyze videotaped data of students working on similar problems during research interviews. The interview data show that students have widely varying approaches to these types of problems. When their intuition conflicts with memorized facts and formulas from class, some students readily abandon their intuition while others trust their intuition more strongly. Only one student in this study attempted to reconcile their intuition and their knowledge from class. In addition, quantitative analysis of students' written responses reveals that despite a full semester of calculus instruction, many students claim that there is no way to interpret rate of change or accumulation from graphical data. This illustrates that students may become proficient at computing derivatives and integrals without an accompanying development of the meanings of these concepts. (Received September 24, 2017)

1135-J5-1711 **Kedar M Nepal\*** ([nepal\\_k@mercer.edu](mailto:nepal_k@mercer.edu)), 103 Aberdeen Cir, Cordele, GA 31015, and **Kailash C Ghimire, Ramjee Sharma and Manoj Thapa.** *How Do Undergraduate Mathematics Students Justify Their Self-assessments in Academic Assignments?* Preliminary report.

Research shows that students generally overestimate their performance on academic assignments. High achieving students, however, tend to underestimate their performance (Nepal, Ghimire, Sharma, & Thapa; 2017). This indicates that many students might not know what they know or do not know. This mixed method study investigates undergraduate mathematics students' justification of their self-assessments. Students from a broad range of mathematics courses at three universities in the southeast United States were asked to predict their expected grades on in-class assignments, and these predictions were compared with the grades assessed by their instructors. They were also asked to justify their self-assessments in writing if they did not give themselves full points. Their written justifications were then analyzed using qualitative techniques. Based on our analysis of the qualitative data, we observed four different student behaviors: 1) knowing about knowing, 2) not knowing about knowing, 3) knowing about not knowing, and 4) not knowing about not knowing. Results show that many high-achieving students demonstrate low confidence in their knowledge and performance, and many low-achieving students demonstrate high confidence, despite their poor performance in the assignments. (Received September 24, 2017)

1135-J5-1779 **Brian Arthur Christopher\***, [brian.christopher@unco.edu](mailto:brian.christopher@unco.edu), and **Gulden Karakok.** *The Relationship Between Pre-service Elementary Teachers' Calibration, Mathematics Anxiety and Achievement.*

The recent research study by Chang and Beilock (2016) indicates that math anxiety is highly prevalent in students' learning, and in fact has significant negative relationship with math achievement. Hence, as educators we need to understand the factors that explain the relationship between math anxiety and achievement, and find ways to help students and enhance students' comprehension and achievement of math. In this presentation, I share results of a study that explores this particular issue through the construct, calibration, and its relationship with math anxiety and achievement in pre-service teachers enrolled in a math content course. Calibration can be described as the alignment between what a person believes s/he is capable of and what the person is actually capable of. Through the correlational analyses, I found significant correlations between calibration and exam performance, calibration accuracy and math anxiety, math anxiety and exam performance. In the linear mixed model, calibration and math anxiety were significant predictor of math exam performance along with the interaction between calibration bias and teacher. These results indicate that as a student becomes more calibrated and less mathematically anxious, his/her exam score increases. (Received September 24, 2017)

1135-J5-1816 **Natalie LF Hobson\*** ([natalie.hobson@sonoma.edu](mailto:natalie.hobson@sonoma.edu)), Mathematics and Statistics Department, 1801 E. Cotati Ave., Rohnert Park, CA 94928. *"The slope is increasing!" – Students' takeaways from Calculus.*

The mathematics of relationships between continuously changing quantities is at the essence of the study of Calculus, rate of change and covariation being central concepts discussed in most Calculus courses. What understandings of rate of change and covariation do successful students come away with? What are the consequences of how students might come to think about these ideas? In this study, using clinical interviews, I explore and describe the covariational reasoning of a mathematics education doctoral student with two years of high school teaching experience. In analyzing the participant's activity, I identify and describe three different ways of reasoning that the participant came to use while engaging in tasks in which the student graphically represented dynamic situations. The ways of reasoning constituting the model of his thinking provide evidence to suggest he did not have a connected meaning of covariation as a relationship between changing quantities. In this talk, I

illustrate these ways of reasoning and present some common curricular treatments of rate of change. In doing so, I highlight certain associations we often teach in Calculus that do not seem to provide students the opportunity to develop ways of reasoning about relationships between varying quantities. (Received September 25, 2017)

1135-J5-2053 **Hayley Milbourne\*** ([hmilbourne@sdsu.edu](mailto:hmilbourne@sdsu.edu)), 6475 Alvarado Rd, Suite 206, San Diego, CA 92129, and **Susan Nickerson**. *The ways the discourse around various teaching practices change as graduate teaching assistants engage with professional development*. Preliminary report.

Across the nation, there is increased national interest in improving the way mathematics departments prepare their graduate teaching assistants (GTAs). At many institutions, given the recommendations of the study of Characteristics of Successful Programs in College Calculus (Bressoud, Mesa, & Rasmussen, 2015), the preparation of the GTAs has a focus on ways to enact student-centered instruction with active-learning strategies. We need to understand how GTAs interpret and make sense of these suggested teaching practices. I report preliminary results on the ways in which the discourse around selected teaching practices changed over a term as Calculus GTAs were engaged in professional development. Using a modified framework based on a socio-cultural learning theory, I analyze the ways in which the discourse around the teaching practices changed and the role faculty and other GTAs had in their interpretation and transformation. With this analysis, we can understand how to better support GTAs with their teaching. The research presented here represents the start of an increased understanding of how GTAs form their own understandings of teaching practices for a student-centered classroom. (Received September 25, 2017)

1135-J5-2071 **William L Hall\*** ([w.hall@wsu.edu](mailto:w.hall@wsu.edu)). *The Role of Context in How Students Majoring in the Biological and Life Sciences Solve Calculus Tasks Involving the Definite Integral*.

Calculus serves students from many fields of study. Investigations into how students from these fields reason about calculus concepts are vital, yet lacking. The biological and life sciences make up 30% of traditional Calculus I students and yet we know very little about how these students utilize context as they reason about calculus ideas like the definite integral. In this study, task-based interviews were conducted with 12 undergraduate students majoring in the biological and life sciences. In this session, I share results from two tasks in which students were asked to reason about the accumulation of a quantity (distance traveled and number of plants in a species) given a rate of change graph. Results indicate that context played a role in the students' mathematical reasoning. Students were more likely to interpret the graph of the rate of change in the plant task as if it represented accumulated quantity than they were for the velocity graph in the distance traveled task. Several students indicated that the context directly influenced how they approached each task and how difficult they felt each task was. These results imply that our approaches to calculus may need to include diverse applications, specifically in courses intended for the biological and life sciences. (Received September 25, 2017)

1135-J5-2158 **Margaret Adams\*** ([drmargaretadams@gmail.com](mailto:drmargaretadams@gmail.com)), Charlotte, NC 28269. *Knowing functions before learning limits: undergraduate students' unique perceptions of limits and compromised foundational knowledge of functions*.

This researcher conducted a qualitative case study using in-depth interviews to investigate how Calculus III students think about limits. A constructivist framework and Skemp's model of instrumental and relational understanding guided the study, which explored how 15 students think about functions, limits at a point, limits at infinity and limits that do not exist using a traditional textbook problems and non-traditional tasks. Students with different ability levels were selected and for the initial analysis which gave rise to an evolving in-depth investigation of how students use their definitions of limit and infinity and role of the domain to construct their responses. Follow up research on how students think about functions revealed deficits with generating real-world examples of functions; misperceptions about one to one correspondences and identifying piecewise functions. Results were interpreted with a constructivist framework, and a model of understanding limits was developed. Students appear to assimilate external mathematical content into mental structures, develop appropriate or inappropriate schemas, and ultimately construct correct or incorrect actions. Pedagogical interventions emphasizing the relation between functions and limits are presented. (Received September 26, 2017)

1135-J5-2277 **Angeliki Mali\*** ([anglmali@umich.edu](mailto:anglmali@umich.edu)) and **Vilma Mesa** ([vmesa@umich.edu](mailto:vmesa@umich.edu)). *Instructors and students' uses of dynamic textbooks: What is new?* Preliminary report.

As part of the NSF funded project *Undergraduate Teaching and Learning in Mathematics with Open Software and Textbooks* (UTMOST), we investigate whether and how instructors and students take advantage of features that are included in dynamic textbooks enhanced with Sage cells. Using the documentational approach

(Gueudet & Trouche, 2009) we analyze two processes: instrumentation (how the resources “affect” the user) and instrumentalization (how the user “affects” the resources) while instructors use the course textbooks (either dynamic or identical PDFs) during planning and teaching. We also describe students’ uses of the course textbooks during class and their reported use outside the classroom. Data sources include biweekly logs, surveys, video recordings of the planning and the enactment of lessons, interviews, and tests of content knowledge with seven instructors—four teaching Linear Algebra and three teaching Abstract Algebra.

Reference: Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, 71, 199-218. (Received September 26, 2017)

1135-J5-2486      **Abraham Edwards\*** ([aedwards@msu.edu](mailto:aedwards@msu.edu)), 919 East Shaw Lane, East Lansing, MI 48825. *Dynamic 3D Imagery in Calculus III: Student responses and learning gains*. Preliminary report.

The ability to visualize certain mathematical objects can be an important contributor to learning. With the rise of free software and online applets, students and instructors can quickly generate dynamic images of three-dimensional objects including vectors, planes, and quadric surfaces. This talk reports on a classroom teaching experiment which took place over the course of a semester, in which students regularly produced and analyzed computer-generated 3D images. Students’ affective responses and learning gains were analyzed in terms of several factors to better understand the benefits and constraints of using such software. (Received September 26, 2017)

1135-J5-2499      **Guadalupe Lozano** ([guada@math.arizona.edu](mailto:guada@math.arizona.edu)) and **Jose Maria Menendez\*** ([jmenendez@pima.edu](mailto:jmenendez@pima.edu)). *Active-learning in pre-calculus and calculus: impact and performance outcomes for 2-year HSI students and 4-years HSI transfers*. Preliminary report.

We will discuss preliminary results of an NSF-sponsored active-learning plus mentorship research study, for improving math performance and STEM persistence of community college students and university transfers from Hispanic Serving Institutions (HSIs). The talk will focus on results, successful products, and limitations tied to assessing performance of this student population using locally valid instruments (i.e., final exams) and also two nationally-used instruments, the Pre-Calculus Concept Assessment (PCA) and the Calculus Concept Inventory (CCI). (Received September 26, 2017)

1135-J5-2512      **Rachel L. Rupnow\*** ([rachr15@vt.edu](mailto:rachr15@vt.edu)). *A Refinement of a Genetic Decomposition for Differentiating a Function to a Function Power*. Preliminary report.

Functions of the form  $f(x) = g(x)^{h(x)}$ , including constant functions, power functions, and exponential functions, are examples of functions that differential calculus students should be able to differentiate. Yet students often struggle to distinguish between these forms. Drawing on APOS (Action-Process-Object-Schema) theory and Piaget and Garcia’s Triad of Schema Development, this talk refines a genetic decomposition of the schemas students build for finding the derivative of a function to a function power. We analyze which differentiation rules students chose to use with different function structures of a function to a function power. A previous genetic decomposition, which was informed by existing literature and a pilot study based on clinical interviews with two students, was refined using 18 differential calculus students’ homework and exam papers. These papers were collected regularly throughout a fall semester course. Central aspects of the genetic decomposition include the necessity of a strong background in functions, logarithms, and other differentiation rules. This genetic decomposition could help inform the teaching of calculus by highlighting the need to develop students’ pre-calculus knowledge of functions and logarithms while teaching calculus concepts. (Received September 26, 2017)

1135-J5-2584      **Irma E. Stevens\*** ([istevens@uga.edu](mailto:istevens@uga.edu)), 110 Carlton St., Aderhold Hall, Athens, GA 30602. *How a Pre-Calculus Student Was Able to Reason about Rates of Change Using Magnitudes*.

Researchers have shown how reasoning about amounts of change in quantities’ magnitudes is productive for constructing a graph illustrating appropriate rates of change. To reason about rates of change using amounts of change in quantities’ magnitudes, a student could manipulate the length of a segment understood to represent a quantity’s magnitude by increasing it by successive equal increments and additively comparing the amounts of change that occur in a second bar representing some other quantity’s magnitude. Alternatively, a student could reason about the ratio of the accumulated amounts of two bars relative to one another at various lengths. Over the course of several sessions in a summer-long teaching experiment, we explored how an undergraduate student, who was studying music education and had just successfully completed a pre-calculus course, could manipulate bars representing the magnitudes of the quantities under consideration using the second approach to

reason about rates of change across several graphing tasks and situations. We argue students' abilities to reason with magnitudes supports their ability to manipulate objects representing quantities' magnitudes in a way that encourages multiple ways of reasoning quantitatively about rates of change. (Received September 26, 2017)

1135-J5-2588 **Deniz Kardes Birinci\*** ([denizkardes@usf.edu](mailto:denizkardes@usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 E Fowler Ave, CMC 342, Tampa, FL 33620, and **Mile Krajevski** and **Ruthmae Sears**. *Linear Algebra Students' Ability to Create a Meaningful Visualization of Objects*. Preliminary report.

In this presentation, we will describe how linear Algebra students exhibited the Visual Literacy Competency Standards (VLCS) (ACRL, 2011). Particularly, we will focus our discussion on the following standard: Design and create meaningful images and visual media. To gain insight into the extent linear algebra students created a meaningful visualization of objects (creation standards), and explicated their mathematical knowledge, we employed a mixed method research design. The data were collected from fourteen linear algebra students at a southeastern university in the United States, via mathematical tasks that embodied rich visual content on a graded quiz, and through semi-structured interviews. The quantitative data, which were garnered from students' scores on the quiz, were analyzed using descriptive statistics. The qualitative data, which were garnered from the semi-structured interviews, were analyzed using a grounded theory approach. Preliminary findings indicate that linear algebra students experienced difficulty, and were generally not competent, in creating models to visualize objects. (Received September 26, 2017)

1135-J5-2641 **Jenna R Van Sickle\*** ([jenna.vansickle@gmail.com](mailto:jenna.vansickle@gmail.com)). *Effects of the Operation STEM Program on Underrepresented Minority Students*.

Operation STEM (OpSTEM) is a NSF grant-funded program that seeks to improve retention and graduation among high-risk students seeking STEM degrees by supporting them through the precalculus-calculus sequence. OpSTEM focuses its attention on students from underrepresented minority (URM) groups, first-generation college students, and women. The OpSTEM program has two levels of treatment—one group receives supplemental instruction while another group receives a comprehensive program. This study considers URM students as compared with their non-URM counterparts and considers how well these groups fare in their precalculus courses. Both of the OpSTEM treatments show all groups making significant gains, with URM students making relatively greater gains. For non-URM students, the majority of the gains in pass rates are seen with supplemental instruction alone. For URM students, however, the comprehensive program increases the pass rates so much that URM students become difficult to distinguish from their non-URM counterparts. We conclude that for URM students in particular, a comprehensive program is necessary in order to narrow the achievement gap between these students and their peers. (Received September 26, 2017)

1135-J5-2664 **Draga Vidakovic\*** ([dvidakovic@gsu.edu](mailto:dvidakovic@gsu.edu)), 25 Park Place, Office: 1434, Atlanta, GA 30303-3083, and **Darryl Chamberlain**, **Aubrey Kemp** and **Valerie Miller**. *Improving undergraduate students' proof capabilities*. Preliminary report.

In an effort to improve students mathematical literacy and ability to read, understand, and write proofs, our department is implementing a particular assessment throughout the semester in the Bridge to Higher Mathematics course. In developing assessment items, mathematics faculty use a framework proposed by Mejia-Ramos, Fuller, Weber, Rhoads, & Samkoff (2012). The students' written work on these assessment items is analyzed by multiple faculty and graduate students. The results of this analysis are discussed and used in revising a particular assessment item as well as in developing the new ones. In this presentation, we focus on a subgroup of students' performances on these assessments. Our particular interest is in identifying students' 'developmental proof trajectories' over the period of at least one semester. (Received September 26, 2017)

1135-J5-2698 **Zackery K. Reed\*** ([reedzac@oregonstate.edu](mailto:reedzac@oregonstate.edu)). *Motivating Function Spaces via Uniform Convergence*.

Function spaces are the foundation for many important and sophisticated topics in real analysis and its applications. While some abstract function spaces might not be introduced to students until advanced analysis courses, most introductory analysis courses present students with basic ideas that can motivate function space structure. Specifically, students often encounter uniform and point-wise convergence of continuous functions in such courses. In a recent study involving two teaching experiments, undergraduate real analysis students produced various metrics and norms on continuous function spaces by generalizing notions of distance from finite-dimensional space. Students were able to leverage the structure of uniform convergence as rich source material from which they could generalize. Moreover, discussions of uniform convergence in various settings facilitated useful explorations such as the completeness of various spaces given certain restrictions. I will present

the students' generalizations in these spaces and discuss the various ways that uniform convergence influenced the students' activity and understanding. (Received September 26, 2017)

1135-J5-2857 **Claire Gibbons\*** ([gibbonsc@oregonstate.edu](mailto:gibbonsc@oregonstate.edu)). *Capturing the Mathematical Content in College Algebra Instruction Through the Lenses of Three Observation Protocols.*

In the United States, approximately half of all students enrolled in College Algebra do not successfully pass on to the next course. Thus, College Algebra can be a barrier for STEM-intending undergraduate students. The instruction offered to College Algebra students has an impact on their achievement, and the mathematics presented during instruction affects student understanding and success. To investigate the mathematical content offered by one university's College Algebra instructors, video clips of direct instruction were observed with three observation protocols: the Mathematical Quality of Instruction (MQI) Protocol, the Reformed Teaching Observation Protocol (RTOP), and the Teaching for Robust Understanding of Mathematics (TRU Math) Protocol. Despite using a common curriculum and maintaining a community of practice, the instructors' presentations of mathematical content exhibited wide variation. This talk will discuss the variation of mathematical content provided during instruction and the ability of the MQI, RTOP, and TRU Math protocols to capture this variation. (Received September 26, 2017)

1135-J5-2967 **Melissa Haire\*** ([melissa.haire@uconn.edu](mailto:melissa.haire@uconn.edu)), Department of Mathematics, University of Connecticut, 341 Mansfield Road U1009, Storrs, CT 03254, and **Fabiana Cardetti**. *How are we meeting their needs? Investigating Students' Use of a Quantitative Learning Center.* Preliminary report.

Given the combination of inadequate preparation for success in college mathematics and a lack of motivation and/or mathematical confidence in students, institutions of higher education have experimented with various approaches to support their students. One response has been the creation of quantitative learning and support centers (QLCs). While this is only one approach to address the problems mentioned, it is significant since these centers often attract many students from across disciplines who need additional mathematics help. Since QLCs have become so prevalent, it is important that we find ways to transfer what we learn from studying them to improving practice not only within the QLCs themselves but also in our teaching, office hours, and general interactions with and support of our students. This presentation will discuss results from a study guided by the following questions: (1) Which mathematical learning challenges do students identify as reasons for visiting a QLC, (2) How do these challenges compare to those identified by tutors, and (3) Which tutoring strategies seem most helpful in addressing these challenges? The results are based on our analyses of survey responses and interview transcripts from students and tutors, as well as observations of tutoring sessions. (Received September 26, 2017)

1135-J5-2973 **Jessica Gehrtz\*** ([gehrtz@math.colostate.edu](mailto:gehrtz@math.colostate.edu)), **Jess Ellis Hagman** and **Natasha Speer**. *Investigating Calculus Instructors' Responsiveness to and Interpretation of Student Thinking.*

Understanding how students think about mathematics has been identified as an integral component in supporting effective instructional practices. Further, there is evidence that utilizing student thinking to support effective instructional practices relies on instructors' responsiveness to student thinking. In this talk, we present preliminary findings from a survey investigating calculus instructors' responsiveness to student thinking from 12 institutions. Additionally, we draw on interviews conducted at one of these institutions to complement the survey findings with qualitative data. The interviews investigated calculus instructors' dispositions towards and interpretation of student thinking as they examined students' work on calculus tasks. The interview transcripts were analyzed using an analytic tool inspired by the work of Jacobs, Lamb, and Philipp's (2010) classification of teachers' ability to attend to, interpret, and decide how to respond to student thinking. We use this tool to categorize instructors' responses as exhibiting robust, limited, or no evidence of interpreting student thinking. This work has implications for those interested in mathematical knowledge for teaching, specifically related to utilizing student thinking, and the development of this knowledge. (Received September 26, 2017)

1135-J5-3053 **Andrew J Krause\*** ([krausea3@msu.edu](mailto:krausea3@msu.edu)). *The Learning Experience Framework: Conceptualizing Student Engagement In and Out of the Classroom.*

Institutions with successful calculus programs collect and use local data to inform their teaching. Many departments carefully track quantitative student achievement data—such as grades, exam scores, and homework completion rates—but lack qualitative data about what students are doing while they learn mathematics. Achievement data is important for evaluating a course and identifying issues, but qualitative data about students' learning experiences reveals how students engage with our courses. Do student use online homework as a learning tool?

How do the resources we provide to help students learn? How does the course facilitate or hinder collaboration? A potential barrier to collecting qualitative data is the lack of a conceptualization that unifies existing literature and frames research about students' learning experiences. I present a framework that highlights distinct features of the mathematical tasks that shape student engagement and explain how that framing facilitates research that can inform teaching. I will present findings from a study about students' experience with online calculus homework to demonstrate the utility of this theoretical approach and explore how other learning contexts might be examined similarly. (Received September 26, 2017)

1135-J5-3067 **V. Rani Satyam\*** ([satyamvi@msu.edu](mailto:satyamvi@msu.edu)). *Developmental and Affective Takes on Undergraduates Learning How to Prove.*

Students struggle in learning how to prove; prior research has looked at their struggles and strategies but not as much is known about how these change during the learning process. In addition, not much is known about the affective (attitudes, beliefs, emotions) side of proving. The purpose of this work is to examine the cognitive and affective factors involved in how undergraduates learn how to prove: how their problem solving relative to proving develops and what kinds of satisfying moments, marked by intense positive emotional reactions, they experience. Interviews were conducted with  $N = 11$  undergraduates in a transition to proof course at four points across a semester. Each session consisted of proof construction tasks administered as a think-aloud and a semi-structured interview about satisfying moments they experienced in relation to the course. Findings suggest growth in what students do when stuck and show various types of satisfying moments with potential eliciting factors. This work has implications for the design of introduction to proof curricula and construction of tasks designed to bring about positive relationships with mathematics. (Received September 26, 2017)

## *Environmental Modeling in the Classroom*

1135-K1-9 **John Roe** ([john.roe@psu.edu](mailto:john.roe@psu.edu)), Department of Mathematics, The Pennsylvania State University, University Park, PA 16802, and **Russell deForest\*** ([russ.f.deforest@gmail.com](mailto:russ.f.deforest@gmail.com)), Department of Mathematics, The Pennsylvania State University, University Park, PA 16802. *Mathematics for Sustainability: A new course and textbook, and experience using it for non technical majors in a large state university.*

We will report on our experience designing and teaching "Math for Sustainability", a course for non mathematical majors at Penn State revolving round six key themes of measuring, flowing, networking, changing, risking and deciding. High school algebra is the only mathematical prerequisite and the course is intended for students who need to meet a general education requirement and will not take additional mathematics in future. Student writing and anonymous peer review form a significant component of the course. A textbook with the same title is being published by Springer-Verlag and we hope to have pre-publication copies available at the session as well as at Springer's booth in the exhibition hall. (Received July 17, 2017)

1135-K1-1018 **Carl Olimb\*** ([carl.olimb@augie.edu](mailto:carl.olimb@augie.edu)), 2001 Summit Ave., Sioux Falls, SD 57197. *Using math modeling to teach mathematics.*

Mathematical modeling provides the use of robust complexity of mathematics to solve real-world problems as well as an opportunity to discover mathematics. We developed scenarios of a power grid in hypothetical settings to demonstrate application of applied mathematical modeling; scenarios are robust and can be scaled from secondary to post-secondary education. (Received September 18, 2017)

1135-K1-1119 **Rikki B. Wagstrom\*** ([rikki.wagstrom@metrostate.edu](mailto:rikki.wagstrom@metrostate.edu)), Metropolitan State University, 700 E Seventh St, Saint Paul, MN 55106. *Environmental Modeling in Lower Division Mathematics Courses.*

In this presentation, I will discuss two sets of environmental mathematics modules that I developed as part of the Engaging Mathematics project (NSF DUE-1322883) as well as curricular initiatives since the completion of the grant. The Engaging Mathematics modules explore declining milkweed population abundance, monarch reproduction, the science and financing of wind energy, and carbon footprints. One set of modules was developed for use in pre-calculus and lower levels of mathematics. The other set of modules was developed for use in calculus courses. In light of the current hostility and divisiveness we are seeing in our country, I have been reflecting on the purpose of integrating environmental issues into mathematics classrooms, and how these experiences can shape the perspective of students. Consequently, I will conclude this presentation with a discussion of how I am beginning to enhance these modules, to encourage students to consider environmental issues from multiple perspectives. (Received September 19, 2017)

1135-K1-2376 **Martha Shott\*** ([shott@sonoma.edu](mailto:shott@sonoma.edu)), Rohnert Park, CA. *Predicting Extreme Rainfall Events in Sonoma County: A Service-Learning Project in Mathematical and Statistical Modeling*. Preliminary report.

Sonoma State University (SSU) is situated about 30 miles inland of the Pacific Ocean, and just east of the first mountains that atmospheric rivers encounter on their journey from the tropics up toward the Northern California coast. Depending on the precise route of these “rivers in the sky,” the region surrounding Sonoma State may be subject to periods of intense rainfall that can lead to flooding. Having any sort of lead time on such a severe storm event may allow the county to begin their flood response efforts and mitigate damage to property and agricultural land.

To this end, students and faculty in the Math and Stats department at SSU partnered with researchers at Fairfield Osborn Preserve to investigate whether rainfall measurements at the coast could predict the rainfall received on campus. This project, developed in 2014 for SSU’s Mathematical and Statistical Modeling course, has continued in subsequent iterations of the course and has led to several student presentations at university symposia. In this talk, we discuss the initial development of this service-learning project, its evolution over four semesters, the primary learning outcomes embedded into the assignments, and the students’ responses to collaborating on the project. (Received September 26, 2017)

1135-K1-2703 **Celes A. Woodruff\*** ([woodruca@jmu.edu](mailto:woodruca@jmu.edu)) and **Yonathan Admassu** ([admassyx@jmu.edu](mailto:admassyx@jmu.edu)). *Identifying Sinkholes in an Introductory Numerical Methods Course*. Preliminary report.

High resolution digital elevation models (DEMs) have proven useful in mapping geomorphic features indicative of past geologic hazards, such as sinkholes. Sinkhole location mapping is necessary for studying factors controlling their development and also for city planning. Although sinkholes are easily discernible on high resolution DEMs by their rounded outlines, their large number has led to the investigation of automated mapping techniques to identify them.

Can the curvature of the boundary be used to determine whether a given depression is a sinkhole? We present a class activity and project for an introductory numerical methods course in which Calculus 1 and 2 are prerequisites. In the classroom we explore curvature of a function in the plane, radius of curvature, and osculating circles, then present a method for calculating curvature using discrete points. Outside of class, groups of students create codes in Matlab to calculate the curvature at each boundary point of some depression. Each group then decides how to use these individual curvatures to determine whether a depression is a sinkhole. Once all projects are complete the students present their methods in class and test their codes on actual geological data. (Received September 26, 2017)

## *Technology and Resources for Teaching Statistics*

1135-K5-21 **Dan Seth\*** ([dseth@wtamu.edu](mailto:dseth@wtamu.edu)), Mathematics, WTAMU, Canyon, TX 79015. *Enhanced student learning and attitudes with bi-weekly MINITAB explorations*. Preliminary report.

The MINITAB program has been integrated into Mathematical and Engineering Statistics at WTAMU the past five years. The course was restructured two years ago to incorporate bi-weekly MINITAB labs in a hybrid format. Labs were posted on the classroom management system and class time replaced with time in a computer laboratory. Students are introduced to MINITAB tools, explore or visualize theories and collect data to apply statistical tools learned. Sample lab activities will be presented along with assessment results. Students visualize Continuous PDF’s and generate data to explore the Central Limit Theorem. They collect fresh data for labs on Hypothesis Tests for the Mean or proportion, apply ANOVA to Real Estate Data, and run Chi-Square tests for Vehicle Data. Incorporation of the labs has enhanced learning and engendered an interest in statistics as a meaningful subject of study. (Received June 17, 2017)

1135-K5-63 **Leon Kaganovskiy\*** ([leonkag@gmail.com](mailto:leonkag@gmail.com)), 1233 East 19th, apt 6J, Brooklyn, NY 11230. *Teaching Statistics with R and Applications to Interdisciplinary Research*.

In this talk, I will briefly discuss contemporary textbooks and approaches which I have used in teaching lower and intermediate level Applied Statistics courses using R software. R is free and powerful software with thousands of packages applicable to both most advanced research and undergraduate teaching. In the remaining time, I will discuss joint projects with Psychology and Ecology departments which were made possible by R. (Received July 17, 2017)



1135-K5-169 **Rasitha R Jayasekare\*** (rjayasek@butler.edu), Dept. of Math, Stat & Actuarial Science, Butler University, 4600 Sunset Ave, Indianapolis, IN 46208. *Learning Statistics through Applications to Community.*

The elementary statistics course offered by the department of mathematics, statistics and actuarial science is a service course intended to meet the needs of students pursuing studies in many different areas within the college as well as for those who will later pursue postgraduate studies in social and natural sciences or professional programs in medicine. The topics include data analysis, descriptive statistics, linear regression, chi-square tests, analysis of variance, and tests and confidence intervals for means and proportions. In this course, the students usually undertake two group projects to analyze data on various topics. The first project is describing and learning from data and the second is an application of statistical procedures to employ inferential statistics. Last summer, the focus was on community related topics. Data on trash, recycling and waste water treatments from the community were used in the class. The student experience was changed from merely learning statistics to learning how it relates to the world outside the classroom. Students' feedback showed that they obtained a greater awareness of their community. In this talk I will present the tools and activities that were implemented for the community based learning approach in statistics. (Received August 07, 2017)

1135-K5-256 **Durga Prasad Dhakal\*** (durgadhakal@kusoed.edu.np), Kathmandu University School of Education, Lalitpur, 3 44700, Nepal, and **David A. Thomas** (mathcenter@ugf.edu), University of Great Falls, 1301 20th St S, Great Falls, MT 59405. *Teaching and learning statistics in education through MOODLE in Nepal.* Preliminary report.

This study aims at analysing the statistical skills of the students of Mathematics Education and English Language Teaching programs of Kathmandu University School of Education. Both the groups were treated with the same curriculum. They were taught through MOODLE software for forty-eight hours. They were continuously assessed through the format of the University and End semester examination. The result exposes that there was no significant difference between the score of statistics of the students of English Language Teaching and Mathematics programs. The study concludes that skills focused subjects can be taught through integrated curriculum even for the post graduate level students in order to have the desired outcomes. Keywords: Achievement, Integrated Curriculum, MOODLE, Statistical skills, Teaching & Learning. (Received August 16, 2017)

1135-K5-261 **Christine Davidson\*** (davidsc@sunysuffolk.edu), 533 College Road, Selden, NY 11784. *The Do's and Don'ts of a Statistics Project.*

At Suffolk County Community College, we offer two statistics courses. MAT103 is an introductory statistics course whereas MAT104 expands on those statistical concepts learned in MAT103 as well as introduces the students to more advanced statistical techniques. One of the course goals for MAT104 is to train students to conduct statistical studies as well as prepare them to analyze and present their results. One of my course requirements for MAT104 is a project that consists of developing a survey, collecting data, summarizing the data, constructing confidence intervals, performing hypothesis tests and writing an analysis report. I have had the opportunity to teach MAT104 four times. Over time, the structure of how I assign this project has changed based on my experience of what has seemed to work. In this presentation, I will share the evolution of this project and provide tips of how to create a successful and manageable class project. (Received August 16, 2017)

1135-K5-535 **Nicholas Long\*** (longne@sfasu.edu) and **Jeremy Becnel.** *Multivariable Calculus in Virtual Reality.*

We created an app that uses a Google Cardboard viewer to allow the user to visualize the concepts of multivariable calculus in a virtual reality setting. The user can specify their own objects for visualization as well go through lessons on the geometry and calculus of multivariable functions and the corresponding surfaces. In addition to this, there are interactive demonstrations where the user can further explore the concepts covered in the lessons. The rendering of these elements in a virtual environment is important since that allows the user to see the depth of these mathematical objects and helps the user to see the multiple facets at play in the study of these mathematics topics. We have also paired these lessons with writing assignments to assess their effectiveness in furthering student understanding of the geometric ideas. (Received September 07, 2017)

1135-K5-692 **Adam F. Childers** and **David G. Taylor\*** (taylor@roanoke.edu). *Classroom Stats: Spice Up Your Classroom with Fun, Live, Data Collection and Analysis.*

Studies show that students are more engaged in a statistics class when the data being used is personal, either because the data supports a cause they care about or because it is about them. In this talk, we demonstrate a new, free, mobile application and website for teaching statistics that allows instructors to collect data (via fun,

small, games), run experiments, and instantly analyze the results in front of the class. Instructors and students can also download the raw data to use in other statistics software such as MINITAB, R, and SPSS. The benefits of using the Classroom Stats system is that it eliminates the tedious process of manually collecting and recording data from students, and it allows the instructor to design and run meaningful experiments to teach statistical inference. We demonstrate how to use this application for both descriptive and inferential statistics along with our experience using it in the classroom. (Received September 13, 2017)

1135-K5-1036     **Howard Troughton\*** ([htroughton@babson.edu](mailto:htroughton@babson.edu)). *Bayes' Theorem and Lie Detector Tests*. At one time lie detector tests were used to help determine the innocence or guilt of a person. It was believed that an impartial machine would be able to objectively assess whether a person was lying or not. As it turned out, lie detectors are not perfect since they rely on bio-metric measures such as heart rate, perspiration, and others. In fact, a lie detector with 95% accuracy has the disturbing side effect that 32% of people it sends to jail are actually innocent! For this reason, lie detector tests are no longer admissible in a court of law. The reason for this side effect has to do with Bayes' Theorem, which is one of the most challenging topics to teach students in introductory statistics courses. Using this lie detector example the presenter will illustrate Bayes' Theorem and also show how this example relates to another challenging topic in statistics: Type I versus Type II errors. (Received September 18, 2017)

1135-K5-1220     **Alana J Unfried\*** ([aunfried@csumb.edu](mailto:aunfried@csumb.edu)), Department of Mathematics and Statistics, 100 Campus Center, Seaside, CA 93955. *Using Interactive R Tutorials and Reproducible Research Practices to Introduce Statistical Learning Ideas to Undergraduate Statistics Students*.

The demand for data-savvy workers has led to an increase in undergraduate students seeking advanced statistical training. Therefore, statistics faculty in my department are bringing prominent statistical software and advanced statistical topics into the undergraduate classroom. In particular, I will discuss an upper-division Linear Models course that heavily utilizes R software for computation. To enhance student understanding through the use of active learning and real, multivariate data, I utilize R Tutorials and R Markdown throughout the course. Interactive R Tutorials are a new tool created by RStudio, available through the `learnr` package. R Markdown, also from RStudio, creates a reproducible document that weaves together text, code, and results. I will discuss my use of R Tutorials to introduce the statistical learning topics of cross-validation and principal components analysis to Linear Models students. This allows students to be actively involved in learning both programming and statistical concepts in class. I will also discuss the implementation of lab assignments through R Markdown on the same topics. R Tutorials and R Markdown can be utilized in any statistics course, introductory or beyond, that incorporates the R programming language. (Received September 20, 2017)

1135-K5-1557     **Allen G Harbaugh\*** ([harbaugh@bu.edu](mailto:harbaugh@bu.edu)). *Clarifying and Reimagining the Empirical Rule: An Introduction to the By-Thirds Rule*.

This talk explores the pedagogic value of the 68-95-99.7 empirical rule (ER) presented in nearly every introductory statistics class. The value of the conventional rule will be examined in parallel with the methods in which it is frequently used, the manner in which it is presented in textbooks, and the historical context from which it evolved. A critical examination of the outcomes and effectiveness of teaching this concept to our students will reveal that the ER is best suited to serve as (1) a reasonable approximation for mental calculations and (2) a bridge to more complicated concepts and calculations. Alas, in its current incarnation, the ER apparently fails to accomplish these goals for most students. Finally, a revised version of the ER—preliminary coined the “By-Thirds Rule”—will be introduced. It will be demonstrated how this revised rule accomplishes the desired outcomes. Additionally, some history of statistics will be revisited to show that the revised ER more naturally mirrors the historical context from which the ER emerged. A key element of this talk is that it addresses introductory statistics instruction for an array of audiences, ranging from courses for mathematics majors to courses found in many social science undergraduate and graduate programs. (Received September 24, 2017)

1135-K5-1803     **Barb Bennie\*** ([bbennie@uwlax.edu](mailto:bbennie@uwlax.edu)), 1725 State Street, LaCrosse, WI 54601, and **Erick Hofacker** ([erick.b.hofacker@uwrf.edu](mailto:erick.b.hofacker@uwrf.edu)), 214C North Hall, River Falls, WI 54022. *Teaching Statistics through Simulation*.

Simulation is a method that is regularly referred to and suggested in the Common Core State Standards (K-12) and the Collegiate GAISE report for teaching statistics and probability. Our presentation will emphasize the use of simulation to assist in the teaching and learning of topics in the areas of Sampling Distributions, the Law of Large Numbers, the Central Limit Theorem, Confidence Intervals, and Hypothesis Testing. We will share how we use the StatKey app in order to teach inferential statistical concepts through the bootstrapping method

and randomization. Through a visual representation, students are exposed to a deeper understanding of the meaning of a p-value or the construction of a confidence interval. A series of tasks and activities will provide an understanding of how to implement this in the classroom, as well as the power of this method on constructing conceptual understanding during the early stages of an introductory statistics course. (Received September 24, 2017)

1135-K5-2443 **Kimberly A Roth\***, Brumbaugh Academic Center, 1700 Moore Street, Huntingdon, PA 16652. *Introducing R to different statistical audiences*. Preliminary report.

This semester all three of my classes, Introduction to Probability and Statistics, Introduction to Data Science, and Statistical Consulting are using R. Each class has many students who have never used R and have a few who have. I will discuss the implementations that I used in each class and what the advantages and challenges of each implementation were. Options such as using an Rstudio server and different graphing tools will be discussed. (Received September 26, 2017)

1135-K5-2594 **Matthew Leingang\*** ([leingang@nyu.edu](mailto:leingang@nyu.edu)), 251 Mercer St, New York, NY 10012.

*Combining auto-multiple-choice and Gradescope for paperless grading*. Preliminary report.

Auto-multiple-choice (AMC) is an application for generating exams with randomized elements. Instructors can give each student a unique exam, with parameters changed, options permuted, and even questions shuffled. Completed exam papers are scanned and scored using optical mark reader (OMR) software. Free-response questions can be included, scored by hand, and read with the OMR software.

Gradescope is a web service that allows digital grading of short-answer and free-response questions. Instructors prepare a PDF template and upload scanned exams. The software presents the grader with student submissions, and the grader can apply a consistent rubric with comments. Gradescope can handle multiple choice questions, but only if they are the same on each exam.

The author will detail a workflow for using both services on the same exam. It involves processing the same scans through each service, then running scripts to merge the scores on the multiple choice items (generated by AMC) with scores and comments on the free response items (collected by Gradescope). (Received September 26, 2017)

1135-K5-2755 **D Brian Walton\*** ([waltondb@jmu.edu](mailto:waltondb@jmu.edu)), Roop Hall, MSC 1911, 60 Bluestone Dr, Harrisonburg, VA 22807. *Web-based apps for practicing algebra and calculus skills*.

Web-based apps for practicing algebra and calculus skills provide a tool with which learners can progress through a collection of problems of varying difficulty while receiving instant feedback. An underlying algebra engine was written in Javascript to enable parsing and evaluation of algebraic expressions for numerical rather than symbolic comparison of expressions. Randomly generated problems are created based on a user-selected difficulty level. Examples include algebraic factoring, function composition, differentiation and antidifferentiation, and evaluation of limits. Work in progress includes defining problems for live exercises in online texts authored using the PreTeXt. (Received September 26, 2017)

1135-K5-2768 **Robin H Lock\*** ([rlock@stlawu.edu](mailto:rlock@stlawu.edu)). *Does the Randomization Method Matter?* Preliminary report.

As we gain experience with using simulation-based methods to teach statistical inference, we note that we can sometimes use more than one randomization method to simulate samples for producing a confidence interval or performing a hypothesis test. For example, in a regression setting, we can generate bootstrap samples by resampling data cases or by resampling residuals. We approach the “Does the method matter?” question on two levels: “Is there a noticeable difference in the statistical result, depending on the randomization method?” and “Should we have students worry about which method to use?”. (Received September 26, 2017)

1135-K5-2860 **Dominic Klyve\*** ([klyved@cwu.edu](mailto:klyved@cwu.edu)). *Teaching P-values from Primary Sources*.

Teaching the concept of the p-value is an important part of a class in elementary statistics, but it is often recognized as an idea concerning which it is difficult to help students build a rich understanding. This talk will discuss a project author has developed for the TRanforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS). In this project, students are asked to read excerptd from 18th century texts which can be viewed as precursors to modern ideas of p values and hypothesis testing together with some of the work of Ronald Fisher, and to answer questions and perform tasks related to their reading. (Received September 26, 2017)

1135-K5-2933 **John D. Ross\*** ([rossjo@southwestern.edu](mailto:rossjo@southwestern.edu)), SU Box 7371, 1001 E. University Avenue, Georgetown, TX 78626. *Implementing R Activities and Projects in Introductory Statistics.*

Using R to teach statistics is not a novel idea. However, R is not often used in an introductory statistics course - especially one that is being used as a service course for other majors. In such a setting, with students who might be math- or stats-averse, adding on an extra layer of material through R can be intimidating. And yet, R is incredibly useful, and such students could benefit greatly in their future courses or research experiences from having some R experience.

This paper presents a series of in-class activities and out-of-class projects that were created to introduce students to R in an Introductory Statistics course in a liberal arts setting. The activities and projects will be discussed, as well as the experiences (and lessons learned) that came with using R in such an environment.

(Received September 26, 2017)

1135-K5-2991 **Kari Lock Morgan\*** ([k1m47@psu.edu](mailto:k1m47@psu.edu)), 323B Thomas, University Park, PA 16802. *Engaging Students by Using Simulations to Address the Question of the Day.*

This talk will discuss a recent redesign of introductory statistics. The first goal of this redesign was to introduce inference using simulation-based methods, capitalizing on modern computing power to promote conceptual understanding of fundamental ideas. The second goal was to design each class and lab to use data to address a real "question of the day," capitalizing on student's natural curiosity and demonstrating the power and relevance of statistics. This talk will discuss the changes, challenges, and resulting improvement on learning. (Received September 26, 2017)

1135-K5-3011 **Charlotte A Bolch\*** ([cbolch@ufl.edu](mailto:cbolch@ufl.edu)), College of Education, School of Teaching and Learning, PO Box 117048, Gainesville, FL 32611, and **Tim Jacobbe** ([jacobbe@coe.ufl.edu](mailto:jacobbe@coe.ufl.edu)). *Teaching Students Data Visualization Skills.*

Students are interacting more with graphical displays and data visualizations presented in both courses and their everyday lives. Educators should consider incorporating data visualizations skills into statistics courses where graphical displays are introduced. A data visualization module that was developed for and used in an introductory statistics course will be presented. This module addresses the skills of learning how to read, interpret, and create data visualizations that make the data stand out, facilitate comparisons, and are information rich. Students were taught to interpret, critique, and construct data visualizations using software including Minitab, Google Sheets, iNZight, and RAWGraphs. Students completed a data visualization assignment, lab assignments, a data visualization group poster, pre/post statistical assessments (LOCUS project, DRL-1118168), pre/post Survey of Attitudes Towards Statistics, and journal entries reflecting on their experiences within the module. Teaching resources, a data visualization assignment, and student feedback will be shared regarding the data visualization module along with how the module was integrated into the course curriculum to encourage students to use multivariate thinking and understand statistics as an investigative process. (Received September 26, 2017)

1135-K5-3015 **Gail F Burrill\***, 12155 W Luther Ave, Hales Corners, WI 53130. *Developing Concept Images Core Statistical Ideas: The Role of Interactive Dynamic Technology.* Preliminary report.

Coming to understand a mathematical or statistical concept involves creating a mental image of that concept. A technology-leveraged approach can help students develop such a mental image and embed that image in a framework for organizing basic statistical ideas. Carefully designed applets where students take a meaningful action and observe and reflect on the consequences can help students develop dynamic mental images of central concepts such as distribution, variability or sampling with an emphasis on the use of simulation techniques. In particular, research has identified typical misconceptions students have related to core statistical concepts, and the applets and accompanying materials focus on helping students confront these misconceptions by creating visual interactive representations (mini "video clips"). The discussion will describe a research project that investigated how these dynamic interactive applets were used in a course for preservice elementary students to help them build robust concept images of key statistical ideas. (Received September 26, 2017)

## *Innovative and Effective Online Teaching Techniques*

1135-L1-58      **M. Reba\*** ([mreba@clemson.edu](mailto:mreba@clemson.edu)), Department of Mathematical Sciences, Martin O-202, Clemson University, Clemson, SC 29631. *Online Calculus I in Five Weeks with Remediation Opportunities.*

Summer Calculus I Online covers in five weeks the same material that we cover in 15 weeks during the academic year. The students enrolled in Calculus I comprise three (unequal) groups: (1) those who failed it before; (2) those who want a review; and (3) those who are taking it for the first time. Among each group are non-traditional or at-risk students who narrowly passed either the placement exam or the prerequisite courses. To keep students working during this intense time-period, there are multiple daily-assignments and almost weekly-proctored exams. There are also optional assignments to remediate algebra and trigonometry skills. I will outline how the remediation process works in conjunction with specific sections of the Calculus text. I will also discuss how the structure of my lectures (using 43 recorded-videos) evolved and how discussion posts aid student understanding by asking them to explain core concepts in their own words. I will explain why I grade scanned and uploaded handwritten text-homework, in conjunction with assignments from an online homework system. Finally, I will discuss the process I use to communicate with remote testing centers (both national and international) for hundreds of exams. (Received July 13, 2017)

1135-L1-405      **Cheryll E Crowe\*** ([cheryll.crowe@asbury.edu](mailto:cheryll.crowe@asbury.edu)). *Opportunities & Challenges: Teaching Mathematics Online to Non-Traditional Pre-Service Teachers.*

Elementary teachers often experience math anxiety and fear which can be heightened in the online environment. This presentation will address opportunities and challenges in meeting the needs of non-traditional pre-service teachers in online mathematics education courses. Strategies to support student success will be discussed including the structure of online class meetings, implementation of virtual tutoring, and mentoring to grow and sustain motivation. Best practices for teaching this student population will be incorporated into this talk. (Received August 31, 2017)

1135-L1-823      **Robert J Krueger\*** ([rkrueger@csp.edu](mailto:rkrueger@csp.edu)). *An Approach to Teaching Online Discrete Mathematics.* Preliminary report.

For the last two years, the author has invested time and energy into an online two-course sequence in Discrete Mathematics for computer science students. The courses are delivered in seven-week sessions with a one hour synchronous session per week. The courses utilize an open-source text supplemented heavily by online resources. The author also created an effective learning environment through weekly practice assignments, quizzes, and graded assignments. In the end, most students found success in the timed final exam. The presentation will cover methodology, engagement techniques, and results. Frustrations and future growth opportunities will also be discussed. (Received September 14, 2017)

1135-L1-862      **Michelle Homp\*** ([mhomp3@unl.edu](mailto:mhomp3@unl.edu)), Center for Science, Math, Computer Education, 251 Avery Hall, University of Nebraska, Lincoln, NE 68588. *Facilitating Active Learning in Online Mathematics Courses.*

The University of Nebraska - Lincoln offers graduate courses for secondary mathematics teachers seeking professional development opportunities or pursuing the Master of Arts for Teachers degree from the Department of Mathematics. The face-to-face sections of these courses are highly interactive, with groups collaborating to complete tasks, present ideas and discuss different solution strategies during class time. This presentation will discuss how the interactive features of the in-person courses have been replicated in an online setting. These features include strategies for engaging students in meaningful small and large group discussions, for managing collaborative tasks, for providing timely feedback, and for orchestrating student-student and teacher-student interactions around the mathematics content. Successes and challenges of the various online active learning strategies will be shared along with feedback from past students. (Received September 20, 2017)

1135-L1-1912      **Susan Marla Orme\*** ([ormes@byui.edu](mailto:ormes@byui.edu)), 232 Ricks, Rexburg, ID 83460-2155. *Online College Algebra Course: Engaging Students in Meaningful Learning Experiences.*

Student engagement usually implies that students are attentive and actively learning the topic being taught. In 2015, my colleague and I were asked to develop an online college algebra course. We were faced with the challenge of providing meaningful learning experiences in an online course where students are engaged in connecting real-world applications of functions and matrices in a college algebra course. The development of the course required collaboration with an online curriculum designer and the campus college algebra committee. This collaboration led to the development and pilot of the course during the fall 2017 semester. One challenge we faced was to

incorporate hands-on labs from the face-to-face course in the online course. Videos and apps were created for students to collect and analyze data from a real-world situation. Students then collaborated in online groups to answer questions, discuss what they learned and what their learning meant in the context of the problem. Using data from the pilot, I will share lessons learned, quantitative data of student performance, insights from students and insights from the online instructor. This quantitative and qualitative data will determine how we improve the course to help students engage in the material in meaningful ways. (Received September 25, 2017)

1135-L1-1949      **Miriam Harris-Botzum\*** ([mharrisbotzum@lccc.edu](mailto:mharrisbotzum@lccc.edu)), 4525 Education Park Drive, Office SH 7D, Schnecksville, PA 18069. *Brave New Worlds: My ongoing journey into the world of online mathematics instruction.* Preliminary report.

I started teaching online classes ten years ago, with a College Trigonometry course. I now teach Intermediate Algebra, College Algebra, College Trigonometry, and Calculus II online. While I am still striving to improve my own online courses, I have discovered both technologies and approaches to online education that have dramatically improved the success rates in my classes. In this talk, I will focus primarily on the ways I help students acclimate to the online learning environment, and help them develop the mindset and skill sets they need to succeed. (Received September 25, 2017)

1135-L1-2021      **Yun Lu\***, Department of Mathematics, Kutztown University of PA, Kutztown, PA 19530. *Student Engagement in an Online Mathematics Course.*

In this talk, I will investigate the students engagement in online mathematics courses, and share my experience about various approaches to improve student engagement in my online mathematics courses that were taught recently. I will discuss some of my successful and unsuccessful approaches, as well as students' feedback and performance if time allows. (Received September 25, 2017)

1135-L1-2078      **RaKissa D Manzanares\*** ([rakissa.manzanares@ucdenver.edu](mailto:rakissa.manzanares@ucdenver.edu)), Campus Box 170, P.O. Box 173364, Denver, CO 802173364. *Best Practices for Teaching an Active and Engaging Online Mathematics Course.*

How do we create an active and engaging online learning environment? How do we engage students in a dialogue about mathematics with their instructor and classmates utilizing an online learning platform? How do we help students decode the classroom culture and expectations in an online course? How do we impart our knowledge of the material to students to support them in their learning of the material? In this talk, I will provide a variety of strategies that have worked for me to address these areas of concerns when teaching an online mathematics course. I will also share sample ice breakers and personal introductions, activities, and resources to make effective learning materials including videos for the online learning environment. (Received September 25, 2017)

1135-L1-2827      **Vesna Kilibarda\*** ([vkilibar@iun.edu](mailto:vkilibar@iun.edu)), 3400 Broadway, Department of Mathematics, Indiana University Northwest, Gary, IN 46408. *Group Projects and Ice-breakers Build Classroom Community in a Finite Math Course.*

We are a commuter campus with diverse student body. Majority of our students work more than 20 hours per week, many have families, and almost a fifth are non-traditional students. For our student population online learning provides many advantages, such as flexible learning environment that caters to a variety of learning styles, savings in transportation cost and time, ability to customize the course to their needs, and plenty of one-on-one contact with instructor. Our students face challenges in an online learning environment, from adapting to CMS to time-management and motivation. We have found that group projects where students study real-world problems that matter to them and ice-breaker discussions at critical points in a semester help students feel a part of a classroom community and help boost their self-motivation. We assign group projects where students solve real-world problems using an Excel macro for solving systems of equations or a simplex method Java applet for solving systems of linear inequalities. After a challenging assignment a "Two-truths, One Fib" ice-breaker discussion about their class experience helps students understand that their peers are in a similar situation and helps an instructor modify the class in a way that can bust students' confidence (Received September 26, 2017)

## *Attracting, Involving, and Retaining Women and Underrepresented Groups in Mathematics—Righting the Balance*

1135-L5-61 **Jenna P Carpenter\*** ([carpenter@campbell.edu](mailto:carpenter@campbell.edu)), PO Box 115, Buies Creek, NC 27506.  
*Using Makerspaces to Attract and Retain Women In STEM.*

Despite decades of effort to recruit and retain women in STEM fields, we have made little progress in the hard sciences (AAUW, 2015). Research suggests connecting STEM to applications that make a difference in the world (NAE, 2008), using hands-on projects (Vas, R., Quinn, P., & A. Heinricher, 2013) and developing a sense of belonging (Stout, Dasgupta, Hunsinger, & McManus, 2011) all help with recruitment and retention of women. Makerspaces offer opportunities for doing all three, through the use of equipment such as 3D printers, vinyl cutters, embroidery machines, etc. These tools can be used to introduce students to CAD software and bolster 3D visualization skills, which improves student success across the curriculum (Sorby, 2009). Makerspaces invite creativity, allow for expression of artistic talent, and permit girls to acquire technical skills in a less-gendered environment, which may increase sense of belonging and fit. All of these suggest ways to use makerspaces to increase the number of women in STEM (Morocz, R., Levy, B., Forest, C. & R. Nagel, 2015; Roldan, W., Hui, J., & E. Gerber, 2017). We will report on our first year of makerspace usage, with suggestions for other programs interested in similar strategies for recruiting and retaining women. (Received July 14, 2017)

1135-L5-133 **Susan E Kelly\*** ([skelly@uwlax.edu](mailto:skelly@uwlax.edu)), Department of Mathematics and Statistics, UW - La Crosse, 1725 State Street, La Crosse, WI 54601. *Undergraduate Research in Mathematics History and Social Justice.*

Several years ago, I began mathematics history research with an emphasis on key figures advancing the roles of women and minorities. These projects work well for including undergraduate students in the research and they can often work on parts of these projects early in their studies. I have worked with several female students on these projects. Through the work, they have gained insight on struggles past women and minorities have faced and have learned how drive and determination gave rises to success. I have seen several of my students gain confidence in their own abilities. Also, I try to often select students considering education careers so that they can bring what they gain to their future students. Some of these projects have been presented at national conferences and two such projects have produced papers published in the Notices of the American Mathematical Society. This talk will present some aspects and values I have found in this work. (Received August 02, 2017)

1135-L5-300 **Jacob Price\*** ([jrp14@uw.edu](mailto:jrp14@uw.edu)). *Cultivating an Inclusive Atmosphere in Scientific Computing through Diverse Historical Perspectives.*

Mathematics, science, and the STEM fields in general suffer from retaining women and underrepresented groups, to the detriment of the field as a whole. One potential reason that may discourage individuals from pursuing a STEM degree is that the cultural conception of a “scientist” may not look like members of these underrepresented groups. This cultural conception is generated through years of being provided examples of scientists that fit a particular profile: typically white and male. In this talk, I describe my efforts to reframe this cultural conception in an introductory scientific computing class. This is a critical juncture in the STEM education of students, as learning to code is becoming more and more necessary in science, and computer programming is often seen as an area hostile to underrepresented groups. There are countless historical examples of revolutionary contributions to scientific computing by people who do not fit the stereotypical scientist image, such as Katherine Johnson and Ada Lovelace. By motivating lectures with anecdotes about inspirational scientists of diverse backgrounds, students are presented with a more realistic picture of what a scientist looks like, and hopefully will be able to see themselves in this picture. (Received August 21, 2017)

1135-L5-390 **Charles Peter Funkhouser\*** ([cfunkhouser@fullerton.edu](mailto:cfunkhouser@fullerton.edu)) and **Miles Ruben Pfahl.**  
*Discovering Undergraduate Mathematics in American Indian Culture.*

This project develops and researches undergraduate mathematics materials based in the culture and mathematics of Native American Peoples for integration into undergraduate courses with an emphasis on social justice and equity in mathematics education. While mathematics topics include probability and statistics, number theory, transformational geometry, and pre-service elementary and secondary education-related content, social concerns are reflected in cultural topics such as public health (diabetes) and Tribal membership (blood quantum). The presenters discuss how standard undergraduate mathematics topics can impact daily life among Tribes. These materials—both paper and electronic—are classroom ready, and are developed, piloted and assessed in consultation

with Tribes in the the Rocky Mountains, the Plains, the Pacific Northwest, and the Southwest. This work is an NSF DUE TUES funded project. (Received August 30, 2017)

1135-L5-620 **L. Marizza Bailey\***, BASIS Scottsdale, Scottsdale, AZ 85259, and **Carrie Diaz Eaton**. *Re-claiming identity through cultural education.*

The *Math Mamas* project is an organically organized effort by a group of academic mothers in mathematics to offer diverse storylines of women in mathematics. The project is managed by Dr. Carrie Diaz Eaton, Dr. Pamela Harris, Dr. Emille Davies Lawrence, and Dr. Becky Hall. A special issue of the *Journal of Humanistic Mathematics* will be devoted as well as an American Mathematical Society blog series. Lastly, *Math Mamas* has partnered with the Association for Women in Mathematics to feature profiles of “math mamas” for its new website effort funded by NSF INCLUDES. This project created the opportunity to examine how we shift our storyline. One way to shift from our designated identities to new identities which allow us to belong is through learning about historical, but hidden storylines, such as *Hidden Figures*. For Latin@s, these stories may come from our ancestors beyond our geographic US borders and may require us to engage in the intentional decolonization of our designated identities. Here, we present a one such storyline of Dr. Luz Antonia Mendizabal Galv3ez de Rodriguez, while examining our own identities as mathematicians, mothers, and Latinas. (Received September 11, 2017)

1135-L5-1359 **Darolyn A. Flaggs\*** (df1170@txstate.edu). *Campus Racial Climate and Sense of Belonging: Psychosocial Factors Impacting Persistence Intentions of Students in Developmental Mathematics.* Preliminary report.

College student persistence and retention is a national challenge, especially for students of underrepresented racial groups. This research attempts to identify immediate psychosocial causes that are student modifiable through educational interventions and support. To expand retention models, particularly for underrepresented groups in developmental mathematics, a conditional indirect effects model was created and used to examine indirect effects from campus racial climate to sense of belonging to intent to persist and the moderating role of resiliency within this indirect path. Findings, results, and implications will be shared to raise awareness of issues surrounding retention and to help guide institutions to explore ways to increase students’ sense of belonging and build a culturally supportive climate. (Received September 21, 2017)

1135-L5-1598 **Guadalupe Lozano\*** (guada@math.arizona.edu). *Measuring the Global Gender Gap in Mathematics and the Natural Sciences: Working Frameworks and a Recent Grant-Funded International Initiative.* Preliminary report.

Women’s representation in mathematics and the natural sciences has globally improved in recent decades but not sufficiently to alleviate concerns about a persistent gender gap in such disciplines. How large is the gender gap in STEM fields? How does it vary across disciplines, countries, and what factors are associated to it?

While recent studies have begun to map gender differences in mathematics research publications and journal editorships across time and countries, a systematic effort to tackle the quantification of the global gender gap in STEM was, to our knowledge, lacking until last year.

In this talk I will frame the complex problem of measuring the global gender gap in STEM, and introduce a new initiative led by the International Mathematical Union (IMU) and the International Union of Pure and Applied Chemistry (IUPAC) to measure the global gender gap in mathematics and the natural sciences. (Received September 23, 2017)

1135-L5-2394 **Talitha M Washington\*** (talitha.washington@howard.edu), Department of Mathematics, Howard University, Washington, DC 20059, and **Vernon R. Morris** (vmorris@howard.edu), Department of Chemistry, Howard University, Washington, DC 20059. *The Role of Professional Societies in STEM Diversity.*

The overall percentages of African American scientists indicates underrepresentation in most science, technology, engineering, and mathematics (STEM) disciplines and the percentages appear to be declining over the last three decades. Despite investments in diversity programs, the observable impact on STEM leadership and the demographics of the science and technology workforce remains quite small. This presentation will highlight some of the challenges and barriers that many students and professionals who seek to pursue careers in these fields face, and the role of professional societies in either exacerbating the perpetuation of monocultures in the various STEM disciplines or proactively working to eliminate barriers and discrimination.

We will present and provide clarity on three common myths that are often articulated in discussions of STEM diversity. We will share insights on how professional societies can directly impact the broadening of participation



as well as the persistence of racial groups in the STEM fields and hence, strengthen and sustain the Nation's future workforce. (Received September 26, 2017)

1135-L5-2563 **Helen Burn** ([hburn@highline.edu](mailto:hburn@highline.edu)), 2400 S. 240th Street, MS 18-1, Des Moines, WA 98198, **Vilma Mesa** ([vmesa@umich.edu](mailto:vmesa@umich.edu)), 610 E. University, Ann Arbor, MI 48109, **J Luke Wood\*** ([luke.wood@sdsu.edu](mailto:luke.wood@sdsu.edu)), 5500 Campanile Drive, San Diego, CA 92182, and **Eboni Zamani-Gallaher** ([ezamanig@illinois.edu](mailto:ezamanig@illinois.edu)), 380 Education Building, 1310 S. Sixth St., Champaign, IL 61820. *Programs, Structures and Instructional Strategies that Facilitate Diverse Learners Transitioning to and through Calculus in Two-Year Colleges.*

Transitioning Learners to Calculus in Community Colleges (TLC3, NSF IUSE 1625918, 1625387, 1625946, 1625891) aims to improve the transition of underrepresented minority (URM) students to and through calculus in two-year colleges by identifying programs, structures, instructional strategies, and key transition points within the mathematics curriculum that influence the success of URM students. Project results are disseminated to a networked community of two-year colleges to increase their readiness to facilitate URM student success in courses leading to calculus. This presentation focuses on findings derived from data collected through a census survey of mathematics department chairs in two-year colleges that captured mathematics course offerings and sequencing, instruction, math placement, in-and-out-of-class student support, use of local data, and faculty professional development. The survey responses are linked to a college's MSI (minority-serving institution) designation. The presentation will highlight various approaches taken by mathematics programs in two-year colleges to improve the success of URM students by two-year institutional type. Findings suggest differences within and between those with an MSI designation in contrast to predominately white institutions (PWI). (Received September 26, 2017)

1135-L5-2610 **Nancy Emerson Kress\*** ([nancy.kress@colorado.edu](mailto:nancy.kress@colorado.edu)), **Matt Voigt** ([mkvoigt@gmail.com](mailto:mkvoigt@gmail.com)), **Jess Ellis Hagman** ([jess.ellis@colostate.edu](mailto:jess.ellis@colostate.edu)) and **Jessica Gehrtz** ([jrgehrtz@gmail.com](mailto:jrgehrtz@gmail.com)). *Learning Assistants and Undergraduate Tutors in Active Learning Precalculus and Calculus Courses: Cultivating a Sense of Belonging Among Students from Marginalized Groups.* Preliminary report.

While the use of active learning in university Precalculus and Calculus courses has been associated with increased levels of persistence in mathematics, research has found that reform based mathematics teaching, of which active learning is one example, is inconsistent in its promotion of equitable access to mathematics learning opportunities (Ball et al., 2005; Boaler, 2002; Delpit, 1988; Lubienski, 2000; Parks, 2010). Early results from our large multi-institution study indicate that significant differences in sense of belonging and inclusion persist between students from underrepresented groups and their white male peers in active learning mathematics courses. This presentation will describe a structure for supporting undergraduate learning assistants and tutors to develop a deeper understanding of teaching for equity and access in active learning contexts. Students in these instructional roles provide a key leverage point in the effort to promote a stronger sense of belonging and positive experience in mathematics for students from underrepresented groups. Using sociocultural theory and a critical perspective to motivate the work we will share how learning assistants and tutors can play a valuable role in increasing participation in mathematics by women and minorities. (Received September 26, 2017)

1135-L5-2968 **Alessandra Pantano\*** ([apantano@uci.edu](mailto:apantano@uci.edu)), **Casey Kelleher** ([ckelleher@math.princeton.edu](mailto:ckelleher@math.princeton.edu)) and **Natalia Komarova** ([komarova@uci.edu](mailto:komarova@uci.edu)). *Diversity in Math Festival: Sharing the experience.* Preliminary report.

Organized in April 2017 by a team on UCI faculty and graduate students, the UCI Diversity in Mathematics Festival was a celebration of the mathematical talent of women and other underrepresented groups in Southern California. The festival featured 24 talks, across three parallel sessions focused on communicating advances in mathematics, promoting career awareness and raising understanding of diversity issues. Two panels and a poster session further enriched the offerings. All the speakers were women and/or from groups typically underrepresented in mathematics. A friendly sense of community pervaded the entire event. The UCI Diversity in Mathematics Festival is a successful example of how a relatively small number of passionate individuals can come together to encourage and support a large group of talented mathematicians whose potential risks to go unnoticed. (Received September 26, 2017)

1135-L5-3022 **Janine E. Janoski\*** ([janinejanoski@kings.edu](mailto:janinejanoski@kings.edu)). *King's College Women in Science and Engineering.*

King's College is a small, catholic, liberal arts school located in Northeastern Pennsylvania. At King's, in almost every STEM field the percent of women professors is above the national average, yet our percentage of female STEM students is not. During Summer 2016 a group of female STEM faculty asked ourselves how we can use

our strong female faculty presence to improve the number of female student we both recruit and retain. This was the start of a King's College Women in Science and Engineering (KC-WiSE) group. This group is a faculty led group of faculty and students, women and men, that support women in science. The group rests on three pillars: professional development, social events to build community, and mentoring. Join me as I tell you about our efforts starting such a group, our successes, and the ways we are working to improve how our group has a positive impact on females in STEM majors. (Received September 26, 2017)

## *Good Math from Bad: Crackpots, Cranks, and Progress*

1135-M1-674 **Hossein Behforooz\*** ([hbehforooz@utica.edu](mailto:hbehforooz@utica.edu)), 34 Emerson Av., Utica, NY 13501. *Playing with Continued Radicals and Iterated Exponents*. Preliminary report.

In real analysis and number theory courses, the study of the Continued Radicals and Continued Fractions and Iterated Exponents are very popular and interesting subjects. In this talk we will present some notes on the calculation of the continued radicals and iterated exponents with integer elements and integer values by using splitting of the integers under radicals in a certain ways which are very bizarre and interesting approaches. Since the time of the lecture is short and the title of the session is "Good Math from Bad: Crackpots, Cranks, and Progress" I promise to follow the title of the session and state the theorems without proofs. That will change my presentation from theoretical approach lecture to a fun and amusement type of lecture. We will see some unexpected contradictory results. Yes, this talk is understandable to high school and undergraduate students. Come and join us. MATH is FUN. (Received September 12, 2017)

1135-M1-928 **Montgomery Link\*** ([mink@suffolk.edu](mailto:mink@suffolk.edu)), Suffolk University, Department of Philosophy, Boston, MA 02108. *Kurt Gödel's Last Work on the Power of the Continuum*. Preliminary report.

There are many purveyors of bad mathematics. Squaring the circle, doubling a cube and trisecting angles are just a few well-known ill-conceived pursuits. Crank mathematicians are not always amateurs, and sometimes good ideas are generated on bad problems. But there is a further case of the first-rate mathematician, who already has an established publication record in a particular field, then has a further valuable insight, but adopts a particular argumentative approach that has a gap. In such a case we might say that the situation is not ultimately hopeless. Such was the case with Gödel's later work on the 'true power of the continuum' in which Gödel endeavored ([1970a], [1970b]) to prove that  $2^{\aleph_0} = \aleph_2$ . Gödel drew on early results on pantachies based on Hausdorff's redefinition. Although Gödel was a preeminent expert in this field, and his new approach to this problem drew on a solid basis, his proof is flawed, as he himself later acknowledged. While his methods did not in and of themselves lead to further progress on this front, Gödel's later position that  $2^{\aleph_0} = \aleph_2$  has been to a certain extent vindicated by later deep arguments that show that this result can be obtained from large cardinal axioms. (Received September 17, 2017)

1135-M1-1190 **John J B Webb\*** ([webbjj@jmu.edu](mailto:webbjj@jmu.edu)), Department of Mathematics & Statistics, 305 Roop Hall, MSC 1911, James Madison University, Harrisonburg, VA 22807. *Can  $1 + 2 + 3 + 4 + 5 + \dots$  really equal  $-1/12$ ?*

In 2014, Numberphile set the mathematical world ablaze when they posted a video to YouTube (<https://www.youtube.com/watch?v=w-I6XIVZXww>) of two physicists claiming to prove that  $1 + 2 + 3 + 4 + 5 + \dots = -1/12$ . The video has over 5 million views to date, which probably means that its one of the most popular math videos ever (or something like that). We will discuss this "proof" and some of the deep connections this divergent series has with the innocuous looking little fraction,  $-1/12$ . (Received September 20, 2017)

1135-M1-1293 **Jason Rosenhouse\*** ([rosenhjd@jmu.edu](mailto:rosenhjd@jmu.edu)), Dept. of Math and Stat., James Madison University, Harrisonburg, VA 22807. *On Mathematical Anti-Evolutionism*.

The teaching of evolution in American high schools has long been a source of controversy. The past decade has seen an important shift in the rhetoric of anti-evolutionists, towards arguments of a strongly mathematical character. These mathematical arguments, while different in their specifics, follow the same general program and rely on the same underlying model of evolution. We shall discuss the nature and history of this program and model, and describe general reasons for skepticism with regard to any anti-evolutionary arguments based upon them. We shall then survey the major arguments used by anti-evolutionists, to show how our general considerations make it possible to quickly identify their weakest points. (Received September 21, 2017)

1135-M1-1413 **Gareth E. Roberts\*** ([groberts@holycross.edu](mailto:groberts@holycross.edu)), Dept. of Mathematics and Computer Science, 1 College Street, Worcester, MA 01610. *Ernő Lendvai and the Bartók Controversy.*

In 1955, Hungarian music theorist Ernő Lendvai published research claiming the existence of the Fibonacci numbers and the golden ratio in several works of the famous Hungarian composer Béla Bartók. Lendvai's discoveries and arguments were remarkable and ground-breaking, leading to a flurry of claims that Bartók and other well-known composers (e.g., Debussy) were consciously incorporating sacred mathematical proportions in their music. Unfortunately, Lendvai made some crucial errors in his analysis, cherry-picked favorable data, and made some questionable assumptions. Following the excellent analysis of Roy Howat, we will discuss the evidence for and against the purported use of the Fibonacci numbers and golden mean in Bartók's music, focusing our attention on *Music for Strings, Percussion and Celesta*. (Received September 22, 2017)

1135-M1-2647 **Elizabeth Theta Brown\*** ([brownet@jmu.edu](mailto:brownet@jmu.edu)), Dept. of Mathematics and Statistics, MSC 1911, 800 South Main, Harrisonburg, VA 22807. *A Good False Proof of the Fundamental Theorem of Finitely Generated Abelian Groups.*

We offer a compelling but false proof of FTFGAG, together with a guided discovery project on detecting the errors suitable for a first course in abstract algebra. This "proof," together with the analysis, is useful in that it offers undergraduates both a decent intuition of why the theorem is true, and practice finding subtle errors. Instructors who doubt that such practice is needed are invited to offer the –false–proof, and see how many students object. (Received September 26, 2017)

## *Discrete Mathematics in the Undergraduate Curriculum—Ideas and Innovations in Teaching*

1135-M5-669 **Kathleen M. Shannon\*** ([kmshannon@salisbury.edu](mailto:kmshannon@salisbury.edu)), Kathleen Shannon, Department of Mathematics and Computer Sci., Salisbury University, Salisbury, MD 21801. *Using WebAssign in Discrete Mathematics.* Preliminary report.

This paper will report on the progress of incorporating WebAssign in four sections, with four different instructors, of a freshman/sophomore-level Discrete Mathematics course serving mathematics and computer science majors, as well as mathematics minors from a number of disciplines, at a comprehensive university. The speaker is the author of *Discrete Mathematics: A Brief Introduction*, which has been used as the primary text for Math 210: Discrete Mathematics at Salisbury University since Fall 2003. Over the 2016-17 academic year, the author worked with WebAssign to create an e-version of the text and to program homework exercises for WebAssign's automatic grading system. Salisbury University is beta testing the text for full release beginning in Spring 2018. In this paper, I will report on: the process of preparing a text and the associated homework problems for WebAssign, the use of WebAssign in the fall 2017 beta test, issues with respect to using WebAssign for Discrete Mathematics, and the general nature and content of the course. (Received September 12, 2017)

1135-M5-964 **Katarzyna Kowal\***, [kkowal@ramapo.edu](mailto:kkowal@ramapo.edu). *Effective instructional strategies for teaching a Discrete Structures proof writing course.* Preliminary report.

In this paper the author will describe some effective instructional strategies for teaching the course based on her 13 years of experience of teaching it. Discrete Structures is a course that has recently been credited with the Writing Intensive course attribute, which is an attribute required by the college for graduation. The author will present her research on how the Writing Intensive assignments and the WI attribute affect student performance in the course as compared to the semesters before the course became WI. Some pedagogical interventions will be discussed, including: why it is beneficial to introduce formal proof writing in the course slightly earlier than usual, what are some common misconceptions in the area of logic and set theory that should be pointed out to students, and how to inspire the students in the course to become math majors or math minors. Some innovative elements of teaching the course will be shown, including the idea of how to use graph theory to effectively introduce nested quantifiers. The author will present some useful strategies of how to implement the philosophy that mathematics is a language and that students should be able to come up with valid and coherent proofs, and to communicate mathematics with precision, clarity, and organization. (Received September 17, 2017)

1135-M5-1246 **Andrew J Simoson\*** ([ajsimoso@king.edu](mailto:ajsimoso@king.edu)), Mathematics Department, King University, 1350 King College Road, Bristol, TN 37620, and **Andrew F Rich** ([africh@manchester.edu](mailto:africh@manchester.edu)), Manchester University, 604 E College Avenue, Manchester, IN 46962. *Conjunctions of runners on a circular track.*

Imagine three runners on a circular track of unit length at constant speeds of  $\alpha$ ,  $\beta$ , and  $\gamma$ . If they all start at point  $A$  on the track, when next will all runners be at point  $A$ ? If  $\alpha$ ,  $\beta$ , and  $\gamma$  are rational numbers then the solution involves the least common multiple of their denominators. However what happens when they are irrational? Given a small positive number  $\epsilon$ , we show how to apply Euclid's algorithm for the greatest common divisor of two integers to find a positive time (where all the runners have completed at least one lap) so that the runners are all within  $\epsilon$  distance of  $A$ . (Received September 20, 2017)

1135-M5-1364 **Edmund A Lamagna\*** ([eal@cs.uri.edu](mailto:eal@cs.uri.edu)), Department of Computer Science and Statistics, Kingston, RI 02881. *Puzzling Through Discrete Mathematics.*

The presenter teaches an applied, yet mathematically rigorous, course on combinatorial problem solving. Algorithmic thinking is emphasized throughout, and the course provides a solid foundation for a follow-on course on the design and analysis of algorithms. Major topics include sets, logic, probability, proofs by induction and contradiction, the pigeonhole principle, arrangements, selections, distributions, binomial identities, inclusion-exclusion, recurrence relations and recursion, and graphs and trees.

Each class begins with a set of puzzles (typically four) that introduce and begin to stimulate thinking about the topic for the day. Students work on the puzzles in small groups for about one-third the period. When puzzles were introduced, it was thought that less material could be covered, but this would be outweighed by an increase in student interest and participation, and the course would be more fun. Unexpectedly, all the original material can still be covered since students are now better prepared and motivated for the more traditional presentation that follows "puzzle time."

The key to this approach is selecting relevant, intriguing puzzles for each topic. Examples covering a variety of subjects that have been successfully utilized are presented in the talk. (Received September 21, 2017)

1135-M5-1495 **Jerry Lodder\*** ([jlodder@msu.edu](mailto:jlodder@msu.edu)), Mathematical Sciences, Dept. 3MB, Box 30001, New Mexico State University, Las Cruces, NM 88003. *Figurate Numbers and Mathematical Induction.*

We outline certain topics in discrete mathematics that form the content of a course taught entirely from primary historical sources. It begins with Nicomachus's construction of the figurate numbers, which count the number of dots in regularly shaped figures, such as triangles, squares, pyramids. In his verbal description of the triangular numbers, Nicomachus identifies two geometrical methods for their construction, which today carry the names of a recursive construction and an iterative one. Other figurate numbers in dimension two, such as the square numbers or the pentagonal numbers can be described in terms of first and second differences. For higher dimensional numbers, patterns in their growth were identified by Pierre de Fermat, and can be motivated by certain quotients. Student exercises include identification of these patterns and a discussion of their validity. The module concludes with the work of Blaise Pascal, who arranged these numbers in one table according to a single recursive principle building on Nicomachus. Pascal generalizes Fermat's work by stating a method to compute the quotient of any two consecutive entries in the same base of his table. To justify why these patterns persist, Pascal develops a reasoning process known today as mathematical induction. (Received September 22, 2017)

1135-M5-1834 **Jeffery D Sykes\*** ([sykesj@obu.edu](mailto:sykesj@obu.edu)), Ouachita Baptist University, Arkadelphia, AR 71998. *Discrete mathematics as a first course for mathematics and computer science majors.* Preliminary report.

At my small liberal arts college, all students majoring in mathematics or computer science take my introductory discrete mathematics course in their first semester. The students engage in a study of discrete topics ranging from counting to primality testing, passing through modular arithmetic and cryptology along the way. Most importantly, they are provided the opportunity to get to know each other and form a community that will enhance their learning experience for the next four years. In this talk, I will share information about the content and structure of my course, the WeBWorK problems I have authored, the web tools I use for cryptology assignments, and some of the outcomes we have observed of our students. (Received September 25, 2017)

- 1135-M5-1890 **Suzanne I Dorée\*** ([doree@augsborg.edu](mailto:doree@augsborg.edu)), Augsburg University, 2211 Riverside Avenue, Minneapolis, MN 55454. *An Inquiry-Based Learning Course in Discrete Mathematics (for mathematics majors, computer science majors, and future teachers).*

How do we teach students to think mathematically? Almost twenty years ago I began teaching a Discrete Mathematical Structures course that simultaneously prepares computer science majors for advanced coursework, meets pre-service teaching standards for middle/high school mathematics, and gives mathematics majors experience writing proofs. Over the years I have moved the course to include more and more active, inquiry-based learning to help students to develop their inner mathematician. Students explore mathematical concepts, check examples, make organized lists, develop conjectures, build counterexamples, and write both careful informal explanations and formal proofs. Plus it is fun. In this talk I will describe the pedagogy and whet your appetite with a couple of particularly effective examples showing how we approach the content. Our findings include increased participation in the mathematics major or minor, continued question-posing and inquiry in subsequent coursework, and improved readiness for undergraduate research. Course materials are being class-tested – and more volunteers needed! (Received September 25, 2017)

- 1135-M5-1923 **Bruce F. Torrence\*** ([btorrenc@rmc.edu](mailto:btorrenc@rmc.edu)), Ashland, VA. *Fibonacci and the Stochastic Abacus.*

In the early 1970s Arthur Engel developed what he later called the “stochastic abacus” to allow fourth graders to calculate probabilities of absorbing states in a Markov chain with rational transition probabilities, simply by moving chips along the edges of a directed graph. His method, widely studied since the mid-1980s, is sometimes called a “chip-firing” algorithm, or an “abelian sandpile” method. Using techniques of elementary discrete mathematics, we will explore how Engel’s algorithm can be artfully employed to study a variety of “pass the buck” games, where the players’ winning probabilities are known to be closely connected to the Fibonacci and Lucas numbers. We will also demonstrate an interactive *Mathematica* interface, allowing students to explore and make discoveries with the stochastic abacus. (Received September 25, 2017)

- 1135-M5-2087 **Elise Lockwood\***, 064 Kidder Hall, Department of Mathematics, Oregon State University, Corvallis, OR 97331, and **Branwen Purdy** and **John S. Caughman**. *Reinventing the Multiplication Principle.*

In an effort to better understand students’ understanding of a fundamental aspect of combinatorial enumeration, we had two undergraduate students reinvent the Multiplication Principle during an eight-session teaching experiment. In this presentation, we report on the students’ progression from a nascent statement to a sophisticated statement of the principle, and we highlight three key mathematical issues to which students attended through this process. We additionally present potential implications and directions for further research. (Received September 25, 2017)

- 1135-M5-2089 **John S Caughman\***, [caughman@pdx.edu](mailto:caughman@pdx.edu), and **Elise Lockwood** and **Zackery Reed**. *Subtleties of the Multiplication Principle.*

The Multiplication Principle is fundamental to combinatorics, underpinning many standard formulas and providing justification for counting strategies. We conducted a textbook analysis to examine ways in which this important principle is presented. In this talk, I report on the (surprisingly) wide variety of statements we found. A number of key elements of the principle emerged, affording a categorization of statement types. Findings from the textbook analysis reveal mathematical subtleties that can be made concrete for students through carefully chosen examples. (Received September 25, 2017)

- 1135-M5-2133 **Colin Starr\*** ([cstarr@willamette.edu](mailto:cstarr@willamette.edu)), 900 State St, Salem, OR 97301. *Problems Inspired by Nordhaus-Gaddum.*

The theorems of Nordhaus and Gaddum connect the chromatic numbers of a graph and its complement in some inequalities. We explore some fun, elementary problems arising from consideration of the extreme cases of these inequalities. (Received September 25, 2017)

- 1135-M5-2254 **Edward Hanson\*** ([hansone@newpaltz.edu](mailto:hansone@newpaltz.edu)), Department of Mathematics, SUNY New Paltz, 1 Hawk Drive, New Paltz, NY 12561. *Motivating examples for teaching discrete mathematics.* Preliminary report.

This talk will be a survey of some examples designed to inspire undergraduate students to learn concepts in discrete mathematics as they cultivate new approaches to problem-solving. Our goal is to emphasize ideas that go beyond those commonly found in textbooks. (Received September 25, 2017)

1135-M5-2384     **Art Duval\*** ([aduval@utep.edu](mailto:aduval@utep.edu)). *Bioinformatics-themed projects in Discrete Mathematics*. Over the course of several years, I have developed a series of bioinformatics-themed projects suitable for an introductory Discrete Mathematics course. The projects are based on real bioinformatics problems, mostly around sequence alignment and phylogeny. Yet each project is self-contained, and students do not require any biology to understand or complete the assignment. I will describe how the projects were developed and refined, and how I use them in class. (Received September 26, 2017)

1135-M5-2862     **Kristin M. Arney\*** ([kristin.arney@usma.edu](mailto:kristin.arney@usma.edu)), West Point, NY, and **Kayla K. Blyman** ([kayla.blyman@usma.edu](mailto:kayla.blyman@usma.edu)), West Point, NY. *Transforming Mathematics Assessments to Drive Better Learning*. Preliminary report.

In mathematics classes, it is common to find students asking when they are going to use the material in the real world. We are taking steps towards removing the sterile facade that often plagues a mathematics classroom during exams by assessing students on their ability to apply course material in a more realistic environment with collaboration and technology available to assist them.

This method of assessment is being piloted in *Mathematical Modeling and Introduction to Calculus*. This course, rooted in mathematical modeling with discrete dynamical systems, is the first mathematics course taken by approximately 900 students at the United States Military Academy.

This method entails weekly assessments in place of major exams. The assessments consist of three parts: a night before read-ahead focused on a new application, an in-class individual portion where students respond to short answer questions, and an in-class group portion where groups of 3-4 students provide team responses to the same questions after discussion, learning, and consensus. We believe this will result in better attainment of higher order learning goals, better preparation for professional collaboration, increased technology skills, and more creative excellence.

Sample assessments are provided. (Received September 26, 2017)

1135-M5-3135     **Filip D Sain\*** ([fsain@exeter.edu](mailto:fsain@exeter.edu)), 20 Main Street, Exeter, NH 03833. *Game Theory for Less Advanced Students*.

A class on game theory was taught to high school students who had finished a year of calculus (BC). Various modifications of the common introductory game theory course were made: the Hotelling game was used as an introduction to the concept of equilibrium; Bayes Theorem was learned in the context of solving Kuhn Poker; properties of the Shapley value and the nucleolus were explored using specific examples from H. Peyton Young. The cumulative effect was a course which emphasized conceptual understanding and intuition over formal proofs. In addition to the applications above, the class read papers from economics, politics, and biology which, while not explored in depth, gave students an appreciation for the breadth of application of game theory concepts. We discuss how to structure a class for students with a limited background in mathematics, which focuses on applications, and which uses problem based learning in order to illustrate concepts. (Received September 26, 2017)

1135-M5-3168     **Oswaldo D. Soto\*** ([osoto@sdsu.edu](mailto:osoto@sdsu.edu)), 3763 33rd St, San Diego, CA 92104, and **Kristofer J Siy** and **Trang Vu**. *The SDSU and Sweetwater Discrete Math Partnership: Developing a high school discrete mathematics curriculum targeting standards for mathematical practice*. Preliminary report.

Discrete mathematics offers a wide variety of contemporary contexts useful for college preparation of secondary students when seen through the lens of the standards for mathematical practice. In this session, we report on early results of a project bringing pedagogically sensitive mathematicians and math educators together with local high school teachers to redesign the curriculum of a high school course whose goals are stated in terms of BOTH content and standards for mathematical practice (SMPs) with an eye on a sophomore level undergraduate discrete mathematics. This session will focus on a description of our unit on graph theory, including a description of the targeted SMPs, a description of pedagogical principles guiding our approach and examples of students' (and in-service teachers') demonstrated ways of understand and ways of thinking. This work can help undergraduate level discrete mathematics instruction by shedding light on the kinds of difficulties undergraduate students may experience, together with their sources, and by providing feedback on what undergraduates have internalized when they become teachers. (Received September 26, 2017)

- 1135-M5-3174 **Kristofer J. Siy\*** ([kristofer.siy@uwaterloo.ca](mailto:kristofer.siy@uwaterloo.ca)), 407 Park St., Basement Apt., Kitchener, ON N2G 1N3, Canada, and **Oswaldo D. Soto** and **Trang K. Vu**. *Experiencing the mathematical process through combinatorial games in a high school discrete math course*. Preliminary report.

Discrete mathematics offers a wide variety of contemporary contexts useful for college preparation if we think about them from the perspective of the standards for mathematical practice. Over the past year, we have brought pedagogically sensitive mathematicians and math educators together with local high school teachers to redesign the curriculum of a high school discrete math course. This course includes a unit on combinatorial games, a topic not usually covered in high school or college discrete math courses. In this session, we will summarize our motivation and goals for doing so, especially with respect to the unique opportunity this affords us to engage students with the standards for mathematical practice, discuss specific pedagogical decisions to attempt to give students an authentic experience of the mathematical process, and present preliminary results about pedagogical implementation. We hope that this can provide a different perspective on what kinds of topics could be implemented in an undergraduate level discrete mathematics course. (Received September 26, 2017)

- 1135-M5-3184 **Oscar Levin\*** ([oscar.levin@unco.edu](mailto:oscar.levin@unco.edu)), 501 20th Street, Campus Box 122, Greeley, CO 80639. *Shifting perspectives for counting*. Preliminary report.

In a continuing effort to try to understand why even basic combinatorics (counting) is so difficult for students, we will investigate the mental gymnastics required to answer simple counting questions that as mathematicians we might too often take for granted. Specifically, this talk will explore the necessary shifts in perspective inherent in combinatorics, as well as in the teaching of combinatorics. We will argue that these perspective shifts are the result of ambiguities that, while difficult to overcome for many students, are worth celebrating and are one reason that a discrete course is an ideal setting for a student's transition to higher mathematics. (Received September 27, 2017)

## *Scholarship on Teaching and Learning in Statistics Education*

- 1135-N1-72 **Robert Lee Nichols\*** ([rob\\_nichols@bullis.org](mailto:rob_nichols@bullis.org)), 10601 Falls Road, Potomac, MD 20854. *The Effectiveness of Inquiry-based vs. Didactic Teaching Methods on Student Performance in Undergraduate Statistics*.

This study explored the impact of instructional style in the teaching of introductory statistics on students' attitudes towards statistics and on students' academic outcomes in statistics courses. Four university statistics instructors were surveyed to identify their instructional style. In addition, their students' ( $n=313$ ) mean course scores and mean scores on the Learning Outcomes for Statistical Methods instrument were analyzed. Based on an independent measure of learning outcomes for students, the data indicate instructional styles that are more inquiry-based may be more effective overall for student achievement on the Learning Outcomes for Statistical Methods instrument. (Received July 19, 2017)

- 1135-N1-337 **Samuel Luke Tunstall\*** ([tunstall1@msu.edu](mailto:tunstall1@msu.edu)). *Investigating college students' reasoning with messages of risk and causation*.

Language of risk and causation pervades modern media sources, and statistical literacy is often framed as a critical means of understanding such discourse. For this exploratory study, roughly 200 students in an introductory university-level mathematics course—one which focuses on science and quantitative information in media—responded to an opinionated news article about cancer risk in relation to processed meats. Analysis of students' responses using Toulmin's framework for argumentation revealed that the majority of the students agreed with the author's misleading message about processed meats. Results suggest that prior knowledge and pre-existing biases serve as nontrivial barriers to the types of reasoning desired for statistical literacy. In this presentation, I will focus on the results of the study and their implications for introductory courses in quantitative reasoning or statistics. (Received August 25, 2017)

- 1135-N1-566 **Elizabeth Grace Arnold\*** ([elizabeth.arnold1@montana.edu](mailto:elizabeth.arnold1@montana.edu)), Department of Mathematical Sciences, Montana State University, P.O. Box 172400, Bozeman, MT 59717-2400. *Using annotated lesson plans to support teaching high school statistics with technology*.

Statistics now comprises a substantial area of study within K-12 mathematics curricula. Research has explored how statistics and mathematics are fundamentally different disciplines and require different styles of learning.

Yet, many teachers and students approach statistics with the same mindset, skills, and tools they use for mathematics. New guidelines recommend teaching statistics content standards using a data-driven and randomization-based curriculum, which may require teachers to use unfamiliar techniques. To implement such a curriculum effectively, teachers need additional support. This research examined the use of specially annotated lesson plans as a means to store knowledge and guide in-service high school teachers' use of technology when implementing statistics content standards, particularly those involving the use of simulation. I will discuss how the teachers in my study used technology and simulation. I will also provide examples of annotated lesson plans and highlight their inherent iterative and collaborative properties. Finally, I will reflect on the use of annotated lesson plans as a means to guide the data-oriented instruction of statistics with technology. (Received September 08, 2017)

1135-N1-777      **Beth Chance\*** (bchance@calpoly.edu), **Nathan Tittle** and **Stephanie Mendoza** (stephanie.2014@yahoo.com). *Student Gains in Conceptual Understanding in Introductory Statistics With and Without a Curriculum Focused on Simulation-Based Inference.*

Using "simulation-based inference" (SBI) such as randomization tests as the primary vehicle for introducing students to the logic and scope of statistical inference has been advocated with the potential of improving student understanding of statistical inference, as well as the statistical investigative process as a whole. Moving beyond the individual class activity, entirely revised introductory statistics curricula centering on these ideas have been developed and tested. In this presentation we will discuss three years of cross-institutional tertiary-level data in the United States comparing SBI-focused curricula and non-SBI curricula (roughly 15,000 students). We examine several pre/post measures of conceptual understanding and student attitudes in the introductory algebra-based course, using hierarchical modelling to incorporate student-level, instructor-level, and institutional-level covariates. (Received September 17, 2017)

1135-N1-1945      **Teneal Messer Pardue\*** (parduet@queens.edu), Charlotte, NC 28274, and **Adalira Sáenz-Ludlow** (sae@unc.edu). *Scaffolding Statistical Argumentation in the Introductory Statistics Classroom: A Teaching Experiment.*

Enculturating students into the practice of statistics requires preparing them to listen, to interpret, to speak, and to write the language of statistics. Statistical argumentation—the process of justifying a claim using evidence based on data, statistical concepts, and reasoning—offers structure to facilitate communication of data analysis results. In a semester-long teaching experiment, postsecondary students took an introductory statistics course that included instruction in statistical argumentation, and they completed a series of tasks designed to support the scaffolding of statistical argumentation. Tasks and instructional tools were developed over three semesters of pilot studies prior to this study, which took place in the fourth implementation. The statistical arguments of three representative students were analyzed qualitatively to determine how their arguments changed over the course of the semester. Results show that over time, students incorporated increasingly advanced statistical content into their arguments while improving in previous statistical conceptualizations. Student feedback at the end of the study indicated students believed the tasks supported their learning of statistical concepts and prepared them to interact with statistics in the future. (Received September 25, 2017)

1135-N1-2726      **Michael D. Miner\*** (michael.miner0@mycampus.apus.edu). *MATH120, Introduction to Statistics: More Than Just an Introduction to Statistics Course for Nurses.* Preliminary report.

In an effort to enhance its Nursing Program, American Public University System (APUS), Nursing Program Curriculum Developers requested the APUS Mathematics Department to develop and administer a basic statistics course for the Nursing Program. Prior to the request, the only available statistics course for Nursing students to take was MATH302, Statistics, a grueling 16 weeks online course. MATH302 is a statistics course geared toward undergraduate students majoring in Business, STEM, and other fields of studies such as Homeland Security and Intelligence Studies that require more than a basic understanding of applications of statistical processes. The product of in-depth research and design strategies resulted in MATH120, Introduction to Statistics. MATH120 is an eight week, online basic statistics course designed to address the needs of the Nursing Program. However, the course exceeded its original objectives! This presentation will discuss the development, design strategy, implementation, resource utilization, and serendipitous results of MATH120, Introduction to Statistics. (Received September 26, 2017)



1135-N1-2740 **Phillip L Poplin\*** (poplinpl@longwood.edu), 201 High Street, Mathematics Department, Farmville, VA 23909, **M Leigh Lunsford** (lunsfordml@longwood.edu), 201 High Street, Mathematics Department, Farmville, VA 23909, and **JoEllen G Pederson** (pedersonjg@longwood.edu), 201 High Street, Department of Sociology, Farmville, VA 23909. *Using Assessment and Early Intervention to Improve Student Success in Introductory Statistics.*

In a previously published study we used an assessment tool to identify students who were at-risk of not being successful in our introductory statistics course (Lunsford and Poplin 2011). In this study, we used the tool to identify at-risk students, and then required those students to attend peer tutoring, early in the semester, as an intervention. While we saw a significant increase in student success for all students in this study compared with the previous study, the at-risk students who completed the required tutoring had a significantly higher increase in success than their peers. (Received September 26, 2017)

1135-N1-2765 **Rachel M Chaphalkar\*** (chaphalr@uwv.edu). *Future elementary teachers' conceptual understanding of statistics.* Preliminary report.

The Guidelines for Assessment and Instruction in Statistics Education (GAISE) report and the Statistics Education for Teachers (SET) report encourage a focus on the statistical process. However, not only do future elementary teachers often spend little time in statistics courses, previous research has shown a lack of textbooks for this demographic consistent with the guidelines. This study looked at the differences in students' conceptual understanding of statistics and probability before and after approximately 15 contact hours within a mathematics content course for preservice elementary teachers, examining whether instruction focused on the statistical process and relating examples to teaching increased students' conceptual understanding compared with the textbook-focused course. Students' conceptual understanding was measured using the Levels of Conceptual Understanding of Statistics Test, as well as regular coursework. (Received September 26, 2017)

1135-N1-2866 **Dilrukshika M Singhabahu\*** (dil.singhabahu@sru.edu). *Projects, Presentations, and Flipping when Teaching College Statistics.*

I started teaching Elementary level statistics about five years ago at college level. As was required, I followed the textbook and got all the theory and the examples driven into my students. What I then observed was these students are incapable of successfully handling/analyzing data from a real dataset. That is when I started requiring projects and presentations as part of my classes using SAS and/or R. The positive feedback from the student's completing internships successfully, and getting involved in research as a result of these classes is rewarding. I also have started teaching Elementary Statistics as flipped. I have the data to show a comparison of the student learning from the flipped and traditional teaching of this course. I will discuss what worked and what did not work in my classes. (Received September 26, 2017)

## *Integrating Research into the Undergraduate Classroom*

1135-N5-433 **Ping Ye\*** (pye@ung.edu). *Connecting students with the outside world.*

Stepping out of the own comfort zone is not easy for both the students and faculty advisor. PIC Math program provides a great opportunity for faculty advisor to integrate research into the undergraduate classroom and for students to collaborate with the world outside of the University. The experience to go through the whole process turns out to be extremely valuable for everybody. The undergraduate research project of the students will be introduced in this paper. (Received September 02, 2017)

1135-N5-510 **Liz Lane-Harvard\*** (elaneharvard@uco.edu), Edmond, OK 73034. *Article Analysis in a Graph Theory Course.* Preliminary report.

Motivated by the desire to expose students with varying abilities to research, I incorporated an article presentation as part of the course assessment in a cross-listed Graph Theory course. In this talk, I will discuss why I chose to go this route, how the journal articles were selected and distributed, strategies for helping students process their articles, strategies for assessing the presentations, and student feedback on the process. I will also discuss how I've incorporated a variant of this process in my junior level Linear Algebra classes. (Received September 06, 2017)

1135-N5-661 **Eric Sullivan\*** (esullivan@carroll.edu). *Sophomores can do Research!*

According to the CUPM recommendations for undergraduate research, "every student should work, independently or in a small group, on a substantial mathematical project that involves techniques and concepts beyond the typical content of a single course." But how do we get our students ready for these experiences? In this talk

we present a research project, used in a differential equations and linear algebra course, which foreshadows and strengthens the mathematical habits of mind needed for undergraduate research. We will discuss the logistical challenges associated with the project, present several student projects, and give suggestions for implementation. (Received September 12, 2017)

1135-N5-1162 **Thomas Philip Wakefield\*** (tpwakefield@ysu.edu), Youngstown, OH 44555. *PIC Math Courses: Facilitating Student Research Projects in Business, Industry and Government.*

For the past three years, through participation in the Preparing for Industrial Careers in Mathematics (PIC Math) program, mathematics students at Youngstown State completed interdisciplinary research projects with the support of industrial sponsors in a mathematical modeling course. In this talk, we highlight these experiences, which include redistricting beats for the Youngstown Police Department, determining the return on investment for services in our Center for Student Progress, conducting healthcare utilization studies, and analyzing neighborhood interventions for the Youngstown Neighborhood Development Corporation and describe how we ran the course and facilitated the research experiences for the students. (Received September 20, 2017)

1135-N5-1341 **Erik Lundberg\*** (elundber@fau.edu). *My experiences at FAU with integrating undergraduate research into the classroom.*

This talk will survey recent efforts at Florida Atlantic University in project-oriented course design, an ongoing venture that was pushed to a new level after we participated in the PIC Math program (preparation for industrial careers in mathematics). The PIC Math course had students work in groups to solve a problem provided by an external industry partner. After offering the course in two consecutive years with participation from Oak Ridge National Laboratory, American Traveler Staffing Professionals, and Moffitt Cancer Research Center, FAU faculty developed a capstone course for Math majors largely based on the PIC Math course. I will discuss challenges, successes, and concerns associated with project-oriented course design. (Received September 21, 2017)

1135-N5-1423 **Tanweer J Shapla\*** (tshapla@emich.edu), Department of Mathematics and Statistics, Eastern Michigan University, Ypsilanti, MI 48109, and **Khairul Islam**. *Engaging Undergraduate Students in Research.*

While undergraduate students' research has many potential benefits, engaging students in research is challenging. Giving students' an early exposure to research may change their life. What should we do to integrate research in undergraduate classroom? Engage in classroom activity following by an open discussion, critically analyze a problem, discussing alternative solutions with limitations and assumptions, etc. often help students think outside of the box. Some students end up presenting their work in undergraduate symposium or state level conferences. In this presentation, we provide some examples and approaches that we have used in the class to engage students in research and how that have impacted their future endeavor. (Received September 22, 2017)

1135-N5-1461 **Kristin Lassonde\*** (prof.lassonde@gmail.com). *Undergraduate Research at the Community College? Preliminary report.*

Incorporating research into undergraduate courses is becoming increasingly common. Faculty recognize the value of these experiences which prepare our students for future success. While these research experiences are common in the junior/senior level of undergraduate courses, it is often difficult to engage lower level freshman/sophomore students, especially at the community college. This talk will discuss some issues and successes I have had in incorporating undergraduate research at the community college level, including my experience with the PIC Math program. (Received September 22, 2017)

1135-N5-1830 **Edwin Herman\*** (edwin.herman@uwp.edu). *A Mathematical Research Methods Course.*

In 2016 I taught a Research Methods course as a capstone for our Mathematics Major. I used resources from the PIC Math program (MAA & SIAM; funded via NSF grant DMS-1345499) to help design a course wherein student groups worked on open-ended research problems drawn from real industrial contacts. Unlike many versions of the course, I spent significant time at the start on smaller projects designed to facilitate teamwork and out-of-the-box thinking. Overall, the course ran surprisingly smoothly. The only major issue I had was timing: groups did not have quite enough time to make as much headway as desired on the main research question.

The following year I made changes to improve my course: I found new industrial contacts and streamlined the early part of the course to allow more time for teams to work on their main research project. Groups accomplished more – one team discovered something completely unanticipated by our contact – but there was also more friction in some groups.

In my talk I will discuss how I set up my course and what I learned along the way, including the process of helping contacts generate viable research problems and the challenges of dealing with a difficult IRB. I will also present student feedback about the projects and about the course in general. (Received September 25, 2017)

1135-N5-2576 **Malcah Effron\*** ([meffron@mit.edu](mailto:meffron@mit.edu)). *Integrating Source Use into Undergraduate Research in Mathematics*. Preliminary report.

My students working on undergraduate math research assignments arrive with basic knowledge about citation and style guides. Yet, they often do not intuitively understand how or when to use, attribute, and acknowledge the range of sources used in mathematics. Dealing with similar—although not explicitly mathematical—issues, the field of writing studies has found that students learn appropriate source use more easily when they first understand sources' roles in research and then learn the forms related to those roles [1]. Building from [1], this paper offers an activity to help students discover the different ways math research uses sources and the reasons for them. Before class, students read math articles and note where sources are used. Then, in class, they sort the results into their kinds and roles, which can be done in small group discussion. Together, students discuss these responses to create a guide about how and when to use different kinds of sources. Additionally, because it focuses on source use, the activity is accessible to a wide range of skill levels and research fields in mathematics. [1] Bizup, J. "BEAM: A Rhetorical Vocabulary for Teaching Research-Based Writing," *Rhetorical Review*, 2008. (Received September 26, 2017)

1135-N5-2890 **Caroline Haddad\*** ([haddad@geneseo.edu](mailto:haddad@geneseo.edu)). *Incorporating Student Research in a Beginning Problem-Solving and Procedural Programming Class*.

One way to incorporate research into a beginning undergraduate course is to require group projects at the end of the semester that apply what students have learned to a "real-world" application. This reinforces what they learned, and demonstrates where it is useful. In this talk I will discuss how I have done this in our procedural programming class for math majors, what worked well, and what did not. I will also present portions of the actual projects. (Received September 26, 2017)

1135-N5-3139 **Malgorzata A Marciniak\*** ([mmarciniak@lagcc.cuny.edu](mailto:mmarciniak@lagcc.cuny.edu)), 4266 Phlox Place, Apt C7, Flushing, NY 11355. *Student work on flexible solar panels for NASA geostationary satellites*.

The topic grew from research projects mentored at LaGuardia Community College. We analyze the dimensions of the cylindrical solar panels placed hypothetically on the NASA geostationary satellite using the model of efficiency based on flux. The topic is suitable for students in Calculus 3 class and can be supported by software (Mathematica) calculations. (Received September 26, 2017)

1135-N5-3188 **Helmut Knaust\*** ([hknaust@utep.edu](mailto:hknaust@utep.edu)), Department of Mathematical Sciences, The University of Texas at El Paso, El Paso, TX 79968. *Guided Inquiry for Undergraduates in a Classroom Setting*.

My department offers a freshman/sophomore level course, modeling Mathematics research as a laboratory science. The course is based on materials developed at Mount Holyoke College and requires only co-enrollment in Calculus I.

Small student teams explore several rich mathematical topics on their own. They perform mathematical experiments (with the help of a computer algebra system), formulate, test and refine conjectures, and finally try to prove some of their conjectures. At the end of each two week laboratory, the student teams write up their findings in a laboratory report. (Received September 27, 2017)

## Quantitative Literacy Across the Curriculum

1135-O1-795 **Paula R. Stickle\*** ([pstickles@millikin.edu](mailto:pstickles@millikin.edu)). *Quantitative Literacy Across the Millikin University Campus*.

Quantitative literacy takes on different meanings based on experience and background. As such, across the curriculum students have various needs for their QL experience. As a result, Millikin University students have a range of choices that more closely fit their future needs. Seven different departments across campus offer QL courses. The overarching goals of our QL courses, how disciplines meet them, and data on student performance will be shared. (Received September 14, 2017)

- 1135-O1-847 **Justine Chasmar Stauffer\*** ([justine.chasmar@goucher.edu](mailto:justine.chasmar@goucher.edu)), 1021 Dulaney Valley Road, Julia Rogers 143- Faculty, Baltimore, MD 21204. *Development of a Quantitative Reasoning Center at a Liberal Arts College.*

There has been growing attention to the support of students in their Quantitative Reasoning (QR) skills as the undergraduate population continues to change. Incoming students enter with a varied level of math knowledge, and a significant portion of the work in QR support on college campuses involves the development of math-focused learning centers and learning center programs to support math-knowledge and remediation. This year, to combat growing math anxiety and low math preparation of incoming students, Goucher College created a QR Center with a newly hired faculty member as the Director. The implementation of this center coincided with a major update in the general education requirements for Goucher students with a shift from QR to Data Analytics (DA). To assist students in their QR and DA skills, the Director has started tutoring, study materials, workshops, and other math-focused learning initiatives. In this session, the presenter will discuss the creation of the QR Center at Goucher from ideation to implementation. First, the beginnings of the center and Director position will be presented; the current QR-programming will be reviewed; and future initiatives will be discussed. (Received September 15, 2017)

- 1135-O1-1254 **Amanda E Francis\*** ([afrancis@carroll.edu](mailto:afrancis@carroll.edu)), Amanda Francis, Department of Mathematics, 1601 N Benton Ave, Helena, MT 59625. *How much should I pay for my Airbnb rental?: Using Large, Real-World data in Math Classes across the Curriculum.*

Using interesting, real-world data sets helps students gain confidence and familiarity with navigating a world full of data. We ask students to apply quantitative reasoning skills to answer real-life questions about rich and complicated (and publicly available) data sets. In this talk I share some strategies for bringing these data sets into various math and statistics classes and share some open data resources. (Received September 20, 2017)

- 1135-O1-2888 **Rachel Grotheer\*** ([rachel.grotheer@goucher.edu](mailto:rachel.grotheer@goucher.edu)) and **Bernadette Tutinas.** *Data Analytics Across the Curriculum: Rethinking Quantitative Literacy at Goucher College.*

Goucher College has recently gone through a campus-wide curriculum change to rethink the meaning of a liberal arts education and how to best prepare students for life after college. As a result of this reflection, our “Mathematical Reasoning” general education requirement has changed into a Data Analytics requirement that requires students to take one semester-long course learning the foundations of data analytics and then another semester-long course learning data analytics techniques in the context of another discipline, usually their major. We call this second class a “data analytics across the curriculum” course. While still in the early stages of this curricular change, we have had many course proposals from a variety of disciplines outlining how they will incorporate data literacy into their classes. This talk will give an overview of some of these ideas, as well as look at one of our foundational data analytics courses, which approaches calculus from an applied, data-driven mindset, drawing heavily on examples and datasets from other STEM fields. We will specifically explain how we combine data analytics and calculus in this course through our unit projects, which require students to work in groups to analyze datasets with the help of the software RStudio. (Received September 26, 2017)

- 1135-O1-3008 **Andrew J Miller\*** ([andrew.miller@belmont.edu](mailto:andrew.miller@belmont.edu)), Dept of Mathematics and Computer Science, Belmont University, 1900 Belmont Blvd, Nashville, TN 37212. *Guest-starring Mathematics: Building an Assignment Library to Highlight Quantitative Applications Across the Curriculum.* Preliminary report.

In August 2016, Belmont University began a major overhaul of the general education core mathematics requirement. For the previous decade, our core requirement had been built on a traditional “liberal arts mathematics” foundation, featuring symbolic logic, problem solving, and contemporary mathematical topics. Faced with a charge to develop a requirement that students would find intellectually stimulating and relevant to a wide variety of disciplines, we chose to develop a new course in quantitative literacy and reasoning. A key feature in the early development of this course was the creation of an assignment library consisting of projects and problems from disciplines across campus that highlight mathematics in important ways. We will describe the process of building this library and highlight some interesting examples and how they are being used in the pilot sections of the new quantitative literacy and reasoning course. (Received September 26, 2017)

- 1135-O1-3173 **Daniel G Kim\*** ([kimd@sou.edu](mailto:kimd@sou.edu)), 1250 Siskiyou Blvd., Mathematics Department, Ashland, OR 97520. *Some Successful Ideas to Teach Statistics as a Quantitative Literacy Course.* Preliminary report.

When students enroll for an introductory statistics course toward a Quantitative Literacy requirement, they are generally not excited about the fact that they are taking a statistics course. This is more of the case if students

are majoring in liberal arts such as language, theater, music, or philosophy. This is because they feel that statistical quantitative literacy is not much of their interests and not needed for their majors. In this talk, we consider some ways to win students' enthusiasm for a successful quantitative literacy course through statistical education. (Received September 26, 2017)

## *Innovative Teaching Practices in Number Theory*

1135-O5-606     **Brian S Chen\*** ([a.tutor@me.com](mailto:a.tutor@me.com)), 13256 Ramona Blvd, Baldwin Park, CA 91706.  
*Divisibility Rules and Proofs: K-12 and Beyond.*

Rigorous math requires rigorous proofs, but what if we change the divisibility rules a bit? From planting orderly forests to utilizing year I algebra to calling on mathematical induction to using synthetic division. All students from K-12 can participate in logical, mathematical proofs when it comes to certain, though possibly reworded, divisibility rules. (Received September 10, 2017)

1135-O5-722     **Benjamin Linowitz\*** ([benjamin.linowitz@oberlin.edu](mailto:benjamin.linowitz@oberlin.edu)), 10 North Professor St, Oberlin, OH 44074. *Teaching a first year seminar on cryptography using IBL.*

During the fall of 2016 I taught a course on elementary number theory / cryptography as part of Oberlin College's first year seminar program. The course assumed no previous mathematical background on the part of the students and even conferred credit towards the college's writing requirement. A novelty of the course was that it was taught using a variety of active learning techniques like IBL. There was no assigned textbook and there were no lectures, for instance. Instead, the students spent every class in small groups working on a series of carefully scaffolded worksheets and presenting their solutions to the class. (The first worksheet begins with the definition of an integer dividing another integer. The last worksheet goes over the RSA cryptosystem.) In this talk I will discuss the structure of the course and what I learned about teaching number theory to first year students and non-math majors. (Received September 13, 2017)

1135-O5-736     **Jeffrey Hatley\*** ([hatleyj@union.edu](mailto:hatleyj@union.edu)). *Heads or tails? Coin-flipping with elementary number theory.*

A standard first course in elementary number theory usually covers the Chinese Remainder Theorem and quadratic residues. We begin by explaining a well-known method of "cryptographic coin-flipping" that relies on these two topics. After this, we describe a classroom assignment that gets students to perform some "coin flips" with their peers via email, and we discuss the challenges encountered in past implementations of this assignment. (Received September 13, 2017)

1135-O5-796     **J J Tattersall\*** ([tat@providence.edu](mailto:tat@providence.edu)), Department of Mathematics, 1 Cunningham Square, Providence, RI 02918. *An extended Euclidean algorithm.*

The greatest common divisor has many significant mathematical applications, finding inverses in modular arithmetic, computing continued fractions, solving linear Diophantine equations, and decrypting and encrypting exponential ciphers. Given two integers, most textbooks illustrate the ancient Euclidean algorithm to find the greatest common divisor, then by working the steps of the algorithm backwards, the greatest common divisor can be expressed it as a linear combination of the two given numbers. While effective, the Euclidean approach can put an algebraic strain on students. A more innovative technique, Saunderson's algorithm, offers a much more efficient approach to the problem. In the 1740 edition of {Elements of Algebra}, Nicholas Saunderson, the blind Lucasian Professor, introduced an extended Euclidean algorithm to determine the greatest common divisor of two positive integers and simultaneously express the greatest common divisor as a linear combination of the two numbers. We explain the method and illustrate it with an example. (Received September 14, 2017)

1135-O5-933     **Frank Sanacory\*** ([sanacoryf@oldwestbury.edu](mailto:sanacoryf@oldwestbury.edu)), 223 Store Hill Road, Department of Mathematics & CIS, College at Old Westbury (SUNY), Old Westbury, NY 11568.  
*Introducing Number Theory in High Schools using Inquiry.* Preliminary report.

As mathematicians we are all familiar with definitions such as  $a \approx b \pmod n \iff n|(b-a)$ . How can students who are unfamiliar with the rigor of mathematics understand such definitions? Here we will explore a method of introducing number theory to high school students (currently in 10th and 11th grade). We build from the familiar clock math description of modular mathematics to exploring how to perform algebra on such a system. We end with the formal definitions (often suggested by the students). At the Institute for Creative Problem Solving we have used this method as the introduction to number theory for the students as well as an introduction to mathematical proof. (Received September 17, 2017)

1135-O5-1446 **Katherine Thompson\*** ([kthompson0721@gmail.com](mailto:kthompson0721@gmail.com)). *Number Theory Courses at Davidson College and Carnegie Mellon University: A Comparison.*

There are many exciting, yet challenging, aspects to teaching an undergraduate number theory course. It can be marketed as a co-requisite of (or even alternate to) “Intro to Proof”. It can be treated as a special topic for students with abstract or analysis. Regardless, number theory is a course where the culture of the students and the university can be highlighted, and where various innovative pedagogical techniques can be employed. This talk will highlight the similarities between two number theory courses which (on the surface) seem like they would be drastically different: a 200-level IBL-style taught at Davidson College, and a 400-level lecture-based course taught at Carnegie Mellon. (Received September 22, 2017)

1135-O5-1448 **Kristin A. Camenga\*** ([camenga@ajcindustries.com](mailto:camenga@ajcindustries.com)), Juniata College, 1700 Moore Street, Huntingdon, PA 16652. *Making a Class Textbook in an IBL Number Theory Course.* Preliminary report.

How can you reduce your grading, free student time to work on more proofs and problems in the course, and have students write for someone other than the instructor? While teaching Number Theory from Marshall, Odell and Starbird’s *Number Theory Through Inquiry*, I have had all students work on proving results and present their work in class and then assigned individual students to write up and revise their proofs to be posted on the course management system for later reference by the class. Therefore, I only read one proof of each result and can give detailed feedback which students use to revise their proofs. I will share the different ways I have approached this process with three groups of students at two liberal arts colleges, including timing, revision process, and grading. I will also discuss the improvements I have seen in student writing as a result. (Received September 22, 2017)

1135-O5-1664 **Patrick X Rault\*** ([rault@email.arizona.edu](mailto:rault@email.arizona.edu)), University of Arizona, 9040 S Rita Rd, Ste 2260, Tucson, AZ 85747. *IBL Quadratic Reciprocity.* Preliminary report.

We will present a sequence of problems which lead up to a proof of Quadratic Reciprocity. This sequence is suitable for an elementary number theory course which employs Inquiry-Based Learning. The chosen proof of quadratic reciprocity is relatively new and very elementary. (Received September 24, 2017)

1135-O5-1835 **Mark Kozek\*** ([mkozek@whittier.edu](mailto:mkozek@whittier.edu)), Department of Mathematics, Whittier College, Whittier, CA 90608-0634. *Using infographics to visualize number theory.* Preliminary report.

In this talk we reflect on the use of infographics as a tool for students to visualize the underlying mathematical structure of an elementary number theory course.

Because of mathematics’ inherent abstractness, its structure is hard to visualize, yet understanding its structure is a desired learning outcome in most of our courses. In our undergraduate number theory course, we had the students attempt to visualize its mathematical structure by constructing infographics to represent it. After each lecture, one student created a daily infographic that included every axiom, definition and theorem used in the day’s lecture. At the end of the semester, the students combined all their daily infographics into a composite infographic for the entire number theory course. Our talk is a reflection on this pedagogical experience. (Received September 25, 2017)

1135-O5-1975 **Brandt Kronholm\*** ([brandt.kronholm@utrgv.edu](mailto:brandt.kronholm@utrgv.edu)), University of Texas Rio Grande Valley, Department of Mathematics, 1201 W. University Dr., Edinburg, TX 78539. *An Invitation to Integer Partitions.*

In this talk I will discuss how I introduce students to integer partitions and the variety of mathematical techniques used to study them. While my main intent is to lay the foundation for the possibilities of undergraduate research, variations of this lesson may be appropriate for an elementary number theory course or a mathematics-for-liberal-arts-majors course.

The entryway to the study of integer partitions is wide open. Another goal of this talk is to share how I invite students from every level, undergraduates in particular, to engage in meaningful number theory research. (Received September 25, 2017)

1135-O5-2238 **Eva G Goedhart\*** ([goedhart@lvc.edu](mailto:goedhart@lvc.edu)), Mathematical Sciences, 101 N. College Ave, Annville, PA 17003. *Secret Mission Assignment: Teaching Number Theory Through Cryptography.*

I teach number theory through cryptography at a small college where most of the majors focus on applied subjects. Appealing to that core focus with secret personas, code names, and mission assignments, my students

learn basic number theory and cryptosystems as they send and intercept secret messages to their classmates. (Received September 25, 2017)

1135-O5-2304 **Karl-Dieter Crisman\*** ([karl.crisman@gordon.edu](mailto:karl.crisman@gordon.edu)). *Number Theory: In Context and Interactive.*

In the typical US curriculum, there are few places where we see the unity inherent in mathematics; each course tends to be a silo with few connections to other courses. But as a junior/senior course, elementary number theory is an excellent vehicle for showing the rich connections inherent in the whole curriculum. In addition, number theory is full of opportunities for interactive exploration, whether by hand or using computer assistance.

For the past dozen years, I've worked to create a truly introductory full-semester course in number theory which has clearly articulated connections to calculus, geometry, and algebra. In addition, I've incorporated dozens of interactive SageMath applets alongside many questions appropriate for in-class inquiry activities; all in a free text (pdf/html using PreTeXt as an authoring tool). In this talk, I hope to convince you that for many instructors, the right way to teach number theory is with a view toward all of math and using computers at many steps; that is, to teach number theory *in context and interactive*. (Received September 25, 2017)

1135-O5-2577 **Janet Heine Barnett\*** ([janet.barnett@csupueblo.edu](mailto:janet.barnett@csupueblo.edu)). *Prime Sources: A Classroom-tested Student Project Approach to Learning Today's Number Theory through the Works of its Historical Masters.*

Number theory has deep historical roots dating back to problems first studied in ancient Greece. The subject began to take its modern form in the hands of Euler. There followed a time of extraordinary change, with nineteenth century number theorists generalizing the seemingly simple concepts of 'integer' and 'prime' as the discipline became more abstract, more formal and more rigorous. This talk describes an approach to learning today's number theory by drawing on this rich and exciting history.

We illustrate this approach with details of a Primary Source Project (PSP) based on Dedekind's 1877 "Theory of Algebraic Integers." A key feature of this text is its careful formulation of a new conceptual framework for studying problems previously treated algorithmically. Through guided reading of select excerpts, students encounter Dedekind's methodology and original motivations, and develop their own understanding of underlying concepts by completing tasks interspersed between the excerpts. Overviews of other number theory PSPs and of the inquiry-based pedagogy guiding the NSF-funded project *Transforming Undergraduate Mathematics via Primary Historical Sources* that is supporting the development of PSPs for topics throughout the undergraduate curriculum will also be shared. (Received September 26, 2017)

1135-O5-3000 **Jordan Schettler\***, [jordan.schettler@sjsu.edu](mailto:jordan.schettler@sjsu.edu). *From Oiler to Air-Dish: Guided Group Projects in Number Theory.*

Number theory has the greatest stories in all of mathematics like the tale of Ramanujan's taxi cab number or the EYPHKA! diary entry of Gauss. In my years of teaching undergraduate number theory at UC Santa Barbara and San Jose State, I have developed and refined a way for students to discover and share those stories and the math behind them through group projects. The projects have some very well-defined parameters including the mandatory explanation of historical context and the use of technology, but there are other aspects left open-ended where students can become creative in describing some deep results. Student groups must present an engaging 15 minute slide presentation which addresses some key points in the project description and also keeps the attention of their classmates. I will describe some sample projects (from Euler to Erdos), positive student feedback, and also some challenges involved. (Received September 26, 2017)

1135-O5-3039 **Lola Thompson\*** ([lola.thompson@oberlin.edu](mailto:lola.thompson@oberlin.edu)). *#quadraticreciprocity: from 140-character tweets to polished student-authored textbooks.*

The only textbooks allowed in my inquiry-based number theory course are the ones that my students author at the end of the semester. When I first designed this course, I found that many of my students' textbooks were far from the polished works that I was anticipating. Many were poorly written and unfocused. One particularly memorable textbook consisted entirely of 140-character tweets which, while creative, proved impossible to follow. I consulted with colleagues who teach writing courses in other departments in order to figure out what went wrong. Thanks to my colleagues' valuable advice, I have re-structured my writing prompts, added in-class discussions of disciplinary writing practices, and incorporated a formal revision process. In this talk, I will give specific examples of these adaptations, which have led to significant improvements in my students' textbooks. (Received September 26, 2017)

1135-O5-3137 **Tom Edgar\*** ([edgartj@plu.edu](mailto:edgartj@plu.edu)), Mathematics Department, Pacific Lutheran University, 12180 Park Avenue S., Tacoma, WA 98447. *An inquiry-based approach to elementary number theory via proofs without words*. Preliminary report.

We discuss the use of “proofs without words” as a device to encourage student exploration of number theoretic ideas. In particular, we suggest the process of removing theorems from published proofs without words and guiding students to discover the theorem and its proof by investigating the relevant picture. We will demonstrate this idea with a few careful examples and describe our use of this technique, which can be adapted to introduction to proofs courses or courses investigating elementary number theory. (Received September 26, 2017)

1135-O5-3212 **Duff G. Campbell\*** ([campbell@hendrix.edu](mailto:campbell@hendrix.edu)), 1600 Washington Ave., Hendrix College, Conway, AR 72032. *Using projects to teach number theory*.

There seems to be no consensus as to where elementary number theory becomes algebraic number theory. When I teach a first course in number theory, I like to use projects as a way to explore topics that are not ordinarily part of such a course: Bernoulli numbers and polynomials, arithmetic generating functions, the structure of  $(\mathbf{Z}/n\mathbf{Z})^\times$ ,  $p$ -adic numbers, patterns in numerical data, etc. I will give a brief overview of several projects and the inspirations that led to them. (Received September 27, 2017)

## *The Advancement of Open Educational Resources*

1135-P1-246 **John W Watson\*** ([jwwatson@atu.edu](mailto:jwwatson@atu.edu)), 1509 N. Boulder, Arkansas Tech University, Russellville, AR 72801. *Using Open Resources in a Freshman General Education Course: A progress report*.

For the past several years I have been using open resources in a freshman general education mathematics course for non-STEM majors. This is a progress report on that endeavor which has resulted in the compilation of those open resources into an eBook. (Received August 15, 2017)

1135-P1-828 **Thomas W. Judson\*** ([judsontw@sfasu.edu](mailto:judsontw@sfasu.edu)), Nacogdoches, TX 75962. *The UTMOST Sage Cell Repository*.

Sage, an open source computer algebra system, can have a somewhat steep learning curve. The best remedy to this situation is the Sage cell, a self-contained Sage calculation that can be embedded in any web page. The Sage code is present in the HTML and no programming is required. Sage commands are executed on a remote server. The commands can also be modified. If a mistake is made, one only needs to reload the web page to restore the original commands. Project UTMOST has created a repository of commonly used Sage cells to assist learners and allow authors to work more efficiently to incorporate Sage into their documents. We will demonstrate the Sage cell repository. (Received September 15, 2017)

1135-P1-1435 **Tricia Muldoon Brown\*** ([patricia.brown@armstrong.edu](mailto:patricia.brown@armstrong.edu)) and **Joshua Lambert**. *Implementing OER materials in a quantitative skills and literacy mathematics course*.

The talk will focus on the adoption of an OER textbook and the accompanying MyOpenMath LMS in a quantitative skills and literacy mathematics course. The course is primarily for first-year, non-science students. We review hurdles to implementation at our mid-size primarily undergraduate state school and discuss the pilot courses, both online and face-to-face. Student success and satisfaction measures will be presented along with anecdotal responses from the students and instructor. This is report on the activities of a 2016-2017 Affordable Learning Georgia (ALG) Textbook Transformation Grant. (Received September 22, 2017)

1135-P1-1740 **Jennifer Nordstrom\*** ([jfirkins@linfield.edu](mailto:jfirkins@linfield.edu)). *Open Source Introduction to Game Theory*.

Game theory provides an engaging context for a liberal arts quantitative reasoning course or a high school enrichment course. *Introduction to Game Theory: a Discovery Approach* is an open access resource for teaching an inquiry-based game theory course to students with little mathematics background. It is a natural topic for introducing students to the role of mathematics in decision-making; improving their ability to deal with quantitative concepts; and improving their ability to convert qualitative situations into quantitative ones. This talk will describe some of the course activities and the available formats for the text, including a detailed instructors guide and an interactive online version. (Received September 24, 2017)



1135-P1-1964 **Kent E Morrison\*** ([morrison@aimath.org](mailto:morrison@aimath.org)), American Institute of Mathematics, 600 East Brokaw Road, San Jose, CA 95112. *Ten Years in Ten Minutes: Surveying the OER Movement in Mathematics.*

In addition to the individual efforts of those writing, developing, adapting, and using open educational resources, there are a number of programs on the part of institutions, foundations, universities, and higher education systems. Examples include University of Minnesota Open Academic Catalog, OpenWashington, BC Campus, SUNY OER Services, OpenStax, and my own American Institute of Mathematics. In this talk I will briefly describe some of these efforts. (Received September 25, 2017)

1135-P1-2123 **David Austin\*** ([austind@gvsu.edu](mailto:austind@gvsu.edu)), Department of Mathematics, Grand Valley State University, Allendale, MI 49401. *A new open-access, open-source linear algebra textbook.*

We will describe and demonstrate a new open-access, open-source linear algebra textbook, which will be available for use in Fall 2018. Currently, the text is suitable for a one-semester introduction to linear algebra though additional material for a second semester is planned. As the source is written in PreTeXt, it will be freely available in both PDF and HTML versions with the source code distributed freely as well. By including numerous activities suitable for in-class use, the text encourages active learning and aims to develop a conceptual and intuitive understanding of linear algebra as well as computational fluency in students. In addition, we hope to develop an appreciation for the usefulness of linear algebra through numerous applications such as computer animation, image compression, and Google's PageRank algorithm. The HTML version of the text encourages active reading through the inclusion of embedded Sage cells and interactive graphics written in Javascript. (Received September 25, 2017)

1135-P1-2433 **Petra Bonfert-Taylor** ([petra.b.taylor@dartmouth.edu](mailto:petra.b.taylor@dartmouth.edu)), **Sarah Eichhorn** ([s.eichhorn@austin.utexas.edu](mailto:s.eichhorn@austin.utexas.edu)), **David Farmer\*** ([farmer@aimath.org](mailto:farmer@aimath.org)) and **Jim Fowler** ([fowler@math.osu.edu](mailto:fowler@math.osu.edu)). *Using CuratedCourses to match OER to other OER.*

Instructors may want to "flip" their classroom with open educational resources (OER), but finding resources that are field-tested in a variety of settings and known to be high-quality is a challenge. The CuratedCourses platform addresses this challenge by associating rich, curated metadata with OER. More specifically, by aligning open content to a carefully designed a tag hierarchy for linear algebra, CuratedCourses assists linear algebra instructors in identifying high-quality related resources. Students see related resources automatically appear in the margin of their OER textbook. This material is based upon work supported by the National Science Foundation under DUE-1505246. (Received September 26, 2017)

1135-P1-2540 **Monica VanDieren\*** ([vandieren@rmu.edu](mailto:vandieren@rmu.edu)), **Paul Seeburger** and **Deborah Moore-Russo**. *Lessons Learned in the Dissemination of the CalcPlot3D OER Applet and Exploration Activities.*

CalcPlot3D (see <http://web.monroec.edu/calcNSF/>) is a freely available applet which aids instructors and students in visualizing multivariable calculus concepts. Accompanying this applet are discovery-learning activities that facilitate the exploration of geometric interpretations of the dot product, cross product, velocity and acceleration vectors, lagrange multipliers, etc. The applet and explorations receive over 150,000 page views annually. This presentation will discuss some of the challenges involved in disseminating these educational resources over the past ten years as Java became obsolete and the user base outgrew the initial methods (SurveyMonkey) for administering the explorations. The applet is now translated into JavaScript, working on tablets and phones as well as computers, and the explorations are delivered in WeBWorK. This project is funded by NSF-IUSE 1524968, NSF-IUSE 1523786, and NSF-IUSE 1525216. (Received September 26, 2017)

1135-P1-2570 **Steven Schlicker\*** ([schlicks@gvsu.edu](mailto:schlicks@gvsu.edu)). *Open Textbooks at Grand Valley State University.*

Several studies provide extensive empirical support that active learning in STEM disciplines and mathematics is superior to traditional lecture while other studies show that active learning differentially benefits students of color and/or students from disadvantaged backgrounds. As a response to those studies, and to the undeniable fact that commercial textbooks are just too expensive, some faculty at Grand Valley State University have written inquiry-based texts in trigonometry, single and multivariable calculus, and transition to proof that are either open source or available as free pdf files. Our hope is to help our students by incorporating active learning strategies in our texts, and help them with their cost of their education with free high-quality textbooks. I will share information about the texts that are currently available at GVSU (at <http://scholarworks.gvsu.edu/books/>), and some of the exciting material that is in the works. (Received September 26, 2017)

1135-P1-2629 **Claude Laflamme\*** (laflamme@ucalgary.ca), 2500 University Dr, Calgary, Alberta T2N1N4, Canada, **Stephanie C. Keyowski** (stephanie@lyryx.com), 1425 Kensington Road NW, Calgary, Alberta T2N 3R1, Canada, and **Tamsyn Murnaghan** (tamsyn@lyryx.com), 1425 Kensington Road NW, Calgary, Alberta T2N 3R1, Canada. *A sustainable publishing model for open educational resources.*

Open Educational Resources (OER) are open and “free”... but suddenly instructors may need to become text editors, may need to brush their programming skills for online homework, and may even have to play a customer support role themselves in assisting students through these pieces.

We propose a flipped publishing model, where an open text is produced and distributed at no cost to students, and editorial & support services and all other material including online homework are provided for an affordable but OPTIONAL \$40 price: “optional” means everything is FREE in all institution computer labs; students only pay for the convenience of completing their work anywhere else, at home or coffee shop for example.

We will discuss the model’s success over these past few years in Mathematics & Statistics and Business & Economics at Lyryx Learning, a spinoff from the Mathematics & Statistics Department at the University of Calgary. (Received September 26, 2017)

1135-P1-2645 **Jeff Zeager\*** (jzeager@lorainccc.edu), 1005 Abbe Road North, Elyria, OH 44035. *Using OER’s to Create a Pre-requisite Course for College Algebra.* Preliminary report.

Lorain County Community College has been using the Stitz-Zeager Precalculus OER textbook for our College Algebra class since 2010 and we have found that many of our students have not only pre-requisite content knowledge gaps, but also lack the proper study habits to fully engage in the text. In this talk we will discuss the pre-requisite course we created using OER material to help address these issues. (Received September 26, 2017)

1135-P1-2663 **D Scott Dillery\*** (dillerys@lindsey.edu), Mathematics, Lindsey Wilson College, 210 Lindsey Wilson Street, Columbia, KY 42728. *Open Source in Teaching Statistics.*

Support and quality of open content has grown in Statistics so that one can have a great deal of flexibility in designing a course. We will examine how open texts, software and web courses have been utilized to go beyond the usual textbook in statistics courses for both majors and non-majors. (Received September 26, 2017)

1135-P1-2690 **Joseph E. Fields\*** (fieldsj1@southernct.edu), SCSU - Mathematics, Engleman Hall (EN D118), 501 Crescent St, New Haven, CT 06515. *A 21st century Foundations text.* Preliminary report.

Progress report on IntroToProof. A *Gentle Introduction to the Art of Mathematics* is a free and open-source text for an “introduction to proof” course. Created in L<sup>A</sup>T<sub>E</sub>X, it has gone through multiple revisions culminating with version 3.1 in 2014. Using the source code for this project and an automated conversion to PreTeXt format as a basis, we are creating a truly modern textbook for the transition to upper level mathematics. IntroToProof is optimized for web-delivery – although hard-copy will be available. The book includes integrated WebWork assignments, Computer Algebra examples using Sage, and hundreds of graphics/illustrations (including some interactive graphics). Like its predecessor, IntroToProof is a free, open-source project licensed under the GFDL. (Received September 26, 2017)

1135-P1-2878 **K. Andrew Parker\*** (kparker@citytech.cuny.edu), Brooklyn, NY. *Asynchronous Online Office Hours with WeBWork.* Preliminary report.

Accessibility is a core tenet behind the development of Open Educational Resources. Technologies like the WeBWork online homework platform have provided students free access to instructional materials. Furthermore, when students struggle with their online homework, WeBWork provides a straightforward process to email their instructors. While this feature is a boon to many students, it can be frustrating for their instructors - particularly on campuses where there are many non-traditional students who may not be able to take advantage of other support services.

This talk will showcase a new “Ask for Help” plug-in that replaces the “Email Instructor” option in WeBWork. Inspired by sites like Math Overflow and StackExchange, our new feature is designed to take what used to be a private (and often repetitive) interaction and instead place it in a semi-public setting. We will share lessons learned from our initial rollout, feedback we’ve received, and future directions for development and release under an open-source license.

This work is supported by a Title V grant from the Dept of Education, “Opening Gateways to Completion: Open Digital Pedagogies for Student Success in STEM”. (Received September 26, 2017)

1135-P1-3182 **Robert R Rogers\*** ([robert.rogers@fredonia.edu](mailto:robert.rogers@fredonia.edu)), Department of Mathematical Sciences, SUNY Fredonia, Fredonia, NY 14063, and **Eugene Boman**, Mathematics Department, Penn State - Harrisburg, 777 W. Harrisburg Pike, Middletown, PA 17057.  
*How We Got From There to Here: An Open Source Introductory Real Analysis Text.*

In 2014, the authors wrote an open source real analysis text for inclusion in the SUNY Open Textbook Program and is available to anyone. The book represents the idea of putting the topics of an introductory real analysis course into its historical context. Even the problems are in context, set in the body of the text where they naturally occur. This presentation will go over some of the aspects of the book and how it can be utilized in standard and IBL settings. (Received September 27, 2017)

1135-P1-3238 **Dana C. Ernst\*** ([dana.ernst@nau.edu](mailto:dana.ernst@nau.edu)). *Open-source course materials for an inquiry-based approach to an introduction to proof course and abstract algebra.*

The speaker has written two open-source inquiry-based learning textbooks. One of the books is intended to be used for an introduction to proof course while the second book is designed for an undergraduate abstract algebra course. Both books are available as free downloads via GitHub. The initial development of the abstract algebra book was funded by a Small Grant from the Academy of Inquiry-Based Learning. In addition to saving students the cost of a textbook, the open-source model allows faculty members to adapt the books to their specific purposes. To our knowledge, roughly 30 different instructors have utilized at least one of the textbooks. In this talk, we will take a tour of both books, discuss their pedagogical approach, and communicate some of the design decisions that were made when creating the books. (Received September 27, 2017)

## *Implementing Recommendations from the Curriculum Foundations Project*

1135-P5-197 **Victor I Piercey\*** ([piercev1@ferris.edu](mailto:piercev1@ferris.edu)), Ferris State U. Department of Mathematics, 820 Campus Drive, ASC 2021, Big Rapids, MI 49307, and **Rhonda L Bishop** ([rhondabishop@ferris.edu](mailto:rhondabishop@ferris.edu)) and **Mischelle T Stone** ([mischellestone@ferris.edu](mailto:mischellestone@ferris.edu)).  
*Applying the Curriculum Foundations Recommendations to Mathematics for Business, Nursing, and Social Work.* Preliminary report.

Ferris State University offers a course for business students entitled Quantitative Reasoning for Professionals. Using the Curriculum Foundations, faculty from nursing, social work, and mathematics are revising the course to broaden the student audience. We will share how the Curriculum Foundations has shaped our discussions. In addition, we will share how the Curriculum Foundations inspired the creation of multidisciplinary case studies and simulations that will be added to the materials for the revised course. (Received August 08, 2017)

1135-P5-932 **Mike May, S.J.\***, [mike.may@slu.edu](mailto:mike.may@slu.edu). *Adjusting Math Courses for Business and Building a Dialog in the Spirit of CRAFTY.*

At Saint Louis University, the students taking the one semester survey of calculus course are almost all business students. Inspired by a presentation that seems to have arisen from the original CRAFTY discussions we started making adjustments to the course by incorporating the spreadsheet Excel as the main computational technology for the course. The change in technology led to adjustments to content, with greater emphasis on application, modeling, and shifts in terminology to better support students' knowledge transfer from a math to business setting.

The text is available at <http://math.slu.edu/~may/ExcelCalculus>.

The business school is very happy with the shift in the course. This led to a more discussion with a happy client discipline and recreating some of the CRAFTY math/business discussions on a local scale. We are currently looking at a developing a college algebra for business and in making adjustments in the courses of both disciplines to make connections clearer to students. (Received September 17, 2017)

1135-P5-1441 **Stella K Hofrenning\*** ([hofrenni@augzburg.edu](mailto:hofrenni@augzburg.edu)) and **Suzanne I Dorée** ([doree@augzburg.edu](mailto:doree@augzburg.edu)). *What Mathematics do Economics Students Need to Know?* Preliminary report.

The MAA CRAFTY Curriculum Foundation Project studied the mathematics that student in the partner disciplines need to know. In the new national SUMMIT-P project, we are re-examining those recommendations and updating them to fit modern curricula. In this talk, we'll share examples from the CFP reports and Economics texts that highlight key areas of Mathematics used in Economics with particular emphasis on the concepts of rate of change, system of equations and optimization. These findings are driving the renovation of

our calculus sequence and were developed in collaboration with the partner disciplines. (Received September 22, 2017)

1135-P5-1993 **Mark Gruenwald\*** (mg3@evansville.edu) and **David Dwyer** (dd4@evansville.edu).  
*Resequencing the Calculus Curriculum.*

Among the conclusions of the Curriculum Foundations Project was that the calculus curriculum needed to be “more appropriate for the needs of the partner disciplines.” With this goal in mind, the presenters set about to revamp the traditional calculus sequence with an emphasis on reordering and re-prioritizing the topics to better suit the needs of students in STEM. The Resequencing Calculus project that grew out of these efforts received funding from two NSF grants and culminated in the development of a textbook that supports the new curriculum. This talk will report on the work of the project team, which included faculty in all STEM disciplines, to align the calculus curriculum with the needs of STEM students and to pilot the curriculum at a variety of universities. (Received September 25, 2017)

1135-P5-2029 **Rebecca A Segal\*** (rasegal@vcu.edu), PO Box 842014, Richmond, VA 23284-2014.  
*Collaboration Conversations for Differential Equations (a SUMMIT-P collaboration).*  
Preliminary report.

Virginia Commonwealth University teaches an average of 28 sections of Differential Equations per year. The majority of the students in the course are majors in Engineering or Sciences. Anecdotally, students are not transfer knowledge content from the math course into courses within their majors. In order to improve knowledge retention, we have worked to establish conversations between Mathematics and the partner disciplines of Chemistry, Biology, Physics, and Engineering. Using the “Curriculum Foundations Project: Voices of the Partner Disciplines” report as source of discussion questions, a Fishbowl activity was held with faculty from Chemistry, Biology, Physics, and Engineering. There was good consensus about having the students explore a variety of application problems within each differential equation techniques. To further prioritize content for the course, a follow-up online survey was used to compile a topics wish list from the partner disciplines. This survey had responses from 38 faculty members from 8 different disciplines. Using this information, we are moving forward with three pilot sections of Differential Equations Fall 2017. (Received September 25, 2017)

1135-P5-2407 **Caroline Maher Maher-Boulis\*** (cmaherboulis@leeuniversity.edu), NASCM Department, Lee University, 1120 N. Ocoee St., Cleveland, TN 37320, and **Jason Robinson** (jrobinson@leeuniversity.edu), College of Education, Lee University, 1120 N. Ocoee St., Cleveland, TN 37320. *‘Why Do I Have to Take This Class?’ How Interdisciplinary Collaborations Can Improve Student Attitudes Toward Mathematics.*  
Preliminary report.

In this session we present the implementation of CF recommendations at Lee University through partnerships with faculty from the social sciences, education and the sciences. An overview of the recommendations chosen by each partner discipline will be given, the mapping of these recommendations into the current syllabi of affected courses and the interventions used for implementations will be discussed. The session will also describe a Student Exchange Program (SEP) between the divisions of mathematics and psychology. SEP is also a CF recommendation encouraging collaboration between students in these disciplines. Results from the implementations in fall 2017 will also be shared. (Received September 27, 2017)

1135-P5-2535 **Janet Bowers\*** (jbowers@mail.sdsu.edu), **Matt Anderson** (manderson@mail.sdsu.edu), **Kathy Williams** (kathy.williams@mail.sdsu.edu), **Antoni Luque** (aluque@mail.sdsu.edu) and **Nicole Tomassi** (nictomassi@gmail.com). *Promoting Active Learning and Modeling in PreCalculus: Design Features for Creating Engaging Labs.*  
Preliminary report.

Recommendations from the CRAFTY documents provide a refreshing change of perspective toward the development of scientific habits of mind. However, precalculus instructors face a challenge: How can we create engaging opportunities for students with diverse skill sets, backgrounds, and career interests? A team of researchers (funded by an NSF Grant entitled “A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships”) has addressed this challenge by creating weekly labs that involve active learning opportunities for modeling each of the function families studied in the course. For example, we have developed physics labs including a quadratic model of a ball rolling down a ramp and a trig model that uses the same ramp and ball to “discover” gravity. We have also developed biology-focused labs such as a logarithmic exploration of pH and an exponential model of a virus spread. This talk will describe design characteristics that guide our development and research including having low floor and high ceiling entry

points, opportunities for small group work and whole-class presentation, quality over quantity, explicit ties to the lecture portion of the class, and conceptually-based homework. (Received September 27, 2017)

1135-P5-2617 **Tao Chen\*** ([tchen@lagcc.cuny.edu](mailto:tchen@lagcc.cuny.edu)), 3110 Thomson Avenue, Long Island city, NY 11101, and **Glenn Henshaw, Soloman Kone** and **Choon Shan Lai**. *Contextualize College Algebra with Economics*.

Mathematics Education is usually geared towards teaching students the skills they need to advance to the next math class. Mathematical needs are rarely discussed between disciplines. To challenge this stereotype, the MAA organized a series of inter-disciplinary workshops, known as the Curriculum Foundation, and made a series of recommendations. In order to implement those recommendations, the LaGuardia Community College team collaborated with 10 other universities/colleges on an NSF project. In particular, the Laguardia team is working on contextualizing MAT115 College Algebra with economics and designing web applets to facilitate students learning. (Received September 26, 2017)

1135-P5-2807 **Mary Beisiegel\*** ([mary.beisiegel@oregonstate.edu](mailto:mary.beisiegel@oregonstate.edu)), Department of Mathematics, 368 Kidder Hall, Corvallis, OR 97330, and **Lori Kayes, Michael Lopez, Richard Nafshun** and **Devon Quick**. *Creating Connections in the Content: Using Curriculum Foundations to Improve College Algebra*.

Faculty from Mathematics, Chemistry, and Biology developed a detailed mapping of the content of College Algebra, Introductory Biology, and Introductory Chemistry. The map consists of mathematical concepts found in each of these courses, with examples of variations in mathematical emphasis, language, representations, and applied problems. The goal of this mapping is to understand the mathematical work in biology and chemistry and how it might read and look different from comparable mathematical work in the college algebra course. With the map completed, new activities are being developed for the college algebra course that present the language and representations from biology and chemistry. In this presentation, we will share the content map and the deepened connections we are creating through applied problems in college algebra. (Received September 26, 2017)

1135-P5-3117 **Karen G. Santoro\*** ([karensantoro@ccsu.edu](mailto:karensantoro@ccsu.edu)) and **M. F. Anton** ([anton@ccsu.edu](mailto:anton@ccsu.edu)). *A Modeling Approach to Developmental Algebra*. Preliminary report.

This is a report on a revision of the developmental math program at Central Connecticut State University with a focus on our new Applied Algebra with Models course. The new course is designed to enhance students' critical thinking and algebraic reasoning abilities as well as their problem solving and communication skills, achieved through a guided inquiry approach. We show how we implement the CRAFTY recommendations in the design of the reformed curriculum, the writing of a textbook, and the creation of professional developmental workshops. (Received September 26, 2017)

## Mathematical Themes in a First-Year Seminar

1135-Q1-23 **Aimee Schwab-McCoy\*** ([aimeeschwab-mccoy@creighton.edu](mailto:aimeeschwab-mccoy@creighton.edu)). *"Life in the Data Deluge": A First-Year Seminar on the Implications of Data Science on Daily Life*.

"Life in the Data Deluge" is a first-year seminar course designed to give students a deeper understanding of how data and statistics are used in the world around them. During this course, students learn to look at data with a critical eye, while picking up some introductory knowledge about concepts from statistics and data science. Students also study the motivations for presenting data in a certain context or collecting it in a specific way, and the influence that data have on decisions at all levels. Interesting applications of data science are presented from sports analytics, political polls, and targeted marketing.

This session will provide a glimpse into how statistics and data science can be incorporated into a first-year seminar through popular media, scientific journal articles, and student data collection projects. This session will also explore the importance of social context in a first-year seminar and how quantitative skills can be used to tackle difficult societal issues. Topics from Life in the Data Deluge can be easily modified to fit into a variety of first-year seminar models (reading-intensive, writing-intensive, or deep study of a single topic) or a traditional quantitative literacy course. (Received June 20, 2017)

1135-Q1-324 **Gizem Karaali\*** ([gizem.karaali@pomona.edu](mailto:gizem.karaali@pomona.edu)). *On Zombies, The Republic, and Mathematics: Teaching First Year Seminars That Humanize Mathematics*.

Teaching at a liberal arts college I have had the opportunity to develop a handful of first year seminars. As a mathematician my inclination is almost always to bring in mathematics to the foreground, one way or another.

In this talk I describe two first year seminars I have developed and taught, which revolve around the philosophical themes of what makes us human and what the goals of education should be, respectively. I discuss explicitly how my students engage with mathematics in these contexts. (Received August 24, 2017)

1135-Q1-515 **Tamara B Veenstra\*** ([tamara\\_veenstra@redlands.edu](mailto:tamara_veenstra@redlands.edu)). *Historical Codes and Ciphers as a First Year Seminar*. Preliminary report.

I have taught many different first year seminars with topics ranging from Mathematics and Art, Mathematics of Cryptology, Fantasy Novels (no mathematical content), and currently Historical Codes and Ciphers. I will primarily discuss the current Historical Codes and Ciphers seminar but will also discuss a number of lessons learned about first year seminars from the other seminars. One of the great advantages of teaching a first year seminar is that there is so much freedom to choose interesting topics. One of the great challenges is that there are so many learning outcomes and requirements based on the general first year experience and transition to college. It is especially challenging to find ways to connect mathematical content to these other goals of strengthening students' skills in reading, writing and speaking, addressing issues related to the transition from high school to college, and the purpose and value of a liberal arts education. I have found that picking a topic that allows students to connect mathematical topics with other areas such as literature, history, science, art, or popular culture has been a good solution to this problem. I will discuss in detail how this works in my Historical Codes and Ciphers class. (Received September 06, 2017)

1135-Q1-641 **Cheryl J. McAllister\*** ([cjmcallister@semo.edu](mailto:cjmcallister@semo.edu)), Southeast Missouri State University, 1 University Plaza, MS6700, Cape Girardeau, MO 63701, and **Laurie W. Overmann** ([lwovermann@semo.edu](mailto:lwovermann@semo.edu)), Southeast Missouri State University, 1 University Plaza, MS6700, Cape Girardeau, MO 63701. *The Mathematics of Art: A First-Year Seminar's Impact on Students and the Instructors that Teach It*. Preliminary report.

As a Mathematics instructor, being assigned a First-year Seminar can be an overwhelming task. Even a mathematics based theme for such a course doesn't make the charge of creating and grading assignments in critical thinking, effective writing, and information literacy any less daunting. In this session two faculty from Southeast Missouri State University will share their best tips on how to provide students from all mathematical backgrounds with a view of mathematics as it relates to art. What has worked, what didn't work, what we've learned, and how this type of course can show students a different view of mathematics will be discussed. (Received September 11, 2017)

1135-Q1-663 **David C. Marshall\*** ([dmarshall@monmouth.edu](mailto:dmarshall@monmouth.edu)). *The Mathematics and Ethics of Infinity*. I teach a First Year Seminar titled *Infinity and Beyond* at Monmouth University. Our First Year Seminars include several required learning outcomes. Two examples I will discuss are the requirements that students (1) demonstrate critical thinking as they actively engage in the course material, and (2) demonstrate awareness of ethical debates pertaining to the course topic. The former is an example of what draws many of us to create and teach First Year Seminars, while the latter is representative of what scares many of us away from these courses. I will discuss my approach to meeting the above (and other) course learning outcomes, highlighting both successful attempts and failed experiments. (Received September 12, 2017)

1135-Q1-763 **Philip K. Hotchkiss\*** ([photchkiss@westfield.ma.edu](mailto:photchkiss@westfield.ma.edu)), Department of Mathematics, Westfield State University, 577 Western Avenue, Westfield, MA 01086. *Mathematics For Liberal Arts In A First-Year Only Course*. Preliminary report.

Many first-year students at Westfield State University are enrolled in a first-year only (FYO) course. These courses, which can be in any discipline, are designed to help create a sense of community among first-year students. In addition, the University also selects a first-year read and encourages FYO courses to use this book. This fall I taught one of our mathematics for liberal arts (MLA) course as a FYO course and the first-year book was *Visual Intelligence* by Amy Herman. In this talk I will describe my experiences in this course, how I used the book (as well as recent movies) to discuss mathematical reasoning, current events, transitioning to college and (hopefully) recruit mathematics majors. (Received September 14, 2017)

1135-Q1-765 **William T Ross\***, University of Richmond, Richmond, VA 23173. *The Nature of Mathematics - First Year Seminar at the University of Richmond*.

Since 2010 I have been teaching in the University of Richmond's First Year Seminar in a course titled The Nature of Mathematics. This course is designed for mathematical novices and is a "gems" course. We cover topics from the Greeks (pre Socratics), Euler, Gauss, the six color theorem, the struggle to understand infinity (Cantor), and the personal side of mathematics (Hardy's Apology). In this talk I will give a survey of this course, how students

reacted to it (surprisingly Hardy's Apology become controversial!), and how I brought debate and writing to my students. (Received September 14, 2017)

1135-Q1-884 **Sarah J Greenwald\***, Department of Mathematics, Appalachian State University, Boone, NC 28697. *Breakthroughs and Controversies in Science and Mathematics: Standards-based Learning in a First-Year Seminar*. Preliminary report.

Want to engage students with mathematical topics in a fun, creative, and intellectually stimulating environment? We employ standards-based learning and delve into diverse and opposing viewpoints on many issues. We focus on what science and mathematics is, strategies for success in these fields, ethical and philosophical considerations, public perceptions, and applications to daily tasks. We also explore a series of interrelated questions about proof and certainty: What is truth? When are we convinced? What are the consequences of certain truths? What is the role of chance and probability? The students choose many of the topics we cover, so they feel a sense of shared ownership in the material as they work toward meeting the course learning goals. The goals are broad and center around critical and creative thinking, effective communication, local to global connections, and the responsibilities of community membership; students must examine a single issue from multiple perspectives, learn to conduct quality research, and analyze the arguments of others as they make connections with students, faculty and the university. We will share the reasons why students choose to take this specific section along with the difficulties and great rewards of teaching it. (Received September 16, 2017)

1135-Q1-914 **Janine Wittwer\***, 1840 South 1300 East, Salt Lake City, UT 84105. *Student Driven Modeling in a first year Game Theory Seminar*.

Westminster College (UT) has recently switched to a new model for its Liberal Arts Core where students take seminar courses without prerequisites, designed to take them deeply into the material and methods in a particular subject which is new to them - not usually one covered in any major. In "Games and Decisions", students learn elements of Game Theory with a focus on discovery, critical thinking and modeling. In this talk, we will describe the process, along with its challenges and shortfalls, of working with groups of first year students who come in expecting to memorize what the teacher tells them, and leading them to active learning and critical thinking. By the end of the semester, students collectively and voluntarily created their own game theory competition, and each student completed an original (and often very creative) project relevant to their personal lives, where they explore how the ideas of game theory can help them understand the world around them. (Received September 16, 2017)

1135-Q1-1068 **Gary Clark Hall\*** ([gary.hall@lipscomb.edu](mailto:gary.hall@lipscomb.edu)), One University Park Dr, Nashville, TN 37204. *Murder, They Wrote: Problem Solving Is Fun!* Preliminary report.

I have been teaching a first year seminar course for over ten years. In this course I have used problem solving and Polya's four step process for problem solving as the main theme for the course. We do not look at only math problems, but also how these ideas really extend to daily life and problems between the genders, the races, in politics, in religion, and even in solving murder mysteries whether fictional or real. (Received September 19, 2017)

1135-Q1-1457 **Jenny G Fuselier\*** ([jfuselie@highpoint.edu](mailto:jfuselie@highpoint.edu)). *An Honors First Year Seminar in Network Science*. Preliminary report.

We share experiences in the development and delivery of a first year course entitled "Graph Theory and the Science of Networks." This course was conceived as part of a newly overhauled Honors Program curriculum at a liberal arts institution. Serving as the "quantitative reasoning" component of honors students' general education requirements, the course must touch on mathematical topics related to data accumulation, categorization, and representation, while also seeking to expose students to new mathematics. In line with the rest of the honors curriculum at the institution, the course is structured in a project-based format and writing is a main point of emphasis. The topics of graph theory and network science proved to be a good fit for a group of students with mixed mathematical backgrounds but also with a variety of academic interests across disciplines. In this talk, we explore the structure of the course and discuss both successes and places for improvement. (Received September 22, 2017)

1135-Q1-1468 **Aaron M. Montgomery\***, [amontgom@bw.edu](mailto:amontgom@bw.edu). *A First-Year Seminar on "Lies, Damned Lies, and Statistics"*.

All first-semester students at Baldwin Wallace University take a "First Year Experience" (FYE) course. The goals of these FYE courses are to promote critical thinking, to foster intellectual curiosity, to engage the students in process writing, and to recognize the University Mission. In the Fall semester of 2016, I taught such a course

on "Lies, Damned Lies, and Statistics" which discussed the use, misuse, and abuse of numerical information by politicians and others. In this talk, I will discuss what went well, what went poorly, and what I learned from the experience. (Received September 22, 2017)

1135-Q1-1911 **Karen E Clark\*** (kclark@tcnj.edu), Department of Mathematics and Statistics, Ewing, NJ 08628. *A First Seminar Course on The Mathematics of Equity.*

At The College of New Jersey first year students are required to take a seminar course that is designated as "writing intensive", and must include regular writing assignments. The students in each section of the seminar course are housed together on the same dormitory floor and our-of-class activities are encouraged. I have developed a first seminar course (FSP) entitled "What's Fair? The Mathematics of Equity". The content of the course is fair division and voting theory, covering topics that might be included in a standard finite mathematics course. However the structure of the class is deliberately more discussion-based than lecture, and students are encouraged to explore mathematical ideas through writing and debate. (Received September 25, 2017)

1135-Q1-1950 **Pamela B. Pierce\*** (ppierce@wooster.edu). *Decisions, Decisions: A First-Year Seminar Incorporating Mathematical Themes.*

How do individuals and groups make decisions? This was the focus of a first year seminar designed to develop students' skills in critical thinking, writing, and constructing effective arguments. Students with diverse mathematical backgrounds were able to engage in a variety of mathematically-themed topics, including voting theory, the Monty Hall Problem, and the Prisoner's Dilemma. Some hands-on in-class activities made our "math-focused" days some of the most memorable ones of the semester. I will share these activities with the audience and discuss possible areas for improvement. (Received September 25, 2017)

1135-Q1-2289 **Maria G Fung\*** (mfung@wochester.edu), Mathematics Department, 486 Chandler Street, Worcester State University, Worcester, MA 01602. *Quantitative Literacy in a First-Year Seminar.*

First-year seminars are a natural venue for developing students' quantitative literacy skills. In this talk, several different activities from three different first-year seminars will be showcased. These projects develop students' communication, critical thinking, and information literacy skills while deepening their understanding of a variety of quantitative literacy topics such as collecting, organizing and analyzing statistical and numerical information, considering bias and reliability of data, and modeling with spreadsheets. (Received September 25, 2017)

1135-Q1-2291 **Jennifer R. Bowen\*** (jbowen@wooster.edu), The College of Wooster, 1189 Beall Ave., Wooster, OH 44691. *The Signal and the Noise: Why Numeracy Really Matters.* Preliminary report.

Required of all first-year students, the First Year Seminar (FYS) in Critical Inquiry at The College of Wooster (OH) focuses on the process of critical inquiry in a writing-intensive, small seminar. Each seminar invites students to engage a set of issues, questions, or ideas that can be illuminated by the disciplinary and interdisciplinary perspectives of the liberal arts. Seminars are designed to enhance the intellectual skills essential for liberal learning and for successful participation in the college's academic program. Above all, however, this course offers an introduction to college writing, reading, research, and discussion. In this session, we will examine the experience of teaching the FYS, "The Signal and the Noise: Why Numeracy Really Matters." Is algebra really necessary? When will I use this? What is math for? The course explored quantitative literacy, quantitative reasoning, and numeracy and innumeracy in the United States. In this context, students examined quantitative experience using Nate Silver's writing, informed by big data, social and natural sciences, and popular culture. In addition, the emergence of MOOCs and Khan Academy as educational technologies in mathematics. (Received September 25, 2017)

1135-Q1-2571 **Lisa Carnell\*** (lcarnell@highpoint.edu). *Chance, Data, and Decision-Making: What the Teacher Learned.*

At my institution, every student takes a first-year seminar as part of the general education curriculum. I center my seminar around the ideas of the production of data, how we might critically evaluate the quality of the data we come across in everyday life, and how our perceptions of chance and bias impact the decisions we make. Major goals of the course are critical thinking and information literacy. I incorporate ideas such as sampling, survey methodology, experimental design, and relative risk through current news topics. Not everything I've tried has been successful. I have taught this seminar multiple times, and every time I change it to reflect what I learned from my students in the previous iteration. (Received September 26, 2017)



1135-Q1-2620 **Michael P. Saclolo\*** (mikeps@stedwards.edu), St. Edward's University, School of Natural Sciences, 3001 South Congress Avenue, Austin, TX 78704. *Order and Disorder: A First-Year Seminar*. Preliminary report.

This presentation reports on the implementation of a first-year seminar course with a mathematical theme. The course is designed to introduce entering college students to a liberal arts thematic course as well as orient students to academic life as a whole. As part of a cluster of four seminars with a general theme of "Utopias and Dystopias", this particular seminar focuses on the concepts of order and disorder within mathematics and their depiction in mathematically themed or oriented texts, both fiction and non-fiction. The readings and classroom activities are partly oriented to the discussion utopias and dystopias as social constructs, but they also tackle mathematical topics such as dimension, classification, symmetry, chaos, entropy, similarity and self-similarity. (Received September 26, 2017)

1135-Q1-2665 **Ann N Trenk\*** (atrenk@wellesley.edu), Mathematics Department, Wellesley College, 106 Central Street, Wellesley, MA 02482. *First Year Seminars as a Gateway to Upper Level Mathematics*.

First year seminars can provide a wonderful opportunity to introduce skills and habits that have been shown to be associated with persistence and success in mathematics and other STEM courses. In our first year seminar courses at Wellesley College, we work to build an inclusive and collaborative environment. This helps to create a community of learners who can support one another throughout their four years of college. In this talk we describe some of the mathematical topics and techniques we use to achieve these outcomes. (Received September 26, 2017)

1135-Q1-2892 **Eddy A Kwessi\*** (ekwessi@trinity.edu), San Antonio, TX 78258, **Merline E Nzenang Tienkoue** (info@efilworldfoundation.org), , Cameroon, and **Mathieu Tienkoue** (info@efilworldfoundation.org), , Cameroon. *Thinking about a First Year Math Seminar*. Preliminary report.

In this talk, we will discuss how first year Math Seminars are run in the Department of Mathematics at Trinity University. I will especially emphasize two things: one, the mathematical requirements for our students, and second, the technology requirement. I will show how topics are selected and present the learning goals for our students. Technology is now used and encouraged in almost all areas of our curriculum, so our students have the unique opportunity to enhance their skills throughout this seminar. I will show how we teach students presentations skills such as Latex and interfacing Latex with computing packages such as Matlab and R. We will also discuss how we plan to deploy this learning goals to less fortunate areas in Africa (cameroon) where students can benefit the most from this learning skills. (Received September 26, 2017)

1135-Q1-3102 **Jason Parsley\*** (parslerj@wfu.edu). *A first-year seminar on the Mathematics of Sports Rankings*.

Teaching a first-year seminar is unlike teaching a more traditional, introductory mathematics course. Through the lens of a recent seminar on the Mathematics of Sports Rankings, we explore how to manage the different pedagogical demands of a first-year seminar – preparation, fostering classroom discussion, teaching with no prerequisites. We discuss its successes (e.g., several students presented at a sports analytics conference) and how to avoid its failures. (Received September 26, 2017)

1135-Q1-3155 **Neelesh Venkat Tiruvilumala\*** (tiruvilu@usc.edu). *Mathematics for Earthling Ambassadors to Outer Space*. Preliminary report.

Let's not mince words. The aliens are coming and they're going to expect answers. Here are some of the things that they might want explained:

- How we organize and retrieve our knowledge (search engine algorithms)
- How we get places (global positioning systems)
- How we are training machines to think and to inevitably destroy us (artificial intelligence)
- What we find beautiful (friezes and symmetries)
- How our brains are tuned (musical frequencies and Fourier analysis)
- How we keep secrets (cryptography)

Wouldn't it be great if we had a socially intelligent (disqualifies mathematicians), friendly (disqualifies engineers), and stylish (disqualifies physicists) group of earth representatives to explain these mysteries to outsiders? USC students would make the perfect candidates!

Except for one thing. It is painfully evident that there is a lot of mathematics inherent in the above topics list and let's face it; you didn't really learn much of that in high school. It will be embarrassing for all of us if, when confronted with important questions, all that you can do is sing the quadratic formula song. Wait. You don't even remember that? We'll need to get to work immediately. (Received September 26, 2017)

## *Math Circle Topics with Visual or Kinesthetic Components*

1135-Q5-268 **Frederick A Peck\*** ([frederick.peck@umontana.edu](mailto:frederick.peck@umontana.edu)), Department of Mathematical Sciences, University of Montana, 32 Campus Drive #0864, Missoula, MT 59812, and **Matt B Roscoe** ([matt.roscoe@umontana.edu](mailto:matt.roscoe@umontana.edu)), Department of Mathematical Sciences, University of Montana, 32 Campus Drive #0864, Missoula, MT 59812. *Let's get cracking: Russian egg roulette.*

Russian egg roulette is a regular feature of The Tonight Show. Two players take turns cracking eggs over the other's head. The players choose eggs from a uniform-looking set that includes four hard boiled and eight raw eggs. A player loses when she has two raw eggs cracked on her head.

Egg roulette is ideally suited to math circles:

- (1) Participants engage in an enjoyable and intense kinesthetic game
- (2) The intensity and jocularly of the game helps to build community
- (3) The game evokes a wide range of "natural" questions
- (4) The game can be easily modified, leading to vastly different questions
- (5) Questions can be addressed experimentally and theoretically
- (6) Investigations often involve multiple probability laws, concepts, and tools

Common questions include, "what is the maximum number of rounds?," "what is the probability of a 'clean cap'—a winner with no eggs broken on her own head?," and "is it better to go first or second?" Variations include parameterizing the number of raw and hard boiled eggs, drawing eggs from a distribution where the number of raw and hard boiled eggs are unknown and even changing the winning conditions (e.g., the person with the third egg broken on her head is the loser). (Received August 16, 2017)

1135-Q5-357 **Joshua P. Bowman\*** ([joshua.bowman@pepperdine.edu](mailto:joshua.bowman@pepperdine.edu)), Natural Science Division, Pepperdine University, 24255 Pacific Coast Highway, Malibu, CA 90263. *Counting and symmetry in conceptual art.*

Several works of conceptual art lend themselves well to mathematical exploration because they are concrete realizations of abstract geometric ideas. I will describe a Math Circle I recently led based on "Incomplete Open Cubes" by Sol LeWitt, using Zometool. This project is related to graph theory, combinatorics, and symmetry groups of polyhedra. (Received August 27, 2017)

1135-Q5-424 **Russell A Gordon\*** ([gordon@whitman.edu](mailto:gordon@whitman.edu)), Department of Mathematics, Whitman College, 345 Boyer Avenue, Walla Walla, WA 99362. *A simple construction problem.*

High school geometry courses usually spend a little time exploring compass and straightedge constructions. The fact that some seemingly simple constructions, such as doubling a cube or trisecting an angle, are impossible with just these tools can be surprising to students. Even a brief discussion of such problems can open the door to higher mathematics for curious students. In this talk, we present a collection of construction problems involving rectangles and determine conditions for which the construction can be carried out with compass and straightedge. The simple pictures point the way to some interesting higher level mathematics such as Diophantine equations and algebraic numbers. (Received September 01, 2017)

1135-Q5-561 **Javier Alfredo Ronquillo Rivera\*** ([jr928412@ohio.edu](mailto:jr928412@ohio.edu)). *Groups, Symmetries and  $i$ Dancing?*

Contradance is an American folk dance where couples dance as they stand across each other in long parallel lines (to get an idea of how it looks/sounds visit <https://www.youtube.com/watch?v=jcYOIXIMfJQ>). A "caller" announces the next moves and dancers cheerfully perform them (for example: <https://www.youtube.com/watch?v=DBvhyVata9I>).

In this session I will discuss my experience using Contradance as part of a one week long class for 7th graders at BEAM, a program to help underserved students enter advanced study in mathematics. This activity helps introduce and explore concepts like identity, inverses, commutative, generators, and symmetries of polygons through fun and dancing. (Received September 08, 2017)

1135-Q5-766 **Whitney George\*** ([wgeorge@uwlax.edu](mailto:wgeorge@uwlax.edu)). *Realizing Seifert Surfaces and More!*

A surface is a mathematical object that looks two dimensional. For instance, a beach ball and an inner tube are two types of surfaces. However, there are a lot of other surfaces. Also, there are many different ways to "see" these surfaces. In this talk, we are going to explore a Math Teacher's Circle lesson that investigates different types of surfaces and some of their bizarre characteristics. Get ready to bend your mind! (Received September 14, 2017)

1135-Q5-1211 **David Honda** ([dhonda@sandi.net](mailto:dhonda@sandi.net)), **Yana Mohanty** ([yana@imathgination.com](mailto:yana@imathgination.com)) and **David Patrick\*** ([patrick@aops.com](mailto:patrick@aops.com)). *Snapology Origami: Folding and Snapping Your Way to Geometry, Art, and More.*

What do you get when you take strips of paper, cut them into small rectangles, and fold them together? You get the building blocks to create fantastic mathematical models. This is the art of Snapology Origami, also known as Knotology, created by Heinz Strobl. These models can be used to explore polyhedra, topology, art, and more. Much like unit origami, Snapology Origami models are created without glue or tape, and models are held together strictly by how the pieces are folded and assembled. We will discuss how the Snapology Origami models created by one of our San Diego Math Teachers' Circle participants, as a hobby, evolved into an entire MTC session. We will show how to create an icosahedron, and participants will be shown more complex models based upon similar methods, including various polyhedra, a Sierpinski Pyramid, a Buckyball, a Klein bottle, tori of various genus, and a few abstract sculptures. (Received September 20, 2017)

1135-Q5-1563 **Thomas J Clark\*** ([tom.clark@dordt.edu](mailto:tom.clark@dordt.edu)), 498 4th Ave. NE, Sioux Center, IA 51250. *Barbie Bungee: Multivariable Edition.*

"Model with mathematics" is an important standard for mathematical practice that can lead to deep inquiry. Providing teachers and students an opportunity to engage in mathematical activities that exemplify the standards for mathematical practice is a goal of many math circles. In this activity, rubber bands are tied together to make a bungee cord for a Barbie doll. The goal is to determine how many rubber bands will give a particular doll an exciting but non-fatal ride. Since this number depends on both the mass of the doll and the height of the drop, an opportunity for a multivariable investigation is at hand – a novel addition to the typical presentation of the problem. The session is especially engaging due to the visual nature of the problem and hands on data collection at the core. (Received September 23, 2017)

1135-Q5-1571 **Judith Covington\***, [judith.covington@lsus.edu](mailto:judith.covington@lsus.edu). *Creating and Justifying Platonic Solids.*

How many platonic solids are there? I expected my MTC to immediately say five, but that was not the case. I will share an activity that I used with my MTC that allowed the teachers to not only leave knowing that there are five platonic solids, but also WHY there were only five. In this session I used the commercially available polydrons to help the teachers visualize what was happening when they joined different polygons to create solids.

In this session I will quickly share some of the basic information my teachers needed to formulate before they began this activity. They then used the polydron shapes along with the definition of a platonic solid to create as many platonic solids as they could. I then asked if they thought they had created all and if they could give a convincing argument that they had all. I will share some of these discussions along with the argument that there are only five platonic solids. (Received September 23, 2017)

1135-Q5-1774 **Joie F. Escudro\*** ([escudroj@juniata.edu](mailto:escudroj@juniata.edu)). *Ready, Set, Eureka!* Preliminary report.

Dr. Eureka is a high-speed game of skill, logic and dexterity. Each player will have two green balls, two red balls, two purple balls and three test tubes. Each test tube can hold up to four balls. With the initial position of each test tube holding two balls of the same color, the object of the game is to position the balls as illustrated on a challenge card without touching or dropping any ball. We will explore the many questions that arise from this game. Most questions are of the counting variety but participants can also investigate function inverses and function compositions. (Received September 24, 2017)

1135-Q5-2112 **Maria S. Nogin\*** ([mnogin@csufresno.edu](mailto:mnogin@csufresno.edu)), 5245 N Backer Ave, M/S PB 108, Department of Mathematics, California State University, Fresno, Fresno, CA 93740, **Adnan H. Sabuwala** ([asabuwala@csufresno.edu](mailto:asabuwala@csufresno.edu)), 5245 N Backer Ave, M/S PB 108, Department of Mathematics, California State University, Fresno, Fresno, CA 93740, and **Jenna Tague** ([jtague@csufresno.edu](mailto:jtague@csufresno.edu)), 5245 N Backer Ave, M/S PB 108, Department of Mathematics, California State University, Fresno, Fresno, CA 93740. *Nim-like games with 2D boards.*

In this presentation, we share a few games that we play in grades 3-9 of the Fresno Math Circle. While the rules are very simple, developing strategies for these games uses deep mathematical concepts such as symmetry, working backwards, divisibility, and base representations. We provide examples of how the same game can be

modified for grade level appropriate challenges. We also demonstrate that some number-theoretic games and geometric games are equivalent, and this correspondence can be used in both directions to develop winning strategies. Through such games, we show our math circle participants a beautiful connection between two different areas of mathematics – number theory and geometry. (Received September 25, 2017)

1135-Q5-2115 **Mark C Hughes\*** ([hughes@mathematics.byu.edu](mailto:hughes@mathematics.byu.edu)). *Knotted mathematics for elementary-aged students.*

Understanding knots in 3-dimensional space has been a goal of topologists for over 100 years. Despite their complexity and the important mathematical concepts they lead to, knots can easily be introduced to elementary students, who are able to quickly begin studying and exploring concepts at a wide range of difficulty levels. In this talk I will discuss several ideas for Math Circle meetings based on concepts from knot theory. After tying their first knot with a piece of string or pipe-cleaner, students are ready to explore ideas such as algorithms and equivalence, all while twisting and bending the knots in their hands. Students learn about topological invariants while coloring diagrams and discussing analogies to popular board games. For older students more complicated concepts can be introduced – like Reidemeister moves and polynomial invariants – which still retain the same visual flavor that makes knot theory so appealing to younger children. (Received September 25, 2017)

1135-Q5-2236 **Sharon Lanaghan\*** ([slanaghan@csudh.edu](mailto:slanaghan@csudh.edu)). *Simon Says, Four Gallons.*

Given a 5 gallon jug and a 3 gallon jug, is it possible to measure out exactly 4 gallons? That's the problem faced by John McClane and Zeus Carver in *Die Hard 3* and studied by many Math Circles. In this session, participants will solve this problem and will explore multiple representations that will lead to a deeper understanding of its solution. Both geometric and algebraic approaches will be employed to answer questions such as: "What measurements can be made from different combination of jugs?", "How can geometry be used to model this problem?", "Is there a way to model this problem with equations?", "How are the algebraic and geometric approaches to this problem related?" There will also be a discussion of source material as well as observations from implementation of this session with teachers at the CSUDH Math Teachers' Circle. (Received September 25, 2017)

1135-Q5-2305 **Jameson C Hardy\*** ([jameson.hardy@dixie.edu](mailto:jameson.hardy@dixie.edu)), 1301 West Indian Hills Dr, 29, St George, UT 84770. *Teaching Group Theory Through Twisty Puzzles.* Preliminary report.

The Molecube is a twisty puzzle, similar to the Rubik's Cube, that contains nine different colors. The goal is to arrange the cube so that each color appears on each face only one. In this talk, we will explore how the Molecube and other twisty puzzles can be used to give a hands-on introduction to group theory. We will explore several introductory group theoretic ideas that I have successfully introduced to children as young as 12 years old, including the parity of permutations and the use of commutators to develop algorithms. (Received September 25, 2017)

1135-Q5-2470 **Erica Bajo Calderon\*** ([ebajo@uco.edu](mailto:ebajo@uco.edu)), **Liz Lane-Harvard** and **Carol Lucas**. *Spatial Reasoning at the Central Oklahoma Math Circle.* Preliminary report.

The Central Oklahoma Math Circle is a partnership between US Grant High School in OKC and the University of Central Oklahoma. Numerous studies have shown that higher levels of spatial reasoning correlate to success in mathematics, and it is usually the males that have a better grasp of spatial reasoning. Since our Circle's target audience is females, we like to incorporate at least one lesson on spatial reasoning. In this talk, we will discuss a progression of activities that incorporate spatial reasoning, like quick draw and thinkcards, with the final activity being Blokus Trigon. We will describe our conversation with the participants about spacial reasoning and its importance. Finally, we will discuss variations for the activities. (Received September 26, 2017)

1135-Q5-2844 **Jenna Tague\*** ([jtague@csufresno.edu](mailto:jtague@csufresno.edu)), 5245 North Backer Avenue M/S PB108, Room PB343, Fresno, CA 93740. *Mathematical Modeling in 3-5th Grade Math Circle.*

While mathematical modeling has clear connections in algebra and high school mathematics, often it is not included as part of Math Circles or enrichment for younger students. In this presentation, I share some mathematical modeling problems that are appropriate and accessible for grades 3, 4, and 5. Besides the tasks, I'll share the mathematics that the students needed to complete the tasks, where the students struggled, and resources for implementing your own mathematical modeling tasks in similar situations. (Received September 26, 2017)

1135-Q5-2935 **Rebin A. Muhammad\*** ([rm775311@ohio.edu](mailto:rm775311@ohio.edu)). *Islamic Geometric Pattern: Point-Construction Method.*

An Islamic geometric pattern (IGP) is a 2D wallpaper that is created by using only using a compass and a ruler, Construction lines are a traditional method for creating Islamic Geometric Patterns (IGP). We have

already used this method in many Math Circle sessions. This time we will go further in terms of abstraction and we will introduce a Construction-Points Method. This approach opens the door to new questions and conjectures, and offers more room for students in Math Circle to show their visual creativity, as well as make connections to some areas of math like graph theory. One advantage of this approach is that the level of complexity is variable. First the students will be given a simple pattern with construction lines, then later they will be asked to find all points that are vital to the pattern. Students will do this individually and then later will be asked to compare their answers in groups. Another advantage of construction-point method is helping students to create a set of axioms for IGP, and realize, most importantly, that these axioms can be different, since the way you interpret the visual pattern can be subjective. (Received September 26, 2017)

## Revitalizing Complex Analysis

1135-R1-109 **Martin Flashman\*** ([flashman@humboldt.edu](mailto:flashman@humboldt.edu)). *Making Sense of Complex Integration: Mapping Diagrams Created with GeoGebra to Visualize Definitions, Theory and Applications*. Preliminary report.

While teaching an undergraduate course in complex analysis in Spring, 2015, the author used GeoGebra 5.0 to introduce a new complex function visualization with dynamic 3 dimensional mapping diagrams. Some of this experience was reported previously in "Visualizing Complex Variable Functions with Mapping Diagrams: Linear Fractional Transformations." MAA Contributed Paper Session on Revitalizing Complex Analysis, Jan.9, 2016.

[<http://users.humboldt.edu/flashman/Presentations/JMM2016/MD.JMM.CV.1.9.16.3.html>]

In the simplest form for real variables mapping diagrams visualize tables and provide a valuable addition to graphs in understanding calculus and real analysis. Similarly, complex mapping diagrams can represent many visual aspects of complex analysis. This presentation will demonstrate recent work using mapping diagrams to explore and make sense of complex integration. Besides examining definitions, examples will illustrate basic theorems and applications. (Received July 27, 2017)

1135-R1-266 **Michael Brilleslyper\*** ([mike.brilleslyper@usafa.edu](mailto:mike.brilleslyper@usafa.edu)), 2354 Fairchild Drive Suite 6D-124, USAF Academy, CO 80908, and **Beth Schaubroeck**. *Locating the Roots of a Harmonic Polynomial*. Preliminary report.

We consider the root locations in the complex plane of the *harmonic* trinomial  $q(z) = z^n + \bar{z}^k - 1$ , where  $n \geq 2$  and  $1 \leq k < n$ . We count the number of roots occurring inside, on, and outside the unit circle in terms of  $n$  and  $k$ . The function  $q(z)$  has roots on the unit circle (unimodular roots) if and only if  $n - k$  is divisible by  $6g$ , where  $g = \gcd(n, k)$ . We also provide a closed formula for the number of roots inside the unit circle (interior roots). In previous work, we analyzed the root locations of the related family of analytic trinomials given by  $p(z) = z^n + z^k - 1$ . For both  $q(z)$  and  $p(z)$ , the unimodular roots are identical though for different conditions on  $n$  and  $k$ . Furthermore, we show the formula for the number of interior roots of  $q(z)$  is closely related to the corresponding formula for  $p(z)$ . Finally, we note that for general harmonic polynomials the Fundamental Theorem of Algebra need not hold, however  $q(z)$  always has exactly  $n$  roots. (Received August 16, 2017)

1135-R1-575 **G. Brock Williams\*** ([brock.williams@ttu.edu](mailto:brock.williams@ttu.edu)), Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409. *Visualization of Complex Functions Using Circle Packings*.

It has been understood since the work of Thurston and many others in the late 1980's that discrete maps produced by circle packings can be used to approximate analytic functions. Thus circle packings have found applications in the intervening decades in areas ranging from Teichmüller theory to image analysis.

We will describe how the visual nature of circle packings provides a unique opportunity to introduce students to the geometric nature of analytic functions. Conformality, the actions of Möbius transformations, Schwarz's Lemma, and the argument principle can all be directly illustrated using circle packings. (Received September 08, 2017)

1135-R1-921 **Qi Han\*** ([qhan@tamusa.edu](mailto:qhan@tamusa.edu)), Assistant Professor, Department of Mathematics, Texas A&M University at San Antonio, San Antonio, TX 78224. *How to identify the the Euler-gamma function and the Riemann-zeta function?* Preliminary report.

In this talk we briefly discuss how to identify the Euler-gamma function and the Riemann-zeta function compared to a general complex analytic function of single complex variable. (Received September 16, 2017)

1135-R1-1453 **Stephan Ramon Garcia\*** ([stephan.garcia@pomona.edu](mailto:stephan.garcia@pomona.edu)), Department of Mathematics, Pomona College, 610 N College Ave, Claremont, CA 91711. *Some remarks about the teaching of complex variables.*

We discuss the manner and order in which certain topics are presented in a course on complex variables. To be more specific, we examine some perspectives on the teaching of the Cauchy–Riemann equations, harmonic functions, and the maximum principle; Möbius transformations, conformal maps, and elementary functions; and the presentation of infinite series. (Received September 22, 2017)

1135-R1-2472 **Beth Schaubroeck\***, [beth.schaubroeck@usafa.edu](mailto:beth.schaubroeck@usafa.edu), and **Julie Barnes**. *From Julia Sets to Coloring Pages.*

Iteration of functions and the associated pictures of Julia sets are often included in an undergraduate complex variables course. In this talk, instead of examining the Julia sets of complex functions, we discuss how we might visualize the third or fourth or fifth iterate of a function. We do this by looking at the level contours of the real part of the iterate. This generates surprisingly beautiful images, reminiscent of pictures found in coloring books for adults. Even more wonderful is the interesting mathematics hidden in the images. This collection of intricate complex function designs led to an MAA-published coloring book, which we hope will inspire students to explore the beauty of complex variables. (Received September 26, 2017)

1135-R1-2601 **Russell W Howell\*** ([howell@westmont.edu](mailto:howell@westmont.edu)), Department of Mathematics & Computer Science, Westmont College, Santa Barbara, CA 93108. *A Web Interface for REU Projects in Complex Analysis.*

Recently *PRIMUS* dedicated a theme issue to the *Revitalizing Complex Analysis* effort. Many of the articles in that publication are suitable for use as springboards to a variety of undergraduate research projects. But what resources might be available as new ideas materialize? Discussion among MAA members is now underway to create a web interface that would delineate proposals suitable for an REU program in complex analysis. When completed it will thus allow research experiences in complex analysis to be conducted locally by faculty members and students at their respective institutions, without the students having to navigate a highly-selective application process. This talk will give examples of such projects in conjunction with a preliminary overview of the website, now under construction. (Received September 26, 2017)

## General Session on Assessment

1135-VA-632 **Alison Reddy\*** ([ared@illinois.edu](mailto:ared@illinois.edu)), 1409 W. Green Street, Department of Mathematics, Urbana, IL 61801. *Predictive Analytics and Intangibles: Using data to improve student success and retention rates in core mathematics courses.* Preliminary report.

Students arrive at the University of Illinois with diverse mathematical backgrounds, and diverse mathematical and psychosocial identities. In 2007 the Department of Mathematics at Illinois implemented a new placement program and the use of predictive analytics to improve success in core mathematics courses and to improve retention rates in the STEM disciplines and on campus in general. Data is also collected to identify to what extent tangible and intangible characteristics and / or commonalities help or hinder student success and retention. The qualitative and quantitative evaluation of student data can be used for (1) improving what we measure and why, (2) targeted support for students (holistic and intrusive advising, mentors, tutors, workshops, support programs), (3) planning for personnel needs (academic advisors, mentors, tutors, workshop presenters, other support staff) for students, and (4) the continuation and future development of services for student success, retain and improved pathways to graduation. Collected data will be shared. (Received September 11, 2017)

1135-VA-1348 **Douglas A Riley\*** ([driley@bsc.edu](mailto:driley@bsc.edu)) and **Bernadette Mullins**. *Departmental assessment at a small liberal arts college.*

Assessment of major programs provides opportunities to gather information on how our students learn mathematics and how best to respond to improve student learning. At small schools, which often lack resources and expertise in assessment, practices can vary widely. In this talk, we discuss how one small liberal arts school has approached the assessment question for the mathematics major. We describe the student learning outcomes for the program, how they are measured, and what the department has done to ensure the consistency and reliability of the data. We also discuss how the department has used the assessment process and the most recent data to motivate change to the major program. (Received September 21, 2017)

1135-VA-1721 **Adam Giambrone\*** ([adam.giambrone@uconn.edu](mailto:adam.giambrone@uconn.edu)). *Using Metacognitive Reading/Writing Assignments in a General Education Mathematics Course*. Preliminary report.

At many institutions, students are required to take a general education course in mathematics. In an effort to increase student engagement and to help students see how such a course fits into their college education, reading/writing assignments about intelligence, learning, and thinking have been woven into a liberal arts mathematics course at the University of Connecticut. In this talk, we will discuss the creation and implementation of these reading/writing assignments, using student feedback and responses to highlight some benefits and challenges to incorporating this type of assessment into a general education mathematics course. (Received September 24, 2017)

1135-VA-1793 **Allen G Harbaugh** ([harbaugh@bu.edu](mailto:harbaugh@bu.edu)), **Andrew S Richman\*** ([asrich@bu.edu](mailto:asrich@bu.edu)) and **Suzanne Chapin** ([schapin@bu.edu](mailto:schapin@bu.edu)). *Evaluating Assessments for Learning in the Mathematics Classroom: An Item Response Theory Primer for Mathematics Educators*.

A number of key issues in psychometric measurement theory extend naturally into the classroom assessment setting. Item response theory (IRT, a latent variable analysis tool) can be used to jointly assess item-level difficulties and individuals' abilities. We argue that this aspect of assessment has too often been overlooked in mathematics classrooms. In particular, IRT allows an educator to (1) determine how well a test targets the intended audience, (2) assess test items for differential item functioning, and (3) evaluate the degree to which a test measures an intended unidimensional construct. Simply put, IRT is a tool allowing educators to confirm which—if any—of the items on their tests are measuring the same thing at varying levels of difficulty. To demonstrate the analysis, findings from The Elementary Pre-Service Teachers Mathematics Project will be presented. In this project, content-specific assessments (e.g., number theory) were administered to measure participants' pre to post gains. Using these data, we will demonstrate how an educator might use the information to revise the test or assess students. Finally, as IRT is often used with larger sample sizes, one of the issues to be discussed is how IRT can be used in classrooms with a reasonable number of students. (Received September 24, 2017)

1135-VA-2343 **Germaine Kamleu -Ndouma\*** ([germainekam@gmail.com](mailto:germainekam@gmail.com)) and **Lorna Holtman** ([lholtman@uwc.ac.za](mailto:lholtman@uwc.ac.za)), Robert Sobukwe Rd, Bellville, Cape Town, 7535, South Africa, and **Bingwen Yan** ([yanb@cput.ac.za](mailto:yanb@cput.ac.za)), Symphony Way, Bellville, Cape Town, 7535, South Africa. *Ordinal Regression to Analyze Postgraduate Students' Attitudes toward the Application of Statistical Procedures in the Western Cape Institutions*.

The ordinal regression method was used to model the relationship between the different levels of students' ability regarding the overall learning experience in the application of statistical procedures in both University of Cape Town (UCT) and University of the Western Cape (UWC), and the explanatory variables concerning demographics, emotions, students' attitude and students learning environment in these institutions. The outcome variable for students' attitudes was measured on an ordered, categorical six-point Likert scale. The major decisions involved in the model building for ordinal regression were deciding which explanatory variables should be included in the model and choosing the link function (e.g. logit link, probit link, cauchit link, negative log-log link and complementary log-log link) that demonstrated the model appropriateness. In addition, the model fitting statistics, the accuracy of the classification results and the validity of the model assumptions, e.g., parallel lines, were essentially assessed for selecting the best model. One of our main finding is that suitable environment for development of student capabilities to learn the skills of solving real life problems are highly significant with the application of statistical procedures. (Received September 26, 2017)

1135-VA-2452 **Jim Fowler** ([fowler@math.osu.edu](mailto:fowler@math.osu.edu)) and **Bart Snapp\*** ([snapp.14@osu.edu](mailto:snapp.14@osu.edu)). *Ximera: Measuring the effectiveness of an open-source interactive textbook*.

Ohio State's Department of Mathematics has begun a long-term project to improve student success and STEM retention. The mathematics department has undertaken multiple interventions (active learning, flipped lectures, open source textbooks), the effects of which are being viewed through many lenses (affective surveys, conceptual pre- and post-tests, and online event logs). This talk focuses on the online, open content that we have built, the platform we designed to deploy that content, and the data gathered through that online platform. (Received September 26, 2017)

1135-VA-2520 **Heather Z Brooks** ([heather@math.utah.edu](mailto:heather@math.utah.edu)), 155 S 1400 E Rm. 233, Salt Lake City, UT 84112, and **Anna Romanov\*** ([romanova@math.utah.edu](mailto:romanova@math.utah.edu)), 155 S 1400 E Rm. 233, Salt Lake City, UT 84112. *Formative assessment as a mechanism to improve calculus persistence through increased STEM major utility perception*. Preliminary report.

We are developing and researching a series of formative assessment tools to increase persistence in the engineering calculus sequence at the University of Utah. Our interventions were inspired by an assessment intervention study of high-risk freshmen biology students at Xavier University. Their study found that data-driven assessment techniques, such as exam mastery reports, were successful in improving student retention, particularly for academically at-risk students. With our formative assessment interventions, which include detailed expected learning outcome lists coupled with practice exam problems and subsequent exam mastery reports, we hope to address the challenges of persistence and gender in these courses. We discuss how these assessments target a variety of persistence challenges by altering students' utility beliefs, that is, by positively shifting the perceived utility and benefit of a STEM degree. We seek to achieve this by encouraging a growth mindset and decoupling mastery and ability from traditional summative assessment metrics. (Received September 26, 2017)

1135-VA-2685 **Erin E. M. Rizzie\*** ([erin.rizzie@uconn.edu](mailto:erin.rizzie@uconn.edu)), 341 Mansfield Rd. U1009, Storrs, CT 06269-1009. *Using specifications grading in a five-week summer calculus I course*. Preliminary report.

Specifications grading is a system of grading put forth by Linda B. Nilson in her 2015 book *Specifications Grading: Restoring Rigor, Motivating Students, and Saving Faculty Time*. Nilson's book gives some broad guidelines in order to allow for applications in any field: specifications grading involves grading individual assignments on a pass/fail basis, while giving students a clear framework of specifications for passable work and a precise explanation of how those passes and fails will be translated into a final letter grade.

In the summer of 2017, I used specifications grading for my five-week Calculus I course. I will discuss the system that I chose to implement, including specifications for passable work and scheme for final letter grades, as well as how it was received by students and how learning outcomes were affected. (Received September 26, 2017)

1135-VA-2896 **Perry Y.C. Lee\*** ([plee@kutztown.edu](mailto:plee@kutztown.edu)), Lytle Hall 267, Kutztown University of Pennsylvania, Kutztown, PA 19530, and **Padraig M. McLoughlin** ([mcclough1@kutztown.edu](mailto:mcclough1@kutztown.edu)), Lytle Hall 265, Kutztown University of Pennsylvania, Kutztown, PA 19530. *Assessment of An On-Going Flip-IBL Study: Comparison of Traditional and F/IBL (Flipped and Inquiry-Based Learning) for College Algebra Classrooms*. Preliminary report.

During the past academic years (2013 – to present), an assessment study to obtain student-learned outcome data to multi-sections of College Algebra for 'large' and 'small' classrooms was conducted. The lead author incorporated the Flipped (or inverted) and the Inquiry Based Learning (IBL) approaches (or the F/IBL method) into his 'large' and 'small' classrooms to engage students in the classroom. During the most recent three semesters (Fall 2016, Spring 2017 and Fall 2017), student assessment data was collected using a different assessment instrument for his 'small' College Algebra classrooms compared to the 'large' classroom.

The instrument of choice for the 'small' classroom assessment study will be briefly discussed, and the summary of student-learned outcome data from the most recent three semesters of College Algebra classrooms will be presented. (Received September 26, 2017)

## General Session on History or Philosophy of Mathematics

1135-VB-134 **Amy Ackerberg-Hastings\*** ([aackerbe@verizon.net](mailto:aackerbe@verizon.net)). *Charles Davies as a Philosopher of Mathematics Education*. Preliminary report.

Charles Davies (1798–1876), who taught at West Point, Hartford's Trinity College, New York University, and Columbia, was one of the most prolific and popular 19th-century compilers of mathematics textbooks in the United States. In addition, in 1850 he published *The Logic and Utility of Mathematics, With the Best Methods of Instruction Explained and Illustrated*, which James K. Bidwell and Robert G. Clason (1970) and Phillip S. Jones and Arthur F. Coxford, Jr., (1970) called the "first American book on mathematics teaching methods." I will provide an overview of the contents of this volume and consider whether it imparts any messages relevant to 21st-century mathematics educators. (Received August 01, 2017)



1135-VB-279 **Shigeru Masuda\*** ([hj9s-msd@asahi-net.or.jp](mailto:hj9s-msd@asahi-net.or.jp)), Heights Esperanza, Room # 305, 100-24 Nishida-Cho, Jodoji, Sakyo-Ku, Kyoto, Kyoto-Fu 606-8417. *The Integral methods of the equations of the partial differential in the mathematical physics by Poisson.*

We discuss integrals of the equations of the partial differential in the mathematical physics, in which Poisson proposes the methods to solve the problems in integration of equation of partial differential. Poisson proposes his academic paradigm of mathematical physics, in which he discusses the essential theories in “A Study of Mathematical Physics.”

We show the following two digressions in the above last two books

- integration of equation of partial differential in the digression of problems of mechanics
- that in the digression of problems of heat.

He postulates at first the linear equation  $L=0$ , and the general solution expressed by the series,

$$u = P\theta^\alpha + Q\theta^\beta + R\theta^\gamma + \text{etc. ;}$$

the coefficients  $P, Q, R$ , etc., will be the functions of  $t$ , and the exponents  $\alpha, \beta, \gamma$ , etc.,  $u$  is expressed various types in accordance with the problems, for example, the equation very simple, linear and at partial differentials of the second order,

$$\frac{du}{dt} = a^2 \frac{d^2u}{dx^2}. \quad (1)$$

He will determine these unknowns in each case. (Received August 25, 2017)

1135-VB-748 **Patricia Baggett\*** ([pbaggett@nmsu.edu](mailto:pbaggett@nmsu.edu)), Department of Mathematical Sciences, MSC 3MB P.O. Box 30001, New Mexico State University, Las Cruces, NM 88003-8001, and **Andrzej Ehrenfeucht**, Computer Science Department, University of Colorado, Box 430, Boulder, CO 80309-0430. *Comparing two first-year algebra books from the 1840's, Warren Colburn's "An Introduction to Algebra" and Joseph Ray's "Algebra: Part First", to today's "modern" first-year algebra curriculum.*

Two American textbook authors, Warren Colburn (1793-1833) and Joseph Ray (1807-1855), wrote radically different first-year algebra books, and at the time, both books were praised in education circles. Colburn's book stopped being used soon after it was published, but Ray's book is still used today by some home-schoolers. In this talk we will compare the contents of the two books by pointing out differences in definitions, formulas, proofs, and examples. We will also compare their contents to the content of today's first-year algebra curriculum recommended by the Common Core State Mathematics Standards. (Received September 13, 2017)

1135-VB-1191 **Samuel M Hansen\*** ([jmm@acmescience.com](mailto:jmm@acmescience.com)), 1114 S Park St #4, Madison, WI 53715. *Word Problems from the Mid-1800s to Now: A Textbook Survey.* Preliminary report.

The University of Wisconsin-Madison Kleene mathematics library has a large collection of historic textbooks, many from as far back as the mid-1800s. In this presentation I will show examples of word problems from the historic and modern textbooks to illustrate how the language and content of word problems has changed over time, as well as how often it has really stayed the same. For while it is unlikely modern textbooks would refer to how one fifth of a general's army was disabled by wounds as one problem did in a textbook from 1892, the next question was about how much money a person had left after shopping which would not look out of place at all today. (Received September 20, 2017)

1135-VB-2083 **James P. Howard, II\*** ([james.howard@jhu.edu](mailto:james.howard@jhu.edu)). *Two Fields Separated by a Common Language.* Preliminary report.

This talk will explore the parallel development of mathematics and statistics as fields. Further, it will show how the language of mathematics permeates statistics. Next, this talk will provide a comparison to physics as field that uses mathematics as a toolkit, but not as a core experience. Finally, this talk will argue that statistics cannot be considered a branch of mathematics and is instead an independent field along with the implications of such a decision on pedagogy. (Received September 25, 2017)

1135-VB-2327 **Clarissa McMillar\*** ([mcmillar\\_c06478@utpb.edu](mailto:mcmillar_c06478@utpb.edu)). *The Mathematics of Eastern Architecture.* Preliminary report.

Egypt's pyramids are often thought of as the single most computationally complex structures of the ancient world. Various studies have been conducted to discuss how the Egyptians managed such feats, but the impressive structures of China are not usually discussed in the same breath. The East's history is far reaching and a concentrated, mathematical study of their structures will reveal the state of mathematics during that time period as well as its distinguishing characteristics. Early architects were mathematicians who used their exceptional understanding of geometry to plan for events as catastrophic as earthquakes. Many of these structures have

stood the test of time. Equally as interesting as the Egyptian pyramids, there will be a lot to learn. (Received September 25, 2017)

1135-VB-2630 **Mariah Birgen\*** ([mariah.birgen@wartburg.edu](mailto:mariah.birgen@wartburg.edu)), 100 Wartburg Blvd., Waverly, IA 50677, and **Jeannie Dees** ([jeannie.dees@wartburg.edu](mailto:jeannie.dees@wartburg.edu)), 100 Wartburg Blvd., Box 0519, Waverly, IA 50677. *Learning the History of Mathematics in the British Isles: A Travel Course.*

There is a great deal of information that can be learned about any particular topic by reading books in the field. However, if you really want to learn something it is more effective to participate in active research. The Historical Roots of Math and Physics in the British Isles is a course taught at Wartburg College (Iowa) that takes students to the same towns and universities that were occupied by the famous practitioners. Students learn about Hershel's astronomy in the townhouse where he did the observations. Students read Hamilton's workbooks on calculus in Dublin. Students experience the working replica of the Colossus used to decrypt the Lorenz cipher at Bletchley Park.

This talk will cover some of the most accessible and affordable places to learn about the history of math and its cousins in England, Ireland, and Scotland. If time permits, you will also learn about some of our mistakes from which you are welcome to learn. (Received September 26, 2017)

1135-VB-2908 **Alexander G. Atwood\*** ([atwooda@sunysuffolk.edu](mailto:atwooda@sunysuffolk.edu)), Department of Mathematics, Suffolk County Community College, 533 College Road, Selden, NY 11784. *The Future Impact of Artificial Intelligence on College Mathematics Education.*

Artificial Intelligence has become increasingly powerful in the past ten years. New techniques such as Deep Learning Networks have been successfully implemented to make meaningful progress in difficult problems in medical diagnoses, in game playing (such as the games of Go and Poker), and in the emerging area of autonomous vehicles. Artificial Intelligence also has the real potential of transforming the workplace by powerfully augmenting human performance. In 2013, Carl Benedikt Frey and Michael Osborne, of the University of Oxford, examined the probability of computerization for 702 occupations and found that 47% of workers in America had jobs at high risk of potential automation in the next 10 to 20 years. What should we be teaching in our math courses if Artificial Intelligence will radically change the nature of employment? What skills will our students need to navigate a world in which many jobs may be transformed or even disappear because of Artificial Intelligence? How will increasingly powerful Artificial Intelligence systems change the way in which mathematics is taught in colleges? Several case studies of the impact of Artificial Intelligence systems in various professions will be presented, and a possible future of mathematics education will be envisioned. (Received September 26, 2017)

1135-VB-3041 **Deepak Basyal\*** ([deepak.basyal@uwc.edu](mailto:deepak.basyal@uwc.edu)), 400 University Drive, West Bend, WI 53095. *Singing sines in Sanskrit slokas.*

Pancha-Siddhantika (The five astronomical cannon), a book on mathematical astronomy written by 6th-century Indian astronomer and mathematician Varahamihira, lists trigonometric sine values in sloka form. In this presentation, I will recite a few of these mesmerizing Sanskrit verses written in arya meter. I will discuss how these verses were used as a stimulating tool in my trigonometry class. (Received September 26, 2017)

## General Session on Interdisciplinary Topics in Mathematics

1135-VC-219 **Rebecca Metcalf\*** ([rmetcalf@bridgew.edu](mailto:rmetcalf@bridgew.edu)) and **Sandra Ciocci.** *Math Language: A First Look at Understanding the Complexities in Elementary Mathematics Curriculum.* Preliminary report.

Mathematics educators maintain math language must be precise (accurate and consistent) and some assert that learning math language can be equated with learning a second language, implying a language that is not learned in the home, but at school. Specifically, it contains three distinct language elements: Symbolic language, content vocabulary, and academic language. To understand mathematics at a deeper conceptual level, students need to develop a strong understanding of these three elements. This is particularly true of pre-service elementary teachers. This session will focus on collaboration with a faculty member in the field of communication sciences and disorders and the subsequent language analysis of mathematics for elementary teachers' textbooks. Topics will include discussion of a rubric designed to analyze mathematical language, symbolism, and visual representations; findings from the analysis of top-selling textbooks in elementary mathematics; and implications for pedagogy. (Received August 12, 2017)

1135-VC-1702 **Feng Fu\***, 27 N. Main Street, Hanover, NH 03755. *Mathematical models of combination cancer immunotherapy based on adoptive cell transfer*. Preliminary report.

Recent continued breakthroughs in antibody research and advances in T-cell engineering techniques have begun to unleash the curative potential of cancer immunotherapy. Here our model takes into account two means of immunotherapy. First, administration of monoclonal antibodies with high specificity, which will result in an increase in the ability of immune cells to detect and eliminate cancer cells – its killing rate or efficacy. Second, adoptive immune cell transfer, characterized by the infusion of in-vitro engineered and personalized immune cells into patients. These two novel treatment methods can be combined to take advantage of the bistability phenomenon in cancer, and may be applied in concert as combination immunotherapy. We provide a quantitative mathematical framework to provide practical guidance for clinical assessment of immunotherapy. (Received September 24, 2017)

1135-VC-1723 **Kevin Gerstle\*** (kgerstle@oberlin.edu) and **Miranda Schaum**. *Studying Harmonic Measure through Brownian Motion Simulation and Teleportation*. Preliminary report.

Harmonic measure distribution functions describe the harmonic measure of portions of the boundaries of two-dimensional domains and give information about the geometry of these domains. Such functions can equivalently be defined by the probability a Brownian particle starting inside the domain will first hit the boundary of the domain within distance  $r$  from its starting point. In this talk, we will describe the process for simulating this two-dimensional Brownian motion and will discuss how “teleportation” of Brownian particles may be used to expedite these simulations by taking large random steps rather than small ones. This talk is based on joint work with M. Schaum. (Received September 24, 2017)

1135-VC-1895 **Benjamin J. Braun** and **Wesley K. Hough\*** (houghw@uw.edu), 800 West Main Street, UW-Whitewater, Mathematics Department, LT2218, Whitewater, WI 53190. *Discrete Morse Theory and Poset Homomorphism Complexes*. Preliminary report.

The main idea of discrete Morse theory is to pair cells in a cellular complex in a manner that permits cancellation via elementary collapses, reducing the complex under consideration to a homotopy-equivalent complex with fewer cells. In this talk, we introduce the notion of a homomorphism complex for partially ordered sets, placing particular emphasis on maps between chain posets and the Boolean algebras. We define an iterative discrete Morse matching for these Boolean complexes and provide formulas for enumerating the number of critical cells arising from this matching as well as for the Euler characteristic. We end with a conjecture on the optimality of our matching derived from connections to 3-equal manifolds. (Received September 25, 2017)

1135-VC-1901 **Immanuel James Williams\*** (jamesijw23@gmail.com), 1156 High St, Santa Cruz, CA 95064. *A Speededness Item Response Model for Associating Ability And Speededness Parameters*.

Test speededness is defined as the failure to attempt all items on an assessment within a specified time frame which is an issue known to undermine assessments (Bejar, 1985). However, an assumption about speededness that is often over-looked in the literature is the relationship between speededness and ability of the examinee in the context of IRT modeling. Previous studies have used modified IRT models to reduce test speededness, but none have evaluated the effect of neglecting the association between speededness and ability. The primary purpose is to examine the impact of ignoring the association between ability and speededness on parameter estimation and to investigate the robustness of the proposed model under conditions when speededness and ability are independent. The Markov Chain Monte Carlo (MCMC) Metropolis Hastings algorithm was implemented to estimate model parameters using C++ and R. The results showed that ignoring the association between ability and speededness does not impact the recovery of the IRT model parameters. In summary, this work allows researchers to further understand the impact of speededness and its association with ability in a variety of conditions. (Received September 25, 2017)

1135-VC-2354 **Sierra Nicole Murphy\*** (sierra.n.murphy@asu.edu). *A New Lens for Prostate Cancer Modeling: Cholesterol's Role in Predicting a De-Differentiating Tumor*. Preliminary report.

Though initially androgen deprivation therapy effectively treats prostate cancer, the tumor eventually becomes resistant. One mechanisms of resistance, the hypersensitivity pathway, bypasses therapy by producing androgen locally from precursors, such as cholesterol. The correlation between high cholesterol and higher incidence of prostate cancer, the increase of ACTH during therapy, and the lower risk of prostate cancer for patients on statins, all support this mechanism of resistance. However, current mathematical models of prostate cancer do not consider cholesterol. Thus, including cholesterol, and related factors, contributes to a new class of mathematical models that are more biologically relevant. This brings the mathematical community closer to

accurately modeling the dynamics of cancer and predicting when the tumor escapes treatment. (Received September 26, 2017)

1135-VC-2693 **Mohamed Allali\*** ([allali@chapman.edu](mailto:allali@chapman.edu)). *Image Analysis Using Mathematical Morphology.*

Mathematical morphology is a theory and technique for the analysis and processing of geometrical structures. This theory can be developed in many different ways. In this talk, through adopting one standard method which uses operations on sets of points, I will show through some examples how morphology and digital images can be incorporated into some mathematics courses. Images are the most effective medium of human communication and, when processed under the control of students and teachers, they put mathematical ideas in an exciting new light. (Received September 26, 2017)

1135-VC-2870 **Nigar Karimli\*** ([nigar.karimli042@topper.wku.edu](mailto:nigar.karimli042@topper.wku.edu)), **Ayush Prasad** and **Richard Schugart**. *Identifying Optimal Sampling Distributions for Individual Patients.*

Because the medical treatment of diabetic foot ulcers remains a challenge for clinicians, a quantitative approach using patient data and mathematical modeling can help researchers understand the physiology of the wounds. In this work, the modified version of the previously developed mathematical model defining the interactions among matrix metalloproteinases, their inhibitors, extracellular matrix, and fibroblasts in the healing process of a diabetic foot ulcer is used (Krishna et al., 2015). We estimate parameter values using ordinary least-squares for the model curve-fitted to individual patient data from Muller et al. (2008). However, these model parameters can be estimated more efficiently and accurately by implementing an optimal design method that calculates optimal observation times for collecting clinical data. We introduce an SE-optimal design (standard-error optimal-design) by using a Fisher Information Matrix (FIM) to determine the optimal time evolution of sensitivity values. The goal of this work is to quantify and understand differences between patients to predict future responses and individualize treatment for each patient. Moreover, additional results using various parameter estimation techniques will also be presented. (Received September 26, 2017)

1135-VC-3130 **Colleen Duffy\*** ([duffycm@uwec.edu](mailto:duffycm@uwec.edu)) and **Paul Thomas**. *Relativity and Differential Geometry: an interdisciplinary course in England.* Preliminary report.

Mathematics and physics are inherently connected and strongly influenced each other as they developed historically. However, we often teach mathematics, physics, and history separately. We are developing an interdisciplinary course on special and general relativity and the related mathematics, incorporating a cultural and historical perspective. The course will be taught abroad in England, allowing field trips to sites such as Isaac Newton's home, Cambridge and Oxford Universities, Greenwich, and Jodrell Bank Observatory. The aim is for students to experience the connectedness of mathematics, physics, culture, and history through this course. (Received September 26, 2017)

## *General Session on Mathematics and Technology*

1135-VD-898 **John C.D. Diamantopoulos\*** ([diamantj@nsuok.edu](mailto:diamantj@nsuok.edu)), 14854 N. Trent St., Tahlequah, OK 74464. *Proofs without words...animated-gif style!*

There have been incredible "proofs without words" efforts over the years. I will endeavor to show some proofs/explanations of famous results, using home-brewed animated gifs. I'll also spend some time explaining how I created these animations, for those interested in creating similar animations for proofs of their own to use in their teaching, etc. (Received September 16, 2017)

1135-VD-1566 **Betty Love\*** ([blove@unomaha.edu](mailto:blove@unomaha.edu)), Mathematics Department, University of Nebraska - Omaha, Omaha, NE 68182, and **Victor Winter, Michael Matthews** and **Michelle Friend**. *Using Bricklayer to fuse mathematical thinking, computational thinking, and art.*

Bricklayer is a tech-centric pedagogical tool suite where visual art provides the domain in which mathematical and computational thinking can be taught in ways that are engaging as well as technically meaningful. Bricklayer programs, written in the functional programming language SML, can produce LEGO artifacts, Minecraft artifacts, and artifacts suitable for 3D printing. Effective use of computers ultimately rests on one's ability to understand computational sequences in terms of their underlying patterns. Given this perspective, programming is the art of structuring computations to expose patterns in ways that can be leveraged by the machine. Visual domains, such as that of Bricklayer, provide tremendous cognitive opportunities for pattern recognition. The discrete nature of the Bricklayer domain provides a setting in which various classes of patterns, including

arithmetic, geometric, and evolutionary, can be understood through abstractions found in algebra as well as discrete mathematics. The abstractions of functional programming languages are closely related to mathematical functions. Therefore, functional programming languages facilitate transfer between algebra and programming because they rely on the same underlying conceptual model as functions in algebra. (Received September 23, 2017)

1135-VD-2475 **Marianna Bonanome\*** ([mbonanome@citytech.cuny.edu](mailto:mbonanome@citytech.cuny.edu)), Massapequa, NY. *Re-imagining STEM gateway courses and the faculty seminar developed to support them.* Preliminary report.

“Opening Gateways to Completion: Open Digital Pedagogies for Student Success in STEM” is a large, cross-institutional collaboration between the mathematics departments at the New York City College of Technology, CUNY and the Borough of Manhattan Community College, CUNY aimed at supporting student success in gateway math courses (supported by the U.S. DOE). We discuss how this project has introduced open-source digital technologies, open educational resources, and active learning strategies into the sequence of high-enrollment mathematics courses required for STEM disciplines at each college. The creation of an intensive seminar for faculty participants offered by the Opening Gateways team in the fall 2016 semester, followed by the implementation of a class structure for a gateway Algebra and Trigonometry course which includes active learning pedagogies, the utilization of WeBWorK (an open-source, online homework system) and OpenLab (City Tech’s online platform) is discussed.” (Received September 26, 2017)

1135-VD-3023 **Katrina Morgan\*** ([katri@live.unc.edu](mailto:katri@live.unc.edu)) and **Francesca Bernardi.** *Engaging Girls Through Math Media: Using Technology to Increase Engagement, Teach Scientific Communication, and Maximize Impact in a High School Outreach Program.* Preliminary report.

Students today are part of an active internet culture where blogs and podcasts are popular media outlets. With this in mind, the program design of Girls Talk Math (a free summer day camp for high schoolers who identify as girls interested in math held at the University of North Carolina at Chapel Hill) incorporates media projects in which campers write blog posts and record podcasts. During camp, participants break into 8 groups of 4-5 campers. Each group completes a lengthy problem set, researches the life and work of a female mathematician, writes a blog post about the problems they solved, and records a podcast about the mathematician they researched. The incorporation of blogs and podcasts serves multiple goals:

- Increasing confidence by providing a chance for participants to make their voices heard and publishing their work online
- Engaging students by incorporating media that is relevant to their lives
- Developing scientific communication skills by having campers summarize what they learned in a forum available to a broad audience
- Maximizing program impact by increasing the number of people who are able to learn about the rich history of women in mathematics.

Analysis of pre- and post- survey data will be presented. (Received September 26, 2017)

1135-VD-3054 **Philip B. Yasskin\*** ([yasskin@math.tamu.edu](mailto:yasskin@math.tamu.edu)), Department of Mathematics, Texas A&M University, 3368 TAMU, College Station, TX 77843-3368, and **Andrew Crenwelge, Joseph Martinsen, Matthew Weihing and Matthew Barry.** *Interactive Animations in MYMathApps Calculus.* Preliminary report.

MYMathApps Calculus is an online text for calculus 2 and 3, with more topics planned.

- The text includes many animations created using Maple in addition to the standard static graphics. Interactive animations are being created using Three.js, Javascript MathLex and Sage.
- Examples in the text have full solutions. Exercises in the text have hints, answers and solutions which gradually appear.
- There are internal and external links for additional information, like proofs, that are available to those students who wish to learn more.
- Exercise pages have links back to the pages where the material for each exercise is discussed.
- Many topics have randomly generated tutorials, animations and exercises based on the Maplets for Calculus.

MYMathApps has grown out of the WebCalc project involving Yasskin, Don Allen and Mike Stecher at Texas A&M University, and the Maplets for Calculus project involving Yasskin and Doug Meade at the University of South Carolina. The text is being written using modern web technologies so it is available on all devices. The work is supported in part by NSF DUE TUES-2 Grant 1123255. (Received September 26, 2017)

1135-VD-3218 **William C Bauldry\*** ([bauldrywc@appstate.edu](mailto:bauldrywc@appstate.edu)), Dept of Mathematical Sciences, Appalachian State University, Boone, NC 28608, and **Michael J Bossé** ([bossemj@appstate.edu](mailto:bossemj@appstate.edu)), NC. *Complex Roots of Real Polynomials & Rational Functions and Dynamic Graphics*. Preliminary report.

We investigate determining complex roots of polynomials and rational functions from geometric features of their graphs. We also show a formula we devised that gives the roots of  $q(x)$ , a quartic polynomial, based on three values  $q(0)$  and  $q(\pm x_0)$  for any nonzero  $x_0$ . (Received September 27, 2017)

## General Session on Mentoring

1135-VE-55 **Ronald L Merritt\*** ([ronald.merritt@athens.edu](mailto:ronald.merritt@athens.edu)), 300 North Beaty Street, Athens, AL 35611. *The Effectiveness of a Mentoring Program at a Small Liberal Arts University*.

In the fall of 2012 Athens State University established an annual mentoring program which included formative orientation sessions, opportunities for the new faculty cohort to discuss issues relevant to new instructional faculty at the university during their first year of employment, and support for the mentor-mentee relationship. While many higher education communities realize the need for mentoring new faculty, particularly in mathematics and other STEM disciplines, Athens State University realizes the need to mentor veteran educators new to the university and those who have been employed in vocations not part of the professoriate. The fundamental purpose of the six orientation sessions over the two-semester duration is to acquaint new faculty with representatives of departments or committees considered influential on university faculty. In addition to the representatives' presentations, new faculty members engage in conversations central to the presentation theme. Near the end of the spring semester for each academic year, the participants complete a survey, the results of which are carefully considered by the coordinator of the program, disseminated to the provost and deans, and the program is adjusted to meet the needs of future participants. (Received July 10, 2017)

1135-VE-154 **Thomas R Hagedorn\*** ([hagedorn@tcnj.edu](mailto:hagedorn@tcnj.edu)) and **Monisha S Pulimood** ([pulimood@tcnj.edu](mailto:pulimood@tcnj.edu)). *Mentoring Students through Computational Science Research Projects: Report on the iPics S-STEM grant program*. Preliminary report.

The Departments of Mathematics and Statistics, and Computer Science at The College of New Jersey are in the fourth year of a NSF S-STEM scholarship grant. Our program recruits and supports mathematics and computer science majors with economic need through a combination of computational science projects, advising, and community building activities. This talk will report on our results to date. (Received August 04, 2017)

1135-VE-2408 **Fumiko Futamura\*** ([futamurf@southwestern.edu](mailto:futamurf@southwestern.edu)), 1001 E University Ave, Georgetown, TX 78626, and **Alison Marr**. *EQUIPping freshmen STEM majors through the EQUIP program*.

This talk describes the EQUIP program, started at Southwestern University in 2016. EQUIP (Embracing Quantitative Understanding and the Inquiry Process) introduces a diverse group of 20 incoming freshmen with interest in the STEM fields to college life, various opportunities for STEM majors at SU and Precalculus through inquiry based learning to prepare them for Calculus I, which they take together with one of the EQUIP professors. It begins for a week on campus over the summer, through the Fall as a 1 credit course and informally throughout their academic career. We describe the successes of the program, including involving EQUIP students in publishable math research, as well as improvements made for 2017. (Received September 26, 2017)

1135-VE-2565 **Emily Gismervig\*** ([emilyg27@uw.edu](mailto:emilyg27@uw.edu)), **Cinnamon Hillyard** ([ch7@uw.edu](mailto:ch7@uw.edu)) and **Kathryn Mitchell** ([kmm1101@uw.edu](mailto:kmm1101@uw.edu)). *A Peer Facilitation Model in Precalculus for Increasing Participation in STEM Fields*. Preliminary report.

The entering freshman class at UW Bothell is very diverse: 49% of first year students will be the first in their families to earn a college degree, and 70% are from diverse backgrounds. Additionally, 45% of entering students indicate an interest in pursuing a STEM degree. Many of these students, however, are unable to successfully progress through the math prerequisites needed to be admitted into a STEM degree program. To increase access to STEM degrees, we are starting to require all students taking precalculus to register for a peer-led support course. This course has three main components. First, math content is covered in parallel with the

main precalculus course. Second, the peer facilitators also cover non-math topics like how to study for exams, how to get help with coursework, learning mindsets, and how to choose and apply for a major. Third, the precalculus faculty, peer facilitators and an academic advisor work as a team to identify at risk students and provide additional support. We will present preliminary data on the effectiveness of this program. (Received September 26, 2017)

1135-VE-2717 **Elizabeth W Schott\*** ([eschott@fsw.edu](mailto:eschott@fsw.edu)), 8099 College Parkway, Ft. Myers, FL 33919, and **Laurice L Garrett** ([lgarrett@fsw.edu](mailto:lgarrett@fsw.edu)), 8099 College Parkway, Ft. Myers, FL 33919. *The Successes and Challenges of Creating a Community of Best Practices for Math Faculty.*

Over the last decade the faculty members of the math department of Florida SouthWestern State College have explored methods to foster a collaborative community for investigating the best techniques for teaching mathematics. This talk will look at the process of establishing a formal community of best practices in our department. We will also look at the challenges of incorporating the community within the professional development activities of the college, the challenges of keeping it “fresh,” and the challenges of incorporating the entire full-time faculty & adjuncts across multiple campuses. Finally, we will share our philosophies of scheduling topics and the “highs and lows” of our past meetings. (Received September 26, 2017)

1135-VE-3024 **Francesca Bernardi\*** ([bernardi@live.unc.edu](mailto:bernardi@live.unc.edu)) and **Katrina Morgan** ([katri@live.unc.edu](mailto:katri@live.unc.edu)). *Association for Women in Mathematics Mentoring Network - Supporting Female Mathematics Majors throughout their Undergraduate Career.* Preliminary report.

The Association for Women in Mathematics chapter at the University of North Carolina at Chapel Hill organizes a *Mentoring Network*, pairing graduate student mentors with undergraduate student mentees. The program has several goals, including helping participants navigate undergraduate education, guiding students through the transition to graduate school or the job market, providing undergraduates with role models, and fostering a diverse and supportive community in the department. The network is set up after a *Speed Mentoring* event where undergraduate students are invited to meet the available mentors and briefly chat with each one of them. Different students have different criteria for choosing a mentor, and by allowing them to meet the mentors beforehand, we try to accommodate everyone’s preference. Some undergraduate students have found sharing a mentor with another mentee to be beneficial: this helps create a community of peers even among students at different stages in their undergraduate education. Both male and female mentors are part of the network, while so far all mentees have been female. Analysis of feedback from last year’s network will be presented. (Received September 26, 2017)

1135-VE-3057 **James S Rolf** ([jim.rolf@yale.edu](mailto:jim.rolf@yale.edu)) and **John Hall\*** ([john.hall@yale.edu](mailto:john.hall@yale.edu)). *Using Undergraduate Coach/Mentors in an Online Bridge Program for QR and STEM Preparation.* Preliminary report.

Online Experiences for Yale Scholars (ONEXYS) is an online bridge program for incoming students who are bright, but could benefit from additional preparation for QR and STEM courses during the summer before their first year. This past summer we had 210 participants with 59% women, 50% from an underrepresented group, and 33% first generation students. In order to most effectively help students prepare for QR and STEM experiences at Yale, we hired 45 undergraduate coach/mentors whom we thought could best 1) facilitate collaborative learning in this online environment, 2) support the transition from high school to college and 3) serve as a mentor beyond the summer experience. Our coaches have a significant impact on both student learning and the motivation to learn. We discuss the hiring, training and support of our coach/mentors as well as data about student learning over the summer, feelings of preparedness, and the downstream impact of ONEXYS. (Received September 26, 2017)

## *General Session on Modeling and Applications*

1135-VF-639 **Esther R. Widiasih\*** ([widiasih@hawaii.edu](mailto:widiasih@hawaii.edu)), UHWO, 91-1001 Farrington Highway, E-217, Kapolei, HI 96815. *Mathematical modeling of climate change.* Preliminary report.

Undoubtedly, Earth’s climate is a complex system, and today it is going through some major changes. Can one understand climate change through the lens of simple math models, accessible to undergraduate students? If so, what do these simple models tell us about the climate? In this talk, I will introduce the *Budyko* zonal energy balance model. I will then illustrate its potential applications, ranging from studies of coral to permafrost, under

the duress of temperature rise. These applications could lead to either classroom materials or undergraduate student research. (Received September 11, 2017)

1135-VF-647 **Nikhil Krishna\*** (nikhil.krishna798@topper.wku.edu) and **Arjun Kanthawar** (arjun.kanthawar248@topper.wku.edu). *Accurately Modeling the Healing Process of Chronic Wounds*. Preliminary report.

In order to formulate a mathematical model that accurately represents the physiology of a wound, the model and its parameters must be identifiable when given actual data. Practical identifiability is a method used to determine whether parameters in a model can be uniquely determined given actual data. This work uses a differential equation model that describes the interactions among matrix metalloproteinases, their inhibitors, the extracellular matrix, and fibroblasts (Krishna et al., 2015). A singular value decomposition technique with a QR factorization combined with a correlation analysis is used to find an identifiable subset of parameters. Subsets are analyzed through model prediction intervals and parameter Markov chains and posterior densities. The goal of this work is to formulate a model that can accurately predict the healing process for individual patients. (Received September 11, 2017)

1135-VF-843 **Hansapani S Rodrigo\*** (hansapani.rodrigo@utrgv.edu) and **Chris P Tsokos** (ctsokos@usf.edu). *Bayesian Artificial Intelligence Neural Networks for Nonlinear Poisson Regression and Survival Modeling*.

With the inspiration originated from biological neuron system, Artificial neural networks (ANN) models are efficiently used for nonlinear modeling. It has been shown that the Bayesian treatment of the ANN provides better prediction accuracies in regression modeling as it avoids the network overfitting associated with maximum likelihood approach. Moreover, Bayesian treatment can be used to identify the relative importance of predictor variables and to determine the effective model complexity utilizing the limited amount data in hand. By incorporating these Bayesian treatments, we have developed a novel nonlinear Poisson regression model using ANN assuming that the log of the expected value of the count responses is nonlinearly related with the predictors. The prediction accuracy of our proposed Poisson regression model has been evaluated using a simulation study. We have utilized this to obtain the survival prediction of lung cancer patients. This was achieved by extending our ANN model to create a piecewise constant hazard model. (Received September 15, 2017)

1135-VF-857 **Sara Shirinkam\*** (sara.shirinkam@utsa.edu), 8902 Cordes Junction, Helotes, TX 78023, and **Adel Alaeddini** and **Ngoc Mai Tran**. *Integrating the method of moments with numerical algebraic geometry and multicomplex Taylor series expansion for parameter estimation in large Gaussian mixture models*.

Gaussian Mixture Models (GMM) are among the most statistically mature methods for clustering and density estimation with numerous successful applications in science and engineering. In this study, we investigate an approach based on numerical algebraic geometry (NAG) and multicomplex Taylor series expansion (MCTSE) to enhance the Method of Moments (MM) for estimating parameters of large-scale GMMs. The proposed methodology uses MCTSE as a numerical method for calculating higher-order partial derivatives of the moment generating function of GMM. Next, it employs NAG for solving the resulting system of polynomial equations to estimate the parameters of GMM. We compare the performance of the proposed approach against popular Expectation Maximization (EM) method using extensive simulation. (Received September 15, 2017)

1135-VF-868 **Samantha J. Brozak\*** (sbrozak@asu.edu). *Modeling Nutrient-Plankton Reactions in Oceanic Chain Vortices*. Preliminary report.

The Agulhas system is a key component in global ocean thermohaline circulation and transports warm, briny water from the Indian Ocean to the Southern Atlantic Ocean via the shedding of eddies, known as Agulhas rings. Using direct numerical simulations, we model simple plankton interactions in simulated chain vortices based on Agulhas rings to investigate anomaly residency and dispersion times, stretching, and the dependence of reaction variability on initial conditions. (Received September 15, 2017)

1135-VF-941 **Srihita Mediboina\*** (srihitamediboina@gmail.com), 191 Hunyadi Avenue, Fairfield, CT 06824, and **Xiaodi Wang** (wangx@wcsu.edu), 310 Lexington Blvd, Bethel, CT 06801. *Wavelet based Option pricing algorithms using machine learning techniques*.

In this paper, we explore wavelet and machine learning based nonparametric methodologies for pricing call options. We first apply wavelet transform to remove noise from raw data. We then apply support vector regression as well as neural networks to predict call option prices. These methods, while being prominent in other fields of study, have not heavily been used for financial econometric applications. The accuracy of these methods are compared to the widely used Black Scholes Model. The empirical analysis has shown promising



results for nonparametric methodologies to further accuracy in accommodating for the stochastic volatility of financial markets. (Received September 17, 2017)

- 1135-VF-984 **Mykhaylo M Malakhov\*** ([mykhaylo@andrews.edu](mailto:mykhaylo@andrews.edu)), **Benjamin MacDonald**, **Shandelle M Henson** and **J M Cushing**. *Backward Bifurcations in a Periodic Matrix Model of Seabird Population Dynamics*. Preliminary report.

Rising sea surface temperatures (SSTs) are associated with food resource reductions for seabirds in the Pacific Northwest and have been correlated with a number of behavioral changes, including increased egg cannibalism and egg-laying synchrony. We study the effect of these changes on the long-term survival and dynamics of the population by considering a simplified, discrete-time proof-of-concept model that tracks the population across multiple breeding seasons. We show that cannibalism and synchrony can lead to backward bifurcations and strong Allee effects, allowing the population to survive at lower resource levels than would be possible otherwise. (Received September 18, 2017)

- 1135-VF-990 **Christiane Gallos** ([christiane@andrews.edu](mailto:christiane@andrews.edu)), **Dorothea Gallos\*** ([dorotheag@andrews.edu](mailto:dorotheag@andrews.edu)) and **Shandelle M. Henson** ([henson@andrews.edu](mailto:henson@andrews.edu)). *Bifurcations in an animal behavior model for egg-laying synchrony in a seabird colony*. Preliminary report.

Glaucous-winged gulls (*Larus glaucescens*) breed in a large colony on Protection Island, Washington, and are known to exhibit every-other-day egg-laying synchrony in dense areas of the colony. We present a discrete-time model of egg-laying behavior and use the Jury Conditions to find the stability criteria of the system as a function of the crowding factor. The system loses stability in a two-cycle bifurcation as the crowding factor increases beyond a critical value. We also explore the effects of synchrony in the presence of egg predation and show that synchrony can be advantageous for individuals. (Received September 18, 2017)

- 1135-VF-1052 **Benjamin MacDonald\*** ([bcmacdon@uvm.edu](mailto:bcmacdon@uvm.edu)), **Mykhaylo M. Malakhov**, **Shandelle M. Henson** and **J. M. Cushing**. *A Model of Population Dynamics and Behavior for Pacific Northwest Seabirds*.

Increases in sea surface temperatures (SSTs) in the Pacific Northwest of the US and Canada are associated with reductions in food resources, as well as marked behavioral changes in seabirds. Higher SSTs lead to an increase of egg cannibalism in gulls, which in turn promotes every-other-day egg-laying synchrony in the colony. We develop a simplified discrete-time model to study the population dynamics both within and between breeding seasons. The model allows us to examine the long-term dynamics and tipping points for the population as a function of SST. (Received September 18, 2017)

- 1135-VF-1107 **Elizabeth W. Fulton\*** ([elizabeth.fulton@montana.edu](mailto:elizabeth.fulton@montana.edu)), Department of Mathematical Sciences, Montana State University, P.O. Box 172400, Bozeman, MT 59717-2400. *Ways that the mathematical modeling cycle differs for different grade levels*.

Mathematical modeling is encouraged in grades K-12 by reports and standards such as CCSSM and the GAIMME report. The process of mathematical modeling is often described with various components, such as Make Assumptions and Define Variables. This talk will describe qualitative case study research that studied four elementary teachers and their implementation of mathematical modeling in classrooms. I will describe how many components of mathematical modeling were present in elementary classrooms, but how some components differed from description in the literature. I propose that young students ask clarification questions, which is related to, but different from, making assumptions and defining variables. (Received September 19, 2017)

- 1135-VF-1546 **Chris McCarthy\***, 199 Chambers Street, New York, NY. *Modeling Adsorption Based Filters: 1 Dimensional Filter Equation (Bio-remediation of Heavy Metal Contaminated Water)*.

I will discuss kinetic models of adsorption, as well as one dimensional models of such filters. These mathematical models have been developed in support of our interdisciplinary lab group. Our group conducts research into bio-remediation of heavy metal contaminated water via filtration. The filters are constructed out of biomass, such as spent tea leaves. The spent tea leaves are available in large quantities as a result of the industrial production of tea beverages. The heavy metals bond with the surfaces of the tea leaves (adsorption). The models involve differential equations, stochastic methods, and recursive functions. I will compare the models' predictions to data obtained from computer simulations and experimentally by our lab group. (Received September 23, 2017)

1135-VF-1676 **Jacquelyn L Rische\*** (jrische@marymount.edu). *Dispersal and the spread of language with frequency-dependent fitness*. Preliminary report.

In this talk, we will look at mathematical modeling of language using computer simulations. We develop a model for a population where language does not exist. We assume that the ability to speak appears in the population as a genetic mutation. Using this model, we study how individuals with language spread through the population of individuals without language. To study how the language group will grow, we focus on the effects of talking and movement. If two individuals with language are next to each other on the grid, they can communicate. We consider their ability to talk to be advantageous, giving them a higher fitness. Individuals are also able to move around on the grid and reproduce within a certain radius, called the jump radius. We are particularly interested in how the jump radius affects the time it takes for the individuals with language to invade the population, and we find that this depends on the shape of the grid. (Received September 24, 2017)

1135-VF-1684 **Brittany Bannish\*** (bbannish@uco.edu). *Exploring Differential Regulation of Blood Clot Degradation*.

Blood clots are physiologically degraded via a biochemical cascade initiated by tissue plasminogen activator (tPA). tPA, which is also used clinically to treat stroke, creates plasmin, the main protein involved in degradation. In this talk, I explore the effects of tPA unbinding and diffusion on clot degradation. I propose that plasmin can “force” tPA to unbind from the clot, which has significant implications for the resulting clot degradation. Using a 3-dimensional stochastic multiscale model, three different regulatory mechanisms are explored when tPA is forced to unbind: 1) tPA is immediately able to rebind to fibrin; 2) tPA is immediately removed from the system; 3) tPA diffuses for some time before being able to rebind to fibrin. I will discuss the contributions of each mechanism in clot degradation, the surprising role of plasmin, and the implications for stroke treatment. (Received September 24, 2017)

1135-VF-1720 **Alexander P Hoover\*** (hoover.alexander@gmail.com), 116 N Murat St, Apt B, New Orleans, LA 70119, and **Ricardo Cortez, Eric Tytell and Lisa Fauci**. *Heave and Flow: Understanding the role of resonance and shape evolution for heaving flexible panels*.

Many animals that swim or fly use their body to accelerate the fluid around them, transferring momentum from their bodies to the surrounding fluid. The emergent kinematics from this transfer are a result of the coupling between the fluid and the material properties of the body. Here we present a computational study of a 3-dimensional flexible panel that is heaved at its leading edge in an incompressible, viscous fluid. These high-fidelity numerical simulations enable us to examine the role of resonance, fluid forces, and panel deformations have on swimming performance. Varying both the passive material properties and the heaving frequency of the panel, we find peaks in trailing edge amplitude and forward swimming speed are determined by a dimensionless quantity, the effective flexibility. Modal decompositions of panel deflections reveal that the strength of each mode is related to the effective flexibility and peaks in the swimming speed and trailing edge amplitude correspond to peaks in the contributions of different modes. Panels of different material properties but with similar effective flexibilities have modal contributions that evolve similarly over the phase of the heaving cycle and agreement in dominant vortex structures generated by the panel. (Received September 24, 2017)

1135-VF-1833 **Judith E Canner\*** (jcanner@csumb.edu), **H T Banks, Kidist Bekele-Maxwell, Rebecca Everett, Jennifer Menda, Lyric Stephenson, Jon Morgenstern, Sijing Shao and Alexis Kuerbis**. *Modeling the Behavior of Problem Drinkers in a Clinical Trial*.

In order to make treatment accessible, psychologists are interested in developing outpatient treatment for problem drinkers who want to reduce, but not necessarily abstain from, alcohol consumption. While previous research demonstrated that medication and cognitive behavioral therapy (CBT) are effective outpatient treatments on average, there are still many factors that may impact an individual’s success in reducing drinking. We use a Hidden Markov Model to demonstrate that different treatments affect not only the overall reduction in drinking, but also the probability of behavior change over time. While the addition of medication to CBT does not appear to increase the overall reduction of problem drinking (compared to CBT alone), the combination of treatments affects the transition rate from heavy drinking to social drinking. In addition, we evaluate the risk factors and dynamic relationships between drinking habits, everyday behaviors, and events that may impact a patient’s success in reducing drinking and the probability of transitioning between behaviors (social and heavy drinking). We hope that our efforts will lead to models that can enhance clinical practice in the treatment of problem drinkers. (Received September 25, 2017)

- 1135-VF-2217 **Peter Agaba\***, Mathematics Department, 1906 College Heights BLVD, Bowling Green, KY 42101. *Optimal Control Theory and Parameter Estimation of Parameters in a Differential Equation Model for Patients with Lupus.*

System Lupus Erythematosus (SLE) is a chronic inflammatory autoimmune disorder that affects many parts of the body including skin, joints, kidneys, brains and other organs. Lupus Nephritis (LN) is a disease caused by SLE. Given the complexity of LN, we establish an optimal treatment strategy based on a previous developed mathematical model. As in Budu-Grajdeanu et al., (2010), our model variables are: Immune Complexes (I), Pro-inflammatory mediators (P), Damaged tissue (D), and Anti-inflammatory mediators (A). The analysis in this research project focuses on analyzing therapeutic strategies to control damage using both parameter estimation techniques (integration of data to quantify any uncertainties associated with parameters) and optimal control with the goal of minimizing time spent on therapy for treating damaged tissue by LN. Our simulated results for LN model will be presented in our talk. (Received September 25, 2017)

- 1135-VF-2524 **Zeinab Bandpey** (zeinab.bandpey@morgan.edu), Baltimore, MD 21251, **Isabelle Kemajou-Brown** (elisabeth.brown@morgan.edu), Baltimore, MD 21251, **Xiao-Xiong Gan**, Baltimore, MD 21251, and **Ahlam Tannouri\*** (Ahlam.Tannouri@morgan.edu), Baltimore, MD 21251. *Urban snow removal - just in time mathematical model and algorithm.*

Winter urban traffic issues and performance constitute an important problem in certain countries. In urban areas, there is a need of efficient methods for snow removal. There have been several discussions on modeling snow removal, for which we notice several approaches. In this project, we plan to develop a new routing problem for snow removal that can complement the existing models. The main questions are design of models for efficient routes for snow removal, optimizing time and cost. Our questions are based on the Maryland State Highway Administration final research report SP007B4N (2002) for snow emergency decision support system. In this report, the authors in their model, considered the constraints that ensure continuity of the truck routes and consideration of a single depot, using a new approach. In our project, we plan to generalize the previous models, not only on constraints extension on truck routes, but also on constraints on budgets and time. We also plan to use a more suitable model or approach to solve the projected snow removal problem which will be introduced as emergency transportation model, using Just In Time model. (Received September 26, 2017)

- 1135-VF-2612 **Benjamin D Knisley\*** (bknisleyhome@gmail.com), 26630 Barton Rd, ATP 2212, Redlands, CA 92373, and **Dr. Linn Carothers** (lcarothers@calbaptist.edu), 8432 Magnolia Ave, Riverside, CA 92504. *How High Impact Undergraduate Health Research Initiatives Can Be Discovered and Implemented in The Statistics Classroom.* Preliminary report.

High-impact learning with undergraduate research in the classroom has tremendous benefits. From undergraduate student to Business Systems Analyst this presentation will discuss undergraduate publications, training, and mentoring that eventually led to a job offer with the County of San Bernardino. This presentation will discuss my trajectory in undergraduate research in obesity with Native Americans and the surprising opportunity to predict high utilizers throughout San Bernardino County. Topics include anthropometric and nutritional measurements with a cross-sectional, epidemiological population (n=183) of Native American students (ages 14-18) from diverse tribal backgrounds at an urban residential high school. Furthermore, we discuss how these methods derived from Native American studies eventually led to a recent Western Users of SAS Software publication which engages County systems to integrate health data sources to stratify, identify, and predict high utilizers of public systems. Additionally, we show advanced mathematical concepts taught in the classroom that foster the construction of a predictive model, using a retrospective cohort with multiple logistic regression, to demonstrate the factors and service utilization patterns that most contribute to high utilizer. (Received September 26, 2017)

- 1135-VF-2782 **Syed Abid Rizvi\*** (srizvi@email.wm.edu). *Investigating the Mechanism of Oscillatory Frequency Changes due to NMDA in the CA3 Neural Network of the Hippocampus.* Preliminary report.

Computational models and in vitro experiments have shown that in response to stimulation, the CA3 network experiences gamma oscillations, which are 40 Hz oscillations of the total field potential. Further more, in vitro studies have shown that the oscillation frequency increases to approximately 60 Hz in response to NMDA's presence during stimulation. We model the CA3 network using the Hodgkin Huxley differential equations to simulate each neuron. We will then analyze the system to determine the cause of the frequency shift. This information will help elucidate how the hippocampus is involved in the formation of long-term memory. (Received September 26, 2017)

1135-VF-3037 **Qing Wang\*** (qwang@shepherd.edu), Dept. of Computer Science, Math & Engineering, Shepherd University, Shepherdstown, WV 25443, **Zhijun Wang** (zwang@shepherd.edu), Dept. of Computer Science, Math & Engineering, Shepherd University, Shepherdstown, WV 25443, and **David J Klinke** (david.klinke@mail.wvu.edu), Dept. of Chemical Engineering, West Virginia University, Morgantown, WV 25606. *A Multi-Scale Model of Tumor Growth in Response to an Anti-Nodal Antibody Therapy Combined with a Chemotherapy.*

Nodal, a member of the transforming growth factor-beta superfamily, is not typically observed in most normal adult tissues but is reactivated in various advanced-stage cancers. Recent research reported that some front-line therapies such as BRAF inhibitors as well as chemotherapy agents Dacarbazine (DTIC) and Doxorubicin (DOX) failed to affect Nodal levels in aggressive cancers such as melanoma and breast cancer tissues. Based on the in vitro work presented by Dr. Hendrix and colleagues, a mathematical model was developed to describe the effect of a combination therapy involving an anti-Nodal antibody and DOX on triple-negative breast cancer (TNBC) cell growth. Model parameters were calibrated to published experimental data using a genetic algorithm. Stability analysis and sensitivity analysis were discussed with biological relevance. This research was supported by the NIGMS of the NIH grant as part of the WV-INBRE (P20GM103434). (Received September 26, 2017)

1135-VF-3209 **Yang Ding, Li Zhang and Longhua Zhao\***, lxz315@case.edu. *Analysis of micro-fluidic tweezers in the Stokes regime.*

Nanowire fluidic tweezers have been developed to gently and accurately capture, manipulate and deliver micro objects. The mechanism behind the capture and release has not been well understood yet. Utilizing the method of regularized Stokeslet, we study a cylindrical nanowire tumbling and interacting with spherical particles in the Stokes regime. The capture phenomenon observed in experiments are reproduced and illustrated with the trajectories of micro-spheres and fluid tracers. The flow structure and the region of capture are precisely examined and quantitatively compared for different sizes of particles and various tumbling rates and dimensions of the tweezers. We found that pure kinematic effects can explain the mechanism of capture and transport of particles. We further reveal the relation between the capture region and the behavior of stagnation points in the displacement field. (Received September 27, 2017)

## General Session on Outreach

1135-VG-953 **Victoria Kofman\*** (drkofman@gmail.com), Buffalo Grove, IL. *Gifted For All.*

Stella Academy is a non-for-profit learning center which provides gifted-level math education for all its students. Student success exclusively defines the center's survival since *Stella* operates on parental contributions only. The students are mostly children of immigrants. Students who come to *Stella* in the 40-85th percentile range on national tests consistently and significantly increase their performance within a year—without any sort of "test prep" in their *Stella* curriculum.

*Stella's* program starts from introducing numbers (see VikaSchool.com/Book) and concludes with Algebra-level material. The beginning of the program teaches the fundamentals of mental computations and problem solving. Later, *Stella* introduces integers utilizing vectors, teaches elements of geometry using drafting and construction problems, teaches pivoting using verbal and number problems, helps students discover *sine*-function by prompting them to draw *y*-projection of a unit vector, and more.

*Stella* constantly improves its math program and fits the needs of students of various abilities. In spite of differences in ability, all our students are engaged in problem solving and develop strong interest in STEM. (Received September 17, 2017)

1135-VG-1045 **Rita M Patel\*** (patelr35@cod.edu), **Amanda Harsy** (harsyram@lewisu.edu) and **Dave Klanderma** (dave.klanderma@trnty.edu). *The Impact of Math Teachers' Circles on Persistence, Confidence, and Implementation of Inquiry-based Learning for K-12 Teachers.*

The Southwest Chicago Math Teachers' Circle is a partnership between college and university professors and local K-12 math teachers. This circle focuses on providing opportunities for teachers to work with deep and interesting mathematical problems which may relate to the topics that they teach in their classrooms. The authors have attempted to determine the impact of participation in this program on the teachers' dispositions toward mathematics, the teaching of mathematics, and their perception of the learning of mathematics. Specifically, we measure the teachers' persistence, confidence, and willingness to implement inquiry-based learning in their K-12 classrooms. This talk will share some of the results gathered from two workshops hosted in August 2016 and August 2017. Data sources include written survey responses in conjunction with Likert-scale items. Join

us to learn more about our math teachers' circle, the impact on participating teachers, and the potential future impact on their students. (Received September 18, 2017)

1135-VG-1134 **Mei Zhu\*** ([zhuma@plu.edu](mailto:zhuma@plu.edu)), Mathematics Department, Pacific Lutheran University, Tacoma, WA 98447, **Belinda Louie** ([blouie@uw.edu](mailto:blouie@uw.edu)), School of Education, University of Washington Tacoma, 1900 Commerce Street, Tacoma, WA 98402, **Jose Rios** ([jrios@uw.edu](mailto:jrios@uw.edu)), School Of Education, University of Washington Tacoma, 1900 Commerce Street, Tacoma, WA 98402, and **Riki Thompson** ([rikitekti@uw.edu](mailto:rikitekti@uw.edu)), School of Interdisciplinary Arts and Sciences, University of Washington Tacoma, 1900 Commerce Street, Tacoma, WA 98402. *Collaborative Effort and Outcome in Providing In-Person and Online Professional Development to High-Need K-12 Schools in Washington State.* Preliminary report.

From 2013 to 2017 a team of faculty members from Literacy and Language Arts Education, Language and Rhetoric, Mathematics, Mathematics Education, and Science Education received three grants from the Washington Student Achievement Council to provide professional development in Common Core State Standards (CCSS) and Next Generation Science Standards (NGSS) to K-12 teachers in high-need schools. We held eight three-day and two one-day in-person workshops and numerous online courses for teachers in seventy two schools across the state. Some of the topics were on teaching CCSS and NGSS practices and content standards, developing lesson plans, understanding the Smarter Balanced Assessment system, selecting and using online resources, and working effectively with students of diverse backgrounds and needs. In addition, we provided teachers with written feedback for their lesson plans. In this talk we will share our program structure, the challenges and successes we had, and the role mathematicians played. We will also discuss how the collaborative working experience has shaped each of us by deepening our understanding of teaching philosophy and pedagogical development as college professors. (Received September 23, 2017)

1135-VG-1153 **Marc Chamberland\*** ([chamberl@grinnell.edu](mailto:chamberl@grinnell.edu)), 1116 8th. Ave., Grinnell, IA 50112. *Popularizing Mathematics with YouTube.* Preliminary report.

How is mathematics being popularized with YouTube? We showcase various math channels, including the speaker's channel Tipping Point Math, and explain how videos can be both informative and entertaining, thus allowing a broader crowd to see the beauty of mathematics. (Received September 19, 2017)

1135-VG-1799 **Erick B Hofacker\*** ([erick.b.hofacker@uwrf.edu](mailto:erick.b.hofacker@uwrf.edu)), 214C North Hall, River Falls, WI 54022, and **Sherrie Serros** and **Ashlee LeGear** ([ashlee.legear@uwrf.edu](mailto:ashlee.legear@uwrf.edu)), 214C North Hall, River Falls, WI 54022. *Teacher Development Through the Mathematical Practice Standard Continuum.*

In 2015 we were awarded a three-year Mathematics Science Partnership grant to work with over 15 school districts in Western Wisconsin. The project focused on the progression of topics through K-12 mathematics, but also the development of the habits of mind that are important for successful teaching and learning of mathematics. Over 100 in-service teachers participate in the project, as well as a dozen apprentices. The apprentices are pre-service teachers that receive the same professional development, but also work in the classroom to assist their cooperating teachers with implementing these habits of mind. As a result of this project and previous projects, our team has developed a continuum scale for analyzing how teachers progress in their appreciation and implementation of the mathematical practice standards with their students. Rich mathematical tasks are used with the participants and apprentices to help develop an appreciation of the mathematical practice standards. Once an appreciation of the practice standards is developed, it provides a sense for teachers on the importance of interpreting and developing them with their own students. Preliminary data from our work will be shared. (Received September 24, 2017)

1135-VG-2369 **Jie Liu\*** ([liu@dixie.edu](mailto:liu@dixie.edu)), **Clare Banks** and **Vinodh Chellamuthu.** *MAGIC (Mathematics Advances Great Intellectual Confidence).*

In summer 2017, the mathematics department of Dixie State University (DSU) held a math summer camp for a group of students who had just completed the 5th grade. The goal of this MAA funded program was to encourage students' math confidence and interests, build a math community for young students and increase community awareness of the importance of mathematics. The summer camp was a great success. In this presentation, we will share our experiences and the results of our MAGIC summer program. (Received September 26, 2017)

1135-VG-2370 **Gangadhar Acharya\*** ([gangadhar.acharya@ttu.edu](mailto:gangadhar.acharya@ttu.edu)), 3299 Adams Street NE, Apt C70, Albuquerque, NM 87110. *Frameworks for different types of mathematics outreach programs: A proposed model.*

Various researchers have emphasized the necessity of mathematics outreach programs at the school level (K-12). Large numbers of outreach programs are being implemented at universities and colleges to improve the K-12 mathematics education in the United States. A rigorous study of over 100 such mathematics outreach programs enables us to categorize university/college level mathematics outreach programs into three categories: motivational, preparational and motivational plus preparational, according to project outcome expectations. Analysis of the structural components of these programs provides a framework for each type of mathematics outreach program and reveals a consistent set of attributes that can be associated with each of these three categories. A model mathematics preparational outreach program to address perceived issues involving the transition to algebra from arithmetic in the early grades is purposed based on the above identified framework. In general it is hoped that these frameworks can serve as guidelines for university/college faculty interested in developing their own mathematics outreach programs. (Received September 26, 2017)

1135-VG-2797 **Claire Merriman\*** ([emerrim2@illinois.edu](mailto:emerrim2@illinois.edu)), 273 Altgeld Hall, 1409 W Green St, Urbana, IL 61801, and **Emily Heath** and **Simone Sisneros-Thiry**. *Summer Illinois Math Camp.*

The Summer Illinois Math Camp is a free, week-long math day camp for middle and high school students established and run by graduate students in the Department of Mathematics at the University of Illinois at Urbana-Champaign. In this talk, we will share our experience translating college-level mathematics into activities for middle and high school students, as well as lessons learned from working with graduate student instructors and undergraduate teaching assistants. (Received September 26, 2017)

1135-VG-2985 **Kyle Evans\*** ([kyle.evans@uconn.edu](mailto:kyle.evans@uconn.edu)) and **Fabiana Cardetti** ([fabiana.cardetti@uconn.edu](mailto:fabiana.cardetti@uconn.edu)). *Mathematics and Global Citizenship: Preparing for the Future.*

As the world becomes increasingly interconnected, the demand for global education has also increased. While mathematics education has addressed the incorporation of social justice issues into the mathematics classrooms, we conducted a study to explore the following questions: (1) In what ways can mathematics content be linked to a more expansive list of objectives associated with global education? and (2) As teacher educators, how can we prepare future mathematics teachers to best support the development of students and future generations into global learners and global citizens? In this presentation we will discuss the results of this study that included interviews, classroom observations, and analyses of teacher-generated tasks. We will present at least one high school lesson plan to help us examine the connections between mathematics content and global citizenship objectives and to discuss the factors associated with its design and implementation. Implications for teacher education will also be offered. (Received September 26, 2017)

## *General Session on Teaching and Learning Developmental Mathematics*

1135-VH-426 **James A Young-Nastally\*** ([janastal@unoh.edu](mailto:janastal@unoh.edu)), 1441 N. Cable Rd., Lima, OH 45805. *Math Tutoring Center's Online Presence Within Virtual Classrooms - Changing Attitudes & Cultures.*

Maintaining presence within an online classroom is of the utmost importance for a virtual college community. The discipline of Mathematics, specifically, requires a strong level of presence as learners navigate themselves through an online course. Various forms of presence has been tested and executed from the UNOH Math Lab Tutoring Center over the past two years. An F-Test and appropriate t-Test were ran to observe the statistical impacts of the ventures that were observed throughout the following study. (Received September 01, 2017)

1135-VH-517 **Anne Cawley\*** ([acawley@umich.edu](mailto:acawley@umich.edu)). *Classroom Instructional Experiences of Latinx Students in a Community College Intermediate Algebra Course.* Preliminary report.

There is little understanding of the ways in which Latinx students experience mathematics courses at community colleges. Latinx students are more likely to enroll in community colleges, have lower success rates in mathematics, and even lower numbers of degree attainment as compared to their White peers (NCES, 2015). Given that the classroom is one of the most important spaces in community colleges (Wood & Harris III, 2015), little research has been done to look within the mathematics classroom to understand Latinx student experience (Crisp, Reyes,

& Doran, 2015). Instruction is an important feature of classroom experiences (Cohen, Raudenbush, & Ball, 2003). This study investigates the instructional experiences of Latinx students in a developmental mathematics course using qualitative methods that rely on interviews, diary entries, classroom observations and classroom artifacts. I aim to understand (1) what is the quality of instruction within a developmental mathematics class at a community college, (2) how Latinx students make sense of particular instructional experiences, and (3) how these experiences shape students' mathematical understanding. (Received September 06, 2017)

1135-VH-962      **Qingxia Li\*** (qli@fisk.edu), 411 Annex Ave, Apt B4, Nashville, TN 37209. *Support Needs in Teaching Developmental Mathematics Courses.*

STEM (science, technology, engineering and mathematics ) retention is a major problem in most colleges and universities, especially HBCUs. A broad model of support systems that includes psychological factors is adopted to address retention in biology and mathematics. The purpose of our study was to develop an instrument to identify the support needs of college students registered in College Algebra and General Biology. We adapted the theoretical model of the performance pyramid to create a 70-item measure called the Student Support Needs Scale. We examined the psychometric properties of our scale, established the reliability and validity of the resulting instrument. This instrument could potentially help our institutional programs to make informed decisions about resource allocation based on students' needs. (Received September 17, 2017)

1135-VH-1250      **Johannes C. Familton\*** (jfamilton@bmcc.cuny.edu), New York, NY. *Supplementary Instruction - A success story for remedial mathematics in three CUNY community colleges.* Preliminary report.

Supplementary Instruction was created at the University of Missouri-Kansas in 1973 by Dr. Deanna Martin. It incorporated ideas from developmental psychology. The original form of Supplementary Instruction required that students were motivated enough to be open to being independent learners. Unfortunately this is not always true for students in remedial mathematics courses.

BMCC, along with other CUNY community colleges has been developing their own version of Supplementary Instruction (SI) to accommodate these types of students. This more flexible form of SI was developed in order to remove the remedial stigma that is often attached to traditional academic assistance programs. The BMCC model does not identify high-risk students, but identifies high risk classes.

In this talk we will talk about three CUNY community colleges, BMCC, Hostos and LaGuardia, present statistics, discuss their best practices and experiences developing Supplementary Instruction for remedial mathematics classes. (Received September 21, 2017)

1135-VH-1394      **Michael E Matthews\*** (michaelmatthews@unomaha.edu), 6001 Dodge St., UNO DSC 231, Omaha, NE 68182, **Michelle Friend** (mefriend@unomaha.edu), 6001 Dodge St., UNO RH 308J, Omaha, NE 68182, **Betty Love** (blove@unomaha.edu), 6001 Dodge Street, UNO DSC 224, Omaha, NE 68182, and **Victor Winter** (vwinter@unomaha.edu), 6001 Dodge St., UNO PKI 174C, Omaha, NE 68182. *Impact of functional programming on visual-spatial ability and functional reasoning of gifted elementary school students.* Preliminary report.

We report on a project in which elementary school students in a mid-sized Midwestern town were provided math/coding lessons for ten weeks. The curriculum used Bricklayer, an open-source, online educational ecosystem that provides an example-rich and problem-dense domain in which students learn to write in the functional programming language SML. When executed, Bricklayer programs can produce artifacts in electronic LEGO®, Minecraft, or 3D printing format. Bricklayer provides a low-threshold environment for elementary students to learn about and use algebraic structures like algebraic expressions, geometrical structures like coordinates or lines, and functional reasoning like repeatedly calling a function with parameters. 62 participants came from nine different schools in an urban setting. Participants were tested for mathematical gains using a pre- and post-test design. Participants took tests that measured their understanding of coordinate systems, their visual-spatial ability, and their understanding of functional concepts. Significant changes between pre- and post-test scores with moderate effect sizes were present in understanding of coordinate systems and visual spatial ability, with small growth reported in functional reasoning. (Received September 21, 2017)

1135-VH-2676 **Victor V. Cifarelli\***, Department of Mathematics & Statistics, University of North Carolina at Charlotte, 9201 University City Boulevard, Charlotte, NC 28223-0001, and **Volkan Sevim** (vsevim@uscb.edu), Department of Mathematics and Computational S, University of South Carolina-Beaufort, Hargray 226 One University Blvd, Bluffton, SC 29909. *A Exploration of College Algebra Students' Understanding of Higher Order Polynomial Functions.*

The mathematics community agrees that the study of non-linear functions must have a prominent place in the school mathematics curriculum (NCTM, 2000). This paper reports on the first in a series of studies we are conducting with College Algebra students, addressing their understanding of non-linear functions. The current study examines students' understanding of polynomial functions of degree  $n > 2$ ,  $f(x) = anx^n + an - 1x(n - 1) + \dots + a_1x^1 + a_0$ , with particular emphasis on how students' conceptions guide their actions to produce and make connections among graphs of polynomial functions that are of degree three or more. The study included interviews with students as they completed mathematical problem tasks, enabling a focus on the students' ongoing cognitive actions. The results seem to support our hypothesis regarding how students' prior experiences with quadratics might limit their developing a sense of invariance when acting on polynomial functions of degree three or higher. Specifically, most students related the graph of the given polynomial functions to parabolas in some way; and all demonstrated some degree of static shape thinking (Moore & Thompson, 2015). The presentation will illustrate some of these findings with results from two students. (Received September 26, 2017)

1135-VH-3018 **Benjamin Atchison\*** (batchison@framingham.edu). *Switching to a Corequisite Remediation Model for GenEd Mathematics and Statistics Courses.*

At Framingham State University, all students must complete one credit-bearing mathematics course as part of their general education requirements. For many years, students who did not achieve a satisfactory placement exam score would have been required to complete a non-credit bearing General Mathematics course prior to fulfilling their general education mathematics requirement. In an effort to improve retention rates and foster student success in the first year of study, the University made a decision to adopt a corequisite remediation model for each of its general education mathematics offerings. Beginning in Fall 2017, students who would have previously been required to complete General Mathematics were instead enrolled into a credit-bearing mathematics course and an accompanying 2-hour weekly mathematics lab, intended to feature just-in-time remediation via an online learning system. This talk will focus on the development, implementation, and initial findings from the adoption of this model. (Received September 26, 2017)

## General Session on Teaching and Learning Introductory Mathematics

1135-VI-137 **Cynthia J Huffman\*** (cjhuffman@pittstate.edu), Mathematics Dept, Pittsburg State University, 1701 S. Broadway, Pittsburg, KS 66762. *Using Activities based on Original Sources in a General Education Mathematics Course.* Preliminary report.

This presentation reports on the results of using short in-class activities based on original sources in a general education mathematics course called Quantitative Reasoning. Several original source activities were used to provide students with active learning experiences and opportunities to engage with the interdisciplinary content (history and mathematics) as well as opportunities to engage with other students in the course, in order to move students from lower level cognitive skills on Bloom's Taxonomy to higher levels. (Received August 02, 2017)

1135-VI-217 **Katie Pridemore\*** (kt.pridemore@gmail.com), 1800 South Kirkman Road, Orlando, FL 32811. *Desmos.com A free online graphing tool for developing classroom activities!*

Desmos is becoming an extremely useful tool due to its intuitive nature, sleek beauty in graphing, and best of all, it is FREE! For anyone not familiar with it, Desmos.com is essentially a free online graphing calculator, but it can be used as so much more and even if you have heard of it, the flip side, using the teacher log-in, has endless possibilities. When I discovered Desmos, I was hooked. I have used it in many different ways in my classes, but for this presentation I would like to showcase the Desmos Classroom Activities feature.

This is a hands-on way to have students connect what they are learning on paper to an interactive visual representation. I will begin the presentation by handing out an activity sheet with instructions on how to access the activity through Desmos.com. I will model how I do these activities in class, as the participants work to complete the activity. This will give them the student experience which I feel is very important and effective!



As they are completing the activity, towards the end, I will transition into explaining about the teacher side of it. For example, one of the most amazing features is the ability for the teacher to see every student's screen from their log in page, without any additional software needed. (Received August 12, 2017)

1135-VI-282 **Javad Namazi\*** (namazi@fdu.edu). *Cryptography as a gateway to core mathematics*. Preliminary report.

In this talk we discuss how an elementary course in cryptography can be used as a gateway to expose students to core mathematical ideas such as group theory, number theory, and linear algebra, among others. (Received August 18, 2017)

1135-VI-423 **Olaseni T. Fadipe\*** (fadip1ot@cmich.edu) and **Katrina L Piatek-Jimenez** (k.p.j@cmich.edu). *Developing Percents Skills in College Students*.

In this study, college students' understanding of percentages involving multiple percent changes was investigated. Twenty-one students completed a preliminary percent task as well as a similar one on the midterm. Of the 21 participants, seven volunteered to participate in three in-depth interviews where they were asked to attempt two more percent tasks during interview 2 and 3 respectively. Results show that students made significant gains in their understanding of percents between the first week and the midterm. For example, 16 (approximately 76% of the students) provided a complete and correct solution on the midterm task compared to just one student on the preliminary task. Furthermore, students demonstrated several quantitative reasoning techniques like using convenient and practical values to disprove some quantitative statements. (Received September 01, 2017)

1135-VI-516 **Shumei C Richman\*** (richmansmc@gmail.com). *1H3W for the Teaching of Beginning Algebra*. Preliminary report.

We all know that partial understanding leads to mistakes. This is the case for most of the mistakes my students make in remedial beginning algebra classes. They are used to doing a math problem by following the steps in a given formula or procedure, without knowing either what is going on or why it works. Because of this "knowing how only" approach, they do not have a good sense about when to use which formulas/procedures, resulting in common mistakes such as misusing the distributive law. To help them avoid mistakes, I have developed this alternative 1H3W approach, which requires students to understand what, why and when, in addition to how. To do so, for each problem being solved a concept approach is added, which reveals what and why, alongside a commonly used formula approach. Furthermore, several problem sets, each including similar-looking problems that use different formulas, are given and discussed, in terms of when to use what formulas. This 1H3W approach is much more time consuming but gives students a comprehensive understanding of a problem or formula or topic, and so helps them to not only avoid mistakes but also build up mathematical sense. In this talk, we will discuss how to teach 1H3W, by examples from the distributive law to exponent properties and beyond. (Received September 17, 2017)

1135-VI-559 **Jay A Malmstrom\*** (jmalmstrom@occc.edu), Mathematics Dept, Oklahoma City Community College, 7777 S May Ave, Oklahoma City, OK 73159. *Using the Coin Jumping Puzzle to Introduce Students to Polya's Four Phases of Problem Solving*. Preliminary report.

A common puzzle is described as follows: Given seven spaces with three pennies in the three spaces on the left and three nickels in the three spaces on the right - move all the pennies to the right and all the nickels to the left. The rules are: 1) pennies may only move to the right, nickels can only move to the left; 2) A coin may move into an empty space in the appropriate direction or jump a single coin of the opposite type in the appropriate direction. This puzzle makes an ideal tool for introducing students to the problem solving process: 1) Understand the Problem, 2) Make a Plan, 3) Carry Out the Plan, and 4) Reflect on the Solution. In a guided activity, students learn to reduce the problem to a smaller problem (fewer coins and spaces) and use to results to solve the original problem. As a final step, the students are asked to generalize how many moves would be required to solve a larger version of the problem. (Received September 08, 2017)

1135-VI-564 **Tumulesh K Solanky\*** (tsolanky@uno.edu), 2000 Lakeshore Drive, Mathematics Department, University of New Orleans, New Orleans, LA 70148. *Enhancing Student Engagement by Using Technology Based Interactive Teaching*.

A number of universities nationally have already adopted a technology based redesigned curriculum. The format University of New Orleans (UNO) has adopted is a variant of the emporium model of Virginia Tech, and it focuses on increased interaction between the students and teacher in the classroom. Under this format, individualized instruction is provided to students who need it to ensure that each student is able to do at least 5-6 problems during the class time itself. This is followed by homework and quizzes, which are done at home for mastery of the

subject material. The mathematics department has documented the impact of the technology based interactive teaching by collecting data on student success and student retention (the students who registered at UNO in the following semester) by comparing these two teaching formats via: impact on student success; impact on drop rate; and impact on student retention. The success of the interactive format at UNO has been reported to Board of Regents under the Grad Act reporting for the past 3 years. The interactive teaching format is easily portable and can be implemented successfully. The interactive format has been extended to three high schools in New Orleans including with successful results. (Received September 08, 2017)

1135-VI-737 **L. Franklin Kemp\*** ([lfkmp@yahoo.com](mailto:lfkmp@yahoo.com)), 3004 Pataula Lane, Plano, TX 75074-8765.  
*(Co)Sine Clock.*

(Co)Sine Clock, a computer generated clock, projects a unit second hand onto the  $x$  and  $y$  axes of a unit circle to manifest cosine and sine hands, respectively. Two mechanical versions are described. Trig Sum Clock forms the complex product of unit second and minute hands in a unit circle to give a unit hand whose position is the sum of the arcs of the two hands, reducing the product to a sum. (Received September 13, 2017)

1135-VI-918 **Rad Dimitric\***, CUNY-CSI, Department of Mathematics, 1S, Staten Island, NY 10314.  
*What is  $1/2$  plus  $1/3$  ?* Preliminary report.

This study attempts to assess the extent of knowledge of basic mathematics in the general population. In particular this one question was used persistently in checking the level of proficiency with fractions. The source of our data consists of A) answers to the question in diagnostic testing for incoming (university) freshmen. B) Interviews with random street people about the question (on a subway, on the street, in a neighborhood, etc) both in the US and some European and Asian countries. We analyse our data from different angles and offer some interpretations and conclusions. (Received September 16, 2017)

1135-VI-1555 **Heather Pierce\*** ([pierceh@emmanuel.edu](mailto:pierceh@emmanuel.edu)). *Helping and Advancing College Algebra Students.*

This summer, I piloted two programs, one to help lower level students in College Algebra, and one to get higher level students out of College Algebra and into Precalculus. I will discuss the logistics of the programs, included what topics were covered. I will also discuss the results of these students in their first level math class. (Received September 23, 2017)

1135-VI-1570 **James Morrow\*** ([jmorrow@ntholyoke.edu](mailto:jmorrow@ntholyoke.edu)). *The Theme of Observation in a Geometry Course.*

I describe structured observation exercises as both a theme and a jumping off point for an introductory geometry course. These observations are designed to encourage students to make detailed observation a habit of mind. Exercises include: Observing and describing a given complex object, finding and describing something interesting encountered while on a walk, discovering and representing a real-life problem to solve, uncovering what they understand and do not understand in a geometry reading, and examining a sketch, painting, or sculpture in an art museum. I illustrate how observation strategies can be the basis for geometric and artistic design and construction and form a strong foundation for further mathematical study and for a reflective life. For students in an introductory mathematics course, becoming a better observers helps them become more independent, better able to ask deeper questions, know what they know and what they don't understand, better able to be creative, have more strategies for problem solving, and be able to distinguish between claims that have a basis in reasoning and those without such a basis. (Received September 23, 2017)

1135-VI-1713 **Igor V Minevich\*** ([minevich@rose-hulman.edu](mailto:minevich@rose-hulman.edu)), Rose-Hulman Institute of Technology, 5500 Wabash Avenue CM129, Terre Haute, IN 47803. *The Importance and Joy of Teaching an "Ideas in Mathematics" Class.* Preliminary report.

The most enjoyable class I have ever taught, called "Ideas in Mathematics", is designed to show students the "spirit, beauty, and vitality of mathematics". I'd like to briefly discuss how important such a class can be in changing the world's perception of mathematics and then give you a few ideas for teaching this kind of class. I will also share some ways that you can implement these ideas in other lower-level classes. For example, in my class students work with NP-complete and other puzzles; these puzzles can be used in calculus as a prequel to complexity theory and O-notation. Students also make a presentation and write a paper on a completely open-ended topic of their own choosing; I have also implemented this in Calculus with pleasing results. (Received September 24, 2017)

1135-VI-2039 **Jeff Poet\*** (poet@missouriwestern.edu). *Use of Sudoku Variations in an Introduction to Proofs Course for Majors*. Preliminary report.

I am now in my second semester of using Sudoku puzzles as a two-week introduction to mathematical proofs. Classroom activities will be shared, along with results from the end of the course. (Received September 25, 2017)

1135-VI-2119 **Michelle Richard-Greer\*** (michelle.richard-greer@sdsmt.edu) and **Debra Bienert** (debra.bienert@sdsmt.edu). *South Dakota School of Mines Math Initiative: Part One*.

It has been the experience of the South Dakota School of Mines & Technology (SDSM&T) that success in mathematics will give students the foundation necessary to succeed in their academics and, ultimately, to graduate as engineers and scientists. To facilitate greater success in mathematics and increase student retention, SDSM&T has initiated a program that addresses proficiency in mathematics on several levels. Various strategies have been used over the summer and during the academic year. During summer break, the *Mathspark* program SDSM&T used online resources to keep students engaged. During the semester, the university used supplemental instruction, recitation, and gateways to assist students with their learning. This presentation is part one: the summer program. (Received September 25, 2017)

1135-VI-2126 **Debra Bienert\*** (debra.bienert@sdsmt.edu) and **Michelle Richard-Greer** (michelle.richard-greer@sdsmt.edu). *South Dakota School of Mines Math Initiative: Part Two*.

It has been the experience of the South Dakota School of Mines & Technology (SDSM&T) that success in mathematics will give students the foundation necessary to succeed in their academics and, ultimately, to graduate as engineers and scientists. To facilitate greater success in mathematics and increase student retention, SDSM&T has initiated a program that addresses proficiency in mathematics on several levels. Various strategies have been used over the summer and during the academic year. During summer break, the *Mathspark* program SDSM&T used online resources to keep students engaged. During the semester, the university used supplemental instruction, recitation, and gateways to assist students with their learning. This presentation is part two: the academic year. (Received September 25, 2017)

1135-VI-2589 **Laura M. Singletary\***, lsingletary@leeuniversity.edu. *Prospective Teachers Analyzing Transcripts of Teaching*.

In this presentation, I discuss “analyzing transcripts of teaching,” an activity developed to support prospective secondary mathematics teachers in developing and using a transcript of his or her teaching practice to analyze and learn from teaching. This activity provides an opportunity for prospective teachers to engage in internal reflection on his or her teaching episode and then provides an external manifestation of his or her internal learning. From research of the enactment of this activity, I examine the opportunities prospective teachers had to learn about teaching through this activity and research findings of their learning. Although exploratory in nature, this research provides evidence for the potential effectiveness of this activity in helping prospective teachers learn about teaching through a detailed analysis of their teaching actions by situating their learning in the context of teaching. (Received September 26, 2017)

1135-VI-2689 **Lynda Wynn\***, lwynn@sdsu.edu, and **Bill Zahner** and **Hayley Milbourne**. *Introducing Linear and Exponential Rates of Change in Linguistically Diverse Secondary Classrooms: Exploring Connections Among Curriculum, Tasks, and Student Understandings*. Preliminary report.

We examine how two 9th grade integrated mathematics teachers from a linguistically diverse school introduced linear and exponential rates of change, and we describe how their students demonstrated learning on a written assessment (n=64) and a set of clinical interviews (n=20). The Common Core State Standards contrast exponential rates, “equal factors over equal intervals,” from linear rates, “equal differences over equal intervals,” earlier in the secondary mathematics curriculum than previous state standards. To teach this new topic, the two teachers used different curriculum materials as the basis for their instructional planning. We found evidence that students learned different concepts. For example, many students in one class successfully used a general formula to fit an exponential function to a table, but struggled to use this formula on a contextualized problem. Our preliminary analysis, using Lobato et al.’s (2013) focusing framework, indicates that students’ performance can be traced back to the teachers’ use of particular tools in the instructional environment such as “ratio tables.” We discuss the connections between the understandings that students demonstrated and the different foci of the classroom activities that may have supported these student interpretations. (Received September 26, 2017)

1135-VI-2695 **Sean Droms\*** ([droms@lvc.edu](mailto:droms@lvc.edu)). *Reading Mathematics is a Learnable Skill*. Preliminary report.

The mathematics department at Lebanon Valley College places a strong emphasis on reading and understanding technical material. I will share some of the ideas and techniques our department uses to teach the reading of mathematics, especially in service courses for non-majors. (Received September 26, 2017)

1135-VI-2722 **Jialing Chan\*** ([jchan4@pacific.edu](mailto:jchan4@pacific.edu)), 3601 Pacific Ave, Stockton, CA 95211. *Students' Attitude Changing towards Statistics After a First Statistics Course*. Preliminary report.

Almost all undergraduate students are required to take an introductory statistics either as a pre-requisite for another course or in fulfillment of general education requirements. Accompanying a lack of mathematical and/or statistical preparation for the statistics course that they are required to complete is negative attitudes and disbelief of statistics. In this study, we will assess how student's attitudes toward statistics change after a first course in statistics. Furthermore, we will investigate whether online homework has positive impact on students' attitudes towards statistics. The preliminary findings will be presented in this talk. (Received September 26, 2017)

1135-VI-2808 **Sher B. Chhetri\*** ([sbchhetri.math@gmail.com](mailto:sbchhetri.math@gmail.com)), Boca Raton, FL 33432. *Teaching an Introductory Statistics Course: A New Partially Flipped Approach*. Preliminary report.

The implementation of active learning within and outside of the classroom is a traditional practice. The use of the inverted (flipped) classroom has gained attention among teachers and educators in recent years. Motivated by some recent partially flipped approach of teaching, we introduced an introductory statistic flipped class so called 'a new partially flipped class'. Survey results show that students prefer the partially flipped approach to the traditional classroom model in our institution. We will also present the course content we cover and some important techniques we used in lectures and homework. Further, we will also highlight core ideas and common pitfalls of the flipped class and the new flipped approach. We hope that this model will be of particular interest to teachers and educators of mathematics/statistics and will bring great attention in the future. (Received September 26, 2017)

1135-VI-2992 **Minal Vora\*** ([mvora@ega.edu](mailto:mvora@ega.edu)), Statesboro, GA. *Improving Student Success Rate*.

Improving student success rate in statistics is very important. I will discuss methods involved in retaining students in introductory statistics classes. (Received September 26, 2017)

1135-VI-3104 **Terry (Tee) Barron\*** ([tbarron@ggc.edu](mailto:tbarron@ggc.edu)), Georgia Gwinnett College, 1000 University Center Lane, Lawrenceville, GA 30043. *Allowing Test Corrections in STEM (Mathematics) Undergraduate Courses: Benefits versus Time and Energy!*

Research typically indicates that allowing students to correct tests in any subject is beneficial for the students as well as the teacher. Students have an opportunity to reach higher levels of mastery, while teachers are able to provide more effective feedback; or at least those are the perceived notions! The benefits of test corrections must outweigh the time energy for both the students and the teacher in the team effort to achieve higher levels of mastery learning. This presentation focuses on the different types of test corrections across mathematics courses, ways to mitigate time despair for the teacher and lessons learned. (Received September 26, 2017)

1135-VI-3146 **Skona Brittain\***, [skona@sbfamilyschool.com](mailto:skona@sbfamilyschool.com). *Once There Was a King Who had Two Sons: Stories to Inspire Topological Exploration for Non-Majors or Children*.

There are a couple of classic stories that can start with the line "Once there was a king who had two sons", which I have embellished in different ways. I use them to introduce different topics in elementary topology classes, including map coloring and Möbius strips. Students become incredibly engaged by playing the role of the wizard trying to help the king, which of course requires doing mathematics. I have used these stories successfully with kids of all ages and with non-math-major adults. Note that there are different versions – not all are PG-rated. (Received September 26, 2017)

## *General Session on Teaching and Learning Calculus*

1135-VJ-8 **Ram Verma\***, International Publications USA, 1200 Dallas Drive Suite 912, Denton, TX 76205. *Hybrid Teaching Method*. Preliminary report.

Hybrid Teaching Method (HTM) differs significantly with Standard Teaching Methods (STM) in the sense that it brings a common-sense role for the methodology of chapterization to sectionization to subsectionization so

that the conceptual understanding of the subject matter must be achieved. Our class setting experiments show that the HTM works extremely well toward achieving that learning excellence goal. (Received May 30, 2017)

1135-VJ-361 **Paul Sisson\*** ([paul.sisson@lsus.edu](mailto:paul.sisson@lsus.edu)) and **Tibor Szarvas** ([tibor.szarvas@lsus.edu](mailto:tibor.szarvas@lsus.edu)).  
*Calculus and Art.*

Calculus is still too often presented as a collection of tools and theorems devoid of human connections and relationships to other topics. The graphic art of calculus is one such often-overlooked connection. Drs. Sisson and Szarvas, both of whom have many years of experience as professors of mathematics and as university administrators, illustrate the art that arises from the search for ever more effective approaches to teaching and from giving students the means to explore calculus while they learn. The images they discuss can serve to motivate students and inspire a deeper appreciation for the subject. Examples include Enneper surfaces, Lissajous and cycloid-type figures, and objects formed in the TNB frame, with special emphasis on lessons learned while creating images for their calculus textbook. (Received August 28, 2017)

1135-VJ-581 **Janet Chen\*** ([jjchen@math.harvard.edu](mailto:jjchen@math.harvard.edu)). *Using Applets to Build Understanding of Infinite Series.*

Students studying infinite series often focus on convergence tests without getting a big picture of why convergence matters. In this talk, I will share applets we use in our second-semester calculus course to motivate the study of convergence and to help students build intuition for ideas such as the interval of convergence of a power series. (Received September 09, 2017)

1135-VJ-789 **Timmy Ma\*** ([timmy@math.uci.edu](mailto:timmy@math.uci.edu)), Department of Mathematics, University of California, Irvine, Irvine, CA 92697. *Using Kahoot! in The Classroom to Engage Calculus Students.*

Online tools are helping educators transition from the whiteboard to the smartphones. We discuss Kahoot! as an online source for formative assessment in a math classroom and how it can be used to evaluate student progress in introductory classes such as Differential and Integral Calculus, Multivariable Calculus, and more. We will also have a demonstration for the audience to get a first-hand look at how Kahoot! can be used in the classroom. (Received September 14, 2017)

1135-VJ-824 **John A. Rock\*** ([jarock@cpp.edu](mailto:jarock@cpp.edu)), Cal Poly Pomona, Mathematics and Statistics, 3801 W Temple Ave, Pomona, CA 91768. *It's Just Parts: A User's Guide for the Tabular Method of Integration by Parts.*

The tabular method for integration by parts is not as limited as its reputation may suggest. By emphasizing the information generated with each row, the tabular method proves to be an efficient bookkeeper for integration by parts. This talk features several examples exhibiting the utility of the technique, referred to as 'row integration by parts' or simply 'RIP', and includes the tic-tac-toe example made famous by the film *Stand and Deliver*. Additionally, the RIP method allows for an elegant derivation of Taylor's Formula with integral remainder as well as the Laplace transform of the  $n$ th derivative of a suitable function. Perhaps most importantly, the RIP method is easy use and easy to learn. A worksheet will be provided for members of the audience who would like to take a RIP at the technique themselves! (Received September 14, 2017)

1135-VJ-974 **Lina Wu\*** ([lwu@bmcc.cuny.edu](mailto:lwu@bmcc.cuny.edu)), 529 West 42nd Street Apt. 5K, New York, NY 10036. *Applying Maple Technology in Calculus Teaching To Create Artwork.* Preliminary report.

Using Maple Software as educational tool to help students create artwork is an effective way to teach math. It brings excitement to mathematic education. The presenter is interested in sharing her research experiences on how to link math and art by using Maple Software. She introduced the use of Project-Based Learning (PBL) Pedagogy in her teaching career. A sequence of Calculus projects of using Maple Software were assigned to her students at Borough of Manhattan Community College. The design of the "2018 Asian Art" and "2017 Sports Art" and "2016 Multi-Cultures Art" and "2015 Geometric Abstract Art" and "2014 Cartoon Art" and "2013 Polar Art" have been completed in pilot Calculus courses. Students' technical skills as well as their mathematical knowledge have been well-recognized in their artwork. Students' creativity and diversity in their math thinking have been fostered in PBL Pedagogy. Projects descriptions and students' artwork will be presented. Benefits to students and challenges to teachers in PBL Pedagogy will be discussed at the end of this presentation. (Received September 18, 2017)

- 1135-VJ-1232 **Abigail L. Higgins\*** ([ahiggins@csum.edu](mailto:ahiggins@csum.edu)), California State University Maritime Academy, 200 Maritime Academy Drive, Vallejo, CA 95490. *A Case of Community, Investment, and Doing in an Active-Learning Business Calculus Course.*

Teaching business calculus exposed me to unique instructional challenges. Pilot interviews revealed that students in this course at this particular institution identified two main concerns: (seemingly) irrelevant content and a lack of opportunities to be active in class. This project involved an instructional redesign of this course that intended to address these concerns. In this talk, I will present a case study of Christina - a very engaged student who was retaking this course. It was clear from analysis that the size of class mediated Christina's access to classroom community, which she perceived to be a necessary condition for her learning. Additionally, Christina drew a distinction between authentically situated mathematics and pseudo-situated problems that fail to invest her in the problem-solving process. She noticeably valued opportunities to do mathematics during class and receive feedback from her instructor and her peers. This project has implications for active-learning in higher education and situated mathematics problems. Furthermore, this work hopes to contribute to the limited body of research on the teaching and learning of business calculus. (Received September 20, 2017)

- 1135-VJ-1339 **Corey Shanbrom** ([corey.shanbrom@csus.edu](mailto:corey.shanbrom@csus.edu)), **Vincent Pigno\*** ([vincent.pigno@csus.edu](mailto:vincent.pigno@csus.edu)), **Jennifer Lundmark** and **Lynn Tashiro**. *Peer Assisted Learning in Calculus*. Preliminary report.

The Peer Assisted Learning (PAL) program at Sacramento State University has over five years of data showing improved student performance (20% bump in course grade) and a closing or narrowing of the achievement gap in Precalculus and Calculus 1, 2, and 3. In optional, 1-unit adjunct sections, students work in small groups on worksheets written by Sacramento State faculty. Highly trained undergraduate Facilitators do not teach, tutor, or even confirm answers. Here, we will describe the structure, philosophy, and history of the PAL program along with recent successes and challenges. (Received September 21, 2017)

- 1135-VJ-1412 **Robert R. Rogers\*** ([robert.rogers@fredonia.edu](mailto:robert.rogers@fredonia.edu)), Department of Mathematical Sciences, SUNY Fredonia, Fredonia, NY 14063. *Riemann Sums Belong at the End of Integral Calculus, Not in the Beginning.*

The typical Calculus II course starts with the formal definition of an integral utilizing Riemann sums, even though the integral sign and Fundamental Theorem of Calculus predate the birth of Riemann by 133 years and the idea of summing infinitesimals is even older. See how a Calculus II course can be taught with Riemann sums put in their proper historical and pedagogical place. (Received September 22, 2017)

- 1135-VJ-1434 **Stephanie A Blanda\*** ([sab466@psu.edu](mailto:sab466@psu.edu)). *Increasing mathematics self-efficacy in calculus students using study packets*. Preliminary report.

Mathematics is a content area recognized as a gatekeeper to college retention and degree completion. As such, it is vital to student success that we promote their mathematical self-efficacy, that is, the confidence in their mathematical ability. In data collected via clicker questions in our calculus classes, more students reported using paid tutoring services as opposed to the free tutoring available at campus learning centers. This is problematic because the material generated by for-profit centers often is not consistent with current syllabus content or uses techniques not covered/approved for use in the given course. In addition, access to these review materials is limited to residential students who can afford to purchase them. Our current project makes superior review materials available to all students. The goal is to enhance student learning outcomes in undergraduate mathematics courses, giving resident and online students free access to practice problem sets aligned with current exams as well as study advice written by peer tutors and vetted by faculty members. We will discuss the results of our limited roll-out taking place during the fall 2017 semester (and welcome feedback for the full pilot set for spring 2018). (Received September 22, 2017)

- 1135-VJ-1587 **Elgin H. Johnston\*** ([ehjohnst@iastate.edu](mailto:ehjohnst@iastate.edu)), Department of Mathematics, Iowa State University, Ames, IA 50011, and **Heather Bolles**, **Travis Peters**, **Craig Ogilvie**, **Alexis Knaub**, **Thomas Holmes**, **Chassidy Bozeman**, **Stefanie Wang** and **Anna Seitz**. *Teaching Large Lecture Calculus Using Team Based Learning*. Preliminary report.

We are in our fourth semester of using Team Based Learning (TBL) in large lecture (150 students) sections of Calculus I. We will give a brief overview of the components of the course and talk about the adjustments we have made to the "standard" TBL model. We will also discuss results of our research into the effectiveness of this approach, including results on the Calculus Concept Inventory, departmental midterms and finals, DF-drop rates, attendance, and performance in downstream courses. (Received September 23, 2017)

1135-VJ-1734 **David R. Burns\*** ([burnsd@wcsu.edu](mailto:burnsd@wcsu.edu)), 181 White Street, Danbury, CT 06810. *Adapting understanding of functions and domain to create 3D printed art.* Preliminary report.

In the spring semester 2017 I created a lab project for my students in calculus III. Students applied previous knowledge of functions and domains to create mathematical sculpture. The project used a transformation on a defined domain that is reasonably consistent in producing 3D printable solids. The only output requirements were the size of the object created. The students were encouraged to work with a domain and transformation of their own design and experiment with different parameters. The talk will detail the project and methods used as well as show examples of student sculptures. (Received September 24, 2017)

1135-VJ-1889 **Houssein El Turkey\*** ([helturkey@newhaven.edu](mailto:helturkey@newhaven.edu)), 300 Boston Post road, West Haven, CT 06515, and **Salam Turki** and **Yasanthi Kottegoda**. *Two Implementations of Pre Class Readings in Calculus.*

Active learning practices highly depend on students' preparation for class in advance. However, reading Calculus can be a challenging task to students. We address this concern by assigning targeted pre-class readings in multiple Calculus courses. We report on two implementations, provide students' feedback, and discuss the lessons learned from these implementations. (Received September 25, 2017)

1135-VJ-2202 **Eugene C. Boman\*** ([ecb5@psu.edu](mailto:ecb5@psu.edu)). *Limits Belong at the End of Differential Calculus, Not at the Beginning.*

The typical first semester Calculus course begins with the formal definition of the limit, which is then used to develop the familiar rules for differentiation. Logically, this makes perfect sense. Historically, it is backwards.

The limit concept was first rigorously stated after the ideas and techniques of Calculus had been fruitfully exploited for 200 years. That is, the limit concept was not a precursor to the use of Calculus it was a postscript. It is the solution to the problem, "Why does Calculus work?" When we teach limits first we are giving our students the solution of this problem, without first stating the problem.

I will discuss my recent attempt to implement this philosophy by reordering of topics in a first semester calculus course. (Received September 25, 2017)

1135-VJ-2230 **Derege H Mussa\*** ([dxm146130@utdallas.edu](mailto:dxm146130@utdallas.edu)), University of Texas at Dallas, Richardson, TX, **Jigar Patel** ([patel1@cims.nyu.edu](mailto:patel1@cims.nyu.edu)), New York University, NYC, NY, and **Changsong Li** ([cxl109120@utdallas.edu](mailto:cxl109120@utdallas.edu)), University of Texas at Dallas, Richardson, TX. *The impact of the Derivatives in Applied Calculus II course: A case study in Applied Calculus II at the University of Texas Dallas.* Preliminary report.

Mathematicians and Mathematics educators provide rich experiences to help students gain a deep understanding of various representations, with and without the use of technology. The formal definition of the derivative in most calculus textbooks relies on both the concepts of limit and function. These two concepts are very critical to understand the derivatives. According to many researchers, students' understanding of function have substantial influence in their understanding of main ideas of calculus such as limits, continuity and the slope of a tangent line. Derivative is fundamental concept for Applied Calculus II course; however students understanding of derivative in the course has significant impact to the course as a whole. The study finds new results on impacts of the derivatives in their performance & their relation with Applied Calculus II course and recommend possible suggestions. (Received September 26, 2017)

1135-VJ-2546 **Mel Henriksen\*** ([henriksenm@wit.edu](mailto:henriksenm@wit.edu)), **Gary Simundza** ([simundzag@wit.edu](mailto:simundzag@wit.edu)) and **Emma Smith Zbarsky** ([smithzbarskye@wit.edu](mailto:smithzbarskye@wit.edu)). *Discovering Calculus through Pasta.*

We present an overview of several exercises for an introductory calculus class designed to engage students in active learning using spaghetti or other pasta to investigate applied contexts. In one exercise, to refresh students' knowledge of functions, students measure the buckling force for a number of different lengths of spaghetti "columns," plot these forces against the column lengths and generate a regression function. In another exercise students gather data for the position of the tip of an oscillating, cantilevered spaghetti noodle, plot these data in a position vs. time graph and fit an appropriate function to the data. They then calculate the average velocity from the position data, plot these new data points vs. time and postulate an instantaneous velocity function that fits the data. Students empirically explore the effect of various function parameters using the online graphing application Desmos as they adjust their functions to best fit the data. Students later explore the chain rule and product rule using these data. (Received September 26, 2017)

1135-VJ-2736 **Amit A Savkar\*** ([amit.savkar@uconn.edu](mailto:amit.savkar@uconn.edu)), 341 Mansfield Road U1009, Storrs, CT 06269. *Mixture model approach of classifying students based on their performance in differential calculus*. Preliminary report.

Over the last three years, we have collected data on each question (item) on each of the summative assessments given in our first-semester differential calculus course. The questions on these assessments test the students' ability to solve problems that involve concepts in limits, derivatives, and applications of derivatives. It has been theorized that these items explain the variance in three factors which are being measured. Using a mixture model with a three-factor structure and latent class analysis to classify students, it is possible to classify the students based on their performance on these items into high, medium, and low ability class. All of the items were graded using partial credit. I will present the results from a simulation study on the effect of using different thresholds for modeling partial credit response in comparison with a dichotomous (multiple choice (right/wrong)) response on the classification of the students in the different classes. (Received September 26, 2017)

1135-VJ-2783 **Ryan Maccombs\*** ([maccomb1@math.msu.edu](mailto:maccomb1@math.msu.edu)) and **Andrew Krause** ([krausea3@math.msu.edu](mailto:krausea3@math.msu.edu)). *Large Lectures of Flipped Calculus*.

We have developed a flipped large lecture calculus course that has provided an effective alternative to small sections. Originally brought on by budget constraints, we have found that it gives students choices in their learning experience and extends the reach of exceptional instructors. These lectures incorporate active learning techniques by using a flipped model: students view videos at home, fill out course notes that accompany the lecture videos, and class time during lecture is focused on doing mathematics with peers, rather than watching mathematics being demonstrated. To help students grow as independent learners, the course is structured to facilitate effective study strategies both intrinsically (classroom norms) and extrinsically (graded components). Local data has informed continual improvements to our course design—high attendance rates, positive experiences with the flipped model, and positive reactions to the lecture videos are all indicators of positive student engagement. DFW rates in our calculus class have been as low as 16.3% (compared to the national average of 20.8%) and statistical analyses of achievement data suggest no significant differences in outcomes between small classes and flipped large lectures. (Received September 26, 2017)

1135-VJ-2893 **Austin Mohr\*** ([amohr@nebrwesleyan.edu](mailto:amohr@nebrwesleyan.edu)), 5000 Saint Paul Avenue, Lincoln, NE 68504. *Using Points-Free Homework to Promote Perseverance*. Preliminary report.

In a points-free grading scheme, one sets criteria for acceptable and unacceptable work as opposed to issuing a numerical score corresponding to partial credit. By setting a clear, discrete level at which credit is obtained, students are required to persevere through challenging work in instances where they might have otherwise accepted partial credit. The safety net of partial credit is replaced with the opportunity for revision of unacceptable work, allowing students to respond to instructor feedback where it is most useful. I will discuss the points-free framework I have used in second- and third-semester Calculus and offer reflections on its effectiveness. (Received September 26, 2017)

1135-VJ-2901 **Tracey Baldwin McGrail\*** ([tracey.mcgrail@marist.edu](mailto:tracey.mcgrail@marist.edu)). *Putting the Logs to the Fire – From Calculus to Algorithmics*. Preliminary report.

In this talk, I will discuss ways to bridge the gap between calculus and a course in algorithms by emphasizing the use of logarithms and other functions common to natural complexity classes. (Received September 26, 2017)

1135-VJ-2936 **Kayla K. Blyman\*** ([kayla.blyman@usma.edu](mailto:kayla.blyman@usma.edu)), West Point, NY, and **Kristin M. Arney** ([kristin.arney@usma.edu](mailto:kristin.arney@usma.edu)), West Point, NY. *An Alternate Assessment Technique - Evaluated*. Preliminary report.

In an attempt to bring a more realistic environment into the classroom during assessments, we are piloting an alternate method of assessment in the *Mathematical Modeling and Introduction to Calculus* course at the United States Military Academy. Approximately 144 of the 900 students enrolled in the course for the fall 2017 semester are in sections receiving weekly assessments in place of major exams.

These assessments are primarily conceptual in nature and consist of three parts: a night before read-ahead introducing a new application, an in-class individual portion, and an in-class group portion.

In order to evaluate this innovative idea, we are collecting data on many fronts. We gain a base-line of the student's knowledge through a pre-test and incorporate a post-test cumulative assessment that resembles a traditional exam. Additionally we examine the outcomes of the weekly assessments through individual and group responses and their corresponding scores. Finally, we use a pre- post-course survey to evaluate changes in attitudes and beliefs over the course of the semester.



Through the evaluation of this assessment technique we hope to determine if it should be expanded to a larger audience in the future. Preliminary findings of this evaluation are presented. (Received September 26, 2017)

1135-VJ-3078 **David C. Webb\*** ([dcwebb@colorado.edu](mailto:dcwebb@colorado.edu)), University of Colorado Boulder, 249 UCB, Boulder, CO 80309. *The Role of Low Instructional Overhead Tasks as Supports for Active Learning in Undergraduate Calculus Courses.*

In active learning classrooms, students are encouraged to offer conjectures, communicate their reasoning, and justify their arguments in the process of solving mathematics problems. The motivation for this approach is research that demonstrates how active learning can result in increased student persistence in the calculus sequence and improved students' dispositions towards mathematics. However, such approaches require more classroom instruction time, and time in calculus courses is a scarce resource. It also takes more time to plan for interactive instruction (or does it?) This session focuses on design principles for tasks that can be used to increase active learning in undergraduate calculus, but designed to involve low instructional overhead (i.e., additional planning time is minimal). These principles for task design include use of multiple representations, organization of related prompts around key concepts, and planning that focuses on the use of student responses to inform instruction. Using examples from a variety of publicly available instructional activities for calculus, this session will highlight how to use, adapt and design calculus activities that are more conducive to student interaction, reasoning, and problem solving in ways that are practical and productive. (Received September 26, 2017)

1135-VJ-3145 **Karen Edwards\*** ([kedwards@math.harvard.edu](mailto:kedwards@math.harvard.edu)) and **Brendan Kelly** ([kelly@math.harvard.edu](mailto:kelly@math.harvard.edu)). *Improving Feedback.* Preliminary report.

Research has shown that feedback is an important driver in students' learning. Thinking about how to maximize the effects of feedback leads us to investigate: What does effective feedback on a mathematics exam look like? How can we grade exams so that assessments become a launching pad for future success? The advent of online grading systems opens up the option for the feedback to be robust, thoughtful, and individualized within realistic time constraints. As we adopt software to provide digital feedback how do we make sure that students engage the feedback in meaningful ways? (Received September 26, 2017)

1135-VJ-3171 **Gianluca Guadagni** ([gg5d@virginia.edu](mailto:gg5d@virginia.edu)), **Bernard Fulgham**, **Stacie Pisano\*** ([sp7h@virginia.edu](mailto:sp7h@virginia.edu)), **Hui Ma**, **Diana Morris**, **Monika Abramenko** and **Julie Spencer**. *New tracks for a Calculus Curriculum in Engineering.* Preliminary report.

This paper describes a calculus redesign project that is in progress at our institution.

The goal involves creating three Engineering Math tracks for incoming engineering students, in which students in all tracks will complete a calculus sequence in two semesters. In addition, a self-paced Math Lab course address knowledge and skill gaps in non-calculus areas.

The Core Engineering Math track is for students whose calculus background is weak. This track provides the minimum necessary calculus foundation, but a strong foundation nonetheless.

The Engineering Math track is for students who have a good calculus background, having completed Single Variable Calculus I (AP AB) in high school.

The Honors Engineering Math track is for students with the strongest math background who would begin with Multivariable Calculus today.

In 2016-2017 we implemented the Honors Engineering Math track. The measurable outcome has been positive and we are now moving forward with a pilot course of the Core Engineering Math I and II. We will report on the results of the first semester of Core Engineering Math, on how the Bridge summer program has supported the first class, and how the two semester structure can benefit all our students. (Received September 26, 2017)

## *General Session on Teaching and Learning Advanced Mathematics*

1135-VK-65 **Christina Starkey\*** ([c.starkey@snhu.edu](mailto:c.starkey@snhu.edu)), School of Arts and Sciences, 2500 N. River Rd., Manchester, NH 03106. *Reflective Journaling as a Tool to Support Learning Mathematical Proof.*

This presentation will discuss a study that investigated how reflective journaling supported students' transition to advanced mathematics and learning to prove in an Introduction to Advanced Mathematics course. Students submitted weekly journal entries that were composed of unstructured prompts and structured, proof-related prompts. Students' reported benefits from the journals were coded using Borasi and Rose's (1989) classification

of student benefits from journaling in mathematics, and their journals were coded according to Raman's (2003) framework of ideas about proof writing. In the unstructured journals, students demonstrated primarily therapeutic, problem solving, and content benefits. However, students reported experiencing mostly problem solving and content benefits, as well benefits related to dialoguing with the instructor. There was a significant positive correlation between the number of journals completed and course grade. Over half the students felt the journals influenced their learning to prove by helping them pin down their understandings and write about proof ideas in their own words, which they then connected to the more formal writing in their proofs. (Received July 17, 2017)

1135-VK-756 **May F Hamdan\***, May Hamdan, LAU POBox 13 5053 F 64, Qoraitem. Beirut, Lebanon, Beirut, Lebanon. *Alignment of Mathematical Objects with Proper attributes*. Preliminary report.

As a way to help students improve their class performance in the introduction to Topology class, and as an opportunity for them to earn extra credit, I proposed that they prepare and deliver an oral presentation on the class current topic in my office. During the presentation, they informally answer questions covering a vast spectrum of concepts relating to their topic, and I would allow myself to interrupt with questions and probes when I had to.

I was surprised with the patterns of language treatise I began to notice. Students were confusing Objects with the wrong or irrelevant Attributes, thus mismatching terms with adjectives; I repeated this experiment in my other classes (linear Algebra and Real analysis as well as Abstract Algebra) and noticed similar results, pointing to the alarming need to ensure the alignment of language with concepts (Received September 14, 2017)

1135-VK-1088 **Kathy A Tomlinson\*** ([kathy.tomlinson@uwrf.edu](mailto:kathy.tomlinson@uwrf.edu)), 410 South Third Street, River Falls, WI 54022. *Rich Mathematical Modeling Projects for the Upper Division Student*.

In our upper division mathematical modeling course students work in teams to complete a semester-long project with the goal of learning about the modeling process, including background research, creation of multiple models, implementation of models with technology, and model testing. The teams select an open-ended problem that they find interesting from a list of choices provided by the instructor. The problems are supported by various modeling methods that are presented in the course. Experience taught us that students have a tendency towards applying only high school level mathematics in their models. A re-design of the problem statements resulted in projects that maintained the original modeling process goals, while promoting the application of university level mathematics. The re-designed projects are rich in the sense that they encourage multiple solutions, a multi-step modeling process, and productive conversation among teammates. We will share sample re-designed problem statements and student approaches. (Received September 19, 2017)

1135-VK-1135 **Dywayne A Nicely\*** ([nicely@ohio.edu](mailto:nicely@ohio.edu)), Ohio University-Chillicothe, 101 University Drive, Chillicothe, OH 45601. *Different Deliveries of Discrete*.

Discrete Mathematics is one of the highest-level mathematics courses that we offer at Ohio University-Chillicothe and it also serves as an introductory proof-writing course. Our students, like others, have difficulty transitioning to an advanced mathematics course such as this. In particular, I see my students struggle with the abstract and logical thought processes that are required to successfully write proofs. In an attempt to help students with this transition, I have experimented with different delivery methods and types of assessment in our discrete mathematics course. This presentation gives details on the three delivery methods (Traditional Lecture, the Ohio University Learning Network (OULN), and a Modified Moore approach) and means of assessing the students accompanied by student-grade analysis. The Traditional Lecture groups are composed of students from the fall semester of 2013 and the spring semester of 2015. In the OULN and Modified Moore groups are students from the fall semester of 2016 and spring semester of 2017, respectively. Lastly student comments on the strengths and weaknesses of each delivery method will be provided along with instructor feelings on the pros and cons of the distinct delivery methods. (Received September 19, 2017)

1135-VK-1302 **Melvin G. Royer\*** ([melvin.royer@indwes.edu](mailto:melvin.royer@indwes.edu)), Indiana Wesleyan University, 4201 S Washington St, Marion, IN 46953. *A Bridge to Everywhere*. Preliminary report.

This presentation describes my experiences developing and teaching a new "bridge" course for the first time. Our department had identified that (1) many students were underprepared for proofs in Abstract Algebra and Real Analysis, (2) depending on their major and chosen electives, some students were graduating with core knowledge gaps in probability, sets/relations, and geometry, and (3) our professional development initiatives for students were sometimes ineffective due to lack of sequenced accountability checkpoints through our four-year curriculum. We decided to address all these issues with a required sophomore level "transitions" course. Using four textbooks

with mathematical content selected from probability, proof technique, number theory, and geometry, I taught a “tilted” class in which lecture was mixed with outside reading reports, in-class groupwork, peer evaluation, and professional development requirements (e.g. describing our library’s math resources). The presentation will describe the structure of the class along with detailed written student perceptions and my own observations. (Received September 21, 2017)

1135-VK-1952     **V N Tran\*** ([thangtran@netzero.net](mailto:thangtran@netzero.net)), 13092 Allard Ave #D, Garden Grove, CA 92840. *Pathway to Riemann Hypothesis. Part I. Preliminary report.*

It has been a while since David Hilbert addressed the 23 unsolved problems in 1900. The Riemann Hypothesis states: “The real parts of the complex zeros of the so-called Riemann Zeta’s function lie on the line  $x = \frac{1}{2}$  of the complex plane  $\mathbb{C}$ .” It has been over 11 decades and a solution is still nowhere to be found. In this series of investigations, we try to reach to the current status of the problem. In this first presentation, we will begin with an investigation on the point at infinity  $\{\infty\}$  as a candidate solution of the equation  $n^2 - (n^2 - 1) = 0$ , where  $n \in \mathbb{N}$ . Note that  $\{\infty\}$  is more well-known in the presence of the construction of the Riemann sphere,  $\mathbb{C} \cup \{\infty\}$ , which is also known to be one-point compactification. Is it possible that Riemann had more in mind, perhaps a deeper connection between the fields of geometry, analysis, algebra, and more? Maybe it takes at least a set of axioms to piece all of these possibilities together. For this, we will learn and teach ourselves how this comes about before we reach to the statement above in this presentation. Parts II, III, . . . , if possible, will be brought about in follow-up presentations, due to limitation of technicality as well as of the organization’s policy. (Received September 26, 2017)

1135-VK-2218     **Jeffrey D Pair\*** ([jeffrey.pair@csulb.edu](mailto:jeffrey.pair@csulb.edu)). *Introducing the IDEA Framework for the Nature of Pure Mathematics.*

Research has revealed that many students view mathematics as an impersonal and uncreative subject—a body of unchanging knowledge. To combat such naive views, our field needs a humanistic vision and explicit goals for what we hope students understand about the nature of mathematics. The goal of this study was to identify humanistic characteristics of pure mathematics that may serve as goals for undergraduate students’ understanding, and to tell real-life stories to illuminate those characteristics. Using the methodological framework of heuristic inquiry, the researcher identified such characteristics by collaborating with a professional mathematician, by co-teaching an undergraduate transition-to-proof course, and being open to mathematics wherever it appeared in life. The results of this study are the IDEA Framework for the Nature of Pure Mathematics and ten corresponding stories that illuminate the characteristics of the framework. The IDEA framework consists of four foundational characteristics: Our mathematical ideas and practices are part of our Identity; Mathematical ideas and knowledge are Dynamic and forever refined; Mathematical inquiry is an emotional Exploration of ideas; and Mathematical ideas and knowledge are socially vetted through Argumentation. (Received September 25, 2017)

1135-VK-2451     **Joyati Debnath\*** ([jdebnath@winona.edu](mailto:jdebnath@winona.edu)), 64 White Oak Court, Winona, 55987. *The Game of Proof.*

Students in courses like Foundation of Mathematics that leads to Abstract Algebra, Number Theory and Real Analysis, often, do not demonstrate the idea of step by step logical and thoughtful analysis of the proof techniques. After several hours of lectures and grading several assigned works, it appears that the students are still vacationing in a faraway island. Students in my class were handing in work that is either copied from somewhere in the internet or copied from each other. In this presentation, I wish to share The Game of Proof that was introduced to the course in the hope of making my students think individually. They learned to write each of the proof techniques in their own words and in thoughtful manner without being persuaded by another fellow friend or the instructor. The Game of Proof made the students participate and kept them engaged in the teaching and learning process. (Received September 26, 2017)

1135-VK-2934     **Po-Ning Chen** ([poningc@ucr.edu](mailto:poningc@ucr.edu)), **Tim McEldowney\*** ([mceldowney@math.ucr.edu](mailto:mceldowney@math.ucr.edu)) and **John Simanyi** ([simanyi@math.ucr.edu](mailto:simanyi@math.ucr.edu)). *Insights from a Graduate Student led Summer Program.*

Many mathematics majors at University of California Riverside have difficulty completing their undergraduate degree and continuing their career in mathematics. This is especially the case for underrepresented minority, female, and first-generation college students who often face additional social and economic obstacles in their careers. Looking to improve undergraduate retention and success, UC Riverside graduate students, with help from faculty, developed the Summer Program in Advanced Mathematics. This program was a three week long FREE summer class on logic, abstract algebra, and real analysis taught by graduate students, with talks on graduate school. Through surveying and talking to our 25 participants, we began to develop a better

understanding of the major difficulties for undergraduates in math, as well as what we can do to help. Of particular note, we found some anecdotal data related to the low continuation rate of female and minority students in math. We plan to talk about what we learned from the program and what we plan to do going forward. We will also discuss some possible approaches to running effective supplemental student programs on a minimal budget. (Received September 26, 2017)

1135-VK-3009 **Shannon R Lockard\*** (slockard@bridgew.edu). *Finding the Right Angle: First Experiences in Teaching Geometry.*

Foundations of Geometry is an upper level course at Bridgewater State University. Taken primarily by future elementary and secondary educators, the course covers a variety of advanced topics in geometry. In Fall 2017 I took on the task of teaching this course for the first time. While this is not unusual, the fact that I haven't thought about geometry in over a decade made it a somewhat daunting task. In addition to tackling this new course, I decided to use an Inquiry Based Learning (IBL) pedagogy, only the second time I have used this teaching method in a course.

In this talk I will share my newbie experiences teaching geometry and my almost-newbie teaching experiences in IBL: the good, the bad, and the ugly. From deciding on a textbook and thinking about how to best present the material for the audience, to creating presentation rubrics and uncovering student misconceptions. I'll share what worked for me and what I learned not to do anymore. (Received September 26, 2017)

1135-VK-3092 **Gianluca Guadagni\*** (gg5d@virginia.edu), **Hui Ma** and **Lindsay Wheeler.** *How Undergraduate Teaching Assistants can change mathematics education.* Preliminary report.

We have been supported by Undergraduate Teaching Assistants [UTAs] for several semesters in the Applied Mathematics program at our Engineering School. UTAs interact with students during class time by guiding them through the assigned material. Students are divided in groups and assigned a worksheet with questions and problems to be solved. UTAs roam the classroom and facilitate discussion in within each group, they follow the development of the task, support and assist groups with suggestions (but not with straight answers or solutions). The result that we will report is positive in terms of students performance, indeed a confirmation of previous similar experiments in STEM. The real novelty is the incredible benefit UTAs receive from their experience as peer educators. Our Department is increasing the participation of UTAs into our curriculum and we will report on the strategies we are adopting to improve the learning experience for UTAs in addition to improving the learning experience of our students as well, in almost all Applied Mathematics courses [Calculus I,II,III, ODE, Linear Algebra, Probability, Statistics, etc.]. Thanks to a recent grant we can now support our UTAs with training in educational methods and techniques to maximize their impact on students. (Received September 26, 2017)

1135-VK-3237 **Christopher A Chudzicki\*** (christopher.chudzicki@gmail.com). *Math3d.org: Create, save, and share 3d visualizations in your web browser—free and open source.*

I will present the webapp Math3d.org, a free, open-source tool that I developed for visualizing 3D mathematics in modern web browsers. From volumes of revolution to parametric curves and surfaces, visualizing mathematics in three dimensions is a hurdle for many students. Existing offline products provide powerful tools for 3d-visualization but often have steep learning curves and can be expensive; most existing online products are limited in scope or focus on two-dimensional visualizations. Math3d.org aims to fill this gap and provide a flexible, intuitive, and free environment to explore three-dimensional mathematics. Learners can create, animate, and share three-dimensional visualizations that run natively in modern browsers. In addition to discussing the capabilities of Math3d.org, I will present some of the ways that my students and I have used this tool in my own Multivariable Calculus and Linear Algebra courses. (Received September 27, 2017)

## General Session on Algebra

1135-VL-895 **Ryan Shifler\*** (rmshifler@salisbury.edu). *Universal Groebner Bases of Circulant Polynomial Systems.*

A circulant system of polynomials is a set of polynomials where the first polynomial generates each of the following polynomials by applying a shift to the coefficients. A past undergraduate research project has produced divisibility conditions for a type of circulant system of polynomials to be a Universal Groebner basis. Other types of circulant systems will also be presented that a future undergraduate mentee may explore. (Received September 16, 2017)

1135-VL-1109 **Michelle Rabideau\*** ([michelle.rabideau@uconn.edu](mailto:michelle.rabideau@uconn.edu)) and **Ralf Schiffer**. *Markov number ordering conjectures*. Preliminary report.

A Markov number is a number in the triple  $(x, y, z)$  of positive integer solutions to the Diophantine equation  $x^2 + y^2 + z^2 = 3xyz$ . Markov numbers are a classical topic in number theory related to many areas of mathematics such as combinatorics and cluster algebras. Markov numbers are related to cluster algebras by Markov snake graphs, where a Markov snake graph is the snake graph of a cluster variable of the once punctured torus. The number of perfect matchings of a Markov snake graph, given by the numerator of the associated continued fraction, is a Markov number. In this talk, we discuss three conjectures given in Martin Aigner's book [A] that provide an ordering on the Markov numbers  $m_{p/q}$  for a fixed numerator  $p$ , fixed denominator  $q$  and a fixed sum  $p + q$ .

[A] M. Aigner, *Markov's theorem and 100 years of the uniqueness conjecture*, Springer 2010 (Received September 19, 2017)

1135-VL-1518 **Jeff Shriver\*** ([jeffrey.shriver@colorado.edu](mailto:jeffrey.shriver@colorado.edu)). *Hardness Results for the Subpower Membership Problem*.

The *subpower membership problem* for a fixed finite algebra  $\mathbb{A}$  is the following combinatorial decision problem:

**SMP**( $\mathbb{A}$ )

Input: A positive integer  $m$  and  $m$ -tuples  $a_1, \dots, a_n, b$  in  $\mathbb{A}^m$ .

Question: Is  $b$  in the subalgebra  $\langle a_1, \dots, a_n \rangle$  of  $\mathbb{A}^m$  generated by  $a_1, \dots, a_n$ ?

In this talk, we will discuss conditions in which we can construct algebras  $\mathbb{A}$  with 'nice' structural properties for which the problem **SMP**( $\mathbb{A}$ ) is as hard as possible. (Received September 22, 2017)

1135-VL-1903 **David A Nash\*** ([nashd@lemoyne.edu](mailto:nashd@lemoyne.edu)), Le Moyne College, 1419 Salt Spring Road, Syracuse, NY 13214, and **Sara Randall**. *How do you fix an oval track puzzle?*

The oval track group,  $OT_{n,k}$ , is the subgroup of the symmetric group,  $S_n$  generated by the basic moves in a generalized oval track puzzle with  $n$  tiles and a turntable of size  $k$ . In this paper we completely describe the oval track group for all possible  $n$  and  $k$  and use this information to answer the following question: If the tiles are removed from an oval track puzzle, how must they be returned in order to ensure that the puzzle is still solvable? As part of this discussion we introduce the parity subgroup of  $S_n$  in the case when  $n$  is even. (Received September 25, 2017)

1135-VL-1978 **Yordanka Aleksandrova Kovacheva\*** ([ykovacheva12@gmail.com](mailto:ykovacheva12@gmail.com)), 1401 E 55 Str Apt 807N, Chicago, IL 60615. *Intersection Pairing and Determinant Line Bundle*.

I construct an intersection pairing of cycles modulo relations and the corresponding determinant line bundle. More specifically, I consider the map  $CH^p(X) \times CH^q(X) \rightarrow Pic(S)$  of Chow groups of a variety  $X$  over a base  $S$ . Here  $p + q = d + 1$ , for  $d$  is the relative dimension of the morphism  $X \rightarrow S$ . I treat the Chow groups  $CH^p(X)$  as categories with the obvious objects and morphisms arising from the  $Z^p(X, 1)$  term in Bloch's complex modulo the image of Tame symbols of  $K_2$ -chains. This pairing coincides with the Knudsen-Mumford determinant line bundle using the structure sheaves of the cycles on  $X$ .

Restricting to cycles that are algebraically trivial on the generic fiber  $X_\eta$ , I show that the image in  $Pic(S)$  does not depend on the rational equivalence of the cycles. However, when working with numerically trivial divisors and zero cycles, the image does depend on the rational equivalence of the zero cycles. In the proof I construct an explicit isomorphism  $H_{et}^1(X, \mathbb{Z}/n) \rightarrow Hom(H_1(Sus_\bullet(X)/n), \mathbb{Z}/n)$ , which in the case of smooth curves is the Weil pairing. In the non-projective case I hope to extend the pairing to a pairing  $Z^d(X, \cdot) \times Sus_\bullet(X) \rightarrow Z^1(F, \cdot)$  between the Bloch's complex and Suslin's complex. (Received September 25, 2017)

1135-VL-1988 **Lance Bryant** and **James Hamblin\*** ([jehamb@ship.edu](mailto:jehamb@ship.edu)). *The Position Vector of a Numerical Semigroup*.

A numerical semigroup is a subset of the natural numbers that is closed under addition. The smallest nonzero element of a numerical semigroup is its *multiplicity*. The least representative in the semigroup from each congruence class modulo the multiplicity is called the *Apéry set*, and the positions of these elements in the semigroup give the *position vector*. We prove that the position vector of a numerical semigroup uniquely determines it, though not every vector determines a semigroup. We also discuss various cases and properties of these position vectors. (Received September 25, 2017)

1135-VL-2135 **Erik Hieta-aho\*** (eh991112@ohio.edu). *Error Correcting Codes within a Frobenius Ambient*. Preliminary report.

Traditionally in Algebraic Coding Theory a code is defined over a finite field as a subspace of a vector space with a specific parameter such as the Hamming distance. A code can then be mapped into an ideal of the corresponding polynomial quotient ring called the Ambient. Given a non-degenerate bilinear form it can be shown that the dual of a code as an ideal can be defined as the ideal's annihilator. By the development and duality of the Kerdock and Preparata families of codes the importance of codes over rings was established. It was shown by J. Wood that for codes over finite Frobenius rings a version of the Mac Williams equivalence theorem holds. Thus our project focuses on codes over Frobenius rings and codes within a specific Ambient ring as well as the properties that they have in common. This talk will focus on our preliminary results as well as open questions which we have developed in the midst of our research. (Received September 25, 2017)

1135-VL-2162 **Bernadette Boyle\*** (boyleb7@sacredheart.edu). *The Index of a Family of Complete Intersection Numerical Semigroup Rings*. Preliminary report.

In 2013, O. Veliche developed a closed formula to find the index of a complete intersection numerical semigroup ring with three generators. There are other ways to compute the index of these rings with more than three generators; however, these methods can be complicated and require one to find other values first, such as the Frobenius number and the order of elements. In this talk, we will compute the index of a family of complete intersection numerical semigroup rings, in particular those associated to the semigroup generated by  $(2^n, 2^n + k, 2^n + 2k, 2^n + 2^2k, \dots, 2^n + 2^{n-1}k)$  where  $k$  is an odd integer. (Received September 25, 2017)

1135-VL-2242 **Joshua Maglione\*** (jmaglione@math.kent.edu), Department of Mathematical Sciences, PO Box 5190, Kent, OH 44242. *A multilinear toolkit for isomorphism*.

We introduce a number of new structures that constrains the possible isomorphisms between groups using multilinear and nonassociative algebra. We use it to improve the complexity of isomorphism in known difficult cases and to improve our performance in practice. We report on individual and joint work with Peter A. Brooksbank, Uriya First, and James B. Wilson. (Received September 25, 2017)

1135-VL-2258 **Maggie Rahmoeller\***, Trexler 270J, 221 College Lane, Salem, VA 24019. *Demazure Crystals of the Quantum Affine Lie Algebra  $U_q(A_{n-1}^{(1)})$* .

In 1968, Victor Kac and Robert Moody defined affine Lie algebras, a class of infinite dimensional Lie algebras. Kashiwara showed that irreducible modules for the  $q$ -deformed universal enveloping algebra of an affine Lie algebra admit crystal bases. In 1991, Kang, Kashiwara, Misra, Miwa, Nakashima and Nakayashiki gave the path realizations of affine crystals as a semi-infinite tensor product of some finite crystals called perfect crystals. In this talk, we use this path realization and results from Kuniba, Misra, Okado, and Uchiyama to show that the union and intersection of certain Demazure crystals for the quantum affine algebra  $U_q(A_{n-1}^{(1)})$  are finite tensors of the corresponding perfect crystals. (Received September 25, 2017)

1135-VL-2737 **R. Scott Williams\*** (rwilliams77@uco.edu), 100 North University Drive, Box 129, Edmond, OK 73034, and **Erin Williams** (ewilliams50@uco.edu), 100 North University Drive, Box 129, Edmond, OK 73034. *Computing the minimal Euclidean function over  $\mathbb{Z}[i]$* . Preliminary report.

In 1949, T. Motzkin provided a recursive definition for the "minimal" Euclidean function in a given Euclidean domain. Motzkin was able to define a closed form for this function over  $\mathbb{Z}$ ; however, finding a closed form over other domains is substantially more difficult. In the case of  $\mathbb{Z}[i]$ , bounds for which the closed form must satisfy have been established, but no explicit formula yet exists. In this talk, we present our results which refine Motzkin's original recursive definition of the function over  $\mathbb{Z}[i]$ , as well as describe how these results can be applied to determine a closed form for the function. (Received September 26, 2017)

1135-VL-2880 **Wade Mattox\*** (wade.mattox@salem.edu), 601 S. Church St., Winston Salem, NC 27101. *Module Theory With Group Von Neumann Algebras*.

There are connections between the structure of a group  $G$  (e.g., amenability) and the module-theoretic properties (e.g., flatness, injectivity, projectivity, homology and cohomology calculations) of the group von Neumann algebra  $N(G)$  and related modules (such as  $L^p(G)$  and the algebra of affiliated operators) over the complex group ring  $\mathbb{C}G$ . For example,  $N(G)$  is flat over  $\mathbb{C}G$  if  $G$  is locally virtually cyclic, and the converse has been established for certain subclasses of elementary amenable groups. Other recent results and accessible open conjectures will be discussed. The flexibility of  $N(G)$  as a  $C^*$ -algebra which is open to both algebraic and analytic investigation will be highlighted. (Received September 26, 2017)

1135-VL-2919 **Tanner J Rosenberg\*** ([tjr268@nau.edu](mailto:tjr268@nau.edu)). *Computing maximal genetic distance in terms of signed permutations*. Preliminary report.

One can model a configuration of genes as a permutation of the numbers 1 through  $n$ , where each number can be right-side-up or upside-down. In this model, one type of mutation corresponds to performing  $180^\circ$  reversals of consecutive subsequences of the permutation. The genetic distance between two configurations of genes is the minimum number of reversals needed to convert one permutation to the other. While there exist algorithms for computing genetic distance between two given permutations, our goal is to determine the maximum genetic distance between any permutation of 1 through  $n$  and the identity permutation. This maximum determines an upper bound for the evolutionary distance between any two gene sequences of the same length. In this presentation, we will discuss our current progress on the gene sorting problem. (Received September 26, 2017)

1135-VL-2966 **Kelli M Karcher\*** ([kkarcher@vt.edu](mailto:kkarcher@vt.edu)), 1743 Autumn Splendor Way, Blacksburg, VA 24060, and **Peter Linnell**. *The Space of Biorders on Some Solvable Groups*. Preliminary report.

A group is said to be biorderable if it has a total order invariant under left and right multiplication. These orders can be given a topology and is called the space of biorders on this group. There has been intensive study on the space of left-orders recently, but less on the space of biorders. We will focus on solvable groups to show under certain conditions the space of biorders is either finite or homeomorphic to the Cantor set. (Received September 26, 2017)

1135-VL-3048 **M. F. Anton\*** ([anton@ccsu.edu](mailto:anton@ccsu.edu)). *Relative Brauer Relations of Abelian  $p$ -Groups*.

The Brauer relations of a finite group  $G$  are virtual differences of non-isomorphic  $G$ -sets  $X - Y$  which induce isomorphic permutation  $G$ -representations  $\mathbb{Q}[X] \simeq \mathbb{Q}[Y]$  over the rationals. These relations have been classified by Tornehave-Bouc and Bartel-Dokchitser. Motivated by stable homotopy theory, a relative version of Brauer relations for  $(G, C_p)$ -bisets which are  $C_p$ -free have been classified by Kahn in case  $G$  is an elementary Abelian  $p$ -group. In this paper we extend Kahn's classification to the case when  $G$  is a finite Abelian  $p$ -group. (Received September 26, 2017)

## General Session on Analysis

1135-VM-87 **Prashanta Majee\*** ([prashanta@maths.iitkgp.ernet.in](mailto:prashanta@maths.iitkgp.ernet.in)), Dept. of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur, WestBengal 721302, India, and **Chandal Nahak** ([cnahak@maths.iitkgp.ernet.in](mailto:cnahak@maths.iitkgp.ernet.in)), Dept. of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur, WestBengal 721302, India. *Inertial proximal method for a system of equilibrium problems and fixed point problems*.

Equilibrium problem is an important mathematical problem which covers a vast range of problems like optimization problem, Nash equilibrium problem, saddle point problem, fixed point problem, variational inequality problem, complementarity problem, convex differential optimization and many other important problems. Several authors have introduced different types of algorithms to find a common solution of this problem along with some fixed point problems.

In this work, we introduce two proximal algorithms (one parallel and another sequential) with inertial effect for approximating a common solution of a system of equilibrium problems and fixed point problems of finite collection of nonexpansive mappings. Under suitable conditions, we establish weak convergence results for the proposed algorithms. In earlier iterative methods, the control parameters has to satisfy some strong conditions. In this work, we describe a modified method to find a common solution of the stated problems under mild conditions. Finally, we give a numerical example to demonstrate the convergence and performance of the proposed algorithms. (Received July 25, 2017)

1135-VM-224 **Mutaz Mohammad\*** ([mutaz.mohammad@zu.ac.ae](mailto:mutaz.mohammad@zu.ac.ae)), Abu Dhabi, United Arab Emirates. *Gibbs Phenomenon in tight framelet expansions*.

we explore several ways to investigate the Gibbs phenomenon in tight framelet representations. We present results concerning the Gibbs phenomenon by expanding functions using the quasi-affine system. This system is generated by the Haar tight framelets. More precisely, we investigate the existence of Gibbs phenomenon in the truncated expansion of a given function which is expanded by some tight framelet representation. The tight frame method is essentially a generalized wavelet based method. It provides various construction methods to expand functions in  $L^2(R)$ . (Received August 13, 2017)





1135-VM-1420 **Laramie Paxton\*** (realtimemath@gmail.com) and **Kevin R. Vixie** (vixie@speakeasy.net). *A Geometric Definition of the Derivative.*

In first-year calculus, students are taught that the derivative, where it exists, is the slope of the tangent line at a point on the graph of a function, and possibly that the derivative is the optimal linear approximation at that point. However there is, among other viewpoints, a very geometric approach to derivatives using tangent cones. In this talk, we shall use detailed and colorful graphics to provide a very visual way of defining the derivative of a function that also carries over readily to multivariable functions. In addition to providing a more intuitive entry point to the derivative for students of first-year calculus, the tangent-cone approach also provides advanced students a pathway into the field of geometric measure theory. (Received September 22, 2017)

1135-VM-1444 **Matthew J Sequin\*** (msequin@saintpeters.edu). *Limit Points of Folding Sequences.*

Imagine a thin strip of paper labeled with the interval  $[0, 1]$ , where 0 is on the very left edge of the paper and 1 is on the right edge. If this strip of paper is folded in a certain way, the creases from the folds will correspond to different numbers in the interval. Folding the paper an infinite number of times will yield a sequence, called a *folding sequence*. In this talk, we will introduce folding sequences and discuss some of their properties. In particular, we will focus on the limit points of these sequences, and briefly discuss how folding sequences can be used as an example to introduce key concepts in an undergraduate point-set topology or analysis course. (Received September 22, 2017)

1135-VM-1645 **Aram Tangboonduangjit\*** (aram.tan@mahidol.edu), Mahidol University International College, 999 Phutthamonthon 4 Road, Nakhonpathom, 73170, Thailand, and **Thotsaporn Thanatipanonda** (thotsaporn.tha@mahidol.ac.th), Mahidol University International College, 999 Phutthamonthon 4 Road, Nakhonpathom, 73170, Thailand. *Harmonic Number Identities via Generalized Bernoulli Polynomials.* Preliminary report.

Using Lagrange inversion formula, we present a direct method (as opposed to an inductive method provided by an early work of the first author) of deriving a closed form in terms of the Stirling numbers of the first kind of the residues (at 0) of functions related to the generating function of the generalized Bernoulli polynomials. We then give a closed form of some generalized Bernoulli polynomials in terms of these residues. One of the applications includes some curious identities involving harmonic numbers. (Received September 24, 2017)

1135-VM-1770 **Benjamin P Russo\***, benjamin.russo@uconn.edu, and **Joel Rosenfeld** and **Warren Dixon**. *A Generalization of the Fock Space.*

In this talk we will introduce a generalized Fock space which uses the Mittag-Leffler function as its reproducing kernel. This space has been featured in the development of a finite difference method. However, it has yet to be investigated as a space of entire functions. We will discuss some preliminary results in comparison to the Fock space and potential applications. This is joint work with Joel Rosenfeld and Warren Dixon. (Received September 24, 2017)

1135-VM-1800 **Derek DeSantis\*** (derek.desantis@huskers.unl.edu). *On Operator Algebras Generated by Left Invertibles.* Preliminary report.

Operator algebras generated by partial isometries and their adjoints form the basis for some of the most well understood classes of  $C^*$ -algebras. The  $C^*$ -algebra generated by the unilateral right shift operator, known as the Toeplitz algebra, is an example. The right shift operator is left invertible – its left inverse being the unilateral left shift operator (the adjoint). Motivated by questions from linear equations in Hilbert spaces (frame theory), we wish to understand particular types of operator algebras generated by left invertible operators. Similar to the Toeplitz algebra, we investigate the norm closed operator algebra generated by a left invertible with a canonical left inverse (the Moore-Penrose inverse). (Received September 24, 2017)

1135-VM-1969 **Robert Booth\*** (rjbooth@live.unc.edu), University of North Carolina at Chapel Hill, Department of Mathematics, CB 3250, 329 Phillips Hall, Chapel Hill, NC 27599, and **Hans Christianson**, **Jason Metcalfe** and **Jacob Perry**. *Local energy decay for wave equations with degenerate trapping.*

Local energy estimates are a fundamental bound in the study of dispersive PDE. For wave equations in the presence of background geometry, it is known that geodesic trapping necessitates a loss in regularity compared to the usual Minkowski space-time estimate. For non-degenerate hyperbolic trapping (such as the photon sphere in Schwarzschild space-time), the loss is logarithmic; while all is lost except a logarithm for elliptic trapped sets. We consider the wave equation on a manifold with degenerate hyperbolic trapping and attain a local energy estimate with sharp polynomial loss. To our knowledge, this is the first such example for the wave equation and is motivated by the work of Christianson-Wunsch for the Schrödinger equation. Proof techniques include a WKB

inspired analysis and a quasimode construction. This is joint work with Hans Christianson, Jason Metcalfe, and Jacob Perry and related to a separate talk given by Jacob Perry. (Received September 25, 2017)

1135-VM-1985 **Jinling Niu\*** ([niu jinling@hit.edu.cn](mailto:niu jinling@hit.edu.cn)) and **Yuming Xing**. *Imbedding Theorems for Composition of Homotopy and Projection Operators.*

We prove both local and global imbedding theorems with  $L^{\varphi}$ -norms for the composition of the homotopy operator and projection operator applied to differential forms. We also establish some  $L^{\varphi}$ -norm inequalities for certain compositions of the related operators. (Received September 25, 2017)

1135-VM-2107 **Daniel Camacho\***, 701 North C Street, Indianola, IA 50125, and **Addison Grant** and **Sara Lawson**. *Mathematical Billiards and the search for a finite number of shapes.*

We follow the methods of Henk Don to find a maximum of 39 polygons produced by a billiard flow on an L-shaped table. We use the result on the number of gaps produced by arc exchanges on the circle which generalizes the Three Gap Theorem for Rotations. (Received September 25, 2017)

1135-VM-2666 **Hoewoon B. Kim\***, Department of Mathematics Kidder Hall 368, Oregon State University, Corvallis, OR 97331-4605. *The Stability of Partial Differential Equations in terms of Duhamel's Principle.*

In 1940, a Polish-American mathematician, S. M. Ulam proposed the stability problem of the linear functional equation  $f(x + y) = f(x) + f(y)$  that can be generalized as "Under what conditions a mathematical object satisfying a certain property approximately must be close to an object satisfying the property exactly?". One year later, the first, affirmative, and partial solution to Ulam's question was provided by D. H. Hyers by explicitly constructing the linear function in Banach spaces directly from a given approximate function. For the last decades, stability problems of various functional equations, not only linear case, have been extensively investigated and generalized by many mathematicians. An extension of the Ulam's stability problems in terms of differential equations was recently proposed for ordinary differential equations and are actively studied by a variety of scholars in various fields now. In this presentation, we will define the stability problem of initial and boundary problems of partial differential equations and instigate the stability of the heat/diffusion equations through the Duhamel's principle argument. (Received September 26, 2017)

1135-VM-2754 **Joshua Brummer\*** ([brummerjd@ksu.edu](mailto:brummerjd@ksu.edu)) and **Virginia Naibo**. *Weighted fractional Leibniz-type rules for bilinear multiplier operators.*

In its simplest form, the product rule can be used to estimate a Lebesgue norm of the first derivative of the product of two functions in terms of Lebesgue norms of the functions themselves and their first derivatives. Motivated by problems in partial differential equations, this notion can be generalized in a number of directions. These include studying higher order derivatives and fractional derivatives, combining the functions using more general operators (instead of simple multiplication), or considering other function space norms. In this talk we will discuss some of those directions and present results on weighted fractional Leibniz-type rules for Coifman-Meyer and bi-parameter Coifman-Meyer multiplier operators. This is joint work with Virginia Naibo. (Received September 26, 2017)

1135-VM-2760 **Waleed K. Al-Rawashdeh\*** ([walrawashdeh@mtech.edu](mailto:walrawashdeh@mtech.edu)), Montana Tech, 1300 West Park Street, Butte, MT 59701. *Weighted Differentiation Composition Operators from Nevanlinna Classes to Weighted-type Spaces.*

Let  $\phi$  be an analytic self-map of open unit disk  $\mathbb{D}$ . Let  $\mathcal{H}(\mathbb{D})$  be the space of all analytic functions on  $\mathbb{D}$ . For a nonnegative integer  $n$ , the weighted differentiation composition operator on  $\mathcal{H}(\mathbb{D})$  is defined as  $D_{\phi,u}^n f(z) = u(z)f^{(n)}(\phi(z))$ , for  $f \in \mathcal{H}(\mathbb{D})$  and  $z \in \mathbb{D}$ . In this talk, we characterize the boundedness and compactness of the weighted differentiation composition operator  $D_{\phi,u}^n$  from the weighted Nevanlinna classes  $\mathcal{N}_{\alpha}^p$  to the weighted-type space  $H_{\mu}^{\infty}$  and the little weighted-type space  $H_{\mu,0}^{\infty}$ . (Received September 26, 2017)

1135-VM-2895 **Colleen Ackermann\*** ([ctackermann@smcm.edu](mailto:ctackermann@smcm.edu)) and **Alastair Fletcher**. *Using Equilateral Hyperbolic Triangles To Characterize Quasiconformal Mappings.* Preliminary report.

Planar quasiconformal mappings are a generalization of conformal mappings in which the analyticity requirement is relaxed. Visually, they take infinitesimal circles to infinitesimal ellipses of uniformly bounded eccentricity. Over the past century, the theory has expanded to increasingly generalized classes of metric spaces and has found applications in many fields including PDE's, complex dynamics and Teichmüller theory. Their usefulness may in part be due to their many definitions of different flavors.

John Hubbard defined the skew of a topological triangle to be the ratio of the distance between the two furthest vertices to the distance between the two closest vertices. He asked if one could give a sufficient condition for a mapping to be quasiconformal in terms of only the skews of the images of equilateral triangles. Hubbard's question was answered in the affirmative by Aimo Hinkkanen, Peter Haïssinsky and myself. I will be discussing a generalization of this project with Alastair Fletcher which uses equilateral hyperbolic triangles to characterize quasiconformal mappings between planar domains. (Received September 26, 2017)

1135-VM-3064 **Timothy I Myers\*** ([timyers@howard.edu](mailto:timyers@howard.edu)), DC. *Lebesgue Integration on a Banach Space with a Schauder Basis.*

This talk will feature the construction of a Lebesgue measure and integral on any Banach space  $\mathcal{B}$  with a Schauder basis. This theory has the advantage that the integral is computable from below as a limit of Lebesgue integrals on Euclidean space as the dimension  $n \rightarrow \infty$ , so that we may evaluate infinite dimensional quantities by means of finite dimensional approximation. We will discuss applications to Gaussian measure. (Received September 26, 2017)

1135-VM-3116 **Swarup N. Ghosh\*** ([swarup.ghosh@swosu.edu](mailto:swarup.ghosh@swosu.edu)), Department of Mathematics, Southwestern Oklahoma State University, 100 Campus Drive, Weatherford, OK 73096, and **Alexander J. Izzo** ([aizzo@bgsu.edu](mailto:aizzo@bgsu.edu)), Department of Mathematics and Statistics, Bowling Green State University, Bowling Green, OH 43403. *Relation between point derivation and Gleason part in uniform algebra.* Preliminary report.

In the theory of uniform algebra, point derivation and Gleason part are two fundamental notions. It is well-known how both these notions are related to other important notions, namely, peak point and isolated point (in the dual space norm). However, the direct relation between point derivation and Gleason part is not known in general. In this talk, we will explore their relation. We will first construct a uniform algebra with a nontrivial Gleason part but with no nonzero point derivations. We will then construct a uniform algebra with a nonzero point derivation but with no nontrivial Gleason parts. (Received September 26, 2017)

1135-VM-3208 **Robert Booth**, NC, and **Hans Christianson, Jason Metcalfe and Jacob S Perry\***, CB 3250, 329 Phillips Hall, 120 E Cameron Ave, Chapel Hill, NC 27599-3250. *Local energy decay for wave equations with degenerate trapping.*

Localized energy estimates for solutions to the wave equation are well studied and provide a global integrability estimate (in both time and space). Analogous local smoothing estimates for the Schrodinger equation are well established and show that locally in space and averaged in time, solutions gain one half of a derivative in regularity compared to the initial data. When considering such estimates for equations on differentiable manifolds, in either case it is known that geodesic trapping necessitates a loss. For non-degenerate hyperbolic trapping, the loss is logarithmic. For elliptic trapping, everything is lost except a logarithm. Recently, Christianson and Wunsch demonstrated an algebraic loss for solutions to the Schrodinger equation on a surface of revolution with degenerate hyperbolic trapping. In this talk, we will review these prior results and consider the analogue for the wave equation on a warped product manifold with degenerate hyperbolic trapping, attaining an algebraic loss of derivative. We will then use a quasimode construction to show that our estimate is sharp. This is a joint work with Robert Booth, Hans Christianson, and Jason Metcalfe. (Received September 27, 2017)

## General Session on Applied Mathematics

1135-VN-220 **Anna Cui\*** ([annacui08@gmail.com](mailto:annacui08@gmail.com)), 7403 Rockfish Road, Fayetteville, NC 28306, and **Zhenlu Cui** ([zcui@uncfsu.edu](mailto:zcui@uncfsu.edu)), 1200 Murchison Rd., Fayetteville, NC 28301. *The Fibonacci-Type Sequence Revisited: A Geometric Progression.* Preliminary report.

We consider a generalized Fibonacci sequence  $A_{n+2} = kA_{n+1} + pA_n$ ,  $n \geq 1$ , where  $k$  and  $p$  are real numbers. Some well-known sequences such as Fibonacci sequence, Lucas sequence, Pell's sequence are special cases of this generalization. We find a closed formula for the  $n$ th term in the sequence by constructing a geometric sequence which is different from the methods in literature. We derive the generating function and establish some generalized identities for the sequence. We also find the limit of the ratio of successive terms, from which the Metallic Means can be obtained. (Received August 24, 2017)

- 1135-VN-417 **Rebecca Conley\***, rconley@saintpeters.edu, and **Tristan J. Delaney** and **Xiangmin Jiao**. *High-Order Adaptive Extended Finite Element Method (AES-FEM) and Direct Treatment of Neumann Boundary Conditions on Curved Boundaries*. Preliminary report.

The finite element methods (FEM) are important and powerful tools for solving partial differential equations. Their high-order generalizations pose significant challenges for curved geometries, because the geometry must be represented accurately. It is very difficult to generate high-quality meshes with curved elements that guarantee positive Jacobian everywhere, which is important for the convergence of FEM. Here we present a high-order Adaptive Extended Stencil Finite Element Method (AES-FEM), which requires only high-order surface elements along with piecewise-linear volumetric elements. AES-FEM is a generalization of the FEM that is insensitive to mesh quality. It replaces the traditional finite-element basis function with a set of generalized Lagrange polynomial basis functions computed using weighed least squares. To impose Neumann boundary conditions, which in general require curved volumetric elements in the isoparametric finite-element methods, we introduce test functions defined over the surface elements so that only high-order surface integrals are involved. This significantly simplifies mesh generation for curved geometry without compromising accuracy. We present numerical results to demonstrate the high-order convergence of AES-FEM for elliptic PDEs in 2D and 3D. (Received September 01, 2017)

- 1135-VN-444 **Tania Hazra\*** (thazra@crimson.ua.edu), 1103 14th Street, Apt 26A, Tuscaloosa, AL 35401, and **Shan Zhao** (szhao@ua.edu), 345 Gordon Palmer Hall, Box 870350, Tuscaloosa, AL 35487-0350. *Stable ADI Scheme with Super-Gaussian Dielectric Distribution and Minimal Molecular Surface*.

The work explores the stability impact of the novel unconditionally stable operator splitting methods for solving the time dependent nonlinear Poisson-Boltzmann (NPB) equation for the electrostatic analysis of solvated biomolecules. We start solving the NPB equation using conditionally stable method, namely Alternating Direction Implicit (ADI) scheme. Our goal is to make the existing ADI schemes stable. Now, for modeling protein electro-statics using implicit methods, we require dielectric properties of the system to be known, in particular, the value of the dielectric constant of protein. In our work, we have applied the Gaussian dielectric constant distribution to ADI scheme. Also we have used the minimal molecular surface (MMS), that minimizes the surface free energy of the macromolecule in the aquatic environment and it is typically free of geometric singularities. Combination of Gaussian dielectric constant distribution and MMS makes the ADI scheme unconditionally stable. By raising the content of the exponent to a higher power (super-Gaussian dielectric constant distribution) ADI scheme can produce more accurate solvation energy for proteins. (Received September 03, 2017)

- 1135-VN-524 **Olusheye A Akinfenwa\*** (akinolu35@yahoo.com), Department of Mathematics, University of Lagos, Akoka, Yaba, Nigeria, and **Ridwanulai I Abduganiy** and **Adebola S. Okunuga**. *Third Derivative Block Multistep Algorithm for solving the Second Order Nonlinear Lane-Emden Type Equations*. Preliminary report.

In this paper we propose a direct solution for some well-known classes of Lane-Emden type equations which are nonlinear second order ordinary differential equations, without converting them into first order system of equations by using a new class of third derivative block multistep method. These methods are derived from a continuous scheme through interpolation and collocation technique and are assemble in block form to produce the numerical solution in the specified interval on the entire range of integration. The properties of the block method is discussed as well as the efficiency of the method by applying them on the some famous Lane-Emden type equations. (Received September 7, 2017)

- 1135-VN-596 **Olaniyi S. Iyiola\*** (niyi4oau@gmail.com), Department of Mathematics, Minnesota State University, Moorhead, MN 56563, and **Emmanuel O. Asante-Asamani** and **Bruce Wade**. *Rational Approximation of the Mittag-Leffler Functions with Real Distinct Poles*.

Mittag-Leffler functions are indispensable in the theory of fractional calculus and many other applications in engineering. However, their computational complexities have made them difficult to deal with numerically. A real distinct pole rational approximation of the two-parameter Mittag-Leffler function is proposed. Under some mild conditions, this approximation is proven and empirically shown to be L-Acceptable. These approximations are especially useful in developing efficient and accurate numerical schemes for partial differential equations of fractional order. Some applications are presented, such as complementary error function and solution of fractional differential equations. (Received September 09, 2017)

1135-VN-655 **Kiwoon Kwon\*** (kwkwon@dongguk.edu), 30, Pildongro 1gil, Jung-Gu, Seoul, 04620, South Korea, and **Sungwhan Moon**. *A numerical method for conical Radon transform with the vertices on a helix.*

Since a Compton camera, also known as an electronically collimated  $\gamma$ -camera, was introduced for use in SPECT, a Radon-type transform assigning the surface integral for a given function over various sets of cones has attracted much interest.

A typical Compton camera consists of a scattering detector and an absorption detector. The recorded energy in each detector gives us to know that the observed photon must have been emitted on the surface of a cone with the vertex on a helix  $\mathbf{u} = (\cos u, \sin u, u)$ , the central axis  $\beta$ , and the scattering angle  $\psi$ , which is called conical radon transform.

Our reconstruction of the density  $f$  from the conical Radon transform  $Cf$  is as follows:

Let  $f \in C^\infty(\mathbf{R}^3)$  have compact support in the cylinder  $\{\mathbf{x} = (x_1, x_2, x_3) : |(x_1, x_2)| < 1\}$ . Then we have

$$f(\mathbf{x}) = \frac{1}{8\pi^4} \int_{S^2} \int_{\mathbf{R}} \int_0^\pi Cf(u, \beta, \psi) \frac{1}{\cos \psi (\mathbf{x} \cdot \beta - g_\beta(u))^3} d\psi dg_\beta(u) dS(\beta),$$

where  $g_\beta(u)$  is some monotone function related with  $k_\beta(u) = \mathbf{u} \cdot \beta$ .

A numerical algorithm and simulation of this reconstruction formula is given. (Received September 18, 2017)

1135-VN-719 **Yingxiang Zhou\*** (yxzhou@udel.edu), 15 Orchard Rd, Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, and **Pak-Wing Fok**. *Inference of transition rates in a birth-death chain from conditional extinction times.*

Consider a birth-death process (BDP) of length  $N + 1$  with general birth-death rates, which has a maximum population of  $N$  and becomes extinct when the population reaches zero. In this talk, a method of recovering the birth-death rates of the BDP from its extinction times (ETs) is presented. Given that the maximum site  $n$  reached by each trajectory is also known, we use the proportion of trajectories that do not exceed  $n$  and corresponding mean ET to recover the birth-death rates sequentially from 1 to  $N$ . In each step, the method focuses on the coefficients of the characteristic polynomial of the matrix that governs the process, relates these coefficients to those in the previous steps in a recurrence relation, and solves for the rates at that site. In general, the initial error will propagate with the site number exponentially. However, with sufficient amount of input data, we can recover the rates with relatively small error. For instance, given 50 million ETs of an 11-site birth-death chain, we can recover the rates with a relative error about 3%. (Received September 13, 2017)

1135-VN-940 **Kevin L. Li\*** (likevin6688@gmail.com), 124 Mitchell Road, Somers, NY 10589, and **Xiaodi Wang** (xiaodiwang1@yahoo.com), 310 Lexington Blvd, Bethel, CT 06801. *Intrusion Detection Algorithm Based On Discrete Wavelet Transform and Support Vector Machines.*

Network security intrusion and attacks are becoming increasingly adaptive. Therefore, an accurate detection system is necessary to protect public or private information. Existing Intrusion Detection Systems (IDSs) are only reliable to detect already known intrusions, as it is a self learning mechanism. Using discrete wavelet transform, we can capture both the frequency of attacks and the location on the server that has been attacked which allows us to categorize the intrusions. By using Support Vector Machines (SVMs) and feature analysis on the NSL-KDD Dataset, the efficiency of the generalization of the SVM can design better classifiers for more efficient IDSs and protect network information. By using a wavelet kernel function, the mechanism has a much more versatile learning curve to increase the chance of decreasing the false negative rate and increase the true positive rate. This method is compared to other models and improvements such as the Radial Basis Function (RBF) with SVMs to analyze the effectiveness of the methodology. (Received September 17, 2017)

1135-VN-982 **Samundra Regmi\*** (sr937857@cameron.edu) and **Parshuram Budhathoki**. *Quantum circuits for arithmetic operations over binary field.* Preliminary report.

Public key cryptography is concerned with cryptographic algorithms that require two separate keys, public and private. Public-key algorithms build on the perceived hardness of certain algorithmic problems, such as the discrete logarithm problem (DLP) and factorization problem. For classical computers these algorithms are assumed to be infeasible for suitably chosen parameters. However, Shor's algorithms offer an efficient solution of these hard problems on quantum computers. One of the critical tasks in implementing these algorithms on quantum computers is the identification (and design) of an efficient quantum circuit and underlying field arithmetic operations.

In this project, we will survey some arithmetic operations over binary field and try to optimize the design of the quantum circuits with respect to the gates, depth and qubits. (Received September 18, 2017)

- 1135-VN-983 **Muhammad Hameed\*** ([mhameed@uscupstate.edu](mailto:mhameed@uscupstate.edu)), Department of Mathematics, USC UPSTATE, 800 University Way, Spartanburg, SC 29316. *Long-wave asymptotic model for deformation and breakup of a fluid thread.*

The influence of surfactant on the breakup of a periodic fluid thread of low viscosity immersed in highly viscous exterior fluid at low Reynolds number is studied. With an aim to better understand the pinch-off dynamics, we use long-wave asymptotic, numerical simulations and experimental studies to investigate the effect of surfactant on the necking and breakup. Evolution equations for the jet interface and surfactant concentration are derived using long wavelength approximations. These one dimensional partial differential equations are solved numerically for given initial interface and surfactant concentration. It is found that the presence of surfactant at the interface retards the pinch-off process. The influence of various physical effects on the breakup process is also investigated. The influence of surface diffusion of surfactant on the thread deformation is studied by varying surface Peclet number. It is found that greater diffusion of surfactant causes the jet to pinch faster. (Received September 18, 2017)

- 1135-VN-1160 **Alrazi M Abdeljabbar\*** ([aabdeljabbar@pi.ac.ae](mailto:aabdeljabbar@pi.ac.ae)), P.O Box 2533, Abu Dhabi, United Arab Emirates. *New double Wronskian solutions for a generalized (2+1)-dimensional Boussinesq system with variable coefficients.*

New generalized (2+1)-dimensional Boussinesq system with variable coefficients has been introduced,

$$u_t + \alpha_1(t)u_{xy} + \alpha_2(t)(uw)_x + \alpha_3(t)v_x = 0,$$

$$v_t + \beta_1(t)(wv_x + 2vu_y + uv_y) + \beta_2(t)(u_xw_y - (u_y)^2) + \beta_3(t)v_{xy} + \beta_4(t)u_{xyy} = 0,$$

where  $w_x = u_y$ .

A double Wronskian solutions has been formulated under certain constraints on the variable coefficients. Hirota differential operator and its properties have been employed to transform the system into a bilinear form. (Received September 20, 2017)

- 1135-VN-1187 **Bryce M. Barclay\*** ([bmbarcl1@asu.edu](mailto:bmbarcl1@asu.edu)), School of Mathematical and Statistical, Sciences Box 871804, Arizona State University, Tempe, AZ 85287-1804. *Comparison of Simulated Models for ADR Systems to Idealized Models with Constant Reaction Propagation Speed.*

In recent years, a theoretical model has been developed to explain the existence of one-sided barriers to reaction front propagation in advection-diffusion-reaction fluid flows. The theoretical model, however, assumes a sharp front and a constant propagation velocity in addition to the background flow. This talk will describe the discrepancies between this theoretical model and a numerically simulated model. We simulate the Fisher-Kolmogorov reaction in a four-gyre doubly periodic background flow, and analyze the difference between the numerical and the theoretical front propagation subject to front curvature and concentration gradient. (Received September 20, 2017)

- 1135-VN-1452 **Mili Shah\*** ([mishah@loyola.edu](mailto:mishah@loyola.edu)). *Calibrating Robotic Systems with Mathematics.*

Robotic systems use a variety of sensors in order to perform tasks that are assigned to them. In order to use the data collected from these sensors, they must be calibrated to a common coordinate frame. This talk will begin with an overview of the mathematical tools needed to setup this calibration problem and will conclude with previous and current research related to solving this problem. (Received September 22, 2017)

- 1135-VN-1528 **Casey J Smith\*** ([cjsmit55@asu.edu](mailto:cjsmit55@asu.edu)). *A Low Dispersion Numerical Scheme for Maxwell's Equations.*

The dispersion errors in numerical solutions to the Maxwell equations depend significantly on the method employed. Third-order Runge-Kutta (RK3) produces fourth-order accuracy in phase errors, whereas the classical Yee scheme generates second-order accuracy. Computational efficiency is also critical when numerically approximating electromagnetic wave propagation over long periods of time. A new time-differencing scheme will be presented. It requires only one evaluation per time-step and produces fourth-order accuracy in phase errors. (Received September 22, 2017)

- 1135-VN-1585 **Lasith Adhikari** ([ladhikari@ucmerced.edu](mailto:ladhikari@ucmerced.edu)), **Reheman Baikejiang** ([rbaikejiang@ucdavis.edu](mailto:rbaikejiang@ucdavis.edu)), **Omar DeGuchy\*** ([odeguchy@ucmerced.edu](mailto:odeguchy@ucmerced.edu)) and **Roummel F. Marcia** ([rmarcia@ucmerced.edu](mailto:rmarcia@ucmerced.edu)). *Non-Convex Shannon Entropy for Photon-Limited Imaging.*

Photon-limited imaging is used in a variety of applications including night vision, astronomy and medical imaging. The key characteristic of the low-photon imaging process is that the measurements at the detector are corrupted by noise and are typically modeled using the Poisson distribution. Under this Poisson model assumption, the

goal is to reconstruct high-dimensional sparse signals from noisy low-dimensional, low-photon count observations. This reconstruction involves a challenging non-linear optimization problem requiring the minimization of the negative Poisson log-likelihood function. In our work, we assume the signal has a sparsity structure we can exploit in the reconstruction. Typically this is done using an  $\ell_1$ -norm penalty. This research builds upon previous work on minimizing the Poisson log-likelihood and incorporates more recent results on the generalized non-convex Shannon entropy function to promote sparsity in solutions. We explore the effectiveness of the proposed approach using numerical experiments involving 1-D signal recovery and medical imaging. (Received September 25, 2017)

1135-VN-1700 **Feng Fu** ([fufeng@gmail.com](mailto:fufeng@gmail.com)), 27 N. Main Street, 6188 Kemeny Hall, Hanover, NH 03755, and **Xingru Chen\***, 27 N. Main Street, Hanover, NH. *Social learning can promote population optimal use of antibiotics.*

Antibiotics treatment of bacterial infections is commonly believed to benefit individual patients and also to suppress the overall epidemic of diseases on the population level. Yet, the presence of high treatment coverage intensifies the selective pressure favoring the emergence of resistant strains. High demand for antibiotics driven by self-interest is not necessarily aligned with the social optimum of antibiotics consumption. Under certain conditions, this can lead to the tragedy of the commons in antibiotics overuse. To address this issue, we propose a mathematical model that incorporates a feedback loop between prescription behavior and the resistance evolution. Our model results provide practical rationale for why we should adjust our prescription usage in response to the underlying dynamics of resistance evolution as well as for how we can increase public awareness of the consequences of resistance in order to take prompt actions to curb the irrational use of antibiotics. (Received September 24, 2017)

1135-VN-1753 **Hum Nath Bhandari\*** ([hum.n.bhandari@ttu.edu](mailto:hum.n.bhandari@ttu.edu)), 2801 Slide Road, Lubbock, TX 79407. *Behavior of the Particle Swarm Optimization Algorithm.* Preliminary report.

We present a deterministic convergence analysis of the particle swarm optimization(PSO) algorithm using contraction mapping principles. Our purpose is to understand the dynamics of the algorithm and the behavior of the particle trajectories under various conditions. We noticed that the PSO update equations can be viewed as a sequence of affine mappings in  $R^K$  with an offset vector based on current personal best and global best positions. We prove that these mappings are in-fact contraction mappings for certain selections of learning parameters and the sequence converges provided personal best and global best positions converge. Our analysis is more general and removes the stagnation assumptions used in previous analyses. We allow the personal best and global best to have infinitely many positions before converging. An example is provided to elucidate the theoretical findings. Moreover, we prove that the example is also a counter example to some of the claims made in previous studies. Currently our analysis is deterministic but we are working to extend the analysis to include stochastic components in the future. (Received September 24, 2017)

1135-VN-1947 **Rachel Antoniette Lewis\*** ([rachel.lewis9312@gmail.com](mailto:rachel.lewis9312@gmail.com)), 2049 Westlake Avenue, Savannah, GA 31405, and **Kira Parker, Sheng Gao, Rong Li and Ehsan Ebrahimzadeh.** *Personalization of Indexed Content via Collaborative Filtering and Topic Modeling.*

The USC Shoah Foundation's Visual History archive hosts a large collection of videotaped interviews with genocide survivors and witnesses, each tagged with a number of keywords describing the content. Existing search tools for the archive rank relevant testimonies with respect to a query regardless of the information about the specific user who makes the query. We designed a collaborative latent semantic model to discover users' underlying preferences, which in turn are used to recommend new relevant testimonies and rank the query results in a personalized fashion. The core idea in ranking testimonies beyond naive relevance matching is to make connections between users' past queries and the testimonies through latent semantic spaces underlying the users' interest and the testimonies' semantic similarities surfaced via keywords. Specifically, we formulate the problem as completing a matrix that describes users' interests in different testimonies by joint factorization of matrices that describe the relevance of index terms to testimonies and user interests respectively. We test our proposed solution on USC Shoah Foundation's historical data and compare our results with the state of the art methods. (Received September 25, 2017)

1135-VN-1953 **Devin Willmott\*** ([devin.willmott@uky.edu](mailto:devin.willmott@uky.edu)), **David Murrugarra** and **Qiang Ye.** *RNA State Inference with Deep Recurrent Neural Networks.*

The problem of determining which nucleotides of an RNA sequence exist in base pairs, which we refer to as RNA state inference, can be approached with a variety of machine learning techniques. Successful state inference of

RNA sequences can be used to gain insights into the related problem of secondary structure inference. Typical tools for this task, such as hidden Markov models, exhibit poor performance in RNA state inference, owing in part to their inability to recognize nonlocal dependencies. Bidirectional long short-term memory (LSTM) neural networks have emerged as a powerful tool that can model global nonlinear sequence dependencies and have achieved state-of-the-art performance on many sequential classification problems. This paper presents a method for RNA state inference centered around convolutions and LSTM networks. Our method achieves highly accurate state inference predictions and significantly outperforms hidden Markov models on a test sets of RNA sequences with a broad range of lengths, MFE accuracies, and nonuniform patterns of paired and unpaired regions. (Received September 25, 2017)

1135-VN-2041 **Parshuram Budhathoki\*** (pbudhath@cameron.edu), 2800 West Gore Blvd., Department of Mathematical Sciences, Cameron University, Lawton, OK. *Quantum Circuits for Multiplication Operation*. Preliminary report.

Quantum circuits for multiplication operation are one of the cost critical tasks in the design of the circuits for many quantum algorithms, such as Shor's algorithm. A basic goal is to keep the qubits, gates, depth and ancilla as low as possible. In the meeting we are going to talk about an efficient quantum circuits for multiplication operation. (Received September 25, 2017)

1135-VN-2058 **Alex Ander Kirvan\*** (akirvan@asu.edu). *A Low Dispersion Numerical Scheme for Nonlinear Electromagnetic Propagation*.

I will describe a low dispersion numerical scheme that approximates solutions to wave equations. The scheme employs spatial and temporal staggering. It is applied to the nonlinear Maxwell Equations. Simulations are performed using two electromagnetic waves propagating through linear and nonlinear media with different frequencies. The nonlinear effect causes secondary wave generation. Comparison of the spectral response of the generated wave with that expected from theoretical arguments demonstrates the accuracy of the scheme. (Received September 25, 2017)

1135-VN-2060 **Melody Alsaker\*** (alsaker@gonzaga.edu), Department of Mathematics, 502 E. Boone Avenue, MSC 2615, Spokane, WA 99258. *Imaging the Human Body using Electrical Impedance Data and a D-bar Algorithm with an Optimized Spatial Prior*.

Electrical Impedance Tomography (EIT) is a promising and versatile imaging technique that uses boundary current and voltage data to reconstruct the internal electrical properties of the body. There are numerous potential applications for EIT, and medical imaging in particular is being studied extensively by research groups around the world. Mathematically, EIT is an extremely ill-posed nonlinear inverse problem, and therefore is extremely sensitive to noise present in real-world data, as well as measurement and modeling errors. This poses many challenges in the reconstruction process, and EIT reconstruction methods are continually studied and improved upon in an effort to stabilize the reconstructions. Direct D-bar methods are one way to perform the mathematical inversion, and recent advances have shown that a carefully constructed spatial prior inserted into the inversion process can improve spatial resolution in the resulting images. In this talk, we present the first-ever human data reconstructions to use a D-bar algorithm with an included spatial prior. Static images and functional EIT videos of human thoracic data will be presented. (Received September 25, 2017)

1135-VN-2091 **Matthew S Mizuhara\*** (mizuharm@tcnj.edu). *Traveling wave solutions in a PDE model of cell motility*.

Motility of cells appears ubiquitously in biological processes ranging from wound healing to cancer metastasis. In particular, the study of crawling eukaryotic cells has been of recent interest to biologists and mathematicians. Their motion is modeled by a 2D phase-field consisting of an Allen-Cahn type PDE coupled with a vectorial reaction-diffusion equation. In the sharp interface limit, the cell's membrane evolves via a non-linear and non-local geometric evolution equation. We establish criteria for both existence and non-existence of traveling wave solutions corresponding to persistently moving cells. This work was completed with Ph.D. adviser Leonid Berlyand in collaboration with Volodymyr Rybalko, Lei Zhang, and Peng Zhang. (Received September 25, 2017)

1135-VN-2095 **Curtis Taylor Peterson\*** (curtistaylorpeterson@gmail.com), AZ, and **Wenbo Tang**. *On the Nature of Advection-Diffusion-Reaction Systems Exhibiting Long-Term Limit Cycles or Stable Asymptotic States at a Bifurcation Point*.

The nutrient-phytoplankton-zooplankton (NPZ) equations describe the time evolution of uniform concentration fields of plankton subject to various degrees of predation and availability of nutrients in the biogeochemical system. This set of equations may exhibit a transition in its long-term behavior at a bifurcation point, such



that the system can either be driven to a stable asymptotic state or a limit cycle. We extend upon the NPZ system by first studying the behavior of interacting chemical species with stable limit cycle behavior subject to a non-uniform background flow field by studying a system of coupled advection-diffusion-reaction (ADR) equations with periodic reaction terms. We show, given some impurity confined in different flow patterns, that our system's long term behavior can reach either a stable asymptotic state or a limit cycle at a bifurcation point. We then investigate our system's behavior in both cases. (Received September 25, 2017)

1135-VN-2102 **Alyssa E. Burgueno\***, 900 S. Palm Walk, Tempe, AZ 85281, and **Rodrigo Platte**.  
*Magnetic Resonance Recovery from Single-Shot Time Dependent Data.*

Magnetic resonance imaging (MRI) captures the alignment of water molecules subject to a magnetic field, which are then computed with an inverse Fourier transform. Traditional MRI, using the standard image recovery model, traverses Fourier space to recover images accurately at the cost of scan time. This project employs a new reconstruction model; though it yields computational inefficiencies, the new model supports fast data acquisition, recovers additional information, and is more suitable to mitigate patient motion. (Received September 25, 2017)

1135-VN-2192 **Alexander Putnam Barnes\*** (apbarnes1@crimson.ua.edu) and **Brendan Ames**.  
*Heuristics of Large-Scale Semidefinite Programming.*

Large-scale semidefinite programming has many applications, including optimal control, computer vision, and machine learning. However, current algorithms for solving semidefinite programs (SDPs) can be time consuming and memory intensive. We look at new heuristics for the solutions of SDPs based on non-convex factorization, the augmented Lagrangian method, and alternating minimization. In particular, we will focus on solutions of semidefinite relaxations for the k-clique and k-cluster programs. We will present numerical results illustrating the efficacy of our approach for clustering of real and simulated data. (Received September 25, 2017)

1135-VN-2213 **Jacob S. Menix\*** (jacob.menix841@topper.wku.edu). *Using Computational Bayesian Statistics to Analyze Parameters in a Differential Equation Model.* Preliminary report.

The purpose of this project is to use Bayesian statistics to analyze values of parameters for a previously developed system of ordinary differential equations which describes the healing process of diabetic foot ulcers. The model describes the relationships between matrix metalloproteinases, their inhibitors, and extracellular matrix. A Bayesian approach is used when the availability of data is sparse, as it is in this case. Delayed Rejection Adaptive Metropolis (DRAM), a MATLAB implementation of a Metropolis-Hastings algorithm, is used to estimate parameters. Using this approach with the individual patient data allows us to refine the parameter estimates, find associated confidence intervals using parameters' posterior distributions, and compare pairwise plots of parameters. This will help improve the wound-healing model in order to better predict wound-healing outcomes for individual patients. (Received September 25, 2017)

1135-VN-2271 **Duane C Harris\*** (dharri31@asu.edu) and **Eric Kostelich**. *Predicting Androgen Resistance in Prostate Cancer Using a Kalman Filter.*

I will describe a study of observing system simulation experiments concerning prostate cancer populations undergoing intermittent androgen suppression therapy. Synthetic data is generated by one model (assumed to be a perfect model of the tumor) and assimilated (using a Kalman filter) by a different model. The talk will assess the effect of systematic model errors on the ability of the filter to predict the progression of the tumor (as measured by PSA levels) over periods of 1-3 months. (Received September 25, 2017)

1135-VN-2274 **Megan K Franke\*** (mkfranke@umail.ucsb.edu) and **Samuel Muthiah** (smuthiah@westmont.edu). *A Convex Realization for an Arbitrary Binary Code.*

Much work has been done to identify which binary codes can be represented by collections of convex open or convex closed sets. While not all binary codes are realizable by collections of either convex open or convex closed sets, in this talk we show that every binary code can be realized by convex sets when there is no restriction on whether the sets are all open or closed. We achieve this by constructing a convex realization for an arbitrary code with  $d$  nonempty codewords in  $\mathbb{R}^{d-1}$ . This result justifies the restriction of the definition of convex neural codes to include only those that can be realized by receptive fields that are all either convex open or convex closed. (Received September 25, 2017)

1135-VN-2346 **Alan Ghazarians\*** (alan.ghazarians@sjsu.edu). *A Numerical Study of the van Roosbroeck System for Semiconductors.* Preliminary report.

In 1950, van Roosbroeck described the fundamental semiconductor device equations as a system of three nonlinear coupled PDEs. The equations include two drift-diffusion equations for electrons and holes and a Poisson equation for the electric potential. This system poses a challenge numerically because of its strong nonlinearity and coupled

equations. Its difficulties lie in simultaneously solving drift-diffusion equations for electrons and holes and using their solutions to solve a Poisson equation. To start off, we will numerically solve the one-dimensional drift diffusion equation with constant velocity using unwinding techniques and illustrate the results using MATLAB for a toy model. Then we will point out the difficulties of solving the drift-diffusion equation when we have a non-constant velocity. This attempt will only complicate solving two drift-diffusion equations and will not suffice in solving the full system. Thus, we will analyze the standard finite difference scheme proposed by Scharfetter and Gummel that deals nicely with the nonlinearity and coupled equations. Then we will compare and contrast the solutions by the standard finite difference scheme proposed by Scharfetter and Gummel and direct discretization of the fluxes in Slotboom variables. (Received September 26, 2017)

1135-VN-2440 **Phylisicia Nicole Carter\*** ([pncarter@crimson.ua.edu](mailto:pncarter@crimson.ua.edu)), 345 Gordon Palmer Hall, Box 870350, Tuscaloosa, AL 35487. *Sparse Regression for Twitter Analysis.*

We consider sparse regression techniques as tools for classification of sentiment within Twitter posts. Analysis of Twitter usage suffers from several unique challenges. For example, the 140-character limit severely limits the amount of information contained in each post; this causes most tweets to contain an extremely small subset of the dictionary, presenting challenges for learning schemes based on dictionary usage. To remedy this undersampling issue, we propose usage of penalized regression. Here, we employ logistic regularization to avoid any degeneracy caused by the sparse usage of the dictionary in each tweet, while simultaneously learning which terms are most associated with each sentiment. As an illustrative example, we employ sparse logistic regression to classify tweets based on the users' perception of a connection between vaccination and autism. Our regression scheme provides a classification function as well as feature selection in the form of a list of words most associated with pro- and anti-vaccination sentiments. (Received September 26, 2017)

1135-VN-2445 **Ghanshyam Bhatt\*** ([gbhatt@tnstate.edu](mailto:gbhatt@tnstate.edu)), 4409 Kirkbrook Ct, Nashville, TN 37221. *From Orthonormal basis to Frames: An introduction.*

Frames generalize an orthonormal basis in a Hilbert space as they are redundant spanning set. Yet the construction of frames is not very straightforward. We present a simple algorithm that produces a set of finite frames. (Received September 26, 2017)

1135-VN-2479 **Sarah Teichman** ([steichman18@amherst.edu](mailto:steichman18@amherst.edu)), **Jing Li\*** ([gracelj@hku.hk](mailto:gracelj@hku.hk)), **Quentin Dubroff** and **Xiaoyu Xie**. *Catching a Falling Ball with Reinforcement Learning.* Preliminary report.

Reinforcement learning is an area of machine learning concerned with teaching an agent how to act so as to maximize a reward within a given environment. In this project, we compared two deep reinforcement learning methods, deep Q-learning and policy based learning, with shallow reinforcement learning, a proposed alternative to these methods that avoids the complicated network architecture of a deep neural network. We applied these methods to design a control strategy for a simple game, in which a ball falls diagonally and a paddle attempts to catch it. We compared the three methods to each other over several combinations of game parameters. We found that with a standard optimizer, Q-learning and policy based learning performed similarly for small game parameters, while shallow learning took more computation to reach a lower performance. For larger game parameters, Q-learning significantly outperformed policy based learning when they both used a standard optimizer, but when we implemented policy based learning with the recently developed Adam optimizer, their performance was comparable. We also visualized the convolutional filters for our deep learning methods and found that the first layer of these networks responded heavily to the direction of the falling ball. (Received September 26, 2017)

1135-VN-2587 **Amanda A Howard\*** ([amanda\\_howard@brown.edu](mailto:amanda_howard@brown.edu)), **Nathaniel Trask** and **Martin R Maxey**. *Simulations of suspension flows with a meshless moving least squares scheme.*

This talk will focus on a meshfree method for simulations of neutrally buoyant, non-Brownian particles in Stokes flow. We will discuss a meshless scheme using Moving Least Squares polynomial reconstructions to provide a computationally efficient method with higher order accuracy for use with general boundary conditions and arbitrary polynomial shapes while maintaining stability. The emphasis will be on applications to dense suspensions of particles, especially particles with polydispersed sizes and non-spherical shapes. Results will be compared to other schemes including the Force Coupling Method. (Received September 26, 2017)

1135-VN-2598 **Kathryn A. Lokken\*** ([klokken@uwm.edu](mailto:klokken@uwm.edu)). *The Firing Squad Synchronization Problem.*

Cellular automata provide a simple environment in which to study global behaviors. One example of a problem that utilizes cellular automata is the Firing Squad Synchronization Problem, first proposed in 1957. This talk

provides an overview of the standard Firing Squad Synchronization Problem, including descriptions of some techniques used in solutions that are known to the problem. Known properties about the problem, such as existing lower bounds to the number of states required in a minimal time solution to the problem are discussed. Current work being done as well as possibilities of future work are discussed, including, but not limited to, work to produce a formal proof of the non-existence of a 4 state solution, constructing a 5 state minimal time solution or proving the non-existence of such a solution, and examining a new variant of the problem where a different type of local neighborhood for the cellular automata is considered. (Received September 26, 2017)

1135-VN-2720     **John T. Nardini\*** ([john.nardini@colorado.edu](mailto:john.nardini@colorado.edu)). *A Biochemically-Structured Fisher's Equation with Applications in Wound Healing.*

Recent biological research has sought to understand how biochemical signaling pathways, such as the mitogen-activated protein kinase (MAPK) family, influence the migration of a population of cells during wound healing. Fisher's Equation has been used extensively to model experimental wound healing assays due to its simple nature and known traveling wave solutions. This partial differential equation with independent variables of time and space cannot account for the effects of biochemical activity on wound healing, however. To this end, we derive a structured Fisher's Equation with independent variables of time, space, and biochemical pathway activity level and prove the existence of a self-similar traveling wave solution to this equation. We also consider a more complicated model with different phenotypes based on MAPK activation and numerically investigate how various temporal patterns of biochemical activity can lead to increased and decreased rates of population migration. (Received September 26, 2017)

1135-VN-2745     **Abdul Hasib Rahimyar\*** ([rahimyar001@connect.wcsu.edu](mailto:rahimyar001@connect.wcsu.edu)), 181 White Street, Danbury, CT 06810, **Hieu Nguyen** ([nguyen084@connect.wcsu.edu](mailto:nguyen084@connect.wcsu.edu)), 181 White Street, Danbury, CT 06810, and **Xiaodi Wang** ([xiaodiwang1@yahoo.com](mailto:xiaodiwang1@yahoo.com)), 181 White Street, Danbury, CT 06810. *Stock Forecasting Using M-Band Wavelet Based Machine Learning Methods.*

The task of predicting future stock values has always been one that is heavily desired albeit very difficult. This difficulty arises from stocks following non-stationary behavior, and thus predictions are best made through analysis of historical stock data rather than through an explicit function. To handle big data sets, current convention involves the use of the Moving Average. However by utilizing the Wavelet Transformed stock signals instead of original stock data or Moving Average-altered data, financial data can be smoothed and more accurately broken down. This newly transformed and more stable stock data set can be followed up by non-parametric statistical methods, such as Principal Component Analysis (PCA), Support Vector Regression (SVR), Correlation and Regression Tree (CART), and Logistic Regression to predict future stock movement. Through the implementation of this method, one is left with a more accurate stock forecast, and in turn, increased profits. (Received September 26, 2017)

1135-VN-2792     **Upama Neupane\*** ([upama.neupane@cameron.edu](mailto:upama.neupane@cameron.edu)), **Gokul Raj Kadel** and **Parshuram Budhathoki**. *Polynomial multiplication over binary field and its implementation.* Preliminary report.

Several cryptographic applications require efficient (time and resources wise) finite field arithmetic specifically arithmetic over binary extension field is often used. In comparison to other arithmetic multiplication contributes most to the total number of bit operations. So, the design of algorithms for binary polynomial multiplication has been of great interest to many mathematician and cryptographer. In this project, we will focus on the state-of-the-art for the polynomial multiplication and its implementation in different applications. (Received September 26, 2017)

1135-VN-2812     **Alfredo N Wetzel\*** ([alfredo.wetzel@wisc.edu](mailto:alfredo.wetzel@wisc.edu)), **Leslie M Smith** and **Samuel N Stechmann**. *Linear Analysis of Moisture Transport Due to Baroclinic Atmospheric Waves.*

In this presentation, the effects of rainfall speed and moisture gradients on the moisture transport of mid-latitude baroclinic waves are examined. These effects are investigated using an idealized quasi-geostrophic model with simplified microphysics. Specifically, a single-phase linearization about a constant velocity shear background state is considered for three fundamental moisture regimes: unsaturated, saturated with no rainfall speed, and saturated with rainfall speed. The relative simplicity of this setup allows us to obtain analytically tractable formulas and determine the dependence of the moisture transport on the moisture parameters. It is shown that the meridional moisture transport, as a function of height, has a mid-column maximum in the case of no rainfall speed and a maximum in the lower or surface troposphere for sufficiently large values of rainfall speed. These

results for different rainfall speed values are then discussed in the context of meridional moisture transport in observational data. (Received September 26, 2017)

1135-VN-2868 **Robben Teufel\*** (robben.teufel1739@myci.csuci.edu), **Xiaofeng Xu** (xxuam@connect.ust.hk), **Guanzhi Wang** (gwangaj@connect.ust.hk) and **Aldo Gael Carranza** (aldogael@stanford.edu). *Intelligent Skincare Assistant: A Deep Learning Approach to Dermatology.*

FitME is an AI company that aims to create an intelligent skincare assistant that makes personalized skin health recommendations based on magnified skin images. We build an image classification network that classifies skin images into six dimensions—water, oil, elasticity, fairness, sensitivity, and porosity—with five gradation levels each. We explore state of the art convolutional neural network (CNN) models in order to achieve an optimal image classification accuracy. Moreover, we utilize image analysis techniques to amplify the distinguishing features of the skin images and improve our classification accuracy. We achieve approximately 70% classification accuracy for non-enhanced oil data and approximately 89% accuracy for enhanced oil data, showing that image enhancement is favorable to increasing accuracy. We also find that using transfer learning to train standard machine learning classifiers achieved equal accuracy, and in some cases better, indicating that CNNs may not offer greater classification ability given a relatively small data set. This work was done at the 2017 Research in Industrial Projects for Students (RIPS) program in Hong Kong and was sponsored by IPAM and the Hong Kong University of Science and Technology. (Received September 26, 2017)

1135-VN-2943 **Susan Margulies\***, margulie@usna.edu, and **Elisabeth Gaar, Daniel Krenn and Angelika Wiegeler.** *Two Optimization-based Approaches for Computational Proofs of Vizing's Conjecture.* Preliminary report.

Vizing's conjecture (open since 1968) relates the size of dominating sets in graphs  $G$  and  $H$  to the size of a dominating set in the product graph  $G \square H$ . In this paper, we formulate Vizing's conjecture itself as two different ideals, thus relating the conjecture to the question of 1) is there a Postivstellensatz or sum-of-squares certificate for a given polynomial, or 2) is the variety associated with the specified ideal empty, i.e., does there exist a Hilbert's Nullstellensatz certificate of infeasibility. In this paper, we present the theoretical structure of both computer-based proofs, and demonstrate the sum-of-squares approach as a problem in semidefinite optimization, and the Hilbert's Nullstellensatz certificate of infeasibility approach as a problem in linear algebra. We also present preliminary computational results. (Received September 26, 2017)

1135-VN-3040 **Brittan Farmer\*** (bafarmer@umn.edu), 127 Vincent Hall, 206 Church Street, Minneapolis, MN 55455, and **Selim Esedoglu and Peter Smereka.** *Crystallization for a Brenner-like potential.*

Graphene is a carbon molecule with the structure of a honeycomb lattice. We show how this structure can arise in two dimensions as the minimizer of an interaction energy with two-body and three-body terms. In the engineering literature, the Brenner potential is commonly used to describe the interactions between carbon atoms. We consider a potential of Stillinger-Weber type that incorporates certain characteristics of the Brenner potential: the preferred bond angles are 180 degrees and all interactions have a cutoff radius. We show that the thermodynamic limit of the ground state energy per particle is the same as that of a honeycomb lattice. We also prove that, subject to periodic boundary conditions, the minimizers are translated versions of the honeycomb lattice. (Received September 26, 2017)

1135-VN-3087 **Khoa D Tran\*** (kdtran3@illinois.edu), 205 E Stoughton St, Unit 22, Champaign, IL 61820. *Stability of Periodic Fixed Points and Invariant Sets of the Modulated Logistic Map.*

In this talk, We are studying the stability of the 1- and 2-periodic fixed points and the invariant sets of the two-dimensional dynamical system proposed by Elhadj and Sprott in [1]. The dynamical system generalizes the logistic map by considering  $F: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $F(x, y) = (ax(1-x), (b+cx)y(1-y))$  for  $a, b$ , and  $c$  in  $\mathbb{R}$ . While the first component is the familiar 1d logistic map with parameter  $a$ , the second component is also a logistic map with a modulated parameter  $(b+cx)$ , dependent on  $x$ . We begin by finding the the periodic fixed points, and determine their stability as a function of  $a, b$ , and  $c$ . In certain parametric regimes, we further analyze the basin of attraction of stable fixed points and fully describe the dynamics of arbitrary initial data. Lastly, we want to numerically and analytically determine the invariant sets of the second component in the domain  $[0, 1]$  as it often represents the valid region for the underlying models. We will detail our analysis using computer graphics and simulations to illustrate our results.

[1]Elhadj, Zeraoulia, and J. C. Sprott. "The effect of modulating a parameter in the logistic map." *Chaos: An Interdisciplinary Journal of Nonlinear Science*. 18.2 (2008): 023119. (Received September 26, 2017)

1135-VN-3114 **Ryan M. Evans\*** ([ryan.evans@nist.gov](mailto:ryan.evans@nist.gov)), NIST, 100 Bureau Drive, Gaithersburg, MD 20899, **Anthony J. Kearsley**, NIST, 100 Bureau Drive, Gaithersburg, PA 20899, **Robert G. Brinson**, IBBR, 9600 Gudelsky Drive, Rockville, MD 20850, **Luke W. Arbogast**, IBBR, 9600 Gudelsky Drive, Rockville, MD 20850, and **Frank Delaglio**, IBBR, 9600 Gudelsky Drive, Rockville, MD 20850. *Classifying Nuclear Magnetic Resonance Spectra of Biologics*.

The term *biologics* refers to a class of medicines derived from living organisms, such as insulin, penicillin, or any vaccine. A very important type of biologic that has recently garnered attention is monoclonal antibodies—these are therapeutic proteins used to treat a wide variety of diseases from rheumatoid arthritis to certain cancers. Recent patent expiration of many name-brand biologics has created the opportunity for pharmaceutical companies to develop less expensive alternatives known as biosimilars. The popularity of biosimilars has created a need for novel analytic techniques to characterize the structure of monoclonal antibodies. Although Nuclear Magnetic Spectroscopy (NMR) has been shown an excellent tool for this task, current analysis of NMR spectra relies on a combination of visual inspection and Principle Component Analysis (PCA). The use of the first Wasserstein metric for comparing spectra will be presented and it will be shown that the pairing of this metric with an appropriate unsupervised learning algorithm enables superior grouping of spectra according to chemical structure. (Received September 26, 2017)

1135-VN-3120 **Ricardo J Cordero-Soto**, 8432 Magnolia Ave, Riverside, CA 92504, **Brittni K Hall\*** ([bkh0018@tigermail.auburn.edu](mailto:bkh0018@tigermail.auburn.edu)), 351 Thach Concourse, Haley Center, Office 5096, Auburn, AL 36849, and **Joseph D Gemignani**, 8432 Magnolia Ave, Riverside, CA 92504. *The Dynamics at the Battle of Kruger: Age Structured Defense in a Buffalo and Lion Predator-Prey Model*. Preliminary report.

Dynamical systems have been used to study predator-prey interactions since Lotka and Volterra both proposed their equations in the 1920's. In some ecological settings, defense from predation by the prey has been observed and modeled. While defense structures have been modeled, age-structured-prey-defense has not been previously modeled to our knowledge. This specific phenomenon has been well-observed with lions and buffalo. The authors present a predator-prey model with adult buffalo, infant buffalo, and lions with age-structured defense by buffalo, by modifying a previous model for buffalo, warthog and lions. The system is analyzed and results, as well as their ecological implications, are subsequently discussed. (Received September 26, 2017)

1135-VN-3186 **Molly Hoch** and **Samuel A Muthiah\*** ([samuel.a.muthiah@gmail.com](mailto:samuel.a.muthiah@gmail.com)), 2056 Sinaloa Avenue, Altadena, CA 91001, and **Nida Obatake**. *On the identification of  $k$ -inductively pierced codes using toric ideals*.

Neural codes are binary codes in  $\{0, 1\}^n$ ; here we focus on the ones which represent the firing patterns of a type of neurons called place cells. There is much interest in determining which neural codes can be realized by a collection of convex sets. However, drawing these convex sets, particularly as the number of neurons in a code increases, can be very difficult. It has been shown that an algorithm for drawing Euler diagrams can be used to draw a class of codes that are said to be  $k$ -inductively pierced for  $k = 0, 1, 2$ . We use the toric ideal to show sufficient conditions for a code to be 1- or 2-inductively pierced, so that we may use the existing algorithm to draw realizations of 2-inductively pierced codes. (Received September 27, 2017)

1135-VN-3194 **Michael T Stobb\*** ([mstobb@ucmerced.edu](mailto:mstobb@ucmerced.edu)), UC Merced, 5200 Lake Rd, Merced, CA 95340, and **Suzanne Sindi** and **Karin Leiderman**. *Using Uncertainty Quantification to Assess the Significance of Product Inhibition in Biochemical Assays*.

Blood coagulation is a complex network of biochemical reactions that is necessary for blood clot formation. Many of the coagulation reactions are mediated by enzymes to accelerate the rate at which they occur. Chromogenic substrates are frequently used to measure the activity of specific enzymes, but often the exact dynamics of the reaction are not well understood. In this work, we combine mathematical modeling with experimental data and determine the product created by the chromogenic substrate inhibits the reaction it was developed to measure. In our approach, we develop an ODE model and incorporate uncertainties in biochemical reaction rates and initial conditions. We use the output from this model to assess the likelihood of our experimental data. The posterior distribution for the parameter corresponding to inhibition suggests product inhibition plays a significant role in dynamics. Our findings suggest that when experiments use chromogenic substrates, in particular long time assays as in coagulation systems where large amounts of product are generated, the role of product inhibition should be considered. This work shows that precise quantitative agreement is attainable between experiments

and mathematical models when both are designed and developed simultaneously. (Received September 27, 2017)

1135-VN-3225 **Emily Hendryx\*** ([emily.hendryx@rice.edu](mailto:emily.hendryx@rice.edu)), **Beatrice Riviere**, **Craig Rusin** and **Danny Sorensen**. *An Extended DEIM Algorithm for Subset Selection*. Preliminary report.

With large data sets becoming more prevalent, there is an increased demand for dimension reduction techniques. One approach to this problem is to select a subset of original samples using the discrete empirical interpolation method (DEIM), preserving the interpretability of the dimension-reduced data set. However, the number of DEIM-selected samples is limited to be no more than the rank of the original data matrix. While this is not an issue for many data sets, there are a number of settings in which this can limit the algorithm's potential for selecting a subset that contains representatives from each class present in the data. In the presented work, we address this issue through an extension of the DEIM algorithm that allows for the selection of a subset with size greater than the matrix rank. (Received September 27, 2017)

1135-VN-3230 **Dimplekumar N Chalishajar\*** ([dipu17370@gmail.com](mailto:dipu17370@gmail.com)), 1227 Landon Dr., Harrisonburg, VA 22801. *Exact and Trajectory Controllability of Nonlinear Fractional order systems with deviated arguments with Infinite delay*.

In this research we studied the existence and controllability results of the nonlinear fractional order system with infinite delay with deviated argument. We have used the monotone operator theory to prove the results and fixed point theorem. Examples are given to illustrate the theory. Numerical estimation is the addition work for the same and we compared the results. (Received September 27, 2017)

## General Session on Geometry

1135-VO-815 **J Mealy\*** ([jmealy@austincollege.edu](mailto:jmealy@austincollege.edu)), 900 North Grand Avenue, suite 61560, Sherman, TX 75090, and **Congling Zhang** ([czhang14@austincollege.edu](mailto:czhang14@austincollege.edu)), 900 North Grand Avenue, suite 60207, Sherman, TX 75090, Peoples Rep of China. *Tailoring geodesics in geometries with smooth metrics, and in geometries with staircase metrics*.

After a brief discussion of the nature and generality of the category, staircase metric geometry, as well as its accompanying methodology, we utilize this framework to set metrics featuring geodesics that, in the parameter space view, mimic curves with familiar properties. We do the same in the differentiable category, and with similarly tracked geodesics; comparisons between the two schemes are discussed. As time allows, a further broadening of the SMG category via the utilization of yet more general metrics on the various sub-domains would be discussed. (Received September 14, 2017)

1135-VO-1345 **L.-H. Huang** and **D. Martin\***, 52 Mount Vernon Drive, Apartment C, Vernon, CT 06066, and **P Miao**. *Static Potentials and Area Minimizing Hypersurfaces*.

We show that if an asymptotically at manifold with horizon boundary admits a global static potential, then the static potential must be zero on the boundary. We also show that if an asymptotically at manifold with horizon boundary admits an unbounded static potential in the exterior region, then the manifold must contain a complete non-compact area minimizing hypersurface. (Received September 21, 2017)

1135-VO-1683 **Yoav Len\*** ([yoav.len@uwaterloo.ca](mailto:yoav.len@uwaterloo.ca)). *Curves, graphs, and tangent lines*.

I will discuss applications of tropical geometry to enumerative problems involving tangency conditions. I will explain how to associate a graph to an algebraic curve, in a way that encodes many of its geometric properties. Using appropriate lifting theorems, statements about intersections of curves can be reformulated in terms of combinatorial problems on graphs. As I will show, such techniques can be used to count bitangent lines to curves, and uncover combinatorial aspects of projective dual curves. (Received September 24, 2017)

1135-VO-1745 **Ralph Morrison\*** ([10rem@williams.edu](mailto:10rem@williams.edu)). *Tropical hyperelliptic curves in the plane*.

Classically, any hyperelliptic curve admits a non-degenerate planar representation with an equation of the form  $y^2 = f(x)$ . I will show that tropically, things are far more restrictive: tropical hyperelliptic curves in the plane are limited to a basic type of graphs called chains, with nontrivial conditions on the achievable metrics. (Received September 24, 2017)

1135-VO-2047 **Vance T. Blankers\*** (blankers@math.colostate.edu). *Hyperelliptic classes are rigid and extremal in genus two.*

A hyperelliptic class on a moduli space of pointed curves is the Chow class of the closure of the locus of hyperelliptic curves with  $\ell$  marked Weierstrass points,  $m$  marked conjugate pairs of points, and  $n$  free marked points. We show that in genus two such classes are rigid and extremal in the cone of effective codimension- $(\ell+m)$  classes on  $\overline{\mathcal{M}}_{2,\ell+2m+n}$  using the rich recursive structure of the relevant moduli spaces. Our result establishes an infinite family of rigid and extremal classes in arbitrarily-high codimension. (Received September 25, 2017)

1135-VO-2094 **Oscar Vega\*** (ovega@csufresno.edu). *Unitals in Figueroa planes.* Preliminary report.

Most of what is known about unitals is on those that can be embedded in  $PG(2, q^2)$ . However, unitals may be found in a wide variety of finite projective planes. Lately, quite a few results have appeared discussing the geometry of unitals embedded in ‘unusual’ planes, like the Figueroa plane of order  $q^6$ ,  $\mathcal{F}_{q^6}$ , which is not even a translation plane.

Although it was known since 1998 (de Resmini and Hamilton) that  $\mathcal{F}_{q^6}$  admitted the so-called Figueroa unital, it was not until 2012 that Hui and Wong were able to prove that this unital is not classical. No other unitals are known to exist in  $\mathcal{F}_{q^6}$ .

In this presentation we will explore the existence of unitals in  $\mathcal{F}_{q^6}$  that are not the Figueroa unital. (Received September 25, 2017)

1135-VO-2198 **John Shier and Douglas Dunham\*** (ddunham@d.umn.edu), Department of Computer Science 320 HH, 1114 Kirby Drive, Duluth, MN 55812-3036. *A Property of Area and Perimeter.* Preliminary report.

We describe an algorithm for filling a region of the plane with progressively smaller copies of a motif. For simplicity we take the region to be a circle and the motifs to be discs, though the algorithm can be naturally modified to work with other shapes. After placing the first  $i$  discs, random locations are tried for a placement of the next disc until a position is found such that the disc does not intersect any previously placed disc. After having placed  $i$  discs, we call the remainder of the bounding circle the *gasket*. At this point we let  $A_i$  and  $P_i$  be the area and perimeter (boundary) of the gasket respectively. Thus  $A_i$  decreases and  $P_i$  increases with increasing  $i$ . We choose the radius of the next disc by  $r_{i+1} = \gamma(A_i/P_i)$ , where  $\gamma$  is a dimensionless parameter between 0 and 2 that is chosen *a priori*. As  $\gamma$  approaches 2, it becomes more likely that the algorithm will halt, but it rarely halts for  $\gamma = 3/2$ . By examining log-log plots of the areas of the discs versus  $i$ , which seems to be linear for large  $i$ , we conjecture that the areas of the discs obey an inverse power law. That power  $c$  seems to be given by the equation  $c = -(4 + 2\gamma)/(4 + \gamma)$  (verified to several significant digits). (Received September 25, 2017)

1135-VO-2633 **Sonja Hohloch, Yohann Le Floch and Joseph Palmer\***, j.palmer@math.rutgers.edu. *Explicit constructions of integrable systems of semitoric type.*

A semitoric integrable system is a four-dimensional integrable system for which one of the integrals is assumed to have periodic Hamiltonian flow, corresponding to a circular symmetry of the system. Since their introduction by Pelayo and Vũ Ngọc semitoric integrable systems have been the subject of a large amount of research, but so far what has been missing a large collection of examples. In this talk we remedy this by both providing explicit examples of such systems and by showing how a large class of semitoric integrable systems can be constructed in general. (Received September 26, 2017)

1135-VO-2637 **Casey Mann** (cemann@uw.edu), School of STEM, 18115 Campus Way N.E., Box 358538, Bothell, WA 98011, and **Jadie Adams, Gabriel Lopez\*** (gabriel.lopez.x15@gmail.com) and **Nhi Tran**. *Your Friendly Neighborhood Voderberg Tile.* Preliminary report.

In 1936, H. Voderberg demonstrated an example of a tile (the Voderberg tile) with the amazing property that two copies can enclose a single copy (or two copies) of the tile without gaps or overlapping. Further, this tile gives rise to periodic tilings of the plane as well as striking nonperiodic spiral tilings. In this talk, we will discuss a generalization of the Voderberg tile having several amazing properties, a few of which answer long-standing open questions from Grünbaum and Shephard's *Tilings and Patterns*. (Received September 26, 2017)

1135-VO-2646 **Andrew Lazowski\*** (lazowskia@sacredheart.edu), Department of Mathematics, SC200F, 5151 Park Ave., Fairfield, CT. *Exploring Exceptional Points For Fuchsian groups.*

Let  $G$  be a Fuchsian group acting on the hyperbolic plane. If  $G$  is cocompact then almost every Dirichlet region has  $12g - 6$  sides where  $g \geq 2$  is the genus of the surface. It is known that exceptional points for  $G$ , points in hyperbolic space where the associated Dirichlet region has less than  $12g - 6$  sides, not only exist but are uncountable. Here we will consider the case where  $G$  is not cocompact. This is joint work with Joe Fera. (Received September 26, 2017)

1135-VO-2661 **Iwan Praton\*** (ipraton@fandm.edu), Department of Mathematics, Franklin & Marshall College, Lancaster, PA 17604. *Maximal tilings with the minimal tile condition*. Preliminary report.

Fix a positive integer  $n \neq 2, 3, 5$ . There are many ways to tile a unit square with  $n$  smaller squares. For each such tiling  $T$ , define  $S(T)$  to be  $s_1 + \dots + s_n$ , where  $s_i$  is the side length of the  $i$ th small square. Erdos and Soifer introduced the problem of finding  $F(n)$ , the maximum value of  $S(T)$ . Although they gave precise conjectured values for  $F(n)$ , the problem is still unsolved. Later on, Alm introduced a minimal tile condition on the tilings  $T$  and asked whether it is possible to determine the maximum value of  $S(T)$  over all tilings  $T$  that satisfy the minimal tile condition; the hope is that the problem becomes more computationally tractable. This talk gives an overview of the problem and focuses on the case  $n = k^2 + 3$ : in that case, under certain technical conditions, the maximum value is  $k + 1/k$ , confirming the Erdos-Soifer value. (Received September 26, 2017)

1135-VO-3077 **Allan Berele** (aberele@depaul.edu), Department of Mathematics, DePaul University, Chicago, IL 60614, and **Stefan Catoiu\*** (scatoiu@depaul.edu), Department of Mathematics, DePaul University, Chicago, IL 60614. *The Perimeter Bisecting Deltoid of a Triangle*.

We introduce a new curve: the “perimeter-bisecting deltoid of a triangle” is the envelope of all lines that bisect its perimeter. This is a six-sided curve in the shape of the Greek letter delta consisting of three line segments and three segments of parabolas. We describe this curve both as analytic and as geometric locus, compute the area enclosed by it, and classify the points of the triangle according to the number of distinct perimeter-bisecting lines through them. (Received September 26, 2017)

## General Session on Graph Theory

1135-VP-284 **Jang Hun Choi\***, Roosevelt Hall 309B, Hempstead, NY 11549, and **Celine Lee, Soohyun Ahn and Junwon Lee**. *The Hadwiger-Nelson problem with two forbidden distances*.

The chromatic number of the plane problem, named after Hugo Hadwiger and Edward Nelson, asks for the minimum number of colors required to color the plane such that no two points at distance 1 from each other have the same color. It is easy to show that at least four colors are needed, while seven colors are sufficient.

We consider the following related question:

For a given  $d > 1$ , what is the minimum number of colors the plane can be colored with such that no two points at distance 1 or  $d$  from each other can be assigned the same color?

In this context, we say that 1 and  $d$  are forbidden distances. We prove that at least five colors are needed if  $d$  takes any of the following values:

$$\sqrt{2}, \sqrt{3}, (\sqrt{5} + 1)/2, (\sqrt{6} + \sqrt{2})/2, 2, 2/\sqrt{3}, (2 + 2\sqrt{3} + 2\sqrt{2} \times 31/4)1/2/2$$

Our methods are constructive: we find finite point sets in the plane which require five colors if no two points at distance 1 or  $d$  from each other can be assigned the same color. We also point out how this problem can lead to a proof of the fact that the chromatic number of the plane is at least 5. (Received August 18, 2017)

1135-VP-328 **Patrick Bahls\*** (pbahls@unca.edu), CPO #2350, One University Heights, Asheville, NC 28804. *The unimodality of the independence polynomials of trees with non-regular structure*.

The *independence polynomial*  $I(G, x)$  of a graph  $G$  is the polynomial in variable  $x$  in which the coefficient  $a_n$  on  $x^n$  gives the number of independent subsets  $S \subseteq V(G)$  of vertices of  $G$ . We say that  $I(G, x)$  is *unimodal* if there is an index  $\mu$  such that  $a_0 \leq a_1 \leq \dots \leq a_{\mu-1} \leq a_\mu \geq a_{\mu+1} \geq \dots \geq a_{d-1} \geq a_d$ . While the independence polynomials of many families of graphs with highly regular structure are known to be unimodal, little is known about less regularly-structured graphs. We analyze the independence polynomials of a large infinite family of trees without regular structure and show that these polynomials are unimodal through a combinatorial analysis of the polynomials' coefficients. (Received August 24, 2017)

1135-VP-335 **Brian G. Kronenthal\*** (kronenthal@kutztown.edu). *Algebraically defined graphs of girth eight*. Preliminary report.

For a field  $\mathbb{F}$  and polynomials  $f, g \in \mathbb{F}[x, y]$ , the partite sets  $P$  and  $L$  of a three-dimensional algebraically defined (bipartite) graph are each copies of  $\mathbb{F}^3$ , and  $(p_1, p_2, p_3) \in P$  and  $[\ell_1, \ell_2, \ell_3] \in L$  are adjacent if and only if  $p_2 + \ell_2 = f(p_1, \ell_1)$  and  $p_3 + \ell_3 = g(p_1, \ell_1)$ . Of interest is whether for a particular field there exist nonisomorphic girth eight algebraically defined graphs; this question was originally motivated by the study of generalized quadrangles. In this talk, we will discuss results over several fields of interest. (Received August 24, 2017)



1135-VP-567 **John Asplund\*** (jasplund@daltonstate.edu), 650 College Drive, Dalton, GA 30736, and **Brad Fox.** *The Minimum Coprime Number and Graph Operations.*

Graph labeling is a large topic of research as evidenced by the seminal 430+ page survey on the topic by Joe Gallian. In this talk, I will focus on one small sector of this larger topic: prime labelings. We say a graph has a *prime labeling* if we can label the vertices of a graph of order  $n$  with distinct labels from  $\{1, 2, \dots, n\}$  so that the labels on adjacent vertices are relatively prime.

Many graphs are not prime, including all but one of the complete graphs. To be as inclusive as possible, we will primarily discuss coprime labelings in this talk. A *coprime labeling* of a graph is the same thing as a prime labeling except we use the labels  $\{1, 2, \dots, m\}$  for some  $m > n$  instead of the labels  $\{1, 2, \dots, n\}$ . To make this more interesting, we care about making  $m$  as small as possible and call a labeling of the vertices of a graph using distinct  $m$  positive integers ( $m$  is minimized) with relatively prime adjacent vertex labels a *minimum coprime labeling*. Finding this minimum coprime labeling will be our main focus in this talk. A secondary focus of this talk will be to discuss some of the faults within the literature on prime labeling. (Received September 08, 2017)

1135-VP-595 **Monsikarn Jansrang\*** (jansr1m@cmich.edu). *Graph Complement Conjecture for Minimum Semidefinite Rank.* Preliminary report.

Given an  $n \times n$  Hermitian matrix  $A = [a_{ij}]$  we associate a graph  $G(A)$  to the matrix  $A$  in such a way that the set of vertices is  $\{v_1, \dots, v_n\}$  and the set of edges is  $E = \{\{v_i, v_j\} : a_{ij} \neq 0, i \neq j\}$ . The diagonal entries of  $A$  do not have an effect on  $G(A)$ . Let  $P(G) = \{A \in M_n(\mathbb{C}) : A^* = A, A \text{ is positive semidefinite, } G(A) = G\}$ . The *minimum semidefinite rank* of  $G$  is defined to be  $mr_{\mathbb{C}}^{\pm}(G) = \min\{\text{rank}(A) : A \in P(G)\}$ . If we restrict to real symmetric positive semidefinite matrices the real minimum semidefinite rank is denoted by  $mr_{\mathbb{R}}^{\pm}(G)$  and it is clear that  $mr_{\mathbb{C}}^{\pm}(G) \leq mr_{\mathbb{R}}^{\pm}(G)$ .

It has been conjectured that  $mr_{\mathbb{R}}^{\pm}(G) + mr_{\mathbb{R}}^{\pm}(\overline{G}) \leq |G| + 2$  where  $\overline{G}$  is the complement of the graph  $G$  and  $|G|$  is the number of vertices in  $G$ . This conjecture is called “Graph Complement Conjecture” and is denoted  $GCC_{\pm}$ . In this talk we will mention some known results on  $GCC_{\pm}$  and some new results about certain bipartite graphs. (Received September 24, 2017)

1135-VP-624 **Sooyeon Lee\*** (slee27@olemiss.edu) and **Haidong Wu.** *Beta invariants of 3-connected matroids.*

Beta invariants have been studied by Crapo, Brylawski, Oxley and others. Crapo showed that a matroid with at least two elements is connected if and only if its beta invariant is greater than zero. Brylawski showed that a connected matroid has beta invariant one if and only if  $M$  is isomorphic to a serial-parallel network. Oxley characterized all matroids with beta invariants 2, 3 and 4. In this paper, we first give a best possible lower bound on the beta invariants of 3-connected matroids, then we characterize all 3-connected matroids attaining the lower bound. We also completely characterize all binary matroids with beta invariants 5, 6, and 7. Lastly, we provide alternate proofs for the known results in polynomial invariants of graphs using the beta invariant. (Received September 11, 2017)

1135-VP-638 **Jason J Moliterno\*** (moliternojs@sacredheart.edu), Department of Mathematics, Sacred Heart University, 5151 Park Avenue, Fairfield, CT 06825. *Entries of the group inverse of the Laplacian matrix for generalized Johnson graphs.*

Given a graph on  $n$  vertices, the Laplacian matrix  $L$  is the matrix  $D - A$  where  $D$  is the diagonal matrix of the vertex degrees and  $A$  is the traditional adjacency matrix. The Laplacian matrix is singular since the vector of all ones is an eigenvector corresponding to the eigenvalue of zero. However, the group inverse  $L^{\#}$  is known to exist. In this talk, we use graph theoretic and combinatorial properties of generalized Johnson graphs to compute the entries of the group inverse of the Laplacian matrix for such graphs. Concepts such as spanning trees and spanning forests for both directed and undirected graphs will be essential in these computations. (Received September 11, 2017)

1135-VP-807 **Katie Anders\*** (kanders@uttyler.edu) and **Kassie Archer.** *Enumerating unimodal rooted forests avoiding the pattern 321.*

We say that an unordered rooted labeled forest avoids the pattern  $\pi \in S_n$  if the sequence obtained from the labels along the path from the root to any vertex does not contain a subsequence that is in the same relative order as  $\pi$ . Forests that avoid  $\{312, 213\}$  are unimodal forests. We enumerate the class of unimodal forests avoiding the pattern 321 via a bijection with the set of ordered partitions of  $[n]$ . (Received September 14, 2017)

- 1135-VP-958 **Sergei Bezrukov, Pavle Bulatovic and Nikola Kuzmanovski\***, kuzmanovski07@hotmail.com. *New families of edge-isoperimetric graphs*. Preliminary report.

We present new infinite families of regular graphs whose all cartesian powers admit nested solutions in the edge-isoperimetric problem. For a given graph the problem is to specify a subgraph of a given order  $m$  that has max number  $I(m)$  of induced edges among all subgraphs of order  $m$ . Our results include as special cases most previously published results in this area. The graphs are specified by delta-sequences of the length given by the number of vertices in the graph. The  $m$ -th element of the sequence is  $d(m)=I(m)-I(m-1)$ . It is known that  $d(m+1)$  does not exceed  $d(m)+1$ . We emphasize on delta-sequences that have several monotonically increasing segments of the same length, for example, 0,1,2,2,3,4,4,5,6 for a sequence with 3 segments of length 3 each. We show that by ordering the vertices of the  $n$ -th cartesian power of our graphs lexicographically, the subgraph induced by any initial segment of this order spans max number of edges. Previously such results were only known for graphs/sequences with just 2 monotonic segments. Based on a special representation of graphs as a union of disjoint cliques we introduce new technique for extending a graph admitting nested solutions in the edge-isoperimetric problem. These results can be applied to the bisection width or wirelength problems. (Received September 17, 2017)

- 1135-VP-1067 **Darren A Narayan\*** (dansma@rit.edu), School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623. *Shortest paths and centrality in circulant graphs*.

The edge betweenness centrality of an edge  $e$  in a graph  $G$ , denoted  $B'_G(e)$ , measures the frequency at which  $e$  appears on a shortest path between two distinct vertices  $x$  and  $y$ . If the values for  $B'_G(e)$  (over all edges in  $G$ ) can be partitioned into  $k$  different groups then  $G$  is said to have  $k$ -uniform edge betweenness centrality. We investigate which circulant graphs have  $k$ -uniform edge centrality where  $1 \leq k \leq 3$ . Furthermore, for certain subclasses, we precisely determine the different edge betweenness values. (Received September 19, 2017)

- 1135-VP-1176 **Éva Czabarka, Kayvan Sadeghi, Johannes Rauh, Taylor Short\*** (shorttay@gvsu.edu) and **László Székely**. *The maximum number of non-zero elements in a joint degree vector*.

The joint degree vector (JDV) of a  $n$ -vertex graph gives the number of edges between vertices of degree  $i$  and degree  $j$  for  $1 \leq i < j \leq n-1$ . The number of non-zero entries in the JDV of a graph provides an upper bound on the number of estimable parameters in the exponential random graph model with bidegree distribution as its sufficient statistic. Determining the maximum number of non-zero entries of the JDV over all  $n$ -vertex graphs seems quite challenging. We find lower and upper bounds for this quantity and discuss room for improvement. (Received September 20, 2017)

- 1135-VP-1233 **Karolyne Fogel\***, kfogel@callutheran.edu, and **Aparna Higgins, William Higgins and John Villalpando**. *Irreducible  $L(2,1)$ -Colorings for Products of Paths and Cycles*. Preliminary report.

An  $L(2,1)$ -coloring of a graph is a labeling of the vertices using non-negative integers such that adjacent vertices differ in label by at least 2 and distance two vertices differ in label. An  $L(2,1)$ -coloring of a graph is irreducible if reducing the label on any vertex violates an  $L(2,1)$ -coloring condition. The invariant icap is the least number of color classes required to create an irreducible  $L(2,1)$ -coloring on a given graph. We determine the value of icap for  $P_2 \square C_n$  and examine bounds for icap of  $P_m \square C_n$  for other values of  $m$  and  $n$ . (Received September 20, 2017)

- 1135-VP-1236 **J Villalpando\*** (jvillalp@callutheran.edu), **K Fogel, A Higgins and W Higgins**. *Minimizing the number of labels for an irreducible  $L(2,1)$ -labeling on the Cartesian product of two cycles*. Preliminary report.

An  $L(2,1)$ -labeling of a graph is a labeling of the vertices using non-negative integers such that adjacent vertices differ in label by at least two and distance two vertices differ in label. An  $L(2,1)$ -labeling of a graph is irreducible if reducing the label on any vertex violates an  $L(2,1)$ -labeling condition. The invariant icap of a graph is the least number of labels required to create an irreducible  $L(2,1)$ -labeling on the graph. We study the icap number of the Cartesian product of two cycles,  $C_n \square C_m$ . We determine the icap number when  $n = 3$  and  $m$  is even and when  $n$  and  $m$  are multiples of five. We determine bounds for other values of  $m$  and  $n$ . (Received September 20, 2017)

1135-VP-1548 **Pallavi Mishra\***, Department of Mathematics, IIT Kharagpur, Kharagpur, India, and **Dharmendra Kumar Gupta**, Department of Mathematics, Indian Institute of Technology, Kharagpur, Kharagpur, India. *A graph-based approach for counting all Sudoku squares of rank  $n$ .*

This paper deals with a graph-based approach for counting all Sudoku squares of rank  $n$ . First, all the S-permutations are generated and an S-permutation graph  $G_s = (V_s, E_s)$  is constructed in which vertices represent S-permutations and two vertices are connected by an edge if and only if their corresponding S-permutations are not disjoint to each other. A set of mutually disjoint S-permutations corresponds to an independent set of  $G_s$ . A vertex  $v \in V_s$  is selected randomly and an induced subgraph  $\hat{G}_s = (\hat{V}_s, \hat{E}_s)$  of  $G_s$  is derived by considering all mutually disjoint vertices to  $v$ . There is a one to one correspondence between a maximum independent set of  $\hat{G}_s$  together with  $v$  and a Sudoku square. Now, an algorithm is developed to count all the maximum independent sets of  $\hat{G}_s$  which are equal to all Sudoku squares of rank  $n$ . The correctness of the algorithm is shown and its time complexity is  $O(3^{\frac{\xi_n}{3}})$ , where  $\xi_n$  is the total number of S-permutations mutually disjoint to an S-permutation. The algorithm is experimentally tested for Sudoku squares of rank up to 3. An upper bound on the total number of Sudoku squares is also derived. (Received September 23, 2017)

1135-VP-1590 **Alexis Byers\*** (alexis.d.byers@wmich.edu). *Graceful Colorings of Graphs.*

A graceful labeling of a graph  $G$  of order  $n$  and size  $m$  is a one-to-one function  $f : V(G) \rightarrow \{0, 1, \dots, m\}$  that induces a one-to-one function  $f' : E(G) \rightarrow \{1, 2, \dots, m\}$  defined by  $f'(uv) = |f(u) - f(v)|$ . A graph that admits a graceful labeling is a graceful graph. A proper coloring  $c : V(G) \rightarrow \{1, 2, \dots, k\}$  is called a graceful  $k$ -coloring if the induced edge coloring  $c'$  defined by  $c'(uv) = |c(u) - c(v)|$  is proper. The minimum positive integer  $k$  for which  $G$  has a graceful  $k$ -coloring is its graceful chromatic number. The graceful chromatic numbers of cycles, wheels and caterpillars are determined. An upper bound for the graceful chromatic number of trees is determined in terms of its maximum degree. We also present recent results and open questions in this area of research. (Received September 23, 2017)

1135-VP-1894 **James M Hammer\*** (jmhammer@cedarcrest.edu), 100 College Drive, Allentown, PA 18104, and **Joshua Harrington**. *Graph Polynomials for a Class of DI-Pathological Graphs.*

Let  $G = (V, E)$  be a simple graph. A dominating set  $D \subseteq V$  is a set such that the closed neighborhood of  $D$  is the entire vertex set. An independence set of a graph  $G$  is a subset of vertices that are pairwise non-adjacent. A DI-pathological graph is a graph where every minimum dominating set intersects every maximal independent set. Let  $d(G, i)$  denote the number of dominating sets of  $G$  of size  $i$ . The domination polynomial of a graph  $G$  is defined by  $D(G, x) = \sum_{i=\gamma(G)}^{|V|} d(G, i)x^i$ . Let  $s(G, i)$  denote the number of independent sets of size  $i$  in a graph  $G$ . The independence polynomial is defined by  $I(G, x) = \sum_{i=0}^{\alpha(G)} s(G, i)x^i$ . We will examine the domination polynomial and the independence polynomial of an extremal family of DI-pathological graphs. We will further define an independent dominating set and examine the corresponding independent domination polynomial for these graphs. (Received September 25, 2017)

1135-VP-2007 **Pamela Kirkpatrick\*** (prk213@lehigh.edu). *Structural considerations for interval orders with length constraints.*

Interval orders are a class of partially ordered sets (posets), each element of which can be represented by an interval on the real line such that the interval corresponding to  $x$  lies completely to the left of the interval corresponding to  $y$  if and only if  $x$  precedes  $y$  under the poset relation. We will look at variations of interval orders which include length constraints for the intervals. In 1985, Fishburn published a list of forbidden suborders which prevent a partially ordered set from having an interval representation with lengths in  $[p, q]$ , for positive integers,  $p, q$ . Using tools from weighted digraphs, we present an alternative approach to this problem and provide a list of *minimal* forbidden suborders for interval lengths in  $[2, q]$ . (Received September 25, 2017)

1135-VP-2141 **Radmila Sazdanovic** and **Daniel Scofield\***, dscofie@ncsu.edu. *Chromatic graph homology: structure and computations.*

Chromatic homology, introduced by Helme-Guizon and Rong, is a way of lifting the chromatic polynomial to a homology theory. Chromatic homology admits a long exact sequence as an analogue of the deletion-contraction formula, and the choice of algebra determines the number of colors. Chromatic homology over the algebra  $\mathbb{Z}[x]/(x^2)$  is determined by the chromatic polynomial. However, over  $\mathbb{Z}[x]/(x^3)$  chromatic homology is strictly stronger than the chromatic polynomial, and we show that it can also distinguish graphs with the same Tutte polynomial and cycle matroid. To provide insight into this phenomenon, we describe some of the structural

properties of chromatic homology in terms of combinatorial data for graphs such as the cyclomatic number and the number of blocks. (Received September 25, 2017)

1135-VP-2143 **Bob Kuo\*** (bob.kuo@coloradocollege.edu), 902 N Cascade Ave, WB 896, Colorado Springs, CO 80946, **Leah S. Miller** (leah.miller@gordon.edu), 122 Porcupine Ct., Bushkill, PA 18324, **Vanessa Gomez** (gomezvga@gmail.com), 507 Glenrock Ave Suite 100, Los Angeles, CA 90024, and **Katie Bruegge** and **Boyang Su**. *Partially Restricted Vertex and Edge Connectivity*.

In this paper, we extend the study of connectivity for optimization of theoretical network reliability measures given specific conditions. Expanding on and combining the ideas of restricted and component order connectivities, we define *Partially Restricted Connectivity* in terms of both edges and vertices. We establish formulas and bounds for the partially-restricted connectivities of different classes of graphs including paths, cycles, trees, and complete graphs. Finally, we present an analysis of the reliability of graphs in the general class  $G(n, m)$  in relation to this new definition. (Received September 25, 2017)

1135-VP-2211 **Caitlin M. Owens\*** (cmo312@lehigh.edu), Lehigh University Math Department, Christmas-Saucon Hall, 14 E Packer Ave., Bethlehem, PA 18015. *HP, or not 2HP, that is the question*.

It is known that 2-trees are Hamiltonian if and only if they are 1-tough. However, the analogous statement for Hamiltonian paths does not hold. I will present an infinite class of 1-path-tough 2-trees, 2-trees with scattering number one, which do not contain a Hamiltonian path. I will further discuss structural qualities of 2-trees which have Hamiltonian paths, by looking at endpoint limitations. These limitations lead us to two variations of the Hamiltonian path problem, 1HP and 2HP, which fix one or two endpoints of a Hamiltonian path, respectively. (Received September 25, 2017)

1135-VP-2358 **Ronald Gould** and **Warren Shull\*** (wshull@emory.edu). *On Spanning Trees with few Branch Vertices*.

A conjecture of Matsuda, Ozeki, and Yamashita posits that, for any positive integer  $k$ , a connected claw-free  $n$ -vertex graph  $G$  must contain either a spanning tree with at most  $k$  branch vertices or an independent set of  $2k + 3$  vertices whose degrees add up to at most  $n - 3$ . In other words,  $G$  has this spanning tree whenever  $\sigma_{2k+3}(G) \geq n - 2$ . We prove this conjecture. This result is best possible, and generalizes a sufficient condition for traceability. (Received September 26, 2017)

1135-VP-2434 **Lisa A Lister\*** (llister@bloomu.edu), 224 Ben Franklin Hall, Dept. of Mathematical and Digital Sciences, Bloomsburg University, Bloomsburg, PA 17815. *Rank Decompositions of  $(0, 1, -1, *)$ -Matrices*. Preliminary report.

In this talk we use bipartite graphs to look at pattern preserving rank decompositions of matrices. In particular, we consider the families of  $(0, 1, -1, *)$ -matrices, giving necessary and conjectured sufficient conditions for these families to support rank decompositions. (Received September 26, 2017)

1135-VP-2457 **Kim A. S. Factor\*** (kim.factor@marquette.edu), 1313 W. Wisconsin Ave., Milwaukee, WI 53201. *Structure of the underlying graph giving a minimum directed restrained domination set*.

Extensions of domination in graphs includes restrained domination, as introduced by Domke, Hattingh, Hedetniemi, Laskar and Markus in the 1990s. Let  $G = (V, E)$  be a graph with vertex set  $V$  and edge set  $E$ . A set  $S \subseteq V$  is a *restrained dominating set* if every vertex in  $V - S$  is adjacent to a vertex in  $S$  and another vertex in  $V - S$ . The cardinality of a minimum restrained dominating set is the *restrained domination number*, denoted  $\gamma_r(G)$ . Here, we give initial results of the directed graph extension based upon this concept, the *directed restrained dominating set* of a digraph  $D = (V, A)$ . Further, the structure of an underlying graph whose orientation can give a minimum restrained domination set is discussed, with a labeling algorithm provided for complete bipartite underlying graphs and extended to complete  $t$ -partite underlying graphs. (Received September 26, 2017)

1135-VP-2636 **Garner Cochran\*** (gcochran@math.sc.edu), **Eva Czabarka** and **Peter Dankelmann**. *Quick Trips: An Improved Bound on the Oriented Diameter of Graphs*.

It was shown by Dankelmann that the oriented diameter of a graph with order  $n$  and minimum degree  $\delta$  can be bounded above by  $11 \frac{n}{\delta+1}$  and below by  $3 \frac{n}{\delta+1}$ . The upper bound was improved by Surmacs to  $7 \frac{n}{\delta+1}$ . In this talk we will show a further improvement of this bound. (Received September 26, 2017)

1135-VP-2660 **Eugene Han** (eugeneh@andrew.cmu.edu) and **David Offner\*** (offnerde@westminster.edu). *A new result on linear polychromatic colorings of the hypercube.*

Let  $Q_n$  denote the  $n$ -dimensional hypercube. For a fixed  $\ell \geq 1$ , a  $Q_\ell$ -coloring of  $Q_n$  is a coloring of the  $\ell$ -dimensional faces. A  $Q_\ell$ -coloring of  $Q_n$  is  $d$ -polychromatic if every  $Q_d$  in  $Q_n$  contains a  $Q_\ell$  of every color. For  $1 \leq \ell \leq d$ , let  $p^\ell(d)$  be the maximum number of colors such that any hypercube has a  $d$ -polychromatic  $Q_\ell$ -coloring with that number of colors. Polychromatic colorings of the hypercube were first studied by Alon, Krech, and Szabó. Their work, along with subsequent work of Offner established the value of  $p^1(d)$  for all  $d \geq 1$ . More recently, Chen introduced the notion of linear  $Q_\ell$ -colorings. We denote by  $p_{lin}^\ell(d)$  the maximum number of colors such that any hypercube has a  $d$ -polychromatic linear  $Q_\ell$ -coloring. Chen proved an upper bound on  $p_{lin}^2(d)$  for sufficiently large  $d$ . In this talk we will describe what is known about linear colorings and present a new theorem: For all  $d \geq 1$ ,  $p_{lin}^d(d+1) = 2$ . (Received September 26, 2017)

1135-VP-2691 **Taylor McMillan\*** (mcmi9872@bears.unco.edu), 825 12th St Apt 5, Greeley, CO 80631, and **Oscar Levin**. *On Locally Harmonious Labelings*. Preliminary report.

A *harmonious labeling* of a graph  $G$  with  $v$  vertices is an injective function from the vertices to  $\{0, 1, \dots, v-1\}$  such that the labels on the edges, induced by taking the sum of the incident vertices, are distinct. The natural extension of this type of labeling to infinite graphs is too restrictive. We consider a variant on harmonious labelings that maintains the harmonious property, but at a local level. In this talk, we define a *locally harmonious labeling*, and investigate this labeling for both finite and infinite graphs.

(Received September 26, 2017)

1135-VP-2819 **Jeannette Ramirez\*** (jeannette.r.831@gmail.com), 15649 Kalisher street, Granada Hills, CA 91344. *Radio  $k$ -labeling of Cycles for Large  $k$ .*

In a graph  $G$ , the distance between vertices  $u$  and  $v$ , denoted  $d(u, v)$ , is the shortest path from  $u$  to  $v$ . A radio  $k$ -labeling of  $G$ , is a function  $f : V \rightarrow \mathbb{N}_0$  that assigns to each vertex a non-negative integer label such that the separation of labels between any two vertices  $u$  and  $v$  is at least  $k+1-d(u, v)$  for some fixed non-negative integer  $k$ . The span of the radio  $k$ -labeling  $f$ , is the difference between the smallest and largest labels assigned by  $f$  and the radio  $k$  number of  $G$  is the smallest possible span for any radio labeling of  $G$ . I will present my results on exact radio  $k$  values for cycle graphs when  $k$  is larger than diameter. (Received September 26, 2017)

1135-VP-2879 **Grant Robinson\*** (grobins4@calstatela.edu) and **Daphne Der-Fen Liu**. *Sequences of Integers with Three Missing Separations.*

Let  $D$  be a set of positive integers. We will examine the maximum density  $\mu(D)$  of integral sequences in which the separation between any two terms does not fall in  $D$ . The sets  $D$  considered in this presentation are mainly of the form  $\{1, j, k\}$ . The closely related function  $\kappa(D)$ , the parameter involved in the “lonely runner conjecture,” is also investigated. Exact values of  $\kappa(D)$  and  $\mu(D)$  are found for many families of  $D = \{1, j, k\}$ . In particular, we prove that the boundary conditions in some earlier results of Haralambis [1977] are sharp. Consequently, our results declaim two conjectures of Carraher et al. [2016], and extend some findings of Haralambis [1972] and Gupta [2000]. The connection of these results to the problem of finding the fractional chromatic number of certain distance graphs will be explained. (Received September 26, 2017)

1135-VP-2971 **Janet Fierson\*** (fierson@lasalle.edu), Dept. of Mathematics and Computer Science, La Salle University, 1900 W. Olney Ave., Philadelphia, PA 19141, and **Robert Scholle**. *Reconfiguration graphs of prime labelings*. Preliminary report.

Motivated by the application of reconfiguration graphs to the vertex coloring problem (for which the reconfiguration graphs are also known as coloring graphs), we investigate reconfiguration graphs as applied to prime labeling.

In a prime labeling of a graph  $G$  of order  $n$ , the vertices of  $G$  are labeled with distinct integers  $1, 2, \dots, n$  such that the labels of adjacent vertices are relatively prime. For a graph  $G$ , we consider all possible prime labelings and create the corresponding reconfiguration graph,  $R(G)$ , in the following way: For each prime labeling of  $G$ , create a distinct vertex in  $R(G)$ . Then, for each pair of vertices in  $R(G)$ , insert an edge if the corresponding prime labelings of  $G$  differ in exactly two vertex labels.

We present results on which graphs may and may not appear as reconfiguration graphs, and we also consider subgraphs of reconfiguration graphs. In addition, we discuss findings on properties of reconfiguration graphs such as order, girth, and connectedness. (Received September 26, 2017)

1135-VP-3007 **Kathleen M Ryan\*** ([kathleen.ryan@desales.edu](mailto:kathleen.ryan@desales.edu)), 2755 Station Avenue, Center Valley, PA 18034, and **Theresa Marlin**. *Degree Sequences of Halin Graphs and Their Subgraphs*. Preliminary report.

Informally speaking, a Halin graph is essentially a tree  $T$  where a cycle has been drawn through the leaves of  $T$ . (A more formal definition will be presented during the talk.) Moreover, a *partial Halin graph* is a spanning subgraph of a Halin graph. In other words, a partial Halin graph can be obtained by removing edges from a Halin graph. In this talk, we characterize the degree sequences of partial Halin graphs, and we also discuss how a related edge-coloring problem gave us impetus to discover this characterization. (Received September 26, 2017)

1135-VP-3096 **Minh T Vu\*** ([minht.vu1993@gmail.com](mailto:minht.vu1993@gmail.com)).  *$L(j, k)$ -labeling for square cycles*.

Let  $G$  be a graph. The distance between two vertices  $u$  and  $v$  is denoted by  $d(u, v)$ . Let  $j, k$  be positive integers with  $j \leq k$ . An  $L(j, k)$ -labeling of  $G$  is a mapping  $f$  from  $V(G)$  to the non-negative integers such that  $|f(u) - f(v)| \geq j$  if  $d(u, v) = 1$ , and  $|f(u) - f(v)| \geq k$  if  $d(u, v) = 2$ . The span of  $f$  is  $\max\{|f(u) - f(v)| : u, v \in V(G)\}$ . The  $L(j, k)$ -labeling number of  $G$ , denoted by  $\lambda_{j,k}(G)$ , is the minimum span of all  $L(j, k)$ -labelings admitted by  $G$ . The  $k$ -power of an undirected graph  $G$  is a graph with the same vertex set as  $G$ , in which two vertices are adjacent if their distance in  $G$  is at most  $k$ . The  $L(j, k)$ -labeling number of square paths has recently been completely determined. In this talk, we show the exact values of  $\lambda_{j,k}(C_n^2)$  for some square cycles  $C_n^2$  and present upper bounds for all other square cycles. We conjecture that these bounds are the exact value for  $\lambda_{j,k}$ . (Received September 26, 2017)

1135-VP-3162 **Rebecca Robinson\*** ([rebrobin@umflint.edu](mailto:rebrobin@umflint.edu)) and **Cameron McLeman**. *Chromatic Polynomials of Graph Subdivisions and the Integral Root Problem*.

The most rudimentary tool used in computing chromatic polynomials of graphs is the multiplication principle. For graphs for which this tool is sufficient, we automatically find chromatic polynomials whose roots are all integers. It is interesting to note that the converse to this observation is false – there exist graphs whose chromatic polynomials have all integer roots, and yet which cannot be computed solely via the multiplication principle. In this talk, we present a theorem on chromatic polynomials of graph subdivisions and use it to shed light on some families of such graphs. (Received September 26, 2017)

## General Session on Linear Algebra

1135-VQ-649 **Mehmet Gumus\*** ([mzg0083@auburn.edu](mailto:mzg0083@auburn.edu)). *A Generalization of Additive  $D$ -Stability*.

A real, square matrix  $A$  is said to be additive  $D$ -stable if  $A + D$  is stable for any nonnegative diagonal matrix  $D$ . This type of matrix stability plays an important role in various applied problems such as diffusion models of biological systems and neural networks. In this talk, we generalize the concept of additive  $D$ -stability and introduce a new notion called additive  $H(\alpha)$ -stability, with  $\alpha$  being a partition of the index set  $\{1, \dots, n\}$ . We present several new results concerning the relationship between additive  $H(\alpha)$ -stability and other types of matrix stability involving  $\alpha$ . Some special cases of additive  $H(\alpha)$ -stability are further investigated. In particular, we provide a sufficient condition for additive  $D$ -stability and discuss an extension of this result. (Received September 11, 2017)

1135-VQ-1374 **Samuel C Schaub\***, 50 S Lincoln St, Washington, PA 15301, and **Ryan S Higginbottom**. *Properties of a generalized determinant*.

In this talk we define a determinant for any (not necessarily square) matrix. This function shares many properties with the traditional determinant, including results about nonzero determinants and linearly independent vectors. We will discuss which of the familiar statements in the Invertible Matrix Theorem have analogues for non-square matrices via this generalized determinant. (Received September 21, 2017)

1135-VQ-1897 **Joshua Boone\*** ([joshua.boone@lmunet.edu](mailto:joshua.boone@lmunet.edu)). *Powers of Arbitrary  $2 \times 2$  and  $3 \times 3$  Matrices*. Preliminary report.

For integers  $n \geq 2$ , a formula is given for  $A^n$ , where  $A$  is an arbitrary  $2 \times 2$  matrix. A similar formula is presented for the  $3 \times 3$  case. Each formula utilizes the complete homogeneous symmetric polynomial evaluated at the eigenvalues of  $A$ . Motivation for formulas of this type is given, as well as some corollaries. Examples and work toward the  $4 \times 4$  case will be discussed as well, time permitting. (Received September 25, 2017)

1135-VQ-2064 **Shulai Yang\*** ([yangs@jay.washjeff.edu](mailto:yangs@jay.washjeff.edu)), 50 S. Lincoln St. #1263, Washington, PA 15301. *Square Roots of  $2 \times 2$  Matrices.*

The square root of a  $2 \times 2$  matrix  $A$  is a matrix  $B$  such that  $B^2 = A$ . We show how the diagonalization method and the Cayley-Hamilton Theorem can be used to find the square root of a  $2 \times 2$  matrix. Along the way, we can determine what matrices have square roots and how to count them. (Received September 25, 2017)

1135-VQ-2411 **Göran Bergqvist\*** ([gober@mai.liu.se](mailto:gober@mai.liu.se)), Department of Mathematics, Linköping University, 58183 Linköping, Sweden. *On the rank of random tensors.*

Random matrices have full rank with probability 1, but for higher-order real random tensors (multi-arrays) several ranks can occur with positive probability. We show how to calculate the only known exact values of such tensor rank probabilities, which are for  $n \times n \times 2$  Gaussian tensors. (Received September 26, 2017)

1135-VQ-2743 **Jane Breen\*** ([breenj3@myumanitoba.ca](mailto:breenj3@myumanitoba.ca)) and **Steve Kirkland**. *Minimising the largest mean first passage time of a Markov chain and the influence of directed graphs.*

For a Markov chain described by an irreducible stochastic matrix  $A$  of order  $n$ , the mean first passage time  $m_{i,j}$  measures the expected time for the Markov chain to reach state  $j$  given that the system begins in state  $i$ , thus quantifying the short-term behaviour of the chain. In this talk, we give a lower bound for the maximum mean first passage time in terms of the stationary distribution vector of  $A$ . We also discuss the characterisation of the directed graphs  $D$  for which any stochastic matrix  $A$  respecting this directed graph attains equality in the lower bound, thus producing a class of Markov chains with optimal short-term behaviour. (Received September 26, 2017)

## General Session on Logic and Foundations

1135-VR-2121 **William J Franczak\*** ([wjf212@lehigh.edu](mailto:wjf212@lehigh.edu)), 203 Van Buren Street, Bethlehem, PA 18015. *K-Triviality in General Settings*. Preliminary report.

Abstract: The area of algorithmic randomness is primarily concerned with using the tools of computability theory to define and quantify notions of randomness formally, typically done in Cantor space. An important concept is that of K-triviality, describing the class of infinite binary strings which are as easy as possible to describe, and thus far from random. This class of K-trivial strings has many characterizations in the standard setting, affirming its robustness as a notion, and Andre Nies has given a generalization of this class that preserves many of its nice properties in general computable metric spaces. We look at other possible characterizations for this notion, in particular in computable measure spaces, and investigate how they compare. (Received September 25, 2017)

1135-VR-2670 **Tucker R. Rhodes\*** ([tuckerrhodes@cedarville.edu](mailto:tuckerrhodes@cedarville.edu)), Tucker Rhodes, 251 N. Main St., P.O. Box #5065, Cedarville, OH 45314, and **Adam J. Hammett** and **Nathan A. Harold**. *Does Logic Help Us Beat Monty Hall?*

The classical Monty Hall problem entails that a hypothetical game show contestant be presented three doors and told that behind one door is a car and behind the other two are far less appealing prizes, like goats. The contestant then picks a door, and the host (Monty) is to open a different door which contains one of the bad prizes. At this point in the game, the contestant is given the option of keeping the door she chose or changing her selection to the remaining door (since one has already been opened by Monty), after which Monty opens the chosen door and the contestant wins the prize which lies behind it. Inspired by the work of Morrow, Oman and Salminen (2016, "Game Show Shenanigans: Monty Hall Meets Mathematical Logic," *Elemente der Mathematik* 71(4), pp. 145-155), we consider several logic-themed variants of this problem. Among these are versions where  $d$  doors and  $p$  prizes reside behind some  $p$  of these doors, and the contestant is permitted to present Monty with  $q$  random true/false questions concerning the location of the prizes, to which Monty must respond truthfully. Our results extend those of the original paper, and involve a combination of probabilistic techniques and exhaustive computation using a computer program. (Received September 26, 2017)

## General Session on Number Theory

1135-VS-12 **Richard Benjamin Gottesman\*** (rgottesm@ucsc.edu), Santa Cruz, CA. *The algebra and arithmetic of vector valued modular forms*. Preliminary report.

Vector valued modular forms are generalizations of modular forms with a character. They form a graded module over the ring of modular forms. I will explain how understanding the structure of the module of vector valued modular forms allows one to show that the component functions of vector valued modular forms are solutions to certain ordinary differential equations. In certain cases, one can use Hauptmoduls and hypergeometric series to solve these differential equations. One then obtains the  $q$ -series expansions of the vector valued modular forms. This perspective gives a viable approach towards proving the unbounded denominator conjecture for modular forms on non-congruence subgroups. (Received August 18, 2017)

1135-VS-221 **Richard F. Ryan\*** (rryan@marymountcalifornia.edu), 30800 Palos Verdes Drive East, Rancho Palos Verdes, CA 90275. *A Generalized Fermat Equation with an Emphasis on Non-Primitive Solutions*.

The equation  $x^m + y^n = z^r$  is considered under the condition that the given integers values for  $m$ ,  $n$ , and  $r$  are greater than one. Solutions of this equation are given for cases in which  $\gcd(mn, r) = 1$ ,  $\gcd(mr, n) = 1$ , or  $\gcd(nr, m) = 1$ . (Received August 12, 2017)

1135-VS-385 **Robert M Sulman\*** (sulmanrm@oneonta.edu) and **Tyler Fedoris**. *Polynomial Orbits of the Ring of Integers Modulo  $n$* . Preliminary report.

Polynomial Orbits of the Ring  $(\mathbb{Z}_n, +n, \bullet n)$  - Robert Sulman and Tyler Fedoris (SUNY Oneonta)

We examine the orbits of polynomials acting on the ring  $(\mathbb{Z}_n, +n, \bullet n)$  for various  $n$ . These orbits are described as graphs, with inverse-pairs of units seen as well. We shall see a great deal of symmetry, and the many structures that arise are analyzed: Modulus and coefficients (of the given polynomial) determine several features of such graphs. Furthermore, when a (polynomial) function  $f$  is fixed and we vary the modulus in a "consistent" manner (such as  $n = 2k$  for  $k = 6, 7, 8, 9, \dots$ ), we see the resulting orbit graph structures change with re-occurring features in a fractal-like way. (Received August 30, 2017)

1135-VS-640 **Bruce G Dearden, Joel E Iiams\*** (joel.iiams@und.edu) and **Jerry M Metzger**, 101 Cornell Street Stop 8376, Witmer Hall 324, University of North Dakota, Grand Forks, ND 58202-8376. *The Rumor conjecture*.

A running modulus recurrence (rumor) is a recurrence relation where  $z_0$ , the initial seed, is any positive integer and successive terms are given by the formula  $z_n = b * z_{n-1} + c \pmod{n+k}$ , where  $b \geq 2$  and  $k \geq 0$ . We will discuss the genesis of the study of rumors, connections with the work of others, and progress on the rumor conjecture. (Received September 11, 2017)

1135-VS-686 **Jeremiah Bartz\*** (jeremiah.bartz@und.edu), **Bruce Dearden** and **Joel Iiams**.  *$\nu$ -Gap Balancing Numbers*. Preliminary report.

Gap balancing numbers were introduced by Panda and Rout as a certain generalization of balancing and cobalancing numbers. In this talk, we give an alternative definition of gap balancing numbers motivated by geometric considerations. Using this viewpoint, several results are presented regarding classes of gap balancing numbers and related sequences. (Received September 13, 2017)

1135-VS-800 **Jack J. Billings\*** (jjbillings@noctrl.edu), 30 N Brainard St., Student Mailroom 1222, Naperville, IL 60540, and **Neil Nicholson** (nrnicholson@noctrl.edu), 30 N Brainard St., Department of Mathematics, Naperville, IL 60540. *Weak Visibility Preserving Functions*. Preliminary report.

A point  $P$  in a set  $S$  of lattice points is weakly visible from the origin  $O$  if no other point in  $S$  lies on the line segment  $OP$ . A function whose domain is  $S$  with co-domain consisting of lattice points is said to be weak visibility preserving if any point  $P$  that is weakly visible in  $S$ , then  $f(P)$  is weakly visible in  $f[S]$ . Two particular types of functions are investigated, with one always being weak visibility preserving and the other never being so. (Received September 14, 2017)

1135-VS-1072 **Ralph P Grimaldi\*** (grimaldi@rose-hulman.edu), Mathematics Department - RHIT, 5500 Wabash Avenue, Terre Haute, IN 47803. *Up-Down Ternary Strings*.

For the alphabet  $\Sigma = \{0, 1, 2\}$  and  $n \geq 2$ , let  $a_n$  count the number of strings in  $\Sigma^n$  of the form  $x_1x_2x_3 \dots x_n$ , where  $x_i < x_{i+1}$ , for  $i$  odd, and  $x_i > x_{i+1}$ , for  $i$  even. For example, for  $n = 3$ , there are  $a_3 = 5$  such strings of



length 3 - namely,

$$010, \quad 021, \quad 020, \quad 121, \quad 120.$$

In general we find that  $a_2 = 3$ ,  $a_3 = 5$ , and for  $n \geq 4$ ,  $a_n = a_{n-1} + a_{n-2}$ . Thus, for  $n \geq 2$ ,  $a_n = F_{n+2}$ , where  $F_n$  denotes the  $n$ th Fibonacci number. For these strings we examine (i) the number of 0's, 1's, and 2's that occur among the  $a_n$  strings; (ii) the sum of all the integers that occur in the  $a_n$  strings; (iii) the sum of all the  $a_n$  strings considered as base 3 integers; (iv) the number of runs of consecutive integers of the same parity; and, (v) the number of inversions that occur among the  $a_n$  strings. (Received September 19, 2017)

1135-VS-1115     **Khang Tran**, Fresno, CA, and **Andres I Zumba Quezada\***  
([andreszumba@mail.fresnostate.edu](mailto:andreszumba@mail.fresnostate.edu)), 4909 N Backer Ave Apart 125, Fresno, CA 93726.  
*Zeros of polynomials with four-term recurrence.*

For any real numbers  $b, c \in \mathbb{R}$ , we form the sequence of polynomials  $\{H_m(z)\}_{m=0}^{\infty}$  satisfying the four-term recurrence

$$H_m(z) + cH_{m-1}(z) + bH_{m-2}(z) + zH_{m-3}(z) = 0, \quad m \geq 1,$$

with the initial conditions  $H_0(z) = 1$  and  $H_{-m}(z) = 0$ ,  $\forall m \geq 1$ . We find necessary and sufficient conditions on  $b$  and  $c$  under which the zeros of  $H_m(z)$  are real for all  $m$ , and provide an explicit real interval on which  $\bigcup_{m=0}^{\infty} \mathcal{Z}(H_m)$  is dense where  $\mathcal{Z}(H_m)$  is the set of zeros of  $H_m(z)$ . (Received September 19, 2017)

1135-VS-1186     **John E. Mosley\*** ([jmosley@exeter.edu](mailto:jmosley@exeter.edu)). *The Greatest Common Divisor of Multinomial Coefficients.*

While studying a certain characteristic number of a class of special unitary manifolds, we noticed the calculation was simply computing a multiple of a multinomial coefficient. We were, at the time, interested in computing the greatest common divisor of these characteristic numbers. From that emerged a fun and interesting number theory problem. In this talk, I will present the solution to this problem, which gives the greatest common divisor of certain multinomial coefficients. (Received September 20, 2017)

1135-VS-1207     **Sebastian I Troncoso\*** ([sitronco@bsc.edu](mailto:sitronco@bsc.edu)), 1236 Greensboro Rd, Birmingham, AL 35208. *Preperiodic hypersurfaces and preperiodic points.*

Let  $K$  be a number field and  $\phi$  be an endomorphism of  $\mathbb{P}^n$  over  $K$  of degree  $d \geq 2$ . Let  $S$  be the set of places of bad reduction for  $\phi$  (including the archimedean places). Let  $HTail(\phi, K, e)$  be the set of  $K$ -rational purely preperiodic hypersurfaces of  $\mathbb{P}^n$  of degree  $e$ .

We give a strong arithmetic relation between  $K$ -rational purely preperiodic hypersurfaces of degree  $e$  and  $K$ -rational periodic points. Indeed, if we consider  $N = \binom{e+n}{e} - 1$  and assume that  $\phi$  has at least  $2N + 1$   $K$ -rational periodic points such that no  $N + 1$  of them lie in a hypersurface of degree  $e$  then we give an effective bound on a large subset of  $HTail(\phi, K, e)$  depending on  $e$  and the number of places of bad reduction  $|S|$ . Finally, we prove that the set  $HTail(\phi, K, e)$  is finite if we assume that  $\phi$  is an endomorphism of  $\mathbb{P}^2$ . (Received September 20, 2017)

1135-VS-1277     **Robert Styer\*** ([robert.styer@villanova.edu](mailto:robert.styer@villanova.edu)) and **Reese Scott**. *Number of solutions to the Diophantine equation  $X + Y = c^z$ .* Preliminary report.

Consider  $X + Y = c^z$  where  $c > 1$  is odd,  $\gcd(X, Y) = 1$ ,  $\gcd(XY, c) = 1$ , with  $XY$  divisible precisely by primes in a given set of  $n$  primes. The number of solutions  $(X, Y, z)$  in positive integers is bounded by  $2^{n-1} + 1$ . When  $n < 4$  the bound in Theorem 1 is precise. This bound is independent of the number of primes dividing  $c$ . As a corollary,  $ra^x + sb^y = c^z$  has at most 4 solutions in positive integers  $(x, y, z)$  except for a family of exceptions. (Received September 20, 2017)

1135-VS-1344     **Frank Patane\*** ([fpatane@samford.edu](mailto:fpatane@samford.edu)), Samford University, 800 Lakeshore Dr, Homewood, AL 35209. *On a Generalized Identity Connecting Theta Series Associated with Discriminants  $\Delta$  and  $\Delta p^2$ .*

When the discriminants  $\Delta$  and  $\Delta p^2$  are idoneal, there is a well known theorem which connects the theta series associated to binary quadratic forms of each discriminant. In this talk we discuss the result of a recent paper which generalizes this theorem by allowing  $\Delta$  and  $\Delta p^2$  to be non-idoneal. We state this theorem and give an example of an identity which connects the theta series associated to a single binary quadratic form of discriminant  $\Delta$  to a theta series associated to a subset of binary quadratic forms of discriminant  $\Delta p^2$ . (Received September 21, 2017)

1135-VS-1553 **Constantin N. Beli, Wai Kiu Chan, Maria Ines Icaza and Jingbo Liu\*** (jliu02@hku.hk). *On a Waring's problem for integral quadratic and hermitian forms.*

For each positive integer  $n$ , let  $g_{\mathbb{Z}}(n)$  be the smallest integer such that if an integral quadratic form in  $n$  variables can be written as a sum of squares of integral linear forms, then it can be written as a sum of  $g_{\mathbb{Z}}(n)$  squares of integral linear forms. We show that as  $n$  goes to infinity, the growth of  $g_{\mathbb{Z}}(n)$  is at most an exponential of  $\sqrt{n}$ . Our result improves the best known upper bound on  $g_{\mathbb{Z}}(n)$  which is in the order of an exponential of  $n$ . We also define an analogous number  $g_{\mathcal{O}}^*(n)$  for writing hermitian forms over the ring of integers  $\mathcal{O}$  of an imaginary quadratic field as sums of norms of integral linear forms, and when the class number of the imaginary quadratic field is 1, we show that the growth of  $g_{\mathcal{O}}^*(n)$  is at most an exponential of  $\sqrt{n}$ . (Received September 23, 2017)

1135-VS-2017 **Eugene Fiorini** (eugenefiorini@muhlenberg.edu), Muhlenberg College, Allentown, PA 18104, **Eric Jovinelly\*** (ej250253@muhlenberg.edu), Muhlenberg College, Allentown, PA 18104, **Edgar Jaramillo-Rodriguez** (ejaramillo@berkeley.edu), University of California, Berkeley, Berkeley, CA 94720, **Tony Wong** (wong@kutztown.edu), Kutztown University of Pennsylvania, Kutztown, PA 19530, **Grant Fickes** (gfick710@live.kutztown.edu), Kutztown University of Pennsylvania, Kutztown, PA 19530, and **Nathaniel Benjamin** (nbenj582@iastate.edu), Iowa State University, Ames, IA 50011. *On Unique Integers in the Catalan Triangle.*

The Catalan triangle is the number triangle whose entries, denoted  $c_{n,k}$ , give the number of strings with  $n$  X's and  $k$  Y's, where  $n, k \in \mathbb{N}$ , such that no initial segment has more Y's than X's. While it is easy to show that every positive integer appears at least once on the Catalan Triangle, very little is known about which integers appear uniquely. This talk investigates the sequence of integers that appear uniquely on the Catalan Triangle (OEIS reference A275481). We present conditions that guarantee uniqueness if met. Specifically, we show that all the prime powers, except for 2, 5, 9, and 27, appear uniquely on the Catalan Triangle. Additionally, we show for  $n, k, x, l \in \mathbb{N}$ ,  $n \geq 2$ , and  $k, l \geq 3$ ,  $c_{n,k} = x^l$  has no solutions. (Received September 25, 2017)

1135-VS-2076 **Catherine L McAdoo\*** (mcadooc1@jay.washjeff.edu), 50 S Lincoln, Washington & Jefferson College, Washington, PA 15301, and **Greg Lu** (lub@jay.washjeff.edu). *Partition of Integers Through Euler and Beyond.*

Our study is inspired by Euler's generating function for integer partitions:

$$\mathcal{E}(x) = \prod_{k=1}^{\infty} \frac{1}{1-x^k} = \sum_{m=0}^{\infty} p(m)x^m.$$

With the Ferrers Diagram as a graphical representation of partitions, we investigate some variations of the original Euler's generating function and their corresponding effects on the several types of partitions and their counts  $p(n)$ . Further on, we outline and explain a few identities of integer partitions. In some of these identities we observe that the graphical representations provide clearer visualization of the partition process and thus offer insights on the proofs, which are backed by algebraic proofs through bijection. (Received September 25, 2017)

1135-VS-2191 **Daniel Disegni\*** (daniel.disegni@gmail.com). *Heegner cycles and derivatives of p-adic L-functions.*

In 1952 Heegner pioneered a method to construct rational solutions to cubic equations in two variables, using modular forms. Heegner's method can be generalised to construct algebraic cycles on other varieties. Moreover these cycles fit into p-adic families. I will present a formula for the p-adic height of the universal family of Heegner cycles, and its applications to the analogue of the Birch and Swinnerton-Dyer conjecture. (Received September 25, 2017)

1135-VS-2247 **Sonny Arora\*** (sza149@psu.edu). *Constructing Picard Curves with Complex Multiplication.*

For applications to cryptography, one is interested in constructing curves of genus less than or equal to 3 over finite fields whose Jacobians have complex multiplication. Several algorithms exist in the cases of elliptic curves and genus 2 curves, however, less is known in the genus 3 case. An interesting test case for such an algorithm is the case of Picard curves which are genus 3 curves of the form  $y^3 = f(x)$  where  $f$  is a separable polynomial of degree 4. We discuss some difficulties encountered in the genus 3 case and present an algorithm to construct Picard curves with complex multiplication using a Chinese Remainder Theorem approach. This is joint work with Kirsten Eisenträger. (Received September 25, 2017)

1135-VS-2283 **Elizabeth M. Reid\*** (emreid2@buffalo.edu) and **Thomas Cusick**. *Permutations between cubic 2-rotation symmetric Boolean functions.*

A Boolean function in  $n$  variables is 2-rotation symmetric if it is invariant under even powers of the cyclic permutation  $\rho(x_1, \dots, x_n) = (x_2, \dots, x_n, x_1)$  of the variables, but not under the first power. We call such a function a 2-function. A 2-function is said to be monomial rotation symmetric (MRS) if it is generated by applying powers of  $\rho^2$  to a single monomial. In 2014 Cusick and Johns developed the theory of cubic MRS 2-functions in  $2n$  variables generated by a monomial  $x_1 x_r x_s$  with  $1 < r < s$  and  $r$  and  $s$  not both odd. They gave a complete description of the affine equivalence classes for these functions. In 2017, Cusick and Reid determined the smallest set of permutations that act on the set of all these cubic MRS 2-functions to give the affine equivalence classes. Here, we develop the theory further by giving a complete description of permutations between two equivalent cubic MRS 2-functions and an exact count of their number. (Received September 25, 2017)

1135-VS-2295 **Sungkon Chang\*** (sungkon.chang@armstrong.edu), Department of Mathematics, Armstrong State University, 11935 Abercorn St, Savannah, GA 31419. *Average Number of Zeckendorf Integers.*

By Zeckendorf's theorem each positive integer is uniquely written as a sum of distinct non-adjacent terms of the Fibonacci sequence. This representability remains true for so called *the  $N$ th order Fibonacci sequence*, and for a further generalization to linear recurrences with positive coefficients. In this talk we consider sequences  $\{G_n\}$  that have the same linear recurrence relations as the  $N$ th order Fibonacci sequence but has different initial values, and investigate the number of positive integers up to  $X$  that are written as a sum of distinct terms of  $G_n$ . (Received September 25, 2017)

1135-VS-2464 **Jeremy Alm** (jal@lamar.edu), Department of Mathematics, Lamar University, P.O. Box 10047, Beaumont, TX 77710, **Robert W. Vallin\*** (robert.vallin@lamar.edu), Department of Mathematics, Lamar University, P.O. Box 10047, Beaumont, TX 77710, and **José Vega-Guzman** (jvegaguzman@lamar.edu), Department of Mathematics, Lamar University, P.O. Box 10047, Beaumont, TX 77710. *Results on 3-Free Tribonacci Sequences.* Preliminary report.

Let  $a_1, a_2$ , and  $a_3$  be any three positive integers none of which are multiples of three. For  $n \geq 4$  define

$$a_n = \frac{a_{n-3} + a_{n-2} + a_{n-1}}{3^k}$$

where  $k$  is the largest non-negative integer such that  $3^k$  divides  $(a_{n-3} + a_{n-2} + a_{n-1})$ . These sequences are called *3-free Tribonacci sequences*. In this talk we look at various properties of these sequences. (Received September 26, 2017)

1135-VS-2471 **Ji Young Choi\*** (jychoi@ship.edu), 1871 Old Main Dr., Department of Math, Shippensburg, PA 17257. *Generalized Collatz functions and Jacobsthal numbers.* Preliminary report.

Let  $g$  be an integer greater than 1, and let  $b = g + 1$ . For any positive integer  $n$ , we consider a generalization of Collatz function:  $f(n) = \frac{n}{g}$  if  $n$  is a multiple of  $g$ ;  $\lceil \frac{bn}{g} \rceil$  otherwise. Using this function, we consider a sequence of the base- $b$  representation of integers, starting with  $b^N$  for an arbitrary large integer  $N$ . This talk will show the number of digits in each repeating string of the sequence generalizes the Jacobsthal numbers. (Received September 26, 2017)

1135-VS-2492 **Thomas Koshy** and **Zhenguang Gao\*** (zgao@framingham.edu), 100 State Street, Framingham, MA 01701. *Polynomial Extensions of a Diminnie Delight.*

Recently, we extended to Fibonacci polynomials a complex, but interesting, recurrence studied by C.R. Diminnie. We then studied the corresponding versions to Lucas, Pell, and Pell-Lucas polynomials, and extracted the respective number-theoretic counterparts. In this presentation, we explore extensions to Jacobsthal, Jacobsthal-Lucas, Vieta, and Chebyshev polynomials. (Received September 26, 2017)

1135-VS-2549 **Amanda Beecher\*** (abeecher@ramapo.edu). *Number Theory Math Fair.*

Number Theory is an excellent source of fun topics that can expose many with limited background to see the beauty of mathematics, including children. In my 200-level Number Theory course, students were required to create a short hands-on learning activity appropriate for young children (elementary or middle school age) to learn about a number theory topic. We hosted a Math Fair on campus to share these activities with young people and the College community. This talk will discuss the logistics of hosting a Math Fair, the benefits to the

students and young participants, and the multifaceted assignments that supported this activity in a 200-level course. (Received September 26, 2017)

1135-VS-2572 **Clifford Blakestad\*** ([clifford.blakestad@colorado.edu](mailto:clifford.blakestad@colorado.edu)), University of Colorado Boulder, Department of Mathematics, UCB 395, Boulder, CO 80309, and **David Grant**. *Universal  $p$ -adic sigma and Weierstrass zeta functions*. Preliminary report.

Mazur and Tate introduced a  $p$ -adic  $\sigma$  function defined on the kernel of reduction of an ordinary elliptic curve defined over a complete discrete valuation domain of residual characteristic  $p > 2$ , which they used to compute  $p$ -adic local heights. The logarithmic derivative of this function is a variant of a Weierstrass  $\zeta$  function. From the perspective of  $p$ -adic integrality, the  $\zeta$  function is the more natural object. For  $p > 3$  we produce a  $\zeta$  function as a Laurant series from a limit of mod  $p^n$  objects on a universal ordinary Weierstrass model, deducing the integrality of the  $\sigma$  function via an explicit comparison between the universal curve and its quotient by the canonical  $p$ -torsion subgroup. (Received September 26, 2017)

1135-VS-2593 **Kirsten Eisentraeger, Sean Hallgren and Travis Morrison\*** ([txm950@psu.edu](mailto:txm950@psu.edu)), Penn State University, Mathematics Department, University Park, PA 16802. *Computing isogenies and endomorphism rings of supersingular elliptic curves*.

NIST is currently soliciting submissions of post-quantum cryptographic protocols, meaning cryptosystems which would be secure even against a quantum computer. Some suggested cryptosystems are believed secure due to the hardness of computing isogenies between supersingular elliptic curves. Breaking these systems reduces to the problems of either computing an isogeny of prime-power degree between two given supersingular elliptic curves, or computing the endomorphism ring  $End(E)$  (meaning return a maximal order of a quaternion algebra isomorphic to  $End(E)$ ) of a supersingular elliptic curve  $E$ . These problems are deeply related, and are often referred to as equivalent problems. In joint work with Kirsten Eisentraeger and Sean Hallgren, we study the size of these objects, which is necessary to have meaningful reductions. Additionally, we give a reduction from the problem of computing a  $\ell$ -power isogeny between supersingular curves for a prime  $\ell$  to the problem of, given a supersingular curve  $E$ , computing both the maximal order isomorphic to  $End(E)$  along with its action on  $E[\ell]$ . Thus we reduce the problem of computing isogenies to a problem of computing endomorphism rings, meaning knowing  $End(E)$  algebraically along with a little bit of its geometric information. (Received September 26, 2017)

1135-VS-2632 **Matthew Krauel\*** ([krauel@csus.edu](mailto:krauel@csus.edu)). *Congruence and noncongruence vector-valued modular forms in the theory of vertex operator algebras*.

That certain trace functions of vertex operators in the theory of vertex operator algebras (VOAs) are congruence vector-valued modular forms has long been conjectured and proved in many cases. However, the story when the trace functions are built from more general intertwining operators has received less attention. In this talk I will briefly discuss some results concerning congruence and noncongruence vector-valued modular forms occurring from minimal model VOAs. Familiarity with VOAs or modular forms is not required. (Received September 26, 2017)

1135-VS-2729 **Tyler J Sullivan** ([tjsulliva@coastal.edu](mailto:tjsulliva@coastal.edu)), 204 Doctors Lane, Myrtle Beach, SC 29579, and **Dominique E Forbes\*** ([deforbes@coastal.edu](mailto:deforbes@coastal.edu)), 458 Saint Charles Circle, Myrtle Beach, SC 29588. *Presentation and Analysis of Modified Fibonacci Sequences, Generalized Golden Ratios and Their Convergence*. Preliminary report.

We determine a family of recursively defined sequences and their growth rates. We prove that these growth rates converge to generalized 'golden ratios', and from there we prove that these resulting sequences of generalized 'golden ratios' converge as well. We also present how these recursive sequences can arise by modifying certain assumptions that generated the famous Fibonacci sequence. (Received September 26, 2017)

1135-VS-2771 **Christopher Richardson\*** ([crichardson@ksu.edu](mailto:crichardson@ksu.edu)), 1404 Yuma St., Manhattan, KS 66502, and **Chris Pinner** and **Todd Cochrane**. *A Generalization of the Goresky-Klapper Conjecture*.

For a fixed integer  $n$ , we show that a permutation of least positive residues of  $f(x) = Ax^k \pmod p$  cannot map a residue class mod  $n$  to just one residue class mod  $n$  for sufficiently large  $p$ , other than the maps  $f(x) = \pm x$  and  $f(x) = \pm x^{\frac{p+1}{2}} \pmod p$ . (Received September 26, 2017)

1135-VS-2803 **David Walter Stoner\*** (applepi2012@gmail.com), 208 Ashwood Drive, Aiken, SC 29801. *On Symmetric but not Cyclotomic Numerical Semigroups.*

A numerical semigroup is called *cyclotomic* if its corresponding numerical semigroup polynomial  $P_S(x) = (1-x) \sum_{s \in S} x^s$  is expressible as the product of cyclotomic polynomials. Ciolan, García-Sánchez, and Moree conjectured that for every embedding dimension at least 4, there exists some numerical semigroup which is symmetric but not cyclotomic. We affirm this conjecture by giving an infinite class of numerical semigroup families  $S_{n,t}$ , which for every fixed  $t$  is symmetric but not cyclotomic when  $n \geq \max(8(t+1)^3, 40(t+2))$  and then verify through a finite case check that the numerical semigroup families  $S_{n,0}$ , and  $S_{n,1}$  yield acyclotomic numerical semigroups for every embedding dimension at least 4. (Received September 26, 2017)

1135-VS-2835 **Tristan Phillips\*** (tp7924@ship.edu), Department of Mathematics, Shippensburg University, Shippensburg, PA 17257. *Galois Groups and Integral Basis for some Lucas Polynomial Sequences.* Preliminary report.

In 1892 Hilbert used his famous irreducibility theorem to prove the existence of irreducible polynomials of each degree  $n$  with Galois group  $S_n$ . Later Schur made this explicit by showing that the Laguerre polynomials were all irreducible and had Galois group  $S_n$ . Since Schur many other families of irreducible polynomials with certain Galois groups have been discovered. As an additional layer, we will consider families of irreducible polynomials which are minimal polynomials for monogenic number fields, that is, number fields which have a power basis for their ring of integers. In this talk we pay special attention to polynomial sequences defined by linear recurrences; these polynomials are called Lucas polynomial sequences. Among other things, we show that the irreducible Fibonacci polynomials have Galois group  $\mathbb{Z}_2 \times \mathbb{Z}_{\frac{p-1}{2}}$  and are minimal polynomials for monogenic number fields. (Received September 26, 2017)

1135-VS-3025 **Kimberly E. Stubbs\*** (stubbske@g.cofc.edu), 196 Curtis Creek Rd, Candler, NC 28715. *Geometric Representations of Dedekind's Proof of Irrationality.* Preliminary report.

In *Essays on the Theory of Numbers*, Richard Dedekind gives a general algebraic proof that if  $D$  is a positive integer that is not the square of an integer, then  $\sqrt{D}$  is irrational. In the 1960's, Stanley Tennenbaum gives the geometric representation of Dedekind's proof for which  $D = 2$ . In this talk we'll look at the geometric representations of Dedekind's proof for which  $D = 3, 5, 6, 8$ , and 12 and their constructions which are similar to the construction for the  $D = 2$  case. (Received September 26, 2017)

1135-VS-3140 **Cam McLeman\*** (mclemanc@umflint.edu), 303 E Kearsley St., Flint, MI 48502. *Missing Class Groups for Imaginary Quadratic Number Fields.*

The Cohen-Lenstra heuristics describe, for  $p$  odd, the distribution of  $p$ -class groups of imaginary quadratic number fields, and when  $p = 2$ , a modification of these heuristics due to Gerth achieves the same thing. In both of these cases, recent progress has allowed us to refine the general picture of the family of such class groups, and in particular articulate broad families of finite abelian  $p$ -groups which should never occur as class groups in this context. The intent of this talk is to showcase some of these results and describe the status of the theory. (Received September 26, 2017)

1135-VS-3147 **Pietro Paparella\***, 18115 Campus Way NE, Bothell, WA 98011. *Eisenstein's criterion, Fermat's last theorem, and a conjecture on powerful numbers.*

Given integers  $\ell > m > 0$ , and a positive integer  $n$ , we define monic polynomials  $X_n$ ,  $Y_n$ , and  $Z_n$  with the property that  $\mu$  is a zero of  $X_n$  if and only if the triple  $(\mu, \mu + m, \mu + \ell)$  satisfies  $x^n + y^n = z^n$ . It is shown that the irreducibility of these polynomials implies Fermat's last theorem. It is also shown, in a precise asymptotic sense, that for a vast majority of cases, these polynomials are irreducible via Eisenstein's criterion. We conclude by offering a conjecture on powerful numbers. (Received September 26, 2017)

1135-VS-3163 **Bir Kafle\*** (bkafle@pnw.edu), 1401 S. U.S. 421, Westville, IN 46391, and **A. Togbé, F. Luca, A. Montejano** and **L. Szalay**. *On the  $x$ -coordinates of Pell equations which are Fibonacci numbers.* Preliminary report.

Let  $d > 1$  be a positive integer which is not a perfect square. Let  $(x_n, y_n)_{n \geq 1}$  be the sequence of positive integer solutions  $(x, y)$  of the Pell equations  $x^2 - dy^2 = \pm 1$ . Let  $\{F_m\}_{m \geq 0}$  be the sequence of Fibonacci numbers. In this talk, we explain when can  $x_n$  be a product of two Fibonacci numbers, which then reduces to the study of Diophantine equation

$$x_n \in \{F_m F_\ell\}. \quad (1)$$

We will show that the above equation has at most one solution  $n$  in positive integers, with a few exceptions in  $d$ . Our proofs use the linear forms in logarithms of algebraic numbers. (Received September 26, 2017)

## General Session on Probability and Statistics

1135-VT-64      **Eddy Kwessi\*** (ekwessi@trinity.edu), 1 Trinity Place, San Antonio, TX.  
*Semi-Parametric Rank Estimation of Partially Linear Models with Penalized Wavelets.*

In this paper, we consider partially linear models. The parametric part is estimated using general linear rank regression techniques. The nonparametric part is considered to be a monotone function and estimated using penalized wavelets. When the design of the monotone function is fixed and known, classical wavelets are employed, whereas when the design is irregular, random, and unknown, irregular or warped wavelets are employed. The resulting wavelets coefficients are thresholded to produce consistent estimators in nonparametric settings. Monte Carlo simulations, real world applications, and comparisons with B-splines are provided (Received July 17, 2017)

1135-VT-76      **Volodymyr Melnykov**, Tuscaloosa, AL 35401, and **Xuwen Zhu\***  
 (xzhu20@crimson.ua.edu), Louisville, KY 40207. *Studying crime trends in the United States in the XXI century.* Preliminary report.

Finite mixture models are famous for their flexibility in modeling heterogeneity in data. A novel approach is proposed and applied to the United States crime data collected between 2000 and 2012 years. A step-by-step model development is provided illustrating differences and improvements associated with every stage of the process. Results obtained by the final model are illustrated and thoroughly discussed. Multiple interesting conclusions have been drawn based on the developed model and obtained partition. (Received July 21, 2017)

1135-VT-138      **Jenkin Tsui\*** (jenkin.tsui@utoronto.ca), Dept. of Computer and Mathematical Sciences, 1065 Military Trail, Toronto, Ontario M1C1A4, Canada, and **Abel Dasyilva** and **Kenneth Chu**. *Optimal Estimating Equation for Logistic Regression with Linked Data.*

Record Linkage is the act of bringing together records from two files, say file X and file Y, that relate to the same individual or entity. A record is unlinked if it has no link to any outgoing record in the other file. A link between two unmatched records is called a false positive. Chipperfield et al. (2011) considered the situation where logistic regression is applied, using linked pairs and a sample of such pairs, which are each known to be matched or unmatched through clerical reviews. Their proposed solution is inspired by the maximum-likelihood framework incorporating separate features for false positives and unlinked records. In the maximum-likelihood framework, under general regularity conditions, the Fisher information is equal to the variance-covariance matrix of the score function. This key property leads to the asymptotic efficiency of maximum likelihood estimators through the Cramer-Rao bound. In what follows, we show that this property is not satisfied by Chipperfield's score function. Then we refine the related estimator to decrease its variance in large samples by proposing an optimal estimating equation for logistic regression with linked data while accounting for false positives, built upon the Quasi-likelihood framework. (Received September 27, 2017)

1135-VT-149      **PANPAN ZHANG\*** (panpan.zhang@uconn.edu) and **DIPAK DEY**. *The degree distribution and Gini index of random caterpillar trees.*

We consider two classes of random caterpillar trees (RCTs) which evolve in different manners: uniform and nonuniform. Particularly for the nonuniform class, we look into the RCTs growing in the fashion of preferential attachment. We determine both the exact and asymptotic joint distributions of the degrees of the vertices on the central path (of the RCTs) for both classes. We also propose a topological index, based on the Gini index, as a measure of disparity of the evolutionary processes of the two classes of RCTs. (Received August 04, 2017)

1135-VT-346      **Xiaonan Zhu\*** (xzhu@nmsu.edu), **Baokun Li**, **Tonghui Wang** and **Arjun K. Gupta**.  
*Sampling distributions of skew normal populations.*

The sample mean and sample variance are commonly used statistics. In this talk, sampling distributions of mean and variance from a skew normal population are derived under closed skew normal (CSN) settings. The relationship between the CSN distribution and other multivariate skew normal distributions is investigated. The noncentral closed skew chi-square distribution is defined, and the distribution of quadratic forms is discussed. Several examples are given for illustration of our results. (Received August 26, 2017)

- 1135-VT-541 **Nehal Adhikari\*** ([nehal.adhikari@coyotes.usd.edu](mailto:nehal.adhikari@coyotes.usd.edu)), Department of Mathematical Sciences, University of South Dakota, 414 East Clark Street, Vermillion, SD 57069, and **Yuhlong Lio** ([yuhlong.lio@usd.edu](mailto:yuhlong.lio@usd.edu)), Department of Mathematical Sciences, University of South Dakota, 414 East Clark Street, Vermillion, SD 57069. *Comparative Study of Quality Control for the Strength of Carbon Fibers*. Preliminary report.

Carbon fibers are ingredients for the rigid composite material used in aerospace and other applications. It is very important to ensure the strength of carbon fibers to meet the required standard. Lio et al. (2014) studied Burr type-X distribution for percentile quality control charts of carbon fibers through maximum likelihood estimator (MLE) and moment method estimation (MME). However, both MLE and MME did not provide close mathematics forms for the estimators of percentiles. Therefore, three additional methods, estimator based on percentile, least square method, and weighted least-square method, is used in this paper. Empirical distribution data were collected through simulation using R language. Stimulation was carried out with different sample sizes, and different percentile of interest. Two thousand bootstrap repetitions,  $B=2000$ , had been used to determine the control limits for each bootstrap chart. The collected ARLs, UCLs and LCLs through stimulation can be compared with all proposed control charts and two existing procedure by Lio, et al.(2014) to monitor carbon fiber strength quality in terms of average running length for in-control and out-control procedures. (Received September 07, 2017)

- 1135-VT-693 **Doo Young Kim\*** ([dkim@astate.edu](mailto:dkim@astate.edu)), PO Box 70, State University, AR 72467. *Risk Based Target Lag Clustering of Time Dependent Information in Finance*.

This study proposes an efficient clustering method in a stock market based on short term risk indexes. The cross-lag time dependencies in volatility between two financial time series are investigated in order to identify lagging stocks to a set of leading stocks. Also, the arc length of log return on the weighted time line is used as another measure of risk in a stock market to cluster stocks based on risk or variability. The concept of the target lag allows a stock investor to have more flexible clustering output based on one's trading strategy. (Received September 13, 2017)

- 1135-VT-783 **Tiffany Nicole Kolba\*** ([tiffany.kolba@valpo.edu](mailto:tiffany.kolba@valpo.edu)). *Minimal Noise-Induced Stabilization of One-Dimensional Stochastic Differential Equations*.

The phenomenon of noise-induced stabilization occurs when an unstable deterministic system is stabilized by the addition of randomness into the system. In this talk, we investigate under what conditions one-dimensional, autonomous ordinary differential equations can be stabilized by noise, where we take the notion of stability to be that of global stochastic boundedness. Specifically, we prove the minimum amount of noise necessary for noise-induced stabilization to occur when the drift and noise coefficients are power, polynomial, exponential, or logarithmic functions. (Received September 14, 2017)

- 1135-VT-832 **Lakshmi Roychowdhury\*** ([lakshmi.roychowdhury@utrgv.edu](mailto:lakshmi.roychowdhury@utrgv.edu)), 1201 West University Drive, University of Texas Rio Grande Valley, Edinburg, TX. *Quantization in Statistics*.

Quantization is a nonlinear, zero-memory operation of converting a continuous signal into a discrete signal that assumes only a finite number of levels. It has broad application in engineering technology such as signal processing and data compression. I will talk about the quantization for some probability distributions. (Received September 15, 2017)

- 1135-VT-901 **Grigory Terlov\*** ([terlov@wisc.edu](mailto:terlov@wisc.edu)), 131 Monroe St, Apt. 2s, Hoboken, NJ 07030. *On the Probability of Random Polynomials with Integer Coefficients Being Irreducible*.

In this article we study asymptotical behavior of the probability that a random monic polynomial with integer coefficients is irreducible over the integers. We consider the cases where the coefficients grow together with the degree of the random polynomials. Our main result is a generalization of a theorem proved by Konyagin in 1999. We also generalize Hilberts Irreducibility Theorem and present an analog of this result with shifted Binomial distributed coefficients. (Received September 16, 2017)

- 1135-VT-917 **Bhikhari P Tharu\*** ([btharu@spelman.edu](mailto:btharu@spelman.edu)) and **Nirajan Dhakal**. *Spatiotemporal trends in daily precipitation extremes and their connection with North Atlantic tropical cyclones for the Southeastern United States*.

In this study, the linear quantile regression method is employed to analyze spatio-temporal trends of extreme precipitation and to study the impact of North Atlantic Tropical Cyclones in the distribution of extreme precipitation for the Southeastern United States. Daily annual maximum precipitation over the period of 62 years (1950 -2011) for 130 sites was used for the analysis. Our results show that changes in upper quantiles of the distributions of the extreme precipitation have occurred in the Southeastern United States and at, a much higher

rate than previously believed. Analysis of the potential changes in the distribution of the extreme precipitation by separating the historical record into two periods, i.e., before and after 1980, reveals that upper-quantile trends have increasing magnitude in most of the sites for the latest time period. Analysis of the impact of tropical cyclones in the extreme precipitation distribution shows that overall, the heavy rainfall events in the recent decades may have been caused by tropical cyclones. (Received September 16, 2017)

1135-VT-978 **John C. Wierman\*** ([jwierma1@gmail.com](mailto:jwierma1@gmail.com)), Dept. of Applied Mathematics & Statistics, 100 Whitehead Hall, Johns Hopkins University, Baltimore, MD 21218. *An unexpected expectation trick for maximums and minimums of two random variables.*

A trivial identity involving the maximum and minimum of two random variables can be used to simplify the calculation expectations of certain functions of the maximum and minimum, such as moments, the moment-generating function, and the covariance. The proof relies on the linearity of expectation, and does not require independence or identical distribution of the random variables. The result is accessible to students in a calculus-based probability course, in which it could be used in in-class examples or as a homework exploration. The trick was discovered and was very beneficial in undergraduate research on the astronaut problem in rendezvous search theory. (Received September 18, 2017)

1135-VT-1012 **Steven B Kim\*** ([stkim@csumb.edu](mailto:stkim@csumb.edu)), 907 Walnut Street B, Pacific Grove, CA 93950. *An Alternative Parameterization for Hormesis Problem in Toxicology.* Preliminary report.

In toxicology, hormesis refers to a phenomenon where exposure to low doses of a harmful agent may result in a beneficial effect. For example, if an outcome of interest is tumor development in an animal-based experiment, hormesis theory assumes that a small dose of carcinogen may reduce the risk of tumor development. Even if hormesis exists, researchers can fail to provide significant evidence for hormesis due to a small number of experimental doses and sparse data. In this talk, we compare hypothesis testing under three- and four-parameter logistic regression models using a simulation design similar to a typical experimental design. In the simulation, we observe high statistical power under correct specification, but the power is extremely low even under slight misspecification. To address the impact of misspecification, we discuss an alternative parameterization to improve the robustness. The alternative parameterization allows discontinuity in the dose-response relationship between the zero dose (i.e., control group) and the first non-zero dose (lowest experimental group), and we can still address the scientific question (testing hormesis). At the end of the talk, we discuss the use of the alternative parameterization for future direction. (Received September 18, 2017)

1135-VT-1116 **William Cipolli\*** ([will@cipolli.com](mailto:will@cipolli.com)), 13 Oak Dr, Department of Mathematics, Hamilton, NY 13346, and **Timothy Hanson.** *Supervised Learning via Smoothed Polya Trees.* Preliminary report.

The goal in any classification scheme is to design a system that classifies new observations into their true class as often as possible. The Bayesian nonparametric approach of the multivariate Polya tree proposed realizes impressive results in simulations and real data analyses, performing similarly to, or better than, current approaches in many cases. The flexibility gained from relaxing certain distributional assumptions from the model can greatly improve the ability to correctly classify new observations; even minor deviations from parametric distributional assumptions could lead to missing an important feature in any one class's density. Completing classification using Bayes Rule and the nonparametric density estimation of the multivariate Polya Tree is quite fast compared to other supervised classifiers and very simple to implement as there are no kernel tricks or initialization steps. (Received September 19, 2017)

1135-VT-1184 **Xiaoya Meyer\*** ([xm930527@cameron.edu](mailto:xm930527@cameron.edu)), Lawton, OK 73505, and **Hong Li** ([lhong@cameron.edu](mailto:lhong@cameron.edu)), Lawton, OK 73505. *A Comparison of Robust Logistic Regression Methods.*

Logistic regression (binary regression) is one of the most popular and widely used models for analyzing the effect of explanatory variables on a binary response variable. The maximum likelihood (ML) method has been generally adopted to estimate the regression parameters. However, the presence of an outliers and/or influential observations greatly reduces the accuracy of parameter estimates of ML method. A few robust regression methods such as Bianco-Yohai robust estimator (BY) and Mallows robust estimator were proposed and have been used in the presence of outliers. Li et. al. introduced a new robust binary regression model (TLRL) and a multinomial regression model in 2014. In this study we compare the performance of TLRL estimates with BY estimates using two real data sets which contain outliers. Besides, we conduct a simulation study to compare the performance of ML, TLRL and BY in the presence of outliers. We identify the outliers using diagnostic graphs and measures. Akaike Information Criterion is computed to assess the goodness of fit for each model. Results indicate that



TLRL method performs better than the BY estimator for the examples we considered. TLRL method is as good as BY estimator in terms of bias and mean squared errors based on the simulation study. (Received September 20, 2017)

1135-VT-1198 **Rajeshwari Majumdar** and **Suman Majumdar\*** ([suman.majumdar@uconn.edu](mailto:suman.majumdar@uconn.edu)), Department of Statistics, University of Connecticut, 1 University Place, Stamford, CT 06901-2315. *Necessary and Sufficient Condition for Asymptotic Standard Normality of the Two Sample Pivot.*

The asymptotic solution to the problem of comparing the means of two heteroscedastic populations, based on two random samples from the populations, hinges on the pivot underpinning the construction of the confidence interval and the test statistic being asymptotically standard Normal. Existing results in the literature establish the convergence of the pivot to the standard Normal distribution assuming that the samples are independent and the ratio of the sample sizes converges to a finite positive number. We show, without any restriction on the asymptotic behavior of the ratio of the sample sizes, that Cesaro convergence of the sequence of cross sample correlation coefficients to 0 is necessary and sufficient for the aforesaid pivotal convergence. We also obtain, without any assumption on the cross sample dependence structure, that both iterated limits of the pivot are standard Normal and if the joint distribution of the standardized sample means converges to a spherically symmetric distribution, then that distribution must be bivariate standard Normal. (Received September 20, 2017)

1135-VT-1303 **Ram Sharan Adhikari\*** ([radhikari@rsu.edu](mailto:radhikari@rsu.edu)), Rogers state University, Claremore, OK 74017. *Mean square stability analysis of a weak modified Euler-Maruyama method based on trapezoidal rule for a class of stochastic differential equations.*

The proposed weak modified Euler-Maruyama method has the potential to overcome some of the numerical instabilities that are often experienced when using explicit Euler method. This work also aims to determine the mean-square stability region of the weak modified Euler-Maruyama method for linear stochastic differential equations with multiplicative noises. In this work, a mean-square stability region of the weak modified Euler-Maruyama scheme is identified, and step-sizes for the numerical method where errors propagation are under control in well-defined sense are given. The main results are illustrated with numerical examples. (Received September 21, 2017)

1135-VT-1309 **Veera Holdai\*** ([vxholdai@salisbury.edu](mailto:vxholdai@salisbury.edu)), **Ben Duklewski** and **Barbara Wainwright**. *On Some Limitations of Financial Models.* Preliminary report.

Thin-tailed probability distributions (like normal) are used in creating investment portfolios and assessing risks. Thick-tailed probability distributions (like Pareto) could be considered instead of thin-tailed distributions under the Brownian Motion model for put option pricing. Unfortunately, such models fall apart due to divergent integrals that arise from the Put-Call Parity. The arguments and results presented might be of interest to instructors and undergraduate students in actuarial science who are familiar with basic probability and interest theories, but not necessarily with stochastic calculus. (Received September 21, 2017)

1135-VT-1365 **Dilli Bhatta\*** ([dbhatta@uscupstate.edu](mailto:dbhatta@uscupstate.edu)), 800 University Way, Spartanburg, SC 29303. *Bayesian Analysis of Contingency Tables With Covariates Under Cluster Sampling.*

We implement a Bayesian approach of testing independence between two categorical variables presented in a two-way contingency table with covariates for a two-stage cluster sample. Under this approach, we convert the cluster sample with covariates into an equivalent simple random sample without covariates which provides a surrogate of the original sample. Then, this surrogate sample is used to compute the Bayes factor to make an inference about independence. We apply our methodology to the data from the Trend in International Mathematics and Science Study (2007) for fourth grade U.S. students to assess the association between the mathematics and science scores represented as categorical variables. We show that if there is strong association between two categorical variables, there is no significant difference between the tests with and without the covariates. We also performed a simulation study to further understand the effect of covariates in various situations. We found that in borderline cases (moderate association between the two categorical variables) there are noticeable differences in the test with and without covariates. (Received September 21, 2017)

1135-VT-1398 **Suttisak Wisadwongsa\***, 239 Huaykaew road, Tumbol Suthep, Amphur muang, Chiang Mai, 50200, Thailand, and **Santi Tasena**. *Bivariate Quadratic Copula Constructions.*

In this work, we present the concept of quadratic copula constructions using two arbitrary copulas. We characterize all quadratic polynomials of four variables whose composition with any two copulas always results in a

copula. We show that these polynomials form a closed convex set in a seven-dimensional vector space. We also apply the result to obtain a new family of copulas. (Received September 21, 2017)

1135-VT-1509 **Steve S. Chung\*** ([schung@csufresno.edu](mailto:schung@csufresno.edu)), Department of Mathematics, California State University, Fresno, 5245 N. Backer Ave. M/S PB 108, Fresno, CA 93740. *Conditional variance estimation using support vector machine.*

Conditional variance plays an important role in finance because it is associated with the risk. This is also called the volatility. A growing body of literature shows that risks associated with volatility are priced in stock, option, bond, and foreign exchange markets. Therefore, accurate estimation is critical in financial markets. The generalized autoregressive conditional heteroskedasticity (GARCH) has been one of the most popular models and the parameters are usually estimated from conditional maximum likelihood estimation (MLE) method. In this work, we attempt to improve the MLE-based GARCH forecast using the support vector machine (SVM). We also compare the SVM-based model with two popular asymmetric models: exponential GARCH (E-GARCH) and Glosten-Jagannathan-Runkle GARCH (GJR-GARCH). We carry out the analysis through simulations and real datasets. The results show that the SVM-based models provide better predictive potential than the existing parametric models. (Received September 22, 2017)

1135-VT-1549 **Ayman A Alzaatreh\*** ([aalzaatreh@aus.edu](mailto:aalzaatreh@aus.edu)), Sharjah, United Arab Emirates, **Indranil Ghosh**, Wilmington, NC, and **Arnold Barry**. *Family of weighted bivariate and multivariate distributions.*

Recently, weighted distributions have received widespread attention over the last two decades because of their flexibility for analyzing skewed data. In this talk, a family of weighted bivariate and multivariate weighted distribution is proposed. For illustrative purposes, some examples of the proposed method are presented. Several structural properties of the bivariate weighted distributions are studied. In addition, we provide some multivariate extensions of the proposed models. A real-life data set is used to show the applicability of the proposed method. (Received September 23, 2017)

1135-VT-1916 **Gaoran Yu\*** ([gyu9@jhu.edu](mailto:gyu9@jhu.edu)), 103 W 39th St, Apt F3, Baltimore, MD 21210, and **John C. Wierman**. *Rigorous Upper Bounds for Bond Percolation Thresholds of 3D Lattices.*

We introduce a growth process approach that provides upper bounds for bond percolation thresholds of 3D lattices. The approach views the open cluster of the configuration of a 3D lattice as a dynamic process. Projecting the 3D process onto a carefully chosen plane results in a projected process on a 2D lattice. The projected process is then related to a canonical bond percolation model on the same 2D lattice by comparing their corresponding probability measures using "step-wise" stochastic ordering. Subsequently, we derive the percolation threshold of the 3D lattice from that of the 2D lattice.

Using this approach, we determined upper bounds for the bond percolation thresholds of the simple cubic lattice, the BCC lattice and the FCC lattice, which are 0.34730, 0.27455 and 0.19333, respectively. The approach can also be applied to stacked lattices.

(Received September 25, 2017)

1135-VT-2106 **Ganesh B. Malla\*** ([ganesh.malla@uc.edu](mailto:ganesh.malla@uc.edu)), 8004 Higgins Ct, Cincinnati, OH 45242. *A New Test for New Better Than Used in Expectation Lifetimes.*

**Abstract:** The mean residual life of a non-negative random variable  $X$  with a finite mean is defined by  $M(t) = E\{X - t \mid X > t\}$  for  $t \geq 0$ . A popular nonparametric model of aging is new better than used in expectation (NBUE), when  $M(t) \leq M(0)$  for all  $t \geq 0$ . The exponential distribution lies at the boundary. There is a large literature on testing exponentiality against NBUE alternatives. However, comparisons of tests have been made only for alternatives much stronger than NBUE. We show that a new Kolmogorov-Smirnov type test is much more powerful than its competitors in most cases. (Received September 25, 2017)

1135-VT-2222 **Rui Liu\***, Department of Mathematics and Statistics, P.O. Box 10384, Louisiana Tech University, Ruston, LA 71272, and **Ioannis Vlachos**. *Statistical Dependency in the Frequency Domain for Application in Biological and Natural Systems.* Preliminary report.

Biological and natural systems comprise of multiple components that are (typically) interacting non-linearly and producing multiple outputs of specific frequency characteristics. Without exact knowledge of the underlying mechanism, we are confined to the study and quantitative analysis of time series (observable outputs) to identify the dependencies and increase our understanding of the system. Analysis of the time series in the frequency domain is achieved through Fourier transform, or other similar decomposition procedures, while dependencies can be estimated through specified models, or more general probabilistic frameworks, such as Mutual Information.

Herein, the background of this type of analyses is presented and discussed. Additionally, by combining ideas from Information Theory and analysis of time series in the frequency domain, a model-free methodology for quantifying nonlinear dependencies between time series with respect to frequency is developed. Results are presented from applications of this methodology to simulated coupled nonlinear systems data, and to real-world electrophysiological data. (Received September 25, 2017)

1135-VT-2310 **Lori McCune\***, Missouri Western State University, 4525 Downs Dr, St. Joseph, MO 64507, and **David McCune**, William Jewell College, 501 College Hill, Liberty, MO 64068. *Count Your Chickens with Markov Chains.*

The cooperative game "Count your Chickens!" teaches preschoolers counting and subtracting by moving a number of chicks in or out of a chicken coop each turn based on each player's spin. Similar to "Chutes and Ladders" or "Monopoly", this game involves movement of a token on a game board in which the movement is determined by pure chance. Such games can be analyzed using the probabilistic tool of Markov Chains. We discuss the use of Markov Chains to determine the win probability for "Count Your Chickens!" and the win probability for variants of the game in which we change the structure of the underlying game board. (Received September 25, 2017)

1135-VT-2406 **Keshav P Pokhrel\***, 4901 Evergreen Road, Dearborn, MI 48128, **Vinod Shidham**, Detroit, MI 48201, and **Sabita Dugu**. *Effectiveness of Cervical Cancer Screening Tests.* Preliminary report.

We present the effectiveness of cervical cancer screening tests in the light of surgical results. Ordinal regression models are used to test the adequacy of human papillomavirus (HPV) and Pap smear tests (PAP) in predicting likelihood of cervical cancer. We compare our results with cervical cancer screening guidelines for average-risk women in different age groups. Our primary focus is to analyze the possible false negative results of HPV tests. (Received September 26, 2017)

1135-VT-2436 **Lasanthi CR Pelawa Watagoda\*** ([lasanthi@appstate.edu](mailto:lasanthi@appstate.edu)), Department of Mathematical Sciences, Appalachian State University, Boone, NC 28607, **Hasthika S Rupasinghe Arachchige Don**, Department of Mathematical Sciences, Appalachian State University, Boone, NC 28607, and **Alan T Arnholt** ([arnholtat@appstate.edu](mailto:arnholtat@appstate.edu)), Department of Mathematical Sciences, Appalachian State University, Boone, NC 28607. *A new regularization and variable selection technique - HRLR.*

This work propose a new variable selection and parameter estimation method for the multiple linear regression model  $Y = \beta_1 x_1 + \dots + \beta_p x_p + e$ . This new method is a hybrid of ridge regression and relaxed lasso regularization. Theoretical and simulated results demonstrate that the new method produces sparser models with equal or lower prediction loss than the regular Lasso and Relaxed Lasso estimators for high dimensional data. (Received September 26, 2017)

1135-VT-2447 **Hasthika S Rupasinghe Arachchige Don\*** ([hasthika@appstate.edu](mailto:hasthika@appstate.edu)), Department of Mathematical Sciences, Appalachian State University, Boone, NC 28607, and **David J Olive** ([dolive@siu.edu](mailto:dolive@siu.edu)), Department of Mathematics, Southern Illinois University Carbondale, Carbondale, IL 62901. *Bootstrapping Analogs of the one way MANOVA test.*

The classical one way MANOVA model is used to test whether the mean measurements are the same or differ across  $p$  groups, and assumes that the covariance matrix of each group is the same. This work suggests using the Olive (2017abc) bootstrap technique to develop analogs of one way MANOVA test. A large sample theory test has also been developed. The bootstrap tests can have considerable outlier resistance, and the tests do not need the population covariance matrices to be equal. The two sample Hotelling's  $T^2$  test is the special case of the one way MANOVA model when  $p = 2$ . (Received September 26, 2017)

1135-VT-2573 **Mehdi Razzaghi\*** ([mrazzagh@bloomu.edu](mailto:mrazzagh@bloomu.edu)). *An Extension of the Log-Lindley Distribution with Application.* Preliminary report.

Since the introduction of the Lindley distribution in the late 1950's several generalizations and modifications have been considered by several authors. Recently, however, the log-Lindley distribution was introduced by some researchers with many interesting properties and features, and with extensive applications in insurance and economics. The advantage of the log-Lindley distribution is that it can be regarded as an alternative to beta distribution. This latter distribution serves as a very flexible model in a variety of set ups and therefore, in theory, the log-Lindley distribution has the potential of being a very important model in various applications. Here, we

introduce the log-Lindley-normal distribution as an alternative to the beta-normal model. The properties of this new proposed model are explored and an application is discussed. (Received September 26, 2017)

1135-VT-2802 **Jessica Jaynes\*** (jjaynes@fullerton.edu). *Using Design of Experiments to Determine Consumer Preference with Applications to Health Science*. Preliminary report.

A discrete choice experiment (DCE) is a survey method that gives insight into individual preferences for particular attributes. They provide a rich source of data to assess real-life decision-making processes, which involve trade-offs between desirable characteristics. Traditionally, methods for constructing DCEs focus on identifying the individual effect of each attribute. However, an interaction effect between two attributes better represents real-life trade-offs, and provides us a better understanding of subjects' competing preferences. The choice of the design for a DCE is critical because it determines which attributes' effects and their interactions are identifiable. We propose the use of blocked fractional factorial designs to construct DCEs and address some identification issues by utilizing the known structure of blocked fractional factorial designs. These designs are easy to construct and for many practical scenarios are readily available in the literature. Further, we discuss the implementation of our design methodology with an application in health sciences pertaining to college students' snack selection and nutritional ingredient attributes. (Received September 26, 2017)

1135-VT-2873 **Luis J Soliz\*** (luis13soliz@gmail.com), 6114 W Grace St, Chicago, IL 60634, and **Maunak Rana**. *The application of Bayes' theorem to justify the use of a triple-phase bone scan (TPBS) in helping diagnose complex regional pain syndrome (CRPS) within select patient populations*.

Complex regional pain syndrome (CRPS) is a poorly understood and complex neuropathic disease process often affecting a single extremity after inciting trauma. Since 2007, the Budapest criteria have been the recommended standard in diagnosing CRPS. Critics have argued that bone scintigraphy (BS) offers little clinical value in supporting the diagnosis of CRPS and cannot be used to confirm its diagnosis. We obtain input data for Bayes' Theorem and calculate the probability of a patient having CRPS given a positive triple-phase bone scan (TPBS). We further utilize a variable base rate probability based on different selected patient populations to illustrate the importance of careful patient population selection when utilizing Bayesian analysis. We found the probability of a patient having CRPS given a positive TPBS was <1% for the general population. In contrast, the probability of a patient having CRPS given a positive TPBS within a patient population of individuals undergoing external fixation of tibial fractures and one of surgically treated distal radius fractures were 74% and 84.4%, respectively. These findings support the judicious use of BS in confirming the diagnosis of CRPS when there is a moderately high pre-test clinical suspicion to avoid delayed treatment. (Received September 26, 2017)

1135-VT-2899 **Gokarna R Aryal\***, aryalg@pnw.edu. *Characterizations of Beta Exponential Pareto Distribution with Applications*. Preliminary report.

In this study we develop the so-called beta exponential Pareto (BEP) distribution by using the composition of the beta distribution and exponential Pareto distribution. Several lifetime distributions including the beta Weibull, beta exponential, beta Rayleigh, generalized Weibull, Weibull among others are embedded in the proposed distribution. Various mathematical properties along with parameter estimation and simulation issues are discussed. The importance and flexibility of the proposed model is illustrated by means of real data analysis (Received September 26, 2017)

1135-VT-2906 **Sheida Riahi\*** (sr1315@msstate.edu) and **Prakash Patil**. *Numerical Approach to Testing Central Symmetry in Bivariate Settings*. Preliminary report.

A concept of symmetry become important many domains such as detection of abnormalities as asymmetrical patterns in the thermographic images which is linked to the concept of symmetric regression functions. Or else when the null distributions of t and F statistics in the univariate general linear model depend on spherical symmetry of the error distribution. For a multivariate distribution one can define different kinds of symmetry, e.g. central, spherical, elliptical symmetry etc. Our interest here is to study these symmetries in bivariate set-up with the intention of quantifying and testing for different kinds of asymmetries. For that here as a primary step we have numerically exhibited that the recently proposed tests for symmetry in Partlett and Patil (2015) could be extended to the case of central symmetry in bivariate settings. (Received September 26, 2017)

1135-VT-2921 **Gene B Kim\***, genebkim@usc.edu. *Distribution of descents in matchings*.

The distribution of descents in a fixed conjugacy class of  $S_n$  is studied, and it is shown that its moments have an interesting property. A particular conjugacy class that is of interest is the class of matchings (also known as fixed point free involutions). This paper provides a bijective proof of the symmetry of the descents and major

indices of matchings and uses a generating function approach to prove an asymptotic normality theorem for the number of descents in matchings. (Received September 26, 2017)

1135-VT-2938 **Luis J Soliz\*** (luis13soliz@gmail.com), 6114 W Grace St, Chicago, IL 60634, and **Maunak Rana**. *The application of Bayes' theorem for diagnosing herniated nucleus pulposus (HNP) based on physical exam findings.*

Primary care physicians routinely utilize physical exam findings to help support a diagnosis of lumbar radiculopathy. These findings can help dictate whether or not a patient will undergo further diagnostic workup or even possible surgical considerations. We obtain input data for Bayes' theorem and calculate the probability of having a lumbar herniated disc (HNP) given positive findings on commonly utilized physical exam maneuvers as part of the physical workup for HNP. The required information to calculate this was found within the orthopedic literature and includes the prevalence of lumbar HNP, the probability of a positive straight leg test (SLT) given that there is no HNP, the probability of a positive SLT given that there is HNP, the probability of a positive contralateral straight leg test (CSLT) given that there is no HNP, and the probability of a positive CSLT given that there is HNP. We found the probability of lumbar HNP given a positive SLT was only 3.8% and the probability of lumbar HNP given a positive CSLT was not much better at 6.9%. Given the low prevalence of lumbar HNP in the general population, one cannot trust pure physical exam findings when diagnosing lumbar HNP. (Received September 26, 2017)

1135-VT-2953 **Ahmad Alzagal\*** (ahmad1.alzagal@farmingdale.edu), NY 11735. *A new gamma-Pareto distribution: Properties and Applications.* Preliminary report.

A new gamma-Pareto distribution is introduced and studied. Some of its properties are discussed. The method of maximum likelihood is used to estimate the new gamma-Pareto distribution parameters. The flexibility of the new gamma-Pareto distribution is illustrated by applying it to real data sets and the results compared with other distributions. (Received September 26, 2017)

1135-VT-2958 **Jewgeni H. Dshalalow** and **Ali Hussein Mahmood Al-Obaidi\*** (aalobaidi2013@my.fit.edu), 1245 Palm Bay Rd, Apt# R102, Palm Bay, FL 32905. *Modulated Random Measures On Hausdorff Topological Spaces.*

In this paper, we will introduce random measures on  $\sigma$ -compact Hausdorff spaces and define their stochastic integrals of functions of one and two variables. We further form a rigorous construction of a random measure perturbed by a stochastic process and target its stationary intensity often occurring in control theory. To obtain that intensity we modify and refine the classic Campbell's theorem for random measures. Examples of modulated Poisson measures are discussed. (Received September 26, 2017)

1135-VT-3062 **Daniel Brumley\***, dbrumley1@uco.edu, and **Tyler Cook**. *Pathway and Gene Selection with Guided Regularized Random Forests.*

Many approaches have been developed in order to model a biological outcome based on microarray data. Much focus has recently been given to incorporating gene interactions via genetic pathway information available in online databases. The additional knowledge of gene relationships may help researchers better understand the biological processes under investigation. In this talk, we outline a method for pathway and gene selection based on guided regularized random forests (GRRF) that allows for the ranking of both pathways and genes in classification problems. In GRRF, variable importance scores from a random forest guide a regularization procedure to identify a subset of significant predictors. Simulation studies, as well as an analysis of a breast cancer dataset, show that our methodology is successful in identifying a compact set of important pathways and genes with a low prediction error rate. (Received September 26, 2017)

1135-VT-3112 **George Mytalas\***, george.mytalas@brooklyn.cuny.edu, and **Ioannis Farmakis**. *Batch arrival queueing system with vacations, disasters and repairs under the N-policy.* Preliminary report.

We consider a M/G/1 queueing system with batch arrivals subject to disasters and server breakdowns under N-policy. The server is turned off as soon as the system empties. When the queue length reaches or exceeds a value N (threshold), the server is turned on and begins to serve the customers. When a disaster occurs the system is cleared of all customers and the server initiates a repair period. During the repair period arriving batches of customers accumulate in the queue without receiving service. Besides, the server has an exponential lifetime in addition to the catastrophe process. By applying the supplementary variables method, we obtain the steady-state solutions for both queueing measures and reliability quantities of interest. (Received September 26, 2017)

- 1135-VT-3180 **Abdul-Nasser El-Kassar\*** ([abdunnasser.kassar@lau.edu.lb](mailto:abdunnasser.kassar@lau.edu.lb)), Lebanese American University, Information Tech. & Oper. Man. Dept, P.O. Box 13- Chouran, Beirut, 1102 2801, Lebanon, and **Noura Yassine** ([noura.yassein@bau.edu.lb](mailto:noura.yassein@bau.edu.lb)), Faculty of Science, Mathematics and Computer Department, P.O. Box 11-5020, Beirut, 1102 2801, Lebanon. *Functions Involving The Maximum And Minimum Of Random Variables Arising From Inventory Models With Quality.*

Applications of the study of functions of several random variables can be found in various fields such as biology, engineering, and operations research. We present several extensions of classical inventory models that incorporate the effects of quality of the various types of products/components considered in the models. Each type of product/component is assumed to have a percentage of perfect quality items, a continuous random variable having a known probability distribution. We show that the optimal solution depends on deriving the probability distribution of a function of the random variables representing the percentages of perfect quality items, and develop a closed form formula approximating the optimal solution. The closed form formula is given in terms of the expected value. The proposed models give rise to functions involving the minimum, maximum and sum of the random variables. We describe how the probability functions and the expected values can be obtained. Expressions in the case of uniform and normal distributions are derived. Difficulties encountered when numerically determining via simulation the expected value of a function of random variables are discussed. For future research, we pose several open problems suggest several extensions of the inventory models . (Received September 26, 2017)

## General Session on Topology

- 1135-VU-720 **Aliaksandra Yarosh\*** ([alex.yarosh@psu.edu](mailto:alex.yarosh@psu.edu)). *Computations in twisted Morava K-theory.*

We introduce Morava K-theory twisted by bundles of Eilenberg-MacLane spaces, and compute twisted Morava homology of all connective covers of the stable orthogonal group and the stable unitary group, their classifying spaces, as well as spheres and Eilenberg-MacLane spaces. Surprisingly, in all the cases considered, there are only two possibilities: either the twisted homology vanishes, or it is isomorphic to untwisted homology. (Received September 13, 2017)

- 1135-VU-836 **Mehmet E Aktas\*** ([maktas@uco.edu](mailto:maktas@uco.edu)), 100 North University Drive Box 129, Edmond, OK 73034, and **Serdar Cellat** and **Hubeys Gurdogan**. *A polynomial invariant for plane curve complements: Krammer polynomials.*

We use the Krammer representation of the braid group in Libgober's invariant and construct a new multivariate polynomial invariant for curve complements: Krammer polynomial. We show that the Krammer polynomial of an essential braid is equal to zero. We also compute the Krammer polynomials of some certain n-gonal curves (Received September 15, 2017)

- 1135-VU-838 **Craig R Guilbault** and **Molly A Moran\*** ([mmoran@coloradocollege.edu](mailto:mmoran@coloradocollege.edu)). *Quasi-Isometric Boundary Swapping.*

Bestvina formalized the concept of a group boundary by introducing the notion of a Z-structure on a group. In his initial paper, Bestvina proved a boundary swapping theorem that can be applied to a group  $G$  with a finite  $K(G, 1)$ . He also suggested that a generalized version of boundary swapping should hold for two groups that are quasi-isometric. We will present a generalization of this result and discuss some of the implications. (Received September 15, 2017)

- 1135-VU-1183 **Peter Feller, Michael Klug, Trent Schirmer and Drew Zemke\***, [drew.zemke@gmail.com](mailto:drew.zemke@gmail.com). *Calculating the classical algebraic topology of a 4-manifold from a trisection diagram.*

The notion a *trisectioned 4-manifold*, introduced in 2012 by Gay and Kirby, allows one to study a smooth, oriented 4-manifold from the perspective of some of its 2- and 3-dimensional submanifolds. In particular, the smooth structure of a 4-manifold can be encapsulated in three sets of isotopy classes of simple closed curves on a closed surface in what is called a *trisection diagram*. I will provide a brief background on trisections and trisection diagrams, and then show how to compute the homology and intersection form of a 4-manifold  $X$  from the homological information in any trisection diagram for  $X$ . (Received September 20, 2017)

1135-VU-1192 **Samantha Pezzimenti\*** (spezziment@brynmawr.edu), Bryn Mawr, PA. *Legendrian Knots and their Lagrangian Fillings.*

Given a knot, what surfaces in the 4-dimensional space can it bound? We ask a version of this classic knot theory question about Legendrian knots, which are knots that satisfy an additional geometric condition imposed by a contact structure. Now the question is: Given a Legendrian knot, what Lagrangian surfaces can it bound? From an invariant polynomial associated to the Legendrian knot, we can extract a great deal of geometric information about the Lagrangian surface. Extending work of Sabloff and Traynor about embedded fillings, I show that this polynomial also encodes restrictions on the minimal number of double points of an immersed filling of a given genus. I will also give some combinatorial constructions of some “minimal” immersed fillings. (Received September 20, 2017)

1135-VU-1218 **Steven Scheirer\*** (sts413@lehigh.edu). *Topological complexity of graph configuration spaces.*

The topological complexity of a path-connected space  $X$ , denoted by  $TC(X)$ , is an integer which can be thought of as the minimum number of continuous “rules” required to describe how to move between any two points of  $X$ . We will consider the case in which  $X$  is a space of configurations of  $n$  points on a graph  $\Gamma$ . There are two such configurations spaces: in the first, denoted by  $C^n(\Gamma)$ , the order of the points on  $\Gamma$  is of importance, while in the second, denoted by  $UC^n(\Gamma)$ , the order of the points is irrelevant. We will discuss methods to determine the topological complexity of these spaces in the case in which  $\Gamma$  is a tree. (Received September 20, 2017)

1135-VU-1488 **Jennifer L. Dalton** (j.dalton@rivers.org), **John B. Etnyre** (etnyre@math.gatech.edu) and **Lisa Traynor\*** (ltraynor@brynmawr.edu). *Legendrian Torus Links.* Preliminary report.

Legendrian knots and links are smooth knots and links that satisfy a geometrical condition imposed by a contact structure. Due to this extra structure, there are multiple (in fact, infinitely many) Legendrian representatives of any knot or link. In this talk, we focus on Legendrian torus knots and links. Etnyre and Honda gave a classification of Legendrian torus knots. I will discuss the classification of unordered and ordered Legendrian torus links. (Received September 22, 2017)

1135-VU-1496 **Jack Arndt\*** (jack.arndt@my.simpson.edu), 701 North C Street, Indianola, IA 50125, and **Malia Jansen** and **Payton McBurney**. *Investigating diagonal knot grid diagrams.* Preliminary report.

In 2003, Ozsváth, Szabó, and Rasmussen introduced the  $\tau$  invariant for knots, and in 2011, Sarkar published a computational shortcut for the  $\tau$  invariant of knots that can be represented by diagonal grid diagrams. We show that all such knots are positive knots, and we conjecture that the set of these diagonal knots is equal to the set of positive braid knots. (Received September 22, 2017)

1135-VU-1882 **Brandon Q Tran\*** (tran2833@stthomas.edu). *Classifying composite links.* Preliminary report.

Mathematical links are appearing in science literature with increasing frequency (for example, proteins). Meanwhile, most mathematical analysis of entanglement, and computational tools to study entanglement, are focused on knotting. Our goal is to create a software chain that can classify link types in diagrams with up to 10 crossings.

While prime links have been classified through 11 crossings, there has been no systematic study of composite links (to the best of our knowledge). We know, to some extent, what all of the composite links should be. However, composing links is more complicated than composing knots, e.g. one needs to specify which components are to be composed.

Cantarella et al. have generated all link diagrams (and knot diagrams too) through 10 crossings. In this talk, we summarize our efforts to compute the link types of these diagrams, the most challenging of which are the composite link types. (Received September 25, 2017)

1135-VU-1936 **Zach Sorenson\*** (sore1324@stthomas.edu) and **Isaac Vraspir**. *Recognizing knot types using neural networks.* Preliminary report.

Artificial intelligence and neural networks seem to be in the news daily. Among the long list of applications, these algorithms can be used for image processing, e.g. to differentiate between pictures of different types of animals.

Meanwhile, the overarching problem in knot theory is to classify the knot type of a given configuration. Can neural nets be used to recognize knot types? We analyze the simple case of six-edge equilateral knots, in

which case only three different knot types are possible: unknot, positive trefoil, and negative trefoil. (Received September 25, 2017)

1135-VU-1937 **Isaac Vraspir\*** (vras0001@stthomas.edu) and **Zach Sorenson**. *Knots in tight confinement*. Preliminary report.

Knots in nature, e.g. in DNA or proteins, are typically formed under some sort of spatial confinement. Recent research has provided insights into how confinement affects the knotting of random chains. Unfortunately, the chain generation algorithms have bounds on their confinement: the confining spheres must have a diameter of twice the edge length. Our goal is to study knots in extreme confinement, where the sphere diameter is only slightly larger than the edge lengths.

We model this situation using cylinders with very large height-to-radius ratios (i.e. long skinny cylinders). We create a roughly equilateral knot by choosing points on the opposite disk ends of the cylinders in an alternating fashion. In this talk, we explore how knotting changes as the confinement becomes more and more extreme. (Received September 25, 2017)

1135-VU-2097 **Malcolm Henry Gabbard\*** (malcolm.gabbard@coloradocollege.edu) and **Sam Kottler** (s.kottler@coloradocollege.edu). *A Study of Metrics on Visual Boundaries*.

Certain metrics on the boundary of CAT(0) spaces have been well studied. However, these metrics are not easily extendable to the interior of the CAT(0) space. We propose a new family of metrics on the boundary of CAT(0) spaces which can be extended to metrics on the interior of the space. We explore whether changing the parameters for these metrics is a quasi-symmetric transformation. The advantage of this family of metrics is that it depends on the entire geodesic ray so it captures more of the geometry of the space. This will hopefully help with problems such as generalizing which boundaries of CAT(0) spaces have a finite linearly controlled dimension. (Received September 25, 2017)

1135-VU-2171 **Vincent Longo\*** (vlongo2@unl.edu) and **Alex Zupan**. *Bridge Trisections of Surfaces in 4-Manifolds*.

In classical knot theory, many knot invariants are derived from a diagram of the knot. When studying knotted surfaces in dimension four, it becomes apparent that producing a diagram of a knotted surface can be complicated, and thus inherently more difficult to produce combinatorial definitions of knot invariants derived from a diagram. In this talk, we explore the idea of generalizing bridge splittings of classical knots to knotted surfaces in dimension four in order to obtain a simple diagram for the knotted surface. We call this generalization a bridge trisection of the knotted surface. (Received September 25, 2017)

1135-VU-2209 **Mike Krebs\*** (mkrebs@calstatela.edu). *Is There a Topology on  $\mathbb{Q}$  That Detects Continuous Extensions to  $\mathbb{R}$ ?*

Let  $\mathbb{Q}$  be the set of rational numbers, and let  $\mathbb{R}$  be the set of real numbers in the usual topology. Consider the function  $f$  from  $\mathbb{Q}$  to  $\mathbb{Q}$  where  $f(x) = 1$  if  $x$  is greater than the square root of 2, and  $f(x) = 0$  otherwise. With the usual topology on  $\mathbb{Q}$ , we have that  $f$  is a continuous function. However,  $f$  does not extend to a continuous function from  $\mathbb{R}$  to  $\mathbb{R}$ . How awful! Can we remedy this situation by changing the topology on  $\mathbb{Q}$ ? In other words, does there exist a topology on  $\mathbb{Q}$  so that a function from  $\mathbb{Q}$  to  $\mathbb{Q}$  is continuous if and only if it extends to a continuous function from  $\mathbb{R}$  to  $\mathbb{R}$ ? In this talk, we answer that question. The solution uses only basic definitions and theorems that appear early in a first course in point-set topology, so this question can be used as a challenge problem or a portfolio problem for an introductory Topology class. One can generalize the question to subsets of  $\mathbb{R}$  other than  $\mathbb{Q}$ . We conclude the talk by formulating a conjecture as to precisely which subsets of  $\mathbb{R}$  possess a topology of the desired form. (Received September 25, 2017)

1135-VU-2299 **Rachel Skipper\***, Department of Mathematical Sciences, Binghamton University, PO Box 6000, Binghamton, NY 13902-6000, and **Matthew C.B. Zaremsky**. *Finiteness Properties of Nekrashevych Groups*.

Given a self-similar group  $G$  acting on a regular rooted  $d$ -ary tree, we consider the subgroup  $V_d(G)$  of almost automorphisms of the tree that “locally look like”  $G$ . This forms a Nekrashevych group and provides a natural way of joining the Higman-Thompson group  $V_d$  with the self-similar group  $G$ .

In this talk, we discuss finiteness properties of certain Nekrashevych groups. This work follows and expands on work of Belk and Matucci who considered the Rover group,  $V_2(G)$  where  $G$  is the Grigorchuk Group. (Received September 25, 2017)



1135-VU-2481 **Ik Jae Lee\*** ([leei@rowan.edu](mailto:leei@rowan.edu)), Department of Mathematics, Rowan University, Glassboro, NJ 08028, and **David N Yetter**. *Dijkgraaf–Witten type invariants of Seifert surfaces in 3-manifolds*.

In this talk, we introduce defects, with internal gauge symmetries, on a knot and Seifert surface to a knot into the combinatorial construction of finite gauge–group Dijkgraaf–Witten theory. The appropriate initial data for the construction are certain three object categories, with coefficients satisfying a partially degenerate cocycle condition. (Received September 26, 2017)

1135-VU-2728 **Alyson Bittner\*** ([alysonbi@buffalo.edu](mailto:alysonbi@buffalo.edu)). *Spaces With Complexity One*.

The inductive construction of a  $CW$ -complex builds spaces out of spheres. This process can be generalized to build  $A$ -cellular spaces out of some fixed space  $A$ . Given such a construction, we can ask if it is the most efficient construction in the sense that it requires the least ordinal number of steps to build the space out of copies of  $A$ , called the  $A$ -complexity of the space. With certain assumptions on  $A$ , every space has  $A$ -complexity less than or equal to one. We will discuss the properties and significance of such spaces  $A$ . (Received September 26, 2017)

1135-VU-2986 **Marla Williams\*** ([marla.williams@huskers.unl.edu](mailto:marla.williams@huskers.unl.edu)). *On Trisections of 4-Manifolds*. Preliminary report.

Trisections of smooth 4-manifolds are a generalization of Heegaard splittings of 3-manifolds, and a trisection can be similarly described by a set of curves on  $\Sigma_g$ , a genus  $g$  surface. This talk will explore an algorithm for finding trisection diagrams for  $\Sigma_g \times S^2$ , and discuss progress on a generalization to  $\Sigma_g \times \Sigma_k$ . (Received September 26, 2017)

1135-VU-3177 **Rosemary K Guzman\*** ([rguzma1@illinois.edu](mailto:rguzma1@illinois.edu)). *On a paradoxical-type decomposition of Culler–Shalen of a Patterson–Sullivan measure for Kleinian groups*.

In this talk, we will review a technique of Culler and Shalen which relates the combinatorial structure of a free group to its action on its limit set. The construction was used to prove their  $\log(3)$ -Theorem, but is intriguing on its own. In particular, we look at certain isometries of hyperbolic space, which act as Mobius transformation on the sphere at infinity. (Received September 26, 2017)

1135-VU-3219 **Peter Sparks\*** ([peter.sparks@marquette.edu](mailto:peter.sparks@marquette.edu)). *Generalized Dunce Hats Do Not Have The Double Collapsible Property*.

We show that no generalized dunce hat can be decomposed into two collapsible subpolygons whose intersection is also collapsible. This is related to the notion of the *double  $n$ -space property* of a contractible open  $n$ -manifold:  $M$  has the double  $n$ -space property if  $M$  can be written as the union of two euclidean  $n$ -spaces meeting in a third euclidean  $n$ -space. (Received September 27, 2017)

## General Session on Other Topics

1135-VV-20 **Knute E Thorsgard\*** ([knutoniansphysics@gmail.com](mailto:knutoniansphysics@gmail.com)), 14416 East Fox Lake Road, Detroit Lakes, MN 56501. *Unhyphenated spacetime*.

Relativity currently calls for more dimensions than are needed. Hyphenated space-time associates time and space but does not unify them. This results in spatial dimensions being separate from temporal dimensions which does not even allow time to be real. The way to lose the hyphen and to reduce the number of dimensions is to grant the same equivalence to time and space as is given to mass and energy. The mathematics is simple. It is the same process used to reduce the two dimensional area of a circle to the curved one dimensional circumference. This process can be applied to Albert Einstein's second postulate. The 2s in Einstein's second postulate move from the superscripted squared position to out front. It goes from working with areas to working with segments which are simpler than areas. Working with a geometric horizon is easier than working with Lorentz transformations. Physicists will squeal that there is not supposed to be a fixed frame of reference. They will settle down when they realize that a center is not a frame and that the horizon is not fixed. They will complain, correctly, that  $-C$  is meaningless.  $-C$  is meaningless like longitude is meaningless.  $-C$  can be assigned meaning like longitude can be assigned meaning.  $E = M(-C)^2$  (Received June 14, 2017)

1135-VV-255 **Sanskar Dhakal\*** ([sanskardhaka182@yahoo.com](mailto:sanskardhaka182@yahoo.com)), SOS Hermann Gmeiner School, Bhaktapur, 3 44811, Nepal. *Role of statistics for socially just*. Preliminary report.

The aim of this poster is to explore the politics of statistics for social justice. This poster is prepared on the basis of a real scenario of a family. The crux of this poster is to present the skill of statistics which is used to settle the financial dispute between husband and wife. Diagrammatic representation and basic mathematical calculation

method are used to analyse the contribution of a housewife. This study derives a finding that there is the more economic contribution of a housewife in a family than that of job holder husband. Keywords: dispute, statistics, social justice, settle. (Received August 16, 2017)

1135-VV-351 **John Risher\*** (riserjt@email.sc.edu) and **Wei-Kai Lai** (laiw@mailbox.sc.edu). *On Some Inequality Problems*. Preliminary report.

In Cvetkovski's book "Inequalities: Theorems, Techniques and Selected Problems" we found some patterns from several inequalities with sums of fractions in three variables. In this talk we first prove a more generalized case using the technique introduced by Cvetkovski. We then provide a different proof of the new result, together with some special cases. (Received August 27, 2017)

1135-VV-499 **Jason Robinson\*** (jrobinson@leeuniversity.edu), 1120 N. Ocoee St., Cleveland, TN 37320, **Caroline Maher-Boulis** (cmaherboulis@leeuniversity.edu), 1120 N. Ocoee St., Cleveland, TN 37320, and **Bryan Poole** (bpoole@leeuniversity.edu), 1120 N. Ocoee St., Cleveland, TN 37320. *Career and Technical Content in High School Mathematics*<sup>2</sup>. Preliminary report.

CATCH Math<sup>2</sup> is a project funded by the Tennessee Higher Education Commission. High school teachers gain insight into mathematical content and career-related skills in Statistics and Functions domains. The project involved a summer workshop where real-world problems, the use of manipulatives and hands-on science experiments enable the teachers, and hence their students, to realize the relevance of mathematics and become aware of career options for mathematics graduates. In this presentation we give an overview of the ideas of the project and share the results of the evaluation instruments used in assessing the project. (Received September 06, 2017)

1135-VV-602 **Ethan D Bolker\*** (ebolker@gmail.com), **Samuel A Feuer** and **Catalin Zara**. *Balance weighing - variations on a theme*.

Let  $w_k$  be an increasing sequence of integer weights. We explore weighing problems when you may use at most one of each kind, but may put some on either side of the balance. The solutions depend on analyzing how arithmetic works when you expand numbers in the mixed base defined by the weights. Much of the material is suitable for elementary school exercises in various grades. (Received September 21, 2017)

1135-VV-672 **Wing Hong Tony Wong\*** (wong@kutztown.edu), 15200 Kutztown Road, Kutztown, PA 19530, and **Diego Manzano-Ruiz**. *Several variations of vertex coloring games*.

Alice and Barbara take turns to color the vertices of a given simple graph, with Alice starting first, so that no adjacent vertices share the same color. The first player who is unable to color a vertex loses the game. We consider the following two versions: 1. Alice uses color  $A$  and Barbara uses color  $B$ ; 2. both of them use a common color  $C$ . The first version is a small generalization of the game of Col, and the second version is called node-kayles in the literature. These games are studied by Berlekamp, Conway, Guy, and many others. We determine the winning strategies on some common families of graphs. We also combine them and form a third version, namely that Alice uses colors  $A$  and  $C$ , and Barbara uses color  $B$  and  $C$ . (Received September 12, 2017)

1135-VV-955 **Sherry Sarkar\*** (sherry.sarkar@outlook.com) and **Simon Rubinstein-Salzedo**. *A Stability Result for Take-Away Games*. Preliminary report.

We investigate the structure of a natural generalization of the following classical take-away game: There are two players who alternate turns and a pile of  $n$  stones in the middle. The first player may take up to  $n - 1$  stones. After that, a player may take up to twice the number of stones removed from the pile by the previous player. The player who takes the last stone wins. This game is better known as *Fibonacci Nim*. This is because the losing positions are the Fibonacci numbers: if a player starts a move in a pile with a Fibonacci number of stones, that player will lose with optimal play, and otherwise the first player will win. More generally, let  $\alpha > 1$ , and allow each player to take up to  $\alpha$  times as many stones as the previous player did. We show that the losing positions of this game satisfy a linear recurrence similar to that of the Fibonacci numbers, and also that there is a surprising stability in the losing positions as  $\alpha$  varies: the set of losing positions is constant on some half-open interval containing  $\alpha$ . (Received September 17, 2017)

1135-VV-961 **Stephen K Liddle\*** (sliddle@gmu.edu), 3083 Covington Street, Fairfax, VA 22031. *Being a scribe for a blind math student*.

Starting in the Fall of 2015, I served as a scribe for a blind student as he progressed from Precalculus through Calculus 2. In this talk, I will discuss some of the pedagogical adjustments made by his instructors, the student,

and me, as we all had to rethink how to effectively communicate mathematics in a non-visual manner. I will then describe the solutions to the problems I faced while serving as a scribe for quizzes, tests, and homework problems. Finally, I will discuss the organizational challenges of working between departments. (Received September 17, 2017)

1135-VV-1161 **Basanta Raj Lamichhane\*** (mebasanta98@gmail.com), Bharatpur Metropolitan City, 22, Patihani, Chitwan, Nepal. *Assessment in Mathematics Education: An Integral Perspective*. Preliminary report.

The perspective of assessment has been changing because of the paradigm shifts in mathematics education. However, positivistic orientation of assessment recognizes externally imposed standardized paper-pencil tests. In this regards, this paper intends to describe the attributes, significance, and strategies of the alternative perspectives of assessment: assessment for/as mathematics learning. Assessment for learning generates the information from the multiple sources and uses to enrich meaningful learning. It mirrors the status of teaching-learning and helps for planning the instructional strategies. Assessment as learning regards assessment as an inbuilt process that focuses on developing metacognitive and higher ordered thinking in students so that they can become self-reflective practitioners, self-assessor of their own learning, and thus making appropriate adjustment to achieve deeper understanding. It signifies that we should rethink the traditional assessment system to transform mathematics education- otherwise, all other efforts that have been endorsed in intended curricula again become constituent factors of the conventional practice perpetuation. Keywords: Assessment for learning; assessment as learning; metacognitive thinking (Received September 20, 2017)

1135-VV-1168 **Kendra Killpatrick\*** (kendra.killpatrick@pepperdine.edu) and **Kristina Garrett**. *Pattern Avoiding Generalized Alternating Permutations*.

In 2011, Lewis proved that the number of alternating permutations of length  $2n$  avoiding the pattern 1234 is counted by the number of standard Young tableaux of shape  $\langle 3^n \rangle$ . Lewis generalized further to the set  $L_{n,k}$  of permutations  $\pi = \pi_{11}\pi_{12}\pi_{13}\cdots\pi_{1k}\pi_{21}\pi_{22}\cdots\pi_{2k}\cdots\pi_{n1}\pi_{n2}\cdots\pi_{nk}$  of length  $nk$  such that  $\pi_{i1} < \pi_{i2} < \cdots < \pi_{ik}$  for  $1 \leq i \leq n$  by proving that the number of permutations in  $L_{n,k}(123\cdots k(k+1)(k+2))$  is counted by the number of standard Young tableaux of shape  $\langle (k+1)^n \rangle$ .

In 2017, Mei and Wang further extended Lewis' results to permutations in  $L_{n,k,I}(123\cdots k(k+1)(k+2))$  for an index set  $I \subseteq [n]$  and proved that these permutations were counted by standard Young tableaux of shape  $\langle (k+1)^n \rangle$  independent of index set  $I$ . In their paper, Mei and Wang pose the question about finding a direct bijection between the pattern avoiding permutations in  $L_{n,k,I}$  for different index sets  $I$  that does not rely on the RSK correspondence and the standard Young tableaux. This talk will answer that open question by giving such a bijection and discuss the connection of this result to certain skew tableaux. (Received September 20, 2017)

1135-VV-1244 **Heather Hunt Elfen\*** (elfen@rmu.edu), 6001 University Blvd., 315 John Jay, Moon Township, PA 15108, and **Allison Boldrick** and **Prasanna K. Sahoo**. *A Few Simple Levi-Civita Functional Equations On Groups*.

We present several generalizations of the multiplicative homomorphism functional equation and the Levi-Civita functional equation defined on a non-abelian group. We focus on solving the functional equations

$$\begin{aligned} f(xy) + g(yx) &= 2h(x)k(y) \\ f(xy) + f(yx) &= 2f(x) + 2f(y) + 2\lambda f(x)f(y) \end{aligned}$$

defined on an arbitrary semi-group  $S$ . (Received September 20, 2017)

1135-VV-1249 **Naama T. Lewis\*** (tleelayl@siu.edu), 1240 Lincoln Drive, Carbondale, IL 62901. *Convex Optimization Techniques: A view point in Item Response Theory Models using I-projections*. Preliminary report.

Item Response Theory (IRT) Models have been used for many years to analyze the properties and behaviors of test items and the interaction between test items and respondents. More recently, with the use of computer algorithms to estimate the parameters, researchers have begun to develop and describe more complex interactions and models for test, test items, and examinee interactions. This dissertation work will place traditional IRT models within the framework of an I-projection problem. We will extend the 1 dimensional I-projection case into multiple dimensions, connect the solution to the 1-Parameter IRT model, estimate item parameters based on simulated data, and propose new models based on the I-projection dual space functional model. By expanding I-projections into multiple dimensions, we are able to recover and uncover properties of IRT models that were otherwise overlooked when considering the same or similar models outside of the dual space and relate our new models to multidimensional IRT data. (Received September 20, 2017)

1135-VV-1621 **Ioana Mihaila\***, Dept of Mathematics and Statistics, Cal Poly Pomona, 3801 W. Temple Ave., Pomona, CA 91768. *Beautiful Problems*.

Beauty is in the eye of the beholder, or in this case, of the problem solver and writer. This presentation takes the audience along the author's one-year journey of solving problems from math journals. (Received September 23, 2017)

1135-VV-1797 **Sherrie Serros\*** ([serross@mtmary.edu](mailto:serross@mtmary.edu)), 2900 River Pkwy, Milwaukee, WI 53222, **Erick B Hofacker** ([erick.b.hofacker@uwrf.edu](mailto:erick.b.hofacker@uwrf.edu)), 214C North Hall, River Falls, WI 54022, and **Ashlee LeGear** ([ashlee.legear@uwrf.edu](mailto:ashlee.legear@uwrf.edu)), 207 North Hall, River Falls, WI 54022. *The Power of Visuals when Teaching Secondary Mathematics*.

One of the eight best teaching practices as advocated in Principles to Action (NCTM, 2014) is that teachers use and connect mathematical representations. An important characteristic for a successful secondary mathematics teacher is one that embraces the use of visuals when they learn new mathematical concepts. In order to develop this sense, mathematics teachers need opportunities to engage in meaningful tasks that present the opportunity to use powerful visuals through either their undergraduate coursework, or through professional development.

We will share a series of rich mathematical tasks that provide teachers the opportunity to use and develop these visuals over multiple domains of study in secondary mathematics. Bar models and linking cubes will be examined for their usefulness when looking at equivalent algebraic expressions and solving algebraic equations. Double number lines provide an important visual when working with situations involving ratios and proportions. Area models can be used throughout a progression of topics involving multiplication and division. By emphasizing these visuals when teaching, it sends the message to students that these are appropriate and important tools that should be regularly used when studying mathematics. (Received September 24, 2017)

1135-VV-2056 **Xueyi Lei\*** ([leix@jay.washjeff.edu](mailto:leix@jay.washjeff.edu)), 50 South Lincoln Street, Washington, PA 15301. *Construction and Completion of a Latin Square*.

An  $n \times n$  Latin square is an  $n \times n$  array of  $n$  symbols, where each symbol occurs once in each row and column. We show several ways to construct a Latin square using permutations and the group structure of  $\mathbb{Z}_n$ . We also investigate the completion of several types of partial Latin squares using the Marriage Theorem. (Received September 25, 2017)

1135-VV-2059 **Denver Stahl\*** ([stahldi@jay.washjeff.edu](mailto:stahldi@jay.washjeff.edu)) and **Roman Wong**. *Any Which Way You Split*.

The splitting problem is an effective way to illustrate the concept of strong induction. We start by splitting a pile of  $n$  objects into two piles of  $r$  and  $s$  objects, respectively. We then compute the product  $r \times s$ . We repeat this process for the smaller piles, splitting each into 2 piles any which way we want until we are left with  $n$  piles of 1 object. Finally, we calculate the sum of all these products,  $\sum r \times s$ . It is a surprising discovery that this sum of products is always the same for a given  $n$  regardless of how the piles are split in the process. Can we generalize this problem to splitting into 3 piles? 4 piles? What about  $k$  piles? Can we find a method of splitting such that, no matter how we split, we find an invariant? In this talk, we delve into this generalization and provide an answer to these questions (Received September 25, 2017)

1135-VV-2248 **Toirna Lewis\*** ([tlewis@cau.edu](mailto:tlewis@cau.edu)), Department of Mathematical Sciences, Clark Atlanta University, Atlanta, GA 30314. *On the Fourier Series of Square Periodic Functions*. Preliminary report.

The "square periodic functions" are derived by inscribing a square whose four vertices intersect the unit circle at the solutions to the equation  $z^4 = 1$ . To this end we were able to calculate their explicit representation in terms of  $\theta$  and give plots for these functions; see, Lewis and Mickens, "Square Functions as a Dynamic System" and Mickens, "Some Properties of Square Periodic Functions". The "square periodic functions" are collectively comprised of the Square Cosine, Square Sine, and Square Dine functions, noted by  $S_c(\theta)$ ,  $S_s(\theta)$ , and  $S_d(\theta)$ , respectively. We derive a number of the fundamental properties of these functions. In particular, we demonstrate how to calculate the various Fourier series representation. (Received September 25, 2017)

1135-VV-2555 **Tamas Szabo\*** ([szabot@uwv.edu](mailto:szabot@uwv.edu)), Tamas Szabo, Whitewater, WI 53190. *The Role of Definitions in Geometry Courses for Prospective Middle and High School Teachers*. Preliminary report.

In a randomized comparative experiment, half of each geometry class was given the task of writing out a key definition on the exams before solving the related problem (usually proofs or constructions). Surprisingly, this did not increase their results on the problems at all. We try to explain why, and suggest corresponding improvements in mathematics teacher preparation. (Received September 26, 2017)

1135-VV-2603 **James A Swenson\*** ([swensonj@uwplatt.edu](mailto:swensonj@uwplatt.edu)) and **Dan Swenson**. *Tweaking the NFL's head-to-head tiebreaker.*

When two or more teams finish a season with the same win-loss record, the National Football League uses a long list of tiebreaking procedures to determine which teams qualify for the playoffs, and which of those earn home playoff games or byes. We show that there is a canonical alternative to the NFL's "head to head tiebreaker" – which seems preferable when three or more teams are tied. (Received September 26, 2017)

1135-VV-2810 **Li Feng** ([li.feng@asurams.edu](mailto:li.feng@asurams.edu)), Department of Mathematics and Computer Sci., Albany State University, Albany, GA 31705, and **Brandie T. Hall\*** ([bhall12@students.asurams.edu](mailto:bhall12@students.asurams.edu)), Department of Mathematics and Computer Sci., Albany State University, Albany, GA 31705. *The Arithmetic Combinations of Four a's.*

The Four 3's puzzle is a very popular and elementary problem. It asks if one can use arithmetic operations (+, -, \* and /) to combine four 3's in different order and come up with 0, 1, 2, 3 ... or 10 as its result. This problem motivates many other similar puzzles such as the Four 2's puzzle, the Four 4's puzzle. In this presentation, we will elevate the puzzle into the general Four a's puzzle. We calculate out all the possible combination values and the corresponding frequencies. We prove the following more advanced result. Except for a finitely many real numbers, every real number a, the arithmetic combinations of four will result 61 different values. Some interesting questions related to the problem will also be raised. (Received September 26, 2017)

1135-VV-3044 **Josh M. Beal\*** ([jobeal@iue.edu](mailto:jobeal@iue.edu)). *A metric for quantifying the accuracy of market indicators.*

*Market indicators*, or asset models, have gained in popularity over the last twenty years. Today, their popularity grows at an unprecedented rate, in large part due to the influx of so called *Robo-advisors*, automated trading engines. In this talk, we develop a metric for quantifying the accuracy of an indicator's future projections. (Received September 26, 2017)

1135-VV-3105 **Andres N. Mejia\*** ([am8248@bard.edu](mailto:am8248@bard.edu)), 65-36 Metropolitan Avenue, New York, NY 11379. *On The Structure of The  $C_2$  Spider.*

The spiders of Kuperberg provide a diagrammatic encoding the combinatorics of the representation theory of  $U_q(\mathfrak{g})$ , where  $\mathfrak{g}$  is a Lie algebra. We restrict our attention to the  $C_2$  spider, which reflects the representation category of  $\mathfrak{sp}(4)$ . A description of this combinatorial object is given in terms of generators and relations for diagrams. We extend results known for other spiders and give a more explicit description of these diagrammatic web spaces. Moreover, triple clasped spaces are explored when  $q$  is a root of unity, recovering aspects of a closely related modular tensor category. (Received September 26, 2017)

1135-VV-3122 **Joseph F. Kolacinski\*** ([jkolacinski@elmira.edu](mailto:jkolacinski@elmira.edu)), Elmira College, 1 Park Pl, Elmira, NY 14901, and **Brandon Payne**. *Mathematical Properties of Semi-Closed Primaries.* Preliminary report.

In a previous talk we explored the differences between open and closed primaries through the mathematical lens of "fairness criteria," a set of desirable characteristics "fair" election systems should satisfy.

In this talk we will extend the comparison to semi-closed primaries. Semi-closed primaries, like the California Democratic Primary, are closed to members of the opposing political party but allow some unaffiliated or third party voters to participate. We will show in particular, that with respect to fairness criteria, semi-closed primaries have no advantages over closed primaries. (Received September 26, 2017)

1135-VV-3133 **John Robert Botzum\*** ([botzum@kutztown.edu](mailto:botzum@kutztown.edu)), 5528 Heather Lane, Orefield, PA 18069. *The Phi-bonacci Sequence.*

Sometimes innocent questions from students can stir long-forgotten memories and stimulate mathematical investigation. Last year I took over my wife's Discrete Mathematics course for a week. I spent much of the first class working out elementary induction proofs and introducing recurrence relations. At the end of the class one of the students asked if there were a recurrence relation for the nth term of the Fibonacci sequence. I was in a hurry to teach a class 45 minutes away so I foolishly replied that I did not think there was. On the drive to my other job I recalled Binet's formula which prompted me to dig up notes on this subject that I had long since forgotten. This paper will discuss my discovery and re-discovery of the beautiful interplay between Binet's recurrence relation, the Golden Mean, and the Fibonacci Sequence as well as discuss whether Binet was the first to discover the formula. (Received September 26, 2017)

1135-VV-3152 **M Beck, J Friedman, M Gagliardo, T Hunter, N Sieben, C Williams and S Wright\***, [swright8@fitchburgstate.edu](mailto:swright8@fitchburgstate.edu). *The Arithmetic and Geometry of (P)s(e)udokus*.

As we set out on this journey, our goal was to develop a counting theory for sudoku puzzles and their relatives. Spoiler alert: this is difficult. While the end of this talk may not wrap up the problem with a nice bow, the layers of pretty paper are still quite nice. This work reminds us that mathematics is not a collection of discrete fields of study; there's something here for everyone and it is all mixed up together. We see the value of the good mathematical habits we aim to instill in our students and the utility of powerful theorems... and ... there are puns. (Received September 26, 2017)

1135-VV-3178 **Stephen Abbott\*** ([abbott@middlebury.edu](mailto:abbott@middlebury.edu)), 157 Chipman Park, Middlebury, VT 05753. *Mathematics as Art in Contemporary Theater*.

During the previous century, a handful of avant-garde playwrights took inspiration from the various revolutions in geometry, logic, and theories of the infinite to challenge the artistic norms of their respective eras. This unexpected synthesis of mathematics and theater eventually found its way to the mainstage with critical successes such as *Arcadia* (1993), *Proof* (2000), and *A Disappearing Number* (2007). For the last 15 years I have been team-teaching an interdisciplinary course that engages the mathematical ideas with the goal of understanding how they contribute to the mission of the artists. Likewise, the course engages the theater in an authentic way, regularly performing scenes in class and, on one occasion, mounting a small production. (Received September 26, 2017)

1135-VV-3211 **Nastasha Johnson\*** ([nejohnson@purdue.edu](mailto:nejohnson@purdue.edu)), 504 West State Street, WALC 3053, West Lafayette, IN 47906. *Publishing Habits of Math Faculty: Where & How Often & Why?*

As library collections are refined and edited for the advent of more open resources, tough decisions have to be made regarding existing collections. A research study was conducted analyzing the publishing habits of leading faculty at 3 prestigious universities to gain an understanding of what journals are most frequented for math research. Their publishing records were gathered, recorded, and analyzed for factors important to scholars and libraries alike. Findings from this research will be discussed, as well as future considerations for math faculty and libraries alike. (Received September 27, 2017)



# 2010

# MATHEMATICS

# SUBJECT

# CLASSIFICATION

Compiled in the Editorial Offices of MATHEMATICAL REVIEWS and ZENTRALBLATT MATH

- 00 General
- 01 History and biography
- 03 Mathematical logic and foundations
- 05 Combinatorics
- 06 Order, lattices, ordered algebraic structures
- 08 General algebraic systems
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- 12 Field theory and polynomials
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- 14 Algebraic geometry
- 15 Linear and multilinear algebra; matrix theory
- 16 Associative rings and algebras
- 17 Nonassociative rings and algebras
- 18 Category theory; homological algebra
- 19  $K$ -theory
- 20 Group theory and generalizations
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- 31 Potential theory
- 32 Several complex variables and analytic spaces
- 33 Special functions
- 34 Ordinary differential equations
- 35 Partial differential equations
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- 39 Difference and functional equations
- 40 Sequences, series, summability
- 41 Approximations and expansions
- 42 Fourier analysis
- 43 Abstract harmonic analysis
- 44 Integral transforms, operational calculus
- 45 Integral equations
- 46 Functional analysis
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- 51 Geometry
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- 62 Statistics
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- 74 Mechanics of deformable solids
- 76 Fluid mechanics
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- 80 Classical thermodynamics, heat transfer
- 81 Quantum theory
- 82 Statistical mechanics, structure of matter
- 83 Relativity and gravitational theory
- 85 Astronomy and astrophysics
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- 90 Operations research, mathematical programming
- 91 Game theory, economics, social and behavioral sciences
- 92 Biology and other natural sciences
- 93 Systems theory; control
- 94 Information and communication, circuits
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