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\* Indicates who will present the paper at the meeting.

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## PAPERS PRESENTED AT MEETINGS

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MEETING #	DATE	PLACE	ABSTRACT DEADLINE	ABSTRACT ISSUE
1142	November 3–4, 2018	Fayetteville, AR	EXPIRED	Vol 39, No. 4
1143	October 20–21, 2018	Ann Arbor, MI	EXPIRED	Vol 39, No. 4
1144	October 27–28, 2018	San Francisco, CA	EXPIRED	Vol 39, No. 4
1145	January 16–19, 2019	Baltimore, MD	EXPIRED	Vol 40, No. 1
1146	March 15–17, 2019	Auburn, AL	January 22	Vol 40, No. 2
1147	March 22–24, 2019	Honolulu, HI	January 29	Vol 40, No. 2
1148	April 13–14, 2019	Hartford, CT	February 5	Vol 40, No. 2
1149	June 10–13, 2019	Quy Nhon City, Vietnam	TBA	NONE
1150	September 14–15, 2019	Madison, WI	July 16	Vol 40, No. 3
1151	October 12–13, 2019	Binghamton, NY	August 20	Vol 40, No. 3
1152	November 2–3, 2019	Gainesville, FL	September 10	Vol 40, No. 4
1153	November 9–10, 2019	Riverside, CA	September 3	Vol 40, No. 4
1154	January 15–18, 2020	Denver, CO	TBA	TBA

## NEWARK, DE, September 29–30, 2018

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Abstracts of the 1141st Meeting.

### 00 ► General

1141-00-60     **Vincent Coll, Grant Fickes, Dylan Green, Jonelle Hook, Colton Magnant and Karen McCready\***, karenmccready@kings.edu, and **Kathleen Ryan, Nathaniel Sauerberg and Jill Stifano**. *Adding color to the search for shortest paths*. Preliminary report.

An edge-colored path is properly colored if adjacent edges receive distinct colors. An edge-colored graph is properly connected if each pair of vertices in the graph is connected by at least one properly colored path. In such a graph we extend the idea of diameter to that of proper diameter, a function of both the graph and its coloring, which is defined to be the maximum length of a shortest properly colored path between any two vertices in the graph. We will explore the relationship between the diameter and proper diameter of certain graph classes and consider how proper diameter values relate to the vertex connectivity of a graph. (Received July 13, 2018)

1141-00-135     **Christian Sampson\*** (christian.sampson@gmail.com), **N. Benjamin Murphy, Elena Cherkhev and Kenneth Golden**. *Bounds on the effective viscoelasticity of an ice covered ocean*.

Wave-ice interactions in the polar oceans comprise a complex but important set of processes influencing sea ice extent, ice pack albedo, and ice thickness. In both the Arctic and Antarctic, the ice floe size distribution in the Marginal Ice Zone (MIZ) plays a central role in the properties of wave propagation through it. Ocean waves break up and shape the ice floes which, in turn, attenuate various wave characteristics. Recently, continuum models have been developed which treat the MIZ as a two-component composite of ice and slushy water. At the heart of these models are effective parameters, namely, the effective elasticity, viscosity, and complex viscoelasticity. In practice, these effective parameters, which depend on the composite geometry and the properties of constituents, are quite difficult to determine. To help overcome this limitation, we employ the methods of homogenization theory, in a quasi-static, fixed frequency regime, to find a Stieltjes integral representation for the complex viscoelasticity. This integral representation involves the spectral measure of a self adjoint operator and provides bounds on the effective viscoelasticity. The bounds themselves depend on the moments of the measure which then depend on the geometry of the floes. (Received July 25, 2018)

1141-00-229 **Iurii Posukhovskiy\*** (i.posukhovskiy@ku.edu), 4100 W 24TH PL, D-23, Lawrence, KS 66047, and **Atanas Stefanov**. *On the ground states of the generalized Ostrovsky equation*. The Ostrovsky (Ostrovsky-Vakhnenko, short pulse) equations are ubiquitous models in mathematical physics. They describe water waves under the action of a Coriolis force as well as the amplitude of a short pulse in an optical fiber. We rigorously construct ground traveling waves for two versions of these models. That is, these waves minimize the Hamiltonian among all waves with fixed  $L^2$  norm. The existence argument proceeds via the method of compensated compactness, but it involves surprisingly heavy Fourier analysis arguments to rule out the non-triviality of the limits of the minimizing sequences. In addition, we show that a special subclass of these waves are (strongly) spectrally stable.

In the case of quadratic nonlinearities, it is known that these waves are unique, by the work of P. Zhang and Y.Liu. Thus, our results in this case imply that the Zhang-Liu waves are spectrally stable. (Received July 30, 2018)

1141-00-294 **Lin Wan\*** (wan@udel.edu), 139 the Green, College of Engineering, Newark, DE 19716, and **Mohsen Badiey**. *Acoustic remote sensing of the ocean environment in shallow water*. Preliminary report.

Around 70% of the Earth's surface is covered by ocean water. Shallow water regions, found on the continental shelf, are important to human activities (e.g. shipping, fishing, and oil production). Studying the shallow water environment is a stimulating and exciting discipline for physicists, oceanographers and mathematicians. Underwater sound can travel greater distances than electromagnetic wave and sample the environment as it propagates. Shallow water acoustics deals with strong ocean bottom and surface interactions and involves complex variability in the water column. While in-situ measurements of the ocean environment are spatially limited and satellite/airborne remote sensing data are severely under sampled in time, acoustic remote sensing can be exploited to estimate the seafloor properties and to monitor spatial and temporal variability of the ocean surface and the water column. This research demonstrates the acoustic and environmental data simultaneously collected on the continental shelf off the eastern coast of the United States and utilizes them for acoustic remote sensing of the ocean environment. (Received July 31, 2018)

## 05 ► Combinatorics

1141-05-15 **Paul W Eloe** and **Catherine M Kublik\*** (ckublik1@udayton.edu), Department of Mathematics, University of Dayton, 300 College Park, Dayton, OH 45469. *A connection between numerical analysis, and Catalan and generalized Motzkin numbers*.

We study a doubly indexed sequence, defined by a linear recursion relation, that contains the Catalan numbers and relates to a class of generalized Motzkin numbers. We obtain an explicit form, a generating function and a nonlinear recursion relation for this sequence. The nonlinear recursion relation is obtained by using standard error analysis techniques from numerical analysis on a finite difference scheme which is exact when applied to a differential operator acting on the Euclidean distance function. This work shows the connection between numerical analysis and well-known sequences from the field of combinatorics. (Received May 18, 2018)

1141-05-23 **Alex Kodessa\*** (kodessa@farmingdale.edu) and **Felix Lazebnik**. *The Isomorphism Problem for Monomial Digraphs  $D(q; m, n)$* .

Let  $p$  be a prime, let  $e$  be a positive integer,  $q = p^e$ , and let  $\mathbb{F}_q$  denote the finite field of  $q$  elements. Let  $m, n$ ,  $1 \leq m, n \leq q - 1$ , be integers. The monomial digraph  $D = D(q; m, n)$  is defined as follows: the vertex set of  $D$  is  $\mathbb{F}_q^2$ , and  $((x_1, x_2), (y_1, y_2))$  is an arc in  $D$  if  $x_2 + y_2 = x_1^m y_1^n$ . We study the question of isomorphism of monomial digraphs  $D(q; m_1, n_1)$  and  $D(q; m_2, n_2)$ . We conjecture that  $D(q; m_1, n_1) \cong D(q; m_2, n_2)$  if and only if  $(m_2, n_2) = k(m_1, n_1)$  for some integer  $k$  coprime with  $(q - 1)$ . While the sufficiency of this condition is known, its necessity remains an open question. We present a number of partial results that support the conjecture. (Received June 25, 2018)

1141-05-40 **Nimrod Kriger**, Achva Academic College, Israel, and **Andrew Woldar\*** (andrew.woldar@villanova.edu), Department of Mathematics and Statistics, Villanova, PA 19085. *Automorphism groups of classical affine association schemes of Latin type*. Preliminary report.

We consider the family of **complete classical affine association schemes**  $\mathcal{A}_p$  of order  $p^2$  and rank  $p+2$  where  $p$  is an odd prime. Each such scheme is known to be amorphic, meaning that every possible merging of its  $p+1$  classes results in a fusion scheme. We refer to such fusion schemes as **classical affine schemes**.

Let  $\mathcal{M}$  be a classical affine scheme of order  $p^2$ . Then the automorphism group  $\text{Aut}(\mathcal{M})$  contains  $\text{Aut}(\mathcal{A}_p) \rtimes K$  where  $K$  is the stabilizer of  $\mathcal{M}$  in  $\text{PGL}(2, p)$ . We are especially interested in the case when  $\text{Aut}(\mathcal{M}) = \text{Aut}(\mathcal{A}_p) \rtimes K$ . We call such schemes **standard**.

In our investigations we make strong use of a bijection between all classical affine schemes  $\mathcal{M}$  and all ordered partitions  $\pi$  of the point set of the projective line  $\text{PG}(1, p)$ . We write  $\mathcal{M} = \mathcal{M}(\pi)$ .

Special attention is paid to schemes of so-called **Latin type**, i.e., schemes  $\mathcal{M}(\pi)$  in which every cell of  $\pi$  has size at least 3. Based on exhaustive computer data for  $p \leq 11$  and partial data for  $p = 13$ , we make the following:

**Conjecture:** *Every scheme of Latin type is standard.* (Received July 06, 2018)

1141-05-49 **Steve Butler\*** ([butler@iastate.edu](mailto:butler@iastate.edu)). *A forest building process.*

Given a graph we can build a forest through the following process: Order the edges in some fashion and then only keep edges which are incident to a vertex not incident to any previous edge.

We consider the problem of finding the expected number of components of the resulting tree for various graphs and establish some basic properties.

Joint work with Zhanar Berikkyzy, Jay Cummings, Misa Hamanaka, Marie Hardt, Kristin Heysse, Paul Horn, Ruth Luo, and Brent Moran (Received July 07, 2018)

1141-05-71 **Wing Hong Tony Wong\*** ([wong@kutztown.edu](mailto:wong@kutztown.edu)), 15200 Kutztown Road, Kutztown, PA 19530, and **Grant Fickes** ([gfick710@live.kutztown.edu](mailto:gfick710@live.kutztown.edu)), 15200 Kutztown Road, Kutztown, PA 19530. *The Edge-Distinguishing Chromatic Number of Spider Graphs with Three Legs or Bounded Leg Lengths.*

The edge-distinguishing chromatic number  $\lambda(G)$  of a simple graph  $G$  is the minimum number of colors  $k$  assigned to the vertices in  $V(G)$  such that each edge  $\{u_i, u_j\}$  corresponds to a different set  $\{c(u_i), c(u_j)\}$ . Al-Wahabi et al. derived an exact formula for the edge-distinguishing chromatic number of a path and of a cycle. We derive an exact formula for the edge-distinguishing chromatic number of a spider graph with three legs and of a spider graph with  $\Delta$  legs whose lengths are between 2 and  $\frac{\Delta+3}{2}$ . (Received July 16, 2018)

1141-05-72 **Allison Ganger, Shannon Golden, Brian Kronenthal\*** ([kronenthal@kutztown.edu](mailto:kronenthal@kutztown.edu)), **Felix Lazebnik** and **Carter Lyons**. *The girth of two-dimensional algebraically defined graphs.* Preliminary report.

The objects of interest in this talk are algebraically defined bipartite graphs, which are constructed as follows. Let  $\mathbb{F}$  denote a field, and consider the bipartite graph whose partite sets  $P$  and  $L$  are copies of  $\mathbb{F}^2$  such that  $(p_1, p_2) \in P$  and  $(\ell_1, \ell_2) \in L$  are adjacent if and only if  $p_2 + \ell_2 = p_1 \ell_1$ . This graph has girth six, and of particular interest is identifying any polynomials  $f \in \mathbb{F}[x, y]$  such that replacing  $p_1 \ell_1$  with  $f(p_1, \ell_1)$  in the adjacency condition produces a girth six graph that is not isomorphic to the original. In addition to discussing some results related to this question, we will also explain the connection between algebraically defined graphs and incidence geometry, which partially motivates this line of inquiry. (Received July 16, 2018)

1141-05-83 **Vladislav Taranchuk\*** ([vladtar@udel.edu](mailto:vladtar@udel.edu)) and **Craig M Timmons** ([craig.timmons@csus.edu](mailto:craig.timmons@csus.edu)). *The Anti-Ramsey Problem for the Sidon equation.*

Ramsey problems on the integers are an important part of Ramsey Theory which includes well known results such as Schur's Theorem and Van der Waerden's Theorem. In this talk, we will discuss a Ramsey problem on the integers that concerns the Sidon equation  $X + Y = Z + T$ . Given a coloring  $c$  of the first  $n$  positive integers, one can ask how many monochromatic solutions must there be to the Sidon equation. This question was answered by Saad and Wolf who initiated a rather general study of such problems. Motivated by Anti-Ramsey numbers in graph theory, we consider the problem of maximizing the number of rainbow solutions so the Sidon equation over all colorings of  $\{1, 2, \dots, n\}$  with  $k$  colors. (Received July 17, 2018)

1141-05-98 **Ju Zhou\*** ([zhou@kutztown.edu](mailto:zhou@kutztown.edu)), Kutztown, PA 19530. *Cycle-forced Hamiltonian Bipartite Graphs.*

A *forced cycle*  $C$  of a graph  $G$  is a cycle in  $G$  such that  $G - V(C)$  has a unique perfect matching. A graph  $G$  is a *cycle-forced graph* if every cycle in  $G$  is a forced cycle. We give a complete characterization of the cycle-forced hamiltonian bipartite graphs. (Received July 20, 2018)

1141-05-101 **Chun-Hung Liu\*** ([chliu@math.princeton.edu](mailto:chliu@math.princeton.edu)). *Packing topological minors half-integrally.*

A classical result of Erdős-Pósa states that the maximum number of disjoint cycles in a graph and the minimum number of vertices that are required to hit all cycles in the same graph are bounded by functions of each other. In other words, the set of cycles has the Erdős-Pósa property. Robertson and Seymour proved a far generalization

of Erdős-Pósa's theorem in terms of graph minors: the set of graphs containing  $H$  as a minor has the Erdős-Pósa property if and only if  $H$  is a planar graph. Thomas conjectured that the planarity of  $H$  can be dropped if half-integral packing is allowed. The main result of this talk is that the set of  $H$ -topological minors has the half-integral Erdős-Pósa property for any graph  $H$ , which easily implies Thomas' conjecture. Namely, we proved that for every graph  $H$ , there exists a function  $f$  such that for every graph  $G$ , either  $G$  contains  $k$   $H$ -topological minors where each vertex of  $G$  is contained in at most two of them, or there exists a set of vertices of  $G$  of size at most  $f(k)$  hitting all  $H$ -topological minors. (Received July 20, 2018)

1141-05-107 **Garth Isaak\*** ([gisaak@lehigh.edu](mailto:gisaak@lehigh.edu)), Lehigh University, and **Matt Prudente**, Alvernia University. *A Two Player Graph Pebbling Game.*

We introduce a two player game graph pebbling game. Based on classical graph pebbling, players take turns removing two pebbles from a vertex and placing a single pebble on an adjacent vertex. Mover wins if a pebble is placed on a specified root and Defender wins if the root is not reached and no pebbling moves remain. In certain cases the problem reduces to another combinatorial game on sets. We can determine the minimum number,  $\eta$ , of pebbles so that Mover wins for any configuration with  $\eta$  pebbles for a large class of diameter two graphs and for path powers. We present bounds for  $\eta$  in the case of paths and cycles, for which it is surprisingly difficult to get an exact value. (Received July 21, 2018)

1141-05-109 **Mahya Ghandehari\*** ([mahya@udel.edu](mailto:mahya@udel.edu)), Department of Mathematical Sciences, University of Delaware, Newark, DE, and **Jeannette Janssen**. *A new parameter for seriation of noisy data.*

A square symmetric matrix is Robinsonian if entries in its rows and columns are non-decreasing when moving towards the diagonal. A Robinsonian matrix can be viewed as the affinity matrix between objects arranged in linear order, where objects closer together have higher affinity. Adjacency matrices of geometric graphs are special cases of Robinsonian matrices. In this talk, we introduce a new parameter,  $\Gamma_{\max}$ , which recognizes Robinsonian matrices that are perturbed by noise. This parameter can therefore be a useful tool in the problem of seriation of noisy data. More precisely, we show that a matrix is Robinsonian exactly when its  $\Gamma_{\max}$  attains zero, and a matrix with small  $\Gamma_{\max}$  is close (in the normalized  $\ell^1$ -norm) to a Robinsonian matrix. Moreover, we show that both  $\Gamma_{\max}$  and the Robinsonian approximation can be computed in polynomial time.

This talk is based on a joint work with Jeannette Janssen. (Received July 22, 2018)

1141-05-110 **James M Hammer\*** ([jmh0036@auburn.edu](mailto:jmh0036@auburn.edu)), 100 College Drive, Curtis 219, Allentown, PA 18104, and **John Asplund**, **Joe Chaffee** and **Matt Noble**.  *$\gamma'$ -Realizability and Other Musings on Inverse Domination.*

This talk will introduce and study  $\gamma'$ -realizable sequences. For a finite, simple graph  $G$  containing no isolated vertices,  $I \subseteq V(G)$  is said to be an *inverse dominating set* if  $I$  dominates all of  $G$  and  $I$  is contained by the complement of some minimum dominating set  $D$ . Define a sequence of positive integers  $(x_1, \dots, x_n)$  to be  $\gamma'$ -realizable if there exists a graph  $G$  having exactly  $n$  distinct minimum dominating sets  $D_1, \dots, D_n$  where for each  $i \in \{1, \dots, n\}$ , the minimum size of an inverse dominating set in  $V(G) \setminus D_i$  is equal to  $x_i$ . In this work, we show which sequences having minimum entry 2 or less are  $\gamma'$ -realizable. We then detail a few observations and results arising during our investigations that may prove useful in future research. (Received July 22, 2018)

1141-05-126 **Michael Tait\*** ([mtait@cmu.edu](mailto:mtait@cmu.edu)). *The Zarankiewicz problem in 3-partite graphs.*

Let  $F$  be a graph,  $k \geq 2$  be an integer, and write  $\text{ex}_{\chi \leq k}(n, F)$  for the maximum number of edges in an  $n$ -vertex graph that is  $k$ -partite and has no subgraph isomorphic to  $F$ . The function  $\text{ex}_{\chi \leq 2}(n, F)$  has been studied by many researchers. Finding  $\text{ex}_{\chi \leq 2}(n, K_{s,t})$  is a special case of the Zarankiewicz problem. We prove an analogue of the Kővári-Sós-Turán Theorem by showing

$$\text{ex}_{\chi \leq 3}(n, K_{s,t}) \leq \left(\frac{1}{3}\right)^{1-1/s} \left(\frac{t-1}{2} + o(1)\right)^{1/s} n^{2-1/s}$$

for  $2 \leq s \leq t$ .

Using Sidon sets constructed by Bose and Chowla, we prove that this upper bound is asymptotically best possible in the case that  $s = 2$  and  $t \geq 3$  is odd, i.e.,  $\text{ex}_{\chi \leq 3}(n, K_{2,2t+1}) = \sqrt{\frac{t}{3}} n^{3/2} + o(n^{3/2})$  for  $t \geq 1$ .

This is joint work with Craig Timmons. (Received July 24, 2018)

1141-05-131 **Jonathan Cutler\*** ([jonathan.cutler@montclair.edu](mailto:jonathan.cutler@montclair.edu)), **JD Nir** and **A. J. Radcliffe**. *Supersaturation for extremal enumeration.*

Turán's theorem states that the maximum number of edges in  $K_{r+1}$ -free graph on  $n$  vertices is attained by the complete  $r$ -partite graph with part sizes as equal as possible. We write the number of edges in this graph as

$\text{ex}(n, K_{r+1})$ , the extremal number of  $K_{r+1}$ . Supersaturation in graphs asks if  $G$  has more than  $\text{ex}(n, K_{r+1})$  edges, how many copies of  $K_{r+1}$  must  $G$  contain? Recently, Alon and Shikhelman introduced a generalization of the extremal number. Given graphs  $H$  and  $G$ , let  $\text{ex}_G(n, H)$  be the maximum number of copies of  $G$  an  $H$ -free graph on  $n$  vertices can contain. It is natural to ask supersaturation questions in this context as well. We present some results in this area. (Received July 25, 2018)

1141-05-143 **Tomasz Tkocz\***, Carnegie Mellon University, Pittsburgh, PA 15213. *Log-concave random graphs.*

I shall talk about connectivity and giant components of random graphs whose (generally dependent) edges are determined by certain log-concave measures (uniform distributions on Orlicz balls). Joint work with Alan Frieze. (Received July 26, 2018)

1141-05-155 **Steven Michael Senger\*** (stevensenger@missouristate.edu) and **Eyvindur Ari Palsson**. *Upper bounds on the number of chains in a point set.* Preliminary report.

Given a large finite point set in two or three dimensions, we give upper bounds on the number of occurrences of  $(k + 1)$ -tuples of points with the  $k$  distances between consecutive pairs of points fixed. This can be seen as a generalization of the Erdős unit distance problem. We compare these estimates to recent work of Bennett, Iosevich, and Taylor, where related questions were studied using analytic methods. (Received July 26, 2018)

1141-05-161 **Christopher Cox\***, Carnegie Mellon University, Pittsburgh, PA, and **Boris Bukh**, Carnegie Mellon University, Pittsburgh, PA. *A fractional version of Haemers' bound.*

We present a fractional version of Haemers' bound on the Shannon capacity of a graph, which is originally due to Blasiak. This bound is a common strengthening of both Haemers' bound and the fractional chromatic number of a graph. We show that this fractional version outperforms any bound on the Shannon capacity that could be attained through Haemers' bound and show also that this bound is multiplicative. As a consequence, the fractional Haemers bound belongs to the "asymptotic spectrum" in the recent duality result of Jeroen Zuiddam on Shannon capacity. (Received July 27, 2018)

1141-05-190 **Novi Herawati Bong\*** (nhbong@udel.edu), 313 Ewing Hall, University of Delaware, Newark, DE 19711, and **Yuqing Lin, Slamin Slamin and Roman Soták**. *On inclusive and non-inclusive vertex irregular  $d$ -distance vertex labelings.*

Let  $k$  be a positive integer. A *distance irregular vertex labeling* of the graph  $G$  with vertex set  $V$  is an assignment  $\lambda : V \rightarrow \{1, 2, \dots, k\}$  so that the weights at each vertex are distinct. The *weight* of a vertex  $x$ ,  $wt(x)$ , in  $G$  is defined as the sum of the labels of all the vertices at distance 1 from  $x$ . Let  $N(x)$  denote the set of neighbors of  $x$ . Formally,

$$wt(x) = \sum_{y \in N(x)} \lambda(y).$$

The *distance irregularity strength* of  $G$ , denoted by  $\text{dis}(G)$ , is the minimum value of the largest label  $k$  over all such irregular assignments.

In this talk, we generalize the notion of distance irregular labeling to vertex irregular  $d$ -distance vertex labeling, for any distance  $d$  up to the diameter. We will introduce the inclusive vertex irregular  $d$ -distance vertex labeling and give the lower bound of the inclusive vertex irregular 1-distance vertex labeling for general graphs. We will show some constructive examples of this labeling for certain family of graphs. Finally, there is a relation between the inclusive vertex irregular 1-distance vertex labeling on cycles and the vertex irregular 1-distance vertex labeling on prisms. (Received July 29, 2018)

1141-05-193 **Michal Adamaszek, Henry Adams, Ellen Gasparovic\*** (gasparoe@union.edu), **Maria Gommel, Emilie Purvine, Radmila Sazdanovic, Bei Wang, Yusu Wang** and **Lori Ziegelmeier**. *Homotopy types and persistence of metric gluings.*

This is a two-part talk that will focus on topological summary information that one can capture from metric wedge sums, gluings, and graphs. We will give a complete characterization of the persistence diagrams in dimension 1 for metric graphs under a particular intrinsic setting. We will look at two persistence-based distances that one may define for metric graphs and discuss progress toward establishing their discriminative capacities. We will show that the Vietoris-Rips (resp., Cech) complex of a wedge sum, equipped with a natural metric, is homotopy equivalent to the wedge sum of the Vietoris-Rips (resp., Cech) complexes. We also provide generalizations for when two metric spaces are glued together along a common isometric subset. As a result, we can describe the persistent homology, in all homological dimensions, of the Vietoris-Rips complexes of a wide class of metric graphs. (Received July 29, 2018)

1141-05-195 **Jason Williford\*** (jwillif1@uwyo.edu). *d-Geodetic Graphs.*

Bondy, Erdős and Fajtlowicz classified all graphs of diameter 2 with no 4-cycles in their paper “Graphs of diameter two with no 4-circuits”, showing that they are Moore graphs, polarity graphs of projective planes, or have a vertex adjacent to all others. We define a graph to be  $d$ -geodetic if it has diameter  $d$ , and there is at most one  $d$ -path between any pair of vertices. We call a  $d$ -geodetic graph ‘degenerate’ if it contains a vertex of eccentricity less than  $d$ . The theorem of Erdős et al. shows the non-degenerate 2-geodetic graphs are precisely Moore graphs and polarity graphs of planes.

In this talk, we discuss joint work with Michael Huntington to classify the non-degenerate  $d$ -geodetic graphs of diameters 3,4 and 5. (Received July 29, 2018)

1141-05-220 **Eugene Fiorini\*** (eugenefiorini@muhlenberg.edu), 2400 Chew Street, Allentown, PA 18104, **Vin de Silva**, 333 N. College Way, Claremont, CA 91711, and **Channing Verbeck, Jr.** *Symmetric class 0 subgraphs of complete graphs.*

In graph pebbling, a simple, connected graph is called *Class 0* if it has a pebbling number equal to the order of the graph. This talk addresses the question of when it is possible to edge-partition a complete graph into  $k$  complementary Class 0 subgraphs. We define the notion of  $k$ -Class 0 graphs: a graph  $G$  on  $n$  vertices is  $k$ -Class 0 if it contains  $k$  edge-disjoint subgraphs of order  $n$ , where each subgraph is Class 0. We next present a family of  $k$ -Class 0 graphs for  $k = 2$ , showing that for  $n \geq 9$ ,  $K_n$  is 2-Class 0. We finally provide a probabilistic argument to prove that  $\forall k \in \mathbb{N}$  such that  $K_n$  can be edge-partitioned into  $k$  cyclically symmetric subgraphs of diameter 2 and connectivity 3: that is,  $K_n$  is  $k$ -Class 0. (Received July 30, 2018)

1141-05-224 **M. DeVilbiss, B. Fain\*** (bfain@udel.edu), **A. Holmes, P. Horn, S. Mafunda** and **K. E. Perry**. *An Extremal Problem on Rainbow Spanning Tress in Graphs.*

Given a simple graph  $G$  and an edge-coloring  $\varphi$ , we define

$$R(G, \varphi) = \{T \subseteq E(G) : T \text{ is a rainbow spanning tree}\}.$$

In this presentation we look at minimizing and maximizing  $|R(G, \varphi)|$  where the coloring is restricted to what are known as JL-colorings. We give some general bounds for all simple connected graphs and also give a sharp lower and upper bound when  $G$  is a complete graph and when  $G$  is a complete bipartite graph. (Received July 30, 2018)

1141-05-226 **Michael R Yatauro\***, Penn State University, Brandywine, 25 Yearsley Mill Rd., Media, PA 19063. *A Probability Polynomial Associated with Edge Covers of a Graph.*

Given a uniform probability  $\rho$ ,  $0 < \rho < 1$ , of selecting edges independently from a graph  $G$ , we define the edge cover probability polynomial  $Ep(G, \rho)$  of  $G$  to be the probability of randomly selecting an edge cover of  $G$ . We provide general, and in some cases specific, formulas for obtaining  $Ep(G, \rho)$ . We then demonstrate the existence of graphs which have either the largest or the smallest  $Ep(G, \rho)$  within its class for all  $\rho$ . The classes we consider are trees, unicyclic graphs, and connected graphs having one more edge than the number of vertices. Thus we determine the optimal constructions with respect to edge covers within the context of these classes. (Received July 30, 2018)

1141-05-242 **Radmila Sazdanovic\***, rsazdanovic@math.ncsu.edu, and **Michal Adamazsek, Henry Adams, Ellen Gasparovic, Maria Gommel, Emilie Purvine, Bei Wang, Yusu Wang** and **Lori Ziegelmeier**. *Homotopy types and persistence of metric gluings.*

This is a two-part talk that will focus on topological summary information that one can capture from metric wedge sums and gluings of metric spaces, metric graphs in particular. We will give a complete characterization of the persistence diagrams in dimension 1 for metric graphs under a particular intrinsic setting. Next we analyze two persistence-based distances defined for metric graphs and discuss progress toward establishing and comparing their discriminative capacities. We will show that the Vietoris-Rips (resp., Cech) complex of a wedge sum, equipped with a natural metric, is homotopy equivalent to the wedge sum of the Vietoris-Rips (resp., Cech) complexes. We also provide generalizations to gluing metric spaces along a common isometric subset. As a result, we can describe the persistent homology, in all homological dimensions, of the Vietoris-Rips complexes of a wide class of metric graphs. (Received July 30, 2018)

1141-05-248 **János Komlós, Emily Sergel\*** (esergel@upenn.edu) and **Gábor Tusnády**. *On the distribution of  $(s, s+1)$ -core partitions with distinct parts.*

An  $s$ -core partition is a partition with no hook length equal to  $s$ . Cores arise naturally in algebraic combinatorics. Recently there has been interest in studying the distribution of sizes among partitions which are simultaneously



s-cores and t-cores for coprime  $s, t$ . We prove that sizes among  $(s, s+1)$ -core partitions with distinct parts are approximately normally distributed, as conjectured by Zaleski (2017). (Received July 30, 2018)

1141-05-258 **Debsoumya Chakraborti, Wenying Gan** and **Po-Shen Loh\*** ([ploh@cmu.edu](mailto:ploh@cmu.edu)), Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, and **Benny Sudakov**. *Extremal Graphs With Local Conditions*.

We will discuss two natural questions in Extremal Graph Theory, which turn out to have similar answers, one conjectured and one proven. Both questions ask for the maximum or minimum number of combinatorial substructures in a graph with a certain condition that holds locally on every vertex. The first question asks for the maximum number of triangles in any  $n$ -vertex graph for which every vertex has degree at most  $t$ , and the second question asks for the minimum number of edges in any  $n$ -vertex graph for which every vertex is in a complete graph of order  $t$ . In both questions, when  $n$  satisfies particular divisibility properties with respect to  $t$ , then the answer is a disjoint union of complete graphs, but becomes more complicated for general values of  $n$  and  $t$ . (Received July 31, 2018)

1141-05-268 **William J. Martin\*** ([martin@wpi.edu](mailto:martin@wpi.edu)), Dept. of Mathematical Sciences, 100 Institute Road, Worcester Polytechnic Institute, Worcester, MA 01609, and **Jason S. Williford** ([jwillif1@uwyo.edu](mailto:jwillif1@uwyo.edu)), Department of Mathematics and Statistics, Dept. 3036, 1000 E. University Ave., Laramie, WY 82071. *Some remarks on the nearest neighbor graph in a  $Q$ -polynomial (cometric) association scheme*. Preliminary report.

Let  $(X, \mathcal{R})$  be a symmetric  $d$ -class association scheme which is  $Q$ -polynomial (cometric) with respect to the ordering  $E_0, E_1, \dots, E_d$  of its primitive idempotents. Order the entries of  $|X|E_1$  in decreasing order as  $Q_{01} > Q_{11} > \dots > Q_{d1}$  and consider the graph  $\Gamma = (X, R_1)$  determined by the basis relation with adjacency matrix  $A_1$  (so that  $A_1 \circ E_1 = \frac{Q_{11}}{|X|} A_1$ ). We study the combinatorics of  $\Gamma$ .

We prove that  $\Gamma$  has  $d + 1$  distinct eigenvalues and provide bounds on both its diameter and its valency. In particular, we apply Terwilliger's balanced set condition, together with a result of Kodalen and Martin, to prove that the rank of  $E_1$  is bounded above by the sum of valencies  $v_1 + v_i$  for any  $i > 1$  with intersection number  $p_{11}^i > 0$  where  $A_i$  has constant row sum  $v_i$ . We then explore extremal cases for some of the inequalities derived. (Received July 31, 2018)

1141-05-298 **Janet Fierson\*** ([fierson@lasalle.edu](mailto:fierson@lasalle.edu)), Dept. of Mathematics and Computer Science, La Salle University, 1900 W. Olney Ave., Philadelphia, PA 19046. *Reconfiguration graphs of vertex problems*. Preliminary report.

Constructing a reconfiguration graph requires the selection of a problem, a base graph on which to solve the problem, and a reconfiguration rule that defines adjacency of solutions. Each solution is represented by a vertex in the reconfiguration graph, and vertices representing adjacent solutions are joined by edges. A 2018 introductory survey by Nishimura emphasizes that it "does not attempt to catalog all research results that can be categorized as reconfiguration, but instead focuses on demonstrating the main themes in the area, the scope of the approach, and promising directions for the future." We share additional work on reconfiguration graphs, specifically reconfiguration graphs of vertex problems. This includes the application of reconfiguration graphs to problems not addressed in the survey, further results for problems that do appear, and manipulation of reconfiguration rules to produce reconfiguration graphs with particular structural characteristics. (Received July 31, 2018)

## 06 ► Order, lattices, ordered algebraic structures

1141-06-19 **Jinha Kim**, , South Korea, **Ryan R Martin\*** ([rymartin@iastate.edu](mailto:rymartin@iastate.edu)), 396 Carver Hall, 411 Morrill Road, Ames, IA 50011-2104, **Tomas Masarik**, , Czech Rep, and **Warren Shull, Heather C. Smith, Andrew Uzzell** and **Zhiyu Wang**. *On difference graphs and the local dimension of posets*.

The dimension of a partially-ordered set (poset) is the minimum number of linear extensions sufficient to ensure that for every incomparable  $x$  and  $y$ , there is one of the extensions that yields  $x < y$ . Introduced by Dushnik and Miller, the dimension is a well-studied parameter. However, in any given realization of the dimension of a poset, a given element might not be in many linear extensions.

Ueckerdt introduced the invariant called local dimension which, instead, uses partial linear extensions and which is bounded above by the Dushnik-Miller dimension. For instance, the dimension of the standard example of order  $n$  is  $n/2$ , but the local dimension is only 3.

In this talk, we study the local dimension of show that the maximum local dimension of a poset of order  $n$  is  $\Theta(n/\log n)$ , the local dimension of the  $n$ -dimensional Boolean lattice is at least  $\Theta(n/\log n)$  and make progress toward resolving a version of the removable pair conjecture for local dimension. We also connect the computation of local dimension of a poset to the decomposition of the edges of a graph into what are called difference graphs. (Received June 14, 2018)

## 13 ► Commutative rings and algebras

1141-13-44 **Irena Swanson** ([iswanson@reed.edu](mailto:iswanson@reed.edu)), 3203 SE Woodstock Boulevard, Portland, MI 97202, and **Robert Marshawn Walker\*** ([robmarsw@umich.edu](mailto:robmarsw@umich.edu)), 530 East Church Street, 2070 East Hall, Ann Arbor, MI 48109. *Tensor-Multinomial Sums of Ideals and Applications.*

This is joint work with Irena Swanson found on arXiv:1806.03545. Given a polynomial ring  $C$  over a field and proper ideals  $I$  and  $J$  whose generating sets involve disjoint variables, we determine how to embed the associated primes of each power of  $I + J$  into a collection of primes described in terms of the associated primes of select powers of  $I$  and of  $J$ . We discuss applications to constructing primary decompositions for powers of  $I + J$ , and to attacking the persistence problem for associated primes of powers of an ideal. (Received July 06, 2018)

1141-13-46 **Jugal Verma\*** ([jkv@math.iitb.ac.in](mailto:jkv@math.iitb.ac.in)), Department of Mathematics, IIT Bombay, Mumbai, Maharastra 400076, India. *Eakin-Sathaye type theorems for joint reductions and good filtrations of ideals.*

Analogues of Eakin-Sathaye theorem for reductions of ideals are proved for  $\mathbb{N}^s$ -graded good filtrations. These analogues yield bounds on joint reduction vectors for a family of ideals and reduction numbers for  $\mathbb{N}$ -graded filtrations. Several examples related to lex-segment ideals, contracted ideals in 2-dimensional regular local rings and the filtration of integral and tight closures of powers of ideals in hypersurface rings are constructed to show effectiveness of these bounds. (Received July 07, 2018)

1141-13-74 **Andras C Lorincz\*** ([alorincz@purdue.edu](mailto:alorincz@purdue.edu)), Department of Mathematics, Purdue University, W Lafayette, IN 47907, and **Claudiu Raicu**. *Iterated local cohomology groups and Lyubeznik numbers for determinantal rings.*

We present a recipe for determining iterated local cohomology groups with support in ideals of minors of a generic matrix in characteristic zero, expressing them as direct sums of indecomposable  $\mathcal{D}$ -modules. For non-square matrices these indecomposables are simple, but this is no longer true for square matrices where the relevant indecomposables arise from the pole order filtration associated with the determinant hypersurface. Specializing our results to a single iteration, we determine the Lyubeznik numbers for all generic determinantal rings, thus answering a question of Hochster. (Received July 16, 2018)

1141-13-144 **Francesca Gandini\*** ([fragandi@umich.edu](mailto:fragandi@umich.edu)) and **Harm Derksen**. *Resolutions of ideals associated to subspace arrangements.*

Suppose that  $W_1, W_2, \dots, W_d$  are subspaces of an  $n$ -dimensional  $\mathbb{K}$ -vector space  $W \cong \mathbb{K}^n$  and let  $I_1, I_2, \dots, I_d \subseteq \mathbb{K}[x_1, x_2, \dots, x_n]$  be the vanishing ideals of  $W_1, W_2, \dots, W_d$ . Conca and Herzog showed that the Castelnuovo-Mumford regularity of the product ideal  $I_1 I_2 \cdots I_d$  is equal to  $d$ . Derksen and Sidman showed that the Castelnuovo-Mumford regularity of the intersection ideal  $I_1 \cap I_2 \cap \cdots \cap I_d$  is at most  $d$  and similar results hold for more general ideals constructed from linear ideals. In this paper we show that analogous results hold when we replace the polynomial ring with the exterior algebra and work over a field of characteristic 0. The proofs of aforementioned theorems rely on the existence of non-zero divisors, so this approach fails for the exterior algebra. Instead, we rely on the functoriality of free resolutions and construct a functor  $\Omega$  from the category of polynomial functors to itself. The functor  $\Omega$  transforms resolutions of ideals in the polynomial ring to resolutions of ideals in the exterior algebra. (Received July 26, 2018)

1141-13-160 **Selvi Kara Beyerslan, Huy Tai Ha** and **Augustine O'Keefe\*** ([aokeefe@conncoll.edu](mailto:aokeefe@conncoll.edu)). *Algebraic properties of toric rings of graphs.*

Let  $G = (V, E)$  be a simple graph. We investigate the Cohen-Macaulayness and algebraic invariants, such as the Castelnuovo-Mumford regularity and the projective dimension, of the toric ring  $k[G]$  via those of toric rings associated to induced subgraphs of  $G$ . (Received July 27, 2018)

1141-13-165 **Dang Tan\*** ([dangt@purdue.edu](mailto:dangt@purdue.edu)). *Gorenstein linkage class of complete intersection of monomial  $m$ -primary ideals*. Preliminary report.

Linkage (Liaison) class of complete intersection has been widely studied before. One generalization of linkage is Gorenstein linkage where many properties can be retained. Some classes of ideals have been shown to be in Gorenstein linkage class of complete intersection (glicci). I will talk about monomial  $m$ -primary ideals and sufficient conditions for those to be glicci in graded setting. (Received July 27, 2018)

1141-13-170 **Sean K. Sather-Wagstaff\*** ([ssather@clemonson.edu](mailto:ssather@clemonson.edu)) and **Jonathan P. Totushek**. *CI-Hom Injective Dimension*.

We introduce an injective version of the complete intersection dimension of Avramov, Gasharov, and Peeva. We show that (a) it characterizes the complete intersection property for local rings, (b) it fits between the classical injective dimension and the  $G$ -injective dimension of Enochs and Jenda, (c) it satisfies a version of Chouinard's formula for injective dimension, (d) it provides modules with Bass numbers that are bounded by polynomials, and (e) it improves Bass' conjecture (which was proved by Roberts) for finitely generated modules of finite injective dimension. (Received July 27, 2018)

1141-13-172 **Paolo Mantero\*** ([pmantero@uark.edu](mailto:pmantero@uark.edu)), **Jason McCullough** and **Lance E Miller**. *Properties and Singularities of Rees-like algebras*.

Recently, J. McCullough and I. Peeva provided the first counterexamples to the Eisenbud-Goto Regularity Conjecture. These examples are obtained by a construction dubbed Rees-like algebra. Since one still hopes that the Regularity Conjecture may hold under additional assumptions (e.g. in the smooth case), it is natural to ask: How "bad" can the singularities of Rees-like algebras be? And, more generally, what is the relation between algebraic and geometric properties of an ideal and its Rees-like algebra?

In this talk, based on joint work with J. McCullough and L. Miller, we will provide some quantitative and qualitative answers to these questions. (Received July 27, 2018)

1141-13-175 **Youngsu Kim** and **Vivek Mukundan\***, University of Virginia, 141 Cabell Drive, Kerchof Hall, Office 131, Charlottesville, VA 22904. *The equations defining the graph of a certain rational map*.

Consider the rational map  $\phi : \mathbb{P}_k^{n-1} \xrightarrow{[f_0 : \dots : f_n]} \mathbb{P}_k^n$  defined by homogeneous polynomials  $f_0, \dots, f_n$  of the same degree  $d$  in a polynomial ring  $R = k[x_1, \dots, x_n]$ . Suppose the ideal  $I = (f_0, \dots, f_n)$  is a height two perfect ideal satisfying  $\mu(I_p) \leq \dim R_p$  for  $p \in \text{Spec}(R) \setminus V(x_1, \dots, x_n)$ . We study the equations defining the graph of  $\phi$  whose coordinate ring is the Rees algebra  $R[It]$ . We provide new methods to construct these equations using work of Buchsbaum and Eisenbud. Furthermore, for certain classes of ideals, we show that our construction is general. These classes of examples are interesting, in that, there are no known methods to compute the defining ideal of the Rees algebra of such ideals. These new methods also give rise to effective criteria to check that  $\phi$  is birational onto its image. (Received July 28, 2018)

1141-13-191 **Rebecca R.G.\*** ([rebeccargmath@gmail.com](mailto:rebeccargmath@gmail.com)). *Characteristic-free test ideals*. Preliminary report.

We define the test ideal of a general closure operation  $\text{cl}$ , and give some of its properties. We highlight connections to the trace ideal and interior operations, and the applications of these viewpoints to the study of singularities of commutative rings. In all characteristics, test ideals coming from big Cohen-Macaulay modules or algebras can take on the role of the tight closure test ideal used in characteristic  $p > 0$  to study singularities. (Received July 29, 2018)

1141-13-221 **Florian Enescu\*** ([fenescu@gsu.edu](mailto:fenescu@gsu.edu)), Department of Mathematics and Statistics, 25 Park Place, Georgia State University, Atlanta, GA 30303. *Generating functions and the twisted construction for graded rings*. Preliminary report.

Prompted by the definition for the Frobenius complexity of a local ring of positive characteristic, we examine generating functions that can be associated to the twisted construction of a graded ring of positive characteristic. We will examine ideas related to these functions and detail some open questions that appear naturally in this context. This work is joint with Yongwei Yao. (Received July 30, 2018)

1141-13-223 **Irina Ilioaea\*** ([iilioaea1@gsu.edu](mailto:iilioaea1@gsu.edu)), 3200 Lenox Rd Ne, Apt F414, Atlanta, GA 30324, and **Florian Enescu**. *Strong Test Ideals associated to Cartier Algebras*. Preliminary report.

In this talk, Cartier algebras are used to produce a large class of strong test ideals for a local  $F$ -finite reduced ring of positive prime characteristic. Results of Vraciu and Takagi are recovered under this new framework.

The main result of the talk states that the number of generators of the test ideal associated to pairs of Stanley-Reisner rings and linear maps is actually the number of facets of the simplicial complex associated to the Stanley-Reisner ideal.

Moreover, we will show how our results motivated us to introduce a new class of rings, called  $n$ -tight rings. (Received July 30, 2018)

1141-13-244 **Patricia J Klein\*** ([triciajk@umich.edu](mailto:triciajk@umich.edu)), **Linquan Ma, Quy Pham Hung, Ilya Smirnov** and **Yongwei Yao**. *A uniform convergence theorem for Koszul homology and the Stückrad–Vogel conjecture.*

Let  $(R, \mathfrak{m})$  be a Noetherian local ring and  $M$  a finitely generated  $R$ -module of dimension  $d$ . Let  $e(I, M)$  be the Hilbert–Samuel multiplicity of  $M$  with respect to  $I$ . In 1996, Stückrad and Vogel conjectured that

$$\sup_{\sqrt{I+\text{Ann}(M)}=\mathfrak{m}} \left\{ \frac{l(M/IM)}{e(I, M)} \right\} < \infty$$

if and only if  $M$  is quasi-unmixed (i.e.,  $\widehat{M}$  is equidimensional) and proved the “only if” direction of their conjecture.

It is well known that for every fixed system of parameters  $x_1, \dots, x_d$  on  $M$ , the ratio  $\frac{l(H_i(x_1^t, \dots, x_d^t; M))}{l(M/(x_1^t, \dots, x_d^t)M)}$  tends to 0 as  $t$  tends to infinity. We prove a uniform convergence result: If  $M$  is quasi-unmixed, then for every  $\varepsilon > 0$ , there exists  $t_0$  such that, for all  $t \geq t_0$ , all systems of parameters  $x_1, \dots, x_d$  of  $M$ , and all  $1 \leq i \leq d$ ,

$$\frac{l(H_i(x_1^t, \dots, x_d^t; M))}{l(M/(x_1^t, \dots, x_d^t)M)} < \varepsilon.$$

We then use this result to settle the Stückrad–Vogel conjecture in the affirmative. (Received July 30, 2018)

1141-13-253 **Ela Celikbas** and **Jai Laxmi\*** ([1axmiuohyd@gmail.com](mailto:1axmiuohyd@gmail.com)), House No. 3065, IIT Kanpur, Kanpur, India, and **Jerzy Weyman**. *Embeddings of Canonical Modules.*

It is well-known that, for a Cohen-Macaulay local ring  $S$  with a canonical module  $\omega_S$ , if  $S$  is generically Gorenstein, then  $\omega_S$  can be identified with an ideal of  $S$ , that is,  $\omega_S$  embeds into  $S$ . In this talk, we are concerned with a specific embedding of a canonical module of  $R/I_{m,n}$  to itself, where  $I_{m,n}$  is an ideal generated by all square-free monomials of degree  $m$  in a polynomial ring  $R$  with  $n$  variables. We discuss how to construct such an embedding using a minimal generating set of  $\text{Hom}_R(R/I_{m,n}, R/I_{m,n})$ .

This talk is based on a joint work with Ela Celikbas and Jerzy Weyman. (Received July 31, 2018)

1141-13-262 **Thomas M Ales\*** ([tales@masonlive.gmu.edu](mailto:tales@masonlive.gmu.edu)), 210 Marcum Ct, Sterling, VA 20164. *Tight closure invariants in Stanley-Reisner rings.* Preliminary report.

In a ring  $R$  of characteristic  $p \geq 0$ , tight closure  $I^*$  of an ideal  $I \subset R$ , named as such because it is a tighter closure operator than integral closure, is in general difficult to compute. However, a pair of results by Hochster and Huneke make computing the tight closure of ideals in Stanley-Reisner rings relatively simple. If we let  $\mathfrak{m}$  be the maximal ideal of a Stanley-Reisner ring generated by the images of the variables of the ring, we will examine all ideals  $I$  such that  $I^* = \mathfrak{m}$ , specifically discussing the minimal number of generators of  $I$ , called the  $*$ -spread of  $\mathfrak{m}$  and the intersection of all such ideals, called the  $*$ -core( $\mathfrak{m}$ ). We also make special mention of the similarities between these notions and their integral closure counterparts. (Received July 31, 2018)

1141-13-280 **Robin Baidya\*** ([rbaidya@utk.edu](mailto:rbaidya@utk.edu)), Department of Mathematics, The University of Tennessee, Knoxville, TN 37996. *Stable range for modules.* Preliminary report.

Bass’s Stable Range Theorem gives a sharp upper bound on the stable rank of a ring, given that the ring is a module-finite algebra over a commutative ring with a finite-dimensional Noetherian maximal spectrum. In this talk, we generalize Bass’s Stable Range Theorem using an analogue of stable rank for a pair of modules. Our result also generalizes theorems by Eisenbud–Evans and Forster–Swan. (Received July 31, 2018)

1141-13-293 **Claudia Miller, Hamid Rahmati\*** ([rahmath@miamioh.edu](mailto:rahmath@miamioh.edu)) and **Rebecca R.G.** *Betti numbers of the Frobenius powers of the maximal ideal over a generic hypersurface.* Preliminary report.

We study the behavior of the Betti numbers of the Frobenius powers of the maximal ideal in the hypersurface ring  $R = k[x, y, z]/(f)$ , where  $k$  is an infinite field of positive characteristic. We show that if  $f$  is chosen generically then high enough Frobenius powers of the maximal ideal have identical graded Betti numbers, up to explicit shifts. (Received July 31, 2018)

## 14 ► Algebraic geometry

1141-14-295 **Bill F Trok\*** (wftr222@g.uky.edu). *Terao's Conjecture and Connections to Geometry of Points.*

Terao's Conjecture is well known conjecture in the field of hyperplane arrangements. It asks whether freeness of the module of derivations is entirely dependent on the combinatorics of the arrangement. We restate this problem, dualizing the hyperplane arrangement, and stating Terao's conjecture in terms of the geometry of points in projective space. Then we proceed by showing this new perspective can be used to settle certain cases of the conjecture. (Received July 31, 2018)

1141-14-296 **Davesh Maulik\***, 77 Massachusetts Avenue, Cambridge, MA 02139. *Enumerative geometry and quantum groups.*

There is a long history of results linking enumerative questions in algebraic geometry with techniques from representation theory. In recent years, an example of this phenomenon has been very actively studied, in which the enumerative geometry of curves on certain symplectic varieties can be understood via constructions from quantum groups and mathematical physics. In this lecture, I will try to survey these developments. (Received July 31, 2018)

## 16 ► Associative rings and algebras

1141-16-124 **Juan Jose Villarreal\*** (jjvillarreal28@gmail.com). *Nilmanifolds and their associated non local fields.*

We attach for some nilmanifolds an affine Kac Moody vertex algebra together with a module  $\mathcal{H}$ . For vectors in  $\mathcal{H}$  we associated fields in such a way that lead us to consider logarithmic fields, this map can be seen as an extension of the fields defined on a vertex affine Kac Moody algebra. This construction is motivated by physics.

In this work we are interested in the singularities of these fields. We study a particular case, we show that when the nilmanifold  $N$  is a  $k$  degree  $S^1$ -fibration over the two torus and a choice of  $l \in \mathbb{Z} \simeq H^3(N, \mathbb{Z})$  the fields associated to the space  $\mathcal{H}$  have tri-logarithm singularities whenever  $kl \neq 0$ . (Received July 24, 2018)

1141-16-198 **Luigi Ferraro\*** (ferrarl@wfu.edu), **Ellen Kirkman**, **Frank Moore** and **Robert Won**. *Hopf algebra actions on some AS-regular algebras of small GK-dimension.*

The classical Chevalley-Shephard-Todd Theorem gives a characterization of when a group acting linearly on the commutative polynomial ring has a ring of invariants that is isomorphic to a polynomial ring. Understanding when group actions (or more generally, Hopf actions) on AS-regular algebras give AS-regular invariant rings has proven to be a difficult problem. We provide some new examples of Hopf actions on some AS-regular algebras such that the ring of invariants is also AS-regular. (Received July 29, 2018)

1141-16-201 **Gabriele La Nave\*** (lanave@illinois.edu). *Fractional Virasoro Algebras.*

In recent work with P. Phillips, we showed that it is possible to construct a generalization of the Virasoro algebra as a central extension of the fractional Witt algebra generated by non-local operators of the form,  $L_n^a \equiv \left(\frac{\partial f}{\partial z}\right)^a$  where  $a \in \mathbb{R}$  and  $\left(\frac{\partial f}{\partial z}\right)^a$  is the fractional holomorphic derivative. The Virasoro algebra is explicitly of the form,

$$[L_m^a, L_n^a] = A_{m,n}(s) \otimes L_{m+n}^a + \delta_{m,n} h(n) c Z^a$$

where  $A_{m,n}(s)$  is a specific meromorphic function depending on Gamma functions,  $c$  is the central charge (not necessarily a constant),  $Z^a$  is in the center of the algebra and  $h(n)$  obeys a recursion relation related to the coefficients  $A_{m,n}$ . In fact, we show that all central extensions which respect the special structure developed here which we term a multimodule Lie-Algebra, are of this form. This result provides a mathematical foundation for non-local conformal field theories, in particular recent proposals in condensed matter in which the current has an anomalous dimension. (Received July 30, 2018)

## 17 ► Nonassociative rings and algebras

1141-17-53 **Suzanne Crifo\*** (secrifo@ncsu.edu). *Some Maximal Dominant Weights and their Multiplicities for Affine Lie Algebra Representations.* Preliminary report.

Affine Lie algebras are infinite dimensional analogs of finite dimensional simple Lie algebras. It is known there are finitely many maximal dominant weights for any integrable highest weight representation of an affine Lie

algebra. However, determining these maximal dominant weights is a nontrivial task. So far only the descriptions of these weights are known for affine Lie algebra  $A_n^{(1)}$ . In this talk we will discuss the maximal dominant weights of the integrable highest weight representation of any affine Lie algebra with highest weight  $k\Lambda_0$  and give some examples of their multiplicities. (Received July 11, 2018)

1141-17-54 **Kailash C. Misra\*** ([misra@ncsu.edu](mailto:misra@ncsu.edu)), Raleigh, NC 27695-8205. *Crystal like bases for some quantized imaginary Verma modules for  $U_q(\widehat{\mathfrak{sl}(2)})$ .*

For the affine Lie algebra  $\widehat{\mathfrak{sl}(2)}$  there exists a closed partition of the root system which is not Weyl group conjugate to the standard partition of the root system. This nonstandard partition of the root system gives rise to a nonstandard Borel subalgebra. The Verma module  $M(\lambda)$  with highest weight  $\lambda$  induced by the nonstandard Borel subalgebra is called the imaginary Verma module. This imaginary Verma module  $M(\lambda)$  can be  $q$ -deformed to the quantized imaginary Verma module  $M_q(\lambda)$ . We define imaginary crystal bases for  $U_q(\widehat{\mathfrak{sl}(2)})$ -modules in certain category  $\mathcal{O}_{\text{red,im}}^q$  and show the existence of such bases for reduced quantized imaginary Verma modules for  $U_q(\widehat{\mathfrak{sl}(2)})$ . Then we show the existence of imaginary crystal basis for any object in the category  $\mathcal{O}_{\text{red,im}}^q$ .

This talk is based on some recent work jointly with Ben Cox and Vyacheslav Futorny. (Received July 12, 2018)

1141-17-61 **Joshua Sussan\*** ([jsussan@mec.cuny.edu](mailto:jsussan@mec.cuny.edu)), CUNY Medgar Evers, Mathematics Department, 1650 Bedford Ave., Brooklyn, NY 11225. *Categorical Heisenberg structures.*

Khovanov constructed a category whose Grothendieck group is conjecturally isomorphic to a Heisenberg algebra. We will formulate a categorical boson-fermion correspondence and explore other structures arising from this Heisenberg category. (Received July 14, 2018)

1141-17-100 **Chad R Mangum\*** ([cmangum@niagara.edu](mailto:cmangum@niagara.edu)), 5795 Lewiston Rd, Dunleavy Hall 333, Niagara University, NY 14109. *Bosonic Free Field Representations of Twisted Toroidal Lie Algebras.*

Lie algebra representation theory has been significant in various areas of mathematics and physics for several decades. In this talk, we will discuss one instance of this theory, namely certain representations of twisted (2-)toroidal (Lie) algebras, which we view as universal central extensions of twisted multi-loop algebras. The usual loop algebra realization generalizes the familiar realization of affine Kac-Moody algebras. To facilitate our study of the representation theory, however, we will discuss a new realization given by generators and relations; this is similar to a realization by Moody, Rao, and Yokonuma in the untwisted case. Subsequently, we will discuss an application, namely bosonic free field representations, which are similar to those of Feingold and Frenkel in the case of affine algebras. (Received July 20, 2018)

1141-17-122 **Robert H. McRae\*** ([robert.h.mcrae@vanderbilt.edu](mailto:robert.h.mcrae@vanderbilt.edu)). *Compact automorphism groups of vertex operator algebras and tensor categories.*

Vertex operator algebras  $V$  based on affine Lie algebras admit complex Lie groups acting as automorphisms; in particular there is a faithful action of the compact real form  $G$  of some Lie group. By a Schur-Weyl duality-type result of Dong, Li, and Mason,  $V$  decomposes semisimply as  $G \times V^G$ -module, where  $V^G$  is the vertex operator subalgebra of  $G$ -fixed points. In this talk, I will describe an equivalence of symmetric tensor categories between the category of finite-dimensional continuous  $G$ -modules and the semisimple category of  $V^G$ -modules generated by the simple modules occurring in  $V$ . The only assumption needed for the tensor equivalence is the existence of suitable vertex algebraic tensor category structure on this category of  $V^G$ -modules. In the simplest non-trivial case (where  $V$  is the simple vertex operator algebra associated to  $\widehat{\mathfrak{sl}(2)}$  at level 1), these results imply the existence of tensor category structure on a certain semisimple category of Virasoro algebra representations at central charge  $c = 1$ , and the equivalence of this tensor category with representations of  $SO(3)$ . (Received July 24, 2018)

1141-17-200 **Fei Qi\*** ([fei.qi@yale.edu](mailto:fei.qi@yale.edu)), 10 Hillhouse Ave., New Haven, CT 06511. *Studies on Meromorphic Open-String Vertex Algebras.*

A meromorphic open-string vertex algebra is an algebraic structure formed by vertex operators that satisfy associativity but do not necessarily satisfy commutativity. It was introduced by Yi-Zhi Huang in 2012. In this brief talk I will introduce the definition and give a brief summary on the current progress of our joint studies on these algebras, their representations and cohomology theory. (Received July 29, 2018)

1141-17-208 **Thomas Creutzig, Shashank Kanade\*** ([shashank.kanade@du.edu](mailto:shashank.kanade@du.edu)) and **Robert McRae**. *Applications of tensor categories*. Preliminary report.

In this talk, I will explain some specific examples regarding how tensor-categorical framework can help us understand relations between representation theories of various vertex operator (super)algebras. The three main instruments that let us derive such equivalences are: (1) The induction functor, (2) Mirror equivalence, and (3) The coset realization theorem for principal  $W$ -algebras of ADE type by Arakawa-Creutzig-Linshaw. In particular, these techniques give us precise equivalences between Virasoro and  $\mathfrak{sl}_2$  categories,  $\mathfrak{osp}(1|2)$  categories and  $N = 1$  super Virasoro categories and so on. This is a joint work in progress with T. Creutzig and R. McRae. (Received July 30, 2018)

1141-17-217 **Jonas T Hartwig\*** ([jth@iastate.edu](mailto:jth@iastate.edu)) and **Nina Yu**. *Simple Whittaker modules over free bosonic orbifold vertex operator algebras*.

We construct weak (i.e. non-graded) modules over the vertex operator algebra  $M(1)^+$ , which is the fixed-point subalgebra of the higher rank free bosonic (Heisenberg) vertex operator algebra with respect to the  $-1$  automorphism. These weak modules are constructed from Whittaker modules for the higher rank Heisenberg algebra. We prove that the modules are simple as weak modules over  $M(1)^+$  and calculate their Whittaker type when regarded as modules for the Virasoro Lie algebra. Lastly, we show that any Whittaker module for the Virasoro Lie algebra occurs in this way. These results are a higher rank generalization of some results by Tanabe. (Received July 30, 2018)

1141-17-219 **Andrew R Linshaw\*** ([andrew.linshaw@du.edu](mailto:andrew.linshaw@du.edu)), Department of Mathematics, University of Denver, 2390 S. York St., Denver, CO 80208. *On  $W_\infty$ -algebras*.

I will discuss vertex algebras defined over commutative rings, and as special cases the universal  $W_\infty$ -algebras of types  $\mathcal{W}(2, 3, 4, \dots)$  and  $\mathcal{W}(2, 4, 6, \dots)$ , which are defined over the polynomial ring in two variables. The existence and uniqueness of these algebras was conjectured in the physics literature, and was recently established in my papers [arXiv:1710.02275](https://arxiv.org/abs/1710.02275) and [arXiv:1805.11031](https://arxiv.org/abs/1805.11031) (joint with S. Kanade). All one-parameter vertex algebras of type  $\mathcal{W}(2, 3, \dots, N)$  or  $\mathcal{W}(2, 4, \dots, 2N)$  for some  $N$  satisfying some mild hypotheses, can be obtained as quotients of these algebras. This includes the principal  $W$ -algebras of types A, B, and C, as well as many others arising as cosets of affine vertex algebras inside larger structures. (Received July 30, 2018)

1141-17-230 **Christopher Sadowski\*** ([csadowski@ursinus.edu](mailto:csadowski@ursinus.edu)) and **Marijana Butorac**. *Combinatorial bases of principal subspaces of modules for twisted affine Lie algebras of type  $A_{2l-1}^{(2)}, D_l^{(2)}, E_6^{(2)}$ , and  $D_4^{(3)}$* .

In this talk, we discuss principal subspaces of standard modules with highest weight  $k\Lambda_0$  for the affine Lie algebras of type  $A_{2l-1}^{(2)}, D_l^{(2)}, E_6^{(2)}$ , and  $D_4^{(3)}$ . Using the theory of vertex operator algebras, we construct combinatorial bases for these principal subspaces, and use these bases to obtain the characters of these principal subspaces. (Received July 30, 2018)

1141-17-232 **Katrina Barron\*** ([kbarron@nd.edu](mailto:kbarron@nd.edu)), 255 Hurley Hall, Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556. *Aspects of higher level Zhu algebras for vertex operator algebras*.

We discuss how to construct higher level Zhu algebras for a vertex operator algebra and how to use these algebras to investigate the modules for the vertex operator algebra. (Received July 30, 2018)

1141-17-238 **Darlayne Addabbo\*** ([daddabbo@nd.edu](mailto:daddabbo@nd.edu)), Notre Dame, IN 46556. *Q-systems and Generalizations in Representation Theory*.

I will introduce hierarchies of difference equations whose solutions, called  $\tau$ -functions, are matrix elements for the action of loop groups,  $\widehat{GL}_n$ , on  $n$ -component fermionic Fock space. In the  $n = 2$  case, the  $\tau$ -functions are determinants of Hankel matrices and by applying the Desnanot-Jacobi identity, one can see that they satisfy a  $Q$ -system of type A.  $Q$ -systems appear in many areas of mathematics, so it is interesting to study the more general,  $n > 2$  hierarchies. I will discuss these hierarchies and the progress we have made in investigating them within the context of other areas of mathematics. A generalization of this work, in which the  $\tau$ -functions are matrix elements for the action of infinite matrix groups on  $n$ -component fermionic Fock space produces more general hierarchies of difference equations, the simplest of which is a  $T$ -system of type A. (Joint with Maarten Bergvelt) (Received July 30, 2018)

1141-17-246 **Corina Calinescu\*** (ccalinescu@citytech.cuny.edu), **Michael Penn** and **Christopher Sadowski**. *Principal Subspaces of Higher Level Standard  $A_2^{(2)}$ -Modules*.

In this talk we discuss presentations of the principal subspaces of higher level standard  $A_2^{(2)}$ -modules. (Received July 30, 2018)

## 20 ► *Group theory and generalizations*

1141-20-91 **Julianne G. Rainbolt\*** (julianne.rainbolt@slu.edu). *Regular Elements in Double Cosets*. Preliminary report.

Let  $\tilde{G}$  be a connected reductive algebraic group defined over  $\bar{\mathbb{F}}_q$  and let  $\tilde{B}$  be a Borel subgroup of  $\tilde{G}$ . Let  $W$  denote the Weyl group of  $\tilde{G}$  and let  $\tilde{w}$  denote an element in the preimage of the natural map from  $N_{\tilde{G}}(\tilde{T})$  to  $W$  where  $\tilde{T}$  is a maximally split torus of  $\tilde{G}$  in  $\tilde{B}$ . An element  $g \in \tilde{G}$  such that  $C_{\tilde{G}}(g)$  has the minimal possible dimension is called a regular element of  $\tilde{G}$ . We investigate the probability that a randomly selected element in the double coset  $\tilde{B}\tilde{w}\tilde{B}$  (called a Bruhat cell) is a regular element of  $\tilde{G}$  (Received July 18, 2018)

## 28 ► *Measure and integration*

1141-28-18 **Silvia Ghinassi\***, ghinassi@math.stonybrook.edu. *Sufficient conditions for  $C^{1,\alpha}$  parametrization and rectifiability*.

We say a measure is  $C^{1,\alpha}$   $d$ -rectifiable if there is a countable union of  $C^{1,\alpha}$   $d$ -surfaces whose complement has measure zero. We provide sufficient conditions for a Radon measure in  $\mathbb{R}^n$  to be  $C^{1,\alpha}$   $d$ -rectifiable, with  $\alpha \in [0, 1]$ . We assume a priori the measure to have positive and finite upper density. The conditions involve a Bishop-Jones type square function and all statements are quantitative in that the  $C^{1,\alpha}$  constants depend on such a function. Key tools for the proof come from Guy David and Tatiana Toro's parametrization of Reifenberg flat sets (with holes) in the Hölder and Lipschitz categories. (Received June 11, 2018)

1141-28-28 **Guy C. David\***, Department of Mathematical Sciences, Ball Sta, Muncie, IN, 47306, and **Kyle Kinneberg**. *Bi-Lipschitz behavior of Lipschitz mappings between metric spaces*.

We will survey some old and recent results and counterexamples on the problem of finding bi-Lipschitz behavior in arbitrary Lipschitz mappings between given metric spaces. We will then discuss new results (joint with Kyle Kinneberg) concerning the situation between spaces supporting Poincaré inequalities and Carnot groups. These address some questions of Semmes. (Received June 28, 2018)

1141-28-132 **Vasilis Chousionis**, **Sean Li** and **Scott Zimmerman\*** (scott.zimmerman@uconn.edu). *The Traveling Salesman Theorem in Carnot groups*.

Peter Jones proved his famous Traveling Salesman Theorem in the plane in 1990. His result classified those sets which are contained in a rectifiable curve via the boundedness of a certain Carleson integral. The methods introduced by Jones have seen applications in harmonic analysis and geometric measure theory, and his theorem has since been generalized to the setting of Hilbert spaces, the Heisenberg group, and the graph inverse limits of Cheeger and Kleiner.

I will present one direction of the Traveling Salesman Theorem for rectifiable curves in any Carnot group. A Carnot group is a class of nilpotent Lie groups whose abelian members are precisely Euclidean spaces, and these groups have been the focus of much recent study in geometric measure theory. As an application, I will show how this theorem may be used to prove uniform boundedness of the singular integral operator associated with a certain non-negative kernel on any set contained in a rectifiable curve. (Received July 25, 2018)

## 30 ► *Functions of a complex variable*

1141-30-31 **Dmitry Khavinson\*** (dkhavins@usf.edu), 4202 E. Fowler Avenue, Tampa, FL 33620. *A thought on uniform approximation by polyanalytic functions*.

The concept of polyanalytic content of a compact set in the complex plane is introduced to study the problem of uniform approximation by polyanalytic functions. It expands the notion of analytic content introduced and studied by the author in the early 1980s in connection with problems of rational approximation on compact subsets of  $\mathbb{C}$ . (Received June 29, 2018)



1141-30-32 **Jeff Lindquist\*** (jvindquistaccts@gmail.com). *Branched Quasisymmetric Mappings and Extensions*. Preliminary report.

(Joint work with Pekka Pankka from the University of Helsinki). We investigate a class of maps called branched quasymmetric mappings (BQS maps) between  $Q$ -regular metric spaces. BQS maps are generalizations of quasymmetric maps that may not be locally injective. We define vertical quasi-isometric mappings (VQI maps) between fixed hyperbolic fillings of bounded turning  $Q$ -regular metric spaces. We relate the classes of BQS maps and VQI maps under some mild assumptions. (Received June 30, 2018)

1141-30-48 **Rebekah Y. Jones\*** (jones3rh@mail.uc.edu). *Modulus of sets of finite perimeter and quasiconformal maps between metric spaces of globally  $Q$ -bounded geometry*.

In Euclidean space, it is well-known that quasiconformal maps quasi-preserve the  $n$ -modulus of curves. Kelly also showed that the  $n/(n-1)$ -modulus of “surfaces” is quasi-preserved. We generalize this result to the setting of complete, Ahlfors  $Q$ -regular metric spaces supporting a 1-Poincaré inequality. In fact, we consider a larger class of so-called surfaces, namely the measure theoretic boundaries of sets of finite perimeter, and so our results are new even in Euclidean space. This talk is based on joint work with Panu Lahti and Nageswari Shanmugalingam. (Received July 07, 2018)

1141-30-77 **Tao Cheng, Huiqiang Shi and Shanshuang Yang\*** (syang05@emory.edu), Department of Mathematics, Emory University, Atlanta, GA 30322. *Harmonic measure on quasicircles and symmetric quasicircles*. Preliminary report.

This talk is concerned with the study of harmonic measure on quasicircles and symmetric quasicircles. We investigate how the harmonic measure changes from one side of a Jordan curve to the other side. In particular, we characterize quasicircles and symmetric quasicircles using a type of harmonic symmetry property. We also explore the connection between quasicircles and the harmonic reflection property. (Received July 17, 2018)

1141-30-85 **Dimitrios Ntalampekos\*** (dimitrios.ntalampekos@stonybrook.edu), Stony Brook, NY 11794. *(Non)Removability of the Sierpiński Gasket*.

Removability of sets for quasiconformal maps and Sobolev functions has applications in Complex Dynamics, in Conformal Welding, and in other problems that require “gluing” of functions to obtain a new function of the same class. We, therefore, seek geometric conditions on sets which guarantee their removability. In this talk, I will discuss some very recent results on the (non)removability of the Sierpiński gasket.

A first result is that the Sierpiński gasket is removable for continuous functions of the class  $W^{1,p}$  for  $p > 2$ . The method used applies to more general fractals that resemble the Sierpiński gasket, such as Apollonian gaskets and generalized Sierpiński gasket Julia sets.

Then, I will sketch a proof that the Sierpiński gasket is non-removable for quasiconformal maps and thus for  $W^{1,p}$  functions, for  $1 \leq p \leq 2$ . The argument involves the construction of a non-Euclidean sphere, and then the use of the Bonk-Kleiner theorem to embed it quasymmetrically to the plane. (Received July 18, 2018)

1141-30-87 **William T. Ross\*** (wross@richmond.edu), University of Richmond, Department of Mathematics and CS, Richmond, VA 23173. *Rethinking Ritt’s theorem*.

In this joint work with Isabelle Chalendar, Pam Gorkin, and Jonathan Partington, we re-examine an old theorem of Ritt which determines when a finite Blaschke product can be written as the composition of two other finite Blaschke products (in a non-trivial way). Our result is in terms of Clark measures. (Received July 18, 2018)

1141-30-104 **Katrin Fassler, Anton Lukyanenko\*** (alukyane@gnu.edu) and **Jeremy T. Tyson**. *Heisenberg quasiregular ellipticity*.

Following the Euclidean results of Varopoulos and Pankka–Rajala, we provide a necessary topological condition for a sub-Riemannian 3-manifold  $M$  to admit a nonconstant quasiregular mapping from the sub-Riemannian Heisenberg group  $\mathbb{H}$ . As an application, we show that a link complement  $S^3 \setminus L$  has a sub-Riemannian metric admitting such a mapping only if  $L$  is empty, the unknot or Hopf link. In the converse direction, if  $L$  is empty, a specific unknot or Hopf link, we construct a quasiregular mapping from  $\mathbb{H}$  to  $S^3 \setminus L$ . (Received July 20, 2018)

1141-30-114 **Richard Fournier\*** (fournier@dms.umontreal.ca), CRM and DMS, CP 6128, Succ. Centre-ville, Montreal, Quebec H3C 3J7, Canada. *An interpolation formula for divided differences of algebraic polynomials and some inequalities following from it*.

Let  $\mathbb{D}$  denote the unit disc of the complex plane and  $\mathcal{P}_n$  the class of polynomials of degree at most  $n$  with complex coefficients. It has been obtained that

$$\max_{z \in \partial \mathbb{D}} \left| \frac{p_k(z) - p_k(\bar{z})}{z - \bar{z}} \right| \leq n^{1+k} \max_{0 \leq j \leq n} \left| \frac{p(e^{ij\pi/n}) + p(e^{-ij\pi/n})}{2} \right|,$$

where  $p_0 := p$  belongs to  $\mathcal{P}_n$  and for  $k \geq 0$ ,  $p_{k+1}(z) := zp'_k(z)$ . We obtain a new proof of a well-known inequality of Duffin and Schaeffer and of some other classical inequalities as the inequality of Schur. (Received July 23, 2018)

1141-30-117 **Vyron S Vellis\*** (vyron.vellis@uconn.edu), Department of Mathematics, University of Connecticut, 341 Mansfield Road U1009, Storrs, CT 06269. *Quasisymmetric uniformization of metric spheres.*

One of the biggest problems in Quasiconformal Analysis is the classification of metric spaces which are quasisymmetric (or quasiconformal) to the unit sphere  $\mathbb{S}^n$ . While settled for  $n = 1$ , the problem is completely open for  $n \geq 2$ . In this talk we present a survey on this question and some recent results regarding on how bad (geometrically and analytically) quasisymmetric spheres can be and, similarly, how good non-quasisymmetric spheres can be. (Received July 23, 2018)

1141-30-159 **Roger W. Barnard, Jerry Dwyer, Erin Williams and G. Brock Williams\*** (brock.williams@ttu.edu). *The Newton Maps of Rational Functions.*

Newton's method is a very well-known process for finding roots of a function  $f$  using the iterative formula  $z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}$ . The associated function  $F(z) = z - \frac{f(z)}{f'(z)}$  is called the Newton map for  $f$ .

Newton maps of polynomials have been an object of study for a considerable time, but far less is known about the Newton maps of rational functions. We will describe those rational functions whose Newton maps are conjugate by Möbius transformations to quadratic polynomials and prove that there are no Newton maps of rational functions which are conjugate to  $z^3$ . (Received July 27, 2018)

1141-30-166 **C. David Minda\***, Department of Mathematical Sciences, Cincinnati, OH 45221-0025. *The hyperbolic metric in geometric function theory.*

The aim is to give an overview of selected results from complex analysis that can be understood geometrically in terms of the hyperbolic metric. Hyperbolic geometry was created axiomatically in the first half of the 1800s. Its role in complex analysis begins with the introduction of the Poincaré model for hyperbolic geometry on the unit disk (or upper half-plane). This model is differential geometric in nature as it starts with the hyperbolic (or Poincaré) metric with curvature  $-1$  on the unit disk. The focus of this talk is the hyperbolic metric, rather than hyperbolic geometry. From this perspective, the first major contribution is due to Georg Pick in 1915, the Schwarz-Pick Lemma. The next major contribution is due to Lars Ahlfors in 1938 who established the fundamental theorem that the hyperbolic metric is maximal among all conformal metrics with curvature at most  $-1$ . The main portion of the talk is on some developments since the 1950s. The hyperbolic metric continues to play an important role in geometric function theory and in suggesting generalizations to other contexts. (Received July 27, 2018)

1141-30-185 **J. E. Pascoe\*** (pascoe@uf1.edu), **Meredith Sargent** and **Ryan Tully-Doyle**. *Geometric aspects of the Julia quotient on sets with controlled tangential approach.*

The Julia quotient measures how fast a function value approaches the boundary of the range as the input approaches the boundary of the domain. Classically, it was shown that boundedness of the Julia quotient for an analytic function near a boundary point implies nice regularity properties, namely the existence of a nontangential limit and derivative. (Julia regarded this as an asymptotic version of the Schwarz lemma.) We will discuss the relationship between the (amortized) Julia quotient on sets with controlled tangential approach, so-called  $\lambda$ -Stolz regions, and certain types of strong regularity which is phrased in terms of integrability of certain functions against a measure arising in a corresponding Nevanlinna representation, so-called  $\gamma$ -regularity, along the lines of the classical theory of the Hamburger moment problem as developed by Nevanlinna. Our goal will be to discuss geometric aspects of the theory fairly concretely, including the need for amortization of the Julia quotient, (which surprisingly can be dropped for extreme forms of  $\gamma$ -regularity where the theory greatly simplifies) and some applications, including moment theory and moment determinacy problems. (Received July 29, 2018)

1141-30-202 **Javad Mashreghi\*** (javad.mashreghi@mat.ulaval.ca), 1960 Boul Laurier, Quebec, QC G1S 1M8, Canada. *Banach algebras and entire functions of exponential type.*

We illustrate a technique from the theory of entire functions by proving a variant of Allan's result about power growth in Banach algebras. This method eventually leads to an inequality about binomial sums, in which the sharp constant is still unknown.

Joint work (the first one!) with T. Ransford. (Received July 30, 2018)

1141-30-228 **Kei Kobayashi, Joan Lind\*** (jlind@utk.edu) and **Andrew Starnes**. *Effect of random time changes on Loewner hulls*.

Loewner hulls are determined by their real-valued driving functions, via conformal maps. We study the geometric effect on the Loewner hulls when the driving function is composed with a random time change, such as the inverse of an  $\alpha$ -stable subordinator. In contrast to SLE, we show that for a large class of random time changes, the time-changed Brownian motion process does not generate a simple curve. Further we develop criteria which can be applied in many situations to determine whether the Loewner hull generated by a time-changed driving function is simple or non-simple. To aid our analysis of an example with a time-changed deterministic driving function, we prove a deterministic result that a driving function that moves faster than  $at^r$  for  $r \in (0, 1/2)$  generates a hull that leaves the real line tangentially. (Received July 30, 2018)

1141-30-276 **Catherine Beneteau\*** (cbenetea@usf.edu), **Myrto Manolaki** and **Daniel Seco**. *Boundary Behavior of Optimal Polynomial Approximants*. Preliminary report.

In this talk, I will discuss certain polynomials that approximate, in some optimal way, inverses of functions in Dirichlet-type function spaces of the unit disk. These polynomials are closely related to classical objects in function theory such as orthogonal polynomials and reproducing kernels in weighted spaces. In particular, I will examine the limiting behavior of optimal approximants of inverses of polynomials on the unit circle. (Received July 31, 2018)

## 31 ► Potential theory

1141-31-151 **Stephan T Grilli\*** (grilli@uri.edu), Department of Ocean Engineering, University of Rhode Island, Narragansett, RI 02882, **J T Kirby**, Center for Applied Coastal Research, University of Delaware, Newark, DE 19716, **M Derakhti**, Applied Physics Lab, University of Washington, Seattle, WA, **M Banner**, School of Mathematics and Statistics, University of New South Wales, Sidney, 2052, Australia, and **J Thompson**, Applied Physics Laboratory, University of Washington, Seattle, WA. *Fully nonlinear modeling of wave breaking induced by bathymetry: review and recent results*.

Fully nonlinear potential flow (FNPF) models based on a Boundary Element Method (BEM) have been used to simulate and study wave overturning induced by bathymetry in shallow water, but are limited to simulating a single overturning wave. Models based on Navier-Stokes (NS) equations have simulated such problems without this limitation. Important properties of waves at the onset of breaking were studied with FNPF-BEM or NS models, and in particular the so-called “breaking criteria” and the dissipation in breaking waves. Recent work has proposed a universal dynamic breaking criterion based on parameter  $B = u/c = 0.85-0.86$  (with  $u$  the water velocity at the crest and  $c$  the crest celerity), and a measure of dissipation in the subsequent breaking wave proportional to the  $dB/dt$  value at this threshold. Here, a 2D FNPF-BEM model is applied to solitary wave propagation over plane slopes and bars, as well as undular bores propagating over constant depth. Values of  $B$  and  $dB/dt$  are computed in these simulations that confirm the predicted threshold. Two cases of particular interest that will be detailed are solitary waves that nearly break as surging breakers on steep slopes, but then end up running up the slope and the largest non-breaking undular bore that can be achieved on a plane slope. (Received July 26, 2018)

## 35 ► Partial differential equations

1141-35-6 **Alexander Pankov\*** (alexander.pankov@morgan.edu), Mathematics Department, Morgan State University, 1700 E Cold Spring Ln, Baltimore, MD 21251. *Periodic Nonlinear Schrödinger equation on periodic metric graphs*.

We consider the nonlinear Schrödinger equation with periodic linear and nonlinear potentials on periodic metric graphs. Assuming that the spectrum of linear part does not contain zero, we prove the existence of finite energy ground state solution which decays exponentially fast at infinity. The proof is variational and makes use of the generalized Nehari manifold for the energy functional combined with periodic approximations. Actually, a finite energy ground state solution is obtained from periodic solutions in the infinite wave length limit. (Received February 12, 2018)

1141-35-34 **Ugur Abdulla** (abdulla@fit.edu), **Vladislav Bukshytynov** (vbukshytynov@fit.edu) and **Ali Hagverdiyev\*** (ahaqverdiyev2011@my.fit.edu), 300 Cornell Ave, Melbourne, FL 32901. *Gradient Method in Hilbert-Besov Spaces for the Optimal Control of Parabolic Free Boundary Problems.*

In this presentation I will talk about computational analysis of the inverse Stefan type free boundary problem, where information on the boundary heat flux is missing and must be found along with the temperature and the free boundary. We pursue optimal control framework introduced in *U.G. Abdulla, Inverse Problems and Imaging*, 7, 2(2013), 307-340; 10, 4(2016), 869-898, where boundary heat flux and free boundary are components of the control vector, and optimality criteria consist of the minimization of the quadratic deviations from the available measurements of the temperature distribution at the final moment, phase transition temperature on the free boundary, and the final position of the free boundary. We develop gradient descent algorithm based on Frechet differentiability in Hilbert-Besov spaces complemented with preconditioning or increase of regularity of the Frechet gradient through implementation of the Riesz representation theorem. Five model examples with various levels of complexity are considered. Extensive comparative analysis through implementation of preconditioning and Tikhonov regularization, calibration of preconditioning and regularization, noisy data, comparison of simultaneous identification of control parameters vs. nested optimization is pursued. (Received July 01, 2018)

1141-35-38 **Ihsan Topaloglu\*** (iatopaloglu@vcu.edu). *Slow Diffusion Limit of Aggregation-Diffusion Energies and Their Gradient Flows.*

For a range of physical and biological processes—from dynamics of granular media to biological swarming—the evolution of a large number of interacting agents is modeled according to the competing effects of pairwise interaction and (possibly degenerate) diffusion. In particular, models with hard height constraints (such as pedestrian crowd motion) attract significant interest. We prove that minimizers and gradient flows of constrained interaction energies arise as the slow diffusion limit of well-known aggregation-diffusion energies. We then apply this to develop numerical insight for open conjectures in a class of geometric variational problems. This is a joint project with Katy Craig. (Received July 05, 2018)

1141-35-47 **Fengyan Li\***, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, 100 8th Street, Troy, NY 12180. *Energy Stable DG Methods for Maxwell's Equations in Nonlinear Media.*

The propagation of electromagnetic waves is modeled by time-dependent Maxwell's equations coupled with constitutive laws that describe the response of the media. In this work, we examine a nonlinear optics model that describes electromagnetic waves in linear Lorentz and nonlinear Kerr and Raman media. To design efficient, accurate, and stable computational methods, we apply high order discontinuous Galerkin discretizations in space, and the resulted semi-discrete methods are proved to be stable. The challenge to achieve provable stability for fully-discrete methods lies in the temporal discretizations of the nonlinear terms. To overcome this, novel modifications are proposed for both the second-order leap-frog and implicit trapezoidal temporal schemes. This work was in collaboration with V. A. Bokil, Y. Cheng, Y. Jiang. (Received July 07, 2018)

1141-35-50 **Oleksandr Misiats\*** (misiats@gmail.com), Richmond, VA 23284. *Long time behavior of stochastic bidomain equations.*

Bidomain equations are widely used in studying the propagation of electrophysiological waves in the myocardium. One of the main applications of this model is to understand, and, consequently, improve the mechanism of heart defibrillation. During defibrillation, various regions of heart tissue are in different, usually random, phases of electrical activity (excited, refractory, partially recovered etc), and the purpose of defibrillation is to give an electric impulse that stimulates the entire heart and returns it to its normal (e.g. stationary) state. Therefore, understanding long time behavior of reaction-diffusion equations with bidomain operators is crucial in order to understand whether such recovery can take place, and how quickly it happens. The major mathematical challenge of this model is that the bidomain operator is nonlocal. In my talk, I will describe the well posedness and the existence of stationary solutions for randomly perturbed reaction-diffusion models involving the bidomain operator. (Received July 09, 2018)

1141-35-57 **J. Douglas Wright\*** (jdoug@math.drexel.edu) and **Mathew Johnson**. *Generalized solitary wave solutions of the capillary-gravity Whitham equation.*

“Whitham” equations have enjoyed a recent resurgence of popularity as models for free surface fluid flows. They are, roughly speaking, obtained by using the full linear part of the appropriate Euler equation together with a simpler “KdV”-type nonlinearity. Generalized solitary waves are traveling wave solutions which are the superposition of a classical solitary wave with a “small beyond all orders” periodic wave. Such waves are known

to exist for the full capillary-gravity wave problem and in this talk we discuss recent work on establishing their existence for the “Whithamized” version. (Received July 13, 2018)

1141-35-64 **Dmitry Pelinovsky\*** ([dmpeli@math.mcmaster.ca](mailto:dmpeli@math.mcmaster.ca)), Department of Mathematics, McMaster University, 1280 Main street West, Hamilton, Ontario L8S 4K1, Canada. *Stationary states on bounded and unbounded graphs in the limit of large mass.* Preliminary report.

In this work, we elaborate the asymptotic representation of the stationary states on a quantum graph in the limit of large mass as *shrinking NLS solitons*. We show that this approach is applicable to rather general bounded and unbounded graphs. In particular, if a bounded graph has a center of symmetry, we find a criterion for selection of a symmetric state as the global minimizer of energy. If a graph is unbounded, we find a criterion for existence of global minimizers. We illustrate numerically the validity of predictions of the asymptotic method. This is a joint work with G. Berkolaiko (Texas A & M University) and J. Marzuola (University of North Carolina at Chapel Hill). (Received July 15, 2018)

1141-35-65 **Wenxiong Chen\*** ([wchen@yu.edu](mailto:wchen@yu.edu)), Department of Mathematical Sciences, Yeshiva University, 2495 Amsterdam Av., New York, NY 10033, and **Congming Li.** *Direct methods on fractional equations.*

In this talk, we will summarize some of the recent developments on the study of qualitative properties of solutions for nonlinear equations involving the fractional Laplacian, the fractional  $p$ -Laplacian, and more general nonlinear non-local operators. We will focus on the ideas and techniques in analyzing qualitative properties of solutions, mainly on the direct methods we introduced in recent years, such as the direct method of moving planes, of moving spheres, and of blowing-re-scaling. We will illustrate how to apply these methods to obtain symmetry, non-existence, and a priori estimates for solutions. (Received July 16, 2018)

1141-35-82 **Panos Kevrekidis\*** ([kevrekid5@gmail.com](mailto:kevrekid5@gmail.com)), 710 N. Pleasant Street, Dept. of Mathematics and Statistics, Amherst, MA 01003. *Dark Solitons: From 1D to 2D and 3D with Some Quantum Touches.*

In the present talk, we will revisit some principal excitations in self-repulsive Bose-Einstein condensates, namely dark solitons in single-component systems, and dark-bright solitons in multi-component systems. Upon introducing them and explaining their existence and stability properties in 1d, we will extend them both in the form of stripes and in that rings in two-dimensions, presenting an alternative (adiabatic-invariant based) formulation of their stability and excitations. We will explore their filamentary dynamics, as well as the states that emerge from their transverse (snaking) instability. Then, we will consider these structures even in three dimensions, in the form of planar, as well as spherical shell solitons and generalize our adiabatic invariant formulation there. Finally, time permitting, we will give some glimpses of how some of these dynamical features in 1d and 2d generalize in a multi-orbital, time-dependent quantum setting. (Received July 17, 2018)

1141-35-89 **David M. Ambrose\*** ([dma68@drexel.edu](mailto:dma68@drexel.edu)), 3141 Chestnut St, Philadelphia, PA 19104. *Global bifurcation theory for periodic interfacial waves.*

We reformulate the traveling wave problem using a parameterized curve so that waves with multi-valued height are permitted. For problems with surface tension or hydroelastic effects, we are able to write the resulting equations in “identity plus compact” form. We then use a global bifurcation theorem to find the existence of continua of traveling waves, with several options for how each branch might terminate. We illustrate with simulations to show that most of the options do in fact occur. This includes joint work with Benjamin Akers, Walter Strauss, David Sulon, and J. Douglas Wright. (Received July 18, 2018)

1141-35-94 **Xiaofeng Ren\*** ([ren@gwu.edu](mailto:ren@gwu.edu)), 801 22nd Street, NW, Room 739, Washington, DC 20052, and **Chong Wang** ([chongwang@gwmail.gwu.edu](mailto:chongwang@gwmail.gwu.edu)), 801 22nd Street, NW, Room 739, Washington, DC 20052. *Primary and secondary structures of inhibitory geometric variational problems.*

In this talk we discuss a geometric variational problem with inhibitory long range interaction. It is a ternary system originally proposed to model triblock copolymers. There exists a morphological phase of a double bubble assembly as a stable stationary point of the variational problem. While the locations of the double bubbles in the assembly are determined in an earlier analytical result, the directions of the double bubbles are studied by a recent numerical computation. One of the conditions for the existence of the double bubble assembly is that the two by two nonlocal interaction matrix parameter is positive definite and a bound is assumed on the ratio of the two eigenvalues of the matrix. A more complete study of the interaction matrix shows that the double bubble assembly may lose to a disc assembly as the two eigenvalues become less comparable. When the matrix becomes

indefinite, there appears a stationary disc assembly whose primary structure is the microscopic disc. It also has a secondary structure that the discs of one type are separated from discs of the second type by a macroscopic interface. (Received July 19, 2018)

1141-35-95 **Mickaël D. Chekroun, Michael Ghil and Honghu Liu\*** (hhliu@vt.edu), 460 McBryde Hall, 225 Stanger St., Blacksburg, VA 24061, and **Shouhong Wang.** *Galerkin approximations of nonlinear delay differential equations.*

Delay differential equations (DDEs) are widely used in many applied fields to account for delayed responses of the modeled systems to either internal or external factors. In contrast to ordinary differential equations (ODEs), the phase space associated even with a scalar DDE is infinite-dimensional. Oftentimes, it is desirable to have low-dimensional ODE systems that capture qualitative features as well as approximate certain quantitative aspects of the DDE dynamics. In this talk, we present a new Galerkin scheme for general nonlinear DDEs. The main new ingredient is the use of a type of polynomials that are orthogonal under an inner product with a point mass. The associated Galerkin scheme enjoys some nice properties that help reduce the derivation of the corresponding convergence results to essentially very basic functional analysis exercises. Analytic formulas are also available within this approach, which simplify the numerical treatment. The efficiency of the method will be illustrated on several examples, one of which has solutions that recall Brownian motion. (Received July 19, 2018)

1141-35-105 **Javier Morales\***, javierm1@cscamm.umd.edu. *The synchronization problem for Kuramoto oscillators and beyond.*

Collective phenomena such as aggregation, flocking, and synchronization are ubiquitous in natural biological, chemical, and mechanical systems—e.g., the flashing of fireflies, chorusing of crickets, synchronous firing of cardiac pacemakers, and metabolic synchrony in yeast cell suspensions. The Kuramoto model introduced by Yoshiki Kuramoto is one of the first theoretical tools developed to understand such phenomena and has recently gained extensive attention in the physical and mathematical community. Moreover, it has become the starting point of several generalizations that have applications ranging from opinion dynamics to the development of human-made interacting multi-agent systems of UAVs and data clustering. In this talk, we will review the state of the art for the synchronization problem of the Kuramoto model at the kinetic and particle level. Additionally, we will introduce new developments and variational techniques for the dynamics of this model and some of its variants and its generalization (Received July 20, 2018)

1141-35-125 **Tao Huang\***, Wayne State University, and **Na Zhao**, Fudan University. *Regularity of weak solutions of a gradient flow of the Landau-de Gennes energy.*

For a gradient flow of the Landau-de Gennes energy, the unique global weak solution of initial and boundary value problem in dimension two has been constructed by Iyer-Xu-Zarnescu 2015 with small initial data. We investigate the regularity of such solution, and prove that the weak small solution constructed in Iyer-Xu-Zarnescu's paper is actually regular. (Received July 24, 2018)

1141-35-154 **Jun Wang\*** (wangmath2011@126.com), Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, China, 212013. *Standing waves of coupled Schrödinger equations with quadratic interactions from Raman amplification in a plasma.* Preliminary report.

The standing wave solutions of a coupled nonlinear Schrödinger equations with quadratic nonlinearities from Raman amplification of laser beam in a plasma are considered. For both the original three-wave system and a reduced two-wave system, the existence/nonexistence, continuous dependence and asymptotic behavior of positive ground state solutions are established. In particular, multiple positive standing wave solutions are found via a combination of variational and bifurcation methods for the attractive interaction case, which has not been found for the conventional nonlinear Schrödinger systems with cubic nonlinearities. This is a joint work with Professor Junping Shi(College of William and Mary). (Received July 26, 2018)

1141-35-156 **Ryan W Murray\*** (rwm22@psu.edu). *Accelerating Stein variational gradient descent.*

Recently machine learning researchers have proposed an algorithm called *Stein variational gradient descent* in order to accurately approximate Bayesian posterior probabilities. The proposed algorithm may be seen as a particle approximation of the non-linear evolution equation

$$\partial_t \rho = \nabla \cdot (\rho(\nabla K * \rho + K * (\nabla V \rho))), \quad \rho(0, x) = \rho_0(x), \quad (1)$$

where the posterior distribution of interest is  $e^{-V}$ ,  $K$  is a positive definite kernel and  $\rho_0$  is initial data. This equation may be seen as a gradient flow of the relative entropy function in an appropriately modified Wasserstein space. This talk will discuss some recent work regarding convergence rates of this equation. Numerical results demonstrating the effectiveness of the method will also be presented. (Received July 26, 2018)

1141-35-158 **Yuhua Li, Fuyi Li and Junping Shi\*** (jxshix@wm.edu), Department of Mathematics, College of William and Mary, Williamsburg, VA 23187. *Ground States of Nonlinear Schrödinger Equation on Star Metric Graphs.*

The existence and nonexistence of the ground state to Nonlinear Schrödinger equation on several types of metric graphs are considered. In particular, for some star graphs with only one central vertex, the existence of ground state solution or positive solutions are shown. It is shown that the structure of the set of positive solution is quite different from the one for corresponding bounded  $n$ -dimensional domain. The proofs are based on variational methods, rearrangement arguments, energy estimates and phase plane analysis. (Received July 27, 2018)

1141-35-174 **Oreoluwa Adekoya** (oadekoya@ou.edu) and **John P Albert\*** (jalbert@ou.edu), Department of Mathematics, University of Oklahoma, Norman, OK 73019. *Stable travelling-wave solutions of the periodic dispersion-managed NLS equation.*

The dispersion-managed nonlinear Schrodinger equation is a model equation for optical pulses in a "dispersion-managed" fiber: one made of lengths of glass with alternating positive and negative dispersion. Typically in such fibers, the average dispersion is nearly zero, which is reflected in the fact that the energy functional associated with the equation lacks coercivity. Nevertheless, Kunze was able to show that minimizing sequences for the energy functional converge to stable solitary-wave solutions.

Here we consider the periodic dispersion-managed NLS equation, which models periodic waves in a dispersion-managed fiber. The minimization problem for the energy is more delicate in the periodic case: for certain ratios of the period of the wave to the period of the fiber, constrained minimizers do not exist. However, we show that for certain other values of the ratio, minimizers do exist, and correspond to stable travelling-wave solutions. (Received July 28, 2018)

1141-35-179 **Hongjie Dong\*** (hongjie\_dong@brown.edu), 182 George St, Providence, RI 02906, and **Tuoc Phan.** *Non-stationary Stokes systems with singular VMO coefficients and applications.*

We prove the mixed-norm Sobolev estimates for solutions to both divergence and non-divergence form time-dependent Stokes systems with unbounded measurable coefficients having small mean oscillations with respect to the spatial variable in small cylinders.

A new  $\epsilon$ -regularity criterion for Leray-Hopf weak solutions of Navier-Stokes equations is also obtained as a consequence of our regularity results, which in turn implies some borderline cases of the well-known Serrin's regularity criterion. (Received July 28, 2018)

1141-35-181 **Tam Do, Alexander Kiselev, Lenya Ryzhik and Changhui Tan\*** (tan@math.sc.edu). *Global regularity for Burgers equation with density dependent fractional dissipation.*

Fractional Burgers equations are a family of equations which connect inviscid and viscous Burgers equations. It is well-known that if the dissipation is strong, the solution is globally regular. On the other hand, if the dissipation is weak (called supercritical case), the solution can lose regularity at a finite time. In this talk, I will introduce a model where the dissipation depends on density. The model is motivated by self-organized dynamics in math biology. Despite that the equation shares a lot of similarities to fractional Burgers equation, the solution is globally regular, even in the supercritical case. I will explain the regularization mechanism that is due to the nonlocal nonlinear modulation of dissipation. (Received July 28, 2018)

1141-35-186 **Gabor Lippner\*** (g.lippner@northeastern.edu) and **Dan Mangoubi.** *Liouville-type theorems on Cayley graphs via absolute monotonicity.*

We show a novel method to understand harmonic functions on Abelian Cayley graphs by studying their growth modulus. The growth modulus turns out to be absolutely monotonic which yields strong convexity properties of harmonic functions. This leads to a simple proof of the strong Liouville theorem, stating that polynomially growing harmonic functions are polynomials.

Though absolute monotonicity pertains only to Abelian groups, it turns out that the methods can be used to prove the strong Liouville theorem for nilpotent groups as well. (Received July 29, 2018)

1141-35-199 **Ming Chen\***, 301 Thackeray Hall, Department of Mathematics, University of Pittsburgh, Pittsburgh, PA 15260, and **Samuel Walsh** and **Miles Wheeler.** *Asymptotic behavior of deep water solitary waves with localized vorticity.*

We consider free-surface solitary waves of an infinitely deep two- or three-dimensional fluid with a localized distributed vorticity, with or without surface tension. We are able to obtain precise asymptotics at infinity, given a very mild algebraic decay assumption. Moreover using such asymptotic information we can prove several

qualitative properties of the wave. This is a joint work with S. Walsh and M. Wheeler. (Received July 29, 2018)

1141-35-203 **Xianpeng Hu** and **Changyou Wang\***, 150 N. University Street, West Lafayette, IN 47907. *Existence and partial regularity of suitable weak solutions of a coupled Navier-Stokes and Q-tensor system.*

In this paper, we will show both the existence and partial regularity of suitable weak solutions of a coupled Navier-Stokes and Q-tensor system, that models the hydrodynamics of nematic liquid crystal materials. This is a joint work with Xianpeng Hu. (Received July 30, 2018)

1141-35-218 **Pei Liu\*** (pul21@psu.edu), **Simo Wu** and **Chun Liu**. *Non-Isothermal Electrokinetics: Energetic Variational Approach.*

Fluid dynamics accompanies with the entropy production thus increases the local temperature, which plays an important role in charged systems such as the ion channel in biological environment and electrodiffusion in capacitors/batteries. In this article, we propose a general framework to derive the transport equations with heat flow through the Energetic Variational Approach. According to the first law of thermodynamics, the total energy is conserved and we can use the Least Action Principle to derive the conservative forces. From the second law of thermodynamics, the entropy increases and the dissipative forces can be computed through the Maximum Dissipation Principle. Combining these two laws, we then conclude with the force balance equations and a temperature equation. To emphasis, our method provide a self consistent procedure to obtain the dynamical equations satisfying proper energy laws and it not only works for the charge systems but also for general systems. (Received July 30, 2018)

1141-35-231 **Shu-Ming Sun\*** (sun@math.vt.edu), Department of Mathematics, Virginia Tech, Blacksburg, VA 24061. *Solitary-wave solutions for higher-order BBM equations with nonlinear part nonhomogeneous.*

The talk discusses the existence of solitary-wave solutions of a general higher-order Benjamin-Bona-Mahony (BBM) equation, whose linear part is a pseudo-differential operator. The nonlinear part of the equation involves the polynomials of solution and its derivatives with different degrees (not homogeneous), which has not been studied before. One of such equations can be derived from water-wave problems as the second-order approximate equation from fully nonlinear governing equations. Under some conditions on the symbols of pseudo-differential operators and the nonlinear terms, it is shown that the equation has solitary-wave solutions. Numerical study of the solitary-wave solutions for some special fifth-order BBM equations will also be discussed. (This is a joint work with J. Bona, H. Chen, and J.-M. Yuan). (Received July 30, 2018)

1141-35-236 **Ana Maria Soane\*** (soane@usna.edu). *Multigrid preconditioners for stochastic optimal control problems with elliptic SPDE constraints.*

In this work we construct multigrid preconditioners to be used in the solution process of pathwise optimal control problems constrained by elliptic partial differential equations with random coefficients. We combine a sparse-grid collocation approach to discretize in the stochastic space with multigrid techniques in the physical space. Numerical results show that the proposed multigrid preconditioners lead to significant computational savings, with the number of preconditioned conjugate gradient iterations decreasing as the resolution increases. (Received July 30, 2018)

1141-35-275 **Ryan Murray Evans\***, 100 Bureau Drive, Mail Stop 8910, Gaithersburg, MD 20899, and **Arvind Balijepalli** and **Anthony Kearsley**. *Transport Phenomena in Field Effect Transistors.*

Tailoring therapies to individuals for personalized care can be safer and yield superior outcomes with lower doses for conditions such as diabetes, Alzheimers disease, or even certain cancers. However, widespread use of personalized care is currently limited by inability to measure pathology and detect biomarkers. Moreover, existing strategies require specialized facilities, can be slow to perform and can be expensive. This has led to the development of a new portable detection tool known as a field effect transistor (FET). Very well-suited for biomarker measurements due their high charge sensitivity and direct signal transduction, FETs allow label-free measurements at physiological concentrations. Chemical reactants are injected at the top of solution-well and diffuse through a well to bind with another chemical reactant immobilized to the well-floor. A resulting response curve allows for biomarker measurement and estimation of key parameters, such as binding affinities. A mathematical model for FET experiments will be presented that takes the form of a diffusion coupled to a nonlinear equation that describes the evolution of the reacting species concentration. (Received July 31, 2018)



1141-35-297 **Dennis Kriventsove\*** ([dennisk@cims.nyu.edu](mailto:dennisk@cims.nyu.edu)). *Spectral optimization and free boundary problems.*

A classic subject in analysis is the relationship between the spectrum of the Laplacian on a domain and that domain's geometry. One approach to understanding this relationship is to study domains which extremize some function of their spectrum under geometric constraints. I will give a brief overview of some of these optimization problems and describe the (very few) explicit solutions known. Then I will explain how to approach these problems more abstractly, using tools from the calculus of variations to find solutions. A key difficulty with this approach is showing that the solutions (which are a priori very weak) are actually smooth domains, which I address in some recent work with Fanghua Lin. Our method revolves around relating spectral optimization problems to certain vector-valued free boundary problems of Bernoulli type (Received July 31, 2018)

1141-35-304 **Maxim Zyskin\*** ([maximzyskin@gmail.com](mailto:maximzyskin@gmail.com)), University of Oxford, Parks Road, Oxford, Oxford, Oxfordshire OX1 3PJ, United Kingdom. *Mathematics of electric batteries modeling.*

In my talk I will describe: a modeling approach to derive transport equations accounting for electric batteries multi-physics; novel analytic and numerical methods of solving those equations; and the relationship with molecular dynamics simulations. This work has important applications, where a number of modeling and simulation questions remain unresolved (Received August 01, 2018)

## 37 ► *Dynamical systems and ergodic theory*

1141-37-20 **Jianyu Chen\*** ([jchen@math.umass.edu](mailto:jchen@math.umass.edu)), Department of Mathematics and Statistics, Lederle Graduate Research Tower 1030, 710 N. Pleasant Street, Amherst, MA 01003, and **Fang Wang** and **Hong-Kun Zhang**. *Markov Partition and Thermodynamic Formalism for Chaotic Billiards.*

This is a joint work with Fang Wang and Hong-Kun Zhang.

In this talk, we investigate the chaotic billiards and other related hyperbolic systems with singularities, and construct a Markov partition of the phase space with countable states. Based on such special structure, we further establish the thermodynamic formalism for the family of geometric potentials, by adapting the inducing scheme developed by Pesin, Senti and Zhang. Stochastic properties of the corresponding equilibrium measures immediately follow, including the existence and uniqueness, the decay rates of correlations and the central limit theorem.

All the results apply to Sinai dispersing billiards, and their small perturbations due to external forces and nonelastic reflections with kicks and slips. (Received June 14, 2018)

1141-37-62 **Alexander Grigo\*** ([alexander.grigo@ou.edu](mailto:alexander.grigo@ou.edu)). *Averaging in billiard-like systems.*

In this talk I will present an averaging theorem for a fully coupled system with singularities. As an application we present a billiard-like model for Haff's cooling law. (Received July 14, 2018)

1141-37-113 **Nicholas J Russell\*** ([nrussell@udel.edu](mailto:nrussell@udel.edu)), 501 Ewing Hall, Newark, DE 19716, and **Louis Rossi**. *Small Organisms Causing Big Problems: Modeling Heterosigma Akashiwo.*

A specific species of phytoplankton, *Heterosigma Akashiwo*, has been the cause of harmful algal blooms (HABs) in waterways around the world causing millions of dollars in damage to farmed animals and destroying ecosystems. Developing a fundamental understanding of their movements and interactions through phototaxis and chemotaxis is vital to comprehending why these HABs start to form and how they can be prevented. In this talk, we attempt to create a complex and biologically accurate mathematical and computational model reflecting the movement of an ecology of plankton, incorporating phototaxis, chemotaxis, and the fluid dynamics that may be affecting the flow. We present and analyze a succession of models together with a sequence of laboratory and computational experiments that inform the mathematical ideas underlying the model. Lastly, we discuss further experiments and research necessary for our continued insight into problems that we are encountering, such as plankton's formation of aggregations, the gaps in-between those aggregations, and the difficulty of expanding our models to higher dimensions biologically, mathematically, and computationally. (Received July 23, 2018)

1141-37-150 **Christopher L. Cox\*** ([c1cox@udel.edu](mailto:c1cox@udel.edu)), 1000 E Lingleville Rd, 3004, Stephenville, TX 76401. *Bounded Orbits of No-Slip Billiards.*

No-slip billiards are a dynamical system based on a rigid body collision model in which total momentum is conserved, with linear and angular momentum exchanged at collisions. Bounded orbits are ubiquitous in dimension two, but less common in dimension three. We also consider systems with an external force, including cases in which the boundedness persists. (Received July 26, 2018)

1141-37-157 **Kien Nguyen\*** ([kien@math.umass.edu](mailto:kien@math.umass.edu)), Department of Mathematics and Statistics, University of Massachusetts Amherst, Amherst, MA 01003, and **HongKun Zhang** ([hongkun@math.umass.edu](mailto:hongkun@math.umass.edu)), Department of Mathematics and Statistics, University of Massachusetts Amherst, Amherst, MA 01003. *Central limit theorem for billiards with flat points.*

In this paper, we constructed stationary martingale difference approximations to certain processes generated by billiards with flat points, using the filtration generated by the first return time function. This leads to the central limit theorem for observables adapted to the filtration. Moreover, we also are able to obtain an explicit formula for the diffusion constant for this class of observables. (Received July 26, 2018)

1141-37-171 **Stanislav M Mintchev\***, [mintchev@cooper.edu](mailto:mintchev@cooper.edu), and **Benjamin Ambrosio**. *Periodically kicked feedforward chains of simple excitable FitzHugh-Nagumo neurons.*

We present results on regular depolarization cascades in periodically-kicked feedforward chains of excitable two-dimensional FitzHugh-Nagumo systems. The study documents a parameter exploration by way of changes to the forcing period, upon which the dynamics undergoes a transition from simple depolarization to more complex behavior, including the emergence of mixed mode oscillations. Both rigorous studies and careful numerical observations are presented. In particular, we provide rigorous proofs for existence and stability of periodic traveling waves of depolarization, as well as the existence and propagation of a simple mixed mode oscillation that features depolarization and refraction in alternating fashion. Detailed numerical investigation reveals a mechanism for the emergence of complex mixed mode oscillations featuring a potentially high number of large amplitude voltage spikes interspersed by an occasional small amplitude reset that fails to cross threshold. Further careful numerical investigation provides insights into the propagation of this complex phenomenology in the downstream, where we see an effective filtration property of the of the network; the latter amounts to a successive reduction in the complexity of mixed mode oscillations down the chain. (Received July 27, 2018)

1141-37-178 **Diana Davis\*** ([dianajdavis@gmail.com](mailto:dianajdavis@gmail.com)), 500 College Avenue, Swarthmore, PA 19081, and **Samuel Lelievre** ([samuel.lelieuvre@gmail.com](mailto:samuel.lelieuvre@gmail.com)), Bâtiment 300, 91405 Orsay, France. *Periodic paths on the pentagon.*

Mathematicians have long understood periodic trajectories on the square billiard table. In the present work, we describe periodic trajectories on the regular pentagon – their geometry, symbolic dynamics, and group structure. Some of the periodic trajectories exhibit a surprising “dense but not equidistributed” behavior. I will show pictures of periodic trajectories, which are very beautiful. (Received July 28, 2018)

1141-37-180 **Robert G. Niemeyer\*** ([robert.niemeyer@uiwtx.edu](mailto:robert.niemeyer@uiwtx.edu)), 4301 Broadway, School of Mathematics, Science and Engineerin, San Antonio, TX 78209, and **Charles C. Johnson** ([ccjohnson@gmail.com](mailto:ccjohnson@gmail.com)), Rawles Hall, 831 East 3rd St., Bloomington, IN 47405-7106. *The geometry of wild, elusive singularities of the T-fractal surface.*

We first provide background on the T-fractal billiard, recent results on periodic orbits of such and a construction of the T-fractal translation surface using what we have called a quad-T surface. We then use such a construction to show that the set of elusive singularities is a Cantor set and that such a set is a set of wild singularities, meaning that they are not finite angle conical singularities nor are they infinite angle conical singularities of the flow on the translation surface. Moreover, we will examine the geometry of such points. We will close our talk with how such results may apply to the Koch snowflake fractal translation surface, a surface yet to be rigorously defined. (Received July 28, 2018)

1141-37-188 **Yao Li\*** ([yaoli@math.umass.edu](mailto:yaoli@math.umass.edu)), 710 N. Pleasant Street, Amherst, MA 01003, and **Lingchen Bu** ([yaoli@math.umass.edu](mailto:yaoli@math.umass.edu)), 710 N Pleasant St, Amherst, MA 01003. *From deterministic billiards to thermodynamic laws.*

Consider a long and thin tube that connects two heat baths with different temperatures at its ends. Assume this tube contains many kinetic gas particles whose only interactions are elastic collisions. Some simplifications are necessary to make this model mathematically tractable. We first localize particles by “trapping” them within a 1D chain of billiard tables. Then we use numerical simulations to reduce the localized billiard system to a stochastic energy exchange model. Many mathematical and computational justification can be made for this stochastic energy exchange model. The fundamental goal is to derive macroscopic thermodynamic laws for the stochastic energy exchange model and its mesoscopic limit equation. (Received July 29, 2018)

1141-37-192 **Federico Bonetto\*** ([bonetto@math.gatech.edu](mailto:bonetto@math.gatech.edu)), School of Mathematics - GaTech, 656 Cherry St, Atlanta, GA 30332, and **Nikolai Chernov**, **Alexey Korepanov** and **Joel Lebowitz**. *Long time behavior for thermostated particles under a weak electric field.*

We investigate a model system consisting of  $N$  particles moving on a  $d$ -dimensional torus  $\mathbb{T}^d$  under the action of an electric field  $E$  with a Gaussian thermostat to keep the total energy constant. The particles are also subject to stochastic collisions which randomize direction but do not change the speed. We prove that in the van Hove scaling limit,  $E \rightarrow 0$  and  $t \rightarrow t/E^2$ , the trajectory of the speeds  $v_i$  is described by a specific stochastic differential equation. (Received July 29, 2018)

1141-37-196 **Renato Feres\***, [feres@wustl.edu](mailto:feres@wustl.edu), and **Tim Chumley**, **Scott Cook** and **Chris Cox**. *No-slip bouncing and discrete non-holonomic systems.*

Non-holonomic mechanical systems can sometimes exhibit surprising behavior. A classical example consists of a ball that rolls against the inner surface of a vertical cylinder; contrary to what might be expected, to the extent that energy dissipation may be neglected and the ball doesn't "peel off," the ball will oscillate vertically up and down without falling. (This is observed, to fair approximation, in lab experiments.) In this talk I will consider a discrete version of such non-holonomic systems, called no-slip billiards. These are dynamical systems that model the bouncing of a perfectly elastic ball with a rough (rubbery) surface. For no-slip billiards in a cylinder with generic initial conditions, the bouncing ball will accelerate downward as one might expect, but if the first bounce satisfies a transverse rolling condition, the ball will again oscillate vertically without falling. I will focus on the differential geometric basis of this dynamical behavior. (Received July 29, 2018)

1141-37-239 **Alex Blumenthal\*** ([alexbl23@math.umd.edu](mailto:alexbl23@math.umd.edu)), 4176 Campus Drive, William E. Kirwan Hall, College Park, MD 20742. *Lagrangian chaos for models in fluid mechanics.*

In models of fluid mechanics, given a possibly time-varying, divergenceless vector field  $u(x, t)$ ,  $x \in M$ ,  $t \geq 0$ , on a manifold  $M$ , the Lagrangian flow  $\phi^t : M \rightarrow M$  on  $M$  is the flow generated by integrating along the streamlines of  $u(x, t)$ . Typically one assumes that  $u(x, t)$  itself evolves according to one of the standard models of fluid mechanics, e.g., the Navier-Stokes equations.

It is anticipated that in many regimes (e.g., high Reynolds number, in the presence of suitably nondegenerate noise) that the dynamics of the Lagrangian flow  $\phi^t$  should be chaotic in the sense of sensitivity with respect to initial conditions. I will present a recent joint work with Jacob Bedrossian and Sam Punshon-Smith in which we rigorously verify this chaotic property (actually, verify the presence of a positive Lyapunov exponent) when  $u$  evolves according to various models in fluid mechanics, including stochastic Navier-Stokes on the 2D torus as well as the Galerkin truncation of Navier-Stokes on the 3D torus.

A consequence of our work is a rigorous verification of Yaglom's law, a scaling law for passive scalar advection analogous to the famous Kolmogorov 4/5 law for turbulence in the Navier-Stokes equations. (Received July 30, 2018)

1141-37-241 **Maria F Correia\*** ([correia@math.umass.edu](mailto:correia@math.umass.edu)), Department of Mathematics and Statistics, Lederle Graduate Research Tower, University of Massachusetts Amherst, 710 N. Pleasant Street, Amherst, MA 01003-9305, and **Christopher Cox** and **Hong-Kun Zhang**. *Chaotic behavior in generalized umbrella-shaped billiards.* Preliminary report.

In this work, we investigated a three-parameter family of billiard tables with circular arc boundaries. These umbrella-shaped billiards may be viewed as a generalization of two-parameter moon and asymmetric and convex lemon billiards, in which the latter classes comprise instances where the new parameter is zero. Like those two previously studied classes, for certain parameters umbrella billiards exhibit evidence of chaotic behavior despite failing to meet certain criteria for defocusing or dispersing, the two most well understood mechanisms for generating ergodicity and hyperbolicity. We characterize the periodic points and investigate the transition to ergodicity in the three-parameter umbrella families. (Received July 30, 2018)

1141-37-256 **Scott Cook\*** ([sccook@tarleton.edu](mailto:sccook@tarleton.edu)). *No Slip Lorentz Gas.*

The no-slip billiard collision law was introduced by Broomhead and Gutkin to model a gas where particles "interact with each other and with the container walls without slipping, in such a way that the impact conserves total energy but mixes the tangential velocity components with the angular components of colliding spheres." Recent work by Cox and Feres have revealed some remarkable contrasts between the behavior of billiard systems with a small number of particles under the no-slip law versus the standard specular law. In this talk, we will extend this comparative analysis of collision laws to the classical Lorentz gas model. We investigate the behavior of the Lorentz gas under the no-slip law and contrast with the standard specular collision law. (Received July 31, 2018)

1141-37-271 **Konstantin Mischaikow\*** ([mischaik@math.rutgers.edu](mailto:mischaik@math.rutgers.edu)), Department of Mathematics, Rutgers, The State University of New Jersey, 110 Frelinghusen Rd, Piscataway, NJ 08854. *A Homological Description of Global Nonlinear Dynamics*. Preliminary report.

I will describe recent work towards developing a database that describes the global dynamics of regulatory networks over large regions of parameter space. I will focus on why this is possible, i.e. why can we get a mathematically rigorous finite combinatorial description of dynamics, and how homology in the form of the Conley index can be used to organize this information. (Received July 31, 2018)

## 41 ► *Approximations and expansions*

1141-41-9 **A Abdurrrhman\*** ([ababdu@ship.edu](mailto:ababdu@ship.edu)), Department of Physics, 1871 Old Main Drive, Shippensburg, PA 17257, and **I Abdurrahman** ([ia4021@uw.edu](mailto:ia4021@uw.edu)), Department of Physics, 3910 15th Ave. NE, Seattle, WA 98195-15. *Series expansion of rational numbers*.

We show that an iterative procedure for computing the center of mass for  $n$  unit masses along the  $x$ -axis, give rise to a simple procedure for expanding rational numbers in powers of  $r/s$ , with  $s > r > 1$ . Using this method we give a constructive proof for the convergence of the geometric series. Furthermore, if every perturbative step is carried out on a line parallel to the  $x$ -axis and located at  $y(x)$ , where  $y(x)$  is an arbitrary function of  $x$ , then the length of the curve,  $l[y(x)]$  tracing the perturbative center of mass, as the center of mass approaches a limit point, goes through a discontinuity (a phase transition) and thus this construction admits a critical scale. (Received April 02, 2018)

## 42 ► *Fourier analysis*

1141-42-25 **Allan Greenleaf** ([allan@math.rochester.edu](mailto:allan@math.rochester.edu)), Department of Mathematics, University of Roch, Rochester, NY 14627, and **Alex Iosevich\*** ([iosevich@gmail.com](mailto:iosevich@gmail.com)), Department of Mathematics, Rochester, NY 14627. *Group actions, similarities and finite point configurations*.

We prove that if the Hausdorff dimension of a compact subset of Euclidean space is sufficiently large, then given any positive real number, one can find many point configurations that are similar up to a dilation by this number. The result is based on group invariances and the analysis behind the recent advances in the Falconer Distance Conjecture. (Received June 27, 2018)

1141-42-69 **Palle Jorgensen\***, Dept Mathematics MLH, University of Iowa, Iowa City, IA, and **Pedersen**. *Spectral analysis is also considered for a family of fractal measures*.

The study of spectral duality for singular measures started with a joint paper, Jorgensen-Pedersen. In the case of affine IFS measures  $\mu$ , when an associated complex Hadamard matrix is further assumed to satisfy an additional symmetry condition; then the  $L^2(\mu)$  Hilbert space will have an orthogonal Fourier basis; in other words we get an associated fractal Fourier transform. In order to appreciate the nature of the spectral duality, note that spectral duality holds for the middle-1/4 Cantor measure, but not for its middle-1/3 cousin. Typically the distribution of the associated Fourier frequencies satisfies very definite lacunary properties, in the form of geometric almost-gap distributions; the size of the gaps grows exponentially, with sparsity between partitions. The probabilistic significance will be explored. Use will be made of reproducing kernel Hilbert spaces of analytic functions. Spectral analysis is also considered for a wider family of fractal measures. (Received July 16, 2018)

1141-42-112 **Eyvindur Ari Palsson\*** ([palsson@vt.edu](mailto:palsson@vt.edu)), Department of Mathematics, McBryde Hall, Virginia Tech, Blacksburg, VA 24061, and **Allan Greenleaf**, **Alex Iosevich** and **Bochen Liu**. *A group-theoretic viewpoint on Falconer type theorems*.

Finding and understanding patterns in data sets is of significant importance in many applications. One example of a simple pattern is the distance between data points, which can be thought of as a 2-point configuration. Two classic questions, the Erdos distinct distance problem, which asks about the least number of distinct distances determined by  $N$  points in the plane, and its continuous analog, the Falconer distance problem, explore that simple pattern. Questions similar to the Erdos distinct distance problem and the Falconer distance problem can also be posed for more complicated patterns such as patterns based off of three points, which can be viewed as 3-point configurations. In this talk I will explore such generalizations and highlight a novel group-theoretic viewpoint which has allowed for much progress recently. The main techniques used come from analysis and geometric measure theory. (Received July 22, 2018)

1141-42-119 **Nikolaos Chatzikonstantinou\*** ([nchatzik@ur.rochester.edu](mailto:nchatzik@ur.rochester.edu)), **Alex Iosevich**, **Sevak Mkrtchyan** and **Jonathan Pakianathan**. *Rigidity, graphs and Hausdorff dimension.*

A set of  $k+1$  points in Euclidean space is called a  $(k+1)$ -point configuration. Two configurations are congruent if they are equal up to an affine isometry. Given a compact subset  $E$  of  $\mathbb{R}^d$ ,  $d \geq 2$  of Hausdorff dimension greater than  $d - \frac{1}{k+1}$  we prove that the Lebesgue measure of noncongruent  $(k+1)$ -point configurations in  $E$  is positive, for  $k > d$ , complementing the results of Greenleaf-Iosevich-Liu-Palsson (2015) for  $k \leq d$ . (Received July 23, 2018)

1141-42-288 **Azita Mayeli\*** ([amayeli@gc.cuny.edu](mailto:amayeli@gc.cuny.edu)), NY. *Recent developments in orthogonal Gabor bases and tiling problems.*

Given  $g \in L^2(\mathbb{R}^d)$ ,  $d \geq 1$ , and a countable set  $\Lambda \subset \mathbb{R}^{2d}$ , the *Gabor system* (also known as *Weyl-Heisenberg system*)  $\mathcal{G}(g, \Lambda)$  is the collection of functions  $\pi(a, b)g$  defined by the time and frequency shifts of  $g$ :

$$\pi(a, b)g(x) = M_b T_a g = e^{2\pi i(b, x)} g(x - a) \quad (a, b) \in \Lambda.$$

The collection  $\mathcal{G}(g, \Lambda)$  is called *orthogonal Gabor basis* for  $L^2(\mathbb{R}^d)$ , if it is complete and mutual orthogonal in  $L^2(\mathbb{R}^d)$ .

In this talk, we assume  $g = \chi_K$  be the characteristic function of a bounded set  $K \subset \mathbb{R}^d$  with positive Lebesgue measure, and study full lattices in  $\mathbb{R}^{2d}$  for which the orthogonality and completeness of the Gabor system  $\mathcal{G}(\chi_K, \Lambda)$  imply the tiling property of the set  $K$  by translations in  $\mathbb{R}^d$ .

This is a joint work with Chun-Kit Lai. (Received July 31, 2018)

## 46 ► Functional analysis

1141-46-8 **Naeem Saleem\*** ([naeem.saleem2@gmail.com](mailto:naeem.saleem2@gmail.com)), C-II, Johar Town, Lahore, Pakistan, Lahore, Punjab 50400, Pakistan. *Best Proximity Point Results in Fuzzy Metric Spaces.* Preliminary report.

In this article, we obtained best proximity points and optimal coincidence element results for non-self mapping in fuzzy metric space. We also mentioned some interesting aspects of best proximity element theory in the setup of fuzzy metric spaces. We provided some examples to explain the obtained results, which shows that obtained results are generalization of already existing results in literature. This article could be viewed as a discussion on extension of recent development on proximal contraction mappings in such spaces. (Received March 30, 2018)

1141-46-11 **Gareth James Speight\*** ([gareth.speight@uc.edu](mailto:gareth.speight@uc.edu)). *Directional derivatives and universal differentiability sets in higher step Carnot groups.*

Rademacher's theorem asserts that Lipschitz functions from  $\mathbb{R}^n$  to  $\mathbb{R}^m$  are differentiable almost everywhere. This may not admit a converse: if  $n > 1$  then there exists a Lebesgue null set  $N$  in  $\mathbb{R}^n$  containing a point of differentiability for every Lipschitz mapping from  $\mathbb{R}^n$  to  $\mathbb{R}$ . Such sets are called universal differentiability sets; their construction relies on the fact that existence of an (almost) maximal directional derivative implies differentiability. We will see that this Euclidean theory extends to certain Carnot groups where the Carnot-Carathéodory distance is suitably differentiable. This includes all step 2 Carnot groups and examples of Carnot groups of arbitrarily high step. However, there exists a Carnot group of step 3 where things go badly wrong: maximality of a directional derivative does not imply differentiability for any horizontal direction. Based on joint work with Andrea Pinamonti and Enrico Le Donne. (Received April 26, 2018)

1141-46-17 **NASEER AHMAD ASIF\*** ([naseerasif@yahoo.com](mailto:naseerasif@yahoo.com)), University of Management and Technology, CII Johar Town, 54770 Lahore, Pakistan, Lahore, 54770, Pakistan. *Positive Solutions of a Fractional Singular Boundary Value Problem.*

Sufficient conditions has been provided to establish the existence of positive solutions of a singular boundary value problem for an ordinary differential equation in presence of Caputo derivative. The ordinary differential equation includes singularities with respect to both dependent and independent variables. Some available results of fixed point theory and functional analysis has been utilized to prove the existence of solutions. (Received June 10, 2018)

1141-46-37 **Basit Ali\*** ([basit.ali@umt.edu.pk](mailto:basit.ali@umt.edu.pk)), Department of Mathematics, School of Science, University of Management and Technology, Lahore, 54770, Pakistan. *Completeness characterizations of some distance spaces via fixed point theorems with applications.*

Banach contraction principle (BCP) and its so called generalizations guarantee the existence of fixed point of mappings of complete metric spaces. But converse of BCP, "if every Banach contraction has a fixed point in a metric space  $X$ ; then  $X$  is complete" does not hold true. In this talk, we consider the problem of completeness of underlying distance spaces via fixed point theorems.

The "completeness problem" (CP) of underlying space is related to the "end problem" (EP) in behavioral sciences, that is, "to know when and where a human dynamics, which starts from an initial point, and followed by transitions, defined as a successive approximations ends somewhere". Further, it appears that behaviors particularly related to fears can lead to the propagation of viral diseases like Ebola. In this talk, we consider an application in connection with behavioral sciences. (Received July 05, 2018)

1141-46-70 **Jeannette Janssen\*** ([jeannette.janssen@dal.ca](mailto:jeannette.janssen@dal.ca)), Dept. of Math & Stats, Dalhousie University, Halifax, NS, Canada. *Infinite random geometric graphs.*

Classical results of Erdős and Rényi establish that countably infinite random graphs are isomorphic with probability 1. We examine the equivalent question for infinite geometric random graphs. Such graphs have vertices identified with points in a metric space, and edges are added with a given probability dependent if their endpoints are within a given threshold distance. The question we consider is whether infinite random geometric graphs are almost always isomorphic. The answer depends on the metric space; In particular, it is closely related to the existence of a *step isometry*, which is a bijective map that preserves distances up to their integer multiples. We answer the isomorphism question in the affirmative for a variety of normed linear spaces, such as  $\ell_\infty^n$  for all finite dimensions  $n$ , certain sequence spaces, and subspaces of  $C[0, 1]$ .

This is joint work with Anthony Bonato and Anthony Quas (Received July 16, 2018)

1141-46-129 **J. E. Pascoe, Meredith Sargent\*** ([sargent@uark.edu](mailto:sargent@uark.edu)) and **Ryan Tully-Doyle.** *Escaping non-tangential approach: amortization and auguries.*

Let  $\phi : D \rightarrow \Omega$  be a complex analytic function. A classical Julia-Carathéodory theorem states that if there is a sequence tending to  $\tau$  in the boundary of a  $D$  along which the Julia quotient is bounded, then the function  $\phi$  can be extended to  $\tau$  such that  $\phi$  is nontangentially continuous and differentiable at  $\tau$  and  $\phi(\tau)$  is in the boundary of  $\Omega$ .

We develop a theory in the case of Pick functions where we consider sequences that approach the boundary in a controlled tangential way and their relationship to higher order regularity. In this talk, we discuss some of the technical details involved, including amortization of the Julia Quotient,  $\gamma$ -regularity, and  $\gamma$ -auguries. (Received July 24, 2018)

1141-46-139 **Ben Li\*** ([bx1292@case.edu](mailto:bx1292@case.edu)), **Carsten Schuett** and **Elisabeth Werner.** *The Loewner Function of Log-concave Functions.*

We introduce the notion of Loewner ellipsoid function for log concave function. The duality of this notion to the John ellipsoid function which is introduced by D. Alonso-Gutierrez et al with also be discussed. This is based on joint work with C. Schuett and E. Werner. (Received July 25, 2018)

1141-46-169 **Thomas A Courtade\*** ([courtade@berkeley.edu](mailto:courtade@berkeley.edu)) and **Max Fathi** ([max.fathi@math.univ-toulouse.fr](mailto:max.fathi@math.univ-toulouse.fr)). *Stability of the Poincaré inequality for log-concave measures.*

We give a dimension-free improvement of a result by De Philippis and Figalli asserting that if a 1-uniformly log-concave measure has almost the same Poincaré constant as the standard Gaussian measure, then it almost splits off a Gaussian factor as measured under the  $W_1$  transport distance. We also prove a similar result when a subset of coordinates have almost unit variance and the Poincaré constant is smaller than one, without any log-concavity assumption. Proofs are based on Stein's method, mass transport, and an approximate integration by parts identity relating measures and approximate optimizers in the associated Poincaré inequality. (Received July 27, 2018)

1141-46-183 **Ben Jaye\*** ([bjaye@clemson.edu](mailto:bjaye@clemson.edu)), **Galyna Livshyts, Grigoris Paouris and Peter Pivovarov.** *On the Rényi entropy of the sum of independent Random Variables.*

We consider the problem of finding the probability distributions of a fixed Rényi entropy for which the Rényi entropy of a sum of independent copies is minimized, a problem motivated by conjectures of Madiman and Wang. Joint work with Galyna Livshyts, Grigoris Paouris, and Peter Pivovarov. (Received July 28, 2018)

1141-46-189 **Ryan Tully-Doyle\*** (rtullydoyle@newhaven.edu), **J. E. Pascoe** and **M. Sargent**.  
*Escaping nontangential approach.*

The classical Julia-Carathéodory theorem describes the differentiability of an analytic function  $f : \mathbb{D} \rightarrow \mathbb{D}$  at a boundary point  $\tau$  in the circle  $\mathbb{T}$  by examining the behavior of a corresponding function  $J_\tau^f$ , the so-called Julia quotient, that measures the growth of  $f$  as  $z \rightarrow \tau$  through a nontangential set. This theorem has been widely generalized in various settings.

In the present work, joint with J. E. Pascoe and M. Sargent, we introduce  $\gamma$ -regularity, a more general notion than linear expansion, that characterizes boundary regularity of a Pick function in terms of integrability of  $\gamma$ . We show how to extend the notion of nontangential approach to relate this regularity to a boundary growth condition on a suitable tangential set at  $\tau$ . We will highlight potential connections with moment theory, as well as the relationship with existing work of Bolotnikov and Kheifets. (Received July 29, 2018)

1141-46-194 **David Herron** and **Abigail Richard\*** (richaab@mail.uc.edu), University of Cincinnati, Mathematics Department, 2600 Clifton Avenue, Cincinnati, OH 45221, and **Marie Snipes**.  
*Convergence Relationships.*

We present relationships between various types of convergence. In particular, we examine several types of convergence of sets including Hausdorff convergence, Kuratowski convergence, etc. In studying these relationships, we also gain a better understanding of the necessary conditions for certain conformal metric distances to pointed Gromov-Hausdorff converge. (Received July 29, 2018)

1141-46-211 **Ray Cheng\*** (rcheng@odu.edu), Department of Mathematics and Statistics, Old Dominion University, 4700 Elkhorn Avenue, Norfolk, VA 23529. *Inner functions and zero sets for  $\ell_A^p$ .*

The space  $\ell_A^p$  is defined to be the class of analytic functions on the open unit disk  $\mathbb{D}$  for which their Taylor coefficients belong to the sequence space  $\ell^p$ . Relatively little is known about these spaces in comparison to, for example, the Hardy spaces  $H^p$ . Some tools from the geometry of Banach spaces, however, make possible some inroads. These tools include a sort of Pythagorean theorem on  $\ell_A^p$ , where orthogonality is in the sense of Birkhoff-James.

A notion of  $p$ -inner function is introduced, for  $1 < p < \infty$ . A complete description of the zero sets of  $\ell_A^p$  is obtained, in terms of an associated sequence of  $p$ -inner functions. This result is used to construct, for instance, a non-Blaschke zero set when  $p > 2$ . Connections are also made to invariant subspaces, multipliers, and canonical factorization. (Received July 30, 2018)

1141-46-264 **Eric A Carlen\***, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019, and **Rupert Frank** and **Elliott H Lieb**. *Inequalities for  $L^p$ -norms that sharpen the triangle inequality and complement Hanner's Inequality.*

In 2006 Carbery raised a question about an improvement on the naïve norm inequality  $\|f + g\|_p^p \leq 2^{p-1}(\|f\|_p^p + \|g\|_p^p)$  for two functions in  $L^p$  of any measure space. When  $f = g$  this is an equality, but when the supports of  $f$  and  $g$  are disjoint the factor  $2^{p-1}$  is not needed. Carbery's question concerns a proposed interpolation between the two situations for  $p > 2$ . The interpolation parameter measuring the overlap is  $\|fg\|_{p/2}$ . We prove an inequality of this type that is stronger than the one Carbery proposed. Moreover, our stronger inequalities are valid for all  $p$ . (Received July 31, 2018)

1141-46-274 **Li Gao**, **Samuel Harris** and **Marius Junge\*** (junge@math.uiuc.edu). *Quantum Teleportation and Super-dense Coding in Operator Algebras.*

We discuss results from our paper Quantum Teleportation and Super-dense Coding in Operator Algebras. (Received July 31, 2018)

1141-46-300 **Mujahid Abbas\*** (abbas.mujahid@gmail.com), Department of Mathematics, Government College University, Lahore, Punjab 54000, Pakistan. *Stability of fixed points in generalized cone metric spaces.*

We define the stability of sequence of fixed points of mappings that satisfying certain generalized contractive conditions in the setup of generalized cone metric space. We also define the new convergence criteria and properties for convergence in generalized cone metric spaces. Our obtained results unify, strengthen and generalize various fixed point results and stability results in the existing literature. (Received August 01, 2018)

## 47 ► Operator theory

1141-47-21 **Matthew Fleeman\*** ([matthew\\_fleeman@baylor.edu](mailto:matthew_fleeman@baylor.edu)), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76798-7328, and **Constanze Liaw** ([liaw@udel.edu](mailto:liaw@udel.edu)). *Hyponormal Toeplitz operators with non-harmonic Symbol acting on the Bergman space.*

The Toeplitz operator acting on the Bergman space  $A^2(\mathbb{D})$ , with symbol  $\varphi$  is given by  $T_\varphi f = P(\varphi f)$ , where  $P$  is the projection from  $L^2(\mathbb{D})$  onto the Bergman space. We present some history on the study of hyponormal Toeplitz operators acting on  $A^2(\mathbb{D})$ , as well as give some results for when  $\varphi$  is a non-harmonic polynomial. (Received June 14, 2018)

1141-47-35 **Pamela Gorkin\*** ([pgorkin@bucknell.edu](mailto:pgorkin@bucknell.edu)), Bucknell University, Department of Mathematics, Olin Science 380, Lewisburg, PA 17837. *Operator theory and applications of interpolation.*

We present results about recent progress in operator theory. Topics discussed include the numerical range and unitary dilations of model space operators, truncated Toeplitz operators and norm estimates, and the relation of these questions to interpolation (Received July 03, 2018)

1141-47-55 **Samuel J Harris\*** ([sj2harri@uwaterloo.ca](mailto:sj2harri@uwaterloo.ca)). *Separating matrix-valued generalizations of the bipartite Tsirelson correlation sets.*

In recent years, there has been much study of various sets of quantum bipartite correlations in a finite-input, finite-output system. These sets are often denoted by  $C_t(m, k)$ , where  $m$  is the number of inputs,  $k$  is the number of outputs and  $t$  represents the model that is being used. Some of the most notable models are the finite-dimensional tensor product model ( $C_q(m, k)$ ) and the tensor product model ( $C_{qs}(m, k)$ ). Recently, J. Stark and A. Codalangelo proved that  $C_q(5, 3) \neq C_{qs}(5, 3)$ , but resolving whether or not  $C_q(m, k) = C_{qs}(m, k)$  for smaller values of  $m, k$  is still open.

In this talk, we consider a matrix-valued generalization of these sets, denoted  $C_t^{(n)}(m, k)$ , where Alice and Bob have access to  $n$  states instead of just 1. We show that, whenever  $m, k \geq 2$  and  $(m, k) \neq (2, 2)$ , there is some  $n \leq 4$  such that  $C_q^{(n)}(m, k) \neq C_{qs}^{(n)}(m, k)$ . (Received July 12, 2018)

1141-47-67 **Michael T. Jury** and **Robert T.W. Martin\***, [rtwmartin@gmail.com](mailto:rtwmartin@gmail.com). *Extreme points of the free Schur class and operators affiliated to the free shift.*

We extend equivalent characterizations of extreme points of the Schur class of contractive analytic functions on the open complex unit disk to the several non-commuting variable Schur class of contractive free holomorphic functions on the multi-variable non-commutative open unit ball,  $\mathbb{B}_N^d := \bigsqcup_{n=1}^{\infty} (\mathbb{C}^{n \times n} \otimes \mathbb{C}^d)_1$ .

As an application, we show that any closed operator,  $T$ , affiliated to the left free shift on the full Fock space over  $\mathbb{C}^d$  acts as left multiplication by a free function,  $T(Z)$ , in the left free Smirnov class of ratios of free functions with outer denominator. Here, the Fock space is identified with the free Hardy space of free holomorphic functions on  $\mathbb{B}_N^d$  with square-summable MacLaurin series coefficients, and the left free shift is the (row) isometry of left multiplication by the independent (non-commuting) variables  $Z_1, \dots, Z_d$ . (Received July 16, 2018)

1141-47-76 **Chang-Pao Chen\*** ([cpchen@math.nthu.edu.tw](mailto:cpchen@math.nthu.edu.tw)), Department of Mathematics, National Tsing Hua University, Hsinchu, 30092, Taiwan, and **Dah-Chin Luo Luo**. *Hölder-type estimates of the modular operator norm of an integral operator over spherical cones.*

In this paper, we introduce a new type of estimates to evaluate the modular operator norm of an integral operator over Lebesgue measurable subsets of  $\mathbb{R}^n$ . We get this estimate by means of the Hölder inequality. Our results not only extend the corresponding ones in Maz'ja's book, also provide better results than the Muckenhoupt-type estimate shown in Linear Multilinear Algebra. **62**(5), 683–713 (2014) for some cases. (Received July 17, 2018)

1141-47-90 **Raphael Clouatre\*** ([raphael.clouatre@umanitoba.ca](mailto:raphael.clouatre@umanitoba.ca)) and **Edward Timko** ([edward.timko@umanitoba.ca](mailto:edward.timko@umanitoba.ca)). *Towards a classification theory for constrained row contractions.*

Allowing for operator-valued analytic functions, it is known that certain abstract row contractions can be classified up to unitary equivalence using concrete multiplication operators. If we are willing to settle for a less faithful description, then we may hope to construct simpler, scalar-valued functional models. For single contractions satisfying an analytic constraint, this vision has been fulfilled and has culminated in a powerful classification theorem.



The aim of this talk is to explore potential extensions of this theorem to the multivariate world. On one hand, we exhibit some higher-dimensional difficulties and identify crucial univariate facts that simply fail to hold in several variables. On the other hand, we explain how interpolating sequences can be exploited to obtain a satisfactory classification for commuting row contractions satisfying appropriate analytic constraints. (Received July 18, 2018)

1141-47-128 **Benjamin Peter Russo\*** ([russobp@farmingdale.edu](mailto:russobp@farmingdale.edu)) and **Arthur J Parzygnat** ([arthur.parzygnat@uconn.edu](mailto:arthur.parzygnat@uconn.edu)). *C\*-algebras and the Category of Stochastic maps.*

Stochastic maps are a generalization of functions in that they assign to each point in the domain a probability measure on the codomain. In this talk we will discuss the category of stochastic maps. In particular, we will explore some generalizations of probabilistic concepts resulting from the existence of a contravariant functor from the category of stochastic maps into the category of  $C^*$ -algebras. This is joint work with Arthur Parzygnat. (Received July 24, 2018)

1141-47-163 **Mark E. Mancuso\***, [mark.mancuso@wustl.edu](mailto:mark.mancuso@wustl.edu). *Noncommutative function theory on operator domains.*

We discuss noncommutative (nc) functions on domains of bounded operators on an infinite dimensional Hilbert space. The main results are inverse and implicit function theorems for nc functions defined on norm-connected domains. The hypotheses of these theorems involve conditions on the derivative of the function, and we will show how to relax these conditions in the case where the function is suitably continuous in the strong operator topology. (Received July 27, 2018)

1141-47-209 **Torrey M. Gallagher\***, [tmg012@bucknell.edu](mailto:tmg012@bucknell.edu). *An overview of mean nonexpansive mappings.*

The notion of a mean nonexpansive mapping was introduced by Goebel and Japon Pineda in 2007 as a generalization to the usual notion of a nonexpansive mapping. Since then, there have been many results pertaining both to the general behavior of mean nonexpansive mappings as well as their fixed point properties. We will present a survey of these results, including basic fixed point theorems, a classification of mean isometries, a demiclosedness principle, and a generalization to the definition with open questions highlighted throughout. (Received July 30, 2018)

1141-47-212 **Joshua D. Jackson\*** ([jdj65@drexel.edu](mailto:jdj65@drexel.edu)). *A Determinantal Representation for Bivariate Polynomials whose Bezoutians admit a Canonical Factorization.*

For every bivariate polynomial  $p(z_1, z_2)$  of bidegree  $(n_1, n_2)$ , with  $p(0, 0) = 1$ , which has no zeros on the bitorus  $\mathbb{T}^2$ , if the bezoutian  $Q(p, \bar{p})$  admits a canonical factorization then we can construct a determinantal representation of the form

$$p(z_1, z_2) = \det(I - KZ),$$

where  $Z$  is an  $(n_1 + n_2) \times (n_1 + n_2)$  diagonal matrix with coordinate variables  $z_1, z_2$  on the diagonal, and  $K$  is a  $J$ -contraction, where  $J = I_{\alpha_+} \oplus I_{\alpha_-}$  and  $\alpha_+$  ( $\alpha_-$ , respectively) is the constant number of eigenvalues of  $Q$  inside (outside) the unit bidisk. (Received July 30, 2018)

1141-47-245 **Greg Knese\*** ([geknese@wustl.edu](mailto:geknese@wustl.edu)), Washington University in St. Louis, One Brookings Drive, Dept. of Mathematics & Statistics, CB1146, Saint Louis, MO 63130. *Global bounds on stable polynomials and the Pólya class.*

A classical inequality of Szász bounds polynomials with no zeros in the upper half plane entirely in terms of their first few coefficients. Borcea-Brändén generalized this result to several variables as a piece of their characterization of linear maps on polynomials preserving stability. In this talk, we use determinantal representations to prove Szász type inequalities in two variables, and then prove that one can use the two variable inequality to prove an inequality for several variables. We shall also discuss applications of these inequalities to the study of an important class of entire functions, the Pólya class. (Received July 30, 2018)

1141-47-254 **Ali S. Kavruk\*** ([askavruk@vcu.edu](mailto:askavruk@vcu.edu)). *Joint Quantum Probability Distributions via Operator Systems.*

Following the rapid development of tensor and nuclearity theory of operator systems, multiple research projects execute the connection of the topics in quantum information theory in this setting. In this talk we first introduce the non-commutative cubes/polygons and then, by engaging different types of tensor products, we obtain joint quantum probability distributions in relativistic, non-relativistic and contextual setting. Time permitting we introduce different types of quantum violations and exhibit some known approximations. In this introductory

presentation, which is accessible to grad students, we also have the opportunity talk about many open questions. (Received July 31, 2018)

1141-47-266 **Stefan Richter\*** (srichter@tennessee.edu), University of Tennessee, Knoxville, TN -37996, and **Faruk Yilmaz** (yilmaz@ahievran.edu.tr), Kirşehir Ahi Evran University, Kirşehir, Turkey. *Regularity for generators of invariant subspaces of the Dirichlet shift.*

Let  $D$  denote the classical Dirichlet space of analytic functions on the open unit disc whose derivative is square area integrable. For a set  $E \subseteq \partial\mathbb{D}$  we write  $D_E = \{f \in D : \lim_{r \rightarrow 1^-} f(re^{it}) = 0 \text{ } q.e.\}$ , where  $q.e.$  stands for "except possibly for  $e^{it}$  in a set of logarithmic capacity 0". We show that if  $E$  is a Carleson set, then there is a function  $f \in D_E$  that is also in the disc algebra and that generates  $D_E$  in the sense that  $D_E = \text{closure}\{pf : p \text{ is a polynomial}\}$ .

We also show that if  $\varphi \in D$  is an extremal function, then the limits of  $|\varphi(z)|$  exist for every  $e^{it} \in \partial\mathbb{D}$  as  $z$  approaches  $e^{it}$  from within any polynomially tangential approach region. (Received July 31, 2018)

1141-47-273 **Michael T. Jury\*** (mjjury@uf1.edu) and **Robert T.W. Martin** (rtwmartin@gmail.com). *Factoring functions in complete Pick spaces via free lifts.*

Any function  $f$  in the Drury-Arveson space  $H_d^2$  over the unit ball in  $\mathbb{C}^d$  can be lifted to a "noncommutative" function  $F$  in the full Fock space  $F_d^2$  (with the same Hilbert space norm). We apply the Arias-Popescu noncommutative inner-outer factorization to the lifted function  $F$  and push this factorization back down to  $H_d^2$ . We discuss several applications of this factorization, and prove the following: 1) Every function  $h$  in a complete Pick space  $\mathcal{H}$  can be expressed as  $h = b/a$  where  $b, a$  are multipliers of  $\mathcal{H}$ ,  $a$  is a cyclic vector, and  $1/a \in \mathcal{H}$ . 2) For many complete Pick spaces  $\mathcal{H}$  (including the Drury-Arveson spaces, and the Dirichlet space  $\mathcal{D}$  in the unit disk), every function  $h$  in the weak product  $\mathcal{H} \odot \mathcal{H}$  can be factored as  $h = fg$  with  $f, g \in \mathcal{H}$ . (Received July 31, 2018)

1141-47-305 **Philip M Gipson\*** (philip.gipson@cortland.edu), NY. *Graphs and Endomorphisms of von Neumann Algebras.*

It is a well-known fact that endomorphisms of  $B(H)$  are intimately connected with families of mutually orthogonal isometries, i.e. with representations of the so-called Toeplitz  $C^*$ -algebras. In this paper we consider a natural generalization of this connection between the representation theory of certain  $C^*$ -algebras associated to graphs and endomorphisms of certain von Neumann subalgebras of  $B(H)$ . Our primary results give criteria by which we may determine if two representations give rise to equal or conjugate endomorphisms. (Received August 01, 2018)

## 49 ► *Calculus of variations and optimal control; optimization*

1141-49-66 **Joe Neeman\*** (joeneeman@gmail.com) and **Emanuel Milman**. *The Gaussian double-bubble.*

The Gaussian isoperimetric inequality states that if we want to partition  $\mathbb{R}^n$  into two sets with prescribed Gaussian measure while minimizing the Gaussian surface area of the interface between the sets, then the optimal partition is obtained by cutting  $\mathbb{R}^n$  with a hyperplane. We prove an extension to more than two parts. For example, the optimal way to partition  $\mathbb{R}^3$  into three parts involves cutting along three rays that meet at  $120^\circ$  angles at a common point. (Received July 16, 2018)

1141-49-286 **David G Costa** and **Daniel Anthony Corral\*** (dacorral@unlv.nevada.edu). *Remarks on Lagrange Multiplier Approach to Kazdan-Warner Equations on a Finite Graph.*

In 2016 Grigoryan, Lin, and Yang studied the finite graph analogue of the Kazdan-Warner equation. Given a finite graph  $V$  and  $c$  a constant, they studied the equations:

$$\Delta_\mu u = c - k(t)e^u \text{ in } V,$$

and

$$\Delta_\mu u = -k(t)e^u \text{ in } V.$$

They determined if and only if conditions for these equations to have a solution in several cases. They followed closely the technique used by J.Kazdan and F. Warner in their 1974 paper "Curvature functions for compact 2-manifolds" for proving existence of solution in the continuous analogue of this equation. Dr. David Costa and I have generalized some of these cases. (Received July 31, 2018)

1141-49-292 **Ugur G Abdulla** (abdulla@fit.edu) and **Evan Cosgrove\*** (ecosgrove2011@my.fit.edu). *On the Optimal Control of the Multiphase Free Boundary Problems for the Nonlinear Parabolic Equations.*

We consider the inverse Stefan type multiphase free boundary problem for the nonlinear parabolic PDE in which information on the boundary heat flux is missing, and must be found along with the temperature. We generalize the method developed in *Abdulla and Poggi, Applied Mathematics and Optimization, 2018* to pursue optimal control framework where control vector consists of the heat flux, and the cost functional is the  $L_2$ -norm declination of the trace of temperature at the final moment of time to the measured data. We pursue discretization of the problem, and prove convergence of the discrete problem to the original problem with respect to both control and functional. (Received July 31, 2018)

1141-49-303 **Steven M Heilman\*** (sheilman@usc.edu). *Symmetric Convex Sets of Minimal Gaussian Surface Area.*

Let  $\Omega \subset \mathbb{R}^{n+1}$  have minimal Gaussian surface area among all sets satisfying  $\Omega = -\Omega$  with fixed Gaussian volume. It is shown that if  $\Omega$  or  $\Omega^c$  is convex, and if a certain integral of the curvature of  $\partial\Omega$  is not close to 1, then  $\partial\Omega$  must be a round cylinder. That is, with a small range of exceptions, we resolve the convex case of a question of Barthe from 2001.

The main tool is the Colding-Minicozzi theory for Gaussian minimal surfaces, which studies eigenfunctions of an Ornstein-Uhlenbeck type operator associated to the surface  $\partial\Omega$ . Some of our results hold without the assumption of convexity.

Time permitting, we will discuss applications of this method to Euclidean partitions of fixed Gaussian volume and minimal Gaussian surface area. (Received August 01, 2018)

## 51 ► Geometry

1141-51-36 **Artem Pulemotov\*** (a.pulemotov@uq.edu.au), School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia. *The prescribed Ricci curvature problem on homogeneous spaces.*

Consider a compact Lie group  $G$  and a closed Lie subgroup  $H < G$ . We will discuss the problem of finding  $G$ -invariant Riemannian metrics with prescribed Ricci curvature on the homogeneous space  $G/H$ . In order to determine when such metrics exist, we will explore the properties of the graph that describes the Lie subgroups of  $G$  and their inclusions. (Received July 04, 2018)

1141-51-197 **Simon Segert\***, simonsegert@gmail.com. *Path Integrals, Laplacians, and geometry.* Preliminary report.

We present a path integral-based formalism for empirical Laplacian estimation. It is shown how a weighted graph gives rise to a measure on an infinite-dimensional path space, and how the entries of the matrix exponential  $e^{-tL}$  may be recovered by appropriate integrations against this measure. The implications of this representation for data analysis are analyzed: we interpret the standard Gaussian kernel weights  $e^{-|x_i - x_j|/\epsilon}$  by comparison with path integral formulas in the smooth case, and give applications to shortest-path computations. (Received July 29, 2018)

## 52 ► Convex and discrete geometry

1141-52-73 **Xin Yang Lu\***, 955 Oliver Road, Thunder Bay, Ontario P7B5E1, Canada. *Geometric complexity of optimal Voronoi tessellations in 3D.*

Centroidal Voronoi Tessellations (CVT) are tessellations using Voronoi regions of their centroids. Gershó's conjecture, first proposed in 1979, states that there exists an asymptotically optimal CVT whose Voronoi regions are all rescaled copies of the same polytope. Straightforward in 1D, and proven in 2D, Gershó's conjecture is still open for higher dimensions. One of the main difficulties is that Gershó's conjecture is a strongly nonlocal, infinite dimensional minimization problem (even in 3D). In this talk we will present some recent results which reduce Gershó's conjecture to a local, finite dimensional problem in 3D. Joint work with Rustum Choksi. (Received July 16, 2018)

1141-52-93 **Hermann Koenig** and **Alexander Koldobsky\*** (koldobskiya@missouri.edu). *On the maximal perimeter of sections of the cube.*

We prove that the  $(n-2)$ -dimensional surface area (perimeter) of central hyperplane sections of the  $n$ -dimensional unit cube is maximal for the hyperplane perpendicular to the vector  $(1, 1, 0, \dots, 0)$ . This gives a positive answer to a question of Pełczyński who solved the three dimensional case. We study both the real and the complex versions of this problem. We also use our result to show that the answer to an analogue of the Busemann-Petty problem for the surface area is negative in dimensions 14 and higher. (Received July 19, 2018)

1141-52-140 **Arnaud Marsiglietti\*** (a.marsiglietti@uf1.edu). *Entropy power inequality and central limit theorem for Rényi entropy.*

The parallels between convex geometry and information theory have been observed in the late eighties with a unified proof of the Brunn-Minkowski inequality and the entropy power inequality, via the sharp Young inequality. These connections have seen an increasing interest over the past few years.

In this talk, we present further connections, and develop entropy power type inequalities for the Rényi entropy, based on the sharp Young inequality and tools from convex geometry. We also study the behavior of Rényi entropy in the central limit theorem. (Received July 25, 2018)

1141-52-145 **Maria Alfonseca-Cubero** (maria.alfonseca@ndsu.edu), **Fedor Nazarov** (nazarov@math.kent.edu), **Dmitry Ryabogin\*** (ryabogin@math.kent.edu) and **Vlad Yaskin** (yaskinv@gmail.com). *On a local version of the fifth Busemann-Petty problem.*

Let  $K$  be an origin-symmetric convex body in  $\mathbb{R}^n$ ,  $n \geq 3$ , satisfying the following condition: there exists a constant  $c$  such that for all directions  $\xi$  in  $\mathbb{R}^n$ ,

$$h_K(\xi) \text{vol}_{n-1}(K \cap \xi^\perp) = c.$$

(here  $\xi^\perp$  stands for a subspace of  $\mathbb{R}^n$  of co-dimension 1 orthogonal to a given direction  $\xi$ , and  $h_K(\xi)$  is the support function of  $K$  in this direction). The fifth Busemann-Petty problem asks if  $K$  must be an ellipsoid. We give an affirmative answer to this question for origin-symmetric convex bodies that are sufficiently close to an Euclidean ball in the Banach-Mazur distance. This is a joint work with Maria Angeles Alfonseca, Fedor Nazarov and Vlad Yaskin. (Received July 26, 2018)

1141-52-147 **Emanuel Milman**, **Vitali Milman** and **Liran Rotem\*** (lrotem@umn.edu). *Reciprocity and indicatrices in convexity.*

Given a convex body  $K \subseteq \mathbb{R}^n$ , we construct the reciprocal body " $K^{-1}$ ", which we denote by  $K'$ . The map  $K \mapsto K'$  is different from the well-known polarity map, but is still a duality (on its image).

We will answer two main questions regarding the new construction:

- Which convex bodies can be obtained as a reciprocal body?
- Given a reciprocal body  $K$ , what can be set about the class  $\{A : A' = K\}$  ?

(Received July 26, 2018)

1141-52-177 **David Alonso-Gutierrez**, **Maria Hernandez-Cifre**, **Michael Roysdon\*** (mroysdon@kent.edu), **Jesus Yepes-Nicolas** and **Artem Zvavitch**. *On New Inequalities Related to the Rogers-Shephard Inequality.* Preliminary report.

A central inequality to the theory of convex bodies is the Brunn-Minkowski inequality which states that, for any convex bodies  $A, B \subset \mathbb{R}^n$ , one has  $|A+B|^{1/n} \geq |A|^{1/n} + |B|^{1/n}$ , where  $|\cdot|$  denotes the  $n$ -dimensional Lebesgue measure. In the 1950's Rogers and Shephard proved a sort of converse to this inequality which states that, for any convex body  $K$ , one has  $|K+(-K)| \leq \binom{2n}{n}|K|$  with equality if, and only if,  $K$  is a simplex. In a joint work with David Alonso-Gutierrez, Maria Hernandez Cifre, Jesus Yepes Nicolas, and Artem Zvavitch, we present an analogue of this inequality in the setting of general measures with certain properties. Another inequality of Rogers and Shephard is that which gives a lower bound of the volume of a  $K$  in terms of its maximal section and projection onto a linear subspace. A functional analogue of this will be presented in the case of finite Borel measure with quasi-concave densities and when  $K$  is selected to satisfy certain conditions. (Received July 28, 2018)

1141-52-184 **Mark Mixer\***, Wentworth Institute of Technology, and **Kostiantyn Drach**, **Yurii Haidamaka** and **Maksym Skoryk**. *Covers of vertex transitive maps on the torus with few flag orbits.* Preliminary report.

A surjective function from a map  $\mathcal{N}$  to a map  $\mathcal{M}$  that preserves adjacency and sends vertices to vertices, edges to edges, and faces to faces is called a covering of the map  $\mathcal{M}$  by the map  $\mathcal{N}$ .

Given a map  $\tau$  on the torus, we will describe other maps  $\phi$  on the torus so that  $\phi$  covers  $\tau$  and  $\phi$  is as symmetric as possible.

In this talk, we will consider the most symmetric toroidal covers of all the vertex transitive (Archimedean) maps on the torus. (Received July 28, 2018)

1141-52-207 **Han Huang\*** (sthhan@umich.edu), **Boaz A. Slomka** (bslomka@umich.edu) and **Elisabeth M. Werner** (elisabeth.werner@case.edu). *Ulam Floating Body.*

We study a new construction of bodies from a given convex body in  $\mathbb{R}^n$  which are isomorphic to (weighted) floating bodies. We establish several properties of this new construction, including its relation to  $p$ -affine surface areas. We show that these bodies are related to Ulam's long-standing floating body problem which asks whether Euclidean balls are the only bodies that can float, without turning, in any orientation. (Received July 30, 2018)

1141-52-247 **Galyna V Livshyts\***, 686 Cherry st NW, Atlanta, GA 30318. *On random rounding.* Preliminary report.

I will discuss an efficient way to discretize the unit sphere, with some applications. This is joint work with Bo'az Klartag. (Received July 30, 2018)

1141-52-249 **Jae Oh Woo\*** (jaeoh.woo@aya.yale.edu), **Mokshay Madiman** and **Liyao Wang**. *Discrete Rényi Entropy Power Inequalities via Sperner Theory.*

The link between the notions of majorization and strongly Sperner posets is used to obtain a variety of consequences, including classical inequalities of Hardy-Littlewood-Polya, as well as new inequalities for the Rényi entropies of sums of independent, integer-valued random variables. (Received July 30, 2018)

1141-52-260 **Piotr Amit Nayar\*** (nayar@mimuw.edu.pl), Banacha 2, 02-097 Warsaw, Poland. *Gaussian mixtures and geometric problems.*

We discuss several applications of the Gaussian mixture method. This technique allows us, in certain situations, to pass from inequalities valid for Gaussian measures to more general situations. One example is the celebrated Gaussian correlation inequality due to Thomas Royen. It turns out that the validity of this inequality implies similar inequalities for other measures, including uniform measure on the Euclidean hemisphere. Another example is the so-called strong B-inequality, valid for the standard Gaussian measure due to the result of Cordero-Erausquin, Fradelizi and Maurey. We shall explain how this fact implies similar statements for exponential measure and certain measures with rotation invariant densities. Finally, Gaussian mixtures allow us to derive relatively simple analytic formulas for the volumes of the sections of cross-polytope of arbitrary codimension. Relations to the so-called B-conjecture will be mentioned.

Based on a joint work with A. Eskenazis and T. Tkocz and on a joint work with T. Tkocz. (Received July 31, 2018)

1141-52-285 **Beatrice-Helen Vritsiou\*** (vritsiou@ualberta.ca). *Regular covering for not necessarily symmetric convex bodies.*

Given a pair of convex bodies  $K, L$ , we denote by  $N(K,L)$  the covering number of  $K$  by  $L$ , namely the least number of copies of  $L$  needed for their union to contain  $K$ . A result by V. Milman, with far-reaching connections in high-dimensional geometry, states that, if one of the two bodies is a Euclidean ball, then we can linearly transform the other one so that it has the same volume and so that both covering numbers  $N(K,L)$  and  $N(L,K)$  are at most exponential in the dimension (this is optimal as can be seen by specific examples).

An important extension by Pisier, proven only for symmetric convex bodies though, shows that, by taking larger and larger dilates of the covering body in the pair, we will have the covering numbers decrease in a regular way.

We will briefly explain how this result can be extended to the setting of not necessarily symmetric convex bodies (although with quantitatively worse bounds). (Received July 31, 2018)

1141-52-289 **Dan I Florentin\*** (danflorentin@gmail.com) and **Alexander Segal**. *A Santaló-type Inequality for the  $\mathcal{J}$  Transform.*

In recent years, it was proven that there exist precisely four order isomorphisms acting in the class of geometric convex functions. These are the Legendre transform  $\mathcal{L}$ , the geometric duality transform  $\mathcal{A}$ , their composition  $\mathcal{J}$ , and the identity. It is known that  $\mathcal{L}$  and  $\mathcal{A}$  satisfy Santaló-type inequalities, e.g. the quantity  $M(f) = \text{Vol}(f) * \text{Vol}(\mathcal{L}f)$  is bounded from above and below (here  $\text{Vol}(f)$  stands for the integral over  $\mathbb{R}^n$  of  $e^{-f}$ ). We prove

similar (asymptotically sharp) bounds for the quantity  $M^{\mathcal{J}}(f) = \text{Vol}(\mathcal{J}f)/\text{Vol}(f)$ , and describe the extremal functions. (Received July 31, 2018)

## 53 ► Differential geometry

1141-53-187 **Hemangi M. Shah\***, Chhatnag Road, Jhunsi, Allahabad, UP 211019, India. *Geometry of asymptotically harmonic manifolds with minimal horospheres.*

$(M^n, g)$  be a complete Riemannian manifold without conjugate points. We show that, if  $M$  is also simply connected, then  $M$  is flat, provided that  $M$  is also asymptotically harmonic manifold with minimal horospheres (AHM). The (first order) flatness of  $M$  is shown by using the strongest criterion:  $\{e_i\}$  be an orthonormal basis of  $T_pM$  and  $\{b_{e_i}\}$  be the corresponding Busemann functions on  $M$ . Then, (1) The vector space  $V = \text{span}\{b_v | v \in T_pM\}$  is finite dimensional and  $\dim V = \dim M = n$ . (2)  $\{\nabla b_{e_i}(p)\}$  is a global parallel orthonormal basis of  $T_pM$  for any  $p \in M$ . Thus,  $M$  is a parallizable manifold And (3)  $F : M \rightarrow R^n$  defined by  $F(x) = (b_{e_1}(x), b_{e_2}(x), \dots, b_{e_n}(x))$ , is an isometry and therefore,  $M$  is flat. Consequently, AH manifolds can have either polynomial or exponential volume growth, generalizing the corresponding result for harmonic manifolds. In case of harmonic manifold with minimal horospheres (HM), the (second order) flatness was proved by Ranjan and Shah by showing that  $\text{span}\{b_v^2 | v \in T_pM\}$  is finite dimensional. (Received July 29, 2018)

## 54 ► General topology

1141-54-13 **Clement B Ampadu\*** (drampadu@hotmail.com), Boston, MA 02132. *A Characterization of the Hardy and Rogers Mapping Theorem in the sense of Berinde.* Preliminary report.

Berinde in 2004 gave a weak form of the "Banach Inequality" and showed any map satisfying such an inequality has a fixed point under certain conditions. Since then many authors have generalized this weak form of the "Banach Inequality" and obtained some existence and/or uniqueness criteria for the fixed point of maps satisfying such inequality.

In the AMS 2018 Sectional Meeting at Northeastern University, Boston, MA, I raised the possibility of obtaining a weak form of the "Hardy and Rogers Inequality" and finding conditions under which the fixed point of a map satisfying such inequality exists, and if the fixed point exists can we guarantee uniqueness?

In this talk we will give answers to the above questions (Received May 05, 2018)

## 55 ► Algebraic topology

1141-55-7 **Chao Chen\*** (chao.chen.cchen@gmail.com). *Optimal Cycles in Persistent Homology and Their Applications in Imaging Data Analysis.*

We propose an efficient algorithm to compute the optimal cycle of homology classes. We show how it will be used in the context of persistent homology and be applied to the analysis of different biomedical imaging data. (Received March 26, 2018)

1141-55-16 **Robert Short\*** (rshort12@jcu.edu). *The Relative Topological Complexity of a Pair.*

Topological complexity is a homotopy invariant introduced by Michael Farber in the early 2000s. Denoted  $\text{TC}(X)$ , it counts the smallest size of a continuous motion planning algorithm on  $X$ . In this sense, it solves optimally the problem of continuous motion planning in a given topological space. In topological robotics, a part of applied algebraic topology, several variants of TC are studied. In a recent paper, I introduced the relative topological complexity of a pair of spaces  $(X, Y)$  where  $Y \subset X$ . Denoted  $\text{TC}(X, Y)$ , this counts the smallest size of motion planning algorithms that plan from  $X$  to  $Y$ .

In this talk, we will discuss some of the spaces where  $\text{TC}(X, Y)$  has been computed, and indicate some of the differences between  $\text{TC}(X)$  and  $\text{TC}(X, Y)$ . (Received May 18, 2018)

1141-55-27 **Michael Robinson\***, 220 Don Myers Building, American University, 4400 Massachusetts Ave NW, Washington, DC 20016. *Consistency filtrations of assignments to sheaves.*

Because sheaves model consistency relationships between local data, they are easily assembled from detailed models of systems. Being topological in nature, sheaves mediate local-to-global inference. By incorporating local geometry from the start, the global "fit" between local data and models can be quantified, which supports robust inferences about missing or inaccurate data. But since sheaf theory encourages abstraction, sheaves may

provide topological invariants that govern model selection, the bias-variance tradeoff, and ultimately the problem of overfitting. This talk will formalize and unify these ideas using the consistency filtration associated to a sheaf of pseudometric spaces and an assignment of data. As a filtration, it has persistence properties – both functorial and geometric – since persistence is one source of strong properties for arbitrarily thresholded data. (Received June 28, 2018)

1141-55-33 **Kelly Spendlove\***, [kelly.spendlove@rutgers.edu](mailto:kelly.spendlove@rutgers.edu), and **Shaun Harker** and **Konstantin Mischaikow**. *A Computational Framework for Connection Matrices*.

Algebraic topology and dynamical systems are intimately related: the algebra may constrain or force the existence of certain dynamics. Morse homology is the prototypical theory grounded in this observation. Conley theory is a far-reaching topological generalization of Morse theory and the last few decades have seen the establishment of a computational version of the Conley theory. The computational Conley theory is a blend of combinatorics, order theory and algebraic topology and has proven effective in tackling problems within dynamical systems.

Within the Conley theory the connection matrix is the mathematical object which transforms the approach into a truly homological theory; it is the Conley-theoretic generalization of the Morse boundary operator. We'll discuss how the connection matrix can be computed efficiently with techniques from effective homology and discrete Morse theory. We will also discuss a software package for such computations. We'll demonstrate the software by making an application to the setting of a Morse theory on spaces of braid diagrams, giving proofs of the existence of stationary and periodic solutions and connecting orbits in a class of parabolic PDEs. Time permitting, we'll discuss intersections with the field of persistent homology. (Received July 01, 2018)

1141-55-41 **Michael Erdmann\***, 5000 Forbes Ave, 6105 Gates Computer Center, Pittsburgh, PA 15213. *Topology of Privacy : Information Lattices and Bubbles for Inference and Obfuscation*.

Dowker's Theorem establishes a homotopy equivalence between two simplicial complexes derived from a relation. From a privacy perspective, one complex describes individuals with common attributes, the other describes attributes shared by individuals. The homotopy equivalence produces a lattice. An element in the lattice consists of two components, one being a set of individuals, the other being a set of attributes. The lattice operations join and meet each amount to set intersection in one component and set union followed by a potentially privacy-puncturing inference in the other component.

Privacy loss appears as simplicial collapse of free faces. Such collapse is local, but the property of fully preserving both attribute and association privacy requires a global condition: a particular kind of spherical hole. By looking at the link of an identifiable individual in its encompassing Dowker complex, one can characterize that individual's attribute privacy via another sphere condition. Even when long-term attribute privacy is impossible, homology provides lower bounds on how an individual may defer identification, when that individual has control over how to reveal attributes. (Received July 06, 2018)

1141-55-56 **Donald R Sheehy\*** ([don.r.sheehy@gmail.com](mailto:don.r.sheehy@gmail.com)). *Fréchet-Stable Signatures Using Persistent Homology*.

For a metric space  $Y$ , the Fréchet distance is a metric on trajectories  $f, g : [0, 1] \rightarrow Y$  that minimizes  $\max_{t \in [0, 1]} d_Y(f(t), g(h(t)))$  over continuous reparameterizations  $h$  of time. One can define the generalized Fréchet distance between more complex objects, functions  $f : X \rightarrow Y$  where  $X$  is some topological space, that minimizes over homeomorphisms from  $X \rightarrow X$ . This more general definition has been studied for surfaces and often leads to computationally hard problems. We show how to compute in polynomial-time signatures for these functions for which the resulting metric on the signatures can also be computed in polynomial-time and provides a meaningful lower bound on the generalized Fréchet distance. Our approach uses persistent homology and exploits the natural invariance of persistence diagrams of functions to homeomorphisms of the domain. Our algorithm for computing the signatures in Euclidean spaces uses a new method for computing persistent homology of convex functions on simplicial complexes which may be of independent interest. (Received July 12, 2018)

1141-55-68 **Nicholas Scoville\*** ([nscoville@ursinus.edu](mailto:nscoville@ursinus.edu)), 601 E Main Street, Math and CS, Collegeville, PA 19426, and **Anastasio Stefanou**. *Strong homotopy interleaving distance*. Preliminary report.

An interleaving distance is an important concept which allows one to measure how far apart two persistence modules are. Recently, de Silva, Munch, and Stefanou extended this idea to include any category equipped with a certain structure called a flow so that any category with a flow induces a pseudo-metric on the objects. This pseudo-metric is a natural extension of the standard interleaving distance. In this talk, we will apply the theory of flows on a category called the category of strong Morse  $\mathbb{R}$ -complexes in order to obtain a pseudo-metric on

this category. For objects, this category has simplicial complexes equipped with a certain kind of discrete Morse function which respects strong homotopy type. We will introduce strong homotopy theory, strong discrete Morse theory, and see how to define a flow on our category. We end with an example and future directions. This is joint work with Anastasio Stefanou. (Received July 16, 2018)

1141-55-79 **Justin Michael Curry\*** (jmc Curry@albany.edu). *Augmentations of Merge Trees.*

Merge trees are a useful tool for summarizing connectivity properties of sub-level sets of a filter function. However, when the sub-level sets themselves have extra structure, augmentations of merge trees are required. In this talk I'll discuss two such augmentations: the chiral merge tree (CMT) and the decorated merge tree (DMT). CMTs are most useful in studying time-series because there is a left-right ordering of sublevel sets of any time series. DMTs exploit the fact that persistent homology naturally factors through connected components and are of use when studying spiral defect chaos and other image data. This talk includes joint work with Rachel Levanger. (Received July 17, 2018)

1141-55-81 **Min-Chun Wu\*** (wmg9721204@gmail.com). *A Topological Approach to Inferring a Tight Bound for the Minimal Embedding Dimension of Data Sensed by Quasi-Convex Functions.*

It is often observed that activity in neural systems, such as hippocampus, can be modeled by quasi-convex functions. Here the domain of these functions is the space of stimuli; in the case of the hippocampal cells, it is the 2 dimensional physical environment for the rat. One can thus ask a natural question: given the data with the activity of a collection of neurons and assuming that the data are from some unknown quasi-convex functions modelling the neurons, what is the minimal embedding dimension of the data?

We establish a statistical model using quasi-convex functions and develop a topological approach to inferring a tight lower bound for the minimal embedding dimension. Specifically, we define a random multi-filtered Dowker complex from the data, and prove its convergence in probability to the multi-filtered Dowker complex induced from the functions in the limit of large data. Then we define estimators based on the random multi-filtered complex and prove their convergence in probability. A lower bound can be statistically derived from the estimators according to whether they converge to 0 or not. Finally, we construct a class of examples in which the equality of the lower bound and the minimal embedding dimension holds, explaining the tightness of the bound. (Received July 17, 2018)

1141-55-92 **Subhrajit Bhattacharya\*** (sub216@lehhigh.edu). *Recent Advances in Topological Path Planning.*

Topological path planning is an emerging area in optimal robot motion planning. In this talk I will introduce and motivate techniques for topological reasoning in various optimal path planning problems, along with their applications to real-world robotics problems. I will describe techniques for computing homology/homotopy invariants that can be used, in conjunction with graph search algorithms, to find optimal paths in different topological classes, with applications to multi-robot systems and tethered robots. Following that I will present some recent results in topological path panning in coordination space of multi-robot systems, and will discuss their applications. (Received July 19, 2018)

1141-55-103 **Hee Rhang Yoon\*** (irishryoon@gmail.com) and **Robert Ghrist.** *Cellular cosheaves for distributed computation of persistent homology.*

We present a distributed computation mechanism of persistent homology using cellular cosheaves. Our construction is an extension of the generalized Mayer-Vietoris principle to filtered spaces obtained via a sequence of spectral sequences. We discuss a general framework in which the distribution scheme can be adapted according to a user-specific property of interest. The resulting persistent homology reflects properties of the topological features, allowing the user to perform refined data analysis. Finally, we apply our construction to perform a multi-scale analysis to detect features of varying sizes that are overlooked by standard persistent homology. This is joint work with Robert Ghrist. (Received July 20, 2018)

1141-55-115 **Pablo Camara\***, Department of Genetics, University of Pennsylvania, 415 Curie Blvd., Philadelphia, PA 19104. *Geometric Feature Selection.* Preliminary report.

We present a general framework for unsupervised feature selection based on the combinatorial Laplacian on simplicial complexes. Our framework generalizes ideas found in spectral network analysis to simplicial complex representations of the data. We illustrate the utility of this framework with some applications in genomics, including the identification of elusive cancer genes. (Received July 23, 2018)



1141-55-130 **Jakob Hansen\*** ([jhansen@math.upenn.edu](mailto:jhansen@math.upenn.edu)), David Rittenhouse Laboratory, Mathematics Department, 209 S. 33rd St., Philadelphia, PA 19130, and **Robert Ghrist** ([ghrist@math.upenn.edu](mailto:ghrist@math.upenn.edu)), David Rittenhouse Laboratory, Mathematics Department, 209 S. 33rd St., Philadelphia, PA 19130. *Toward a spectral theory of cellular sheaves.*

This talk sketches a program extending concepts from spectral graph theory to cellular sheaves. Using the techniques of combinatorial Hodge theory, we produce a series of Laplacians associated to a cellular sheaf of inner product spaces whose kernels compute the cohomology of the sheaf. The spectra of these Laplacians interact in interesting ways with the sheaf structure and sheaf operations, and can represent notions of interest in applications. We will explore initial theoretical results and discuss potential applications for the theory. (Received July 24, 2018)

1141-55-167 **Mark C Agrios\*** ([magrioso@email.wm.edu](mailto:magrioso@email.wm.edu)), 3726 Prince William Drive, Fairfax, VA 22031. *Topological structure in the study of ablation and desynchronization in neural networks.*

Here we explore burst-synchronizing neural networks using tools from simplicial homology. We show that the algebraic and topological features of a biological neural network can provide helpful information about which synapses might be important in maintaining the synchrony of the network and which can be removed with negligible impact on the behavior of the system. We do this by constructing a simplicial complex from the connection matrix of a given neural network and by looking at the structure of this network in this new perspective we can see which synapses can be ablated but still preserve the homology of the simplicial complex and explore how synapses essential to the homology of the complex may also play an important role in maintaining the synchrony of the network. In collaboration with Prof. Sarah Day and Prof. Drew LaMar we model the synchronized bursting behavior of the pre-Bötzinger complex and demonstrate examples of how the topological features of the network may tell us how the organization of connections is essential to the system's global behavior. (Received July 27, 2018)

1141-55-205 **Alex B Kunin\*** ([abk170@psu.edu](mailto:abk170@psu.edu)), **Vladimir Itskov** ([vladimir.itskov@psu.edu](mailto:vladimir.itskov@psu.edu)) and **Zvi Rosen**. *The Polar Complex of Hyperplane Codes.*

The firing patterns of neurons in the brain give rise to combinatorial codes, i.e. subsets of the boolean lattice. In sensory systems, these firing patterns often represent the abstract intersection patterns of convex regions of a Euclidean space. One-layer feed-forward networks give rise to a class of these codes, hyperplane codes, in which these regions are open half-spaces intersected with a convex set. We investigate the distinguishing characteristics of hyperplane codes via the polar complex, a simplicial complex associated to any code. We demonstrate several necessary conditions for a polar complex to arise from a hyperplane code, and demonstrate that for non-degenerate hyperplane codes, this complex is shellable. (Received July 30, 2018)

1141-55-251 **Carina Curto\***, [ccurto@psu.edu](mailto:ccurto@psu.edu). *A graphical calculus for inhibitory network dynamics.*

Many networks in the nervous system possess an abundance of inhibition, which serves to shape and stabilize neural dynamics. The neurons in such networks exhibit intricate patterns of connectivity, whose structure controls the allowed patterns of neural activity. In this work, we examine inhibitory threshold-linear networks whose dynamics are dictated by an underlying directed graph. We develop a set of parameter-independent graph rules that enable us to predict features of the dynamics from properties of the graph. The resulting graphical calculus provides a direct link between the structure and function of these networks, and provides new insights into how connectivity may shape dynamics in real neural circuits. (Received July 31, 2018)

1141-55-269 **Vladimir Itskov\***, [vladimir.itskov@psu.edu](mailto:vladimir.itskov@psu.edu), and **Aliaksandra Yarosh**. *Directed complexes, sequence dimension and inverting a neural network.* Preliminary report.

What is the embedding dimension, and more generally, the geometry of a set of sequences? This problem arises in the context of neural coding and neural networks. Here one would like to infer the geometry of a space that is measured by unknown quasiconvex functions. A natural object that captures all the inferable geometric information is the directed complex. It turns out that the embedding dimension as well as some other geometric properties of data can be inferred from the homology of an associated directed complex. Moreover each such directed complex gives rise to a multi-parameter filtration that provides a dual topological description of the underlying space. I will also illustrate these methods in the neuroscience context of understanding the "olfactory space". (Received July 31, 2018)

1141-55-281 **Steve Huntsman\*** ([steve.huntsman@baesystems.com](mailto:steve.huntsman@baesystems.com)) and **Michael Robinson** ([michaelr@american.edu](mailto:michaelr@american.edu)). *Topology of basic blocks.* Preliminary report.

Dowker's theorem gives a homotopy equivalence between the two natural simplicial complexes formed from a relation between two finite sets. The topology of these complexes encodes information about the underlying

relation. In the special case of homology with  $\mathbb{F}_2$  coefficients, the complexes do not even need to be explicitly constructed. These observations have by now a long tradition of applications spanning Q-analysis (from the 1970s!), topological navigation and mapping, and computing lower bounds in privacy analyses. In many cases of potential interest, one or both sets underlying a relation to be analyzed is a topological space in its own right, and this leads to a cosheaf structure and attendant notion of local homology, which to our knowledge has not yet been applied. Here, we sketch the application of these ideas to the analysis of basic blocks, i.e., computer programs without control flow. In particular, using the example of matrix multiplication, we give evidence that topological invariants can capture salient information about *algorithms*, versus merely about functions or programs. (Received July 31, 2018)

## 57 ► *Manifolds and cell complexes*

1141-57-141 **Antun Milas\***, Department of Mathematics and Statistics, SUNY-Albany, Albany, NY 12222. *Quantum invariants of 3-manifolds and quantum modular forms*. Preliminary report.

We discuss certain quantum invariants of 3-manifolds. We formulate and and prove, in several special cases, a conjecture of Gukov pertaining to quantum modularity of these invariants. Joint work with K. Bringmann and K.Mahlburg. (Received July 26, 2018)

1141-57-265 **J. Elisenda Grigsby\***, grigsbyj@bc.edu. *Braids, surfaces, and homological invariants*.

I will describe some invariants for braids, knots, and links arising from ideas in quantum topology, and connect their study to classical questions about complexity of smoothly-imbedded surfaces in the 4-ball. Along the way, we will survey the ways in which key representation-theoretic features of the invariant track topological information. (Received July 31, 2018)

1141-57-278 **Daniel Kling\*** (dan@foldstar.com).  *$S_k$ -Holonomy on Coloring Complexes of  $M^n$  with Applications to the Poincaré Conjecture and 4-Color Theorem*.

A natural class of coloring complexes  $X$  on closed manifold  $M^n$  is investigated that gives a holonomy map  $\text{Hol}_X : \pi_1(M) \rightarrow S_{n+1}$ . By a  $k$ -multilayer complex construction the holonomy map may be defined to any finite permutation group  $\text{Hol}_X : \pi_1(M) \rightarrow S_{n+k}$ ,  $k > 0$ . Under isotopy of  $X$  and surgery on  $B^n \subset M^n$  a holonomy class of complexes  $[X]$  is defined with  $[X] = [Y] \iff \text{Hol}_X = \text{Hol}_Y$ . It is also shown that for any homeomorphism  $f : \pi_1(M) \rightarrow S_{n+1}$  there is a complex  $X$  on  $M$  with  $\text{Hol}_X = f$ . These results are applied to express the 4-color Theorem and the Poincaré Conjecture as the existence and uniqueness, respectively, of a certain holonomy class. Other applications are suggested. (Received July 31, 2018)

## 58 ► *Global analysis, analysis on manifolds*

1141-58-24 **Hartmut Führ** and **Isaac Z. Pesenson\*** (pesenson@temple.edu), Department of Mathematics, Temple University, Philadelphia, PA 19122. *Poincaré and Plancherel-Polya inequalities in harmonic analysis on weighted combinatorial graphs*.

We prove Poincaré and Plancherel-Polya inequalities for weighted  $\ell^p$ -spaces on weighted graphs in which the constants are explicitly expressed in terms of some geometric characteristics of a graph. We use Poincaré type inequality to obtain some new relations between geometric and spectral properties of the combinatorial Laplace operator. Several well known graphs are considered to demonstrate that our results are reasonably sharp.

The Plancherel-Polya inequalities allow for application of the frame algorithm as a method for reconstruction of Paley-Wiener functions on weighted graphs from a set of samples. The results are illustrated by developing Shannon-type sampling in the case of a line graph. (Received June 26, 2018)

## 60 ► *Probability theory and stochastic processes*

1141-60-42 **Tvrtko Tadic\*** (tvrtko@math.hr), Microsoft Corporation, One Microsoft Way, (CCP Bellevue 4337), Redmond, WA 98052. *Can one make a laser out of cardboard?*

We consider two dimensional and higher dimensional semi-infinite tubes made of "Lambertian" material, so that the distribution of the direction of a reflected light ray has the density proportional to the cosine of the angle with the normal vector. If the light source is far away from the opening of the tube then the exiting rays are

(approximately) collimated in two dimensions but are not collimated in higher dimensions. Using the properties of the arccosine distribution we are able to analyze the tail of the movement in the direction of the higher dimensional tube and to obtain asymptotic results on the exiting properties for these high dimensions. Further, an observer looking into the higher dimensional tube will see "infinitely bright" spot at the center of vision. In other words, in high dimensions, the light brightness grows to infinity near the center as the light source moves away. Joint work with Krzysztof Burdzy. (Received July 06, 2018)

1141-60-58 **Andrey Sarantsev\*** (asarantsev@unr.edu) and **Soumik Pal** (soumikpal@gmail.com). *Concentration Inequalities for Reflected Diffusions*. Preliminary report.

Talagrand concentration inequalities compare Wasserstein distance and relative entropy. They are related to other functional inequalities, such as log-Sobolev and Poincare. These inequalities have been proved for solutions of stochastic differential equations. We prove them for versions of such equations with normal reflection in convex domains. (Received July 13, 2018)

1141-60-59 **Andrey Sarantsev\*** (asarantsev@unr.edu) and **Davar Khoshnevisan** (davar@math.utah.edu). *Concentration Inequalities for Stochastic Partial Differential Equations*. Preliminary report.

Talagrand concentration inequalities provide estimates for deviations of a Lipschitz functional from its median. They are related to other functional inequalities, such as log-Sobolev and Poincare. They have been previously proved for random variables, Markov chains, and stochastic differential equations. We prove them for stochastic partial differential equations. (Received July 13, 2018)

1141-60-78 **Khem Ghusinga** and **Abhyudai Singh\*** (absingh@udel.edu), University of Delaware, Newark, DE 19713. *Exact lower and upper bounds on moments of biochemical systems*.

Biochemical systems often comprise of constituents that are present in small numbers, and consequently are more accurately characterized as stochastic systems. In the stochastic description, the time evolution of statistical moments for species population counts is described by a linear dynamical system. However, except for some ideal cases (such as zero- and first-order reaction kinetics), the moment dynamics is underdetermined as lower-order moments depend upon higher-order moments. We propose a method to find exact lower and upper bounds on stationary moments for a given arbitrary system of biochemical reactions. Here we utilize the fact that statistical moments of any positive-valued random variable must satisfy some constraints that are compactly represented through the positive semidefiniteness of moment matrices. Our analysis shows that solving moment equations at steady state in conjunction with constraints on moment matrices provides exact lower and upper bounds on the moments. Furthermore, the accuracy of the bounds improves as moment equations are expanded to include higher-order moments. Our results provide avenues for development of methods that provide explicit bounds on moments for nonlinear stochastic systems that are otherwise analytically intractable. (Received July 17, 2018)

1141-60-118 **Matthew Junge\*** (jungem@math.duke.edu), Physics Bldg 120, Science Drive, Durham, NC 27710. *Parking*.

A car or parking spot is placed at each vertex of an infinite graph. Cars drive in search of unoccupied spots. When a car finds one, both the car and spot are removed. Three distinct traffic flow phases occur as we vary the initial ratio of cars to spots. There is particularly intriguing behavior at criticality. (Received July 23, 2018)

1141-60-134 **Miklos Z Racz\*** (mracz@princeton.edu). *High-dimensional random geometric graphs*.

I will talk about two natural random geometric graph models, where connections between vertices depend on distances between latent  $d$ -dimensional labels. We are particularly interested in the high-dimensional case when  $d$  is large. We study a basic hypothesis testing problem: can we distinguish a random geometric graph from an Erdos-Renyi random graph (which has no geometry)? We show that there exists a computationally efficient procedure which is almost optimal (in an information-theoretic sense). The proofs will highlight new graph statistics as well as connections to random matrices. This is based on joint work with Sebastien Bubeck, Jian Ding, Ronen Eldan, and Jacob Richey. (Received July 25, 2018)

1141-60-142 **Leonid Petrov\*** (lenia.petrov@gmail.com), Department of Mathematics, 141 Cabell Drive, Kerchof Hall, Charlottesville, VA 22904. *Nonequilibrium particle systems in inhomogeneous space*.

I will discuss stochastic interacting particle systems in the KPZ universality class evolving in one-dimensional inhomogeneous space. The inhomogeneity means that the speed of a particle depends on its location. I will focus on integrable examples of such systems, i.e., for which certain observables can be written in exact form suitable for asymptotic analysis. Examples include a continuous-space version of TASEP (totally asymmetric simple

exclusion process), and the pushTASEP (=long-range TASEP). For integrable systems, density limit shapes can be described in an explicit way. We obtain asymptotics of fluctuations, in particular, around slow bonds and infinite traffic lights. (Received July 26, 2018)

1141-60-168 **Timothy Chumley\*** (tchumley@ntholyoke.edu) and **Renato Feres**. *Entropy production in random billiards and the second law of thermodynamics.*

A random dynamical system is said to be time-reversible if the statistical properties of orbits do not change after reversing the arrow of time. The degree of irreversibility of a given system is captured by the notion of entropy production rate. We describe a general formula for entropy production that applies to a class of random billiard systems on Riemannian manifolds with boundary for which it is meaningful to talk about energy exchange between billiard particle and boundary. This formula establishes a relation between the purely mathematical concept of entropy production rate and physics textbook thermodynamic entropy. In particular, it recovers Clausius formulation of the second law of thermodynamics: the system must evolve so as to transfer energy from hot to cold. We also demonstrate the relationship between entropy production rate and certain geometric and thermodynamic parameters of systems for some explicit examples. (Received July 27, 2018)

1141-60-173 **Diane Holcomb, Gaultier Lambert, Elliot Paquette\*** (paquette.30@osu.edu), **Bálint Vető** and **Bálint Virág**. *Random matrix point processes via stochastic processes.*

In 2007, Virág and Váiko introduced the Brownian carousel, a dynamical system that describes the eigenvalues of a canonical class of random matrices. This dynamical system can be reduced to a diffusion, the stochastic sine equation, a beautiful probabilistic object requiring no random matrix theory to understand. Many features of the eigenvalues of the random matrix can then be studied via this stochastic process. We will sketch how this stochastic process is connected to eigenvalues of a random matrix and how problems relating to the eigenvalues of this process (such as a functional CLT or deviation estimates for the eigenvalue counting function) can be tackled using this stochastic process. (Received July 27, 2018)

1141-60-176 **Sivan Leviyang\*** (sr286@georgetown.edu), Georgetown University, Department of Mathematics and Statistics, Washington, DC. *The Effects of Heavy Tailed Offspring Distributions on Viral Infection Dynamics and Evolution.*

During infection, viral particles enter cells and produce offspring viral particles that can go on to infect other cells. Over the past decade, it has become increasingly clear that for many viruses the offspring distribution of an infected cell is strongly skewed and likely best modeled by a heavy tailed distribution. Most current models of viral dynamics and evolution do not account for this phenomena. I will present a model of HIV dynamics demonstrating the effect of heavy tailed offspring size on viral population genetics. (Received July 28, 2018)

1141-60-182 **James N MacLaurin\*** (james.maclaurin@gmail.com), 323 Dr Martin Luther King Jr Blvd, Newark, UT 07102, and **Paul C Bressloff** (bressloff@math.utah.edu), LCB Building, President's Circle, SALT LAKE CITY, UT 84111. *Phase Reduction and Synchronization in Stochastic Biology.*

Self-sustained oscillations in biological, physical and chemical systems are often described in terms of limit cycle oscillators where the timing along each limit cycle is specified in terms of a single phase variable. Traditional models take the oscillators to be driven by common Gaussian noise. However, it has also been shown that common Poisson-distributed random impulses, dichotomous or telegrapher noise, and other types of noise generally induce synchronization of limit-cycle oscillators. In this talk I demonstrate that this noise can also induce synchronization in the fast switching limit. I also find the Lyapunov exponent, that gives the leading order decay towards the limit cycle, and compare it to the exponent predicted by the quasi-steady-state approximation of the switching. (Received July 28, 2018)

1141-60-206 **Janko Gravner, Alexander Holroyd** and **David Sivakoff\*** (dsivakoff@stat.osu.edu), Department of Statistics, 1958 Neil Ave, Columbus, OH 43210. *Polluted Bootstrap Percolation in Three Dimensions.*

In  $r$ -neighbor bootstrap percolation, the vertices of  $Z^d$  are initially occupied independently with probability  $p$  and empty otherwise. Occupied vertices remain occupied forever, and empty vertices iteratively become occupied when they have at least  $r$  occupied neighbors. It is a classic result of van Enter ( $r = d = 2$ ) and Schonmann ( $d > 2$  and  $2 \leq r \leq d$ ) that every vertex in  $Z^d$  eventually becomes occupied for any initial density  $p > 0$ .

In polluted bootstrap percolation, vertices of  $Z^d$  are initially closed with probability  $q$ , occupied with probability  $p$  and empty otherwise. The  $r$ -neighbor bootstrap rule is the same, but now closed vertices act as obstacles, and remain closed forever. This model was introduced 20 years ago by Gravner and McDonald, who studied the case  $d = r = 2$  and proved a phase transition exists for this model as  $p$  and  $q$  tend to 0. We prove a similar

phase transition occurs when  $d = r = 3$ , and we identify the polynomial scaling between  $p$  and  $q$  at which this transition occurs for the modified bootstrap percolation model. Our proof relies on duality methods in Lipschitz percolation to find a blocking structure that prevents occupation of the origin. (Received July 30, 2018)

1141-60-225 **Tobias Johnson\*** (tobias.johnson@csi.cuny.edu), **Moumanti Podder** and **Fiona Skerman**. *Fixed points of random tree recursions.*

Let  $\mathcal{B}$  be the set of rooted trees that contain an infinite binary subtree starting at the root. This set satisfies the metaproperty of containing a tree if and only if it contains at least two of its root child subtrees. Suppose we wish to know the probability that a Galton–Watson tree falls in  $\mathcal{B}$ . The metaproperty forces this probability to satisfy a fixed-point equation, which can have multiple solutions. One of these is the probability we seek, but what is the meaning of the other solutions? In particular, are they probabilities of the Galton–Watson tree falling into some other set satisfying the same metaproperty? We create a framework that lets us answer all questions of this sort. Our proofs use spine decompositions of Galton–Watson trees and the analysis of Boolean functions. (Received July 30, 2018)

1141-60-252 **Erik Slivken\***, 16 Rue Michel Lecomte, 75003 Paris, France. *Neighborhood growth on the Hamming plane.*

We consider a family of discrete growth models on  $\mathbb{Z}^2$  with nearest neighbors giving by the Hamming distance. Vertices have one of two initial states, occupied or unoccupied. At subsequent steps the state of a vertex is updated to occupied if certain conditions are met. Once a vertex becomes occupied it remains occupied. This can be viewed as a generalization of bootstrap percolation on the Hamming graph. We give scaling limits for the probability that a finite rectangle becomes fully occupied. We also consider extremal problems. This is based on joint work with Janko Graver, J.E. Paguyo, and David Sivakoff. (Received July 31, 2018)

1141-60-284 **Mykhaylo Shkolnikov\*** (mshkolni@gmail.com) and **Sergey Nadtochiy**. *Particles interacting through their hitting times: neuron firing, supercooling and systemic risk.*

I will discuss a class of particle systems that serve as models for supercooling in physics, neuron firing in neuroscience and systemic risk in finance. The interaction between the particles falls into the mean field framework pioneered by McKean and Vlasov in the late 1960s, but many new phenomena arise due to the singularity of the interaction. The most striking of them is the loss of regularity of the particle density caused by the self-excitation of the system. In particular, while initially the evolution of the system can be captured by a suitable Stefan problem, the following irregular behavior necessitates a more robust probabilistic approach. Based on joint work with Sergey Nadtochiy. (Received July 31, 2018)

1141-60-287 **James Melbourne\*** (jamescmelbourne@gmail.com), UMN Electrical and Computer Engineering, 200 Union St SE, Minneapolis, MN 55455. *Concavity and entropic inequalities.*

We will introduce the Renyi entropy as an interface between information theory, probability, and convex geometry, and discuss some relevant inequalities. In particular recent extensions of the classical Shannon–Stam entropy power inequality to the Renyi setting will be presented. Special attention will be paid to the role of log-concave random variables (and more generally  $s$ -concave) in entropic inequalities. (Received July 31, 2018)

1141-60-301 **Vu Dinh\*** (vucdinh@udel.edu). *Regularized estimators for inferring non-bifurcating evolutionary trees.*

Phylogenetic tree inference using deep DNA sequencing is reshaping our understanding of rapidly evolving systems, such as the within-host battle between viruses and the immune system. Densely sampled phylogenetic trees can contain special features, including sampled ancestors in which we sequence a genotype along with its direct descendants, and polytomies in which multiple sampled descendants arise simultaneously. These features are apparent after identifying zero-length branches in the tree, however, current maximum-likelihood based approaches are not capable of revealing such zero-length branches. In this work, we introduce adaptive-lasso-type regularization estimators to find these zero-length branches, deriving their properties, and showing regularization to be a practically useful approach for phylogenetics (Received August 01, 2018)

1141-60-302 **Hao Shen\*** (pkushenhao@gmail.com), 309 West Johnson Street, Madison, WI 53703. *Stochastic quantization of gauge theories.*

”Stochastic quantization” refers to a formulation of quantum field theory as stochastic PDEs. Interesting progress has been made these years in understanding solutions of these stochastic PDEs, one of the remarkable examples being Hairer and Mourrat–Weber’s results on the  $\Phi_3^4$  equation. In this talk we will discuss stochastic quantization of quantum field theory with gauge symmetries, with focus on an Abelian example but also provide prospects

of non-Abelian Yang-Mills theories. We address issues regarding Wilson's lattice regularization, dynamical gauge fixing, renormalization, Ward identities, and construction of dynamical loop and string observables. (Received August 01, 2018)

## 62 ► *Statistics*

1141-62-43 **Yuliy Baryshnikov** and **Robert Ghrist\*** ([ghrist@math.upenn.edu](mailto:ghrist@math.upenn.edu)). *Signal Processing in Topological Time.*

This talk will survey recent methods for managing time-series and signals in which the temporal information is known only up to an orientation-preserving homeomorphism. Under such circumstances, methods from topology, invariant under deformation, are ascendant. (Received July 06, 2018)

1141-62-123 **Anthea Monod\***, [am4691@cumc.columbia.edu](mailto:am4691@cumc.columbia.edu), and **Sara Kalisnik, Juan A. Patino-Galindo** and **Lorin Crawford**. *Tropical Sufficient Statistics for Persistent Homology.*

We show that an embedding in Euclidean space based on tropical algebraic geometry generates stable sufficient statistics for barcodes. Conventionally, barcodes are multiscale summaries of topological characteristics that capture the "shape" of data; however, in practice, they have complex structures which make them difficult to use in statistical settings. The sufficiency result presented in this work allows for classical probability distributions to be assumed on the tropical-geometric representation of barcodes. This makes a variety of parametric statistical inference methods amenable to barcodes, all while maintaining their initial interpretations. More specifically, we show that exponential family distributions may be assumed, and that likelihood functions for persistent homology may be constructed. We conceptually demonstrate sufficiency and illustrate its utility in persistent homology dimensions 0 and 1 with concrete parametric applications to HIV and avian influenza data. (Received July 24, 2018)

1141-62-215 **Ben Cassidy\*** ([bcassidy@eskatos.com](mailto:bcassidy@eskatos.com)). *Statistical topology of brain activity networks.*

Brain connectivity, particularly based on resting-state fMRI data has been the subject of intense study for over a decade, stimulated by growing evidence of network involvement in brain diseases. But such study needs methods of analysis that can compare networks with differing numbers of nodes and links and provide results that are stable over different spatial and temporal data resolutions. Current graph analysis methods cannot do this. Enter topological data analysis; in particular persistent homology. By focussing on network 'shape' these new methods can attack those questions. Using our recent development of frequency domain persistent homology, we discuss: network interpretation and comparison of sparsely and densely connected brain networks; multi-scale (spatial and temporal) network analyses; data quality diagnostics, and defining of a core brain architecture. We illustrate results with fMRI data. (Received July 30, 2018)

## 65 ► *Numerical analysis*

1141-65-10 **Gang Chen, Bernardo Cockburn, John Singler** and **Yangwen Zhang\*** ([ywzfg4@mst.edu](mailto:ywzfg4@mst.edu)). *Group HDG methods for nonlinear PDEs.*

In this work, we propose a new group hybridizable discontinuous Galerkin (Group HDG) method for some nonlinear PDEs. We adopt ideas from the group finite element method and extend to the HDG method. The resulting Group HDG method is applicable for more general nonlinear PDEs, and we obtained optimal convergence rates. However, we lost the superconvergence after an element-by-element processing. We present recent work to recover the superconvergence. (Received April 02, 2018)

1141-65-39 **Jacob Jacavage\*** ([jjacav@udel.edu](mailto:jjacav@udel.edu)) and **Constantin Bacuta**. *Least squares discretization and multilevel preconditioning for mixed variational formulations.*

We consider a least squares method for discretizing boundary value problems written as primal mixed variational formulations. A stability LBB condition and a data compatibility condition is assumed at the continuous level. For the proposed discretization method, a discrete inf-sup condition is automatically satisfied by natural choices of test and trial spaces. For the proposed discrete spaces and solvers, bases are needed only for the test spaces and assembly of a global saddle point system is avoided. A multilevel preconditioning approach, that could take into consideration discontinuous coefficients and the coupled physics of the problem to be solved, is also

presented. Applications include discretizations of second order PDEs with oscillatory or rough coefficients and first order systems of parametric PDEs, such as the time-harmonic Maxwell equations. (Received July 06, 2018)

1141-65-45 **Qi Wang\***, 134 Land Stone Cir, Irmo, SC 29063-9297, and **Xueping Zhao** (xzha@email.sc.edu), Dept of Math, Univ of South Carolina, Columbia, SC 29208. *thermodynamically consistent hydrodynamic phase field models for compressible fluid mixtures.*

We will present a systematic derivation of compressible, quasi-incompressible and incompressible models for fluid mixtures of various components using the generalized Onsager principle. Then, energy stable numerical approximations will be discussed and new high order numerical algorithms will be given to solve the hydrodynamic phase field models. Examples in polymer mixing and liquid-gas dynamics will be presented. (Received July 06, 2018)

1141-65-52 **Wen Yan\*** (wyan@flatironinstitute.org), 162 5th Ave, New York, NY 10010. *Numerical experiments of active matter: efficient algorithms for long-range and short-range interactions.*

Active matter systems often show intriguing phenomena in large spatial scales and long time scales, due to various interactions between the building-block particles. The long-range interactions are usually through Stokes flow and electrostatic field, while the steric interaction is usually the dominant effect at short-range. We develop an extension to the Kernel Independent Fast Multipole Method to allow adaptive and flexible treatment of long-range interactions with various boundary conditions. We demonstrate the application of this algorithm with a new Stokeslet image system for half-space Stokes flow. To handle the short-range steric interactions, we propose a new method based on constrained minimization to circumvent the stiffness of pairwise repulsive potential. All the discussed algorithms are parallel and scalable, and we demonstrate the applications with a few active matter systems, including microtubule network and growing and dividing cells. (Received July 10, 2018)

1141-65-75 **Mingchang Ding\*** (dmcvamos@udel.edu), **Xiaofeng Cai** and **Jingmei Qiu**. *Efficient and highly accurate semi-Lagrangian discontinuous Galerkin method for convection-diffusion problems.*

We propose to organically combine semi-Lagrangian discontinuous Galerkin (SLDG) method to convection terms with local DG (LDG) scheme to diffusion terms for convection-diffusion problems. In particular, we apply a weak formulation of the SLDG method for the convection term (Cai, Guo and Qiu, JSC, 2017), and use high order implicit Runge-Kutta method for a LDG discretization of the diffusion term along characteristics. The proposed scheme is shown to be mass conservative, high order accurate in both space and in time, and highly efficient due to large time stepping sizes allowed from the semi-Lagrangian and implicit nature of time discretization. The scheme can be straightforwardly extended to 2D problems without operator splitting in the truly multi-D SLDG framework previously proposed. The performance of the scheme will be showcased by several classical linear test problems such as rigid body rotation, as well as incompressible Navier-Stokes equations. (Received July 16, 2018)

1141-65-120 **Yanxiang Zhao\*** (yxzhao@email.gwu.edu), 801 22nd St. NW, Phillips Hall room 739, Washington, DC 20052. *A New Phase-Field Approach to Variational Implicit Solvation of Charged Molecules with the Coulomb-Field Approximation.*

We construct a new phase-field model for the solvation of charged molecules with a variational implicit solvent. By introducing a new phase-field term in the description of the solute-solvent van der Waals and electrostatic interactions, we can keep the phase-field values closer to those describing the solute and solvent regions, respectively, making it more accurate in the free-energy estimate. We first prove that our phase-field functionals Gamma-converge to the corresponding sharp-interface limit. We then develop and implement an efficient and stable numerical method to solve the resulting gradient-flow equation to obtain equilibrium conformations and their associated free energies of the underlying charged molecular system. Our numerical method combines a linear splitting scheme, spectral discretization, and exponential time differencing Runge-Kutta approximations. Applications to the solvation of single ions and a two-plate system demonstrate that our new phase-field implementation improves the previous ones by achieving the localization of the system forces near the solute-solvent interface and maintaining more robustly the desirable hyperbolic tangent profile for even larger interfacial width. (Received July 23, 2018)

- 1141-65-133 **José C. Garay, Frédéric Magoulés and Daniel B. Szyld\*** (szyld@temple.edu), Department of Mathematics, Temple University, 1804 N Broad Street, Philadelphia, PA 19122. *Convergence results for asynchronous optimized Schwarz methods for the solution of PDEs.*

Asynchronous methods refer to parallel iterative procedures where each process performs its task without waiting for other processes to be completed, i.e., with whatever information it has locally available and with no synchronizations with other processes. In this talk, an asynchronous version of the optimized Schwarz method is presented for the solution of differential equations on a parallel computational environment. Convergence is proved under very mild conditions on the size of the subdomains, when approximate (non-optimal) interface conditions are utilized for Poisson's equation (and others) on the plane and on bounded rectangular domains. Numerical results are presented on large three-dimensional problems illustrating the efficiency of the proposed asynchronous parallel implementation of the method. (Received July 25, 2018)

- 1141-65-148 **Louis Rossi\*** (rossi@udel.edu), Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, and **Zhenyu He**. *Achieving high order accuracy with smoothed particle hydrodynamic methods.*

Smooth particle hydrodynamic methods approximate fluid flow density and velocity as a moving collection of interacting particles. Originally developed for astrophysical simulations, these methods have proven to be robust, naturally adaptive mesh-free methods for challenging fluid flow problems. Unfortunately, many implementations are limited to low orders of accuracy, and a systematic means of analyzing and addressing these issues remains elusive. This paper will provide an overview of the state of the art in boosting the accuracy of these methods and provide simple examples of improved rates of convergence. (Received July 26, 2018)

- 1141-65-153 **Leslie F Greengard\***, Flatiron Institute, 162 Fifth Avenue, New York, NY 10010. *Linear and nonlinear inverse problems in imaging.*

We will begin with a brief review of linear inverse problems that arise in medical imaging. Following that, we turn to inverse acoustic scattering and protein structure determination from cryo-electron microscopy data (cryo-EM). These are computationally intensive tasks that are typically formulated as non-convex optimization problems. In cryo-EM, the raw data is extremely noisy and existing methods are generally based on some version of maximum likelihood estimation, with a low resolution starting guess. In inverse acoustic scattering, the underlying physical problem is ill-posed and requires both regularization and high-order methods to solve a sequence of forward scattering problems.

We will present some algorithms for accelerating image reconstruction in all these settings, illustrate their performance with several examples, and discuss open problems in the field. (Received July 26, 2018)

- 1141-65-164 **Jue Yan\***, 396 Carver Hall, Department of Mathematics, Iowa State University, Ames, IA 50010, and **Xinghui Zhong**, zhongxh@zju.edu.cn. *Positivity preserving high order direct DG Methods for Keller-Segel chemotaxis equations.* Preliminary report.

We develop a new direct discontinuous Galerkin (DDG) method to solve Keller-Segel Chemotaxis equations. One unique feature of our method is that we introduce no extra variables to approximate the gradient of the chemical concentration and solve the system directly with DDG method. We obtain optimal  $(k+1)$ th order convergence with  $k$ th degree piecewise polynomials approximations, even on random none uniform meshes. Furthermore, we prove the cell density solution is maintained positive at all time levels with at least third order of accuracy. Cell density blow up phenomena is captured well. (Received July 27, 2018)

- 1141-65-204 **L Ma\*** (lina.ma@trincoll.edu), MECC 251, Trinity College, Hartford, CT 06106, and **X Li and C Liu**. *Model Reduction Technique on Langevin Dynamics.*

Langevin equation is the generic equation that can give rise to Fokker Planck equation. In this talk, we will look at the numerical treatment in the microscopic system. Techniques such as coarse graining and model reduction will be employed to solve the common challenges. Also, the important fluctuation dissipation theorem is carefully studied to make sure the new system has correct equilibrium result. (Received July 30, 2018)

- 1141-65-210 **Matthew W Seiders\*** (matthew.seiders@afit.edu), 2950 Hobson Way, Building 641, Wright-Patterson AFB, OH 45433, and **Benjamin F Akers**. *Dimension-Breaking Bifurcations for Overturned Water Waves.* Preliminary report.

Computations of fully three-dimensional traveling solutions to the water wave problem are discussed. Dimension-Breaking as a continuation procedure is evaluated. The Kadomtsev-Petviashvili equation is used as a toy model. Comparisons are made to the full water wave problem for overturned three-dimensional solutions. The vortex-sheet formulation is used for these computations. (Received July 30, 2018)



1141-65-214 **Dong Wang\*** ([dongwang@udel.edu](mailto:dongwang@udel.edu)), Robinson Hall 210, University of Delaware, NEWARK, DE 19711, and **Tobias Kukulka**, **Brandon G. Reichl**, **Tetsu Hara**, **Isaac Ginis** and **Peter P. Sullivan**. *Langmuir turbulence in the tropical cyclone conditions.*

Wind-driven Eulerian currents interact with surface waves, generating Langmuir turbulence (LT) through the Craik-Leibovich vortex force. LT plays an important role in turbulent momentum transport and has been observed even in the tropical cyclone (TC) conditions that characterize complex wind wave forcing. Based on a large eddy simulation approach, this study investigates the response of LT to TC. The Stokes drift vector that drives LT is determined from spectral wave simulations. LT features strong downwelling and upwelling velocities whose spatial autocorrelation illustrates elongated structure with a principal axis that is used to specify LT's direction. In spite of misaligned wind and wave propagation directions under TCs, the direction of LT is found to be aligned with the wind mostly. This is because Lagrangian shear, that is the sum of Eulerian shear and Stokes drift shear, determines LT's direction and is aligned with the wind. In the presence of LT, the upper OSBL is well-mixed with weak Eulerian shear and, thereby, Lagrangian shear is dictated by Stokes drift shear that is dominated by wind-aligned short waves. Conditional averages reveal that larger scale coherent structures due to LT occupy the upper one-third of the OSBL and efficiently transport heat and momentum. (Received July 30, 2018)

1141-65-227 **Kyle T Mandli\*** ([ktm2132@columbia.edu](mailto:ktm2132@columbia.edu)), 500 W. 120th St., Mudd 200, MC 4701, New York, NY 10027. *Solving the Multi-Layer Shallow Water Equations in the Context of Storm Surge Prediction.*

Coastal hazards related to strong storms are one of the most frequently recurring and wide spread hazards to coastal communities today. In particular storm surge, the rise of the sea surface in response to wind and pressure forcing from these storms, can have a devastating effect on the coastline. Furthermore, with the addition of climate change related effects, the ability to predict these events quickly and accurately is critical to the protection and sustainability of these coastal areas.

Computational approaches to this problem must be able to handle its multi-scale nature while remaining computationally tractable and physically relevant. This has commonly been accomplished by solving a depth-averaged set of fluid equations and by employing non-uniform and unstructured grids. These approaches, however, have often had shortcomings due to computational expense, the need for involved model tuning, and missing physics.

In this talk we will outline some of the approaches to and difficulties that arise when using the multi-layer shallow water equations to model storm surge in an effort to better capture the vertical stratification present in storm surge scenarios. (Received July 30, 2018)

1141-65-235 **Tobin A. Driscoll\*** ([driscoll@udel.edu](mailto:driscoll@udel.edu)), Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, and **Kevin Aiton**. *Preconditioning nonlinear equations with domain decomposition.*

Additive Schwarz methods can precondition the linearization of a Newton algorithm for a nonlinear PDE. More recently it has been shown by Cai, Keyes, Dolean, and others how to generalize this preconditioning to the original nonlinear problem. If separate discretizations are allowed in overlapping regions, it becomes simple to minimize cross-interpolation and communication, coarsen locally for a two-level implementation, and generalize to least-squares formulations. Numerical experiments show good behavior for overlapping spectral discretizations. (Received July 30, 2018)

1141-65-237 **Benjamin Seibold\*** ([seibold@temple.edu](mailto:seibold@temple.edu)), Department of Mathematics, 1805 N. Broad Street, Philadelphia, PA 19022. *Overcoming Order Reduction via Weak Stage Order.*

Order reduction, i.e., the convergence of the solution at a lower rate than the formal order of the time-stepping scheme, is a fundamental problem in stiff ODEs, and particularly in PDE IBVPs. Runge-Kutta schemes with high stage order provide a remedy, but unfortunately high stage order (above two) is incompatible with DIRK schemes. In this talk we present the concept of weak stage order, and demonstrate (a) how it can address order reduction in important PDE problems; and (b) that it can recover up to fourth order convergence with DIRK schemes. (Received July 30, 2018)

1141-65-243 **Peter Monk\*** ([monk@udel.edu](mailto:monk@udel.edu)), Mathematical Sciences, University of Delaware, Newark, DE 19716, and **Shixu Meng** and **Christopher Lackner**. *A finite element method for determining electromagnetic Bloch modes.* Preliminary report.

Photonic crystals, used to manipulate light, often involve periodic arrays of elements. In order to evaluate the design of a photonic crystal, it is useful to determine the Bloch variety for the structure. This represents the

relationship between the wave vector of a propagating mode and the corresponding angular frequency of the mode.

If the material is not frequency dependent, the problem of computing the Bloch variety reduces to finding resonant frequencies for Maxwell's equations with quasi-periodic boundary conditions (given from the wave vector). However, if the constituent parts of the photonic crystal have different frequency dependent electromagnetic properties, this direct approach is no longer applicable. Instead we propose to compute the wave vector as a function of frequency. For a give frequency (and hence given electromagnetic parameters) this results in a quadratic eigenvalue problem for the magnitude of the wave vector.

We show that this approach can be formulated as a mixed variational problem and, after linearization, becomes the problem of finding the generalized eigenvalues of a compact operator. Numerical results show that our approach can indeed compute eigenvalues for frequency dependent structures. (Received July 30, 2018)

1141-65-255 **Lise-Marie Imbert-Gerard\***, [lmig@math.umd.edu](mailto:lmig@math.umd.edu), and **Felipe Vico, Leslie Greengard** and **Miguel Ferrando**. *Integral equation methods for electrostatics, acoustics and electromagnetics in smoothly varying, anisotropic media.*

Our goal is to develop integral equation based numerical methods for the solution of electrostatic, acoustic or electromagnetic scattering problems involving anisotropic, inhomogeneous media. We will present a collection of well-conditioned integral equation formulations and will illustrate their performance using iterative solution methods coupled with an FFT-based technique to discretize and apply the relevant integral operators. (Received July 31, 2018)

1141-65-259 **Ricardo M Campos\*** ([riwave@gmail.com](mailto:riwave@gmail.com)), **Vladimir Krasnopolsk** ([vladimir.krasnopolsky@noaa.gov](mailto:vladimir.krasnopolsky@noaa.gov)), **Stephen G. Penny** ([stephenpenny@gmail.com](mailto:stephenpenny@gmail.com)) and **Jose-Henrique Alves** ([henrique.alves@noaa.gov](mailto:henrique.alves@noaa.gov)). *Nonlinear Wave Ensemble Averaging using Neural Networks.*

Artificial neural networks (ANNs) applied to nonlinear ensemble averaging are developed to improve wave forecasts. This is an approach that expands the conservative arithmetic ensemble mean (EM) from the NCEP Global Wave Ensemble Forecast System (GWES) to a nonlinear mapping that better captures the differences among the ensemble members and reduces the systematic and scatter errors of the forecasts. The goal is to improve the long-range predictability of significant wave height (Hs), peak wave period (Tp), and 10-m wind speed (U10). Several ANNs with different architectures and growing complexity have been tested and a detailed assessment of GWES has been performed. The first experiment was based on ANN training at two locations (Atlantic and Pacific Oceans) using NDBC buoys. A second experiment was conducted in the Gulf of Mexico, using a spatial approach. Results show that a small number of neurons are sufficient to reduce the bias, while 35 to 50 neurons are optimum to reduce both the scatter and systematic errors. The correlation coefficient for forecast Day 10 was increased from 0.39 to 0.61 for U10, from 0.50 to 0.76 for Hs, and from 0.38 to 0.63 for Tp. We are currently running the third experiment using altimeter data to train the ANN for the whole globe. (Received July 31, 2018)

1141-65-279 **Tonatiuh Sanchez-Vizuet\***, 251 Mercer Street, 1010 Warren Weaver Hall. Courant Institute, New York University, New York, NY 10012-1185, and **Manuel E Solano** and **Antoine J Cerfon**. *An h-adaptive HDG solver for Dirichlet boundary value problems in curved domains using embedded polygonal grids: an application to plasma equilibrium.*

The magnetic equilibrium in axisymmetric fusion reactors can be described in terms of the solution to a semi-linear elliptic Dirichlet boundary value problem posed in a domain whose boundary is a piecewise smooth curve, corresponding to the plasma confinement region. Previously, the authors had proposed a high order HDG solver on an unfitted polygonal mesh embedded within the confinement region. The curvature of the domain is handled by a high order transfer scheme that preserves the order of approximation as long as the gap between the computational domain and the true boundary remains of the order of the mesh parameter. The transferring algorithm however may fail to resolve sharp gradients close to the boundaries and local refinement becomes necessary. We propose an h-adaptive method that relies on a residual type estimator on the embedded computational mesh. The refinement is driven by a combination of Dörfler marking and the constraint that the gap between the mesh and the curved boundary must remain of the order of the local mesh diameter. This results on a nested sequence of unfitted grids that "grow" towards the physical boundary as refinement progresses. This is an ongoing collaboration with Antoine Cerfon (NYU) and Manuel Solano (University of Concepción). (Received July 31, 2018)

1141-65-282 **Rayanne Luke\***, University of Delaware, 15 Orchard Rd, Newark, DE 19716, and **R. J. Braun, T. A. Driscoll, D. Antwi and C. G. Begley**. *Parameter Estimation for Tear Film Breakup Dynamics*.

The tear film forms after a blink and serves to protect the ocular surface as well as promote clear vision. Tear film breakup (TBU) occurs when a dry spot appears on the eye, and is often evaporation-driven. Many parameters effect the film thickness and fluorescent intensity distributions over time; exact values or ranges for some are not well known. We used dry lab simulation to test and validate parameter estimation via least squares minimization of the difference between data and computed answers; Levenberg-Marquardt appeared to be the best approach that we tested. Preliminary determination of parameters for selected experimental instances of TBU have yielded sensible values within experimental ranges for variables that can be measured in vivo. (Received July 31, 2018)

1141-65-290 **Linwan Feng\*** (lf46@njit.edu), 108 S 3rd St., Harrison, NJ 07029, and **David Shirokoff and Wooyoung Choi**. *Numerical Methods for Dispersive Shallow Water Equations*.

In this talk, we discuss numerical time-stepping approaches for solving the dispersive shallow water wave equations. The equations take the form of nonlocal evolution equations where an elliptic operator is applied to one of the time derivatives. We examine two approaches for handling the nonlocal operator: (i) iterative methods that must be performed at each time step; and (ii) semi-implicit (ImEx-type) time stepping methods that avoid inverting the full nonlocal operator (and also do not require sub-iterations). Guaranteeing stability for the semi-implicit approach is a nontrivial issue due to the fact that certain stiff terms in the equations are treated explicitly. We provide a stability theory which outlines how to choose the semi-implicit terms in such a way to guarantee numerical stability. (Received July 31, 2018)

## 70 ► *Mechanics of particles and systems*

1141-70-29 **Marek Stastna\*** (mmstastn@uwaterloo.ca), Department of Applied Mathematics, University of Waterloo, 200 University Ave. West, Waterloo, Ontario N2L 3G1, Canada, and **Justin Shaw**. *Mathematical models of hydrodynamically triggered zooplankton swimming: drivers and consequences*.

The swimming of zooplankton is a complex phenomenon with a considerable number of mathematical models available in the published literature. After a brief review of these, I will present in detail a model of shear-triggered swimming that is objective in the sense of continuum mechanics, and that leads to zooplankton patchiness in the vertical. This patchiness occurs at depths that are determined, at least in part, by the hydrodynamics, as opposed to pure light preference levels. I will subsequently discuss the differences between shear triggered and acceleration triggered behaviour, in the context of a simple set up that is analogous to materials characterization in rheology. I will conclude with some speculation on future directions. (Received June 28, 2018)

1141-70-127 **Hansen Pei\*** (hansenp@udel.edu), 15 Orchard Ave, 501 Ewing Hall, Newark, DE 19711, and **Louis Rossi, Pak-Wing Fok and John Pae**. *Modeling of Karlodinium Predation*. Preliminary report.

The predatory plankton species Karlodinium emits toxins as part of their hunting behavior, which can lead to toxic water conditions and fish-kills. One of the main function of toxin production is to immobilize prey, but how this affects predator and prey populations distribution has yet to be discussed mathematically. Our overarching goal is to understand the interactions between predator and prey swimming, toxin emissions and predation rates. A simplified 1-D model based on Broadwell PDEs is built to account for plankton's run-and-tumble movement pattern, and toxin concentrations effect on velocity. In a case where the toxin's source is not moving, we solved the coupled PDE analytically and matched the solution to Monte-Carlo simulations. In a case where the emitting source is moving, simulations are built to show the effect of global parameters on population density, possibly forming attracting and repelling points. (Received July 24, 2018)

## 76 ► *Fluid mechanics*

1141-76-30 **Sarah D Olson\*** (sdolson@wpi.edu). *Navigation in complex environments*.

Microorganisms can swim in a variety of environments, interacting with chemicals and other proteins in the fluid. Some of these extra proteins or cells may act as friction, possibly preventing or enhancing forward progression of swimmers. The homogenized fluid flow is assumed to be governed by the incompressible Brinkman equation, where a friction term with a resistance parameter represents a sparse array of obstacles. Representing the

swimmers with a centerline approximation, we employ regularized fundamental solutions to investigate swimming speeds, trajectories, and interactions of swimmers. Although attraction of two swimmers is more efficient in the Stokes regime, we find that attraction does not occur for larger resistance. Additionally, we study interactions of swimmers in a channel. (Received June 28, 2018)

1141-76-99 **Luc Deike\*** (ldeike@princeton.edu), 41 olden street, Princeton, NJ 08540, and **Wouter Mostert, W.K. Melville and Stephane Popinet.** *Direct numerical simulations of breaking waves in deep and shallow water.*

Wave breaking in the ocean is of fundamental importance in order to quantify ocean atmosphere interaction. Here, we present direct numerical simulations of breaking waves in deep water, solving for the full two-phase Navier Stokes equations in two or three dimensions. Numerical results are carefully validated against laboratory experiments and we discuss the two-phase turbulence related to breaking waves. We discuss air entrainment, transition from two to three dimensional flow during breaking and the Lagrangian drift of passive particles related to breaking. Finally we present high fidelity breaking waves on a sloping beach in shallow water and discuss the related energy dissipated and the run-off. (Received July 20, 2018)

1141-76-102 **Bryan Quaife\*** (bquaife@fsu.edu), Department of Scientific Computing, Florida S, 400 Dirac Science Library, Tallahassee, FL 32311. *The Role of Adhesion in Vesicle Suspensions.*

Vesicle suspensions are an example of a complex Stokesian fluid that is often used to study capillary flow. In addition to the hydrodynamics, other physics that are important to incorporate include adhesion. I will propose an adhesion model, describe the related numerical methods, and describe the consequences of the adhesion model. (Received July 20, 2018)

1141-76-106 **Hoa Nguyen\*** (hnguyen5@trinity.edu), **Christian Oakes, M. A. R. Koehl and Lisa Fauci.** *Effects of cell morphology and surface attachment on the hydrodynamic performance of unicellular choanoflagellates.*

Choanoflagellates are unicellular organisms whose intriguing morphology includes a set of collar/microvilli emanating from the cell body, surrounding the beating flagellum. As the closest living relative to animals, they are important for both ecological and evolutionary studies. We consider two unicellular types: slow swimmers and thecate cells (attached to a wall by a stalk). Assuming they have similar morphologies, we use the method of regularized Stokeslets to (i) simulate cell-fluid interactions of the slow swimmers and thecate cells with the surrounding environment and (ii) show hydrodynamic effects on the amount of fluid flow across a capture zone around the collar (net flux). The results shed light on how each morphological feature of the cell aids in bacteria captures during feeding. We have found that the existence of the collar not only attracts more fluid particles but also impedes the fluid flow close to the microvilli. Among the two choanoflagellate types, slow swimmers gain the most net flux which shows an advantage of being motile. Due to the wall effect, thecate cells have less net flux but the interactions of cell-fluid-wall-stalk create small eddies around the stalk which can be used to explain bacterial gathering in that area. (Received July 20, 2018)

1141-76-138 **James T Kirby\*** (kirby@udel.edu), Center for Applied Coastal Res., 259 Academy St., Newark, DE 19716, and **Morteza Derakhti and Michael L Banner.** *Predicting the breaking strength of gravity water waves in intermediate and deep water.*

The Phillips-Duncan scaling for breaking waves gives a total dissipation  $\epsilon = b\rho c^5/g$ , where  $\epsilon$  is the total wave energy dissipation,  $c$  is the phase speed of the breaking wave crest, and  $b$  is a dimensionless breaking strength parameter.  $b$  is usually related to a spectral wave steepness  $S$ , which is difficult to construct without full knowledge of the wave field and is not well constrained by properties of the local breaking wave. Barthelemy et al. (2018) showed that highest non-breaking waves were clearly separated from marginally breaking waves using a parameter  $B = u/c$  evaluated at the wave crest, with  $B = 0.85$  providing a robust threshold in deep or intermediate water depths. During LES/VOF simulations of comparable events, we observed a strong correlation between  $dB/dt$  near the threshold value of  $B$  and the progressive increase in breaking strength from weak spilling to plunging. We propose a nondimensional parameter  $\Gamma = T_b dB/dt$  to describe this variation, where  $T_b$  is the local wave period of the breaking crest. Results show that  $b$  is well described by a relation  $b = 0.034(\Gamma - 0.3)^{\{5/2\}}$ . The extension to depth-limited breaking is discussed in the talk by Grilli et al. (Received July 25, 2018)

1141-76-149 **Hua Chen\*** (chenhua@udel.edu), **Robert P. Gilbert and Philippe Guyenne.** *A continuum model for gravity waves on an ice-covered ocean.* Preliminary report.

The recurrent interactions between ocean waves and sea ice are a widespread feature of the polar regions, and their impact on sea-ice dynamics and morphology has been increasingly recognized as evidenced by the surge of research activity during the last two decades. The rapid decline of summer ice extent that has occurred in the

Arctic Ocean over recent years has contributed to the renewed interest in this subject. Continuum models have recently gained popularity to describe wave propagation in various types of ice cover and across a wide range of length scales. In this talk, we propose a continuum wave-ice model where the floating ice cap is described as a homogeneous poroelastic material and the underlying ocean is viewed as a slightly compressible fluid. The linear dispersion relation for time-harmonic wave solutions of this coupled system is established and compared to predictions from existing theories. (Received July 26, 2018)

1141-76-152 **Malik Chabane\*** ([mcc38@njit.edu](mailto:mcc38@njit.edu)), Dept. of Mathematical Sciences, University Heights, Newark, NJ 07102, and **Wooyoung Choi**. *On resonant interactions of gravity-capillary waves without energy exchange.*

We consider resonant triad interactions of gravity-capillary waves and investigate in detail special resonant triads that exchange no energy during their interactions so that the wave amplitudes remain constant in time. Using the amplitude equations, it is shown that any resonant triad can interact without energy exchange if the initial wave amplitudes and relative phase satisfy the two conditions for fixed point solutions of the amplitude equations to exist. Furthermore, it is shown that the symmetric resonant triad exchanging no energy forms a transversely-modulated traveling wave field, which can be considered a two-dimensional generalization of Wilton ripples. (Received July 26, 2018)

1141-76-213 **Arkadz Kirshtein\*** ([azk194@psu.edu](mailto:azk194@psu.edu)), 109 McAllister Building, University Park, PA 16802, **Chun Liu** ([c1iu124@iit.edu](mailto:c1iu124@iit.edu)), Rettaliata Engineering Center, Room 208, 10 W. 32nd St., Chicago, IL 60616, and **James Brannick** ([azk194@psu.edu](mailto:azk194@psu.edu)), 109 McAllister Building, University Park, PA 16802. *Modelling and computations of ternary fluid flow.*

In this talk I will introduce the systematic energetic variational approach for dissipative systems applied to multi-component fluid flows. These variational approaches are motivated by the seminal works of Rayleigh and Onsager. The advantage of this approach is that we have to postulate only energy law and some kinematic relations based on fundamental physical principles. The method gives a clear, quick and consistent way to derive the PDE system. I will discuss different approaches to three-component flows using diffusive interface method. The diffusive interface method is an approach for modeling interactions among complex substances. The main idea behind this method is to introduce phase field labeling functions in order to model the contact line by smooth change from one type of material to another. Further I will introduce an efficient energy stable numerical method for the introduced system. (Received July 30, 2018)

1141-76-222 **Fengyan Shi\*** ([fyshi@udel.edu](mailto:fyshi@udel.edu)), Center for Applied Coastal Research, Department of Civil and Environmental Engineering, Newark, DE 19716, **Jian Shi** ([fyshi@udel.edu](mailto:fyshi@udel.edu)), Nanjing, Jiangsu 210098, and **James T Kirby**, Newark, DE 19716. *Interplay between grid resolution and pressure decimation in non-hydrostatic modeling of internal waves.*

The Pressure Decimation and Interpolation (PDI) method is an effective numerical technique to improve the computational efficiency of a non-hydrostatic model. A model configuration associated this method was usually based on trial-and-error, the effects of the configuration on wave dispersion properties, numerical dissipation and diffusion are not well explored. In this study, we carried out an analysis of the wave dispersion relation and quantitative measurements of numerical dissipation and diffusion affected by the PDI scheme in modeling of internal waves. The linear analysis on the wave dispersion relation shows that the accuracy in predicting wave dispersion highly depends on how well the vertical structure of non-hydrostatic pressure is represented over depth. The accuracy is not only dependent on the resolution of the pressure grid, but also on the interpolation method which is used to interpolate the non-hydrostatic pressure from the coarse pressure grid onto the fine velocity grid. (Received July 30, 2018)

1141-76-233 **Wouter Mostert\*** ([wmostert@princeton.edu](mailto:wmostert@princeton.edu)), Mechanical & Aerospace Engineering, Princeton University, Princeton, NJ 08544, and **Luc Deike** ([ldeiße@princeton.edu](mailto:ldeiße@princeton.edu)), Mechanical & Aerospace Engineering, Princeton University, Princeton, NJ 08544. *Nonlinear breaking mechanism and energetics in shallow-water breaking waves.* Preliminary report.

We present numerical results of breaking waves in shallow water. Using solitary wave solutions as initial conditions to a full Navier-Stokes simulation with two-phase flow, we aim to characterize the nonlinear breaking mechanism on the one hand, and dissipative processes in the energetics on the other, as functions of the essential parameters of the initial solitary wave and the bathymetric slope. We present preliminary results along these lines and discuss the role of potential dissipative terms to be added in the shallow water models. (Received July 30, 2018)

1141-76-234 **Anand U Oza\*** (oza@njit.edu), and **Eva Kanso** and **Michael Shelley**. *Traveling waves in a continuum model of 1D schools.*

We construct and analyze a continuum model of a 1D school of flapping swimmers. Our starting point is a delay differential equation that models the interaction between a swimmer and its upstream neighbors' wakes, which is motivated by recent experiments in the Applied Math Lab at NYU. We coarse-grain the evolution equations and derive PDEs for the swimmer density and variables describing the upstream wake. We study the equations both analytically and numerically, and find that a uniform density of swimmers destabilizes into a traveling wave. Our model makes a number of predictions about the properties of such traveling waves, and sheds light on the role of hydrodynamics in mediating the structure of swimming schools. (Received July 30, 2018)

1141-76-261 **Quentin Brosseau\*** (qb3@nyu.edu), 251 Mercer Street, New York, NY 10012, and **Florencio Balboa-Usabiaga**, **Enkeleida Lushi**, **Yang Wu**, **Leif Ristorph**, **Jun Zhang**, **Mike Ward** and **Mike Shelley**. *Dynamics and interactions of asymmetric bimetallic microswimmers.*

We explore the impact of loss of symmetry in bimetallic Au-Pt rod-like microswimmers. These swimmers are known to exhibit complex individual and collective behaviors. As a proxy for change in swimmer type, e.g. pushers and pullers, we conduct experiments on swimmers with different relative lengths of their two metallic segments. We model the rods' reactive region as a region of fluid slip. Numerical simulations show that a non-centered position of the slip-region along the rod allow for a transition from an extensile to contractile force dipole in the disturbance fluid flow. The changes in the generated flow field, which affect interactions with other rods and boundaries, are here evidenced by the analysis of the swimmers rheotactic abilities and their motion near obstacles. (Received July 31, 2018)

1141-76-283 **R J Braun\*** (rjbraun@udel.edu), Dept of Mathematical Sciences, University of Delaware, Newark, DE 19716, **Lan Zhong** (lanzhang@udel.edu), Dept of Mathematical Sciences, University of Delaware, Newark, DE 19716, **T A Driscoll** (driscoll@udel.edu), Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, **C G Begley** (cbegley@indiana.edu), Indiana University School of Optometry, 800 E. Atwater Avenue, Bloomington, IN 47405, and **P E King-Smith** (king-smith.1@osu.edu), College of Optometry, The Ohio State University, 338 W 10th Ave Room 558, Columbus, OH 43210. *Models for Tear Film Breakup (TBU).*

This talk is about mathematical models for what happens to the thin layer of liquid, the tear film, that forms on the front of the eye as a result of a blink. The tear film is important for ocular surface health, but it can fail in localized areas of TBU. The conditions in TBU are thought to be quite adverse for the ocular surface but they cannot be measured with current technology. We discuss mathematical models and computational approaches for computing the conditions in two different types of TBU. The conditions in the two kinds of TBU are quite different, contrary to what many believe in the ophthalmic and optometric communities. (Received July 31, 2018)

1141-76-291 **Fabrice Veron\*** (fveron@udel.edu), University of Delaware, Newark, DE 19716, and **Luc Mieussens**, Bordeaux, France. *A kinetic approach to estimate air-sea exchanges driven by sea spray in high winds.*

Sea-spray is known to be a fundamental component of this air-sea heat flux in high wind speed conditions where water drops are ejected from the sea surface because of breaking waves and breaking related phenomena such as bubble entrainment and whitecaps. Once ejected from the ocean surface, these drops are transported and dispersed in the Atmospheric Boundary Layer (ABL) where they interact and exchange momentum, heat, and moisture with the ambient atmosphere. However, understanding of these spray fluxes pathways, and our ability to model them remains limited.

In this work, we borrow the framework from established kinetic gas theory, and apply these mathematical tools to model the transport of spray droplets and the exchanges of heat, momentum, and moisture between the drops and the atmosphere. With this framework, one of the most vexing component of this spray flux problem, i.e. the feedback from the drops on the atmosphere, is relatively straightforward to account for.

This work is largely exploratory and in the early stages of development. We will present a brief overview of the approach as well as preliminary results. (Received July 31, 2018)

## 81 ► Quantum theory

1141-81-63 **Brad Lackey\*** (bclackey@umd.edu) and **Nishant Rodrigues** (ngrodrig@umd.edu).  
*Nonlocal games with synchronous correlations.*

A nonlocal game with a synchronous correlation is the ideal protocol for quantum key distribution. In this work we examine analogues of Bell's inequalities for synchronous correlations. We prove that unlike in the nonsynchronous case (e.g. with the CHSH inequality) there can be no quantum Bell violation among synchronous correlations with two measurement settings. However we exhibit explicit analogues of Bell's inequalities for synchronous correlation with three measurement settings and two outputs that do admit quantum violations. We also provide an analogue of Tsirl'son's bound in this setting, exhibit this bound is sharp, and prove rigidity of the quantum states achieving the bound. (Received July 15, 2018)

1141-81-84 **David de Laat\*** (mail@daviddelaat.nl), **Sander Gribling** and **Monique Laurent**.  
*Entanglement dimensions and noncommutative polynomial optimization.*

Quantum entanglement allows for nonclassical correlations between distant parties. One measure for the necessary amount of entanglement is the smallest possible dimension of the quantum state used to obtain the correlation. In this talk I will discuss the minimal average entanglement dimension, which measures the minimal entanglement dimension in the situation where the parties have unlimited access to shared randomness. I will explain how techniques from noncommutative polynomial optimization can be used to approximate this number by a sequence of semidefinite programming problems. (Received July 18, 2018)

1141-81-86 **M. Lupini** (lupini@caltech.edu), **L. Mancinska** (mancinska@math.ku.dk), **V. I. Paulsen** (vpaulsen@uwaterloo.ca), **D. E. Roberson** (davidroberson@gmail.com), **G. Scarpa\*** (gscarpa@ucm.es), **S. Severini** (s.severini@ucl.ac.uk), **I. G. Todorov** (i.todorov@qub.ac.uk) and **A. Winter** (andreas.winter@uab.cat). *Perfect strategies for non-signalling games.*

We unify and consolidate various results about non-signalling games, a subclass of non-local two-player one-round games, by introducing and studying new families of games and establishing general theorems about them, which extend a number of known facts in a variety of special cases. Among these families are *reflexive games*, which are the hardest non-signalling games that can be won using a given set of strategies. We introduce *imitation games*, in which the players display linked behaviour, and which contains as subclasses variable assignment games, binary constraint system games, synchronous games, many games based on graphs, and *unique* games. We associate a C\*-algebra to any imitation game and show that its properties characterise the existence of perfect quantum commuting (resp. quantum, local) strategies, extending known results about synchronous games. We single out a subclass of imitation games, which we call *mirror games*, and provide a characterisation of their quantum commuting strategies that has an algebraic flavour, showing that their approximately quantum perfect strategies arise from amenable traces on the encoding C\*-algebra. We describe the main classes of non-signalling correlations in terms of states on operator system tensor products. (Received July 18, 2018)

1141-81-240 **Iana I Anguelova\*** (anguelovai@cofc.edu), Department of Mathematics, College of Charleston, Charleston, SC 29424. *Boson-fermion correspondences, bicharacter construction and vacuum expectation values identities.*

One of the aspects of any boson-fermion correspondence is the vacuum expectation values identities resulting from such a correspondence. In this talk we discuss how to use the Borchers bicharacter construction of vertex algebras and the associated formulas for the vacuum expectation values, to prove identities such as the Borchardt's determinant-permanent Identity and its pfaffian-hafnian analogue. (Received July 30, 2018)

1141-81-270 **Se-Jin Kim**, **Vern I. Paulsen** and **Christopher Schafhauser\***  
(cschafhauser@uwaterloo.ca). *A synchronous game for binary constraint systems.*

Recently, W. Slofstra proved that the set of quantum correlations is not closed. We prove that the set of synchronous quantum correlations is not closed, which implies his result, by giving an example of a synchronous game that has a perfect quantum approximate strategy but no perfect quantum strategy. We also exhibit a graph for which the quantum independence number and the quantum approximate independence number are different. We prove new characterizations of synchronous quantum approximate correlations and synchronous quantum spatial correlations. We solve the synchronous approximation problem of Dykema and the second author, which yields a new equivalence of Connes' embedding problem in terms of synchronous correlations. (Received July 31, 2018)

## 82 ► *Statistical mechanics, structure of matter*

1141-82-121 **C. Boutillier** and **Zhongyang Li\*** ([zhongyang.li@uconn.edu](mailto:zhongyang.li@uconn.edu)), 341 Mansfield Road, Unit 1009, Storrs, CT 06269. *Limit shape and height fluctuations of perfect matchings on square-hexagon lattice.*

We study asymptotics of perfect matchings on a large class of graphs called the contracting square-hexagon lattice, which is constructed row by row from either a row of a square grid or a row of a hexagonal lattice. We assign the graph periodic edge weights with period  $1 \times n$ , and consider the probability measure of perfect matchings in which the probability of each configuration is proportional to the product of edge weights. We show that the partition function of perfect matchings on such a graph can be computed explicitly by a Schur function depending on the edge weights. By analyzing the asymptotics of the Schur function, we then prove the Law of Large Numbers (limit shape) and the Central Limit Theorem (convergence to the Gaussian free field) for the corresponding height functions. We also show that the distribution of certain type of dimers near the turning corner is the same as the eigenvalues of Gaussian Unitary Ensemble, and explicitly study the curve separating the liquid region and the frozen region for certain boundary conditions. (Received July 23, 2018)

1141-82-267 **Peter Nandori\***, [pnandori@math.umd.edu](mailto:pnandori@math.umd.edu). *Toward the rare interaction limit in hard ball models.*

An important problem in Statistical Mechanics is to derive basic laws of Physics (such as heat equation) from microscopic deterministic dynamics. I will review some recent preliminary results in simple models, mostly involving elastic collisions between rigid balls. I will also discuss the hydrodynamic limit in related stochastic models, and mention how I expect to recycle some ideas in the deterministic world. The talk assumes no background knowledge. (Received July 31, 2018)

## 86 ► *Geophysics*

1141-86-88 **Helga S. Huntley\*** ([helgah@udel.edu](mailto:helgah@udel.edu)), **A. D. Kirwan**, **Henry Chang** and **Gregg Jacobs**. *Lagrangian Evolution of Kinematic Properties of Ocean Surface Currents Across Scales*. Preliminary report.

Regional ocean models are generally considered reasonably reliable: They are able to reproduce observed features of ocean flows. Necessarily, models are limited by their spatial resolution. For Eulerian evaluations at sufficiently large scales, this can often be ignored. Water particles, however, are subjected to the currents at all scales, and dispersion at small scales can often impact the larger scale distribution patterns as well. Thus, the question arises how significant the small-scale variability of divergence and strain is.

We break this question into two components. First, we analyze the trajectories of a large group of surface drifters deployed in the northern Gulf of Mexico. The use of rapidly deployed, dense grids of drifters permits the efficient estimation of the kinematic properties (in particular divergence and strain rate) at a range of scales and how these evolve following the particles.

In a second step, we consider a state-of-the-art regional ocean model forecast for the same area. Its finite resolution means that the variability in the kinematic properties is dampened for small scales. We explore how this impacts the statistics of the kinematic properties at larger scales and whether these differences matter for drift predictions. (Received July 18, 2018)

## 91 ► *Game theory, economics, social and behavioral sciences*

1141-91-146 **Igor Erovenko\*** ([igor@uncg.edu](mailto:igor@uncg.edu)) and **Mark Broom**. *Public goods game in mobile populations on stochastic multiplayer networks.*

We create stochastic simulations of a finite evolving population of individuals on a network. Individuals move around the network following a Markov process and interact with each other via a public goods game. We investigate how the population size, movement cost, exploration time, and network structure affect the evolution of cooperation. This modeling framework allows to extend the analytic approach of Pattni, Broom, and Rychtar (2018) for complete graphs to arbitrary networks. (Received July 26, 2018)



## 92 ► *Biology and other natural sciences*

- 1141-92-22 **Hye Won Kang\*** ([hwkang@umbc.edu](mailto:hwkang@umbc.edu)). *Multiscale stochastic reaction-diffusion algorithms for biochemical networks.*  
 A Markov chain model has become popular to present the discrete nature of the molecular copy numbers and inherent stochasticity in reaction-diffusion systems, but its computation can be expensive. A possible approach to reduce computational cost is to approximate a part of the model by some coarse-grained methods. In this talk, I will introduce two multiscale algorithms coupling the suitably discretized stochastic partial differential equations (SPDEs) and the Markov chain model, which provide good approximations to the solutions obtained by the Markov chain model applied in the entire spatial domain. Two coupling methods of the Markov chain model and the SPDEs across the interface will be discussed. This is joint work with Radek Erban at the University of Oxford. (Received June 15, 2018)
- 1141-92-51 **Enkeleida Lushi\*** ([elushi@flatironinstitute.org](mailto:elushi@flatironinstitute.org)), Newark, NJ 07102. *Motion of micro-swimmers in confinement.*  
 Interactions between motile microorganisms and solid boundaries play an important role in many biological and technological processes. I will discuss recent advances in experiments and simulations that aim to understand the motion of micro-swimmers such as bacteria, micro-algae or spermatozoa in confinements or structured environments. Our results highlight the complex interplay of the fluidic and contact interactions of the individuals with each-other and the boundaries to give rise to intricate behavior. (Received July 10, 2018)
- 1141-92-80 **Felix Xiaofeng Ye\***, 307 Davage Ln, Towson, MD 21286. *Dynamic Looping of a Free-Draining Polymer.*  
 We revisit the celebrated Wilemski-Fixman (WF) treatment for the looping time of a free-draining polymer. The WF theory introduces a sink term into the Fokker-Planck equation for the  $3(N+1)$ -dimensional Ornstein Uhlenbeck process of the polymer dynamics, which accounts for the appropriate boundary condition due to the formation of a loop. The assumption for WF theory is considerably relaxed. A perturbation method approach is developed that justifies and generalizes the previous results using either a Delta sink or a Heaviside sink. For both types of sinks, we show that under the condition of a small dimensionless  $\epsilon$ , the ratio of capture radius to the Kuhn length, we are able to systematically produce all known analytical and asymptotic results obtained by other methods. This includes most notably the transition regime between the  $N^2$  scaling of Doi, and  $N^{3/2}/\epsilon$  scaling of Szabo, Schulten, and Schulten. The mathematical issue at play is the non-uniform convergence of  $\epsilon \rightarrow 0$  and  $N \rightarrow +\infty$ , the latter being an inherent part of the theory of a Gaussian polymer. Our analysis yields a novel term in the analytical expression for the looping time with small  $\epsilon$ , which is previously unknown. (Received July 17, 2018)
- 1141-92-96 **Virginia Eirini Kilikian\*** ([eirkili@udel.edu](mailto:eirkili@udel.edu)), 15 Orchard Rd, Ewing Hall, Newark, DE 19716. *An ODE model for the aerotactic response of the Caulobacter Crescentus at the motile stage.* Preliminary report.  
 Caulobacter Crescentus is a flagellated motile swarmer cell in the initial stage of its cell cycle. The direction of rotation (clockwise or counter-clockwise) of the flagellum motor determines the swimming behavior of the cell (forward or reverse running). We suggest a mathematical model for the signaling pathway of the motor switches, in response to a temporally varying concentration of oxygen. (Received July 20, 2018)
- 1141-92-97 **Pavol Bokes\*** ([pavol.bokes@fmph.uniba.sk](mailto:pavol.bokes@fmph.uniba.sk)), Department of Applied Mathematics, and Statistics, Comenius University, 84248 Bratislava, Slovak Rep, and **Michal Hojcka** and **Abhyudai Singh.** *Effects of feedforward regulation by microRNA on mRNA copy-number distribution.*  
 Cells use various regulatory motifs, including feedforward loops, to control the intrinsic noise that arises in gene expression at low copy numbers. Here we study one such system, which is broadly inspired by the interaction between an mRNA molecule and an antagonistic microRNA molecule encoded by the same gene. The two reaction species are synchronously produced, individually degraded, and the second species (microRNA) exerts an antagonistic pressure on the first species (mRNA). Using linear-noise approximation, we show that the noise in the first species, which we quantify by the Fano factor, is sub-Poissonian, and exhibits a nonmonotonic response both to the species lifetime ratio and to the strength of the antagonistic interaction. Additionally, we use the Chemical Reaction Network Theory to prove that the first species distribution is Poissonian if the first species is much more stable than the second. Finally, we identify a special parametric regime, supporting a broad range of behaviour, in which the distribution can be analytically described in terms of the confluent hypergeometric

limit function. We verify our analysis against large-scale kinetic Monte Carlo simulations. (Received July 20, 2018)

1141-92-108

**Daniel B Cooney\*** (dcooney@math.princeton.edu), Program in Applied and Computational Math, Princeton University, Fine Hall, Floor 2, Princeton, NJ 08540. *The Replicator Dynamics for Multilevel Selection in Evolutionary Games.*

We consider a stochastic model for evolution of group-structured populations in which selection operates at two organization levels: individuals compete with individuals in their group, while groups compete with other groups. Payoff is obtained from the Prisoner's Dilemma or the Hawk-Dove game. In the limit of infinite population size, we derive a non-local PDE describing the probability distribution of groups in the population. We characterize the long-time behavior of our system, with an emphasis on understanding the most frequent group compositions at steady state.

When average payoff of groups is maximized by all-cooperator groups, steady state composition ranges from all-defector groups when individual-level selection dominates to all-cooperator groups when group-level selection dominates. When group payoff is maximized by a mix of cooperators and defectors, then the steady state features a fewer cooperators than required for the mix optimizing group payoff, even in the limit where group-level selection is infinitely stronger than individual-level selection. In such cases, the conflict between the two levels of selection cannot be decoupled, and cooperation cannot be sustained in when between-group competition favors perfect coexistence of cooperators and defectors. (Received July 21, 2018)

1141-92-111

**Lauren M Childs\*** (lchilds@vt.edu), Department of Mathematics, Virginia Tech, Blacksburg, VA 24061, and **Olivia Prosper**, Department of Mathematics, University of Kentucky, Lexington, KY. *Stochastic model of within-mosquito generation of malaria parasite diversity.*

*Plasmodium falciparum*, the malaria parasite causing the most severe disease in humans, undergoes an asexual stage within the human host, and a sexual stage within the mosquito host. Because mosquitoes may be superinfected with parasites of different genotypes, this sexual stage of the parasite life-cycle presents the only opportunity in the full life cycle for genetic mixing of parasites. To investigate the role that mosquito biology plays for parasite diversity, we constructed a stochastic model of parasite development within the mosquito, generating a distribution of parasite densities at five parasite stages over the lifespan of a mosquito. We then coupled a stochastic model of sequence diversity generation via recombination and reassortment between genotypes to the population model. Our two-part stochastic model framework shows that early bottlenecks decrease diversity from the initial parasite population in a mosquito's blood meal, but diversity increases later in the parasite life cycle within the mosquito. Furthermore, beginning from only two distinct parasite genotypes, the probability of transmitting more than two unique genotypes from mosquito to human is high ( $> 65\%$ ) for many realistic infecting parasite densities. (Received July 22, 2018)

1141-92-116

**J M Conway\***, jmconway@psu.edu, and **T T Immonen, C Deleage and B F Keele.** *SIV infection dynamics in vaginal tissues.*

Understanding the events that occur following HIV exposure is critical to the development of interventions. Recent nonhuman primate (NHP) experiments with SIV from the Keele Lab shed light on the dynamics of early viral replication and development of systemic dissemination following vaginal exposure. Importantly, the infecting inoculum was a synthetic viral swarm permitting distinction between genotypically separate, but phenotypically identical variants. We are interested in using this dataset with modeling to gain insight into replication dynamics in local tissues and systemic infection. We first focus on events preceding systemic infection. The data reveals that SIV form foci of local infection in the female genital tract (FGT). Viral variants from multiple viral lineages were found within each focus of infection in the FGT. We will discuss results from our investigation, in which we model viral dynamics in the FGT via multitype continuous time branching processes, using observations of viral variants in foci to validate model predictions. We will explore in particular the role of the infected cell burst size, and use model-predicted time-dependent extinction probabilities, to examine the survival of viral lineages in the tissues during systemic dissemination. (Received July 23, 2018)

1141-92-136

**Yao Li\*** (yaoli@math.umass.edu), 710 N. Pleasant Street, Amherst, MA 01003, and **Hui Xu** (xuhui\_lisa@163.com), 31 Quadrangle Drive, Amherst, MA 01002. *Multiple firing events in a spiking neural field model.*

The multiple firing event (MFE) is an emergent dynamical behavior appearing in many neural network models, in which neuronal spikes are partially synchronized. It is believed that such partial synchronization is related to gamma oscillations. In this talk I will report some recent progress based on the stochastic neural field model

developed in my joint work with Chariker and Young. (See Logan Chariker's talk for details). We extend this model to a spatially heterogeneous setting to study the spatial and temporal properties of multiple firing events. Then we mathematically justify the mechanisms of a number of phenomena, including the formulation of MFEs, the spatial correlation of MFEs, and the decay of spatial correlations. Our result is consistent with experimental results that the gamma oscillation is a relative local phenomenon. This is joint work with my undergraduate student Hui Xu. (Received July 25, 2018)

1141-92-137 **Yao Li** and **Logan Chariker\*** (bortkiew@gmail.com), 1 Washington Square Village, Apt 4G, New York City, NY 10012, and **Lai-Sang Young**. *How well do reduced models capture the dynamics in models of interacting neurons?*

This talk investigates emergent dynamics of local neuronal populations in cortex using stochastic models of varying complexity levels. First, we introduce a class of models of interacting neurons which encodes a reasonable degree of biological realism and exhibits emergent dynamics as seen in cortex while remaining mathematically tractable. For example, existence and uniqueness of nonequilibrium steady states can be rigorously proven. Next, these network models are compared to very simple reduced models driven by the same mean excitatory and inhibitory currents. Discrepancies in firing rate between the network and reduced models are investigated and explained by correlations in spiking, or partial synchronization, working in concert with nonlinearities in the time evolution of membrane potentials. The use of simple random walks and their first passage times to simulate fluctuations in neuronal membrane potentials and interspike times is also considered.

This is joint work with Yao Li and Lai-Sang Young. (Received July 25, 2018)

1141-92-162 **YN Young\*** (yyoung@njit.edu), 519 Cullimore Hall Dept. Math. Sci., NJIT, University Heights, Newark, NJ 07102, and **Y Mori** and **M Miksis**. *A two-phase flow model for a poroelastic drop actuated with swimming gaits*. Preliminary report.

In this work a two-phase flow model is constructed to study the combined effects of interfacial slip, permeability and elasticity of the porous skeleton inside a viscous drop under simple linear flows. This two-phase flow model describes a viscous fluid filling a deformable elastic skeleton inside a drop whose interface deforms according to the balance of traction on the interface. When the viscous dissipation of the interior porous flow is negligible (compared to the friction between the fluid and the skeleton), the two-phase flow is reduced to a poroelastic Darcy fluid instead. At the interface between such an interior poroelastic fluid and an exterior Stokesian fluid, both slip and permeability are taken into account. The permeating flow induces dissipation that depends on the elastic stress of the interior solid. Actuation on the drop surface gives rise to swimming, and analysis gives insight to possible flow patterns of a system of self-propelling swimmers with porous flow (such as intracellular cytosol) inside. (Received July 27, 2018)

1141-92-250 **Victor Matveev\*** (matveev@njit.edu), University Heights, Newark, NJ 07102. *Mass-Action vs Stochastic Simulations of Ca<sup>2+</sup> Dependent Vesicle Release*. Preliminary report.

Synaptic vesicle exocytosis is known to exhibit high variability. Stochastic Ca<sup>2+</sup> channel gating is a major source of this stochasticity, as are fluctuations in the Ca<sup>2+</sup> binding to buffers and sensors. This leads to a widely-held assumption that the mass-action approach does not provide sufficient accuracy for modeling Ca<sup>2+</sup>-dependent processes. However, recent comparative studies showed a surprising close agreement between deterministic and stochastic simulations of vesicle exocytosis when Ca<sup>2+</sup> channel gating is not Ca<sup>2+</sup> dependent. We present a deeper analysis and comparison of stochastic and mass-action simulations, focusing on Ca<sup>2+</sup> dynamics downstream of Ca<sup>2+</sup> channel gating. We show that the discrepancy between the two approaches can be surprisingly small even when only as few as 30 ions enter per single channel-vesicle complex. We argue that the accuracy of mass-action approach is determined by the size of correlations between reactant molecule numbers rather than their variance, explaining this close agreement. Further, contrary to naive intuition, mass-action description provides an estimate of the release time probability density, and not just its first moments. Supported by NSF DMS-1517085. (Received July 31, 2018)

1141-92-257 **Cheng Ly\*** (cly@vcu.edu), 1015 Floyd Avenue, P.O. Box 843083, Richmond, VA 23284, and **Andrea K Barreiro**, Dallas, TX. *Practical approximation method for firing rate models of coupled neural networks with correlated inputs*.

Rapid experimental advances now enable simultaneous experimental recordings of biological activity at single-cell resolution across large regions. Models of this network activity will necessarily increase in size and complexity, thus increasing the computational cost of simulating them and the challenge of analyzing them. Here we present a novel method to approximate the activity and firing statistics (1<sup>st</sup> and 2<sup>nd</sup> order) of a general neural network

model. The method requires solving a system of transcendental equations and is fast compared to Monte Carlo simulations of coupled stochastic differential equations. We implement the method with several examples of coupled neural networks and show that the results are quantitatively accurate even with moderate coupling strengths and an appreciable amount of heterogeneity in many parameters. This work should be useful for investigating how various neural attributes qualitatively effect the spiking statistics of coupled neural networks. Matlab code implementing the method is freely available (see my webpage). (Received July 31, 2018)

1141-92-277 **Marcella Torres, Rebecca A. Segal, Shobha Ghosh and Angela M. Reynolds\*** (areynolds2@vcu.edu). *Parameter estimation and sensitivity analysis for a model of peritonitis focusing on the sequential immune cell response.*

Macrophages can be activated to a more inflammatory M1 phenotype or to an M2-like phenotype, which promotes the resolution of inflammation. Problems with this phenotypic switch can result in a population imbalance that leads to chronic wounds or disease. We have developed a model for the sequential influx of immune cells in the peritoneal cavity in response to a bacterial stimulus that includes macrophage polarization. With this model we are able to reproduce the expected timing of sequential influx of immune cells and mediators in a general inflammatory setting. Sensitivity analysis and numerical simulations were used to explore which dynamics give rise to changes in outcome. This model is the core framework for a model of plaque formation in atherosclerosis. (Received July 31, 2018)

## 94 ► *Information and communication, circuits*

1141-94-216 **Kathryn Haymaker\***, kathryn.haymaker@villanova.edu. *Absorbing sets of codes from incidence graphs.*

An absorbing set is a graph substructure that can impact iterative decoding algorithms for codes on graphs. We examine the presence of absorbing sets, fully absorbing sets, and elementary absorbing sets in low-density parity-check codes arising from certain classes of finite geometries. In particular, we prove the parameters of the smallest absorbing sets for finite geometry codes using a tree-based argument. Moreover, we obtain the parameters of the smallest absorbing sets for a special class of codes whose graphs are  $d$ -left-regular with girth  $g$ . (Received July 30, 2018)



# 2010 MATHEMATICS

## SUBJECT

## CLASSIFICATION

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- 00 General
- 01 History and biography
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- 14 Algebraic geometry
- 15 Linear and multilinear algebra; matrix theory
- 16 Associative rings and algebras
- 17 Nonassociative rings and algebras
- 18 Category theory; homological algebra
- 19  $K$ -theory
- 20 Group theory and generalizations
- 22 Topological groups, Lie groups
- 26 Real functions
- 28 Measure and integration
- 30 Functions of a complex variable
- 31 Potential theory
- 32 Several complex variables and analytic spaces
- 33 Special functions
- 34 Ordinary differential equations
- 35 Partial differential equations
- 37 Dynamical systems and ergodic theory
- 39 Difference and functional equations
- 40 Sequences, series, summability
- 41 Approximations and expansions
- 42 Fourier analysis
- 43 Abstract harmonic analysis
- 44 Integral transforms, operational calculus
- 45 Integral equations
- 46 Functional analysis
- 47 Operator theory
- 49 Calculus of variations and optimal control; optimization
- 51 Geometry
- 52 Convex and discrete geometry
- 53 Differential geometry
- 54 General topology
- 55 Algebraic topology
- 57 Manifolds and cell complexes
- 58 Global analysis, analysis on manifolds
- 60 Probability theory and stochastic processes
- 62 Statistics
- 65 Numerical analysis
- 68 Computer science
- 70 Mechanics of particles and systems
- 74 Mechanics of deformable solids
- 76 Fluid mechanics
- 78 Optics, electromagnetic theory
- 80 Classical thermodynamics, heat transfer
- 81 Quantum theory
- 82 Statistical mechanics, structure of matter
- 83 Relativity and gravitational theory
- 85 Astronomy and astrophysics
- 86 Geophysics
- 90 Operations research, mathematical programming
- 91 Game theory, economics, social and behavioral sciences
- 92 Biology and other natural sciences
- 93 Systems theory; control
- 94 Information and communication, circuits
- 97 Mathematics education



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