ABSTRACTS

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Editorial Committee

Abstracts for

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* Indicates who will present the paper at the meeting.

SUBSCRIPTION INFORMATION

PAPERS PRESENTED AT MEETINGS

THIS CALENDAR lists meetings of the Society which have been approved by the Council at which papers may be presented. Programs of Annual Meetings appear in the *Notices* and on the AMS website; programs for sectional meetings appear on the AMS Web pages in the Meetings & Conferences section, and are electronically archived in the *Notices* section on the AMS website.

MEETING #	DATE	PLACE	ABSTRACT DEADLINE	ABSTRACT ISSUE
1164	March 13–14, 2021	Virtual Meeting	January 19	Vol. 42, No. 2
1165	March 20–21, 2021	Virtual Meeting	January 19	Vol. 42, No. 2
1166	April 17–18, 2021	Virtual Meeting	February 16	Vol. 42, No. 2
1167	May 1–2, 2021	Virtual Meeting	February 16	Vol. 42, No. 2
1168	July 5–9, 2021	Grenoble, France	TBA	NONE
1169	July 19–23, 2021	Buenos Aires, Argentina	TBA	NONE
1170	September 18–19, 2021	Buffalo, NY	July 27	Vol. 42, No. 3
1171	October 9–10, 2021	Omaha, NE	August 10	Vol. 42, No. 3
1172	October 23–24, 2021	Albuquerque, NM	August 24	Vol. 42, No. 4
1173	November 20–21, 2021	Mobile, AL	September 21	Vol. 42, No. 4
1174	January 5–8, 2022	Seattle, WA	TBA	TBA
1175	March 11–13, 2022	Charlottesville, VA	January 18	TBA
1176	March 19–20, 2022	Medford, MA	January 18	TBA
1177	March 26–27, 2022	West Lafayette, IN	January 25	TBA
1178	May 14–15, 2022	Denver, CO	March 15	TBA

Virtual Joint Mathematics Meetings, January 6–9, 2021

Abstracts of the 1163rd Meeting.

00 ▶ General

1163-00-16 **Andrea Nahmod*** (nahmod@math.umass.edu), University of Massachusetts Amherst.
Propagation of randomness under the flow of nonlinear dispersive equations.
The study of partial differential equations (PDEs) with randomness has become an important and influential subject in the last few decades. In this talk we focus on the time dynamics of solutions of nonlinear dispersive equations with random initial data. It is well known that in many situations, randomization improves the behavior of solutions to PDEs: the key underlying difficulty is in understanding how randomness propagates under the flow of nonlinear PDEs. In this context, starting with an overview of J. Bourgain's seminal work on the invariance of Gibbs measures for nonlinear Schrödinger equations we describe new methods that offer deeper insights. We discuss in particular the theory of random tensors, a powerful new framework that we developed with Yu Deng and Haitian Yue, which allows us to unravel the propagation of randomness beyond the linear evolution of random data and probe the underlying random structure that lives on high frequencies/fine scales. This enables us to show the existence and uniqueness of solutions to the NLS in an optimal range relative to the probabilistic scaling. A beautiful feature of the solution we find is its explicit expansion in terms of multilinear Gaussians with adapted random tensor coefficients.

We conclude with some future directions and open problems.   (Received July 6, 2020)

1163-00-21 **Linda J. S. Allen*** (linda.j.allen@ttu.edu), Texas Tech University. *Modeling of Viral Zoonotic Infectious Diseases from Wildlife to Humans.*
Zoonotic diseases are infectious diseases transmitted from animals to humans. It is estimated that over 60% of human infectious diseases are zoonotic. Viral pathogens represent a large proportion of emerging and re-emerging infectious diseases, including coronaviruses, ebolaviruses, and orthohantaviruses. Spillover of infection from animals to humans depends on a complex transmission pathway that includes a natural wildlife reservoir and sometimes an intermediate host before transmission to humans. The natural reservoir for SARS and MERS

coronaviruses and ebolaviruses is bats and for orthhantaviruses it is rodents (rats, voles, mice). In this presentation, we discuss a few of the modeling efforts to better understand the spread of infection in the natural reservoir and the spillover to humans. The impacts of demographic and seasonal variations on timing of spillover are discussed as well as public health interventions in the prevention and control of zoonotic infectious diseases. (Received July 2, 2020)

1163-00-24   **Talitha Washington***. *Leveraging Data Science at HBCUs to Advance Innovation*
With the abundance of data, new skills will be needed to prepare students for jobs that do not yet exist. As we develop new data science tools and platforms, innovations in research are needed to advance the field of data science. Historically Black Colleges and Universities (HBCUs) have the unique opportunity to develop novel ways to address ethics and bias issues associated with data science research. As mathematics is a cornerstone of data science, this presentation will share recent advancements in data science at HBCUs and how policy encourages advancement in the data science innovation ecosystem.   (Received June 26, 2020)

1163-00-25   **Chelsea Walton***. *An Invitation to Noncommutative Algebra.*
This talk will delve into the wonderful world of Noncommutative Algebra, primarily discussing the roles of Symmetry, Representations, and Deformations in this area of mathematics. The talk will contain a mix of classical results, the speaker's research contributions, as well as open questions and entertaining anecdotes in this very active area of research. It will be based on the speaker's survey article of the same title, published in the 2019 EDGE program volume for the AWM Springer series.   (Received June 26, 2020)

1163-00-63   **Paul L. Goethals*** (`paul.goethals@westpoint.edu`), Department of Mathematical
                Sciences, United States Military Academy, West Point, NY 10996. *Threat Analytics:*
                Establishing Measures for Security.
The contemporary threats we observe come in a wide array of forms and purpose – external and internal, deliberate and accidental, benign and malignant, and with varying degrees of impact and complexity. The models used to evaluate security threats for analysis and interpretation in defense studies are just as diverse across many of the mathematical sub-fields and sciences. This presentation provides an illustration of the breath of threat analytics for security and defense while outlining the gaps or weaknesses in their formulation or measurement. Several new or emerging constructs for representation within the threat research space are highlighted. Finally, extensions and implications of this work are presented to give the audience an understanding of the nature and direction toward future threat analytics research.   (Received August 04, 2020)

1163-00-213   **Keisha J Cook*** (`kcook7@tulane.edu`), **Sherry Scott** and **Kamal Barley**. *The Effects*
                of Pre-symptomatic Spread on COVID-19 Mortality. Preliminary report.
Reports have shown that the spread of COVID-19 involves individuals passing through a pre-symptomatic stage in addition to the incubation period following infection. We develop a 10-compartment system, including both exposed and exposed but pre-symptomatic individuals. We explore topics related to how pre-symptomatic and asymptomatic individuals affect the mortality population given different scenarios. We are interested in the affects before lockdown, during lockdown, after lockdown, and throughout the entirety of the pandemic. Our model simulations, using the COVID-19 Data Repository from the Center for Systems and Engineering at Johns Hopkins University, show the dynamics of the system when including the pre-symptomatic stage for New York State, Florida, Washington State, and Arizona. With the results, we discuss how to use this information to determine and guide post-lockdown strategies.   (Received August 27, 2020)

1163-00-265   **John A. Rock*** (`jarock@cpp.edu`), 3801 W Temple Ave, Pomona, CA 91768. *BAMM!:*
                Bolstering the Advancement of Masters in Mathematics.
The NSF-funded BAMM! program provides financial support and mentoring for Master's students who wish to pursue a Ph.D. in the mathematical sciences. BAMM! is a fulfilling, cohort-based program in which participants receive annual scholarships of up to $10,000 for a maximum of two years at any of the three BAMM! sites (California State Universities in Fresno, San Francisco, and Pomona). Key features include a supportive community of fellow students and mentors, advanced coursework in the mathematical sciences, research experiences, continual guidance, and opportunities to travel to attend conferences in which students can network and gain experience presenting their results. This presentation will discuss the current status of BAMM!, which is working with its first cohort of scholars in Fall 2020, and lessons learned with a forced transition to a virtual program.   (Received August 31, 2020)

1163-00-295   **Rosalie Dance*** (`radance@gmail.com`). *Tensor-SUMMA: Outreach to Strengthen the Mathematics Community.*

The MAA's Tensor-SUMMA Program supports outreach by college and university mathematics faculty to undergraduates, middle school and high school students from demographic backgrounds under-represented in mathematics: African-American, Latinx, Native American, and Pacific Islander. In this session, we'll look at a few exemplary projects and consider some successful approaches to outreach to the communities we wish to attract to mathematics.   (Received September 01, 2020)

1163-00-334   **Johnny L Houston*** (`jlhouston602@gmail.com`). *James A. Donaldson, PhD, An Internationally Recognized Mathematical Scholar and Academic Leader with Distinction!*

For 50 years, I had the great honor, distinct in deed, to have developed and maintained a cherished professional relationship with Jim Donaldson. It began in January 1969 and lasted until his passing in October 2019. We were contemporaries. We were both born in in 1941 in the Southern part of the USA; we both attended an HBCU; we both earned a PhD degree in mathematics from a Big Ten university; and we both, by choice, decided to spend the majority of our professional careers as faculty and academic leaders at an HBCU. In this presentation I wish to share with the audience some remarkable activities, impactful achievements and recognitions that constitute the Legacy of my Distinguished Colleague, Jim Donaldson, as well as some cherished moments we shared.   (Received September 02, 2020)

1163-00-443   **Edel Reilly*** (`ereilly@iup.edu`). *S-COAM: Scholarships for Creating Opportunities for Applying Mathematics.*

This presentation reports on an ongoing NSF funded project in the Department of Mathematical and computer Sciences at Indiana University of Pennsylvania (IUP). The goals of the project include increasing the number of students graduating with a major, minor, or Master's degree in mathematics; strengthening the academic culture in the department; and strengthening the relationships within the broader STEM community within and beyond the university. Ways in which the goals of the project are being met will be discussed including recruiting strategies used to get students into the program; offering of activities in the form of presentations and workshops to help students prepare for careers the STEM industry; peer-led tutoring sessions to help with academic success in mathematics classes, and monthly meetings in which participants present original research. Data collected from student surveys at the end of each semester will be reported. Finally, the impact of transitioning to online learning as a result of COVID-19 in the middle of a semester on a project that focuses on community development will be shared. In addition, to students' responses to online learning data collected from faculty teaching in STEM disciplines will also be shared.   (Received September 07, 2020)

1163-00-631   **Candice R Price***, 10 Elm St, Smith College, Burton Hall 312, Northampton, MA 01060. *Increasing the Rate of Change: The Impact of Broadening the Visibility of Mathematicians of Color.*

Black Americans, American Indians/Alaska Natives, and Latinx– who have historically comprised a minority of the U.S. population– are growing in size and influence. Currently, while constituting 30 percent of the U.S. population, by 2050, these groups together will account for greater than 40 percent of the U.S. population. Yet, these groups have been historically excluded from the STEM fields—especially mathematics. Lately, there has been a growing discussion around this issue and its effect on the growth and innovation needed in these disciplines to solve the most complex issues humanity faces. My service mission is to support those underrepresented in STEM by creating and supporting programs that increase visibility and amplify the voices of women and people of color while creating networks and community in STEM to provide opportunities and share resources. In this presentation, I will discuss my involvement in programs that are working towards broadening the visibility of mathematicians of color.   (Received September 10, 2020)

1163-00-664   **Bogumił Kamiński*** (`bkamins@sgh.waw.pl`), **Tomasz Olczak** (`tolczak@gmail.com`), **Paweł Prałat** (`pralat@ryerson.ca`) and **François Théberge** (`theberge@ieee.org`). *Artificial Benchmark for Community Detection (ABCD).*

The standard and extensively used method for generating artificial networks is the LFR graph generator. This model has some scalability limitations and it is challenging to analyze it theoretically. Moreover, the main parameter of the model guiding the strength of the communities, has a non-obvious interpretation and so can lead to unnaturally-defined networks. In this talk we provide an alternative random graph model with community structure and power-law distribution for both degrees and community sizes, the Artificial Benchmark for Community Detection (ABCD graph). We show that the new model solves the issues identified above. Indeed, it is fast, simple, and can be easily tuned to allow the user to make a smooth transition between the two extremes:

pure (independent) communities and random graph with no community structure. In the presentation we discuss the properties of the algorithm and its scalability both in serial and parallel implementations. (Received September 11, 2020)

1163-00-673   **JT Halbert***, jthalbe@tycho.ncsc.mil. *Trustworthy Data Science in the Department of Defense.* Preliminary report.

The Office of the Director of National Intelligence (ODNI) published "The AIM Initiative – A Strategy for Augmenting Intelligence using Machines" in 2019. It outlines a strategy and priorities for closing the gap between data collection and decisions. In this talk I will lay out how some very successful data science projects have organized themselves and how we are progressing toward that vision. I will pay particular attention to our focus on data quality management and exploratory analysis as necessary pre-conditions for ML/AI. I will also address the importance of accountability structures around data use and purpose. I will lay out how we structured our most successful teams and how we chose what to work on and when. (Received September 11, 2020)

1163-00-704   **Steve Sashihara*** (ssashihara@princeton.com), 2 Research Way, Princeton, NJ 08540. *The Princeton 20: the Most Common Risk Factors in Implementing Optimization/AI into Production (Spoiler alert: It is usually not the Math).*

Princeton Consultants has over 40 years worked with innovators in many industries to improve their decision-making using advanced analytics. What has historically been called expert systems, decision support, and operations research is today often simply called Artificial Intelligence (AI). During this time, the core techniques have vastly expanded–from bases in statistics, simulation, and mathematical optimization–to include machine learning and new classes of hybrid algorithms. These advances have been multiplied by improvements in supporting hardware.

We have found consistent and repeatable factors that govern whether a given analytics project will make it into successful production. These factors also apply to the latest AI projects. We developed the "Princeton 20" – a list of 10 environmental and 10 technical risk factors to help anticipate issues and increase the likelihood of a successful transition from promising research to production deployment and wide usage. Princeton Consultants CEO Steve Sashihara will discuss these risk factors, and tips and techniques to successfully deploy solutions. (Received September 11, 2020)

1163-00-744   **Jeanette Shakalli*** (jshakall@gmail.com), , Panama. *Math Outreach in Panama During the Pandemic.* Preliminary report.

Due to the pandemic, the Panamanian Foundation for the Promotion of Mathematics (FUNDAPROMAT) had to become more creative to continue fulfilling its mission. Therefore, we started running virtual events since the end of March 2020. These virtual events are free and open to the general public, which means that kids and adults of all ages are welcome to attend. The invited speakers include well-known international mathematicians who love to share their passion for mathematics with a general audience. In this presentation, we will briefly describe the different kinds of virtual events that we have organized. We will also discuss some of the logistics, the marketing strategy and the publicity that are necessary to get people from all over the world to attend, and in general, what has worked and what has not. FUNDAPROMAT's mathematical outreach efforts are an example of extracurricular activities that others might be interested in replicating in their own communities. (Received September 12, 2020)

1163-00-959   **Sowmya Muniswamy*** (sowmyam@rvce.edu.in) and **Aghalaya S Vatsala**. *Mixed Generalized Iterative Method for Nonlinear Caputo Fractional Differential Equations with initial conditions with Applications.*

Qualitative and quantitative study of differential equations with initial and boundary conditions plays an important role in applications, since the dynamic equations arise from modeling problems in science and engineering. Here we develop a methodology to find the solution of nonlinear Caputo fractional differential equations by generalized iterative methods, when the non-linear function is the sum of convex, concave, increasing and decreasing functions, using coupled lower and upper solution. We use generalized iterative methods such as generalized monotone method and/ or generalized quasilinearization method coupled with lower and upper solutions or coupled lower and upper solutions. To start with we will use the natural lower and upper solution to start with, since they are relatively easy to compute. We will use these natural lower and upper solutions to compute the coupled lower and upper solutions. Further, using these coupled lower and upper solution, we will construct monotone sequences on the desired interval, which converge uniformly and monotonically to coupled minimal

and maximal solutions. These minimal and maximal solutions will converge to a unique solution if the nonlinear term satisfies the Lipshitz condition.   (Received September 14, 2020)

1163-00-978   **Elina Gaile-Sarkane** (`elina.gaile-sarkane@rtu.lv`), 6 Kalnciema Street, Riga, LV-1048, Latvia, and **Inese Suija-Markova*** (`inese.suija@videsinstituts.lv`), 6 Kalnciema Street, Riga, LV-1048, Latvia. *Multitasking and Its Effects on an Individual in Study Process.* Preliminary report.
Nowadays, people multitask at work, in their cars, on the street, and while being at home. Multitasking is omnipresent in many work domains such as command and control, information and knowledge work, aviation, medicine, and others. The term "multitasking" originated in the computer engineering industry in the middle of 1960s to refer to the ability of a microprocessor to process several tasks simultaneously. Since then, the term has been applied to study human behaviour. As part of this study, the authors analysed 17 definitions of the term "multitasking" and revealed that they could be clustered along two main dimensions. First, most of the definitions reviewed define multitasking in relation to task concurrency. That implies the human behaviour of doing more than one task in parallel. Second, multitasking is defined in the context of task interconnectedness. The current research study employed an online survey methodology. Within the research multi-tasking behaviour is analysed, positive and negative effects of multitasking are evaluated, multitasking and individual time management orientation is described.   (Received September 14, 2020)

1163-00-985   **Delaram Kahrobaei*** (`dk2572@nyu.edu`). *Quantum-safe E-Voting Schemes.*
A carefully set-up homomorphic encryption scheme can be self-tallying, in that given a specific set of encrypted values, certain summation operation results can be carried out without requiring the decryption key. Such self-tallying homomorphic encryption schemes are quite desirable since they enable certain applications with minimal trust assumptions, e.g.e-voting systems without trusted tallying authorities and privacy-preserving smart metering systems that do not require trusted third parties. However, the security of current self-tallying homomorphic encryption schemes is based on mathematical problems such as the discrete logarithm problem which can be feasibly solved if quantum computers are built; hence, many current schemes are not 'quantum-safe', whereas applications require so-called ever-lasting privacy. Recent proposals for quantum-safe schemes include public-key encryption schemes that use semidirect product of (semi)groups. In this talk, we show how one such scheme can be adapted to provide a self-tallying homomorphic encryption scheme and how our proposed scheme can be used to realise a verifiable e-voting system without tallying authorities and a privacy-preserving smart metering system. This a joint work with Simons (Oxford) and Shahandashti (York).   (Received September 14, 2020)

1163-00-1028   **Michael D Bolt*** (`mdb7@calvin.edu`), 1740 Knollcrest Circle SE, Grand Rapids, MI 49546. *S-STEM: Building Foundations for Success in STEM.* Preliminary report.
The S-STEM program at Calvin is aimed at improving retention in STEM via scholarships to cover a critical funding gap, an implementation of research-based classroom interventions and co-curricular supports, and a leveraging of existing university supports including the Calvin LifeWork program and research and internship opportunities. Each year a scholarship cohort of 15 students participates in an extended first-year seminar and benefit from proactive advising and mentoring. They also are assigned to special sections of introductory courses in computer science, chemistry, and mathematics. This talk is an overview of the program at Calvin including lessons learned during the first year and adjustments made for the second year.   (Received September 14, 2020)

1163-00-1313   **Shelby Stanhope*** (`shelby.stanhope@afacademy.af.edu`). *Cryptography Outreach Activity for High School or Middle School Students.*
In the typical high school curriculum, students are rarely exposed to interesting topics in mathematics outside the realms of algebra, trigonometry, geometry, and calculus. Despite this fact, they already have the mathematical background to explore mathematics beyond these traditional areas. One particularly accessible topic is cryptography. Students can learn to encrypted and decrypt secret messages using ciphers which rely on modular arithmetic. In this presentation, I will talk about a cryptography outreach event held for students at Philadelphia High School for Girls by faculty members, postdocs, graduate, and undergraduate students from Temple University.   (Received September 15, 2020)

1163-00-1381   **Tai Melcher*** (`melcher@virginia.edu`). *Math Ambassadors: training our graduate students to do math outreach.*
We'll discuss ways to incentivize and facilitate graduate student participation in outreach efforts, particularly to encourage them to incorporate outreach as part of their professional identity.   (Received September 15, 2020)

1163-00-1456   **Raymundo Navarrete*** (raymundo@math.arizona.edu). *Multi-Perspective, Simultaneous Embedding.*

We describe MPSE: a Multi-Perspective Simultaneous Embedding method for visualizing high-dimensional data, based on multiple pairwise distances between the data points. Specifically, MPSE computes positions for the points in 3D and provides different views into the data by means of 2D projections (planes) that preserve each of the given distance matrices. We consider two versions of the problem: fixed projections and variable projections. MPSE with fixed projections takes as input a set of pairwise distance matrices defined on the data points, along with the same number of projections and embeds the points in 3D so that the pairwise distances are preserved in the given projections. MPSE with variable projections takes as input a set of pairwise distance matrices and embeds the points in 3D while also computing the appropriate projections that preserve the pairwise distances. The proposed approach can be useful in multiple scenarios: from creating simultaneous embedding of multiple graphs on the same set of vertices, to reconstructing a 3D object from multiple 2D snapshots, to analyzing data from multiple points of view.   (Received September 15, 2020)

1163-00-1550   **Aaron Hutchinson*** (a5hutchi@uwaterloo.ca). *Optimizations of the CSIDH Key Exchange Protocol.*

The Commutative Supersingular Isogeny-based Diffie-Hellman (CSIDH) protocol was proposed in 2018 as a post-quantum key exchange algorithm analogous to the original Diffie-Hellman algorithm. To perform the exchange, CSIDH makes use of isogenies between supersingular elliptic curves defined over a finite field $\mathbb{F}_p$, where $p = 4\ell_1 \cdots \ell_n - 1$ with $\ell_1, \ldots, \ell_n$ being distinct small odd primes. In this talk, we give an overview of the CSIDH algorithm and detail optimization methods which reduce the computational cost associated to computing the isogenies required within the protocol for both parties. In particular, we see how optimal strategies proposed for SIDH can be adapted for use in CSIDH, and we examine how permuting the primes ℓ_i within the protocol can yield increased performance.   (Received September 15, 2020)

1163-00-1584   **Roselyn E. Williams*** (roselyn.williams@famu.edu), 2114 Broad Street, Tallahassee, Florida 32301, Tallahassee, FL 32301. *Celebrating the Service of Dr. James A. Donaldson through the National Association of Mathematicians.*

Through leadership by example, Dr. Donaldson has demonstrated to students, faculty, and mathematicians of all walks of life, the value of scholarship, service, leadership and character. This talk will reflect on Dr. Donaldson's involvement with the National Association of Mathematicians and the positive impact that he had on the growth and development of this Corporation.   (Received September 15, 2020)

1163-00-1678   **Beverly Wright*** (beverly.wright@sc.edu). *Forgotten Origins: The Criticality of Understanding Equations for Data Science.* Preliminary report.

The analytics and data science discipline has grown tremendously in the past few decades. Application of analytics techniques, development of processes and tools, as well as specialized consulting services represent partial manifestation of this growth. However, are we losing sight of the math behind the magic in data science? Our reliance on others to properly program tools that enable a push-button style of data science to solve complex problems could raise concerns for the future of the data science field. Learn the cautions, projections, and suggestions for healthy progression and advancement in data science, with strong acknowledgement of the importance of equations.   (Received September 16, 2020)

01 ► *History and biography*

1163-01-31   **Brittany Anne Carlson*** (bcarl005@ucr.edu). *The Narrative, the Visual, and Material Culture: Mediating the Humanistic Mathematical Discovery Process of the Nineteenth Century.*

This paper offers a brief history of the complex, mediated relationships between the narrative, the visual, and the mathematical in material culture to suggest a human-centered approach to mathematical discovery in the 19th century. This relationship emerges as a rejection of early nineteenth-century math pedagogy, which was steeped in rote memorization and corporal punishment. As the century progressed, pedagogues recognized the importance of active student involvement in their education, which was facilitated by material culture. Beginning with Friedrich Froebel's Gifts and expanding to include Mary Boole's curve stitching, Edwin Abbott's Flatland, and Sundara Row's Geometric Exercises in Paper Folding, I lay out a nineteenth-century historical overview of the role of material culture and mathematical discovery. Through this historical survey, I conclude that the complex relationships between the learner and their cultural capital, the narrative, and the visual work together

to construct a grand narrative of the learner's observations and mathematical understanding. As a result, I argue that it is more productive to think of mathematics in the nineteenth-century as a quantitative literacy with history, culture, and narratives that are shaped by the discovery process. (Received July 10, 2020)

1163-01-60   **Amy Ackerberg-Hastings*** (`aackerbe@verizon.net`). *Analysis and Synthesis in Robert Simson's The Elements of Euclid (1756).*
In the 18th and 19th centuries, three understandings of the terms 'analysis' and 'synthesis' were particularly influential with the creators and readers of elementary geometry textbooks in Western Europe and North America: as perceived contrasts in styles of mathematical practice in Great Britain and France, as contemporary appeals to ancient methods of proof, and as approaches to mathematics education. One widely-used textbook arose from the attempt by University of Glasgow mathematics professor Robert Simson to restore Euclid's text, which appeared in 1756 as *The Elements of Euclid*, in simultaneous English and Latin versions. The talk will explore what we can learn about the book's preparation and reception by examining it through the lenses of analysis and synthesis. (Received August 03, 2020)

1163-01-73   **Glen R Van Brummelen*** (`glen.vanbrummelen@twu.ca`), Faculty of Natural and Applied Sciences, Trinity Western University, 22500 University Drive, Langley, BC V2Y 1Y1, Canada. *The Emergence of Auxiliary Astronomical Tables in Medieval Europe.* Preliminary report.
Auxiliary astronomical tables were a substantial and extensive tradition in medieval Islam, beginning as early as the 9th century. These tables, computing functions that are more complicated than primitive trigonometric quantities but with no direct astronomical application, arise naturally in the context of spherical astronomy where solutions to different problems often share mathematical elements. We are fortunate to have two treatises with the same title — the Tabulae primi mobilis — that allow us to trace the gradual birth of the idea of auxiliary tables in the works of their European inventor, the Italian astronomer Giovanni Bianchini, leading to their fullest realization in his Tabulae magistrales. Repeating the evolution in medieval Islam, one of these original auxiliary tables evolved into what we now call the tangent function. Regiomontanus copied Bianchini's idea in his Tabulae directionum but took the notion much further in his single giant auxiliary table, his Tabula primi mobilis, a table whose idea would be rediscovered several times in following centuries. We shall trace the development of auxiliary tables from its European origin in the 15th century through the end of the 16th century. (Received August 08, 2020)

1163-01-158   **David E. Dunning*** (`david.dunning@maths.ox.ac.uk`). *"Essentially Physical": Deduction, Writing, and the Reflexive Turn in Modern Logic.*
Mathematical logic is a paragon of abstraction; and yet, reflecting in 1938 on his discipline's recent breakthroughs, American logician Emil L. Post insisted on its materiality. "Modes of symbolization and processes of deduction are themselves essentially physical," he wrote, "and hence subject to formulations in a physical science." In this talk I explore the physicality of deduction as experienced and exhibited in the classic reflexive arguments of 1930s mathematical logic. I focus especially on Alan Turing's famous 1937 paper that used the metaphor of a "universal machine" to argue for a negative answer to the decision problem for first-order logic. I cast a comparative eye toward the works of Gödel and Church, and put all these texts in dialogue with Post, their forerunner and also an insightful commentator upon them. In all these milestones of modern logic we find a productive tension between soaring heights of abstraction on one hand and, on the other, attention lavished on the mundane facts of the writing of mathematics. By foregrounding mathematical logicians' efforts to take physicality seriously, we can understand logic's transformation from a discipline that used symbolic systems to one that took such systems as its fundamental concern. (Received August 24, 2020)

1163-01-212   **Brigitte Stenhouse*** (`bstenhouse17@gmail.com`). *Preparing a mathematical translation: Mary Somerville's 1831 Mechanism of the Heavens.*
Mary Somerville's 1831 work, Mechanism of the Heavens, was widely recognised for its importance in bringing analytical mathematics, and its applications to physical astronomy, to wider attention in early-19th-century Britain. The single volume work was ostensibly a translation of Laplace's Traité de Mécanique Céleste, which had been published in 5 volumes between 1799 and 1825. One of the many arguments given for the perception of a decline in British mathematics at the time was the small number of British mathematicians sufficiently literate in analysis and algebra to read and understand Laplace's work. Therefore, when producing her translation Somerville was required to act as both interpreter of the French language, and of the mathematical language and methods employed by Laplace; moreover, she incorporated numerous improvements that had been made since their original publication. I will investigate these changes made by Somerville through a consideration of the

work itself, alongside contemporary reviews and correspondence with John Herschel and John William Lubbock. My investigation will illuminate the state of analysis and physical astronomy in Britain during the early 19th century, and the accessibility of mathematical texts published on the continent at that time.   (Received August 27, 2020)

1163-01-326   **Kevin Lambert***, Klambert@fullerton.edu. *The Malthus Library: The Library as Cognitive Instrument in the Making of the Malthusian Population Principle.* Preliminary report.

Thomas Robert Malthus (1766-1834) is infamous for his argument, made in the first edition of his An Essay on the Principle of Population (1798) that population growth would always outstrip food production. In this paper, I will investigate how Malthus justified his population principle as a universal bio-mathematical law in the larger and more complex later editions of the Essay. Most of the materials Malthus assembled to write the second 1803 edition of the Essay are preserved at the Old Library, Jesus College, Cambridge. Malthus used that assembly of books and maps as an instrument with which to look across the early nineteenth European, American and Pacific worlds. Malthus' close friends, William Otter and Edward Daniel Clarke were key figures in the founding of the first lasting Cambridge scientific society, the Cambridge Philosophical Society (CPS). George Peacock, a fellow CPS founding member, would justify his principle of equivalent forms as the foundation for English algebra through the collection of materials and reports from around the world. I will argue that when Malthus assembled his library of materials, he was employing a characteristically eighteenth- and early nineteenth-century Cambridge mixed-mathematical practice.   (Received September 02, 2020)

1163-01-423   **Maria Zack*** (`mzack@pointloma.edu`), Mathematical, Information & Computer Sciences, 3900 Lomaland Drive, San Diego, CA 92106. *Early Computations on the Cycloid - Some Interesting Puzzles.*

Many well-known mathematicians of the seventeenth and eighteenth centuries studied the cycloid. These include Galileo, Roberval, Descartes, Pascal, Wallis, Huygens, Fermat, Newton and Leibniz and more than one Bernoulli. This talk will consider the quadrature computations of Galileo's student Evangelista Torricelli. The computations were completed sometime before April 1643 when Calvalieri sent a letter to Torricelli congratulating him on the findings. Torricelli's results were published in an appendix in his Opera Geometrica (1644). Some of the computations are relatively easy to follow, but one of them provides an interesting puzzle.   (Received September 06, 2020)

1163-01-456   **Tom Archibald*** (`tarchi@sfu.ca`). *Charles Hermite's Publications in non-French journals.* Preliminary report.

Between his first publication in 1842 and his death in 1901, Hermite had roughly 200 papers in journals (a few were published in two different places). Of these, 117 were published outside France, though all were in the French language. Almost all are research papers, though there are a couple of problem solutions. Many of these are communicated in the older epistolary format, though surely in almost every case the result would have been intended for publication, even if it appears as an excerpt from a letter containing other material.

Thus we have a list of journals, nations or regions with which they are associated, and in some cases individual correspondents. We raise questions about how the journals function in the communities in which they participate, and about Hermite's aims.

In an incipiently international and professional mathematics, we idenify some variety and some shifts in the ways the journals function. By thinking about why Hermite places papers where he does, we hope to get some articulated ideas about the ways in which mathematics journals work and interact during the second half of the nineteenth century.   (Received September 07, 2020)

1163-01-648   **Louise A. Raphael*** (`lraphael@howard.edu`). *James Donaldson: Research Mathematician and Activist.*

James Donaldson founded and led the first PhD program in mathematics at a HBCU - Howard University. His research papers in differential equations appeared in the *AMS Bulletin*, *AMS Proceedings* and the *Proceedings of the National Academy of Sciences.* He was also:

- Dean of Howard's College of Arts and Sciences and Acting President of Lincoln University
- co-founder of NAM
- activist in the Mathematicians Action Group
- AMS Council Member and VP of the MAA.

Highlights of Jim's career will be presented, including:

- recruitment of research mathematicians to join the Howard's math department, and

- range of his earlier research publications on differential equations to his articles on Black Americans in mathematics.

(Received September 10, 2020)

1163-01-650 **Daniel J Fairbanks*** (`daniel.fairbanks@uvu.edu`), 800 W. University Parkway, Orem, UT 84058, and **Bob Palais** (`bob.palais@uvu.edu`), 800 W. University Parkway, Orem, UT 84058. *Gregor Mendel and Combinatorial Mathematics at the DNA Level.*

One of the great mathematical achievements in the history of biology was Gregor Mendel's employment of combinatorial mathematics to derive a theory of inheritance that has stood the test of time. Mendel was student of Andreas von Ettingshausen, who was a renowned physicist and mathematician at the University of Vienna, and also author of the 1826 book, Die combinatorische Analysis, a standard work on combinatorial mathematics. Though Mendel did not know it at the time, his mathematical theory at its most fundamental level is based in the nature of DNA replication, and the segregation and assortment of DNA molecules. This presentation examines the combinatorial mathematical symmetry of Mendel's dihybrid experiments, depicting them, however, at the level of DNA molecules replicating, segregating, and re-assorting.   (Received September 10, 2020)

1163-01-682 **Ellen Abrams*** (`ema85@cornell.edu`), Cornell University, Ithaca, NY. *'Logical Fate' and 'Intellectual Freedom': Cassius Keyser and the Humanism of Mathematics.*

As historians have shown, mathematicians in the early-twentieth-century United States dealt with an ongoing tension between the 'autonomy' garnered through abstract mathematics and the potential to provide a 'service' through teaching and applications. Columbia mathematician Cassius Keyser, however, essentially collapsed this autonomy-service dichotomy by considering abstract mathematics itself as a service to humanity. In this talk, I offer a close reading of Keyser and his ideas in order to examine the historical value and values of American mathematics. In his 1922 Mathematical Philosophy: A Study of Fate and Freedom, Keyser detailed his conceptions of modern mathematics as well as his broader concerns about American culture. On one hand, Keyser worked to defend mathematics from critiques that its modern, axiomatic form had become a lifeless trick of mechanics, detached from both the physical world and human spirit. On the other hand, he used postulate systems and doctrinal functions to define mathematics and to promote its claims to human concern. Perhaps because technoscientific justifications for mathematics became especially powerful in the aftermath of World War II, Keyser's humanistic conception of mathematics, though well-regarded at the time, has since been overlooked. (Received September 11, 2020)

1163-01-687 **Raymond L Johnson*** (`rlj2@rice.edu`). *James Donaldson and the Howard doctoral program.*

A large part of Jim Donaldson's mathematical legacy is the doctoral program at Howard University, which I believe is the first established at a historically black university. I will discuss my interactions with Jim while he was setting up the program, my two years as a faculty member at Howard after the program started and general reflections on Jim's accomplishments.   (Received September 11, 2020)

1163-01-706 **Sloan E Despeaux** (`despeaux@wcu.edu`) and **Robert Manzo*** (`rorobe15@email.unc.edu`). *A New Resource for the History of Mathematics: The Educational Times Online Database of Mathematical Questions and Answers.* Preliminary report.

The *Educational Times* (*ET*) began in 1847. While it initially focused on pedagogical themes, mathematical questions soon infiltrated the monthly journal. The format of the *ET*, in which mathematicians posed questions for others to answer, proved to be a highly dynamic form of communication between mathematicians (both male and female) with diverse backgrounds and training. By 1918, the *ET* had published over 18,000 questions. While these questions and their answers provide an invaluable window inside the inner workings of mathematical communication in Britain, it has been difficult for historians of mathematics to search the journal in any systematic way. However, Jim Tattersall has painstakingly catalogued the contents of the *ET* in spreadsheets. A team then embarked on a multiyear project that converted these spreadsheets into a searchable, online database, now available for use: `http://educational-times.wcu.edu`. In this talk, we will describe the capabilities of the database and give examples of the types of questions this tool can help historians of mathematics explore. (Received September 12, 2020)

1163-01-763   **Abdul-AzizW Yakubu*** (`ayakubu@howard.edu`), Howard University, Department of
Mathematics, 2441 6th Street NW, Howard University, DC 20059. *The Mathematics
Department of Howard University and Legacy Of Dr. James A. Donaldson as Faculty,
Chair and Dean.*
Howard University was fortunate to have had Dr. James A. Donaldson as a faculty member and administrator.
In this memorial talk, I'll discuss my personal reflections on the legacy and positive contributions of Dr. James
A. Donaldson to the Mathematics Department of Howard University, where he served as a faculty member,
Department Chair and Dean of College of Arts and Sciences.   (Received September 12, 2020)

1163-01-766   **Isom H. Herron*** (`herroi@rpi.edu`), Department of Mathematical Sciences, RPI, 110 8th
St., Troy, NY 12180. *Jim Donaldson: A man for all seasons.*
The dictionary of idioms calls a man for all seasons, "a man who is ready to cope with any contingency, and
whose behavior is always appropriate for every occasion". During my more than thirty-year association with
Jim, that to me well describes him. I will mention how his mathematical influencers played an important role in
his vision for the Department of Mathematics at Howard University. I will recount his sense of the importance
of people of the African Diaspora having a place where they could flourish as mathematicians. I will use my own
experiences as a younger colleague of his to illustrate these points.   (Received September 12, 2020)

1163-01-777   **Eisso J Atzema*** (`eisso.atzema@maine.edu`), Department of Mathematics & Statistics,
University of Maine, Orono, ME 04469. *Proving the Converse of Ptolemy's Theorem: A
Case Study of Geometrical Research in the First Half of the 19th Century.* Preliminary
report.
In the course of the first few decades of the 19th century, quadrilaterals were the subject of numerous mathe-
matical publications. The cyclic quadrilateral in particular received much attention. Many of the results on this
special kind of quadrilateral were only minor variations of earlier findings or simplification of the proofs involved.
One exception was the converse of Ptolemy's Theorem, which was a non-trivial extension of a classical result. In
this note, I will outline the history of this converse and the context in which its various proofs were formulated.
Specifically, I will discuss how this result went from its first rather cumbersome proofs to its inclusion in the
standard high school geometry curriculum of the era.   (Received September 12, 2020)

1163-01-820   **Laura E. Turner*** (`lturner@monmouth.edu`), Department of Mathematics, Monmouth
University, 400 Cedar Avenue, West Long Branch, NJ 07764. *Women as data and as
individuals: public dialogues on sexism in mathematics during the 1970s, 1980s, and
1990s.* Preliminary report.
The Association for Women in Mathematics (AWM) came of age in the 1980s, emerging by the early 1990s as
a serious mathematics organization which sought to engage members in a shared effort to improve the status
of women in mathematics. The same period saw ongoing resistance to affirmative action measures within
the mathematical establishment; controversial conclusions regarding gender and science, most notably those
of Benbow and Stanley on sex differences in mathematical reasoning ability; and high-profile claims of sexism
in mathematical practice, including the widely-publicized tenure case of Jenny Harrison. Through the public
dialogues surrounding these episodes and against the backdrop of feminism, we will explore aspects of how the
AWM and its members addressed the complex issue of sexism within mathematical practice, and how deeply-held
and widespread belief in the inability of women to do mathematics, exacerbated by the myths of the canonical
feminine woman and the possibility for objectivity in mathematical practice, simultaneously emphasized the
importance of treating each woman mathematician as an individual, instead of as a representative of her sex,
and left her vulnerable to the very same discrimination the AWM sought to combat.   (Received September 13,
2020)

1163-01-827   **Duncan J Melville*** (`dmelville@stlawu.edu`), Dept. of Mathematics, St. Lawrence
University, Canton, NY 13617. *The Mathematical Papers of RFA Lee.* Preliminary report.
In the early 1800s, Rachel Frances Antonina Lee (1774–1829) spent a considerable amount of time and effort in
the preparation of a "Course of Mathematics". Although the work went through three partial drafts covering
hundreds of pages over a period of several years, it was never completed and never published. Its survival, and
that of her other mathematical papers, is due to accidental and fortuitous circumstances.

We will explain the background and life of the author and discuss the content, both mathematical and
philosophical, of the "Course of Mathematics". Given the absence of any comment on her mathematical education
or attainments in other biographical sources, the survival of such subterranean mathematics suggest historians
should be cautious in our assessment of the range and depth of female mathematical knowledge and interests in
the early 19th century.   (Received September 13, 2020)

1163-01-867   **Johannes C Familton*** (`jfamilton@bmcc.cuny.edu`), 199 Chambers Street, New York,
                   NY 10007. *Olinde Rodrigues' most famous work.*
In 1840 Rodrigues wrote a paper about the laws of geometry that control the displacement of a solid system in space. This work preceded Hamilton's quaternions. Rodrigues' work went unnoticed until 1846 when Cayley acknowledged Euler's and Rodrigues' priority describing orthogonal transformations in a letter to the Editors of the Philosophical Magazine.

This paper was originally written in French. It may be considered Rodrigues' most famous piece of scientific work, yet in the 178 years since its publication, except perhaps for excerpts, it has yet to be translated into English as a full transcript, as far as we know. Dr. Lucio Prado and Dr. Johannes Familton started translating this paper in 2017, but never completed the task. Dr. Richard Friedberg, one of Dr. Familton's former advisors, has now been translating the entire paper. Dr. Familton has been intimately and continuously involved in the process of translation. Through this process we have uncovered just how deep and self-contained the paper is.

In this talk, Dr. Familton will discuss the history of Rodrigues' work, and his 1840 paper. He will also give some of the insights that he and Dr. Friedberg have discussed and discovered as a result of struggling through this paper in its original form.   (Received September 13, 2020)

1163-01-878   **Roger Howe*** (`rogerhowe@tamu.edu`), 308 Harrington Hall, College of Education and
                   Human Development, Texas A&M University, College Station, TX 77843. *Dick Askey and
                   Mathematics Education.*
This talk will review some of the contributions Dick Askey made to the debate on how to improve mathematics education in the US. One of his lasting contributions was to help found, and later to host, the MathEd discussion group, whose membership included a significant number of mathematicians seriously working on mathematics education,as well as some notable mathematics education researchers, and some excellent teachers. Dick also participated in a number of significant debates and events, and some examples will be discussed.   (Received September 14, 2020)

1163-01-919   **Thomas Drucker*** (`druckert@uww.edu`). *Bounded in a Nutshell of Straightedge and
                   Compass.* Preliminary report.
David Richeson's 'Tales of Impossibility' is a recent reminder of the long history of straightedge and compass constructions. Richeson points to the uncertainty surrounding the origin of that practice, but its influence did persist for thousands of years. Even those who came up with constructions that went beyond those tools recognized that this involved 'breaking the rules'. By the time the classical problems were solved in the eighteenth and nineteenth centuries, the limitations were being taken for granted. It can be asked whether there was any philosophical basis for the original selection of those tools, at least based on what we know of Plato's comments on geometry. Then one can speculate on whether there was any residual philosophical basis hundreds or thousands of years later. We shall suggest some philosophical basis for the original specification of those tools and then a time by which that had run its course.   (Received September 14, 2020)

1163-01-942   **Joseph W. Dauben*** (`jdauben@gc.cuny.edu`). *The Students' Lament: Three
                   Multi-Tasking Problems in the Suan shu shu, a Western Han Dynasty Mathematical Work
                   from Ancient China.*
In 1983–1984, a mathematical work dating to the second century BCE was discovered in the tomb of a government bureaucrat excavated from a burial site near Zhangjiashan, in Jiangling county, Hubei Province, China. Archaeologists and historians of mathematics, however, agree that the contents of the *Suan shu shu* in general reflect the state of mathematics in the pre-Qin and primarily that of the Warring States period (475–221 BCE). The *Suan shu shu* is not a printed book but an ancient work written on bamboo strips, which when tied together originally formed a bamboo roll. The recovered pieces thus constitute an authentic document from the very time at which it was created. This presentation will begin with a brief discussion of certain special features of ancient Chinese bamboo texts in general and the challenges researchers face in dealing with these ancient materials. It will then focus on three closely related problems in the *Suan shu shu* that exhibit some especially interesting features, particularly in the methods applied for their solution. In conclusion, questions related to the possible pedagogical significance of these three problems will also be considered.   (Received September 14, 2020)

1163-01-953   **John McCleary*** (`mccleary@vassar.edu`), Department of Mathematics and Statistics,
                   Vassar College, Poughkeepsie, NY 12604-0069. *On what is called the Dehn invariant.*
                   Preliminary report.
In his solution to Hilbert's third problem Dehn considered various quantities associated to a decomposition of a polyhedron into subpolyhedra. What we call today the *Dehn invariant* of a polyhedron lies in an algebraic

object not available to Dehn. I will discuss the contexts of various papers treating scissors congruence from Dehn to the mid 20th-century to follow the evolution of this idea, and in this way consider how the conceptual image of this invariant changed in the hands of various researchers.   (Received September 14, 2020)

1163-01-1093   **Brenda Davison*** (`bdavison@sfu.ca`), 8888 University Drive, Burnaby, B.C. V5A 1S6, Canada. *The Influence of the 1856 Stokes Paper on the Mathematics of Divergent Series.* Preliminary report.

In a paper published in 1856, G.G. Stokes (1819-1903) introduced, via three examples, a method of using divergent series in order to quickly compute the values of convergent integrals. This talk will examine who reponded to this paper in the years immediately following its publication. And which physicists and/or mathematicians adopted this method and what problems that allowed them to solve. The fact that divergent series, while not rigorously understood at this time, were useful focused attention on them in a manner that made it important to understand how to correctly handle them.   (Received September 14, 2020)

1163-01-1100   **Jessica M Otis*** (`jotis2@gmu.edu`). *"The Diverse Wits of Man": Multiplistic Numeracy in Early Modern England.*

The Oxford English Dictionary defines a numerate person as someone "competent in the basic principles of mathematics, esp. arithmetic; able to understand and work with numbers." But for the people of early modern England, this definition would have seemed almost trivial; they believed that God had endowed all humans with the ability to calculate, and those who could not were legally condemned as mentally incompetent. While numbers were thus an almost-universal technology of knowledge, symbolic systems for recording and manipulating numbers were far more diverse. There were performative, object-based, and literate options, and people regularly transcoded between multiple symbolic systems as they went about their daily lives. This talk will examine how the various symbolic systems of early modern England functioned in symbiosis with one another before the widespread introduction of Arabic numerals, and demonstrate that early modern patterns in Arabic numeral adoption are consistent with multiplistic numeracy. Over the course of the sixteenth and seventeenth centuries, English men and women did not adopt Arabic numerals wholesale but rather in a context-specific fashion, employing them side-by-side and interchangeably with other symbolic systems.   (Received September 14, 2020)

1163-01-1110   **Atsuhiro Nakamoto*** (`nakamoto@ynu.ac.jp`), Yokohama National University, 79-1 Tokiwadai, Hodogaya-ku, Yokohama, 2408502, Japan. *Spanning quadrangulations in triangulations.*

Every triangulation on any surface admits a spanning quadrangulation, since every bridgeless cubic graph is known to have a perfect matching. When the surface is a sphere, then the quadrangulation obtained must be bipartite, since every quadrangulation on the sphere is bipartite. However, in general, it seems to be difficult to control the bipartiteness of quadarnglations in a triangulation, and some triangulations do not have bipartite ones. In our talk, we introduce a problem for finding a spanning bipartite quadrangulation in a triangulation on a surface, and describe some result on it. Furthermore, we introduce an attempt to extend our problem to a higher dimension.   (Received September 14, 2020)

1163-01-1116   **Della Dumbaugh*** (`ddumbaugh@richmond.edu`), Math and Computer Science, 221 Richmond Way, Richmond, VA 23173. *"Extensive cooperation with rugged individualism": George Mackey's guide for practitioners of mathematics.*

The mathematician George Mackey (1916-2006) is often remembered for his academic contributions and his methodical, solitary work habits, tempered by an eager affinity for discussing mathematics with all who took an interest. His broad view of the subject inspired his contributions in infinite dimensional group representations, ergodic theory and mathematical physics. He adhered to a disciplined lifestyle that began with focus on his mathematical research each morning. In the afternoons, he would often walk the mile or so to Harvard (to his office or the faculty club for lunch). He ended his days with an early bedtime. He carried a clipboard at all times. He wore a seersucker jacket in warm months and a tweed jacket in cooler ones. For Mackey, the advancement of mathematics hinged on what he described as an "extensive cooperation with rugged individualism." He seemed to protect time for the "rugged individualism" in the morning and foster "extensive cooperation" in the form of teaching and mathematical discussions later in the day. This talk provides an introduction to George Mackey, including the critical geography of his youth that set his mathematical education in motion, and aims to shed new light on the life and contributions of this celebrated American mathematician.   (Received September 14, 2020)

1163-01-1260   **Fern Yvette Hunt*** (`fern.hunt@nist.gov`). *James Donaldson as Faculty Mentor and Activist Ally.*

The past 3 decades have ushered in a vast increase in the number of African American mathematicians and statisticians who have been able to have not just normal careers but even brilliant careers with notable achievements. That is in no small part due to those pioneers in the mathematical sciences who came of age in the 1950's and 1960's. Among such leaders, 6 foot 5 James Ashley Donaldson stands tall. Alongside his roles in the National Association of Mathematics (NAM) and the American Mathematical Society (AMS), Donaldson's most significant achievement was building the Howard Mathematics PhD program after its founding in 1971. We will discuss the years when Hunt was a faculty member in the department and Donaldson was chairman. We next turn to a discussion of Donaldson's participation in the anti- Vietnam war and women's movement and his importance as a bridge between the groups of largely non-minority mathematicians who participated in them and African American mathematicians. We show how the cause of human rights in the mathematics community benefited from this alliance.   (Received September 15, 2020)

1163-01-1268   **Sandro Caparrini*** (`sandro.caparrini@polito.it`), via Tagliamento 6, 10040 Rivalta di Torino, Italy. *An abstract definition of the delta function from the 1830s.*

While today Giovanni Plana (1781-1864) has been forgotten except by specialists, in his time he was considered a leading international expert in theoretical astronomy and mathematical physics. He studied in Paris, at the prestigious Ecole Polytechnique, and during his entire life he remained in contact with the chief exponents of French science. For his achievements, in 1834 he was awarded the Copley Medal by the Royal Society. Late in life, he was given the title of 'Baron' for scientific services rendered to the Kingdom of Sardinia. The Plana Collection of manuscripts consists of more than 5.000 pages of mathematical manuscripts. They are essentially Plana's notebooks. Up to the late 1980s, these documents were stored away, unknown to scholars, in the vaults of the Turin Academy of Sciences. Written over a period of about half a century, from about 1810 to 1864, they reveal the inner workings of the mind of a representative scientist, and the influences that shaped his thought. The manuscripts contain a kind of generalized Fourier transform and an abstract definition of the Dirac delta function, invented by the physicist Paul Dirac in 1929. While, from the modern point of view, the mathematics is hopelessly muddled, this was no mean achievement in the 1830s.   (Received September 15, 2020)

1163-01-1300   **Philip Beeley*** (`philip.beeley@history.ox.ac.uk`), History Faculty, Oxford University, George Street, Oxford, OX1 2RL, United Kingdom. *Dr Gregory's Scheme. Reforming the Instruction of Mathematics at Oxford circa 1700.*

The turn of the eighteenth century witnessed the emergence of a number of initiatives to reform the teaching and examination of mathematics at the University of Oxford. These initiatives responded partly to the success of knights' academies in France and Germany in attracting young members of the English gentry and nobility with a curriculum that included fencing, dancing, law, rhetoric, and mathematics. At the same time, members of Isaac Newton's wider scientific circle such as John Arbuthnot, author of 'An Essay on the Usefulness of Mathematical Learning' contributed to contemporary discussion, while David Gregory devised a scheme for reforming the instruction of mathematics around a sophisticated framework comprising Euclid's Elements (Books I-VI, XI, and XII), plain trigonometry, algebra (including Diophantine equations), and mechanics. The paper will sketch out the largely forgotten background and motivation to these reforms and consider the extent of their success and impact into the early 1700s.   (Received September 15, 2020)

1163-01-1324   **Edray Herber Goins*** (`edray.goins@pomona.edu`), 610 North College Avenue, Claremont, CA 91711. *The Black Mathematician Chronicles: Our Quest to Update the MAD Pages.*

In 1997, Scott Williams (SUNY Buffalo) founded the website "Mathematicians of the African Diaspora," which has since become widely known as the MAD Pages. Williams built the site over the course of 11 years, creating over 1,000 pages by himself as a personal labor of love. The site features more than 700 African Americans in mathematics, computer science, and physics as a way to showcase the intellectual prowess of those from the Diaspora.

   Soon after Williams retired in 2008, Edray Goins (Pomona College), Donald King (Northeastern University), Asamoah Nkwanta (Morgan State University), and Weaver (Varsity Software) have been working since 2015 to update the Pages. Edray Goins led an REU of eight undergraduates during the summer of 2020 to write more biographies for the new MAD Pages.

   In this talk, we discuss the results from Pomona Research in Mathematics Experience (PRiME), recalling some stories of the various biographies of previously unknown African American mathematical scientists, and reflecting on some of the challenges of running a math history REU. This project s funded by the National Science Foundation (DMS-1560394).   (Received September 15, 2020)

1163-01-1467   **Brit Shields*** (bshields@seas.upenn.edu). *Mathematics within an Engineering Education: The Second World War US Engineering, Science, and Management War Training Program.*

During the Second World War, the US Office of Education sponsored the education of American citizens to serve the industrial and engineering demands of the war effort. These emergency training programs were run on college and university campuses across the country. Within the broad range of disciplines, mathematics played an important role as a fundamental subject. This talk will explore how mathematics became an important part of this training program and how the mathematics courses taught through this program created new career opportunities and trajectories for both their instructors and students.   (Received September 15, 2020)

1163-01-1524   **Abram Kaplan*** (adkaplan@fas.harvard.edu), 78 Mount Auburn Street, Cambridge, MA 02138. *Between Scholasticism and Scholarship: Figures and Equations in John Wallis' Treatise on Conic Sections.*

The mathematical investigations of John Wallis (1616-1703) exemplify the productive diversity of intellectual traditions characteristic of seventeenth century learning. I discuss the traditions that inform the innovative figures in Wallis' treatise on conic sections (1655). Wallis wrote the treatise as an introduction to the algebraic study of the conics. Wallis' construction of the conic sections and his reduction of the sections to equations reflect his awareness of contemporary traditions of infinitesimal mathematics and symbolic algebra. They also, I argue, draw on the metaphysical disputations of Francisco Suárez and earlier Renaissance debates about the angle of contact. Wallis used arguments about situs (situation) elaborated by Suárez, and measurement techniques employing it developed by the sixteenth century algebraist Jacques Peletier, to establish the priority of symbolic representations of the conic sections over their geometric, in-diagram instantiations. In other works he used erudite scholarship to explain why Greek mathematicians preferred geometry and to extract ideals of mathematical practice. He then used these same ideals to justify his own use of symbols on grounds he claimed to share with antiquity.   (Received September 15, 2020)

1163-01-1541   **Toke L Knudsen*** (toke.knudsen@oneonta.edu). *Hans Christian Andersen and the Pythagorean Theorem.*

Most people will know at least some of the fairy tales of Hans Christian Andersen (1805–75). They have been translated to some 160 languages and include The Princess and the Pea, The Ugly Duckling, and The Emperor's New Clothes. But it is not at all well known that Andersen in 1831 wrote a poem on mathematics. The poem, entitled Formens evige magie (The Eternal Magic of Form), presents the Pythagorean theorem along with a proof, based on that found in Euclid's Elements. While the poem has been largely isolated from the world of mathematics and the history of mathematics, it is nonetheless a fascinating piece of literature, which tells us a lot about Andersen's mathematical education and attitude to mathematics. The talk will present an English translation of the poem along with a discussion of its context, including Andersen's own recollections of his interactions with mathematics when he was in school.   (Received September 15, 2020)

1163-01-1603   **Eunsoo Lee*** (agwaves@gmail.com). *As shown in the diagram.* Preliminary report.

A formulaic phrase, as shown in the diagram, is a typical channel in modern mathematics through which the text invites the reader to see the visual. This deictic phrase traces back through early modern science at least to the late Middle Ages when some authors pointed out the visual through a similar Latin phrase, "sicut patet in hac figura (as manifest in this figure)." Can we find a corresponding phrase in ancient Greek mathematical science? If so, to what extent did the phrase play a similar role as its modern correspondent? This paper explores how ancient Greek mathematical science convened the visual in the text. To this end, the paper analyzes some spots where ancient mathematical texts incorporated the visual through the phrase, "as in the figure." The case studies in this paper show various aspects of the relationship between the text and the visual in ancient science. Thus, the paper confirms different mindsets in using visuals between ancient and modern mathematics but at the same time questions the simple dichotomy between mentalite par les yeux and par l'oreille.   (Received September 15, 2020)

1163-01-1605   **Julia C Tomasson*** (jct2182@columbia.edu), 523 W 112, Apt 33, New York, NY 10025. *The Lockean Foundations of Newton's Calculus in The Analyst Controversy.*

George Berkeley's (1685-1753) critique of Newton's calculus in his The Analyst: A DISCOURSE Addressed to an Infidel Mathematician (1734) provoked many responses, collectively referred to as "The Analyst Controversy." Understandably, the controversy is often discussed in adversarial terms, all being some variation of "Berkeley vs. Isaac Newton (1643-1727)." In this talk, I present evidence that this debate might be better summarized as "Berkeley vs. John Locke (1632-1704)." I argue that 1) Berkeley is much more sympathetic to Newton and

Newtonianism and much more antagonistic towards Locke than has been previously recognized; 2) Berkeley's critique of the product rule does not reject Newton's results, but rather the basis for accepting them; 3) this basis is thoroughly Lockean (and not Newtonian) and contemporary mathematicians realized this. Understanding a Lockean epistemology of mathematics provides insight into the strange character of eighteenth-century British mathematics, between the invention of the calculus and its rigorization over a century later.   (Received September 15, 2020)

03 ▶ *Mathematical logic and foundations*

1163-03-86   **Mehmet M. Dalkilic*** (`dalkilic@iu.edu`), 700 North WoodLawn Dr, Office 3056, Bloomington, IN 47408, and **Kurban Hasan** (`hakurban@gmail.com`), 700 North Woodlawn Dr, Bloomington, IN 47408. *Teaching an Old Dog New Tricks: Making EM work with Big Data using Heaps.*
Contemporary data mining algorithms are easily overwhelmed with truly big data. While parallelism, improved initialization, and ad hoc data reduction are commonly used and necessary strategies, we note that (1) continually revisiting data and (2) visiting all data are two of the most prominent problems–especially for iterative learning techniques like expectation-maximization algorithm for clustering (EM-T). To the best of our knowledge, there is no freely available software that specifically focuses on improving the original EM-T algorithm in the context of big data. We demonstrate the utility of CRAN package DCEM that implements an improved version of EM-T which we call EM* (EM star). DCEM provides an integrated and minimalistic interface to EM-T and EM* algorithms, and can be used as either (1) a stand-alone program or (2) a pluggable component in existing software. We show that EM* can both effectively and efficiently cluster data as we vary size, dimensions, and separability.   (Received August 11, 2020)

1163-03-307   **Russell Miller*** (`russell.miller@qc.cuny.edu`), Mathematics Dept., Queens College – CUNY, 65-30 Kissena Blvd., Flushing, NY 11355. *Computable structure theory with noncomputable structures.*
From its inception, computable model theory has used the notion of a computable structure: a structure with domain ω whose functions and relations can all be computed by Turing machines. This notion enables a logician to focus on the complexity of various aspects of these structures – new relations on them, isomorphisms between them, interpretations of one structure in another – without allowing distractions from complexities that could be baked into the structure.

   We describe another approach: treat the atomic diagram of a structure as an oracle, via a Gödel coding. Even if the structure is not itself computable, one can then ask which aspects can be computed by a Turing functional endowed with such an oracle. On its face, this seems unlikely to yield results much different from those using traditional computable structures. Surprisingly, though, many properties that were quite complex under the traditional approach become far more tractable when oracles for noncomputable structures are considered this way. If anything, the presence of noncomputable structures makes life easier! We will provide several examples of this phenomenon, due to many researchers, illustrated to make them accessible even to logicians with no background in this area.   (Received September 01, 2020)

1163-03-365   **Karen Lange** and **Alexandra Shlapentokh*** (`shlapentokha@ecu.edu`), Department of Mathematics, East Carolina University, Greenville, NC 27858. *Truncation closed embeddings into fields of power series.* Preliminary report.
We show that any field not algebraic over a finite field can be embedded into a field of power series in a truncation closed fashion. Further, the cardinality of the set of such non-equivalent embeddings is at least the cardinality of the field.
(Received September 04, 2020)

1163-03-366   **Katalin Bimbó*** (`bimbo@ualberta.ca`), University of Alberta, Department of Philosophy, Assiniboia Hall 2–40, Edmonton, Alberta T6G2E7, Canada, and **J. Michael Dunn** (`dunn@indiana.edu`), Luddy School of Informatics, Computing and, Engineering, and Department of Philosophy, Indiana University, 901 East Tenth Street, Bloomington, IN 47408. *Entailment and (restricted) mingle.*
The logic of entailment ($E_\rightarrow$) was formulated as a sequent calculus by Kripke (*Journal of Symbolic Logic* 24 (1959):324). *RM*, the (full) logic of relevant implication R with (M), the mingle axiom $A \rightarrow (A \rightarrow A)$ has been thoroughly investigated in the literature. $E_\rightarrow$ can be (non-equivalently) extended with (M) or $(\overrightarrow{M})$, the restricted

mingle axiom $(A \to B) \to ((A \to B) \to (A \to B))$. Anderson and Belnap (*Entailment. The Logic of Relevance and Necessity*, 1975, p. 94) posed the question (attributing it to S. McCall) whether $E_\to = R_\to \cap E\overrightarrow{M}_\to$. We use sequent calculus formulations of these logics to prove that the set of theorems of $E_\to$ is indeed the intersection of the set of theorems of relevant implication and that of $E\overrightarrow{M}_\to$. We also consider a version of the problem with (M), and we use a counter example to prove that $E_\to \neq R_\to \cap EM_\to$.   (Received September 04, 2020)

1163-03-451   **Chris Hall**, Western University, London, Ontario , Canada, **Julia Knight**, University of Notre Dame, Notre Dame, IN , and **Karen Lange*** (`karen.lange@wellesley.edu`), Wellesley College, Wellesley, MA. *Complexity of well-ordered subsets in algebraic structures.* Preliminary report.

In [1], Knight, Lange, and Solomon bound the computational complexity of the root-taking process over Puiseux and Hahn series, two kinds of generalized power series. But it is open whether the bounds given are optimal. By looking at the most basic steps in the root-taking process, we became interested in the complexity of problems associated with well-ordered subsets of a fixed ordered abelian group. Here we report on our progress so far.

[1]   J. F. Knight, K. Lange, and D.Ṙ. Solomon "Roots of polynomials in fields of generalized power series", To appear in <u>Proceedings for Aspects of Computation</u>, World Scientific.   (Received September 07, 2020)

1163-03-609   **Chris Conidis*** (`chris.conidis@csi.cuny.edu`). *An algebraic characterization of the Tree Antichain Principle.*

The Tree Antichain Principle (TAC) says that every extendible binary tree with infinitely many splittings contains an infinite antichain, and arises in the study of Noetherian Commutative Algebra. We will give an algebraic characterization of TAC over $RCA_0+BSigma_2$ by showing that it is equivalent to saying "every Noetherian ring in which every minimal prime ideal is maximal contains finitely many prime ideals" (NFCP). We will also introduce a weakening of TAC, WTAC, such that TAC $\to$ NCP $\to$ WTAC over RCA_0.   (Received September 10, 2020)

1163-03-621   **Anton Bernshteyn*** (`bahtoh@gatech.edu`). *Descriptive combinatorics and distributed algorithms.*

Descriptive combinatorics is the study of combinatorial problems (such as graph coloring) under additional topological or measure-theoretic regularity restrictions. It turns out that there is a close relationship between descriptive combinatorics and distributed computing, i.e., the area of computer science concerned with problems that can be solved efficiently by a decentralized network of processors. In this talk, I will outline this relationship and present a number of applications.   (Received September 10, 2020)

1163-03-646   **Leigh Evron**, **Reed Solomon*** (`david.solomon@uconn.edu`) and **Rachel Stahl**. *Weakly cop-win graphs and dominating orders.*

The game of cops and robbers is a vertex pursuit game in which two players take turns moving on a graph. Player 1 (the cop) wins if she eventually lands on the same vertex as Player 2. For a finite graph G, Player 1 has a winning strategy if and only if there is a dominating order on G. This connection breaks down for infinite graphs and various attempts have been made to alter the rules of the game to restore it. In 2016, Lehner proposed a notion of a weakly cop-win graph, proved that if G has a dominating order, then G is weakly cop-win, and asked whether the converse holds. We show that the converse does not hold by constructing a family of weakly cop-win graphs that do not have dominating orders, and explore some computability theoretic properties of this variant of the game of cops and robbers.   (Received September 10, 2020)

1163-03-683   **Gabriel Conant***, University of Cambridge, Centre for Mathematical Sciences, Cambridge, CB3 0WB, United Kingdom. *Model theoretic tameness in multiplicative combinatorics.*

In combinatorics, an "inverse theorem" is a result in which mathematical objects exhibiting approximate structure are proved to be close to objects that are perfectly structured. A celebrated example is the structure theorem for approximate subgroups due to Breuillard, Green, and Tao, which built on work of Hrushovski.

   This talk is about related results in the context of model-theoretic tameness. For example, Martin-Pizarro, Palacín, and Wolf showed that under a local stability assumption, a finite approximate subgroup can be approximated by a bounded number of cosets of a finite subgroup, up to error $\epsilon > 0$. Their proof combines local stability theory with the stable arithmetic regularity lemma for finite groups due to C., Pillay, and Terry, but gives ineffective bounds. I will first discuss a new proof of this result, which yields polynomial bounds in $1/\epsilon$. This also provides the first quantitative account of stable arithmetic regularity for arbitrary finite groups, and

improves the previous exponential bound in the abelian case (due to Terry and Wolf). I will then describe joint work with Pillay on analogous qualitative results in the setting of bounded VC-dimension, which is motivated by previous work on NIP arithmetic regularity.   (Received September 11, 2020)

1163-03-795   **Patrick Lutz*** (pglutz@berkeley.edu). *Recent progress on Martin's conjecture.*
Martin's conjecture is an attempt to classify all functions on the Turing degrees under strong set-theoretic hypotheses. Roughly, it says that every function from the Turing degrees to the Turing degrees is either eventually constant or eventually a (transfinite) iterate of the Turing jump. The conjecture has also been proposed as a partial explanation for the absence of natural Turing degrees strictly between 0 and $0'$. I will explain the conjecture and report on some recent progress, including joint work with Benny Siskind and with Vittorio Bard. (Received September 12, 2020)

1163-03-829   **Rumen Dimitrov**, **Valentina Harizanov**, **Andrey Morozov** and **Paul Shafer***
                (p.e.shafer@leeds.ac.uk), School of Mathematics, University of Leeds, Leeds, LS2 9JT,
                United Kingdom, and **Alexandra Soskova** and **Stefan Vatev**. *Cohesive powers of linear orders.*
A cohesive power of a computable structure is an effective analog of an ultrapower where a cohesive set acts as an ultrafilter. We study cohesive powers of computable copies of ω, which are computable linear orders that are isomorphic to $(\mathbb{N}, <)$, but not necessarily by computable isomorphisms.

Every cohesive power of the standard presentation of ω has order-type $\omega + \zeta\eta$, which is expected because $\omega + \zeta\eta$ is the familiar order-type of countable non-standard models of PA. We show that it is possible for cohesive powers of computable copies of ω to exhibit a variety of order-types:

- There is a computable copy of ω with a cohesive power of order-type $\omega + \eta$.
- For every finite, non-empty $X \subseteq \mathbb{N} \setminus \{0\}$ (thought of as a set of finite order-types), there is a computable copy of ω with a cohesive power of order-type $\omega + \sigma(X)$. Here σ denotes the shuffle operation.
- For every $X \subseteq \mathbb{N} \setminus \{0\}$ that is either Σ_2^0 or Π_2^0, there is a computable copy of ω with a cohesive power of order-type $\omega + \sigma(X \cup \{\omega + \zeta\eta + \omega^*\})$.

(Received September 13, 2020)

1163-03-855   **Liling Ko*** (lko@nd.edu). *Bounding lattices below fickle degrees.* Preliminary report.
The ability to embed lattices below a computably enumerable (c.e.) Turing degree seems to be characterized by the fickleness of that degree. By earlier work it is known that the fickleness at the ω and ω^ω levels are characterized by critical triples (or L_7) and the 1-3-1 lattice, respectively. However no lattice has been found to characterize the ω^2 or ω^n levels. We explore candidate lattices, including infinite ones, and seek to understand the challenges faced in finding an ω^n level lattice and in embedding infinite lattices.   (Received September 13, 2020)

1163-03-916   **Noam Greenberg**, **Matthew Harrison-Trainor***
                (matthew.harrisontrainor@vuw.ac.nz), **Ludovic Patey** and **Dan Turetsky**. *Computing sets from all infinite subsets.*
A set is introreducible if it can be computed from all of its infinite subsets. Such a set can be thought of as coding all of its information in a redundant way. The two most natural examples are the set of initial segments of a given infinite binary string, and the range of the modulus of a c.e. set. We prove a number of results about introreducibility, including answering two questions from Jockusch from the 60's.   (Received September 14, 2020)

1163-03-968   **Johanna Franklin*** (johanna.n.franklin@hofstra.edu), Department of Mathematics,
                Room 306, Roosevelt Hall, Hofstra University, Hempstead, NY 11549, and **Alexander Melnikov** and **Dan Turetsky**. *The complexity of the index set of Banach spaces isomorphic to $C[0,1]$.*
The complexity of the index set of the computably presented Banach spaces that are isomorphic to $C[0,1]$ can be crudely bounded by Σ_1^1; Brown had demonstrated that an arithmetical bound could be obtained if the signature for Banach spaces was extended. Here, we show that there is a Δ_5^0 isomorphism between any computable presentation of $C[0,1]$ and the standard presentation using only the usual signature plus addition, giving us a tighter bound on the complexity of this index set.   (Received September 14, 2020)

1163-03-970   **Wesley Calvert*** (`wcalvert@siu.edu`), School of Mathematical & Statistical Sciences,
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and **Daniel Turetsky** (`dan.turetsky@vuw.ac.nz`), School of Mathematics and Statistics,
Victoria University of Wellington, Gotton Building, Gate 7, Kelburn Parade, Wellington,
New Zealand. *Degrees High for Isomorphism, and Related Classes of Degrees.* Preliminary
report.

For some structures, called computably categorical, any two computable isomorphic copies are isomorphic by a
computable isomorphism. For other structures, of course, a higher oracle is needed to compute the isomorphisms.
We aim to characterize the degrees that we call "high for isomorphism" — the degrees relative to which every
computable structure is computably categorical.

Along the way to characterizing these degrees, we describe several other classes of degrees, each having
interesting properties from the perspective of computable structures, and each having the property that any
member is "not much below" Kleene's $\mathcal{O}$.   (Received September 14, 2020)

1163-03-993   **Barbara F Csima***, Department of Pure Mathematics, University of Waterloo, Waterloo,
Ontario N2L 3G1, Canada. *Understanding Frameworks for Priority Arguments in
Computability Theory.*

Priority Arguments are a common proof technique used in Computability Theory. A theorem is broken down
to being equivalent to a list of requirements. These requirements are given a priority order, and a strategy is
devised to meet all the requirements, making use of the priority order.

Those who know a Computability Theorist know that we love our priority arguments! In this talk, we will
discuss why Computability Theory lends itself so well to this proof technique, and discuss at a high level the
types of strategies used in priority arguments.

As soon as one first learns of priority arguments, one asks, can we save repeating ourselves, and have a
framework for this? In this talk, we will discuss, again at a high level, existing frameworks for priority arguments,
with a particular focus on Ash's α-systems and Montalban's η-systems. We discuss the general idea of how the
frameworks work, their power, and their limitations.   (Received September 14, 2020)

1163-03-1045   **Daniel Turetsky*** (`dan.turetsky@vuw.ac.nz`). *Relations between preservation properties.*
Preservation properties are a tool for separating the reverse mathematical strength of various statements. As
an example, if I is a Turing ideal and X is a set outside I, then there is an ideal J containing I but omitting
X and which models WKL_0. The same holds with RT_2^2 in place of WKL_0, but this fails for RT_2^3, thus showing
that WKL_0 and RT_2^2 do not prove RT_2^3.

In fact, for both WKL_0 and RT_2^2, the above holds not just for a single set X, but for countably many sets
simultaneously. In both cases, the proofs for one set and for countably many sets are more or less the same. It
turns out there's a reason for this: any reverse mathematical principal (of the appropriate form) which can be
satisfied while avoiding a single set can be satisfied while avoiding countably many.

This is an example of a relationship between preservation properties. We investigate similar relationships
between various preservation properties.   (Received September 14, 2020)

1163-03-1081   **Valentina Harizanov*** (`harizanv@gwu.edu`), Department of Mathematics, George
Washington University, Washington, DC 20052. *On the isomorphism problem.* Preliminary
report.

Isomorphism is one of the most important equivalence relations between structures for the same language.
For certain natural algebraic classes $\mathcal{K}$, we determine the exact complexity of the isomorphism problem for
its computable members. More precisely, the <u>computable isomorphism problem</u> for a class $\mathcal{K}$ of structures,
closed under isomorphism, is the set of pairs of effective codes (computable indices) for computable structures
in $\mathcal{K}$, which are isomorphic. We focus on classes that have Σ_1^1-complete computable isomorphism problem,
which is of maximal complexity. These classes have isomorphic structures that are not hyperarithmetically
isomorphic. Each of these classes also has a structure M with two tuples of elements of the same length, $\bar{a}$ and
$\bar{b}$, such that $(M, \bar{a})$ and $(M, \bar{b})$ are isomorphic but not hyperarithmetically isomorphic. The method we use is
based on uniform effective interpretations of computable binary relations into computable structures from the
corresponding classes. This is joint work with S. Lempp, A. Morozov, C. McCoy and R. Solomon.   (Received
September 14, 2020)

1163-03-1088   **Peter Cholak*** (`cholak@nd.edu`), cholak@nd.edu, and **Peter Gerdes**
(`gerdes@invariant.org`), gerdes@invariant.org. *The Collapse of an REA hierarchy.*
Preliminary report.

Consider the following statement $S(m,n)$: If C is any set which is $(m+1)$-REA and not of m-REA degree, there exists a set A which is n-r.e. in C such that $A \oplus C$ is not of $(m+n)$-REA degree. Soare and Stob [1982] showed this statement holds for $m=0$ and $n=1$. Cholak and Hinman [1994] showed that this statement hold for $m=0,1$ and arbitrary $n \geq 0$. They also conjectured it holds for all n and m. However, we will show that this statement fails for $n=2$ and $m=1$.   (Received September 14, 2020)

1163-03-1162   **Dana Bartosova*** (`dbartosova@ufl.edu`). *Non-metrizable universal minimal flows.*

For a topological group G, a continuous action of G on a compact Hausdorff space is called a G-flow. A G-flow is minimal if every orbit is dense. The universal minimal G-flow has every minimal G-flow as a quotient and it is unique up to an isomorphism of flows. Universal minimal flows of infinite-dimensional groups have received considerable attention in the past 15 years due to their connection with finite combinatorics. On the other hand, locally compact, non-compact groups have non-metrizable universal minimal flows, which means the failure of the finitary combinatorial principles. However, other methods are available for locally compact groups and we encounter interesting connections with set theory in our investigation of discrete groups and their products. This is in part a joint work with Aleksandra Kwiatkowska.   (Received September 14, 2020)

1163-03-1187   **Haoyang Cao** and **Xin Guo*** (`xinguo@berkeley.edu`), CA. *Convergence of GANs training: an SDE approach.*

Generative Adversarial Networks (GANs) have celebrated great empirical success, especially in image generation and processing. There is a recent surge of interest in GANs to financial applications, including asset pricing, portfolio optimization, and multi-agent market simulation. In this talk, we will discuss some recent progress in mathematical analysis of GANs: approximating GANs training in the form of SDEs. This SDEs approximation provides, for the first time, an analytical tool to understand some well-recognized issues in the machine learning community for GANs training.   (Received September 15, 2020)

1163-03-1189   **Jun Le Goh**, **Joe Miller**, **Mariya Soskova** and **Linda Westrick*** (`westrick@psu.edu`). *Reducing information with few changes.*

Given a real X of effective dimension s and some $t < s$, what density of changes to X are necessary and sufficient to bring the effective dimension down from s to t? We give precise bounds, answering a question of Greenberg, Miller, Shen and W., 2018. We also present a more detailed picture of how reals of various effective dimensions are arranged in Cantor space.   (Received September 15, 2020)

1163-03-1358   **Charles Steinhorn*** (`steinhorn@vassar.edu`), Department of Mathematics and Statistics, Vassar College, 124 Raymond Ave, Poughkeepsie, NY 12604. *Asymptotic and multidimensional asymptotic classes of finite structures.* Preliminary report.

Asymptotic classes of finite structures and measurable structures were introduced by D. Macpherson and the speaker in an effort to develop a model theory for classes of finite structures that reflects contemporary infinite model theoretic themes. This talk surveys work on that topic, including contributions of several others, and on current research that generalizes those concepts to what are called multidimensional asymptotic classes and generalized measurable structures. This most recent work is joint with Macpherson, S. Anscombe, and D. Wolf. (Received September 15, 2020)

1163-03-1394   **Oscar Levin*** (`oscar.levin@unco.edu`), School of Mathematical Sciences, 501 20th Street, Campus Box 122, Greeley, CO 80639-0001. *Partitions in graphs and hypergraphs with too much freedom.* Preliminary report.

We will consider two apparently unrelated questions from computable graph theory. First, a domatic partition in a graph is a partition of the vertices into dominating sets (so that every vertex is in or is adjacent to a vertex in each set). We ask how small the size of computable domatic partition can be in a highly computable graph. Second, a weak coloring of a hypergraph is a partition of its vertices into independent sets (so that no set in the partition contains any hyperedge). We ask how large the size of a computable vertex coloring can be in a highly computable 3-uniform hypergraph.

The two questions are connected only by their apparent difficulty in obtaining tight answers. In this talk we will explore how sub-optimal partitions in these structures possess too much freedom, resulting in difficulties to both diagonalization and computation.   (Received September 15, 2020)

1163-03-1464   **Joachim Mueller-Theys*** (`mueller-theys@gmx.de`). *Named Model Theory.*
Elementarily equivalent structures need not be isomorphic. We found out that the situation changes when structures are considered the objects of which are all *named* by closed terms. Then even equivalence at atomic sentences suffices for isomorphy, and our proof moreover shows that a named structure is homomorphic to another structure if all closed atomic formulæ holding in the former hold in the latter. As a consequence, arithmetical structures are isomorphic to the standard model already if they are likewise named and have the same atomic theory.

Furthermore, (the theory of) any named structure is named axiomatized by its basic theory, comprising the true atomic and negated-atomic sentences only.

If there are but $n > 0$ closed terms, named structures are finite, and any sentence is named equivalent to a PL-combination of atomic sentences, whence variables and quantifiers become redundant (in the infinite case, this may be accomplished by infinitary operations). Validity for such *finitary* structures will even be decidable—unlike finite first-order validity (Trachtenbrot's theorem).

As suggested by P. Maier-Borst, the characterization theorem may be generalised. That is true for the axiomatization theorem as well.   (Received September 15, 2020)

1163-03-1626   **Rachael Alvir*** (`ralvir@nd.edu`). *Relations Between various Scott Ranks.* Preliminary
                  report.
Over the years, many different definitions of "Scott Rank" have been proposed. To help unify them, the "external" notion of the Scott Complexity of a countable structure Ahas been proposed: the least complexity of an infinitary formula which can characterize the structure up to isomorphism among countable structures. The speaker, along with Matthew Harrison-Trainor, Dan Turetsky, and Noam Greenberg has previously argued that this is a good notion since it is maximally "robust," i.e., is the finest notion of "rank" which has many different and desirable equivalent characterizations. While most existing notions of Scott Rank can be computed given the Scott Complexity, there are one or two exceptions. In this talk, we give an overview of all the previously explored notions of Scott Rank, explicitly show how to compute one notion of Scott Rank given another, and also explore other "internal" notions of Scott Rank. In this talk, we discuss why those "internal" notions of Scott Rank have sometimes been preferred in computable structure theory because of, rather than in spite of, their lack of "robust" characterizations.   (Received September 15, 2020)

05 ▶ *Combinatorics*

1163-05-61   **Josephine Brooks*** (`j.brooks@mail.utoronto.ca`), Department of Mathematics, Bahen
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                and **Joseph Vargas** (`varg0261@fredonia.edu`), Mathematical Sciences Department, 223
                Fenton Hall, SUNY Fredonia, Fredonia, NY 14063. *Removing Symmetry in Circulant
                Graphs and Point-Block Incidence Graphs.*
A vertex v in a graph G is *fixed* if it is mapped to itself under every automorphism of G. The fixing number of a graph G is the minimum number of vertices, when fixed, fixes all of the vertices in G. Fixing numbers were introduced by Laison, Gibbons, Erwin, Harary, and Boutin. A *circulant graph* is a graph of n vertices in which the i-th vertex is adjacent to the $(i + j)$th and $(i - j)$th graph vertices for each j in a list L. We determine the fixing number for multiple classes of circulant graphs, showing in many cases the fixing number is 2. However, we show that circulant graphs with *twins*, which are pairs of vertices with the same open neighborhoods, have higher fixing numbers. A *point-block incidence graph* is a bipartite graph $G = (P, B)$ with a set of point vertices $P = \{p_1, p_2, ..., p_r\}$ and a set of blocks $B = \{B_1, B_2, ..., B_s\}$ where $p_i \in P$ is adjacent to $B_j \in B \Leftrightarrow p_i \in B_j$. We show that symmetries in certain block designs cause the fixing number to be as high as $\frac{|V(G)|}{4}$. We also present several infinite families of graphs in which fixing any one vertex in G fixes every vertex in G, thus removing all symmetries from the graph.   (Received August 04, 2020)

1163-05-78 **Henry L Fleischmann*** (henryfl@umich.edu), **Daniela A Elizondo** (delizondo@hmc.edu) and **Rachel H Thornton** (rachelhthornton@utexas.edu). *Efficient (j,k)-Domination on Chess Graphs.*

Graphs defined by the legal moves of a chess piece are a classical setting for efficient domination. For a graph G, a function $f : V(G) \to \{0, 1, \ldots, j\}$ is an *efficient (j,k)-dominating function* if, for all $v \in V(G)$,

$$\sum_{w \in N[v]} f(w) = k,$$

where $N[v]$ is the closed neighborhood of v (introduced by Rubalcaba and Slater, 2007).

Generalizing our complete characterization of efficient (j,k)-domination on King's graphs, we prove by construction that $G \boxtimes H$ is efficiently (j,k)-dominatable if and only if both G and H are. Additionally, we describe several necessary conditions for efficient (j,k)-domination, following our observations on Bishop's graphs.

On the torus, the Queen's and Bishop's graphs are realizable as Cayley graphs. We apply character theory to calculate the spectra of these graphs, through which we determine their efficient (j,k)-dominating functions.

For the standard $n \times n$ Queen's graph, we exploit an equitable partition to show computationally that, for $4 \le n \le 556$, efficient (j,k)-domination occurs only when $n = 10$. Expanding this approach, we construct an infinite class of graphs with an efficient (j,k)-dominating function from analogous equitable partitions. (Received August 10, 2020)

1163-05-110 **Jonathan S Turner*** (jonathan.s.turner.10@gmail.com), WrightPatterson AFB, OH. *Simultaneous Decompression Based Cipher.*

Let $|\boldsymbol{u}^1| = \delta_1$ and $|\boldsymbol{u}^2| = \delta_2$ where $\gcd(\delta_1, \delta_2) = 1$. The simultaneous decompression of $(\boldsymbol{u}^1, \boldsymbol{u}^2)$ is defined as the set of binary vectors $\boldsymbol{v} \in \boldsymbol{V}$ satisfying $u_j^1 = \sum_{k=0}^{\delta_2 - 1} v_{k\delta_1 + j}$ for each $j \in \mathbb{Z}_{\delta_1}$ and $u_j^2 = \sum_{k=0}^{\delta_1 - 1} v_{k\delta_2 + j}$ for each $j \in \mathbb{Z}_{\delta_2}$. We exploit the isomorphism $\mathbb{Z}_{\delta_1 \delta_2} \cong \mathbb{Z}_{\delta_1} \times \mathbb{Z}_{\delta_2}$ to formulate these constraints as a multi-dimensional Subset Sum problem with binary matrix solutions. The set of these matrices allows a graph representation, $\mathbb{G}_{\boldsymbol{V}}$, that is conjectured to contain a Hamiltonian cycle. It follows each $\boldsymbol{v}$ may be uniquely represented by linearly-independent indices of $(\boldsymbol{u}^1, \boldsymbol{u}^2)$ and integer distance along this cycle. We construct a small example on $\delta_1 \delta_2 = 21$ for demonstration, and discuss the pros and cons of this structure with respect to the principles of cryptography. (Received August 16, 2020)

1163-05-116 **Manjil Saikia***, School of Mathematics, 21 - 23 Senghennydd Road, Cardiff, Cardiff CF244AG, United Kingdom, and **Hassan Izanloo**. *Signed Roman Domination on Cartesian Product of Some Graphs.* Preliminary report.

A signed Roman dominating function (SRDF) on a graph $G = (V, E)$ is a function $f : V \to \{-1, 1, 2\}$ satisfying the conditions that, the sum of its function values over any closed neighborhood is at least one and every vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight of a SRDF is the sum of $f(v)$ over all vertices v and the signed Roman domination number (SRDN) of G is the minimum weight of a SRDF in G. In this talk we will study the SRDN of cartesian products of some graphs. This is joint work with Hassan Izanloo. (Received August 17, 2020)

1163-05-145 **Beate G. Zimmer*** (beate.zimmer@tamucc.edu) and **Reid B. Porter**. *Rand Error Calculations for Watershed Segmentations and Connected Component Segmentations.* Preliminary report.

To segment images, a notion of affinities between adjoining pixels is helpful. We assume that affinities are real-valued and unique, with positive affinities denoting similarity and negative affinities denoting dissimilarity. With those affinities, an image becomes as a 4-connected weighted graph. Kruskal's algorithm greedily constructs a maximum spanning tree. Both connected components (CC) and watershed (WS) segmentations can be done through deciding which edges of an MST to cut. For CC, the negative MST edges are cut, for WS only edges, all of whose MST neighbors on at least one side have a lower weight than the edge in question are cut. In using edge-errors to assess a segmentation, one misclassified edge might connect two large segments but still have a very low error. The Rand error calculates the error over all pixel pairs and is then normalized by the number of pixel pairs. We show how to adapt Kruskal's algorithm to keep track of how many correct and incorrect new connections each edge that is added to the MST makes. We then approximate the Rand error with a differentiable loss function and use deep learning to learn the affinities in a new image with this loss function. This method also extends to a hierarchy of a watershed cut followed by a connected component cut. (Received August 22, 2020)

1163-05-146   **Hannah Alpert**, **Russell Barnes**, **Seth Bell**, **Annika Mauro*** (amauro@stanford.edu), **Na'ama Nevo**, **Nataya Tucker** and **Hanna Yang**. *Routing by matching on convex pieces of grid graphs.* Preliminary report.

In this talk, we present an upper bound for the routing number of graphs that are the intersection of a convex polygon with the square lattice grid. The routing number is a graph invariant introduced in 1993 by Alon, Chung, and Graham and has been studied for path, tree, and star graphs, but not for classes of graphs chosen according to a geometric criterion. The geometric motivation for this goal is a continuous version of the problem: studying the set of arrangements of n disjoint identical disks within a polygon. The routing number is a discrete analogue of the diameter of such a configuration space. We show that, consistent with the bound for rectangular grid pieces, the routing number of grid pieces contained within convex polygons grows linearly in the diameter of the graph. (Received August 22, 2020)

1163-05-190   **Theo McKenzie*** (mckenzie@math.berkeley.edu), **Peter Rasmussen** and **Nikhil Srivastava**. *Support of Closed Walks and Second Eigenvalue Multiplicity of Regular Graphs.*

Spectral expansion can be used to bound the expected support of a walk of a given length in a graph. However, no such bound is known when conditioning on the walk being closed. We prove such a result by proving a new bound on the minimum entry of the eigenvector corresponding to the top eigenvalue of the adjacency matrix of a graph. We then use our result to improve the multiplicity of a given eigenvalue to $n/log^{O(1)}n$. (Received August 25, 2020)

1163-05-198   **Andrew Beveridge*** (abeverid@macalester.edu), Department of Mathematics, 1600 Grand Avenue, St Paul, MN 55105, and **Ian Calaway** (icalaway@stanford.edu). *Constructing Admissible Voter Preferences with the Voter Basis.*

When making simultaneous decisions, our preference for the outcomes on one subset can depend on the outcomes on a disjoint subset. In referendum elections, this gives rise to the separability problem, where a voter must predict the outcome of one proposal when casting their vote on another. A set $S \subset [n]$ is separable for preference order $\succeq$ when our ranking of outcomes on S is independent of outcomes on its complement $[n]\backslash S$. The admissibility problem asks which subsets $S \subset [n]$ can arise as the collection of separable subsets for some preference order. We introduce the 2^n-dimensional voter basis, and use it to construct voter preferences whose Hasse diagram of separable sets has a tree structure, or is closed under intersections and unions. (Received August 26, 2020)

1163-05-201   **Vladyslav Oles*** (vlad.oles@protonmail.com). *Two-sided estimation of a Gromov–Hausdorff distance.*

The Gromov–Hausdorff distances measure the difference in shape between metric spaces. While computing the distances poses a hard combinatorial problem, their approximations are used for matching deformable shapes, comparing brain and computer networks, and analyzing hierarchical clustering trees. We propose an approach to approximating one of the Gromov–Hausdorff distances by solving a sequence of tractable assignment problems. The performance and error bounds of the method are demonstrated using real-world data. (Received August 26, 2020)

1163-05-202   **Joshua Carlson**, **Robin Eagleton**, **Jesse Geneson**, **John Petrucci**, **Carolyn Reinhart*** (reinh196@iastate.edu) and **Preetul Sen**. *The damage throttling number of a graph.*

The cop throttling number of a graph, introduced by Breen et al., optimizes the balance between the number of cops used and the number of rounds required to catch the robber in a game of Cops and Robbers. Cox and Sanaei studied a variant of Cops and Robbers in which the robber tries to occupy (or damage) as many vertices as possible and the cop tries to minimize this damage, the minimum number of vertices damaged in this way is called the emphdamage number. We introduce the emphdamage throttling number of a graph, denoted $operatorname{th}_d(G)$, which optimizes the balance between the number of cops used and the number of vertices damaged in the graph. We formalize the definition of emph$k$-damage number, which extends the damage number to games played with k cops. We prove that the damage throttling number is tightly bounded above by one less than the cop throttling number but the parameters exhibit interesting differences. Infinite families of examples and non-examples of tightness in this bound are given. For most families of connected graphs G of order n that we consider, we find that $operatorname{th}_d(G) = O(sqrt{n})$. However, we find an infinite family of connected graphs G of order n for which $operatorname{th}_d(G) = Omega(n^{2/3})$. (Received August 26, 2020)

1163-05-214   **Jacob Brown*** (`jbrown18@ithaca.edu`), Department of Mathematics, Ithaca College,
                 Ithaca, NY 14850, and **Adam Giammarese**, School of Mathematical Sciences, Rochester
                 Institute of Technology, Rochester, NY 14623. *A Networks Perspective on the Sensitivity of
                 the Global Climate to Perturbations over the Amazon.*

The Amazon rain forest is one of the tipping elements of the global climate system; widespread deforestation
is predicted to lead to permanent changes in rainfall patterns, hotter temperatures, and forest dieback over
the region. Network analysis has been used to study several local and global climate systems; in this work, we
employ it to study the impact of the deforestation of the Amazon rain forest on the stability of the global climate
system. We analyze surface air temperature from the NCEP/NCAR reanalysis data set and transform this data
into networks on various time scales. Employing a multitude of measures from network analysis and dynamical
systems, we show that the Amazon rain forest is a significant component of the global climate. We simulated
deforestation as an attack on the global climate network and discovered that removal of the Amazon-produced
a 27.0% decrease in average communicability in the network, as compared to an 8.6% decrease produced via
random removal of any other region of the same size on the planet. Using Lyapunov exponents and the spectra
of adjacency matrices and graph Laplacians, we also analyze the stability of the dynamics of the global climate
under deforestation of the Amazon.   (Received August 27, 2020)

1163-05-227   **Mark E Watkins*** (`mewatkin@syr.edu`), NY. *Vertex-transitive non-Cayley graphs of
                 connectivity 1.* Preliminary report.

A VTNCG is a vertex-transitive non-Cayley graph. While Cayley graphs are vertex-transitive, the converse
is false, and a number of papers (see, for example, C. Praeger or B. McCay) have been written searching for
VTNCGs. We present a variety of conditions for an infinite graph of connectivity 1 to be a VTNCG. A typical
theorem is the following: For some integer $n \geq 2$, let Γ be a graph of connectivity 1 whose lobes (i.e., maximal
biconnected subgraphs) are copies of the Petersen graph (resp., the dodecahedral graph), and each vertex is
incident with exactly n such copies. If n is even, then Γ is a Cayley graph; if n is odd, then Γ is a VTNCG.
More elaborate examples involve lobes that are edge-transitive bipartite graphs.   (Received August 28, 2020)

1163-05-258   **Nicholas Scoville*** (`nscoville@ursinus.edu`), 601 E Main Street, Math And CS,
                 Collegeville, PA 19465, and **Matt Zaremsky**. *Higher connectivity of the Morse complex.*

The Morse complex $\mathcal{M}(\Delta)$ of a finite simplicial complex Δ is the complex of all gradient vector fields on Δ. In
this talk we study higher connectivity properties of $\mathcal{M}(\Delta)$. For example, we prove that $\mathcal{M}(\Delta)$ gets arbitrarily
highly connected as the maximum degree of a vertex of Δ goes to ∞, and for Δ a graph additionally as the
number of edges goes to ∞. We also classify precisely when $\mathcal{M}(\Delta)$ is connected or simply connected. Our main
tool is Bestvina–Brady Morse theory, applied to a "generalized Morse complex."   (Received August 31, 2020)

1163-05-291   **Anthony Bonato*** (`abonato@ryerson.ca`), Department of Mathematics, Ryerson
                 University, Toronto, Ontario M5B2K3, Canada. *The localization number of a graph.*

Graph searching investigates combinatorial models for the detection or neutralization of an adversary's activity
on a network. One such model is the {localization game}, where agents use distance probes to capture an invisible
intruder. We present new results on the {localization number} of a graph, which is the minimum number of
agents needed to capture the intruder. We give bounds on the localization number of incidence graphs of designs,
Kneser graphs, and polarity graphs. In many cases, these bounds are tight.   (Received September 01, 2020)

1163-05-346   **Matthias Beck**, San Francisco, CA, **Benjamin Braun**, Lexington, KY , and **Andres R
                 Vindas Melendez*** (`andres.vindas@uky.edu`), Department of Mathematics, 719
                 Patterson Office Tower, Lexington, KY 40506. *Decompositions of Ehrhart h^*-Polynomials
                 for Rational Polytopes.*

The Ehrhart quasipolynomial of a rational polytope P encodes the number of integer lattice points in dilates
of P, and the h^*-polynomial of P is the numerator of the accompanying generating function. We provide two
decomposition formulas for the h^*-polynomial of a rational polytope. The first decomposition generalizes a
theorem of Betke and McMullen for lattice polytopes. We use our rational Betke–McMullen formula to provide
a novel proof of Stanley's Monotonicity Theorem for the h^*-polynomial of a rational polytope. The second
decomposition generalizes a result of Stapledon, which we use to provide rational extensions of the Stanley and
Hibi inequalities satisfied by the coefficients of the h^*-polynomial for lattice polytopes. Lastly, we apply our
results to rational polytopes containing the origin whose duals are lattice polytopes.   (Received September 03,
2020)

1163-05-360   **Sinan Aksoy**, **Mark Kempton*** (`mkempton@mathematics.byu.edu`) and **Stephen Young**. *Spectral threshold for extremal cyclic edge-connectivity.*

The cyclic edge-connectivity of a graph is the smallest number of edges whose removal disconnects the graph into components where every component contains a cycle. The cyclic condition is natural in applications, such as in network reliability, as cycles are necessary to guarantee multiple paths between vertices. We will present a bound on the cyclic edge-connectivity of regular graphs, and a spectral condition for when this bound is tight. (Received September 03, 2020)

1163-05-440   **Iain Moffatt*** (`iain.moffatt@rhul.ac.uk`), Deprtment of Mathematics, Royal Holloway, University of London, Egham, TW20 0EX, United Kingdom. *The Tutte polynomial and graphs in surfaces.*

The Tutte polynomial is a probably the best-studied graph polynomial. It arises in many applications of graph theory and also has many applications within combinatorics. There have been various extensions of the Tutte polynomial from graphs to graphs embedded in surfaces. The literature on such "topological Tutte polynomials" has mostly focussed on cellularly embedded graphs. In this talk, however, I will argue that to fully understand these topological Tutte polynomials we need to move beyond cellular embeddings in surfaces, and that is is most natural to consider non-cellular embeddings in pseudo-surfaces. I'll give a run through of what is and is not know about these graph polynomials, as well as describing a number of open problems and research directions. (Received September 07, 2020)

1163-05-472   **Jonathan L Gross*** (`gross@cs.columbia.edu`), Department of Computer Science, Columbia University, New York City, NY 10027, and **Toufik Mansour** and **Thomas W Tucker**. *Partial-Tuality Polynomials, Part 1.*

The compositions of Poincaré duality ($*$) and Petrie duality ($\times$) yield a group of operators on maps that include Wilson duality ($*\times*$) and two triality operators ($*\times$ and $\times*$) that are inverses of each other. Ellis-Monaghan and Moffatt have generalized Chmutov's partial Poincaré duality construction for ribbon graphs to the other operators in this group, for which they coined the term *tualities*. The present authors have previously introduced the *partial-$*$ Euler-genus polynomial* that enumerates all the partial-$*$ duals according to Euler-genus. Here we introduce the partial-$\bullet$ polynomials for the other four tualities. Part 1 of this two-part presentation focuses on the effect of the various partial tualities on the monodromy and on how the Euler-genus of a ribbon graph is easily determined directly from the monodromy. We present the symmetric embedding of $K_5 \to S_1$ as a counterexample to our conjecture in a predecessor paper that all partial-$*$ polynomials are log-concave.   (Received September 07, 2020)

1163-05-533   **Lowell E Abrams*** (`labrams@gwu.edu`) and **Lindsy-Kay Lauderdale**. *The Wiener Index for Embedded Graphs.*

An invariant of graphs measuring distance between all pairs of vertices.

   Consider the ratio (more applicable to embedded graphs) between
      the index of an embedded graph and
      the index of its topological dual.

   Choose the surface in which to embed the graphs – what does this imply?

   Designate "primal" and "dual" so the ratio is in (0 to 1] – infinite families have ratios
      converging to 0
      converging to 1,
      converging to various values between and

never changing.
(Received September 08, 2020)

1163-05-602   **Doron Zeilberger*** (`doronzeil@gmail.com`), Dept. of Mathematics, Rutgers University, Piscataway, NJ 08544. *How Richard Askey Inspired the Happy Marriage of Special Functions and Combinatorics.*

The Discrete and Continuous used to hate each other. Thanks, in large part, to Richard Askey, they made up and live happily ever after.   (Received September 10, 2020)

1163-05-610   **Austin P Allen*** (`apallen@andrew.cmu.edu`), apallen@andrew.cmu.edu, **Veronica Dobbs**, veronica.dobbs@marquette.edu, **Lino Yoshikawa**, lino.yoshikawa@gmail.com, **Christopher Soto**, csotonyc@gmail.com, and **Sebastian Papanikolaou-Costa**, spapanikolaou1@email.uagm.edu. *Enumerating "Good" Permutations.*

Two permutations π and σ are said to be order isomorphic if they are equivalent after "pattern" reduction. We call a permutation good" if the first ℓ entries are order isomorphic to the last ℓ entries. Given a k, we wish to enumerate all good permutations on $[k]$ which overlap consecutively. We do this for whenever $\ell \leq k/2$, and via experimentation we conjecture that whenever $\ell > k/2$ the number of good permutations is polynomial in k. We also make a connection of enumerating good permutations to the problem of explicitly determining the expected number of distinct permutation patterns contained in a random permutation.   (Received September 12, 2020)

1163-05-627   **Blake Dunshee** and **Mark Ellingham*** (`mark.ellingham@vanderbilt.edu`). *A Fano framework for graph embeddings.*

One useful representation of embedded graphs is by *graph-encoded maps* or *gems*, which represent a cellularly embedded graph as a cubic graph with a proper 3-edge-colouring. A number of graph embedding properties, such as orientability or bipartiteness, correspond to parity conditions on closed walks in the gem. Using this correspondence we can map seven natural embedding properties to points in the Fano plane (the projective plane of order 2), in such a way that there is a theorem associated with every set of three properties. The type of theorem depends on whether the associated points lie on a line in the Fano plane or not. The seven properties also correspond to the existence of certain orientations or signings of the embedded medial graph. This allows us to construct theorems on whether embeddings have a property after taking partial duals or partial Petrie duals (twists) with respect to certain edges. This generalizes work by Huggett and Moffatt and by Deng and Jin. We also discuss other interpretations of the seven properties, and extensions to other properties.   (Received September 10, 2020)

1163-05-634   **Mark Bilinski*** (`mark.bilinski@navy.mil`). *Naval applications of combinatorics.*

The Navy operates in a vast and complex ocean, far from established infrastructure. This brings with it a unique perspective to a number of common military problems – which are an endless font of research opportunities for academics and industry alike. This talk provides an overview of just a small sample of such problems, specifically some that I have studied thusfar while at the Naval Information Warfare Center Pacific that have a combinatorial flavor. First I discuss challenges with communication in what is called a DIL environment (Disconnected, Intermittent, Limited bandwidth). This led to a practical encoding/decoding scheme for synchronizing large datasets in such tactical Navy environments – from a theory perspective the scheme may seem suboptimal, but it more directly addresses practical Naval concerns. Second, I discuss broad concepts in cyber defense and how graph convolutional neural networks and game theory can be used to better understand adversary behavior. Third, I will present 3-D point cloud data of ships and discuss multiple applications stemming from its analysis, such as computer vision, augmented reality, preventative maintenance, and robotics.   (Received September 10, 2020)

1163-05-653   **Brigitte Servatius*** (`bservat@wpi.edu`). *Equitable edge partitions and Kirchhoff graphs.* Preliminary report.

Kirchhoff graphs are geometric graphs whose edge vectors satisfy a cycle-vertex cut orthogonality condition. They arise naturally as circuit diagrams for chemical reaction networks. We define equitable arc partitions on multidigraphs and show that the partition obtained from an embedded Kirchhoff graph by edge vector equality is equitable. For equitable arc partitions of multidigraphs, realization questions as Kirchhoff graphs arise. This is joint work with Joseph Fehribach, Tyler Reese and Randy Paffenroth.   (Received September 10, 2020)

1163-05-689   **Joseph Alameda*** (`jalameda@iastate.edu`). *Comparing the domination number and the k-power domination number in hypergraphs a preliminary report.* Preliminary report.

In this presentation both the domination number and the k-power domination number are compared in hypergraphs. In particular, I explore the question "given an upper bound for the domination number, is there a related bound for the k-power domination number?" Various examples are given that suggest this question is true. Furthermore, I prove that given a hypergraph $\mathcal{H}$ with $n \geq k + 3$ vertices and edge size at least 3, that $\gamma_p^k((H)) \leq \frac{n}{k+3}$. In doing so, I prove a conjecture given by Bjorkman stating $\gamma_p^1(\mathcal{H}) \leq \frac{n}{4}$ for hypergraphs on $n \geq 4$ vertices with edge size at least 3.   (Received September 11, 2020)

1163-05-697   **Heather M. Russell*** (`hrussell@richmond.edu`). *Symmetries in graph coloring reconfiguration.* Preliminary report.

A reconfiguration system consists of a collection of states and a rule for transitioning between states. By constructing a corresponding transition graph which represents states as vertices and transitions as edges, one can use tools from graph theory to analyze the structure of these systems. In this talk, we discuss a reconfiguration system which has a set of states consisting of the proper vertex k-colorings of a fixed underlying graph and a transition rule given by recoloring a single vertex of that graph. We consider the transition graph for this system as well as an associated locally CAT(0) cube complex for which the transition graph is the one-skeleton. Finally, we discuss structural properties of graph coloring reconfiguration systems that become apparent by considering the action of permuting colors on the transition graph and cube complex.   (Received September 11, 2020)

1163-05-700   **Blake Dunshee*** (`blake.dunshee@belmont.edu`), **Mark Ellingham** and **Joanna A Ellis-Monaghan**. *Directed embeddings with partially predetermined facial walks.*

A directed embedding is a digraph embedded in a surface in such a way that all the faces of the embedding are bounded by directed walks. This is equivalent to an embedding of a digraph where half-arcs alternate between in and out around a vertex. First, we characterize when an embedded graph can be given an orientation such that the resulting embedding of a digraph is a directed embedding. We then characterize when a digraph and a collection of its closed directed walks can be extended to a directed embedding such that the collection of closed directed walks is a subcollection of the facial walks. Furthermore, we characterize when such a directed embedding can be chosen to be orientable. Širáň and Škoviera gave analogous results in the case of undirected graphs by characterizing when a graph and a collection of its closed walks can be extended to an embedding such that the collection of closed walks is a subcollection of the facial walks. The necessary and sufficient conditions are stronger in the case of orientable directed embeddings than those for orientable embeddings of undirected graphs.   (Received September 11, 2020)

1163-05-701   **Steve Butler*** (`butler@iastate.edu`). *On work of Ron Graham.*

Ron Graham spent nearly 2/3 of his mathematical career working at Bell Labs and during that time was involved in a variety of research that blended together his combinatorial interests together with the applied interests of the Labs. We will discuss some of these papers, their motivation, results, and their impact on the field.   (Received September 11, 2020)

1163-05-707   **Thomas W. Tucker*** (`ttucker@colgate.edu`), **Jonathan L. Gross** and **Toufik Mansour**. *Partial-Tuality Polynomials, Part 2.* Preliminary report.

Given partial-tuality operator for a single ribbon e, the effect of that operator on the type (proper, untwisted loop, twisted loop) of edge e and its dual edge e^* is given. This analysis is used to show that if A is a spanning tree of the ribbon graph G, all partial-tualities with respect to A have one vertex (a bouquet), except Petrie duality $\times$. Partial-$*, \times$ polynomials are given for six families of ribbon graphs derived from the cycle in the sphere or projective plane. A recursion is given for the partial-$\times$ polynomials of the linear family of ladders L_n and these polynomials are shown to be log-concave. The partial-$* \times *$ polynomial for L_n is shown to be $(1 + z)(1 + 3z + 4z^2)^n$. This same formula also applies to any graph obtained from a single edge by adding n trisected parallel edges, a subfamily of series-parallel networks.   (Received September 11, 2020)

1163-05-751   **Thomas J. Savitsky** and **Steven Schluchter*** (`steven.schluchter@gmail.com`), Department of Mathematical Sciences, George Mason University, 4400 University Drive, MS: 3F2, Fairfax, VA 22030. *Some excluded minors of the spindle surface.*

We will discuss how we identified, via a computer search, 143 excluded minors of the spindle surface, the space formed by the identification of two points of the sphere. We also identified 847 topological obstructions. We give evidence to support our conjecture that our list is complete: Per our search, any additional excluded minors or topological obstructions must have at least 12 vertices and 28 edges.   (Received September 12, 2020)

1163-05-755   **Dennis Stanton***, School of Mathematics, 206 Church St SE, University of Minnesota, Minneapolis, MN 55455. *Combinatorics of type R_I orthogonal polynomials.* Preliminary report.

In 1995 Ismail and Masson defined a new general family of orthogonal polynomials by perturbing the 3-term recurrence relation. We give the combinatorial theory for these polynomials which corresponds to Viennot's theory. Included are path models for the polynomials and moments, continued fractions and Hankel determinants. Examples are given in the Askey scheme, including Askey-Wilson and q-Racah polynomials. This is joint work with Jang Soo Kim.   (Received September 12, 2020)

1163-05-756   **Bogumil Kaminski**, Warsaw, Poland, **Pawel Pralat*** (`pralat@ryerson.ca`), Toronto, ON , Canada, and **Francois Theberge**, Ottawa, ON , Canada. *An Unsupervised Framework for Comparing Graph Embeddings.*

The goal of many machine learning applications is to make predictions or discover new patterns using graph-structured data as feature information. In order to extract useful structural information from graphs, one might want to try to embed it in a geometric space by assigning coordinates to each node such that nearby nodes are more likely to share an edge than those far from each other. There are many embedding algorithms (based on techniques from linear algebra, random walks, or deep learning) and the list constantly grows. As a result, selecting the best embedding is a challenging task and very often requires domain experts. Our general framework assigns the divergence score to each embedding which, in an unsupervised learning fashion, distinguishes good from bad embeddings. In order to benchmark embeddings, we generalize the Chung-Lu random graph model to incorporate geometry.   (Received September 12, 2020)

1163-05-785   **Lina Fajardo Gómez**, **Margherita Maria Ferrari*** (`mmferrari@usf.edu`), **Nataša Jonoska** and **Masahico Saito**. *Homology of directed graphs with application to DNA recombination.* Preliminary report.

A double-occurence word (DOW) is a word in which every symbol appears exactly twice. We consider the so called repeat patterns ($\alpha\alpha$) and return patterns ($\alpha\alpha^R$), with gaps allowed between the α's; these patterns generalize square and palindromic factors of DOWs, respectively. In the context of genomics, pattern deletions on DOWs have been used to study DNA recombination in certain species of ciliates. We model these reduction processes with a directed graph where vertices are DOWs, and an edge from w to w' exists if w' is obtained from w through a pattern deletion. On this graph, we consider the cell complex consisting of products of directed simplices and define a new boundary operator. This allows the computation of homology groups, which will help in identifying rearrangement pathways that may be of interest.   (Received September 12, 2020)

1163-05-790   **Bogumil Kaminski**, Warsaw, Poland, **Pawel Pralat*** (`pralat@ryerson.ca`), Toronto, ON , Canada, and **Francois Theberge**, Ottawa, ON , Canada. *Community Detection Algorithm Using Hypergraph Modularity.*

Modularity is designed to measure the strength of division of a network represented as a graph into clusters. Graphs with high modularity have dense connections between the vertices within clusters but sparse connections between vertices of different clusters. As a result, modularity is often used in optimization methods for detecting community structure in networks. However, many networks that are currently modelled as graphs would be more accurately modelled as hypergraphs. Unfortunately, the theory and tools are still not sufficiently developed to allow most problems, including clustering, to be tackled directly within this context. We propose one of the very first community detection algorithms for hypergraphs after generalizing the graph modularity function to hypergraphs. The main feature of our algorithm is that it can be adjusted to various scenarios depending on how often vertices in one community share hyperedges with vertices from other community.   (Received September 12, 2020)

1163-05-799   **Stephen J Young***, stephen.young@pnnl.gov. *Ramanujan Graphs and Supercomputing Topologies.* Preliminary report.

In modern supercomputing systems a significant fraction of the overall computation time is dedicated to interprocess communication. As a consequence, significant amount of hardware and algorithmic resources are devoted to decreasing the impact of communication congestion on process runtime. In this work we evaluate the potential applicability of Ramanujan graphs to design supercomputing topologies which are more robust to communication congestion. We specifically consider the ability of supercomputing topologies inspired by the explicit Ramanujan graph constructions given by Margulis as well as Lubotzky, Phillips, and Sarnak, to minimizing the communications overhead of supercomputing topologies. Joint work with Sinan Aksoy, Kevin Barker, Paul Bruillard, Tobias Hagge, Mark Kempton, Joseph Manzano, Carlos Ortiz Marrero, Mark Raugas, and Joshua Suetterlein.   (Received September 13, 2020)

1163-05-811   **Joanna A. Ellis-Monaghan***, jellismonaghan@gmail.com, and **M. N. Ellingham**. *Balanced edge-outer embeddable graphs.* Preliminary report.

A graph is <u>edge-outer embeddable</u> if it has an orientable embedding with a special face whose boundary uses every edge at least once. While every graph has such an edge-outer embedding, finding one with a minimum size special face is NP-hard. For an Eulerian graph however, there is always an edge-outer embedding with a smallest possible face, namely one whose boundary is an Euler circuit. However, this says nothing about the number or sizes of the remaining faces. Thus, the interesting question for Eulerian graphs becomes minimizing the total

number of faces, ideally by finding a <u>balanced</u> edge-outer embedding, that is, one with exactly two faces where each is bounded by an Euler circuit. We put this problem in the context of DNA self-assembly, compatible Euler circuits, and tournament embeddings, and then present some necessary and some sufficient conditions for graphs and digraphs to have balanced edge-outer embeddings.   (Received September 13, 2020)

1163-05-832   **Benjamin Braun** and **Julianne Vega*** (`jvega30@kennesaw.edu`). *Hajos-type Constructions and Neighborhood Complexes.*

Any graph G with chromatic number k can be constructed by iteratively performing certain graph operations on a sequence of graphs starting with K_k, resulting in a variety of Hajos-type constructions for G. Finding such a construction for a given graph or family of graphs is a challenging task. In this talk, I will present our results which imply that for a graph G with a highly-connected neighborhood complex, the end behavior of the construction is quite restricted. Time permitting I will share results of computational experiments.   (Received September 13, 2020)

1163-05-909   **Upama Nakarmi*** (`unakarmi@usf.edu`). *Analyzing Critical Components of Power Grids During Cascading Failures Using Community Structures in Interaction Graphs.*

Cascading failures in high-voltage transmission networks of power grids can result in large-scale blackouts that cause massive economic and social disruption. Analyses and mitigation of cascading failures are challenging problems due to the large number of components in power grids and their complex interactions during cascades. In this study, cascading failure data is used to construct interaction graphs, which capture the local as well as the complex at-distance interactions caused due to the physics of electricity. Next, community structures in interaction graphs are studied to show that communities contain pertinent information about the behavior of cascade processes. In general, the likelihood that a cascade entering a community will spread within the same community is higher compared to the likelihood of it spreading to other communities. Hence, communities act as traps for failures during cascades. Furthermore, a community-centrality metric is defined to identify critical components of cascade processes. Various evaluation techniques including data-driven, graph-based, epidemic simulation-based, and power system simulation-based are used to verify the importance of the components and compare with ones identified using traditional centrality metrics.   (Received September 14, 2020)

1163-05-944   **Axel Gomez*** (`axel.gomez@upr.edu`), **Lucy A Martinez**, **Jan Carrasquillo-Lopez**, **Heidi D. Perez**, **Sebastian Papanikolaou** and **Lino Yoshikawa**. *Minimum Rank of Regular Bipartite Graphs.*

The minimum rank of a graph G is the smallest possible rank of a matrix A over any field with the same off–diagonal, nonzero pattern as the adjacency matrix of G. In this talk, we show the true minimum rank of a class of $n-1$ regular bipartite graphs where $|V_1| = |V_2| = n$ using zero forcing sets and linear recursive sequences. We also discuss the relation between the minimum rank of G and the possible dimension of a Locally Recoverable Code whose recovery sets are the neighborhoods of G.   (Received September 14, 2020)

1163-05-986   **Radmila Sazdanovic*** (`rsazdan@ncsu.edu`), Department of Mathematics NC State University, PoBox 8205, Raleigh, NC 27695, and **Alex Chandler**, **Salvatore Stella** and **Martha Yip**. *On the strength of Khovanov-type categorification of the Stanley symmetric function.*

We investigate the strength of chromatic symmetric homology as a graph invariant. Chromatic symmetric homology is a Khovanov-type categorification of the chromatic symmetric function for graphs. Its Frobenius characteristic is a q,t generalization of the chromatic symmetric function. We exhibit three pairs of graphs where each pair has the same chromatic symmetric function but distinct homology. We also show that integral chromatic symmetric homology contains torsion, and based on computations, conjecture that $\mathbb{Z}_2$-torsion in bigrading (1,0) detects nonplanarity in the graph.   (Received September 14, 2020)

1163-05-1077   **Emily Cairncross**, **Joshua Carlson**, **Peter Hollander**, **Benjamin Kitchen**, **Emily Lopez** and **Ashley Zhuang*** (`azhuang@college.harvard.edu`). *Throttling for standard zero forcing on directed graphs.*

Zero forcing is a process of coloring in a graph that initially has blue and white vertices. The *color change rule* is applied repeatedly: a blue vertex u can force a white vertex w to become blue if w is the only white neighbor of u. This coloring procedure has connections to the inverse eigenvalue problem, and it has applications in engineering and physics. The idea of throttling is to give an optimal balance of resources and time when zero forcing. In particular, the *throttling number* of a graph minimizes the sum of the number of vertices initially blue and the number of time steps needed to color every vertex blue. The process of zero forcing can also be extended to directed graphs (digraphs), in which a white vertex w can be forced if it is the only white out-neighbor of a

blue vertex u. This talk will present recent results on throttling for directed graphs.   (Received September 14, 2020)

1163-05-1097   **Jason T Suagee*** (`jsuagee@gmail.com`). *Generalized Schnyder woods for surface triangulations.*

A Schnyder wood on a planar triangulation is a choice of edge orientations and edge colorings in 3 colors for which the incident edges to each vertex all exhibit the same particular cyclic pattern of edge direction and color assignment around the vertex. They can be used to produce planar drawings and geodesic graph embeddings, as well as to encode and uniformly sample planar triangulations at random in linear time. Schnyder Woods have been generalized to higher genus surfaces where it is hoped that they will also provide similar encoding and random sampling capabilities, however their existence and complete characterization properties are only known at this time in the toroidal case. In this talk we will outline a method for producing generalized Schnyder woods on a large family of surfaces of arbitrarily high genus, and a theorem that guarantees the existence of these generalized Schnyder woods on higher genus triangulations provided that the edge-width of the triangulation is large enough.   (Received September 14, 2020)

1163-05-1120   **John Maharry*** (`maharry.1@osu.edu`). *Graph structures and flexibility of surface embeddings.*

We will give a survey of some results on the flexibility of graph embeddings on surfaces. The parameters involved include connectivity, representativity (face-width) and the genus of the surface. All flexibility of embeddings on the Plane and on the Projective plane are known [Whitney; M. and Slilaty]. Robertson, Zha and Zhao showed that representativity $\rho(\sigma) \geq 4$ on guarantees a unique labeled embedding on the Torus except for a single case. With Zha, we show that $\rho(\sigma) \geq 4$ guarantees uniqueness of labeled embeddings on the Klein bottle. Finally, if a graph has a 4-representative embedding on the Torus it must be orientably simple meaning that it does not have an embedding on the Klein bottle.   (Received September 14, 2020)

1163-05-1125   **Jose H Ceniceros*** (`jcenicer@hamilton.edu`), 198 College Hill Rd, Clinton, NY 13323, and **I. Churchill**, **M. Elhamdadi** and **M. Hajij**. *Cocycle Invariants of Oriented Singular Knots.* Preliminary report.

We extend the quandle cocycle invariant to oriented singular knots and links using algebraic structures called oriented singquandles and assigning weight functions at both regular and singular crossings. This invariant coincide with the classical cocycle invariant for classical knots but provides extra information about singular knots and links.   (Received September 14, 2020)

1163-05-1267   **Sean English*** (`senglish@illinois.edu`), **Calum MacRury** and **Pawel Pralat**. *Zero Forcing Randomly on the Random Graph.*

The zero forcing process is an iterative graph coloring process in which at each timestep a colored vertex with a single uncolored neighbor can force this neighbor to become colored. In this talk, we will consider probabilistic zero forcing, where a blue neighbor has some non-zero probability of forcing a white vertex at each step.

We study probabilistic zero forcing on the Erdős-Réyni random graph, $G(n, p)$ and determine bounds on the propagation time, or the total number of rounds necessary to color the entire graph blue, when starting with a single blue vertex.   (Received September 15, 2020)

1163-05-1269   **Paul Bruillard*** (`pbruillard@expedia.com`), 1111 Expedia Group Way W, Seattle, WA 98119. *A Survey of Ranking and Recommendation Systems.*

Online search engines and market places have shown the importance of ranking systems. From Netflix, to Google, to Amazon, providers that are able to offer the best and most expedient recommendations to customers are able to improve their market share and increase customer loyalty. This arguably began with PageRank, and its success allowed Google to dominate the search engine marketplace. Often these ranking problems are most readily expressed combinatorially, but not as a smooth loss function for conventional optimization schemes. Thus it is common for proxy loss functions to be derived or problem relaxations to be considered. Furthermore, in many cases the optimal ordering for a customer is at odd with the optimal ordering for a business. This is particularly pronounced in two-sided market places. In this talk we will review ranking problems faced by industry, with a focus on problems arising in online travel agencies. We will discuss how the optimization problems are formulated and tackled in real world settings and highlight differences between the optimization problem being solved and the optimal problem. Time permitting we will touch on problems related to customer personalization and ranking inconsistencies that arise in two-sided market places.   (Received September 15, 2020)

1163-05-1306   **James Garrett**, **Natasha Jonoska*** (`jonoska@mail.usf.edu`), **Hwee Kim** and
Masahico Saito. *Monoids associated with DNA origami.*

We initiate an algebraic approach to study DNA origami structures by associating an element from a monoid to each structure. We identify two types of basic building blocks and describe a DNA origami structure by their composition. These building blocks are taken as generators of a monoid, which we call the *origami monoid*. Motivated by the well studied Jones monoids, we identify a set of relations that characterize the origami monoid. With the aid of a *GAP* program, we prove the finiteness of the origami monoid, and propose a normal form of the elements. We study a connection between the Green's classses of the origami monoid and the Green's classes of a direct product of Jones monoids.   (Received September 15, 2020)

1163-05-1327   **Boris Brimkov*** (`boris.brimkov@sru.edu`). *Computational methods and heuristics for zero forcing.*

This talk surveys computational methods for zero forcing. Several heuristics and exact methods based on integer programming, Boolean satisfiability, and dynamic programming are presented and computationally compared on various types of graphs.   (Received September 15, 2020)

1163-05-1354   **Derek D Young*** (`dyoung@mtholyoke.edu`). *The Maximum Nullity and Zero Forcing Number of a Graph.*

The maximum nullity of a simple graph is the largest possible nullity over all symmetric matrices which can be described by the graph. The zero forcing number of a simple graph is the minimum number of blue vertices needed to force all vertices of the graph blue by applying the color change rule. In 2008, it was shown that the maximum nullity of a graph could be bounded above by the zero forcing number of the graph. During this time, the problem of characterizing graphs whose maximum nullity is the same as the zero forcing number was also posed. In this talk, we will discuss some techniques which can be used to determine if the maximum nullity is the same as the zero forcing number for some families of graphs.   (Received September 15, 2020)

1163-05-1373   **Aric Hagberg*** (`hagberg@lanl.gov`), **Jeffrey D Hyman**, **Dave Osthus**, **Shriram Srinivasan**, **Hari Viswanathan** and **Gowri Srinivasan**. *A bipartite graph model for backbone discovery in discrete fracture networks.*

We present a graph-based method to identify primary flow and transport subnetworks in three-dimensional discrete fracture networks (DFNs). We develop a bipartite graph representation as a course-scale representation of a DFN that integrates fracture network topology, fracture geometry, and hydraulic properties. The two most common graph-representations of DFNs, vertices representing intersections and vertices representing fractures, are projections of this bipartite graph thereby providing a generalization of existing modeling approaches. Primary subnetworks are identified by running a heuristic algorithm that determines the edge-disjoint shortest paths through the graph which correspond to the regions where the fastest transport occurs. Using the identified primary subnetworks computational estimates of first-passage times for transport can be computed an order of magnitude faster.   (Received September 15, 2020)

1163-05-1384   **Daniel Slilaty*** (`daniel.slilaty@wright.edu`), Department of Mathematics and Statistics, Wright State University, Dayton, OH 45435. *Properties of Voltage Graphs Embedded in Surfaces.*

Given a multiplicative group Γ, a <u>voltage graph</u> (also called a <u>gain graph</u>) is a triple (G, φ) in which G is a graph and φ is a labeling of the oriented edges of G for which $\varphi(e^{-1}) = \varphi(e)^{-1}$. The labeling φ extends to any walk $e_1, \ldots, e_n$ by setting $\varphi(e_1, \ldots, e_n) = \varphi(e_1) \cdots \varphi(e_n)$. A closed walk w is said to be <u>balanced</u> when $\varphi(w) = 1$. An <u>embedding</u> of a voltage graph (G, φ) in a surface S is an embedding of G in S in which all facial boundary walks are balanced. Embedded voltage graphs have many interesting properties. We will survey some known results and state some unanswered questions.   (Received September 15, 2020)

1163-05-1390   **Sarah E. Anderson**, University of St. Thomas, St. Paul, MN, **Karen L. Collins**, Wesleyan University, Middletown, CT, **Daniela Ferrero*** (`dferrero@txstate.edu`), Department of Mathematics, Texas State University, San Marcos, TX 78666, **Leslie Hogben**, American Institute of Mathematics, San Jose, CA, and Iowa State University, Ames, IA, **Carolyn Mayer**, Worcester Polytechnic Institute, Worcester, MA, and Sandia National Laboratories, Albuquerque, NM, **Ann N. Trenk**, Wellesley College, Wellesley, MA , and **Shanise Walker**, University of Wisconsin-Eau Claire, Eau Claire, WI. *Product Throttling for Power Domination.*

Power domination is a particular form of graph searching in which an initial step of standard graph domination is followed by a zero forcing process. Power domination was introduced independently from zero forcing, as a model

for the monitoring process of electrical power networks. As electrical power systems become increasingly more complex, so does their monitoring process. Initially aimed at preventing blackouts and power surges, nowadays the infrastructure used for monitoring power systems is also used to enhance the quality of their service. As a result, new questions about the power domination problem in graphs have also appeared.In particular, the interest on the trade-off between time and costs involved in a power domination process, leads to the study of product throttling for power domination. In this talk, we introduce the power domination problem and present the concept of power throttling number of a graph, which we present for certain graph families. Recent results, including bounds and a characterization of graphs with extremal product throttling numbers, will also be presented.   (Received September 15, 2020)

1163-05-1482   **Ashwin Sah*** (`asah@mit.edu`) and **Mehtaab Sawhney** (`msawhney@mit.edu`). *Local limit theorems for subgraph counts.*
We introduce a general framework for studying anticoncentration and local limit theorems for random variables, including graph statistics. Our methods involve an interplay between Fourier analysis, decoupling, hypercontractivity of Boolean functions, and transference between "fixed-size" and "independent" models. We also adapt a notion of "graph factors" due to Janson.

As a consequence, we derive a local central limit theorem for connected subgraph counts in the Erdős-Rényi random graph $G(n, p)$, building on work of Gilmer and Kopparty and of Berkowitz. These results improve an anticoncentration result of Fox, Kwan, and Sauermann. We also derive a local limit central limit theorem for induced subgraph counts, as long as p is bounded away from a set of "problematic" densities. We then prove these restrictions are necessary, resolving anticoncentration conjectures of Fox, Kwan, and Sauermann in the negative.

Finally, we also examine the behavior of counts of k-term arithmetic progressions in subsets of $\mathbb{Z}/n\mathbb{Z}$ and deduce a local limit theorem wherein the behavior is Gaussian at a global scale but has nontrivial local oscillations (according to a Ramanujan theta function).   (Received September 15, 2020)

1163-05-1539   **Naiomi Cameron*** (`naiomi.cameron@spelman.edu`) and **Kendra Killpatrick**. *Inversion generating functions for signed pattern avoiding permutations.*
We consider the classical Mahonian statistics on the set $B_n(\Sigma)$ of signed permutations in the hyperoctahedral group B_n which avoid all patterns in Σ, where Σ is a set of patterns of length two. In 2000, Simion gave the cardinality of $B_n(\Sigma)$ in the cases where Σ contains either one or two patterns of length two and showed that $|B_n(\Sigma)|$ is constant whenever $|\Sigma| = 1$, whereas in most but not all instances where $|\Sigma| = 2$, $|B_n(\Sigma)| = (n + 1)!$. We answer an open question of Simion by providing bijections from $B_n(\Sigma)$ to S_{n+1} in these cases where $|B_n(\Sigma)| = (n + 1)!$. In addition, we extend Simion's work by providing a combinatorial proof in the language of signed permutations for the major index on $B_n(21, \bar{2}\bar{1})$ and by giving the major index on $D_n(\Sigma)$ for $\Sigma = \{21, \bar{2}\bar{1}\}$ and $\Sigma = \{12, 21\}$. The main result of this paper is to give the inversion generating functions for $B_n(\Sigma)$ for almost all sets Σ with $|\Sigma| \leq 2$.   (Received September 15, 2020)

1163-05-1553   **Noah Kravitz*** (`nkravitz@princeton.edu`). *Barely lonely runners and very lonely runners: a refined approach to the Lonely Runner Problem.*
We introduce a sharpened version of the well-known Lonely Runner Conjecture of Wills and Cusick. Let $\|x\|$ denote the distance from a real number x to the nearest integer. For each set of positive integer speeds $v_1, \ldots, v_n$, define the associated maximum loneliness

$$\mathrm{ML}(v_1, \ldots, v_n) = \max_{t \in \mathbb{R}} \min_{1 \leq i \leq n} \|tv_i\|.$$

The Lonely Runner Conjecture asserts that

$$\mathrm{ML}(v_1, \ldots, v_n) \geq \frac{1}{n + 1}$$

for all choices of $v_1, \ldots, v_n$. If this conjecture is true, then the quantity $1/(n+1)$ is the best possible, for there are known equality cases with $\mathrm{ML}(v_1, \ldots, v_n) = 1/(n+1)$. A natural but hitherto unasked question is: If $v_1, \ldots, v_n$ satisfy the Lonely Runner Conjecture but are not an equality case, must $\mathrm{ML}(v_1, \ldots, v_n)$ be uniformly bounded away from $1/(n + 1)$?

We conjecture that, surprisingly, the answer is yes. More precisely, we conjecture that for each choice of $v_1, \ldots, v_n$, we have either $\mathrm{ML}(v_1, \ldots, v_n) = s/(ns + 1)$ for some $s \in \mathbb{N}$ or $\mathrm{ML}(v_1, \ldots, v_n) \geq 1/n$. Our main results are: confirming this stronger conjecture for $n \leq 3$; and confirming it for $n = 4$ and $n = 6$ in the case where one speed is much faster than the rest.   (Received September 15, 2020)

1163-05-1562   **Sinan Aksoy**, **Jackson Earl**, **Emily Heath*** (`eheath3@illinois.edu`), **Cliff Joslyn**,
                 Terran Mott, **Brenda Praggastis** and **Emilie Purvine**. *Low-Order Approximations of
                 Hypergraphs.* Preliminary report.

While researchers have long used graphs to model networks, hypergraphs allow edges containing any number of
nodes and hence can serve as more natural and effective models for complex data with multiway relations. Given
the many ways to represent the same data with different graphs and hypergraphs, it is important to consider how
the models we use affect our analysis. How can we manage the tradeoffs which arise from reducing hypergraph
models to simpler, but smaller, graph models? In order to understand the loss of information that accompanies
one such simplification of a hypergraph to a graph, we explore the set of hypergraphs with the same line graph,
which records qualitative information about overlapping hyperedges.   (Received September 15, 2020)

1163-05-1621   **Tayler Fernandes Nunez*** (`tfernandesnunez@smith.edu`), Department of Mathematics,
                 Smith College, Northampton, MA 01063, and **Hope Pungello**. *Generalized splines on
                 dual graphs to triangulations.* Preliminary report.

Splines are a fundamental tool across applied math and analysis, used in fields from computer graphics to data
interpolation. We work with an algebraic- combinatorial generalization of splines, in which we start with an edge-
labeled graph. A spline on this graph is a labeling of vertices so that adjacent vertex-labels differ by a multiple
of the edge-label. We will study particular families of graphs (especially those dual to planar triangulations) in
order to find a basis (and/or the dimension) of the space of splines when using polynomial labels of degree at
most 2. This is part of a longstanding open problem sometimes called the "upper-bound conjecture"—which,
curiously, is completely solved in the case of degree 4 or larger, has a well-known and well-tested conjectural
formula in degree 3, and completely mysterious in degree 2.   (Received September 15, 2020)

1163-05-1629   **Jack Kendrick**, **Meredith Pan** and **Charlotte Steinhardt*** (`csteinhardt@smith.edu`),
                 Department of Mathematics, Smith College, Northampton, MA 01063. *Springer fibers,
                 webs, and tableaux.* Preliminary report.

Springer fibers are a family of geometric objects that are a kind of "eigenspace" for an object called the flag
variety (which consist of nested sequences of lines inside planes inside 3-dimensional spaces, etc.). The pieces
of Springer fibers are called cells and can be described explicitly by matrices. The cells are also known to be
counted by various combinatorial objects, including objects called webs. We describe recent work using webs to
describe the topology of Springer fibers and to describe promotion, a combinatorial operation on Young tableaux
(a filled array of boxes akin to Sudoku puzzles).   (Received September 15, 2020)

1163-05-1646   **Puck Rombach*** (`puck.rombach@uvm.edu`). *Computational Complexity and Forbidden
                 Graph Patterns.*

Many graph problems are NP-hard in general, but may be in P for certain classes of graphs. Instead of classifying
problems, we can therefore fix a problem and classify graphs into "hard" and "easy" classes. This is often done
with respect to classes closed under the minor relation, which is facilitated by the fact that these are well-
founded. However, classifying graphs based on forbidden subgraphs or induced subgraphs (such as bipartite
graphs, graphs of bounded degree, etc) is more appropriate in many applications. In this case, the "hard" and
"easy" classification is more complicated.   (Received September 15, 2020)

1163-05-1659   **Juergen Kritschgau*** (`jkritsch@iastate.edu`) and **Josh Carlson**. *Forbidden induced
                 subgraph characterizations for throttling.*

Zero forcing can be described as a graph process that uses a color change rule in which vertices change white
vertices to blue. The throttling number of a graph minimizes the sum of the number of vertices initially colored
blue and the number of time steps required to color the entire graph. This talk presents results that characterize
the graphs on n vertices with throttling number at least $n - k$ using a finite family of induced subgraphs. These
forbidden subgraphs are constructed for standard throttling.   (Received September 16, 2020)

06 ▶ *Order, lattices, ordered algebraic structures*

1163-06-870   **Ulysses A Alvarez*** (`alvarezs@math.binghamton.edu`), 85 Grand Blvd, Binghamton, NY
                 13905, Binghamton, NY 13905, and **Ross Geoghegan**. *The Up Topology for Mirror
                 Topological Posets.*

For a discrete poset $\mathcal{X}$, McCord proved that there exists a weak homotopy equivalence from the order complex
$|\mathcal{X}|$ to $\mathcal{X}$ where $\mathcal{X}$ has the Up topology. Much later, Živaljević defined the notion of the order complex of
a topological poset. For a large class of such topological posets we prove the analog of McCord's theorem,

namely that there exists a weak homotopy equivalence from the order complex to the topological poset with the Up topology. An interesting example of a topological poset of said class is the truncated Grassmann poset $\widetilde{\mathcal{G}}_{n+1}(\mathbb{R})$, that is, the collection of nonzero proper linear subspaces of $\mathbb{R}^n$, whose order complex is understood to be homotopy equivalent to the m-sphere for some m. In particular, there is a weak homotopy equivalence from the m-sphere to $\widetilde{\mathcal{G}}_{n+1}(\mathbb{R})$ with the Up topology.   (Received September 13, 2020)

1163-06-1185   **Deveena Banerjee**, **Sara Chari*** (schari0301@gmail.com) and **Adriana Salerno**.
                  Higher dimensional origami constructions. Preliminary report.
When performing origami folds on a piece of paper, we may view the paper as the complex plane and the folds as lines in the plane. We start with two seed points and make a fold through each, generating a new intersection point, and by iterating this process for each pair of points formed, we generate a subset of the complex plane. The structure of this set depends on the starting points and on the set of angles at which we create our folds. We extend previously known results about the algebraic and geometric structure of these sets to higher dimensions; in particular, instead of the complex plane, we may start with a quaternion algebra.   (Received September 15, 2020)

1163-06-1236   **Shuhong Gao**, **Ryann Cartor*** (rcartor@clemson.edu), **Benjamin Case**, **Yan Ren**
                  and **Hui Xue**. *Lattice-based Signature Scheme based on M-SIS.*
The 3rd round candidates of the NIST post-quantum cryptography include two signature schemes based on lattices, namely Falcon and Dilithium. Falcon is based on NTRU and has much smaller signature sizes, while Dilithium is based on M-RLWE and is fast and easy to implement. The current paper presents a signature scheme based on M-RLWE and has smaller signature size than Dilithium's under the same security levels. The new signature scheme has signature sizes only slightly bigger than Falcon's but does not require expensive Gaussian sampling.   (Received September 15, 2020)

08 ▶ *General algebraic systems*

1163-08-273   **Jasmine Elder*** (westljasmine@gmail.com). *Post Quantum Cryptography on Reed-Muller code based McEliece Encryption Scheme Variant.*
Cryptography is the science of converting a plaintext, or message into a ciphertext, or scrambled message in order to provide secrecy. With the development of quantum computers on the horizon, it poses a threat to the security of the current cryptosystems that are in place. Quantum computers are able to process complex algorithms and perform advance calculations like integer factorization and discrete logarithm problem that are expected to break all the current cryptosystems. It is already well-known that the McEliece Encryption scheme with Reed Muller codes is not considered as a secure system for both classical and quantum computers. We introduce and study the Reed-Muller code-based RLCE scheme. These successful attacks on the Reed Muller code based McEliece encryption scheme, namely, the Minder-Shokrollahi's attack, the Chizhov-Borodin's attack, and the Square Code attack, are proven to be unsuccessful for the proposed Reed Muller code-based RLCE scheme. We determine the optimal method in preventing these known attacks against the new encryption scheme. We suggest parameters needed for the 128, 192, and 256 bits security level.   (Received August 31, 2020)

1163-08-655   **Sultan Catto*** (scatto@gc.cuny.edu), 365 Fifth Avenue, New York, NY 10016-4309. *Role of Quaternions, Octonions, Jordan and related algebras in physics.*
Elegant algebras, specifically quaternions and octonions, Jordan and related algebras have arisen in a concerted manner in unified theories of fundamental interactions in nature. We shall give a survey of these exceptional structures, and their general aspects interwoven with supersymmetry (realized linearly and nonlinearly), with superstrings, supermembranes, and M-theory. This talk will deal with the historical aspects of algebraic, group theoretical, functional, geometrical, and topological aspects of quaternions and octonions. If time permits, we will conclude with an example and application to color algebras and dynamical supersymmetry leading to multiquark bags in nature.   (Received September 10, 2020)

1163-08-788   **Catherine M Hsu***, chsu2@swarthmore.edu, and **Holley Friedlander**, **Elena Fuchs**,
                  Piper H, **Katherine Sanden**, **Damaris Schindler** and **Katherine Stange**. *Prime components in Apollonian packings.*
An Apollonian circle packing is a fractal arrangement formed by repeatedly inscribing circles into the interstices in a Descartes configuration of four mutually tangent circles. The curvatures of the circles in such a packing are often integers, and so it is natural to ask questions about their arithmetic properties. For example, it is known

by work of Bourgain-Fuchs that a positive fraction of integers appear as curvatures in any integral Apollonian circle packing. In this talk, we investigate the arithmetic properties of the collection of integers appearing in "thickened prime components" of Apollonian circle packings.   (Received September 12, 2020)

11 ▶ Number theory

1163-11-29   **Michael David Fried*** (`michaeldavidfried@gmail.com`). *Every finite group challenges extending Falting's Theorem.*

Consider finite group G; ℓ a prime dividing $|G|$ ($|G$ has no $\mathbf{Z}/\ell$ quotient); and $\mathbf{C} = \{\mathrm{C}_1 \ldots \mathrm{C}_r\}$ any $r \geq 4$ conjugacy classes of order prime to ℓ elements. Ex: $G = A_5$, $\mathbf{C}$ is 4 repetes of the 3-cycle conjugacy class, and $\ell = 2$.

For $(G, \mathbf{C}, \ell)$, $M' \in I$, $|I| < \infty$ gives a $\mathbf{Z}_\ell[G]$ lattice $L_{M'}$ as kernel of an ℓ-Frattini cover $\tilde{G}_{M'} \to G \implies$ a moduli space series

$$\cdots \to \mathcal{H}(G, \mathbf{C}, \ell, L)_k \to \cdots \to \mathcal{H}(\ldots)_1 \to \mathcal{H}(G, \mathbf{C}, \ell, L)_0 \to J_r.$$

Terms are quadi-projective varieties. When $r = 4$ all are upper half plane quotients; J_4 is the classical j-line, minus ∞.

Only for G "close to" dihedral ($r = 4$) are these modular curves.

Main Conjecture: Let K be any number field. For k large, projective normalization of $\mathcal{H}(G, \mathbf{C}, \ell, L)_k$ has general type, and $\mathcal{H}(G, \mathbf{C}, \ell, L)_k$ has no K points.

For $r = 4$, there are two proofs (myself/Cadoret-Tamagawa). We compare these results and how even this presents an unproven challenge to extending Falting's Theorem.   (Received July 08, 2020)

1163-11-95   **Garen Chiloyan***, Department of Mathematics, University of Connecticut, Storrs, CT 06269, and **Álvaro Lozano-Robledo**, Department of Mathematics, University of Connecticut, Storrs, CT 02629. *A Classification of Isogeny-Torsion Graphs over* $\mathbb{Q}$.

An isogeny graph is a nice visualization of the isogeny class of an elliptic curve. A theorem of Kenku shows sharp bounds on the number of distinct isogenies that a rational elliptic curve can have (in particular, every isogeny graph has at most 8 vertices). In this talk, we classify what torsion subgroups over $\mathbb{Q}$ can occur in each vertex of a given isogeny graph of elliptic curves defined over the rationals. This is joint work with Álvaro Lozano-Robledo.   (Received August 13, 2020)

1163-11-101   **Alexandru Buium***, 4101 Lara Dr NE, Albuquerque, NM 87111, and **Lance E. Miller**. *Perfectoid spaces arising from arithmetic jet spaces.* Preliminary report.

Using arithmetic jet spaces we attach perfectoid spaces to smooth schemes and we attach morphisms of perfectoid spaces to δ-morphisms of smooth schemes. We also study perfectoid spaces attached to arithmetic differential equations defined by some of the remarkable δ-morphisms appearing in the theory such as the δ-characters of elliptic curves and the δ-period maps on modular curves.   (Received August 14, 2020)

1163-11-147   **Haochen Wu*** (`wuh15@wfu.edu`), 214 Crowne View Dr, Winston-Salem, NC 27106. *On the Number of Representations by Primitive Positive-Definite Integer-Valued Quaternary Quadratic Forms.* Preliminary report.

Let $\{Q_1, Q_2, \ldots, Q_s\}$ be a finite set of primitive positive-definite integer-valued quaternary quadratic forms. We show that there exists a primitive positive-definite integer-valued quaternary quadratic form Q and a positive integer n such that Q represents n more times than Q_i for all $1 \leq i \leq s$.   (Received August 22, 2020)

1163-11-181   **Xander Faber*** (`awfaber@super.org`). *Benedetto's Trick.*

There are precisely 7 directed graphs that can occur as the portrait of rational preperiodic points for **infinitely many** conjugacy classes of quadratic polynomials over $\mathbb{Q}$. We prove this statement using a refinement of a technique in p-adic analysis due to Robert Benedetto.   (Received August 25, 2020)

1163-11-197   **Andrei S. Rapinchuk** (`asr3x@virginia.edu`), Department of Mathematics, University of Virginia, Charlottesville, VA 22903, and **Igor A. Rapinchuk*** (`rapinchu@msu.edu`), Department of Mathematics, Michigan State University, East Lansing, MI 48824. *Algebraic groups with good reduction.*

Techniques involving reduction are very common in number theory and arithmetic geometry. In particular, elliptic curves and general abelian varieties having good reduction have been the subject of very intensive investigations over the years. The purpose of this talk is to report on recent work that focuses on good reduction in the context of reductive linear algebraic groups over finitely generated fields. In addition, we will highlight

some applications to the study of local-global principles and the analysis of algebraic groups having the same maximal tori. (Parts of this work are joint with V. Chernousov.) (Received August 26, 2020)

1163-11-204 **Arseniy Sheydvasser*** (sheydvasser@gmail.com), 92 Mount Vickery Road, Southborough, MA 01772. *Twists on Quaternion Algebras and Related Orbifolds.*
The accidental isomorphism between $PSL(2, \mathbb{C})$ and the orientation-preserving isometry group of hyperbolic 3-space gives a way of relating number-theoretic information about imaginary quadratic fields to geometric information about hyperbolic 3-orbifolds. In this talk, we will discuss a similar accidental isomorphism between the orientation-preserving isometry group of hyperbolic 4-space and an algebraic group constructed from the quaternions together with a particular choice of involution. We will show that, very similarly, this will allow us to relate number-theoretic information about rational quaternion algebras to geometric information about hyperbolic 4-orbifolds. (Received August 26, 2020)

1163-11-207 **YoungJu Choie*** (yjc@postech.ac.kr), Dept of Mathematics, POSTECH, 77 Cheongam-Ro. Nam-Gu, Pohang, Gyeongbuk. 37673, South Korea. *Modular forms and Period polynomials.*
In 1991, Zagier found a striking identity, which relates a generating function containing all Hecke eigenforms together with all critical values of their corresponding L-functions to products of Jacobi theta series. The L-values are packaged into period polynomials coming from Eichler-Shimura theory of automorphic forms. In this talk we study various extensions of Zagier's identity. (Received August 28, 2020)

1163-11-249 **Jeremy Rouse*** (rouseja@wfu.edu). *Integers represented by positive-definite quadratic forms and Petersson inner products.*
Let Q be a positive-definite quaternary quadratic form with integer coefficients. We study the problem of giving bounds on the largest positive integer n that is locally represented by Q but not represented. Assuming that n is relatively prime to $D(Q)$, the determinant of the Gram matrix of Q, we show that n is represented provided that

$$n \gg \max\{N(Q)^{3/2+\epsilon} D(Q)^{5/4+\epsilon}, N(Q)^{2+\epsilon} D(Q)^{1+\epsilon}\}.$$

Here $N(Q)$ is the level of Q. We give three other bounds that hold under successively weaker local conditions on n.

These results are proven by bounding the Petersson norm of the cuspidal part of the theta series, which is accomplished using an explicit formula for the Weil representation due to Scheithauer. (Received August 30, 2020)

1163-11-262 **Ryan Carpenter** and **Charles L Samuels*** (charles.samuels@cnu.edu). *Linear Programming and the Metric Mahler Measures.*
The metric Mahler measures, denoted by $m_t(\alpha)$ for each $t \in \mathbb{R}$ and $\alpha \in \overline{\mathbb{Q}}$, form a parametrized family of heights on $\overline{\mathbb{Q}}$ that are closely related to the classical Mahler measure. While these heights have a superficial relationship to Lehmer's problem, the more compelling reason to study them arises from a connection between the map $t \mapsto m_t(\alpha)$ and the arithmetic properties of α. Nevertheless, this map is notoriously difficult to study even when $\alpha \in \mathbb{Q}$. We apply techniques from linear programming and operations research to understand its behavior in certain special cases. Our work leads to a polynomial time algorithm for computing $m_t(\alpha)$ in those cases, a large improvement over the previously best known exponential time algorithm. This work is joint with Ryan Carpenter. (Received August 31, 2020)

1163-11-271 **Ellina Grigorieva*** (egrigorieva@twu.edu), PO BOX 425464, Denton, TX 76204. *Distance learning methods for successful teaching number theory course.*
Many advanced math courses are difficult to teach remotely. For example, how to teach a student to solve complex problems in number theory or methods of rigorous proof if personal communication in class and discussion of the results face- to face are not possible? The author will demonstrate some teaching methods practiced by her in distance learning environment. The presentation will be useful to anyone who wants to help students deeply understand the subject of number theory. Student's feedback and their results confirm the importance of the proposed teaching method (Received August 31, 2020)

1163-11-313 **Neil Dummigan** and **Dan Fretwell*** (daniel.fretwell@bristol.ac.uk). *(Real Quadratic) Arthurian Tales.*
In recent years there has been a lot of interest in explicitly identifying the global Arthur parameters attached to certain automorphic forms. In particular Chenevier and Lannes were able to completely identify and prove the full lists of Arthur parameters in the case of level 1, trivial weight automorphic forms for definite orthogonal

groups of ranks 8, 16 and 24 (not a simple task!). One finds interesting modular forms hidden in these parameters (e.g. Delta and a handful of special Siegel modular forms of genus 2) along with information on the degrees of non-vanishing of certain linear combinations of Siegel theta series. Comparing Arthur parameters mod p proves/reproves various Eisenstein congruences for these special modular forms, e.g. the famous 691 congruence of Ramanujan and, more importantly, an example of a genus 2 Eisenstein congruence predicted by Harder (which, up to then, had not been proved for even a single modular form!).

In this talk I will discuss recent work with Neil Dummigan on extending the above to definite orthogonal groups over certain real quadratic fields and try to tell the analogous Arthurian tales (mysteries included). (Received September 02, 2020)

1163-11-408   **Paula Maria Machado Cruz Catarino*** (`pcatarin@utad.pt`), Department of Mathematics, School of Science and Technology, University of Tras-os-Montes e Alto Douro, 5001-801 Vila Real, Portugal. *Sequences of quaternions: A summary on my recent research journey.*

A recurrence relation is a "mathematical technique" which allows us to define sequences, sets, even operations or algorithms, from particular cases to general cases. When we use this "technique", it is important to have attention, firstly, at the initial condition(s) - which must be known - and, on the other hand, the "recurrence equation" - which is not more than the rule that will calculate the next terms in the light of predecessors. In this talk, we introduce some special sequences whose terms are quaternions defined by a recurrence relation, presenting some of its properties, generating function and generating matrix.   (Received September 05, 2020)

1163-11-409   **Sumit Chandra Mishra*** (`sumitcmishra@gmail.com`). *Local-global principles for norm one tori and multinorm tori over semi-global fields.*

Let K be a complete discretely valued field with the residue field κ. Let F be the function field of a smooth, projective, geometrically integral curve over K and $\mathcal{X}$ be a regular proper model of F such that the reduced special fibre X is a union of regular curves with normal crossings. Suppose that the graph associated to $\mathcal{X}$ is a tree (e.g. $F = K(t)$). Let L/F be a Galois extension of degree n. Assume that $\operatorname{char}(\kappa)$ does not divide n. Suppose that κ is an algebraically closed field or a finite field containing a primitive n^{th} root of unity. Then we show that the local-global principle holds for the norm one torus associated to the extension L/F with respect to discrete valuations on F, i.e., an element in $F^{\times}$ is a norm from the extension L/F if and only if it is a norm from the extensions $L \otimes_F F_{\nu}/F_{\nu}$ for all discrete valuations ν of F. We also prove that such a local-global principle holds for certain classes of multinorm tori over F with some assumptions on the residue field κ.   (Received September 08, 2020)

1163-11-428   **Hemar Godinho** and **Michael P. Knapp*** (`mpknapp@loyola.edu`), Department of Mathematics and Statistics, Loyola University Maryland, Baltimore, MD 21093. *Counterexamples to a Conjecture of Norton.*

Let $\Gamma^*(k)$ be the smallest integer s such that the equation

$$a_1 x_1^k + \cdots + a_s x_s^k = 0$$

has a nontrivial solution in every p-adic field $\mathbb{Q}_p$, regardless of the values of the (rational integer) coefficients. An old conjecture of Norton was that we should have $\Gamma^*(k) \equiv 1 \pmod{k}$ for all degrees k. This was disproved in 1974 by Bovey, who showed that $\Gamma^*(8) = 39$, but until a few years ago this was the only known counterexample. In this talk, we show that there are infinitely many counterexamples to Norton's conjecture.   (Received September 06, 2020)

1163-11-463   **David J. Grynkiewicz***, Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152. *Finite Elasticities in Krull Domains with Finitely Generated Class Group.*

Our main motivating goal is the study of factorization in Krull Domains H with finitely generated class group G. While factorization into irreducibles, called atoms, generally fails to be unique, there are various measures of how badly this can fail. One of the most important is the elasticity

$$\rho(H) = \lim_{k \to \infty} \rho_k(H)/k,$$

where $\rho_k(H)$ is the maximal number of atoms in any re-factorization of a product of k atoms. Having finite elasticity is a key indicator that factorization, while not unique, is not completely wild. This talk will describe, in brief, parts of our recent characterization of finite elasticity for any Krull Domain with finitely generated class group G by means of Convex Geometry. Indeed, most of our results are valid for the more general class of Transfer Krull Monoids (over a subset G_0 of a finitely generated abelian group G). We will focus on some of

the arithmetic consequences of the characterization, including the implied finiteness of the set of distances $\Delta(H)$ and catenary degree $\mathsf{c}(H)$, as well as the Structure Theorem for Unions holding, and also broadly describe some of the key aspects of Convex Geometry involved in the proofs. (Received September 07, 2020)

1163-11-512 **Jason Bell**, **Nils Bruin**, **Keping Huang**, **Wayne Peng** and **Thomas J Tucker*** (tjtucker@gmail.com), Math Department, University of Rochester, Rochester, NY 14607. *A Tits alternative for rational functions.*
We prove some variants of the Tits alternative for rational functions. In particular, we show that in characteristic 0 there are no semigroups of rational functions of intermediate growth (that is, all have either polynomial or exponential growth). (Received September 08, 2020)

1163-11-551 **Wade Hindes*** (wmh33@txstate.edu). *Counting points of bounded height in orbits.*
Given a suitable set of endomorphisms on a projective variety, we'll discuss some asymptotics for the number of points of bounded height in the associated semigroup orbits. (Received September 09, 2020)

1163-11-553 **Alexander J Carney*** (alexanderjcarney@rochester.edu). *Heights, dynamics, and isotriviality over finitely generated fields.*
Expanding on work of Moriwaki and Yuan-Zhang, we show how to define vector-valued arithmetic intersections and heights relative to any finitely generated field extension K/k. When $K = \mathbb{Q}$ or K/k has transcendence degree one, these reproduce the usual number field and geometric heights, respectively. Letting X be a projective variety over K, we prove that these heights have a Northcott property, provided either that $k = \mathbb{Q}$ or $\mathbb{F}_q$, or that X is totally non-isotrivial over k, a necessary condition which is slightly stronger than not isotrivial. This generalizes previous work of Baker, Chatzidakis-Hrushovski, and the Lang-Néron Theorem. Now let $f, g : X \to X$ be two polarizable dynamical systems on X. Using the above vector-valued intersections and the Northcott property for heights, we prove the following rigidity theorem for preperiodic points: If $\mathrm{Prep}(f) \cap \mathrm{Prep}(g)$ is Zariski dense in X, then $\mathrm{Prep}(f) = \mathrm{Prep}(g)$. By the Lefschetz principle, this holds over any field. (Received September 10, 2020)

1163-11-555 **Minsik Han*** (minsik_han@brown.edu). *Misiurewicz polynomials for rational maps.*
For a 1-parameter family of rational maps, we can ask which parameter values make the corresponding map post-critically finite with a certain dynamical portrait. Most studies were about the family of unicritical polynomial maps $z \mapsto z^d + c$, where such parameter values are the roots of a polynomial called Gleason polynomial, or Misiurewicz polynomial in strictly preperiodic cases. In this talk, we construct Misiurewicz polynomials for a family of rational maps of degree $d \geq 2$ with an automorphism group containing the cyclic group of order d, and consider the irreducibility of those polynomials in certain cases. (Received September 09, 2020)

1163-11-581 **Florian Breuer*** (florian.breuer@newcastle.edu.au), **Pete L Clark**, **Paul Pollack** and **Andry N Rabenantoandro**. *Torsion bounds for CM Drinfeld modules.*
Let A be the ring of rational functions regular away from a fixed closed point on a smooth projective geometrically integral curve over a finite field, and let F be its field of fractions. We prove that the number of L-rational torsion points on a rank r Drinfeld A-module with complex multiplication is bounded by $C_{A,r} d \log \log d$, where $d = [L : F]$ and the constant $C_{A,r}$ depends only on A and r, in two cases. Firstly in the case where the endomorphism ring is a maximal order, and secondly in the case where $r = 2$. (Received September 10, 2020)

1163-11-584 **Kaoru Sano*** (kaosano@mail.doshisha.ac.jp), 1-3 tataramiyakodani, kyotrnabe-shi, kyoto, 610-0394, Japan. *Zariski density of rational points with the maximal arithmetic degree.*
For an algebraic dynamical system defined over a number field, it is a natural question that how quickly does the Weil height of rational points grow? On this question, Kawaguchi-Silverman introduced the arithmetic degree, which measures the exponential growth rate of the Weil height of rational points. On the other hand, the dynamical degree, a classical invariant of algebraic dynamics, measures the geometric complexity of the dynamical system. Kawaguchi-Silverman conjectured that the arithmetic degree at a point with Zariski dense orbit is equal to the dynamical degree. Matsuzawa proved that the arithmetic degree is always less or equal to the dynamical degree. Recently, we proved that there is a Zariski dense set of a rational point with the maximal arithmetic degree, which has disjoint orbits for any surjective endomorphisms on a projective variety. Moreover, we proved the same thing over a number field when the variety is potentially dense. In this talk, I will introduce these problems and explain the idea of the proof. This is joint work with Takahiro Shibata. (Received September 10, 2020)

1163-11-587   **Travis Scholl*** (traviswscholl@gmail.com) and **Stefano Marseglia**
                 (stefano.marseglia89@gmail.com). *Products and Polarizations of Super-Isolated Abelian*
                 Varieties.

We call an abelian variety over a finite field <u>super-isolated</u> if its (rational) isogeny class contains a single isomorphism class. In this talk, we will study the super-isolated varieties that form a super-isolated product, and those that admit a principal polarization.   (Received September 10, 2020)

1163-11-591   **Robert L. Benedetto*** (rlbenedetto@amherst.edu) and **Junghun Lee**. *Hyperbolicity*
                 and J-stability in non-archimedean dynamics.

Let K be a complete and algebraically closed non-archimedean field, and let $f in K(z)$ be a rational function of degree $d geq 2$. The map f is said to be hyperbolic if there is some metric on its Julia set with respect to which it is expanding. We prove that if f is hyperbolic, then a certain stability property of its Julia set holds in some neighborhood of f in the moduli space M_d.   (Received September 10, 2020)

1163-11-615   **Adrienne I Sands***, sands265@umn.edu. *Automorphic Hamiltonians, Epstein zeta*
                 functions, and Kronecker limit formulas.

We generalize the quantum harmonic oscillator from the real line to spaces of automorphic forms. More precisely, we construct a perturbed Laplacian which has purely discrete spectrum on $L^2(SL_r(\mathbb{Z})\backslash SL_r(\mathbb{R})/SO(r,\mathbb{R}))$, identify its ground state, and show how it can characterize a nuclear Fréchet automorphic Schwartz space. The construction of this so-called automorphic Hamiltonian connects certain degenerate Eisenstein series, Epstein's zeta functions, and Kronecker's first limit formula. We provide enough background to make this talk accessible to advanced undergraduates.   (Received September 10, 2020)

1163-11-618   **Adrienne I Sands*** (sands265@umn.edu). *Automorphic Hamiltonians, Epstein zeta*
                 functions, and Kronecker limit formulas.

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1163-11-656   **Rachel Pries*** (pries@colostate.edu) and **Douglas Ulmer**. *All BT_1's occur as a direct*
                 factor of the p-torsion group scheme of a Jacobian. Preliminary report.

An elliptic curve in characteristic p can be ordinary or supersingular. This difference can be measured by its p-torsion group scheme. More generally, the p-torsion group scheme of an abelian variety in characteristic p is a BT_1 group scheme. We prove that every BT_1 group scheme occurs as a direct factor of the p-torsion group scheme of a Jacobian.   (Received September 10, 2020)

1163-11-660   **Mingming Zhang*** (mingmiz@okstate.edu), Stillwater, OK 74074, and **Lucas**
                 Pottmeyer and **Paul Fili**. *Mahler measure and its behavior under iteration.*

For an algebraic number α we denote by $M(\alpha)$ the Mahler measure of α. As $M(\alpha)$ is again an algebraic number (indeed, an algebraic integer), $M(\cdot)$ is a self-map on $\overline{\mathbb{Q}}$, and therefore defines a dynamical system. The <u>orbit</u> <u>size</u> of α, denoted $\#\mathcal{O}_M(\alpha)$, is the cardinality of the forward orbit of α under M. In this talk, we will start by introducing the definition of Mahler measure, briefly discuss results on the orbit sizes of algebraic numbers with degree at least 3 and non-unit norm, then we will turn our focus to the behavior of algebraic units, which are of interest in Lehmer's problem. We will mention the results regarding algebraic units of degree 4 and discuss that if α is an algebraic unit of degree $d \geq 5$ such that the Galois group of the Galois closure of $\mathbb{Q}(\alpha)$ contains A_d, then the orbit size must be 1, 2 or ∞. Furthermore, we will show that there exists units with orbit size larger than 2!   (Received September 11, 2020)

1163-11-699   **Myrto Mavraki***, mavraki@math.harvard.edu, and **Pierre Le Boudec**. *Statistics in*
                 dynamics of rational maps. Preliminary report.

We investigate conjectures regarding points of small Call-Silverman canonical height with respect to rational maps defined over $\mathbb{Q}$ from a statistical perspective. This is joint work with Pierre Le Boudec. (Received September 11, 2020)

1163-11-737   **Padmavathi Srinivasan*** (padmavathi.srinivasan@uga.edu), Athens, GA 30601, and
                 Mark Kisin, **Joshua Lam** and **Ananth Shankar**. *Finiteness results for reductions of
                 Hecke orbits.* Preliminary report.
I will talk about two finiteness results for reductions of Hecke orbits of abelian varieties defined over finite
extensions of $\mathbb{Q}_p$, as well as applications to CM lifts of abelian varieties defined over finite fields. This is joint
work with Mark Kisin, Joshua Lam and Ananth Shankar.   (Received September 12, 2020)

1163-11-741   **Padmavathi Srinivasan*** (padmavathi.srinivasan@uga.edu), Athens, GA 30601, and
                 Wanlin Li, **Daniel Litt** and **Nick Salter**. *The section conjecture at the boundary of
                 moduli space.* Preliminary report.
Grothendieck's section conjecture predicts that over number fields, rational points on a smooth projective curve
of genus at least 2 are in bijection with sections of a natural exact sequence arising from fundamental groups.
We construct infinitely many curves of each genus satisfying the section conjecture in interesting ways, building
on work of Stix, Harari, and Szamuely. The main input is an analysis of the degeneration of certain torsion
cohomology classes on the moduli space of curves at various boundary components. This is (preliminary) joint
work with Wanlin Li, Daniel Litt and Nick Salter.   (Received September 12, 2020)

1163-11-745   **Katherine Thompson*** (kthomps@usna.edu) and **Jeremy Rouse**. *Almost Universal
                 Forms of Prime Discriminant.* Preliminary report.
We consider positive-definite quaternary quadratic forms with prime discriminant. These forms are almost
universal; that is, they represent all positive integers with finitely-many exceptions. We give a formula for
the last excepted value as a function of the discriminant. This work is joint with Jeremy Rouse (Wake Forest
University).   (Received September 12, 2020)

1163-11-747   **Hanson Smith*** (hanson.smith@uconn.edu). *Ramification in Division Fields and
                 Sporadic Points on Modular Curves.*
Consider an elliptic curve E over a number field K. Suppose that E has supersingular reduction at some prime
$\mathfrak{p}$ of K lying above the rational prime p and that $E(K)$ has a point of exact order p^n. To describe the minimum
necessary ramification at $\mathfrak{p}$, one can classify the valuations of the p^n-torsion points of E by the valuation of a
coefficient of the pth division polynomial. In particular, if E does not have a canonical subgroup at $\mathfrak{p}$, one can
show that $\mathfrak{p}$ has ramification index at least $p^{2n} - p^{2n-2}$ over p.

   In this talk we will briefly outline how to achieve such a classification. We will then apply our work to show
that sporadic points on the modular curve $X_1(p^n)$ cannot correspond to supersingular elliptic curves without a
canonical subgroup. Our methods are generalized to $X_1(N)$ with N composite.   (Received September 12, 2020)

1163-11-753   **Zachary L Scherr*** (scherr@susqu.edu) and **Katherine Thompson**
                 (kthomps@usna.edu). *Integral Polynomial Pell Equations.*
In his work on the Pell equation, Euler discovered several interesting polynomial identities included among them
that $(2n^2 + 1)^2 - (n^2 + 1)(2n)^2 = 1$ for every n. Motivated by Euler's examples, one can ask whether it is
possible to classify all such identities. In particular, for which polynomials $d(x) \in \mathbb{Z}[x]$ do there exist non-trivial
solutions to $f(x)^2 - d(x)g(x)^2 = 1$ with $f(x), g(x) \in \mathbb{Z}[x]$? Yokota and Webb classified all such quadratic $d(x)$
and asked about the situation with $d(x)$ of degree at least 4. In this talk we'll classify all monic, quartic, $d(x)$
which give rise to non-trivial solutions to Pell's equation. In particular, we'll show that other than the previously
known examples there is exactly one infinite family of such $d(x)$. The resolution of this problem is connected to
work by Mazur and Kubert on rational torsion points on elliptic curves over $\mathbb{Q}$.   (Received September 12, 2020)

1163-11-784   **Terrence Richard Blackman*** (tblackman@ias.edu), 75 Meritt Ln, Princeton, NJ 08540.
                 Spectral Correspondences for Mass Waveforms on Quaternion Groups.
We prove that in most cases the Jacquet-Langlands correspondence between newforms for Hecke congruence
groups and newforms for quaternion orders is a bijection. Our proof covers almost all cases where the Hecke
(Received September 12, 2020)

1163-11-847   **Sarah Arpin*** (sarah.arpin@colorado.edu), **Catalina Camacho-Navarro**, **Kristin
                 Lauter**, **Joelle Lim**, **Kirstina Nelson**, **Travis Scholl** and **Jana Sotáková**. *Adventures
                 in Supersingularland: An investigation into the Structure of Supersingular Isogeny Graphs.*
In this work we study isogeny graphs of supersingular elliptic curves. Supersingular isogeny graphs were intro-
duced as a hard problem into cryptography by Charles, Goren, and Lauter for the construction of cryptographic
hash functions [CGL06]. We consider two related graphs that help us understand the structure: the 'spine'
S, which is the subgraph of the usual l-isogeny graph given by the j-invariants in Fp, and the graph in which

both curves and isogenies must be defined over Fp. We show how to pass from the latter to the former. The graph S is relevant for cryptanalysis because routing between vertices in Fp is easier than in the full isogeny graph. We provide an analysis of the distances of connected components of S. We study the involution on the l-isogeny graph that is given by the Frobenius of Fp and give heuristics on how often shortest paths between two conjugate j-invariants are preserved by this involution (mirror paths). We also study the related question of what proportion of conjugate j-invariants are l-isogenous for l=2,3. This data is related to later work of Eisentraeger, Hallgren, Leonardi, Morrison, and Park.   (Received September 13, 2020)

1163-11-859   **Sabine J Lang***, sabine.lang@du.edu. *Selected results in the Theta correspondence:*
                    oscillator representation and exceptional groups. Preliminary report.
This talk presents some new results in the Theta correspondence. We introduce minimal representations, dual pairs, and recall the context of the Theta correspondence. First, we consider the oscillator representation of a real symplectic group and study its restriction to dual pairs of type I. We present a criterion on the size of the groups in the dual pair such that the restriction of the oscillator representation to the smallest group is a projective Harish-Chandra module. We also introduce some results in the Theta correspondence for exceptional groups, which bring us one step closer to the strong duality for these groups.   (Received September 13, 2020)

1163-11-863   **Jennifer Balakrishnan**, Boston, MA, **William Craig**, Dept MathCS, Charlottesville,
                    VA 22901, **Ken Ono*** (ken.ono691@gmail.com), Dept Mathematics, University of Virginia,
                    Charlottesville, VA 22901, and **Wei-Lun Tsai** (tsaiwlun@gmail.com), University of
                    Virginia, Charlottesville. *Variants of Lehmer's Conjecture on Ramanujan's tau-function.*
Modular forms are generating functions of important quantities in arithmetic geometry, combinatorics, number theory, and physics. Despite many deep developments in the arithmetic geometric and analytic aspects (e.g. Deligne's proof of the Weil Conjectures, the development of Galois representations, Birch and Swinnerton-Dyer Conjecture, to name a few), some of the seminal questions about them remain open. Perhaps the most prominent of these is Lehmer's Conjecture on the nonvanishing of modular form coefficients such as Ramanujan's tau-function. In joint work with J. Balakrishnan, W. Craig, and W.-L. Tsai, the speaker has obtained the first results that establish that many integers are never modular form coefficients. This lecture is geared to a general audience.   (Received September 13, 2020)

1163-11-883   **Daniel Alvey*** (dalvey@wesleyan.edu). *Approximation on Affine Subspaces: A*
                    Khintchine Type result.
A conjecture of Mahler birthed a rich tradition of investigating the Diophantine properties of subspaces of Euclidean space. Mahler's original conjecture was concerned with the extremality of a specific manifold. Another such property is that of Khintchine type, or whether the zero-one law of Khintchine's Theorem is inherited by the measure on a subspace. I will present a Khintchine type for divergence result for affine subspaces which satisfy a certain multiplicative Diophantine condition.   (Received September 13, 2020)

1163-11-891   **Pavel Guerzhoy**, **Michael Mertens** and **Larry Rolen*** (larry.rolen@vanderbilt.edu).
                    Periodicities for Taylor coefficients of half-integral weight modular forms.
Congruences of Fourier coefficients of modular forms have long been an object of central study. By comparison, the arithmetic of other expansions of modular forms, in particular Taylor expansions around points in the upper-half plane, has been much less studied. Recently, Romik made a conjecture about the periodicity of coefficients around the point i for the classical Jacobi theta function. Here we prove this conjecture and generalize the phenomenon observed by Romik to a general class of modular forms of half-integral weight.   (Received September 13, 2020)

1163-11-917   **Laura DeMarco*** (demarco@math.harvard.edu) and **N. Myrto Mavraki**. *Elliptic*
                    surfaces, heights, and geometry. Preliminary report.
In joint work with Myrto Mavraki, we study the arithmetic intersection of sections of elliptic surfaces, defined over number fields.   (Received September 14, 2020)

1163-11-918   **Francesc Fité**, **Kiran S. Kedlaya** and **Andrew V. Sutherland*** (drew@math.mit.edu).
                    Sato-Tate groups of abelian threefolds. Preliminary report.
Let A be an abelian variety of dimension g defined over a number field K. As defined by Serre, the Sato-Tate group $\mathrm{ST}(A)$ is a compact subgroup of the unitary symplectic group $\mathrm{USp}(2g)$ equipped with a map that sends each Frobenius element of the absolute Galois group of K at primes $\mathfrak{p}$ of good reduction for A to a conjugacy class of $\mathrm{ST}(A)$ whose characteristic polynomial is determined by the zeta function of the reduction of A at $\mathfrak{p}$.

Under a set of axioms proposed by Serre that are known to hold for $g \leq 3$, up to conjugacy in Usp(2g) there is a finite list of possible Sato-Tate groups that can arise for abelian varieties of dimension g over number fields.

For $g = 1$ there are 3 possibilities for $\mathrm{ST}(A)$, for $g = 2$ there are 52, and last year it was shown that for $g = 3$ there are 410. In this talk I will give a brief overview of this classification and discuss ongoing efforts to produce explicit examples that realize these 410 possibilities. (Received September 15, 2020)

1163-11-946 **Caroline L Matson*** (caroline.matson@colorado.edu). *Ramified abelian extensions of p-adic fields via formal groups.*
Let p be a prime integer and let F be a formal group law over the field of p-adic rational numbers, $\mathbb{Q}_p$. For $n \geq 1$ we obtain a field extension $K_n/\mathbb{Q}_p$ by adjoining the roots of the multiplication-by-p^n endomorphism $[p^n]_F$ to the ground field $\mathbb{Q}_p$. We can study the ramification degree of K_n by determining the p-adic valuations of the roots of $[p^n]_F$. In dimension one, this can be accomplished using Newton polygons. In multiple dimensions we can use similar techniques to compute the valuations of the roots of $[p^n]_F$, ultimately reducing to a question of whether two certain polynomials in p share any integer factors. (Received September 14, 2020)

1163-11-975 **Byungchul Cha*** (cha@muhlenberg.edu), 2400 Chew st, Allentown, PA 18104. *Berggren's theorem and intrinsic Diophantine approximation.*
An old theorem of Berggren says that there exist three square matrices M_1, M_2, M_3 of size 3 with integer coefficients which satisfy the following property; every Pythagorean triple, that is, an integral zero of the Pythagorean form $x^2 + y^2 - z^2$, is obtained in a unique way by multiplying either $(3,4,5)$ or $(4,3,5)$ by $M_{d_1} \cdots M_{d_k}$ for some $d_j \in \{1,2,3\}$. More recently, a similar theorem was found for the integral zeros of several other indefinite quadratic forms. In this talk we will apply these theorems to obtain Lagrange spectra arising from intrinsic Diophantine approximation of circles. A large part of this talk is based on joint work with Dong Han Kim. We will also highlight contributions of undergraduate collaborators. (Received September 14, 2020)

1163-11-1001 **Brandon Boggess** (bboggess@math.wisc.edu) and **Soumya Sankar*** (ssankar@msri.org). *Counting rational points on stacks.*
The presence of non-trivial cohomology can often be used to construct obstructions to the representability of moduli problems. In particular, some very nice moduli spaces are stacks instead of schemes. A classical example of this is the moduli stack of elliptic curves with an N-isogeny. In this talk, I will describe how this poses a problem when attempting to count rational points on this moduli space and talk about some ways to overcome this problem. I will discuss joint work with Brandon Boggess, where use these methods to answer the following classical question for certain N: how many elliptic curves of bounded naive height have a rational N-isogeny? (Received September 14, 2020)

1163-11-1057 **Matthew C. Welsh*** (matthew.welsh@bristol.ac.uk). *Bounds for theta sums.*
For a quadratic form Q in n variables and a real number $M > 0$, we consider the following theta sum,

$$\sum_{\mathbf{m} \in \mathbb{Z}^n} \chi(\frac{1}{M}\mathbf{m})e^{2\pi i Q(\mathbf{m})},$$

where χ is the indicator function of the unit cube $(0,1)^n$. Cosentino and Flaminio (2015) have shown that for almost all Q this theta sum is $\ll_Q M^{\frac{n}{2}}(\log M)^a$ for an explicit $a > 0$ using properties of abelian actions on compact nilmanifolds. Here we present a new approach (joint with Jens Marklof, Soren Mikkelsen, and Gene Kopp) to producing bounds of the same form for almost all Q using theta functions defined via the Segal-Shale-Weil representation and the geometry of $\mathrm{Sp}(n,\mathbb{Z})\backslash\mathrm{Sp}(n,\mathbb{R})$. (Received September 14, 2020)

1163-11-1063 **Nicole R. Looper*** (nicole_looper@brown.edu) and **Joseph H. Silverman** (jhs@math.brown.edu). *Intersection theory on surfaces and small points on Jacobians.*
In this talk, we will revisit certain aspects of intersection theory on arithmetic surfaces and situate them in a dynamical framework. We will discuss applications to open problems in dynamics, including uniform versions of the Manin-Mumford conjecture. (Received September 14, 2020)

1163-11-1072 **Edna Jones*** (elj44@math.rutgers.edu). *The Local-Global Principle for Integral Crystallographic Sphere Packings.* Preliminary report.
We will discuss a local-global principle for certain integral crystallographic sphere packings, such as Soddy sphere packings. (A Soddy sphere packing is a 3-dimensional analogue of a Apollonian circle packing.) Sometimes each sphere in a crystallographic sphere packing has a bend (1/radius) that is an integer. When all the bends are integral, which integers appear as bends? Using quadratic forms and the circle method, we attempt to answer this question. (Received September 14, 2020)

1163-11-1083   **Giacomo Micheli***, University of South Florida, Tampa, FL. *Understanding polynomial maps over finite fields.*

Polynomial maps appear everywhere in cryptography and coding theory. In this talk we will describe an advanced number theoretical tool to address problems arising from applications. We will also show some examples where the method is most effective.   (Received September 14, 2020)

1163-11-1089   **Arda H. Demirhan*** (`ademir3@uic.edu`), 851 S Morgan St (MC 251), University of Illinois at Chicago, Chicago, IL 60607, and **Ramin Takloo-Bighash**, 851 S Morgan St (MC 251), University of Illinois at Chicago, Chicago, IL 60607. *Distribution of Rational Points on Toric Varieties.* Preliminary report.

Manin's conjecture for Fano varieties predicts an asymptotic formula for the number of rational points of bounded height with respect to the anti-canonical height function on a small enough Zariski open set with a dense set of rational points. In the case of toric varieties, Manin's conjecture was verified by Victor Batyrev and Yuri Tschinkel. In this talk, we will explain a multi-height variant of the Batyrev-Tschinkel theorem proposed by Emmanuel Peyre in his paper "Liberté et accumulation" where one considers at height boxes, instead of a single height function, as a way to get rid of accumulating subvarieties. This is our main result: Let X be an arbitrary toric variety over a number field F, and let H_i, $1 \leq i \leq r$, be height functions associated to the generators of the cone of effective divisors of X. Fix positive real numbers a_i, $1 \leq i \leq r$. Then the number of rational points $P \in X(F)$ such that for each i, $H_i(P) \leq B^{a_i}$ as B gets large is equal to $CB^{a_1+\cdots+a_r} + O(B^{a_1+\cdots+a_r-\epsilon})$ for an $\epsilon > 0$. Our result is a first example of a large family of varieties along the lines of Peyre's idea.   (Received September 14, 2020)

1163-11-1168   **Bella Tobin*** (`bella.tobin@okstate.edu`), 401 Math Sciences, Oklahoma State University, Stillwater, OK 74075. *Applications of Dynamical Belyi Polynomials in Arithmetic Dynamics.* Preliminary report.

We will discuss application of dynamical Belyi polynomials in arithmetic dynamics. Dynamical Belyi maps are conservative polynomials defined over $\mathbb{Q}\mathbb{Q}$ with critical points at 0,1 and ∞. The family of dynamical Belyi polynomials are a building block for the family of bicritical polynomials and they prove useful in determining necessary and sufficient conditions for when post-critically finite polynomials can have potential good reduction at a given prime.   (Received September 14, 2020)

1163-11-1173   **Abbey Bourdon**, **David R. Gill**, **Jeremy Rouse** and **Lori D. Watson*** (`watsonl@wfu.edu`). *On isolated points of odd degree on $X_1(N)$.*

For a curve C defined over a number field K, we say that a closed point $x \in C$ of degree d is isolated if it does not belong to an infinite family of degree d points paramaterized by either the projective line or a positive rank abelian subvariety of the curve's Jacobian. In this talk, we characterize elliptic curves with rational j-invariant that give rise to an isolated point of odd degree on $X_1(N)/\mathbb{Q}$ for some positive integer N.   (Received September 14, 2020)

1163-11-1186   **Olivia Beckwith*** (`obeck@illinois.edu`), **Martin Raum** and **Olav Richter**. *Congruence Relations for Hurwitz Class Numbers.*

Hurwitz class numbers $H(N)$ count weighted classes of quadratic forms of discriminant $-N$ and are the Fourier coefficients for a weight $\frac{3}{2}$ mock modular form. This talk will discuss recent work with Martin Raum and Olav Richter, in which we study certain congruence relations for $H(N)$ which behave quite differently from known congruence relations for coefficients of mock theta functions.   (Received September 15, 2020)

1163-11-1192   **Ayse Alaca***, aysealaca@cunet.carleton.ca. *Representations by certain non-diagonal positive-definite quaternary quadratic forms.*

We will discuss the representations of positive integers by certain positive-definite integral non-diagonal quaternary quadratic forms. Explicit formulas will be presented for the representation numbers of several quaternary quadratic forms.   (Received September 15, 2020)

1163-11-1217   **Taylor Dupuy*** (`tdupuy@uvm.edu`), University of Vermont, Department of Mathematics and Statistics, 82 University Pl, Burlington, VT 05405. *New and Old Results in Wittferential Algebraic Geometry.*

In the 90's Buium and Joyal introduced an arithmetic analog of derivatives on number rings called a "p-derivations". These operations behave like "derivatives over the field with one element" and have become ubiquitous in Arithmetic Geometry and Algebraic Topology and have many applications.

Sample application include –effective versions of Manin-Mumford –prismatic cohomology –carving out isogeny classes by wittferential varieties (and getting diophantine results) –an "arithmetic Kodaira-Spencer theory" – theta operations –an arithmetic variant of Lie theory

In this talk we will give a general introduction to the standard Buiumisms (Jackson Morrow's word) and touch on recent developments with an eye toward themes of the session.   (Received September 15, 2020)

1163-11-1228   **Olivia Beckwith** (`obeck@illinois.edu`) and **Gene S Kopp***
(`gene.kopp@bristol.ac.uk`). *Gauss composition with level structure.*
The Gauss composition law famously describes the class group of an order in a quadratic field purely by an operation on classes of binary quadratic forms modulo integer linear transformations. We extend the notion of Gauss composition to describe the ray class group of a quadratic order modulo a conductor. This is done by imposing a stricter equivalence relation on binary quadratic forms.   (Received September 15, 2020)

1163-11-1243   **Angelica Babei***, babeiangelica@gmail.com, and **Larry Rolen** and **Ian Wagner**. *On the partition function modulo 3.* Preliminary report.
Ramanujan's congruences for the partition function and generalizations of the form $p(an + b) \equiv 0 \pmod{m}$ have been of special interest in the past century. In 2010, Ono showed that such congruences exist modulo all primes $m \geq 5$, which was further extended to all m coprime to 6. However, the cases $m = 2, 3$ has been more elusive. In 2010, Ono constructs a generating function using generalized Borcherds' products to show that given $D \equiv -1 \pmod{24}$, if there exist n coprime to 6 such that $p\left(\frac{Dn^2+1}{24}\right) \equiv 0 \pmod{2}$ (respectively, $p\left(\frac{Dn^2+1}{24}\right) \equiv 1$ (mod 2)), then there are infinitely many such n. In this talk, we use a similar construction to examine the case modulo 3.   (Received September 15, 2020)

1163-11-1246   **Adriana Salerno***, asalerno@bates.edu, and **Ursula Whitcher**. *Clausen's formula and high Picard rank K3 surfaces.*
Clausen's formula is a classical identity characterizing certain hypergeometric series as squares of other hypergeometric series. Evans-Greene and Fuselier-Long-Ramakrishna-Swisher-Tu have described finite field analogues of this identity. Clausen's formula also arises in the context of Picard-Fuchs equations satisfied by holomorphic forms on geometrically natural one-parameter families of K3 surfaces. We discuss the implications for point counting on such K3 surfaces over finite fields.   (Received September 15, 2020)

1163-11-1258   **Darleen S Perez-Lavin*** (`darleenpl@uky.edu`), 715 Patterson Office Tower, Lexington, KY 40506-0027. *Plus-Minus Davenport Constant on Finite Abelian Groups.*
Let G be a finite abelian group, written additively. The plus-minus Davenport constant, $D_{\pm}(G)$, is the smallest positive number s such that for any set $\{g_1, g_2, \ldots, g_s\}$ of s elements in G, with repetition allowed, there exists a subset $\{g_{i_1}, g_{i_2}, \ldots, g_{i_t}\}$ such that $g_{i_1} \pm g_{i_2} \pm \cdots \pm g_{i_t} = 0$. We define $De_{\pm}(G)$ similarly but we require our subset to have even length. In this talk, we discuss the connections between $De_{\pm}(G)$ and $D_{\pm}(G)$ for when $G = C_2 \oplus C_3^n$.   (Received September 15, 2020)

1163-11-1280   **Sachi Hashimoto**, **Katrina Honigs**, **Alicia Lamarche** and **Isabel Vogt***
(`ivogt.math@gmail.com`). *Transcendental Brauer-Manin obstructions on a Calabi-Yau threefold.*
In this talk, I will discuss arithmetic aspects of a family Calabi-Yau threefolds originally studied by Hosono and Takagi in the context of mirror symmetry. These varieties come equipped with a natural 2-torsion Brauer class, and we show that under mild conditions on the threefold, this gives a transcendental Brauer-Manin obstruction to weak approximation.   (Received September 15, 2020)

1163-11-1287   **Enrique Treviño*** (`trevino@lakeforest.edu`), 555 N Sheridan Rd, Lake Forest College, Lake Forest, IL 60045. *Partitioning powers into sets of equal sum.* Preliminary report.
For integers $k \geq 1$ and $m \geq 2$, we explore for which integers n can the set $\{1^k, 2^k, \ldots, n^k\}$ be partitioned into m sets of equal sum. Most of the literature on the problem focuses on finding the least n for which a partition is possible. In our work we focus on finding all n given k and m.   (Received September 15, 2020)

1163-11-1335   **Ana Caraiani*** (`a.caraiani@imperial.ac.uk`), 53 Redcliffe Close, Old Brompton Road, London, SW59HZ, United Kingdom. *An excursion through the land of shtukas.*
The Langlands program is an intricate network of conjectures that connect number theory to other areas of pure mathematics and even parts of theoretical physics. These conjectures play a fundamental role in our understanding of arithmetic over global fields, such as the field of rational numbers or the field of rational functions on a curve over a finite field.

Vincent Lafforgue made a deep breakthrough in the global Langlands program over function fields: he gave a general construction of the "automorphic to Galois" direction of the Langlands correspondence. This connects spectral data attached to Hecke operators on the automorphic side with representations of the absolute Galois group of the function field. Lafforgue dreamed up additional symmetries, known as excursion operators, on the automorphic side, and used them as a guide towards the correct Galois representation. I aim to explain what all of this means and then discuss several exciting, even more recent developments in the field.   (Received September 15, 2020)

1163-11-1346   **Jeffrey Hatley*** (`hatleyj@union.edu`) and **Antonio Lei**. *Comparing positive rank Iwasawa modules.*

Iwasawa theory provides a powerful method of studying Selmer groups associated to elliptic curves and modular forms. For a long time, many results have assumed that these Selmer groups were (co)torsion modules for the Iwasawa algebra $\Lambda = \mathbb{Z}[\![T]\!]$. This assumption frequently holds case when dealing with the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$. On the other hand, when considering an anticyclotomic $\mathbb{Z}_p$-extension K_∞/K of an imaginary quadratic field $K/\mathbb{Q}$, these Selmer groups often have positive (co)rank. This talk will discuss recent work with Antonio Lei which develops tools for studying this situation in a manner akin to the (co)torsion setting.   (Received September 15, 2020)

1163-11-1430   **Marc Hindry*** (`marc.hindry@imj-prg.fr`), UFR Mathematiques, Universite de Paris (Paris Diderot), 5 rue Thomas Mann, 75013 Paris, France. *Analogues of Brauer-Siegel theorem for surfaces over finite fields.* Preliminary report.

We will explain analogies between the classical Brauer-Siegel theorem, a statement relating asymptotically the class number, regulator of units and discriminant of a number field, and similar statement involving arithmetic invariants of algebraic varieties over a finite or global field. We present precisely the analogy for surfaces over a finite field and for abelian varieties over a global field (i.e. a number field or the function field of a cuve over a finite field), surveying some recent results.The proof of Brauer-Siegel theorem relies on the class number formula and analytical estimates for the Dedekind zeta function, the analogy draws on formulae predicted by the Birch & Swinnerton-Dyer conjecture, (resp. Artin-Tate conjecture) and analytical estimates for the relevant L-series. (Received September 15, 2020)

1163-11-1439   **Harald Andres Helfgott** and **Lola Thompson*** (`l.thompson@uu.nl`). *Summing mu(n): an even faster elementary algorithm.*

We present a new-and-improved elementary algorithm for computing $M(x) = \sum_{n \leq x} \mu(n)$, where $\mu(n)$ is the Möbius function (an older version of this paper, with a somewhat different approach and larger running time, was presented at the 2020 Joint Mathematics Meetings). Our algorithm takes

$$\text{time } O\left(x^{\frac{3}{5}} \log \log x\right) \quad \text{and space } O\left(x^{\frac{3}{10}} \log x\right),$$

which improves on existing combinatorial algorithms. While there is an analytic algorithm due to Lagarias-Odlyzko with computations based on integrals of $\zeta(s)$ that only takes time $O(x^{1/2+\epsilon})$, our algorithm has the advantage of being easier to implement. The new approach roughly amounts to analyzing the difference between a model that we obtain via Diophantine approximation and reality, and showing that it has a simple description in terms of congruence classes and segments. This simple description allows us to compute the difference quickly by means of a table lookup.   (Received September 15, 2020)

1163-11-1457   **Heidi Goodson*** (`heidi.goodson@brooklyn.cuny.edu`). *Sato-Tate Distributions of Jacobian Varieties.*

Let C be a smooth projective curve defined over $\mathbb{Q}$. We would like to study the limiting distributions of the coefficients of the normalized L-polynomial for C. To determine the distributions, we study the Sato-Tate groups of the Jacobians of the curves. In this talk we give both general results and explicit examples of Sato-Tate groups for certain curves C. We will use these groups to determine the limiting distributions of the coefficients of the normalized L-polynomial.   (Received September 15, 2020)

1163-11-1459   **Sandie Han**, **Ariane M. Masuda**, **Satyanand Singh** and **Johann Thiel*** (`jthiel@citytech.cuny.edu`), 300 Jay St., Brooklyn, NY 11201. *Subgroups of $SL_2(\mathbb{Z})$ characterized by certain continued fraction representations.*

For positive integers u and v, let $L_u = \begin{bmatrix} 1 & 0 \\ u & 1 \end{bmatrix}$ and $R_v = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix}$. Let $S_{u,v}$ be the monoid generated by L_u and R_v, and $G_{u,v}$ be the group generated by L_u and R_v. In this talk we will show an extension of a characterization

of matrices $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in $S_{k,k}$ and $G_{k,k}$ when $k \geq 2$ given by Esbelin and Gutan to $S_{u,v}$ when $u, v \geq 2$ and $G_{u,v}$ when $u, v \geq 3$. We will present a simple algorithmic way of determining if M is in $G_{u,v}$ using a recursive function and the short continued fraction representation of b/d. (Received September 15, 2020)

1163-11-1481 **Maryam Khaqan*** (`mkhaqan@emory.edu`). *Elliptic Curves and Moonshine.*
Moonshine began as a series of numerical coincidences connecting finite groups to modular forms. It has since evolved into a rich theory that sheds light on the underlying structures that these coincidences reflect.

We prove the existence of one such structure, a module for the Thompson group, whose graded traces are specific half-integral weight weakly holomorphic modular forms. We then proceed to use this module to study the ranks of certain families of elliptic curves. In particular, this serves as an example of moonshine being used to answer questions in number theory. (Received September 15, 2020)

1163-11-1494 **Nils-Peter Skoruppa*** (`nils.skoruppa@gmail.com`). *Theta blocks in several variables.*
Theta blocks were introduced in a paper of the same title by V. Gritsenko, the speaker, and D. Zagier (see arxiv). The study of the known infinite families of theta blocks suggested the development of a more systematic theory of theta blocks in several variables, and it led as a first highlight of such a theory to a new and short prove of the Macdonald identities. In this talk we sketch the basics of a theory of theta blocks in several variables, hint to some key problems, and discuss shortly the chances of finding new Macdonald-type identities as one of the main outcomes of such a theory. (Received September 15, 2020)

1163-11-1503 **Lassina Dembele*** (`lassina.dembele@gmail.com`). *Computing Hilbert modular forms with characters.*
We discuss an update to our algorithm which allows for the computation of Hilbert modular forms with characters. We also discuss the implementation in Magma. (Received September 15, 2020)

1163-11-1508 **Alia Hamieh** and **Naomi Tanabe*** (`ntanabe@bowdoin.edu`). *The moments of L-functions.*
In this talk, we study the asymptotics of the moments of various L-functions, with a focus on that of Rankin-Selberg L-functions. (Received September 15, 2020)

1163-11-1513 **Paul Bottinelli**, **Victoria de Quehen*** (`victoria.dequehen@isara.com`), **Chris Leonardi**, **Anton Mosunov**, **Filip Pawlega** and **Milap Sheth**. *On the Separation between Isogeny Assumptions.*
Many isogeny-based cryptosystems rely on the hardness of the Supersingular Decision Diffie-Hellman (SSDDH) problem. However, most cryptanalytic efforts have targeted the more generic supersingular ℓ^e-isogeny problem — an established hard problem in number theory. In this work, we question if the two additional pieces of information given in practical SSDDH instances — the action on a torsion subgroup, and the domain elliptic curve's endomorphism ring — can lead to improved attacks on such cryptosystems. We show that SIKE/SIDH are secure against our techniques. However, in certain settings, e.g. multi-party protocols, our results may suggest a larger gap between the security of these cryptosystems and the ℓ^e-isogeny problem. Our analysis relies on the ability to find many endomorphisms on the base curve that have special properties. We give a correspondence between these endomorphisms and solutions to quadratic forms. We informally discuss the parameter sets where these endomorphisms should exist. Finally, we present a minor variation of the SIKE protocol that avoids exposing a known endomorphism ring. (Received September 15, 2020)

1163-11-1608 **Kristin E Lauter*** (`klauter@microsoft.com`), **Travis Morrison**, **Christophe Petit**, **Kirsten Eisentraeger** and **Sean Hallgren**. *Supersingular isogeny graphs and endomorphism rings.*
Progress on developing quantum computers at scale forces us to consider what hard problems in mathematics our next generation of cryptographic systems will be based on. Supersingular Isogeny Graphs were proposed for use in cryptography in 2006 by Charles, Goren, and Lauter. The hard problem is finding paths in these graphs, i.e. routing. There are no known subexponential algorithms to solve this problem, either classically or on a quantum computer. For this reason, cryptosystems based on the hardness of problems on Supersingular Isogeny Graphs are currently under consideration for standardization in the NIST Post-Quantum Cryptography (PQC) Competition. This talk will introduce these graphs, explain how the security of the Key Exchange protocol depends on the hardness of the path-finding problem, and show an equivalence with the problem of computing endomorphism rings of supersingular elliptic curves. (Received September 15, 2020)

12 ► *Field theory and polynomials*

1163-12-791   **Christina Eubanks-Turner*** (`ceturner@lmu.edu`), 1 LMU Drive, University Hall, Suite 2714, Los Angeles, CA 90045, and **Kathryn Cole** and **Megan Lee**. *Interlace Polynomials of Lollipop and Tadpole Graphs.*

In this presentation, we examine interlace polynomials of lollipop and tadpole graphs. The lollipop and tadpole graphs are similar in that they both include a path attached to a graph by a single vertex. We give both explicit and recursive formulas for each graph, which extends the work of Arratia, Bollobas and Sorkin, among others. We also give special values, examine adjacency matrices and behavior of coefficients of these polynomials. (Received September 12, 2020)

1163-12-952   **Kristen L Hallas*** (`kristen.hallas01@utrgv.edu`), **Joan M Mattle** (`jmattle@ithaca.edu`), **Deanna C Perez** (`dcp@csu.fullerton.edu`) and **Aklilu Zeleke** (`zeleke@msu.edu`). *Recursive Polynomials.* Preliminary report.

In this talk we present some properties of Fibonacci-type recursive polynomials. After introducing the classical Fibonacci-like polynomials and the so-called Golden polynomials, we introduce recursive polynomial sequences defined by

$$G_{n+1}(x) = x^k G_n(x) + x^l G_{n-1}(x), \ k, l \text{ positive integers,}$$

with $G_0 = -1$, $G_1 = x - 1$

We discuss Binet forms, Pascal-like triangle representations and matrix representations for G_n, We derive interesting sequences and identities.

Lastly, we present analytic and numerical results on the nature of the real roots of G_n. Our work extends known results for Fibonacci-like polynomials. (Received September 14, 2020)

1163-12-1564   **Jean-Louis Colliot-Thélène**, **David Harbater**, **Julia Hartmann**, **Daniel Krashen*** (`daniel.krashen@rutgers.edu`), **Parimala** and **Suresh**. *Local-global principles for reductive groups over semiglobal fields.*

In this talk I will discuss recent progress on deciding when local-global principles hold for torsors for reductive groups over semiglobal fields. While a complete characterization of when such principles hold is currently open, I will present results that illustrate how the combinatorics of models for the semiglobal field, and the rationality of the group seem to play a key role. (Received September 15, 2020)

1163-12-1595   **Cameron Byer**, **Tyler Dvorachek**, **Emily Eckard**, **Joshua Harrington**, **Lindsey Wise*** (`wiselm1@appstate.edu`) and **Tony W. H. Wong**. *Properties of Fibotomic Polynomials.*

Fibonacci polynomials are defined recursively in the following manner: $U_0(x) = 0$ and $U_1(x) = 1$, and for all $n \geq 2$, $U_n(x) = U_{n-2}(x) + xU_{n-1}(x)$. In this talk, we consider the irreducible factors of Fibonacci polynomials, which are called the Fibotomic polynomials. The Fibotomic polynomials are known to share similar root structures with cyclotomic polynomials, which makes them an especially interesting class of polynomials to study. We prove several analogous properties for the Fibotomic polynomials that are well-known for the cyclotomic polynomials. Our investigation includes the study of the discriminants of the Fibotomic polynomials, as well as the factorization of Fibotomic polynomials modulo a prime. (Received September 15, 2020)

13 ► *Commutative rings and algebras*

1163-13-109   **Jason Boynton**, Fargo, ND 58102, **Jim Coykendall*** (`jcoyken@clemson.edu`), School of Mathematical & Statistical Sciences, Clemson University, Clemson, SC 29634, and **Chelsey Morrow**, Fargo, ND 58102. *Half-factoriality and the boundary function.*

An atomic integral domain, R, is said to be a half-factorial domain (HFD) if given the irreducible factorizations

$$\alpha_1 \alpha_2 \cdots \alpha_n = \beta_1 \beta_2 \cdots \beta_m$$

then $n = m$.

If R is an HFD with quotient field K then the boundary map is a function $\partial_R : K^* \longrightarrow \mathbb{Z}$ such that $\partial_R(xy) = \partial_R(x) + \partial_R(y)$ for all $x, y \in K^*$ and $\partial_R(\pi) = 1$ for all $\pi \in \mathrm{Irr}(R)$ (∂_R is well-defined precisely when R is an HFD).

The boundary map has been used with some success to determine stability of the half-factorial property in some special ring extensions, and in this talk we will describe how the boundary map can play a utilitarian role in investigating (complete) integral closures of HFDs. We will also explore the open question as to whether nonunits are always contained in the support of the boundary function in the case of atomic integral extensions. (Received August 16, 2020)

1163-13-114   **Cole Brower**, **Scott Chapman**, **Travis Kulhanek**, **Joseph McDonough**, **Christopher ONeill*** (`cdoneill@sdsu.edu`), **Vody Pavlyuk** and **Vadim Ponomarenko**. *Length density and numerical semigroups.*

Length density is a recently introduced factorization invariant, assigned to each element x of a cancellative commutative atomic semigroup, that measures how far the set of factorization lengths of x is from being a full interval. In this talk, we examine length density of elements of numerical semigroups (that is, additive subsemigroups of the non-negative integers).   (Received August 16, 2020)

1163-13-117   **Thomas G. Lucas** and **Abdeslam Mimouni*** (`amimouni@kfupm.edu.sa`), , Saudi Arabia. *"Trace Properties and the Rings $R(X)$ and $R<X>$".*

An integral domain R has the radical trace property (or is an RTP domain, resp. an LTP domain), if $I(R : I)$ is a radical ideal for each nonzero noninvertible ideal I (resp. $I(R : I)R_P = PR_P$ for each minimal prime P of $I(R : I)$). Clearly each RTP domain is an LTP domain, but whether the two are equivalent is open except in certain special cases. In this project, we will study the descent of the trace properties from special overrings of and integral domain R to R itself. Also we will investigate the transfer of the trace properties to the Nagata ring $R(x)$ and Serre's conjecture ring $R(< x >$ in different contexts of integral domains such as integrally closed domains, Noetherian and Mori domains, pseudo-valuation domains and more.   (Received August 17, 2020)

1163-13-119   **Scott T. Chapman*** (`scott.chapman@shsu.edu`), Department of Mathematics and Statistics, Box 2206, Huntsville, TX 77341, and **Christopher O'Neill** and **Vadim Ponomarenko**. *On Length Densities.*

For a commutative cancellative monoid M, we introduce the notion of the *length density* of both a nonunit $x \in M$, denoted $LD(x)$, and the entire monoid M, denoted $LD(M)$. This invariant is related to three widely studied invariants in the theory of non-unit factorizations, $L(x)$, $\ell(x)$, and $\rho(x)$. We consider some general properties of $LD(x)$ and $LD(M)$ and give a wide variety of examples using numerical semigroups, Puiseux monoids, and Krull monoids. While we give an example of a monoid M with irrational length density, we show that if M is finitely generated, then $LD(M)$ is rational and there is a nonunit element $x \in M$ with $LD(M) = LD(x)$ (such a monoid is said to have accepted length density). While it is well-known that the much studied asymptotic versions of $L(x)$, $\ell(x)$ and $\rho(x)$ (denoted $\overline{L}(x)$, $\overline{\ell}(x)$, and $\overline{\rho}(x)$) always exist, we show the somewhat surprising result that $\overline{LD}(x) = \lim_{n \to \infty} LD(x^n)$ may not exist. We also give some finiteness conditions on M that force the existence of $\overline{LD}(x)$.   (Received August 17, 2020)

1163-13-120   **Aqsa Bashir** (`aqsa.bashir@uni-graz.at`), Heinrichstraße 36, 8010 Graz, Austria, **Alfred Geroldinger** (`alfred.geroldinger@uni-graz.at`), Heinrichstraße 36, 8010 Graz, Austria, and **Andreas Reinhart*** (`andreas.reinhart@uni-graz.at`), Heinrichstraße 36, 8010 Graz, Austria. *On the arithmetical advantages of stability.*

Let D be an integral domain with quotient field K and I an ideal of D. Let $\mathcal{R}(I) = \{x \in K \mid xI \subseteq I\}$ be the ring of multipliers of I. Then I is said to be stable if I is invertible in $\mathcal{R}(I)$ and D is called stable if every nonzero ideal of D is stable. For $X \subseteq K$ set $X^{-1} = \{x \in K \mid xX \subseteq D\}$ and $X_v = (X^{-1})^{-1}$. The ideal I is called divisorial if $I_v = I$ and D is said to be divisorial if every nonzero ideal of D is divisorial. We say that D is an order in a Dedekind domain if D is a noetherian, one-dimensional domain with nonzero conductor in its integral closure. Every stable order in a Dedekind domain is divisorial and every order in a quadratic number field is a stable order in a Dedekind domain.

Let D be a stable order in a Dedekind domain. The main purpose of our paper is to elaborate the advantages of stability in orders of Dedekind domains. Among other things, we show that the monoid of invertible ideals of D is half-factorial if and only if the monoid of nonzero ideals of D is half-factorial. We also give an example of a divisorial order in a Dedekind domain whose monoid of invertible ideals is half-factorial, but whose monoid of nonzero ideals fails to be half-factorial.   (Received August 18, 2020)

1163-13-124   **Nicholas R Baeth*** (`nicholas.baeth@fandm.edu`) and **Daniel Smertnig** (`daniel@smertnig.at`). *Monoids of modules over Bass rings.*

Direct-sum decompositions of modules over a ring can often be modeled by the arithmetic of the monoid of their isomorphism classes, with operation induced by the direct sum. We present two models for studying direct-sum

decompositions of finitely generated torsion-free modules over Bass rings — one-dimensional reduced noetherian rings with module-finite integral closure and where every ideal is 2-generated. Using results of Levy-Wiegand and Levy-Odenthall, we show that when the ring is semilocal, the monoid is isomorphic to a diophantine monoid. In the case when the ring is not semilocal, we find a transfer homomorphism to a diophantine monoid. A second model provides more insight into the arithmetic of this monoid. To a ring R we associate the graph $\mathcal{G}_R$ of prime ideal intersections with vertices the minimal prime ideals of R and with an edge between two minimal primes $\mathfrak{p}$ and $\mathfrak{q}$ for each maximal ideal containing both of them. The monoid of agglomerations on $\mathcal{G}_R$ is an additive monoid on the set of all possible labelings of $\mathcal{G}_R$ such that each edge label is at most the labels of its adjacent vertices. We show that there is a transfer homomorphism to the monoid of agglomerations on $\mathcal{G}_R$. (Received August 18, 2020)

1163-13-169   **Nicholas R. Baeth** (nicholas.baeth@fandm.edu) and **Daniel Smertnig***
               (daniel.smertnig@uni-graz.at). *Monoids of graph agglomerations.*
Let G be a finite graph (possibly with multiple edges but no loops). An agglomeration on G is a function f assigning a nonnegative integer to every vertex and every edge, in such a way that $f(e) \leq f(v)$ whenever an edge e is incident with a vertex v. Under pointwise addition, the agglomerations on G form a reduced, finitely generated, Krull monoid. These monoids of graph agglomerations appear as codomains of transfer homomorphisms for monoids of modules over Bass rings. Hence direct-sum decompositions for finitely generated, torsion-free modules over Bass rings may be studied through the arithmetic of monoids of graph agglomerations. In this talk, we discuss several results on the factorization theory of monoids of graph agglomerations. In particular, we demonstrate that their factorization theory is tied to the structure of the underlying graph in a natural way, and that several arithmetical invariants may be expressed, or at least estimated, in terms of graph-theoretical invariants. A characterization of half-factorial monoids of graph agglomerations immediately yields a corresponding characterization of Bass rings over which direct-sum decompositions of finitely generated torsion-free modules have a unique factorization length. (Received August 25, 2020)

1163-13-215   **Eric Swartz** and **Nicholas Werner*** (wernern@oldwestbury.edu). *Covering Numbers of
               Commutative Rings.*
A cover of a unital, associative ring R is a collection of proper subrings of R whose set-theoretic union equals R. If such a cover exists, then the covering number $\sigma(R)$ of R is the cardinality of a minimal cover. In this paper, we show that if R has a finite covering number, then the calculation of $\sigma(R)$ can be reduced to the case where R is a finite ring of characteristic p and the Jacobson radical J of R has nilpotency 2. A ring R is called σ-elementary if $\sigma(R) < \sigma(R/I)$ for every nonzero two-sided ideal I of R. We classify all commutative σ-elementary rings with a finite covering number. Using this, we prove that if R has a finite covering number and R/J is commutative, then either $\sigma(R) = \sigma(R/J)$, or $\sigma(R) = p^d + 1$ for some $d \geq 1$. In particular, this result characterizes the integers that occur as the covering number of a commutative ring. This is joint work with Eric Swartz. (Received August 27, 2020)

1163-13-242   **Gyu Whan Chang*** (whan@inu.ac.kr), Incheon, 22012, South Korea, and **Jun Seok
               Oh**, Incheon, 22012. *The monoid of regular elements in commutative rings with zero
               divisors.* Preliminary report.
Let R be a commutative ring with identity, $R^\bullet$ be the multiplicative monoid of regular elements in R, t be the so-called t-operation on R or $R^\bullet$. A Marot ring is a ring whose regular ideals are generated by their regular elements. The Marot ring was introduced by J. Marot in 1969 and has been playing a key role in the study of rings with zero divisors. The notion of Marot rings can be extended to t-Marot rings such that Marot rings are t-Marot rings. In this talk, we study some ideal-theoretic relationships between a t-Marot ring R and $R^\bullet$. We first construct an example of t-Marot rings that are not Marot. This also serves as an example of rank-one DVRs of reg-dimension ≥ 2. Let R be a t-Marot ring, t-$spec(A)$ be the set of prime t-ideals of A, and $Cl(A)$ be the class group of A for $A = R$ or $R^\bullet$. Then, among other things, we prove that the map $\varphi : t$-$spec(R) \to t$-$spec(R^\bullet)$ given by $\varphi(P) = P^\bullet$ is bijective; $Cl(R) = Cl(R^\bullet)$; R is a Krull ring if and only if $R^\bullet$ is a Krull monoid; and R is a factorial ring if and only if R is a Krull ring with $Cl(R) = \{0\}$. (Received August 30, 2020)

1163-13-332   **Justin Chen*** (justin.chen@math.gatech.edu). *Towards numerical primary
               decomposition: Noetherian operators.* Preliminary report.
Primary decomposition is a fundamental problem in computational algebraic geometry. For reduced schemes, numerical irreducible decomposition has been fairly successful, but additional techniques are needed to capture the data in the non-reduced case. To this end, one may turn to Noetherian operators, which are polynomial differential operators that encode the multiplicity structure of an arbitrary ideal. I will discuss algorithms

(implemented in Macaulay2) to compute Noetherian operators, which coupled with numerical irreducible decomposition, achieves numerical primary decomposition for unmixed ideals.   (Received September 02, 2020)

1163-13-370   **Susan Loepp*** (`sloepp@williams.edu`), 33 Stetson Ct., Williams College, Williamstown, MA 01267, and **Teresa Yu**. *Completions of Uncountable Local Rings with Countable Spectra.*

We present necessary and sufficient conditions for a complete local (Noetherian) ring to be the completion of an uncountable local domain with a countable spectrum. In addition, we present necessary and sufficient conditions for a complete local ring containing the rationals to be the completion of an uncountable excellent local domain with a countable spectrum. These results suggest that uncountable local domains with countable spectra and uncountable local excellent domains with countable spectra are more common than previously thought.   (Received September 04, 2020)

1163-13-379   **Lee Klingler*** (`klingler@fau.edu`), **Alan Loper**, **Warren McGovern** and **Matthew Toeniskoetter**. *Semi-clean group rings.*

We call the commutative ring R a *clean ring* if every element of R can be written as the sum of a unit and an idempotent. The notion of a clean ring was defined by Nicholson [1977] in a study of exchange rings and lifting idempotents. Ye [2003] introduced the notion of semi-clean rings: R is called a *semi-clean ring* if every element of R can be written as the sum of a unit and a periodic element, where $r \in R$ is called *periodic* if there are natural numbers $k < n$ such that $r^k = r^n$. In joint work with Alan Loper, Warren McGovern, and Matthew Toeniskoetter, we show that, if R is a local ring and G is a torsion abelian group, then the group ring $R[G]$ is semi-clean.   (Received September 04, 2020)

1163-13-398   **Caitlyn Booms** and **Daniel Erman***, University of Wisconsin, Madison, WI 53706, and **Jay Yang**. *Heuristics for torsion in high degree syzygies.*

To what extent should we expect the syzygies of Veronese embeddings of projective space to depend on the characteristic of the field? I will present a heuristic model for thinking about this question as well as some related conjectures.   (Received September 04, 2020)

1163-13-452   **Ayah Almousa*** (`aka66@cornell.edu`), **Kuei-Nuan Lin** (`kul20@psu.edu`) and **Whitney Liske** (`whitney.liske@stvincent.edu`). *Rees Algebras of Closed Determinantal Facet Ideals.*

Using SAGBI basis techniques, we find Gröbner bases for the presentation ideals of the Rees algebra and special fiber ring of a closed determinantal facet ideal. In particular, we show that closed determinantal facet ideals are of fiber type and their special fiber rings are Koszul. Moreover, their Rees algebras and special fiber rings are normal Cohen-Macaulay domains and have rational singularities.   (Received September 07, 2020)

1163-13-486   **Eamon Quinlan-Gallego*** (`equinlan@umich.edu`). *D-module constructions in nonzero characteristics.*

Given a smooth k-algebra R, its ring of differential operators D is a noncommutative ring that acts on R and many R-modules of interest. When k has characteristic zero the theory of D-modules and many of its constructions have found numerous applications in singularity theory and commutative algebra. The goal of this talk will be to explain some analogous constructions when k is a field of positive characteristic and, if time permits, when k is a DVR of mixed characteristic.   (Received September 08, 2020)

1163-13-490   **Alessio Caminata***, Dipartimento di Matematica, Universita' di Genova, via Dodecaneso 35, 16146 Genova, GE, Italy. *Realization of semigroups of modules in dimension greater than two.*

The set of isomorphism classes of finitely generated modules over a local ring equipped with the direct sum has a natural structure of Krull monoid. In this talk, we consider the following problem. Given a Krull monoid Λ and a positive integer d does there exist a d-dimensional local ring R such that Λ is isomorphic to the semigroup of isomorphism classes of finitely generated R-modules? We give a positive answer to the previous question for any $d \geq 3$. This generalizes a result by Roger Wiegand who proved it for $d = 2$.   (Received September 08, 2020)

1163-13-546   **Victor Fadinger*** (`victor.fadinger@uni-graz.at`) and **Daniel Windisch** (`dwindisch@math.tugraz.at`). *A characterization of weakly Krull monoid algebras.* Preliminary report.

In 1981, Chouinard gave a characterization of when the monoid algebra $D[S]$ is a Krull domain in terms of properties of the domain D and the monoid S. In 2010, Chang investigated when a monoid algebra is weakly factorial (i.e. every non-zero non-unit is a product of primary elements) and in 2016, El Baghdadi and Kim

proved a characterization of when a monoid algebra is generalized Krull. A domain resp. monoid is called weakly Krull, if the intersection of all localizations at height-one prime ideals is equal to the domain resp. monoid and every non-zero element is contained in only finitely many height-one prime ideals. We investigate the weakly Krull property of monoid algebras and use our result to characterize the weakly Krull domains among the affine monoid algebras.

This is joint work with Daniel Windisch.   (Received September 09, 2020)

1163-13-576   **Tolulope Oke***, Department of Mathematics, Texas A&M University, College station, TX, and **Pablo S Ocal** and **Sarah Witherspoon**. *Homotopy liftings and Hochschild cohomology of some twisted tensor products.*

The Hochschild cohomology of a tensor product of algebras is isomorphic to a graded tensor product of Hochschild cohomology algebras, as a Gerstenhaber algebra. A similar result holds when the tensor product is twisted by a bicharacter. We present new proofs of these isomorphisms, using Volkov's homotopy liftings that were introduced for handling Gerstenhaber brackets expressed on arbitrary bimodule resolutions. These results illustrate the utility of homotopy liftings for theoretical purposes.   (Received September 09, 2020)

1163-13-586   **Thomas Polstra*** (tp2tt@virginia.edu) and **Karl Schwede**. *Compatible ideals in Gorenstein rings.*

Compatible ideals in prime characteristic rings play a role similar to those of multiplier ideals in complex birational algebraic geometry. Compatible ideals arise naturally as follows: if $R \to S$ is a finite map of local prime characteristic rings, then the ideal $I \subseteq R$ which is the sum of images of all R-linear maps $S \to R$ is a compatible ideal of R. We show that if R is $\mathbb{Q}$-Gorenstein of index relatively prime to the characteristic then every compatible ideal of R must arise this way. Namely, if $I \subseteq R$ is a compatible ideal, then there exists a finite extension $R \to S$ such that I is the sum of all images of R-linear maps $S \to R$. This is joint work with Karl Schwede.   (Received September 10, 2020)

1163-13-593   **Ela Celikbas*** (ela.celikbas@math.wvu.edu), Morgantown, WV 26506. *Weakly Arf Rings.*

In 1971, Lipman proved that, if $(R, \mathfrak{m})$ is a complete, one-dimensional local domain with an algebraically closed field of characteristic zero, and R is saturated, then R has minimal multiplicity, that is, the embedding dimension of R is equal to the multiplicity of R. In the proof, Lipman used the fact that such a ring R is an Arf ring, i.e., R satisfies a certain condition that was studied by Arf in 1949 pertaining to a certain classification of curve singularities. The defining condition of an Arf ring is easy to state: if R is as above, then R is Arf provided, whenever $0 \neq x \in \mathfrak{m}$ and $y/x, z/x \in \text{Frac}(R)$ are integral over R, one has that $yz/x \in R$.

In this talk, we introduce weakly Arf rings, which is a generalization of Arf rings. We give characterizations of weakly Arf rings and explore the relation between weakly Arf and Arf rings. We also give several examples. This talk is based on the recent joint work with O. Celikbas, C. Ciupercă, N. Endo, S. Goto, R. Isobe, and N. Matsuoka.   (Received September 10, 2020)

1163-13-620   **Daniel Duarte** and **Luis Núñez-Betancourt*** (luisnub@cimat.mx), Guanajutao, Mexico. *Nash blowups in prime characteristic.*

The Nash blowup is a natural modification of algebraic varieties that replace singular points by limits of certain vector spaces associated to the variety at non-singular points. For several decades, It has been studied whether it is possible to resolve singularities of algebraic varieties by iterating Nash blowups. This problem has mostly been treated in character- istic zero due to an example given by Nobile. In this talk, we will discuss a new approach in prime characteristic using differential operators. This work is inspired by a characterization of strongly F-regular rings via differential operators done by Karen E. Smith.   (Received September 10, 2020)

1163-13-623   **Sarasij Maitra*** (sm3vg@virginia.edu). *Discussions on Berger's Conjecture.*

There is a long standing conjecture of R.W. Berger concerning the torsion of the module of differentials for a reduced one dimensional analytic algebra over a perfect field. In this talk, we will discuss recent results on Berger's conjecture which involve studying co-lengths of isomorphic copies of ideals, as well as studying the ring in terms of valuations induced from the integral closure. Some of this is joint work with Craig Huneke and Vivek Mukundan.   (Received September 10, 2020)

1163-13-625   **Alexandra Seceleanu*** (`aseceleanu@unl.edu`). *Representation theoretic methods for studying symbolic powers of ideals.*

Symbolic powers of ideals that carry a group action can be analyzed using methods pertaining to representation theory or group invariant theory. This talk will present several illustrations and applications of this principle. (Received September 10, 2020)

1163-13-636   **Claudiu Raicu** and **Steven Sam***, ssam@ucsd.edu. *Bi-graded Koszul modules, K3 carpets, and Green's conjecture.*

We extend the theory of Koszul modules to the bi-graded case, and prove a vanishing theorem that allows us to show that the Canonical Ribbon Conjecture of Bayer and Eisenbud holds over a field of characteristic zero or at least equal to the Clifford index. Our results confirm a conjecture of Eisenbud and Schreyer regarding the characteristics where the generic statement of Green's conjecture holds. They also recover and extend to positive characteristics the results of Voisin asserting that Green's Conjecture holds for generic curves of each gonality. (Received September 10, 2020)

1163-13-640   **K. R. Goodearl*** (`goodearl@math.ucsb.edu`) and **M. T. Yakimov** (`yakimov@math.lsu.edu`). *Poisson Cluster Algebras and Unique Factorization.*

Two phenomena frequently encountered in cluster algebras are the existence of a compatible Poisson structure and unique factorization of the Poisson-normal elements of the algebra. It is expected that semiclassical limits of quantum algebras share these properties, an expectation that has been confirmed in many cases.

We will discuss the reverse direction – how combinations of Poisson structures and unique factorization lead to cluster algebra structures. The results apply to algebras from a large class of Poisson UFDs. [Joint work with Milen Yakimov]   (Received September 10, 2020)

1163-13-661   **Giulio Peruginelli*** (`gperugin@math.unipd.it`), Padova, Italy, and **Dario Spirito** (`spirito@math.unipd.it`), Padova, Italy. *Extending valuation domains through pseudo-monotone sequences, I.*

Let V be valuation domain with quotient field K. In case V has rank one, in 1935 Ostrowski introduced the notion of pseudo-convergent sequence in order to describe all the possible rank one extensions of V to the field of rational functions $K(X)$, when K is algebraically closed. The same notion was used a few decades later by Kaplansky to characterize immediate extensions of a general valuation domain.

In 2010, Chabert generalized the concept of pseudo-convergent sequence through the definition of pseudo-monotone sequence, in order to describe the polynomial closure of a subset S of a rank one valuation domain V in the context of the rings of integer-valued polynomials.

In this talk, we will show how Ostrowski's result can be generalized for general valuation domains by means of pseudo-monotone sequences. Loosely speaking, we will show which extensions of V to $K(X)$ can be approximated by pseudo-monotone sequences. We also characterize algebraic properties of these valuations.   (Received September 11, 2020)

1163-13-665   **Dario Spirito*** (`spirito@math.unipd.it`), Via Trieste, 63, 35121 Padova, Italy, and **Giulio Peruginelli** (`gperugin@math.unipd.it`), Via Trieste, 63, 35121 Padova, Italy. *Extending valuation domains through pseudo-monotone sequences, II.*

Let V be a valuation domain and K its quotient field. Pseudo-monotone sequences allow to define extensions of V to the field $K(X)$ of rational functions with precise algebraic properties; furthermore, if K is algebraically closed, all extensions of V to $K(X)$ can be described in this way.

In this talk, I will discuss from a topological point of view the space of extensions of V obtained through pseudo-monotone sequences, viewed as a subspace of the Zariski space $\mathrm{Zar}(K(X)|V)$ under the Zariski and the constructible topology, and some subspaces defined through some algebraic properties of the pseudo-monotone sequences. In particular, I will consider the spaces of extensions obtained through pseudo-convergent sequences that have the same fixed breadth (which give examples of ultrametric subspaces of $\mathrm{Zar}(K(X)|V)$), and the spaces of extensions obtained through pseudo-convergent sequences sharing one pseudo-limit (which give examples of spaces similar to $\mathbb{R}$ with the lower limit topology). Finally, I will show how these spaces can be used to obtain counterexamples, e.g. in determining when the Zariski space (under the constructible topology) is metrizable.

This is a joint work with Giulio Peruginelli.   (Received September 11, 2020)

1163-13-671   **Janet C Vassilev*** (`jvassil@math.unm.edu`), Department of Mathematics and Statistics, Albuquerque, NM 87131. *Generalized Stanley-Reisner rings.* Preliminary report.
We will introduce the notion of a generlaized Stanley-Reisner ring. In work with Berkesch, Chan, Klein, Matusevich and Page, we determine the ring of differential operators of certain generalized Stanley-Reisner rings. We will touch on this work as well as describing the test ideal of these rings.   (Received September 11, 2020)

1163-13-691   **Alana Huszar*** (`huszara@umich.edu`) and **Harm Derksen** (`ha.derksen@northeastern.edu`). *Non-commutative Rank and Stability of Quiver Representations.* Preliminary report.
We start with $n \times n$ matrices $A_1, \ldots, A_m$, and consider the matrix $A = x_1 A_1 + \ldots + x_m A_m$. We would like to determine the rank of A, where $x_1, ..., x_m$ are viewed as non-commuting generators of a free skew-field. This is the non-commutative rank of A. Finding this rank is related to the stability of a representation of a Kronecker quiver. We generalize this non-commutative rank to any acyclic quiver representation.   (Received September 11, 2020)

1163-13-727   **Jesse Elliott*** (`jesse.elliott@csuci.edu`), Camarillo, CA. *Unique factorization in rings with zerodivisors.*
There are several inequivalent notions of what "unique factorization" can mean in a ring with zerodivisors. In this talk I will discuss a few of these notions, and I will add one to the list in order to correct an error I made in Section 2.8 of my recent book, *Rings, Modules, and Closure Operations.* The t-closure semistar operation and a slight modification of it play a central role in this discussion.   (Received September 12, 2020)

1163-13-732   **Zhibek Kadyrsizova**, **Jennifer Kenkel**, **Janet Page**, **Jyoti Singh**, **Karen E Smith**, **Adela Vraciu*** (`vraciu@math.sc.edu`) and **Emily Witt**. *Bounds for F-pure threshold and extremal singularitites in positive characteristic.*
For a hypersurface over a field in positive characteristic, the F-pure threshold can be viewed as a measure of the singularity. More singular hypersurfaces have lower F-pure thresholds. The degree also gives a measure of the singularity and it provides a bound for the F-pure threshold. We describe and classify hypersurfaces that have the lowest possible F-pure threshold, taking the degree into consideration.   (Received September 12, 2020)

1163-13-740   **Craig Huneke*** (`huneke@virginia.edu`). *Expected Resurgence for Gorenstein Ideals.* Preliminary report.
This talk will report on recent joint work with E. Grifo and V. Mukundan. We prove that if S is a regular ring and I is an ideal of S defining a Gorenstein quotient ring, then the resurgence of I is expected if the symbolic powers of I are the saturations of the powers, and if either S has positive characteristic or is equicharacteristic with I having Noetherian symbolic blowup.   (Received September 12, 2020)

1163-13-746   **Nicholas O Cox-Steib*** (`noc3md@mail.missouri.edu`). $\mathfrak{m}$-*adic Perturbations and F-Invariants.*
Suppose $(R, \mathfrak{m}_R)$ is a Noetherian local ring that is complete with respect to the $\mathfrak{m}_R$-adic topology. Given an ideal, $I \subset R$, we investigate quotients of the form

$$\frac{R}{(g_1, \ldots, g_c)}$$

where $g_1, \ldots, g_c \in R$ are $\mathfrak{m}_R$-adically close to a minimal generating set for I.
We discuss new techniques for studying these quotients when R has equal characteristic and $I \subset R$ satisfies certain technical conditions. Applying these methods to F-finite local rings yields interesting new results about F-invariants.   (Received September 12, 2020)

1163-13-774   **Carmelo Antonio Finocchiaro*** (`cafinocchiaro@unict.it`), Department of Mathematics and Computer Scienc, Città Universitaria, viale Andrea Doria 6, 95125 Catania, Catania, Italy, and **Dario Spirito**. *Suprema in spectral spaces and applications.*
Let X be a spectral space. It is well known that the spectral topology of X can be refined to another topology, called the constructible (or patch) topology, which remains spectral and becomes Hausdorf. Being any spectral space X a T_0 space, a natural partial order on X can be defined by setting, for every $x, y \in X$, $x \le y$ if $y \in \overline{\{x\}}$. We will provide conditions for a subset Y of the partially ordered set $(X, \le)$ in order that the supremum of Y (in X) exists and belongs to the closure of Y in the constructible topology. Algebraic applications of such topological results will concern density properties of some spaces of rings and ideals. Moreover, we will provide topological characterizations of distinguished classes of domains, in terms of certain properties of their ideals. Joint paper with D. Spirito.   (Received September 12, 2020)

1163-13-814   **Marco Fontana** (`fontana@mat.uniroma3.it`), **Evan Houston*** (`eghousto@uncc.edu`) and
Mi Hee Park (`mhpark@cau.ac.kr`). *Idempotence and divisorialty in Prüfer-like domains.*
Let D be a Prüfer $\star$-multiplication domain, where $\star$ is a semistar operation on D. We show that certain ideal-theoretic properties related to idempotence and divisoriality hold in Prüfer domains, and we use the associated semistar Nagata ring of D to show that the natural counterparts of these properties also hold in D.   (Received September 13, 2020)

1163-13-831   **Carmelo A. Finocchiaro**, **Sophie Frisch** and **Daniel Windisch***
(`dwindisch@math.tugraz.at`). *Prime Ideals in Infinite Products of Commutative Rings.*
Preliminary report.
In this talk, we present descriptions of prime ideals and in particular of maximal ideals in products $R = \prod D_\lambda$ of families $(D_\lambda)_{\lambda \in \Lambda}$ of commutative rings. Every maximal ideal is induced by an ultrafilter on the Boolean algebra $\prod \mathcal{P}(\max(D_\lambda))$. If every D_λ is in a certain class of rings including finite character domains and one-dimensional domains, then this leads to a characterization of the maximal ideals of R. If every D_λ is a Prüfer domain, we depict all prime ideals of R. Moreover, we give an example of a (optionally non-local or local) Prüfer domain such that every non-zero prime ideal is of infinite height.
This is joint work with Carmelo A. Finocchiaro and Sophie Frisch.   (Received September 13, 2020)

1163-13-866   **Rankeya Datta*** (`rankeya@uic.edu`) and **Takumi Murayama**
(`takumim@math.princeton.edu`). *Are excellent F-pure rings Frobenius split?*
A folklore question in prime characteristic commutative algebra, which can be traced back to when Hochster and Roberts introduced the notion of F-purity, is whether an excellent F-pure ring is always Frobenius split. In this talk, which is based on joint work with Takumi Murayama, we will use a simple construction from rigid analytic geometry to give a negative answer to this question, even for excellent Euclidean domains.   (Received September 13, 2020)

1163-13-868   **Irina Georgeana Ilioaea*** (`irina.ilioaea@lsus.edu`), Louisiana State University in
Shreveport, One University Place, Shreveport, LA 71115. *On the Frobenius Complexity of
Stanley-Reisner Rings.*
The Frobenius complexity of a local ring R measures asymptotically the abundance of Frobenius operators of order e on the injective hull of the residue field of R. It is known that, for Stanley-Reisner rings, the Frobenius complexity is either $-\infty$ or 0. This invariant is determined by the complexity sequence $\{c_e\}_e$ of the ring of Frobenius operators on the injective hull of the residue field. We will show that $\{c_e\}_e$ is constant for $e \geq 2$, generalizing work of Àlvarez Montaner, Boix and Zarzuela. Our result settles an open question mentioned by Àlvarez Montaner in one of his papers.   (Received September 13, 2020)

1163-13-876   **Florian Enescu*** (`fenescu@gsu.edu`), Department of Mathematics and Statistics, 25 Park
Place, 1421, ATLANTA, GA 30303, and **Yongwei Yao**. *Graded rings of rational twist in
prime characteristic.* Preliminary report.
We study the generating function associated to complexity sequence of the twisted construction of an $\mathbb{N}$-graded ring. We regard this as an object reflecting the properties of the ring and its grading and perform a detailed analysis of the case of the polynomial ring with general $\mathbb{N}$-grading. Applications to the Frobenius complexity of determinantal rings are provided.   (Received September 13, 2020)

1163-13-880   **Irena Peeva*** (`ivp1@cornell.edu`), Cornell University, Mathematics Department, Ithaca,
NY 14853. *Linear strand supported on a poset.* Preliminary report.
We discuss a linear strand of a multigraded minimal free resolution over an exterior algebra, which is finite and supported on a poset.   (Received September 13, 2020)

1163-13-886   **Karl Schwede*** (`schwede@math.utah.edu`), Department of Mathematics, University of
Utah, Salt Lake City, UT 84112, and **Bhargav Bhatt**, **Linquan Ma**, **Zsolt Patakfalvi**,
Kevin Tucker, **Joe Waldron** and **Jakub Witaszek**. *Applications of commutative
algebra to mixed characteristic birational geometry.*
This talk, aimed at a commutative algebra audience, will present some ongoing work on mixed characteristic birational algebraic geometry that uses the recent commutative algebra breakthroughs in mixed characteristic, such as the direct summand conjecture, existence of big Cohen-Macaulay algebras, and the fact that R^+ is a big Cohen-Macaulay algebra in mixed charcteristic.   (Received September 13, 2020)

1163-13-896   **Ela Celikbas**, **Emilie Dufresne**, **Louiza Fouli*** (`lfouli@nmsu.edu`), **Elisa Gorla**, **Kuei-Nuan Lin**, **Claudia Polini** and **Irena Swanson**. *Rees algebras of determinantal ideals of sparse matrices.* Preliminary report.
Let $X = (x_{ij})$ be an *mimesn* sparse matrix of indeterminates, that is a matrix whose entries are either zero or indeterminates, and let I be the ideal of maximal minors of X in a polynomial ring $R = k[x_{ij}]$ over a field k. The Rees algebra of I can be described as the quotient of a polynomial ring by an ideal known as the defining ideal of the Rees algebra. Using the theory of SAGBI bases we obtain information about the Rees algebra of I from the Rees algebra of the initial ideal of I with respect to a monomial order. In particular, we will describe the defining ideal of the Rees algebra of I when X is any $2 \times n$ sparse matrix. We will also discuss various properties of the Rees algebra and the special fiber cone of these ideals. This is joint work with Ela Celikbas, Emilie Dufresne, Elisa Gorla, Kuei-Nuan Lin, Claudia Polini, and Irena Swanson.   (Received September 13, 2020)

1163-13-924   **Keller VandeBogert*** (`kellerlv@math.sc.edu`). *Trimming Complexes and Their Applications.*
In this talk, we will introduce trimming complexes and study their applications to a wide range of ideals. These complexes, at their most basic, are used to resolve the ideal obtained by "trimming" a generator of an ideal and rescaling by a new ideal. The resulting resolution only depends on the resolution of the original ideal and the rescaling ideal. As a result, we obtain consequences for the structure theory of certain compressed Artinian k-algebras, determinantal facet ideals, and classes of (rainbow) equigenerated monomial ideals.   (Received September 14, 2020)

1163-13-957   **Hannah Klawa*** (`hklawa@gmu.edu`), 4400 University Drive, MSN:3F2, Fairfax, VA 22030. *Globally perinormal domains.* Preliminary report.
In 2016, Epstein and Shapiro introduced the concept of perinormality and global perinormality. A domain R is globally perinormal (resp. perinormal) if every going-down overring (resp. every local going-down overring) is a localization of R. Perinormality is close to normality but little has been established for globally perinormal domains. We show that in a globally perinormal domain the radical of any invertible ideal is the radical of a principal ideal. We also show that any almost locally factorial Krull domain that is globally perinormal must have torsion class group and, in a particular pullback construction, the pullback of a globally perinormal semi-local PVMD of t-dimension one is globally perinormal.   (Received September 14, 2020)

1163-13-974   **Benjamin Briggs*** (`briggs@math.utah.edu`) and **Srikanth Iyengar**. *Hochschild cohomology and André-Quillen cohomology.*
Hochschild cohomology is an important tool across algebra, especially in representation theory, while André-Quillen cohomology belongs (mostly) to commutative algebra. They are known to be intimately connected. I will try to explain how one can further exploit this connection to simultaneously prove a (now known) conjecture of Vasconcelos on the conormal module, and a conjecture of Quillen on the vanishing of André-Quillen cohomology (originally established by Avramov). All of this is joint work with Srikanth Iyengar.   (Received September 14, 2020)

1163-13-1018   **Monica Lewis*** (`malewi@umich.edu`). *Galois actions and finiteness properties of local cohomology in positive characteristic.* Preliminary report.
Let A be a regular local ring of prime characteristic $p > 0$. Let $\mathcal{L}$ be a finite Galois extension of the fraction field $\mathcal{K}$ of A, whose degree over $\mathcal{K}$ is not divisible by p. Let R be the integral closure of A in $\mathcal{L}$, and suppose that R is Cohen-Macaulay (which, due to a result of Roberts, is automatic if the extension is abelian). What can be said about the finiteness properties of the local cohomology of R? If G denotes the Galois group of $\mathcal{L}$ over $\mathcal{K}$, and I is an ideal of R, then certain direct sums of local cohomology modules associated with (the k-fold sums of) the ideals $\{gI\}_{g \in G}$ carry both a Galois action and a compatible Frobenius action. In this talk, we make use of this rich structure to investigate questions about support and associated primes.   (Received September 14, 2020)

1163-13-1059   **Özgür Esentepe*** (`ozgur.esentepe@uconn.edu`). *Annihilators of cluster tilting modules in singularity categories.*
We consider a commutative Gorenstein ring and the ideal consisting of elements which annihilate every sufficiently high Ext-module between any two modules. This ideal defines the singular locus of the ring. We investigate the equality between this ideal and annihilators of cluster tilting modules in the singularity category.   (Received September 14, 2020)

1163-13-1102   **Hugh R. Geller*** (`hgeller@clemson.edu`). *DG-Structures on Minimal Free Resolutions of Fiber Products.* Preliminary report.

A construction of Tate shows that every algebra over a ring R possess a DG-algebra resolution over R. These resolutions are not always minimal and Avramov even shows that certain algebras cannot have a minimal resolution with a DG-algebra structure. In this talk, I give an explicit construction of the minimal resolution of the fiber product $k[\underline{x}]/\mathcal{I} \times_k k[\underline{y}]/\mathcal{J}$ over $k[\underline{x},\underline{y}]$ where $\mathcal{I} \subseteq \langle\underline{x}\rangle^2$ and $\mathcal{J} \subseteq \langle\underline{y}\rangle^2$. I then show how to put a DG-structure on these resolutions. (Received September 14, 2020)

1163-13-1119   **Kevin Tucker*** (`kftucker@uic.edu`) and **Rankeya Datta**. *On some permanence properties of (derived) splinters.*

Perhaps owing to their simple definition, basic questions about splinters are often devilishly difficult to answer. Following André's celebrated proof of Hochster's direct summand conjecture, it is natural to ask whether splinters satisfy some basic permanence properties enjoyed by other classes of singularities. We show that Noetherian splinters ascend under essentially étale homomorphisms. Along the way, we also prove that the henselization of a Noetherian local splinter is always a splinter and that the completion of a local splinter with geometrically regular formal fibers is a splinter. Finally, we give an example of a (non-excellent) Gorenstein local splinter with mild singularities whose completion is not a splinter. This talk is based on joint work with Rankeya Datta. (Received September 14, 2020)

1163-13-1126   **Gabriel E Sosa Castillo*** (`gsosacastillo@colgate.edu`), **Kuei-Nuan Lin** and **Selvi Kara**. *Koszulness of multi-Rees algebras of the direct sum of strongly stable (0-Borel) ideals.*

In this talk we will survey results concerning the koszulness of Rees and multi-Rees algebras of strongly stable (0-Borel) ideals. Particular attention will be given to (1) how the number, and degree, of the minimal Borel generators of the ideals involved affects the Koszulness, and (2) how combinatorial tools such as fiber graphs and Noetherian reduction relations allow us to detect Koszulness of these algebras. (Received September 14, 2020)

1163-13-1129   **James Edward Gossell***, 155 Anderson Hwy apt. 823, Clemson, SC 29631. *The Power Edge Ideal of a Finite Simple Graph.* Preliminary report.

Every electric power system can be modeled by a graph G whose vertices represent electrical buses and whose edges represent power lines. A phasor measurement unit (PMU) is a monitor that can be placed at a bus to observe the voltage at that bus as well as the current and its phase through all incident power lines. The problem of monitoring the entire electric power system using the fewest number of PMUs is closely related to vertex covering and dominating set problems in graph theory.

In this talk, we will give an overview of the PMU placement problem and its connections to commutative ring theory. By defining the power edge ideal I_G^P of a graph G, we will show how to use graphs of electric power grids to generate polynomial rings with desired algebraic properties. In particular, we will classify the trees G for which I_G^P is Cohen-Macaulay and prove that every such ideal is also a complete intersection. (Received September 14, 2020)

1163-13-1175   **Zhan Jiang*** (`zoeng@umich.edu`), Univ of Michigan, 2074 East Hall, 530 Church St, Ann Arbor, MI 48109. *Test elements for tight closures in equal-characteristic.*

In this talk, we present some new results for tight closure on complete local domains of equal-characteristic. We first talk about the characteristic p case. Then we will briefly introduce the construction of tight closure in characteristic 0 and present some results on the test elements. (Received September 14, 2020)

1163-13-1180   **Felix Gotti*** (`fgotti@mit.edu`), MIT Department of Mathematics, Cambridge, MA 02139. *Bounded and Finite Factorization Properties in Integral Domains.*

Let R be an atomic integral domain. We say that R satisfies the bounded factorization property or that R is a BFD if for each nonzero nonunit $x \in R$, there exists a positive integer n such that x cannot be factored in R as a product of more than n irreducibles (counting repetitions). In addition, we say that R satisfies the finite factorization property or that R is an FFD if each nonzero nonunit in R can be factored into irreducibles in only finitely many ways (up to permutation and associates). The bounded and finite factorization properties were introduced by D. D. Anderson, D. F. Anderson, and M. Zafrullah in 1990 and have been systematically investigated since then. We will discuss the bounded and finite factorization properties in some classes of monoid domains and further related classes of integral domains. (Received September 15, 2020)

1163-13-1183   **J. R. Juett*** (`jjuett@dbq.edu`), **Christopher Park Mooney** and **Lois W. Ndungu**.
                 Unique factorization of ideals in commutative rings with zero divisors.
"Unique factorization" was central to the initial development of ideal theory. We update this topic with several
new results concerning notions of "unique ideal factorization rings" with zero divisors. Along the way, we obtain
new characterizations of several well-known kinds of rings in terms of their ideal factorization properties and
examine when monoid rings satisfy various kinds of "unique ideal factorization."   (Received September 15, 2020)

1163-13-1227   **Oana Veliche*** (`o.veliche@northeastern.edu`), 360 Huntington Avenue, Boston, MA
                 02115, and **Lars W Christensen** and **Jerzy Weyman**. *On the structure of grade 3*
                 almost complete intersection ideals.
The combinatorial approach of Kunth (1996) to overlapping Pfaffians of a skew-symmetric matrix allowed us to
reprove and clarify a formula of Brill (1904) on the minors of a skew-symmetric matrix. As a consequence, we give
a new proof to the structure of grade 3 perfect almost complete intersection ideals. The original, independent,
proofs were given by Avramov (1981) and Brown (1984) using linkage and the structure of grade 3 Gorenstein
ideals given by Buchsbaum and Eisenbud (1977). We also the equivariant construction of generic of grade 3
perfect almost complete intersection ideals.   (Received September 15, 2020)

1163-13-1232   **Austin Antonoiu**, **Ranthony A.C. Edmonds*** (`edmonds.110@osu.edu`), **Bethany**
                 Kubik, **Christopher O'Neill** and **Shannon Talbott**. *On Atomic Density of Numerical*
                 Semigroup Algebras.
A numerical semigroup S is a cofinite, additively-closed subset of the nonnegative integers that contains 0. In this
paper, we initiate the study of atomic density, an asymptotic measure of the proportion of irreducible elements in
a given ring or semigroup, for semigroup algebras. For a fixed field $\mathbb{F}$ an a numerical semigroup S, the numerical
semigroup algebra $\mathbb{F}[S]$ is the subring of $\mathbb{F}[x]$ consisting only of terms of the form x^a for $a \in S$.

    It is known that the atomic density of the polynomial ring $\mathbb{F}_q[x]$ is zero for any finite field $\mathbb{F}_q$. We prove that
the numerical semigroup algebra $\mathbb{F}_q[S]$ also has atomic density zero for any numerical semigroup S. We also
examine the particular algebra $\mathbb{F}_2[x^2, x^3]$ in more detail, providing a bound on the rate of convergence of the
atomic density as well as a counting formula for irreducible polynomials using Möbius inversion, comparable to
the formula for irreducible polynomials over a finite field $\mathbb{F}_q$.   (Received September 15, 2020)

1163-13-1247   **Ashley K Wheeler*** (`awheeler@mtholyoke.edu`), Mount Holyoke College, Department of
                 Mathematics & Statistics, 415A Clapp Laboratory, South Hadley, MA 01075, and **Jessica**
                 Sidman and **Will Traves**. *Geometric equations for matroid varieties.* Preliminary report.
Each point x in $\mathrm{Grass}(r, n)$ corresponds to an $r \times n$ matrix A_x which gives rise to a matroid M_x on its columns.
Gel'fand, Goresky, MacPherson, and Serganova showed that the sets $\{y in \mathrm{Grass}(r, n) | M_y = M_x\}$ form a strati-
fication of $\mathrm{Grass}(r, n)$ with many beautiful properties. However, results of Mnëv and Sturmfels show that these
strata can be quite complicated, and in particular may have arbitrary singularities. We study the ideals I_x of
matroid varieties, the Zariski closures of these strata. We construct several classes of examples based on theorems
from projective geometry and describe how the Grassmann-Cayley algebra may be used to derive non-trivial
elements of I_x geometrically when the combinatorics of the matroid is sufficiently rich.   (Received September
15, 2020)

1163-13-1282   **Haydee Lindo*** (`hlindo@hmc.edu`). *Trace Modules and Rigidity Conjectures.* Preliminary
                 report.
I will speak on some recent developments in the theory of trace modules over commutative Noetherian rings. This
will include applications of trace modules in understanding endomorphism rings and a discussion of ongoing work
examining the relationship between trace ideals and modules having no self-extensions.   (Received September
15, 2020)

1163-13-1294   **Francesca Gandini*** (`francesca.gandini@kzoo.edu`), 1200 Academy St, Kalamazoo, MI
                 49006. *Degree bounds for invariants in the exterior algebra.*
When we consider the action of a finite group on a polynomial ring, a polynomial unchanged by the action is
called an invariant polynomial. A famous result of Noether states that in characteristic zero the maximal degree
of a minimal invariant polynomial is bounded above by the order of the group. Our work establishes that the
same bound holds for invariant skew polynomials in the exterior algebra. Our approach to the problem relies
on a theorem of Derksen that connects invariant theory to the study of ideals of subspace arrangements. We
adapt his proof over the polynomial ring to the exterior algebra, reducing the question to establishing a bound
on the Castelnuovo-Mumford regularity of intersections of linear ideals in the exterior algebra. We prove the
required regularity bound using tools from representation theory. In particular, the proof relies on the existence

of a functor on the category of polynomial functors that translates resolutions of ideals of subspace arrangements over the polynomial ring to resolutions of ideals of subspace arrangements over the exterior algebra.   (Received September 15, 2020)

1163-13-1405   **Thomas Polstra** (`tp2tt@virginia.edu`) and **Austyn Simpson*** (`awsimps2@uic.edu`). *F-purity deforms in $\mathbb{Q}$-Gorenstein rings.*
Given a local ring $(R, \mathfrak{m})$ of prime characteristic $p > 0$ and a non-zero-divisor $f \in \mathfrak{m}$ such that $R/(f)$ is F-pure, is it necessarily the case that R is F-pure? Fedder answered this question affirmatively for Gorenstein rings, but constructed a counterexample which is non-$\mathbb{Q}$-Gorenstein. In this talk, we present an affirmative answer to this deformation question provided that R is $\mathbb{Q}$-Gorenstein. This is joint work with Thomas Polstra.   (Received September 15, 2020)

1163-13-1431   **Neil Epstein*** (`nepstei2@gmu.edu`). *The Ohm-Rush content function and its applications.*
For an R-algebra S, the (Ohm-Rush) <u>content</u> $c(f)$ of an element $f \in S$ is the intersection of all ideals I such that $f \in IS$. If there is always a smallest such ideal (i.e. $f \in c(f)S$), we call S an <u>Ohm-Rush algebra</u>. Further content-related properties carry their own names and implications. The theory examines algebraic properties of polynomial extensions $R \to R[x]$ and what can be generalized from them.

I will report on some results regarding the Ohm-Rush content function, along with applications to apparently disparate areas of commutative algebra. For instance,

- a new criterion for regularity in Noetherian reduced local rings of characteristic p.
- Given a regular field extension L/K, a Noetherian K-algebra R, and a zero-divisor $g \in S := L \otimes_K R$, some nonzero element of R kills g.
- (w/Shapiro) With R, S be as above, if S is locally a UFD, so is R.
- (w/Shapiro) $R \to \hat{R}$ (R Noetherian local) is Ohm-Rush if and only if every ideal of $\hat{R}$ is extended from R.
- (w/Carchedi) for any ring map $R \to S$, an algebraic characterization of when the map of topological spaces $\operatorname{Spec} S \to \operatorname{Spec} R$ is open

(Received September 15, 2020)

1163-13-1568   **Olga Kashcheyeva*** (`kolga@uic.edu`), 851 S Morgan Street, M/C 249, Chicago, IL 60607. *Generating sequences vs key polynomials of valuations centered in 3-dimensional polynomial rings.* Preliminary report.
Let k be a field and $K = k(x, y)$. Let ν be a valuation centered on $k[x, y, z]_{(x,y,z)}$ such that the rank of ν is 1 and the residue field if ν is k. We provide a construction of a generating sequence for ν in $k[x, y, z]$ independent of the rational rank of ν. We then use it to illustrate the difference between generating sequences of valuation ν in $k[x, y, z]$ and complete sequences of key polynomials of ν in $K[z]$.   (Received September 15, 2020)

14 ▶ *Algebraic geometry*

1163-14-26   **Andrew Obus*** (`andrewobus@gmail.com`) and **Tony Shaska** (`shaska@oakland.edu`). *Superelliptic curves with many automorphisms and CM Jacobians.*
Let $\mathcal{C}$ be a smooth, projective, genus $g \geq 2$ curve, defined over $\mathbb{C}$. Then $\mathcal{C}$ has <u>many automorphisms</u> if its corresponding moduli point $p \in M_g$ has a neighborhood U in the complex topology, such that all curves corresponding to points in $U \setminus \{p\}$ have strictly fewer automorphisms than $\mathcal{C}$. We compute completely the list of superelliptic curves having many automorphisms. For each of these curves, we determine whether its Jacobian has complex multiplication. As a consequence, we prove the converse of Streit's complex multiplication criterion for these curves.   (Received July 02, 2020)

1163-14-128   **Alice Garbagnati*** (`alice.garbagnati@unimi.it`), Dipartimento di Matematica, via Saldini, 50, I-20133 Milan, Mi, Italy. *Symplectic automorphisms of order 3 on K3 surfaces: action on cohomology and related "Shioda Inose structures".*
The symplectic automorphisms of finite order on a K3 surface induce an essentially unique isometry on the second cohomology group of the K3 surface, i.e. the lattice $\Lambda_{K3} \simeq U^3 \oplus E_8^2$, as proved by Nikulin at the end of Seventies. In the particular case of the involutions this isometry is very well known and described by Morrison: it switches the two copies of E_8 and acts as the identity on the three copies of U. By using this, one is able to state several interesting results: the existence of the Shioda Inose structures; the description of the relations between the

Picard groups of K3 surfaces with a symplectic involution and the one of their quotient; the presence of infinite towers of isogenous K3 surfaces. The aim of this talk is to present similar results for symplectic automorphisms of order 3 on K3 surfaces. We will describe explicitly the action of the isometry induced by such an automorphism on the second cohomology group of a K3 surface (as Morrison did for the involutions) by giving a different basis for the Λ_{K3} and then we will deduce results analogue to the ones mentioned in the case of the involutions; for example we will generalize the Shioda-Inose construction to our case. The talk is based on a joint project with Y. Prieto.   (Received August 19, 2020)

1163-14-129   **Isabelle Shankar*** (`isabelle_shankar@berkeley.edu`), **Serkan Hosten** and **Alexander Heaton**. *Symmetry Adapted Gram Spectrahedra.*
Sum of squares (SOS) relaxations are often used to certify nonnegativity of polynomials and are equivalent to solving a semidefinite program (SDP). The feasible region of the SDP for a given polynomial is the Gram Spectrahedron. For symmetric polynomials, there are reductions to the problem size that can be done using tools from representation theory. In recent work with Serkan Hosten and Alexander Heaton, we investigate the geometric structure of the spectrahedra that arise in the study of symmetric SOS polynomials, the Symmetry Adapted PSD cone and the Symmetry Adapted Gram Spectrahedron.   (Received August 19, 2020)

1163-14-141   **Frank Sottile***, Department of Mathematics, Blocker Hall TAMU 3368, College Station, TX 77843-3368. *General witness sets.*
Numerical algebraic geometry has a close relationship to intersection theory from algebraic geometry. We deepen this relationship, explaining how rational or algebraic equivalence gives a homotopy. We present a general notion of witness set for subvarieties of a smooth complete complex algebraic variety using ideas from intersection theory. Under appropriate assumptions, general witness sets enable numerical algorithms such as sampling and membership. These assumptions hold for products of flag manifolds. We introduce Schubert witness sets, which provide general witness sets for Grassmannians and flag manifolds.   (Received August 21, 2020)

1163-14-152   **Andreas Malmendier** (`andreas.malmendier@usu.edu`) and **Noah Braeger*** (`noahbraeger2@gmail.com`). *An isogeny between certain K3 surfaces of Picard rank 18.*
We construct a geometric two-isogeny between two algebraic K3 surfaces obtained as quartic projective hypersurfaces explicitly. One is the Kummer surfaces of the Jacobian of a genus-two curve with elliptic involution, the other is the Shioda-Inose surface associated with the Kummer surface of two non-isogeneous elliptic curves. The construction uses a classical result due to Jacobi and Morrison on Nikulin involutions.   (Received August 25, 2020)

1163-14-183   **Humberto Diaz*** (`humberto@wustl.edu`), St. Louis, MO 63130. *Galois descent for higher Brauer groups.*
In this talk, I will discuss a Galois descent result for the higher Brauer group of a smooth projective variety. We show that the obstruction to Galois descent in this case is at worst a finite group (after inverting the exponential characteristic). This generalizes earlier work of Colliot-Th'el'ene and Skorobogatov.   (Received August 25, 2020)

1163-14-203   **Mee Seong Im*** (`meeseongim@gmail.com`), Chauvenet Hall, Office 342, Department of Mathematics, United States Naval Academy, Annapolis, MD 21402, **Chun-Ju Lai** (`cjlai@gate.sinica.edu.tw`), Institute of Mathematics, Academia Sinica, Taipei, 10617, Taiwan, and **Arik Wilbert** (`arik.wilbert@uga.edu`), Department of Mathematics, University of Georgia, Athens, GA 30602. *Nakajima quiver varieties and irreducible components of Springer fibers.*
Springer fibers and Nakajima quiver varieties are amongst the most important objects in geometric representation theory. While Springer fibers can be used to geometrically construct and classify irreducible representations of Weyl groups, Nakajima quiver varieties play a key role in the geometric representation theory of Kac–Moody Lie algebras.

I will begin by first recalling some background on the objects of interest mentioned above. I will then connect Springer fibers and quiver varieties by realizing the irreducible components of two-row Springer fibers inside a suitable Nakajima quiver variety and describing the resulting subvariety in terms of explicit quiver representations.

Next, consider certain fixed-point subvarieties of these quiver varieties, which were studied by Henderson–Licata and Li with the goal of developing the geometric representation theory for certain coideal subalgebras. By applying this machinery, I will give an explicit algebraic description of the irreducible components of all two-row Springer fibers for classical types, thereby generalizing results of Fung and Stroppel–Webster in type A.

This is joint with C.-J. Lai and A. Wilbert.   (Received August 26, 2020)

1163-14-220   **Izzet Coskun***, 851 S Morgan St, Chicago, IL 60607, and **Eric Riedl**. *Recent progress in algebraic hyperbolicity.*

I will give a survey of recent work with Eric Riedl on algebraic hyperbolicity. I will discuss our proof of the algebraic hyperbolicity of the very general quintic surface. I will explain a classification of 1-clustered families in the Grassmannian and give applications to Lang-type conjectures on hypersurfaces.   (Received August 28, 2020)

1163-14-289   **Yuri G. Zarhin*** (`zarhin@math.psu.edu`), Pennsylvania State University, Department of Mathematics, University Park, PA 16802. *Eigenvalues of Frobenius endomorphisms of Abelian varieties over finite fields.*

Let X be a positive-dimensional abelian variety over a finite field of characteristic p, Fr_X the Frobenius endomorphism of X, and $P_X[t]$ the characteristic polynomial of Fr_X, which is a monic polynomial with integer coefficients. Its roots are eigenvalues of Fr_X with respect to its action on the ℓ-adic Tate module of X (for all primes $\ell \neq p$). We discuss multiplicative relations between eigenvalues of Fr_X. As an application we obtain the following result.

Theorem. Let g be a positive integer. Then there exists a positive integer $N = N(g)$ that enjoys the following properties.

Let X be a g-dimensional abelian variety over a finite field k such that there exist a positive integer n and a prime $l \neq char(k)$ such that the self-product X^n of X carries an exotic l-adic Tate class.

Then the self-product X^N of X carries an exotic ℓ-adic Tate class for all primes $\ell \neq char(k)$.

Recall that a Tate class is called exotic if it cannot be presented as a linear combination of products of divisor classes.   (Received September 06, 2020)

1163-14-333   **Cicero Carvalho*** (`cicero@ufu.br`), Faculdade de Matematica, Av. J.N. Avila, 2121, Uberlandia, MG 38408-100, Brazil. *On codes of Reed-Muller type defined over higher dimensional scrolls.*

In 1988 Lachaud introduced the class of projective Reed-Muller codes, defined by evaluating the space of homogeneous polynomials of a fixed degree on the points of a projective space over a finite field. Since then other classes of codes have been obtained by replacing the points of the projective space by the points of a projective variety. In this talk we would like to present results on a class of codes obtained in this way, where the projective variety is a higher dimensional normal scroll. In a joint work with Victor G.L. Neumann, Xavier Ramirez-Mondragon and Horacio Tapia-Recillas we have determined a formula for the dimension of these codes, and the exact value of the minimum distance in a special case.   (Received September 02, 2020)

1163-14-411   **Steven Dale Cutkosky*** (`cutkoskys@missouri.edu`), Dept. Math., University of Missouri, Columbia, MO. *Essential finite generation of valuation rings.*

Let K be a characteristic zero algebraic function field with a valuation ν. Let L be a finite extension of K and ω be an extension of ν to L. We establish that the valuation ring V_ω of ω is essentially finitely generated over the valuation ring V_ν of ν if and only if the initial index $\epsilon(\omega|\nu)$ is equal to the ramification index $e(\omega|\nu)$ of the extension. This gives a positive answer, for characteristic zero algebraic function fields, to a question posed by Hagen Knaf.   (Received September 05, 2020)

1163-14-458   **Ivan Horozov***, City University of New York - BCC, 2155 University Ave, New York City, NY 10453, and **Pavel Sokolov**. *Dedekind Polylogarithms.*

In this talk, first I will present some results on multiple Dedekind zeta values (a generalizations of multiple zeta values to number fields) and their relations to algebraic geometry and motives. I will also mention more resent progress (joint work with Pavel Sokolov) on relations between multiple Dedekind zeta values, abelian Artin L-functions and mixed Tate motives via Dedekind polylogarithms.   (Received September 07, 2020)

1163-14-476   **Timothy Duff*** (`tduff3@gatech.edu`), 686 Cherry St NW, School of Mathematics, Tim Duff, Atlanta, GA 30308, and **Viktor Korotynskiy**, **Tomas Pajdla** and **Margaret Regan**. *Galois/monodromy groups in 3D reconstruction.* Preliminary report.

In computer vision, the study of minimal problems is critical for many 3D reconstruction tasks. Solving minimal problems comes down to solving systems of polynomial equations of a very particular structure. "Structure" of minimal problems may be understood in terms of the Galois/monodromy group of an associated branched cover. We compute these groups for many examples using numerical homotopy continuation methods. Classical problems such as five-point relative pose, planar calibrated homography estimation, and perspective absolute pose give rise to imprimitive Galois groups, and solutions to these problems typically exploit a corresponding

decomposition of the associated branched cover. Beside analyzing these cases, we find also several novel minimal problems whose Galois groups are imprimitive and may be reasonable to solve in practical applications. (Received September 07, 2020)

1163-14-479   **Jean-Louis Colliot-Thélène** and **David Harbater*** (`harbater@math.upenn.edu`),
Department of Mathematics, University of Pennsylvania, Philadelphia, PA 19104-6395, and
Julia Hartmann, **Daniel Krashen**, **R. Parimala** and **V. Suresh**. *Local-global
principles over semi-global fields.* Preliminary report.
Local-global principles are central in the study of algebraic objects such as quadratic forms and central simple algebras, over global fields. Such principles can typically be reformulated in terms of the existence of rational points on homogeneous spaces under algebraic groups, especially on principal homogeneous spaces (torsors). This talk will discuss analogous principles over semi-global fields, i.e. function fields of curves over complete discretely valued fields. In a number of cases we show that local-global principles hold in this situation; in other cases we compute the obstruction, which is analogous to the Tate-Shafarevich group. This work is joint with J. Hartmann, D. Krashen, J-L. Colliot-Thélène, R. Parimala, and V. Suresh.   (Received September 07, 2020)

1163-14-482   **Jose Gonzalez*** (`jose.gonzalez@ucr.edu`) and **Zhixian Zhu** (`zhixian.zhu@ucr.edu`).
Generation of jets and Fujita's jet ampleness conjecture on toric varieties.
A line bundle is k-jet ample if it has enough global sections to separate points, tangent vectors, and also their higher order analogues called k-jets. For example, 0-jet ampleness is equivalent to global generation and 1-jet ampleness is equivalent to very ampleness. We give sharp bounds guaranteeing that a line bundle on a projective toric variety is k-jet ample in terms of its intersection numbers with the invariant curves, in terms of the lattice lengths of the edges of its polytope and in terms of the higher concavity of its piecewise linear function. As an application, we prove the k-jet generalizations of Fujita's conjectures on toric varieties with arbitrary singularities.   (Received September 08, 2020)

1163-14-496   **Samantha N. Sherman*** (`ssherma1@nd.edu`), **Jonathan D. Hauenstein**
(`hauenstein@nd.edu`) and **Charles W. Wampler** (`charles.w.wampler@gm.com`). *A method
to determine the complete set of planar cognate linkages.*
This talk will describe a method for determining all coupler curve cognates for planar linkages with rotational joints. Previously, Dijksman presented a list of six-bar cognates but without proof it was complete. We use arguments concerning the singular foci of the coupler curve to constrain a cognate search and then use polynomial continuation to complete it and show a complete set. Our approach not only confirms that Dijksman's list is in fact comprehensive but provides an approach to compute cognates of mechanisms greater than 6 bars.   (Received September 08, 2020)

1163-14-498   **Michael Burr**, **Frank Sottile** and **Elise Walker*** (`walkere@math.tamu.edu`). *Numerical
homotopies from Khovanskii bases.*
Homotopies are useful numerical methods for solving systems of polynomial equations. Embedded toric degenerations are one source for homotopy algorithms. In particular, if a projective variety has a toric degeneration, then linear sections of that variety can be optimally computed using the polyhedral homotopy. Any variety whose coordinate ring has a finite Khovanskii basis is known to have a toric degeneration. We provide embeddings for this Khovanskii toric degeneration to compute general linear sections of the variety. This is joint work with Michael Burr and Frank Sottile.   (Received September 08, 2020)

1163-14-532   **Michael T Schultz*** (`michael.schultz@usu.edu`). *Connecting the differential geometry of
the Siegel Modular threefold & Hilbert Modular surfaces with algebro-arithmetic data of
certain K3 surfaces.*
Principally polarized abelian surfaces A over $\mathbb{C}$ are realized either as the Jacobian of a genus two curve or as the product of two elliptic curves. One constructs a K3 surface from A as the minimal resolution of dividing A by the minus identity involution, resulting in a Jacobian elliptic surface of Picard rank 17 or 18, respectively, called the Kummer surface Kum(A). Accordingly, the moduli space of such surfaces is isomorphic to the Siegel modular threefold or a Hilbert modular surface, respectively. In this talk, I will explain how the process of degenerating families of these elliptic K3 surfaces can be understood in two different ways: on one hand, it can be viewed as restricting to special subvarieties within the moduli space of abelian surfaces where the endomorphism ring of the abelian surface has special properties, for example, it is a CM field, an indefinite quaternion algebra, or a real quadratic field. Alternatively, it can be understood in terms of certain differential geometric conditions imposed on the conformal curvature tensor on the moduli space using the associated Picard-Fuchs equations. In this

way, we connect a robust differential geometric structure on the moduli space with important algebro-arithmetic data.   (Received September 08, 2020)

1163-14-541   **Andrew Obus*** (andrewobus@gmail.com) and **Padmavathi Srinivasan**.
                 Conductor-discriminant inequality for superelliptic curves.
Let K be a complete discretely valued field with residue characteristic p. For a superelliptic curve X given by an affine equation $y^n = f(x)$ with $p \nmid n$, we show that the (negative) Artin conductor of the minimal regular model of X over $\mathcal{O}_K$ is bounded above by $(n-1)v_K(\mathrm{disc}(f))$. This generalizes earlier work by the authors in the hyperelliptic case, as well as the thesis of Kohls where a similar inequality is given for the conductor exponent rather than the Artin conductor.   (Received September 08, 2020)

1163-14-604   **Rohini Ramadas*** (rohini_ramadas@brown.edu) and **Rob Silversmith**. *Quadratic
                 rational maps with a five-periodic critical point.*
We study the moduli space $\mathrm{Per}_5^{\mathrm{cm}}(0)$ of degree-2 rational maps $\mathbb{P}^1 \to \mathbb{P}^1$ that have a marked 5-periodic critical point. We show that $\mathrm{Per}_5^{\mathrm{cm}}(0)$ is an elliptic curve $\mathcal{C}_5$ punctured at 10 points, and we identify the isomorphism class of $\mathcal{C}_5$ over $\mathbb{Q}$. In order to do so, we develop techniques for using compactifications of Hurwitz spaces to study subvarieties of the moduli space of degree-d rational maps defined by critical orbit relations. We carry out an experimental study of the interaction between dynamically defined points of $\mathrm{Per}_5^{\mathrm{cm}}(0)$ (such as PCF points or punctures) and the group structure of $\mathcal{C}_5$.   (Received September 10, 2020)

1163-14-619   **Federico Scavia***, scavia@math.ubc.ca. *On the Grothendieck ring of algebraic stacks.*
The Grothendieck ring of algebraic stacks was introduced by T. Ekedahl in 2009, following up on work of other authors. It is a generalization of the Grothendieck ring of varieties. For every linear algebraic group G, we may consider the class BG in this ring. It is an interesting problem to compute BG in terms of classes of varieties. There are many similarities with the rationality problem for fields of invariants (Noether's problem). I will give a brief introduction to the Grothendieck ring of stacks, and then state some of my results in the area.   (Received September 10, 2020)

1163-14-714   **Silviana Amethyst*** (amethyst@uwec.edu), **Samantha Maurer** and **William T
                 O'Brien**. *A 3D printed Arduino powered electronic Barth Sextic.* Preliminary report.
Algebraic surfaces are two-dimensional objects defined by polynomials. With modern software, algebraic surfaces can be visualized by rendering beautiful 3D images. This project stems from research that started at the Institute for Computational and Experimental Research in Mathematics (ICERM) during their Fall 2019 program entitled <u>Illustrating Mathematics</u>. The goal of this project was to produce a fully parameterized OpenSCAD model for a snap-together Barth Sextic – a nodal algebraic surface with 65 double points. This digital model enables a variety of puzzles, toys, art, and much larger 3D printed objects, none of which can be done using a one-piece model.

   Two undergraduate students at the University of Wisconsin - Eau Claire were engaged with this specific project during the summer of 2020. The students were to research and program an interactive electronic piece of art. The model they influenced has ports for wires between pieces, and coin-slot plugs for carrying Neopixel Jewel 7-pixel RGBW chips. The controller of the model allows a participant to explore the symmetry group of the icosahedron, including rotations and reflections.   (Received September 11, 2020)

1163-14-778   **Eoin Mackall*** (eoinmackall@gmail.com), 4176 Campus Dr, William E. Kirwan Hall,
                 College Park, MD 20742. *Curves on Severi–Brauer varieties and nontriviality of reduced
                 Whitehead groups.*
Suslin's conjecture on the reduced Whitehead group predicts that central simple algebras of index divisible by the square of a prime have generically nontrivial reduced Whitehead groups (i.e. even if these groups are trivial over the base field, there is some field extension where the reduced Whitehead group of the base-changed algebra is nontrivial). For algebras having index divisible by 4, Suslin's conjecture is known to hold by work of Merkurjev. More generally, Merkurjev has shown that Suslin's conjecture holds under the assumption that the Chow groups of some low dimensional cycles on a Severi–Brauer variety are torsion free. In this talk, we consider the next nontrivial case of Suslin's conjecture (for algebras with index divisible by 9), and relate the torsionfree-ness of these Chow groups to the existence of a family of curves on certain Severi–Brauer varieties. (Received September 12, 2020)

1163-14-823   **Matthew Ballard**, **Alexander Duncan**, **Alicia Lamarche** and **Patrick McFaddin***
                   (`pmcfaddin@fordham.edu`). *Derived categories and rationality of twisted forms of toric
                   varieties.*
Toric varieties defined over the complex numbers provide a rich testing ground for computing algebro-geometric invariants (e.g., the coherent derived category associated to a variety), as many computations of interest may be phrased in terms of combinatorial data such as fans, cones, polytopes. Over general fields, we consider twisted forms of such objects called "arithmetic toric varieties", whose analysis is naturally Galois-theoretic. In this talk, we will present results on the structure of derived categories of arithmetic toric varieties via exceptional collections and how this data reflects rationality of these varieties.   (Received September 13, 2020)

1163-14-853   **Simon Telen*** (`simon.telen@kuleuven.be`). *Cox homotopies: tracking homogeneous
                   coordinates on toric varieties.*
In homotopy algorithms for solving systems of polynomial equations, it is standard practice to work in (multi-)projective space to avoid diverging paths. At each point on a path, a solution is represented by a set of (multi-)homogeneous coordinates. In this talk, we investigate how this approach and many of its advantages generalize to the setting of polyhedral homotopies, where the natural compact solution spaces are more general toric varieties. This is joint work with Timothy Duff, Elise Walker and Thomas Yahl.   (Received September 13, 2020)

1163-14-892   **Benjamin Antieau** and **Akhil Mathew*** (`amathew@math.uchicago.edu`), 5734 S
                   University Ave, Department of Mathematics, University of Chicago, Chicago, IL 60637, and
                   Matthew Morrow. *K-theory and perfectoid rings.*
A classical result of Hiller-Kratzer-Quillen states that for a perfect $\mathbb{F}_p$-algebra, the p-adic K-theory vanishes above degree zero. We prove a p-adic analog of this result. Given a perfectoid ring R over a perfectoid valuation ring, we show that the fiber of the map from the p-adic K-theory of R and $R[1/p]$ is concentrated in degrees ≤ 0. We also generalize a result of Nizioł which extends the statement to smooth algebras. Our observation is that the tilting correspondence reduces this result to characteristic p.   (Received September 13, 2020)

1163-14-903   **Daxin Xu*** (`daxin.xu@amss.ac.cn`), Morningside Center of Mathematics, Chinese
                   Academy of Sciences, 55 Zhongguancun East Road, Beijing, Beijing 100190, Peoples Rep of
                   China. *Bessel F-isocrystals for reductive groups.*
I will first review the Frobenius structure on the Bessel differential equation

$$(x\frac{d}{dx})^2 u - xu = 0,$$

whose Frobenius traces are the Kloosterman sums

$$\mathrm{Kl}(a) := \sum_{xy=a\in\mathbb{F}_p^\times} \exp(\frac{2\pi i}{p}(x+y)).$$

Recently, there are two generalizations of this story (corresponding to GL_2-case) for reductive groups: one is due to Frenkel and Gross from the viewpoint of the Bessel differential equation; another one, due to Heinloth, Ngô and Yun, uses the geometric Langlands correspondence to produce generalized Kloosterman sheaves on $\mathbb{G}_m$. I will report my joint work with Xinwen Zhu, where we study the p-adic aspect of this theory and unify previous two constructions. If time permits, I will talk about the ramification of generalized Kloosterman sheaves at infinity from the p-adic aspect.   (Received September 14, 2020)

1163-14-910   **Jose Israel Rodriguez*** (`jrodriguez43@wisc.edu`), Van Vleck, 480 Lincoln Dr, Madison,
                   WI 53706. *Galois groups in statistics.*
Galois groups encode the internal structure of field extensions. Less well-known is that (families) of systems of polynomial equations also have Galois groups that encode the internal structure of the equations.

A strong interest in the subject comes from applying numerical continuation methods to compute Galois groups of polynomial systems appearing in applications. This is in addition to developments in enumerative geometry, computer vision, number theory, and sparse polynomial system solving.

The aim of this talk is to make give three distinct connections of Galois groups in statistics. The first is to Gaussian mixture models, the second is to the maximum likelihood degree of toric varieties, and the third is to data "at infinity".   (Received September 14, 2020)

1163-14-931   **Maria L Macaulay**, Raleigh, NC , and **Dávid Papp*** (`dpapp@ncsu.edu`), Raleigh, NC.
                   Rational Sum-of-squares Decompositions and Dual Certificates.
We revisit the problem of computing rational certificates for lower bounds of polynomials. Polynomial and sum-of-squares (SOS) optimization problems are typically solved numerically using conic optimization algorithms.

Turning high-precision numerical solutions to semidefinite programming formulations of SOS optimization into exact (verifiable in rational arithmetic) certificates is a challenging problem that has been studied by Peyrl and Parrilo, Kaltofen et al., and others. We present a new, dual approach to rounding numerical certificates to exact ones that avoids the explicit numerical solution of semidefinite programs and does not require high-precision numerical solutions of the SOS problem.   (Received September 14, 2020)

1163-14-1038   **Rajesh S. Kulkarni** and **Charlotte Ure*** (cu9da@virginia.edu). *A Parametrization of Brauer Classes on Elliptic Curves by Binary Cubic Forms.*
The classical notion of Clifford algebras associated to quadratic forms naturally generalizes to forms of higher degree. In this talk, I will focus on the Clifford algebra associated to a binary cubic form, which defines a Brauer class on an elliptic curve. I will discuss how this connection gives rise to a bijection between the set of orbits of binary cubic forms with respect to a GL_2-action and isomorphism classes of pairs (E, α), where E is an elliptic curve of j-invariant 0 equipped with an automorphism θ of order 3, and α is a θ-invariant 3-torsion Brauer class on E.   (Received September 15, 2020)

1163-14-1132   **Daniel Bragg*** (braggdan@berkeley.edu). *Derived invariants of varieties in positive characteristic.*
A fundamental derived invariant associated to a smooth projective variety is its Hochschild homology. In positive characteristic, topological Hochschild homology gives another canonical derived invariant. We will explain how to compute with this object in practice by relating it to crystalline and de Rham-Witt cohomology. As a consequence, we obtain some new restrictions on the Hodge numbers of derived equivalent varieties in positive characteristic. We will also present an example of two derived equivalent 3-folds in characteristic 3 with different Hodge numbers. This is joint work with Benjamin Antieau and Nick Addington.   (Received September 14, 2020)

1163-14-1152   **Jennifer Paulhus*** (paulhus@math.grinnell.edu). *Using computation to study automorphism groups of curves.*
Classification of automorphism groups of Riemann surfaces is a classical problem which has seen a resurgence over the last few decades as computational power has allowed much broader exploration in the field. We will discuss recent computational work on questions of which groups act in certain ways, including a surprising connection to simple groups.   (Received September 14, 2020)

1163-14-1176   **Hiram Lopez** (h.lopezvaldez@csuohio.edu), **Beth Malmskog** (bmalmskog@coloradocollege.edu) and **Gretchen Matthews*** (gmatthews@vt.edu), Department of Mathematics, Virginia Tech, Blacksburg, VA 24061, and **Fernando Pinero-Gonzales** (fernando.pinero1@upr.edu) and **Mary Wootters** (marykw@stanford.edu). *Hermitian-lifted codes.*
Algebraic geometry codes, defined in the 1980s, have variants that are well suited to local recovery. In some applications, it is necessary to recover information by accessing only a few other positions (or nodes) rather than reading an entire received word. Being able to do in many different ways leads to greater availability of information. In this talk, we demonstrate the utility of the Hermitian curve in this scenario by defining Hermitian-lifted codes. Hermitian-lifted codes have the same evaluation set as one-point Hermitian codes, but the functions to be evaluated are not the usual ones. Instead, they are a special set of monomials which restrict to low degree polynomials on lines intersected with the Hermitian curve. As a result, the positions corresponding to points on any line through a given point act as a recovery set for the position corresponding to that point. This yields codes for local recovery of erasures with high availability and constant-bounded rate from the Hermitian curve.   (Received September 14, 2020)

1163-14-1197   **Asher Auel*** (asher.auel@dartmouth.edu), Department of Mathematics, Dartmouth College, Kemeny Hall, Hanover, NH 03755, and **V. Suresh** (suresh.venapally@emory.edu), Department of Mathematics, Emory University, Mathematics & Science Center, Atlanta, GA 30322. *The local-global principle for quadratic forms over function fields.*
The Hasse-Minkowski theorem says that a quadratic form over a global field admits a nontrivial zero if it admits a nontrivial zero everywhere locally. Over more general fields of arithmetic and geometric interest, the failure of the local-global principle is often controlled by auxiliary structures of interest, such as torsion points of the Jacobian and the Brauer group. I will explain work with V. Suresh on the failure of the local-global principle for quadratic forms over function fields varieties of dimension at least two in characteristic zero. The counterexamples we construct are controlled by higher unramified cohomology groups and involve the study of Calabi-Yau varieties of generalized Kummer type that originally arose from number theory. Along the way, we need to develop an

arithmetic version of a result of Gabber on the nontriviality of certain unramified cohomology classes on products of elliptic curves.   (Received September 15, 2020)

1163-14-1250   **Amanda Knecht*** (`amanda.knecht@villanova.edu`), 800 Lancaster Ave, St Augustine Ctr Rm 305, Villanova, PA 19085. *Degree of Unirationality of Cubic Surfaces over Finite Fields.*
A cubic surface is the zero set of a degree three homogeneous polynomial in four variables. For example, the Fermat cubic surface is defined by the vanishing of the equation $x^3 + y^3 + z^3 = w^3$. It has been known for more than 100 years that for any smooth cubic surface X there is a one-to-one map between projective three space and X when the surface is defined over an algebraically closed field like the complex numbers. This is not true over non-closed fields like the real numbers. In 2002 Kollár proved that over any field there is a finite-to-one map from projective three space to X as long as there is at least one solution to the defining polynomial equation over that field. In this talk we will address what is known about the degree of that finite-to-one map for surfaces defined over finite fields. We are still working on finding the lowest degree.   (Received September 15, 2020)

1163-14-1265   **Jennifer Berg*** (`jsb047@bucknell.edu`) and **Jack Petok** (`jack.petok@dartmouth.edu`). *The Brauer group and moduli spaces of Enriques surfaces.* Preliminary report.
The Brauer group is an algebro-geometric object that is a subtle but important birational invariant of a projective surface. In this talk, we will discuss ongoing work about moduli spaces of Enriques surfaces, a class of surfaces whose geometry is of intermediate complexity and whose Brauer group always contains a unique nonzero element. This is joint work with Jack Petok.   (Received September 15, 2020)

1163-14-1289   **Hulya Arguz*** (`nuromur-hulya.arguz@ucsq.fr`). *Advances on Gross–Siebert mirror symmetry and log Gromov–Witten theory.*
Mirror symmetry is a phenomenon arising from dualities in string theories, which is used by mathematicians to obtain predictions on Gromov–Witten invariants of Calabi–Yau varieties. An algebro-geometric approach to mirror symmetry, based on tropical and log geometric techniques, is provided by the Gross–Siebert program. In this talk we will review the most recent advances in this program, and report on joint work with Mark Gross (arxiv:2007.08347) in which we provide new algorithmic tools to compute (log) Gromov–Witten invariants of higher dimensional (log) Calabi–Yau varieties.   (Received September 15, 2020)

1163-14-1315   **James Phillips*** (`james.phillips@wellesley.edu`). *Deformations of wildly ramified one-point covers.*
Abhyankar's Conjecture gives explicit conditions for the existence of a G-cover of a given curve in characteristic p with a given number of branch points. It does not, however, give a description of the possible ramification groups above the branch points. In this talk, I will generalize techniques used by Pries to study the conductors of one-point covers of $\mathbb{P}^1$ when G has a cyclic Sylow p-subgroup. These techniques involve the study of deformations of covers over equicharacteristic discretely valued rings.   (Received September 15, 2020)

1163-14-1318   **Katherine Harris** (`keharri4@ncsu.edu`), North Carolina State University, Campus Box 8205, Raleigh, NC 27695, **Jonathan D Hauenstein** (`hauenstein@nd.edu`), Department of Applied and Computational, Mathematics and Statistics, 102G Crowley Hall, Notre Dame, IN 46556, and **Agnes Szanto*** (`aszanto@ncsu.edu`). *Smooth Points on Semi-algebraic Sets.* Preliminary report.
Many algorithms for determining properties of real algebraic or semi-algebraic sets rely upon the ability to compute smooth points. In this talk, I present a simple procedure based on computing the critical points of some well-chosen function that guarantees the computation of smooth points in each connected bounded component of a real atomic semi-algebraic set. Our technique is intuitive in principal, performs well on previously difficult examples, and is straightforward to implement using existing numerical algebraic geometry software. I also present the application of our method to design an efficient algorithm to compute the real dimension of algebraic sets, the original motivation for this research.   (Received September 15, 2020)

1163-14-1345   **Paul Breiding***, p.breiding@tu-berlin.de. *Certifying roots of polynomial systems using interval arithmetic.*
I will demonstrate a new function in the software HomotopyContinuation.jl that certifies if a true isolated solution of a system of polynomial equations is near an approximation given in 64-bit floating point numbers. The implementation is based on Krawczyk's method and uses Arb for interval arithmetic.   (Received September 15, 2020)

1163-14-1378   **Cordian B Riener*** (cordian.riener@uit.no), UiT University, 9007 Tromsoe, Norway, and **Sebastian Debus**. *Polynomial opzimization problems invariant by finite reflection groups.* Preliminary report.

We consider polynomial optimization problems invariant by finite reflection groups. These groups are related in a beautiful way to combinatorics and the corresponding invariant theory and representation theory can be well understood combinatorically. In the talk, we will present some results on sums of squares representations in this setup and show some more general results which enable to reduce the complexity in polynomial optimization in the context of functions invariant by these groups   (Received September 15, 2020)

1163-14-1428   **Rajesh S Kulkarni*** (kulkarni@math.msu.edu), 619 Red Cedar Rd, Department of Mathematics, Wells Hall, East Lansing, MI 48824, and **Charlotte Ure**, 141 Cabell Drive, Kerchof Hall, P.O. Box 400137, Charlottesville, VA 22904. *Curves of genus 1 and generic Clifford algebras.*

We consider the moduli problem of pairs of j-invariant 0 elliptic curves together with 3-torsion Brauer classes that are invariant under a special order 3 automorphism. In particular, we show that there is a bijection between such pairs and GL_2 orbits of an affine open subset in $\mathbb{A}^4$. The generic Clifford algebra plays the key role in the bijection. We will discuss this connection as well.   (Received September 15, 2020)

1163-14-1458   **Renee Bell*** (rhbell@math.upenn.edu), David Rittenhouse Laboratory, 209 S. 33rd Street, Office 4N61, Philadelphia, PA 19104, and **Jeremy Booher**, **William Chen** and **Yuan Liu**. *Tamely ramified covers of the projective line with alternating and symmetric monodromy.*

Let k be an algebraically closed field of characteristic p and let X the projective line over k with three points removed. We investigate which finite groups G can arise as the monodromy group of étale covers of X that are tamely ramified over the three removed points. This provides new information about the tame fundamental group of the projective line. In particular, we show that for each prime $p \geq 5$, there are families of tamely ramified covers with monodromy the symmetric group S_n or alternating group A_n for infinitely many n. These covers come from the moduli spaces of elliptic curves with $PSL_2(\mathbb{F}_\ell)$-structure, and the analysis uses work of Bourgain, Gamburd, and Sarnak, and adapts work of Meiri and Puder, about Markoff triples modulo ℓ. (Received September 15, 2020)

1163-14-1573   **Edray Herber Goins*** (edray.goins@pomona.edu), 610 North College Avenue, Claremont, CA 91711. *Visualizing Toroidal Belyĭ Pairs.* Preliminary report.

A Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; we may assume these values are $\{0, 1, \infty\}$. A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from 0 to 1. Such graphs can be drawn on the sphere by composing with stereographic projection: $\beta^{-1}([0,1]) \subseteq \mathbb{P}^1(\mathbb{C}) \simeq S^2(\mathbb{R})$.

Replacing $\mathbb{P}^1$ with an elliptic curve E, there is a similar definition of a Belyĭ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$. The corresponding Dessin d'Enfant can be drawn on the torus by composing with an elliptic logarithm: $\beta^{-1}([0,1]) \subseteq E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$. In this project, we use the open source `Sage` to write code which takes an elliptic curve E and a Belyĭ map β to return the Dessin d'Enfant of this map – both in two and three dimensions. We focus on several examples of Belyĭ maps which appear in the *L*-Series and Modular Forms Database (LMFDB).   (Received September 15, 2020)

1163-14-1615   **Sarah Frei*** (sarah.frei@rice.edu) and **Anthony Várilly-Alvarado**. *Reductions of Brauer classes on a K3 surface.*

For a K3 surface over Q of Picard rank 1, it is well-understood that the Picard rank jumps upon reduction modulo a prime. This jumping in the Picard rank is countered by a drop in the size of the Brauer group. In this talk, I will report on joint work with Anthony Várilly-Alvarado, in which we consider the following natural question: Given a non-trivial Brauer class on a K3 surface over Q of Picard rank 1, how often does this class become trivial upon reduction modulo various primes?   (Received September 15, 2020)

1163-14-1685   **Xinwen Zhu*** (xzhu@caltech.edu), 1200 E. California Blvd, Pasadena, CA. *Arithmetic and geometric Langlands program.*

The Langlands program, proposed by Robert Langlands in 1960s, unifies many questions in number theory and representation theory, and has found significant applications to solving classical Diophantine equations. Its geometric version, formulated by Drinfeld and Laumon in 1980s, enlarges the scope of the Langlands philosophy and makes it contact with other subjects such as physics. Interestingly, in recent years, some ideas from the

geometric theory also inspire and lead developments of the traditional arithmetic theory and related problems. I will give an impression of some of these recent developments.   (Received September 16, 2020)

15 ► *Linear and multilinear algebra; matrix theory*

1163-15-35   **Udita N. Katugampola*** (dkatugampola@floridapoly.edu), Department of Applied
                    Mathematics, Florida Polytechnic University, Lakeland, FL 33805. *Finding Eigenvectors:
                    Fast and Nontraditional Approach.*
According to the classical *Gauss-Jordan* elimination approach, we would need five additional steps to find eigenvectors after finding the corresponding eigenvalues. However, we show that we only need one additional step for matrices with a spectrum, $|\sigma(A)| \leq 2$ since those vectors already appear as nonzero columns of the *eigenmatrices*, a term defined in this work. We further generalize this for matrices with $|\sigma(A)| > 2$ and show that eigenvectors lie in the column spaces of eigenmatrices of the complementary eigenvalues.

We introduce two major results, namely, the *2-Spectrum Lemma* and the *Eigenmatrix Theorem*. As a conjecture, we further generalize the Jordan canonical forms for a new class of generalized eigenvectors that are produced by repeated multiples of certain eigenmatrices. We also provide several shortcut formulas to find eigenvectors that does not use echelon forms. The method discussed in this work may be summarized with the mnemonic "Find your puppy at your neighbors'!" argument, where puppy is the eigenvector and the neighbors are the complementary eigenmatrices.   (Received July 17, 2020)

1163-15-108   **Laura Cossu***, Institute of Mathematics and Scientific, Computing, Heinrichstrasse
                    36/III, 8010 Graz, Austria. *Idempotent factorizations of singular 2×2 matrices over
                    quadratic integer rings.*
Let D be the ring of integers of a quadratic number field $\mathbb{Q}[\sqrt{d}]$. Addressing the classical open problem of the characterization of integral domains R such that every singular (*i.e.*, with zero determinant) matrix over R is a product of idempotent matrices, we investigate the idempotent factorization of 2×2 singular matrices over D. We show that when $d < 0$ there exist singular matrices that do not admit an idempotent factorization, while in case $d > 0$ we use Vaseršteǐn's result (1972) that $SL_2(D)$ is generated by transvections to prove that any 2×2 matrix with either a null row or a null column is a product of idempotents. As a consequence, every dimension 2 column-row matrix over a real quadratic integer ring decomposes into idempotent factors. Based on a joint work with P. Zanardo.   (Received August 16, 2020)

1163-15-244   **Malena I Espanol***, School of Math and Stats Sciences, Arizona State University, Tempe,
                    AZ. *Computational Methods for Solving Inverse Problems in Imaging.*
Discrete linear and nonlinear inverse problems arise from many different imaging systems. These problems are ill-posed, which means, in most cases, that the solution is very sensitive to the data. Because the data usually contain errors produced by the different imaging system parts (e.g., cameras, sensors, etc.), robust and reliable regularization methods need to be developed for computing meaningful solutions. In many imaging systems, massive amounts of data are produced which makes the storage of data and the computational cost of the inversion process intractable. In this talk, we will look at different imaging systems, formulate the corresponding mathematical models, develop regularization methods, and show some numerical results.   (Received August 30, 2020)

1163-15-337   **Charles R. Johnson*** (crjohn@wm.edu). *Current Topics in the Nonnegative Inverse
                    Eigenvalue Problem.*
As a practical matter, the very difficult Nonnegative Inverse Eigenvalue Problem has become a bundle of more particular problems. Time permitting, we report on two of these:

1) The doubly stochastic single eigenvalue problem asks which individual complex numbers occur as an eigenvalue of a doubly stochastic matrix. This problem, first discussed in the 1960's, remains open, though its row stochastic analog enjoyed its first "solution" about 70 years ago and has received refinements since. We report on the intriguing progress that is partly empirical.

2) Spectra with repeated eigenvalues may be nonnegatively realizable with some Jordan structures and not others. We sort out what is currently known and what is likely true about the Jordan NIEP.

The speaker's last talk at an AMS meeting on the NIEP was an hour talk about 35 years ago.   (Received September 03, 2020)

1163-15-431   **Pietro Paparella*** (`pietrop@uw.edu`), 18115 Campus Way NE, Bothell, WA 98011-8246. *Perron similarities and the nonnegative inverse eigenvalue problem.*

The *nonnegative inverse eigenvalue problem (NIEP)* is to determine which multisets of n complex numbers occur as the eigenvalues of some n-by-n entry-wise nonnegative matrix.

An invertible matrix S is called a *Perron similarity* if there is a non-scalar diagonal matrix D such that SDS^{-1} is entry-wise nonnegative. Each Perron similarity gives a cone, called its *spectracone*, of NIEP-realizable spectra (thought of as vectors in $\mathbb{C}^n$). Of course, the union of these spectracones is the solution of the DNIEP. By considering the spectracones that come from the Perron similarities associated with certain realizations of the Karpelevič arcs, large portions of NIEP-realizable spectra are generated for a given n. Further results concerning the theory of complex Perron similarities for the diagonalizable NIEP (DNIEP) are discussed along with implications for further research.

This is joint-work with Charles R. Johnson.   (Received September 06, 2020)

1163-15-432   **Ela Celikbas**, **Jai Laxmi*** (`laxmiuohyd@gmail.com`) and **Jerzy Weyman**. *Spinor structures on free resolutions of codimension four Gorenstein ideals.*

Buchsbaum and Eisenbud stated that every Gorenstein ideal of codimension three is generated by the submaximal Pfaffians of an odd order skew-symmetric matrix. Kustin and Miller studied structure of codimension four Gorenstein ideals. We explore spinor structures on free resolutions of codimension four Gorenstein ideals. For such ideals with 6, 7, 8 and 9 generators, we present the number of minimal generators of ideals among spinor coordinates.   (Received September 06, 2020)

1163-15-510   **Tim Marrinan*** (`timothy.marrinan@umon.ac.be`). *Improved sufficient conditions for identifiable nonnegative matrix factorization.* Preliminary report.

Given a nonnegative matrix, $X \in \mathbb{R}_+^{m \times n}$, and a factorization rank, $r \leq \min\{m, n\}$, nonnegative matrix factorization (NMF) identifies two matrices, $W \in \mathbb{R}_+^{m \times r}$ and $H \in \mathbb{R}_+^{r \times n}$, such that $X = WH$. With appropriate normalization, sufficient conditions exist under which this decomposition is identifiable, that is, unique up to permutation and scaling of the columns of W and the corresponding rows of H. Some of these conditions are easy to check, but are rarely satisfied in practice. Other sufficient conditions are often satisfied in practical applications, but checking whether they are satisfied is NP-hard. In this talk we discuss progress on formulating sufficient conditions for identifiability that sit in the sweet spot between existing conditions, that is, ones that are commonly satisfied in practice and can be checked efficiently for large problems.   (Received September 08, 2020)

1163-15-565   **Shaun M Fallat*** (`shaun.fallat@uregina.ca`), Department of Mathematics and Statistics, University of Regina, Regina, Sask. S4S0A2, Canada. *On Graphs Admitting Certain Multiplicity Partitions.*

Given a graph G, we let $S(G)$ denote the set of all real symmetric matrices whose pattern of off-diagonal entries are governed by the adjacency of G. If G has n vertices, then the multiplicities of the eigenvalues of any matrix in $S(G)$ forms a partition of n; this is called a <u>multiplicity partition</u>.

In this talk, we discuss graphs that realize certain restricted multiplicity partitions, including those with 2 integers, and tie this concept together with existing spectral parameters (key to the IEP-G) such as $q(G)$ and $M(G)$.   (Received September 09, 2020)

1163-15-566   **Shaun M Fallat*** (`shaun.fallat@uregina.ca`), Department of Mathematics and Statistics, University of Regina, Regina, Sask. S4S0A2, Canada. *On the ranks of submatrices of totally nonnegative matrices.*

A matrix is called totally nonnegative (TN) if all of its minors are nonnegative. Since the pioneering work of Gantmacher and Krein, it is known that the distribution of ranks among both principal and non-principal submatrices of a TN matrix depends on a number of factors, such as the relative position of the base submatrix.

In this talk, we will survey some well-known results along these lines, including, row/column inclusion and shadowing. In addition, we present a new approach by taking into account the so-called Cauchon algorithm applied to a TN matrix.   (Received September 09, 2020)

1163-15-585   **Ana I. Julio** and **Carlos Marijuán*** (`marijuan@mat.uva.es`), Departamento de Matemática Aplicada, ETS Ingeniería Informática, Campus Miguel Delibes, calle Belén,15, Valladolid, Spain, and **Miriam Pisonero** and **Ricardo L. Soto**, , Chile. *On universal realizability of spectra.*

The nonnegative inverse eigenvalue problem (NIEP) is the problem of characterizing all possible spectra of entrywise nonnegative matrices. If there exists a nonnegative matrix with spectrum Λ for each possible Jordan

canonical form allowed by Λ, we say that Λ is universally realizable (UR). It is well known that an $n \times n$ nonnegative matrix A is co-spectral to a nonnegative matrix B with constant row sums. We extend the co-spectrality between A and B to a similarity between A and B, when the Perron eigenvalue is simple. We ask whether certain properties of the NIEP, such as the three rules that characterize the C-realizability of lists (one the strongest sufficient conditions for the NIEP), extend or not to the UR. In particular, we show that if a list of complex numbers $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, with Perron eigenvalue λ_1, is UR, then $\{\lambda_1 + \epsilon, \lambda_2, \ldots, \lambda_n\}$ is also UR for any $\epsilon > 0$. We also consider the universal realizability of the Guo perturbation $\{\lambda_1 + \epsilon, \lambda_2 - \epsilon, \lambda_3, \ldots, \lambda_n\}$, and of the union of two UR lists Λ_1 and Λ_2.   (Received September 10, 2020)

1163-15-649   **H Tracy Hall*** (`h.tracy@gmail.com`). *The Boustrophedon Conjecture: Using vertex orderings to strengthen the Graph Complement Conjecture for maximum nullity of a graph.*
A graph nullity parameter $M_s(G, \sigma)$ is introduced that depends not only on the simple graph G but on an ordering σ of its vertices. (The subscript stands for <u>sequential</u>.) Maximizing $M_s(G, \sigma)$ over all orderings σ yields $M_s(G)$, which is shown to be a lower bound for maximum semidefinite Strong Arnold Property nullity: $M_s(G) \leq \nu(G)$.

Boustrophedon Conjecture. *Let G be a graph with $|V(G)| = n$. Then there exists an ordering σ of $V(G)$ such that*
$$M_s(G, \sigma) \, + \, M_s(\overline{G}, \overset{\leftarrow}{\sigma}) \; \geq \; n - 2,$$
where $\overline{G}$ denotes the complement of G and $\overset{\leftarrow}{\sigma}$ denotes the reversal of σ.

This would imply for example GCC_ν, the Graph Complement Conjecture for $\nu(G)$. While not quite combinatorial, $M_s(G, \sigma)$ does have the advantage that it can be computed fairly easily for any given graph and vertex ordering. It thus lends itself to computerized explorations, validating particular cases of the Boustrophedon Conjecture.   (Received September 14, 2020)

1163-15-693   **Bryan A Curtis***, bcurtis6@uwyo.edu. *Extremal Matrices with the Strong Inner Product Property.* Preliminary report.
The strong inner product property is useful for constructing sign patterns that allow orthogonality. An $n \times n$ matrix with the strong inner product property has at most $n(n-1)/2$ zero entries. In this talk we discuss how to construct matrices with the strong inner product property and $n(n-1)/2$ zero entries, and applications of such matrices. The construction techniques that are presented have interesting ties to tournaments and weighing matrices.   (Received September 11, 2020)

1163-15-729   **Raphael Loewy*** (`loewy@technion.ac.il`). *On the spectra of nonnegative symmetric 5 x 5 matrices.*
Given a list $\sigma = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ of complex numbers the Nonnegative Inverse Eigenvalue Problem (NIEP) asks when is σ the spectrum of an nxn nonnegative matrix. When σ consists of real numbers the Symmetric Nonnegative Inverse Eigenvalue Problem (SNIEP) asks when is it the spectrum of an nxn nonnegative, symmetric matrix. Both problems are currently unsolved for $n \geq 5$.

We consider SNIEP in the case $n = 5$. Assume the elements of σ are arranged in decreasing order, and define $s_1(\sigma) = \sum_{i=1}^5 \lambda_i$ and $s_3(\sigma) = \sum_{i=1}^5 \lambda_i^3$. The solution is known for $\lambda_3 \leq s_1(\sigma)$. When $y := \lambda_3 - s_1(\sigma) > 0$, we obtain a new inequality involving y, $s_1(\sigma)$ and $s_3(\sigma)$. This enables us to show that certain lists σ, previously unknown to be realizable, are not the spectra of a $5x5$ nonnegative, symmetric matrix.   (Received September 12, 2020)

1163-15-804   **Jephian C.-H. Lin*** (`jephianlin@gmail.com`), **Polona Oblak** and **Helena Šmigoc**. *The strong spectral property for graphs.*
A symmetric matrix A is said to have the strong spectral property if $X = O$ is the only symmetric matrix that satisfies $A \circ X = O$, $I \circ X = O$, and $AX - XA = O$. Here the operation $\circ$ is the entrywise product. If a matrix has the strong spectral property, then one may perturb the matrix slightly to create more nonzero entries without changing its spectrum. This behavior has been used widely for constructing matrices in the inverse eigenvalue problem of a graph. In this talk, we will show that if the nonzero pattern of the matrix is described by certain graphs, then it always has the strong spectral property.   (Received September 13, 2020)

1163-15-838   **Mike Boyle*** (`mmb@math.umd.edu`). *The stable algebra of (nonnegative) matrices.*
Realization questions for nonnegative matrices over the reals (for the spectrum, or Jordan form) have more tractable stable versions (for the nonzero spectrum, or the Jordan form away from zero). I'll review the status of these problems, some motivation, and the deeper stable relations one is forced to consider when studying

matrices over the integers and more general rings. This leads to algebraic K-theory, as well as fundamental open problems for nonnegative real matrices. Primary references for this talk are "Symbolic dynamics and the stable algebra of matrices" with Scott Schmieding, https://arxiv.org/abs/2006.01051 , and "Path methods for strong shift equivalence of positive matrices", with Kim and Roush, Acta Applic. Math. 126, 65-115 (2013). (Received September 13, 2020)

1163-15-846   **C Marijuán** and **M Pisonero*** (`mpisonero@uva.es`). *Eigenvalues of Symmetric Nonnegative Matrices of Small Order.* Preliminary report.

The SNIEP (Symmetric Nonnegative Inverse Eigenvalue Problem) is the problem of characterizing all possible real spectra of entrywise symmetric nonnegative matrices. When the symmetric condition is omitted the problem is named as the RNIEP (Real Nonnegative Inverse Eigenvalue Problem). A (symmetric) nonnegative matrix can be seen as the adjacency matrix of a weighted (graph) digraph. In this talk we make a brief historical overview of both problems applied to graphs and we mention that they remain open for families of size greater than or equal to 5. The first result on the RNIEP is from Suleimanova in 1949 and it is a sufficient condition, while the origin of the SNIEP is due to Fiedler in 1974. Hershkovich, in his thesis in 1978, raises the question whether both problems, RNIEP and SNIEP, are the same. It took until 1996 when Johnson, Laffey and Loewy proved that the two problems are different. It is known that for lists of size less than or equal to 4 the RNIEP and the SNIEP are equivalent problems, and Egleston, Lenker and Narayan in 2004 proved that for lists of size 5 they are different. We focus on the SNIEP in small dimensions and we give the characterizations that are known, as well as the matrices that have these families as spectrum.   (Received September 13, 2020)

1163-15-851   **Alan Krinik** (`ackrinik@cpp.edu`), Mathematics & Statistics, Cal Poly Pomona, 3801 West Temple Avenue, Pomona, CA CA 91768, and **Gerardo Rubino*** (`gerardo.rubino@inria.fr`), Campus de Beaulieu, 35042 Rennes, France. *The power-dual and the exponential-dual of a matrix.* Preliminary report.

In this talk we introduce two transformations of a matrix M into a new matrix, its power-dual $pd(M)$ and its exponential-dual $ed(M)$. Their interest is that the powers of M (resp. its exponential $\exp(M)$) can be obtained by evaluating those of its power-dual (resp. that of its exponential-dual). The cost of finding, for instance, $\exp(M)$ in terms of $\exp(ed(M))$, is low (linear). Matrix M can be finite or infinite. These concepts were born as improved versions of the pseudo-dual of a matrix that the authors introduced recently, itself a generalization of the dual as defined by Sigmund and developed by Anderson in the context of Markov processes. The main applications so far have been in Markov chain analysis, but the transformations presented here are general, they cover any kind of linear differential system or of linear difference equations. In the talk, we will describe how they can make the evaluation of powers or of exponentials easier than working with the original matrix. Examples will also include a few cases in queueing theory.   (Received September 13, 2020)

1163-15-893   **Mary K Flagg*** (`flaggm@stthom.edu`), 3800 Montrose, Houston, TX 77006. *Rigid linkage forcing and eigenvalue multiplicities.*

Zero forcing was introduced as an upper bound on the maximum multiplicity of an eigenvalue of a symmetric matrix determined by a graph. Partial zero forcing, or forcing without coloring the whole graph, potentially provides information on the multiplicities of all the eigenvalues. Partial zero forcing is generalized and connected to vital linkages in the rigid linkage forcing process. The rigid linkage forcing process produces rigid linkages, and spanning rigid linkages are vital linkages, and also determined by a standard zero forcing process. However, when not spanning, rigid linkages, and related rigid shortest linkages give a lower bound on the number of distinct eigenvalues of a graph, or more generally the number of eigenvalues of multiplicity at least i. In this talk i will explain the rigid linkage forcing process and show that it is a useful tool in the study of the inverse eigenvalue problem for a graph.   (Received September 13, 2020)

1163-15-900   **Zi Yang*** (`ziy109@ucsd.edu`) and **Jiawang Nie**. *Hermitian Tensor Decompositions.*

Hermitian tensors are generalizations of Hermitian matrices, but they have very different properties. Every complex Hermitian tensor is a sum of complex Hermitian rank-1 tensors. However, this is not true for the real case. We study basic properties for Hermitian tensors such as Hermitian decompositions and Hermitian ranks. For canonical basis tensors, we determine their Hermitian ranks and decompositions. For real Hermitian tensors, we give a full characterization for them to have Hermitian decompositions over the real field. In addition to traditional flattening, Hermitian tensors specially have Hermitian and Kronecker flattenings, which may give different lower bounds for Hermitian ranks. We also study other topics such as eigenvalues, positive semidefiniteness, sum of squares representations, and separability.   (Received September 14, 2020)

1163-15-1112   **Kennett L. Dela Rosa*** (`pld43@drexel.edu`) and **Hugo J. Woerdeman** (`hugo@math.drexel.edu`). *Location of Ritz values in the numerical range of normal matrices.*

Let μ_1 be a complex number in the numerical range $W(A)$ of a normal matrix A. In the case when no eigenvalues of A lie in the interior of $W(A)$, we identify the smallest convex region containing all possible complex numbers μ_2 for which $\begin{bmatrix} \mu_1 & * \\ 0 & \mu_2 \end{bmatrix}$ is a 2-by-2 compression of A.   (Received September 14, 2020)

1163-15-1210   **Helena Šmigoc*** (`helena.smigoc@ucd.ie`), **Stephen Kirkland** and **Thomas Laffey**. *On the Karpelevič Region.*

A classic result of Karpelevič describes $\Theta_n \subset \mathbb{C}$, the set consisting of all eigenvalues of all stochastic matrices of order n. We will provide an alternative characterisation of Θ_n that for each $\theta \in [0, 2\pi)$ identifies the point on the boundary of Θ_n with argument θ. We will further prove that if $n \in \mathbb{N}$ with $n \geq 2$, and $t \in \Theta_n$, then t is a subdominant eigenvalue of some stochastic matrix of order n. Finally, we will discuss some properties of stochastic matrices with eigenvalues on the boundary of Θ_n.   (Received September 15, 2020)

1163-15-1341   **Michael A. Ostroski*** (`michaelostroski1@gmail.com`). *Dynamic Graph Edge Clustering: The Art of Conversation... Mining.* Preliminary report.

One approach to network security is attempting to find anomalies in network traffic. Netflow data provides records for how a network communicates and a natural way to model this data is with a dynamic multi-graph. We often know which nodes in our graph are important and need to answer the question of "when" we should be paying attention to those nodes. To accomplish this we look for a way to group the record data itself, effectively clustering the edges of the graph in order to identify anomalies. An interesting cluster of edges might be, for example, a pattern of behavior that is emblematic of a breach in security.

The Line Graph of a graph G is another graph that represents the adjacencies between edges of G. Forming the line graph for record data and clustering would result in a grouping of records. Unfortunately, the line graph can be several orders of magnitude larger than the G, which is unacceptable for many data sets. We present a scalable approach to edge clustering that uses an approximate line graph, filtered by time, as a way to capture potential causal relationships between records. Analysis of the time-filtered line graph captures higher-level phenomenology that are not explicitly coded in the data, suggesting that it is an effective model for these problems.   (Received September 15, 2020)

1163-15-1387   **Selcuk Koyuncu*** (`skoyuncu@ung.edu`), 4266 Suwanee Brook Ct, Buford, GA 30518, and **Lei Cao**. *A short note on multilevel Toeplitz matrices.*

Chien, Liu, Nakazato and Tam proved that all $n \times n$ classical Toeplitz matrices (one-level Toeplitz matrices) are unitarily similar to complex symmetric matrices via two types of unitary matrices and the type of the unitary matrices only depends on the parity of n. In this paper we extend their result to multilevel Toeplitz matrices that any multilevel Toeplitz matrix is unitarily similar to a complex symmetric matrix. We provide a method to construct the unitary matrices that uniformly turn any multilevel Toeplitz matrix to a complex symmetric matrix by taking tensor products of these two types of unitary matrices for one-level Toeplitz matrices according to the parity of each level of the multilevel Toeplitz matrices. In addition, we introduce a class of complex symmetric matrices that are unitarily similar to some p-level Toeplitz matrices.   (Received September 15, 2020)

1163-15-1526   **Hein van der Holst*** (`hvanderholst@gsu.edu`). *Some signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 3$.*

Arav et al. introduced the Colin de Verdiere-type parameter ν on signed graphs. They proved that the class of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 1$ coincides with the class of bipartite signed graphs (signed graph with no odd cycles), and gave a complete characterization of the class of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 2$. A signed graph (G, Σ) is projective if G can be embedded in the projective plane and the odd cycles are exactly those that are orientation-reversing. We show that if a signed graph (G, Σ) is projective, then $\nu(G, \Sigma) \leq 3$. (Received September 15, 2020)

1163-15-1666   **Ray A Perlner*** (`ray.perlner@nist.gov`). *MinRank and its Application to Cryptanalysis.*
The MinRank problem is an important problem in multivariate and code-based cryptography, representing a major avenue for attacks against several encryption and signature schemes, including the third round candidates in the NIST PQC standardization process, Rainbow and GeMSS, as well as the second round candidates ROLLO and RQC. It involves finding a low rank linear combination of a collection of matrices over a finite field. In most cases the instance of MinRank involved in key recovery has additional structure, which may or may not be

exploitable by the attacker. Various algebraic and combinatorial approaches have been explored for solving the problem. This talk will focus on joint work with Bardet, Bros, Cabarcas, Gaborit, Smith-Tone, Tillich, and Verbel appearing at Asiacrypt 2020, which resulted in a major change in the estimated security of the schemes ROLLO and RQC and a major change in the estimated complexity of MinRank attacks against Rainbow and GeMSS.   (Received September 16, 2020)

16 ▶ Associative rings and algebras

1163-16-191   **Ellen Kirkman**, **Robert Won*** (robwon@uw.edu) and **James J Zhang**. *Degree bounds for Hopf actions on Artin–Schelter regular algebras.*

In 1916, Noether proved that over a field of characteristic 0, if a finite group G acts linearly on the polynomial ring $k[x_1, \ldots, x_n]$, then the invariant subring can be generated by polynomials of degree at most $|G|$. The Noether bound was extended to the non-modular case (where $|G|$ is invertible in k) independently by Fleischmann, Fogarty, and Derksen–Sidman. In the modular case (where $|G|$ is not invertible in k), the Noether bound does not hold, but Symonds proved that the invariant subring can be generated by polynomials of degree at most $n(|G| - 1)$.

We consider semisimple Hopf actions on noncommutative Artin–Schelter regular algebras, proving several upper bounds on the degrees of the minimal generators of the invariant subring, and on the degrees of syzygies of modules over the invariant subring. This work is joint with Ellen Kirkman and James J. Zhang.   (Received August 25, 2020)

1163-16-238   **Pablo S. Ocal***, pablosanchezocal@gmail.com. *The Gerstenhaber bracket in relative Hochschild cohomology.*

The Gerstenhaber bracket in the usual Hochschild cohomology was introduced by Gerstenhaber, who together with Schack used it to study deformations of algebras. Along the way, they essentially claimed that everything that can be done in Hochschild cohomology can also be done in relative Hochschild cohomology. However, they required a separability condition to obtain relative projective resolutions when working with diagrams of algebras. This additional requirement motivates contextualizing our work to relative homological algebra. This is a less general context but it has multiple advantages: we can remove the separability condition, proofs are approachable, computations can be carried out, and an there is an interpretation of the bracket as a dg Lie algebra structure on a complex. Recent results by Kaygun, who constructed a Jacobi-Zariski long exact sequence, and by Cibils, Lanzilotta, Marcos, Schroll, and Solotar, who described aspects of the Hochschild cohomology of bounded quiver algebras using relative cohomological tools, strongly suggests that this context may be adequate for a better understanding of the cohomology of associative algebras.   (Received August 29, 2020)

1163-16-275   **P. D. Beites*** (pbeites@ubi.pt). *Pleasant Encounters with Quaternions.*

In an algebraic setting, pleasant encounters with quaternions will be provided by joint work with colleagues from Portugal and Spain. More concretely, an overview of research directly or indirectly related to quaternions, and their relatives, will be presented.   (Received August 31, 2020)

1163-16-309   **Luigi Ferraro*** (lferraro@ttu.edu), **Frank Moore** and **Josh Pollitz**. *Support varieties and symmetry of complexity for quotients of skew polynomial rings.*

Building on ideas present in work of Avramov, Buchweitz, Iyengar, and Pollitz, we use color differential graded homological algebra to compute the derived Hochschild cohomology of a skew complete intersection ring R, i.e. a skew polynomial ring modulo an ideal generated by a regular sequence of normal elements. Our calculation uses derivations, which seems to be a new approach even in the commutative case. In addition, we prove that for color modules M and N over R, $\text{Ext}_R(M, N)$ is a finitely generated module over a (potentially different) skew polynomial ring. When the parameters defining the original skew polynomial ring are roots of unity, this allows us to define the support variety of a pair of color modules over such a ring, and we extend many commutative results to this new context.   (Received September 01, 2020)

1163-16-361   **Aleks Kleyn*** (aleks_kleyn@mailaps.org), aleks_kleyn@mailaps.org. *Calculus over quaternion algebra.*

The map $f : H \to H$ is called differentiable, if there exists differential form $\frac{df}{dx}$ such that

$$f(x + h) - f(x) = \frac{df}{dx} \circ h + o(h)$$

where
$$o : H \to H$$
is such continuous map that
$$\lim_{a \to 0} \frac{\|o(a)\|}{\|a\|} = 0$$
Linear map $\frac{df(x)}{dx}$ is called derivative of map f.

The differential form
$$\omega : H \to \mathcal{L}(D; H \to H)$$
is called integrable, if there exists a map
$$f : H \to H$$
such that
$$\frac{df(x)}{dx} = \omega(x)$$
Then we use notation
$$f(x) = \int \omega(x) \circ dx$$
and the map f is called indefinite integral of the differential form ω.

Let $U \subseteq A$ be open set. Let
$$\gamma : [a, b] \to U$$
be a path of class C^1 in U. We define the integral of the differential 1-form ω along the path γ by the equality
$$\int_\gamma \omega = \int_a^b dt \omega(\gamma(t)) \frac{d\gamma(t)}{dt}$$

Differential form
$$\omega : H \to \mathcal{L}(D; H \to H)$$
is integrable iff
$$d\omega(x) = 0$$
For any quaternions a, b, we define definite integral by the equality
$$\int_a^b \omega = \int_\gamma \omega$$
which dos not depend on a path γ from a to b.   (Received September 08, 2020)

1163-16-363   **Daniel Rogalski*** (`drogalski@ucsd.edu`), **Susan J. Sierra** and **J. Toby Stafford**. *Some Noncommutative Birational Transformations.*
In a series of papers joint with Sierra and Stafford, we have studied the birational geometry of projective noncommutative surfaces containing a smooth elliptic curve as a divisor. In particular, we have defined notions of blowing up a point and blowing down a curve of self-intersection -1 in this setting. In this talk we explain how these notions can be used to give two explicit birational transformations modeled on important examples in the commutative case. The first shows that a quadric surface blown up at a point can then be blown down twice to obtain a noncommutative projective plane. The second is a version of the Cremona Transform, which blows up three points on the projective plane and then blows down the three new lines of self-intersection -1 that appear. Both constructions are based on a result of independent interest, which shows how to characterize surfaces that are a blowup of the projective plane at two points, in terms of intersection theory.   (Received September 03, 2020)

1163-16-578   **Garri Davydyan*** (`garri.davydyan@gmail.com`). *Coquaternion as a metafunctional structure of a genome.* Preliminary report.
Stability of cell renewal cycles, which include apoptosis and cells divisions is an essential feature of biologic objects determining advantages in survival. It implies additional structural features of encoding and releasing genetic information from chromosomes. Positive, negative and reciprocal (PNR) feedback mechanisms are considered as a functional basis to form biologic systems as structures with stable functional organization. All three feedback mechanisms are realized by synthesizing signalling compounds which must be in the affinity to the receptors of other systems in order to activate or inhibit their functions. Elements of non-coding DNA may be responsible for regulation of syntheses of specific proteins serving as linking substances for feedback regulatory loops. Matrices of PNR correspond to imaginary part of coquaternion and as elements of a non-commutative ring may represent metafunctional regulatory structures for cell renewal cycles encoded in chromosomes.   (Received September 10, 2020)

1163-16-589   **Daniel Yee*** (`yeedo@uwec.edu`), **Xingting Wang** and **Jason Bell**. *Dixmier-Moeglin Equivalence and Morita Invariance.*

We say that a Noetherian algebra R over a field satisfies the Dixmier-Moeglin equivalence (DME) if the annihilators P of simple R-modules are locally closed in the spectrum of R and the center of the Goldie ring of quotients of R/P is an algebraic extension of the base field. We show that DME is Morita invariant: if two Noetherian algebras are Morita equivalent, and if one satisfies DME then the other algebra satisfies DME. We also generalize the notion of Morita equivalence and obtain a similar result.   (Received September 10, 2020)

1163-16-630   **Tevian Dray*** (`tevian@math.oregonstate.edu`), Department of Mathematics, Oregon State University, Corvallis, OR 97330, and **Corinne A. Manogue** and **Isabella Johnson**. *A visual description of quaternionic multiplication and $\mathfrak{so}(4)$.*

The rotation groups $SO(n)$ are generated by rotations in every pair of planes in $\mathbb{R}^n$. The unitary groups $SU(n) \subset SO(2n)$ can be interpreted as correlated rotations of pairs of planes in $\mathbb{R}^{2n}$. We describe a graphical representation of these rotations at the Lie algebra level, culminating in the well-known decomposition $\mathfrak{so}(4) = \mathfrak{su}(2) + \mathfrak{su}(2)$, expressed in terms of quaternionic multiplication. Our results are not new, but their presentation is somewhat nontraditional.   (Received September 10, 2020)

1163-16-639   **Birge Huisgen-Zimmermann*** (`birge@math.ucsb.edu`). *Dualities induced by tilting modules.*

It is well known that arbitrary tilting bimodules over finite dimensional algebras give rise to partial equivalences of the corresponding module categories. The main point of the talk is to supplement these thoroughly explored covariant functors by dualities. In fact, we show that all dualities defined on certain types of representation-theoretically relevant subcategories of module categories are induced by tilting modules. We discuss the impact of these dualities and outline applications.   (Received September 10, 2020)

1163-16-644   **Hongdi Huang*** (`h237huan@rice.edu`), 2601 S Braeswood Blvd, apt 1104, TX. *Zariski Cancellation Problem.*

The Zariski cancellation problem ask: is an affine variety X over an algebraically closed field k having the property that $X \times \mathbb{A}^1 \cong \mathbb{A}^{n+1}$ necessarily isomorphic to $\mathbb{A}^n$? Ring theoretically, one can ask more generally: for R some specific k-algebra, when does $R[x_1, x_2, \cdots, x_n] \cong S[x_1, x_2, \cdots x_n]$ imply that R and S are isomorphic as k-algebras? Recently, there has been a great amount of successful study towards this Zariski cancellation problem. In this talk, we will give an overview of the rich literature and talk about the noncommutative analogues of a cancellation theorem of Abhyankar, Eakin, and Heinzer.   (Received September 10, 2020)

1163-16-678   **Martin Lorenz***, Dept. of Mathematics, Temple University, Philadelphia, PA 19122. *On the importance of being rational.* Preliminary report.

The title is a nod to a recent excellent survey article by Jason Bell: On the importance of being primitive, Rev. Colombiana Mat. 53 (2019). The first part of the talk will concern the main theme of that article: the so-called Dixmier-Moeglin equivalence, which can be seen as a version of Hilbert's Nullstellensatz in a noncommutative setting. I plan to discuss this in the context of algebras that are equipped with an action by a Hopf algebra. I also hope to discuss semicenters in that context.   (Received September 11, 2020)

1163-16-711   **Kenneth Chan**, **Jason Gaddis*** (`gaddisj@miamioh.edu`), **Robert Won** and **James Zhang**. *Reflexive hull discriminants of quantum generalized Weyl algebras.*

We introduce the reflexive hull discriminant as a tool to study invariant theory questions for algebras that are finitely generated, but not necessarily free, over their centers. We compute this discriminant for quantum generalized Weyl algebras and use this to determine their automorphism groups. This is joint work with Kenneth Chan, Robert Won, and James Zhang.   (Received September 11, 2020)

1163-16-822   **Ana Elena Berrizbeitia*** (`aberrizbeitia@coloradomesa.edu`). *Invariants of Hopf actions on path algebras of quivers.*

Invariant theory has its roots in groups acting on algebraic varieties, where the goal is to describe the polynomial functions that are fixed by the group action. A classic question in the study of group actions is whether the invariant ring is finitely generated, and if so, can we find a nice description for a minimal set of generators. Actions, however, are not limited to group actions, and in this talk, we will show under which circumstances a Hopf Algebra, namely a Taft Algebra, can act on the path algebra of a quiver, extending the work of Kinser and Walton published in 2016. Furthermore, given an action where the group-like element $g \in T(n)$ acts transitively on Q_0, we provide a description of the invariant ring of the action.   (Received September 13, 2020)

1163-16-936   **Jason Gaddis** and **Xingting Wang*** (`xingting.wang@howard.edu`), Washington, DC.
Zariski cancellation for Poisson algebras. Preliminary report.

We study the Zariski cancellation problem for Poisson algebras asking whether $A[x]$ is isomorphic to $B[y]$ implies A is isomorphic to B when A and B are Poisson algebras. We resolve this affirmatively in the cases when A and B are both connected graded Poisson algebras finitely generated in degree 1 without degree one Poisson central elements and when A is a Poisson integral domain of Krull dimension two with nontrivial Poisson bracket. We further introduce the Poisson analogues of Makar-Limanov invariant and discriminant to deal with the Zariski cancellation problem for some families of Poisson algebras. This is joint work with Jason Gaddis.   (Received September 15, 2020)

1163-16-996   **Georgia Benkart*** (`benkart@math.wisc.edu`), Department of Mathematics, University of
                    Wisconsin-Madison, Madison, WI 53706. *Tensor Representations for the Drinfeld Double
of the Taft Algebra.*

The Drinfeld double D_n of the Taft algebra A_n is a quasi-triangular Hopf algebra, which Kauffman and Radford have shown has a unique ribbon element υ when n is odd and $n \geq 3$. We determine an explicit expression for υ and use that and the R-matrix of D_n to construct an action of the Temperley-Lieb algebra $\mathsf{TL}_k(\xi)$ as D_n-module endomorphisms on the k-fold tensor power $\mathsf{V}^{\otimes k}$ of any two-dimensional simple D_n-module V. The parameter $\xi = -(q^{\frac{1}{2}} + q^{-\frac{1}{2}})$, where q is the nth root of unity used to define D_n.

When V is the unique self-dual two-dimensional simple module, there is a diagrammatic algorithm for computing the $\mathsf{TL}_k(\xi)$-action. We show that this action is faithful for any $k \geq 1$ and that $\mathsf{TL}_k(\xi)$ is isomorphic to the centralizer algebra $\mathsf{End}_{\mathsf{D}_n}(\mathsf{V}^{\otimes k})$ for $1 \leq k \leq 2n - 2$.

This is joint work with Rekha Biswal, Ellen Kirkman, Van C. Nguyen, and Jieru Zhu.   (Received September 14, 2020)

1163-16-1066   **Nicolas Andruskiewitsch**, **Ivan Angiono** and **Milen Yakimov***
                    (`yakimov@math.lsu.edu`). *Poisson geometry of large quantum groups.*

In a celebrated sequence of works from the 1990s, De Concini, Kac and Procesi constructed a Poisson geometric framework for the study of the irreducible representations of big quantum groups at roots of unity. We will describe an extension of this framework to a large family of Drinfeld doubles arising in the classification of pointed Hopf algebras, which includes as special cases the families of all big quantum groups and supergroups at roots of unity. This is done by a new method, based on perfect pairings between restricted and non-restricted integral forms, which does not rely on any direct computations of Poisson brackets and reductions to low rank cases. We will provide an intuitive introduction to all of the above notions.   (Received September 14, 2020)

1163-16-1086   **Peter D Goetz*** (`peter.goetz@humboldt.edu`) and **Andrew Conner**. *Global section
rings and certain twisted tensor products.* Preliminary report.

Let A be a connected, $\mathbb{N}$-graded algebra over an algebraically closed field $\mathbb{K}$. Following the seminal work of M. Artin, J. Tate and M. Van den Bergh one can associate to A a certain inverse system of projective schemes: $\Gamma = \{\Gamma_n\}_{n \geq 1}$. The closed points of Γ_n are in one-to-one correspondence with the truncated point modules of A of length $n + 1$. Let $B(\Gamma) = \mathbb{K} \oplus \bigoplus_{n \geq 1} H^0(\Gamma_n, \mathcal{O}_{\Gamma_n}(1))$. Then $B(\Gamma)$ can be given the structure of a graded $\mathbb{K}$-algebra. We call $B(\Gamma)$ *the global section ring associated to* Γ.

In this talk I will first discuss a theorem that characterizes, in terms of local cohomology, when $B(\Gamma)$ is generated in degree 1. In the second part, I will determine a presentation of the ring $B(\Gamma)$ in the case of a certain non-Artin-Schelter regular quadratic twisted tensor product of $\mathbb{K}[x, y]$ and $\mathbb{K}[z]$. Presentations of the global section rings of all quadratic twisted tensor products of $\mathbb{K}[x, y]$ and $\mathbb{K}[z]$ have recently been determined by the authors.   (Received September 14, 2020)

1163-16-1121   **Padmini Veerapen*** (`pveerapen@tntech.edu`), **Jason Gaddis** and **Xingting Wang**.
On Reflection Groups of the Homogenized n^{th} Weyl Poisson Algebra, H_n.
Preliminary report.

In this talk, we will explore the work of Kirkman, Kuzmanovich, and Zhang, on rigidity of graded regular algebras and discuss applications to the homogenized n^{th} Weyl Poisson algebra. In particular, we examine the fixed ring of H_n, under a nontrivial group of automorphisms of H_n, and show that it is not isomorphic to H_n itself. That is, the homogenized n^{th} Weyl Poisson algebra is rigid.   (Received September 14, 2020)

1163-16-1171   **Yevgenia Kashina*** (`ykashina@depaul.edu`) and **Yorck Sommerhäuser**
                    (`sommerh@mun.ca`). *On Biproducts and Extensions.*

In this talk, we will discuss the biproducts of certain cocommuative Yetter-Drinfel'd Hopf algebras from the point of view of extensions. There are exactly four different ways to write them as extensions, which will be

described in the talk. Two of these extensions are abelian, but not cocentral. The other two extensions are not abelian and involve a nontrivial 16-dimensional Hopf algebra. In particular, these Hopf algebras cannot be constructed as extensions of a group of order 2 by a large commutative Hopf algebra.   (Received September 14, 2020)

1163-16-1205   **Siu-Hung Ng*** (rng@math.lsu.edu). *On Hopf algebras of prime dimensions.*
A Hopf algebra of prime dimension p over an algebraically closed field $\Bbbk$ of characteristic zero was proven to be isomorphic to a group algebra by Zhu. The same result was established by Etingof and Gelaki when the characteristic q of $\Bbbk$ is greater than p. However, if $\Bbbk$ is of characteristic p, there are three isomorphism classes of Hopf algebras of dimension p. It is more surprising that the technique for the classification of Hopf algebras of dimension pq over $\mathbb{C}$ reincarnated in the classification of Hopf algebras of dimension p over $\Bbbk$ of characteristic q when $p < 4q$. In this talk, we discuss some background and approach of this result. The talk is based on a joint work with Xingting Wang.   (Received September 15, 2020)

1163-16-1208   **Georgia Benkart**, **Rekha Biswal**, **Ellen Kirkman**, **Van Nguyen** and **Jieru Zhu***,
                jieruzhu699@gmail.com. *Double centralizer properties for the Drinfeld double of the Taft
                algebras.*
The Drinfeld double of the taft algebra D_n, whose ground field contains n-th roots of unity, has a list of 2-dimensional irreducible modules. For each of such module V, we show that there is a well-defined action of the Temperley-Lieb algebra TL_k on the k-fold tensor product of V, and this action commutes with that of D_n. We further establish that when V is self-dual and when $kleq2(n-1)$, the centralizer algebra $operatornameEnd_{D_n}(V^{otimesk})$ is isomorphic to TL_k. Our inductive argument uses a rank function on the TL diagrams, which is compatible with the nesting function introduced by Russell-Tymoczko. This is joint work with Georgia Benkart, Rekha Biswal, Ellen Kirkman and Van Nguyen.   (Received September 15, 2020)

1163-16-1220   **Mátyás Domokos*** (domokos.matyas@renyi.hu), Alfréd Rényi Institute of Mathematics,
                Reáltanoda utca 13-15., Budapest, 1053, Hungary. *Computing in the ring of generic 3×3
                skew-symmetric matrices.*
The ring of generic $n \times n$ skew-symmetric matrices is a noncommutative associative algebra. It appears naturally in the study of polynomial identities of representations of simple Lie algebras. In the special case $n = 3$ a precise combinatorial description of this ring is possible, thanks to its close relation to a commutative ring, whose structure is well known by classical invariant theory.

The talk is based on joint results with Vesselin Drensky.   (Received September 15, 2020)

1163-16-1255   **Andrew B. Conner*** (abc12@stmarys-ca.edu) and **Peter D. Goetz**. *Noncommutative
                Projective Geometry of Certain Twisted Tensor Products.* Preliminary report.
Let T denote the free associative k-algebra on $n + 1$ generators of degree 1. Let $A = T/I$ be the quotient by a finitely-generated, homogeneous ideal I. For $d \geq 1$, let $Z_d \subset (\mathbb{P}^n)^{\times d}$ be the scheme of common zeros of elements of I_d, viewed as functions $(T_1^*)^{\otimes d} \to k$. From the geometric data of the schemes Z_d, one can define a ring structure on $B = \bigoplus_d H^0(Z_d, i^*\mathcal{O}_{(\mathbb{P}^n)^{\times d}}(1))$ where $i : Z_d \to (\mathbb{P}^n)^{\times d}$ is the inclusion.

If A is an Artin-Schelter (AS) regular algebra on three generators, then $Z_d \cong Z_2$ for all $d \geq 2$, and Z_2 is the graph of an automorphism σ on a scheme X. In this case, B is isomorphic to a twisted homogeneous coordinate ring on the data $(X, \sigma, \mathcal{L})$ where $\mathcal{L}$ is an invertible sheaf. If A is not AS-regular, the sequence $\{Z_d\}$ need not stabilize.

In this talk we describe the schemes Z_d, some of their geometric properties, and the associated ring structure of B in the case where A is a quadratic twisted tensor product of $k[x, y]$ and $k[z]$. Such twisted tensor products were recently classified by the authors, and the classification includes both AS-regular and non-noetherian examples. (Received September 15, 2020)

1163-16-1419   **Susan Montgomery*** (smontgom@usc.edu), Department of Mathematics, KAP 104,
                University of Southern California, 3620 S. Vermont Ave, Los Angeles, CA 90089. *Actions of
                pointed Hopf algebras on matrix rings.*
Let H be a finite dimensional pointed Hopf algebra with an abelian group G of group-like elements, over a field k which contains all the nth roots of 1, for n =$|G|$. We determine all possible actions of H on matrices $M_m(k)$.

Our techniques use the classification of group gradings of matrices by Bahturin, Sehgal, and Zaicev.

This work is joint with Yuri Bahturin.   (Received September 15, 2020)

1163-16-1454   **Zachary K Cline*** (`z.cline@bucknell.edu`) and **Jason Gaddis** (`gaddisj@miamioh.edu`). *Actions of quantum linear spaces on quantum algebras.*

Some noncommutative algebras exhibit few classical symmetries. That is, their automorphism group is small and/or uninteresting. This leads us to study the quantum symmetry of such algebras, i.e. actions of Hopf algebras on them. Here, we classify linear actions of quantum linear spaces on quantum affine spaces and quantum matrix algebras, with auxiliary results for actions of quantum linear spaces on quantum exterior algebras, quantized Weyl algebras, and quantum general and special linear groups. (The topic of this talk is subject to change.) (Received September 15, 2020)

1163-16-1530   **Peter Goetz**, **Ellen Kirkman**, **W. Frank Moore*** (`moorewf@wfu.edu`) and **Kent Vashaw**. *Artin-Schelter Regular Algebras From Dual Reflection Groups.*

Let G be a group that coacts on an Artin-Schelter regular algebra A homogeneously and inner-faithfully. When the identity component A_e is also Artin-Schelter regular, providing a generalization of the Shephard-Todd-Chevalley Theorem, we say that G is a dual reflection group for A. We provide two examples of dual reflection groups of order 16, and study properties of the associated Artin-Schelter regular algebras.   (Received September 15, 2020)

1163-16-1532   **Naomi Krawzik*** (`krawzik@shsu.edu`). *Drinfeld Hecke for the symmetric group acting in positive characteristic.*

In the case where the underlying field is zero, deformations of skew group algebras have been well studied, and every Lusztig graded Hecke algebra is isomorphic (as filtered algebras) to a Drinfeld Hecke algebra. However, new algebras arise in the modular case, specifically when the symmetric group acts on a polynomial ring in positive characteristic. We present a classification of the resulting graded Hecke algebras using PBW conditions. (Received September 15, 2020)

1163-16-1534   **J T Hartwig*** (`jth@iastate.edu`). *Modules over orders in smash products.*

For an integral domain Λ with fraction field L, we study a class of noncommutative Λ-orders F in a smash product of L by a Hopf algebra. Specifically we give a sufficient condition for there to be only finitely many isoclasses of simple F-modules that are locally finite for Λ and are supported on a given maximal ideal of Λ. This generalizes a "finiteness of fibers" theorem of Futorny and Ovsienko for Galois orders. We point out some connections to Gelfand-Tsetlin theory for $\mathfrak{gl}_n$, Hopf Galois extensions, Cherednik algebras, and noncommutative Kleinian singularities.   (Received September 15, 2020)

1163-16-1596   **Bach Nguyen***, bnguy38math@gmail.com, and **Martin Lorenz** and **Ramy Yammine**. *Representations of Hopf Algebra and noncommutative prime spectra.*

Let H be a Hopf algebra and A be a left H-module algebra. Then the spectrum of A admits a stratification with its strata indexed by "H-prime" ideals of A. Using this stratification to study A was initiated by K. Goodearl and E. Letzter and then M. Lorenz, where they considered group actions. In this talk, we will provide a description of the H-strata for any A when H is cocommutative. This is a joint work with M. Lorenz and R. Yammine. (Received September 15, 2020)

17 ▶ *Nonassociative rings and algebras*

1163-17-412   **Tolulope N Oke***, Department of Mathematics, Texas A&M University, College Station, TX 77840. *Gerstenhaber bracket on Hochschild cohomology of Koszul algebras using homotopy liftings.*

We present the Gerstenhaber algebra structure on Hochschild cohomology of Koszul algebras defined by quivers and relations using the idea of homotopy liftings. We present explicit examples of homotopy lifting maps for degree 1 and 2 cocycles. For Koszul algebras defined by quivers and relations, there is a natural way of constructing their minimal (graded) projective resolution over the enveloping algebra. This resolution was shown to have a comultiplicative structure. Our presentation involves the use of the resolution and the comultiplicative structure on it.   (Received September 05, 2020)

1163-17-1082   **Juan J. Villarreal*** (`jjvillarrealm@vcu.edu`), 18310 Wisset Terrace, Hagerstown, MD 21740, and **Bojko Bakalov**. *Logarithmic vertex algebras.*

In this work we study and define logarithmic vertex algebras. In particular, we generalized some standard results in the theory of vertex algebras for logarithmic vertex algebras. And, we give different motivations that lead to logarithmic vertex algebras.   (Received September 14, 2020)

1163-17-1134   **Lilit Martirosyan*** (`martirosyanl@uncw.edu`), Department of Mathematics and
Statistics, Osprey Hall 2007G, 601 South College Road, Wilmington, NC 28403-5970. *Braid
rigidity for path algebras.*
Path algebras are a convenient way of describing decompositions of tensor powers of an object in a tensor
category. If the category is braided, one obtains representations of the braid groups B_n for all $n \in \mathbb{N}$. We
say that such representations are rigid if they are determined by the path algebra and the representations of
B_2. We show that besides the known classical cases also the braid representations for the path algebra for the
7-dimensional representation of G_2 satisfies the rigidity condition, provided B_3 generates $\text{End}(V^{\otimes 3})$. We obtain
a complete classification of ribbon tensor categories with the fusion rules of $\mathfrak{g}(G_2)$ if this condition is satisfied.
(Received September 14, 2020)

1163-17-1254   **Dwight Anderson Williams II*** (`dwight@iastate.edu`). *Relations, parity, and super
representation theory.*
Maps can tell us how to get from one place to another; maps can also encode various types of information.
In the spirit of (super) representation theory, the maps in this talk provide new ways to consider the complex
orthosymplectic Lie superalgebras. That is, we have new families of infinite-dimensional representations of
$\mathfrak{osp}(1|2n)$. Keys to this result include using super differential operators to describe our maps, more precisely,
homomorphisms of associative superalgebras, and being mindful of both relations and parity throughout explicit
computation. Come for the wonder—stay for the calculus! (Received September 15, 2020)

1163-17-1275   **Bojko Bakalov** and **Samantha Kirk***, 2108 SAS Hall, Box 8205, Raleigh, NC. *A
Construction of Representations of Toroidal Lie Algebras via Vertex Algebras.*
Given a simple finite-dimensional Lie algebra and an automorphism of finite order, one can construct a twisted
toroidal Lie algebra. Similar to twisted affine Lie algebras, which are well-studied in the literature, we can create
representations of twisted toroidal Lie algebras with the help of vertex algebras. In this talk, I will discuss
twisted modules of vertex algebras and I will show how representations of twisted toroidal Lie algebras can be
constructed from such twisted modules. (Received September 15, 2020)

1163-17-1447   **Darlayne Addabbo*** (`addabbo@math.arizona.edu`) and **Katrina Barron**
(`kbarron@nd.edu`). *Higher level Zhu algebras for Vertex Operator Algebras.* Preliminary
report.
We will discuss the structure of the level $n \geq 2$ Zhu algebras for the Heisenberg vertex operator algebra as well
as techniques used in determining the structure of higher level Zhu algebras in general. We will give an example
to clarify the necessity of certain conditions in defining higher level Zhu algebras, and discuss applications of
these results. (Received September 15, 2020)

1163-17-1579   **Katherine A Maxwell*** (`maxwe204@umn.edu`). *The Neveu-Schwarz algebra and the super
Krichever map.*
In 1986, Manin conjectured that representations of the Neveu-Schwarz (NS) algebra must be related to the
moduli space of SUSY curves. We explain the compatibility of these representations and the super Krichever
map, which maps the moduli space of SUSY curves into the super Sato grassmannian. We find that the Lie
superalgebra of global superconformal vector fields on an affine SUSY curve is perfect, and that the NS algebra
acts on the super Mumford isomorphism with zero central charge. Also discussed is a NS uniformization of the
moduli space of SUSY curves. (Received September 15, 2020)

18 ▶ *Category theory; homological algebra*

1163-18-219   **Joshua R Edge*** (`edgej@denison.edu`). *Classification of symmetric trivalent categories.*
Building on the work of Morrison, Peters, and Snyder, we classify all non-trivial planar algebras of (perhaps
non-planar) trivalent graphs coming from symmetric trivalent categories. For each of these planar algebras,
we also classify when the sub-planar algebra generated by only the trivalent vertex is braided. An interesting
example of this arises from Deligne's S_t. (Received August 28, 2020)

1163-18-230   **Nicolle Gonzalez*** (`nicolle@math.ucla.edu`) and **Matt Hogancamp**. *A skein theoretic
Carlsson-Mellit algebra.* Preliminary report.
The Carlsson-Mellit algebra, or $A_{q,t}$ algebra, originally arose in the proof of the celebrated Shuffle conjecture.
This algebra, built from Hecke algebra generators and a family of raising and lowering operators, has a par-
ticularly interesting representation, known as the polynomial representation, on which its action is given by

complicated plethystic operations. In this talk I will discuss how this algebra (specialized at $t = q^{-1}$) and its polynomial representation can be formulated skein theoretically, as certain braids on an annulus, and consequently categorified. This is joint work with Matt Hogancamp.   (Received August 29, 2020)

1163-18-489   **Siu-Hung Ng**, **Yilong Wang*** (`yilongwang@lsu.edu`) and **Qing Zhang**. *On the classification of transitive modular categories.*
In this talk, we will introduce transitive modular categories. We will explain how we use the representation theory of SL(2, Z/nZ) to obtain a full classification of these modular categories.   (Received September 08, 2020)

1163-18-574   **Roberto Hernandez Palomares*** (`hernandezpalomares.1@osu.edu`). *Classifying unitary modules for generalized TLJ ∗-2-categories.*
Generalized Temperley–Lieb–Jones (TLJ) ∗-2-categories associated to weighted bidirected graphs were introduced by Morrison and Walker. We introduce unitary modules for these generalized TLJ ∗-2-categories as strong ∗-pseudofunctors into the ∗-2-category of separable bigraded Hilbert spaces. We classify these modules up to ∗-equivalence in terms of weighted bi-directed Γ-fair and balanced graphs in the spirit of Yamagami's classification of fiber functors on TLJ categories and DeCommer and Yamashita's classification of unitary modules for $\mathrm{Rep}(SU_q(2))$. Based on joint work with G. Ferrer.   (Received September 09, 2020)

1163-18-670   **Vasily A Dolgushev*** (`vald@temple.edu`), 1805 N. Broad St., Wachman Hall, Rm 638, Philadelphia, PA 19122. *Is there a "wormhole" to elements of the Grothendieck-Teichmueller Lie algebra of high degree?* Preliminary report.
The famous Deligne-Drinfeld-Ihara conjecture states that Deligne-Drinfeld (DD) elements generate the Grothendieck-Teichmueller Lie algebra grt. Modern computer algebra systems are not very helpful for verifying this conjecture because the sizes of matrices for the equations from the definition of grt grow very fast. So far the conjecture was verified (by numerical experiments) up to degree 16 and there is some evidence for degrees < 30. In my talk, I will recall grt and its "action" on Poisson structures. I will speculate about a possibility to use this action to find a "wormhole" to elements of grt of high degree.   (Received September 11, 2020)

1163-18-835   **A Czenky*** (`aczenky@uoregon.edu`) and **J Plavnik**. *On odd-dimensional modular tensor categories.*
In this talk we will discuss odd-dimensional modular tensor categories and maximally non-self dual (MNSD) modular tensor categories of low rank. We will give lower bounds for the ranks of modular tensor categories in terms of the rank of the adjoint subcategory and the order of the group of invertible objects. As an application of these results, we will see that MNSD modular tensor categories of ranks 13 and 15 are pointed. In addition, we will show that MNSD tensor categories of ranks 17, 19, 21 and 23 are either pointed or perfect. This talk is based on arXiv:2007.01477.   (Received September 13, 2020)

1163-18-854   **Kent B Vashaw*** (`kvasha1@lsu.edu`). *Balmer spectra and Drinfeld centers.* Preliminary report.
The Balmer spectrum of a monoidal triangulated category is an important geometric invariant which can be used in many cases to obtain a classification of the thick ideals of the category. Many examples of monoidal triangulated categories arise as stable module categories of finite-dimensional Hopf algebras. We will describe the relationship between the Balmer spectra of stable module categories associated to a Hopf algebra and its Drinfeld double. In particular, we show that there always exists a continuous map between them, and describe how this map relates to classifications of thick ideals.   (Received September 13, 2020)

1163-18-989   **Cain Edie-Michell*** (`cain.edie-michell@vanderbilt.edu`). *Symmetries of Modular Categories, and Quantum Subgroups.*
Since the problem was introduced by Ocneanu in the late 2000's, it has been a long-standing open problem to completely classify the quantum subgroups of the simple Lie algebras. This classification problem has received considerable attention, due to the correspondence between these quantum subgroups, and the extensions of WZW models in physics. A rich source of quantum subgroups can be constructed via symmetries of certain modular tensor categories constructed from Lie algebras. In this talk I will describe the construction of a large class of these symmetries. Many exceptional examples are found, which give rise to infinite families of new exceptional quantum subgroups.   (Received September 14, 2020)

1163-18-1026   **Siu-Hung Ng**, **Eric Rowell**, **Yilong Wang** and **Qing Zhang***, zhan4169@purdue.edu.
Higher central charges and Witt groups.
We introduce the notion of the signature of a braided fusion category and prove its invariance under Witt equivalence. Moreover, the higher central charges of a pseudounitary modular category can be expressed in terms of its signature. We use this fact to prove that the Ising modular categories have infinitely many square roots in the Witt group. This result is further applied to prove a conjecture of Davydov-Nikshych-Ostrik on the super-Witt group. (Received September 14, 2020)

1163-18-1199   **Petter A. Bergh**, **Julia Plavnik*** (jplavnik@iu.edu) and **Sarah Witherspoon**.
Support varieties for finite tensor categories.
In this talk, we will consider the support variety theory for finite tensor categories. We will start by presenting the basic definitions and properties of these varieties. Then, we will discuss some deeper results involving complexity, realization, and connectedness of the varieties. If time allows, we will analyze some examples. (Received September 15, 2020)

1163-18-1262   **Eric Carson ROwell*** (rowell@math.tamu.edu), Mathematics Department, Texas A&M
University, College Station, TX 77843, and **Colleen Delaney**, **Cesar Galindo**, **Julia
Plavnik** and **Qing Zhang**. *Ribbon zesting of braided fusion categories.*
I will describe a construction of new fusion categories from a given G-graded braided fusion category known as zesting. Zesting fits into the general theory of G-graded extensions and so are subject to the cohomological yin and yang of obstructions and parameterization torsors. On the other hand, zesting has several computational advantages. To name a few: the fusion rules are immediately available, we may easily explore braiding and pivotal structures, and when the resulting category is modular the data can be expressed succinctly in terms of the original data. These ideas will be illustrated with examples. See https://arxiv.org/abs/2005.05544. (Received September 15, 2020)

1163-18-1331   **Colleen Delaney*** (crdelane@iu.edu), **Sung Kim** and **Julia Plavnik**. *Zesting link
invariants goes beyond modular data.*
Following the discovery of modular categories not determined by their modular data there have been several invariants proposed as alternatives, like the Whitehead matrix, Borromean tensor, traces of higher genus mapping class group representations, and many others. This proliferation of invariants which go beyond modular data suggests there may be nothing special about any particular invariant and begs the question of how knot theory can be best applied for classifying modular categories. By realizing Mignard and Schauenburg's modular isotopes as examples of the ribbon zesting construction, we are able to explain the phenomenology of link invariants that go beyond modular data and identify specific knots and links which are powerful for studying modular categories related by zesting. (Received September 15, 2020)

1163-18-1364   **Dmitri Nikshych*** (dmitri.nikshych@unh.edu), Department of Mathematics and
Statistics, University of New Hampshire, Durham, NH 03824. *On minimal extensions of
symmetric fusion categories.* Preliminary report.
A <u>minimal extension</u> of a symmetric fusion category $\mathcal{E}$ is a non-degenerate braided fusion category $\mathcal{C}$ such that $\dim(\mathcal{C}) = \dim(\mathcal{E})^2$ together with an embedding $\mathcal{E} \hookrightarrow \mathcal{C}$. Lan, Kong, and Wen observed that minimal extensions of $\mathcal{E}$ form a group $\mathbf{Mext}(\mathcal{E})$. Minimal extensions of Tannakian categories are well understood. Namely, for a finite group G, one has $\mathbf{Mext}(\mathsf{Rep}(G)) = H^3(G, \mathbb{C}^\times)$ and minimal extensions are the twisted group doubles $\mathsf{Rep}(D^\omega(G))$. An interesting open problem is to compute $\mathbf{Mext}(\mathcal{E})$ for a super-Tannakian category $\mathcal{E}$. In this talk, I will explain how one can use the theory of graded braided extensions to compute this group for a pointed super-Tannakian $\mathcal{E}$. This is based on joint work with Alexei Davydov. (Received September 15, 2020)

1163-18-1371   **Peter Schauenburg*** (peter.schauenburg@u-bourgogne.fr). *Morita invariants of
pointed fusion categories.* Preliminary report.
In principle, the problem as to when two pointed fusion categories are categorically Morita equivalent is completely solved. Results of Gelaki and Naidu tell us exactly how to compute all the pointed fusion categories (i.e. underlying group plus cohomology class) Morita equivalent to a given one (by looking at all the abelian normal subgroups giving rise to a Morita equivalent group-theoretical fusion category that is pointed, and computing its group and cohomology class through explicit formulas). In practice, this may fast turn out to be a difficult or heavy task. In particular, calculations on the level of explicit cocycles (maps depending on several variables in the group) are needed, rather than arguments on the abstract level of cohomology groups. We discuss some rather simple to use tools that allow in some cases to distinguish (most of the time pointed) fusion categories up to Morita equivalence. (Received September 15, 2020)

1163-18-1627   **Henry Tucker*** (`htucker@ucr.edu`). *Frobenius-Schur indicators for some families of quadratic fusion categories.* Preliminary report.

Quadratic categories are the fusion categories with a unique non-trivial orbit under the tensor product action of the group of invertible objects. Familiar examples are the near-group fusion categories (with exactly one non-invertible object) and the generalized Haagerup fusion categories (with one non-invertible object for each invertible object). Frobenius-Schur indicators are an important invariant of fusion categories generalized from the theory of finite group representations. We compute these indicators for a class of group-theoretical quadratic categories and study the relationship of the indicators with the structure of the group of invertible objects. (Received September 15, 2020)

19 ▶ K-theory

1163-19-735   **Sophie Frisch*** (`frisch@math.tugraz.at`), Institut für Analysis und Zahlentheorie, Technische Universität Graz, Kopernikusgasse 24, 8010 Graz, Austria. *The stable rank of the ring of integer-valued polynomials.* Preliminary report.

The stable rank of a ring R, denoted by s.r.(R), is an important invariant in algebraic K-theory with the property that, when s.r.$(R) = n$, then every invertible matrix over R can be reduced to an $n \times n$ matrix by elementary row and column operations. For commutative R, s.r.$(R) - 1$ compares to $\dim(R)$, the Krull dimension. In fact, for commutative Noetherian R, s.r.$(R) - 1 \leq \dim(R)$.

The ring of integer-valued polynomials $\mathrm{Int}(\mathbb{Z})$, a non-Noetherian 2-dimensional Prüfer domain, consists of those polynomials in $\mathbb{Q}[x]$ that map every integer to an integer:

$$\mathrm{Int}(\mathbb{Z}) = \{f \in \mathbb{Q}[x] \mid f(\mathbb{Z}) \subseteq \mathbb{Z}\}.$$

Now $\mathrm{Int}(\mathbb{Z})$, lying between $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$, behaves like a polynomial ring over a field in some respects (interpolation, for instance, and Nullstellensatz properties), and like $\mathbb{Z}[x]$ in others (for instance, Krull dimension).

We show that, with respect to the stable rank, $\mathrm{Int}(\mathbb{Z})$ behaves like $\mathbb{Q}[x]$, namely, s.r.$(\mathrm{Int}(\mathbb{Z})) = 2$.   (Received September 12, 2020)

20 ▶ Group theory and generalizations

1163-20-403   **Alexandru Chirvasitu*** (`achirvas@buffalo.edu`), 244 Mathematics Building, University at Buffalo, Buffalo, NY 14260, and **Benjamin Passer**. *On the (non)contractibility of compact quantum groups.*

It is a well known result in algebraic topology that compact topological groups cannot be contractible. We examine this same question in the context of compact quantum groups, i.e. particularly well-behaved non-commutative cosemisimple Hopf algebras mimicking the algebras of continuous functions on classical compact groups. The problem has natural links to topics familiar to geometric group theorists, such as the connection between C^*-algebraic K-theory and K-homology and the Baum-Connes conjecture, and the main result will be a non-contractibility statement in the very particular case when the compact quantum group is the Pontryagin dual of a discrete group meeting certain homological criteria.

(joint w/ Benjamin Passer)   (Received September 05, 2020)

1163-20-684   **Dandrielle C. Lewis*** (`dlewis1@highpoint.edu`), **Maggie Reardon** and **Bridget Lee**. *Centralizer-like Subgroups Associated with the n-Engel Word Inside a Direct Product of Groups.* Preliminary report.

Using the ideas Kappe and Ratchford presented in their article, On Centralizer-like Subgroups Associated with the n-Engel Word, my students and I studied centralizer-like subgroups associated with n-Engel words inside of a direct product of two groups. In this talk, we will explore centralizer-like subgroups and right n-Engel words of groups and a direct product of groups.   (Received September 11, 2020)

1163-20-879   **Vidya Venkateswaran*** (`vvenkat@idaccr.org`). *Quasi-polynomial representations of double affine Hecke algebras.*

In the 1990's, Cherednik introduced a Y-induced, cyclic representation of the double affine Hecke algebra on the space of polynomials, the so-called basic representation. In addition to its importance in the representation theory of DAHA, this representation plays an integral role in the theory of Macdonald polynomials.

In this talk, we present a generalization of this picture. We study a class of Y-induced cyclic representations of DAHA, and show that they admit explicit realizations on the space of quasi-polynomials. We establish several

properties about these representations, which parallel the basic representation, and we define a new family of quasi-polynomials which generalize Macdonald polynomials. We will also discuss some connections to recent work on Weyl group multiple Dirichlet series and metaplectic Whittaker functions. This talk is based on joint works with Siddhartha Sahi and Jasper Stokman.   (Received September 13, 2020)

1163-20-894   **Kasia Jankiewicz*** (`kasia@math.uchicago.edu`) and **Kevin Schreve**
                 (`kschreve@math.uchicago.edu`). *A generalization of the Tits conjecture on right-angled
                 Artin subgroups of Artin groups.*
This is a joint work with Kevin Schreve. The Tits conjecture, proved by Crisp and Paris, states that the subgroup of an Artin group generated by powers of the standard generators is the 'obvious' right-angled Artin group. We generalize for certain families of Artin groups, and show that the subgroup generated by a larger collection of naturally distinguished elements is a RAAG.   (Received September 13, 2020)

1163-20-943   **Ricardo Jesus Ramos*** (`rjesusrc@gmail.com`), Avenida Venezuela 5197, San Miguel,
                 Lima 15086, Peru. *On $S(1,n)$ supergroups.* Preliminary report.
We describe a character over the super group $S(1,n)$, with kernel its reduced part. Using this, we describe properties for super curves with a trivial Berezinian bundle.   (Received September 14, 2020)

1163-20-1333   **Edgar A. Bering*** (`edgar.bering@temple.edu`), **Yulan Qing** and **Derrick
                 Wigglesworth**. *An algorithm to decide if a free-by-cyclic group is a 3–manifold group.*
                 Preliminary report.
In 1992, Bestvina and Handel gave an algorithm to decide if an irreducible outer automorphism of a free group could be realized as a homeomorphism of a punctured surface. In turn, this allows one to recognize some 3–manifold groups. In recent joint work, Y. Qing, D. Wigglesworth, and I give an algorithm to decide, in general, whether an outer automorphism of a free group can be realized by a surface homeomorphism. As an application we obtain an algorithm to decide whether a given free-by-cyclic group is a 3–manifold group. (Received September 15, 2020)

1163-20-1360   **Anisah N. Nu'Man*** (`anisah.numan@spelman.edu`). *Intrinsic Tame Filling Functions for
                 Groups.*
Intrinsic tame filling functions for finitely presented groups are a quasi-isometry invariant refinement of the intrinsic diameter (isodiametric) function. In contrast to the intrinsic diameter function, it is unknown if every pair of a finitely presented group G with presentation $\mathcal{P}$, $(G, \mathcal{P})$, has a finite-valued intrinsic tame filling function. Within this talk, I will discuss work done in determining intrinsic tame filling functions for a pair $(G, \mathcal{P})$. (Received September 15, 2020)

22 ▶ *Topological groups, Lie groups*

1163-22-272   **Christian Rosendal***, Mathematics, Statistics and Computer Science, University of
                 Illinois at Chicago, Chicago, IL 60607. *Groups with bounded geometry.*
Topological and, in particular, Polish groups with bounded geometry form a near perfect geometric generalisation of the locally compact second countable groups. With outset in the recently developed framework for geometric group theory for general topological groups, we shall present a number of results about this specific subclass of Polish groups with a particular focus on coarse embeddings, equivalences and topological couplings.   (Received August 31, 2020)

1163-22-1188   **Camen Galaz-Garcia***, carmengg@math.ucsb.edu. *Zariski dense surface subgroups in
                 $SL(n, \mathbb{Q})$.*
In this talk I will explain how to construct a path $\rho_t \colon \pi_1(S) \to SL(n, \mathbb{R})$ for odd n of discrete, faithful and Zariski dense representations of a surface group such that $\rho_t(\pi_1(S)) \subset SL(n, \mathbb{Q})$ for every $t \in \mathbb{Q}$. As exemplified in the work of Long, Reid and Thistlethwaite, constructing Zariski dense surface subgroups in $SL(n, \mathbb{Q})$ has close connections to finding *thin groups*, which are infinite index subgroups of a lattice in $SL(n, \mathbb{R})$ that are Zariski dense.   (Received September 15, 2020)

26 ► *Real functions*

1163-26-57   **George A Anastassiou*** (ganastss@memphis.edu), University of Memphis, Department
                    of Mathematical Sciences, Memphis, TN 38152. *E.R. Love type left fractional integral
                    inequalities.*
Here first we derive a general reverse Minkowski integral inequality. Then motivated by the work of E.R.
Love (1985) on integral inequali- ties we produce general reverse and direct integral inequalities. We apply
these to ordinary and left fractional integral inequalities. The last in- volve ordinary derivatives, left Riemann-
Liouville fractional integrals, left Caputo fractional derivatives, and left generalized fractional derivatives. These
inequalities are of Opial type.   (Received August 03, 2020)

1163-26-250   **Paul Eloe*** (peloe1@udayton.edu). *The method of quasilinearization applied to boundary
                    value problems for fractional differential equations.*
We apply the method of quasilinearization to several families of boundary value problems for fractional differential
equations. We shall consider boundary value problems for both Riemann–Liouville type fractional equations and
Caputo type fractional equations. The key issues to apply the particular algorithm we employ are that i) the
boundary value problem admits a unique solution and ii) an upper solution is, in fact, bigger than a lower
solution. We shall outline the quasilinearization algorithm; however, the primary purpose of this presentation is
to address issue ii) for each of the Riemann–Liouville fractional derivative and the Caputo fractional derivative.
(Received August 30, 2020)

1163-26-548   **Katharine Ott*** (kott@bates.edu) and **Russell Brown**. *Estimates for Brascamp-Lieb
                    forms in L^p-spaces with power weights.*
We establish a set of necessary conditions and a set of sufficient conditions for boundedness of a family of
multi-linear forms in Lorentz spaces and L^p-spaces with power weights.   (Received September 09, 2020)

1163-26-938   **Alan Krinik*** (ackrinik@cpp.edu), **Gerardo Rubino**, **Hubertus von Bremen**,
                    Jeremy J. Lin, **Thuy Vu Dieu Lu**, **Mark Dela**, **David Perez**, **David Beecher** and
                    Weizhong Wong. *Explicit expressions for Mk for k = 2,3,4, etc. and exp(Mt) for certain
                    matrices M.*
We are interested in explicit eigenvalue and eigenvector formulas for different families of matrices. In particular,
we seek exact eigenvalue and eigenvector formulas that scale up as the dimension of our matrices, M, increase.
For simplicity, we usually assume that our n x n matrices, M, have real entries and distinct eigenvalues. We seek
exact expressions for Mk for k = 2,3,4, etc. and exp(Mt).

Our applications are mainly determining the probability of sample paths of Markov (or sub Markov) chains
and processes under different conditions. This includes finding the transient probability distributions of certain
types of Markov chains (or processes) and finding the solution of the generalized ballot box problem.

We start with tridiagonal, Toeplitz matrices and generalize to tridiagonal matrices having alternating entries
along the sub and super diagonals. Using duality theory, we are able to extend our results to a family of
non-tridiagonal n x n matrices having catastrophe-like matrix entries. Connections to nonnegative matrices are
discussed as time allows.   (Received September 14, 2020)

28 ► *Measure and integration*

1163-28-760   **Tepper L. Gill*** (tgill@howard.edu). *A family of Banach spaces over $\mathbb{R}_I^\infty$.*
In recent work, the topology of $\mathbb{R}^\infty$ was replaced with a new topology denoted by $\mathbb{R}_I^\infty$. This space was then used
to construct Lebesgue measure on $\mathbb{R}_I^\infty$ in a manner that, is no more difficult than the same construction on $\mathbb{R}^n$.
More important, a new class of separable Banach spaces $KS^p[\mathbb{R}^n]$, $1 \le p \le \infty$, for the HK-integrable functions
were introduced. These spaces contain the L^p spaces and the Schwartz space of distributions as continuous dense
embeddings.

In this talk I will extend the work on $KS^p[\mathbb{R}^n]$ to $KS^p[\mathbb{R}_I^\infty]$.   (Received September 12, 2020)

1163-28-1368   **Saurabh Verma*** (`saurabh331146@gmail.com`), Department of Mathematics, Indian
Institute of Technology Delhi, Hauz Khas, New Delhi, Delhi 110016, India, and **Mrinal K.
Roychowdhury** (`mrinal.roychowdhury@utrgv.edu`), School of Mathematical and
Statistical Scienc, The University of Texas Rio Grande Valley, 1201 West University Drive,
Edinburg, TX 78539-2999. *Concerning quantization for probability measures in complete
metric spaces.*

The theory of quantization for probability measures mu supported on a compact set in $mathbbR^n$ has received considerable attention in the literature. The existing literature on quantization concentrates primarily on an invariant measure mu associated with an iterated function system. Building on the literature, this contribution aims to extend the theory of quantization in a complete metric space. In particular, this provides a formula for quantization dimension of an invariant measure mu associated with an iterated function system consisting of a finite number of contractive infinitesimal similitudes in a complete metric space. Not unexpectedly, this constitutes a generalization of the known result on quantization dimension of a self-similar measure in the Euclidean space. This note also records the continuous dependence of quantization dimension of mu on the parameters involved in its definition.   (Received September 15, 2020)

1163-28-1611   **Timothy I Myers*** (`timyers@howard.edu`), 110 Camrose Avenue, Baltimore, MD 21225.
A Constructive Definition of the Fourier Transform on a Separable Banach Space.

Gill and Myers proved that every separable Banach space, denoted $\mathcal{B}$, has an isomorphic, isometric embedding in $\mathbb{R}^\infty = \mathbb{R} \times \mathbb{R} \times \cdots$. They used this result and a method due to Yamasaki to construct a sigma-finite Lebesgue measure $\lambda_{\mathcal{B}}$ for $\mathcal{B}$ and defined the associated integral $\int_{\mathcal{B}} \cdot \, d\lambda_{\mathcal{B}}$ in a way that equals a limit of finite-dimensional Lebesgue integrals.

The objective of this talk is to apply this theory to developing a constructive definition of the Fourier transform on $L^1[\mathcal{B}]$. Our approach is constructive in the sense that this Fourier transform is defined as an integral on $\mathcal{B}$, which, by the aforementioned definition, equals a limit of Lebesgue integrals on Euclidean space as the dimension $n \to \infty$. Thus with this theory we may evaluate infinite-dimensional quantities, such as the Fourier transform on $\mathcal{B}$, by means of finite-dimensional approximation.   (Received September 15, 2020)

30 ► *Functions of a complex variable*

1163-30-137   **Eddy Kwessi*** (`ekwessi@trinity.edu`), 1 Trinity Place, San Antonio, TX 78212. *On the
equivalence between Weak BMO and the space of derivatives of the Zygmund class.*
Preliminary report.

In this presentation, we will discuss the space of function of weak bounded mean oscillation BMO^w. In particular , we will show that this space is the dual space of the so-called special atom space B^1, whose dual space was already known to be the space of derivative of functions (in the sense of distribution) belonging to the Zygmund class Λ_*. We show in particular this a proof that the Hardy space H^1 strictly contains the space B^1.   (Received August 20, 2020)

1163-30-401   **Javad Mashreghi*** (`javad.mashreghi@mat.ulaval.ca`), 1960 Boul. Laurier, Quebec, QC
G1S 1M8, Canada. *Carleson Measures for the Dirichlet Space.*

We show that a finite measure μ on the unit disk is a Carleson measure for the Dirichlet space if it satisfies the Carleson one-box condition $\mu(S(I)) = O(\phi(|I|))$, where $\phi : (0, 2\pi) \to (0, \infty)$ is an increasing function such that $\int_0^{2\pi} \phi(x)/x \, dx < \infty$. We also show that the integral condition on ϕ is sharp.
Joint work with O. El-Fallah, K. Kellay, T. Ransford.   (Received September 04, 2020)

1163-30-421   **Michael Antoine Lorenzo Bush*** (`mikebush@udel.edu`), **Constanze Liaw** and **Robert
T.W. Martin**. *Applications of Matrix-Valued Clark Measures to Differential Operators.*
Preliminary report.

The theory of finite rank perturbations (FRP) allows for the determination of spectral information for broad classes of operators using the tools of analytic function theory. This work applies FRP to several examples of differential operators in order to investigate the spectral measures corresponding to those differential operators. In particular, the support and weights of the Clark measures are computed via the connection between matrix-valued contractive analytic functions and matrix-valued nonnegative measures through the Herglotz Representation Theorem.   (Received September 06, 2020)

1163-30-429   **Andreas Hartmann*** (`andreas.hartmann@math.u-bordeaux.fr`), Institut de Mathématiques de Bordeaux, Université de Bordeaux, 351 cours de la Libération, 33405 Talence, France, and **Dantouma Kamissoko**, **Siaka Konate** and **Marcu-Antone Orsoni**. *Dominating sets in Bergman spaces and sampling constants.*
Given a Banach space of holomorphic functions X on a domain Ω, a dominating set E is a subset of Ω which allows to recover any arbitrary function of X from its values on E with norm control. In view of the applications, the space which had attracted the most interest in the past was the Paley-Wiener space for which it was shown that relatively dense sets are dominating. Relative density turns out to characterize domination also in other prominent spaces like for instance Fock and Bergman spaces.

A natural question is to establish a link between the density of E and the norm estimates. Indeed, this is important in applications when one has to decide on the trade-off between the cost of the sampling and the accuracy of the estimates. In the early 2000's Kovrijkine gave precise and optimal estimates for the Paley-Wiener space. His method, involving Bernstein inequalities, was exploited later on in different other spaces like for instance model spaces.

In this talk I will present estimates for sampling constants in Bergman spaces. The proof is in the spirit of Kovrijkine's, but does not require Bernstein inequalities simplfying thereby the method. Another ingredient are Remez type inequalities by Andrievskii-Ruscheweyh on planar domains.   (Received September 08, 2020)

1163-30-487   **Catherine Beneteau*** (`cbenetea@usf.edu`). *A survey of optimal approximants and connections to digital filter theory.* Preliminary report.
In this talk, I will discuss polynomials that approximate inverses of functions in analytic function spaces such as the Hardy, Bergman, and Dirichlet spaces. Various aspects of these polynomials have been studied over the last ten years in relation to their growth, zeros, and universality properties, among others. It turns out that these polynomials were also of interest in the 70s and 80s in the engineering literature in connection with the development of ideal digital filters. We will examine this connection and discuss some open problems. This work is joint with R. Centner.   (Received September 08, 2020)

1163-30-535   **Andrea Pinamonti**, **Gareth Speight** and **Scott Zimmerman*** (`zimmerman.416@osu.edu`). *Whitney extensions for curves in the Heisenberg group.*
Sub-Riemannian manifolds are mathematical models of settings in which motion is restricted. The simplest non-trivial example of a sub-Riemannian manifold is the Heisenberg group.

I will discuss a version of Whitney's classical extension theorem for smooth, admissible curves in this setting. Whitney provided necessary and sufficient conditions under which a real valued function defined on a compact subset of Euclidean space may be extended to a smooth function defined everywhere. I will provide similar conditions for the extension of smooth curves in the Heisenberg group.   (Received September 08, 2020)

1163-30-848   **Timothy J. Ferguson*** (`tjferguson1@ua.edu`). *Bergman and Szegö projections, Extremal Problems, and Square Functions.*
We study estimates for Hardy space norms of analytic projections. We first find a sufficient condition for the Bergman projection of a function in the unit disc to belong to the Hardy space H^p for $1 < p < \infty$. We apply the result to prove a converse to an extension of Ryabykh's theorem about Hardy space regularity for Bergman space extremal functions. We also prove that the H^q norm of the Szegö projection of $F^{p/2}\overline{F}^{(p/2)-1}$ cannot be too small if F is analytic, for certain values of p and q. We apply this to show that the best analytic approximation in L^p of a function in both L^p and L^q will also lie in L^q, for certain values of p and q.   (Received September 13, 2020)

31 ▶ *Potential theory*

1163-31-728   **Willi Freeden*** (`freeden@rhrk.uni-kl.de`), Kaiserslautern, Rhineland, Germany. *Multiscale Decorrelative Gravimetry.*
The lecture highlights arguments that, coming from Mathematics, have fostered the advancement of Geodesy, as well as those that, generated by geodetic problems, have contributed to the enhancement in Mathematics. We particularly focus the attention on inverse problems of Geodesy and multiscale mollifier regularization strategies. Two examples are studied in more detail: (i) Vening Meinesz multiscale surface mollifier regularization to determine locally the Earth's disturbing potential from deflections of vertical, (ii) Newton multiscale volume mollifier regularization of the inverse gravimetry problem to derive locally the density contrast distribution from

functionals of the Newton integral and to detect fine particulars of geological relevance.   (Received September 13, 2020)

32 ▶ *Several complex variables and analytic spaces*

1163-32-288   **Meredith Sargent*** (sargent@uark.edu) and **Alan A Sola**. *Optimal approximants and orthogonal polynomials in several variables.*

In recent years, optimal polynomial approximants have been used to study cyclicity of functions in Dirichlet-type spaces on the complex unit disk, with particular interest being paid to the connection between the location of the zero sets of the optimal approximants and cyclicity, as well as a correspondence between optimal approximants and orthogonal polynomials. In this talk we discuss generalizing the concept of optimal approximants to several variables, including to the cases of Dirichlet-type spaces on the bidisk, and to a scale of Drury-Arveson-type spaces on the ball, as well as some of the inherent complications.   (Received September 01, 2020)

1163-32-801   **Anne-Katrin Gallagher**, **Purvi Gupta*** (purvigupta@iisc.ac.in), **Loredana Lanzani** and **Liz Vivas**. *Holomorphic Hardy Spaces Inherited by Variety-Deleted Domains in $\mathbb{C}^n$.*

In this talk, we will motivate and present a construction of holomorphic Hardy spaces for a class of domains — the so-called variety-deleted domains, which are derived from domains with a prior Hardy space structure. We will show, via certain illuminating examples, how this approach is particularly suited for obtaining new boundary integral representation formulas for holomorphic functions (with certain allowable singularities).

This is joint work with A.-K. Gallagher, L. Lanzani and L. Vivas.   (Received September 13, 2020)

1163-32-1004   **Bingyuan Liu*** (bingyuan.liu@utrgv.edu). *Geometry's Influence in the Analysis of Several Complex Variables.*

The boundary geometry of domains plays an important role in mathematics and the sciences. From its shape, one can predict various phenomena. For example, by examining the physical shapes of continents and their coastlines, scientists discovered that Earth's continents were once connected. In mathematics, the shape of a domain is fundamental to many problems related to complex analysis. In this talk, I will address questions motivated from partial differential equations and complex analysis. We will see how the boundary shape influences those answers in a natural way.   (Received September 14, 2020)

1163-32-1478   **Debraj Chakrabarti** (chakr2d@cmich.edu), 214 Pearce Hall, Mt Pleasant, MI 48859, and **Anirban Dawn*** (dawn1a@cmich.edu), 201A Pearce Hall, Mt Pleasant, MI 48859. *Laurent Series in Spaces of Holomorphic Functions and Cohomologies.*

We consider the Fourier expansion in locally convex spaces admitting a continuous linear action of the torus group. Special cases of this include classical Laurent series, Fourier series and series expansions of Dolbeault cohomologies of Reinhardt domains. We prove a Fejér type summability theorem for such series. We also prove the unconditional convergence of Laurent series of holomorphic functions smooth up to the boundary of Reinhardt domains, and of Dolbeault cohomology elements.   (Received September 15, 2020)

1163-32-1517   **Tyler J Gonzales*** (gonzaltj9215@uwec.edu), Department of Mathematics, Hibbard Humanities Hall 508, 124 Garfield Avenue, Eau Claire, WI 54701, and **Kamryn Spinelli**, **Gabe Udell** and **Henry Bosch**. *CR-embeddability of Quotients of the Rossi Sphere via Spectral Theory.* Preliminary report.

CR manifolds are smooth manifolds which generalize the notion of boundaries of domains in $\mathbb{C}^n$. Many abstract CR manifolds cannot be globally embedded as CR submanifolds of $\mathbb{C}^n$ for any n (Burns and Epstein 1990) but there are few well-known examples of non embeddable CR manifolds. The Rossi sphere, which is defined as the regular sphere $\mathbb{S}^2 \subset \mathbb{C}^2$ endowed with a perturbed CR structure, is the canonical example of a non-embeddable abstract CR manifold. A modern result states that one can detect CR-embeddability for certain CR manifolds by analyzing the bottom of the spectrum of the Kohn Laplacian. We study similar questions for spherical 3-manifolds that are obtained by taking the quotient of the sphere by left actions of finite subgroups of $SU(2)$. Using spectral-theoretic techniques, we prove that the quotient of the Rossi sphere by the antipodal map is CR-embeddable. We further generalize this result, proving that a quotient of the Rossi sphere which can be understood as a lens space $L(p, p-1)$ with a perturbed CR structure is CR-embeddable if and only if p is even. This characterization gives an infinite family of explicit examples of non-embeddable CR manifolds.   (Received September 15, 2020)

1163-32-1522   **Henry Bosch***, 153 Kirkland Mail Center, 95 Dunster Street, Cambridge, MA 02138, and
Tyler Gonzales, **Gabe Udell** and **Kamryn Spinelli**. *A Tauberian approach to an analog of Weyl's Law for the Kohn Laplacian.* Preliminary report.

In Riemannian geometry, Weyl's Law relates the growth of eigenvalues of the Laplace-Beltrami operator to geometric information about the underlying manifold. In particular, the leading coefficient in the asymptotic expansion is proportional to the volume of the manifold. For CR geometry, although no statement quite as simple as Weyl's Law is known, in 1984 Stanton and Tartakoff obtained an analog of Weyl's Law for eigenvalues of the Kohn Laplacian acting on $(0, q)$-forms ($q \geq 1$) on hypersurfaces in $\mathbb{C}^n$. A recent paper (Bansil and Zeytuncu 2019) found the leading coefficient in the asymptotic expansion for functions on spheres. In this talk we present a new computation of the leading coefficient using Karamata's Tauberian theorem. We conjecture that this representation can be generalized to an analog of Weyl's Law for functions on general CR manifolds. (Received September 15, 2020)

33 ▶ *Special functions*

1163-33-513   **Bonita V. Saunders*** (bonita.saunders@nist.gov), **Sean Brooks**, **Ron Buckmire** and
Rachel E. Vincent-Finley. *Validated Computations of Special Functions I: Overview.*

During the late 1930s, 40s and 50s accurate tables of function values were calculated by human 'computers' to facilitate the evaluation of functions by interpolation. In addition to logarithmic and trigonometric functions, these reference tables included values for gamma, Legendre, Jacobian, Bessel, Airy, and other high level or 'special' functions important for applied and physical applications.

The advent of reliable computing machines, computer algebra systems, and computational packages diminished the need for such reference tables, but today's researchers and software developers still need a way to confirm the accuracy of numerical codes that compute mathematical function values.

This talk introduces the field of validated computations of mathematical functions, which is the development of codes that compute certifiably accurate function values that can be used to test the accuracy of values produced by personal, commercial, or publicly available codes. We give a brief overview, discuss a few examples that motivate the research, and discuss how our research relates to other work in the field.   (Received September 08, 2020)

1163-33-590   **Bruce C Berndt*** (berndt@illinois.edu). *Ramanujan's Beautiful Integrals.*

During his entire life, Ramanujan loved to evaluate integrals. In his published papers, earlier notebooks, lost notebook, and letters to G. H. Hardy, Ramanujan offered a bewildering variety of beautiful integrals, many leading to further important results. In letters to the speaker, Richard Askey helped to bring a fuller understanding to some of these integrals. A survey of some of Ramanujan's integral evaluations will be given.   (Received September 10, 2020)

1163-33-770   **Warren P Johnson*** (wpjoh@conncoll.edu), Connecticut College, Department of
Mathematics and Statistics, New London, CT 06320. *Functional Equations in the Research and Teaching of Dick Askey.*

I will present several of my favorite examples of Dick Askey's use of functional equation arguments, in the evaluation of integrals and in proving series/product identities. I will also discuss a determinant that extends the Sylvester determinant from Dick's paper "Evaluation of Sylvester Type Determinants Using Orthogonal Polynomials".   (Received September 12, 2020)

1163-33-836   **George E. Andrews*** (gea1@psu.edu), 306 McAllister Bldg, Mathematics Department,
Penn State University, University Park, PA 16802. *Chebyshev polynomials and Compositions.*

The Theory of Compositions of integers has mostly been relegated to the very basic aspects of combinatorics. The object of this paper is to reveal their close relation to the Chebyshev polynomial $T_n(x)$ and $U_n(x)$. As a result, interesting combinatorial questions arise for compositions that have not been examined previously. (Received September 13, 2020)

1163-33-912   **Tom H Koornwinder*** (thkmath@xs4all.nl), University of Amsterdam, Korteweg-de
Vries Institute, P.O. Box 94248, 1090 GE Amsterdam, Netherlands. *Charting the Askey and q-Askey schemes.* Preliminary report.

The author's paper [1[will be reconsidered in view of the interesting paper [2] by Verde-Star.

[1] T. H. Koornwinder, The Askey scheme as a four-manifold with corners, Ramanujan J. 20 (2009), 409–439.

[2] L. Verde-Star, A unified construction of all the hypergeometric and basic hypergeometric families of orthogonal polynomial sequences, arXiv:2002.07932 [math.CA], 2020.   (Received September 14, 2020)

1163-33-913   **Hjalmar Rosengren*** (hjalmar@chalmers.se), Mathematical Sciences, Chalmers University of Technology, 43896 Gothenburg, Sweden. *On the Kanade–Russell identities.*
Kanade and Russell conjectured several Rogers–Ramanujan-type identities for triple series. Some of these conjectures are related to characters of affine Lie algebras, and they can all be interpreted combinatorially in terms of partitions. Many of the conjectures were settled by Bringmann, Jennings-Shaffer and Mahlburg. We describe a new approach to the Kanade–Russell identities, which leads both to new proofs of previously known identities and to conjectures that were still open. The new cases are based on quadratic transformations for Askey–Wilson polynomials.   (Received September 14, 2020)

1163-33-940   **Luc Vinet*** (vinet@crm.umontreal.ca) and **Alexei Zhedanov**. *A unified algebraic underpinning for the Hahn polynomials and rational functions.*
An algebra denoted $m\mathfrak{H}$ with three generators is introduced and shown to admit embeddings of the Hahn algebra and the rational Hahn algebra. It has a real version of the deformed Jordan plane as a subalgebra whose connection with Hahn polynomials is established. Representation bases corresponding to eigenvalue or generalized eigenvalue problems involving the generators are considered. Overlaps between these bases are shown to be bispectral orthogonal polynomials or biorthogonal rational functions thereby providing a unified description of these functions based on $m\mathfrak{H}$. Models in terms of differential and difference operators are used to identify explicitly the underlying special functions as Hahn polynomials and rational functions and to determine their characterizations. An embedding of $m\mathfrak{H}$ in $\mathcal{U}(\mathfrak{sl}_2)$ is presented. A Padé approximation table for the binomial function is obtained as a by-product.   (Received September 14, 2020)

1163-33-950   **Jacob Stordal Christiansen*** (stordal@maths.lth.se), **Barry Simon** and **Maxim Zinchenko**. *Residual Polynomials.*
Let $\mathsf{E} \subset \mathbb{R}$ (or $\mathbb{C}$) be a compact, not finite, set and fix a point $x_0 \notin \mathsf{E}$. We denote by $R_n := R_{x_0,n}^{(\mathsf{E})}$ the nth residual polynomial of E normalized at x_0, that is, the degree at most n polynomial with $R_n(x_0) = 1$ which minimizes the sup norm on E. Such polynomials appear in numerical analysis and have applications, e.g., to Krylov subspace iterations.

In the talk, I'll present upper (and lower) bounds on the norms $\|R_n\|_{\mathsf{E}}$ and strong (aka Szegő–Widom) asymptotics of $R_n(z)$. Several examples will also be discussed. The talk is based on joint work with B. Simon and M. Zinchenko dedicated to the memory of Dick Askey.   (Received September 14, 2020)

1163-33-1033   **Erik Koelink*** (e.koelink@math.ru.nl), **Maarten van Pruijssen** and **Pablo Román**. *Multivariable matrix valued orthogonal polynomials from representation theory.*
The classical spherical functions on compact symmetric spaces form sets of orthogonal polynomials, which can be identified with subclasses of Jacobi polynomials in the so-called rank 1 case and with Heckman-Opdam polynomials with special values for the parameters for higher rank cases. We discuss a general way for obtaining matrix valued orthogonal polynomials related to compact symmetric spaces under some specific conditions. For the group case corresponding to $SU(n)$, we make this explicit. In this way we find matrix valued polynomial analogs of Koornwinder's 2-variable polynomials orthogonal on the interior of Steiner's hypocycloid corresponding to $n = 3$.   (Received September 14, 2020)

1163-33-1055   **Al Cuoco*** (acuoco@edc.org), 60 Merrimac st, suite 204, Hatter's Point, Amesbury, MA 01913. *Adventures with Dick in Mathematics Education.*
Over a span of 25^+years, I had hundreds of discussions with Dick Askey about (mainly high school) mathematics. I came to learn about some distinguishing features of his taste in all of mathematics—things like his love for parsimony, his passion for simplicity, and his deep dislike for special-purpose cleverness.

We argued often. But we agreed (and laughed) equally as often. Dick's approach to mathematics and mathematics teaching greatly influenced our curriculum work at EDC, even though I didn't always take his advice about "the right way" do do this or that.

This talk will recount some stories about Dick and his influence on education in the US. I'll include some curated segments of our email and in-person discussions.   (Received September 16, 2020)

1163-33-1221   **Kathy A Driver*** (`kathy.driver@uct.ac.za`), University of Cape Town, Private Bag 3, Rondebosch, Cape Town, W Cape 7701, South Africa. *Zeros of Jacobi polynomials.*
The interlacing of zeros of Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ and $P_{n+1}^{(\alpha,\beta+1)}(x)$, $\alpha, \beta > -1$, are investigated. We also consider the interlacing properties of the zeros of $P_n^{(\alpha,\beta)}(x)$ and $P_{n+1}^{(\alpha+1,\beta+1)}(x)$, $\alpha, \beta > -1$, The special case $\alpha = \beta = \lambda - 1/2$ is also considered.   (Received September 15, 2020)

1163-33-1321   **Persi Diaconis*** (`diaconis@math.stanford.edu`), Department of Statistics, Sequoia Hall, 390 Serra Mall, Stanford, CA 94305, and **Chenyang Zhong** (`cyzhong@stanford.edu`), Department of Statistics, Sequoia Hall, 390 Serra Mall, Stanford, CA 94305. *Orthogonal Polynomials and the Burnside Process.*
Let X be a finite set and G a finite group acting on X. This splits X into orbits. The Burnside process gives a method of choosing a uniformly random orbit. This includes problems of choosing unlabeled objects (e.g., trees) or partitions. The process is simple. From x choose a random element g fixing x and then a random element y fixed by g (both choices uniform). This gives a Markov chain $K(x,y)$ being the chance of moving from x to y in one step. This Chain, suggested by Mark Jerrum, has stationary distribution $pi(x)$ proportional to $1/|orbit(x)|$. It is a special case of the 'hit and run algorithm' which includes the Swedsen-Wang data augmentation and many other special cases. Most of these have resisted analysis, even though they are widely used.

As usual, one may ask for rates of convergence for this Markov chain to stationarity. If X is the set of binary n-tuples, G is the symmetric group $S(n)$, then orbits are indexed by $0, 1, ..., n$ and the chain has a Bose-Einstein stationary distribution. It turns out that the chain is diagonalizable by dual Hahn polynomials and sharp rates of convergence ensue–a finite number of steps suffice, no matter how large n is. There should be many other cases where orthogonal polynomials can be used.   (Received September 15, 2020)

1163-33-1409   **Mourad E. H. Ismail*** (`mourad.eh.ismail@gmail.com`), Department of Mathematics, University of, Central Florida, Orlando, FL 32816. *Mathematical Reminisce about Dick Askey.*
We will share with the audience some personal stories about Dick Askey. Although these reminisces reflect my personal experience, they are typical. I will discuss how Askey treated, and mentored young people and how he continued to help, nurture and influence them throughout their careers.   (Received September 15, 2020)

1163-33-1558   **Shaun Cooper*** (`s.cooper@massey.ac.nz`). *Some elliptic integrals in Ramanujan's lost notebook.*
In the "lost notebook" [2, p. 53] Ramanujan stated a result equivalent to

$$\int_0^x t\, f(-t)\, f(-t^5)\, t(-t^7)\, f(-t^{35})\, \mathrm{d}t = \int_0^v \frac{1}{\sqrt{\left(\frac{1}{t}+1-t\right)\left(\frac{1}{t^3}-\frac{5}{t^2}-9-5t^2-t^3\right)}}\, \frac{\mathrm{d}t}{t}$$

where

$$f(-t) = \prod_{j=1}^{\infty}(1-t^j)$$

and

$$v = x\, \frac{f(-x)\, f(-x^{35})}{f(-x^5)\, f(-x^7)}.$$

In this talk I will describe how this result fits into a larger theory, and indicate how to obtain the corresponding result involving $f(-t)\, f(-t^3)\, t(-t^7)\, f(-t^{21})$ which is new. This is joint work with T. Anusha, E.N. Bhuvan and K.R. Vasuki, [1].

References:

[1] T. Anusha, E.N. Bhuvan, S. Cooper and K.R. Vasuki, <u>Elliptic integrals and Ramanujan-type series for $1/\pi$ associated with $\Gamma_0(N)$, where N is a product of two small primes,</u> J. Math. Anal. Appl., **472** (2019) 1551–1570.

[2] S. Ramanujan, <u>The Lost Notebook and Other Unpublished Papers,</u> Narosa, New Delhi, 1988.   (Received September 15, 2020)

1163-33-1658   **Ian Marquette**, **Sarah Post*** (`spost@hawaii.edu`) and **Lisa Ritter**. *Exceptional Orthogonal Polynomials and rational solutions of Painlevé Equations.*
Exceptional orthogonal polynomials (EOP), classical orthogonal polynomials with gaps in their degree, have recently been the subject of flourishing new research connected to classical, univariate orthogonal polynomials. One area in particular is the connection between rational solutions to Painlevé equations and EOPs. In this presentation, we will discuss the connection between exceptional Hermite polynomials and rational solutions to

the fourth Painlevé equation, which was previously known, and then extend the method to the sixth Painlevé equation via exceptional Jacobi polynomials. Throughout, we will see how superintegrable Hamiltonian systems and polynomial algebras play a role.   (Received September 16, 2020)

34 ▶ Ordinary differential equations

1163-34-40   **Min Wang*** (`mwang23@kennesaw.edu`). *On the resilience of a fractional compartment model.*

In this talk, we consider the resilience problem for a type of fractional compartment models by studying an equivalent boundary value problem. A series of criteria on the existence, uniqueness, and multiplicity of solutions or positive solutions are obtained. Examples are given to demonstrate the applications of the results as well. (Received July 24, 2020)

1163-34-84   **Qimin Huang*** (`qxh119@case.edu`), **David Gurarie**, **Anirban Mondal** and **Martial Ndeffo-Mbah**. *Individual-based modeling of COVID-19 in local community settings.* Preliminary report.

Individual based modeling (IBM) of disease transmission offers an attractive alternative to population-based approaches, as it allows a detailed account of biological (risk) factors, environment, and behavior. This is particularly relevant in local community settings (hospital, workplace, school), where finite population size and host heterogeneity, in terms social interactions and disease progression, make a population-based approach impractical.

We develop such IBM methodology to simulate COVID-19 outbreaks in local settings, and explore different control-mitigation strategies. Our models feature multiple disease pathways (asymptomatic, mild and severe) typical of COVID-19, as well as heterogeneous host communities with different susceptibility levels and structured social contacts.

Two typical IBM examples include (i) hospital, made of interacting healthcare workers and patients, (ii) school/college, where students aggregate in classrooms, dorms, and engage in other (social, dining, sport) activities. In both cases, we used available data to set up and calibrate our models. Different control strategies were explored. The efficacy of each intervention and resources required to prevent or mitigate the outbreak were assessed.   (Received August 11, 2020)

1163-34-122   **Jeffrey W Lyons*** (`jlyons3@citadel.edu`), **Paul Eloe** and **Jeffrey T. Neugebauer**. *Differentiation of Solutions of Caputo Initial Value Problems with respect to Initial Data.*

Let $n \in \mathbb{N}$. In this paper, the authors consider the solution of an initial value problem of a Caputo fractional differential equation of order $\alpha \in (n-1, n)$. This solution is differentiated with respect to the initial time and with respect to the initial conditions. The differentiated solution is shown to solve an associated Caputo variational equation with interesting initial conditions.   (Received August 18, 2020)

1163-34-174   **Henry Zhao** and **Zhilan Feng*** (`fengz@purdue.edu`), 150 N. University Street, Department of Mathematics, Purdue University, West Lafayette, IN 47907. *Staggered Release Policies for COVID-19 Control: Costs and Benefits of Relaxing Restrictions by Age and Risk.*

Lockdown and social distancing restrictions have been widely used as part of policy efforts aimed at controlling the ongoing COVID-19 pandemic. Since these restrictions have a negative impact on the economy, there exists a strong incentive to relax these policies while protecting public health. Using a multigroup SEIR epidemiological model, we explore the costs and benefits associated with the sequential release of specific groups based on age and risk from isolation. The results suggest that properly designed staggered-release policies can do better than simultaneous-release policies in terms of protecting the most vulnerable members of a population, reducing health risks overall, and increasing economic activity. This work is based on Zhao and Feng, Mathematical Biosciences (2020), https://doi.org/10.1016/j.mbs.2020.108405.   (Received August 25, 2020)

1163-34-186   **Imelda Trejo*** (`imelda@lanl.gov`), Theoretical Biology and Biophysics, Los Alamos National Laboratory, Los Alamos, NM 87545. *A modified Susceptible-Infected-Recovered model for observed under-reported incidence data.* Preliminary report.

Fitting Susceptible-Infected-Recovered (SIR) models to incidence data is problematic when a fraction q of the infected individuals are not reported. Assuming an underlying SIR model with general but known distribution for the time to recovery, we introduce a system of differential-integral equations to quantify the fraction of asymptomatic individuals during an epidemic outbreak. Using these equations, we develop a simple stochastic

model for the observed incidence, and propose a Bayesian estimate of model parameters. We use our model to estimate the infection rate and fraction of asymptomatic individuals for the current Coronavirus 2019 outbreak in some European Countries.   (Received August 25, 2020)

1163-34-193   **Brian J Winkel*** (`brianwinkel@simiode.org`), SIMIODE, 26 Broadway, Cornwall, NY 12518. *Modeling in Class - LSD, Falling Water, and Yeast.*

We describe three Modeling Scenarios, their development, and how they can be effectively used to motivate and teach differential equations in context. Each of these activities involve the use and/or acquisition of data and the full modeling cycling from assumptions and building differential equation models to parameter estimation and model validation. All are readily available at www.simiode.org.   (Received August 26, 2020)

1163-34-209   **Johnny Henderson*** (`johnny_henderson@baylor.edu`), Department of Mathematics, Baylor University, Waco, TX 76798-7328, and **Kadda Maazouz** (`mzkadda@gmail.com`), Department of Mathematics, University of Tiaret, 14000 Tiaret, Algeria. *Existence and uniqueness for a Caputo-Katugampola fractional thermistor problem.*

A thermistor is a type of electrical device whose resistance is dependent on temperature. In this talk, existence and uniqueness results are obtained, using fixed point theory, for solutions of a Caputo-Katugampola fractional thermistor problem.   (Received August 27, 2020)

1163-34-210   **Johnny Henderson*** (`johnny_henderson@baylor.edu`), Department of Mathematics, Baylor University, Waco, TX 76798-7328, and **Kadda Maazouz** (`mzkadda@gmail.com`), Department of Mathematics, University of Tiaret, 14000 Tiaret, Algeria. *Existence results for Katugampola fractional differential equations for boundary value problems.*

This talk deals with the existence and uniqueness of solutions for fractional differential equations for boundary value problems involving the Katugampola fractional derivative by using the Banach contraction principle, Schauder's fixed point theorem and Schaefer's fixed point theorem.   (Received August 27, 2020)

1163-34-286   **M. N. Islam*** (`mislam1@udayton.edu`), Department of Mathematics, University of Dayt, Dayton, OH 45469, and **Jeff Neugebauer**, Eastern Kentucky University, Richmond, KY 40475. *Fractional Differential Equation of Riemann-Liouville Type.*

We consider an initial value problem for a fractional differential equation of Riemann-Liouville type. By studying an equivalent Volterra integral equation, we show the existence of a continuous solution on $(0, T]$ for some $T > 0$. We then show for a special case that if a continuous solution exists on $(0, \infty)$, then it is absolutely integrable on that interval.   (Received September 01, 2020)

1163-34-400   **Eric Ngang Che*** (`ericngangche@yahoo.com`), Washington, DC, and **Yeona Kang** (`yeona.kang@howard.edu`) and **Abdul-Aziz Yakubu** (`ayakubu@howard.edu`). *Risk Structured Model of Cholera Infections In Cameroon.*

In this talk, we introduce a risk-structured ODE cholera model of Cameroon with no spatial structure [1]. We use a "fitted" demographic equation (disease-free equation) to capture the total population of Cameroon, and then use a fitted low-high risk structured cholera differential equation model to study reported cholera cases in Cameroon from 1987-2004. The basic reproduction number of our fitted cholera model, $\mathcal{R}_0$, is bigger than 1 and our model predicted cholera endemicity in Cameroon. In addition, the fitted risk structured model predicted a decreasing trend from 1987 to 1994 and an increasing trend from 1995 to 2004 in the pre-intervention reported number of cholera cases in Cameroon from 1987 to 2004. Using the fitted risk structured cholera model, we study the impact of vaccination, treatment and improved sanitation on the number of cholera infections in Cameroon from 2004 to 2022. Furthermore, we use our fitted model to predict future cholera cases.

Reference

[1] E. Che, Y. Kang and A. Yakubu, Risk structured model of cholera infections in Cameroon, Mathematical Biosciences **320** (2020), 108303.
(Received September 04, 2020)

1163-34-435   **Anca Radulescu** (`radulesa@newpaltz.edu`), **Cassandra Williams*** (`williamc18@hawkmail.newpaltz.edu`) and **Kieran Cavanagh**. *Management strategies in a SEIR-type model of COVID-19 community spread.*

The 2019 Novel Corona virus infection (COVID-19) is an ongoing public health emergency of international focus. Significant gaps persist in our knowledge of COVID-19 epidemiology, transmission dynamics, investigation tools and management, despite (or possibly because of) the fact that the outbreak is an unprecedented global threat. On the positive side, enough is currently known about the epidemic process to permit the construction of

mathematical predictive models. We construct and analyze here one first step: that of adapting a traditional SEIR epidemic model to the specific dynamic compartments and epidemic parameters of COVID-19, as it spreads in an age-heterogeneous community. We analyze management strategies of the epidemic course (as they were implemented through lockdown and reopening procedures in many of the US states and countries worldwide); however, to more clearly illustrate ideas, we focus on the example of a small scale college town community. We generate predictions, and assess the efficiency of these control measures (closures, mobility restrictions, social distancing), in a sustainability context.   (Received September 06, 2020)

1163-34-613   **John R. Graef***, Department of Mathematics, University of Tennessee at Chattanooga, Chattanooga, TN 37403. *Existence of Solutions to Fractional q-Difference Inclusions.*

Existence of solutions to a class of boundary value problems for Caputo fractional q-difference inclusions of the form

$$\begin{cases} (^{C}D_q^{\alpha}u)(t) \in F(t, u(t)), \ t \in I := [0, T], \\ L(u(0), u(T)) = 0, \end{cases}$$

are proved. Here, $q \in (0,1)$, $\alpha \in (0,1]$, $T > 0$, $F : I \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a multivalued map, $\mathcal{P}(\mathbb{R})$ is the family of all nonempty subsets of $\mathbb{R}$, $^{C}D_q^{\alpha}$ is the Caputo fractional q-difference derivative of order α, and $L : \mathbb{R}^2 \to \mathbb{R}$ is a continuous function.

The technique of proof involves using set-valued analysis, fixed point theory, and the method of upper and lower solutions. This appears to be the first time the method of upper and lower solutions has been applied to Caputo q-fractional difference equations.   (Received September 10, 2020)

1163-34-633   **Govinda Pageni*** (`govinda.pageni1@louisiana.edu`), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504, and **Aghalaya S Vatsala**. *Study of System of Caputo Fractional Differential Equations with initial conditions via Laplace Transform Method.* Preliminary report.

In this work, we will provide an analytical method to compute the solution of the linear coupled system of Caputo fractional differential equations with initial conditions. The standard method adopted for the system of ordinary differential equations using the exponential of a matrix will not be useful, since the Mittag-Leffler function do not have the nice property of the exponential function. In this work, we have used the Laplace transform method for the system of Caputo fractional differential equations when the order of the derivative is q, such that $0 < q < 1$. The method yields the integer result as a special case. Solutions of sequential Caputo fractional differential equations can be obtained as a special case of our main result. We have obtained some numerical results.   (Received September 10, 2020)

1163-34-643   **Aghalaya S. Vatsala*** (`vatsala@louisiana.edu`), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504. *Study of Sequential Caputo Fractional Differential Equations with Fractional Initial Conditions and Fractional Type Boundary Conditions.* Preliminary report.

It is well known that qualitative study of Caputo fractional differential equation has an advantage over the integer order differential equations. This is certainly true, if the order of the fractional derivative q, when $0 < q < 1$, which can be used as a parameter. This is also possible for Caputo fractional higher order differential equations if we assume that Caputo fractional order derivative of order 'nq' is sequential of order 'q'. In this work, we present some basic results of Caputo sequential differential equations with fractional initial conditions and Caputo sequential boundary value problems. Our results yield the integer results as a special case.   (Received September 10, 2020)

1163-34-654   **Dat Cao**, Department of Mathematics and Statistics, Minnesota State University, Mankato, **Luan Hoang*** (`luan.hoang@ttu.edu`), Department of Mathematics and Statistics, Texas Tech University, 1108 Memorial Circle, Lubbock, TX 79409-1042, and **Thinh Kieu**, Department of Mathematics, University of North Georgia, Gainesville Camp. *Asymptotic Expansions for Decaying Solutions of Dissipative Differential Equations.*

We study the precise asymptotic behavior of a non-trivial solution that converges to zero, as time tends to infinity, of dissipative systems of nonlinear ordinary differential equations. The nonlinear term of the equations may not be smooth. We obtain an infinite series asymptotic expansion, as time goes to infinity, for for the solution.This series expansion provides large time approximations for the solution with the errors decaying exponentially at any given rates. Our result applies to different classes of non-linear equations that have not been dealt with previously.   (Received September 10, 2020)

1163-34-702   **Therese Shelton***, shelton@southwestern.edu. *Student Engagement in Modeling Drugs, Disease, and More.*

We present how students have engaged with introductory modeling activities to complement a more traditional teaching approach. We include plans for remote engagement this semester.   (Received September 11, 2020)

1163-34-710   **Stéphanie Abo*** (`sabo@uwaterloo.ca`) and **Anita Layton** (`anita.layton@uwaterloo.ca`). *Sex-Specific Effects of Shiftwork-Mediated Disruptions of Circadian Rhythms on the Inflammatory Response.*

Shift workers often experience circadian misalignment as their irregular work schedules disrupt the natural sleep-wake cycle, which in turn causes serious health problems associated with alterations in genetic expressions of clock genes. These alterations are sex-specific; however, the underlying mechanisms that govern the immune response to these effects remain unclear. To address this question, we have constructed a mathematical model of the mammalian pulmonary circadian clock coupled to an acute inflammation model. Shiftwork was simulated by an 8h-phase advance of the circadian system with sex-specific modulation of clock genes. The model reproduces the clock gene expression in the lung and the immune response to various doses of lipopolysaccharide (LPS). Our model predicts a reversal of the times of lowest and highest sensitivity to LPS, with males and females exhibiting an exaggerated response to LPS at circadian time (CT) 0, which is countered by a blunted immune response at CT12. Overall, females produce fewer pro-inflammatory cytokines than males and suffer more severe sequelea from shiftwork. Our model also predicts interesting IL-10 dynamics closer to CT0 at baseline. (Received September 11, 2020)

1163-34-844   **Selenne Baneulos**, **Hayriye Gulbudak**, **Mary Ann Horn**, **Qimin Huang**, **Aadrita Nandi*** (`aadrita.nandi5@gmail.com`), **Hwayeon Rye** and **Rebecca Segal**. *Evaluating the effect of immune system on the efficacy of phage therapy in combination with antibiotic for multi-drug resistant bacteria.*

Antimicrobial resistance is one of the biggest concerns of global health. Bacteria and fungi can change themselves to defeat antimicrobial drugs which aim to kill them. With time, overuse of antibiotic is accelerating the spread of antimicrobial resistance. Only in US, 2 million infections and 23,000 deaths are reported every year according to CDC. Research suggests that the phage therapy has the potential to be used to treat multi-drug resistant bacteria. Bacteriophage is administered to infect and lysis the bacteria in phage therapy. We have modeled an ODE system to investigate the effect of immune system on combination treatment of the phage and antibiotic. Our result shows the frequency and concentration of dose as well as the timing of phage administration are important factors of the combination phage therapy.   (Received September 13, 2020)

1163-34-850   **Nourridine Siewe*** (`nourridine@aims.ac.za`), **Suzanne Lenhart** and **Abdul-Aziz Yakubu**. *Ebola Outbreaks and International Travel Restrictions: Case Studies of Central and West Africa Regions.*

Ebola outbreaks in Africa have occurred mostly in the Central and West Africa regions that are politically identified as the Economic Community of Central African States (ECCAS) and Economic Community of Western African States (ECOWAS), respectively. In the ECOWAS region, people and goods are allowed to travel freely across national borders of all the 15 member countries, but in the ECCAS region such regional travel across the national borders of its 10 member countries is limited. In this paper, we use parameterized mathematical models of Ebola to investigate the effects of free international travel, and the timing of border closings, on the high number of Ebola infection cases and deaths of the recent 2014-2016 Ebola outbreaks in Guinea, Liberia and Sierra Leone (ECOWAS); as compared to previous and current outbreaks in Democratic Republic of Congo (ECCAS, 1976-2018). We obtain that international travel restrictions and timing of border closings can play important roles in mitigating against the spread of future fatal infectious disease outbreaks.   (Received September 13, 2020)

1163-34-882   **Nsoki Mavinga*** (`nmaving1@swarthmore.edu`) and **Yusuf Qaddura** (`qaddura.1@osu.edu`). *Delay Differential Equations with Applications to the Analysis of the Spread of Vector-Borne Diseases.*

We are concerned with the stability of equilibrium solutions for a two-lag delay differential equation which models the spread of vector-borne diseases where the lags are incubation periods in humans and vectors. We show that there are some values of the transmission and recovery rates for which either the disease dies out or it spreads into an endemic state. The approach is based on the linearization method and the analysis of roots of transcendental equations. Through MATLAB simulations, we also observe that the solution could possess uncontrolled behavior.   (Received September 13, 2020)

1163-34-889   **Sougata Dhar*** (`sougata.dhar@uconn.edu`), 341 Mansfield Road, Room 406, Department
                   of Mathematics, The University of Connecticut, Storrs, CT 06269, and **Jessica S Kelly**
                   and **Qingkai Kong**. *Lyapunov-type inequalities for quasi-linear eigenvalue problems with
                   indefinite weights.* Preliminary report.

In this paper we consider a second-order quasilinear problems with indefinite weights. We first obtain several
Lyapunov-type inequalities. Then, we apply this inequalities to obtain the lower bound of the first eigenvalue.
Furthermore, we discuss spacing between zeros of a solution. Our result is an improvement of some results in
the literature.   (Received September 13, 2020)

1163-34-990   **Sujani Ambahera**, Murfreesboro, TN , and **Zachariah Sinkala***
                   (`zachariah.sinkala@mtsu.edu`), Deparment of Mathematical Sciences, Middle Tennessee
                   State University, Murfreesboro, TN 37132. *Mathematical Modeling of Car T Cell Therapy
                   in Glioblastoma.*

Mathematical modeling has been used to study immune therapy on different cancer treatments. Chimeric antigen
receptor (CAR)-engineered T cells is a promising immune therapy for treating cancers like Hodgkin Lymphoma.
Some models have used to study the treatment of Hodgkin Lymphoma using Car T cell therapy, but not in
glioblastoma. In our study, we use a system of ordinary differential equations to study CAR T therapy in the
treatment of glioblastoma.   (Received September 14, 2020)

1163-34-1164   **Michael A Karls*** (`mkarls@bsu.edu`), Ball State University, Muncie, IN 47306. *Adapting
                   SIMIODE Projects to an Online ODE Course.* Preliminary report.

In Fall 2017 I incorporated several SIMIODE projects into a traditional ODE course. For Fall 2020, I used a
similar approach, however this time the course was taught in an asynchronous online format. Just as was done
the first time, I wrote accompanying project assignments to help guide students through the SIMIODE projects.
I will look at examples of these accompanying projects, compare how they worked or didn't work in each class
setting, and consider what modifications could be made for future versions of the course.   (Received September
14, 2020)

1163-34-1231   **Jeffrey T. Neugebauer*** (`jeffrey.neugebauer@eku.edu`) and **Aaron Wingo**. *Positive
                   Solutions for a Fractional Boundary Value Problem with Lidstone Like Boundary
                   Conditions.*

We consider a higher order fractional boundary value problem with Lidstone like boundary conditions. By using
a convolution to construct the Green's function for the higher order problem, we are able to apply a fixed point
theorem to show the existence of positive solutions of the boundary value problem.   (Received September 15,
2020)

1163-34-1290   **Enahoro Iboi*** (`enahoroiboi@spelman.edu`), 350 Spelman Ln SW, Atlanta, GA 30314.
                   The Fight to Curtail the Spread of the Novel Coronavirus in the U.S.

The novel coronavirus (COVID-19) that emerged from the Wuhan city of China in late December 2019 continues
to pose devastating public health and economic challenges across the world. This talk uses mathematical model-
ing techniques to assess the impact of the community-wide implementation of non-pharmaceutical intervention
measures, such as social distancing, quarantine of suspected COVID-19 cases, isolation of confirmed cases, use of
face masks in public, contact tracing, testing, and the prospect of using a hypothetical imperfect anti-COVID-19
vaccine on the control of COVID-19 in the United States.   (Received September 15, 2020)

1163-34-1347   **Joan Ponce*** (`ponce0@purdue.edu`), 150 Arnold Dr Apt 12, West Lafayette, IN 47906, and
                   Sheng Zhang. *Transmission dynamics of COVID-19 in Ecuador and age-dependent
                   control strategies.*

As countries affected by the pandemic of coronavirus disease 2019 (COVID-19) begin the reopening process,
a good understanding of the consequences of relaxing restrictions is indispensable to avoid high peaks, which
may overwhelm the health-care systems. We use an age-structured, modified, SEIR model to reconstruct the
dynamics of COVID-19 in Ecuador between February and August 2020 across four different periods defined by
interventions. This paper explores the benefits associated with the release of specific age groups from lockdown
before others. The results suggest that we can prevent cases and deaths with properly designed release policies
to protect the most vulnerable individuals and increase economic activity.   (Received September 15, 2020)

1163-34-1435   **Christopher S Goodrich*** (`c.goodrich@unsw.edu.au`), School of Mathematics and Statistics, UNSW Sydney, The Red Centre, Room 2070, Sydney, 2052, Australia. *A Topological Approach to Nonlocal Boundary Value Problems.*
We consider nonlocal convolution equations of the general form

$$A\left(\int_{[0,1]} (g \circ u)(s) \, d\alpha(s)\right) (a * u'')(t) + \lambda f\big(t, u(t)\big) = 0, \quad t \in (0,1)$$
$$u'(0) = 0$$
$$u(1) = T. \tag{1}$$

The existence of at least one positive solution is shown by means of topological fixed point theory, facilitated by the use of a nonstandard order cone. Applications to fractional differential equations are considered.   (Received September 15, 2020)

1163-34-1578   **Rob Thompson*** (`rthompson@carleton.edu`), Department of Mathematics and Statistics, 100 N College St, Northfield, MN 55057. *Simulating liquid lenses.*
Put a tiny droplet of oil on water. It will form a small lens-like shape. What is this shape, exactly? In this talk we discuss the equations that determine the shape of a fluid lens formed by depositing droplets of one fluid onto another. Reduction of the governing physical equation (the Young-Laplace equation) to a set of ordinary differential equations for the lens shape will be discussed along with methods for numerical solution of these equations. We'll also share the success (and failure) of these simulated lens shapes at predicting the shapes of actual polymer lenses produced in the research lab of Prof. Marty Baylor at Carleton College. This work is done in collaboration with Prof. Baylor, '19 Carleton graduates Emily Schwartz and Sam Stevenson, and Zack Johnson '22.   (Received September 15, 2020)

1163-34-1644   **Chris McCarthy*** (`cmccarthy@bmcc.cuny.edu`), BMCC City University of New York, New York, NY 10007. *Building Bridges with Differential Equation Models.*
One project I give my students is designing a suspension bridge. I have them use differential equations to model the shape of the suspension cable. The mathematics of the parabolic cable are fairly easy. The talk will emphasize what students learn from this and other modeling projects.   (Received September 15, 2020)

1163-34-1687   **Zhilan J. Feng***. *Staggered Release Policies for COVID-19 Control: Costs and Benefits of Relaxing Restrictions by Age and Risk.*
Lockdown and social distancing restrictions have been widely used as part of policy efforts aimed at controlling the ongoing COVID-19 pandemic. Since these restrictions have a negative impact on the economy, there exists a strong incentive to relax these policies while protecting public health. Using a multigroup SEIR epidemiological model, we explore the costs and benefits associated with the sequential release of specific groups based on age and risk from isolation. The results suggest that properly designed staggered-release policies can do better than simultaneous-release policies in terms of protecting the most vulnerable members of a population, reducing health risks overall, and increasing economic activity.   (Received September 17, 2020)

35 ▶ *Partial differential equations*

1163-35-6   **Ryan Hynd*** (`rhynd@math.upenn.edu`), 209 South 33rd St., Philadelphia, PA 19104. *The Hamilton-Jacobi equation, past and present.*
Nearly two centuries ago, William Rowan Hamilton observed that the equations of motion in classical mechanics can be derived by finding stationary points of a certain integral. In addition, he identified an equation satisfied by the integral itself, which is now known as the Hamilton-Jacobi equation. These ideas were later expanded upon by scientists who grappled with the challenges of understanding how to regulate machines, control spacecrafts, and optimize the production of goods. In recent years, mathematicians have made tremendous progress in developing a theory of control which prominently features a Hamilton-Jacobi equation. We will discuss the highlights of this theory, some applications, and a few theoretical issues of interest today.   (Received September 14, 2020)

1163-35-37   **Yannick Sire***, 3400 N. Charles Street, Baltimore, MD 21218. *Aspects of Fractional Schrodinger operators.*
I will report on several recent works dealing with models arising in mathematical physics and involving fractional Schrodinger operators. I will discuss the state of the art and several open problems   (Received July 24, 2020)

1163-35-39   **Nadejda V. Drenska*** (`ndrenska@umn.edu`), **Robert V. Kohn** (`kohn@cims.nyu.edu`) and **Jeff Calder** (`jcalder@umn.edu`). *A PDE Interpretation of Prediction with Expert Advice.*

We study the problem of prediction of binary sequences with expert advice in the online setting, which is a classic example of online machine learning. We interpret the binary sequence as the price history of a stock, and view the predictor as an investor, which converts the problem into a stock prediction problem. In this framework, an investor, who predicts the daily movements of a stock, and an adversarial market, who controls the stock, play against each other over N turns. The investor combines the predictions of n experts in order to make a decision about how much to invest at each turn, and aims to minimize their regret with respect to the best-performing expert at the end of the game. We consider the problem with history-dependent experts, in which each expert uses the previous d days of history of the market in making their predictions. The prediction problem is played (in part) over a discrete graph called the d dimensional de Bruijn graph. In the first part of the talk we focus on an appropriate continuum limit and using methods from optimal control, graph theory, and partial differential equations, we discuss strategies for the investor and the adversarial market. The proposed strategies are asymptotically optimal for n=2 and d at most 4. We prove that the value function for this game, rescaled appropriately, converges as N goes to infinity at a rate of $O(N^{-1/6})$ (for C^4 payoff functions) to the viscosity solution of a nonlinear degenerate parabolic PDE. It can be understood as the Hamilton-Jacobi-Issacs equation for the two-person game. As a result, we are able to deduce asymptotically optimal strategies for the investor. For Lipschitz payoff functions the rate of convergence is $O(N^{-1/6} \log N)$. Our results extend those in the first part of the talk (for n=2, d at most 4, where the convergence rate is $O(N^{-1/2})$). In the last part of the talk we show that the optimality conditions over the de Bruijn graph correspond to a graph Poisson equation, and we establish $O(N^{-1/2})$ optimal strategies for all n and d. This is joint work with Robert Kohn and Jeff Calder.   (Received July 24, 2020)

1163-35-53   **Antonio De Rosa*** (`derosa@cims.nyu.edu`), Department of Mathematics, William E. Kirwan, 4176 Campus Dr, College Park, MD 20742. *Uniqueness of critical points of the anisotropic isoperimetric problem.*

The anisotropic isoperimetric problem consists in enclosing a prescribed volume in a closed hypersurface with least anisotropic energy. Although its solutions, referred to as Wulff shapes, are well understood, the characterization of the associated critical points is more subtle. In this talk we present a uniqueness result: Given an elliptic integrand of class $C^{2,\alpha}$, we prove that finite unions of disjoint (but possibly mutually tangent) open Wulff shapes with equal radii are the only volume-constrained critical points of the anisotropic surface energy among all sets with finite perimeter and reduced boundary almost equal to its closure. To conclude, we will discuss a quantitative stability for this rigidity theorem.   (Received August 02, 2020)

1163-35-56   **Farhan Abedin*** (`abedinf1@msu.edu`) and **Jun Kitagawa**. *Inverse Iteration for the Monge-Ampère Eigenvalue Problem.*

We will present an iterative method for solving the Monge-Ampère eigenvalue problem,

$$\begin{cases} \det D^2 u = \lambda_{MA} |u|^n & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ u \text{ convex.} \end{cases}$$

By a result of Lions, $\lambda_{MA} > 0$ is unique, and all convex solutions u are positive multiples of each other. We show that the iterates $\{u_k\}_{k=0}^{\infty}$ generated by our method converge to a non-trivial solution of the eigenvalue problem, and that $\lim_{k\to\infty} R(u_k) = \lambda_{MA}$, where the Rayleigh quotient $R(u)$ is defined as

$$R(u) := \frac{\int_\Omega |u| \det D^2 u}{\int_\Omega |u|^{n+1}}.$$

The method converges for a wide class of initial choices u_0 that can be constructed explicitly, and does not rely on prior knowledge of the Monge-Ampère eigenvalue λ_{MA}.   (Received August 03, 2020)

1163-35-134   **Ovidiu Savin** and **Hui Yu*** (`huiyu@math.columbia.edu`), Department of Math, Columbia University, New York, NY 10027. *Free boundary regularity in the multiple membrane problem.*

The N-membrane problem is the study of shapes of elastic membranes being pushed against each other. The main questions are the regularity of the functions modeling the membranes, and the regularity of the contact regions between consecutive membranes.

These are classical questions in free boundary problems. However, very little is known when N is larger than 2. In this case, there are multiple free boundaries that cross each other, and most known techniques fail to apply.

In this talk, we discuss, for general N, the optimal regularity of the solutions in arbitrary dimensions, and a classification of blow-up solutions in 2D. Then we focus on the regularity of the free boundaries when N=3.

This talk is based on two recent joint works with Ovidiu Savin (Columbia University).   (Received August 19, 2020)

1163-35-149   **Antoine Mellet** and **Yijing Wu*** (`yijingwu@umd.edu`), 4176 Campus Dr, College Park, MD 20742. *An isoperimetric problem with a competing nonlocal singular term.*

we investigate the minimization of a functional in which the usual perimeter is competing with a non local singular term such as the fractional perimeter. The motivation for this problem is a cell motility model. We establish several facts about global minimizers with a volume constraint. In particular we prove that minimizers exist for small mass, and are radially symmetric. Though we do not fully determine whether the minimizers exist for large mass, we prove that either the minimizing sequences splits into smaller sets or develop fingers. Which of these two alternatives occurs depends on a related minimization problem for the optimal constant in a classical interpolation inequality (a Gagliardo-Nirenberg type inequality for fractional perimeter).   (Received August 23, 2020)

1163-35-154   **Daniela De Silva***, desilva@math.columbia.edu, and **Ovidiu Savin**. *On the Boundary Harnack Inequality.*

We present a direct analytic proof of the classical Boundary Harnack Principle for solutions to linear uniformly elliptic equations in either divergence or non-divergence form.   (Received August 24, 2020)

1163-35-195   **Lingju Kong*** (`lingju-kong@utc.edu`). *Uniqueness of weak solutions for a biharmonic system.*

We study the biharmonic system

$$\begin{cases} \Delta\left(|\Delta u|^{p(x)-2}\Delta u\right) = a(x)|u|^{p(x)-2}u + f(x,u,v) & \text{in } \Omega, \\ \Delta\left(|\Delta v|^{q(x)-2}\Delta v\right) = b(x)|v|^{q(x)-2}v + g(x,u,v) & \text{in } \Omega, \\ u = \Delta u = 0, \ v = \Delta v = 0 & \text{on } \partial\Omega, \end{cases}$$

Using monotone operator theory, we prove that, under some suitable conditions, the above system has a unique weak solution. We also discuss some properties of the solution.   (Received August 26, 2020)

1163-35-196   **Ryan Hynd** and **Francis Seuffert*** (`seuffert@sas.upenn.edu`), David Rittenhouse Lab., 209 South 33rd Street, Philadelphia, PA 19104. *Extremals in nonlinear potential theory.*

We consider the PDE $-\Delta_p u = \rho$, where ρ is a signed Borel measure on $\mathbb{R}^p$. For each $p > n$, we characterize solutions as extremals of a generalized Morrey inequality determined by ρ.   (Received August 26, 2020)

1163-35-283   **Juliette Leblond*** (`juliette.leblond@inria.fr`), INRIA, B.P. 93, 06902 Sophia Antipolis, France. *Inverse moment estimation problems for Poisson-Laplace PDE in dimensions 2 and 3, and best constrained approximation in Hardy spaces; applications to paleomagnetism.*

From joint works with Laurent Baratchart, Eduardo A. Lima, Sylvain Chevillard, Elodie Pozzi.

We consider families of inverse potential problems for Poisson-Laplace PDE, in domains of the plane $R^2 \simeq C$, complex plane, or R^3, with source term in divergence form, div S. They arise for instance from Maxwell's equation in magnetostatics. The given data corresponds to pointwise values of components of the magnetic field taken at some distance from the support of the magnetization S, in some specific geometry. Actually, this concerns magnetized rock samples (like meteorites), whose magnetic field is recorded by a magnetometer. The aim is to recover some characteristics of the magnetization S, like its net (0 order) moment, in order to gain preliminary information for solving the full issue of estimating S itself. These are ill-posed problems. They are regularized by solving corresponding best constrained approximation issues that we state and solve in Hardy spaces.   (Received September 07, 2020)

1163-35-306   **Mouhamadou Sy** and **Xueying Yu***, 77 Massachusetts Ave, Cambridge, MA. *Almost sure global well-posedness for the energy supercritical NLS on the unit ball of $\mathbb{R}^3$.*

In this talk, we present two almost sure global well-posedness (GWP) results for the energy supercritical nonlinear Schrödinger equations (NLS) on the unit ball of $\mathbb{R}^3$ using two different approaches. First, for the NLS with algebraic nonlinearities with the subcritical initial data, we show the almost sure global well-posedness and the invariance of the underlying measures, and establish controls on the growth of Sobolev norms of the solutions. This global result is based on a deterministic local theory and a probabilistic globalization. Second, for the NLS with generic power nonlinearities with critical and supercritical initial conditions, we prove the almost sure

global well-posedness, and the invariance of the measure under the solution flows. This global result is built on a compactness argument and the Skorokhod representation theorem.   (Received September 01, 2020)

1163-35-312   **Aida Britton** and **Xiaolong Han*** (`xiaolong.han@csun.edu`), Department of Mathematics, California State University, Northridge, CA 91330, and **Michael Murray** and **Chuong Tran**. *Nodal sets of Laplacian eigenfunctions.*
The nodal set of a Laplacian eigenfunction is the set on which the function vanishes. The nodal sets have co-dimension one. Their size and distribution in relation to the eigenvalues are the fundamental questions in mathematics and physics. We will survey some recent progress on these problems.   (Received September 02, 2020)

1163-35-323   **Gung-Min Gie**, **James P Kelliher** and **Anna L Mazzucato*** (`alm24@psu.edu`). *On Euler equations with in-flow and out-flow boundary conditions.* Preliminary report.
I will discuss several results concerning the incompressible Euler equations when in-flow and out-flow boundary conditions are imposed on parts of the boundary.   (Received September 02, 2020)

1163-35-325   **Bjoern Bringmann*** (`bringmann@math.ucla.edu`). *Invariant Gibbs measure for the three-dimensional wave equation with a Hartree nonlinearity.*
In this talk, we discuss the invariance of the Gibbs measure for the three-dimensional wave equation with a Hartree nonlinearity. The most interesting aspect is the singularity of the Gibbs measure with respect to the Gaussian free field. The singularity has several consequences in both measure-theoretic and dynamical aspects of our argument. Most importantly, we rely on a novel adaptation of Bourgain's globalization argument.   (Received September 02, 2020)

1163-35-350   **Seongmin Jeon**, **Arshak Petrosyan** and **Mariana Smit Vega Garcia*** (`mariana.smitvegagarcia@wwu.edu`). *Almost minimizers for obstacle problems.*
In the applied sciences one is often confronted with free boundaries, which arise when the solution to a problem consists of a pair: a function u (often satisfying a partial differential equation), and a set where this function has a specific behavior. Two central issues in the study of free boundary problems and related problems in the calculus of variations and geometric measure theory are: (1) What is the optimal regularity of the solution u? (2) How smooth is the free boundary (or how smooth is a certain set related to u)? The study of the classical obstacle problem - one of the most renowned free boundary problems - began in the '60s with the pioneering works of G. Stampacchia, H. Lewy, and J. L. Lions. During the past decades, it has led to beautiful developments, and its study still presents very interesting and challenging questions. In contrast to the classical obstacle problem, which arises from a minimization problem (as many other PDEs do), minimizing problems with noise lead to the notion of almost minimizers. In this talk, I will introduce obstacle type problems and overview recent developments in almost minimizers for the thin obstacle problem, illustrating techniques that can be used to tackle questions (1) and (2) in various settings.   (Received September 03, 2020)

1163-35-368   **Nikolay Tzvetkov*** (`nikolay.tzvetkov@u-cergy.fr`), 95000 Cergy, France. *Transport of gaussian measures by the flow of Hamiltonian PDE.*
The study of the transport of gaussian measures on Wiener spaces by transformations is a classical topic in probability theory. The Cameron-Martin theorem (published in 1944) provides a necessary and sufficient condition concerning the absolute continuity of the transported measure in the case when the transformation is a translation. The Cameron-Martin results was later extended to more general translations and also to more general transformations.

Unfortunately, the above mentioned abstract stochastic analysis results do not apply in the context of the flows of many classical Hamiltonian PDE (even in 1d) if one wishes to study the absolute continuity of the transport by the flow of natural gaussian measures. We recently developed an approach to solve the problem of the absolute continuity of the transported measure in several significant situations. In this talk we plan to survey these results. We also plan to present several remaining issues in this set of problems.   (Received September 04, 2020)

1163-35-387   **Hussain Ibdah*** (`hibdah@math.tamu.edu`), Department of Mathematics, Texas A&M University, College Station, TX. *Lipschitz regularity of solutions to various nonlinear, nonlocal parabolic PDEs.*
In this talk, I will discuss Lipschitz continuity of solutions to various nonlinear, nonlocal parabolic PDEs, building on the idea of propagation of moduli of continuity as introduced by Kiselev, Nazarov, Shterenberg and Volberg. I will start by showing that strong solutions to a modified Michelson-Sivashinsky equation remain Lipschitz for all

time, leading to a global regularity result. I will also extend such ideas to drift-diffusion systems in the presence of nonlocal terms and/or incompressibility constraints. In particular, I will reduce the global regularity problem of the incompressible Navier-Stokes equations to a one dimensional, nonlocal viscous Burgers-type boundary value problem on the half-line. I emphasize this is merely a "regularity criterion" and not a solution to the global regularity problem. To demonstrate the applicability of the method, I will analyze a multi-dimensional, viscous, Burgers-Hilbert problem and obtain a global regularity result in this case. If time permits, I will also explain how to upgrade supercritical (or critical) Holder type regularity assumptions on solutions to the incompressible NSE to logarithmic integrability (or integrability) in time of the Lipschitz constant (respectively), extending a previous result of Silvestre and Vicol.   (Received September 04, 2020)

1163-35-395   **Michael Goldberg** and **William Green*** (`green@rose-hulman.edu`), 5500 Wabash Ave, Terre Haute, IN 47803. *On the L^p boundedness of the wave operators for fourth order Schrödinger operators.*

We consider the fourth order Schrödinger operator $H = \Delta^2 + V(x)$ in three dimensions with real-valued potential V. With $H_0 = \Delta^2$, the wave operators are defined by

$$W_\pm = s - \lim_{t \to \pm\infty} e^{itH} e^{-itH_0}$$

The L^p boundedness of the wave operators is of interest in part due to the intertwining identity, which allows one to deduce L^p based estimates for operator-valued functions $f(H)P_{ac}(H)$ based on estimates for the simpler operator $f(\Delta^2)$.

   We show that if V decays sufficiently and there are no eigenvalues or resonances in the absolutely continuous spectrum of H then the wave operators extend to bounded operators on $L^p(\mathbb{R}^3)$ for all $1 < p < \infty$.   (Received September 04, 2020)

1163-35-410   **Sean D Brooks*** (`sbrooks@coppin.edu`), 3517 Meadowdale Drive, Windsor Mill, MD 21244. *Linear Shallow Water Theory: An investigation at the beach.*

One of Professor James A. Donaldson's major contributions to mathematics is his work in Linear Shallow Water Theory. Here, we discuss some of his contributions to the theory. We further investigate a problem that was of interest to him. In particular, the problem involving a singularity arising in Shallow Water Theory.   (Received September 05, 2020)

1163-35-427   **Mimi Dai*** (`mdai@uic.edu`). *Blow-up of dyadic models of Hall MHD with intermittency parameter.*

Dyadic models for the Hall magnetohydrodynamics with intermittency parameter are derived. For such models, existence of local strong solution is obtained; while global strong solution is obtained in the case of high intermittency dimension. Moreover, we show that positive solution with large initial data develops blow-up in finite time provided the intermittency dimension is lower than a threshold.   (Received September 06, 2020)

1163-35-459   **Tadele Mengesha*** (`mengesha@utk.edu`), Then University of Tennessee, Knoxville. *Calderon-Zygmund type estimates for nonlocal PDE with Holder continuous kernel.*

In this talk I will present a result on L^p-regularity of weak solutions to linear nonlocal equation. To be precise, we study solutions of $\mathcal{L}_K^s u = f$ where the nonlocal operator is given by $\mathcal{L}_K^s u(x) = - \int_{\mathbb{R}^n} K(x,y) \frac{u(x) - u(y)}{|x-y|^{n+2s}} \, dy$. We prove that for $s \in (0,1)$, $t \in [s, 2s]$, $p \in [2, \infty)$, K an elliptic, symmetric, and $K(\cdot, y)$ is <u>uniformly Hölder</u> <u>continuous</u>, the solution u belongs to $H_{loc}^{2s-t,p}(\Omega)$ as long as $2s - t < 1$ and $f \in \left\{ H_{loc}^{t,p'}(\mathbb{R}^d) \right\}^*$. The increase in differentiability and integrability is independent of the Hölder coefficient of K. For example, in the event that $f \in L_{loc}^p$, we can deduce that the solution $u \in H_{loc}^{2s-\delta,p}$ for any $\delta \in (0,s]$ as long as $2s - \delta < 1$. The proof uses a perturbation argument where regularity of solutions of a simpler equation is systematically used to obtained a desired estimate.

   This is a joint work with Armin Schikorra and Sasikarn Yeepo at University of Pittsburgh.   (Received September 07, 2020)

1163-35-547   **Daniela De Silva***, desilva@math.columbia.edu. *Perturbative estimates for the one-phase Stefan Problem.*

We provide perturbative estimates for the one-phase Stefan free boundary problem. We obtain the regularity of flat free boundaries via a linearization technique in the spirit of the elliptic counterpart we developed in an earlier paper, which has proved very useful in a variety of settings.   (Received September 09, 2020)

1163-35-579   **Nalin Fonseka*** (fonsekan@carolinau.edu), 1825 Sherwood St, Apt C, Greensboro, NC
                     NC 27403, and **Ratnasingham Shivaji**, **Jerome Goddard** and **Byungjae Son**. *A
                     diffusive weak Allee effect model with U-shaped emigration and matrix hostility.*
We study positive solutions to steady-state reaction-diffusion equations of the form:

$$-\Delta u = \lambda f(u);\ \Omega$$
$$\alpha(u)\frac{\partial u}{\partial \eta} + \gamma\sqrt{\lambda}[1 - \alpha(u)]u = 0;\ \partial\Omega$$

where u is the population density, $f(u) = \frac{1}{a}u(u + a)(1 - u)$ represents a weak Allee effect type growth of the
population with $a \in (0, 1)$, $\alpha(u)$ is the probability of the population staying in the habitat Ω when it reaches the
boundary, and positive parameters λ and γ represent the domain scaling and effective exterior matrix hostility,
respectively. In particular, we analyze the case when $\alpha(s) = \frac{1}{[1+(A-s)^2+\epsilon]}$ for all $s \in [0, 1]$, where $A \in (0, 1)$
and $\epsilon \geq 0$. In this case $1 - \alpha(s)$ represents a U-shaped relationship between density and emigration. Existence,
nonexistence, and multiplicity results for this model are established via the method of sub-super solutions.
(Received September 10, 2020)

1163-35-597   **Zachary T Hilliard*** (hilliardzt@cofc.edu), Charleston, SC 29401, and **Alex Kasman**
                     (kasmana@cofc.edu), Charleston, SC 29401. *Tail Behavior and Non-Uniqueness of
                     Quaternion-Valued KdV 1-Solitons.* Preliminary report.
A recent paper developed methods for producing quaternion-valued solutions to the KdV Equations, but many
open questions of interest remain. This talk will address two questions concerning the 1-soliton case: one about
the behavior of the tails of breather solutions and the other on the choice of two sets of parameters that produce
the same solution. The main results thus far have been the development of formulas for the tail behavior and the
identification of sets of algebraic equations for the parameters whose solution resolve many of the open questions.
(Received September 10, 2020)

1163-35-607   **Robert Stephen Cantrell*** (rsc@math.miami.edu), Department of Mathematics, The
                     University of Miami, 1365 Memorial Drive, Room 515C, Coral Gables, FL 33146, and
                     King-Yeung Lam. *On the evolution of slow dispersal in multi-species communities.*
For any $N \geq 2$, we show that there are choices of diffusion rates $d_i, 1 \leq i \leq N$, such that for a model for N
competing species in a closed habitat patch, which are ecologically identical and have distinct diffusion rates,
the slowest disperser is able to competitively exclude the remainder of the species. In fact, the choices of such
diffusion rates are open in the Hausdorff topology. Our result provides evidence in the affirmative direction
regarding the well-known 1998 conjecture of Dockery, Hutson, Mischaikow and Pernarowksi. The main tools
include the Morse decomposition of the semi-flow associated with the model and the theory of a normalized
principal bundle for linear parabolic equations.   (Received September 10, 2020)

1163-35-652   **Jinkai Li*** (jklimath@m.scnu.edu.cn) and **Zhouping Xin** (zpxin@ims.cuhk.edu.hk).
                     *Entropy-bounded solutions to the compressible Navier-Stokes equations with far field
                     vacuum.*
The entropy is one of the fundamental physical states of a fluid. For the ideal gases, it can be expressed as
a certain linear combination of the logarithms of the density and temperature in the non-vacuum region, and,
in the viscous case, it satisfies an equation of highly singular in the region close to the vacuum. Due to the
singularity of the logarithmic function at zero and the singularity of the entropy equation near the vacuum
region, the mathematical analyses on the behavior of the entropy near the vacuum region, were rarely carried
out; in particular, in the presence of vacuum, it was unknown if the entropy remains its boundedness. It will
be shown in this talk that the ideal gases retain their uniform boundedness of the entropy, locally or globally in
time, if the vacuum occurs at the far field only and the density decays slowly enough at the far field. Precisely, we
consider the Cauchy problem to the full compressible Navier-Stokes equations, with or without heat conductivity,
and establish the local and global existence and uniqueness of solutions with uniformly bounded entropy in space
at each time slice, in the presence of vacuum at the far field only. These are joint works with Prof .Zhouping
Xin.   (Received September 10, 2020)

1163-35-692   **B. Dodson**, **T. Spencer** and **Avy Soffer*** (soffer@math.rutgers.edu). *Evolution of
                     NLS with Bounded Initial Data.*
We study the nonlinear Schrodinger equation (NLS) with bounded initial data which does not vanish at infinity.
Examples include periodic, quasi-periodic and random initial data. On the lattice we prove that solutions are
polynomially bounded in time for any bounded data. In the continuum, local existence is proved for real analytic
data by a Newton iteration scheme. Global existence for NLS with a regularized nonlinearity follows by analyzing

a local energy norm (arXiv:2003.08849 [math.AP], J.Stat.Phys, 2020). This is a joint work with Ben Dodson and Tom Spencer.   (Received September 11, 2020)

1163-35-705   **Satoshi Masaki**, **Jason Murphy*** (`jason.murphy@mst.edu`) and **Jun-ichi Segata**.
               Asymptotic stability of solitary waves for the 1d NLS with an attractive delta potential.
We consider the 1d nonlinear Schrödinger equation with an attractive delta potential and mass-supercritical nonlinearity. This equation admits a one-parameter family of solitary wave solutions in both the focusing and defocusing cases. We will discuss some recent progress on the problem of asymptotic stability for these solitary waves. This is joint work with S. Masaki and J. Segata.   (Received September 11, 2020)

1163-35-759   **Rachidi Salako**, **Wenxian Shen*** (`wenxish@auburn.edu`) and **Shuwen Xue**. *Can chemotaxis speed up or slow down the spatial spreading in parabolic-elliptic Keller-Segel systems with logistic source?*
This talk is concerned with the spatial spreading speed of the following Keller-Segel chemoattraction system,

$$\begin{cases} u_t = u_{xx} - \chi(uv_x)_x + u(a - bu), & x \in \mathbb{R}, \\ 0 = v_{xx} - \lambda v + \mu u, & x \in \mathbb{R}, \end{cases}$$

where χ, a, b, λ, and μ are positive constants, and $u(t,x)$ and $v(t,x)$ represent the population densities of a mobile species and a chemo-attractant, respectively. It is well known that, in the absence of chemotaxis (i.e. $\chi = 0$), the population of the mobile species spreads at the asymptotic speed $c_0^* = 2\sqrt{a}$. It will be shown in this talk that the chemotaxis neither speeds up nor slows down the spatial spreading of the mobile species provided that the logistic damping constant b is large relative to the chemotaxis sensitivity coefficient χ.   (Received September 12, 2020)

1163-35-764   **Chris Cosner*** (`gcc@math.miami.edu`). *Movement models with switching.*
In classical models for the dispersal of animals in continuous time and space a single advection-diffusion operator or nonlocal integral operator is typically used to model dispersal. Actual animals are often observed to switch between two or more different movement modes for large scale search to locate resources and for small scale search and exploitation once they are located. This talk will give an overview of several papers on models for populations with two or more movement modes. A novel feature of these models is that when combined with population dynamics they can lead to reaction-diffusion systems which are cooperative at low densities but competitive at higher densities, which creates some mathematical challenges in using monotone dynamical systems theory to study them.   (Received September 12, 2020)

1163-35-775   **Shijun Zheng*** (`szheng@georgiasouthern.edu`), Department of Mathematical Sciences, Georgia Southern University, P.O.Box 8093, Statesboro, GA 30460, and **Ting-Jian Luo** and **Shi-Hui Zhu**. *Stability of Solitary Waves for Higher-order Hamiltonians.*
The soliton phenomenon for higher-order PDE arises in wave propagation in intense laser beams for quasi-particles in magnetic medium as well as vibration of beams modeling. We will mainly address the orbital stability problem for such solitons. We use profile decomposition method to give rigorous construction of ground state solutions as well as the threshold dynamics for the bi-harmonic NLS and rotating Bose-Einstein condensation. The modeling equation can be derived from Noether's theorem. The proof relies on profile decomposition and scaling, translation and rotation invariance that are intrinsic symmetries of the system. Numerical simulations will also be presented.   (Received September 12, 2020)

1163-35-813   **Chenjie Fan*** (`fancj@amss.ac.cn`), Academy of Mathematics and Systems Science, Beijing, Peoples Rep of China, and **Weijun Xu** and **Zehua Zhao**. *On wellposedness of mass critical stochastic NLS.*
We will present some recent results of the wellposedness of mass critical stochastic NLS, including both local and global behavior. The talk is based on joint work with Weijun Xu and joint work with Zehua Zhao.   (Received September 13, 2020)

1163-35-824   **Albert Ai**, **Mihaela Ifrim** and **Daniel Tataru*** (`tataru@math.berkeley.edu`). *Low regularity well-posedness for water waves.*
The aim of this talk will be to provide a brief overview of recent results on low regularity well-posedness results for gravity waves in deep water. This is a fully nonlinear, nonlocal evolution which describes the motion of an incompressible, inviscid, irotational fluid with a free boundary. This is joint work with Albert Ai and Mihaela Ifrim.   (Received September 13, 2020)

1163-35-872   **Jill Pipher*** (`jill_pipher@brown.edu`), Dept. of Mathematics, 151 Thayer St.,
Providence, RI 02912. *Boundary value problems for elliptic complex coefficient operators
and systems in the presence of p-ellipticity.*

Let $\mathcal{L} = \mathrm{div}A\nabla$ be a second order elliptic operator, where A is a matrix of bounded measurable complex-valued functions. With M. Dindos, we formulated a condition, *p-ellipticity*, on complex-valued matrices in order to study regularity of solutions to operators like $\mathcal{L}$, borrowing the term from Carbonaro and Dragičević who simultaneously introduced this condition in their study of bilinear embeddings. Our formulation was inspired by work of Cialdea and Mazya on L^p-dissipativity, and we were able to prove higher integrabilty and regularity of solutions via a Moser iteration argument. In this talk we explain the role of $p - ellipticity$ in obtaining solvability of boundary value problems for these complex-valued divergence form equations, and a recent extension of this concept to elliptic systems that is joint work with Martin Dindos and Jungang Li   (Received September 13, 2020)

1163-35-874   **Luke Anthony McLennan***, 2800 West Gore Blvd, Lawton, OK 73505, and **Narayan
Thapa**, 2800 W. Gore Blvd, Lawton, OK 73505. *Numerical Solutions to Navier-Stokes
Initial and Boundary Value Problems with External Force.* Preliminary report.

The Navier-Stokes Equations (NSE) are important partial differential equations which govern fluid dynamics. In this presentation, we discuss a method for numerically solving the 2D Navier-Stokes equations with an external force term. We use the vorticity-stream formulation of the NSE. The Finite Difference Method will be employed in order to solve the equations. We develop computational algorithms and display numerical results. In addition, we present and discuss the accuracy and stability of the implemented methods.   (Received September 13, 2020)

1163-35-885   **Nsoki Mavinga*** (`nmaving1@swarthmore.edu`), **Quinn A. Morris**
(`morrisqa@appstate.edu`) and **Stephen B. Robinson** (`sbr@wfu.edu`). *Existence Results
for Nonlinear Perturbations of Asymmetric Spectrum with Weights.*

We consider a nonlinear elliptic equations with nonlinear boundary conditions and prove existence theorems for both the resonance and nonresonance cases relative to the asymmetric Spectrum with weights. For the resonance case, we provide a sufficient condition, the so-called generalized Landesman-Lazer condition, for the solvability. The proofs are based on variational methods and rely strongly on the variational characterization of the Spectrum.   (Received September 13, 2020)

1163-35-925   **Tracey Balehowsky*** (`tracey.balehowsky@helsinki.fi`), **Antti Kujanpää**, **Matti
Lassas** and **Tony Liimatainen**. *Determining a Lorentzian metric from the
source-to-solution map for the relativistic Boltzmann equation.* Preliminary report.

In this talk, we consider the following inverse problem: Given the source-to-solution map for a relativistic Boltzmann equation on a neighbourhood V of an observer in a Lorentzian spacetime (M, g) and knowledge of $g|_V$, can we determine (up to diffeomorphism) the spacetime metric g on the diamond of causal influence for the set V?

We will show that the answer is yes for certain cases. We first will introduce the relativistic Boltzmann equation and the concept of an inverse problem. We then will highlight the key ideas of the proof of our main result. One such key point is that the nonlinear term in the relativistic Boltzmann equation which describes the behaviour of particle collisions captures information about a source-to-solution map for a related linearized problem. We use this relationship together with an analysis of the behaviour of particle collisions by classical microlocal techniques to determine the set of locations in V where we first receive light particle signals from collisions in the unknown domain. From this data we are able to parametrize the unknown region and determine the metric.   (Received September 14, 2020)

1163-35-927   **Dan Andrei Geba***, University of Rochester, Department of Mathematics, 806 Hylan
Building, Rochester, NY 14627, and **Bai Lin**, University of Rochester, Department of
Mathematics, 910 Hylan Building, Rochester, NY 14627. *Unconditional well-posedness for
the Kawahara equation.*

This talk is concerned with the unconditional well-posedness for the Kawahara equation on the real line and shows that this holds true for initial data in $L^2(\mathbb{R})$. This is achieved by applying an infinite iteration scheme of normal form reductions.   (Received September 14, 2020)

1163-35-956   **Gong Chen*** (`gc@math.toronto.edu`) and **Qingtang Su** (`qingtang@usc.edu`). *Local
well-posedness of 2d water waves with moving obstacles.* Preliminary report.

We prove the local well-posedness of 2d water waves with moving obstacles.   (Received September 14, 2020)

1163-35-961   **K.D. Chu**, **D.D. Hai** and **Ratnasingham Shivaji*** (`r_shivaj@uncg.edu`), Dept of Mathematics & Statistics, University of North Carolina at Greensboro, 116 Petty, 317 College Ave, Greensboro, NC 27412. *A uniqueness result for infinite semipositone p-Laplacian problems in a ball.*

We prove uniqueness of positive radial solutions to a class of singular p-Laplacian equations in a ball with Dirichlet boundary condition when a parameter is large. The reaction term exhibits infinite semipositone structure at zero and is not necessarily increasing or concave on $(0, \infty)$.   (Received September 14, 2020)

1163-35-988   **Allaberen Ashyralyev*** (`allaberen.ashyralyev@neu.edu.tr`), Near East University, Near East Boulevard, 99138 Nicosia / TRNC Mersin 10, Turkey, Nicosia, Turkey, **Maksat Ashyraliyev** (`maksat.ashyralyyev@eng.bau.edu.tr`), Istanbul, Turkey, and **Maral A. Ashyralyyeva** (`ashyrmaral2010@mail.ru`), Ashgabat, Turkmenistan. *Stability of identification problems for the hyperbolic-parabolic equation.*

Authors: Allaberen Ashyralyev, Near East University, M.Ashyraliyev, Bahcheshir University and M. A. Ashyralyyeva, Turkmen State University Title: Stability of identification problems for the hyperbolic-parabolic equation Abstract: The theory of partial differential equations of mixed type with boundary conditions originated in the fundamental research of Tricomi [1]. Local and nonlocal problems for mixed type partial differential equations have been investigated by many scientists. However, identification problems for mixed type of differential equations have not been investigated [2]. In the present paper, the stability of identification hyperbolic-parabolic problems is established. Absolute stable difference schemes for the numerical solution of identification hyperbolic-parabolic problems are presented. The stability of these difference scheme are proved. Numerical results are given. [1]. F. G. Tricomi, Atti Accad. Naz. dei Lincei, 14 (5), 133-247(1923). [2]. M. Ashyraliyev, A. Ashyralyev, M. Ashyralyyeva, Computational Mathematics and Mathematical Physics, 60 (8) 49–60 (2020). Russian. (Received September 14, 2020)

1163-35-1058   **Helge Kristian Jenssen** (`jenssen@math.psu.edu`) and **Charis Tsikkou*** (`tsikkou@math.wvu.edu`). *Amplitude blowup in radial isentropic Euler flow.*

We show that the compressible Euler system for isentropic gas flow admits unbounded solutions. The examples are radial flows of similarity type and describe a spherically symmetric and continuous wave moving toward the origin. At time of focusing, both the density and the velocity become unbounded at the origin. This is followed by an expanding shock wave which slows down as it interacts with the incoming flow.

   While unbounded radial Euler flows have been known since the work of Guderley (1942), those are at the borderline of the regime covered by the Euler model: the upstream pressure field vanishes identically (either because of vanishing temperature or vanishing density there). In contrast, the solutions we build exhibit an everywhere strictly positive pressure field, demonstrating that the geometric effect of wave focusing is strong enough on its own to drive the primary flow variables to infinity.   (Received September 14, 2020)

1163-35-1061   **Allaberen Ashyralyev** and **Abdullah S. Erdogan*** (`aserdogan@gmail.com`), Palm Beach State College, Palm Beach Gardens, FL 33410, and **M. Zuhair Nashed**. *Investigation of regularization methods for a source identification problem in the heat equation using operator approach.*

In this talk, an inverse problem for identifying a space-dependent heat source in a multidimensional heat equation is considered. Simplified version of the Tikhonov regularization method and modified version of the Lavrentiev regularization method are proposed and studied by the operator theory approach. The theoretical results are supported by an example.   (Received September 14, 2020)

1163-35-1068   **Stefania Patrizi*** (`spatrizi@math.utexas.edu`), TX. *From the Peierls-Nabarro model to the equation of motion of the dislocation continuum.*

We consider a semi-linear integro-differential equation in dimension one associated to the half Laplacian whose solution represents the atom dislocation in a crystal. The equation comprises the evolutive version of the classical Peierls-Nabarro model. We show that for a large number of dislocations, the solution, properly rescaled, converges to the solution of a well-known equation called in the physics literature "the equation of motion of the dislocation continuum". The limit equation is a model for the macroscopic crystal plasticity with density of dislocations. In particular, we recover the so called Orowan's law which states that dislocations move at a velocity proportional to the effective stress. This is a joint paper with Tharathep Sangsawang.   (Received September 14, 2020)

1163-35-1087   **Jerome A. Goldstein*** (jgoldste@memphis.edu), Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152. *Jim Donaldson, and old and new results on singular Cauchy problems.* Preliminary report.

The story behind my joint paper with Jim Donaldson (PAMS, 1976) is an interesting one. I shall briefly recall that, then mention some newer and some brand new results on Various issues related to singular linear and nonlinear PDE, including instantaneous blowup.   (Received September 14, 2020)

1163-35-1090   **Yoichiro Mori*** (y1mori@sas.upenn.edu), Philadelphia, PA 19104. *The 2D Peskin Problem of an Immersed Thin Elastic Structure in Stokes Flow.*

The Peskin problem, in which a thin elastic structure interacts with the surrounding fluid, is a prototypical problem in fluid-structure interaction, and is arguably one of the simplest of such problems. In this talk, I will discuss our efforts on the well-posedness and behavior of the solution for different variants of this problem in near-critical and critical function spaces. The work to be presented is a result of collaborations with Analise Rodenberg, Daniel Spirn, Eduardo Garcia-Juarez and Robert Strain.   (Received September 14, 2020)

1163-35-1101   **Zaher Hani***, Mathematics Department, University of Michigan, 530 Church Street, Ann Arbor, MI 48109. *On the derivation of the wave kinetic equation for nonlinear dispersive equations.*

Wave turbulence theory conjectures that the long time behavior of (suitably) generic solutions of nonlinear dispersive PDE is governed by a kinetic equation (called the wave kinetic equation) in a similar fashion to how Boltzmann's equation describes the long time effective dynamics of a particle gas. We will survey some recent progress on this topic starting with a joint work with Yu Deng (USC), and touching on recent progress with J. Shatah and S. Rydin Myerson.   (Received September 14, 2020)

1163-35-1113   **Nicolas Garcia Trillos*** (garciatrillo@wisc.edu), 1664 Monroe Street, Apt D, Madison, WI 53711. *Regularity theory and uniform convergence in the large data limit of graph Laplacian eigenvectors on random data clouds.*

Graph Laplacians are omnipresent objects in machine learning that have been used in supervised, unsupervised and semi supervised settings due to their versatility in extracting local and global geometric information from data clouds. In this talk I will present an overview of how the mathematical theory built around them has gotten deeper and deeper, layer by layer, since the appearance of the first results on pointwise consistency in the 2000's, until the most recent developments. This line of research has found strong connections between PDEs built on proximity graphs on data clouds and PDEs on manifolds, and has given a more precise mathematical meaning to the task of "manifold learning". I will give particular emphasis to recent work with Jeff Calder and Marta Lewicka, where we use newly developed regularity theory for graph Laplacians in order to obtain uniform and almost $C^{\{0,1\}}$ convergence rates of eigenvectors of graph Laplacians on proximity graphs towards eigenfunctions of Laplace-Beltrami counterparts.   (Received September 14, 2020)

1163-35-1131   **Michele Coti Zelati**, **Nathan Glatt-Holtz** and **Konstantina Trivisa*** (trivisa@umd.edu). *Invariant measures for the stochastic one-dimensional compressible Navier-Stokes equations.*

We investigate the long-time behavior of solutions to a stochastically forced one-dimensional Navier-Stokes system, describing the motion of a compressible viscous fluid, in the case of linear pressure law. We prove existence of an invariant measure for the Markov process generated by strong solutions. We overcome the difficulties of working with non-Feller Markov semigroups on non-complete metric spaces by generalizing the classical Krylov-Bogoliubov method, and by providing suitable polynomial and exponential moment bounds on the solution, together with pathwise estimates.   (Received September 14, 2020)

1163-35-1133   **Zachary Bradshaw*** (zb002@uark.edu), **Igor Kukavica**, **Tai-Peng Tsai** and **Dallas Albritton**. *Fluid equations in weighted spaces.*

The scaling properties of many fluid models imply scaling invariant initial data do not exist within the classical Lebesgue space framework. Weighted spaces provide a large and powerful setting to analyze large scaling invariant and generic solutions. We survey results in this direction on global existence and eventual regularity of infinite energy solutions for the Navier-Stokes equations with non-decaying data (joint work with Igor Kukavica and Tai-Peng Tsai) as well as global existence of generic and special solutions to the critical dissipative surface quasi-geostrophic equations (joint work with Dallas Albritton).   (Received September 14, 2020)

1163-35-1178   **Cecilia F Mondaini*** (`cf823@drexel.edu`) and **Nathan Glatt-Holtz**. *Mixing for Hamiltonian Monte Carlo in infinite dimensions.*

The analysis of convergence/mixing rates for testing sampling efficiency in Markov Chain Monte Carlo (MCMC) algorithms, a fundamental question for establishing their range of applicability, has gained increased attention recently. Of particular interest are MCMC methods that are designed to be well-defined in infinite dimensions, a property that allows them to overcome the curse of dimensionality when applied to corresponding finite-dimensional approximations. We analyze such question for an infinite-dimensional version of the Hamiltonian/Hybrid Monte Carlo algorithm, for which mixing rates had been an open problem until being very recently addressed via an exact coupling approach. Our proof uses the weak Harris theorem together with a generalized coupling argument, providing a flexible methodology to establish mixing rates for other MCMC algorithms. Furthermore, as an application of our general result, we show that all required assumptions can be verified in the context of a Bayesian inversion approach to advection-diffusion type PDEs. This is a joint work with Nathan Glatt-Holtz (Tulane U). (Received September 15, 2020)

1163-35-1182   **Svetlana Roudenko***, Department of Mathematics and Statistics, Florida International University, Miami, FL 33199. *Behavior of solutions to the 1D focusing stochastic nonlinear Schrödinger equation.*

We discuss solutions behavior to the focusing stochastic nonlinear Schrödinger equation in 1D with perturbations driven by various noises in the L^2-critical and supercritical settings. We consider additive and multiplicative perturbations driven by space-time white noise and multiplicative noise driven by a Wiener process white in time and colored in space. While the Hamiltonian is no longer conserved in the stochastic setting, the mass is conserved in the multiplicative case due to the Stratonovich integral definition. We describe the influence of the noise on the global dynamics measuring the probability of blow-up versus scattering behavior depending on various parameters, e.g., correlation kernels. We then discuss the effect of the noise on the blow-up behavior: our numerical studies show that such random perturbations do not influence the blow-up dynamics, except for shifts of the blow-up center location being a Gaussian random variable. This is a joint work with Annie Millet, Alex D. Rodriguez and Kai Yang. (Received September 15, 2020)

1163-35-1206   **Alemdar Hasanoglu*** (`alemdar.hasanoglu@gmail.com`), Sehit Ekrem Dsitrict, Altunşehir Str., 41030, Kocaeli, Turkey. *Reconstruction of the principal coefficient in the damped wave equation from Dirichlet-to-Neumann operator.*

A novel approach is proposed for studying the inverse coefficient problem of identifying the principal coefficient $r(x) > 0$ in the damped wave equation $m(x)u_{tt} + \mu(x)u_t = (r(x)u_x)_x$, $(x,t) \in \Omega_T := (0,\ell) \times (0,T)$ subject to the boundary conditions $u(0,t) = s(t)$, $u(\ell,t) = 0$, from the Neumann boundary output $f(t) := r(0)u_x(0,t)$, $t \in (0,T]$. We propose detailed microlocal analysis of the regularity of the solution of the wave equation in each subdomain $D_n \subset \Omega_T$ defined by the characteristics of the wave equation. Based on this analysis we prove the compactness and Lipschitz continuity of the Dirichlet-to-Neumann operator corresponding to the inverse problem. The last property allows us to prove an existence of a quasi-solution of the inverse problem defined as a minimum of the Tikhonov functional and also Fréchet differentiability of this functional. A uniqueness theorem is derived. An explicit formula for the Fréchet gradient of the Tikhonov functional and its justification are derived by making use of the unique solution to corresponding adjoint problem. (Received September 15, 2020)

1163-35-1213   **Ajay Chandra**, **Augustin Moinat** and **Hendrik Weber*** (`h.weber@bath.ac.uk`), Department of Mathematical Sciences, University of Bath, Bath, BA2 7AY, United Kingdom. *A priori bounds for Singular Stochastic PDEs.*

Initiated in groundbreaking works by Hairer and Gubinelli, the theory of "Singular Stochastic PDEs" has made spectacular progress over the last few years. A systematic solution theory for various interesting and previously intractable equations from Mathematical Physics is now available. Examples include the KPZ equation and the stochatic quantisation equations for the 3D Φ^4 and Yang Mills measures.

    The main focus of these works so far was the systematic description of solutions on small scales, and the construction of solutions for short times. In this talk I will show how to obtain matching a priori bounds that rule out the possibility of finite time blow up in the example of the Φ^4 equation. (Received September 15, 2020)

1163-35-1214   **Charles Collot*** (`ccollot@cyu.fr`), 2 avenue Adolphe Chauvin, UFR Sciences et
Techniques, Laboratoire AGM, 95302 Cergy-Pontoise, France, and **Pierre Germain**. *The
role of the dispersion relation in the derivation of the kinetic wave equation.*
We consider the derivation of the kinetic wave equation, as an effective equation from the nonlinear Schrodinger
equation (NLS) for the microscopic description of a system. The regime is weakly nonlinear, on a torus in any
dimension greater than two, and for highly oscillatory random Gaussian fields as initial data. A conjecture
in statistical physics is that there exists a kinetic time scale on which, statistically, the Fourier modes evolve
according to the kinetic wave equation.

Several parameters are involved. First, the oscillation length of the field, and the strength of the nonlinearity,
and it is still unknown in which regimes is the kinetic wave equation rigorously valid. Second, the coefficients of
the dispersion relation - alternatively, the geometry of the torus. The distribution properties of the associated
quadratic form on integer points are directly related to the structure of the resonant terms in the dynamics.

We approach the problem of the validity of the kinetic wave equation via the convergence and stability of
the corresponding Dyson series. We prove that for certain regimes and dispersion relations the series diverges
before the kinetic time. In others, we show its convergence and control the full solution almost up to the kinetic
time.   (Received September 15, 2020)

1163-35-1293   **Jared P Whitehead*** (`whitehead@mathematics.byu.edu`). *Data Assimilation in Large
Prandtl Rayleigh-Benard Convection from Thermal Measurements.*
This work applies a continuous data assimilation scheme—a framework for reconciling sparse and potentially
noisy observations to a mathematical model—to Rayleigh–Benard convection at infinite or large Prandtl numbers
using only the temperature field as observables. These Prandtl numbers are applicable to the earth's mantle
and to gases under high pressure. We rigorously identify conditions that guarantee synchronization between the
observed system and the model, then confirm the applicability of these results via numerical simulations. Our
numerical experiments show that the analytically derived conditions for synchronization are far from sharp; that
is, synchronization often occurs even when sufficient conditions of our theorems are not met. We also develop
estimates on the convergence of an infinite Prandtl model to a large (but finite) Prandtl number generated set of
observations. Numerical simulations in this hybrid setting indicate that the mathematically rigorous results are
accurate, but of practical interest only for extremely large Prandtl numbers.   (Received September 15, 2020)

1163-35-1299   **Sarah Strikwerda*** (`slstrikw@ncsu.edu`) and **Lorena Bociu** (`lvbociu@ncsu.edu`).
Optimal Control in Fluid Flows through Deformable Porous Media. Preliminary report.
We consider an optimal control problem subject to a nonlinear poro-visco-elastic model with applications to fluid
flows through biological tissues. In particular, our goal is to optimize the fluid pressure using either distributed or
boundary control. We present results on the existence of optimal control and the associated necessary optimality
conditions.   (Received September 15, 2020)

1163-35-1316   **J. T. Cronin** (`jcronin@lsu.edu`), Department of Biological Sciences, Louisiana State
University, Baton Rouge, LA 70803, **J Goddard II** (`jgoddard@aum.edu`), Department of
Math. & Stat., C. Comp., Auburn University Montgomery, Montgomery, AL 6124-023, **A
Muthunayake*** (`akmuthun@uncg.edu`), Department of Mathematics and Statistics,
University of North Carolina at Greensboro, 206 Petty Building, 317 College Ave,
Greensboro, NC 27412, and **R Shivaji** (`r_shivaj@uncg.edu`), Department of Mathematics
and Statistics, University of North Carolina at Greensboro, Greensboro, NC 27412.
Modeling the effects of trait-mediated dispersal on coexistence of mutualists.
We analyse positive solutions (u, v) to the steady state reaction diffusion system:

$$\begin{cases} -\Delta u = \lambda u(1-u); \ \Omega \\ -\Delta v = \lambda rv(1-v); \ \Omega \\ \frac{\partial u}{\partial \eta} + \sqrt{\lambda}g(v)u = 0; \ \partial\Omega \\ \frac{\partial v}{\partial \eta} + \sqrt{\lambda}h(u)v = 0; \ \partial\Omega \end{cases}$$

where $\lambda > 0$, $r > 0$ are parameters and $g, h \in C^1([0, \infty), (0, \infty))$ are decreasing functions. This system models
the steady states of two species living in a habitat where the interaction is limited to the boundary. Here,
λ is directly proportional to the size of the habitat and we will study the ranges of λ where coexistence and
nonexistence occurs. Namely, we will consider three cases: (a) $E_1(1, g(0)) = E_1(r, h(0))$, (b) $E_1(1, g(0)) >
E_1(r, h(0))$, (c) $E_1(1, g(0)) < E_1(r, h(0))$. Here $E_1(r, K)$ denotes the principal eigenvalue of: $-\Delta z = rEz$; Ω,
$\frac{\partial z}{\partial \eta} + K\sqrt{E}z = 0$; $\partial\Omega$.   (Received September 15, 2020)

1163-35-1356   **Terrance Pendleton*** (`terrance.pendleton@drake.edu`), **Ryan Hynd** (`rhynd@msri.org`) and **Dennis Ikpe** (`dikpe@msri.org`). *Two Critical Times for The SIR Model.* Preliminary report.

We consider the SIR model and study the first time the number of infected individuals begins to decrease and the first time this population is below a given threshold. We interpret these times as functions of the initial susceptible and infected populations and characterize them as solutions of a certain partial differential equation. This al- lows us to obtain integral representations of these times and in turn to estimate them precisely for large populations.   (Received September 15, 2020)

1163-35-1357   **Gigliola Staffilani*** (`gigliola@mit.edu`), 77 Massachusetts Avenue, Cambridge, MA 02139, and **Minh-Binh Tran** (`tranmbinh85@gmail.com`), Dallas, TX 75275. *On the wave turbulence theory for a random multidimensional KdV equation.* Preliminary report.

In this work we considered a multidimensional KdV type equation and we study the correlation function related to this equation in higher dimension.   (Received September 15, 2020)

1163-35-1396   **Mark Alun Lewis*** (`mark.lewis@ualberta.ca`), Dept Math/Stat Sciences and Dept Biological S, 632 CAB, University of Alberta, Edmonton, AB T6G 2G1, Canada. *Population Dynamics in Moving Environments.*

Classical population dynamics problems assume constant unchanging environments. However, realistic environments fluctuate in both space and time. My lecture will focus on the analysis of population dynamics in environments that shift spatially, due either to advective flow (eg., river population dynamics) or to changing environmental conditions (eg., climate change). The emphasis will be on the analysis of nonlinear advection-diffusion-reaction equations and related models in the case where there is strong advection and environments are heterogeneous. I will use methods of spreading speed analysis, net reproductive rate and inside dynamics to understand qualitative outcomes. Applications will be made to river populations in one- and two-dimensions and to genetic structure of populations subject to climate change as well as to small-scale experimental systems that have been developed to test the mathematical theory.   (Received September 15, 2020)

1163-35-1437   **Vincent R Martinez*** (`vrmartinez@hunter.cuny.edu`), 695 Park Ave, New York, NY 10065. *Higher-order synchronization for a nudging algorithm for 2D Navier-Stokes with nodal observables.*

The analytical study of a nudging algorithm in the infinite-dimensional setting of PDEs was initially carried out by Azouani, Olson, and Titi for the two-dimensional (2D) incompressible Navier-Stokes equations (NSE). In their seminal work, convergence of the approximating solution to the true solution was shown to take place at least in the topology of the Sobolev space H^1. However, this does not include uniform convergence. This talk will discuss convergence in stronger topologies, including the uniform topology, of this nudging based algorithm for data assimilation in the context of the 2D NSE when observations are given as on nodal values of the velocity field. This is joint work with Animikh Biswas (University of Maryland-Baltimore County) and Ken Brown (Hunter College).   (Received September 15, 2020)

1163-35-1450   **April K Tran*** (`chitran17@augustana.edu`), 628 42nd Street, Rock Island, IL 61201. *On the Stability of the Alternative Finite Difference Schemes.* Preliminary report.

While the well-researched Finite Difference Method (FDM) discretizes every independent variable into algebraic equations, Method of Lines discretizes all but one dimension, leaving an Ordinary Differential Equation (ODE) in the remaining dimension. That way, ODE's numerical methods can be applied to solve Partial Differential Equations (PDEs). In this project, Linear Multistep Methods and Method of Lines are used to numerically solve the Heat Equation. Specifically, the Adams-Bashforth method, Adams-Moulton method, and Backward Differentiation Formulas are implemented as the Alternative Finite Difference Schemes. Additionally, I examine their Region of Stability using the extended version of Von Neumann Stability Analysis for multiple timesteps. (Received September 15, 2020)

1163-35-1453   **Maria Ntekoume*** (`mntekoume@math.ucla.edu`). *Integrable dispersive PDE at low regularity.*

In this talk, we will discuss recent results in the intersection of dispersive PDE and completely integrable systems at low regularity.   (Received September 15, 2020)

1163-35-1476   **Daniel A Williams*** (dawilliams@howard.edu), 2627 Chivalry Court, Silver Spring, MD
20902. *My 35 year friendship with James Donaldson and collaboration with him on a
series of two papers on shallow water waves.*

In this talk I will make observations on my perspectives of James Donaldson during the period that I personally interacted with him as colleague in the mathematics department, as a fellow administrator, and as a collaborator in mathematical work on shallow water waves.   (Received September 15, 2020)

1163-35-1538   **José D Pastrana Chiclana*** (jose.pastrana@northwestern.edu). *Non-uniform
continuous dependence for Euler equations in Besov space.*

The motion of an ideal and incompressible fluid, in a domain $\Omega \subseteq \mathbb{R}^d$, is governed by the system of partial differential equations: $\partial_t u + \nabla_u u + \nabla p = 0$. Where incompressibility translates to div $u = 0$ and $u_0(x) := u(0, x)$ is the initial velocity field.

We will discuss the failure of uniform continuity (in time) of the solution map $u_0 \to u$ via a construction due to Himonas and Misiołek which utilizes the approximate solutions technique. The latter traces back to Kenig, Ponce and Vega when working on KdV type equations.   (Received September 15, 2020)

1163-35-1560   **Yu Deng**, **Andrea Nahmod** and **Haitian Yue*** (haitiany@usc.edu). *Random tensors,
propagation of randomness, and nonlinear Schrödinger equations.*

In this talk, we will discuss recent developments on the propagation of randomness, in the setting of random data problems for nonlinear Schrödinger equations. In particular, I will present a new framework called the theory of random tensors that proves almost-sure local well-posedness in the optimal ("probabilistic subcritical") range of regularity. This is work with Yu Deng (USC) and Andrea Nahmod (UMass Amherst).   (Received September 15, 2020)

1163-35-1590   **Shyla R Kupis*** (skupis@g.clemson.edu), Clemson University Martin Hall, M-306,
Clemson, SC 29634, **Vincent Barra** (vincent.barra@isima.fr), Aubiere, France, and
Taufiquar Khan (taufiquar.khan@uncc.edu), Charlotte, NC. *Machine Learning
Applications to the Electrical Impedance Tomography Inverse Problem.* Preliminary report.

Electrical impedance tomography (EIT) is a non-invasive and radiation-free imaging modality for the electrical conductivity distribution of a body using a set of electrodes to measure voltages on the boundary. We address the exponentially ill-posed and highly sensitive EIT inverse problem using machine learning applications. We solve the forward problem using convolutional neural networks and the inverse problem using deep neural networks. Our machine learning algorithm is then trained against a set of randomly simulated data using the complete electrode model for the forward problem with inclusions of various sizes at different locations. Our findings are then compared to the recovered solutions from a modified Gauss-Newton method.   (Received September 15, 2020)

1163-35-1594   **A Hasan**, **N Rodriguez** and **L Wong*** (lyndsey.wong@colorado.edu). *Finding Pattern
Formation in a Model for Wealth and Amenities.*

Gentrification refers to the influx of income into a community leading to the improvement of an area through renovation or the introduction of local amenities. This is usually accompanied by an increase in the cost of living, which displaces lower income populations. To better understand this problem, we will introduce a model for the dynamics of wealth and amenities. In order to find when we have inhomogeneous solutions to this model, we present two approaches. The first is to perform a linear stability analysis in order to find when small perturbations of constant equilibrium solutions become unstable. The second is to prove the existence of a global bifurcation of these solutions from the constant equilibrium solution.   (Received September 15, 2020)

37 ▶ *Dynamical systems and ergodic theory*

1163-37-142   **Michail E Filippakis*** (`mfilip@unipi.gr`), Univercity of Piraeus, Department of Digital Systems, Piraeus, Greece, 18534 Piraeus, Greece, and **Maria eleni Poulou**. *Global Attractor for a stochastic System of Klein - Gordon - Schrödinger Type.*

Let us consider the Cauchyproblem of acoupled system of a Schrödinger equation with fractionalLaplacian and fractional Klein Gordon equation of different order through Yukawa coupling.

$$idu + (\kappa(-\Delta)^a u + i\alpha u - uv)dt = fdt + \sum_{j=1}^{m} \phi_j d\omega_j,$$

$$dv_t - ((-\Delta)^b v + v + \lambda v_t + Reu_x)dt = gdt + \sum_{j=1}^{m} \phi_j d\omega_j. \tag{1}$$

where $a, b \in (1/2, 1)$ and the functions $\{\phi_j\}_{j=1}^{m} \in H^2(\mathbb{R}) \cap W^{2,p}(\mathbb{R})$ for some $p > 1$ and $\{\omega_j\}_{j=1}^{m}$ are independent twosided realvalued Wiener processes on a complete probability space. So the intoduction of the disspative mechanisms are necessary to force the energy to decay to zero when t goes to infinity.Our aim is to prove with the help of the a priori estimates the existence and uniqueness of a solution of the stochatic fractional system as well as the existence of a global attractor.

(Received August 21, 2020)

1163-37-222   **Joseph H. Silverman***, Mathematics Department, Brown University, Box 1917, Providence, RI 02912. *The Pentagram Map on Cubic Surfaces.* Preliminary report.
The classical pentagram map is a dynamical system on the space of pentagons, or more generally n-gons, in the projective plane $\mathbb{P}^2$. We consider an analogous dynamical system on the set of n-gons lying on a cubic surface in $\mathbb{P}^3$, for which we describe some elementary properties and pose some questions.   (Received August 28, 2020)

1163-37-229   **Jasper Weinburd*** (`jweinburd@hmc.edu`) and **Andrew J Bernoff**. *Emergent patterns in locust swarms using agent-based and continuous models.*
Locusts are devastating pests that infest and destroy crops in regions of the world already afflicted by drought and food shortage. Swarms of flightless juvenile locusts take on distinctive shapes that appear to serve ecological processes. How does individual locust behavior lead to these various shapes and help the swarm to cope with its current environment? We model this phenomenon using two approaches in tandem; an agent-based model that tracks individuals and a partial differential equation for the mean locust density. The agent-based model allows direct comparison to empirical data and captures the systems inherent stochasticity. The PDE provides a framework for theoretical analysis and allows extensive exploration of the model's sensitivity to changes in input parameters. In this talk, we discuss the effects of social forces and attraction to resources on the collective behavior of the band.   (Received August 28, 2020)

1163-37-353   **Wayne Peng*** (`junwen.wayne.peng@gmail.com`), 88 Surrey Hill Way, Rochester, NY 14623, **Thomas Tucker** (`thomas.tucker@rochester.edu`), Rochester, NY 14627, **Dang-Khoa Nguyen**, Calgary, AB T2N 4T4 , Canada, and **Fedor Pakovich** (`pakovich@math.bgu.ac.il`), Beer Sheva, Israel. *Toward dynamical isogeny theorem.* Preliminary report.
Let E be an elliptic curve and p be a prime. The Tate module $T_p(E)$ is the inverse limit being taken with respect to the natural maps

$$E[p^{n+1}] \xrightarrow[[p]]{]} E[p^n]$$

where $E[p^n]$ is the p^n-torsion subgroup of E and $[p]$ is an isogeny multiplication by m. The Tate's isogeny theorem then says the natural map

$$\mathrm{Hom}_K(E_1, E_2) \otimes \mathbb{Z}_p \to \mathrm{H0m}_K(T_p(E_1), T_p(E_2))$$

is an isomorphism if K is a finite field or a number field.

The natural analogy of the Tate module for a dynamical system is a dynamical trees. However the isogeny for dynamical systems is missing. In this talk we are going to present a possible way to define the dynamical isogeny and formulate the dynamical isogeny conjecture.   (Received September 03, 2020)

1163-37-433   **Nandor J Simanyi*** (simanyi@uab.edu), University Hall, Room 4005, 1402 10th Avenue
South, Birmingham, AL 35294-1241, and **Michael Hofbauer** (hofbauer@uab.edu),
University Hall, Room 4005, 1402 10th Avenue South, Birmingham, AL 35294-1241.
Asymptotic Homotopical Complexity of a Sequence of 2D Dispersing Billiards. Preliminary
report.

We are studying the asymptotic homotopical complexity of a sequence of billiard flows on the 2D unit torus T^2
with n circular obstacles. We get asymptotic lower and upper bounds for the radial sizes of the homotopical
rotation sets and, accordingly, asymptotic lower and upper bounds for the sequence of topological entropies.
The obtained bounds are rather close to each other, so this way we are pretty well capturing the asymptotic
homotopical complexity of such systems. Note that the sequence of topological entropies grows at the order of
log(n), whereas, in sharp contrast, the order of magnitude of the sequence of metric entropies is only log(n)/n.

Also, we prove the convexity of the admissible rotation set AR, and the fact that the rotation vectors obtained
from periodic admissible trajectories form a dense subset in AR.   (Received September 06, 2020)

1163-37-473   **Anila Yadavalli*** (anilayad@umn.edu) and **Bojko Bakalov** (bojko_bakalov@ncsu.edu).
Darboux Transformations and Fay Identities for the Extended Bigraded Toda Hierarchy.

Integrable hierarchies arise by starting with a differential equation that models a real-life process and construct-
ing a system of infinitely many differential equations that can be solved simultaneously. Some classical examples
of integrable hierarchies are the KP, KdV, and Toda hierarchies. Solutions to such hierarchies can be studied
algebraically using tools such as Darboux transformations.

I will begin this talk by introducing the Toda hierarchies, with emphasis on the extended bigraded Toda
hierarchy (EBTH). Darboux transformations for the EBTH were determined by Li and Song in 2016. I will show
that the action of these Darboux transformations on solutions to the EBTH is given by a vertex operator. As a
consequence of this result, I will derive a formula for generalized Fay identities of the EBTH.

This is joint work with B. Bakalov.   (Received September 07, 2020)

1163-37-507   **Stephanie Dodson*** (sadodson@ucdavis.edu), **Briana Abrahms**, **Steven J Bograd**,
Jerome Fiechter and **Elliott L Hazen**. *Using Agent-Based Models to Understand
Drivers of Migration in Northern Pacific Blue Whales.*

In general, both abiotic and biotic factors are thought to influence migratory behavior, but their relative roles
are difficult to disentangle. For migratory marine predators, both temperature and prey availability have been
shown to be significant predictors of space use, though often physical proxies are relied on due to the lack
of data on dynamic prey fields. To evaluate the relative roles of abiotic (sea surface temperature) and biotic
(prey availability) factors in driving blue whale (Balaenoptera musculus) movement decisions and migratory
behavior in the eastern North Pacific, we developed a spatially explicit agent-based movement model which
uses data from a coupled regional ocean model that explicitly includes a dynamic prey model. Our agent-
based movement model helps elucidate the mechanisms underlying migration and demonstrates how fine-scale
individual decision-making can lead to emergent migratory behavior at the population level. Understanding
the drivers of movement, migration and distribution of individuals is important for insight into how species will
respond to changing environmental conditions.   (Received September 08, 2020)

1163-37-552   **Gangaram S. Ladde*** (gladde@usf.edu), Department of Mathematics and Statistics,
University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700.
A Foundation for Dynamic Binary State Processes. Preliminary report.

By introducing the concepts of forward and backward dynamic flows, important structural, quantitative and
qualitative features are investigated in systematic and coherent manner. Several examples are give to illustrate
role and scope of the concepts. The binary state potential energy and activation processes are special cases of
dynamic flow plows processes. The byproduct of this initiates a study of conceptual dynamic network systems in a
coherent manner. In fact, it provides a basis for dynamic algorithm for investigating binary digital state dynamic
processes in a systematic and unified way. The modeling dynamic approach is based on historical development
of classical models of McCulloch-Pitts, biological and social processes coupled with feedback process in neural
network. This in general provides an alternate approach.   (Received September 10, 2020)

1163-37-561   **Xavier Buff**, **Bill Floyd**, **Sarah Koch*** (kochsc@umich.edu) and **Walter Parry**.
Factoring Gleason polynomials. Preliminary report.

The polynomial $G_n \in \mathbb{Z}[c]$ that defines the centers of the hyperbolic components of period n in the Mandelbrot set is called the Gleason polynomial of period n. We investigate G_n modulo 2 and find something rather curious. (Received September 09, 2020)

1163-37-583   **Tamara Kucherenko** (tkucherenko@ccny.cuny.edu), **Anthony Quas** (aquas@uvic.ca) and **Christian Wolf*** (cwolf@ccny.cuny.edu). *Multiple phase transitions on compact symbolic systems.*

Let $\phi : X \to \mathbb{R}$ be a continuous potential associated with a symbolic dynamical system $T : X \to X$ over a finite alphabet. Introducing a parameter $\beta > 0$ (interpreted as the inverse temperature) we study the regularity of the pressure function $\beta \mapsto P_{\text{top}}(\beta\phi)$ on an interval $[\alpha, \infty)$ with $\alpha > 0$. We say that ϕ has a phase transition at β_0 if the pressure function $P_{\text{top}}(\beta\phi)$ is not differentiable at β_0. This is equivalent to the condition that the potential $\beta_0\phi$ has two (ergodic) equilibrium states with distinct entropies. For any $\alpha > 0$ and any increasing sequence of real numbers (β_n) contained in $[\alpha, \infty)$, we construct a potential ϕ whose phase transitions in $[\alpha, \infty)$ occur precisely at the β_n's. In particular, we obtain a potential which has a countably infinite set of phase transitions. (Received September 10, 2020)

1163-37-734   **Kenneth Njengele Dukuza***, University of Pretoria, Department of Mathematics, P/bag x 20, Pretoria, South Africa. *Nonstandard Finite Difference (NSFD) methods for some continuous models of ordinary differential equations with a local bifurcation.*

We propose some procedures for the nonstandard discretization of some models of ordinary differential equations that have different bifurcation properties at certain parameter values. The main objective is to ensure that, under the same conditions, the constructed NSFD schemes capture the bifurcation properties of the continuous models. (Received September 12, 2020)

1163-37-884   **Katherine J. Meyer***, kjmeyer@carleton.edu. *Bridging between continuous and discrete disturbances in ecological models.* Preliminary report.

To incorporate repeated disturbances into a differential equation (DE) model of ecological processes, one might embed the disturbance continuously in the DE or resolve the disturbance discretely using a flow-kick framework or impulsive DE. For example, do harvests from a logistic population appear continuously as $x' = x(1-x) - h(x)$ or do individual harvests periodically kick the state x as it flows according to $x' = x(1-x)$? Are fires always smoldering in a model savannah, or do they burn at discrete timepoints? In this talk we'll examine the dynamic implications of such modeling choices. We'll position continuous disturbances as limits of repeated, discrete ones, and discuss how the dynamics of a continuous disturbance model do and don't predict those of a flow-kick model. (Received September 13, 2020)

1163-37-902   **David Krumm*** (dkrumm@reed.edu) and **John R. Doyle**. *Classification of preperiodic portraits for quadratic polynomials over quadratic fields.*

By a quadratic pair we mean a pair (K, c), where K is a quadratic number field and $c \in K$. To every such pair one can associate the directed graph $G(K, c)$ whose vertices are the K-rational preperiodic points for the polynomial map $x \mapsto x^2 + c$. In this talk we discuss the problem of classifying all such graphs $G(K, c)$ up to isomorphism. In particular, we describe a new result identifying all directed graphs which can occur as $G(K, c)$ for infinitely many quadratic pairs (K, c). (Received September 14, 2020)

1163-37-933   **William Ott*** (ott@math.uh.edu), **Edward Stout** and **Zijie Zhou**. *Quantization of fractal sets and measures in Banach spaces.*

To what extent can finite-dimensional data be used to make inferences about infinite-dimensional objects? Consider a set X or a measure μ in a Banach space $\mathfrak{B}$. A nonlinear map $f : \mathfrak{B} \to \mathbb{R}^m$ produces a quantized image of the set or measure, $f(X)$ or $f_*(\mu)$. In this talk, we use prevalence, the theory of projection constants, and thickness exponents to investigate how well a typical nonlinear map f preserves the structure of X or μ. Does a typical f embed X? If so, what is the Hölder regularity of the inverse? Does a typical f preserve the Hausdorff dimension of X? For the embedding problem, we present a new theorem that improves on recent work of Margaris and Robinson. Our theorems on preservation of Hausdorff dimension build upon work that has been done in the Hilbert space setting. This is joint work with Edward Stout and Zijie Zhou. (Received September 14, 2020)

1163-37-964   **Sergey Bezuglyi**, **Palle E. T. Jorgensen** and **Shrey Sanadhya***
(shrey-sanadhya@uiowa.edu). *Substitution dynamical system on infinite alphabets and Bratteli diagrams.* Preliminary report.

In this work we build Bratteli-Vershik models for a certain class of substitutional dynamical systems (known as left determined substitutions) on a countable infinite alphabet. Further, we provide new versions of Rokhlin's Lemma for such substitutional dynamical systems. Using our Bratteli-Vershik model we construct invariant measures and ergodic measures (both finite and infinite). We provide examples of such systems, as well as explicit formulas for invariant and ergodic measures.   (Received September 14, 2020)

1163-37-979   **Sangita Jha*** (jhasa@nitrkl.ac.in), Depatment of Mathematics, National Institute of Technology Rourkela, Rourkela, 769008, India, and **Arya Kumar Bedabrata Chand** (chand@iitm.ac.in). *α-Fractal Zipper Rational Cubic Fractal Interpolation Functions.*

In this talk, we propose an interpolation method using a binary factor called signature. The proposed interpolant is constructed such that the graph of the interpolant is an attractor of a zipper rational iterated function system. Using the scale factor α and the binary parameter ϵ, we produce a wide variety of interpolants. In particular, we introduce a class of α-fractal zipper rational cubic spline $S_{epsilon}^{alpha} \in \mathcal{C}^1$ and investigate its shape preserving aspects. At the end we will discuss the convergence aspects of the proposed interpolant.   (Received September 14, 2020)

1163-37-998   **Irina Popovici*** (popovici@usna.edu), Annapolis, MD 21402. *On the Existence of Multiple Bifurcations in A Piece-Wise Smooth Planar Dynamical System Modelling Cardiac Rhythm.*

The talk concerns the bifurcations of a dynamical system modelling the response to stimulation of a cardiac cell (or electrically coupled multicellular preparation). The two-dimensional system is controlled by four kinetic parameters and the stimulus period, T; it is continuous but only piece-wise smooth. Earlier work (with M. Kidwell, M. Baker, R. Kline) showed that the dynamics transitions from simple to complex as T decreases: large T correspond to global contractions, mid-range T have co-existing stable periodic orbits, and for small T we have multiple unstable cycles.

The first bifurcation, of period-doubling type, results from the collision of two cycles with a switching manifold. The second type of bifurcations captures the transition of 1-, 2-, . . . n-escalators from stability to instability; their largest (i.e. first) bifurcation times follow the 1, 1/2 , 1/3, . . . 1/n pattern detected experimentally. Observations and reduced models suggested that the bifurcation from stability to instability is unique.

This talk gives a few rigorous results re. the existence of kinetic parameters for which escalators have multiple transitions from stability to instability and gives conditions for parameter subsets to have unique bifurcations. (Received September 14, 2020)

1163-37-1009   **Salisu M Garba*** (salisu.garba@up.ac.za), Department of Mathematics, University of Pretoria, Pretoria, Gauteng 0002, **Jean M-S Lubuma** (jean.lubuma@up.ac.za), Department of Mathematics, University of Pretoria, Pretoria, Gauteng 0002, and **Berge Tsanou** (berge.tsanou@up.ac.za), Department of Mathematics, University of Pretoria, Pretoria, 0002. *Nonstandard Finite Difference Models of the COVID-19 Pandemic.*

Since the emergence late in 2019 of the COVID-19 pandemic, several deterministic models have been designed for its transmission dynamics and control. Due to many factors that are still unknown on SARS-CoV-2, existing models exhibit interesting features (e.g. direct and indirect) about the manner in which the disease is spread. Moreover, the incorporation into the models of non-pharmaceutical interventions (NPIs) to curtail the disease, brings additional features. In this work, we construct nonstandard finite difference (NSFD) schemes for some SEIR-type models for COVID-19 and show that they are dynamically consistent. In particular, we show that our NSFD schemes are discrete dynamical systems, which are elementary stable. That is: (a) their fixed-points coincide with the continuum of disease-free-equilibria (DFE) of the continuous models, and (b) the fixed-points replicate the asymptotic stability property of the continuum of DFE whenever the control reproduction number is less than unity. Moreover, we show that the NSFD schemes preserve the effectiveness of NPIs (e.g. self-isolation, social-distancing, lockdown, wearing a facemask, disinfecting objects / surfaces, etc.) to combat the COVID-19 pandemic, as demonstrated for continuous models.   (Received September 14, 2020)

1163-37-1161   **Kelsey Gasior*** (kgasior@fsu.edu), **Kirill Korshunov**, **Paul Trombley** and **Richard Bertram**. *Modeling Neuron Response to Ramped Input Current.*

Recently, novel experiments were performed in olfactory bulb dopamine-secreting neurons (OBDA neurons) wherein current applied to the individual neurons was ramped to mimic biologically realistic neuronal input (at

Florida State University, the Trombley lab performed). However, this new stimulus protocol raises the questions of what is the proper way to interpret these data and how can mathematical analysis help? We have developed a methodology to study transient dynamics in the electrical activity of single neurons while maintaining a collaboration with the biologists who carry out these experiments. This work allows us to understand how different ion channels shape the transient response dynamics in OBDA neurons. In particular, we are using linear regression to model a slow-activating M-type Potassium channel and bifurcation analysis to understand the influence of both the applied current and ramp duration in the spiking behavior. These mathematical tools we developed can be used to explore the behavior of other cell types as it is our belief that the ramping technique could be extended to study the dynamics of all neuron types. Ultimately, this work helps close the gap between mathematical modeling and biological data in computational neuroscience.   (Received September 14, 2020)

1163-37-1193   **Jason Atnip*** (`j.atnip@unsw.edu.au`), **Gary Froyland**, **Cecilia Gonzalez-Tokman** and **Sandro Vaienti**. *Thermodynamic Formalism for Random Piecewise-Monotone Maps of the Interval.*
In this talk we develop a quenched thermodynamic formalism for random dynamical systems generated by countably branched, piecewise-monotone mappings of the interval that satisfy a random covering condition. We consider a general random contracting potential (in the sense of Liverani-Saussol-Vaienti) and we prove there exists a unique random conformal measure and a unique random equilibrium state for this potential. Further, we prove quasi-compactness of the associated transfer operator cocycle and exponential decay of correlations for the unique equilibrium state. We will give several examples of our general theory. In particular, our results apply to random beta-transformations, random Gauss-Renyi maps, and random dynamics of non-uniformly expanding maps such as intermittent maps and maps with contracting branches.   (Received September 15, 2020)

1163-37-1286   **Razvan Teodorescu*** (`razvan@usf.edu`), CMC 314, University of South Florida, 4202 E Fowler Ave, Tampa, FL 33620, and **Iuliana Teodorescu** and **Antonino Travia**. *Effective dimensions of fractal dynamical systems and applications.*
We consider the effective dimensions of dynamical systems of integrable type, subject to projections (and random projections) to vector subspaces. The main motivating problem has to do with the emergence of scale-free structures in effective 2D systems based on noncommutative gauge theories. In 2D, the absence of a definite lengthscale leads to conformal invariance, and therefore a rich algebraic structure for the effective theory. We explore the fundamental result and its possible applications to 2D turbulent flows.   (Received September 15, 2020)

1163-37-1297   **Cara Jill Sulyok***, csulyok@vols.utk.edu, and **Judy Day** and **Suzanne Lenhart**. *A Mathematical Framework to Augment the Q-MARSH Score in the Diagnosis of Celiac Disease.* Preliminary report.
Celiac disease (CeD) is a hereditary autoimmune disease that affects approximately 1 in 133 Americans. It is caused by a reaction to the protein gluten found in wheat, rye, and barley. After ingesting gluten, a patient with CeD may experience a range of unpleasant symptoms while small intestinal villi, essential to nutrient absorption, are destroyed in an immune-mediated process. The only known treatment for this disease is a lifelong gluten-free diet.

   This preliminary work provides a mathematical framework to better understand the effects of immune activation on gut health. This mathematical model uses a system of ordinary differential equations to track changes in villus and crypt cell densities as well as intraepithelial lymphocytes to better understand the dynamics of small intestinal destruction and relates these cell densities to the Q-MARSH score, a criterion used in the diagnosis of CeD. The model will be used to investigate and analyze various theories behind the progression of this disease by focusing on understanding the dynamics of the small intestine in situations mirroring healthy function and CeD. By doing so, we can assist in further quantifying and augmenting diagnostic measures and investigate potential therapies to mitigate the negative effects of CeD.   (Received September 15, 2020)

1163-37-1311   **Katharine Gurski** and **Kathleen A Hoffman*** (`khoffman@umbc.edu`), 1000 Hilltop Circle, Baltimore, MD 21250. *An HIV Model with Longterm Partnerships and Disease Progression.* Preliminary report.
The dynamics of HIV infections depends on many factors such as sexual behavior, mixing strategies, cultural norms, stages of disease progression, treatment options, and longterm and casual partnerships. In this talk, I will focus on including longterm partnerships into a model of HIV that incorporates a non-constant population, multiple sexual behaviors, and three stages of disease progression: acute, chronic, and virally suppressed. Incorporating longterm partnerships requires a model that accounts for the impact of an infected individual on

subsequent partnerships which is mathematically represented by a nonlocal term in the model. The nonlocal term takes the form of an expected value, which we then linearize to a more tractable form. We develop the model to study the effect of intervention strategies that minimize the amount of disease in the population and the reproductive number of HIV.   (Received September 15, 2020)

1163-37-1395   **Scott Kaschner*** (`skaschne@butler.edu`). *Bifurcation Phenomena in Quadratic Rational Maps.* Preliminary report.
This talk will survey known results regarding bifurcation in a family, $\{f_\lambda, t\}_{t \in \mathbb{C}}$, of quadratic rational maps with a fixed point multiplier of λ; similar phenomena in another family will be presented. We will also discuss variety of attempts to describe resonance phenomena in these bifurcations.   (Received September 15, 2020)

1163-37-1401   **Ivan Sudakov*** (`isudakov1@udayton.edu`), 300 College Prk., Dayton, OH 45469. *Complex bifurcations in fast-slow climate-ecosystem dynamics.*
Understanding the effects of the climate on ecosystems and biodiversity has been a focus of intense theoretical and empirical research recently. Much less attention has been paid to the possible feedback that ecosystems and the biosphere more generally can have on the climate. Meanwhile, the is evidence that such feedbacks do exist: one example is readily given by the dependence of the ocean albedo on the phytoplankton abundance. In this paper, we consider the stability of the global climate system by linking a conceptual climate model to a generic population dynamics model with random parameters. We first show that the dynamics of the corresponding coupled system possesses multiple timescales and hence falls into the class of slow-fast dynamics. We then investigate the properties of a general dynamical system to which our model belongs and prove that the feedbacks from the population dynamics cannot break the system's stability as long as the biodiversity is sufficiently high. That may explain why the climate is apparently stable over long time intervals. Interestingly, our coupled climate-biosphere system can lose its stability if biodiversity decreases; in this case, the evolution of the biosphere under the effect of random factors can lead to a global climate change.   (Received September 15, 2020)

1163-37-1403   **Subhadip Chowdhury*** (`schowdhury@wooster.edu`). *Rationality and Rigidity of Extremal Actions of Free Group on the Circle.*
We will discuss some interesting rationality and rigidity properties of Calegari-Walker ziggurats, which are graphs of extremal rotation numbers associated with positive words in free groups. We will provide an explicit formula for the fringe fractals and prove conjectured bounds affirmatively for interior points in specific cases. We will also look into strategies for exploring the case of a general word.   (Received September 15, 2020)

1163-37-1426   **Suddhasattwa Das*** (`iamsuddhasattwa@gmail.com`), 205 21st Street, Apt 2, Brooklyn, NY 11232, and **Dimitrios Giannakis** and **Joanna Slawinska**. *The spectral measure of a dynamical system.*
Many dynamical systems are described by a flow Φ^t on an ambient manifold M. Instead of the trajectories of this flow, the operator theoretic framework studies the dynamics induced on the space of observables. This gives rise to a unitary group U^t called the Koopman group. It describes the time-evolution of measurements, such as the state-space variables of an ODE. Many problems in theoretical and applied dynamics can be restated in terms of the Koopman group. A fundamental notion for such groups is that of a spectral measure, which is an operator valued, Borel measure on the complex plane. The spectral measure completely characterizes U^t and hence the trajectories of the flow. I will discuss many diverse ways in which the spectral measure manifests itself analysis of data generated by the dynamical system, such as spectral analysis, decay of correlations, periodic approximation of dynamical systems, and visibly as coherent spatiotemporal patterns. Each of these topics are of great interest of their own, and thus an accurate determination and computation of the spectral measure is of great value. I will finally describe a data-driven means of approximating the spectral measure, which relies on a number of tools from functional analysis.   (Received September 15, 2020)

1163-37-1498   **Eugen Andrei Ghenciu*** (`ghenciue@uwstout.edu`), 1296 River Ridge Rd., River Falls, WI 54022. *Iterated function systems associated with Reny-like continued fraction expansions.*
We look at families of conformal iterated function systems associated with Reny-like continued fraction expansions. Their Hausdorff dimension spectrum is investigated and we show that a very large subfamily has a full dimension spectrum. Several other related examples are given.   (Received September 15, 2020)

1163-37-1511   **Mrinal K Roychowdhury***, 1201 West University Drive, Edinburg, TX 78539.
Quantization for Probability Distributions.

Quantization for probability distributions refers to the idea of estimating a given probability by a discrete probability supported by a set with no more than n points. It has broad application in signal processing and data compression. Quantization dimension gives the speed how fast the specified measure of the error goes to zero as n approaches to infinity. Quantization dimension is also connected with other dimensions of dynamical systems such as Hausdorff, packing and box counting dimensions. It is also connected with the temperature function that arises in the thermodynamic formalism in multifractal analysis. In the discretization process it is much more difficult to find the optimal sets of n-means than to calculate the quantization dimension. I will talk about it.   (Received September 15, 2020)

1163-37-1604   **Andrew Bridy**, **Rafe Jones*** (rfjones@carleton.edu), **Gregory Kelsey** and **Russell Lodge**. *Periodic points of quadratic rational functions in towers of finite fields.*
Preliminary report.

Let $\mathbb{F}_q$ be a finite field with q elements and ϕ a rational function with coefficients in $\mathbb{F}_q$. For each $n \geq 1$, the orbit of every $x \in \mathbb{F}_{q^n}$ under ϕ is either periodic or strictly preperiodic. Little is known about how the proportion of such x that are periodic changes as n grows. Part of what makes the problem difficult is that, since ϕ is defined over a finite field, it is necessarily post-critically finite. We study the case where ϕ is quadratic, and show that in all but a few special cases the lim inf of this proportion is zero. The proof begins by using the Chebotarev density theorem over function fields and a result of Pink on lifting to characteristic zero. But the real guts of it are group-theoretic and complex dynamical: we make a careful study of the iterated monodromy groups of post-critically finite quadratic rational functions over $\mathbb{C}$, including several new results about the fixed-point proportion of the natural action of these groups on the infinite rooted binary tree.   (Received September 15, 2020)

1163-37-1619   **Sunmi Lee**, Department of Applied mathematics, Kyunghee University,, Yongin, 446-701, South Korea, and **Minseok Kim** and **Yun Kang*** (yun.kang@asu.edu), Mesa, AZ 85212. *Optimal control of a discrete-time plant-pest model with bistability and fluctuating environments.* Preliminary report.

Discrete-time plant-pest models with two different constant control strategies (i.e., removal versus reduction strategies) have been investigated to understand how to regulate pest population. The corresponding optimal control problem has been explored on three scenarios of bistability of plant-pest dynamics where these dynamics are determined by the growth rate of the plant and the damage rate inflicted by pest. Through analysis and simulations, we identify and evaluate the optimal controls and their impacts of fluctuating environments on the plant-pest dynamics. There are critical factors to characterize the optimal controls and the corresponding plant-pest dynamics such as the control upper bound (the effectiveness level of the implementation of control measures) and the initial conditions of the plant and pest. The results show that the pest is hard to be eliminated when the control upper bound is not large enough or the initial conditions are chosen from the inner points of the basin of attractions. However, as the control upper bound is increased or the initial conditions are chosen from near the boundary of the basin of attractions, then the pest can be manageable regardless of fluctuating environments.   (Received September 15, 2020)

1163-37-1686   **Hau-Tieng Wu*** (hauwu@math.duke.edu). *Exploring manifold learning theory to study non-stationary time series for clinical application.*

While manifold learning has been widely applied, however, there are several open problems on the way toward statistical inference and handling big dataset. We discuss some topics in this direction, like robustness to high dimensional noise, eigenvalue distribution, and Roseland to speed up computation. Its clinical application to analyzing modern long-term physiological signal will be demonstrated.   (Received September 16, 2020)

39 ▶ *Difference and functional equations*

1163-39-74   **xinyun zhu*** (zhu_x@utpb.edu), Odessa, TX 79762. *Solvability for a discrete fractional mixed type sum-difference equation boundary value problem in a Banach space.*
Preliminary report.

In this paper, by means of Darbo's fixed point theorem, we establish the existence of solutions to a nonlinear discrete fractional mixed type sum-difference equation boundary value problem in a Banach space. Additionally, as an application, we give an example to demonstrate the main result.   (Received August 09, 2020)

1163-39-138   **Eddy Kwessi*** (`ekwessi@trinity.edu`), 1 Trinity Place, San Antonio, TX 78212, and **Lloyd Edwards** (`ljedward@uab.edu`), Birmingham, AL. *A Nearly Exact Discretization Scheme for the FitzHugh-Nagumo Model.*

Human brain contains a large number of neurons that often evolve in large neural networks representing groups of neural populations where each element interacts under excitement impulses with other elements. Often, systems of continuous differential equations are used to model these ensembles of neurons. In the presence of data however, discrete models are preferred and it has been well documented that without proper care, discrete and continuous models do not always yield the same dynamics. The non-standard methods and later additions aim to address the discrepancies between continuous and discrete models. One model often used in neuroscience to represent ensembles of neurons is the FitzHugh-Nagumo system. This system consists of two ordinary differential equations linking an activator and an inhibitor and represents the excitability of the neural network. In this paper, we propose a nearly exact discretization scheme for the FitzhHugh-Nagumo model. We prove that the scheme preserves qualitatively and quantitatively the dynamics and features of the original continuous system. (Received August 20, 2020)

1163-39-194   **Lingju Kong*** (`lingju-kong@utc.edu`) and **Min Wang** (`mwang23@kennesaw.edu`). *Multiple nontrivial solutions for a nonlinear discrete problem of the second order.*

We study the existence of multiple nontrivial solutions of the second order discrete problem

$$\begin{cases} -\Delta^2 u(k-1) = f(k, u(k)), \ k \in [1, N]_{\mathbb{Z}}, \\ u(0) = 0, \ u(N+1) = \mu u(N). \end{cases}$$

Our first theorem provides criteria for the existence of at least two nontrivial solutions of the problem, and also finds conditions under which the two solutions are sign-changing. Our second theorem proves, under some appropriate assumptions, that the problem has at least three nontrivial solutions, one of which is positive, one is negative, and one is sign-changing. As applications of our theorems, we further obtain several existence results for an associated eigenvalue problem. We include two examples in the paper to show the applicability of our results. Our theorems are proved by employing variational approaches, combined with the classic mountain pass lemma and a result on the invariant sets of descending flow. (Received August 26, 2020)

1163-39-319   **Min Wang*** (`mwang23@kennesaw.edu`). *A second order discrete boundary value problem with mixed periodic boundary conditions.*

In this talk, we study the sufficient conditions on the existence of solutions of a second order discrete boundary value problem with mixed periodic boundary conditions by the critical point theory. A particular Banach space and an associated functional are presented to handle the asymmetry due to the mixed periodic boundary conditions. Examples are given to illustrate the applications of the results as well. (Received September 02, 2020)

1163-39-335   **Evelina Giusti Lapierre*** (`elapierre@jwu.edu`) and **Wirot Tikjha** (`wirottik@psru.ac.th`). *On the Global Behavior of a System of Piecewise Linear Difference Equations.*

In a previous paper we considered the system $x_{n1} = |x_n| - y_n - 1$ and $y_{n1} = x_n |y_n| - 1$ and showed by mathematical induction that when the initial condition is an element of the closed second or fourth quadrant, the solution to the system is either a prime period-3 solution or one of two prime period-4 solutions. In this paper we complete the study of the global behavior of the system. We show that when the initial condition is an element of $mathbbR^2$ then the solution is the equilibrium point, one of two prime period-3 solutions, or one of two prime period-4 solutions. (Received September 02, 2020)

1163-39-359   **Michael Alexander Radin*** (`michael.a.radin@gmail.com`), Rochester Institute of Technology, College of Science, School of Mathematical Sciences, Rochester, NY 14623. *Periodic and Eventually Periodic Solutions and Patterns of Transient Terms of Max-Type Difference Equations.* Preliminary report.

Our first aim is to detect the patterns of periodic and eventually periodic solutions of Max-Type Difference Equations.Our second aim is to determine the existence of patterns of transient terms and how many transient terms to expect. We will show graphical examples of transient terms as descending triangular-shapes or as step-shapes. In addition, we will trace the transient terms' existence on specific intervals. (Received September 03, 2020)

1163-39-397   **Abdul-Aziz Yakubu** (ayakubu@howard.edu) and **Eric Ngang Che***
(ericngangche@yahoo.com), Washington, DC. *A Discrete-Time Risk Structured Model of*
Cholera Infections In Cameroon. Preliminary report.

In this talk, unlike the ODE model of Che et. al. [1], we introduce a discrete-time cholera model of Cameroon with risk structure and no spatial structure. We use our discrete-time demographic equation to "fit" the total annual population census data of Cameroon. Furthermore, we use our fitted discrete-time cholera model to capture the annually reported cholera cases in Cameroon from 1987-2004. As in the ODE model, we obtain that the basic reproduction number of our fitted discrete-time cholera model, $\mathcal{R}_0 \simeq 1.1832$. The $\mathcal{R}_0$ of the ODE model with a similar risk structure is 1.1803. That is, the two models have approximately the same value for $\mathcal{R}_0$ and both predicted cholera endemicity in Cameroon. As in the ODE model, we use our fitted discrete-time cholera model to study the impact of vaccination, treatment and improved sanitation on the number of cholera infections in Cameroon from 2004 to 2019. Furthermore, we use our fitted model to predict future cholera cases.

Reference

[1] E. Che, Y. Kang and A. Yakubu, Risk structured model of cholera infections in Cameroon, Mathematical Biosciences **320** (2020), 108303.
(Received September 04, 2020)

1163-39-467   **Danielle Burton*** (dlburton@wisc.edu), **Suzanne Lenhart**, **Frank Hilker** and **Daniel Franco**. *Optimal control of harvest timing in difference equations.*

Harvest plays an important role in management decisions. Discrete models enable us to explore the importance of timing of management decisions including the order of events of particular actions. We derive novel mechanistic models featuring explicit within season harvest timing and level. Our models feature explicit discrete density independent birth pulses, continuous density dependent mortality, and density independent harvest level at a within season harvest time. We explore optimization of within-season harvest level and timing through optimal control of these population models. We maximize an objective functional which includes management goals of maximizing yield, maximizing stock, and minimizing costs associated with both harvest intensity and harvest timing.   (Received September 07, 2020)

1163-39-564   **Candace Marie Kent*** (cmkent@vcu.edu), 3510 Hanover Avenue, Richmond, VA
23221-2208. *A modified second-order Collatz equation as a mathematical model of bipolar*
disorder.

We propose, for the sake of dialogue, that the following system of difference equations serve as a phenomenological model of bipolar disorder, a psychiatric illness characterized by cycles or recurrent episodes of severe disturbances in mood (i.e., in being happy or sad, emotions at opposite poles of the spectrum):

$$\begin{cases} x_{n+1} = (ax_n + b) \bmod m, \\\\ z_{n+1} = \begin{cases} \dfrac{-z_n - z_{n-1}}{2}, & \text{if } z_n + z_{n-1} \text{ is even,} \\ -z_n - z_{n-1}, & \text{if } z_n + z_{n-1} \text{ is odd} \end{cases} + s\delta(x_n), \end{cases}$$

and

$$\delta(x) = \begin{cases} 0, & \text{if } x \neq d \in \{0, 1, \ldots, m-1\}, \\\\ 1, & \text{if } x = d. \end{cases}$$

The first equation in the system is a linear congruential sequence; and the second equation is a modified version of one of the sixteen mostly eventually periodic Collatz difference equations. We observe (and conjecture) that every solution $\{z_n\}_{n=0}^{\infty}$ of the system above is also eventually periodic. Thus, a solution $\{z_n\}_{n=0}^{\infty}$ of the system is intended to represent the recurrent episodes of mood disturbance seen in an individual with bipolar disorder. (Received September 09, 2020)

1163-39-616   **Vlajko L Kocic*** (vkocic@xula.edu), Xavier University of Louisiana, Department of
Mathematics, 1 Drexel Dr., New Orleans, LA 70125-1056. *The dynamics of Williamson's*
discrete population model. Preliminary report.

In this paper we present the results related to the study of oscillations, structure of semicycles, periodicity, and attractivity in discrete discontinuous Williamson's population model. The connections to discrete West Nile epidemics model were also established.   (Received September 10, 2020)

1163-39-716   **Mustafa Kulenovic***, 165 little rest road, Unversity of Rhode Island, Kingston, RI 02881.
Global Behavior of Certain Nonautonomous Linearizable Three Term Difference Equations.
We investigate the nonautonomous difference equation

$$x_{n+1} = g_0 x_n + g_1 x_{n-1}, \quad n = 0, 1, \ldots .$$

with real initial conditions and coefficients $g_i, i = 0, 1$ which are in general functions of n and/or the state variables $x_n, x_{n-1}, \ldots$ and satisfy $g_0 + g_1 = 1$. We also obtain some global results about the behavior of solutions of the nonautonomous non-homogeneous difference equation

$$x_{n+1} = g_0 x_n + g_1 x_{n-1} + g_2, \quad n = 0, 1, \ldots .$$

where $g_i, i = 0, 1, 2$ are functions of n and/or the state variables $x_n, x_{n-1}, \ldots$ with $g_0 + g_1 = 1$. Our results are based on the explicit formulas for solutions. Our applications will include some discontinuous and piecewise difference equations.   (Received September 11, 2020)

1163-39-742   **Ronald Mickens*** (`rmickens@cau.edu`), Clark Atlanta University, Atlanta, GA 30314.
What is a Nonstandard Finite Difference Scheme?
Since its creation by Mickens, the NSFD methodology "stated rules" have evolved to an ever decreasing set of requirements. At the present time, there is only one restriction that is needed and it is the "Principle of Dynamic Consistency (PDC)", appropriately interpreted. This presentation considers issues related to "What is a valid NSFD scheme?" and the interplay among knowledge of the original system, its mathematical models, and the construction of discretizations via the PDC. An elementary, but nontrivial example is used to fully illustrate the fundamental issues and their resolution.   (Received September 12, 2020)

1163-39-743   **Talitha Washington*** (`twashington@cau.edu`), Clark Atlanta University, Atlanta, GA
30314. *Positivity Preserving Nonstandard Finite Difference Scheme for a Modified
Parabolic Reaction-Advection-Diffusion PDE.*
The nonstandard finite difference (NSFD) scheme methodology can be employed to construct a discretization for a parabolic partial differential equation (PDE) that has linear advection and diffusion and with a nonlinear reaction term. We will show how to construct a scheme that preserves the positivity of solutions and, as a consequence, a functional relationship exists between the space and time step-sizes.   (Received September 12, 2020)

1163-39-806   **Candace Kent** and **David Chan*** (`dmchan@vcu.edu`), 1015 Floyd Avenue, Box 842014,
Richmond, VA 23284-2014, and **William Strasser**. *Parameter Exploration of an Epilepsy
Model.* Preliminary report.
In this talk we examine a difference equation model, involving the maximum function, for neuron excitation that mimics seizure activity in the brain. We explore the model's parameter space for complex behavior. Bifurcation diagrams and phase portraits are presented where seizures are prevalent.   (Received September 13, 2020)

1163-39-1130   **Ferhan M. Atici*** (`ferhan.atici@wku.edu`), 1906 College Height Blvd., Department of
Mathematics, Western Kentucky University, Bowling Green, KY 42101, **Samuel Chang**
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Mohan Jonnalagadda** (`j.jaganmohan@hotmail.com`), Birla Institute of Technology, and
Science Pilani, Department of Mathematics, Hyderabad- Telangana, India. *Linear
fractional order h-difference equations.*
In this talk, we introduce linear fractional order h-difference equations, where the order of the equation is any non-integer positive real number. The nabla fractional operators are used in the sense of Riemann-Liouville definition. We obtain the general solution of the fractional order equation by means of Mittag-Leffler type functions. Several properties of the Mittag-Leffler type functions are obtained. As an application, an eigenvalue problem with Dirichlet boundary condition is considered. We give a method for explicit calculation of the eigenvalues of the boundary value problem.   (Received September 14, 2020)

1163-39-1170   **Harold M Hastings*** (`hhastings@simons-rock.edu`) and **Tai Young-Taft**. *Vector
difference equations, Gerschgorin's theorem, and design of multi-networks for human
interactions to reduce spread of epidemics.* Preliminary report.
We start with the SIR model (susceptible, infected, removed) on a network. Since the goal is to make $I = 0$ a (Lyapunov) stable equilibrium, we linearize the discrete-time SIR model to obtain difference equations of the form $I_{new} = I(1 + aS - b)$ at each node before including infections derived from other nodes. We assume S

equal to its initial value at that node. Here a depends upon the infectivity and contact rate, $b = 1/\tau$ where τ = duration of infectivity and the traditional $Rt = aS/b$ ($Rt < 1$ corresponds to $aS < b$). This yields a vector difference equation $\mathbf{I_{new}} = \mathbf{MI}$. Since all entries in M are assumed non-negative, one expects that the maximum row sum is a relatively tight bound on the maximum eigenvalue by the Gerschgorin circle theorem. Interpretation: for 0 to be a stable equilibrium (the infection dies out), the total flow into any node must be less than the value of $aS - b$ at that node. The entries of M may vary in time, even discontinuously as flows between nodes are turned on and off. This may yield useful design constraints for a multi-network composed of weak and strong interactions between pairs of nodes representing interactions within and among cities. (Received September 15, 2020)

1163-39-1512   **Saber Elaydi*** (`selaydi@trinity.edu`), One Trinity Place, San Antonio, TX 78216. *A Mathematical model for the aggregation of Amyloid-beta.*
Whether Amyloid-Beta (Aβ) is the causal factor of Alzheimer's Disease (AD) or a downstream response to some as yet unidentified causative agent, the association of Aβ aggregation with Alzheimer's Disease means that understanding this process is of considerable importance. Consequently, in this talk, we focus on modeling the process of the aggregation of Aβ.

In its simplest forms, β-amyloid plaque formation can be described by protein aggregation, involving the misfolding of Aβ into soluble and insoluble assemblies. Kinetic studies have suggested that the misfolding of monomeric Aβ has been shown to precede the formation of oligomers, which then serve as seeds for accelerated fibril growth. We will investigate the dynamics of all the stages of the aggregation of Amyloid-Beta, from monomers to dimers, trimers,...oligomers, and fibrils.
(Received September 15, 2020)

1163-39-1520   **Saber Elaydi*** (`selaydi@trinity.edu`), One Trinity Place, San Antonio, TX 78212. *Mathematical Models of the Evolution of Species.*
In this talk, we apply a new approach to a special class of discrete-time evolution models and establish a solid mathematical foundation to analyze them. We propose new single and multi-species evolutionary competition models using the evolutionary game theory that require a more advanced mathematical theory to handle effectively. The new approach is to consider the discrete models as non-autonomous difference equations and alternatively as triangular maps. We embed the non-autonomous difference equations in autonomous discrete dynamical systems in a higher dimension space, which is the product space of the phase space and the space of the functions defining the non-autonomous system. Our current approach applies to two scenarios. In the first scenario, we assume that the trait equations are decoupled from the equations of the populations. This requires specialized biological and ecological assumptions. In the second scenario, we do not assume decoupling, but we assume that the dynamics of the trait is known. For instance, in the latter case, we assume that the trait approaches a positive stable equilibrium point. This latter case may apply to a much broader evolutionary dynamics. (Received September 15, 2020)

1163-39-1533   **Saber Elaydi***, One Trinity Place, San Antonio, TX. *Discretization of some neuroscience continuous models.*
We will consider neuroscience models describing the aggregation process of the tau and beta-amyloid. These models arise in the study of the causes of Alzheimer's Disease (AD). The dimension of these models ranges from 5 to 7 depending on the number of stages of aggregation considered. (Received September 15, 2020)

40 ► *Sequences, series, summability*

1163-40-1410   **Andrzej K Brodzik*** (`andrzej.k.brodzik@gmail.com`) and **Richard Tolimieri**. *On the Jacobi symbol and certain binary and binary-like sequences with good autocorrelation properties.*
Using the Chinese Remainder Theorem, it can be shown that the Jacobi symbol is an eigenvector of the $N \times N$ DFT matrix, where N is a product of distinct odd primes. This result facilitates the construction of a binary sequence, based on a simple modification of the Jacobi symbol, that has a two-level autocorrelation, provided $N = pq$, with $q = p + 2$. Further modification of the modified Jacobi sequence yields a binary-like complex-valued sequence with ideal autocorrelation, referred to as the Björck or the Golomb sequence. The modified Jacobi sequences are of interest in cryptography, due to their favorable sequence-theoretic properties, and, independently, in number theory, due to their close relationship with the Gauss sums. The Björck-Golomb sequences are of

interest in radar. Apart from these special cases, a general setting of multiplicative characters for the design of sequences with good correlation properties is briefly considered.   (Received September 15, 2020)

41 ▶ *Approximations and expansions*

1163-41-126   **Nabin Kumar Sahu*** (`nabinkumar_sahu@daiict.ac.in`), DA-IICT, Gandhinagar, India.
Frame expansion and construction of tight frames.
The notion of compression is given enormous attention in recent years because of its necessity in terms of the computational cost and other applicable features. But many a times the notion expansion appears to be quite useful. Tight frames are quite useful in signal reconstruction, signal and image de-noising, compressed sensing because of the availability of a simple, explicit reconstruction formula. So in this presentation, we discuss about the expansion of a frame by including some new vectors so that the new frame become a tight frame. We do the frame expansion in finite dimensional Hilbert spaces to construct tight frames. We formulate constructive algorithms to do the aforementioned task. The algorithms guarantee us to produce tight frames with very less computational cost. They also do not disturb the vectors in the given frame. We also present some applications of the aforementioned concept.   (Received August 19, 2020)

1163-41-971   **Laurent Baratchart**, **Herbert Stahl** and **Maxim L. Yattselev***
(`maxyatts@iupui.edu`). *Distribution of Poles of Optimal Rational Approximants.*
Preliminary report.
Early in the 20th century, Walsh has shown that

$$\limsup_{n\to\infty} \inf_{r\in\mathcal{R}_n} \|f - r\|_A \leq \inf_B \exp\left\{-1/\mathrm{cap}(A, B)\right\},$$

where f is holomorphic in a neighborhood of a continuum A, $\mathcal{R}_n$ is the set of rational functions of type (n, n), $\mathrm{cap}(A, B)$ is the condenser capacity, and the infimum on the right is taken over all compact sets B such that f is holomorphic in the complement of B (the complement must be connected and necessarily contain A). In general this bound is sharp. Elaborating on the work of Stahl, Gonchar and Rakhmanov have shown that

$$\lim_{n\to\infty} \inf_{r\in\mathcal{R}_n} \|f - r\|_A = \inf_B \exp\left\{-2/\mathrm{cap}(A, B)\right\}$$

if f is a multi-valued function meromorphic outside of a compact polar set. For a subclass of such functions, asymptotic distribution of poles of sequences of rational approximants $\{r_n\}$ such that

$$\lim_{n\to\infty} \|f - r_n\|_A = \inf_B \exp\left\{-2/\mathrm{cap}(A, B)\right\},$$

where A is a continuum, will be discussed.   (Received September 14, 2020)

1163-41-1029   **Adel Faridani*** (`faridani@oregonstate.edu`) and **Hussain Al-Hammali**
(`h.hammali@ytit.uz`). *Sampling of bandlimited functions with faster convergence.*
A family of sampling theorems for the reconstruction of bandlimited functions from their samples is presented. The theorems apply to both uniform sampling and a large class of non-uniform sampling sets. Taking a few additional samples is shown to yield more rapidly convergent series with lower truncation errors. The additional samples may be values of the function itself and/or its derivatives. Numerical illustrations are presented. (Received September 14, 2020)

1163-41-1191   **Xin Li*** (`xin.li@ucf.edu`). *A weighted max-min-max problem on the unit circle.*
For $n = 1, 2, 3, ...$, let $\{t_j\}$ be a set of n points such that

$$0 \leq t_1 < t_2 < \cdots < t_n < 2\pi. \tag{1}$$

A related extremal problem is to find

$$m_n := \max_{\{t_j\}} \min_{1\leq j\leq n} \left(\max_{t_j\leq t\leq t_{j+1}} \left| \prod_{j=1}^{n} (e^{it} - e^{it_j}) \right| \right). \tag{2}$$

Khrushchev (2009) proved that $m_n = 2$ and the optimal $\{t_j\}$ are equally spaced. Recently, Erdélyi, Hardin, and Saff (2015) obtained this using their inverse Bernstein inequality with the Gauss-Lucas theorem. We solve the following problem: Let $w(z)$ be a monic polynomial of degree n with no zero on the unit circle and let $\{t_j\}$ satisfy (1). Find

$$m_{n,w} = \max_{\{t_j\}} \min_{1\leq j\leq n} \left(\max_{t_j\leq t\leq t_{j+1}} \left| \frac{\prod_{j=1}^{n} (e^{it} - z_j)}{w(e^{it})} \right| \right).$$

This is a weighted version of (2). Although there is a version of inverse Bernstein inequality, but there is no analogue of Gauss-Lucas theorem for our situation. Indeed, we find that a zero-counting argument is enough.

As a by-product, we provide an alternative, more elementary proof even for the polynomial case.   (Received September 15, 2020)

42 ▶ *Fourier analysis*

1163-42-98   **Yugesh Shanmugam*** (`yugeshs@ssn.edu.in`), Assistant Professor, Department of Mathematics, SSN College of Engineering,Kalavakkam-603 110, Chennai, 603 110, India. *Generalized average sampling and reconstruction for shift-invariant spaces.*
Sampling theorem is one of the significant results in signal analysis and modern digital data processing. Average sampling is motivated by realistic needs. As an extension of the average sampling, we analyze generalized average sampling and reconstruction problem over shift-invariant space V_0. We show that for any $f \in V_0$ can be uniquely and stably reconstructed from its generalized average samples. The optimal upper bound for the support length of averaging functions can also be investigated and the reconstruction procedure could be discussed.   (Received August 14, 2020)

1163-42-127   **Theresa C Anderson*** (`tcanderson@purdue.edu`) and **Bingyang Hu**. *A classification of distinct dyadic systems.*
Distinct dyadic systems are ubiquitous in harmonic analysis and related fields as they allow one to decompose a continuous operator or object into a sum or intersection of (easier to handle) dyadic counterparts – as long as those counterparts form a distinct dyadic system. This topic had showed up in many places in the literature, and in joint work with Hu, Jiang, Olson, and Wu, we were able to completely classify these systems on the real line. With Hu, we were able to extend this work to Euclidean space. Both of these works are closely related (surprisingly?) to number theory, but in $\mathbb{R}^n$, certain interesting geometric structures also play a key role. (Received August 19, 2020)

1163-42-245   **John J. Benedetto*** (`jjb@umd.edu`) and **Michael R. Dellomo**. *Reactive sensing and multiplicative frame super-resolution.*
The problem is to evaluate the behavior of an object when primary sources of information about the object become unavailable, so that any information must be obtained from the intelligent use of available secondary sources. This evaluative process is reactive sensing. Reactive sensing is initially viewed in terms of spatial super-resolution. The theory of reactive sensing is based on two equivalent ideas, one physical and one mathematical. The physical idea models volume, e.g., engine volume in the case of analyzing engine health, and the sensitivity of sensors to such volume. The mathematical idea of multiplicative frames provides the factorization theory to compare quantitatively such volume and sensitivity. This equivalence is the foundation for reactive sensing theory and its implementation. Simulations for airplane mechanical vibration problems are given.   (Received August 30, 2020)

1163-42-276   **Jessica Bennett*** (`jessica_bennett@brown.edu`) and **Penelope Fiaschetti** (`pmf2022@bu.edu`). *Robust and Efficient Phase Retrieval from Magnitude-Only Windowed Fourier Measurements.* Preliminary report.
We propose and analyze a new generalization of an existing algorithm to reconstruct a complex vector (up to a global phase factor) from the squared magnitude of its windowed discrete Fourier transform. This is more commonly referred to as a phase retrieval problem, since this process requires the recovery of critically important phase information from magnitude-only measurements. This is a challenging yet fascinating non-linear inverse problem since there are often several possible solutions. The proposed algorithm utilizes results from discrete Fourier analysis to linearize the governing equations and obtain a highly structured Fourier based linear system. This linear system of equations can be efficiently inverted using the fast Fourier transform (FFT) algorithm. This provides relative phase information which we use to construct a special class of banded matrices, on which we perform spectral analysis to retrieve individual phase information. In addition to developing an efficient reconstruction algorithm, we provide mathematically rigorous theoretical error bounds in the case of noisy measurements, and provide numerical simulations demonstrating that this algorithm is computationally efficient and able to recover data in the presence of noise.   (Received August 31, 2020)

1163-42-506   **Árpád Bényi***, benyia@wwu.edu, and **Tadahiro Oh**. *Modulation spaces with scaling symmetry.*
We indicate how to construct a family of modulation spaces that have a scaling symmetry and illustrate the behavior of the Schrödinger multiplier on such function spaces.   (Received September 08, 2020)

1163-42-921   **Alexander Kegeles*** (`alexander.kegeles@geophysik.tu-freiberg.de`),
Gustav-Zeuner-Str. 12, 09599 Freiberg, Germany. *Harmonic continuation of vector fields on Lipschitz surfaces.* Preliminary report.

On the sphere, every square-integrable vector field can be written as a sum of three: one, that has a harmonic extension to the inside of the sphere, one, that has a harmonic extension to the outside of the sphere, and one that is divergence-free on the sphere. This decomposition is known as the Hardy-Hodge decomposition. This decomposition allows us to use harmonic analysis to study some properties of square-integrable vector fields on a sphere. In this talk, I present our result in which we show how this decomposition generalizes to hold on Lipschitz surfaces. (Received September 14, 2020)

1163-42-1122   **Evan Randles*** (`erandles@colby.edu`) and **Huan Bui**. *A generalized polar-coordinate integration formula with applications to local (central) limit theorems and operator semigroups.*

The classical polar-coordinate integration formula is an essential tool in harmonic analysis and, especially, the analysis of radial functions in $\mathbb{R}^d$. In this talk, we discuss a generalization of the polar-coordinate integration formula where integration with respect to spherical measure $d\theta$ over the unit sphere $\mathbb{S}^{d-1}$ is replaced by integration with respect to a certain surface-carried measure $d\sigma$ over a compact (non-smooth and non-convex) hypersurface S and where the isotropic dilation $x \mapsto r \cdot x$ is replaced by a generally anisotropic dilation $x \mapsto T_r x$. Using oscillatory integral techniques, we will then discuss applications of this work to local (central) limit theorems and operator semigroups. (Received September 14, 2020)

1163-42-1256   **Betsy Stovall*** (`stovall@math.wisc.edu`). *Fourier restriction to degenerate hypersurfaces.*
We will discuss some recent progress on the restriction theory of certain hypersurfaces with vanishing, varying, or negative curvature. The talk will include a discussion of work joint with Jeremy Schwend, Benjamin Bruce, and Diogo Oliveira e Silva. (Received September 15, 2020)

1163-42-1521   **Ellen Urheim*** (`urheim@math.upenn.edu`). *Oscillatory integral operators and geometric stability.*
We will discuss recent work, joint with P. T. Gressman, which proves best-possible decay estimates for a family of multilinear oscillatory integral operators of a form inspired by the general framework of Christ, Li, Tao, and Thiele. These estimates are geometrically stable in the sense that this family of operators is closed under small, smooth perturbations of both the phase and the projections. The proof relies on a novel frequency space decomposition which nearly diagonalizes the problem. (Received September 15, 2020)

1163-42-1525   **Taryn C Flock*** (`tflock@macalester.edu`). *Sharp inequalities for the X-ray Transform.*
Inequalities play a central role in harmonic analysis. However, in many cases the fundamental question "When and how can one achieve equality?" is left unanswered. Answering these questions opens the door to proving stronger versions of the inequality. In this talk, we'll consider this question in the context of L^p-L^q inequalities for the X-ray transform. I'll discuss how exploring the problem with undergraduate researchers Connor Bass and Arthur DressenWall has shaped my perspective. (Received September 15, 2020)

1163-42-1554   **Wojciech Czaja*** (`wojtek@math.umd.edu`), Norbert Wiener Center, Department of Mathematics, University of Maryland, College Park, MD 20742. *Fourier Scattering Transforms as Efficient Feature Extractors.*
We present a construction of a family of feature extractors which combine Mallat's scattering transform framework with the benefits of different time-frequency representations. We do this by introducing a class of frames, called uniform covering frames, which includes a variety of semi-discrete Gabor systems and other Fourier-based representations. We then incorporate these frames into an iterative neural network-like structure, to generate our candidate features, which we aggregate into a new scattering transformation. This approach proves advantageous in several data-related applications, in particular in the context of hyperspectral imaging and similar imaging modalities. We also explore several other mathematically-inspired ideas, including those of composite wavelets and rotationally invariant Fourier frames. This talk presents a body of joint work with several co-authors: Weilin Li, Ilya Kavalerov, Mike Pekala, and Rama Chellappa. (Received September 15, 2020)

1163-42-1634   **Wojciech Czaja*** (`wojtek@math.umd.edu`), Norbert Wiener Center, Department of Mathematics, University of Maryland, College Park, MD 20742, and **Weilin Li**, **Yiran Li** and **Mike Pekala**. *MaxFun Pooling - maximal function-inspired pooling operation.*
Convolutional neural networks (CNNs), as a popular type of architecture in deep learning, have shown outstanding performances in various applications. Many examples of CNNs have used pooling as a layer in their

networks. Pooling is a dimension reduction technique that divides an image into subregions and returns only one pixel value as the representative of each subregion. Max pooling and average pooling are widely used classical pooling strategies which have been popular in many application tasks. Inspired by the Hardy and Littlewood's maximal function from harmonic analysis, we introduce a novel pooling strategy, which we call MaxFun pooling, and we compare it to both max pooling and local averaging. We illustrate the applicability of the new method with an example from convolutional sparse coding.   (Received September 15, 2020)

43 ▶ *Abstract harmonic analysis*

1163-43-125   **Lillian B Pierce*** (pierce@math.duke.edu). *On superorthogonality.*
We will survey how superorthogonality in a sequence of functions results in direct or converse inequalities for an associated square function. We distinguish between three main types of superorthogonality, which we demonstrate arise in a wide array of settings in harmonic analysis and number theory. This perspective gives clean proofs of central results, and unifies topics including Khintchine's inequality, Walsh-Paley series, discrete operators, decoupling, counting solutions to systems of Diophantine equations, multicorrelation of trace functions, and the Burgess bound for short character sums.   (Received August 18, 2020)

1163-43-187   **Xuan Thinh Duong,**, Department of Mathematics, Macquarie University, NSW, 2109, Australia, **Ji Li**, Department of Mathematics, Macquarie University, NSW, 2109, Australia, **Eric T Sawyer**, Hamilton, Ontario , Canada, **Naga Manasa Vempati*** (m.vempati@wustl.edu), Department of Mathematics, Washington University, One Brookings Drive, Saint Louis, MO 63130-4809, **Brett D. Wick** (wick@math.wustl.edu), Department of Mathematics, Washington University in Saint Louis, One Brookings drive, Saint Louis, MO 63130-4809, and **Dongyong Dongyong Yang**, Department of Mathematics, Xiamen University, Xiamen, 361005. *A two weight inequality for Calderón-Zygmund operators on spaces of homogeneous type with applications.*
Let (X, d, μ) be a space of homogeneous type in the sense of Coifman and Weiss, i.e. d is a quasi metric on X and μ is a positive measure satisfying the doubling condition. Suppose that u and v are two locally finite positive Borel measures on (X, d, μ). Subject to the pair of weights satisfying a side condition, we characterize the boundedness of a Calderón-Zygmund operator T from $L^2(u)$ to $L^2(v)$ in terms of the A_2 condition and two testing conditions. For every cube $B \subset X$, we have the following testing conditions, with $\mathbf{1}_B$ taken as the indicator of B

$$\|T(u\mathbf{1}_B)\|_{L^2(B,v)} \leq \mathcal{T}\|1_B\|_{L^2(u)},$$

$$\|T^*(v\mathbf{1}_B)\|_{L^2(B,u)} \leq \mathcal{T}\|1_B\|_{L^2(v)}.$$

The proof uses stopping cubes and corona decompositions originating in work of Nazarov, Treil and Volberg, along with the pivotal side condition.   (Received August 25, 2020)

1163-43-1680   **Hans G. Feichtinger***, hans.feichtinger@univie.ac.at. *Time-Frequency Analysis and Suitable Function Spaces.*
The key-point of this talk will be some exploration of function spaces concepts arising from time-frequency analysis respectively Gabor Analysis. Modulation spaces and Wiener amalgams have proved to be indispensable tools in time-frequency analysis, but also for the treatment of pseudo-differential operators or Fourier integral operators.

More precisely, we will recall a short summary of the concepts of Wiener amalgam spaces and modulation spaces, as well as the concept of Banach Gelfand Triples, with the associated kernel theorem (in the spirit of the L.Schwartz kernel theorem). We will indicate in which sense these spaces allow to capture more precisely the mapping properties of operators which may be unbounded in the Hilbert space setting. The subfamily of translation and modulation invariant spaces plays a specific role, with naturally associated regularization operators involving smoothing by convolution and localization by pointwise multiplication.   (Received September 16, 2020)

45 ▶ *Integral equations*

1163-45-1351   **Rolf J Ryham*** (`rryham@fordham.edu`), Fordham University, Department of Mathematics, JMH 401, Bronx, NY 10458, and **Szu-Pei Fu**, **Yuan-Nan Young** and **Bryan Quaife**. *Hydrodynamics of small unilamellar vesicles (sUVs) simulated using a hybrid approach.* Preliminary report.

In this talk we simulate the hydrodynamics of small unilamellar vesicles (sUVs) using a hybrid approach that is shown to capture the formation of sUVs in a solvent (SIAM J Multiscale Model. Simul., vol 18, pp. 79-103). In this hybrid formulation, the non-local interactions between the coarse-grained lipid molecules are described by a hydrophobicity functional, giving rise to forces and torques (between lipid particles) that dictate the motion of both particles and the fluid flow in the viscous solvent. Both the hydrophobic and hydrodynamic interactions between the coarse-grained amphiphilic particles are formulated into integral equations, which allow for accurate and efficient numerical simulations in both two- and three-dimensions. We validate our hybrid coarse-grained model by reproducing various physical properties of a lipid bilayer membrane, and use this simulation tool to examine how a small unilamellar vesicle behaves under a planar shear flow, and investigate the collective dynamics of sUVs under a shear flow. Finally we also examine the possibility of membrane rupture by extreme flowing conditions.   (Received September 15, 2020)

46 ▶ *Functional analysis*

1163-46-118   **Javad Mashreghi**, **Pierre-Olivier Parisé** and **Thomas Ransford*** (`ransford@mat.ulaval.ca`). *Failure of approximation of odd functions by odd polynomials.*
We construct a Hilbert holomorphic function space H on the unit disk such that the polynomials are dense in H, but the odd polynomials are not dense in the odd functions in H. As a consequence, there exists a function f in H that lies outside the closed linear span of its Taylor partial sums $s_n(f)$, so it cannot be approximated by any triangular summability method applied to the $s_n(f)$. We also show that there exists a function f in H that lies outside the closed linear span of its radial dilates f_r, $r < 1$.   (Received August 17, 2020)

1163-46-143   **Raymond Cheng*** (`rcheng@odu.edu`), Department of Mathematics and Statistics, Old Dominion University, 4700 Elkhorn Avenue, Norfolk, VA 23529. *Inner functions, zero sets, and reproducing kernel spaces.*
By comparison with the familiar shift operator on the Hardy space H^2, we will ponder what it could mean for a vector to be "inner" with respect to an arbitrary operator on a Hilbert space. We'll look at some things we can do with this concept (e.g., describe zero sets of reproducing kernel spaces), and some things we cannot do (e.g., insist on a canonical factorization). Joint work with J. Mashreghi and W. Ross.   (Received August 21, 2020)

1163-46-175   **Kari Eifler*** (`keifler@math.tamu.edu`), TX. *Non-local games and quantum metric spaces.*
We will look at quantum metric spaces, which are a non-commutative analogue of finite metric spaces. Banica has defined the quantum symmetry group of a finite metric space, and I will talk about how to capture Banica's definition using the Weaver-Kupperberg framework of quantum metric spaces. I will also connect this extension to the theory of non-local games.   (Received August 25, 2020)

1163-46-200   **Jurij Volčič*** (`volcic@math.tamu.edu`). *Positive trace polynomials.*
Trace polynomials are polynomials in noncommuting variables and traces of their products. While originating in invariant theory as equivariant maps between tuples of matrices, trace polynomials more recently received attention in free probability and quantum information theory. This talk addresses positive semidefiniteness of their evaluations on matrices or operators from von Neumann algebras. New algebraic certificates for positivity (Positivstellensätze) in terms of sums of squares and their traces are presented. In the dimension-fixed setting, these certificates are proved using invariant theory, polynomial identities and real algebraic geometry; on the other hand, functional analysis and operator algebras are applied in the dimension-free setting. As a consequence, optimization of trace polynomials over von Neumann algebras subject to tracial constraints can be tackled using semidefinite programming.

The talk is based on joint work with Igor Klep, Victor Magron and Špela Špenko.   (Received August 26, 2020)

1163-46-300   **Marcel Bischoff**, **Ian Charlesworth**, **Samuel Evington**, **Luca Giorgetti**, **André Henriques** and **David Penneys*** (`penneys.2@osu.edu`). *Distortion for multifactor bimodules and representations of multifusion categories.*

We call a von Neumann algebra with finite dimensional center a multifactor. We introduce an invariant of bimodules over II_1 multifactors that we call modular distortion, and use it to formulate two classification results.

We first classify connected, finite index, finite depth II_1 hyperfinite multifactor inclusions $A \subset B$ in terms of the standard invariant (a unitary planar algebra), together with the restriction to A of the unique Markov trace on B. The latter determines the modular distortion of the associated bimodule. Three crucial ingredients are Popa's uniqueness theorem for such inclusions which are also homogeneous, for which the standard invariant is a complete invariant, a generalized version of the Ocneanu Compactness Theorem, and the notion of Morita equivalence for inclusions.

Second, we classify fully faithful representations of unitary multifusion categories into bimodules over hyperfinite II_1 multifactors in terms of the modular distortion. Every possible distortion arises from a representation, and we characterize the proper subset of distortions that arise from connected II_1 multifactor inclusions.   (Received September 01, 2020)

1163-46-311   **Kathryn McCormick*** (`kathryn.mccormick@csulb.edu`). *Holomorphic subalgebras of n-homogeneous C^*-algebras.* Preliminary report.

There is a long tradition of analyzing a C^*-algebra through some topological invariant. One such result is Tomiyama and Takesaki's 1961 proof that an n-homogeneous C^*-algebra A is determined up to $*$-isomorphism by an underlying continuous matrix bundle, and A is an algebra of continuous cross-sections of the bundle. Suppose that the base space of the bundle is a bordered Riemann surface with finitely many smooth boundary components. Then for each such realization of an n-homogeneous C^*-algebra, one can define a subalgebra of holomorphic cross-sections. We will describe a partial result towards classifying these subalgebras up to complete isometric isomorphism based on topological invariants.   (Received September 01, 2020)

1163-46-347   **Chris Bruce***, chris.bruce@glasgow.ac.uk. *C*-algebras of ring-theoretic origin.*

C*-algebras can be realised as operator-norm closed, self-adjoint algebras of operators on Hilbert space. They comprise one of the most important classes of operator algebras. Recently, there have been several constructions of C*-algebras from ring theory. Indeed, out of an integral domain R together with multiplicative monoid M of non-zero elements in R, one can naturally construct several C*-algebras. C*-algebras arising this way provide an interesting class of operator algebras, whose structure is complicated yet sufficiently tractable. I will explain some of these constructions, and then discuss possible directions for future work   (Received September 03, 2020)

1163-46-504   **Douglas Farenick**, **Farrah Huntinghawk**, **Adili Masanika** and **Sarah Plosker*** (`plosker@brandonu.ca`), Brandon, Manitoba R7A6A9, Canada. *Complete Order Equivalence of Spin Unitaries.*

In this talk, we study spin systems, which are finite sets of anticommuting selfadjoint unitary matrices. We focus on complete order isomorphisms between these linear spaces of matrices: linear isomorphisms such that the matricial order within these spaces is preserved. We extend our work to the case of countable many unitaries. We also connect our findings to recent developments on the topic of free spectrahedra and matrix convex sets (in particular, maximum spin balls).   (Received September 08, 2020)

1163-46-522   **Palle E. T. Jorgensen**, **Sooran Kang** and **Myung-Sin Song*** (`msong@siue.edu`), Department of Mathematics & Statistics, Southern Illinois University Edwardsville, Edwardsville, IL 62026, and **Feng Tian**. *Data Dimension Reduction using Kernel Principal Component Analysis.*

In linear data case, Principal Component Analysis is used for data dimension reduction. In nonlinear data dimension reduction , kernel-Principal Component Analysis is used instead with manifold and feature space transforms. The results extend earlier work for probabilistic Karhunen-Loève transforms on compression of wavelet images which were algorithms for optimization, selection of efficient bases, or components, which serve to minimize entropy and error; and hence to improve digital representation of images, and hence of optimal storage, and transmission. Several new theorems for data-dimension reduction will be presented, and with the use of frames in Hilbert space, and a new Hilbert-Schmidt analysis, we identify when a choice of Gaussian kernel is optimal.   (Received September 08, 2020)

1163-46-577   **Daniel Aron Alpay*** (`alpay@chapman.edu`), Schmid College of Science and Technology, Chapman University, One University Drive, Orange, CA 92866. *Reproducing kernel spaces in hypercomplex analysis: quaternions, split quaternions and beyond.*

Reproducing kernel Hilbert spaces of analytic functions in one and several complex variables play an important role in analysis and operator theory. The Bargmann-Fock-Segal space and the Drury-Arveson space are two important instances of such spaces. We will discuss the counterpart of these spaces in hypercomplex analysis. We discuss in particular the slice-hyperholomorphic setting, the case of Fueter series and the case of split quaternions.   (Received September 09, 2020)

1163-46-721   **Priyanga Ganesan*** (`priyanga.g@tamu.edu`), 500 Wellborn Rd N, Apt 1203-C, College Station, TX 77840. *Quantum Graph Homomorphisms.*

Quantum graphs are an operator space generalization of classical graphs that have emerged in operator systems theory, non-commutative topology and quantum information theory. In this talk, I will review the literature on quantum graphs and introduce a notion of quantum graph homomorphisms, using a non-local game with quantum inputs and classical outputs. We will discuss how the winning strategies of the game correspond to different versions of quantum homomorphisms in the literature and lead to a unified theory. This is based on joint work with Michael Brannan and Samuel Harris.   (Received September 11, 2020)

1163-46-748   **Roy Araiza*** (`raraiza@purdue.edu`) and **Travis Russell**. *An Abstract Characterization for Projections in Operator Systems.*

Given an abstract operator system V it is not clear how one would go about defining the notion of a projection. During this talk I will present an answer and some recent results on this question. This is done by first considering abstract compression operator systems associated with a positive contraction in V and then determining when we have a realization of V in such an abstract compression operator system. It then follows that there is a one-to-one correspondence between abstract and concrete projections, and in particular, that every abstract projection is a concrete projection in the C*-envelope of V. I will then conclude with some applications to quantum information theory. In particular, the study of certain correlation sets.   (Received September 12, 2020)

1163-46-758   **Tepper L. Gill*** (`tgill@howard.edu`). *An Extension of the Yosida Approximation.*

When I first came to Howard, Jim Donaldson was interested in C_o semigroups of operators and we had discussions on the approximation of unbounded linear operators by a sequence of bounded operators. It is known that if a closed densely defined linear operator A on a separable Banach space, is the generator of a C_o semigroup of contraction operators then resolvent set $\rho(A) \supset (0, \infty)$, and for each λ, with $Re(\lambda) > 0$, $A_\lambda = \lambda A(\lambda - A)^{-1}$ is bounded and $\lim_{\lambda \to \infty} A_\lambda f = Af$, for $f \in D(A)$ (Yosida approximation).

Vernice Steadman replaced contraction operators by uniformly bounded ones for a restricted class of Banach spaces. I will prove that, if A a closed densely defined linear operator on a separable Banach space, there always exists bounded linear operators A_n, with $\lim_{\lambda \to \infty} A_n f = Af$, for $f \in D(A)$.   (Received September 12, 2020)

1163-46-761   **Sara Azzali**, **Sarah Browne**, **Maria Paula Gomez Aparicio**, **Lauren Ruth*** (`lcruth222@gmail.com`) and **Hang Wang**. *The Baum-Connes correspondence for pure braid groups.* Preliminary report.

We calculate the left-hand side and the right-hand side of the Baum-Connes correspondence for the pure braid group on n strands, each side relying on different techniques. Our long-term goal is to elucidate Schick's abstract proof of the correspondence for braid groups by explicitly describing the assembly map.   (Received September 12, 2020)

1163-46-765   **James Fletcher**, **Elizabeth Gillaspy*** (`elizabeth.gillaspy@mso.umt.edu`) and **Aidan Sims**. *Homotopy of product systems, and K-theory for k-graph algebras.*

One can model the C^*-algebra of a higher-rank graph (k-graph) via a product system, which is a higher-dimensional version of a C^*-correspondence. Just as for the Cuntz–Pimsner algebra associated to a C^*-correspondence, there is a 6-term exact sequence for the K-theory of the Cuntz–Nica–Pimsner algebra of a product system. In this talk, I will explain the compatibility of this 6-term exact sequence with the new notion of a homotopy of product systems, and discuss the applications to higher-rank graphs. Our results imply that certain questions about the K-theory of k-graph C^*-algebras reduce to questions about the path-connectedness of certain spaces of matrices.   (Received September 12, 2020)

1163-46-834   **Roy Araiza*** (`raraiza@purdue.edu`) and **Travis Russell**. *A Look into the Abstract
                 Theory of Operator Systems and Some Applications to Quantum Information Theory.*
Concretely an operator system $\mathcal{V}$ is a self-adjoint unital subspace of a C*-algebra $\mathcal{A}$. In particular one may take
$\mathcal{A}$ to be $B(H)$, the bounded operators on some Hilbert space H. The abstract definition of an operator system
is much more involved and this, along with other abstract properties of operator systems were first investigated
in the 70's by Choi and Effros. It was realized that operator system techniques can be used to attack problems
in C*-algebra theory and quantum information theory, though the latter was realized much later. Focusing
on the abstract setting, since operator systems are not ∗-algebras then it is not clear how one would go about
defining a projection in such a space. I will give an answer to this problem and discuss how one may use these
abstract projections to characterize certain correlation sets that arise in quantum information theory.   (Received
September 13, 2020)

1163-46-865   **Raul E Curto*** (`raul-curto@uiowa.edu`), Dept. of Mathematics, University of Iowa, Iowa
                 City, IA 52242, and **In Sung Hwang** and **Woo Young Lee**. *The Beurling-Lax-Halmos
                 Theorem for Infinite Multiplicity.*
We consider several questions emerging from the Beurling-Lax-Halmos Theorem, which characterizes the shift-
invariant subspaces of vector-valued Hardy spaces. The Beurling-Lax-Halmos Theorem states that a backward
shift-invariant subspace is a model space $\mathcal{H}(\Delta) \equiv H_E^2 \ominus \Delta H_E^2$, for some inner function Δ. Our first question
calls for a description of the set F in H_E^2 such that $\mathcal{H}(\Delta) = E_F^*$, where E_F^* denotes the smallest backward
shift-invariant subspace containing the set F. This is related to a canonical decomposition of operator-valued
strong L^2-functions.

   Next, we ask: Is every shift-invariant subspace the kernel of a (possibly unbounded) Hankel operator? We
study a new notion of "Beurling degree" for an inner function, and establish a deep connection between the
spectral multiplicity of the model operator (the truncated backward shift) and the Beurling degree of the corre-
sponding characteristic function. We also consider the notion of meromorphic pseudo-continuations of bounded
type for operator-valued functions, and use this notion to study the spectral multiplicity of model operators
between separable complex Hilbert spaces. In particular, we consider the multiplicity-free case.   (Received
September 13, 2020)

1163-46-898   **Marcel Bischoff*** (`bischoff@ohio.edu`), **Simone Del Vecchio** and **Luca Giorgetti**.
                 Compact Hypergroups from Discrete Subfactors.
We show that to any local braided discrete subfactor $N \subset M$ of type III one can associate a "compact hypergroup"
acting by extremal ucp maps on M, such that N is given by the fixed point algebra under this action. If the
subfactor is also of depth two, then the hypergroup is exactly a compact group G and N is the fixed point under
a minimal action of G. The motivation is to obtain an invariant and understand discrete inclusions of conformal
nets.   (Received September 13, 2020)

1163-46-1020   **Ishan Ishan** (`ishan.ishan@vanderbilt.edu`), **Jesse Peterson***
                 (`jesse.d.peterson@vanderbilt.edu`) and **Lauren Ruth** (`lruth@mercy.edu`). *Von
                 Neumann equivalence and properly proximal groups.*
We will introduce the notion of von Neumann equivalence between groups and von Neumann algebras. This
is a non-commutative generalization of measure equivalence for groups. We show that a number of analytic
properties such as amenability, the Haagerup property, and property (T) are preserved by this equivalence. We
also show that proper proximality, which was introduced recently by R. Boutonnet, A. Ioana, and the speaker,
is preserved by von Neumann equivalence. We then obtain examples of non-inner amenable groups that are not
properly proximal. This is joint work with I. Ishan, and L. Ruth.   (Received September 14, 2020)

1163-46-1099   **Gayatri Pany*** (`gayatripany@gmail.com`), Department of Mathematics, University of
                 Central Florida, Orlando, FL 32816, and **Ram Narayan Mohapatra**
                 (`ram.mohapatra@ucf.edu`), Department of Mathematics, University of Central Florida,
                 Orlando, FL 32816. *A study on vector variational-like inequalities with convexi ficator
                 approach.*
This work deals with the vector variational-like inequalities, namely Stampacchia and Minty type under invexity
in the framework of convexificators. Equivalence between both the problems along with the link to vector
optimization problem is analyzed. Further the bilevel version of these problems are formulated followed by the
study on the solution procedure involving the auxiliary principle technique. An iterative algorithm is constructed.
It is proved that the approximates obtained converge strongly to the exact solution.   (Received September 14,
2020)

1163-46-1174   **James A Mingo*** (`mingo@queensu.ca`) and **Pei-Lun Tseng** (`16plt@queensu.ca`).
Infinitesimal Freeness and Non-commutative Functions. Preliminary report.

Free probability has shown itself to be the probability theory of random matrices. Infinitesimal freeness is finer type of independence used to analyze spike models in random matrix theory. I will show how non-commutative functions are used to find the functional relations for infinitesimal freeness. This is joint work with Pei-Lun Tseng.   (Received September 14, 2020)

1163-46-1487   **Tepper L. Gill** and **Douglas Mupasiri*** (`mupasiri@uni.edu`), 220 Wright Hall,
Department of Mathematics, University of Northern Iowa, Cedar Falls, IA 50614-0506. *An
application of Kuelb's Hilbert Representation of a separable Banach space to the problem of
determining whether a given subspace is complemented in the space.*

It is well known that c_0 is not complemented in ℓ_∞. One way to prove this result is to use Kuelb's Hilbert Representation of ℓ_∞. The approach suggests a method that seems to be widely applicable to the determination of whether a given subspace of a separable Banach space is complemented in the space. In this talk, we will describe the method.   (Received September 15, 2020)

1163-46-1616   **Aqeeb A Sabree*** (`sabreea@xavier.edu`), 108 Hinkle Hall, Xavier University, Cincinnati,
OH 45207. *Positive Definite Kernels, Harmonic Analysis, and Boundary Spaces:
Drury-Arveson Theory, and Related.*

A reproducing kernel Hilbert space (RKHS), $\mathcal{H} \subset \mathcal{F}(X, \mathbb{C})$, is a Hilbert function space where the values $f(x)$, for $f \in \mathcal{H}, x \in X$, are reproduced from the inner product $< \cdot, \cdot >_\mathcal{H}$:

$$f(x) = < f(\cdot), k_x(\cdot) >_\mathcal{H}, \quad \text{where } k_x \in \mathcal{H}.$$

In this setting, we have an explicit correspondence between reproducing kernel Hilbert spaces and reproducing kernel functions—also called positive definite functions. My research studies the duality between positive definite functions and their boundary spaces for specific RKHSs. We will define these notions and cover an overview of the subject area. Ultimately, this presentation will provide the audience with a look at applications of RKHSs to harmonic analysis, signal/image analysis, mathematical physics, and machine learning algorithms.   (Received September 15, 2020)

1163-46-1669   **Götz Eduard Pfander*** (`pfander@ku.de`), Ostenstrasse 26, 85072 Eichstätt, Germany,
and **Dae Gwan Lee** (`dae.lee@ku.de`), Ostenstrasse 26, 85072 Eichstätt, Germany.
Sampling of restricted classes of operators.

Classical sampling theory considers the recovery of a bandlimited function from its values on an sufficiently fine lattice in the real numbers. This methodology can be extended to allow for the recovery of so-called bandlimited operators from their action on a single input signal, a distribution supported on a lattice in the real numbers.

After a short recap of sampling of operator theory we shall discuss the recovery of operators that are known to satisfy linear constraints. Further, we discuss the problem of transmitting information through an unknown channel operator that has not been identified/sampled yet.   (Received September 16, 2020)

47 ▶ *Operator theory*

1163-47-179   **Constanze D Liaw*** (`liaw@udel.edu`), Department of Mathematical Sciences, University
of Delaware, 311 Ewing Hall, Newark, DE 19716, and **Sergei Treil** and **Alexander
Volberg**. *Dimension of the exceptional set in the Aronszajn–Donoghue theorem for finite
rank perturbations.*

The classical Aronszajn–Donoghue theorem states that for a rank one perturbation of a self-adjoint operator (by a cyclic vector) the singular parts of the spectral measures of the original and perturbed operators are mutually singular. As simple direct sum type examples show, this result does not hold for finite rank perturbations. However, the set of exceptional perturbations is pretty small.

Namely, for a family of rank d perturbations $A_\alpha := A + B\alpha B*$, $B : \mathbb{C}^d \to \mathcal{H}$, with Ran B being cyclic for A, parametrized by $d \times d$ Hermitian matrices α, the singular parts of the spectral measures of A and A_α are vector mutually singular for all α except for a small exceptional set E.

It was shown earlier, that E is a subset of measure zero of the space of $d \times d$ Hermitian matrices. We now learn that the set E has small Hausdorff dimension, $\dim(E) \le d^2 - 1$.

This presentation is based on joint work with Sergei Treil and Alexander Volberg.   (Received August 25, 2020)

1163-47-182   **Kelly Bickel**, **Anne Greenbaum**, **Pamela Gorkin*** (`pgorkin@bucknell.edu`), **Thomas Ransford**, **Felix Schwenninger** and **Elias Wegert**. *Crouzeix's Conjecture and Related Problems.*

Let $W(A)$ denote the numerical range of an $n \times n$ matrix A:

$$W(A) = \{\langle Ax, x \rangle : \|x\| = 1\}.$$

Michel Crouzeix showed that there is a constant C such that

$$\|f(A)\| \leq C \sup_{z \in W(A)} |f(z)|,$$

for all f analytic on a neighborhood of $W(A)$ and he conjectured that the best constant is $C = 2$. In this talk, we present a short history of work on the conjecture, some recent results, and open questions related to this conjecture.   (Received August 25, 2020)

1163-47-185   **Emmanuel Fricain*** (`emmanuel.fricain@univ-lille.fr`). *Backward shift invariant subspaces in reproducing kernel Hilbert spaces.*

In this talk, we will discuss the backward shift invariant subspaces for an abstract class of reproducing kernel Hilbert space. This result is inspired by a result of Sarason concerning the de Branges-Rovnyak space (in the non-extreme case). Furthermore, we give new applications in the context of the range space of co-analytic Toeplitz operators and sub-Bergman spaces.   (Received August 25, 2020)

1163-47-188   **Joseph A Ball*** (`joball@math.vt.edu`), Department of Mathematics, Virginia Tech, Blacksburg, VA 24061. *The Arveson extension theorem for completely positive noncommutative kernels.*

A <u>completely positive (cp) kernel</u> on Ω to $\mathcal{L}(\mathcal{A}, \mathcal{L}(\mathcal{Y}))$ (bounded linear operators from the C^*-algebra $\mathcal{A}$ to bounded linear operators on the Hilbert space $\mathcal{Y}$) in the sense of Barreto-Bhat-Liebscher-Skeide is a function $k \colon \Omega \times \Omega \to \mathcal{L}(\mathcal{A}, \mathcal{L}(\mathcal{Y}))$ satisfying a certain complete positivity condition. A <u>free noncommutative (nc) cp kernel</u> is a quantized version of the BBLS cp kernel, whereby one allows the point set to include matrices over the level-1 set of points Ω and demands that the kernel function satisfy natural compatibility relations with respect to intertwinings via complex scalar matrices of the matrix-point arguments. The notion of cp nc kernel contains that of cp map between C^*-algebras (or more generally operator systems) as a special case. We present a natural extension of the Arveson extension theorem for cp maps to the level of nc cp kernels. This is joint work with Gregory Marx and Victor Vinnikov.   (Received August 25, 2020)

1163-47-253   **Edward John Timko*** (`edward.timko@umanitoba.ca`) and **Raphael Clouatre**. *Function Algebras and d-tuples of Model Operators.*

In this talk, we look at algebras generated by d-tuples of model operators associated with certain complete Nevanlinna-Pick spaces. The central question we investigate is when these algebras fail to have completely isometric Gelfand transforms. We connect this with properties of Arveson's noncommutative Choquet boundary, and find a variety of conditions, often on the joint spectrum of the model d-tuple, under which the Gelfand transforms of the algebras are not completely isometric. This is joint work with Raphaël Clouâtre.   (Received August 30, 2020)

1163-47-266   **Igor Klep*** (`igor.klep@fmf.uni-lj.si`). *Noncommutative partial convexity.*

Motivated by classical notions of partial convexity, and bilinear matrix inequalities, we present the theory of Γ-convexity in the free noncommutative setting. Given a tuple of polynomials Γ, a free set is called Γ-convex if it is closed under isometric conjugation by isometries intertwining Γ. We establish an Effros-Winkler Hahn-Banach separation theorem for Γ-convex sets; they are delineated by linear pencils in the coordinates of Γ and the variables x. We shall also consider partial convexity for nc functions. For instance, we will explain that nc rational functions that are partially convex admit butterfly-type realizations that necessitate square roots.

This is based on joint works with Michael Jury, Mark Mancuso, Scott McCullough, James Pascoe and Jurij Volčič.   (Received August 31, 2020)

1163-47-269   **Jeffrey S Geronimo** and **Hugo J Woerdeman*** (`hjw27@drexel.edu`), Department of Mathematics, Drexel University, 3141 Chestnut Street, Philadelphia, PA 19104, and **Chung Y Wong**. *The autoregressive filter problem for multivariable degree one symmetric polynomials.* Preliminary report.

The multivariable autoregressive filter problem asks for a polynomial $p(z) = p(z_1, \ldots, z_f)$ without roots in the closed d-disk based on prescribed Fourier coefficients of its spectral density function $1/|p(z)|^2$. The necessary and sufficient conditions derived in this paper for the existence of a degree one symmetric polynomial reveal a major divide between the case of at most two variables vs. the the case of three or more variables. The latter

involves multivariable elliptic functions, while the former only involve polynomial conditions. The three variable case is treated with more detail, and entails hypergeometric functions. (Received August 31, 2020)

1163-47-299 **Anna Skripka*** (`skripka@math.unm.edu`). *Approximation of functions of differential operators.* Preliminary report.

We will discuss recent advancements in approximation of operator functions that arise in problems of mathematical physics and noncommutative geometry. In those problems, natural perturbations are not compact and do not commute with initial operators while traces of approximation errors should be small and nicely bounded. The talk is based on joint work with Teun van Nuland. (Received September 01, 2020)

1163-47-301 **Robert V Huben***, rhuben@huskers.unl.edu. *Gauge Invariant Uniqueness and Amenable Reductions.*

P-graph C*-algebras are a generalization of graph C*-algebras and k-graph C*-algebras where the paths in the graph are given a "length" from P, the set of positive elements of a group G under some weak quasi-lattice ordering. We introduce a certain kind of quotient on an ordered group (G,P) called a reduction, and show that if (G,P) has a reduction into an amenable group, then there is a gauge invariance uniqueness theorem for P-graph algebras. We additionally show that the property "has an amenable reduction" is preserved under direct and free products. (Received September 01, 2020)

1163-47-314 **Srivatsav Kunnawalkam Elayavalli***
(`srivatsav.kunnawalkam.elayavalli@vanderbilt.edu`), Stevenson Center, Nashville, TN 37209, and **Isaac Goldbring**. *Embeddings of II_1 factors into their ultrapowers.*

K. Jung showed in 2006 that a II_1 factor is amenable if and only if there is exactly 1 embedding into R^ω up to unitary equivalence. In a recent preprint, S.Atkinson, I. Goldbring and myself study a more general question, where one replaces unitary equivalence with arbitrary automorphism equivalence in the ultrapower. We show several results in this setting, in particular, a generalized Jung theorem: An R^ω embeddable II_1 factor N is amenable if and only if there is exactly 1 embedding into N^ω up to automorphic equivalence. In light of the recent refutation of the Connes embedding problem, we are also able to obtain the existence of non R^ω embeddable generalized Jung factors, i.e, one that satisfies the condition that there is exactly 1 embedding into N^ω up to automorphism equivalence. These theorems are proved using a combination of operator algebraic and model theoretic ideas. As an application we obtain continuum many II_1 factors that satisfy the statement of Popa's factorial commutant problem, which is a long standing open problem in operator algebras. In this talk I will show some of these results and discuss the ideas used in the proof. (Received September 02, 2020)

1163-47-343 **William T Ross*** (`wross@richmond.edu`), Department of Mathematics and CS, University of Richmond, Richmond, VA 23173. *The shift and its adjoint: served three ways.*

In this talk I will present the classical unilateral shift and its adjoint combined three ways: the sum, the direct sum, and the tensor product. In particular, I focus on the invariant subspaces for these operators. (Received September 03, 2020)

1163-47-356 **Caleb G Parks*** (`cgparks@uark.edu`) and **Daniel H Luecking**. *Generalized Volterra-Type Operators between Hardy Spaces.*

We discuss a generalized Volterra-Type operator V_g involving higher-order derivatives of g. Using tent space methods, we give an affirmative answer to the conjecture of Nikolaos Chalmoukis in 2019. This result completes the characterization of V_g mapping between different Hardy spaces. Along the way, we characterize the pointwise multipliers between the analytic tent spaces. As a result, we recover the known characterizations of multipliers between Hardy and Bergman spaces. (Received September 03, 2020)

1163-47-378 **Erin Griesenauer*** (`grieseel@eckerd.edu`), **Paul Muhly** and **Baruch Solel**. *Noncommutative Functions, Matrix Bundles, and Azumaya Algebras.*

We study noncommutative function algebras as subalgebras of n-homogeneous C*-algebras. These algebras may be viewed as cross sections of certain holomorphic matrix bundles which arise naturally in noncommutative function theory and geometric invariant theory. We describe the connection between these algebras and bundles, and show that the function algebras we consider are Azumaya algebras. (Received September 04, 2020)

1163-47-447 **Wing Suet Li*** (`li@math.gatech.edu`). *The enumeration of extreme rigid honeycombs.*

The rigid honeycombs were used to solve the Horn conjecture, which characterize the eigenvalues of the sum of self-adjoint matrices. The intricate combinatorial relations of rigid extreme honeycombs sometimes allow us to observe very complicated geometrical relationships between the sum of two self-adjoint matrices and the

summands. In this talk I will present ways to grow these rigid extreme trees and provide a scheme to enumerate them. This is a joint work with H. Bercovici.   (Received September 07, 2020)

1163-47-516   **Alberto Dayan*** (`alberto.dayan@wustl.edu`), 1 Brookings Drive, Campus Box 1146, St Louis, MO 63130. *Interpolating Matrices.*
In the first part of the talk, we will extend Carleson's characterization of interpolating sequences to sequences of matrices with spectra in the unit disk. In the second part of the talk, we will give some necessary and sufficient conditions for a sequence of d-tuples of matrices to be interpolating: we will describe the commutative and the noncommutative setting separately.   (Received September 08, 2020)

1163-47-572   **Roberto Hernandez Palomares*** (`hernandezpalomares.1@osu.edu`). *Realizations of unitary tensor categories over GJS C*-algebras.*
We show that an arbitrary countably generated unitary tensor category (aka rigid C*-tensor category) $\mathcal{C}$ acts on some simple separable monotracial C*-algebra B. This is, we realize $\mathcal{C}$ as a full subcategory of the finitely generated projective bimodules over B. The C*-algebra B depends only on $\mathcal{C}$ and is constructed using diagrammatic techniques from Gionnet-Jones-Shlyakhtenko. We recover the the realization of Brothier-Hartglass-Penneys as bifinite bimodules over an interpolated free group factor by means of a monoidal functor between these bimodule categories. Finally, we construct a simple separable monotracial C*-algebra that admits an action from every unitary fusion category. Based on joint work with M. Hartglass.   (Received September 09, 2020)

1163-47-599   **Shuaibing Luo**, **Caixing Gu** and **Stefan Richter*** (`srichter@utk.edu`). *Higher order local Dirichlet integrals and de Branges-Rovnyak spaces.*
Let $B = (b_1, \ldots, b_N)$ be a row vector of analytic functions such that $\|B(z)\|^2 = \sum_j |b_j(z)|^2 < 1$ for all z in the open unit disc $\mathbb{D}$. The de Branges-Rovnyak space $H(B)$ is defined by the reproducing kernel $\frac{1 - B(z)B(w)^*}{1 - \overline{w}z}$. Assume that M_z acts boundedly on $H(B)$ and let $\mathcal{M}$ be the largest subspace of $H(B)$ such that M_z is isometric on $\mathcal{M}$. We show that all b_i's are rational, if and only if $\mathcal{M}^\perp$ is finite dimensional, and that the degree of the rational tuple B equals the dimension of $\mathcal{M}^\perp$. Furthermore, in this case $\mathcal{M}^\perp$ is invariant for the backward shift L, and there is a constant c such that $a = c\frac{\tilde{p}}{\tilde{q}}$ satisfies $|a|^2 + \|B\|^2 = 1$ a.e. on $\partial\mathbb{D}$. Here p and q are the characteristic polynomials of $M_z^*|\mathcal{M}^\perp$ and $L|\mathcal{M}^\perp$, and $\tilde{r}(z) = z^n r(1/z)$, $n = \dim \mathcal{M}^\perp$.

If B is rational and if $M_z : H(B) \to H(B)$ is a $2m$-isometric operator, then all the zeros of p lie in the unit circle and the norm of the functions in $H(B)$ can be expressed by use of m-th order local Dirichlet integrals. (Received September 10, 2020)

1163-47-657   **Samir Raouafi*** (`szr0067@auburn.edu`), Department of Mathematics and Statistics, Auburn University, Auburn, AL 36832. *Positive Semi-Definite Block Matrices and Norm Inequalities.*
Positive semi-definite block matrices occur as an efficient tool in matrix analysis and quantum information theory. In particular, those partitioned in two by two blocks allow to derive important norm inequalities. Hiroshima's 2003 result, which provides a majorization criteria for distillability of a bipartite quantum state, shed light on an interesting norm inequality in matrix analysis. In this talk, we will discuss some extension of this inequality. Furthermore, we will constructively provide a counter-example of Bourin-Lee-Lin's conjecture. (Received September 11, 2020)

1163-47-695   **Jonathan H Brown**, **Adam H Fuller**, **David R Pitts*** (`dpitts2@unl.edu`) and **Sarah A Reznikoff**. *Quotients of some C*-algebras by Regular Ideals.* Preliminary report.
For $S \neq \emptyset$ in a C^*-algebra $\mathcal{A}$, let $S^\perp := \{a \in \mathcal{A} : aS = Sa = 0\}$. An ideal J in $\mathcal{A}$ is *regular* if $(J^\perp)^\perp = J$.

Given an inclusion of C^*-algebras $\mathcal{D} \subseteq \mathcal{C}$ and an ideal $J \subseteq \mathcal{C}$, desirable properties need not pass to the quotient inclusion, $\mathcal{D}/(\mathcal{D} \cap J) \subseteq \mathcal{C}/J$. For example, if $\mathcal{D}$ is the C^*-algebra generated by the range projections of finite paths in the C^*-algebra, $C^*(E)$, of a row-finite directed graph E, the Cuntz-Krieger uniqueness theorem holds for the pair $(C^*(E), \mathcal{D})$ when E satisfies "condition (L)". But condition (L) need not be preserved under quotients.

In this talk, I will discuss the following.

Theorem 1: *Let E be a row-finite directed graph satisfying condition (L). If $J \subseteq C^*(E)$ is a* REGULAR *ideal, then J is invariant under the gauge action and $C^*(E)/J \simeq C^*(F)$, where F is a row-finite directed graph satisfying condition (L).*

Theorem 2: *Suppose $\mathcal{D}$ is a Cartan MASA in the C^*-algebra $\mathcal{C}$ and J is a regular ideal in $\mathcal{C}$. Then $\mathcal{D}/(\mathcal{D} \cap J)$ is a Cartan MASA in $\mathcal{C}/J$.   (Received September 11, 2020)*

1163-47-731   **Kristin E Courtney*** (kcourtne@uni-muenster.de) and **Wilhelm Winter**. *Images of completely positive order zero maps.*

A completely positive (cp) map is called order zero when it preserves orthogonality. Such maps enjoy a rich structure, which has made them a key component of completely positive approximations of nuclear C*-algebras. In this talk, we will consider a few nice properties of the image of a cp order zero map and see how these properties actually characterize those self-adjoint subspaces of a C*-algebra that are the image of such a map. We will also discuss implications for generalized inductive sequences of C*-algebras.   (Received September 12, 2020)

1163-47-805   **Wolfram Bauer*** (bauer@math.uni-hannover.de), Leibniz Universität Hannover, Institut für Analysis, Welfengarten 1, 30167 Hannover, Germany. *Berger-Coburn theorem, localized operators and the Toeplitz algebra.*

We consider Toeplitz operators on the p-Fock space ($1 \leq p \leq \infty$) over the complex n-space $\mathbb{C}^n$. In the first part of the talk we present an extension of the classical Berger-Coburn theorem on the boundedness of Toeplitz operators via the heat flow to the non-Hilbertian case $p \neq 2$. Even in the classical situation $p = 2$ our proof is a simplification and slightly extends the original statement. In the second part we give a short survey on compactness characterizations via the Berezin transform in the framework of bounded operators on Bergman spaces and the p-Fock space. Based on some new observations combined with a result by J. Xia we then discuss various new characterizations of the Banach algebra generated by Toeplitz operators having bounded measurable symbols. If time permits we will comment on R. Werners *correspondence theory*, which, adapted to the Fock space setting, is a suitable tool in the study of Toeplitz algebras. This talk is based on a recent joint work with R. Fulsche (Hannover).   (Received September 13, 2020)

1163-47-842   **Anna Duwenig**, **Elizabeth Gillaspy**, **Rachael Norton*** (rnorton9@fitchburgstate.edu), **Sarah Reznikoff** and **Sarah Wright**. *Cartan subalgebras of twisted groupoid C*-algebras.*

Since J. Renault initiated the study of Cartan subalgebras of C^*-algebras in 1979, the theory has been advanced by many authors including A. Kumjian, X. Li, and J. Renault. In this talk we focus on groupoid C^*-algebras whose multiplication is twisted by a circle-valued 2-cocycle. We identify sufficient conditions on a subgroupoid $S \subset G$ under which the twisted C^*-algebra of S is a Cartan subalgebra of the twisted C^*-algebra of G. Time permitting, we describe (in terms of G and S) the so-called Weyl groupoid and twist that J. Renault defined in 2008, which give us a different groupoid model for our Cartan pair.   (Received September 13, 2020)

1163-47-984   **J E Pascoe*** (pascoej@ufl.edu). *Algebraic geometry and topology arising from tracial and determinantal free noncommutative functions.*

We give an elementary method determining whether or not two expressions in noncommuting indeterminants have the same determinant when evaluated on matrices. Our method uses the canonical implementation of the Weil divisor given by the logarithmic derivative. The divisor of a free rational function is always the difference of two polynomial divisors, and hence, via factorization results of Helton-Klep-Volcic, the set of rational divisors form a free abelian group generated by the polynomials. Determining which functions arise as principal divisors, or more generally as gradients of tracial free functions, motivates a theory of tracial cohomology and fundamental groups. The natural fundamental group is given by the equivalence classes of "paths" taking some base point to itself which can be distinguished by analytic continuation. Surprisingly, the tracial fundamental group is abelian and isomorphic to a direct sum of copies of the rationals. Finally, time permitting, we will note that our methods can be adapted to a sheaf theoretic formulation and work on thin sets, such as noncommutative varieties, and hence bear insight into problems on domains of commuting tuples of matrices.   (Received September 14, 2020)

1163-47-1006   **Thomas Peebles*** (tpeebles@albany.edu) and **Michael Stessin** (mstessin@albany.edu). *Spectral Hypersurfaces for Operator Pairs and Hadamard Matrices.*

Given a pair of $n \times n$ matrices A and B, if they have a projective joint spectrum $\sigma(A, B, AB, I)$ is $\{[x, y, z, t] \in \mathbb{CP}^3 : x^n + y^n + (-1)^{n-1}z^n - t^n = 0\}$ then this pair is unitary equivalent to a pair associated with a complex Hadamard matrix. Since there is a complete description of Hadamard matrices of order $n = 3, 4, 5$, we give a list of those that generate the pair. Furthermore, if we consider the projective joint spectrum of $\sigma(A, B, AB, BA, I)$ and it is of the form $\{[x, y, z_1, z_2] \in \mathbb{CP}^4 : x^n + y^n + (-1)^{n-1}(e^{2\pi i}z_1 + z_2)^n - t^n = 0\}$ then the Hadamard matrix is exactly the Fourier matrix F_n. Under some mild conditions, we can extend these results to pairs of operators on a Hilbert space.   (Received September 14, 2020)

1163-47-1011   **Sedi Bartz** and **Rubén Campoy*** (ruben.campoy@ua.es), C/ Pablo Picasso 4, 2A, 03660 Novelda, Alicante, Spain, and **Hung M. Phan**. *Demiclosedness Principles for Generalized Nonexpansive Mappings*.

Demiclosedness principles are powerful tools in the study of convergence of iterative methods. For instance, a multi-operator demiclosedness principle for firmly nonexpansive mappings is useful in obtaining simple and transparent arguments for the weak convergence of the shadow sequence generated by the Douglas–Rachford algorithm. We provide extensions of this principle, which are compatible with the framework of more general families of mappings such as cocoercive and conically averaged mappings. As an application, we derive the weak convergence of the shadow sequence generated by the adaptive Douglas–Rachford algorithm.   (Received September 14, 2020)

1163-47-1036   **Nicole Tuovila*** (n2vila@ufl.edu). *More on Automorphisms of Free Spectrahedra.*
Spectrahedra are the intersection of an affine linear space with the cone of positive semidefinite matrices. They arise in convex optimization and real algebraic geometry. Free spectrahedra are the fully matricial analog of spectrahedra and are related to finite dimensional operator systems spanned by matrices. In this talk we will discuss recent results on automorphisms of some classes of free spectrahedra.   (Received September 14, 2020)

1163-47-1064   **Joel A Rosenfeld*** (rosenfeldj@usf.edu), Tampa, FL. *Generalization of Weighted Compositions Operators and Applications in Dynamical Systems Theorem.*
In this talk we present several generalizations of weighted composition operators, and detail how they can be applied in modeling nonlinear systems relevant to control theory.

Specifically, we will present recent work on Dynamic Mode Decompositions (DMD) and occupation kernels, which use observed trajectory data from a dynamical system to construct finite rank representations of operators for spectral decomposition and reconstruction of nonlinear systems. Modally, the operators leveraged in DMD are unbounded.

However, with the proper selection of weighted composition operator and Hilbert space provides a compact operator for spectral decomposition with minimal adjustment of the observed data. These generalizations utilize reproducing kernel Hilbert spaces, both scalar and vector valued, where the vector valued spaces are integral to the study of control systems.   (Received September 14, 2020)

1163-47-1098   **Rachael Norton** (rnorton9@fitchburgstate.edu), **Sarah Reznikoff*** (sarahrez@ksu.edu) and **Sarah Wright** (swright8@fitchburgstate.edu). *A picture of Cartan subalgebras in twisted k-graph algebras.*
In 2008, Renault proved that every topologically principal groupoid C*-algebra contains a Cartan subalgebra. We recall work with Duwenig and Gillaspy that shows certain non-topologically principal twisted groupoid C*-algebras contain Cartan subalgebras, and in the case that the groupoid is the path groupoid of a k-graph we describe certain subgroupoids that give rise to these subalgebras.   (Received September 14, 2020)

1163-47-1166   **Minh N. Bui*** (mnbui@ncsu.edu), North Carolina State University, Department of Mathematics, Raleigh, NC 27695-8205, and **Patrick L. Combettes**. *Multivariate Monotone Inclusions in Saddle Form.*
We introduce the notion of a saddle operator for highly structured multivariate monotone inclusions involving a mix of set-valued, cocoercive, and Lipschitzian monotone operators, as well as various monotonicity-preserving operations among them. The properties of this saddle operator are discussed, and asynchronous block-iterative algorithms to find its zeros are presented. In turn, this allows us to solve the original system via a novel block-iterative splitting algorithm of great flexibility in terms of processing the constituent operators individually and exploiting their specific attributes. Comparisons with the state-of-the-art in monotone operator splitting will be carried out. Joint work with Patrick L. Combettes.   (Received September 14, 2020)

1163-47-1557   **Meric L Augat*** (maugat@wustl.edu). *Differentiation and antidifferentiation in free analysis.*
In recent years, several classical theorems about analytic functions have been extended to the setting of free analysis e.g. the inverse function theorem has been generalized to matrix and operator settings. In particular, the free inverse function theorem has a stronger statement than its classical counterpart, partly due a remarkable property of differentiation in free analysis: the free derivative can be realized via point evaluation.

In this talk, we first discuss the free derivative, its remarkable properties and their consequences. Next, we investigate necessary and sufficient conditions for antidifferentiation of free functions. We generalize two classical theorems to free analysis; if F is a differentiable vector field equal to the gradient of some potential function,

then F is curl free and conversely, if F is a curl free vector field on a simply connected domain, then F is the derivative of a potential function.

Our two main results are as follows: the derivative of an analytic free map must be free-curl free and when we are on a connected free domain, every free-curl free map can be antidifferentiated.

Recall that a free function in g freely noncommuting variables sends g-tuples of matrices (of the same size) to h-tuples of matrices (of the same size).   (Received September 15, 2020)

49 ▶ *Calculus of variations and optimal control; optimization*

1163-49-38   **Hossein Dabirian**, **Jiwen He**, **Robert Azencott** and **Andreas Mang*** (andreas@math.uh.edu), Philip Guthrie Hoffman Hall, 3551 Cullen Blvd, Room 614, Houston, TX 77204. *Statistical Analysis of Shapes and Shape Deformations in 3D.* Preliminary report.

We present research on statistical analysis on infinite-dimensional shape spaces for classification and clustering of 3D-shapes and shape deformations. Our ultimate goal is to provide computational methods that allow us to automatically discriminate between clinically distinct patients groups through the lens of anatomical shape variability. In a Riemannian setting, we can express the similarity between two shapes s_0, s_1, in terms of an energy minimizing diffeomorphism y such that $y(s_0) = s_1$. We use an optimal control formulation, in which the diffeomorphism y is parameterized by a smooth, time-dependent velocity field v (the control variable of our problem). After computing an optimal $v^\star$ and the associated $y^\star$, we derive the strain distribution of $y^\star$ as well as a Hilbert norm of $v^\star$ to characterize the dissimilarities between s_0 and s_1. Using these features, as well as standard anatomic measurements, we implement machine learning techniques to achieve the automatic classification of shapes extracted from cardiac imaging (mitral valves, to be precise).   (Received July 24, 2020)

1163-49-99   **Cheikh Birahim Ndiaye*** (cheikh.ndiaye@howard.edu), Washington, DC 20059. *Non-rectangular constrained Willmore minimizers.*

We will discuss the conformally constrained Willmore problem for tori of non-rectangular conformal class. By estimates of Li-Yau and Montiel-Ros, it is known that the Clifford torus minimizes the Willmore energy in its conformal class. Moreover, from a previous work with R. M. Schaetzle, it is known that the homogeneous tori are solutions of the conformally constrained Willmore problem for conformal classes close to the square one. In this talk, we will explain how the $(1, 2)$-equivariant tori with intrinsic period 1 are solutions of the conformal Willmore problem for non-rectangular conformal classes close to the square one discussing a joint work with L. Heller.   (Received August 14, 2020)

1163-49-111   **Melanie Weber*** (mw25@math.princeton.edu) and **Suvrit Sra**. *Riemannian Frank-Wolfe Methods and Applications.*

Many applications involve non-Euclidean data, such as graphs, strings, or matrices, where exploiting Riemannian geometry can deliver algorithms that are computationally superior to standard nonlinear programming approaches. We introduce Riemannian Frank-Wolfe (RFW) methods as a class of projection-free algorithms for constraint geodesically convex and nonconvex optimization. In contrast to the projected-gradient approaches considered in the previous literature, RFW is guaranteed to stay in the feasible region, circumventing the need to compute potentially costly projections. At the heart of RFW lies a Riemannian "linear" oracle that determines the update conditioned on geodesically convex constraints. While in general a nonconvex semi-definite problem, we discuss matrix-valued tasks where the solution can be computed in closed form. RFW extends naturally to stochastic settings, where we discuss both purely stochastic and finite-sum problems, including empirical risk minimization. We show that RFW converges at a nonasymptotic sublinear rate, recovering the best-known guarantees for its Euclidean counterpart. Finally, we discuss the computation of Riemannian centroids and Wasserstein barycenters via RFW, both of which are crucial subroutines in many machine learning methods. (Received August 29, 2020)

1163-49-132   **Tuyen Tran*** (ttran18@luc.edu), **Mau Nam Nguyen**, **Wondi Geremew** and **Samuel Reynolds**. *DC Progamming for Hierarchical Clustering.*

Multilevel hierarchical clustering has a long history and enormous important applications in data mining and statistics. In this talk, we consider a different formulation of the bilevel hierarchical clustering problem, a commonly used model in designing optimal multicast networks and a discrete optimization problem which can be

shown to be NP-hard. Our approach is to reformulate the problem as a continuous optimization problem by making some relaxations on the discreteness conditions. Then, Nesterov's smoothing technique and a numerical algorithm for minimizing difference of convex functions called the DCA are applied to cope with the nonsmoothness and nonconvexity of the problem. Numerical examples are provided to illustrate our method.   (Received September 15, 2020)

1163-49-167   **Levon Nurbekyan*** (`lnurbek@math.ucla.edu`). *Spectral and machine-learning methods for mean-field games.*
I will present some of the recent developments in computational methods for mean-field game (MFG) systems. In particular, I will discuss Fourier (feature space) expansion, deep neural network (DNN) methods, and their combination for solving MFG systems.   (Received August 25, 2020)

1163-49-168   **Alexander J Zaslavski*** (`ajzasl@technion.ac.il`), 32000 Haifa, Israel. *The Projected Subgradient Algorithm with Computational Errors.*
We study an extension of the subgradient projection algorithm for minimization of convex and nonsmooth functions, under the presence of computational errors, taking into consideration the fact that for our algorithm every iteration consists of several steps and that computational errors for different steps are different, in general. For this extension, instead of the projection on the feasible set it is used a quasi-nonexpansive retraction on this set. We show that our algorithms generate a good approximate solution, if computational errors are bounded from above by a small positive constant. Moreover, for a known computational error, we find out what an approximate solution can be obtained and how many iterates one needs for this.   (Received August 25, 2020)

1163-49-176   **Maryam Yashtini*** (`my496@georgetown.edu`), 2320 Wisconsin Ave NW, Washington, DC 20007. *A fast relaxed normal two split method and an effective weighted TV approach for Euler's elastica image inpainting.*
In this talk, I introduce two numerical algorithms for solving Euler's elastica-based image inpainting model. The minimizing functional is non-smooth, non-convex, and involves high-order derivatives, that traditional gradient descent based methods converges very slowly. Recent alternating minimization methods show fast convergence when a good choice of parameters is used. The objective of this paper is to introduce efficient algorithms which have simple structures with a fewer parameters. These methods are based on operator splitting and alternating direction method of multipliers, and subproblems can be solved efficiently by Fourier transforms and shrinkage operators. For the first method, we relax the normal vector in the curvature term of the Euler's elastica model and exploit two operator splitting techniques to propose a Relaxed Normal two Split (RN2Split) method. The second method, κ-weighted Total Variation (κTV), solves the Euler's elastica minimization problem as a weighted total variation. We present the analytical properties of each algorithm. Various numerical experiments, including comparison with some existing state-of-art algorithms, are presented to show the efficiency and the effectiveness of the proposed RN2Split and κTV methods.   (Received August 25, 2020)

1163-49-296   **Ashkan Mohammadi*** (`amohamm@umn.edu`). *Composite optimization in normed spaces with applications in semi-infinite programming and calculus of variations.*
This talk is devoted to developing and applications of a generalized differential theory of variational analysis that allows us to work in incomplete normed spaces, without employing conventional variational techniques based on completeness and limiting procedures. The main attention is paid to generalized derivatives and subdifferentials of the Dini-Hadamard type with the usage of mild qualification conditions revolving around metric subregularity. In this way we develop calculus rules of generalized differentiation in normed spaces without imposing restrictive normal compactness assumptions and the like and then apply them to general problems of constrained optimization. Most of the obtained results are new even in finite dimensions. Finally, we derive refined necessary optimality conditions for nonconvex problems of semi-infinite and calculus of variation. (This is a joint work with Boris Mordukhovich).   (Received September 15, 2020)

1163-49-348   **Max Engelstein**, **Robin Neumayer*** (`neumayer@math.northwestern.edu`) and **Luca Spolaor**. *Quantitative stability for minimizing Yamabe metrics.*
The Yamabe problem asks whether, given a closed Riemannian manifold, one can find a conformal metric of constant scalar curvature (CSC). An affirmative answer was given by Schoen in 1984, following contributions from Yamabe, Trudinger, and Aubin, by establishing the existence of a function that minimizes the so-called Yamabe energy functional; the minimizing function corresponds to the conformal factor of the CSC metric.

We address the quantitative stability of minimizing Yamabe metrics. On any closed analytic Riemannian manifold we show—in a quantitative sense—that if a function nearly minimizes the Yamabe energy, then the

corresponding conformal metric is close to a CSC metric. Generically, this closeness is controlled quadratically by the Yamabe energy deficit. However, we construct examples demonstrating that this quadratic estimate is false in the general. This is joint work with Max Engelstein and Luca Spolaor.   (Received September 03, 2020)

1163-49-415   **Tan Cao**, **Giovanni Colombo**, **Boris Mordukhovich** and **Dao Nguyen***
(`gc9683@wayne.edu`), 4500 Cass Ave, Apt 718, Detroit, MI 48201. *Optimization and discrete approximation of sweeping processes and its applications.*
This talk addresses a new class of optimal control problems for perturbed sweeping processes which are governed by the maximal monotone mappings. We develop a constructive discrete approximation procedure, employ advanced tools of first-order and second-order variational analysis and generalized differentiation, derive numerical algorithms to compute second-order constructions, and necessary optimality conditions for discrete optimal solutions under fairly general assumptions formulated entirely in terms of the given data. The obtained results give us efficient suboptimality ("almost optimality") conditions for the original sweeping control problem that are illustrated by several numerical examples.

This is based on joint work with Tan Cao, Giovanni Colombo, Boris Mordukhovich.   (Received September 05, 2020)

1163-49-466   **Shaofeng Deng*** (`sfdeng@math.ucdavis.edu`), **March Boedihardjo** and **Thomas Strohmer**. *A Performance Guarantee for Spectral Clustering.*
The two-step spectral clustering method, which consists of the Laplacian eigenmap and a rounding step, is a widely used method for graph partitioning. It can be seen as a natural relaxation to the NP-hard minimum ratio cut problem. In this talk we consider the central question: when is spectral clustering able to find the global solution to the minimum ratio cut problem? First we provide a condition that naturally depends on the intra- and inter-cluster connectivities of a given partition under which we may certify that this partition is the solution to the minimum ratio cut problem. Then we give a deterministic two-to-infinity norm perturbation bound for the the invariant subspace of the graph Laplacian that corresponds to the k smallest eigenvalues. Finally by combining these two results we give a condition under which spectral clustering is guaranteed to output the global solution to the minimum ratio cut problem, which serves as a performance guarantee for spectral clustering.   (Received September 07, 2020)

1163-49-767   **Enrique Alvarado*** (`enrique.alvarado@wsu.edu`), **Bala Krishnamoorthy** (`kbala@wsu.edu`) and **Kevin R Vixie** (`vixie@speakeasy.net`). *The Maximum Distance Problem and Minimum Spanning Trees.*
Given a compact $E \subset \mathbb{R}^n$ and $s > 0$, the maximum distance problem seeks a compact and connected subset of $\mathbb{R}^n$ of smallest one dimensional Hausdorff measure whose s-neighborhood covers E. For $E \subset \mathbb{R}^2$, we prove that minimizing over minimum spanning trees that connect the centers of balls of radius s, which cover E, solves the maximum distance problem.

The main difficulty in proving this result is overcome by the proof of a Lemma which states that one is able to cover the s-neighborhood of a Lipschitz curve Γ in $\mathbb{R}^2$ with a finite number of balls of radius s, and connect their centers with another Lipschitz curve Γ_*, where $\mathcal{H}^1(\Gamma_*)$ is arbitrarily close to $\mathcal{H}^1(\Gamma)$.

We also present an open source package for computational exploration of the maximum distance problem using minimum spanning trees, available at https://github.com/mtdaydream/MDP_MST.

A preprint is available at https://arxiv.org/abs/2004.07323.   (Received September 15, 2020)

1163-49-773   **Nicolás García Trillos** and **Ryan Murray***, rwmurray@ncsu.edu, and **Matthew Thorpe**. *Clustering on point clouds: consistency guarantees for Cheeger cuts.*
Separating points into clusters based upon geometry is a central problem in data science. This talk will address the problem of partitioning point clouds which are random samples of an underlying manifold. In particular, I will discuss recent work with Nicolás García Trillos and Matthew Thorpe which provides quantified, geometric consistency guarantees between minimizers of discrete Cheeger cut problems and a limiting continuum Cheeger set problem. These consistency guarantees rely upon novel energy estimates and recent results from Riemannian geometry about the stability of isoperimetric sets.   (Received September 12, 2020)

1163-49-890   **Melissa Tong**, **Jerome Gilles** and **Luminita Aura Vese*** (`lvese@math.ucla.edu`). *A Joint Variational Model for Atmospheric Distortion Correction.* Preliminary report.
We study the problem of restoring images distorted by atmospheric turbulence, which is a type of inverse problem. Geometric distortions and blur are the two main components of degradations due to atmospheric turbulence. Prior work has been done to address these components separately. We propose two variants of a combined deblurring and geometric distortion correction model, in a variational setting. The minimization problem seeks

to find the restored image and the geometric correction in the same formulation. The Euler-Lagrange equations associated with the joint minimization problem are discretized using finite difference schemes and the gradient descent approach. We present numerical results for image restoration using synthetic and real data.   (Received September 13, 2020)

1163-49-969   **Priyanka Mishra*** (`priyankamshr122@gmail.com`), Department of Mathematics, Indian Institute of Technology Patna, Patna, Bihar 801103, India, and **B. B. Upadhyay** (`bhooshan@iitp.ac.in`), Department of Mathematics, Indian Institute of Technology Patna, Patna, Bihar 801103, India. *On Minty Variational Principle for Nonsmooth Interval-Valued Multiobjective Programming Problems.*

In this paper, we consider a class of nonsmooth interval-valued multiobjective programming problems and a class of approximate Minty and Stampacchia vector variational inequalities. Under generalized approximate LU-convexity hypotheses, we establish the relations between the solutions of approximate Minty and Stampacchia vector variational inequalities and the approximate LU-efficient solutions of the nonsmooth interval-valued multiobjective programming problem. The results of this paper extend and unify the corresponding results of Mishra and Upadhyay (2013), Mishra and Laha (2016), Upadhyay et al. (2017) and Gupta and Mishra (2018) for nonsmooth interval-valued multiobjective programming problems. Keywords:Approximate LU-convexity;Approximate LU-efficient solutions; Intervalvalued multiobjective programming   (Received September 14, 2020)

1163-49-982   **B. B. Upadhyay*** (`bhooshan@iitp.ac.in`), Depatment of Mathematics, Indian Institute of Technology Patna, Patna, Bihar 801103, India. *On generalized vector variational inequalities and nonsmooth multiobjective programming problems in Asplund spaces.*

In this work, we derive the relations between generalized Minty and Stampacchia vector variational inequalities in terms of limiting subdifferential and nonsmooth multiobjective programming problem in Asplund spaces. Under approximate convexity hypotheses, equivalence among the solutions of these vector variational inequalities and local quasi efficient solution of nonsmooth multiobjective programming problem are studied. We also introduce the weakformulations of considered vector variational inequalities in terms of limiting subdifferential and investigate the relations between their solution and local weak quasi efficient solution of nonsmooth multiobjective programming problem. Furthermore, by employing generalized version of KKM-Fan theorem, we derive existence results for considered generalized Minty and Stampacchia vector variational inequalities. The results presented in this paper extend, generalize and sharpen several known results in literature such as Giannessi (1997), Lee (2000), Lee and Lee (2005) and Fang and Hu (2009) to more general spaces namely Asplund spaces and to a class of nonconvex functions using powerful tool of limiting subdifferential.

   Keywords: Minty vector variational inequality; Approximate convex functions; Quasi efficiency; KKM-Fan theorem   (Received September 14, 2020)

1163-49-1092   **Georg Stadler*** (`stadler@cims.nyu.edu`), **Shanyin Tong** (`shanyin.tong@nyu.edu`) and **Eric Vanden-Eijnden** (`eve2@cims.nyu.edu`). *Extreme event probability estimation using tools from PDE-constrained optimization.*

We propose methods for the estimation of extreme event probabilities in complex systems governed by PDEs. Our approach is guided by ideas from large deviation theory (LDT) and PDE-constrained optimization. The systems under consideration involve random parameters and we are interested in quantifying the probability that a scalar function of the system state is at or above a threshold. We first compute parameters that minimize the LDT-rate function over the set of parameters leading to extreme events. These solutions provide asymptotic information about small probability events. We propose several methods to refine these estimates, namely methods based on importance sampling and on geometric approximation of the extreme event sets. Theoretical and numerical arguments show that the performance of our methods is insensitive to the extremeness of the event. We illustrate the application of our approach to quantify the probability of extreme tsunami events on shore. Tsunamis are typically caused by a sudden, unpredictable change of the ocean floor elevation during an earthquake. We model this as random process and use the one-dimensional shallow water equation to model tsunamis. We present a comparison of the methods for extreme event probability estimation.   (Received September 14, 2020)

1163-49-1103   **Boris Mordukhovich*** (`aa1086@wayne.edu`), Detroit, MI 48202. *Parabolic Regularity in Geometric Variational Analysis.*

The talk is devoted to systematic developments and applications of geometric aspects of second-order variational analysis and optimization that are revolved around the concept of parabolic regularity of sets. This concept has been known in variational analysis for more than two decades while being largely underinvestigated. We

discover here that parabolic regularity is the key to derive new calculus rules and computation formulas for major second-order generalized differential constructions of variational analysis. The established results of second-order variational analysis and generalized differentiation, being married to the developed calculus of parabolic regularity, allow us to obtain novel applications to both qualitative and algorithmic aspects of constrained optimization including second-order optimality conditions, augmented Lagrangians, SQP and Newton-type methods, etc.

Based on the joint paper with Ashkan Mohammadi and Ebrahim Sarabi to appear in the Transactions of the American Mathematical Society.   (Received September 14, 2020)

1163-49-1150   **Ryan Hynd** and **Dennis Ikpe*** (`ikpedenn@msu.edu`), 1230 Garden City Road, East Lansing, MI 48823, and **Terrance Pendleton**. *An eradication time problem for the SIR model.*

We consider a susceptible, infected, and recovered infectious disease model which incorporates a vaccination rate. In particular, we study the problem of choosing the vaccination rate in order to reduce the number of infected individuals to a given threshold as quickly as possible. This is naturally a problem of time-optimal control. We characterize the optimal time as a solution of a Hamilton-Jacobi-Bellman equation and give necessary and sufficient conditions for a vaccination rate to be optimal.   (Received September 14, 2020)

1163-49-1184   **Ouayl Chadli*** (`ochadli@gmail.com`), Department of Economics, Ibn Zohr University, Agadir, Morocco. *Noncoercive mixed equilibrium problems under monotonicity-type conditions with applications to nonlinear evolutions problems and hemivariational inequalities.*

In this talk, we present some results on the existence of solutions for noncoercive mixed equilibrium problems described by the sum of a maximal monotone bifunction and a pseudomonotone (or quasimonotone) bifunction in the sense of Brézis. The approach developed is based on recession analysis and on some recent results established on the existence of solutions of equilibrium problems with pseudomonotone perturbations. As applications, we consider nonlinear evolution equations associated to a noncoercive time dependent pseudomonotone (or quasimonotone) operator and hemivariational inequalities with lack of coercivity. The talk cover some new results by the author and his collaborators recently published.   (Received September 15, 2020)

1163-49-1226   **Minh N. Dao*** (`m.dao@federation.edu.au`), PO Box 663, Ballarat, Victoria 3353, Australia. *Extrapolated proximal subgradient algorithms for nonconvex and nonsmooth fractional programs.* Preliminary report.

In this work, we consider a broad class of nonsmooth and nonconvex fractional programs including composite optimization problems and encompassing many important modern fractional optimization problems arising from signal processing, discriminant analysis, and finance. We propose extrapolated proximal subgradient algorithms for solving this optimization model and analyze their convergence properties. The choice of our extrapolation parameter is flexible and includes the popular extrapolation parameter adopted in the restarted Fast Iterative Shrinking-Threshold Algorithm (FISTA). By providing a unified analysis framework of descent methods, we establish the convergence of the full sequence under the assumption that a suitable merit function satisfies the Kurdyka–Lojasiewicz property. Our theoretical results are illustrated with both analytical and simulated numerical examples. This is based on joint work with R.I. Bot (Uni. Vienna) and G. Li (UNSW Sydney). (Received September 15, 2020)

1163-49-1234   **Liming Cai**, **Lanjing Bao**, **Logan Rose**, **Jeffery Summers** and **Wandi Ding***, 1301 E. Main Street, MTSU Box 34, Murfreesboro, TN 37132. *Mathematical Modeling and Optimal Control for Malaria Transmission Using Sterile Insect Technique and Insecticide-Treated Nets.*

We consider a malaria transmission model with SEIR (susceptible-exposed-infected-recovered) classes for the human population, SEI (susceptible-exposed-infected) classes for the wild mosquitoes and an additional class for sterile mosquitoes. We derive the basic reproduction number of infection. We formulate an optimal control problem in which the goal is to minimize both the infected human populations and the cost to implement two control strategies: the release of sterile mosquitoes and the usage of insecticide-treated nets to reduce the malaria transmission. Adjoint equations are derived and the characterization of the optimal controls is established. Finally, we quantify the effectiveness of the two interventions aimed at limiting the spread of Malaria. A combination of both strategies leads to a more rapid elimination of the wild mosquito population that can suppress Malaria transmission. Numerical simulations are provided to illustrate the results.   (Received September 15, 2020)

1163-49-1263   **Rayanne A. Luke*** (rayanne@udel.edu), **Richard J. Braun**, **Tobin A. Driscoll**, **Carolyn G. Begley** and **Deborah Awisi-Gyau**. *Parameter Identification for Tear Film Thinning.*

Dry eye syndrome is thought to be caused by breakdown of a uniform tear film, which occurs when the tear film experiences breakup. The dynamics of the tear film can be studied using fluorescence imaging. Many parameters affect tear film thickness and fluorescent intensity distributions over time; exact values or ranges for some are not available. We estimate breakup parameters by fitting to experimental fluorescent intensity data gathered from normal subjects' tear films *in vivo*. The fitting is done with thin film fluid PDE models for the tear film thickness, osmolarity and fluorescein concentration with circular (spot) or linear (streak) geometry. The corresponding fluorescent intensity is computed from the tear film thickness and fluorescein concentration. The parameters are determined by a nonlinear least squares minimization between computed and experimental fluorescent intensity. The results vary across subject trials. Optimal values for variables that cannot be measured *in vivo* within tear film breakup often fall within accepted experimental ranges for related tear film dynamics; however, some instances suggest that a wider range of parameter values may be acceptable. This new understanding of tear breakup may lead to better understanding of dry eye.   (Received September 15, 2020)

1163-49-1369   **X. Wang*** (shawn.wang@ubc.ca), Department of Mathematics, University of British Columbia, Kelowna, BC V1V1V7, Canada, **H. Bauschke**, Department of Mathematics, University of British Columbia, Kelowna, BC V1V1V7, Canada, **X. Li**, Department of Mathematics, Southwest Petroleum University, Chengdu, Sichuan , Peoples Rep of China, and **Z. Wang**, Department of Mathematics, University of British Columbia, Kelowna, BC V1V1V7, Canada. *The Bregman Proximal Average.*

Proximal average is important in optimization algorithms. We provide a proximal average with respect to Legendre functions. This is a joint work with H. Bauschke, X. Li and Z. Wang.   (Received September 15, 2020)

1163-49-1380   **Akhtar A. Khan*** (aaksma@rit.edu), School of Mathematical Sciences, 85 Lomb Memorial Drive, Rochester, NY 14623. *Stochastic approximation framework for inverse problems of identifying random parameters in stochastic partial differential equations.* Preliminary report.

The primary objective of this work is to study the inverse problem of identifying a parameter in partial differential equations with random data. We explore the nonlinear inverse problem in a variational inequality framework. We propose a projected-gradient-type stochastic approximation scheme for general variational inequalities and give a complete convergence analysis under weaker conditions on the random noise than those commonly imposed in the available literature. The proposed iterative scheme is tested on the inverse problem of parameter identification. We provide a derivative characterization of the solution map, which is used in computing the derivative of the objective map. By employing a finite element based discretization scheme, we derive the discrete formulas necessary to test the developed stochastic approximation scheme. Preliminary numerical results show the efficacy of the developed framework.   (Received September 15, 2020)

1163-49-1424   **Quoc Tran-Dinh*** (quoctd@email.unc.edu), Chapel Hill, NC 27599. *Generalized Newton Methods with Polynomial-Time Iteration-Complexity for A Class of Variational Inequalities.*

In this talk, we will discuss some generalized Newton-type methods for solving a class of monotone variational inequalities that can achieve polynomial-time iteration-complexity. This problem class covers three common convex optimization problems as special cases. Our approach relies on a combination of interior-point and proximal-Newton methods. First, we will propose a two-phased algorithm and estimate its iteration-complexity. Our complexity is polynomial-time and remains the same order as in existing interior-point methods. Next, we will also specify our method to solve constrained convex optimization as well as convex-concave saddle-point problems. Finally, we will provide some numerical examples to illustrate our methods and compare them with some existing solvers.

This talk is based on the work collaborated with T. Sun (UNC) and S. Lu (UNC).   (Received September 15, 2020)

1163-49-1479   **Stefan Sremac*** (ssremac@puc.edu), Department of Mathematics, Pacific Union College, 1 Angwin Ave., Angwin, CA 94508, and **Hugo J. Woerdeman** and **Henry Wolkowicz**. *Error Bounds and Singularity Degree in Semidefinite Programming.*

Optimization algorithms for semidefinite programs (SDPs) terminate at a 'solution' where the residual of the optimality conditions is sufficiently small. However, such proposed solutions have been known to be wildly inaccurate. The Euclidean distance from the 'solution' to the optimal set can be several orders of magnitude

greater than the residual of the optimality conditions. Since the distance to the optimal set is generally unknown, such a discrepancy is very undesirable. In 2001, Jos Sturm introduced *singularity degree* as a way to explain this pathology. In particular, he showed that large singularity degree is a necessary property of SDPs that exhibit the pathology. In this presentation we show that, in some sense, large singularity degree is also a sufficient property for this type of poor convergence.   (Received September 15, 2020)

1163-49-1500   **Summer R. Atkins*** (srnatkins@ufl.edu), **Maia Martcheva** (maia@ufl.edu) and **W. W. Hager** (hager@ufl.edu). *Regularization of an Optimal Fishery Harvesting Problem via Bounded Variation.*

We consider an optimal harvesting problem using a spatially explicit model with Dirichlet boundary conditions assumed on the state variable. The optimal control problem is linear with respect to the harvesting control. Previous presentations of this problem demonstrated complications associated with numerically solving for the problem due to the presence of a singular subarc. In those numerical simulations, wild oscillations occurred along the singular region, which resembled a phenomenon called "chattering".

As a means of generating a more realistic optimal harvesting strategy, we consider regularizing this problem by adding a bounded variation term to the cost functional. We numerically solve for this regularized problem by use of a Polyhedral Active Set Algorithm called PASA that is used for solving general nonlinear optimization problems with sparse polyhedral constraints. In solving for the regularized problem, we generate a harvesting strategy that does not oscillate along the singular region. Additionally, the numerical results obtained indicate the need for marine reserves.   (Received September 15, 2020)

1163-49-1502   **Milagros Loreto*** (mloreto@uw.edu), **Ana Custodio**, **Thomas Humphries** and **Bradford Halter**. *Adjusting a Gaussian Process Model to the GLODS algorithm.* Preliminary report.

The Global and Local Optimization using Direct Search (GLODS) algorithm was proposed for computing all the problems. local minima, from which the global minimum can be easily identified in a constrained case.For the application of this method, no assumptions regarding the smoothness of the function defining the problem are required. The numerical results have shown that GLODS is competitive when compared to commonly-used global derivative-free optimization solvers. In this work, we want to improve the numerical performance of GLODS by incorporating a Gaussian Process (GP) Model that allows reusing the functions evaluations calculated during the poll and search steps. The proposed method includes approaches to calibrate the GP model and to initialize GLODS. We describe the new algorithm structure and report encouraging numerical results.   (Received September 15, 2020)

1163-49-1535   **Cornelia Mihaila*** (cmihaila@math.uchicago.edu). *A problem in shape optimization.* Preliminary report.

We will discuss a problem in shape optimization.   (Received September 15, 2020)

1163-49-1556   **Ebrahim Sarabi*** (sarabim@miamioh.edu), 301 S. Patterson Ave, Oxford, OH 45056. *Twice epi-differentiability of extended-real-valued functions ands its remarkable applications.*

This talk is devoted to the study of the twice epi-differentiablity of extended-real-valued functions, with an emphasis on functions satisfying a certain composite representation. First, we present conditions under which this property can be ensured for extended-real-valued functions. We then provide remarkable applications of this property in parametric optimization, second- order variational analysis, and local convergence analysis of numerical algorithms including the Newton method and the augmented Lagrangian method. This talk is based on joint works with Nguyen T. V. Hang, Ashkan Mohammadi, and Boris Mordukhovich.   (Received September 15, 2020)

1163-49-1623   **Bijaya Kumar Sahu*** (bks10@iitbbs.ac.in), College of Basic Sciences, Indian Institute of Technology, Bhubaneswar, Bhubaneswar, Orissa 752050, India. *Existence of solutions for extended generalized complementarity problems.*

In this paper we introduce extended generalized complementarity problems in Hausdorff topological vector spaces in duality and study the existence of their solutions. We use a different method than those in the literature on the existence of solutions of complementarity problems, which are usually based on arguments from generalized monotonicity. This leads us to obtain new results and improve many existing results in the literature.We also

prove some existence results for extended generalized complementarity problems in reflexive Banach spaces by means of a Tikhonov regularization procedure under a copositivity assumption and arguments from the recession analysis.   (Received September 15, 2020)

51 ▶ Geometry

1163-51-247   **Marshall A. Whittlesey*** (`mwhittle@csusm.edu`), Department of Mathematics, California State University San Marcos, 333 S. Twin Oaks Valley Road, San Marcos, CA 92096. *Using quaternions to prove theorems in spherical geometry.*

We show how to use quaternions to prove theorems in spherical geometry. In doing so we feature the parametrization of rotations in real three-dimensional space with quaternions. This technique is featured in the speaker's recent book "Spherical Geometry and its Applications," with CRC Press.   (Received August 30, 2020)

1163-51-318   **Taylor Brysiewicz*** (`taylor.brysiewicz@mis.mpg.de`), Inselstrasse 22, 04103 Leipzig, Germany, and **Michael Burr** (`burr2@clemson.edu`). *Alternative trace tests.* Preliminary report.

The trace test is a fundamental tool in numerical algebraic geometry which verifies the completeness of a witness set. Despite its long history and widespread use, many consequences of the trace test remain unexplored. We establish a collection of results pertaining to the trace test and explain how these give rise to new alternative trace tests. This is joint work with Michael Burr.   (Received September 02, 2020)

1163-51-388   **Michael Kirby**, **Xiaofeng Ma** and **Chris Peterson*** (`peterson@math.colostate.edu`). *Flag Manifolds in Data Analysis.*

This talk will describe Flag Manifolds, their geometry, and examples of their use in data analysis.   (Received September 04, 2020)

1163-51-545   **Madeline M Brown*** (`mmbrown4200@gmail.com`), 8469 E Gilded Perch Dr, Scottsdale, AZ 85255. *Nested Links, Linking Matrices, and Crushtaceans.*

If two knots have homeomorphic complements, then they are isotopic; however, this is not true for links. Geometry and topology can be combined to show when certain hyperbolic links have homeomorphic complements. What remains is to determine if the links themselves are isotopic. We determine an algorithm to find the linking numbers of two types of hyperbolic links known as fully augmented links (FALs) and nested links from their respective graphical representations, known as crushtaceans. We will show that some links constructed from the same crushtacean are not isotopic. The algorithm shows that topological information can be obtained directly from the crushtacean's combinatorial data. One useful application of the algorithm is to distinguish links with homeomorphic complements.
(Received September 09, 2020)

1163-51-754   **Khanh Le*** (`khanh.q.le@temple.edu`) and **Rebekah Palmer**. *Totally geodesic surfaces in twist knot complements.*

The study of surfaces has been essential in studying the geometry and topology of the 3-manifolds that contain them. In particular, there has been considerable work in understanding the existence of totally geodesic surfaces in hyperbolic 3-manifolds. Most recently, Bader, Fisher, Miller, and Stover showed that having infinitely many maximal totally geodesic surfaces implies that the 3-manifold is arithmetic. In a joint work with Rebekah Palmer, we present examples of infinitely many non-commensurable (non-arithmetic) hyperbolic 3-manifolds that contain exactly k totally geodesic surfaces for every positive integer k. In this talk, I will discuss some ideas in the proof of the main results. I will also talk about applications of these ideas in showing that a family of twist knot complements is not right-angled and examples of maximal Fuchsian subgroups of infinite covolume in hyperbolic 3-manifolds.   (Received September 12, 2020)

1163-51-789   **Mark D Meyerson*** (`mdmeyerson@yahoo.com`). *Halving It All.*
In 1990, Mark Kidwell and I were somewhat surprised to realize that although a triangular plate would balance on every line through its centroid, only the three median lines both balance the plate and halve its area. In fact, drawing the halving lines produced curves, as envelopes, that we showed were hyperbolas. At Mark's suggestion, we published these ideas in a new journal called Quantum, intended for gifted high school students, recently launched by the National Science Teaching Association.   (Received September 12, 2020)

1163-51-812   **Hang Huang*** (hhuang235@tamu.edu), 1601 Holleman Dr Apt 1013, College Station, TX 77840, and **Mateusz Michalek** and **Emanuele Ventura**. *Vanishing Hessian and Wild Polynomials.*

Notions of ranks and border rank abound in the literature. Polynomials with vanishing hessian and their classification is also a classical problem. Motivated by observation of Ottaviani, we will discuss why when looking at concise polynomials of minimal border rank, being wild, i.e. their smoothable rank is strictly larger than their border rank, are the same as having vanishing Hessian. The main tool we are using here is the recent work of Buczynska and Buczynski relating the border rank of polynomials and tensors to multigraded Hilbert scheme. From here, we found two infinite series of wild polynomials and we will try to describe their border varieties of sums of powers, which is an analog of the variety of sums of powers.   (Received September 13, 2020)

1163-51-857   **Noelle Sawyer*** (sawyern@southwestern.edu). *Marked Length Spectrum Rigidity with Partial Data.* Preliminary report.

The marked length spectrum of a metric on a compact Riemannian manifold records the length of the shortest closed curve in each free homotopy class. Suppose we know that a metric m on a compact surface is negatively curved and Riemannian, and we know the marked length spectrum of m, excluding a set of homotopy classes that grows slowly. My results show that is enough to uniquely identify the metric m.   (Received September 13, 2020)

1163-51-965   **Elin Farnell*** (elinfarnell@gmail.com). *Encoding intrinsic variation in data: a survey of the use of Grassmann, flag, and Stiefel manifolds in data science.* Preliminary report.

As the extraction of information from large, high-dimensional datasets has become an ever more critical ingredient to scientific progress, it has become increasingly clear that finding the correct geometric framework to capture intrinsic structure in data can make or break analysis. One geometric framework that has been successfully exploited to capture intrinsic structure in data is the Grassmann manifold.

In this talk, we will provide background and context for the Grassmann manifold. We will also survey the broad range of applications of Grassmann manifolds and their generalizations in data science. Our ultimate hope for this work is that by compiling a single source that summarizes the way that fundamental ideas originating in "pure math" have improved analysis of real-world datasets, we will increase awareness and access to these tools in the data science community.   (Received September 14, 2020)

1163-51-1056   **Anthony Sanchez*** (asanch33@uw.edu). *Translation covers of platonic solids and their monodromy groups.* Preliminary report.

Platonic solids have been studied for thousands of years. By unfolding a platonic solid we can associate to it a translation surface. Interesting information about the underlying platonic solid can be discovered in the cover where more (dynamical and geometric) structure is present. The translation covers we consider have a large group of symmetries that leave the global composition of the surface unchanged. However, the local structure of paths on the surface is often sensitive to these symmetries. The Kontsevich-Zorich mondromy group keeps track of this sensitivity.

In joint work with D. Lee, we study the mondromy groups of translation covers of some platonic solids. (Received September 14, 2020)

1163-51-1237   **Yaim Cooper*** (yaim@math.ias.edu). *The critical locus of the loss function of deep neural networks.*

The mathematical heart of deep learning is gradient descent on a loss function L. If gradient descent converges, it will converge to a critical point of L. Thus the geometry of the locus of critical points is of great interest. We will discuss what is known about the critical points of L, including dimension estimates and connectedness results.   (Received September 15, 2020)

1163-51-1240   **Richard Evan Schwartz***, 177 Weyl Lane, Princeton, NJ 08540. *Rectangles, Curves, and Klein bottles.*

The rectangular peg problem is a variant of the notorious square peg problem of Toeplitz, which goes back to to 1911 and asks if every continuous loop contains an inscribed square - i.e., 4 points which form the vertices of a square. The square peg problem has long been known to be true for smooth or polygonal loops, and the case of wild fractal curves is unsolved and is what keeps people up at night.

One can make an even bolder conjecture: Every continuous loop contains an inscribed rectangle of every possible aspect ratio. Until the very recent work of Greene and Lobb, this had been unknown even for smooth loops, though Cole Hugelmeyer had made some inspiring progress on it.

In this talk I will exposit the work of Greene and Lobb as best I can. Their basic idea is to look at the problem in "symplectic coordinates" and, through a surgery construction, reduce the problem to the statement that a certain kind of Klein bottle cannot be embedded in 4 dimensions. This is a bit peculiar because Klein bottles famously do have 4-dimensional embeddings. The key is that the extra constraints make it impossible. (Received September 15, 2020)

1163-51-1314   **Rebekah Palmer*** (`rebekah.palmer@temple.edu`) and **Khanh Le** (`khanh.q.le@temple.edu`). *Totally geodesic surfaces in twist knot complements II.*
The study of surfaces has been essential in studying the geometry and topology of the 3-manifolds that contain them. In particular, there has been considerable work in understanding the existence of totally geodesic surfaces in hyperbolic 3-manifolds. Most recently, Bader, Fisher, Miller, and Stover showed that having infinitely many maximal totally geodesic surfaces implies that the 3-manifold is arithmetic. In this talk, we will conversely present examples of infinitely many non-commensurable (non-arithmetic) hyperbolic 3-manifolds that contain a unique totally geodesic surface and then extend that result to 3-manifolds that contain exactly k totally geodesic surfaces via covering space techniques. This is joint work with Khanh Le.   (Received September 15, 2020)

1163-51-1471   **Eric Geiger*** (`edgeiger@ncsu.edu`) and **Irina A. Kogan** (`iakogan@ncsu.edu`). *Non-congruent curves with identical signatures.* Preliminary report.
This talk will focus on using the Euclidean Signature to determine whether two smooth planar curves are congruent under the Special Euclidean group. Work done by Emilio Musso and Lorenzo Nicolodi emphasize that signatures must be used with caution by constructing 1-parameter families of non-congruent curves with degenerate vertices (curve segments of constant curvature) with identical signatures. We address the claim made by Mark Hickman, that the Euclidean Signature uniquely identifies curves without degenerate vertices. While the claim is true for simple, closed curves with simple signatures, it fails for curves with non-simple signatures. For curves with non-simple signatures, we associate a directed graph (a signature quiver) with the signature and show how various paths along the quiver give rise to a family of non-congruent, non-degenerate curves with identical Euclidean Signatures. Using this additional structure, we formulate congruence criteria for non-degenerate, closed, simple planar curves.   (Received September 15, 2020)

1163-51-1546   **Orsola Capovilla-Searle*** (`ocapovilla@gmail.com`), Durham, NC 27701. *Complements of smoothed toric divisors of centered toric 4-manifolds.* Preliminary report.
Weinstein manifolds are symplectic manifolds with a handle decomposition compatible with the symplectic structure. Examples of Weinstein manifolds include complex affine spaces, and cotangent bundles of smooth manifolds. The symplectic topology of Weinstein manifolds is encoded in the attaching spheres of the handles. In joint work with Acu, Gadbled, Marinković, Murphy, Starkston and Wu we give an algorithm for the construction of the Weinstein handlebody diagrams for complements of smoothed toric divisors in centered toric 4-manifolds. (Received September 15, 2020)

1163-51-1643   **C. Ashley*** (`caleb.ashley@bc.edu`), **N. Cameron**, **E. Goins**, **E. Lawrence**, **T. McKenzie,** and **K. Pershell**. *Monodromy of Compositions of Belyi Maps.* Preliminary report.
Among all hyperbolic structures on a Riemann surface S_g of fixed genus $g \geq 2$, those that are also arithmetic are special. Grothendieck's famous CNRS research proposal, "Esquisse d'un Programme" outlines a link between combinatorial data of "dessin d' enfants," the complex geometry of algebraic curves, and their corresponding algebraic number field of coefficients. We describe some fundamental objects and results which outline this theory. Our main goal is to describe $Mon(\beta \circ \alpha)$, the monodromy group of the composition of two dynamical Belyi maps α and β, of a certain broad subclass. This is joint work with N. Cameron, E. Goins, E. Lawrence, T. McKenzie, K. Pershell.   (Received September 15, 2020)

52 ▶ *Convex and discrete geometry*

1163-52-52   **Jeff M. Phillips***, 50 S. Central Campus Dr., Salt Lake City, UT 84108. *A Primer on the Geometry in Machine Learning.*
Machine Learning is a discipline filled with many simple geometric algorithms, the central task of which is usually classification. These varied approaches all take as input a set of n points in d dimensions, each with a label. In learning, the goal is to use this input data to build a function which predicts a label accurately on new data drawn from the same unknown distribution as the input data. The main difference in the many

algorithms is largely a result of the chosen class of functions considered. This talk will take a quick tour through many approaches from simple to complex and modern, and show the geometry inherent at each step. Pit stops will include connections to geometric data structures, duality, random projections, range spaces, and coresets. (Received August 01, 2020)

1163-52-68   **Tom Needham*** (`tneedham@fsu.edu`) and **Samir Chowdhury**. *Applications of Gromov-Wasserstein distance to network science.*

Recent years have seen a surge of research activity in network analysis through the lens of optimal transport. This perspective boils down to the following simple idea: when comparing two networks, instead of considering a traditional registration between their nodes, one instead searches for an optimal 'soft' or probabilistic correspondence. This perspective has led to state-of-the-art algorithms for robust large-scale network alignment and network partitioning tasks. A rich mathematical theory underpins this work: optimal node correspondences realize the Gromov-Wasserstein (GW) distance between networks. GW distance was originally introduced, independently by K. T. Sturm and Facundo Mémoli, as a tool for studying abstract convergence properties of sequences of metric measure spaces. In particular, Sturm showed that GW distance can be understood as a geodesic distance with respect to a Riemannian structure on the space of isomorphism classes of metric measure spaces (the 'Space of Spaces'). In this talk, I will describe joint work with Samir Chowdhury, in which we develop computationally efficient implementations of Sturm's ideas for network science applications. We also derive theoretical results which link this framework to classical notions from spectral network analysis.   (Received August 05, 2020)

1163-52-358   **Jesus A. De Loera*** (`deloera@math.ucdavis.edu`), Dept. of Mathematics, University of California, One Shields Avenue, Davis, CA 95616. *Combinatorial Challenges arising from the Simplex Method.*

Dantzig's Simplex method is a work horse of modern optimization. But despite its success we do not understand its complexity. To bound the number of iterations of the Simplex method we take a geometric point of view and investigate the lengths of monotone paths inside the oriented graphs of polyhedra (oriented by the objective function). We consider the shortest and the longest monotone paths possible and estimate the (monotone) diameter and the height of some famous combinatorial polyhedra (such as TSP, fractional matching polytopes, and others). Surprisingly, as we look at all monotone paths put together we see a rich topological CW-space structure which can be used to count how many are there or to generate them randomly. Our main enumerative results include bounds on the number of monotone paths, and on the the diameter of the CW-complex of monotone paths (how far are two monotone paths from each other?). The picture is fairly complete in dimension three, but an open problem for high dimensional polytopes.

Joint work with Moise Blanchard (MIT) and Quentin Louveaux (U. Liege) and Christos Athanasiadis (U. Athens) and Zhenyang Zhang (UC Davis).   (Received September 03, 2020)

1163-52-1325   **Randall Balestriero*** (`randallbalestriero@gmail.com`), 6100 Main Street, Houston, TX 77005, **Romain Cosentino**, 6100 Main Street, Houston, TX 77005, **Behnaam Aazhang**, 6100 Main Street, Houston, TX 77098, and **Richard G Baraniuk**, 6100 Main Street, Houston, TX 77098. *The Geometry of Deep Networks: Power Diagram Subdivision.*

We study the geometry of deep (neural) networks (DNs) with piecewise affine and convex nonlinearities. The layers of such DNs have been shown to be max-affine spline operators (MASOs) that partition their input space and apply a region-dependent affine mapping to their input to produce their output. We demonstrate that each MASO layer's input space partitioning corresponds to a power diagram (an extension of the classical Voronoi tiling) with a number of regions that grows exponentially with respect to the number of units (neurons). We further show that a composition of MASO layers (e.g., the entire DN) produces a progressively subdivided power diagram and provide its analytical form. The subdivision process constrains the affine maps on the (exponentially many) power diagram regions to greatly reduce their complexity. For classification problems, we obtain a formula for a MASO DN's decision boundary in the input space plus a measure of its curvature that depends on the DN's nonlinearities, weights, and architecture. Numerous numerical experiments support and extend our theoretical results.   (Received September 15, 2020)

53 ▶ *Differential geometry*

1163-53-503   **Samuel Pérez-Ayala*** (`sperezay@nd.edu`). *Maximal Metrics for the Conformal Laplacian.*

Let (M^n, g) be a closed Riemannian manifold of dimension $n \geq 3$. Assume $[g]$ is a conformal class for which the Conformal Laplacian $L_g := -\Delta_g + c_n R_g$ has at least two negative eigenvalues and $0 \notin \mathrm{Spec}(L_g)$. If we define $\Lambda_2(M^n, [g])$ as the supremum of the second eigenvalue over generalized conformal metrics with unit volume, then we show that there is a nonnegative and nontrivial function $\bar{u} \in C^\alpha(M^n) \cap C^\infty(M^n \setminus \{\bar{u} = 0\})$, $\alpha \in (0, 1)$, such that "$g_{\bar{u}} = \bar{u}^{\frac{4}{n-2}} g$" attains this supremum. As an application, and depending on the multiplicity of $\lambda_2(\bar{u})$, we find either nodal solutions to a Yamabe type equation, or a harmonic map into a sphere.   (Received September 08, 2020)

1163-53-557   **Jacob Bernstein** (`bernstein@math.jhu.edu`) and **Lu Wang*** (`drluwang@caltech.edu`). *Relative expander entropy in the presence of a two-sided obstacle and applications.*

We study a notion of relative entropy motivated by self-expanders of mean curvature flow. In particular, we obtain the existence of this quantity for arbitrary hypersurfaces trapped between two self-expanders that are asymptotic to the same cone and bound a domain. This allows us to begin to develop the variational theory for the relative entropy functional for the associated obstacle problem. We also obtain a version of the forward monotonicity formula for mean curvature flow proposed by Ilmanen.   (Received September 09, 2020)

1163-53-559   **Thomas E. Mark*** (`tmark@virginia.edu`), PO Box 400137 Kerchof Hall, Charlottesville, VA 22904, and **Bülent Tosun**. *On hypersurfaces of contact type in $\mathbb{R}^4$.*

In recent joint work with Bülent Tosun, we proved that no Brieskorn homology sphere can be embedded as a hypersurface of contact type in standard symplectic 4-space. This implies that no rationally convex domain in C^2 has boundary diffeomorphic to a Brieskorn sphere. I will provide some background and context for these results, including some hints toward the proof, which is based on an obstruction coming from Heegaard Floer homology.   (Received September 09, 2020)

1163-53-828   **Pedro Valentin De Jesus*** (`pedro-valentindejesus@uiowa.edu`), The University of Iowa, 14 MacLean Hall, 2 West Washington Street, Iowa City, IA 52242. *On a Sharp Isoperimetric Inequality for a Constrained Region.* Preliminary report.

In this work we focus on a sharp Isoperimetric Inequality for convex domains of arbitrary dimension with the constraint of having its boundary contained in a prescribed minimal n-spherical shell. The scale-free Isoperimetric deficit of a region within the n-Euclidean space is a measure of how deformed this region is from the n-ball. This invariant has been extensively studied, we depart from B. Fuglede's contribution. He proves that for nearly spherical or convex domains one can bound from below the Isoperimetric deficit by a function in terms of the n-spherical shell's thickness containing the boundary of such domain. In this talk, we will discuss our attempt to improve his results where only an order estimate for a function bounding below the isoperimetric deficit is given.   (Received September 13, 2020)

1163-53-1023   **Ryan Vaughn*** (`rv2152@nyu.edu`), **Tyrus Berry** and **Harbir Antil**. *Diffusion Maps for Manifolds with Boundary.*

Manifolds with boundary are a natural class of objects to consider for manifold learning applications, including data-driven solvers for boundary value problems. Despite their relative importance, the geometry of such manifolds presents several obstacles to proving pointwise consistency results for manifold learning methods based on estimating the Laplacian (such as diffusion maps). In this work, we show that several of these issues can be resolved by viewing the Laplacian in a weak (variational) sense. We also derive new asymptotic estimates to show pointwise consistency of the bias of graph Laplacian estimators on manifolds with boundary. These theoretical results are then verified experimentally by numerically solving several boundary value problems on point cloud data sampled from manifolds with boundary.   (Received September 14, 2020)

1163-53-1284   **Enrique Alvarado*** (`enrique.alvarado@wsu.edu`), **Zhu Liu**, **Michael J Servis**, **Bala Krishnamoorthy** (`kbala@wsu.edu`) and **Aurora E Clark** (`auclark@wsu.edu`). *A Geometric Measure Theory Approach to Identify Complex Structural Features on Soft Matter Surfaces.*

The structural features that protrude above or below a soft matter interface are well-known to be related to interfacially mediated chemical reactivity and transport processes. It is a challenge to develop a robust algorithm for identifying these organized surface structures, as the morphology can be highly varied and they may exist on top of an interface containing significant interfacial roughness. A new algorithm that employs concepts from

geometric measure theory, algebraic topology, and optimization, is developed to identify candidate structures at a soft matter surface, and then using a probabilistic approach, to rank their likelihood of being a complex structural feature. The algorithm is tested for a surfactant laden water/oil interface, where it is robust to identifying protrusions responsible for water transport against a set identified by visual inspection. To our knowledge, this is the first example of applying geometric measure theory to analyze the properties of a chemical/materials science system.

A preprint is available at https://doi.org/10.26434/chemrxiv.11988048.v1. (Received September 15, 2020)

1163-53-1308   **Jeffrey S Meyer*** (`jeffrey.meyer@csusb.edu`), **Sara Lapan** and **Benjamin Linowitz**.
Systoles of Arithmetic Manifolds in Dimensions 3 and 4.
The systole of a manifold is the minimal length of a non-contractible closed loop. Systoles in arithmetic manifolds have many fascinating relationships with deep problems in number theory, such as Lehmer's Mahler measure problem. In recent years, there have been many papers studying systoles, their bounds, and their growth up covers as you vary the underlying manifolds. In this talk, I will survey these results and discuss recent work with collaborators Sara Lapan and Benjamin Linowitz as it pertains to manifolds in dimensions 3 and 4. (Received September 15, 2020)

1163-53-1449   **Ratikanta Behera*** (`ratikanta.behera@ucf.edu`), Department of Mathematics,
University of Central Florida, Orlando, FL. *Tensor Operations and Computations.*
Tensors have significantly impacted many areas of science and engineering in recent years. However, operations with tensors is still a major challenge and preoccupation for the scientific community. Therefore, it is appropriate to develop an infrastructure that supports reasoning about tensor computations. In this talk, we will discuss a closed multiplication operation between tensors with the concept of transpose, and inverse as well as the identity of a tensor. A significant motivation for considering this new type of tensor multiplication is to devise new kinds of factorization of tensors. The technique allows us an excellent environment for learning about tensor computations. (Received September 15, 2020)

54 ▶ *General topology*

1163-54-381   **Jun Zhang*** (`jun.zhang.3@umontreal.ca`), Department of Mathematics and Statistics,
University of Montreal, Montreal, Quebec H3C 3J7, Canada. *Persistent homology and its
applications in symplectic topology.*
Recent years have seen a significant development of persistent homology theory, especially its applications in various subjects in mathematics. In this talk, I will present several updated applications in symplectic topology, in particular, based on the Floer theory that is often viewed as an infinite-dimensional analogue of Morse theory. Remarkably, some fundamental questions can be solved by a skillful play of barcodes. Inspired by these applications, I will mention a high-tech viewpoint called triangulated persistence category (TPC), based on an in-progress work with P. Biran and O. Cornea, aiming to apply persistent homology theory to the study of homological mirror symmetry. (Received September 04, 2020)

1163-54-738   **Brandon Bavier*** (`bavierbr@msu.edu`). *Geometric Properties of Weakly Generalized
Alternating Knots.*
A goal of knot theory is to relate geometric properties of the knot complement to diagrammatic conditions of the knot. When we are working with alternating knots in S^3, this can be straightforward. It is possible, though, to generalize alternating knots and get similar results. In this talk, we will introduce a type of generalization, and relate the volume of the knot to diagrammatic quantities by way of skein theory. (Received September 12, 2020)

1163-54-1420   **Nickolas A. Castro*** (`nacastro@uark.edu`) and **Jason Joseph** (`jason.joseph@rice.edu`).
Relative group trisections and smooth 4-manifolds with boundary. Preliminary report.
A trisection of a smooth, compact, 4-manifold is a decomposition into three diffeomorphic pieces, where the complexity of the 4-manifold lies in how these pieces are attached to one another. In the case of a manifold with boundary, a *relative trisection* induces a structure on the boundary known as an open book decomposition. In this talk, we will provide a correspondence between relative trisections of 4-manifolds with boundary and commutative cubes of groups, known as *relative group trisections*. This extends group trisections of closed 4-manifolds, due to work of Abrams, Gay, and Kirby, to the relative setting. The key difference in the relative

case is that a relative group trisection also encapsulate the data of the induced open book decomposition. We will briefly discuss open questions relating to relative group trisections.   (Received September 15, 2020)

1163-54-1543   **Candice R Price*** (`cprice@smith.edu`), **Catherine Chalikian**, **Elaona Lemoto** and **Zoe Dillion-Davidson**. *Classes of 2-dimensional projections of knots.* Preliminary report.
A core question in knot theory is: How do we identify whether two knots are equivalent under a specific set of operations? Knot theory deals not only with the curious variations in the underlying topological structure (e.g. strands, loops, choice of operations) of knots but also with invariants (e.g. Alexander, Jones, link homologies). Knot theory has been used in various fields of mathematics and has become an essential resource for understanding the topology and the geometry of DNA. In this presentation, we will apply combinatorial tools to interpret 2-dimensional projections of knots and share methods for categorizing these projections.   (Received September 15, 2020)

55 ▶ *Algebraic topology*

1163-55-69   **Gunnar Carlsson** and **Benjamin Filippenko*** (`benfilip@stanford.edu`). *The space of sections of a smooth function and the evasion path problem.*
Given a compact manifold X with boundary and a submersion $f : X \to Y$ whose restriction to the boundary of X has isolated critical points with distinct critical values and where Y is $[0, 1]$ or S^1, the connected components of the space of sections of f are computed from π_0 and π_1 of the fibers of f. This computation is then leveraged to provide new results on a smoothed version of the evasion path problem for mobile sensor networks: From the time-varying homology of the covered region and the time-varying cup-product on cohomology of the boundary, a necessary and sufficient condition for existence of an evasion path and a lower bound on the number of homotopy classes of evasion paths are computed. No connectivity assumptions are required.   (Received August 06, 2020)

1163-55-131   **Justin Michael Curry***, 1400 Washington Ave., Albany, NY 12222, and **Jordan T DeSha**, 1400 Washington Ave., Albany, NY 12222. *Counting Problems in Persistence.*
This talk will begin with a review of elementary constructions in topological data analysis (TDA), such as merge trees and the Elder Rule, which is the procedure for determining a barcode/persistence diagram from a merge tree. Previous work on how many merge trees determine the same barcode will be reviewed. The talk will conclude with recent combinatorial results obtained with Jordan DeSha and others on counting embedded spheres with the same level-set barcode and other combinatorial problems pertinent to persistent homology. (Received August 19, 2020)

1163-55-136   **Mikael Vejdemo-Johansson*** (`mvj@math.csi.cuny.edu`), **Jie Chu**, **Denis Khryashchev** and **Ping Ji**. *Homotopy path enumeration for the evasion problem.*
The evasion problem asks: Given a family of moving sensors and a movement pattern over time, can there be a path that evades discovery? Adams and Carlsson gave an algorithmic sufficient and necessary condition for evasion with the assumption of some extra capabilities on part of each sensor. From the Adams-Carlsson algorithm, we can recover an enumeration of homotopy classes of possible evasion patterns. In this talk we will describe two methods of this homotopy path enumeration.   (Received August 20, 2020)

1163-55-150   **Henry Adams***, Colorado State University, Department of Mathematics, 1874 Campus Delivery, Fort Collins, CO 80523. *Applied topology: From global to local.*
Through the use of examples, I will explain one way in which applied topology has evolved since the birth of persistent homology in the early 2000s. The first applications of topology to data emphasized the *global* shape of a dataset, such as the three-circle model for 3×3 pixel patches from natural images, or the configuration space of the cyclo-octane molecule, which is a sphere with a Klein bottle attached via two circles of singularity. More recently, persistent homology is being used to measure the *local* geometry of data. How do you vectorize geometry for use in machine learning problems? Persistent homology, and its vectorization techniques including persistence landscapes and persistence images, provide popular techniques for incorporating geometry in machine learning. I will survey applications arising from machine learning tasks in agent-based modeling, shape recognition, archaeology, materials science, and biology.   (Received August 23, 2020)

1163-55-151   **Henry Adams*** (`henry.adams@colostate.edu`), Colorado State University, Department of
Mathematics, 1874 Campus Delivery, Fort Collins, CO 80523. *Evasion paths in mobile
sensor networks.*

Suppose ball-shaped sensors wander in a bounded domain. A sensor doesn't know its location but does know
when it overlaps a nearby sensor. We say that an evasion path exists in this sensor network if a moving intruder
can avoid detection. Relative homology or zigzag persistence give a necessary condition, depending only on the
time-varying connectivity data of the sensors. However, no method with time-varying connectivity data (i.e.
Cech complexes) as input can give necessary and sufficient conditions for the existence of an evasion path. Indeed,
the existence of an evasion path depends on more than just the fibrewise homotopy type of the region covered
by sensors. In the setting of planar sensors that also measure weak rotation information, we provide necessary
and sufficient conditions for the existence of an evasion path, and we pose an open question concerning Cech and
alpha complexes. Joint with Gunnar Carlsson, with recently computed statistics for the probability of mobile
coverage under different motion models joint with Deepjyoti Ghosh, Clark Mask, William Ott, and Kyle Williams
at the University of Houston (code at https://github.com/elykwilliams/EvasionPaths).   (Received September
09, 2020)

1163-55-163   **Michael Robinson*** (`michaelr@american.edu`), 220 Don Myers Building, 4400
Massachusetts Ave NW, Washington, DC 20016. *The Category of Binary Relations,
Dowker complexes, Cosheaves, and Functoriality.*

The Dowker complex is an abstract simplicial complex that is constructed from a binary relation in a straight-
forward way. As motivation, this talk will consider the problem of determining a consensus file format from
the behavior of programs that purport to read compliant files. Although there are two ways to perform the
Dowker construction – vertices for the complex are either the rows (programs) or the columns (files) of the
matrix representing the relation – the two constructions are homotopy equivalent. This talk explains how the
construction of a Dowker complex from a relation is a non-faithful covariant functor. Furthermore, this functor
can be made faithful by enriching the construction into a cosheaf on the Dowker complex. The cosheaf can be
summarized by an integer weight function on the Dowker complex that is a complete isomorphism invariant for
the relation. The cosheaf representation of a relation actually embodies both Dowker complexes, and the talk
will describe a duality functor that exchanges the two complexes.   (Received August 24, 2020)

1163-55-189   **Henry Adams*** (`henry.adams@colostate.edu`), Colorado State University, Department of
Mathematics, Fort Collins, CO 80523. *Descriptors of Energy Landscapes using Topological
Analysis (DELTA).*

Many of the properties of a chemical system are described by its energy landscape, a real-valued function defined
on a high-dimensional domain. I will explain how topology, and in particular persistent homology, can be used
in order to describe some of the pertinent features of an energy landscape. Whereas a merge tree encodes how
connected components of an energy landscape evolve as the energy level increases, persistent homology can
also quantify the shape of these connected components. As a motivating example, we completely describe the
sublevelset persistent homology of the n-alkanes. Our proof of the number of sublevelset persistent homology bars
is completely combinatorial; indeed we proceed by induction on the number of carbons in the alkane chain. Our
proof of the lengths of the bars is more topological, relying on the Künneth formula. In a recently funded NSF
Harnessing the Data Revolution project, the DELTA team is learning how to identify and leverage changing
topological features of energy landscapes across a range of chemical conditions in order to predict reactivity.
(Received August 25, 2020)

1163-55-199   **Jose A Perea*** (`joperea@msu.edu`). *Synchronization problems in applied topology.*

Modern technologies have made commonplace the asynchronous collection of data, and its later storage in
distributed systems. This has prompted a growing class of data science questions where localized measurements
need to be globally aligned or assembled. In this talk we present recent work towards computing algebraic and
topological constraints to general synchronization in several data science questions, and how to do local-to-global
inference in the presence of non-trivial topological obstructions.   (Received August 26, 2020)

1163-55-237   **Erin Wolf Chambers**, **Elizabeth Munch*** (`muncheli@msu.edu`) and **Tim Ophelders**.
The Truncated Interleaving Distance for Reeb Graphs.

Reeb graphs and main other related graphical signatures have extensive use in applications, but only recently
has there been intense interest in finding metrics for these objects. In this talk, we introduce an extension of
smoothing on Reeb graphs, which we call truncated smoothing; this in turn allows us to define a new family of
metrics which generalize the interleaving distance for Reeb graphs. Intuitively, we "chop off" parts near local

minima and maxima during the course of smoothing. After formalizing truncation as a functor, we show that when applied after the smoothing functor, this prevents extensive expansion of the range of the function, and yields particularly nice properties. Further, for certain choices of the truncation parameter, we can construct a categorical flow for any choice of slope $m \in [0, 1]$, which gives a family of interleaving distances. While the resulting metrics are not stable, we show that any pair of these for $m, m' \in [0, 1)$ are strongly equivalent metrics, which in turn gives stability of each metric up to a multiplicative constant.   (Received August 29, 2020)

1163-55-263   **Bradley J. Nelson*** (`bradnelson@statistics.uchicago.edu`). *A fibration model for d-dimensional image patches.*

Data sets of natural image patches have motivated the creation of several interesting topological models with non-trivial structure. In this work, we formulate a fibration model for arbitrary d-dimensional image patches, encompassing the Klein bottle for 2-dimensional image patches. We use this model to study patches in different 3-dimensional data sets, and show how different types of data produce different distributions of patches on the fibration.   (Received August 31, 2020)

1163-55-571   **Martin Helmer** and **Vidit Nanda***, nanda@maths.ox.ac.uk. *Computational Topology in Intersection Theory.*

Associated to any pair of complex projective varieties (X,Y), with X irreducible inside Y, is a positive integer called the Hilbert-Samuel multiplicity of Y along X. Not only does this number measure the type of singularity which X forms inside Y, but it also features prominently in a host of other intersection-theoretic contexts (including recursive formulas for MacPherson's Euler obstruction). The typical method for computing these multiplicities relies on Grobner basis computations. In this talk, I will describe a new and far more efficient topological algorithm, which can be used directly with dense point samples and does not require knowledge of the defining polynomials for X and Y.   (Received September 09, 2020)

1163-55-580   **Yasuaki Hiraoka*** (`hiraoka.yasuaki.6z@kyoto-u.ac.jp`), Kyoto, Kyoto 606-8501, Japan. *Characterizing rare events in persistent homology.*

Indecomposables obtained through decompositions of persistent homology are regarded as topological summary of real data. However, as is well known, there exist pathologically complicated indecomposables in multi-parameter persistent homology in purely algebraic setting, and this fact makes it difficult to build mathematical theory on that setting. Our fundamental question is, how much should we care about such complicated indecomposables in the real data, and what is a suitable framework to study this question? To this aim, we will show several ongoing results, especially, (1) large deviation principle on 1 parameter persistent homology, and (2) law of large numbers on multi-parameter persistent homology. Then we will discuss how these two results (partially) answer to the original question.   (Received September 10, 2020)

1163-55-647   **Dustin Arendt**, **Matthew Broussard*** (`matthew.broussard@wsu.edu`), **Bala Krishnamoorthy** and **Nathaniel Saul**. *Steinhaus Filtration and Stable Paths in the Mapper.*

Two key concepts of topological data analysis are persistence and the Mapper. Persistence uses a sequence of objects built on data called filtration. Mapper produces insightful summaries of data, and is applied widely.

We define a new filtration called the cover filtration built from a single cover using a generalized Steinhaus distance (which generalizes Jaccard distance). We prove a stability result: cover filtrations of two covers are a/m interleaved when bottleneck distance between covers $\leq a$ and size of smallest set in either cover is m. We show our construction is equivalent to Čech filtration under some settings. Also, Vietoris-Rips filtration completely determines the cover filtration in all cases. We develop a theory for stable paths within this filtration.

We show our framework can be employed in many applications when a metric is not given but a cover is readily available. We present a new model for recommendation systems using cover filtration. Stable paths on a movies data set give gentle transitions from one genre to another. In explainable machine learning, we apply Mapper for model induction, and provide explanations of a predictive model using paths between subsets of images.

A preprint is available at https://arxiv.org/abs/1906.08256.   (Received September 12, 2020)

1163-55-939   **Radmila Sazdanovic*** (`rsazdan@ncsu.edu`), Department of Mathematics NC State University, PO Box 8205, Raleigh, NC 27695, and **Javier Arsuaga**, **Gina Gonzalez** and **Arina Ushakova**. *Computational topology in cancer genomics.*

Cancer is a polygenic disease in which genomic events are selected in order to produce a sophisticated and coordinated outcome. Determining when two events are co-occurring is an important open question in data science. This work focuses on addressing that question in breast cancer genomics by developing topology-based

methods for analyzing how Copy Number Aberrations (CNAs) relate to the breast cancer types and prognosis. (Received September 14, 2020)

1163-55-1019   **Gunnar Erik Carlsson*** (`aihomotopy@gmail.com`), 998 Cottrell Way, Stanford, CA 94305. *Unstable parametrized homotopy theory for evasion detection and motion planning.*
We will be discussing the use of techniques from unstable homotopy theory, in the context of parametrized homotopy theory, i.e. homotopy theory of spaces X equipped with a reference map $f : X \to B$ to a fixed base space B, to study time varying problems in the theory of sensor networks and motion planning. In this case the space B will be the real line or an interval within it. It is key that the map f is not assumed to be a vibration. It will be interesting to determine when the map f does not admit a section, as well as to understand the space of sections. The techniques involve interesting extensions of unstable homotopy theory to these settings. This is joint work with Ben Filippenko and Wyatt Mackey.   (Received September 14, 2020)

1163-55-1030   **Ellen Gasparovic**, **Maria Gommel**, **Emilie Purvine*** (`emilie.purvine@pnnl.gov`), **Radmila Sazdanovic**, **Bei Wang**, **Yusu Wang** and **Lori Ziegelmeier**. *Homology of Graphs and Hypergraphs.* Preliminary report.
Graphs and hypergraphs are typically studied from a combinatorial perspective. A graph being a collection of vertices and pairwise relationships (edges) among the vertices, and a hypergraph capturing multi-way or group-wise relationships (hyperedges) among the vertices. But both of these objects have topological structure in addition to their well-studied combinatorial aspects. Graphs, being inherently pairwise objects, can be considered either as a one dimensional simplicial complex or as a metric space using shortest path distance. Hypergraphs, on the other hand, capture group-wise interactions and thus do not have a simple pairwise metric space that captures the complex structure. While hypergraphs do have an associated simplicial complex, there are multiple hypergraphs consistent with the same simplicial complex. In this talk I will survey some recent results on the homology of both graphs and hypergraphs, including persistent homology of metric graphs and a variety of notions of homology for hypergraphs.   (Received September 14, 2020)

1163-55-1160   **Kerry M. Luse*** (`lusek@trinitydc.edu`). *A few knotty problems with Mark Kidwell.*
Mark and I began working on knotty problems together in 2012. In this talk I will highlight a few of the problems that we worked on, including our most recent result about the Alexander polynomial for rational links and a related conjecture.   (Received September 14, 2020)

1163-55-1251   **Daniel Scofield*** (`daniel.scofield@fmarion.edu`) and **Radmila Sazdanovic** (`rsazdanovic@ncsu.edu`). *Extremal Khovanov homology and the girth of a knot.*
The maximal girth among the all-A graphs of a link L determines certain gradings in which the Khovanov homology of L is trivial. We describe these gradings and obtain a simple characterization of extreme Khovanov homology groups and the associated extreme coefficients of the Jones polynomial. Defining the girth of L to be the maximal girth of its all-A graphs, we describe possible values for this invariant and show that if L has an all-A graph whose girth is equal to $\ell > 2$, then the girth of L is equal to ℓ.   (Received September 15, 2020)

1163-55-1261   **Nina Otter*** (`otter@math.ucla.edu`), **Kristian Strommen**, **Matthew Chantry** and **Joshua Dorrington**. *The topology of climate data.* Preliminary report.
We use persistent homology, a method in topological data analysis that allows to study the "shape of data", to analyse some well-known data sets of weather regimes. Finding patterns in these measurements is believed to be crucial for our understanding of climate, and standard data analysis methods currently fail at this task. (Received September 15, 2020)

1163-55-1273   **Barbara Giunti*** (`barbara.giunti@unimore.it`), Reggio Emilia, Italy, **Claudia Landi** (`clandi@unimore.it`), Reggio Emilia, Italy, **Wojciech Chachólski** (`wojtek@kth.se`), Stockholm, Sweden, and **Alvin Jin** (`alvinj@kth.se`), Stockholm, Sweden. *Persistence algorithms from filtered chain complexes point of view.*
Persistence homology has proved to be a useful tool to extract information from real-world data sets. However, homology may be an overkill, forgetting too much information, and several papers have recently focused on lifting persistence from homology to homotopy. This motivates interest in studying tame parametrized chain complexes and a special subclass of them: the filtered chain complexes and their decomposition. We present an algorithm to decompose filtered chain complexes into indecomposables by splitting off interval spheres. We show that these splittings yield a unified explanation of the clearing and compress optimization techniques used in persistence algorithms. We also derive some other innovations for persistence computation such as processing the boundary matrix in almost random order.   (Received September 15, 2020)

1163-55-1279   **Cliff A Joslyn*** (`cliff.joslyn@pnnl.gov`), 1100 Dexter Ave. # 500, Seattle, WA 98109.
Finite Topologies and Hypergraphs: Essential Tools for Network Science.
Complex systems and network science researchers are increasingly turning to higher levels of mathematical abstraction in order to faithfully capture the properties of complex systems. Specifically, hypergraphs and abstract simplicial complexes (along with labeled, directed, attributed, and ordered versions of these) are increasingly popular to model multidimensional structures and high order interactions (i.e., more than binary relations and primary effects) frequently present in complex systems. As mathematical objects, these multi-dimensional graph structures stand poised to connect applied network science to computational topology, in light of the deep connections between hypergraphs as finite set systems, finite partial orders and distributive lattices, and finite topologies. Conversely, topological structures useful in computational topology applications can be interpreted in a combinatorial perspective in the context of hypergraphs and complex networks. I will present a perspective on the finite topologies associated with hypergraphs, and how our research group is using them for both homological analysis of relational data and data-driven topological sheaves.   (Received September 15, 2020)

1163-55-1303   **Wojciech Chachólski**, **Barbara Giunti** and **Claudia Landi***
(`claudia.landi@unimore.it`). *Invariants for tame parametrised chain complexes.*
The aim of this presentation is to explain why extracting persistence invariants in Topological Data Analysis is an example of the homotopical process of approximating complicated objects by simpler ones. It can be comprehensively applied to several cases that standard persistence theory handles separately, such as persistence modules, zigzag modules, and commutative ladders. On the one hand, this unifying framework helps to understand more thoroughly known aspects of persistence. On the other hand, it allows us to extract computable invariants also in certain cases, such as commutative ladders, that have not been covered by more standard approaches.   (Received September 15, 2020)

1163-55-1406   **Justin Michael Curry***, 1400 Washington Ave., ES 120C, Albany, NY 12222. *Discrete*
Approximations and Multi-Scale Analysis in Sheaves and Persistence. Preliminary report.
This talk will tie together various efforts to use sheaves as a tool for proving approximation theorems in persistence, dating back to the pioneering results on Mapper by Elizabeth Munch and Bei Wang, as well as more recent results by Masaki Kashiwara and Pierre Schapira. A recently developed "Theory of Relative Interleavings" (https://arxiv.org/abs/2004.14286) will show how the interleaving distance itself can be approximated by relativizing over a discrete poset or lattice. Additional results on multi-scale analysis in persistence will be presented as well if time permits.   (Received September 15, 2020)

1163-55-1434   **Shelley B Kandola*** (`kandola2@msu.edu`) and **Elizabeth Munch** (`muncheli@msu.edu`).
Algorithms for the Lusternik-Schnirelmann category of finite topological spaces.
Topological complexity (TC) is a homotopy invariant rooted in the robot motion planning problem. Given a path-connected space X that represents a robot's space of configurations, $TC(X)$ gives the minimum number of continuous motion planning rules required to program that robot to move from one position into another position. It is not unreasonable to assume a robot is only capable of obtaining finitely many positions. When the space is T_0, we can model that finite space by a poset, P. In an absence of algorithms for directly computing the TC, there has been interest in the upper- and lower-bounds of TC. A popular bound is determined by the Lusternik-Schnirelmann category, which is the minimal number of open sets covering a space whose inclusion is nullhomotopic. For a finite space P, this yields the upper-bound

$$TC(P) \leq cat(P)^2.$$

In this talk, we present original theorems and algorithms used in determining $cat(P)$, including a Python class in which we have implemented these results.   (Received September 15, 2020)

1163-55-1485   **Casey Necheles***, clnechel@syr.edu, and **Stephan Wehrli**. *An odd annular Bar-Natan*
category and equivalent Temperley Lieb supercategory.
We introduce an odd version of the annular Bar-Natan category and show it is equivalent to a dotted version of the odd Temperley Lieb supercategory $\mathcal{STL}_{\bullet,\,\delta=0}$ defined by Brundan and Ellis. After setting dots equal to zero, $\mathcal{STL}_{\bullet,\,\delta=0}$ embeds into the representation category of the Lie superalgebra $\mathfrak{gl}(1|1)$. This gives us an interpretation of the $\mathfrak{gl}(1|1)$ action on odd Khovanov homology shown as shown by Grigsby and Wehrli in 2018. (Received September 15, 2020)

1163-55-1492   **Bei Wang*** (`beiwang@sci.utah.edu`). *Sheaves for Visual Encodings.* Preliminary report.
Sheaves are general and abstract objects that connect local data to global ones. Visual encodings are the mappings of data attributes to visual structures, upon which we create a visualization on a screen. In this

talk, we explore the potential of using a sheaf-theoretical language to study visual encodings. In particular, we demonstrate that sheaves provide a unifying framework for visual encodings as well as a new perspective on theoretical foundations of visualization.   (Received September 15, 2020)

1163-55-1663   **Gregory Arone***, gregory.arone@math.su.se, and **Alvin Jin**. *Applying calculus of functors to the evasion path problem.* Preliminary report.

The traditional evasion path problem can be formulated mathematically as follows: suppose we have a map (not necessarily a fibration) $p\colon E \to B$. Determine whether p has a section. The problem is non-trivial even in the case when B is an interval.

Traditional obstructions are defined in terms of sheaf cohomology of B. One obvious limitation of the cohomological approach is that it is determined by the fiberwise stable homotopy type of p. There is a specific example, due to Henry Adams, of two spaces over the interval that are fiberwise stable homotopy equivalent, but where one of them has a section and the other one does not. Traditional cohomological methods do not distinguish the two maps.

We use the quadratic approximation to the identity functor to define a "metastable" obstruction to the existence of a section in the case when cohomological obstructions vanish. Our obstruction does distinguish the two map in Adams's example.

In the process of defining the obstruction we give an explicit description of the quadratic stage of the Goodwillie tower of the identity of *unpointed* spaces. This may be of independent interest.

This is joint work with Alvin Jin.   (Received September 16, 2020)

57 ▶ *Manifolds and cell complexes*

1163-57-19   **Ciprian Manolescu*** (`cm5@stanford.edu`). *Khovanov homology and surfaces in four-manifolds.*

Over the last forty years, most progress in four-dimensional topology came from gauge theory and related invariants. Khovanov homology is an invariant of knots in R^3 of a different kind: its construction is combinatorial, and connected to ideas from representation theory. There is hope that it can tell us more about smooth 4-manifolds; for example, Freedman, Gompf, Morrison and Walker suggested a strategy to disprove the 4D Poincare conjecture using Rasmussen's invariant from Khovanov homology. It is yet unclear whether their strategy can work, and I will explain some of its challenges. I will also review other topological applications of Khovanov homology, with regard to smoothly embedded surfaces in 4-manifolds.   (Received September 02, 2020)

1163-57-27   **Blake Mellor*** (`blake.mellor@lmu.edu`) and **Riley Smith**. *Finite N-quandles of knots, links and graphs.*

In this talk, we will investigate a generalization of the n-quandle for a knot, link or spatial graph. If a link or spatial graph has k components or edges, and given a k-tuple $N = (n_1, \ldots, n_k)$ of positive integers, we define the N-quandle by adding relations $x^{y^{n_i}} = x$ to the full quandle whenever y is an arc of edge i. The usual n-quandle is the case when $n_i = n$ for every i. In this case, Hoste and Shanahan provided a complete list of links where the n-quandle is finite, proving a conjecture of Przytycki. We extend the conjecture to spatial graphs and links where the entries of N are not all the same, and provide evidence for the extended conjecture by computing finite N-quandles associated to a number of links and spatial graphs.   (Received July 04, 2020)

1163-57-75   **Vladimir Chernov*** (`vladimir.chernov@dartmouth.edu`), 6188 Kemeny Hall, Dartmouth College, Hanover, NH 03755, **Gage Martin** (`martaic@bc.edu`), Department of Mathematics, Boston College, Chestnut Hill, MA 02467, and **Ina Petkova** (`ina.petkova@dartmouth.edu`), 6188 Kemeny Hall, Dartmouth College, Hanover, NH. *Khovanov homology and causality in spacetimes.*

We observe that Khovanov homology detects causality in $(2 + 1)$-dimensional globally hyperbolic spacetimes whose Cauchy surface is homeomorphic to $\mathbb{R}^2$. We use this result to discuss how Khovanov homology can be used to detect causality in globally hyperbolic spacetimes whose Cauchy surface is different fron $\mathbb{R}^2$.   (Received September 04, 2020)

1163-57-103   **Jason Manning** (`jfmanning@cornell.edu`), **Saul Schleimer**
                     (`s.schleimer@warwick.ac.uk`) and **Henry Segerman*** (`segerman@math.okstate.edu`).
                     From veering triangulations to Cannon-Thurston maps. Preliminary report.

Agol introduced veering triangulations of mapping tori, whose combinatorics are canonically associated to the pseudo-Anosov monodromy. In previous work, Hodgson, Rubinstein, Tillmann and I found examples of veering triangulations that are not layered and therefore do not come from Agol's construction.

However, non-layered veering triangulations retain many of the good properties enjoyed by mapping tori. For example, Schleimer and I constructed a canonical circular ordering of the cusps of the universal cover of a veering triangulation. Its order completion gives the veering circle; collapsing a pair of canonically defined laminations gives a surjection onto the veering sphere.

In work in progress, Manning, Schleimer, and I prove that the veering sphere is the Bowditch boundary of the manifold's fundamental group. As an application we produce Cannon-Thurston maps for all veering triangulations. This gives the first examples of Cannon-Thurston maps that do not come, even virtually, from surface subgroups.   (Received August 15, 2020)

1163-57-135   **Erica Flapan*** (`ericaflapan@gmail.com`) and **Song Yu** (`syu@math.columbia.edu`).
                     Symmetries of Spatial Graphs in 3-*manifolds.* Preliminary report.

We consider when automorphisms of a graph can be induced by homeomorphisms of embeddings of the graph in a 3-manifold. In particular, we prove that every automorphism of a graph is induced by a homeomorphism of some embedding of the graph in a connected sum of one or more copies of $S^2 \times S^1$, yet there exist automorphisms which are not induced by a homeomorphism of any embedding of the graph in any orientable, closed, connected, irreducible 3-manifold. We also prove that for any 3-connected graph G, if an automorphism σ is induced by a homeomorphism of an embedding of G in an irreducible 3-manifold M, then G can be embedded in an orientable, closed, connected 3-manifold M' such that σ is induced by a finite order homeomorphism of M', though this is not true for graphs which are not 3-connected. Finally, we show that many symmetry properties of graphs in S^3 hold for graphs in homology spheres, yet we give an example of an automorphism of a graph G that is induced by a homeomorphism of some embedding of G in the Poincaré homology sphere, but is not induced by a homeomorphism of any embedding of G in S^3.   (Received August 19, 2020)

1163-57-139   **Sergei Gukov** and **Ciprian Manolescu*** (`cm5@stanford.edu`). *GPV invariants and knot
                     complements.*

Gukov, Putrov and Vafa predicted (from physics) the existence of some 3-manifold invariants that take the form of power series with integer coefficients, converging in the unit disk. Their radial limits at the roots of unity should recover the Witten-Reshetikhin-Turaev invariants. Further, they should admit a categorification, in the spirit of Khovanov homology. Although a mathematical definition of the GPV invariants is lacking, they can be computed in many cases. In this talk I will discuss what is known about the GPV invariants, and their behavior with respect to Dehn surgery. The surgery formula involves associating to a knot a two-variable series, obtained by parametric resurgence from the asymptotic expansion of the colored Jones polynomial.   (Received August 20, 2020)

1163-57-140   **David Boozer*** (`adboozer@princeton.edu`). *Computer Bounds for Kronheimer-Mrowka
                     Foam Evaluation.*

Kronheimer and Mrowka recently suggested a possible approach towards a new proof of the four color theorem that does not rely on computer calculations. Their approach is based on a functor $J^\sharp$, which they define using gauge theory, from the category of webs and foams to the category of vector spaces over the field of two elements. They also consider a possible combinatorial replacement $J^\flat$ for $J^\sharp$. Of particular interest is the relationship between the dimension of $J^\flat(K)$ for a web K and the number of Tait colorings $\text{Tait}(K)$ of K; these two numbers are known to be identical for a special class of "reducible" webs, but whether this is the case for nonreducible webs is not known. We describe a computer program that strongly constrains the possibilities for the dimension and graded dimension of $J^\flat(K)$ for a given web K, in some cases determining these quantities uniquely. We present results for a number of nonreducible example webs. For the dodecahedral web W_1 the number of Tait colorings is $\text{Tait}(W_1) = 60$, but our results suggest that $\dim J^\flat(W_1) = 58$.   (Received August 20, 2020)

1163-57-155   **Scott Taylor***, scott.taylor@colby.edu. *(Non)Additivity of Spatial Graph Invariants.*

Knot invariants such as bridge number and tunnel number can be extended to be invariants of spatial graphs. I'll survey recent joint work showing how tools coming from Heegaard splitting theory can be adapted to study the additivity and non-additivity properties of such invariants under connected sum and trivalent vertex sum.

Some of this is based on joint work with Maggy Tomova and other aspects are based on joint work with Qidong He.   (Received August 24, 2020)

1163-57-162   **Jake Pichelmeyer*** (jacobpichelmeyer@gmail.com). *Obstructing sliceness in $\mathbb{C}P^2$ using homological degrees.*

The $\mathbb{C}P^2$ genus of a knot K, denoted $g_{\mathbb{C}P^2}(K)$, is the least genus among all orientable surfaces Σ properly embedded in $\mathbb{C}P^2 \setminus B^4$ with boundary $\partial\Sigma = K$. We say a knot K is *slice in $\mathbb{C}P^2$* if $g_{\mathbb{C}P^2}(K) = 0$. One can show that a knot is slice in $\mathbb{C}P^2$ by explicitly constructing a slice disc using diagrammatic methods. But what does one do if no such slice disc exists? In a recently submitted paper, the speaker obstructed some knots from bounding a slice disc by obstructing all possible homological degrees of the disc. These degree obstructions were generated using a variety of results from 4-dimensional smooth topology. In this talk, we will examine these obstructions.   (Received August 24, 2020)

1163-57-180   **Elizabeth Denne***, Mathematics Department, Washington & Lee University, Lexington, VA 24450. *Crossing Number and Ribbonlength of Folded Ribbon Knots.*

We study Kauffman's model of a folded ribbon knot: a knot made from a thin strip of paper folded flat in the plane. The folded ribbonlength is the length to width ratio of such a ribbon. We give upper bounds on ribbonlength and relate these to crossing number for various families of knots. We also give a construction showing ribbonlength is sublinear in crossing number for torus knot families. This is joint work with undergraduate students.   (Received August 25, 2020)

1163-57-211   **Nicolas Petit***, Boston College, Chestnut Hill, MA 02467. *The Multi-variable Affine Index Polynomial for tangles.* Preliminary report.

We will discuss our recent generalization of the Affine Index Polynomial to links and tangles. After introducing the invariant for links, we will discuss how the notion naturally extends to tangles, and show that it is compatible with tangle addition. We will conclude with some questions about the invariant we're currently investigating. (Received August 27, 2020)

1163-57-217   **Tye Lidman** and **Allison H Moore*** (moorea14@vcu.edu), Virginia Commonwealth University, Richmond, VA , and **Claudius Zibrowius**. *Essential Conway spheres and Floer homology via immersed curves.*

We will consider the problem of whether Dehn surgery along a knot in the three-sphere produces an L-space, which is a Floer-theoretic generalization of a lens space. The geometric characterization of these manifolds remains a difficult outstanding problem, and it is natural to ask whether the existence of certain essential surfaces in the complement of a knot can obstruct non-trivial surgeries yielding L-spaces. We will prove any knot in the three-sphere with a nontrivial L-space surgery admits no essential Conway spheres. As a corollary, we recover a classic result of Wu that states that if a knot K has an essential Conway sphere, then the fundamental group of rational Dehn surgery along K is never finite. Our proof uses the technology of peculiar modules, a Floer theoretic invariant for tangles due to Zibrowius, and the geometric realization of these modules as certain decorated immersed curves on the four-punctured sphere. This is joint work with Lidman and Zibrowius. (Received August 27, 2020)

1163-57-224   **John A Baldwin***, john.baldwin@bc.edu, and **Steven Sivek**. *Progress and questions in instanton Floer homology.*

Instanton Floer homology stands out among Floer homological invariants of 3-manifolds for several reasons. For one, it is transparently related to the fundamental group in a way that isn't true of Heegaard Floer homology, monopole Floer homology, and embedded contact homology. Moreover, its connection with the latter three invariants (which are isomorphic to one another) remains elusive. Finally, and in a related vein, instanton Floer homology is generally very difficult to compute. We will discuss progress towards computing framed instanton homology, some topological applications, and some natural questions and speculation that arise from this line of research. Much of this is joint work with Steven Sivek.   (Received August 28, 2020)

1163-57-231   **Vijay Higgins*** (vijay@ucsb.edu). *Triangular decomposition of SL_3 skein algebras.*
The Kauffman bracket skein algebra of a surface is spanned by link diagrams on the surface. If the surface has an ideal triangulation, the algebra admits a decomposition into skein algebras of triangles after we pass to the finer stated skein algebra introduced by Le. A key ingredient is the fact that the canonical basis of the Kauffman bracket skein algebra can be extended to a canonical basis of the stated skein algebra. In this talk, we will introduce a stated version of the SL_3 skein algebra of trivalent webs on a surface and prove that the situation here is analogous to the SL_2 case.   (Received August 29, 2020)

1163-57-240   **Ikshu Neithalath***, ikshu@ucla.edu, and **Ciprian Manolescu**, Stanford University.
                Skein Lasagna Modules for 2-handlebodies.
Morrison, Walker, and Wedrich recently introduced a generalization of Khovanov-Rozansky homology to links
L in the boundary of a 4-manifold W. For 4-manifolds with only $0, 2$, and 4-handles, we describe the simplest
piece of their invariant, the skein lasagna module $\mathcal{S}_0^N(W; L)$, in terms of the Khovanov-Rozansky homology of
infinitely many cables on the attaching link of the 2-handles. We obtain explicit formulae when W is a disc
bundle over S^2 and when $W = \mathbb{CP}^2$ or $\overline{\mathbb{CP}}^2$. This is joint work with Ciprian Manolescu.   (Received August 30,
2020)

1163-57-268   **Allison N Miller*** (`allison.miller@rice.edu`). *The satellite operation and knot
                concordance.*
The classical satellite construction associates to a pattern knot P in a solid torus and a companion knot K in S^3
a satellite knot $P(K)$, the image of P when the solid torus is 'tied into' the knot K. This operation descends to
a well-defined map on the set of (smooth or topological) concordance classes of knots. Many natural questions
about these maps remain open: when are they surjective, injective, or bijective? How do they behave with
respect to measures of 4-dimensional complexity? How do they interact with additional group or metric space
structure on the concordance set? I will discuss work giving partial progress towards answering these questions,
including joint work with Piccirillo and with Feller–Pinzón-Caicedo.   (Received August 31, 2020)

1163-57-321   **Irving Dai**, **Jennifer Hom*** (`hom@math.gatech.edu`), **Matthew Stoffregen** and **Linh
                Truong**. *Knot concordance invariants and homomorphisms.*
Knots under the operation of connected sum form a monoid. This set fails to be a group, since it lacks inverses.
However, modulo a certain equivalence relation called concordance, we do obtain a group, called the knot
concordance group. We will discuss some knot concordance invariants coming from knot Floer homology. This
is joint work with I. Dai, M. Stoffregen, and L. Truong.   (Received September 02, 2020)

1163-57-344   **Ka Ho Wong*** (`daydreamkaho@math.tamu.edu`), Department of Mathematics, Texas A&M
                University, College Station, TX 77843, and **Tian Yang** (`tianyang@math.tamu.edu`),
                Department of Mathematics, Texas A&M University, College Station, TX 77843. *Relative
                Reshetikhin-Turaev invariants, hyperbolic cone metrics and discrete Fourier transforms.*
Recently, Tian Yang and I made a volume conjecture for the relative Reshetikhin-Turaev invariants of a closed
oriented 3-manifold M with a colored framed link L inside it. We propose that their asymptotic behavior is
related to the volume and the Chern-Simons invariant of the hyperbolic cone metric on the manifold with singular
locus the link and cone angles determined by the coloring. In this talk, I will discuss how this conjecture can
be understood as an interpolation between the Kashaev-Murakami-Murakami volume conjecture of the colored
Jones polynomials and the Chen-Yang volume conjecture of the Reshetikhin-Turaev invariants. Besides, I will
introduce the change-of-pair operation that changes a pair (M, L) without changing the complement $M\backslash L$ and
discuss its meanings in quantum topology. Finally, I will summarize our recent progress on this conjecture.
(Received September 03, 2020)

1163-57-352   **Diana Hubbard*** (`diana.hubbard@brooklyn.cuny.edu`), **Dongtai He** and **Linh Truong**.
                Right-veering open books and the Upsilon invariant.
Fibered knots in a three-manifold Y can be thought of as the binding of an open book decomposition for Y.
One can ask how properties of the open book decomposition relate to properties of the corresponding knot. In
this talk I will describe work that explores this question: we give a sufficient condition for the monodromy of an
open book decomposition of a fibered knot to be right-veering from the concordance invariant Upsilon. I will
discuss an application of this work to the Slice-Ribbon conjecture.   (Received September 03, 2020)

1163-57-355   **Inanc Baykur*** (`baykur@math.umass.edu`) and **Noriyuki Hamada**
                (`hamada@math.umass.edu`). *Lefschetz fibrations and symplectic geography.*
We will discuss how to build symplectic Lefschetz fibrations with prescribed signatures and spin type, along with
novel applications to the geography of symplectic 4-manifolds. This is joint work with N. Hamada.   (Received
September 03, 2020)

1163-57-372   **Sam Gunningham*** (`samgunningham@gmail.com`), School of Mathematics, University of
                Edinburgh, James Clerk Maxwell Building, Edinburgh, EH9 3FD. *The finiteness conjecture
                for skein modules.*
The Kauffman bracket skein module of an oriented 3-manifold M is a vector space (depending on a parameter q)
which is generated by framed links in M modulo certain skein relations. The goal for the talk is the explain our

recent proof (joint with David Jordan and Pavel Safronov) that the skein module of a closed 3 manifold is finite dimensional for generic q, confirming a conjecture of Witten. The proof involves understanding skein modules in terms of deformation quantizations of SL(2,C)-character varieties.   (Received September 04, 2020)

1163-57-373   **Gage Martin*** (`martaic@bc.edu`). *Khovanov homology and link detection.*
Khovanov homology is a combinatorially defined link homology theory. Due to the combinatorial definition, many topological applications of Khovanov homology arise via connections to Floer theories. A specific topological application is the question of which links Khovanov homology detects. In this talk, we will give an overview of Khovanov homology and link detection, mention some of the connections to Floer theoretic data used in detection results, and sketch a proof that Khovanov homology detects the torus link T(2,6).   (Received September 04, 2020)

1163-57-420   **Neil R Hoffman*** (`neil.r.hoffman@okstate.edu`), 401 Math Science Building, Stillwater, OK 74078. *Cusp types of quotients of hyperbolic knot complements.*
This work completes a classification of the types of orientable and non-orientable cusps that can arise in the quotients of hyperbolic knot complements. In particular, $S^2(2,4,4)$ cannot be the cusp cross-section of any orbifold quotient of a hyperbolic knot complement. I will also discuss how this reduces the complexity of problem of finding orbifolds irregularly covered by knot complements, which is relevant to joint work with Chesebro, Deblois, Millichap, Mondal, and Worden.   (Received September 05, 2020)

1163-57-422   **Joshua Wang*** (`jxwang@math.harvard.edu`). *The cosmetic crossing conjecture for split links.*
Given a nontrivial band sum of two knots, we may add full twists to the band to obtain a family of knots indexed by the integers. In this talk, I'll show that the knots in this family have the same Heegaard and instanton knot Floer homology but distinct Khovanov homology, generalizing a result of M. Hedden and L. Watson. A key component of the argument is a proof that each of the three knot homologies detects the trivial band. The main application is a verification of the generalized cosmetic crossing conjecture for split links.   (Received September 06, 2020)

1163-57-424   **Hans U Boden**, **Micah Chrisman*** (`chrisman.76@osu.edu`) and **Homayun Karimi**. *The Gordon-Litherland pairing for knots in thickened surfaces.*
The Gordon-Litherland pairing of knots in S^3 is a symmetric bilinear form that unifies the quadratic forms of Trotter and Goeritz. The Gordon-Litherland pairing was extended to knots in $\mathbb{Z}_2$-homology 3-spheres by Greene. Here we extend the Gordon-Litherland pairing to knots in thickened surfaces $\Sigma \times I$, where Σ is closed and oriented. Our extended pairing is defined for all $\mathbb{Z}_2$-homologically trivial knots in $\Sigma \times I$. The resulting signature invariants depend only on the S^*-equivalence class of the (unoriented) spanning surface F. Previously, Im-Lee-Lee defined signature invariants for checkerboard colorable knots in thickened surfaces using Goeritz matrices. For $\mathbb{Z}$-homologically trivial knots in $\Sigma \times I$, signature invariants were defined by Boden-Chrisman-Gaudreau via a modification of the Seifert pairing. Our Gordon-Litherland pairing likewise unifies both these signature invariants for knots in $\Sigma \times I$. We show that the extended Gordon-Litherland pairing can be realized as a relative intersection form of a twofold branched cover of F pushed into a thickened 3-manifold $W \times I$, where $\partial W = \Sigma$. Geometric applications of these results to virtual knots are discussed.   (Received September 06, 2020)

1163-57-445   **Samantha Allen*** (`samantha.g.allen@dartmouth.edu`) and **Charles Livingston**. *Unknotting with a single twist.*
Ohyama showed that any knot can be unknotted by performing two full twists, each on a set of parallel strands. We consider the question of whether or not a given knot can be unknotted with a single full twist, and if so, what are the possible linking numbers associated to such a twist. It is observed that if a knot can be unknotted with a single twist, then some surgery on the knot bounds a rational homology ball. Using tools such as classical invariants and invariants arising from Heegaard Floer theory, we give obstructions for a knot to be unknotted with a single twist of a given linking number. In this talk, I will discuss some of these obstructions, their implications (especially for alternating knots), many examples, and some unanswered questions. This talk is based on joint work with Charles Livingston.   (Received September 07, 2020)

1163-57-461   **John Etnyre** and **Caitlin Leverson*** (`cleverson@bard.edu`). *Lagrangian Realizations of Ribbon Cobordisms.* Preliminary report.
Similarly to how every smooth knot has a Legendrian representative (in fact, infinitely many different representatives), in this talk we will discuss why every ribbon cobordism has a Legendrian representative. Meaning, if C is a ribbon cobordism in $[0,1] \times S^3$ from the link K_0 to K_1, then there are Legendrian realizations Λ_0 and Λ_1

of K_0 and K_1, respectively, such that C may be isotoped to a decomposable Lagrangian cobordism from Λ_0 to Λ_1. We will also give examples of some interesting constructions of such decomposable Lagrangian cobordisms. This is joint work with John Etnyre.   (Received September 07, 2020)

1163-57-462   **Nicholas Owad*** (`owad@hood.edu`), Frederick, MD , and **Anastasiia Tsvietkova**.
                Random meander links.
In the recent years, there has been an increased interest in using probabilistic methods in topology and more specifically, knot theory. Here we present a new model of random links, called random meander links, and discuss some of its properties. In particular, we will show that the expected hyperbolic volume of a random meander link is bounded between linear functions of the number of crossings and how hypergeometric series have come up in the research.   (Received September 07, 2020)

1163-57-474   **Jessica Purcell** and **Anastasiia Tsvietkova*** (`a.tsviet@rutgers.edu`). *The number of
                surfaces of fixed genus embedded in a 3-manifold.* Preliminary report.
It was noticed before that presence of embedded essential surfaces in a 3-manifold can give information about that manifold. However to construct, classify or count such surfaces is a non-trivial task. If 3-manifold is complement of an alternating link with n crossings in a 3-sphere, we previously showed that the number of genus-g surfaces is bounded by a polynomial in n. This was the first polynomial bound. This was joint work with Joel Hass and Abigail Thompson. In the talk, I will discuss a generalization that concerns any cusped 3-manifold that is complement of a link alternating on some embedded surface in an arbitrary 3-manifold.   (Received September 07, 2020)

1163-57-475   **Hugh N Howards***, Math Department, Manchester Hall, Winston Salem, NC 27109, and
                Erica Flapan. *Intrinsic Chirality of Graphs.*
We examine joint results of Erica Flapan and Hugh Howards on the intrinsic chirality of graphs. The main result is that for every closed, connected, orientable, irreducible 3-manifold M, there is an integer n_M such that if γ is a graph with no involution and a 3-connected minor λ with genus $\lambda > n_M$ then every embedding of γ in M is chiral. By contrast, we also show that for every graph γ there are infinitely many closed, connected, orientable, irreducible 3-manifolds M such that some embedding of γ in M is pointwise fixed by an orientation reversing involution of M.   (Received September 07, 2020)

1163-57-483   **Rhea Palak Bakshi*** (`rhea_palak@gwu.edu`). *Counterexamples in Kauffman bracket
                skein modules of 3-manifolds.*
Skein modules were introduced by Józef H. Przytycki as generalisations of the Jones and HOMFLYPT polynomial link invariants in S^3 to arbitrary 3-manifolds. The Kauffman bracket skein module (KBSM) is the most extensively studied of all. However, computing the KBSM of a 3-manifold is known to be notoriously hard, especially over the polynomial ring $\mathbb{Z}[A^{\pm 1}]$. With the goal of finding a definite structure of the KBSM over this ring, several conjectures and theorems were stated over the years for KBSMs. We show that some of these conjectures, and even theorems, are not true. In this talk I will discuss my counterexample to Julien Marché's generalisation of Witten's conjecture. I also show that the theorem stated by Przytycki about the KBSM of the connected sum of two handlebodies does not hold.   (Received September 08, 2020)

1163-57-491   **Charles Frohman*** (`charles-frohman@uiowa.edu`) and **Adam Sikora**
                (`asikora@buffalo.edu`). *Coordinatizing trivalent graphs embedded in finite type surfaces.*
A web is an trivalent graph whose edges have been oriented so that every vertex is a source or sink, embedded in a surface F so that no exterior regions are monogons, bigons, or quadrigons. If F is a finite type surface with at least one puncture having negative Euler characteristic then F admits an ideal triangulation. We give coordinates for isotopy classes of webs embedded in such a finite type surface based on the intersection of the web with an ideal triangulation.   (Received September 08, 2020)

1163-57-494   **Uwe Kaiser*** (`ukaiser@boisestate.edu`), Department of Mathematics, Boise State
                University, 1910 University Drive, Boise, ID 83725. *Kashaev and extended Kauffman state
                summation.* Preliminary report.
For each integer $N \geq 2$ we define a map from the set of Kashaev N-states for the Kashaev invariant of links into a set of states, which extends the set of Kauffman states for the Alexander polynomial. We discuss various properties of these maps, in particular the relation to the Kashaev state sum contributions. We consider the special case of 3-braids and find explicit formulas for weave knots. We also discuss the relation with state summation for the colored Jones polynomials.   (Received September 08, 2020)

1163-57-495   **Heather A Dye*** (heateranndye@gmail.com), IL , and **Aaron Kaestner**. *Using virtual knots to define groups.* Preliminary report.
Aaron Kaestner and I defined a virtual knot group that incorporated information about the parity of the classical crossings and used this group to define an Alexander type polynomial. This work was motivated by the paper: Virtual knot groups and almost classical knots by Boden et al that describes several different knot groups obtained from virtual knots. In this work, we use knot diagrams to define quotients of the free groupThen we examine specializations and properties of these groups.   (Received September 08, 2020)

1163-57-508   **Maggie Miller*** (maggiehm@mit.edu) and **Alexander Zupan** (zupan@unl.edu). *Characterizing handle-ribbon knots.*
Kauffman conjectured that a knot K is slice if and only if it bounds a genus-g Seifert surface containing a framed g-component slice link as a cut system. It is easy to show that a knot is ribbon if and only if it bounds a genus-g Seifert surface containing a g-component zero-framed unlink as a cut system. I will describe why an intermediate statement is true: a knot is handle-ribbon (aka strongly homotopy-ribbon) if and only if it bounds a genus-g Seifert surface containing a framed g-component R link L as a cut system – i.e. surface-framed surgery on L yields $\#_g S^1 \times S^2$. This gives a 3-dimensional definition of a 4-dimensional property.   (Received September 08, 2020)

1163-57-509   **Ramin Naimi**, Occidental College, Los Angeles, CA 90041, **Andrei Pavelescu**, University of South Alabama, Mobile, AL 36688, and **Elena Pavelescu***, University of South Alabama, Mobile, AL 36688. *Bounds for maximal linkless graphs.*
A linklessly embeddable graph G is *maxnil* if it is not a proper subgraph of a linklessly embeddable graph of the same order. The property of being maxnil is, in a way, analogous to the property of being maximal planar. While it is well known that every maximal planar graph with n vertices has $3n-6$ edges, an analogous statement for maxnil graphs does not exist. In this talk we discuss properties of maxnil graphs, and we present two new families of maximal linklessly embeddable graphs on n vertices: one family with $3n-5$ edges for all $n \geq 10$, and another family with n vertices and $m < \frac{25n}{12}$ edges for all $n \geq 13$.   (Received September 08, 2020)

1163-57-511   **Jozef H. Przytycki*** (przytyck@gwu.edu), George Washington Universuty, Phillips Hall, Washington, DC 20052-0001. *Motivated by Mark Kidwell - from Hoste-Kidwell multivariable HOMFLYPT polynomial to Witten conjecture.*
In Fall of 1986, after coming from Poland in July 1986, I attended conferences in Santa Cruz and in Santa Barbara. At one of them (or both?) I heard the talk by Mark Kidwell on his effort, with Jim Hoste, of finding many variable HOMFLYPT polynomial. I will start my talk describing this period, between July 1986 and April 1987 (when the idea of skein modules became clear for me). I will sketch the history of skein modules of 3-dimensional manifolds stressing influence of Hoste-Kidwell work.   (Received September 08, 2020)

1163-57-514   **Ramin Naimi**, **Andrei Pavelescu*** (andreipavelescu@southalabama.edu) and **Elena Pavelescu**. *Topological Properties of Graphs and Their Complements.*
In this talk, we present some connections between the order and the topological properties of a simple graph and the topological properties of its complement. In 1962, J. Battle, F. Harary, and Y. Kodama, J.R. Ball, and W. T. Tutte independently proved that the complement of a planar graph of order 9 is not planar, thus proving that K_9 is not bi-planar. In this talk, we discuss similar statements about linkless embeddable graphs, intrinsic knotted graphs, and several other hereditary classes of graphs.   (Received September 08, 2020)

1163-57-517   **David Freund*** (dfreund@math.harvard.edu), 1 Oxford St, Department of Mathematics, Cambridge, MA 02138. *Boundary Surfaces for Virtual Links.* Preliminary report.
Virtual links can be realized as stable equivalence classes of links in thickened surfaces. Given a virtual link L, a natural problem is to determine what surfaces in the ambient thickened surface have boundary L. In classical knot theory, every link bounds an orientable surface that encodes the Alexander polynomial and can be used to define additional link invariants, including linking numbers. However, as we will discuss, not every virtual link bounds a surface and only the family of almost-classical virtual links bound orientable surfaces. In this talk, we will define a construction which provides a geometric description for the linking numbers of virtual links and use it to build toward a geometric classification of virtual links that bound surfaces.   (Received September 08, 2020)

1163-57-519   **Anna Beliakova**, **Matthew Hogancamp**, **Krzysztof K. Putyra** and **Stephan M. Wehrli*** (`smwehrli@syr.edu`), 215 Carnegie, Syracuse University, Syracuse, NY 13244. *On categorifications of the colored Jones polynomial.* Preliminary report.
Around 2010, Cooper and Krushkal introduced categorified Jones-Wenzl projectors and used them to define a new categorification of the colored Jones polynomial. In this talk, I will show that this categorification agrees with the one defined by Khovanov, as long as one works in a quantum annular setting. I will first discuss this result for the case of the colored unknot and then use satellite operations to deal with the general case. (Received September 08, 2020)

1163-57-529   **Thang T. Q. Le** (`letu@math.gatech.edu`), School of Mathematics, Georgia Institute of Technology, 686 Cherry St., Atlanta, GA 30332, and **Adam S. Sikora*** (`asikora@buffalo.edu`), 244 Math Bldg, University at Buffalo, Buffalo, NY 14260. *Stated SU(n)-skein modules.* Preliminary report.
We introduce stated SU(n)-skein modules $\mathcal{S}_n(M)$ of 3-manifolds M which extend the Reshetikhin-Turaev SU(n)-quantum invariant of links to arbitrary 3-manifolds and quantize the SL(n)-character varieties of M (and their generalizations).

   We prove several properties of our skein modules; In particular, a splitting theorem which relates the stated SU(n)-skein module of M to the tensor product of the pieces of M cut along a disk.

   In the case of thickened marked surface $\Sigma \times I$, the skein module of it, denoted by $\mathcal{S}_n(\Sigma)$ is a non-commutatuve algebra. We prove that for the bigon it is isomorphic with the quantum group $\mathcal{O}_q sl(n)$. We also show that for any ideally triangulated marked surface Σ, the splitting theorem defines an embedding of $\mathcal{S}_n(\Sigma \times I)$ into the tensor product of stated skein algebras of the ideal triangles and, consequently, into a quantum torus.   (Received September 08, 2020)

1163-57-534   **Irving Dai**, **Matthew Hedden** and **Abhishek Mallick***, mallicka@msu.edu. *Corks, involutions and Heegaard Floer homology.*
We introduce and study a set of Floer-theoretic invariants aimed at detecting corks. Our invariants obstruct the extension of a given involution of a 3-manifold over any homology ball (rather than a particular contractible manifold). We call such a 3-manifold with involution a strong cork. Our method does not explicitly reference closed 4-manifolds or contact structures, instead, we utilize the formalism of local equivalence coming from involutive Heegaard Floer homology. As an application, we define a modification of the homology cobordism group $\Theta_{\mathbb{Z}}^{\tau}$ which takes into account involutions acting on homology spheres, and prove that this admits a $\mathbb{Z}^{\infty}$-subgroup generated by strong corks. The group $\Theta_{\mathbb{Z}}^{\tau}$ can be viewed as a refinement of the classical bordism group of diffeomorphisms. Using our invariants, we establish several new families of strong corks and prove that various known examples are also strong.   (Received September 08, 2020)

1163-57-539   **Juanita Pinzon-Caicedo*** (`jpinzonc@nd.edu`), **Peter Feller** and **Allison Miller**. *The topological 4-genus of satellite knots.*
A satellite knot $P(K)$ is obtained by tying a given knot P inside a solid torus V along another knot K. The winding number w of the satellite operation is given by the algebraic intersection number of P with a meridional disk of the solid torus V. A result of Schubert states that for any pattern P with winding number w, there exists a constant $g_3(P)$ such that for any nontrivial knot K in S^3 we have $g_3(P(K)) = g_3(P) + |w|g_3(K)$. In this talk I will show that in the 4-dimensional smooth category an analogous formula holds, but in the topological category the winding number of the pattern is no longer pivotal. This is joint work with Allison Miller and Peter Feller. (Received September 08, 2020)

1163-57-543   **Krzysztof Putyra** and **Alexander Shumakovitch***, AShumakovitch@gmail.com. *Unified Khovanov homology for links and its properties.* Preliminary report.
Unified Khovanov homology combines even and odd Khovanov homology theories into a single algebraic object that carries the structure of a module over the group ring $\mathbb{Z}\mathbb{Z}_2$. It was previously shown by the authors that the unified Khovanov homology is often a stronger knot invariant than the even and odd Khovanov homology combined. In this talk, we consider a similar question for links with 2 and 3 components and present evidence that two links with the same unified Khovanov homology might nonetheless have different even and/or odd Khovanov homology. This is remarkable since one can easily obtain even and odd Khovanov chain complexes from the unified one.   (Received September 08, 2020)

1163-57-544   **Hyunki Min*** (`hmin38@gatech.edu`), 180 Jackson st. NE, ATP 5308, Atlanta, GA 30312, and **John Etnyre** and **Anubhav Mukherjee**. *Which 3-manifolds bound exotic 4-manifolds?*

It has been a long standing question that which closed 4-manifolds admit exotic smooth structures. In this talk, we consider the relative version of this question: given a 3-manifold, does it bound a compact 4-manifold which admits infinitely many smooth structures? We give several criteria when a 3-manifold bounds an exotic 4-manifold. This is a joint work with John Etnyre and Anubhav Mukherjee.   (Received September 09, 2020)

1163-57-568   **Hugh Morton**, **Alex Pokorny** and **Peter Samuelson*** (`psamuels@ucr.edu`). *Skein algebras of the torus.*

The skein algebra of a surface is spanned by links in the thickened surface modulo (some version of) the skein relations. In this talk we give explicit descriptions of the skein algebra of the torus for the Homflypt and Kauffman skein relations. If time permits we will briefly mention how these algebras arise in other areas of mathematics. (Based on joint works with Morton and Pokorny.)   (Received September 09, 2020)

1163-57-573   **Matthias Goerner*** (`enischte@gmail.com`). *Verifying of and raytracing in finite hyperbolic triangulations.*

Casson's Geo and Heard's Orb can find a hyperbolic structure on a closed 3-manifold using finite triangulations. However, the methods are numerical and do not mathematically prove that the manifold is hyperbolic.

   I will show how to use interval arithmetic to rigorously verify hyperbolicity using a finite triangulation. This required a new theoretical result for finite triangulations akin to a theorem by Neumann-Zagier and Moser for ideal triangulations upon which HIKMOT is based.

   I will also show how finite triangulations can be used to render cohomology fractals for closed manifolds (joint work with David Bachmann, Saul Schleimer and Henry Segerman) to visualize Cannon-Thurston maps. (Received September 09, 2020)

1163-57-582   **Robin T Wilson*** (`robinwilson@cpp.edu`), Department of Mathematics and Statistics, 3801 W. Temple Ave, Pomona, CA 90043, and **Emille D. Lawrence** and **Erica Flapan**. *The Topological Symmetry Groups of the Heawood Graph.*

Although, motivated by chemistry, spatial graph theory has now become a subfield of low dimensional topology closely related to knot theory. In particular, the study of topological symmetry groups of graphs embedded in S^3 can be thought of as a generalization of the study of symmetries of knots and links. For a given embedding, we are interested in the automorphisms of the graph that are induced by a homeomorphism of the 3-sphere. This subgroup of the automorphism group of the graph is known as the topological symmetry group of that embedding. We will discuss recent results classifying which groups can occur as the topological symmetry group of some embedding of the Heawood graph in S^3.   (Received September 10, 2020)

1163-57-596   **Hannah Turner*** (`hannahturner@math.utexas.edu`). *Links all of whose branched cyclic covers are L-spaces.*

For a fixed link in a rational homology sphere and a positive integer, there is a unique three-manifold called the n-fold branched cover of the link. There are examples of links for which the n-fold branched cyclic cover is a Heegaard Floer L-space for any n. I'll discuss techniques to confirm or obstruct this property for links with a free 2-periodic symmetry. Some of this is joint work with Ahmad Issa.   (Received September 10, 2020)

1163-57-603   **Melissa Zhang*** (`melissa.zhang@uga.edu`) and **Linh Truong**. *On Upsilon-like invariants from Khovanov homology.*

Υ is a concordance invariant defined by Ozsváth, Stipsicz, and Szabó from knot Floer homology. By 'mixing' the Maslov grading and Alexander filtration, one obtains a 1-parameter family of filtrations, which are used to 'measure' a distinguished homology class. The resulting measurements generalize τ.

   Khovanov homology's construction is quite different from knot Floer homology's, but we've learned that the two theories can yield similar types of information. This talk will discuss the constructions in Khovanov homology that resemble that of Υ. This is partly based on joint work with Linh Truong.   (Received September 10, 2020)

1163-57-611   **Priyam Patel*** (`patelp@math.utah.edu`), **Carolyn Abbott** and **Nicholas Miller**. *Infinite-type mapping class groups and their actions on hyperbolic graphs.* Preliminary report.

Given a finite-type surface, there are two important objects naturally associated to it. The first is the mapping class group: the group of homeomorphisms of the surface up to isotopy. The second is the curve graph: an

infinite-diameter hyperbolic graph on which the mapping class group acts via isometries. This action is well understood and has been extremely useful in understanding the algebraic and geometric properties of mapping class groups. There has been a recent surge of interest in surfaces of infinite type, and in this talk we shift our focus to their mapping class groups. I will first discuss the problem of choosing an appropriate graph for the mapping class group to act on in this setting, and then describe recent joint work with Carolyn Abbott and Nicholas Miller explicitly constructing mapping classes with interesting (loxodromic) actions on this graph. If time permits, I will discuss some results about 3-manifolds arising as mapping tori for these mapping classes. (Received September 10, 2020)

1163-57-612   **Priyam Patel*** (`patelp@math.utah.edu`). *Infinite-type surfaces.*
A surface is of finite type if it is homeomorphic to a compact surface with at most finitely many points removed. Otherwise, the surface is of infinite-type. There has been a recent surge of interest in infinite-type surfaces and their mapping class groups (the group of homeomorphisms of the surface up to a natural equivalence called isotopy), which arise naturally in a variety of contexts. In this talk, I will give a crash course on infinite-type surfaces and highlight some of the main avenues of research in this area. I will end by discussing some recent joint work with Tarik Aougab and Nick Vlamis regarding the isometry groups of infinite-type surfaces. (Received September 10, 2020)

1163-57-663   **Scott A Taylor***, scott.taylor@colby.edu, and **Maggy Tomova**, maggy-tomova@uiowa.edu. *A new look at bridge number.* Preliminary report.
Bridge number is a classical knot invariant which can be defined as the minimum number of maxima of a knot, with the minimum taken over all height functions on the knot. By drawing on connections with thin position theories, we recently gave a different, but equivalent, definition of bridge number. As a result of this new perspective, we are able to give new proofs of Schubert's theorems that bridge number is "-1 additive" under connected sum and that the bridge number of a satellite knot is bounded below by the product of the bridge number of the companion and the wrapping number of the pattern. (Received September 11, 2020)

1163-57-676   **Zhenkun Li*** (`zhenkun@stanford.edu`), 450 Jane Stanford Way, Stanford, CA 94305, and **Fan Ye** (`pagasus4u@gmail.com`), Peking University, Beijing, Beijing 100871, Peoples Rep of China. *Instanton Floer homology of (1,1)-knot.*
Instanton knot homology was first introduced by Floer around 1990 and was revisited by Kronheimer and Mrowka around 2010. It is built based on the solution to a set of partial differential equations and is very difficult to compute. On the other hand, Heegaard diagrams are classical tools to describe knots and 3-manifolds combinatorially and are also the basis of Heegaard Floer homology, which was introduced by Ozsváth and Szabó around 2004. In this talk, I will explain how to extract some information about the instanton theory from Heegaard diagrams. In particular, we study the (1,1)-knots, which are known to have simple Heegaard diagrams. We provide an upper bound for the dimension of instanton knot homology for all (1,1)-knots. Also, we prove that, for some families of (1,1)-knots, including all torus knots, the upper bound we obtained is actually sharp. This is a joint work with Fan Ye. (Received September 11, 2020)

1163-57-725   **Jason Joseph** and **Puttipong Pongtanapaisan*** (`puttipong-pongtanapaisan@uiowa.edu`). *Bridge numbers of knotted surfaces in the four-sphere.* Preliminary report.
Meier and Zupan showed that every smoothly embedded surface in the 4-sphere can be put in bridge position. This is a 4-dimensional analog of bridge position for a knot in the 3-sphere, and gives rise to a complexity for knotted surfaces called the bridge number. In this talk, we discuss computations of the bridge numbers for various families of knotted surfaces. The lower bound involves calculations of the meridional rank, which is the minimum number of meridional generators taken over all presentations of the fundamental group of the surface's exterior in the 4-sphere. (Received September 12, 2020)

1163-57-757   **Tarik Aougab**, **Max Lahn**, **Marissa Kawehi Loving*** (`mloving6@gatech.edu`) and **Yang Xiao**. *Covers and Curves.*
It is a celebrated result of Scott that every closed curve on a hyperbolic surface lifts to a simple closed curve on some finite cover. In the spirit of this work we pose the following question,

What information about two covers X and Y of S can be derived by understanding how curves on S lift simply to X and Y?

In this talk, we will give our motivation for asking this question and explore its answer for regular finite covers of a closed hyperbolic surface. This work is joint with Tarik Aougab, Max Lahn, and Yang (Sunny) Xiao (Received September 12, 2020)

1163-57-769   **Jennifer Hom*** (`hom@math.gatech.edu`). *Getting a handle on the Conway knot.*
When does a knot bound a disk? In three dimensions, the only knot that bounds a smoothly embedded disk is the unknot. However, if one considers disks in the four-ball, the answer becomes significantly more difficult.

A knot is called slice if it bounds a smooth disk in the four-ball. For 50 years, it was unknown whether a certain 11 crossing knot, called the Conway knot, was slice or not, and until recently, this was the only one of the thousands of knots with fewer than 13 crossings whose slice-status remained a mystery. In this talk, we will describe Lisa Piccirillo's proof that the Conway knot is not slice. The main idea of her proof is given in the title of this talk.   (Received September 12, 2020)

1163-57-771   **Neil Hoffman**, **Christian Millichap** and **William Worden***
                (`william.worden@rice.edu`). *Symmetries and hidden symmetries of (ϵ, d_L)-twisted knot complements.*
It is a longstanding conjecture of Neumann and Reid that exactly three knot complements can irregularly cover a hyperbolic orbifold—the figure-8 knot and the two Aitchison–Rubinstein dodecahedral knots. This conjecture, when combined with work of Boileau–Boyer–Walsh, implies a more recent conjecture of Reid and Walsh, which states that there are at most 3 knot complements in the commensurability class of any hyperbolic knot complement. We give a Dehn filling criterion that is useful for producing large families of knot complements that satisfy both conjectures. This work is joint with Hoffman and Millichap.   (Received September 12, 2020)

1163-57-780   **Moshe Cohen*** (`cohenm@newpaltz.edu`) and **Keith F. Grover**. *Random knots obtained from finite continued fractions in +1 and -1.* Preliminary report.
A knot is a circle embedded in 3-space. A 2-bridge knot can be described by a finite sequence of nonzero integers counting crossings in alternating twist regions; such a knot is also called a rational knot. A Chebyshev knot diagram has in its finite continued fraction only +1 and -1, giving way to fewer knot diagrams for the same knot.

Schubert translated results on continued fractions into results on 2-bridge knots, and Koseleff and Pecker, in formalizing Chebyshev knots, built on these results for continued fractions in +1 and -1.

Together with Sunder Ram Krishnan and then Chaim Even-Zohar, the first author developed this into a model for random knots. In this talk the first author presents extensions of this work, including new work with the second author, an undergraduate student.   (Received September 12, 2020)

1163-57-782   **Michelle Chu*** (`michu@uic.edu`) and **Alan W Reid**. *Embedding closed hyperbolic 3-manifolds in small volume hyperbolic 4-manifolds.*
The smallest volume cusped hyperbolic 3-manifolds, the figure-eight knot complement and its sister, contain many immersed but no embedded closed totally geodesic surfaces. In this talk we discuss the existence or lack thereof of codimension-1 closed embedded totally geodesic submanifolds in minimal volume cusped hyperbolic 4-manifolds.   (Received September 12, 2020)

1163-57-783   **Rima Chatterjee*** (`rchatt1@lsu.edu`). *Links in overtwisted contact manifolds.*
Knots/links in overtwisted contact structures have been less explored. I'll start with a discussion of the two types of knots/links in overtwisted contact manifolds , namely- loose and non-loose. Next I'll state my recent work on coarse classification of loose null-homologous Legendrian and transverse links and close by showing coarse equivalence class of loose null-homologous Legendrian links has support genus zero.   (Received September 15, 2020)

1163-57-787   **Julian Chaidez**, **Jordan Cotler** and **Shawn Cui***, Department of Mathematics, Office 646, 150 N. University Street, West Lafayette, IN 47907. *Trisection invariants of 4-manifolds from Hopf algebras.*
The Kuperberg invariant is a topological invariant of closed 3-manifolds based on finite-dimensional Hopf algebras. Here we initiate the program of constructing 4-manifold invariants in the spirit of Kuperberg's 3-manifold invariant. We utilize a structure called a Hopf triplet, which consists of three Hopf algebras and a bilinear form on each pair subject to certain compatibility conditions. In our construction, we present 4-manifolds by their trisection diagrams, a four-dimensional analog of Heegaard diagrams. The main result is that every Hopf triplet yields a diffeomorphism invariant of closed 4-manifolds. In special cases, our invariant reduces to Crane-Yetter invariants and generalized dichromatic invariants, and conjecturally Kashaev's invariant. As a starting point,

we assume that the Hopf algebras involved in the Hopf triplets are semisimple. We speculate that relaxing semisimplicity will lead to even richer invariants. (Received September 12, 2020)

1163-57-794   **Priyadip Mondal*** (`prm50@pitt.edu`). *Hidden symmetries and Dehn fillings on links in the tetrahedral census.*
A hidden symmetry of a hyperbolic three manifold M is an isometry between finite index covers of M which is not a lift of any self-isometry of M. Neumann and Reid, in 1992, asked whether there are hyperbolic knots apart from the figure-eight knot and the two dodecahedral knots of Aitchison and Rubinstein whose complements have hidden symmetries. This question has motivated a fair amount of recent research on hidden symmetries. In this talk, I will expound the relation of hidden symmetries with certain families of hyperbolic knots obtained from Dehn filling all but one component of tetrahedral links in the Fominykh-Garoufalidis-Goerner-Tarkaev-Vesnin census. (Received September 15, 2020)

1163-57-800   **Joshua A Howie*** (`jahowie@ucdavis.edu`). *Alternating genera of knots.*
The alternating genus of a knot is the minimum genus of a surface onto which the knot has an alternating diagram satisfying certain conditions. We study spanning surfaces for knots, and define a related invariant from the extremal spanning surfaces. This gives a lower bound on the alternating genus and can be calculated exactly for torus knots. (Received September 13, 2020)

1163-57-816   **Radmila Sazdanovic*** (`rsazdan@ncsu.edu`), Department of Mathematics NC State University, PO Box 8205, Raleigh, NC 27695, and **John A. Baldwin**, **Nathan Dowlin**, **Adam Simon Levine** and **Tye Lidman**. *Khovanov homology detects the figure-eight knot.*
Knot classification is an important and difficult problem. Therefore, showing that a particular invariant detects some knots or classes of knots is important. In this work we show that Khovanov homology, a categorification of the Jones polynomial, can detect certain knots. (Received September 13, 2020)

1163-57-819   **Timothy M Morris***, 795 Geiger Rd, Fernandina Beach, FL 32034. *Minimal Index Non-Abelian Covers of Knot Complements: A survey.*
We say a cover of a knot complement is non-abelian if either the cover is irregular or the associated deck group is non-abelian. In 1987 John Hempel, as a consequence of residual finiteness, showed that such a cover exists for every knot. Moving forward, a natural question to ask: given a knot complement, what is the minimal index of a non-abelian cover? In this talk we will survey all the work done to answer this question, propose some directions moving forward, and some constructions of such covers. (Received September 13, 2020)

1163-57-877   **Jonathan C. Johnson*** (`jonjohnson@utexas.edu`). *The Bi-Orderability of Knot Groups and Its Applications.*
The orderability of 3-manifold groups has become a topic of interest in the last couple of decades. In this talk, I will discuss recent results related to the bi-orderability of the fundamental groups of knot complements and relate bi-orderability to questions concerning the L-space conjecture. (Received September 13, 2020)

1163-57-922   **Paolo Aceto**, **Corey Bregman**, **Christopher W Davis**, **JungHwan Park** and **Arunima Ray*** (`aruray@mpim-bonn.mpg.de`), Vivatsgasse 7, 53111 Bonn, Germany. *Isotopy and equivalence of knots in 3-manifolds.*
We show that in a prime, closed, oriented 3-manifold M, equivalent knots are isotopic if and only if the orientation preserving mapping class group is trivial. In the case of irreducible, closed, oriented 3-manifolds we show the more general fact that every orientation preserving homeomorphism which preserves free homotopy classes of loops is isotopic to the identity. In the case of $S^1 \times S^2$, we give infinitely many examples of knots whose isotopy classes are changed by the Gluck twist. (Received September 14, 2020)

1163-57-1003   **Radmila Sazdanovic*** (`rsazdan@ncsu.edu`), Department of Mathematics NC State University, PO Box 8205, Raleigh, NC 27695, and **Mikhail Khovanov**. *Bilinear pairings on two-dimensional cobordisms and generalizations of the Deligne category.*
The Deligne category of symmetric groups is the additive Karoubi closure of the partition category. It is semisimple for generic values of the parameter t while producing categories of representations of the symmetric group when modded out by the ideal of negligible morphisms when t is a non-negative integer. The partition category may be interpreted, following Comes, via a particular linearization of the category of two-dimensional oriented cobordisms. The Deligne category and its semisimple quotients admit similar interpretations. This viewpoint coupled to the universal construction of two-dimensional topological theories leads to multi-parameter

monoidal generalizations of the partition and the Deligne categories, one for each rational function in one variable. (Received September 14, 2020)

1163-57-1008   **Radmila Sazdanovic*** (`rsazdan@ncsu.edu`), Department of Mathematics NC State University, PO Box 8205, Raleigh, NC 27695, and **Carmen Caprau**, **Nicolle Gonzalez**, **Christine Ruey Shan Lee**, **Adam M. Lowrance** and **Melissa Zhang**. *On Khovanov Homology and Related Invariants.*

Rich algebraic structure of Khovanov-type theories also carries a lot of topological information about knots. In this talk we focus on two new applications. We extend Levine and Zemke's ribbon concordance obstruction and produce bounds on the alternation number and Turaev genus of a knot.   (Received September 14, 2020)

1163-57-1040   **Thomas Kindred*** (`thomas.kindred@unl.edu`), 2844 Washington St., Lincoln, NE 68502. *Symmetries of alternating link exteriors, spatial graphs, and branched surfaces.* Preliminary report.

Menasco-Thistlethwaite observed, in the 1991 announcement of their proof of the flyping theorem, that "any element of the mapping class group of the pair $(S^3, [\text{an alternating link } L])$ must be "obvious", in the sense that it must arise from flypes and symmetries of the planar graph underlying the diagram" of L. I will unpack some of the geometric content of this observation by relating the symmetries of alternating link exteriors to symmetries of certain spatial graphs and branched surfaces.   (Received September 14, 2020)

1163-57-1065   **Charles D Frohman** and **Joanna Kania-Bartoszynska***, jkaniaba@nsf.gov, and **Thang Q Le**. *Geometric Kauffman bracket invariant of closed 3-manifolds.*

Kauffman bracket skein module of a 3-manifold M is defined by taking the linear combinations of isotopy classes of framed links in M with complex coefficients, and dividing by a submodule spanned by the Kauffman bracket skein relations. We define a reduced Kauffman bracket skein module that depends on a choice of an irreducible representation of the fundamental group of M into SL(2,C). We show that if the 3-manifold is closed then the reduced Kauffman bracket skein module is isomorphic to complex numbers. This can be interpreted as extending the Kauffman bracket invariant of links in a 3-sphere to a geometric invariant in an arbitrary closed, oriented 3-manifold.   (Received September 14, 2020)

1163-57-1071   **Colin C. Olson*** (`colin.olson@nrl.navy.mil`), 4555 Overlook Ave. SW, Washington, DC 20375. *Spatial-Spectral Data and Unsupervised Autoencoder Training for Hyperspectral Anomaly Detection.*

We discuss a new method for training an autoencoder (AE) as an unsupervised hyperspectral anomaly detector that leverages a percentile loss that can be defined to reliably construct an accurate background manifold model while limiting the erroneous inclusion of anomalous data. We describe the sampling theoretic considerations that drive loss function design and additionally improve detection performance and reliability by exploiting a novel cumulative detection score that incorporates statistics calculated from the ensemble of AE models generated over the history of the training process. We have previously considered only the spectral information present in a hyperspectral pixel at a given spatial location, that is, each datum as a one-dimensional spectral vector. Here we show improvements in detection performance by extending each datum to include spatial information present in a local neighborhood as long as the loss function is properly modified. We show improved detection performance on two data sets relative to the original algorithm, a graph-learning method, and a baseline algorithm that looks for deviations relative to an assumed statistical model.   (Received September 15, 2020)

1163-57-1094   **Louis H Kauffman*** (`kauffman@uic.edu`), 5530 South Shore Drive, Apt 7C, Chicago, IL 60637-1946. *States in Formal Knot Theory.* Preliminary report.

This talk will review the state of the art for the presenter in working with the formal knot theory states (single cycle smoothing states) of link diagrams. The states were originally introduced by Kauffman to support a state summation model for the Alexander-Conway polynomial. They can be used to support the Jones polynomial and for generalizations to knotoids. They figure in models for the Heegard-Floer knot homology. We will discuss the structure of these states and conjectures about them.   (Received September 14, 2020)

1163-57-1096   **Helen Wong*** (`hwong@cmc.edu`), Claremont, CA 91711, and **Han-Bom Moon** (`hmoon8@fordham.edu`), New York, NY. *Roger-Yang skein algebra and decorated Teichmuller space.*

In the case of a closed surface, there is a rich body of work describing how the Kauffman bracket skein algebra can be regarded as a quantization of the Teichmuller space. We will show how analogously for punctured surfaces,

there is a new skein algebra defined by Roger and Yang that is the quantization of Penner's decorated Teichmuller space.   (Received September 14, 2020)

1163-57-1123   **Keiko Kawamuro*** (`keiko-kawamuro@uiowa.edu`), 14 MacLean Hall, Iowa City, IA 52242. *Flype, pseudo-Anosov braids and the fractional Dehn twist coefficient.*
I will describe behavior of the fractional Dehn twist coefficient of pseudo-Anosov braid under flype moves. This is joint work with Elaina Aceves.   (Received September 14, 2020)

1163-57-1136   **Sherilyn Tamagawa*** (`shtamagawa@davidson.edu`). *The Cobweb Interpretation of the B_2 Spider.* Preliminary report.
Kuperberg spiders are a diagrammatic interpretation of representations of quantum Lie algebras. These spiders have been described for several classes of Lie algebras. Recently, work has been done to reinterpret some of these spiders as simpler diagrams with fewer relations, known as cobwebs. Cobwebs are formed using the combinatorial information from the root systems of some Lie algebras. They are also diagrammatic interpretations of representations of certain quantum Lie groups, but without trivalent vertices. We extend these results to a new class of spider. We established a well-defined, non-trivial map from the original Kuperberg spider to the newly de fined cobwebs.   (Received September 14, 2020)

1163-57-1139   **Chandrika Sadanand***, Sadanand@illinois.edu. *Heegaard splittings and square complexes.* Preliminary report.
A construction of Stallings encodes the information of a Heegaard splitting as a continuous map between 2-complexes. We investigate this construction from a more geometric perspective and find that irreducible Heegaard splittings can be encoded as square complexes with certain properties.   (Received September 14, 2020)

1163-57-1159   **Dimitrios Giannakis*** (`dimitris@cims.nyu.edu`), 251 Mercer St, New York, NY 10012, and **Tyrus Berry** (`tberry@gmu.edu`), 4400 University Dr, Fairfax, VA 22030. *Data-driven approximation of vector fields and differential forms with the Spectral Exterior Calculus.*
We present a data-driven framework for exterior calculus on manifolds. This framework is based on a representations of vector fields, differential forms, and operators acting on these objects in frames (overcomplete bases) for L^2 and higher-order Sobolev spaces built entirely from the eigenvalues and eigenfunctions of the Laplacian of functions. Using this approach, we represent vector fields either as linear combinations of frame elements, or as operators on functions via matrices. In addition, we construct a Galerkin approximation scheme for the eigenvalue problem for the Laplace-de-Rham operator on 1-forms, and establish its spectral convergence. We present applications of this scheme to a variety of examples involving data sampled on smooth manifolds and the Lorenz 63 fractal attractor.   (Received September 14, 2020)

1163-57-1207   **Carmen L Caprau*** (`ccaprau@csufresno.edu`), **Gabriel Coloma** and **Marguerite Davis**. *The L-move and Markov theorems for trivalent braids.*
The L-move for classical braids, introduced by Sofia Lambropoulou, extends naturally to trivalent braids. We follow the L-move approach to the Markov Theorem, to prove a one-move Markov-type theorem for trivalent braids. We also reformulate this L-Move Markov theorem and prove a more algebraic Markov-type theorem for trivalent braids and knotted trivalent graphs.   (Received September 15, 2020)

1163-57-1216   **Agnese Barbensi*** (`barbensi@maths.ox.ac.uk`), University of Oxford, Radcliffe Observatory, Andrew Wiles Building, Oxford, OX2 6GG, United Kingdom. *Network of Grid Diagrams to Study the Knots Space and Applications.* Preliminary report.
The configuration space of knots can be modelled as an infinite network whose vertices are knot diagrams and edges are local moves on diagrams. This setting provides a topological framework to investigate on statistical properties of knotted biopolymers.   (Received September 15, 2020)

1163-57-1224   **Biji Wong*** (`biji.wong@cirget.ca`), Max Planck Institute for Mathematics, Vivatsgasse 7, 53111 Bonn, Germany. *Twisted Mazur pattern satellite knots and bordered Floer theory.*
Bordered Floer theory has proven quite useful for studying satellite knots. In this talk, I will discuss how to use gradings in bordered Floer theory to study the Floer thickness, 3-genus, and fiberedness of twisted Mazur pattern satellite knots. Time permitting, I will also show that for certain families of companion knots, Mazur pattern satellite knots satisfy the Cosmetic Surgery Conjecture. This is joint work with Ina Petkova.   (Received September 15, 2020)

1163-57-1271   **Dylan P Thurston***, Rawles Hall, 831 E Third St, Bloomington, IN 47405-7106. *Hopf algebras and framed sutured 3-manifolds.* Preliminary report.

We strengthen the relationship between Hopf algebras and 3-manifolds (following Greg Kuperberg and Rohit Thomas), by giving a correspondence between, on the algebraic side, open Hopf algebra expressions (with some free variables), and, on the geometric side, sutured 3-manifolds with markings. Among other benefits, this gives a purely geometric interpretation of the Drinfel'd Double construction.

This is joint work-in-progress with Roland van der Veen.   (Received September 15, 2020)

1163-57-1330   **Jim Hoste*** (jhoste@pitzer.edu), 1011 Fuller Drive, Claremont, CA 91711, and **Patrick D. Shanahan** and **Cornelia A. Van Cott**. *Crosscap number of 2-bridge knots and the partial order on prime knots.*

Suppose K and J are distinct knots in the 3-sphere and that $K > J$, meaning that there is an epimorphism of the knot group of K onto the knot group of J. Then, $\gamma(K) \geq 3\gamma(J) - 4$, where $\gamma(K)$ is the *crosscap number*, or *non-orientable genus* of the knot K. Furthermore, the inequality is sharp.   (Received September 15, 2020)

1163-57-1377   **Katherine Vance***, katherine.vance@simpson.edu. *Random graph grid diagrams.* Preliminary report.

Many different models of random knotting have been used to study a wide variety of properties of knots and links. Random grid diagrams are one such model. In 2017, Harvey and O'Donnol defined graph grid diagrams, a generalization of knot and link grid diagrams, which represent transverse spatial graphs. We use computational methods to generate samples of random graph grid diagrams and investigate properties of the spatial graphs they represent.   (Received September 15, 2020)

1163-57-1388   **Kyle Hayden*** (hayden@math.columbia.edu). *Corks, complex curves, and 4-manifolds.*

Corks are certain contractible 4-manifolds that play an important role in the study of exotic phenomena in dimension four. In this talk, I will describe a couple of applications of corks to the study of surfaces in 4-manifolds, including the construction of smoothly (indeed, holomorphically) embedded disks in the 4-ball that are isotopic through ambient homeomorphisms but not through ambient diffeomorphisms.   (Received September 15, 2020)

1163-57-1506   **Jiayi Chen**, **Jinghan Gao**, **Dayln Gillentine**, **Laetitia Huang** and **Sophia Marx*** (smarx@smith.edu). *Extending knot colorings over surfaces.* Preliminary report.

A knot in the 3-sphere is slice if it bounds a disk in the 4-ball. Suppose K is p-colorable and slice, with p odd and square-free. Given a slice disk D for K, some p-coloring of K must extend over D. If K is not slice, one can instead ask when some p-coloring of K extends over a surface of genus g in the 4-ball, where g is the smooth 4-genus of K. We investigate this question from a combinatorial and diagrammatic perspective, and give several examples.   (Received September 15, 2020)

1163-57-1593   **Kyle A Miller*** (kmill@berkeley.edu). *A 2D TQFT approach to topological graph polynomials and graphs in thickened surfaces.* Preliminary report.

Many invariants of graphs or ribbon graphs — such as the chromatic polynomial — satisfy a deletion-contraction formula and, perhaps after a normalization factor, are multiplicative under disjoint unions and wedge sums. Such invariants are specializations of the Tutte–Whitney polynomial or, more generally, the Bollobás–Riordan polynomial. I will describe a 2D TQFT approach to such invariants and explain their classification in terms of symmetric Frobenius algebras. From this perspective, we can obtain a short graphical proof of the result of Dasbach, et al., that the Jones polynomial is the Bollobás–Riordan polynomial of a ribbon graph associated to a Turaev state surface of a link diagram.

The Yamada polynomial is a polynomial invariant of spatial graphs that also satisfies deletion-contraction, and as an application, I will discuss multivariable generalizations to ribbon graphs in thickened surfaces and virtual ribbon graphs.   (Received September 15, 2020)

1163-57-1614   **Andrew Ducharme** and **Emily Peters*** (epeters3@luc.edu). *Combinatorial Random Knots.*

We explore free knot diagrams, which are projections of knots into the plane which don't record over/under data at crossings. We consider the combinatorial question of which free knot diagrams give which knots and with what probability. Every free knot diagram is proven to produce trefoil knots, and certain simple families of free knots are completely worked out. We make some conjectures (supported by computer-generated data) about bounds on the probability of a knot arising from a fixed free diagram being the unknot, trefoil, or figure eight knot.   (Received September 15, 2020)

1163-57-1667   **Siddhi Krishna***, krishna@math.gatech.edu. *Taut foliations and Dehn surgeries along positive braid knots.* Preliminary report.

The L-Space Conjecture is taking the low-dimensional topology community by storm. It aims to relate seemingly distinct Floer homological, algebraic, and geometric properties of a closed 3-manifold Y. In particular, it predicts a 3-manifold Y isn't "simple" from the perspective of Heegaard-Floer homology if and only if Y admits a taut foliation. The reverse implication was proved by Ozsváth and Szabó. In this talk, we'll present some new results supporting the forward implication. Namely, we'll build taut foliations in manifolds obtained by Dehn surgery along positive braid closures. Our construction is concrete and combinatorial in nature. No background in Floer homology or foliation theories will be assumed.   (Received September 16, 2020)

58 ▶ *Global analysis, analysis on manifolds*

1163-58-907   **Maria Trnkova*** (mtrnkova@math.ucdavis.edu), One Shields Ave, Davis, CA 95616. *Geodesics of hyperbolic manifolds.*

A computer program "SnapPea" and its descendant "SnapPy" compute many invariants of a hyperbolic 3-manifold M. Some of their results can be rigorous but some not. In this talk we will discuss computation of geodesics length and will mention a number of applications when it is crucial to know the precise length spectrum up to some cut off.

C.Hodgson and J.Weeks introduced a practical length spectrum algorithm implemented in SnapPea. The algorithm uses a tiling of the universal cover by translations of a Dirichlet domain of M by elements of a fundamental group. In theory the algorithm is rigorous but in practice its output does not guaranty the correct result. One of the obstacles is the requirement to use the exact data for the Dirichlet domain which is available only in some special cases. We show that under some assumptions on M an approximate Dirichlet domain can work equally well as the exact Dirichlet domain. Our result explains the empirical fact that the program "SnapPea" works surprisingly well despite it does not use exact data.   (Received September 14, 2020)

1163-58-1049   **Sherry Gong*** (ggyiran@gmail.com) and **Marco Marengon**. *Non-orientable cobordisms and torsion in Floer homology.* Preliminary report.

In this paper, we use unoriented versions of instanton and knot Floer homology to give bounds on the number of critical points appearing in not-necessarily orientable cobordisms, extending results of a recent paper by Juhasz, Miller, and Zemke concerning orientable cobordisms. The versions of instanton and knot Floer homology that induce maps for non-orientable cobordisms require a decoration–much of the subtlety in our arguments lies in choosing the necessary decorations. We introduce unoriented versions of the band unknotting number and the refined cobordism distance and use our results to give bounds on these based on the torsion orders of the Floer homologies.   (Received September 14, 2020)

1163-58-1452   **John D Ross***, SU Box 7371, 1001 E. University Avenue, Georgetown, TX 78626. *On curves that generate symmetric λ-hypersurfaces.*

This talk showcases some recent work on discovering hypersurfaces $\Sigma^n \subseteq \mathbb{R}^{n+1}$ that satisfy a mean curvature condition $H = \frac{1}{2}\vec{x} \cdot \vec{n} + \lambda$. Such surfaces, named λ-hypersurfaces, are natural generalizations of mean curvature flow self-shrinkers and act as critical points for the Gaussian surface area. It turns out that rotationally symmetric λ-hypersurfaces can be generated by a planar curve that satisfies a natural system of ODEs. Thus, new λ-hypersurfaces can be discovered by understanding solutions to the ODE system. We examine the system and highlight some strategies towards finding solutions.   (Received September 15, 2020)

60 ▶ *Probability theory and stochastic processes*

1163-60-232   **Aubain Hilaire Nzokem*** (hilaire77@gmail.com) and **Neal Madras** (madras@yorku.ca). *SIS Epidemic Model : Birth-and-Death Markov Chain Approach.*

We are interested in describing the infected size in the SIS Epidemic model using Birth-Death Markov processes. The Susceptible-Infected-Susceptible (SIS) model is defined within a population of constant size (M); the size is maintained by replacing each death with a new born healthy individual. The life span of each individual in the susceptible population (S) and the infected population (I) is modelling by an exponential distribution with parameter α; and the disease spreads within the population with the transmission rate (β). As methodology, we use both numerical and analytical approaches; The analysis relies on the limiting stationary distribution (π_M) of Markov chain developed by Taylor and Karlin (1975) for Birth and Death processes.The numerical method

uses sample path simulations to show the relationship between infected size and Reproduction number ($R = \frac{\beta}{\alpha}$); some stable statistical characteristics of the infected size will be estimated. The analytical method uses the poisson distribution proprieties, the Taylor series technique and the local central limit theorem to show that the infected size follows a normal distribution with mean $\mu = (1 - \frac{1}{R})M$ and variance $\sigma^2 = \frac{M}{R}$ when M becomes big.   (Received August 29, 2020)

1163-60-317   **Konstantinos Spiliopoulos*** (`kspiliop@bu.edu`). *Mean field analysis of neural networks: typical events and fluctuations.*
Machine learning has revolutionized fields such as image, text, and speech recognition. Important real-world applications are driven by neural networks, e.g. in finance, engineering, robotics, and medicine. Despite their immense success in practice, there is limited mathematical understanding of neural networks. Our work shows how neural networks can be studied via stochastic analysis, and develops approaches for addressing some of the technical challenges which arise. We analyze neural networks in the asymptotic regime of simultaneously (A) large network sizes and (B) large numbers of stochastic gradient descent training iterations. We establish the limiting behavior of the neural network and we show that, under suitable assumptions on the activation functions and the behavior for large times, the limit neural network recovers a global minimum (with zero loss for the objective function). We rigorously prove a central limit theorem, which describes the neural network's fluctuations around its mean-field limit. The fluctuations have a Gaussian distribution and satisfy a stochastic partial differential equation. We demonstrate the theoretical results in the study of the evolution of parameters in the well known MNIST and CIFAR10 data sets. This is joint work with Justin Sirignano,   (Received September 02, 2020)

1163-60-328   **Oleksii Mostovyi*** (`oleksii.mostovyi@uconn.edu`), University of Connecticut, Department of Mathematics, U1009, 341 Mansfield Road, Storrs, CT 06269-1009, and **Mihai Sirbu** and **Thaleia Zariphopoulou**. *On the analyticity of the value function in optimal investment.*
We study the analyticity of the value function in optimal investment with expected utility from terminal wealth. We identify both a class of utilities and a class of semi-martingale models for which we establish analyticity. Specifically, these utilities have completely monotonic inverse marginals, while the market models have a maximal element in the sense of infinite-order stochastic dominance. We construct two counterexamples, themselves of independent interest, which show that analyticity fails if either the utility or the market model does not belong to the respective special class. We also provide explicit formulas for the derivatives, of all orders, of the value functions as well as their optimizers. This talk is based on the joint work with Mihai Sîrbu and Thaleia Zariphopoulou.   (Received September 02, 2020)

1163-60-364   **Mohammad Rafiqul Islam** and **Nguyet Nguyen*** (`ntnguyen01@ysu.edu`), 1 University Plaza, Youngstown, OH 44555. *Comparison of Financial Models for Stock Price Prediction.*
Time series analysis of daily stock data and building predictive models are complicated. In this talk, we will present a comparative study for stock price prediction using three different methods, namely autoregressive integrated moving average, artificial neural network, and stochastic process-geometric Brownian motion. Each of the methods is used to build predictive models using historical stock data collected from Yahoo Finance. Finally, output from each of the models is compared to the actual stock price. Empirical results show that the conventional statistical model and the stochastic model provide better approximation for next-day stock price prediction compared to the neural network model.   (Received September 03, 2020)

1163-60-436   **Xiangchan Zhu***, Chinese Academy of science, Beijing, Peoples Rep of China. *Singular HJB Equations with Applications to KPZ on the real line.*
This talk is devoted to studying the Hamilton-Jacobi-Bellman equations with distribution-valued coefficients, which is not well-de ned in the classical sense and shall be understood by using paracontrolled distribution method introduced in [GIP15]. By a new characterization of weighted Holder space and Zvonkin's transformation we prove some new a priori estimates, and therefore, establish the global well-posedness for singular HJB equations. As an application, the global well-posedness for KPZ equations on the real line in polynomial weighted Holder spaces is obtained without using Cole-Hopf's transformation.   (Received September 06, 2020)

1163-60-624   **Yujia Ding*** (`yujia.ding@cgu.edu`) and **John Angus**. *On the ratio of current age to total life for null recurrent renewal processes.*
A number of open problems associated with determining the limit distribution of the ratio of current age to total life for a null recurrent renewal process (i.e. where inter-arrival times have infinite mean) are solved. In particular, when the survival function for the inter-arrival times satisfies $\bar{F}(t) \sim t^{-\alpha}L(t)$ as $t \to \infty$ with L slowly

varying and $0 \leq \alpha \leq 1$, we prove that the limit distribution corresponds to that of $U^{1/\alpha}$ where U is uniformly distributed on $(0, 1)$, with the limit distribution taken to be degenerate at 0 when $\alpha = 0$. By using direct methods instead of appealing to strong renewal theorems, we are able to prove this result without regard to whether the inter-arrival time distribution is latticed or not, and without extraneous constraints on the renewal function.   (Received September 10, 2020)

1163-60-626   **Sarah Miracle**, **Amanda Pascoe Streib*** (`ampasco@super.org`) and **Noah Streib**.
                    Iterated Decomposition of Markov Chains.
Markov chains are useful in a variety of areas of scientific computing, where they can provide a way to sample from complex probability distributions. The decomposition method allows one to analyze a Markov chain M in terms of simpler constituent Markov chains. This idea can be particularly useful when M can be decomposed iteratively into simpler and simpler pieces. However, for certain applications, the overhead from the classical decomposition theorem prohibits applying it iteratively. In this work, we develop a new decomposition theorem. We apply our new theorem iteratively and obtain nearly optimal bounds on the spectral gap of a widely studied Markov chain over biased permutations, providing a dramatic improvement on the analysis of this chain.   (Received September 10, 2020)

1163-60-628   **Marek Kimmel***, kimmel@rice.edu. *Drivers and passengers: Analysis and simulations of
                    modes of competition of cancer cells.*
In a series of publications, McFarland and co-authors introduced the tug-of-war model of evolution of cancer cell populations. In the present paper, we put the tug-of-war in the context of multitype Moran model and multitype branching process, which serve as mathematical framework for two different types of selection in cell populations. Moran model philosophy can be viewed as "competitive replacement",by which individual cells face each other and inhibit each other's right to be replaced by a direct descendant. Branching process represents the so-called "crowding out" in which a faster-growing clone makes the slower-growing one rare to the extent of nonexistence. We begin with mathematical definitions of the Moran model and branching process versions of tug-of-war. Then we present simulation results, which demonstrate the differences between the long-term behavior of the two versions. We also use some typical population genetics non-neutrality tests to see how the effects of tug-of-war competition are reflected in testing. Finally, in the Discussion section, we discuss the tug-of-war process behavior vis-'a-vis recent simulation and experimental studies of long-term cell population growth.   (Received September 10, 2020)

1163-60-923   **Milan Stehlik*** (`mlnstehlik@gmail.com`), Department of Applied Statistics, Johannes
                    Kepler University Linz, Altenberger Straße 69, Linz, – A4040. *Modelling and prediction of
                    COVID-19 outbreaks.*
We formulate the ill-posedness of inverse problems of estimation and prediction for COVID-19 outbreaks from statistical and mathematical perspectives. These leave us with a plenty of possible statistical regularizations, thus generating plethora of sub-problems. We can mention the as examples stability and sensitivity of peak estimation, starting point of exponential growth curve, or estimation of parameters of SIR model. In a specific country, one can define several social groups which can contribute in a heterogeneous way to whole country epidemiological curves. Moreover, each parameter has its own specific sensitivity, and naturally, the more sensitive parameter deserves a special attention. E.g. in SIR (Susceptible-Infected-Removed) model, parameter β is more sensitive than parameter γ. In simple exponential epidemic growth model, b parameter is more sensitive than a parameter. We provide sensitivity and illustrate it on the country specific data. We also discuss on statistical quality of COVID-19 incidence prediction, where we justify an exponential curve considering the microbial growth in ideal conditions for epidemic. We model number of infected in Iowa State, USA, Hubei Province in China, New York State, USA.   (Received September 14, 2020)

1163-60-941   **Masilamani Sambandham*** (`msambandham@yahoo.com`), 8998 Betony Wood Trail,
                    Jonesboro, GA 30236-5268. *A short survey of recent results on the zeros of random
                    polynomials.* Preliminary report.
There are various random polynomials including algebraic polynomials, binomial polynomials, orthogonal polynomials, trigonometric polynomials, Weyl polynomials, hyperbolic polynomials, etc. This presentation will contain a survey of selected results in random polynomials on the real zeros and distribution of zeros of random polynomials. A few application also will be presented.   (Received September 14, 2020)

1163-60-1053   **Joseph Paul Squillace*** (josephps@uri.edu), 138 Tyler Hall, 9 Greenhouse Road, University of Rhode Island, Kingston, RI 02881. *On the dependence of the component counting process of a uniform random variable.*

We are concerned with proving the existence of joint distributions of discrete random variables M and N subject to constraints of the form $\mathbb{P}(M = i, N = j) = 0$. In particular, the variable M has an infinite range and consists of an independent component counting process $(Z_k)_k$, and the other variable N is uniformly distributed and consists of a dependent component counting process $(C_k)_k$. The constraints placed on the joint distributions of M and N will require, for all but one j in the range of N, $\mathbb{P}(M = i, N = j) = 0$ for infinitely many values of i in the range of M, and the corresponding values of i depend on j. The constraints imposed on our joint distribution are $\mathbb{P}(M = i, N = j) = 0$ whenever $\sum_k (C_k - Z_k)^+ > 1$ for any realization of $M = i$ and $N = j$. To prove the existence of such joint distributions, we introduce the notion of pivot mass which is then combined with a theorem proved by Strassen on the existence of specified joint distributions with known marginals. We are providing a partial answer to the question "how much dependence is there in the process $(C_i(n))_{i \leq n}$?" (Received September 14, 2020)

1163-60-1105   **Mihai Stoiciu*** (mstoiciu@williams.edu), Department of Mathematics and Statistics, 33 Stetson Ct, Williams College, Williamstown, MA 01267. *Convergent Point Processes and Their Rates of Convergence.* Preliminary report.

We investigate various families of random point processes and their rates of convergence. Motivated by concrete examples of eigenvalue point processes associated to ensembles of random matrices, we construct general families of random point processes that converge to the Poisson point process and to the picket fence point process. We study the rate of convergence of these point processes and apply these methods to the Circular Beta Ensemble. (Received September 14, 2020)

1163-60-1107   **Chenmin Sun**, **Nikolay Tzvetkov** and **Weijun Xu*** (weijunxu@bicmr.pku.edu.cn), Peking University, 5 Yi He Yuan, Beijing, Beijing 100871, Peoples Rep of China. *Higher order nonlinear approximations to 2D cubic fractional wave equation and its Gibbs measure.*

We consider a class of higher order nonlinear approximations to both the 2D cubic fractional wave equation with random initial data and its associated Gibbs measure. These considerations are motivated from the weak universality questions in the weakly nonlinear regime in singular parabolic stochastic PDEs. Similar to the parabolic case, the coupling constant of the limiting equation and the Gibbs measure depend on the cutoff function in the approximation. Another interesting feature is that a strict positivity condition of the approximating potential is needed in order for the Gibbs measures to converge. This condition (for the approximation nonlinearity) is stronger than that for the global convergence for the corresponding wave dynamics, and we also show that it is almost necessary. Joint work with Chenmin Sun and Nikolay Tzvetkov.   (Received September 14, 2020)

1163-60-1146   **Gopinath Panda*** (gopinath.panda@gmail.com). *Decision making under uncertainty in G-queues with server vacations.* Preliminary report.

We consider a Markovian queueing system with heterogeneous customers and server vacations. There are two classes of customers, namely, positive customers and negative customers. Positive customers are ordinary customers that require service whereas negative customers do not. Upon arrival negative customers destabilize the system by breaking the server and killing the positive customer currently under service. Once the server is broken, it is sent to repair immediately and resumes service after repaired. The server operates under two different service rates: faster service (busy period) and slower service (working vacation period). The positive customers are uncertain about the number of customers in the system and the state of server. They are charged a fee for their waiting and are rewarded after service completion. Each positive customer decides whether to join the queue or balk based on the above reward-cost structure. The decision of a customer is affected by the decision made by other customers and each one tries to maximize his/her benefit out of the system. We analyze the system under steady-state and obtained the Nash equilibrium joining/balking strategy of customers. We also solved the selfish optimization problem and social optimization problem.   (Received September 14, 2020)

1163-60-1209   **Debasmita Banerjee*** (debasmitabanerjee12@gmail.com), **Bapi Dutta**, **Debashree Guha** and **Luis Martínez**. *Implementation of stochastic multiobjective acceptability analysis in combined MCHP-QUALIFLEX method to handle multi-criteria decision-making problems.*

QUALIFLEX is a convenient outranking technique to handle multi-criteria decision-making problems due to its less complexity and more user-friendly nature, whereas the multiple criteria hierarchy process (MCHP) allows decision-makers to deal with the hierarchical structure of criteria set where decision-makers can even estimate

results for a particular sub-criterion at some intermediate level of the hierarchy. In this talk, we discuss the fusion of the MCHP and the QUALIFLEX methodology with special emphasis on modeling interaction among the set of criteria using the concept of bipolar Choquet integral. To give the decision-makers more freedom for expressing their cognition, the q-rung orthopair fuzzy (q-ROF) environment is adopted to expresses the criteria measurement. In addition, this study aims to establish a new framework by implementing stochastic multiobjective acceptability analysis (SMAA) in the proposed extended QUALIFLEX method to take into account a variety of parameters compatible with the descriptive information regarding the relative importance and interaction of different criteria provided by the decision-maker. At last, a numerical example based on the supplier selection problem is presented to inspect the feasibility of the proposed methodology. (Received September 15, 2020)

1163-60-1218   **Isabelle Kemajou-Brown*** (`elisabeth.brown@morgan.edu`), 1700 E Cold Spring Ln, Baltimore, MD 21251. *Some results of a partially observed risk sensitive optimal control.* Preliminary report.

In this talk, we present an investigation of computational result of a partially observed risk sensitive optimal control portfolio choice problem under Markovian regime switching. This is a result used to solve a fully observed risk sensitive Markov regime switching optimal portfolio choice problem. (Received September 15, 2020)

1163-60-1233   **Andrew Papanicolaou*** (`apapani@ncsu.edu`), apapani@ncsu.edu. *Trade Signals In VIX Futures.*

VIX curves are used as input data for constructing trade signals. We assume that VIX futures curves come from a stochastic model, which provides us with a distribution of returns from trading. We use historical VIX data to estimate the model's parameters, and then we use a deep neural network to estimate the expected value of a reward function. We train this deep neural network with simulation-data generated from the VIX model. Training with simulation implies that historical data can only impact learning through the VIX model's parameters, yet even in this highly specified setting we can still observe trade signals that perform well when tested on out-of-sample data. Similar out-of-sample performance from deep-learning predictors have been observed in problems outside of finance, and is one of the reasons why machine learning and modern non-parametric statistics have recently gained so much attention. The contribution of this paper is a convincing application of deep learning in a financial setting and the demonstration of an improvement in trade signals when deep learning is applied. (Received September 15, 2020)

1163-60-1238   **Michal Branicki*** (`m.branicki@ed.ac.uk`), Department of Mathematics, University of Edinburgh, Edinburgh, United Kingdom, and **Animikh Biswas** (`abiswas@umbc.edu`), Department of Mathematics & Statistics, University of Maryland Baltimore County, Baltimore, MD. *Data assimilation for dissipative PDEs based on approximate Gaussian filters and sparse time-varying sets of nodal observations.* Preliminary report.

Key challenges in data assimilation for PDE-driven dynamics stem from model error in the approximate finite-dimensional forward dynamics, as well as sparse space-time observations. We consider a prototypical time-sequential Bayesian technique, 3DVAR, known to be accurate for filtering dissipative systems with a suitably inflated 'background' covariance and spectral observations when enough low frequency modes are observed independently. We derive rigorous criteria for the accuracy of 3DVAR estimates from spatially sparse and noisy observations which inevitably alias the dynamics of the spectral modes; moreover, we consider situations where the observation operator accounts for evolution of the set of observation locations and it has a time-dependent rank. (Received September 15, 2020)

1163-60-1319   **William A Massey*** (`wmassey@princeton.edu`), ORFE Department, Sherrerd Hall, Princeton University, Princeton, NJ 08544. *Bessel Functions, Cauchy Problems, and Stochastic Processes.*

Many special functions and partial differential equations of mathematical physics can be reinterpreted using probability through the use of fundamental stochastic processes. As an example, we show that modified Bessel functions arise from the difference of two independent Poisson processes. Moreover, we can also solve the Cauchy problem for a parabolic heat equation the same way that we would solve the same for a hyperbolic reduced wave equation. We do this by averaging over the random "characteristic curves" of Brownian motion.

This talk is dedicated to the life and mathematics of Professor James Ashley Donaldson (1941-2019). He was Chair of the Mathematics Department at Howard University and later Dean of the College of Arts and Sciences. As chair, he oversaw the development of the first PhD degree program in mathematics at a historically Black college or university. He was also a founder of the National Association of Mathematicians and an inaugural

speaker at the first Conference for African American Researchers in the Mathematical Sciences.   (Received September 15, 2020)

1163-60-1355   **Tomoyuki Ichiba***, South Hall, University of California, Santa Barbara, CA 93106. *Filtering degenerate rank-based particle systems.*
In stochastic portfolio theory, the rank-based market models describe the long-term stability of ranked market weights. Each stock can be seen as a particle in the particle systems of the whole market. We study stochastic filtering of systems of interacting particles, in which drifts and variances are assigned by rank. The filtered systems are degenerate: the variances corresponding to one or two ranks can vanish, and so the corresponding ranked motions become ballistic rather than diffusive. We study the system of filtering equations, in particular, when the number of stocks is large. The stability properties for the resulting processes of gaps between successive ranks are also studied.   (Received September 15, 2020)

1163-60-1393   **Eric Truslow*** (`eric.truslow@ll.mit.edu`), MIT-Lincoln Laboratory, 244 Wood St., Lexington, MA 02421, and **Dimitris Manolakis** (`dmanolakis@ll.mit.edu`), MIT-Lincoln Laboratory, 244 Wood St., Lexington, MA 02421. *Modeling Patterns of Life Using Count Time Series Models.*
Recent advances in image processing and object recognition have made it possible to rapidly extract objects of interest from imagery. The counts of objects in an area can be used to understand the pattern of life within an area and between disparate areas. Tracking object counts over time leads to count time series of the area and of the different object types. Two challenges in modeling this type of data are: irregular sampling in time due to collection timing, and differing sampling times among various areas. Our main goals in this work are to adequately model observed count time series as well as find relationships among different areas. Both goals require handling the irregular time sampling of the data and the mismatch in sampling times among different areas. First, we examine modifications to discrete-valued models that incorporate non-uniform time steps by treating the time steps as a random variable. Then, we use a coupled model to simulate two sets of irregular, unsynchronized measurements and explore the conditions on the sampling pattern needed to determine if the patterns are related. Finally, we explore how these techniques could be used in a workflow for finding areas with related patterns of life in large datasets   (Received September 15, 2020)

1163-60-1438   **Barbara H Margolius***, Dept Mathematics, 1515 RT, 2121 Euclid Ave, 2121 Euclid Ave, Cleveland, OH 44115-2214. *Using generalized eigenvalues to analyze queueing models with Erlang arrivals and service and time-varying periodic transition rates.* Preliminary report.
We consider a stochastic process with Erlang arrivals and service. This is a quasi-birth-death process, that is, a stochastic process with a two-dimensional state space $X(t), J(t)$, where $X(t) in mathbbZ$ is the level of the process and $(J(t) in 1, 2, dots, K)$ is the phase. Transitions are only possible within the current level or to an adjacent level. We are interested in such processes in which transition rates vary periodically and where the process is ergodic. In this case, we can analyze the system by studying the generalized eigenvalues of the generating function for the unbounded process over a single time period. This generating function will be a Laurent series whose coefficients are matrices. The (i, j)th component of the coefficient of z^k represents the probability of a transition from phase i to phase j and a net change of k levels in a single time period. Those values of z such that the determinant of this generating function are zeros are the generalized eigenvalues of the system. The probability generating function for the queue-length process and other performance measures of the system can be determined in terms of this generating function for the unbounded process. The generalized eigenvalues provide information about system asymptotics.   (Received September 15, 2020)

1163-60-1484   **Serges Love Teutu Talla*** (`seteu1@morgan.edu`), 1700 East Cold Spring Lane, Baltimore, MD 21251, and **Isabelle Kemajou-Brown** (`elisabeth.brown@morgan.edu`), 1700 East Cold Spring Lane, Baltimore, MD 21251. *Computation of a risk-sensitive optimal portfolio problem with Regime switching.* Preliminary report.
In this work, we consider a risk-sensitive stochastic maximum principle problem in the financial market, motivated by a space-time Poisson process: Markov regime-switching. Applying this principle under some optimal conditions, we assume the wealth dynamic follows a stochastic differential equation, and use the Forward-Backward Stochastic Differential Equation (FBSDE) method to obtain the first and second adjoint variables, and to compute the Hamiltonian of our system. Using the Python programming language, we find some computational results to illustrate our approach.   (Received September 15, 2020)

1163-60-1507 **Linden Yuan*** (`lyuan12@umd.edu`) and **Wai-Tong Louis Fan**. *Spectral gaps of Markov chains on strings.* Preliminary report.

Let $\mathcal{A}$ ($\mathcal{A}$ for "alphabet") be a finite set of cardinality at least two, and let $\mathcal{S} = \bigcup_{M=0}^{\infty} \mathcal{A}^M$ be the finite-length sequences over $\mathcal{A}$. Motivated by preexisting models of DNA evolution, we study a certain family of continuous-time Markov chains with state space $\mathcal{S}$. Each of these Markov chains is ergodic, meaning there is a unique invariant distribution, call it Π, and the transition probabilities $P_t(x, \cdot) \to \Pi$ (converge to Π) as $t \to \infty$. Using tools from optimal transport, we derive an explicit formula for the "convergence rate" ε_1 of $P_t(x, \cdot) \to \Pi$. In addition, existing results imply that $\varepsilon_1 = \lambda_1$, where λ_1 is the spectral gap. This equality yields an information-theoretic interpretation of our result, since λ_1 is (essentially) known to govern a phase transition in a couple of inference problems. Finally, after introducing two real-world models of DNA evolution and discussing implications of our results for these models, we outline alternative models and alternative inference problems for future research. This work was done at the Mathematics REU at Indiana University Bloomington. (Received September 15, 2020)

1163-60-1559 **Padmanabhan Sundar*** (`psundar@lsu.edu`), Department of Mathematics, Lockett Hall, Louisiana State University, Baton Rouge, LA 70803. *The Boltzmann Process.* Preliminary report.

A stochastic differential equation (SDE) is constructed that corresponds to a suitably linearized Boltzmann equation. Under certain regularity conditions, its solvability is established. The law of the solution of the SDE is shown to coincide with the solution of the Boltzmann equation whenever the latter is unique. This is a joint work with Albeverio and Ruediger. (Received September 15, 2020)

1163-60-1613 **Aziz Issaka***, Department of Mathematics and Statistics, University of North Carolina at Charlotte, 9201 University City Blvd., Charlotte, NC 28223, and **Indranil SenGupta**. *Analysis of variance based financial instruments: swaps and price indices.*

In this presentation, various financial applications of a stochastic model are studied. Firstly, a number of aspects of the variance swap in connection to the Barndorff-Nielsen and Shephard model (BN-S) are studied. A partial integrodifferential equation that describes the dynamics of the arbitrage-free price of the variance swap is formulated. Secondly, under appropriate assumptions for the first four cumulants of the driving subordinator, a "Vecer-type theorem" is proved. The bounds of the arbitrage-free variance swap price are also found. Finally, a price-weighted index modulated by market variance is introduced. The large-basket limit dynamics of the price index and the "error term" are derived. Empirical data-driven numerical examples are provided in support of the proposed price index. (Received September 15, 2020)

1163-60-1653 **Scott A McKinley*** (`scott.mckinley@tulane.edu`), 6823 St Charles Ave, New Orleans, LA 70125. *Modeling and Inference for the Movement of Microparticles in Biological Fluids.*

Micro-scale agents in biological media – viruses, organelles, bacteria, and manufactured microparticles – display a wide variety of movement behaviors. An ongoing challenge is to first develop models that capture these varied behaviors and then to develop complementary statistical tools that can take a given path and determine which model mostly like produced the behavior. In this talk, we will look at recent progress that considers not only the underlying biophysical mechanisms, but also aspects of the observation process that bias and limit our inference goals. (Received September 16, 2020)

1163-60-1668 **Martin Larsson*** (`martinl@andrew.cmu.edu`), **Aaditya Ramdas**, **Johannes Ruf** and **Wouter Koolen**. *Martingale methods for sequential learning.*

In a sequential learning problem one wishes to draw inference from data that is gathered gradually through time. This is the typical situation in many applications, including finance. A sequential learning procedure is 'anytime-valid' if the decision to stop or continue an experiment can depend on anything that has been observed so far, without compromising statistical error guarantees. A recent approach to anytime-valid inference views a test statistic as a bet against the null hypothesis. These bets are constrained to be supermartingales - hence unprofitable - under the null, but designed to be profitable under the relevant alternative hypotheses. This perspective opens the door to tools from financial mathematics. In this talk I will discuss how notions such as supermartingale measures, fork-convexity, and the optional decomposition theorem shed new light on anytime-valid sequential learning. (This talk is based on joint work with Wouter Koolen (CWI), Aaditya Ramdas (CMU) and Johannes Ruf (LSE).) (Received September 16, 2020)

1163-60-1681   **Sebastian Jaimungal*** (sebastian.jaimungal@utoronto.ca), Toronto, Ontario M5S3G3, Canada, and **Arvind Shrivats** and **Dena Firoozi**. *A Mean-Field Game Approach to Equilibrium Pricing in Renewable Energy Certificate Markets.*

Renewable Energy Certificate markets are a market-based system designed to incentivize solar energy generation. A regulatory body imposes a lower bound on the energy each regulated firm must generate via solar means, providing them with a certificate for each MWh generated. Regulated firms minimize the cost imposed on them, by modulating their SREC generation and trading activities. As such, the market can be viewed as a large stochastic game with heterogeneous agents, where agents interact through the market price of the certificates. We study this stochastic game by solving the mean-field game (MFG) limit with heterogeneous agents. Our market participants optimize costs accounting for trading frictions, cost of generation, non-linear non-compliance penalty, and generation uncertainty. We endogenize SREC price through market clearing. Using techniques from variational analysis, we characterize firms' optimal controls as the solution of McKean-Vlasov (MV) FBSDEs and determine the equilibrium SREC price. We establish the existence and uniqueness and prove that the MFG strategies have the ϵ-Nash property. Finally, we develop a numerical scheme for solving the MV-FBSDEs and conclude by demonstrating how firms behave in equilibrium using simulated examples.   (Received September 16, 2020)

1163-60-1688   **Philippe Casgrain*** and **Anastasis Kratsios**. *Optimizing Optimizers: Generating regret-optimal gradient descent algorithms.*

The need for fast and robust optimization algorithms are of critical importance in all areas of machine learning. This paper treats the task of designing optimization algorithms as a stochastic control problem. Using the expected future regret as a metric for an algorithm's performance, we derive the necessary and sufficient dynamics that regret-optimal algorithms must satisfy as a discrete-time FBSDE. We study the existence and uniqueness of regret-optimal algorithms and derive bounds on rates of convergence to solutions of convex optimization problems. Though closed-form optimal dynamics cannot be obtained in general, we present fast numerical methods for approximating them, generating optimization algorithms which directly optimize their long-term regret. Lastly, these are benchmarked against various other commonly used optimization algorithms to demonstrate their effectiveness.   (Received September 18, 2020)

1163-60-1693   **Thaleia Zariphopoulou***. *Human-machine interaction models and stochastic optimization.*

I will introduce a family of human-machine interaction (HMI) models in optimal asset allocation, risk management and portfolio choice (robo-advising). Modeling difficulties stem from the limited ability to quantify the human's risk preferences and describe their evolution, but also from the fact that the stochastic environment, in which the machine optimizes, adapts to real-time incoming information that is exogenous to the human. Furthermore, the human's risk preferences and the machine's states may evolve at different scales. This interaction creates an adaptive cooperative game with both asymmetric and incomplete information exchange between the two parties. As a result, challenging questions arise on, among others, how frequently the two parties should communicate, what information can the machine accurately detect, infer and predict, how the human reacts to exogenous events, how to improve the inter-linked reliability between the human and the machine, and others. Such HMI models give rise to new, non-standard optimization problems that combine adaptive stochastic control, stochastic differential games, optimal stopping, multi-scales and learning. (Received October 21, 2020)

62 ▶ *Statistics*

1163-62-15   **Lenka Zdeborova***, EPFL École Polytechnique Fédŕale de Lausanne. *What Physics Teaches us about Computation in High Dimensions.*

Computational questions in high-dimensional problems are ubiquitous, yet we still lack a satisfying theoretical framework able to answer most of them. Corresponding problems often map to certain statistical-physics systems where the high dimension translates into a large number of interacting elements. Theoretical tools of statistical physics can then be deployed to study computational problems and inform both mathematical and algorithmic developments. We will highlight some key results in this field with examples of applications in artificial neural networks, and in signal processing. We will discuss results that have led to new mathematical development and well as others that are still open and inspiring current research.   (Received April 22, 2020)

1163-62-32   **Caroline Uhler*** (`cuhler@mit.edu`). *Multi-Domain Data Integration: From Observations to Mechanistic Insights.*

Massive data collection holds the promise of a better understanding of complex phenomena and ultimately, of better decisions. An exciting opportunity in this regard stems from the growing availability of perturbation / intervention data (manufacturing, advertisement, education, genomics, etc.). To obtain mechanistic insights from such data, a major challenge is the integration of different data modalities (video, audio, interventional, observational, etc.). Using genomics and in particular the problem of identifying drugs for the repurposing against COVID-19 as an example, I will first discuss our recent work on coupling autoencoders in the latent space to integrate and translate between data of very different modalities such as sequencing and imaging. I will then present a framework for integrating observational and interventional data for causal structure discovery and characterize the causal relationships that are identifiable from such data. We end by a theoretical analysis of overparameterized autoencoders, characterizing their implicit bias and showing that such networks trained using standard optimization methods implement associative memory. Collectively, our results have major implications for planning and learning from interventions in various application domains.   (Received July 15, 2020)

1163-62-33   **Ivo D Dinov*** (`statistics@umich.edu`), 426 N. Ingalls Str, Ann Arbor, MI 48109. *Data Science, Time Complexity, and Spacekime Analytics.* Preliminary report.

Human behavior, communication, and social interactions are profoundly augmented by the rapid immersion of digitalization and virtualization of all life experiences. This process presents important challenges of managing, harmonizing, modeling, analyzing, interpreting, and visualizing complex information. There is a substantial need to develop, validate, productize, and support novel mathematical techniques, advanced statistical computing algorithms, transdisciplinary tools, and effective artificial intelligence applications. Spacekime analytics is a new technique for modeling high-dimensional longitudinal data. This approach relies on extending the notions of time, events, particles, and wavefunctions to complex-time (kime), complex-events (kevents), data, and inference-functions. We will illustrate how the kime-magnitude (longitudinal time order) and kime-direction (phase) affect the subsequent predictive analytics and the induced scientific inference. The mathematical foundation of spacekime calculus reveal various statistical implications including inferential uncertainty and a Bayesian formulation of spacekime analytics. Many 4D Minkowski spacetime processes are extended to a 5D spacekime manifold, where a number of interesting mathematical problems arise.   (Received July 15, 2020)

1163-62-41   **Afonso S Bandeira**, **Ben Blum-Smith**, **Joe Kileel**, **Jonathan Niles-Weed*** (`jnw@cims.nyu.edu`), **Amelia Perry** and **Alexander S Wein**. *Statistical estimation under group actions.*

A common challenge in the sciences is the presence of heterogeneity in data. Motivated by problems in signal processing and computational biology, we consider a particular form of heterogeneity where observations are corrupted by random transformations from a group (such as the group of permutations or rotations) before they can be collected and analyzed. We establish the fundamental limits of statistical estimation in such settings and show that the optimal rates of recovery are precisely governed by the invariant theory of the group. As a corollary, we establish rigorously the number of samples necessary to reconstruct the structure of molecules in cryo-electron microscopy. We also give a computationally efficient algorithm for a special case of this problem, and discuss conjectured statistical-computational gaps for the general case.   (Received July 26, 2020)

1163-62-50   **Lizhe Sun**, **Mingyuan Wang**, **Yangzi Guo** and **Adrian Barbu*** (`abarbu@stat.fsu.edu`). *A Novel Framework for Online Supervised Learning with Feature Selection.*

Current online learning methods suffer from lower convergence rates and limited capability to recover the support of the true features compared to their offline counterparts. In this work, we present a novel online learning framework based on running averages and introduce online versions of some popular existing offline methods such as Elastic Net, Minimax Concave Penalty and Feature Selection with Annealing. The framework can handle an arbitrarily large number of observations as long as the data dimension is not too large, e.g. p<50,000. We prove the equivalence between our online methods and their offline counterparts and give theoretical true feature recovery and convergence guarantees for some of them. In contrast to the existing online methods, the proposed methods can extract models of any sparsity level at any time. Numerical experiments indicate that our new methods enjoy high accuracy of true feature recovery and a fast convergence rate, compared with standard online and offline algorithms. We also show how the running averages framework can be used for model adaptation in the presence of model drift. Finally, we present applications to large datasets where again the proposed framework shows competitive results compared to popular online and offline algorithms.   (Received July 29, 2020)

1163-62-316   **James D. Servidea*** (jdservi@nsa.gov). *Why Statistics Isn't Mathematics.*
It's not uncommon to lump statistics in with mathematics. My agency does it too: I'm a statistician in charge
of our math group. But statistics is fundamentally not mathematics, and I think it's important to understand
the difference. In this talk, I'll make the case for why these are two different disciplines and argue that anyone
claiming to be a mathematician ought to know some statistics, too.   (Received September 02, 2020)

1163-62-331   **Javier E Flores*** (javier-flores@uiowa.edu), 108 McLean St, Apt 13, Iowa City, IA
52246, and **Joseph E Cavanaugh**. *A-I-(don't)-C the Future: A New Class of Selection
Criteria Geared Towards (Better) Prediction.*
Like some brand of mathematical mentalist, the skilled statistician instills awe in their collaborators with eerily
accurate predictions obtained from carefully curated statistical models. However, without informed selection, the
statistician's predictive performance almost surely falls flat. For this reason, an integral part of any statistician's
box of tricks are information criteria used for model selection. Of these criteria, it is well known that the Akaike
information criterion, AIC, and its corrected counterpart, AICc, facilitate the selection of models with good
predictive ability. However, the development of these and other AIC variants presume a panel of validation data
that follows an identical distribution to that of the fitting sample, which is conceptually misaligned with the
objective of predicting future or new observations, latent data, or missing data. This presentation introduces
a new class of selection criteria that addresses this potential shortcoming and allows for the selection of good
predictive models, regardless of the nature of data one would like to predict. We demonstrate the utility of
our criteria through comparisons of predictive efficacy to models selected by AIC, BIC, and AICc in simulation
settings.   (Received September 02, 2020)

1163-62-338   **Maxime G Pouokam*** (mpouokam@ucdavis.edu), 1 Shields Ave, Davis, CA 95616, **Javier
Arsuaga** (jarsuaga@ucdavis.edu), 1 shield ave, DAVIS, CA 95616, and **Prabir Burman**
(pburman@ucdavis.edu), 1 Shield ave, Davis, CA 95616. *Statistical Topology of Genome
Analysis in Three Dimension.*
The three-dimensional (3D) configuration of chromosomes within the eukaryote nucleus is an important factor
for several cellular functions, including gene expression regulation, and has also been linked with many diseases
such as cancer-causing translocation events. Recent adaptations of high-throughput sequencing to chromosome
conformation capture (3C) techniques, allows for genome-wide structural characterization for the first time with a
goal of getting a 3D structure of the genome. In this study, we present a novel approach to compute entanglement
in open chains in general and apply it to chromosomes. Our metric is termed the linking proportion (Lp). We
use the Lp in two different settings. We use the Lp to show that the Rabl configuration, an evolutionary
conserved feature of the 3D nuclear organization, as an essential player in the simplification of the entanglement
of chromatin fibers. We show how the Lp incorporates statistical models of inference that can be used to
determine the agreement between candidate 3D configuration reconstructions. In the last part of our work,
we present Smooth3D, a novel 3D genome reconstruction method via cubic spline approximation.   (Received
September 03, 2020)

1163-62-404   **Wen-Hao Chiang**, **Xueying Liu** and **George Mohler*** (gmohler@iupui.edu), 105 E
Westfield Blvd, Indpls, IN 46220. *Hawkes process modeling of COVID-19 with mobility
leading indicators and spatial covariates.* Preliminary report.
Hawkes processes are used in machine learning for event clustering and causal inference, while they also can
be viewed as stochastic versions of popular compartmental models used in epidemiology. Here we show how to
develop accurate models of COVID-19 transmission using Hawkes processes with spatial-temporal covariates.
We model the conditional intensity of new COVID-19 cases and deaths in the U.S. at the county level, estimating
the dynamic reproduction number of the virus within an EM algorithm through a regression on Google mobility
indices and demographic covariates in the maximization step. We validate the approach on short-term forecasting
tasks, showing that the Hawkes process outperforms several benchmark models currently used to track the
pandemic, including an ensemble approach and an SEIR-variant. We also investigate which covariates and
mobility indices are most important for building forecasts of COVID-19 in the U.S.   (Received September 05,
2020)

1163-62-450   **Natalie C. Gasca*** (ncgasca@uw.edu) and **Robyn L. McClelland**. *Sparse Partial Least
Squares Regression to Model the Relationship Between Correlated Diet Data and Risk of
Heart Disease Events.* Preliminary report.
When relating the concurrent impact of foods on heart disease risk factors, nutritional researchers encountered
frequent unintuitive results due to the strong correlations between some foods. Supervised dimension reduction

methods such as sparse partial least squares (PLS) can reduce diet data in a tailored way to heart disease by selecting a few dietary patterns and foods while conserving predictive ability of the risk factor. To directly relate diet to the risk of incident disease, the Cox proportional hazards model is a standard approach that models the time to first event as a function of observed covariates. We propose incorporating sparse PLS into the Cox proportional hazards model to study the relationship between correlated data and heart disease risk while utilizing variable selection and imposing fewer model assumptions than existing methods. We compare sparse and non-sparse PLS Cox methods in their ability to create interpretable and predictive patterns, via simulations. We also apply the methods to diet data and time-to-coronary-disease in the Multi-Ethnic Study of Atherosclerosis. We will discuss in which settings this proposed method can construct interpretable patterns related to time-to-disease-events by leveraging variable selection and dimension reduction.   (Received September 07, 2020)

1163-62-549   **Maxwell D Lovig*** (maxwelllovig@gmail.com), 78 Arrowhead Way, Woodbury, CT 06798, and **Sumaita Rahman** (sumaitasr@gmail.com), 421 E Hendrix St. Apt C, Greensboro, NC 27405. *A Mixture Binary RRT Model With A Unified Measure Of Privacy And Efficiency.*
In this study, we introduce a mixture binary Randomized Response Technique (RRT) model by combining the elements of the Greenberg Unrelated Question model and the Warner Indirect Question model. We also account for untruthful responding in the proposed model. A unified measure of model efficiency and respondent privacy is also presented. Finally, we present a simulation study to validate the theoretical findings.
(Received September 09, 2020)

1163-62-562   **Victoria Belotti*** (vbelotti@hawk.iit.edu), **Matt Jester**, **Kaitlin Keegan**, **Yu-Min Chung** and **Sarah Day**. *Topological estimation of image data via subsampling.* Preliminary report.
We develop a novel statistical approach to estimate topological information from large, noisy images. Our main motivation is to measure pore microstructure in 3-dimensional X-ray micro-computed tomography (micro-CT) images of ice cores. The pore space in these samples is where gas can move and get trapped within the ice column and is of interest to climate scientists. While the field of topological data analysis offers tools (e.g. lifespan cutoff and PD Thresholding) for estimating topological information in noisy images, direct application of these techniques becomes infeasible as image size and noise levels grow. Our approach uses image subsampling to estimate the number of holes of a prescribed size range in a computationally feasible manner. In applications where holes naturally have a known size range on a smaller scale than the full image, this approach offers a means of estimating Betti numbers, or global counts of holes of various dimensions, via subsampling of the image.   (Received September 09, 2020)

1163-62-668   **Bogumił Kamiński**, **Paweł Prałat** and **François Théberge*** (theberge@ieee.org). *A Framework for Comparing Graph Embeddings.*
Graph embedding is a mapping from vertices of a graph into real vector space. A good embedding should capture the graph topology, vertex-to-vertex relationship, and other relevant information about the graph, its subgraphs, and vertices. If these objectives are achieved, an embedding is a meaningful, understandable, and provides a compressed representation of a network. Moreover, vector operations are usually simpler and faster than comparable operations on graphs. Unfortunately, selecting the best embedding is a challenging task and very often requires domain experts. We propose a divergence score that can be assigned to embeddings to help distinguish good ones from bad ones. This general framework provides a tool for an unsupervised graph embedding comparison. In order to achieve it, we needed to generalize the well-known Chung-Lu model to incorporate geometry which is an interesting result in its own right. This framework has quadratic complexity in the number of vertices, so it is only suitable for small networks. We also present a landmark-based version of our framework which allows for much greater scalability. Detailed quality and speed benchmarks are provided. (Received September 11, 2020)

1163-62-720   **Venkateswara Rao Mudunuru*** (vmudunur@mail.usf.edu), 4202 E. Fowler Ave, CMC 342, Tampa, FL 33620. *Fast Forest Quantile Regression Approach in Predicting Breast Cancer Tumor Sizes.*
When dealing with large datasets, random forest regression provides robust and powerful models with an accurate approximation of the conditional mean of the output variable. On the contrary, quantile random forest regression models provide a way to estimate conditional quantiles for the output variables. A fast forest quantile regression (FFQR) is a new and powerful machine learning model which improves the prediction accuracy of the models.

This paper presents a comparison of traditional regression models and quantile regression and fast forest quantile regression models applied to breast cancer data in predicting the tumor sizes.   (Received September 11, 2020)

1163-62-726   **Vidya Bhargavi M*** (`vidya.msc05@gmail.com`), Chapel Road, Abids, Hyderabad, 500001, India, **Sireesha V**, Department of Applied Mathematics, GITAM Institute of Science, GITAM (Deemed to be University), Visakhapatnam, 530 045, India, and **Venkateswara Rao Mudunuru** (`vmudunur@mail.usf.edu`), 4202 E. Fowler Ave CMC 342, Tampa, FL 33620. *Colon Cancer Survival Analysis with Quantile Regression.*

Quantile regression emerged as an alternative and robust technique to the commonly used regression models. Even in the survival analysis, quantile regression is offering more flexible modeling of survival data without any constraints attached. Unlike traditional Cox hazards models or accelerated failure models, quantile regression does not restrict the variation of the coefficients for different quantiles. In this research we modeled and compared traditional survival regression method with quantile regression applied to colon cancer data.   (Received September 12, 2020)

1163-62-809   **Mihai Cucuringu*** (`mihai.cucuringu@stats.ox.ac.uk`), mihai.cucuringu@stats.ox.ac.uk, Oxford, United Kingdom. *Spectral methods for clustering signed and directed graphs, and constrained clustering.*

We study problems that share an important common feature: they can all be solved by exploiting the spectrum of their corresponding graph Laplacian. We first discuss a simple spectral approach to the well-studied constrained clustering problem, which captures constrained clustering as a generalized eigenvalue problem with graph Laplacians. The proposed algorithm works in nearly-linear time, provides guarantees for the quality of the clusters for 2-way partitioning, and consistently outperforms existing spectral approaches both in speed and quality. This line of work extends to the setting of clustering signed networks, where the edge weights between the nodes of the graph may take either positive or negative values, for which we provide theoretical guarantees in the setting of a signed stochastic block model. Finally, we discuss a spectral clustering algorithm for directed graphs based on a complex-valued representation of the adjacency matrix, which is provably able to capture the underlying cluster-structures, for which the information given by the direction of the edges is crucial.   (Received September 13, 2020)

1163-62-899   **David M Ruth*** (`druth@usna.edu`), druth@usna.edu. *Graph-theoretic approaches to change detection, with defense applications.*

Detecting a distribution change with respect to sets or sequences of observations is a classic problem in statistics. The problem is especially challenging in multivariate settings where underlying probability distributions are unknown; this is the case in myriad defense applications. Graph-theoretic methods have proven useful in detecting subtle distribution changes in such instances. Data are modeled as complete graphs, where each observation constitutes a vertex and each vertex pair has an undirected edge weighted by interpoint dissimilarity. Successful approaches have used minimum spanning trees, nearest neighbor graphs, minimum-weight non-bipartite matchings, and other minimum-weight regular graphs. We review key concepts associated with these approaches, share some recent work that counts edges ordered by weight in a manner that is computationally inexpensive and also has impressive power to detect distribution changes in a two-sample setting, then extend this recent work to the sequential change-detection setting. Defense applications will be considered.   (Received September 15, 2020)

1163-62-1326   **Maxime G Pouokam*** (`mpouokam@ucdavis.edu`), 1 Shield ave, Davis, CA 95616, and **Javier Arsuaga** and **Prabir Burman**. *Statistical Topology of Genome Analysis in Three Dimension.*

The three-dimensional (3D) configuration of chromosomes within the eukaryote nucleus is an important factor for several cellular functions, including gene expression regulation, and has also been linked with many diseases such as cancer-causing translocation events. Recent adaptations of high-throughput sequencing to chromosome conformation capture (3C) techniques, allows for genome-wide structural characterization for the first time with a goal of getting a 3D structure of the genome. In this study, we present a novel approach to compute entanglement in open chains in general and apply to chromosomes. Our metric is termed the linking proportion (Lp). We use the Lp in two different settings. First, we use the Lp to show that the Rabl configuration reduces the incidence of topological entanglement of chromosomes in budding yeast. Second, we show how the Lp incorporates statistical models of inference that can be used to determine the agreement between candidate 3D configuration reconstructions. In the last part of our work, we present Smooth3D, a novel 3D genome reconstruction method via cubic spline approximation.   (Received September 15, 2020)

65 ► *Numerical analysis*

1163-65-9   **Douglas N. Arnold***. *Structure preservation in the discretization of partial differential equations.*

Many of the most significant and fundamental partial differential equations are associated with underlying geometric and algebraic structures, such as elliptic complexes and their cohomology and Hodge theory. These structures strongly govern behaviors of the equations such as their well-posedness. Such equations arise in numerous applications, including electromagnetics, solid mechanics, fluid mechanics, and general relativity, and their accurate numerical solution is of great importance. But, in many cases, finding accurate numerical methods has proven difficult or impossible. Over the last decade it has become clear that, when the PDEs are discretized for numerical computation, the structures underlying the PDEs must be preserved at the discrete level in order to obtain stable and consistent discretizations needed for convergence. This realization has given rise to structure-preserving discretization, an approach to the discretization of differential equations which brings fields as diverse as geometry, topology, homological algebra, functional analysis, and representation theory to bear on numerical analysis. The new viewpoint has led to remarkable progress, resulting in effective new numerical methods for problems where they were previously unavailable.   (Received September 1, 2020)

1163-65-83   **Gus I. Argyros** (gus.argyros@cameron.edu), **Michael I. Argyros** (michael.argyros@cameron.edu), **Ioannis K. Argyros*** (iargyros@cameron.edu), **Samundra Regmi** (samundra.regmi@cameron.edu) and **Santhosh George** (sgeorge@nitk.edu.in). *Extended local convergence of Newton's algorithm for solving strongly regular generalized equations.*

This study presents a new one-parameter family of the well-known fixed point iteration method for solving nonlinear equations numerically. The proposed family is derived by implementing approximation through a straight line. The presence of arbitrary parameters in the proposed family improves convergence characteristic of the simple fixed-point iteration as it has a wider domain of convergence. Further, we propose many two-step predictor-corrector iterative schemes for finding fixed points which inherits the advantages of the proposed fixed-point iterative schemes. Finally, several examples are given to further illustrate their efficiency.   (Received August 11, 2020)

1163-65-277   **Ethan Jensen*** (ejensen18@georgefox.edu) and **Corban Harwood**. *Existence and Shape of Numerical Oscillations in Solving Parabolic PDE.*

In this talk, we employ linear algebra and functional analysis to determine necessary and sufficient conditions for oscillation-free and stable solutions to linear and nonlinear parabolic partial differential equations. We apply singular value decomposition and Fourier analysis to various finite difference schemes to extract patterns in the eigenfunctions (sampled by the eigenvectors) and the shape of their eigenspectrum. Through these, we share how the initial and boundary conditions affect the frequency and long term behavior of numerical oscillations, as well as the solution regions most sensitive to them.   (Received August 31, 2020)

1163-65-351   **Maria Irene Falcão*** (mif@math.uminho.pt), Universidade do Minho, Departamento de Matematica, Campus de Gualtar, 4710-057 Braga, Portugal, and **Fernando Miranda** (fmiranda@math.uminho.pt), **Ricardo Severino** (ricardo@math.uminho.pt) and **Maria Joana Soares** (jsoares@math.uminho.pt). *Comparative study of quaternionic root-finding methods.*

Since the well-known work of Niven in the 1940's, there has been a growing interest in studying the problem of characterizing and computing the zeros (or roots) of quaternionic polynomials, mainly from the theoretical point of view. Nevertheless, in the last decade several authors proposed algorithms for finding the zeros of one-sided left quaternionic polynomials (i.e. polynomials whose coefficients are located only on the left-hand side of the powers). Most of these root-finding methods rely on the connection between the zeros of a quaternionic polynomial and the zeros of a certain real polynomial, usually with multiple zeros, and as such, they face the usual difficulties associated with the computation of multiple zeros or clusters of zeros.

In this talk we revisit some of the available numerical root-finding methods and present an iterative method entirely based on quaternionic arithmetic, which does not suffer from the aforementioned drawbacks. We also propose a collection of selected polynomials to test the performance of each method and make a comparative study based on several indicators.   (Received September 03, 2020)

1163-65-380   **Ron Buckmire***, 1600 Campus Road, Los Angeles, CA 90041, and **Treena Basu**, 1600
                Campus Road, Los Angeles, CA 90041. *Using Approximations of Unity in the
                Construction of Positivity-Preserving NSFD Schemes.*

We present examples of nonstandard finite difference (NSFD) schemes that preserve positivity while simultane-
ously exploiting various ways unity (i.e. the number "1") can be represented algebraically in these expressions.
NSFD schemes (also known as "Mickens discretizations") use nonstandard numerical techniques to approximate
derivatives and other features in ordinary and partial differential equations. These NSFD schemes can often be
used to produce numerical solutions to differential equations that have particular desired properties like increased
accuracy, preserving positivity, and maintaining boundedness.   (Received September 04, 2020)

1163-65-434   **Jonathan Hauenstein*** (`hauenstein@nd.edu`), **Laura Matusevich**, **Chris Peterson**
                and **Samantha Sherman**. *Numerically testing binomiality via a witness set.* Preliminary
                report.

Sparse polynomials that vanish on algebraic sets are preferred in many computations since they are easy to
evaluate and often arise from underlying structure. Due to the structure of algebraic sets defined by binomials,
there has been several approaches to determine if a given ideal is generated by binomials. This talk will consider
a new numerical algebraic geometric approach for deciding binomiality of the ideal vanishing on a given algebraic
set described by a witness set. Several examples will be presented demonstrating this new numerical approach.
This is joint work with Laura Matusevich, Chris Peterson, and Samantha Sherman.   (Received September 06,
2020)

1163-65-485   **Roumen Anguelov*** (`roumen.anguelov@up.ac.za`), Department of Mathematics and
                Applied, Mathematics, University of Pretoria, Pretoria, Gauteng 0028, South Africa.
                *Constructing topologically dynamically consistent numerical methods for continuous
                dynamical systes.*

The interest in numerical methods adequitely representing the properties of the model they approximate, comes
from different directions: conservation of Hamiltonian, preservation of volume, reversibility of the flow, mimetic
discretisation, dynamic consistency. Preserving qualitative properties is a specific goal of the nonstandard finite
difference method. There is substantial intersection between these approaches. For example, often discretization
aimed preserving particular property, e.g. conservation law, yields a method utilizing the tools defining the
nonstandard finite difference method, i.e. nonlocal approximation and renormalization denominator. In this talk,
we discuss the concept of topological dynamic consistency of a discrete dynamical system (a numerical method)
and a continuous dynamical system (system of ODEs) using topological conjugacy. Since the properties of interest
in a dynamical system are typically of topological nature, this concept indeed captures well the expectations from
qualitative point of view that a numerical method should satisfy. We discuss appropriate tools and techniques
for practical application of this definition to characterise existing numerical methods and to construct numerical
schemes which are topologically dynamically consistent.   (Received September 08, 2020)

1163-65-515   **Alexander Cloninger*** (`acloninger@ucsd.edu`), 9500 Gilman Rd, La Jolla, CA 92093,
                and **Caroline Moosmueller** (`cmoosmueller@ucsd.edu`), 9500 Gilman Rd, La Jolla, CA
                92093. *Fast Pairwise Optimal Transport and Linear Classification for Nonlinear Problems.*

Discriminating between distributions is an important problem in a number of scientific fields. This motivated the
introduction of Linear Optimal Transportation (LOT), which has a number of benefits when it comes to speed
of computation and to determining classification boundaries. In this paper, we characterize a number of settings
in which the LOT embeds families of distributions into a space in which they are linearly separable. This is true
in arbitrary dimension, and for families of distributions generated through perturbations of shifts and scalings of
a fixed distribution. The transform is defined by computing the optimal transport of each distribution to a fixed
reference distribution, and considering distances between the transport maps. We also prove conditions under
which LOT between two distributions is nearly isometric to Wasserstein-2 distance between those distributions.
This is of significant computational benefit, as one must only compute K optimal transport maps to define
the K^2 pairwise distances between the K distributions. We demonstrate the benefits of LOT on a number of
distribution classification problems.   (Received September 08, 2020)

1163-65-526   **Ron Buckmire*** (`ron@oxy.edu`), **Sean Brooks** (`sbrooks@coppin.edu`), **Bonita V.
                Saunders** (`bonita.saunders@nist.gov`) and **Rachel E. Vincent-Finley**
                (`rachel_finley@subr.edu`). *Validated Computation of Special Functions II: Error Analysis.*

One of the key features of the validated computation of special functions is the ability to control the error of the
calculations, i.e. to validate the computation. Two of the most important types of error involved in the validated
computation of special functions using electronic devices are truncation error and round-off error. In this talk,

we discuss some of the techniques and methods used to estimate and control errors involved in implementing mathematical operations to user-specified numerical precision on a computer.   (Received September 08, 2020)

1163-65-558   **Michael Burr** and **Anton Leykin***, School of Mathematics, Georgia Tech, Atlanta, GA. *Inflation of poorly conditioned zeros of systems of analytic functions.*
Given a system of analytic functions and an approximate zero, we transform it into a system with a regular quadratic zero. This results in a method for isolating a cluster of zeros of the given system.   (Received September 10, 2020)

1163-65-592   **Wuchen Li***, 586 Eagles Rest Drive, Chapin, SC 29036. *High order MCMC methods via Transport information geometry.*
In AI and computational statistics, the MCMC method is a classical model-free method for sampling target distributions. A fact is that the optimal transport first-order method (gradient flow) forms the MCMC scheme, known as Langevin dynamics. A natural question arises: Can we propose high order optimization techniques for MCMC methods? We positively answer this question by considering second-order optimization methods combining optimal transport and information geometry, known as transport information geometry. Here we introduce a theoretical framework for Newton's flows in probability space with the Wasserstein-2 metric. Several numerical examples are provided to demonstrate the effectiveness of the proposed Newton's method.   (Received September 10, 2020)

1163-65-632   **Margaret H Regan*** (mregan@math.duke.edu). *Machine learning the discriminant locus.*
Parameterized systems of polynomial equations arise in many applications in science and engineering with the real solutions describing, for example, equilibria of a dynamical system, linkages satisfying design constraints, and scene reconstruction in computer vision. Since different parameter values can have a different number of real solutions, the parameter space is decomposed into regions whose boundary forms the real discriminant locus. In this talk, I will discuss a novel sampling method for multidimensional parameter spaces and how it is used in various machine learning algorithms to locate the real discriminant locus as a supervised classification problem, where the classes are the number of real solutions. Examples such as the Kuramoto model will be used to show the efficacy of the methods. Finally, an application to real parameter homotopy methods will be presented. This project is joint work with Edgar Bernal, Jonathan Hauenstein, Dhagash Mehta, and Tingting Tang.   (Received September 10, 2020)

1163-65-637   **John Harlim*** (jharlim@psu.edu), 109 McAllister Bldg, University Park, PA 16802. *Ghost points diffusion maps for solving PDEs on manifolds with classical boundary conditions.*
In this talk, I will discuss recent efforts in using the Diffusion Maps (DM) algorithm to solve elliptic PDEs on unknown manifolds using point clouds data. The key idea rests on the fact that away from the boundary, the second-order elliptic differential operators can be approximated by integral operators defined with appropriate Gaussian kernels. The key advantage of such an approximation is that one can avoid parameterizing the manifold, which can be complicated if the manifold is embedded in a high-dimensional ambient space. On manifolds with boundary, however, such an approximation is only valid for functions that satisfy the Neumann boundary condition. Motivated by the classical ghost-point correction in the finite-difference method for solving Neumann problems, we extend the diffusion maps algorithm with ghost points such that it is a consistent estimator in the pointwise sense even near the boundary. Applying the proposed algorithm, which we called the Ghost Points Diffusion Maps, to solve the well-posed elliptic PDEs with Dirichlet, Neumann, or Robin boundary conditions, we establish the convergence of the approximate solution under appropriate smoothness assumptions. Supporting numerical examples of problems on various known and unknown manifolds will be shown.   (Received September 10, 2020)

1163-65-677   **Guillaume Bal**, **Emil Sidky** and **Fatma Terzioglu*** (fterzioglu@uchicago.edu). *Uniqueness and stability of reconstruction in Multi-Energy Computed Tomography.* Preliminary report.
Multi-Energy Computed Tomography (ME-CT) is a medical imaging modality aiming to reconstruct the spatial density of materials from the attenuation properties of probing x-rays. For each line in two or three dimensional space, ME-CT measurements may be written as a nonlinear mapping from the integrals of the unknown densities of a finite number of materials (typically bone, water and contrast agents) along said line to an equal or larger number of energy-weighted integrals corresponding to different x-ray source energy spectra. Image reconstruction from ME-CT data may thus be achieved in two steps: first the reconstruction of line integrals of the material densities from their energy-weighted integrals, and then the reconstruction of material densities from their line

integrals. The second step is the standard linear x-ray CT problem whose invertibility is well-known. The first step is however a nonlinear map with no known analytical inverse. In this talk, we focus on the first step and present a sufficient criteria that guarantee global uniqueness and stability of ME-CT reconstructions. We also present a new ME-CT material reconstruction algorithm whose convergence is ensured using our criteria. (Received September 11, 2020)

1163-65-712   **Rachel E. Vincent-Finley*** (`rachel_finley@subr.edu`), **Sean Brooks** (`sbrooks@coppin.edu`), **Ron Buckmire** (`ron@oxy.edu`) and **Bonita V. Saunders** (`bonita.saunders@nist.gov`). *Validated Computation of Special Functions III: DLMF Tables.*

DLMF Tables, short for DLMF Standard Reference Tables on Demand project, is a spin-off of the NIST Digital Library of Mathematical Functions (DLMF). The project is a collaborative effort between the National Institute of Standards and Technology (NIST) and the University of Antwerp to develop an online software testing system that generates validated tables of special function values computed to user-specified precision with an error certification. In this talk we look at the DLMF Tables beta site where techniques explained in our earlier talks (Parts I and II) were previously implemented. We also discuss our plans for a multi-level interface that will make the system accessible to various classes of users, from those with only a casual interest to scientific researchers, personal or commercial software developers, and numerical analysts.   (Received September 11, 2020)

1163-65-797   **Taufiquar R Khan*** (`taufiquar.khan@uncc.edu`), Department of Mathematics and Statistics, University of North Carolina at Charlotte, Fretwell 360E, 9201 University City Blvd., Charlotte, NC 28223, **Vincent Barra**, University of Clermont-Auvergne, Clermont-Ferrand, France, and **Shyla Kupis**. *Deep Neural Networks for Image Reconstruction in Electrical Impedance Tomography.*

We will present deep learning algorithm for image reconstruction in electrical impedance tomography. We also compare the results from the machine learning approach to the deterministic approach using Gauss Newton method.   (Received September 12, 2020)

1163-65-803   **Sascha Timme*** (`timme@math.tu-berlin.de`). *Catastrophes in Elastic Tensegrity Frameworks.*

Elastic tensegrity frameworks are structures made from rigid bars and elastic cables. These structures can be effectively modelled using the theory of elasticity and energy minimization. In particular, they always position themselves in a state of (local) minimal energy. A change of control parameters of the framework can yield a catastrophe, a discontinuous jump in the location of local minima. In this talk we develop an algebraic relaxation of the aforementioned problem and determine a semi-algebraic set, the catastrophe set, which governs the possibility of a catastrophe happening. We then show how to compute it using numerical nonlinear algebra. This is joint work with Alex Heaton.   (Received September 13, 2020)

1163-65-906   **Isao Yamada*** (`isao@sp.ce.titech.ac.jp`), Ookayama 2-12-1-S3-60, Meguro-ku, Tokyo, 152-8550, Japan, and **Masao Yamagishi** (`myamagi@sp.ce.titech.ac.jp`), Ookayama 2-12-1-S3-60, Meguro-ku, Tokyo, 152-8550, Japan. *A pair of fixed point strategies with proximal splitting operators for unified applications to certain inverse problems in data science.*

In this talk, we demonstrate how the fixed point theoretic strategies can be enhanced with the proximal splitting operators for powerful applications to increasingly complex inverse problems desired to be tackled in data science. As such typical examples, we introduce a pair of key ideas established recently for (i) the hierarchical convex optimization with the hybrid steepest descent method, and for (ii) the Linearly involved Generalized Moreau Enhanced (LiGME) models designed specially to utilize nonconvex penalties in wide range of the sparsity-rank-aware signal processing.   (Received September 14, 2020)

1163-65-949   **Felix Dietrich*** (`felix.dietrich@tum.de`), **Daniel Lehmberg**, **Severin Reiz** and **Tobias Neckel**. *Efficient Manifold Learning with Diffusion Maps.*

Kernel-based approximations of the Laplace-Beltrami operator, such as Diffusion Maps, can be employed to construct meaningful, low-dimensional representations of data sets by embedding them into the first few eigenfunctions of the operator. In this approach, it is common to use the entire data set simultaneously to form the kernel matrix. This is demanding with respect to computer memory and computation time if the number of data points is very large and the ambient space dimension is very high—to the point where it becomes prohibitive to employ kernel methods entirely.

We discuss a technique to alleviate this challenge and make kernel-based approaches for manifold learning applicable to large data sets. We employ a Multi-Level Monte Carlo approach to obtain a numerical estimate of the number of randomly chosen points necessary to accurately estimate the operator. We show convergence of the algorithm in several controlled environments, and demonstrate that manifold learning with the estimate is faster than training an auto-encoder network on a real data set.   (Received September 14, 2020)

1163-65-958   **Antonio Leitao*** (`acgleitao@gmail.com`), Department of Mathematics, Federal Univ. of St. Catarina, P.O.Box 476, Florianopolis, SC 88040-900, Brazil. *On range-relaxed strategies and iterative type methods for obtaining stable solutions to ill-posed operator equations.*
In this talk we discuss a novel range-relaxed strategy for choosing the Lagrange multipliers in several iterative type methods for solving ill-posed operator equations (e.g. iterated Tikhonov (iT), Levenberg-Marquardt (LM), and the corresponding Kaczmarz versions (iTK and LMK).

Convergence analysis is presented (monotonicity, convergence, stability, semi-convergence). Numerical experiments are shown for some applications (both linear and nonlinear). The obtained numerical results validate the efficiency of the proposed approach.   (Received September 14, 2020)

1163-65-1024   **Thomas Schuster*** (`thomas.schuster@num.uni-sb.de`), Department of Mathematics, Saarland University, 66123 Saarbruecken, Germany, and **Clemens Meiser** and **Anne Wald**. *Data driven acceleration of a parameter identification problem associated with the eikonal equation.*
We deal with the inverse problem of computing the refractive index from time-of-flight measurements which is of great importance, e.g., in terahertz tomography. The mathematical model is given by the eikonal equation. Using training data we accelerate the evaluation of the forward operator, i.e. the solution of the eikonal equation, significantly compared to standard techniques such as marching schemes. This leads also to a more efficient solution of the inverse problem, e.g. by using Landweber's method.   (Received September 14, 2020)

1163-65-1073   **Hristo V Kojouharov***, The University of Texas at Arlington, Department of Mathematics, 411 South Nedderman Drive, PKH 478, Arlington, TX 76019-0408, and **Souvik Roy**, **Madhu Gupta**, **Fawaz Alalhareth** and **John M Slezak**. *Higher-order modified nonstandard finite difference methods for autonomous differential equations.*
A new class of nonstandard finite difference methods for solving autonomous differential equations are constructed and analyzed. The new methods are based on the nonstandard versions of the theta method and the second-order Runge-Kutta method, among others; but, in addition, they have higher-order of accuracy. A set of numerical simulations is presented that supports the theoretical results.   (Received September 14, 2020)

1163-65-1104   **Kee-Myoung Nam*** (`kmnam@g.harvard.edu`). *Robustness and parameter geography in post-translational modification systems.*
Biological systems are often acknowledged to be robust to perturbations, but a rigorous understanding of this has been elusive. In a mathematical model, perturbations often exert their effect through parameters, so sizes and shapes of parametric regions offer an integrated, global view of robustness. Here, we explore this "parameter geography" for the property of bistability in post-translational modification (PTM) systems. In particular, we exploit two recent mathematical advances: first, we use the "linear framework" for timescale separation to describe the steady-state of a two-site PTM system as the solutions of two polynomial equations in two variables, with eight non-dimensional parameters. Importantly, this approach allows us to accommodate arbitrarily complex enzyme mechanisms, beyond the conventional Michaelis-Menten scheme, which unrealistically forbids product rebinding. Furthermore, we use the numerical algebraic geometry tools Bertini, Paramotopy, and alphaCertified to compute and statistically assess the solutions to these equations at billions of parameter points. Combining these developments in theory and software, we are able to uncover mathematical conjectures and questions regarding parameter geography through a high-dimensional, data-centric analysis.   (Received September 14, 2020)

1163-65-1223   **Phindile Dumani*** (`phindile.dumani@up.ac.za`), Lynnwood Road, Hatfield, Pretoria, Gauteng 0002, South Africa, and **Michael Chapwanya**. *Numerical study of microbial dormancy under environmental stress.* Preliminary report.
Dormancy is a strategy for microbial populations to survive various unfavourable conditions until the environment allows for proliferation. This state is a phase of minimal to no physical or/and metabolic activity. We present a modeling concept for the growth and decay of microorganisms that explicitly accounts for their ability to switch between active and dormant states in response to a limiting-nutrient. We then show that in conditions of nutrient limitation, oscillations can be observed, thus giving a natural description of the boom and bust phenomenon. A

nonstandard finite difference scheme, dynamically consistent with the differential system is proposed. Numerical simulations supporting the effectiveness of the scheme in comparison to classical numerical methods will be provided.   (Received September 15, 2020)

1163-65-1225   **M Chapwanya*** (`m.chapwanya@up.ac.za`), University of Pretoria, Department of Mathematics & Applied Mathemati, Pretoria, Gauteng 0002, South Africa, and **H Banda**. *Pattern formation in the Holling-Tanner predator-prey model with prey-taxis.*

The pioneering work on the Lotka-Volterra model gave rise to rich literature on the interaction of two or more species. In this paper, the pattern formation in the Holling-Tanner predator-prey model with prey-taxis is investigated. We construct a dynamically consistent nonstandard finite difference scheme for the proposed model. Numerical simulations are provided to support our findings.   (Received September 15, 2020)

1163-65-1433   **Yifei Lou***, University of Texas at Dallas, FO35, 800 West Campbell Road, Richardson, TX 75080. *Blind Hyperspectral Unmixing Based on Graph Total Variation Regularization.*

Remote sensing data from hyperspectral cameras suffer from limited spatial resolution, in which a single pixel of a hyperspectral image may contain information from several materials in the field of view. Blind hyperspectral image unmixing is the process of identifying the pure spectra of individual materials (i.e., endmembers) and their proportions (i.e., abundances) at each pixel. In this paper, we propose a novel blind hyperspectral unmixing model based on the graph total variation (gTV) regularization, which can be solved efficiently by the alternating direction method of multipliers (ADMM). To further alleviate the computational cost, we apply the Nyström method to approximate a fully-connected graph by a small subset of sampled points. Furthermore, we adopt the Merriman-Bence-Osher (MBO) scheme to solve the gTV-involved subproblem in ADMM by decomposing a grayscale image into a bit-wise form. A variety of numerical experiments on synthetic and real hyperspectral images are conducted, showcasing the potential of the proposed method in terms of identification accuracy and computational efficiency.   (Received September 15, 2020)

1163-65-1463   **Elias M. Adamu** and **Kailash C. Patidar*** (`kpatidar@uwc.ac.za`), Department of Mathematics and Applied Mathema, University of the Western Cape, Private Bag X17, Bellville, 7535, South Africa, and **Andriamihaja Ramanantoanina**. *A robust nonstandard finite difference scheme to solve a mathematical model for Visceral Leishmaniasis.*

Leishmaniasis is a vector-borne disease, caused by a protozoan parasite which is transmitted to humans by the bite of infected female phlebotomine sandflies. There are four main types of this disease: Visceral Leishmaniasis (also known as Kala-Azar); Post-Kala-Azar Dermal Leishmaniasis; Cutaneous Leishmaniasis; and Mucocutaneous Leishmaniasis. In this work, our focus is on a mathematical model of Visceral Leishmaniasis. The model incorporates three populations: the human, the reservoir and the vector host populations. The resulting model is a system of highly nonlinear differential equations. We carefully analyze this model to understand essential qualitative dynamics of the solution. We then design a robust NSFD scheme. A rigorous theoretical analysis of this scheme and comparative numerical results will be presented at the conference.   (Received September 15, 2020)

1163-65-1491   **Samuel F. Potter*** (`sfpotter@umd.edu`). *Jet marching methods for numerical geometric acoustics.*

Precomputed sound propagation based on solving the wave equation followed by real-time auralization has recently become feasible as a method for simulating sound in the sorts of large, complicated environments arising in e.g. virtual reality. However, its effectiveness is limited to relatively low-frequency sound not encompassing the entire audible spectrum. To address this issue, we propose a novel approach for modeling high-frequency sound propagation based on solving the eikonal equation and computing the amplitude along with it using paraxial ray tracing. This approach involves solving a tree of eikonal problems describing all orders of reflection and diffraction above a chosen amplitude threshold. Crucially for this approach, we have developed a family of high-order, semi-Lagrangian direct solvers for the eikonal equation, which use compact stencils. These solvers march the partial derivatives of the eikonal up to some order in addition to the eikonal itself, using Hermite interpolation to locally approximate the eikonal and minimizing rays when computing semi-Lagrangian updates. We present numerical and theoretical results pertaining to our solvers and show how these solvers can be used in practice to efficiently carry out realistic acoustic simulations.   (Received September 15, 2020)

1163-65-1501   **Alex Townsend***, townsend@cornell.edu, and **Matt Colbrook** and **Andrew Horning**.
                  Infinite-dimensional linear algebra.

Spectral methods for solving differential eigenproblems usually follow the "discretize-then-solve" paradigm. Discretize first, and then solve the matrix eigenproblem. The discretize-then-solve paradigm can be tricky for differential eigenproblems as the spectrum of matrix discretizations may not converge to the spectrum of the differential operator. Moreover, it is impossible to fully capture the continuous part of the spectrum with a finite-sized matrix eigenproblem. In this talk, we will discuss an alternative "solve-then-discretize" paradigm for differential eigenproblem. To compute the discrete spectrum, we will discuss a continuous analogue of FEAST by approximating the action of the resolvent operator. For the continuous spectra, we will use a Cauchy-like integral to calculate a smoothed version of the spectral measure.   (Received September 15, 2020)

1163-65-1581   **Mirjeta Pasha*** (mpasha3@asu.edu), Arizona State University, Tempe, AZ 85280, and
                  Lothar Reichel and **Alessandro Buccini**. *Krylov meets Bregman: Sparse image
                  reconstruction with nonnegativity constraint.*

Bregman-type iterative methods have received considerable attention in recent years due to their ease of implementation and the high quality of the computed solutions they deliver. However, these iterative methods may require a large number of iterations and this reduces their usefulness. In this talk we will discuss a computationally attractive linearized Bregman algorithm by projecting the problem to be solved into an appropriately chosen low-dimensional Krylov subspace. The projection reduces the computational effort required for each iteration. A variant of this solution method, in which nonnegativity of each computed iterate is imposed, also is described. Extensive numerical examples illustrate the performance of the proposed methods.   (Received September 15, 2020)

1163-65-1592   **Anna Fitzpatrick*** (akfitzpatrick@wpi.edu), **Molly Folino** (mjfolino@wpi.edu) and
                  Andrea Arnold (anarnold@wpi.edu). *Fourier Series Approximation of Time-Varying
                  Parameters using the Ensemble Kalman Filter.*

In this work we utilize Fourier series approximation within the ensemble Kalman filtering framework to estimate time-varying system parameters. We demonstrate the capability of this approach in estimating high and low frequency sinusoidal forcing functions, as well as polynomial and step functions, in a mass-spring system. Results emphasize the importance of the choice of Fourier series terms in the corresponding parameter estimates. (Received September 15, 2020)

1163-65-1677   **Adrianna Gillman***, 526 UCB, Boulder, CO 803090526. *A high order accurate solution
                  techniques for Stokes equations.* Preliminary report.

This talk presents a high order discretization technique for variable coefficient steady Stokes' equation which comes with an efficient direct solver. The discretization is based on local spectral collocation. The local approximations are "glued" together by enforcing the continuity of the traction and the velocity along shared interfaces. The computational cost of constructing the associated nested dissection inspired direct solver is $O(N^{1.5})$ where N is the number discretization points. The cost for applying the inverse scales $O(Nlog(N))$ with a small constant prefactor. The combination of high order approximations and efficient solves make this technique ideal to be combined with time-stepping methods for unsteady Stokes' equation. Numerical results will illustrate the performance of the unsteady Stokes' solver.   (Received September 16, 2020)

68 ▶ *Computer science*

1163-68-93   **Nektarios A Valous*** (nek.valous@nct-heidelberg.de), **Rodrigo Rojas Moraleda**,
                  Eckhard Hitzer, **Dragoş Duşe**, **Meggy Suarez-Carmona**, **Anna Berthel**, **Dirk
                  Jäger**, **Inka Zörnig** and **Niels Halama**. *Processing color information in natural and
                  biomedical images.*

Color pixels can be encoded by a linear combination of the three basis vectors in a hypercomplex algebra framework; this encoding provides the opportunity to process color images in a geometric way. The proposed approach is based on a rapid and flexible method of recolorizing, decolorizing, and separating colors in natural and biomedical (histology) images using quaternions. This pixel-based approach is computationally efficient thus taking advantage of parallel architectures in modern computing systems. This computational method has applications either as a standalone module or as part of automated processing pipelines, and essentially it demonstrates that feature-rich mathematical frameworks provide efficient solutions for color image processing. (Received August 13, 2020)

1163-68-104   **Hannah Guan*** (hguan000@gmail.com). *Predicting biological age: Neural network and algorithmic models for high dimensional DNA methylation data.*

While the traditional formula of DNA methylation aging is based on linear models, few works have explored the effectiveness of neural network models, which can learn more complex relationships from data. DNA methylation data usually consists of hundreds of thousands of feature space, which is much greater than the number of biological samples. This introduces the problem of overfitting which leads to a poor generalization of the neural network model. We propose a neural network model called Correlation Pre-Filtered Neural Network (CPFNN). CPFNN uses Spearman Correlation to pre-filter the features before feeding them into a neural network. We compare CPFNN with the Statistical Regression models (e.g, Horvath, and Hannum's formula), the Basic Neural Network, and the Dropout Neural Network. CPFNN outperforms these models by at least 1 year in terms of mean average error (MAE), with an MAE of 2.7 years. We also test for association between the epigenetic age using CPFNN with Schizophrenia and Down syndrome (p=0.024 and <0.001, respectively). We discover that for a large number of candidate features, such as in genome-wide DNA methylation data, a key factor in improving prediction accuracy is how to appropriately weight features that are highly correlated with the outcome of interest.   (Received August 15, 2020)

1163-68-261   **James M Murphy*** (jm.murphy@tufts.edu). *Data-Dependent Distances for Hyperspectral Images.*

Approaches to unsupervised clustering and semisupervised learning with data-dependent distances are proposed. By considering metrics derived from data-driven graphs, robustness to noise, class geometry, and dimensionality is achieved. The proposed algorithms enjoy theoretical guarantees on flexible data models, and also have quasilinear computational complexity in the number of data points. Applications to remotely-sensed hyperspectral images are emphasized. Portions of this work are joint with Anna Little (Utah), Mauro Maggioni (Johns Hopkins), and Shukun Zhang (Tufts).   (Received August 31, 2020)

1163-68-303   **Jonathan Mosheiff**, **Nicolas Resch**, **Noga Ron-Zewi**, **Shashwat Silas** and **Mary Wootters***, marykw@stanford.edu. *LDPC Codes Achieve List-Decoding Capacity.*

We show that Gallager's ensemble of Low-Density Parity Check (LDPC) codes achieve list decoding capacity. These are the first graph-based codes shown to have this property. Previously, the only codes known to achieve list-decoding capacity were completely random codes, random linear codes, and codes constructed by algebraic (rather than combinatorial) techniques. This result opens up a potential avenue towards truly linear-time list-decodable codes which achieve list-decoding capacity.

Our result on list decoding follows from a much more general result: any local property satisfied with high probability by a random linear code is also satisfied with high probability by a random LDPC code from Gallager's distribution. Local properties are properties characterized by the exclusion of small sets of codewords, and include list-decoding, list-recovery and averageradius list-decoding. Along the way, we give a characterization of sets of codewords that are likely to appear in a random linear code, which may be of independent interest.

This is joint work with Jonathan Mosheiff, Nicolas Resch, Noga Ron-Zewi, and Shashwat Silas.   (Received September 01, 2020)

1163-68-315   **Constantino Carlos Reyes-Aldasoro*** (reyes@city.ac.uk), Northampton Square, London, EC1V 0HB, United Kingdom, and **Cefa Karabag** and **Martin L Jones**. *Semantic Segmentation of Cancerous Cells observed with Electron Microscopy: An Objective Comparison between one Image Processing Algorithm and Four Deep-Learning Architectures.*

In recent years, advances on computational techniques, have provided excellent results in many tasks, in some cases outperforming human levels. Those techniques are sometimes categorised as Artificial Intelligence or Deep Learning with Convolutional Neural Networks being very popular. In this presentation I will explore the objective quantitative comparison between four deep learning models (VGG16, ResNet18, Inception-ResNet-v2, and U-Net) and one traditional image processing algorithm as applied in a particular biomedicine task, namely, the semantic segmentation of cancerous cells as observed with electron microscopy. The segmentations were compared by measuring pixel-based segmentation accuracy and Jaccard index against a labelled ground truth. The results indicated a superior performance of the traditional algorithm (Accuracy = 0.99, Jaccard = 0.93) over the deep learning architectures: VGG16 (0.93, 0.90), ResNet18 (0.94, 0.88), Inception-ResNet-v2 (0.94, 0.89), and U-Net (0.92, 0.56).   (Received September 02, 2020)

1163-68-341   **Tamar Schlick*** (`schlick@nyu.edu`), Department of Chemistry, 100 Washington Square,
Silver Building, New York University, New York, NY 10003. *Structure-Altering Mutations
of the SARS-CoV-2 Frame Shifting RNA Element.*
Due to the high sequence conservation of the SARS-CoV-2 viral genome, its close evolutionary relationship
to other viruses, and the rise of gene editing and RNA-based vaccines, studies focused on the RNA genome
form a complement to work focusing on the viral proteins. Here we apply our graph-theory-based framework for
representing RNA secondary structures "RAG" (RNA-As Graphs) to destroy key structural features of the frame-
shifting element of the SARS-CoV-2 virus, one of three highly conserved regions of coronaviruses, potentially
inhibiting protein synthesis. Specifically, using RAG machinery of genetic algorithms for inverse folding adapted
for pseudoknots, we computationally predict minimal mutations that destroy a stem and/or the pseudoknot of
the frame-shifting RNA element. Such transformations potentially dismantle the virus against translation of the
polyproteins that start the viral replication cycle. These findings not only advance our computational design of
RNAs containing pseudoknots; they pinpoint to key residues of the virus as targets for anti-viral drugs and gene
editing approaches.   (Received September 03, 2020)

1163-68-369   **Austin R Benson*** (`arb@cs.cornell.edu`), **Jon Kleinberg** and **Nate Veldt**.
Hypergraph Cuts with General Splitting Functions.
The minimum s-t graph cut is a fundamental combinatorial optimization problem, and graph cuts underlie
algorithms throughout applied mathematics. While graphs are a standard model for pairwise relationships, hy-
pergraphs provide the flexibility to model multi-way relationships, and are now a standard model for complex
data and systems. However, when generalizing from graphs to hypergraphs, the notion of a "cut hyperedge" is
less clear, as a hyperedge's nodes can be split in several ways. Here, we develop a framework for s-t hypergraph
cuts by considering various models for penalties at cut hyperedges and use this to develop new local hyper-
graph clustering algorithms for mining data from product reviews and online question-and-answer platforms.
(Received September 04, 2020)

1163-68-446   **Manuel E Lladser*** (`manuel.lladser@colorado.edu`), Department of Applied
Mathematics, University of Colorado, Boulder, CO 80309-0526. *Low-dimensional
representation of genomic sequences.*
Numerous data analysis and data mining techniques require that data be embedded in a Euclidean space. When
faced with symbolic datasets, particularly biological sequence data produced by high-throughput sequencing
assays, conventional embedding approaches like binary and k-mer count vectors may be too high dimensional or
coarse-grained to learn from the data effectively. Other representation techniques such as Multidimensional Scal-
ing (MDS) and Node2Vec may be inadequate for large datasets as they require recomputing the full embedding
from scratch when faced with new, unclassified data. To overcome these issues we amend the graph-theoretic
notion of "metric dimension" to that of "multilateration." Much like trilateration can be used to represent points
in the Euclidean plane by their distances to three non-colinear points, multilateration allows us to represent any
node in a graph by its distances to a subset of nodes. Specializing to Hamming graphs, which are particularly
well suited to representing biological sequences, we can readily generate low-dimensional embeddings to map
sequences of arbitrary length to a real space. This work is in collaboration with Richard C. Tillquist, and has
been partially funded by the NSF grant 1836914.   (Received September 07, 2020)

1163-68-708   **Bill Kay***, 1 Bethel Valley Rd, Oak Ridge, TN 37830, and **Pasanna Date**, **Catherine
Schuman** and **Jade O'Connor**. *Graph Algorithms in Neuromorphic Computing.*
Neuromorphic computing architectures are spike based processors that differ from traditional (i.e., GPU, CPU)
compute nodes. In particular, they are build on top of a (weighted, directed) graph and function very much
like brains. Their unique architecture makes them well suited for a number of tasks, including implementation
of graph algorithms in a low energy, highly parallel way. In this talk, I will provide a primer on neuromorphic
computing, and explain how one implements several graph algorithms on neuromorphic machines. No background
on neuromorphic computing (or programming in general) is required.

This is joint work with Prasanna Date, Catherine Schuman, and Jade O'Connor   (Received September 11,
2020)

1163-68-724   **Shi Bai***, Florida Atlantic University, 777 Glades Road, Boca Raton, FL 33431. *Concrete
hardness in lattice-based cryptography.*
The learning with errors (LWE) problem introduced by Regev (STOC'05) is one of the fundamental problems
in lattice-based cryptography. It has been used extensively as a security foundation, for public-key encryption,
signatures, fully homomorphic encryption, pseudo-random functions and many others.

A lattice-based strategy to solve the LWE problem often reduces the LWE problem to a unique SVP (uSVP) problem via Kannan's embedding and then applies a lattice reduction to solve the underlying uSVP problem. In this talk, we will discuss and compare several lattice-based strategies for solving LWE, and give some concrete estimates for breaking variants of LWE. Furthermore, we will discuss some recent developments in lattice reduction algorithms and discuss refined concrete hardness analysis.   (Received September 11, 2020)

1163-68-1002   **Rodrigo Rojas Moraleda*** (rodrigo.moraleda@nct-heidelberg.de), AG. Jäger, Im Neuenheimer Feld 460, 69120 Heidelberg, 69120 Heidelberg, Germany, and **Bénédicte Lenoir**, **Wei Xiong**, **Nektarios A. Valous** and **Niels Halama**. *Detection and segmentation of capillary structures in biomedical images based on a computational topology framework.* Preliminary report.

In histology, identifying and classifying 3D structures such as lymphatic capillaries is not a trivial task. This is mainly due to the wide variability in shapes and sizes adopted by these capillaries after the biological sample is prepared on a glass slide and digitized as a 2D image. On the other hand, topology and homology are mathematical frameworks well known for providing tools to classify spaces, regardless of their geometry.

This work is a continuation of the progress made in the development of computerized analysis algorithms based on the principles of persistent homology to filter and classify cell patterns in histological images. In particular, this work focuses on cell patterns describing walls of lymphatic capillaries.

By changing the representation of a histological image into a collection of simplicial complexes where cells represent vertices and edges represent relationships between pairs of cells, homology classes dimensions one and two are computed to perform the segmentation and classification of lymphatic capillaries.

For performance evaluation, images acquired from histological sections of ovarian tissue are used. Accuracy is verified against expert annotations.   (Received September 14, 2020)

1163-68-1022   **Jessica C De Silva*** (jdesilva1@csustan.edu), **Darcy Brunk**, **Juan Valencia** and **Talita Perciano**. *Image Segmentation via Hypergraph-based MRF Models.* Preliminary report.

X-ray micro-tomography (μ-CT) is a non-destructive 3D imaging technique often used to image material samples. The Advanced Light Source at Berkeley National Laboratory houses a synchroton-based μ-CT instrument which produces high volumes of data at a fast rate. This has led to the need for image processing techniques capable of extracting valuable information in large complex data sets. Image segmentation is an important processing step which separates various components in an image. Graph-based segmentation algorithms have been used for many years, with current interest primarily in designing algorithms which can handle large data sets. Recent approaches using Markov Random Fields (MRFs) exploit local properties of MRFs to run computations in parallel. We have developed an image segmentation algorithm using a hypergraph-based MRF model. The algorithm is coded in C++ and preliminary results indicate that this generalized model improves the precision of the segmentation of μ-CT images.   (Received September 14, 2020)

1163-68-1044   **Anna Lysyanskaya*** (anna_lysyanskaya@brown.edu), Brown University, Providence, RI 02912. *Mercurial Signatures.*

A canonical digital signature scheme consists of three algorithms: key generation, signing, and verifying. It needs to be (1) correct: verification accepts (PK,M,σ) if σ is the output of the signing algorithm on input (SK,M), for SK corresponding to PK, and (2) unforgeable: (informally) a signature that verifies under PK can only be produced by PK's owner.

In a <u>mercurial</u> signature, public keys and messages are partitioned into equivalence classes using relations $\equiv_k$ and $\equiv_m$, and there are additional algorithms:

* ConvertSig: On input (PK,M,σ) where σ is a valid signature on M under public key PK, output (PK',M,σ'), where PK' $\equiv_k$ PK, and σ' is a valid signature on M under public key PK'.

* ChangeRep: On input (PK,M,σ) where σ is a valid signature on M under public key PK, output (PK,M',σ'), where M' $\equiv_m$ M, and σ' is a valid signature on M' under public key PK.

Further, for an appropriate choice of message space and public key space, the new message M' cannot be linked to the original M, and the new public key PK' cannot be linked to PK.

In this talk we will go over constructions and applications of mercurial signatures and open problems related to them.   (Received September 15, 2020)

1163-68-1195   **Rachid Ait Malem Lahcen*** (`rachid@ucf.edu`), Depatment of Mathematics, University
of Central Florida, Orlando, FL 32816, and **R. N. Mohapatra** (`ram.mohapatra@ucf.edu`),
Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Assessment
of Human Factors to Mitigate Insider Threat.*

Despite the importance of human error, research focus is still on computer systems to combat cyber breaches. Considering human error and insider risks is an under-explored or under-researched area in cybersecurity. Humans remain the weakest link in threats to cybersecurity and insider threat causes some of the major confidentiality breaches. Leading industries in human factors aim at tight levels of security because of their inclusion of human error in their system design. In this presentation, we attempt to shine light on understanding human factors in cybersecurity and propose the use of Object Measurement to understand and mitigate insider threat. (Received September 15, 2020)

1163-68-1277   **Erica Blum**, **Jonathan Katz**, **Chen-Da Liu-Zhang** and **Julian Loss***
(`lossjulian@gmail.com`). *Asynchronous Byzantine Agreement with Subquadratic
Communication.*

Byzantine agreement (BA) is a fundamental problem in distributed computing. In this context, n parties agree on a common output by running a distributed protocol. This should hold even if a powerful adversary adaptively corrupts some f of those parties and makes them deviate from the protocol description arbitrarily. As applications of BA typically involve large numbers of parties, it is critical to understand how the communication complexity of BA scales with n. Protocols with $o(n^2)$ communication complexity (CC) have been obtained under the assumption that all messages are delivered within some worst-case message delay. However, no solution exists for the asynchronous model where delays can be arbitrary, as long as messages are eventually delivered. This leads us to ask: Are there asynchronous BA protocols with $o(n^2)$ CC tolerating $f < n/3$ adaptive corruptions? We give the first positive and negative answers to this question: 1) We show asynchronous BA protocols with (expected) subquadratic CC tolerating an adaptive adversary that corrupts $f < n/3$ parties. Our protocol uses cryptographic setup by a trusted dealer. 2) We show that some form of setup is inherent for subquadratic BA tolerating $O(n)$ corrupted parties.   (Received September 15, 2020)

1163-68-1320   **Anamaria Costache*** (`anamaria.costache@ntnu.no`). *Fully Homomorphic Encryption.*
In 1978, Rivest, Adleman and Dertouzos introduce the concept of Fully Homomorphic Encryption (FHE). Dubbed as the "holy grail" of cryptography, the problem will remain unsolved for another 30 years when Gentry offered the first construction in 2009. In this talk, we will discuss FHE, its mathematical foundations and (some of) its challenges.   (Received September 15, 2020)

1163-68-1432   **John M. Schanck*** (`jschanck@uwaterloo.ca`). *The impact of quantum computation on
the security of lattice based encryption schemes.* Preliminary report.
There is a close relationship between the security of lattice-based encryption schemes and the computational complexity of finding a short vector in a lattice. Asymptotically, the fastest known algorithm for finding a short vector is based on a technique called sieving. In practice, however, speed records have been held by a variety of asymptotically inferior techniques, and sieving has only recently become a practical tool. A similar situation might play out again with asymptotically inferior quantum algorithms outperforming quantum sieving in practice. We will discuss how likely this is based on recent work on the non-asymptotic complexity of quantum algorithms for the shortest vector problem.   (Received September 15, 2020)

1163-68-1548   **Matthew J Hirn*** (`mhirn@msu.edu`), 428 South Shaw Lane, East Lansing, MI 48824.
Spectral Theory and Geometric Deep Learning.
Neural networks on graphs and manifolds generalize convolutional neural networks for images and other signals by arranging their artificial neurons according to the underlying structure imposed by the graph or manifold. Ideas from spectral graph theory and spectral geometry play a prominent role here, particularly for so-called "spectral-based" networks. In this talk we'll consider a wide class of networks, including those that would not be classified as spectral-based, and we'll show that nevertheless ideas from spectral graph theory can still be used to obtain insights into the nature of the network, including properties related to invariance and stability. In many cases analogous results on manifolds hold as well. Time permitting, we'll discuss some problems for the case when the graph approximates a manifold, and the role manifold learning can play here.   (Received September 15, 2020)

70 ► *Mechanics of particles and systems*

1163-70-1278   **Lorenzo Sala***, 180 Queen's Gate, London, SW7 2AZ, United Kingdom, and **Pierre Degond**, **Angelika Manhart**, **Sara Merino-Aceituno** and **Diane Peurichard**. *Sperm motility pattern formation study via a swimmer-obstacle-fluid interactions model.* Preliminary report.

Roughly 15% of UK couples has difficulty conceiving, and in about one quarter of these cases, the cause of infertility cannot be identified. One of the main challenges is the lack of information in the processes involved in the reproduction of mammalians, in particular in the sperm selection operated by the female reproductive tract. One important element involved in this mechanism is mucus, the complex hydrogel composed of a protein called mucin arranged in a network structure and immersed in water through which the sperm cells must swim. We propose a mathematical framework that captures the discrete interactions between swimming sperm and the mucin network within water and use this model to understand the physical mechanisms of sperm selection in the female tract through large-scale simulation and multiscale analysis. We propose a preliminary sensitivity analysis, which led to the uncovering of different swimmer patterns forming. Although it is still unclear exactly how the mucin network differentiates the healthy, viable sperm from the abnormal cells, experiments have provided clear evidence that active selection is occurring. Thus, this is a first step to understand these underlying mechanisms of sperm selection, in order to better diagnose infertility going forwards.   (Received September 15, 2020)

76 ► *Fluid mechanics*

1163-76-567   **Animikh Biswas** and **Jing Tian***, 7800 york road, Towson, MD , and **Suleyman Ulusoy**. *Error Estimates for Deep Learning Methods in Fluid Dynamics.*

In this study, we provide error estimates and stability analysis of deep learning techniques for certain partial differential equations including the incompressible Navier-Stokes equations. In particular, we obtain explicit error estimates (in suitable norms) for the solution computed by optimizing a loss function in a Deep Neural Network approximation of the solution, with a fixed complexity.   (Received September 09, 2020)

1163-76-569   **Tej-eddine Ghoul**, **Slim Ibrahim** and **Quyuan Lin*** (`abellyn@math.tamu.edu`), 155 Ireland St, Blocker Building, College Station, TX 77840, and **Edriss S. Titi**. *Three-dimensional Inviscid Primitive Equations with Rotation.*

Large scale dynamics of the oceans and the atmosphere are governed by the primitive equations (PEs). It is well-known that the $3D$ viscous primitive equations are globally well-posed in Sobolev spaces. In this talk, I will discuss the ill-posedness in Sobolev spaces, the local well-posedness in the space of analytic functions, and finite-time blowup of solutions to the $3D$ inviscid PEs with rotation (Coriolis force). Moreover, I will also show, in the case of "well-prepared" analytic initial data, the regularizing effect of the Coriolis force by providing a lower bound for the life-span of the solutions that grows toward infinity with the rotation rate. Joint work with Tej-eddine Ghoul (New York University in Abu Dhabi), Slim Ibrahim (University of Victoria), and Edriss S. Titi (Texas A&M and University of Cambridge).   (Received September 14, 2020)

1163-76-1060   **Yuan-Nan Young*** (`yyoung@njit.edu`), 519 Cullimore Hall Dept. of Math. Sci., University Heights, NJIT, Newark, NJ 07102, and **James Feng**, Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, Canada. *Modeling and Simulating Hydrogel as a Viscous Fluid in a Deformable Porous Medium.*

Hydrogels consist of a polymer skeleton hydrated by an aqueous solvent, and their hydrodynamics is often described by a coarse-grained poroelasticity model where the boundary conditions between the hydrogel and a surrounding solvent require careful consideration. Young et al. (Phys. Rev. Fluids, 4, 063601, 2019) used the energy dissipation principle to derive a set of boundary conditions regarding the velocity jumps at the interface. However, when applied to an external shear flow over a gel layer, these conditions predict no entrained flow inside the gel, in contrast to the prediction of a previous model by Minale (Phys. Fluids 26, 123102, 2014). We adapt Young et al.'s procedure to derive an alternative set of boundary conditions that does allow an external shear flow to induce shear inside the gel, and compare the velocity profile to that of Minale. We also derive the limiting form of the boundary conditions in a Darcy medium.   (Received September 14, 2020)

1163-76-1085   **Wai-Tong Fan**, **Michael Jolly*** (`msjolly@indiana.edu`) and **Ali Pakzad**. *Three-dimensional shear driven turbulence with noise at the boundary.*

We study the effect of noise in the movement of a boundary wall in three-dimensional shear flow on the energy dissipation rate. Upper bounds on the expected value of the dissipation rate and on its variance are found.

The expected value estimate recovers the bound by Doering-Constantin [Phys. Rev. Lett. 69, 1992] for the deterministic case. The movement of the boundary is given by an Ornstein–Uhlenbeck process   (Received September 15, 2020)

1163-76-1141   **Malgorzata Peszynska***, Oregon State University, Corvallis, OR 97330, and **Azhar Alhammali**, **Lisa Bigler** and **Choah Shin**. *Biofilm with multiple species: variational inequality versus other approaches.*
In the talk we consider a model for biofilm embedded in a surround liquid. Biofilms are complex structures composed of gel-like polymeric substance called EPS, and of microbial cells which produce this EPS. Given access to sufficient nutrient resources, the microbes multiply until their maximum density is achieved, after which the biofilm domain expands through the interface with the surround liquid. The liquid may provide nutrient as well as medium for microbial cells at low concentrations, i.e., planktonic cells. The liquid and biofilm are separated by a sharp or diffuse interface, and together they form a fluid with very complex properties.

Our model consists of a coupled system of partial differential equation model under constraints describing the dynamics of biofilm plus EPS and nutrient. In our prior work we studied using a system of parabolic variational inequalities for which we proved convergence of a finite element approximation scheme. However, we considered no advection. This talk is devoted to extensions of the model to multiple species, allowing for advection, as well as to a comparison with a phase field model. We evaluate practical use of our model compared to other models in the setting when the biofilm is considered as part of porous medium.   (Received September 14, 2020)

1163-76-1181   **Jon Wilkening*** (`wilkening@berkeley.edu`), 970 Evans Hall, Department of Mathematics, University of California, Berkeley, CA 94720-3840. *Quasi-periodic water waves.*
We present a framework to compute and study two-dimensional gravity-capillary water waves that are quasi-periodic in space and/or time. This means they can be represented as periodic functions on a higher-dimensional torus by evaluating along irrational directions. In the spatially quasi-periodic case, we consider both traveling waves and the general initial value problem. The former are a generalization of the classical Wilton ripple problem. In both cases, the nonlocal Dirichlet-Neumann operator is computed using conformal mapping methods and a quasi-periodic variant of the Hilbert transform. We devise a shooting method to compute temporally quasi-periodic water waves that are either hybrid traveling-standing waves that return to a spatial translation of their initial condition at a later time or are nonlinear superpositions of several standing waves with irrationally related periods. Many examples will be given to illustrate the types of behavior that can occur.   (Received September 15, 2020)

1163-76-1322   **Lucia Carichino*** (`lcsma1@rit.edu`) and **Sarah D Olson** (`sdolson@wpi.edu`). *Modeling sperm motility in a complex fluid environment.*
Sperm are navigating in a complex three-dimensional (3D) fluid environment in order to achieve fertilization. Sperm trajectories vary from planar to helical depending on species, on external fluid properties and on proximity to walls. Biochemical signaling along the sperm flagellum, such as changes in calcium, regulates sperm trajectories and flagellar beat patterns. We present a fluid-structure interaction model of the sperm flagellum 3D motion that accounts for calcium signaling in the flagellum, interactions with a planar wall, and sperm-sperm interactions. The fluid is modeled as a Newtonian viscous fluid and the flagellum is modeled as an elastic rod with preferred curvature and twist, using the Kirchhoff rod model. The fluid-structure interaction problem is solved using the regularized Stokeslets method, and the effect of a planar wall is implemented via the method of images. The calcium dynamics, represented as a reaction-diffusion model on the moving flagellum, is coupled to the sperm motility via the flagellum curvature. Model results of 3D emergent waveforms and trajectories are compared to the planar case, and to experimental data.   (Received September 15, 2020)

78 ▶ *Optics, electromagnetic theory*

1163-78-438   **Cristian Gutierrez** and **Henok Z. Mawi*** (`henok.mawi@howard.edu`). *Numerical Method for the Near Field Refractor Problem.*
In this presentation we will discuss a numerical method to approximate a solution to the near field refractor problem. This inverse problem involves determining an interface between two media that is capable of refracting a light beam of a given illumination intensity emanating from a punctual source in one medium, to rays that will illuminate a certain target set located in another medium while forming a prescribed irradiance distribution on the target set. A mathematical model to the problem and a method of iteratively constructing the solutions for

the problem by using a set of primitive surface elements will be shown. More specifically the convergence of the iterative method will be discussed. This is joint work with Cristian Gutiérrez.   (Received September 06, 2020)

1163-78-1304   **David Messinger*** (`messinger@cis.rit.edu`), Center for Imaging Science, Rochester Institute of Technology, 54 Lomb Memorial Dr, Rochester, NY 14623, and **S. Huang** (`sh1452@rit.edu`), Center for Imaging Science, Rochester Institute of Technology, 54 Lomb Memorial Dr, Rochester, NY 14623. *Radiometrically Accurate Spatial Resolution Enhancement of Spectral Imagery for Improved Exploitation.*

The remote sensing community has a long history of developing methods to fuse high spatial resolution information from panchromatic sensors with the spectrally diverse information, but generally of lower spatial resolution, from spectral sensors. However, these methods have typically had as their primary motivation the creation of color imagery of high visual quality for visual interpretation. These methods do not in general seek to preserve the quantitative spectral information in the data and can sacrifice the radiometric fidelity of the spectral sensor. This presentation will describe current work in the area of spectrally accurate spatial resolution enhancement of hyperspectral imagery, with a goal of ensuring good performance from spectral exploitation algorithms such as target detection. A new algorithm based on a learning framework for MSI - HSI fusion will be presented which leverages unique loss functions to achieve both spatial and spectral accuracy in the sharpened product. Experimental results will be shown for several hyperspectral experimental collections.   (Received September 15, 2020)

81 ► *Quantum theory*

1163-81-251   **Bely Rodriguez Morales*** (`belymorales@id.uff.br`), R. Prof. Marcos Waldemar de Freitas, Bloco G, Niteroi, Rio de Janeiro, 24210-201, Brazil. *Logarithmic modules over vertex algebras and nilmanifolds.*

While studding chiral differential operators over certain nilmanifolds often some logarithmic singularities appear. We will briefly discus the theory of logarithmic quantum fields and logarithmic modules as a tool to handle this type of singularities. Finally, we will present a highly non-trivial example of logarithmic module by generalizing the construction of vertex operators in terms of exponentiated scalar fields to Jacobi theta functions naturally appearing in these nilmanifolds.   (Received August 30, 2020)

1163-81-256   **Hao Li*** (`hli29@albany.edu`), 1400 Washington Avenue, Albany, NY 12222, and **Antun Milas**. *Jet schemes, Quantum dilogarithm and Feigin-Stoyanovsky's principal subspaces.* Preliminary report.

We analyze the structure of the infinite jet algebra, or arc algebra, associated to level one Feigin-Stoyanovsky's principal subspaces. For A-series, we show that their Hilbert series can be computed either using the quantum dilogarithm or as certain generating functions over finite-dimensional representations of A-type quivers. In particular, we obtain new fermionic character formulas for level one A-type principal subspaces, which implies that they are classically free.   (Received August 30, 2020)

1163-81-260   **Anna Vershynina*** (`anna@math.uh.edu`), 2912 Jackson Street, Houston, TX 77004. *How close is "close" is quantum mechanics?*

The talk will start with a broad crash course in quantum mechanics and quantum information theory. After introducing several measures of "closeness" between quantum states, we will present a notoriously simple Pinsker inequality relating some of these measures. A reverse inequality has been proven to be difficult to obtain and in some cases difficult to calculate. A result of the speaker provides a reversed Pinsker inequality in its most simple form. The result holds for any quasi-relative entropy and qubit states, or for finite-dimensional states and regular relative entropy or Tsallis q-entropy.   (Received August 31, 2020)

1163-81-322   **Bin Gui*** (`bin.gui@rutgers.edu`), Piscataway, NJ 08854. *Vertex algebraic approach and functional analytic approach to 2d rational conformal field theories.*

I will present recent progress in the interplay between the two approaches to 2d rational CFT mentioned in the title. I will show how certain problems in one approach can be (or perhaps can only be) solved using results and techniques from the other one.   (Received September 02, 2020)

1163-81-418   **Adam Bene Watts** and **Bill Helton*** (helton@math.ucsd.edu). *3XOR games with perfect quantum strategies.*

Computing the entangled value of a kXOR game amounts to optimizing a particular k linear noncommutative hermitian function b of hermitian operators S_j on Hilbert space H satisfying $S_j^2 = I$ and simple commutation relations. Always $b \preceq I$ and a set S of S_j making $\|b(S)\| = 1$ is called a perfect "quantum strategy". This paper concerns 3XOR in the case where a perfect Qstrategy exists. Tsirelson completely handled Qstrategies for two players, 2XOR, in 1987. Not perfect 3XOR Qstrategies have potentially ∞ advantage over classical strategies, but approximating the entangled value of 3XOR games is at least NP hard (2013), and may be undecidable. We show whether or not a 3XOR game has a perfect quantum strategy is decidable in polynomial time. We do this with a constructive proof: if a perfect Qstrategy exists, it is achievable in 8 dimensions, but the Q advantage over a classical strategy is bounded.   The proof introduces an intermediate algebra $\mathcal{A}$ which satisfies the physical relations above plus the requirement that pairs S_iS_j of hermitian operators commute. Then it shows (unexpected and not easy) that $\max b$ over $\mathcal{A}$ is 1 iff the original game is perfect.   (Received September 05, 2020)

1163-81-442   **Tai-Danae Bradley*** (tai.danae@math3ma.com). *Exploring Language with Linear Algebra.*

The linear algebra of quantum mechanics provides a good starting point for investigating mathematical structure found in natural language. In this talk, I'll give a brief tour of these ideas, showing how a probability distribution on a finite set of sequences can be modeled by a rank 1 density operator, how to assign reduced density operators to expressions in language, and how this assignment fits nicely within the framework of category theory. (Received September 07, 2020)

1163-81-1569   **Du Pei***, Center for Math. Sciences and Applications, 20 Garden St, Cambridge, MA 02138. *Quantization of Coulomb Branches and Double Affine Hecke Algebras.*

The study of Coulomb branches of supersymmetric quantum field theories have in recent years led to many interesting insights into geometry and representation theory. In this talk, I will discuss the A-models with Coulomb branch targets, and show how to use them to better understand the representation theory of the double affine Hecke algebra.   (Received September 15, 2020)

85 ▶ *Astronomy and astrophysics*

1163-85-1353   **S. James Gates*** (sylvester_gates@brown.edu), Brown Theoretical Physics Center, Director's Office, Rm. 110, 340 Brook St., Providence, RI 02912. *Linking Supersymmetry in Physics to Permutations in Mathematics.*

The permutation group S_4 is a well known structure in mathematics. On the physics side the concept of "supersymmetry" as applied to elementary particles beyond the Higgs boson has been a source of vigorous investigation for over a decade. The "permutahedron" provides a representation of the group and can be connected to hypothetical superpartners in beyond the Standard Model physics speculations.   (Received September 15, 2020)

86 ▶ *Geophysics*

1163-86-367   **Amanda Ziemann***, ziemann@lanl.gov, and **Christopher Ren** and **James Theiler**. *Detecting anomalous changes across remote sensing images from different satellites.*

Combining multiple satellite remote sensing sources can provide a far richer, more frequent view of the earth than that of any single source; the challenge is in distilling this large volume of heterogeneous sensor imagery into meaningful characterizations of the imaged areas. The traditional approach to change detection involves difference-based techniques, but these do not naturally extend to image pairs captured by sensors with different designs and modalities. To leverage imagery in this multi-sensor context, algorithms are being developed to effectively combine different kinds of sensor imagery that can identify subtle but important changes among the intrinsic data variation, e.g., multispectral to synthetic aperture radar. Here, we implement a joint-distribution framework for anomalous change detection that can effectively "normalize" for these changes in modality. Results are shown using satellite imagery from different sensor platforms over time.   (Received September 04, 2020)

1163-86-493   **Mikhail Gilman** and **Semyon Tsynkov*** (`tsynkov@math.ncsu.edu`), Department of
Mathematics, North Carolina State University, Box 8205, Raleigh, NC 27695. *Synthetic
Aperture Radar Imaging through a Turbulent Ionosphere.*

Earth's ionosphere causes distortions of spaceborne synthetic aperture radar (SAR) images. They depend on the state of the ionosphere and parameters of the sensor. In particular, turbulent ionosphere presents a major challenge because image distortions acquire a random component. The performance in azimuth of low-frequency systems (P-band) can be particularly vulnerable to random fluctuations of the propagation phase.

We consider the size of the synthetic array relative to the scale of ionospheric turbulence and characterize image distortions in terms of blurring and azimuthal shift. Specifically, we show that in the case of large-scale turbulence, a high level of eikonal fluctuations can coexist with the low degree of image distortions, and that blurring becomes significant at much higher levels of fluctuations than the shift. In the small-scale case, a low level of eikonal fluctuations is a precondition for imaging, while the magnitude of distortions depends on the ratio between the eikonal correlation radius and the length of the synthetic aperture.

We also discuss the potential mitigation strategies based on autofocus.   (Received September 08, 2020)

1163-86-1172   **Julien Chaput*** (`jachaput@utep.edu`), 500 W University ave, El Paso, TX 79968, and
Richard Aster (`rick.aster@colostate.edu`), Warner College of Natural Resources, Fort
Collins, CO 80523. *Seismic resonances in dynamic snow environments: modeling climate
forcing through singing ice.*

Significant pushes in the pace of Antarctic data collection have occurred over the past decade in light of accelerating climate change. Multiple unknowns currently impede models describing ice mass evolution and stability, and there exists a hierarchy of effects that cascade into the disintegration of these systems. Of those effects, firn, the uppermost porous snow that progressively compacts into solid ice, is viewed as the most sensitive link in the chain, and loss of the firn due to melt may cause ice shelf destruction through melt water ponding and hydrofracture. Here, we present the results of ongoing work on the Ross Ice Shelf, whereby spectrograms of passively recorded seismic signals across a broad array reveal dissonant resonance patterns caused by trapped waves in the top few meters of snow. These ghostly songs can be leveraged to model a slew of surface climate forcing effects, from variations in surface snow forms due to passing storms, to direct melt related to periods of anomalously high temperatures, to variable patterns of crevassing and tensile strain. Spectrograms of this data thus provide information-rich images of the "state of mind" of the cryosphere and could be learned to provide near real-time estimates of stress undergone by such environments.   (Received September 14, 2020)

90 ▶ *Operations research, mathematical programming*

1163-90-70   **Ram Verma*** (`ramverma114@yahoo.com`). *SEMIINFINITE FRACTIONAL PROGRAMS.*
Preliminary report.

Semiinfinite fractional programs have a wide range of robotic -engineering models for Automobile Assembly Plants and other industries, based on future of sensing robots, future of humanoid robots, and future virus-testing -robots for robust testing and contact tracking capabilities involving Medical Doctors/Medical Scientists. (Received August 06, 2020)

1163-90-156   **Elena Constantin*** (`constane@pitt.edu`), University of Pittsburgh at Johnstown,
Mathematics Department, 450 Schoolhouse Road, Johnstown, PA 15904. *Primal Necessary
Conditions for a Weak Minimum for Nonsmooth Degenerate Multiobjective Problems.*

In this talk we provide primal first and second-order necessary conditions for the existence of a local weak minimum for nonsmooth multiobjective optimization problems with inequality constraints and an arbitrary constraint set. Our conditions are formulated in terms of first and second-order tangent cones in Pavel-Ursescu sense. We give a constructive characterization of the second-order tangent cone to a degenerate equality constraint set. Our characterization allows us to derive primal necessary conditions for nonsmooth multiobjective optimization problems with inequality constraints and degenerate equality constraints. In our primal necessary conditions we do not suppose any constraint qualifications or regularity conditions or any kind of differentiability of any order of the objective and inequality constraint functions.The effectiveness of our results is illustrated on an example. (Received August 24, 2020)

1163-90-226   **Daniel Bartl** (`daniel.bartl@univie.ac.at`), **Samuel Drapeau**
(`sdrapeau@saif.sjtu.edu.cn`), **Jan Obloj** (`jan.obloj@maths.ox.ac.uk`) and **Johannes
Wiesel*** (`jw4043@columbia.edu`). *Robust uncertainty sensitivity quantification.*
We consider sensitivity of a generic stochastic optimization problem to model uncertainty. We take a non-parametric approach and capture model uncertainty using Wasserstein balls around the postulated model. We provide explicit formulae for the first order correction to both the value function and the optimizer and further extend our results to optimization under linear constraints. We present applications to statistics, machine learning, mathematical finance and uncertainty quantification. In particular, we prove that LASSO leads to parameter shrinkage, propose measures to quantify robustness of neural networks to adversarial examples and compute sensitivities of optimised certainty equivalents in finance. We also propose extensions of this framework to a multiperiod setting. This talk is based on joint work with Daniel Bartl, Samuel Drapeau and Jan Obloj. (Received August 28, 2020)

1163-90-384   **Lawrence D Stone*** (`stone@metsci.com`), 1818 Library Street, Suite 600, Reston, VA
20190. *Optimal Bayesian Search for Moving Targets.*
Bayesian search theory is used by the US Coast guard to find people and boats lost at sea. In Bayesian search theory, one starts with a prior distribution on the location of the target, a constrained amount of search effort, and a detection function that relates effort as a function of location to the probability of detecting the target given it is in that location . The basic search problem is to find an allocation of effort over locations that maximizes the probability of detecting the target within the effort constraint. For stationary targets, this problem has been solved in great generality over a range of location spaces (discrete and continuous) and detection functions. In the case where the target is moving, the prior distribution becomes a stochastic process giving a probabilistic description of the target's location as function of time. The optimal search for a moving target problem is to find an allocation of effort as a function of time that maximizes the probability of detecting the target by time T while satisfying an effort constraint at each time in [0,T]. We show that a certain class of moving target problems can be solved by solving a sequence of stationary target problems and how this result leads to an efficient algorithm for find optimal plans.   (Received September 04, 2020)

1163-90-449   **Lynesia R Taylor*** (`lrtaylo2@ncsu.edu`). *Portfolio Rebalancing with Illiquid Assets.*
Consider a portfolio consisting of three types of assets, a bond, a liquid risky asset, and an illiquid risky asset. Liquid assets can be traded continuously whereas an illiquid asset can only be traded at specific pre-specified times or randomly. The investor's liquid and illiquid wealth are modeled as stochastic differential equations (SDE). We consider a control problem governed by these SDEs. We will model the rebalancing of the portfolio, transferring between liquid and illiquid wealth, as an impulsive control problem. Using a dynamic programming approach we will derive an integro-differential equation to characterize the solution as well as an explicit form for the value function. Numerical examples are given to validate the correctness for our results.   (Received September 07, 2020)

1163-90-560   **Jiawang Nie** (`jnie@ucsd.edu`), 9500 Gilman Drive #0112, La Jolla, CA 92093-0112, and
Xindong Tang* (`xit039@ucsd.edu`), 9500 Gilman Drive, La Jolla, CA. *Nash Equilibrium
Problems of Polynomials.*
This talk discusses Nash equilibrium problems that are given by polynomial functions. We formulate efficient polynomial optimization problems for computing Nash equilibria. The Lasserre type Moment-SOS relaxations are used to solve them. Under genericity assumptions, the method can find a Nash equilibrium if there is one. Moreover, it can find all Nash equilibria if there are finitely many ones of them. The method can also detect nonexistence if there is no Nash equilibrium.   (Received September 09, 2020)

1163-90-598   **Gábor Pataki** and **Aleksandr Touzov*** (`touzov@live.unc.edu`). *A normal form to
construct all weakly infeasible semidefinite programs and bad projections of the PSD cone.*
Preliminary report.
A weakly infeasible semidefinite program (SDP) has no feasible solution, but it has nearly feasible solutions that approximate the constraint set to an arbitrary precision. These SDPs are ill-posed and numerically often unsolvable. They are also closely related to "bad" linear projections that map the cone of positive semidefinite matrices to a nonclosed set. We develop a simple normal form of weakly infeasible SDPs with the following traits: it is obtained mostly by elementary row operations (inherited from Gaussian elimination); it makes weak infeasibility evident; and using it we can construct any weakly infeasible SDP or bad linear projection by an elementary algorithm.   (Received September 10, 2020)

1163-90-642   **Victoria C. P. Chen*** (`vchen@uta.edu`), **Yuan Zhou**, **Jay M. Rosenberger**, **Alireza Fallahi**, **Amith Viswanatha**, **Jingmei Yang**, **Yasaman Ghasemi**, **Nilabh S. Ohol** and **Askhan A. Farahani**. *Balancing the Protection of COVID-19 At-Risk Populations While Reopening Communities.* Preliminary report.

As communities reopen during the COVID-19 pandemic, they are facing two conflicting objectives. The first is to minimize cases of severe illness leading to hospitalizations and potential fatalities. The second is to revive the U.S. economy and the livelihoods of millions of Americans. This research presents a study of the delicate balance between the expected fatality rate and the level of normalcy in the community. Given the disproportionate fatality characteristics of COVID-19 among those in different age groups or with an underlying medical condition or those living with crowding, the key is a framework focused on "key contacts" that separate individuals at higher risk from the rest of the population. Given the lack of pharmaceutical solutions, i.e., a vaccine or cure, we seek to optimize the use of non-pharmaceutical interventions, namely COVID-19 testing; personal protective equipment; and social precautions, such as distancing, hand-washing, and face coverings. Simulations using an agent-based SEIR epidemic model are presented, and extensions to vaccine deployment are also discussed. (Received September 10, 2020)

1163-90-685   **Arnold Greenland*** (`arnie.greenland@gmail.com`), College Park, MD 20742. *Introduction to Driving Transformation Through Advanced Analytics.* Preliminary report.

The field of that has come to be known as .analytics. has experienced a huge revolution in the last decade. It brings together mathematics, statistics, operations research, multiple fields of engineering, and many other application areas that are data rich, yet in need of efficient ways to leverage that data for good. This special session contains a diverse variety of topics showing how advanced analytics, in many cases using advanced mathematical approaches such as machine learning, optimization, simulation, and related fields to bring important applications to society and business. This talk includes an introduction to advanced analytics for those who are newcomers to the concept and provides a window into how the session is organized and what are some of key areas that are covered in the remaining talks of the Session. Examples in this talk will be taken from the area of interest for the author, which is machine learning applied to fraud detection.   (Received September 11, 2020)

1163-90-696   **Brenda L Dietrich*** (`bld34@cornell.edu`), 217 Rhodes Hall, Ithaca, NY 14850-9597. *Navigating the data analytics minefield.*

"Data is the new resource = we should guard, refine, and sell products based on it." " Consumer data should be owned by the consumer." "You can't predict that". "Advanced algorithms can predict anything, with great precision". " That's a great insight about these clients. We should exploit it" "That's creepy. We don't want to be known for computing that."

Heard any of these? Functional groups within enterprises, in for example marketing, HR, and procurement, are of at least two minds about the use of data to drive decision making, especially operational decision making. This talk will explore the range of viewpoint of the use of data, beliefs on the capabilities of analytics, and perceptions on the value of both. It will offer some thoughts on building confidence in the use of analytics, both for routine decision, and for the design of data and insight products and services.   (Received September 11, 2020)

1163-90-709   **Karla L. Hoffman*** (`khoffman@gmu.edu`), George Mason University, Systems Eng. and Oper. Res. Dept., Mail Stop 4A6, 4400 University Blvd., Fairfax, VA 20124, and **Brian B. Smith** (`brianbsmith@gmail.com`), ICF International, Inc., 9300 Lee Highway, Fairfax, VA 22031. *Operations Research consulting for the U.S. government: A telecommunications success story.*

In this presentation, we present how the optimization team at the Federal Communications Commission (FCC) has been using mathematical optimization to:

(1) Assist in the design and running of the highly-successful Incentive Auction that resulted in revenues of close to $20 Billion.

(2) Schedule the reassignment of over 1000 broadcast TV stations to new channels for over-the-air broadcasting in the United States and Canada in order to free up spectrum for mobile use and 5G and

(3) Describe our continued work for the FCC for both auctions and spectrum availability mapping.

We will present "lessons learned" and suggest how optimization can be used within government settings to assist in policy decisions.   (Received September 11, 2020)

1163-90-715   **R. W. R. Darling*** (`rwdarli@nsa.gov`) and **Jacob D. Baron** (`jdbaron@nsa.gov`).
K-Nearest Neighbor Approximation Via the Friend-of-a-Friend Principle.

Suppose V is an n-element set where for each $x \in V$, the elements of $V \setminus \{x\}$ are ranked by their similarity to x. The K-nearest neighbor graph is a directed graph including an arc from each x to the K points of $V \setminus \{x\}$ most similar to x. Constructive approximation to this graph using far fewer than n^2 comparisons is important for the analysis of large high-dimensional data sets. <u>K-Nearest Neighbor Descent</u> is a parameter-free heuristic where a sequence of graph approximations is constructed, in which second neighbors in one approximation are proposed as neighbors in the next. Run times in a test case fit an $O(nK^2 \log n)$ pattern. This bound is rigorously justified for a similar algorithm, using range queries, when applied to a homogeneous Poisson process in suitable dimension. However the basic algorithm fails to achieve subquadratic complexity on sets whose similarity rankings arise from a "generic" linear order on the $\binom{n}{2}$ inter-point distances in a metric space.   (Received September 14, 2020)

1163-90-796   **Jane Ye*** (`janeye@uvic.ca`), Department of Mathematics and Statistics, University of
Victoria, Victoria, BC V8W 2Y2, Canada. *Second-order optimality conditions for
non-convex set-constrained optimization problems.*

In this paper we study second-order optimality conditions for non-convex set-constrained optimization problems. For a convex set-constrained optimization problem, it is well-known that second-order optimality conditions involve the support function of the second-order tangent set. In this paper we propose two approaches for establishing second-order optimality conditions for the non-convex case. In the first approach we extend the concept of the support function so that it is applicable to general non-convex set-constrained problems, whereas in the second approach we introduce the notion of the directional regular tangent cone and apply classical results of convex duality theory. Besides the second-order optimality conditions, the novelty of our approach lies in the systematic introduction and use, respectively, of directional versions of well-known concepts from variational analysis.   (Received September 12, 2020)

1163-90-963   **Diego Cifuentes***, 77 Massachusetts Avenue, Room 2-246C, Cambridge, MA 02139, and
Ankur Moitra. *Polynomial time guarantees for the Burer-Monteiro method.*

The Burer-Monteiro method is one of the most widely used techniques for solving large-scale semidefinite programs (SDP). The basic idea is to solve a nonconvex program in Y, where Y is an $n \times p$ matrix such that $X = YY^T$. We show that this method can solve SDPs in polynomial time in an smoothed analysis setting. More precisely, we consider an SDP whose domain satisfies some compactness and smoothness assumptions, and slightly perturb the cost matrix and the constraints. We show that if $p \gtrsim \sqrt{(2 + 2\eta)m}$, where m is the number of constraints and $\eta > 0$ is any fixed constant, then the Burer-Monteiro method can solve SDPs to any desired accuracy in polynomial time, in the setting of smooth analysis. Our bound on p approaches the celebrated Barvinok-Pataki bound in the limit as η goes to zero, beneath which it is known that the nonconvex program can be suboptimal.   (Received September 14, 2020)

1163-90-1075   **Gabor Pataki*** (`gabor@unc.edu`). *How do exponential size solutions arise in semidefinite
programming?* Preliminary report.

It is known that in semidefinite programming (SDP) exponential size solutions may arise. We show that a suitable transformation reveals a certain hierarchy among the variables in any SDP that satisfies Slater's condition, in many cases yielding exponential size solutions. We also look at how exponential size solutions appear in practical problems.   (Received September 14, 2020)

1163-90-1143   **Ludovic Tangpi***, ludovic.tangpi@princeton.edu, and **Daniel Bartl**,
daniel.bartl@univie.ac.at. *Non-Asymptotic rates for the estimation of risk measures.*

Consider the problem of computing the riskiness of a financial position F written on the underlying S with respect to a general law invariant risk measure (for instance the average value at risk). In practice the true distribution of S is unknown and one needs to resort to historical data for the computation. In this talk we present rates of convergence results to the riskiness of F(S) when the distribution of S is estimated by its empirical measure given N observations. We will present (sharp) non-asymptotic rates for both the deviation probability and the expectation of the estimation error. This talk is based on a join work with Daniel Bartl.   (Received September 14, 2020)

1163-90-1194   **Michael P Johnson*** (`michael.johnson@umb.edu`), 100 Morrissey Blvd., McCormack Hall, Room 3-428A, Dept of Public Policy and Public Affairs, Boston, MA 02125-3393. *Community-engaged operations research: Quantitative and qualitative problem-solving for local impact and social change.* Preliminary report.

In these days of 'analytics', 'big data' and 'smart cities', many of us are aware of the importance of data and quantitative analytic methods for program design, implementation and evaluation. However, many mission-driven but resource-constrained nonprofit organizations face problems that may not match well with conventional quantitative methods, at least initially.

'Community-engaged operations research' can enable such community-based organizations to collect and use data for creative decision-making. I will discuss ways that qualitative and quantitative data can support program design through community involvement in problem identification, data collection and analysis, and recommendations for new services, programs, and infrastructure. This approach can support engagements with community-based organizations as well as directly with community members; it can generate model-based prescriptions that are Pareto-improving, as well as insights about problem situations based mostly on qualitative analysis that can by themselves yield valuable insights for planning and operations. This approach is compatible with efforts to infuse principles of diversity, equity and inclusion, as well as social and racial justice, into structured decision-making and decision support.   (Received September 15, 2020)

1163-90-1230   **Grigoriy Blekherman*** (`greg@math.gatech.edu`), School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30332. *Locally Positive Semidefinite Matrices.*

The cone of positive semidefinite matrices plays a prominent role in optimization, and many hard computational problems have well-performing semidefinite relaxations. In practice, enforcing the constraint that a large matrix is positive semidefinite can be expensive. We introduce the cone of k-locally posiitive semidefinite matrices, which consists of matrices all of whose k by k principal minors are positive semidefinite. We consider the distance between the cones of positive and locally positive semidefinite matrices, and possible eigenvalues of locally positive semidefinite matrices. Hyperbolic polynomials play a role in some of the proofs. Joint work with Santanu Dey, Marco Molinaro, Kevin Shu and Shengding Sun.   (Received September 15, 2020)

1163-90-1567   **Mareike Dressler*** (`mdressler@ucsd.edu`), Department of Mathematics, University of California, San Diego, La Jolla, CA 92093, and **Janin Heuer**, **Helen Naumann** and **Timo de Wolff**. *Global Optimization via the Dual SONC Cone and Linear Programming.*

Using the dual cone of sums of nonnegative circuits (SONC), we provide a relaxation of the global optimization problem to minimize an exponential sum and, as a special case, a multivariate real polynomial. Our approach builds on two key observations. First, that the dual SONC cone is contained in the primal one. Hence, containment in this cone is a certificate of nonnegativity. Second, we show that membership in the dual cone can be verified by a linear program. We implement the algorithm and present initial experimental results comparing our method to existing approaches.   (Received September 15, 2020)

91 ▶ *Game theory, economics, social and behavioral sciences*

1163-91-97   **Joseph A Balderas** (`joseph.balderas@students.tamuk.edu`) and **Aden O Ahmed*** (`aden.ahmed@tamuk.edu`). *Linear Programming Solutions of Finite, Noncooperative, Two Player Games.* Preliminary report.

In this talk, we present a Maple package that computes all the pure and mixed Nash equilibria of finite bimatrix games with real entries. The Maple package comprises two procedures: when a finite bimatrix is prompted by the user, one procedure outputs the pure and mixed Nash equilibria of the game and the other procedure takes in these equilibria as inputs and outputs the payoffs to the players.   (Received August 13, 2020)

1163-91-107   **Moirangthem Tiken Singh** and **Surajit Borkotokey*** (`surajitbor@yahoo.com`), Department of Mathematics, Dibrugarh University, India, Dibrugarh, Assam 786004, India, and **Rachid Lahcen** and **Ram N Mohapatra**. *A Generic Scheme for Cyber Security in Resource Constraint Network Using Incomplete Information Game.*

We propose a model that efficiently activates the defending mechanism of a resource constraint network where activation of the defending system otherwise consumes a significant amount of resource. Our model is de- signed so that it executes the activation of the defending system only when it is needed. It is a standard practice in

this literature that the defender has incomplete information about the attacker's strategy. As the resource is scarce and precious for the defender, she needs to learn the behavior of the attacker to identify the malicious network agents. Triggered by the lack of unavailability of the information about the attacker's action, the attacker leverage the information asymmetry to misinform and mislead any defense system. Thus, we model the interaction between the defender and the attacker as an incomplete stochastic game. We develop a learn- ing algorithm for incomplete information game to update the strategy for playing the game. The results show that it converges and leads to a Nash equilibrium.   (Received August 16, 2020)

1163-91-159   **Igor Cialenco*** (`cialenco@iit.edu`), 10 W 32nd Str, Bld RE, Room 220, Chicago, IL
60616, and **Tomasz R. Bielecki** and **Tao Chen**. *Time-inconsistent Markovian control
problems under model uncertainty with application to the mean-variance portfolio selection.*
We study a class of time-inconsistent terminal Markovian control problems in discrete time subject to model uncertainty. We combine the concept of the sub-game perfect strategies with the adaptive robust stochastic control method to tackle the theoretical aspects of the considered stochastic control problem. As an application of the theoretical results, by applying a machine learning algorithm, we solve numerically the mean-variance portfolio selection problem under the model uncertainty.   (Received August 24, 2020)

1163-91-208   **Eunjung Noh** and **Kim Weston*** (`kimberly.weston@gmail.com`). *Price impact
equilibrium with transaction costs and TWAP trading.*
In this talk, I will discuss an equilibrium model with transaction costs and price impact where two agents are incentivized to trade towards a target. The two types of frictions – price impact and transaction costs – lead the agents to two distinct changes in their optimal investment approach: price impact causes agents to continuously trade in smaller amounts, while transaction costs cause the agents to cease trading before the end of the trading period. As the agents lose wealth because of transaction costs, the exchange makes a profit. I will also discuss the existence of a strictly positive optimal transaction cost from the exchange's perspective.   (Received August 27, 2020)

1163-91-216   **Ibrahim Ekren*** (`iekren@fsu.edu`), Tallahassee, FL. *Kyle-Back Models with risk aversion
and non-Gaussian beliefs.*
We show that the problem of existence of equilibrium in Kyle's continuous time insider trading model can be tackled by considering a system of quasilinear parabolic equation and a Fokker-Planck equation coupled via a transport type constraint. We show the well-posedness of solutions and study the properties of the equilibrium obtained for small enough risk aversion parameter. In our model, the insider has exponential type utility and the belief of the market on the distribution of the price at final time can be non-Gaussian. Based on a joint work with S. Bose.   (Received August 27, 2020)

1163-91-223   **Jetlir Duraj*** (`jetlirduraj@gmail.com`). *Bargaining with Endogenous Learning.*
I study a discrete-time dynamic bargaining game in which a buyer can choose to learn privately about her value of the good. Information generation takes time and is endogenous. After learning, the buyer can disclose verifiable evidence of her valuation to the seller. Examples include venture capital negotiations or procurement of new technologies, which sometimes feature significant delay due to endogenous costly learning. The buyer receives informational rents for any period-length only if learning is costly. The high-frequency limits of stationary equilibria result in a folk-theorem type of result about the delay until agreement. Maximal delay is achieved in equilibria with mixed pricing. Near the high-frequency limit, all stationary equilibria feature non-extreme prices and non-extreme payoffs. The analysis allows for closed-form solutions and for comparative statics.   (Received August 28, 2020)

1163-91-228   **Ruimeng Hu*** (`rhu@ucsb.edu`). *Convergence Of Deep Fictitious Play For Stochastic
Differential Games.*
In this talk, I will focus on the convergence analysis for deep fictitious play, which is a novel machine learning algorithm for finding Markovian Nash equilibrium of large N-player asymmetric stochastic differential games. By incorporating the idea of fictitious play, the algorithm decouples the game into N sub-optimization problems, and identifies each player's optimal strategy with the deep backward stochastic differential equation method parallelly and repeatedly. I will show the proof of convergence of the algorithm to the true Nash equilibrium, and show that the strategy based on DFP forms an ϵ-Nash equilibrium. I will also discuss some generalizations by proposing a new approach to decouple the games and present numerical results of large population games showing the empirical convergence of the algorithm beyond the technical assumptions in the theorems.   (Received August 28, 2020)

1163-91-274   **Maxim Bichuch** and **Zachary Feinstein*** (`zfeinste@stevens.edu`). *Endogenous Price Impacts and Inverse Demand Functions.*

In this talk we present an equilibrium formulation for price impacts. This is motivated by the Bühlmann equilibrium in which assets are sold into a system of market participants and can be viewed as a generalization of the Esscher premium. Existence and uniqueness of clearing prices for the liquidation of a portfolio are studied. Additional properties of the liquidation value of a portfolio are studied, e.g., monotonicity and concavity. Price per portfolio unit sold is also presented. In special cases we study price impacts generated by market participants who follow the exponential utility and power utility.   (Received August 31, 2020)

1163-91-279   **William L. He***, WilliamHe@u.northwestern.edu, and **Christopher M. Lee** and **Alexandria Volkening**. *Mathematical Modeling to Forecast U.S. Elections.*

Forecasting the outcomes of U.S. elections is a pertinent and complex task that has been approached in many ways, most commonly incorporating statistical methods. We take a differential equations approach to forecasting elections by adapting a compartmental model commonly employed in epidemiology. We model the evolution of Democrat or Republican political affiliation across states. We use R to fit our parameters based on polling data and MATLAB to run simulations with the model. Through thousands of simulations of our stochastic differential equations with correlated noise, we forecast a range of election outcomes at the state level, specifically focused on swing states. Our model's final forecasts for past presidential, senatorial, and gubernatorial elections have been comparable to those of popular forecasting sites like FiveThirtyEight. We have examined the accuracy of our model's forecasts of past presidential elections in detail, as well as applied the model to create forecasts for the 2020 U.S. elections. Our work demonstrates the effectiveness of data-driven forecasting from a mathematical-modeling perspective and suggests additional research in this field.   (Received August 31, 2020)

1163-91-588   **Thilini V Mahanama*** (`thilini.v.mahanama@ttu.edu`) and **Abootaleb Shirvani**. *A Natural Disasters Index.*

Natural disasters, such as tornadoes, floods, and wildfire pose risks to life and property, requiring the intervention of insurance corporations. One of the most visible consequences of changing climate is an increase in the intensity and frequency of extreme weather events. The relative strengths of these disasters are far beyond the habitual seasonal maxima, often resulting in subsequent increases in property losses. Thus, insurance policies should be modified to endure increasingly volatile catastrophic weather events. We propose a Natural Disasters Index (NDI) for the property losses caused by natural disasters in the United States based on the "Storm Data" published by the National Oceanic and Atmospheric Administration. The proposed NDI is an attempt to construct a financial instrument for hedging the intrinsic risk. The NDI is intended to forecast the degree of future risk that could forewarn the insurers and corporations allowing them to transfer insurance risk to capital market investors. This index could also be modified to other regions and countries.   (Received September 10, 2020)

1163-91-629   **Maxim Bichuch*** (`mbichuch@jhu.edu`), **Benjamin Hobbs** (`bhobbs@jhu.edu`) and **Xinyue Song** (`xsong11@jhu.edu`). *Optimal Electricity Distribution Pricing under Risk and High Photovoltaics Penetration.*

We model a hierarchical Stackelberg game in a competitive power market under high behind-the-meter Photovoltaics (PV) penetration and demand-side uncertainty, with emphasis on the feedback loop between distributed generation via PV and power prices. The Stackelberg leader, who is the government regulator, attempts to define a set of network tariffs that results in maximal overall system net benefits with consideration of consumer utility, cost recovery and renewable energy promotion. The Stackelberg followers, who are rational consumers of electricity, choose their individual PV investments in order to maximize their personal utilities. With the consumers' demand evolution described by a discretized Ornstein–Uhlenbeck process, we find a closed form approximation to consumer's utility, and existence of a game equilibrium between all the consumers and the regulator. Numerical results are calibrated to PJM power market data, and illustrate the market participants' coupled decisions. Results suggest that consumers tend to rely more on PV when the market demand is more volatile, with potential risks of the utility death spiral where the high electricity retail price resulting from increased distributed generation incentivizes further PV investment.   (Received September 10, 2020)

1163-91-776   **Yuanying Guan*** (`yguan8@depaul.edu`), 1 E. Jackson Blvd., Chicago, IL 60604, and **Jacob Jakubowicz** and **Micah Pollak**. *Cyber Risk in Heterogeneous Networks and Implications for Cyber Insurance.*

Cyber risk recently became the top risk concern for risk managers in the United States according to the 2018 Allianz Risk Barometer report. However, the research in cyber risk and cyber insurance is still very limited. The nature of cyber risk is different from many other traditional types of risk. In traditional insurance products,

insurers measure and control the risk by aggregating the individual risk faced by policyholders. This feature usually does not hold for cyber risk because individual policyholders may be connected in various ways. For example, multiple organizations could rely on the same cloud service provider. Therefore, interdependent risk needs to be considered when insurers evaluate cyber risk and design related cyber insurance products. The accumulative/systemic cyber risk caused by various connections calls for a better understanding of different types of real-world network structures. This paper analyzes interdependent cyber risk in different types of networks, especially networks with high heterogeneity, and investigates the implications for both insurers and policyholders on cyber insurance.   (Received September 14, 2020)

1163-91-807   **David Chan*** (`dmchan@vcu.edu`), 1015 Floyd Avenue, Box 842014, Richmond, VA 23284-2014, and **Hollee McGinnis**, **Michael Broda**, **Jeremy Winslow**, **Haya Hamid** and **Joy Ma**. *The impact of prime supporters within undergraduate students' social and support networks on their education.*

In this talk I describe the collection of support network data, and present preliminary data collected in the fall of 2019 from undergraduate students who were interviewed about their social and support networks. Half of the egocentric graphs produced from the interviews included at least one prime supporter. We examine the impact having a prime supporter has on students' educational success and mental health.   (Received September 13, 2020)

1163-91-818   **Eyal Neuman** and **Alexander Schied*** (`aschied@uwaterloo.ca`). *Protecting Pegged Currency Markets from Speculative Investors.*

We consider a stochastic game between a trader and a central bank in a target zone market with a lower currency peg. This currency peg is maintained by the central bank through the generation of permanent price impact, thereby aggregating an ever increasing risky position in foreign reserves. We describe this situation mathematically by means of two coupled singular control problems, where the common singularity arises from a local time along a random curve. Our first result identifies a certain local time as that central bank strategy for which this risk position is minimized. We then consider the worst-case situation the central bank may face by identifying that strategy of the strategic investor that maximizes the expected inventory of the central bank under a cost criterion, thus establishing a Stackelberg equilibrium in our model.   (Received September 13, 2020)

1163-91-960   **Sergey Nadtochiy*** (`snadtochiy@iit.edu`), Rettaliata Engineering Center, 10 West 32nd Street, Chicago, IL 60616, and **Mykhaylo Shkolnikov**. *Credit network game as a de-centralized network flow problem.*

I will present a de-centralized version of the network flow problem, in which the individual nodes of a network decide strategically on how much connectivity is optimal for each of them. I will show how to construct a Nash equilibrium in this problem by solving a system of linear equations in the max-plus algebra. The specific choice of the optimization objective of the nodes, chosen in this work, corresponds to a model of credit network, where the nodes represent (risk-neutral) firms and the connections represent credit exposures (e.g., lending and borrowing). Remarkably, the resulting equilibrium is very explicit and allows us to compute numerically the equilibrium interest rates and the credit exposures of all participants. Treating the firms as financial institutions, one can also use the proposed model to determine the optimal capital injection strategy in case of a crisis: i.e., for a fixed size of a capital injection, the model allows one to determine which financial institutions should receive it, so that the overall flow of capital to the real economy is maximized. Based on a joint work with M. Shkolnikov. (Received September 14, 2020)

1163-91-1048   **Beatrice Acciaio*** (`beatrice.acciaio@math.ethz.ch`), ETH Zurich, Raemistrasse 101, HG G 54.3, 8092 Zurich, Switzerland. *PQ-GAN: a market generation model consistent with observed spot prices and derivative prices.*

In this talk I will present a model for market generation that is consistent with both the observed spot prices and the market prices of derivatives. The structure used to learn the evolution of asset prices is that of a conditional GAN for time series generation (such as conditional COT-GAN and conditional Sig-Wasserstein GAN), while derivative prices are used to learn the change of measure from the real-world one (P) to the pricing one (Q). The latter can be done in two different ways: either within the same GAN structure, or in a separate structure via supervised learning.   (Received September 14, 2020)

1163-91-1067   **Daewa Kim*** (`daewa.kim@mail.wvu.edu`), Morgantown, WV , and **Annalisa Quaini**.
Coupling kinetic theory approaches for pedestrian dynamics with disease contagion model.
Preliminary report.

The goal of this talk is to study infectious disease spreading in a medium-size population occupying a confined environment. For this purpose, we consider a kinetic theory approach to model crowd dynamics in bounded domains and couple it to a kinetic equation to model contagion. The interactions of a person with other pedestrians and the environment are modeled by using tools of game theory. The pedestrian dynamics model allows to weight between two competing behaviors: the search for less congested areas and the tendency to follow the stream unconsciously in a panic situation. Each person in the system has a contagion level that is affected by the people in their neighborhood. For the numerical solution of the coupled problem, we propose a numerical algorithm that at every time step solves one crowd dynamics problem and one contagion problem, i.e. with no sub-iterations between the two. We test our coupled model on a problem involving a small crowd walking through a corridor. Future developments include a more realistic contagion model and extension of the coupled problem to two spatial dimensions, which will allow us to test real-world scenarios and draw more interesting conclusions. (Received September 14, 2020)

1163-91-1106   **Heather Zinn Brooks*** (`hzinnbrooks@g.hmc.edu`), **Franca Hoffmann**, **Alexander Pan**
and **Mason A Porter**. *Bounded-confidence models for media impact on online social
networks.* Preliminary report.

Online social media networks have become extremely influential sources of news and information, and content that spreads on online social networks can have important consequences on public opinion, policy, and voting. To better understand the online content spread, mathematical modeling of opinion dynamics is becoming an increasingly popular field of study. In this talk, I will introduce an agent-based model of media impact on opinion dynamics on online social networks. I will then discuss the emergence of consensus versus multiple ideological states (e.g., 'echo chambers') in this model. To further understand these qualitative dynamics, we derive and study a mean-field integro-differential equation of the full network model, which we can use to gain deeper insight into the stationary states and bifurcations in the distribution of opinion states. (Received September 14, 2020)

1163-91-1127   **Robert Stephen Cantrell**, **Chris Cosner** and **King-Yeung Lam***, 100 Math Tower,
231 W 18th Avenue, Columbus, OH 43210. *Ideal Free Dispersal in Spatio-temporally
Heterogeneous Habitats.* Preliminary report.

A population is said to have an ideal free distribution in a spatially heterogeneous but temporally constant environment if each of its members have chosen a fixed spatial location in a way that optimizes its individual fitness, allowing for the effects of crowding. In this paper, we extend the idea of individual fitness associated with a specific location in space to account for the full path that an individual organism takes in space and time over a periodic cycle, and extend the mathematical formulation of an ideal free distribution to general time periodic environments. We find that, as in many other cases, populations using dispersal strategies that can produce a generalized ideal free distribution have a competitive advantage relative to populations using strategies that do not produce an ideal free distribution. A sharp criterion on the environmental functions is found to be necessary and sufficient for such ideal free distribution to be feasible. In the case the criterion is met, we showed that there exist dispersal strategies that can be identified as producing a time-periodic version of an ideal free distribution, and such strategies are evolutionarily steady and are neighborhood invaders from the viewpoint of adaptive dynamics. (Received September 14, 2020)

1163-91-1219   **Robert Ghrist*** (`ghrist@math.upenn.edu`). *Sheaves of Opinions over Social Networks.*

A classical approach to the dynamics of opinions over social networks involves using the graph Laplacian to modulate opinions with those of immediate neighbors using a distributed diffusion. More recent entries in the literature tweak these systems to accommodate polarized or multiple opinions, using weighted Laplacians or selective detachment. This talk will outline work [joint with Jakob Hansen] on using sheaves of vector spaces to provide a fully customizable model for the spread and control of opinions over social networks. In this model, the Hodge Laplacian induces a heat equation in which harmonic opinion distributions are expressions of agreeable consensus, with sheaf cohomology acting as an obstruction to harmonic extension. Work in progress on extensions to more sophisticated data types [representing joint work with Hans Riess] will also be given. (Received September 15, 2020)

1163-91-1334   **Martin Le Doux Mbele Bidima*** (mbelebidima@gmail.com), University of Yaounde I,
Department of Mathematics, PO Box 812, Yaounde, Cameroon, and **Tesfamariam
Tadesse Welemical** (tesfatade@gmail.com), Askum University, Ethiopia. *A New
Two-factor Commodity Model of Schwartz-CIR Type for Futures Contracts Pricing.*

The two-factor model for storable commodity developed by Schwartz (1997) has the main drawback that the
second factor i.e., the net convenience yield, which is modeled as an Ornstein-Uhlenbeck (OU) process, can be
negative, which may lead to cash-storage arbitrage opportunity as proved a decade ago by Liu and Tang (2010).
In this talk, we propose a new model overcoming this drawback by modifying the diffusion part of spot prices
dynamics and by assuming that the net convenience yield follows the Cox-Ingersoll-Ross (CIR) process instead
of an OU process. With this new model, we derive a new pricing formula for futures contracts on the storable
commodity. Finally, using Kalman Filter method, we calibrate the new model to crud oil real data taken for US
Energy Information Administration, showing that our proposed model presents less volatile futures prices than
the Schwartz (1997)'s.   (Received September 15, 2020)

1163-91-1633   **Dashiell E.A. Fryer*** (dashiell.fryer@sjsu.edu), One Washington Square, San Jose,
CA 95192. *Analyzing Fairness of Social Structures using Game Theory.*

Game theory is the study of decision making when the outcomes for each individual player is dependent the
choices of all the players. If a graph is added, where vertices are players and edges represent interaction between
players, the game theory model can be used to analyze social structure.

A key concern of social structure is the notion of fairness to the constituents. A recent model, suggested a
definition of fairness based on average payoff across the population of players per game could be used to compare
fairness of different network structures. A recent student and I show that under equivalent circumstances we
show the same increase in average payoff to players given the same graphs, but also show a corresponding
increase in payoff variance. In simplest terms, rich players (well-connected) became richer, while less wealthy
players (almost isolated) became poorer in these same graphs. We suggest that a definition of fairness should
account for payoff variance as well as average at the very least.

I will also survey some current projects looking at similar issues by our undergraduate and master students.
(Received September 15, 2020)

1163-91-1639   **Moritz Voss*** (voss@math.ucla.edu), University of California Los Angeles, Department of
Mathematics, 520 Portola Plaza, Los Angeles, CA 90095. *Trading with the Crowd.*

We formulate and solve a multi-player stochastic differential game between financial agents seeking to cost-
efficiently liquidate their position in a risky asset in the presence of jointly aggregated transient price impact
on the risky asset's execution price along with taking into account a common general price predicting signal.
In contrast to an interaction of the agents through purely permanent price impact as it is typically considered
in the existing literature on multi-player price impact games, accrued transient price impact does not persist
but decays over time. The unique Nash-equilibrium strategies of our game reveal how each agent's liquidation
policy compensates the predictive price signal for the accumulated transient price distortion induced by all other
agents' price impact and thus unfolds a direct and natural link in equilibrium between the agents' jointly followed
trading signal and their overall trading activity. We also formulate and solve the resulting mean field game in
the limit of infinitely many agents and show how the finite-player Nash equilibrium strategies converge to the
unique mean field game solution.

This is joint work in progress with Eyal Neuman (Imperial College London).   (Received September 15, 2020)

92 ► *Biology and other natural sciences*

1163-92-51   **Angela M. Jarrett*** (ajarrett@utexas.edu), **David A. Hormuth**, **Guillermo
Lorenzo**, **Julie C. DiCarlo**, **John Virostko**, **Anna G. Sorace**, **Russell C. Rockne**
and **Thomas E. Yankeelov**. *Preclinical and clinical studies of mathematical modeling for
individualizing therapeutic regimens in breast cancer.*

One of the great challenges in treating cancer is the optimization of therapeutic regimens for the individual
patient. Current experimental and clinical approaches are fundamentally limited to trial and error. To overcome
this issue, we posit the use of a data-driven mathematical framework to design personalized optimal treatment
plans. We present three mathematical models of breast cancer informed by experimental and imaging data
ranging from cellular to tissue scales. First, we discuss a preclinical ODE model that was calibrated with in vitro
microscopy data to quantify the synergistic potential of combined cytotoxic and targeted therapies based on the
order and timing of doses. Second, we present a clinically oriented 3D PDE system constrained by patient-specific

MRI data for predicting tumor response to neoadjuvant therapies and demonstrate its potential for optimizing chemotherapeutic dosing and scheduling for individual patients. Third, we discuss preliminary modeling efforts translating our understanding of breast cancer therapeutic regimens in vitro with clinical precision imaging in vivo to improve and enhance our therapeutic outcome predictions.

1163-92-102   **Eileen Figueroa*** (`exf7951@g.rit.edu`), School of Mathematical Sciences, Rochester Institute of Techology, Rochester, NY 14623, **James Garrison** (`jamesgarrison99@gmail.com`), Mathematics & Computer Science Department, Hampden-Sydney College, Hampden-Sydney, VA 23943, and **Michelle Hewson** (`michellelhewson@gmail.com`), Mathematics and Computer Science Department, Western Carolina University, Cullowhee, NC 28723. *A new coupling model of out-of-phase regions of electrical wave propagation in the heart.*

Ventricular fibrillation (VF) is a cardiac rapid rhythm disorder that is a leading cause of death in the United States. Discordant alternans, an out-of-phase spatial pattern of electrical waves within the heart, renders the heart susceptible to VF. Many mathematical models fail to replicate the observed spatial scale of this pattern. An "ephaptic" model of intercellular coupling, which can theoretically reduce the discordant alternans spacing, was used to describe the connectivity between cells. A computer simulation, based on the ephaptic model, was used to study discordant alternans spacing. Wave velocities and length scales were also obtained from the model, and simplified circuits were created to study the characteristics of the ephaptic connection. Linear differential equations and Fourier analysis were used to identify characteristic time scales in these circuits. The velocities obtained were consistent with these time scales, and were comparable to those observed in the heart. The length scales compared favorably to the theory of Echebarria and Karma and with experimental observations. The components of the ephaptic connection within the model are thus understood and can be manipulated to reduce the spatial scale of the discordant alternans pattern.   (Received August 14, 2020)

1163-92-112   **Alexandria Volkening*** (`alexandria.volkening@northwestern.edu`). *Agent-based modeling and analysis of zebrafish skin patterns.*

Zebrafish and their evolutionary relatives feature a range of colorful skin patterns, including, stripes, spots, and labyrinth curves. Zebrafish patterns form due to the interactions of pigment cells, and several models have been developed to describe or predict the cell behaviors that are involved. By adjusting parameters in these models, we can search for the cell interactions that produce different fish patterns. However, agent-based models have many parameters, and empirical descriptions of zebrafish patterns are largely qualitative and variable. To help address these challenges, we draw on topological data analysis to quantitatively describe pattern features in an interpretable, cell-based way. By applying our techniques to both simulated data and real fish images, we show how to model and quantitatively distinguish different cell-based patterns.   (Received August 16, 2020)

1163-92-115   **Wasiur R. KhudaBukhsh*** (`khudabukhsh.2@osu.edu`), **Hye-Won Kang**, **Eben Kenah** and **Grzegorz A. Rempala**. *Incorporating age and delay into models for biophysical systems.*

In many biological systems, chemical reactions or changes in a physical state are assumed to occur instantaneously. For describing the dynamics of those systems, Markov models that require exponentially distributed inter-event times have been used widely. However, some biophysical processes such as gene transcription and translation are known to have a significant gap between the initiation and the completion of the processes, which renders the usual assumption of exponential distribution untenable. We consider relaxing this assumption by incorporating age-dependent random time delays into the system dynamics. We do so by constructing a measure-valued Markov process on a more abstract state space, which allows us to keep track of the "ages" of molecules participating in a chemical reaction.

We study the large-volume limit of such age-structured systems. We show that, when appropriately scaled, the stochastic system can be approximated by a system of Partial Differential Equations (PDEs) in the large-volume limit, as opposed to Ordinary Differential Equations (ODEs) in the classical theory. We show how the limiting PDE system can be used for the purpose of further model reductions and for devising efficient simulation algorithms.   (Received August 17, 2020)

1163-92-144   **Stanca M Ciupe*** (stanca@vt.edu), 416 McBryde Hall, Blacksburg, VA 24060. *Inoculum size, immune responses, and the nonlinear host-virus interactions.*
Experimental studies have shown that immune protection and disease severity do not correlate linearly with the size and infectious-stage of viral inoculum. We use mathematical models in connection with data to better represent the relationship between the inoculum dose and disease outcome. In this talk, I will present two case studies: simian immunodeficiency virus infection in rhesus macaques and hepatitis B virus infection in non-human primates to provide hypotheses on when different inoculum doses trigger immune responses that provide protection, induce immune tolerance and chronic disease, and/or lead to pathogenesis. Such results can guide our understanding of the virus-host dynamics that control virus infections or permit a transition to chronic disease. (Received August 21, 2020)

1163-92-148   **Maria-Veronica Ciocanel***, 117 Physics Bldg Science Drive, Box 90320, Durham, NC 27708. *Modeling the dynamics and organization of intracellular proteins.*
Actin filaments are polymers that interact with motor proteins inside cells and play important roles in cell motility, shape, and development. Depending on its function, this dynamic network of interacting proteins reshapes and organizes in a variety of structures, including bundles, clusters, and contractile rings. Motivated by observations from the reproductive system of the roundworm C. elegans, we use an agent-based modeling framework to simulate interactions between actin filaments and myosin motor proteins inside cells. We also develop tools based on topological data analysis to understand time-series data extracted from these filamentous network interactions. Our analysis reveals potential mechanistic differences between motor proteins that are believed to shape the organization of various structures inside cells. In addition, we show that changes in actin filament treadmilling may significantly regulate protein organization during cell cycle progression. (Received August 22, 2020)

1163-92-153   **Marissa Renardy*** (renardy@umich.edu), **Marisa Eisenberg** and **Denise Kirschner**. *Predicting the second wave of COVID-19 in Washtenaw County, MI.*
We study and predict the spread of COVID-19 in Washtenaw County, MI, using a discrete and stochastic network-based modeling framework. In this framework, we construct contact networks based on synthetic population datasets specific for Washtenaw County that are derived from US Census datasets. We assign individuals to households, workplaces, schools, and group quarters (such as prisons or long term care facilities). In addition, we assign casual contacts to each individual at random. Using this framework, we explicitly simulate Michigan-specific government-mandated workplace and school closures as well as social distancing measures. We perform sensitivity analyses to identify key model parameters and mechanisms contributing to the observed disease burden in the three months following the first observed cases of COVID-19 in Michigan. We then consider several scenarios for relaxing restrictions and reopening workplaces to predict what actions would be most prudent. In particular, we consider the effects of 1) different timings for reopening, and 2) different levels of workplace vs. casual contact re-engagement. Through simulations and sensitivity analyses, we explore mechanisms driving the magnitude and timing of a second wave of infections upon re-opening. (Received August 24, 2020)

1163-92-157   **Heiko Enderling*** (heiko.enderling@moffitt.org), Integrated Mathematical Oncology, Moffitt Cancer Center, 12902 USF Magnolia Dr, Tampa, FL 33612. *Quantitative personalized radiation oncology.*
HPV+ oropharyngeal cancer is one of the few malignancies that is increasing in incidence, both in the United States and world-wide. At Moffitt Cancer Center, much like other major centers, treatment of early stage HPV+ oropharyngeal cancer consists of radiotherapy alone or in combination with chemotherapy. As outcomes with standard dosing of radiotherapy are excellent, we hypothesize that HPV+ oropharyngeal cancer represents the ideal opportunity to establish a paradigm of individualized radiotherapy based on mathematical models. We have developed a variety of mathematical models to help analyze patient specific tumor growth dynamics and responses to therapy. These models are now deployed to predict which patients will be controlled with the given approach, and who will not. Based on these models, calibrated and validated with historic data of individual patients, we propose the first mathematical modeling biomarker to intensify therapy when needed, and de-escalate therapy without compromising outcomes when possible. (Received August 24, 2020)

1163-92-170   **Rasa Giniunaite*** (`rasa.giniunaite@maths.ox.ac.uk`), Oxford, OX1 3DW, United Kingdom, and **Ruth E Baker** (`ruth.baker@maths.ox.ac.uk`) and **Philip K Maini** (`philip.maini@maths.ox.ac.uk`). *Comparing Xenopus and chick neural crest cell migration.*

There is an ongoing debate between experimentalists working on Xenopus and chick neural crest (NC) cells about the key mechanisms that drive invasion. In particular, cell-cell repulsion and attraction are thought to be the key driving factors for cranial NC cells in Xenopus but these interactions appear not to be observed in chick cranial NC cells, which are demonstrated to be guided by a cell-induced chemoattractant gradient. There are some physical differences between Xenopus and chick cranial NC cells, for example, Xenopus NC cells are much larger than chick NC cells, and the migratory domain is much longer in chick than Xenopus. However, it is an open question as to whether these physical differences are the reason why different biological mechanisms are required to ensure successful invasion in these organisms.

To address this question, we use a single individual-based stochastic model to replicate the results of a successful invasion in Xenopus and chick. We perform parameter sensitivity analysis to explore under what parameter regimes invasion is most robust. We will compare the results for Xenopus and chick and suggest reasons for the observed differences and similarities.   (Received August 25, 2020)

1163-92-218   **Maria R D'Orsogna*** (`dorsogna@csun.edu`), **Renaud Dessalles** and **Tom Chou**. *How T cell receptor abundance distributions are shaped by heterogeneous thymic output and proliferation.*

T-cell receptors (TCR) populate the surface of T-cells. They recognize fragments of foreign proteins, activate the T-cells where they reside and initiate immune responses. The group of T cells that express the same TCR sequence form a clone. Clonal diversity arises from the stochastic recombination of the V(D)J gene segments during T cell development, and since recombination rates are not the same for all sequences, certain TCRs may be more abundant than others. Furthermore, clone-dependent interactions between TCRs and self-antigens may lead to differential proliferation rates. As a result, clonal abundance distributions display non-trivial shapes. We present a mean-field birth-death-immigration model to investigate TCR-dependent heterogeneity in both T cell production and proliferation rates on the overall clone abundance distributions. We also compare predicted clone abundances with experimentally sampled ones. We find that the mechanism underlying the observed clone abundance distributions is most likely heterogeneity in proliferation rates rather than heterogeneity in thymic output rates.   (Received August 27, 2020)

1163-92-252   **Heyrim Cho**, **Allison L Lewis*** (`lewisall@lafayette.edu`) and **Kathleen M Storey**. *Bayesian information-theoretic calibration of patient-specific radiotherapy sensitivity parameters for informing effective scanning protocols in cancer.*

For any proposed model of tumor growth and treatment, we observe large variability among individual patients' parameter values, particularly those relating to treatment response; thus, exploiting the use of various metrics such as tumor volume for model calibration can be helpful to infer such parameters both accurately and early. However, taking measurements can be costly and invasive. As such, the determination of optimal times and metrics for which to collect data in order to best inform treatment protocols is of great assistance to clinicians. In this investigation, we employ a Bayesian information-theoretic calibration protocol for experimental design to identify optimal times at which to collect data for informing treatment parameters. Data collection times are chosen sequentially to maximize the reduction in parameter uncertainty with each added measurement, so that a budget of n experimental measurements results in maximum information gain about the model parameter values. In addition to investigating the optimal temporal pattern for data collection, we also develop a framework for deciding which metrics should be utilized at each data collection point. We illustrate this framework with a variety of toy examples, each utilizing a radiotherapy treatment regimen.   (Received August 30, 2020)

1163-92-255   **Tin Phan*** (`tin.t.phan@asu.edu`), **Justin Bennett**, **Taylor Patten**, **Yang Kuang** and **Eric J Kostelich**. *Model Preselection in Precision Medicine for Prostate Cancer.*

The representation of a patient's characteristics as the parameters of a model is a key component in many studies of personalized medicine. In this context, the underlying mathematical models are used to describe, explain, and forecast the course of treatment. On the other hand, useful insights obtained from these models by means of analysis are deemed too abstract to be utilized as a part of its application. By incorporating known observations of mathematical models within the existing framework of Markov Chain Monte Carlo for the study of parameter identifiability, we develop a new approach to predict the effectiveness of treatment specific to a patient. We demonstrate this approach using several well-studied models of prostate cancer.   (Received August 30, 2020)

1163-92-264   **Pengcheng Xiao*** (`pxiao4@kennesaw.edu`), 850 Polytechnic Lane, D 204, MD 9085, Marietta, GA 30060. *Long term stress hormones modeling and application.*
In this talk, we propose a mathematical model for the stress reaction system Hypothalamic-Pituitary-Adrenal (HPA) axis to address the long term stress hormones secretion within the timescale of weeks. Applications related to neural plasticity with patients with chronic depression and PTSD will be discussed.   (Received August 31, 2020)

1163-92-287   **Churni Gupta*** (`churnibidisha@ufl.edu`), 2915 SW 13 ST, Gainesville, FL 32608, and **Necibe Tuncer** and **Maia Martcheva**. *A Network Immuno-epidemiological HIV Model.* Preliminary report.
In this talk we formulate a multi-scale nested immuno-epidemiological model of HIV on complex networks. The system is described by ordinary differential equations coupled with partial differential equations. First, existence and uniqueness results of the immunological system is proved. Next, wellposedness of the multi-scale system is established. We derive an explicit expression of the basic reproduction number of the immuno-epidemiological model. Existence and stability of disease free equilibrium and endemic equilibrium is established. Numerical simulations suggest that with all things held fixed, if the number of nodes in the network is increased, so is the basic reproduction number. Further, we find that for a scale free network the number of infected individuals at equilibrium is a hump-like function of the within-host reproduction number; however, the dependence becomes monotone if the network has predominantly low connectivity nodes or high connectivity nodes.   (Received September 01, 2020)

1163-92-293   **Christopher E Miles*** (`christopher.miles@cims.nyu.edu`) and **Alex Mogilner**. *Understanding mitotic spatiotemporal trajectories with modeling and data science.*
For cells to divide, they must spatially organize their copied DNA (chromosomes) quickly and precisely. This process, called mitosis, is one of the most fundamental processes in biology, but due to its complexity, remains far from fully understood. Recent experimental advances provide 3D spatial trajectories of chromosomes during mitosis. These trajectories can be thought of as non-stationary stochastic paths, leading to challenges in applying tools typically applied to particle trajectories. I will discuss our attempts to use classical data science and topological analyses to explore whether these trajectories reveal anything about models and mechanisms driving them.   (Received September 01, 2020)

1163-92-310   **Zhuolin Qu*** (`zhuolin.qu@utsa.edu`), Department of Mathematics, One UTSA Circle, San Antonio, TX 78249, and **Benjamin H. McMahon**, **Douglas J. Perkins** and **James M. Hyman**. *Staged progression epidemic model for the transmission of invasive nontyphoidal Salmonella (iNTS) with treatment.*
We develop and analyze a stage-progression compartmental model to study the emerging invasive nontyphoidal *Salmonella* (iNTS) epidemic in sub-Saharan Africa. iNTS bloodstream infections are often fatal, and the diverse and non-specific clinical features of iNTS make it difficult to diagnose. We consider immune-compromised subpopulations in the region with major risk factors for mortality, such as malaria and malnutrition among children and HIV infection and ART coverage in both children and adults. We parameterize the progression rates between infection stages using the branching probabilities and estimated time spent at each stage. We interpret the basic reproduction number $\mathcal{R}_0$ as the total contribution from an infinite infection loop produced by the asymptomatic carriers in the infection chain. The results indicate that the asymptomatic HIV+ adults without ART serve as the driving force of infection for the iNTS epidemic, and the worst disease outcome is among the pediatric population. Our sensitivity analysis indicates that the most effective strategies to reduce iNTS mortality in the studied population are to improve the ART coverage among high-risk HIV+ adults and reduce malnutrition among children.   (Received September 01, 2020)

1163-92-327   **Suzanne Sumner*** (`sumner@umw.edu`), UMW Mathematics Department, 1301 College Avenue, Fredericksburg, VA 22401, and **Noah Carpenter** (`ncarpen2@umw.edu`), UMW Mathematics Department, 1301 College Avenue, Fredericksburg, VA 22401. *Harvesting and Stocking in the Caughley Elephant-Forest System Model.* Preliminary report.
Graeme Caughley proposed in 1976 a predator-prey differential equation model for elephant-tree dynamics which hypothesized an outcome of stable limit cycles. Duffy et al investigated Caughley's model and discovered that stable limit cycles were unlikely when using realistic values of the parameters.

Our modifications to Caughley's model include the effects of harvesting and stocking on both species' density for elephants and trees. Both harvesting and stocking are assumed to occur at rates proportional to the density

size. In this way, the model can now account for elephant poaching or restocking, as well as harvesting or planting of trees.

With these modifications to Caughley's model, not only are limit cycles improbable, but coexistence between elephant and trees is unlikely as well for the realistic parameter values offered by Duffy et al. The best hope for coexistence in the system with harvesting and stocking is in the scenario of planting trees when the elephant decay rate outweighs their stocking rate. Then elephants and trees are successful at coexistence. (Received September 02, 2020)

1163-92-336   **Hongying Shu**, **Wanxiao Xu** and **Xiang-Sheng Wang***, Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70503, and **Jianhong Wu**. *Complex dynamics in a delay differential equation with two delays in tick growth with diapause.*

We consider a delay differential equation for tick population with diapause, derived from an age-structured population model, with two time lags due to normal and diapause mediated development. We derive threshold conditions for the global asymptotic stability of biologically important equilibria, and give a general geometric criterion for the appearance of Hopf bifurcations in the delay differential system with delay-dependent parameters. By choosing the normal development time delay as a bifurcation parameter, we analyze the stability switches of the positive equilibrium, and examine the onset and termination of Hopf bifurcations of periodic solutions from the positive equilibrium. Under some technical conditions, we show that global Hopf branches are bounded and connected by a pair of Hopf bifurcation values. This allows us to show that diapause can lead to the occurrence of multiple stability switches, coexistence of two stable limit cycles, among other rich dynamical behaviours. (Received September 03, 2020)

1163-92-342   **Tamar Schlick*** (schlick@nyu.edu), Department of Chemistry, 100 Washington Square, Silver Building, New York University, New York, NY 10003. *In silico drug discovery targeting the SARS-CoV-2 Frame Shifting RNA Element.*

With the rapid rate of Covid-19 infections and deaths, urgently needed are treatments and cures besides hand washing, social distancing, masks, isolation, and quarantines. The treatments and vaccines rely on the basic biophysics of the complex viral apparatus. While proteins are serving as main drug and vaccine targets, the 30,000 nucleotide RNA viral genome also defines important targets. Using mutation and tertiary analysis of a small gene region in the SARS-Cov-2 RNA genome, we define key residues of the virus as targets for anti-viral drugs and gene editing approaches within the frame-shifting element. These leads are pursued for virtual drug screening as an approach for discovering potential anti-viral therapies from known chemical libraries. (Received September 03, 2020)

1163-92-362   **Yunshyong Chow** (chow@math.sinica.edu.tw), Taipei, Taiwan, and **Sophia Jang*** (sophia.jang@ttu.edu), 01672176502170, Lubbock, TX 79424. *Neimark-Sacker bifurcations in a host-parasitoid system with a host refuge.*

In this talk we introduce a discrete model of host-parasitoid interactions with a constant proportion of host refuge. The model is built upon the classical Nicholson-Bailey system by assuming in each generation a constant proportion of the host is free from parasitism. We derive a sufficient condition based on the model parameters for both populations to coexist. We show that it is possible for the system to undergo a supercritical and then a subcritical Neimark-Sacker bifurcation or for the system only to exhibit a supercritical Neimark-Sacker bifurcation. It is illustrated numerically that a constant proportion of host refuge can stabilize the host-parasitoid interaction. (Received September 03, 2020)

1163-92-386   **Jeffrey Shaman*** (jls106@cumc.columbia.edu), 722 W. 168th St., New York, NY 10032. *Transmission Dynamics of SARS-CoV-2: Inference and Projection.*

Dynamic models of infectious disease systems are often used to study the epidemiological characteristics of disease outbreaks, the ecological mechanisms and environmental conditions affecting transmission, and the suitability of various mitigation and intervention strategies. In recent years these same models have been employed to generate probabilistic forecasts of infectious disease incidence at the population scale. Here I present research from my own group describing development of model systems and combined model-inference frameworks capable of simulation, inference and projection of SARS-CoV-2. (Received September 04, 2020)

1163-92-393   **Christina J Edholm*** (cedholm@scrippscollege.edu). *Investigating the Role of "Silent Spreaders" in COVID-19 Dynamics.*

The dynamic nature of the COVID-19 pandemic has demanded a public health response that is constantly evolving due to the novelty of the virus. Many jurisdictions in the US, Canada, and across the world have adopted social distancing and recommended the use of face masks. Considering these measures, it is prudent

to better understand the contributions of subpopulations – such as "silent spreaders" – to disease transmission dynamics in order to better inform public health strategies in a jurisdiction-dependent manner. Our model includes two classes of individuals: silent spreaders, who neither have symptoms nor disease-induced mortality; and symptomatic spreaders, who experience symptoms and a resultant mortality rate. We fit the model to real-time COVID-19 confirmed cases and deaths to derive the transmission rates, death rates, and other relevant parameters for geographically-defined outbreaks. We seek to answer whether a blanket public health strategy in which all individuals are required to engage in social distancing and encouraged to wear masks is appropriate, or whether a response that targets symptomatic individuals and those at high-risk of disease-induced mortality is more efficient.   (Received September 04, 2020)

1163-92-394   **Miranda Ijang Teboh-Ewungkem*** (`mit703@lehigh.edu`), Chandler Ullman Hall, Department of Mathematics, Lehigh University, Bethlehem, PA 18015, and **Gideon Akumah Ngwa** (`akumhed@yahoo.com`), Department of Mathematics, Buea. *A long term mathematical model used to investigate the roles of masks, social distancing, self-quarantining and human behaviors in impeding the spread of the SARS-CoV-2 virus.* Preliminary report.
The use of face masks and adherence to general social distancing, protocols, are methods that have been pushed to curb the spread of the SARS-Cov-2 virus. These intervention methods can also potentially reduce the severity of a new infection, especially if the viral particles a healthy individual comes in contact with is minimal. Additionally, A well enacted contact tracing procedure can also help identify potential new cases which, in the event that the identified individuals adhere to the social distancing and mask wearing intervention strategies, and self-quarantine could potentially reduce or eliminate the SARS-Cov-2 viral spread to healthy individuals. Here, we present a long-term mathematical model, used to understand how the combinations of the aforementioned intervention methods-contact tracing, mask wearing, self-quarantining can impact the long term dynamics of SARS-CoV-2 spread.   (Received September 04, 2020)

1163-92-399   **Suzanne Lenhart*** (`slenhart@utk.edu`), University of Tennessee, Dopartment of Mathematics, Knoxville, TN 37996-1320. *Economic Modeling of Free-Roaming Cats in Knox County, TN.* Preliminary report.
Unowned free-roaming cats are a global problem due to predation on wildlife and the spread of infectious diseases. Programs such as Trap, Neuter, and Return and Trap-Euthanize have been used to control cat populations. This talk presents a data-driven bioeconomic model that weighs the benefits and costs of free-roaming cat control programs. The benefits of the control programs are illustrated by a mathematical model of the dynamics of a free-roaming cat population. The economic costs associated with possible management strategies include the costs of trapping, neutering, euthanasia, the monetary value of wildlife killed by free-roaming cats, and the value of those cats. This model includes a term for caretaker cooperation with managers which reflects caretakers actively reducing the carrying capacity of a colony, to reduce the population.   (Received September 04, 2020)

1163-92-406   **Maia Martcheva*** (`maia@ufl.edu`), Gainesville, FL , and **Aziz Yakubu**. *A Discrete-Continuous ODE Hybrid Model for Interaction of Two Species.* Preliminary report.
Discrete models are used for species with non-overlapping generations while continuous ODE models are best suited for species with continuous births and deaths. When species with discrete and continuous generations interact the most adequate mathematical model is a hybrid discrete-continuous ODE model. We develop such a model and the analytical techniques for studying such models. We use the techniques to compute the reproduction number of the hybrid system. We show that the disease-free equilibrium is locally asymptotically stable if the reproduction number is less than one and unstable, if the reproduction number is grater than one.   (Received September 05, 2020)

1163-92-407   **Churni Gupta** (`churnibidisha@ufl.edu`), **Necibe Tuncer** (`ntuncer@fau.edu`) and **Maia Martcheva*** (`maia@ufl.edu`), Department of Mathematics, University of Florida, Gainesville, FL 32601. *A Network Immuno-epidemiological Model of HIV and Opioid Epidemics.* Preliminary report.
We introduce a network immuno-epidemiological model of HIV and opioid epidemics where the jointly affected class is structured by the within-host dynamics. We fit the within-host model to data, collected in monkeys. We compute the reproduction numbers of the HIV and opioid epidemics. We show that the disease-free equilibrium is locally stable if both reproduction numbers are below one, and unstable if at least one of the reproduction numbers is above one. The HIV-only equilibrium exists if the reproduction number of HIV is larger than one.

The opioid-use only equilibrium exists if the reproduction number of opioid use is larger than one. The HIV-only equilibrium is locally asymptotically stable if the invasion number of the opioid epidemic is below one and unstable if the invasion number of opoioid epidemic is above one. The opoioid-only equilibrium is locally asymptotically stable if the invasion number of the HIV epidemic is below one and unstable if the invasion number of HIV epidemic is above one. Simulation suggest that larger networks lead to higher reproduction numbers.   (Received September 05, 2020)

1163-92-416   **Maryann E Hohn*** (`maryann.hohn@pomona.edu`), **Christina J Edholm** and **Ami E Radunskaya**. *To open or not to open.*
In-person college classes require students, faculty, and staff to congregate together in indoor spaces creating a higher risk for possible COVID-19 infection. Small residential colleges such as the five Claremont Colleges (5Cs), where a majority of the students live on campus, present a relatively closed campus environment, curtailing students' interactions with their greater community. However, the close knit quarters in which students live may contribute to a rise in infections that may ultimately reach other more vulnerable populations on the campuses such as faculty and staff.

In this talk, we present a model of COVID-19 spread consisting of several interconnected modified SEIR differential equations to investigate the dynamics between different populations at the 5Cs and the influence of mitigation techniques such as students adhering to health protocols and contact tracing. We then present an app which allows the user to vary parameters in our model, providing intuition and guidance on campus openings under a range of conditions.   (Received September 05, 2020)

1163-92-419   **Hannah Thompson*** (`hthomp15@vols.utk.edu`), **Gregory Wiggins**, **Tom McAvoy** and **Suzanne Lenhart**. *Data Driven Models of Hemlock Woolly Adelgid Impacts and Biological Control.* Preliminary report.
We present a preliminary report of two models of hemlock woolly adelgid, an invasive insect pest in the eastern US that kills hemlock trees. Using data collected in the Great Smoky Mountains National Park, we model the population dynamics of adelgids and two biocontrol species. Using data collected in Virginia, we model the interaction of adelgid density and hemlock health. The models are systems of ordinary differential equations and include seasonality and life stage structure.   (Received September 05, 2020)

1163-92-425   **Peter C Jentsch***, pjentsch@uwaterloo.ca, and **Chris T Bauch** and **Madhur Anand**. *Interactions between social distancing and vaccination in an age-structured model of COVID-19.* Preliminary report.
The COVID-19 pandemic has produced an unprecedented worldwide closure of schools and workplaces, and created widespread awareness of the importance of contact patterns. To this end, recommendations by the World Health Organization (WHO) include physical distancing, and avoiding large gatherings. The effects of school closure, workplace closure, and physical distancing, vary significantly with age in a population, and age has been identified as a significant risk factor for poor prognosis in COVID-19 infections. Furthermore, perceived importance of physical distancing, and facility closure, is affected by the perceived severity of COVID-19 infection, and this results in feedback with disease dynamics. In this work, we combine these observations in an age-structured model of COVID-19 spread coupled with behavioural dynamics, parameterized with data from Ontario, Canada. We use this model to test the efficacy of various age-structured vaccination strategies with respect to the date of vaccine availability, projected vaccination rate and other variables. We find that with respect to minimizing long-term mortality, the best vaccination strategy depends significantly on these factors. (Received September 06, 2020)

1163-92-454   **Nina H Fefferman***, 447 Hesler Biology Building, Knoxville, TN 37996. *How Mass Incarceration Affects Outbreaks of Infectious Disease.*
Certain activities and populations lead to increased challenge in combating outbreaks of infectious disease. For influenza, has focused on young children in schools. For COVID-19, attention to nursing homes and large gatherings ranging from religious services to sporting events has been shown to be critical. There is also a less widely discussed but equally potentially challenging population: those incarcerated in jails and prisons. Medical research has clearly demonstrated the variety of ways that incarceration compromises health, leaving populations both more susceptible to transmission of infection and also more likely to suffer severe outcomes from infection. Not only is this of concern for managing the health of incarcerated populations, but just as with nursing homes, schools, churches, concerts, and sporting events, these facilities do not operate separately from the rest of society. In the United States, approximately 2.2 million (%0.7) of the entire population is incarcerated in 2020 and the institutions surrounding their management and care involve millions more. In this talk, we will explore a model

of the impact of jailed populations on community management of COVID-19 and discuss the general implications for management of all infectious diseases.   (Received September 07, 2020)

1163-92-465   **Lauren M Childs*** (`lchilds@vt.edu`) and **Olivia F Prosper** (`oprosper@utk.edu`). *The impact of within-vector parasite development on the extrinsic incubation period.*
Mosquito-borne diseases, in particular malaria, have a significant burden worldwide leading to nearly half a million deaths each year. The malaria parasite requires a vertebrate host, such as a human, and a vector host, the *Anopheles* mosquito, to complete its full life cycle. Here, we focus on the parasite dynamics within the vector to examine the first appearance of sporozoites in the salivary glands, which indicates a first time of infectiousness of mosquitoes. The timing of this period of pathogen development in the mosquito until transmissibility, known as the extrinsic incubation period, remains poorly understood. Here, we develop compartmental models of within-mosquito parasite dynamics fitted with experimental data on oocyst and sporozoite counts. We find that only a fraction of oocysts burst to release sporozoites and bursting must be delayed either via a time-dependent function or a gamma-distributed set of compartments. We use Bayesian inference to estimate distributions of parameters and determine that bursting rate is a key epidemiological parameter. A better understanding of the factors impacting the extrinsic incubation period will aid in the development of interventions to slow or stop the spread of malaria.   (Received September 07, 2020)

1163-92-468   **Najat Ziyadi***, 1700 East Cold Spring Lane, Baltimore, MD 21251. *A mathematical model of human papillomavirus (HPV) and cervical cancer.*
In this talk, we will introduce a mathematical model of human papillomavirus (HPV) and cervical cancer using a system of ordinary differential equations. We will use the next generation method to compute the basic reproduction number. Sensitivity analysis will be used to illustrate the impact of model parameters on the basic reproduction number.   (Received September 07, 2020)

1163-92-470   **Keisha Cook*** (`kcook7@tulane.edu`) and **Scott McKinley** (`smckinl3@tulane.edu`). *Methods for Analyzing Movement in Single Particle Tracking.* Preliminary report.
Single particle tracking techniques allow us to analyze the behavior of lysosomes in human lung cells. Intracellular transport is essential to cell health and the success of the processes that they carry out. This transport is carried out in membrane-bound vesicles through the use of motor proteins. We are interested in mathematically quantifying aspects that influence lysosome movement. With regard to cellular internalization, experiments were conducted where lysosomes either contained or did not contain titanium dioxide (TiO_2) nanoparticles. With regard to location in the cell, the movement of lysosomes was recorded for those located in the perinuclear and peripheral regions. We developed a biophysical model that allows us to simulate the movement of lysosomes, specifically transitioning from inactive to active states over time. Our statistical analysis methodology allows us to infer information about the lysosome trajectories. The results show that lysosomes in the presence of TiO_2 and those not in their presence move in a similar manner, however the location of lysosomes in the cell affect movement significantly.   (Received September 07, 2020)

1163-92-480   **Jim M Cushing*** (`cushing@math.arizona.edu`), The University of Arizona, Department of Mathematics, 617 N Santa Rita, Tucson, AZ 85719-1108. *Does evolution select against chaos?*
Despite the ubiquity of chaotic attractors in many theoretical equations of population dynamics, unequivocal evidence of its occurrence in biological populations is sparse and is, for the most part, limited to populations manipulated in laboratory settings. One of the numerous hypotheses offered to explain this is that evolution selects against complex dynamics in favor of equilibrium dynamics. We investigate this hypothesis by means of a Darwinian dynamics version of the iconic Ricker difference equation. We investigate how the threshold e^2 for the onset of complexity (i.e. the destabilization of an equilibrium and a period-doubling bifurcation cascade to chaos) is affected by allowing the model coefficients to evolve according to Darwinian principles. We find that when evolution is slow, the Darwinian Ricker equation has an onset of complexity threshold larger than e^2 and that, in this sense evolution, selects against complexity. On the other hand, when evolution is fast the threshold can be less than $\exp(2)$ and, in this sense, evolution selects for complexity. In the latter case, the onset of complexity is by means of a Naimark-Sacker bifurcation, not a period-doubling bifurcation.   (Received September 07, 2020)

1163-92-492   **Adrianne L Jenner***, adrianne.jenner@umontreal.ca, Montreal, Quebec , Canada, and **Munisha Smalley**, **Aaron Goldman**, **Paul Macklin** and **Morgan Craig**. *Exploring the impact of glioblastoma stromal heterogeneity on oncolytic virotherapy using agent-based modelling.* Preliminary report.

Oncolytic viruses (OV) are an exciting immunotherapeutic modality being investigated for the treatment of glioblastoma multiforme (GBM), an aggressive brain cancer with a poor clinical prognosis. Unfortunately, promising pre-clinical investigations of immunotherapies have led to disappointing trail results. Recapitulating the tumour microenvironment (TME) and finding useful pre-clinical models to elucidate the efficacy of OVs is, therefore, crucial to improve OV treatments. Leveraging pre-clinical GBM spheroids, we constructed an agent-based representation for the infiltration of an OV in patient GBM samples. The model was developed in PhysiCell, an open-source cell-based simulator, and used to determine OV characteristics that optimized therapeutic efficacy with respect to the stromal density. Overall, our results showed that the intracellular viral replication rate is the primary driver of OV infiltration patterns observed in patient samples. In addition, we quantified the relationship between stromal density and treatment efficacy and found a threshold above which treatment was no longer as effective. This work has implications on the development of OVs for the treatment of GBM and in our understanding of the impact of spatial heterogeneity on new treatment approaches.   (Received September 08, 2020)

1163-92-527   **Shanshan Chen**, **Junping Shi**, **Zhisheng Shuai** and **Yixiang Wu*** (yixiang.wu@mtsu.edu). *Spectral Monotonicity of Perturbed Quasi-positive Matrices with Applications in Population Dynamics.*

Threshold values in population dynamics can be formulated as spectral bounds of matrices, determining the dichotomy of population persistence and extinction. For a square matrix $\mu A + Q$, where A is a quasi-positive matrix describing population dispersal among patches in a heterogeneous environment and Q is a diagonal matrix encoding within-patch population dynamics, the monotonicy of its spectral bound with respect to dispersal speed/coupling strength/travel frequency μ is established via two methods. The first method is an analytic derivation utilizing a graph-theoretic approach based on Kirchhoff's Matrix-Tree Theorem; the second method employs Collatz-Wielandt formula from matrix theory and complex analysis arguments. It turns out that our established result is a slightly strengthen version of Karlin-Altenberg's Theorem, which has previously been discovered independently while investigating reduction principle in evolution biology and evolution dispersal in patchy landscapes. Nevertheless, our result provides a new and effective approach in stability analysis of complex biological systems in a heterogeneous environment. We illustrate this by applying our result to well-known biological models of single species, predator-prey and competition.   (Received September 08, 2020)

1163-92-550   **Nalin Fonseka** (fonsekan@carolinau.edu), **James T. Cronin** (jcronin@lsu.edu) and **Jerome Goddard II*** (jgoddard@aum.edu), Department of Mathematics, P.O. Box 244023, Montgomery, AL 36124-4023, and **Ratnasingham Shivaji** (r_shivaji@uncg.edu). *Can density dependent emigration enhance or even counteract a patch-level Allee effect?*

The relationship between conspecific density and the probability of emigrating from a patch can play an essential role in determining the population-dynamic consequences of an Allee effect. In this talk, we will employ a theoretical model based upon the reaction diffusion framework to answer the question: "can density dependent emigration enhance or even counteract a patch-level Allee effect?" The model assumes that a population is diffusing and growing according to a weak Allee effect growth rate inside a focal patch, but the organism's emigration probability is dependent on conspecific density. The habitat patch is one-dimensional and is surrounded by a tuneable hostile matrix. In particular, we consider five different forms of density dependent emigration (DDE) that have been noted in previous empirical studies. Our results are obtained mathematically through the method of sub-super solutions, time map analysis, and numerical computations using Wolfram Mathematica. (Received September 09, 2020)

1163-92-554   **Tina Huang*** (huangyu@lafayette.edu) and **Allison Lewis**. *Predicting tumor response to radiotherapy based on pre-treatment parameter estimates.* Preliminary report.

Due to continuous improvement in medical technology, it is now possible for clinicians to collect detailed information about a variety of tumor characteristics as the tumor evolves. As a result, a variety of cancer treatments have been developed to inhibit tumor growth dynamics. However, it remains difficult to predict the efficacy of a given treatment prior to administration. Additionally, the process of collecting information about the tumor may be invasive and expensive. Thus, the creation of a framework for predicting patient response to treatment using only limited data collected prior to the start of the treatment regimen is invaluable to clinicians in designing a

targeted treatment protocol for each individual. In this study, we employ ODE models for tumor growth to simulate tumor dynamics and utilize synthetic data from a cellular automaton model for calibration. We investigate which model parameters drive a tumor's response to radiotherapy by clustering model simulations according to the final tumor volume after treatment and comparing the associated parameter distributions. Additionally, we develop a framework for determining the probability of observing complete tumor remission following radiotherapy based only on a patient's pre-treatment parameter values.   (Received September 09, 2020)

1163-92-556   **Melody Walker*** (melody@vt.edu), Department of Mathematics, 460 McBryde Hall, Virginia Tech 225 Stanger S, Blacksburg, VA 24061, and **Lauren M Childs**. *The Role of Testing in COVID-19 Outbreak Control.* Preliminary report.

The on-going pandemic of novel coronavirus disease (COVID-19) caused by infection with SARS-CoV-2 has caused a significant burden on individuals and society. As COVID-19 is known to be transmitted by non-symptomatic individuals, travel restrictions and social distancing measures have been placed on society as a whole in order to decrease transmission. An alternative to these restrictive measures, testing along with contact tracing could be used to slow the spread of COVID-19. While some countries, such as South Korea and Iceland, have successfully utilized random or broadly applied testing strategies, testing remains limited in scope in the US. Individual localities may have restrictions on the number of tests due to access to supplies, lab equipment and personnel. Here, using ordinary differential equations we model how different testing strategies impact disease spread. We build in limits to testing capacity and delays to receiving results. Once tested, we assume that individuals respond to knowledge of their disease status through changes in behavior. We examine strategies for optimally implementing testing to decrease spread of COVID-19, and, thus, minimize socially and economically restrictive mitigation measures. Keywords: COVID-19, testing, ODE model, disease dynamics   (Received September 09, 2020)

1163-92-606   **Meghan C Ferrall-Fairbanks**, Tampa, FL 33612, and **Gregory J. Kimmel** and **Philipp M. Altrock*** (philipp.altrock@moffitt.org), Tampa, FL 33612. *Uncovering time-dependence of intra-tumor heterogeneity.* Preliminary report.

We have developed a pipeline to quantify intra-tumor heterogeneity at the transcriptomic level using a generalized diversity index (GDI). We here ask at which time points in a tumor's evolutionary trajectory GDI can be used prognostically. We can show that an order of diversity parameter, q, allows us to control for different population properties. Richness emerges at low values of q, while high q shifts the emphasis to the phenotypic drivers of the adapting tumor population. We then use an evolutionary game theoretic approach to understand how GDI changes over time. Using replicator-mutator dynamics, we explore the effects of constant and frequency-dependent selection. Our analyses suggest that GDI undergoes non-monotonic changes as the population evolves. We explore these dynamic features further in sequential single-cell RNA sequencing samples of fused breast cancer cells, where at earlier passages after fusion, GDI approaches a maximum, and later returns to levels similar to the initial state. Overall, GDI as a means to quantify intra-tumor heterogeneity is a powerful tool to understand eco-evolutionary dynamics in cancer under uncertainty of the precise adaptive forces.   (Received September 10, 2020)

1163-92-608   **Tricia Phillips*** (tmphilli@bsc.edu), **Suzanne Lenhart**, **W. Christopher Strickland** and **Orou Gaoue**. *Modeling Lethal and Non-lethal Harvest of African Mahogany.* Preliminary report.

Khaya senegalensis (African Mahogany) is a tree species in Western Africa that is harvested non-lethally for its non-timber forest products, as well as lethally. A preliminary report will be given on the formulation of a size-structured, seasonal system of difference equations with time-varying lethal and non-lethal harvest for this tree species. An application of this model to the Soassararou population will be discussed in order to better understand the effects of harvesting on these trees.   (Received September 10, 2020)

1163-92-645   **David Murrugarra*** (murrugarra@uky.edu), Department of Mathematics, University of Kentucky, Lexington, KY 40506. *Improving RNA secondary structure prediction via state inference with machine learning and deep learning methods.*

RNA state inference is the task of determining which nucleotides of an RNA sequence are paired or unpaired in the secondary structure of an RNA, which can be studied by different machine learning techniques. The state inference is a binary classification task on each nucleotide which is different from determining the full secondary structure consisting of sets of nested base pairs. Successful state inference of RNA sequences can be used to generate auxiliary information for data-directed RNA secondary structure prediction. This talk will discuss different approaches for improving RNA secondary structure prediction via the Nearest Neighbor

Thermodynamic Model (NNTM) using machine learning and deep learning methods for state inference. This talk will also highlight the challenges such as overfitting and the need of data for state inference from using methods such as hidden Markov models (HMM) and recurrent neural networks (RNN) as well as the benefits of using these methods on different classes of RNA.   (Received September 10, 2020)

1163-92-658   **Bob Palais*** (`bob.palais@uvu.edu`), 800 West University Parkway, MS 261, Orem, UT 84058. *Math for rapid identification of SARS-CoV-2 and other disease causing pathogens and mutations using high-resolution melting analysis.*

We discuss mathematical methods used to model and analyze high-resolution DNA melting, that are incorporated in a widely used platform for rapid detection and identification of infectious agents including SARS-CoV-2, and in systems used to diagnose a variety of genetic disorders.   (Received September 11, 2020)

1163-92-659   **Javier Arsuaga***, Department of Molecular and Cellular Biology, Department of Mathematics, On Shields Avenue, Davis, CA 95616. *Mathematical analysis of DNA packing in bacterial viruses.*

Bacterial viruses pack their genome in a small protein structure called capsid. Inside the capsid the dsDNA molecule is found at a concentration of 200 mg/ml-800mg/ml and an osmotic pressure of 60 atmospheres. How DNA organizes under these extreme conditions remains to be understood.

In this talk I will present results from three different mathematical approaches to study the problem of dsDNA packing in bacteriophages. The first approach complements the cryoEM observations and uses the formation of knots inside viral capsids as a probe for DNA packing. These results suggest that DNA knots are highly likely upon confinement and that the DNA molecule is chirally organized inside the viral capsid. The second approach aims at identifying the possible sources of the chiral organization of the genome and employs methods from random knotting and brownian dynamics. Our third approach uses continuum mechanics models to describe cryoEM observations as the minima of a liquid crystalline phase. The emergent picture of these approaches suggest that DNA is in a chirally organized liquid crystalline phase in which knots may be the product of liquid crystal defects.   (Received September 11, 2020)

1163-92-667   **Chuang Xu*** (`chuang.xu@ma.tum.de`). *Criteria for dynamics of one-dimensional continuous time Markov chains with applications.*

I will talk about our recent results on the dynamics of continuous time Markov chains (CTMCs) of possibly unbounded jumps on non-negative integers with polynomial transition rate functions. These dynamics include explosivity, transience, recurrence, certain absorption, positive recurrence as well as ergodicity of quasi-stationary distributions. Sharp criteria for these dynamics are provided in terms of at most four easily computable parameters of the CTMC. To demonstrate the wide applicability, we apply our main results to stochastic reaction networks, extended branching processes, a gene expression model, as well as a bursty population process. This is a joint work with Mads Christian Hansen and Carsten Wiuf.   (Received September 11, 2020)

1163-92-680   **Adrianne Jenner*** (`adrianne.jenner@umontreal.ca`), **Sofia Alfonso**, **Rosemary Aogo**, **Courtney Davis** (`courtney.davis2@pepperdine.edu`), **Penelope Morel**, **Amber Smith** (`amber.smith@uthsc.edu`) and **Morgan Craig** (`morgan.craig@umontreal.ca`). *Predicting immune response dynamics in COVID-19 through mathematical modeling.* Preliminary report.

The novel coronavirus SARS-CoV-2 is the source of a global pandemic and ongoing public health crisis. Mounting evidence points to dysregulated and hyper-reactive inflammatory responses, including hyperinflammation and cytokine storms, as particular presentations in severe COVID-19. Much remains to be uncovered about the mechanisms that lead to disparate outcomes in coronavirus disease 2019. Here, quantitative approaches, especially mechanistic mathematical models, can be leveraged to improve our understanding of the immune response to SARS-CoV-2 infection. We have developed a quantitative framework to interrogate open questions about the innate and adaptive immune reaction in COVID-19. This talk will outline our mechanistic model of SARS-CoV-2 viral and immune response dynamics at both the tissue and systemic levels. A portion of this work is done as part of an international and multidisciplinary coalition working to establish a comprehensive tissue simulator (physicell.org/covid19). Ultimately, by improving our understanding of SARS-CoV-2 infection and immune responses to novel coronavirus, our results help to understand the orchestration of the immune response after infection and to identify mechanisms defining differential clinical manifestations of COVID-19.   (Received September 11, 2020)

1163-92-694   **Folashade B. Agusto***, Ecology and Evolutionary Biology, Lawrence, KS 66045. *To isolate or not to isolate: The impact of changing behavior on COVID-19 transmission.* Preliminary report.

COVID-19 is a respiratory disease caused by a recently discovered novel coronavirus, SARS-COV 2. Furthermore, disease transmission are often driven by public perception of risk/fear of the disease. In this talk I will present a model developed for COVID-19 using a system of ordinary differential equation following the natural history of the infection. The model uniquely incorporate the behavior of susceptibles and symptomatic individuals; the susceptible in the community are willing to support all social distancing efforts including lockdown while symptomatic are will to self-isolate. Using appropriate payoff functions relating to the perception of risk measured using disease incidence and severity of infection the model is coupled to a series of human behaviors including violating self-isolation rules. Analysis and simulations of the model show the possibility of multiple waves of infections. The results also show the importance of incentivizing self-isolation as a means to reduce disease transmission.   (Received September 11, 2020)

1163-92-713   **Gerardo Chowell***, gchowell@gsu.edu. *Combating the COVID-19 pandemic with mathematical and statistical modeling tools.*

The devastating COVID-19 pandemic represents an unprecedented opportunity to test and apply mathematical and statistical modeling approaches to infer key epidemiological and transmission characteristics of the novel coronavirus as well as evaluate the performance of different theoretical models for forecasting the trajectory of the pandemic at various spatial scales. In this context, I will present results from multiple ongoing collaborations involving interdisciplinary quantitative scientists, doctoral students, and public health officials.   (Received September 11, 2020)

1163-92-717   **Mustafa Kulenovic***, 165 little rest road, Unversity of Rhode Island, Kingston, RI 02881. *Asymptotic Behavior of a Discrete-Time Density-Dependent SI Epidemic Model With Constant Recruitment.*

We use the epidemic threshold parameter, $\Re_0$, and invariant rectangles to investigate the global asymptotic behavior of solutions of the density-dependent discrete-time SI epidemic model

$$\begin{cases} S_{n+1} = aS_n e^{-I_n} + B \\ I_{n+1} = aS_n \left(1 - e^{-I_n}\right) + bI_n \end{cases} \quad n = 0, 1, 2, \ldots$$

where the parameters a, b and B and the initial conditions S_0 and I_0 satisfy

$$a \in (0,1), \ b \in [0,1), \ B \in (0,\infty), \quad S_0 \geq 0, \quad I_0 \geq 0.$$

The variables S_n and I_n represent the populations of susceptibles and infectives at the n-th generation, respectively. The constant survival "probabilities" of susceptible and infective individuals are a and b, respectively. B is the constant recruitment per generation into the susceptible class. We compute the basic reproductive number, $\Re_0$, and use it to predict the local persistence or extinction of the infective population.   (Received September 11, 2020)

1163-92-722   **Nicole Pagane**, **Devany West**, **Quinn MacPherson**, **Bruno Beltran**, **Andrew J Spakowitz** and **Viviana I Risca*** (vrisca@rockefeller.edu). *A Monte Carlo simulation framework for interpreting sub-kilobase chromatin folding data from RICC-seq.* Preliminary report.

The sub-kilobase structure of chromatin—the nucleosome-bound state of DNA found in the nuclei of eukaryotic cells—has undergone a recent re-evaluation in light of experiments showing it is more heterogeneous than previously thought. Chromatin structure at this length scale determines the local geometry and accessibility of binding sites on DNA for proteins driving critical processes like transcription. It has been difficult to probe in cells because it is densely packed and the weak interactions that shape it are easily perturbed. We have recently developed RICC-seq, a method for probing chromatin structure in a minimally perturbative way using ionizing radiation that generates population-averaged measurements. To generate chromatin structure ensembles for comparison with and interpretation of the pairwise DNA-DNA contact data RICC-seq provides, we describe a chromatin Monte Carlo simulation framework based on our previous code {wlcsim} that includes a stretchable-shearable worm-like chain approximation of DNA, explicit twist and steric constraints, and coarse-grained rigid body nucleosome representations based on crystal structures. We discuss how chromatin fiber geometry affects compaction and compare results with data from RICC-seq experiments in cultured cells.   (Received September 11, 2020)

1163-92-723   **Tingting Tang*** (`ttang2@sdsu.edu`), CA , and **Naveen Vaidya**. *A coupled four-stage compartment model to understand the unusual spread of COVID-19 at Imperial Valley County.*

Imperial valley is a rural county of California state which had the highest positive rate for months in the State. In this talk, a coupled four-stage compartment model is developed to incorporate the factors such as border transportation and larger family structure. The four stages are susceptible individuals, asymptomatic infected individuals, symptomatic infected individuals, and recovered individuals. Two pair of such four-stage compartment model for different average household size are used to understand the impact of family size and poverty. The model is then analyzed to find the reproduction number, disease free equilibrium, and endemic equilibrium. Sensitivity analyze is used to investigate the impact of border transportation and large family cultural on the equilibrium. Finally, a case study of Imperial Valley is presented.   (Received September 11, 2020)

1163-92-730   **Jacqueline M Dresch*** (`jdresch@clarku.edu`), Clark University, Worcester, MA. *Using bioinformatic and mathematical analyses to predict protein-DNA binding preferences.*

When mathematically modeling gene regulation from DNA sequence information, a critical first step is to predict where on the DNA sequence transcription factor (TF) proteins might bind, and the biophysical affinity of such binding. Since the 1980s, one of the most popular bioinformatic approaches for predicting TF binding sites involves constructing a Position Weight Matrix (PWM) and scanning a DNA sequence for subsequences with a high probability of binding the TF of interest. The question remains whether the bioinformatic scores obtained using PWM-based approaches relate directly to the binding affinities of the TF binding sites. Focusing our efforts on a set of TFs known to drive gene expression during early fruit fly development, we have used known binding sites within the genome, bioinformatic tools, clustering and statistical analyses to investigate and predict the relative TF-DNA binding preferences.   (Received September 15, 2020)

1163-92-752   **Anna S Frank**, **Kamila Larripa**, **Hwayeon Ryu*** (`hryu@elon.edu`), **Ryan G Snodgrass** and **Susanna Roblitz**. *Bifurcation and sensitivity analysis reveal key drivers of multistability in a model of macrophage polarization.*

We analyze a mathematical model for polarization of a single macrophage which, despite its simplicity, exhibits complex dynamics in terms of multistability. In particular, we demonstrate that an asymmetry in the regulatory mechanisms and parameter values is important for observing multiple phenotypes. Bifurcation and sensitivity analyses show that external signaling cues are necessary for macrophage commitment and emergence to a phenotype, but that the intrinsic macrophage pathways are equally important. Based on our numerical results, we formulate hypotheses that could be further investigated by laboratory experiments to deepen our understanding of macrophage polarization.   (Received September 12, 2020)

1163-92-768   **Robert Stephen Cantrell**, **Chris Cosner*** (`gcc@math.miami.edu`) and **Salome Martinez**. *Persistence for a two-stage reaction-diffusion system.*

It is well known that in bounded domains logistic reaction-diffusion models predict that slower diffusion rates are advantageous relative to faster diffusion. More broadly, a wide class of models arising in population genetics, population dynamics, and related areas display some version of the reduction principle, which says that dispersal, which causes faster mixing, typically reduces the rate of population growth. However, the situation seems to be quite different in the case of stage structured populations. We study a reaction-diffusion model with stage structure to explore whether faster diffusion is harmful or advantageous for a stage structured population. We find the answer a depends on the details of the spatial distributions of habitats that are favorable for reproduction by adults and those that are favorable for survival by juveniles. The analysis is mainly based on estimates for the principle eigenvalue of the system linearized around zero.   (Received September 12, 2020)

1163-92-779   **Abba Gumel***, School of Mathematical and Statistical Sci., Tempe, AZ, **Enahoro Iboi**, Spelman College, and **Calistus Ngonghala**. *To mask or not to mask: that's the question for the COVID-19 pandemic.*

The novel coronavirus that pandemic, which started as an outbreak of pneumonia of unknown cause in the city of Wuhan in December 2019, has become the most important public health and socio-economic challenge humans have faced since the 1918 Spanish flu pandemic. Within weeks of emergence, the highly-transmissible and deadly COVID-19 pandemic spread to almost every part of the world, so far accounting for nearly 30 million confirmed cases and over 900,000 deaths, in addition to causing severe economic burden globally.

   This talk is focused on the use of mathematical models, of the form of deterministic systems of nonlinear differential equations, to provide insight into transmission dynamics and control of COVID-19 in the US, and to

use the models to assess the impact of various non-pharmaceutical interventions for controlling and mitigating the spread of the pandemic in the United States. It will be shown that, like its coronavirus cousins (SARS and MERS), COVID-19 is a respiratory disease that is controllable using basic public health interventions. (Received September 12, 2020)

1163-92-786   **Zhilan Feng**, **Katharine Gurski*** (`kgurski@howard.edu`), **Margaret Grogan**, **Olivia Prosper** and **Miranda I Teboh-Ewungkem**. *A vector-borne disease model with non-exponentially distributed infection and treatment stages.*

Most models for vector-borne disease assume exponentially distributed residence times in disease stages in order to simplify the model formulation and analysis. However, tackling drug resistance in malaria causing parasites requires an accurate description of the interaction between drug concentration and parasite load within hosts. For example, how long a human host has been infected is likely to influence their parasite load and their ability to transmit the parasite to a mosquito, especially among the immunologically naive individuals. The onset of clinical symptoms is likely to dictate when treatment begins. How long an individual has been undergoing treatment for an infection will determine their current blood-drug concentration and parasite load, and therefore, their susceptibility to re-infection with drug resistant parasites, along with their ability to transmit. Thus, we formulate a model by considering arbitrarily distributed sojourn for various disease stages. The model formulation is presented using integral equations. We present examples with malaria using data driven distributions.   (Received September 12, 2020)

1163-92-808   **David Chan*** (`dmchan@vcu.edu`), 1015 Floyd Avenue, Box 842014, Richmond, VA 23284-2014, and **Julie Zinnert**, **Greg Robson**, **Eric Schoen** and **Harold Reed Ogrosky**. *Vegetation effects on barrier island evolution.*

A cellular model of a barrier island is described that updates regularly according to different physical and biological processes. These processes include marine, aeolian, and marsh processes, as well as plant processes that represent growth, death, and seed dispersal. The evolution of various barrier islands are demonstrated. (Received September 13, 2020)

1163-92-817   **Glenn F. Webb*** (`glenn.f.webb@vanderbilt.edu`), Mathematics Department, Vanderbilt University, Nashville, TN 37240. *Predicting the Development of COVID-19 Epidemics from Reported Case Data.*

Mathematical models are developed to provide predictions for COVID-19 pandemics. The models incorporate asymptomatic and symptomatic transmission. The models incorporate reported and unreported cases. Reported case data is used to parameterise the models. The models are used to project the epidemic forward with varying public health measures.   (Received September 13, 2020)

1163-92-821   **Wencel W Valega-Mackenzie*** (`wvalegam@vols.utk.edu`) and **Suzanne Lenhart**. *Resource Allocation in a PDE Ecosystem model.* Preliminary report.

The effect of heterogeneous habitat on a diffusing population is crucial to understand population dynamics. We formulate a reaction-diffusion population model to study the effect of resource allocation in an ecosystem with resources having their own dynamics in space and time. This is more realistic than only assuming the resource level is not changing as the population changes. Moreover, we solve an optimal control problem of our ecosystem model to maximize the abundance of a single species while minimizing the cost of inflow resource allocation. (Received September 13, 2020)

1163-92-826   **Christine Heitsch*** (`heitsch@math.gatech.edu`) and **Svetlana Poznanovic**. *Some combinatorics of RNA branching.*

Understanding the folding of RNA sequences into three-dimensional structures is one of the fundamental challenges in molecular biology. For example, the branching of an RNA secondary structure is an important molecular characteristic yet difficult to predict correctly. However, recent results in geometric combinatorics (both theoretical and computational) yield new insights into the distribution of optimal branching configurations, and suggest new directions for improving prediction accuracy.   (Received September 13, 2020)

1163-92-833   **P. Charoentong** and **A. Konstorum*** (`konstorum@uchc.edu`). *Using tensor decomposition for multi-omics data in ovarian cancer.*

Tensor decomposition is an unsupervised learning method that can identify patterns in multi-dimensional data arrays. The CANDECOMP/PARAFAC (CP) decomposition factorizes a tensor into the sum of rank-1 tensors (components). We use a CP decomposition on a tensor representing multi-omics data from ovarian cancer tissue samples. Each face of the tensor corresponds to one omics experiment, such as gene expression or gene copy

number variation, across all samples. The resulting component patterns identified correspond to known and novel patterns in the data, showing that tensor decomposition can serve as a useful tool to probe multi-omics datasets.   (Received September 13, 2020)

1163-92-840   **Avner Friedman** (`nourridine@aims.ac.za`) and **Nourridine Siewe***
(`nourridine@aims.ac.za`). *Overcoming Drug Resistance to BRAF Inhibitor.*
One of the most frequently found mutations in human melanomas is in the B-raf gene, making its protein BRAF a key target for therapy. However, in patients treated with BRAF-inhibitor (BRAFi), although the response is very good at first, relapse occurs within 6 months, on the average. In order to overcome this drug resistance to BRAFi, various combinations of BRAFi with other drugs have been explored, and some are being applied clinically, such as a combination of BRAF and MEK inhibitors. Experimental data for melanoma in mice show that under continuous treatment with BRAFi, the pro-cancer MDSCs and chemokine CCL2 initially decrease but eventually increase to above their original level, while the anti-cancer T cells continuously decrease. In this paper we develop a mathematical model that explains these experimental results. The model is used to explore the efficacy of combinations of BRAFi with anti-CCL2, anti-PD-1 and anti-CTLA-4, with the aim of eliminating or reducing drug resistance to BRAFi.   (Received September 13, 2020)

1163-92-862   **Azmy S. Ackleh*** (`azmy.ackleh@louisiana.edu`), Department of Mathematics, University
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Departamento de Matemática, Universidad de Buenos Aires, (1428) Pabellón I - Ciudad
Universitaria, Buenos Aires, Argentina. *A Structured Coagulation-Fragmentation Equation
in the Space of Radon Measures.*
We present a structured coagulation-fragmentation model which describes the population dynamics of oceanic phytoplankton. This model is formulated on the space of Radon measures equipped with the bounded Lipschitz norm. We prove that the model is well-posed using a fixed-point approach. We also show that the model reduces to the classical discrete and continuous models for certain choices of parameters. We study the interplay between the physical processes of coagulation and fragmentation and biological processes including growth and reproduction to understand how these processes contribute to the regularity of solutions. We also present a numerical approximation for the coagulation-fragmentation equation in the space of Radon measures and test its performance.   (Received September 13, 2020)

1163-92-869   **Azmy S. Ackleh*** (`azmy.ackleh@louisiana.edu`), Department of Mathematics, University
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(`amy.veprauskas@louisiana.edu`), Department of Mathematics, University of Louisiana at
Lafayette, Lafayette, LA 70504-1010. *Global stability and bifurcation results for
discrete-time predator-prey models.*
We consider a two-dimensional discrete-time predator-prey model that was developed by Ackleh et. al in 2019. Utilizing an approach which is based on nullcline analysis, we derive conditions for the global stability of the interior equilibrium. Then, we study a three-dimensional evolutionary counterpart developed in Ackleh et. al (2019) which couples the population dynamics with the dynamics of an evolving phenotypic trait. We extend the global stability results to the interior equilibrium of the three-dimensional predator-prey model with frequency-independent evolution. We also show that when evolution in the prey is frequency-dependent then a Neimark-Sacker bifurcation is possible.   (Received September 13, 2020)

1163-92-887   **Shamreen Iram**, **Emily Dolson**, **Jacob Scott*** (`scottj10@ccf.org`) and **Michael
Hinczewski**. *Controlling the speed and trajectory of evolution with counterdiabatic driving.*
The pace and unpredictability of evolution are relevant in a variety of modern challenges, such as combating drug resistance in pathogens and cancer, understanding how species respond to environmental perturbations like climate change and developing artificial selection approaches for agriculture. Progress has been made in quantitative modelling of evolution using fitness landscapes, allowing a degree of prediction for future evolutionary histories. Yet fine-grained control of the speed and distributions of these trajectories remains elusive. We propose an approach to achieve this using ideas originally developed in a completely different context—counterdiabatic driving to control the behaviour of quantum states for applications like quantum computing and manipulating ultracold atoms. Implementing these ideas for the first time in a biological context, we show how a set of external control parameters (that is, varying drug concentrations and types, temperature and nutrients) can guide the

probability distribution of genotypes in a population along a specified path and time interval. While this control has many possible applications, we focus here on application to evolutionary cancer and pathogen therapy. (Received September 13, 2020)

1163-92-895   **Elizabeth Gross*** (`egross@hawaii.edu`), Honolulu, HI 96822, and **Cvetelina Hill** (`cvetelina.hill@gatech.edu`). *Mixed volumes of steady-state systems.*
The steady-state degree of a chemical reaction network is the number of complex solutions to the steady-state system for generic parameters. In general, the steady-state degree may be difficult to compute, but it can be bounded above by the mixed volume of the system. In this presentation, using tools from combinatorial polyhedral geometry, we compute the mixed volume for three infinite families of networks, each generated by joining smaller networks to create larger ones. Each of these examples illustrate a different relationship between the steady-state degree and the mixed volume of the steady-state system.   (Received September 13, 2020)

1163-92-897   **Yanxiang Zhao*** (`yxzhao@email.gwu.edu`), 801 22nd St. NW, Phillips Hall room 739, Washington, DC 20052. *Cell motility dependence on adhesive wetting.*
Adhesive cell–substrate interactions are crucial for cell motility and are responsible for the necessary traction that propels cells. These interactions can also change the shape of the cell, analogous to liquid droplet wetting on adhesive substrates. To address how these shape changes affect cell migration and cell speed we model motility using deformable, 2D cross-sections of cells in which adhesion and frictional forces between cell and substrate can be varied separately. Our simulations show that increasing the adhesion results in increased spreading of cells and larger cell speeds. We propose an analytical model which shows that the cell speed is inversely proportional to an effective height of the cell and that increasing this height results in increased internal shear stress. The numerical and analytical results are confirmed in experiments on motile eukaryotic cells.   (Received September 13, 2020)

1163-92-901   **Jimin Zhang**, **Jude Kong** and **Junping Shi*** (`jxshix@wm.edu`), Department of Mathematics, 200 Ukrop Way, Williamsburg, VA 23187-8795, and **Hao Wang**. *Phytoplankton competition for nutrients and light in a stratified lake: a mathematical model connecting epilimnion and hypolimnion.*
We introduce a new mathematical model connecting epilimnion and hypolimnion to describe the growth of phytoplankton limited by nutrients and light in a stratified lake. Stratification separates the lake with a horizontal plane called thermocline into two zones : epilimnion and hypolimnion. The epilimnion is the upper zone which is warm (lighter) and well-mixed; and the hypolimnion is the bottom colder zone which is usually dark and relatively undisturbed. The growth of phytoplankton in the water column depends on two essential resources: nutrients and light. The critical thresholds for settling speed of phytoplankton cells in the thermocline and the loss rate of phytoplankton are established, which determine the survival or extirpation of phytoplankton in epilimnion and hypolimnion.   (Received September 14, 2020)

1163-92-908   **Chris McCarthy*** (`cmccarthy@bmcc.cuny.edu`), BMCC City University of New York, 199 Chambers Street, New York, NY 10007, and **Johannes Familton** (`jfamilton@bmcc.cuny.edu`), BMCC City University of New York, 199 Chambers Street, New York, NY 10007. *Quasispecies and error catastrophe: some of the mathematics, history, and implications.*
Quasispecies refers to the paradigm of viewing a species as a distribution of mutating genotypes. Error catastrophe refers to deleterious effects due to excessive mutation rates. We will discuss some of the interesting mathematics, implications, and history of the quasispecies concept and the error catastrophe theory.   (Received September 15, 2020)

1163-92-914   **Shelby N. Wilson*** (`shelby.wilson@jhuapl.edu`). *Social Organization and its Effects on Disease Spread.*
Individuals living in social groups are susceptible to disease spread through their social networks. The network's structure, including group stability, clustering, and an individual's behavior and affiliation choice all have some impact on the effect of disease spread. Moreover, under certain scenarios, a social group may change its own structure to suppress the transmission of infectious disease. Evidence that social organization may protect populations from pathogens in certain circumstances prompts the question as to how social organization affects pathogenic spread on dynamic networks. We will introduce discrete-time dynamic social network model and discuss the effects of both pathogenic and parasitic epidemics. In each case, we highlight the bi-directional influence between social structure and infection dynamics.   (Received September 14, 2020)

1163-92-934 **Brody H Foy*** (`bfoy1@mgh.harvard.edu`), Center for Systems Biology, Simches Research Building, 185 Cambridge Street, Boston, MA 02114, and **John M Higgins**. *Using mass balance equations to quantify properties of red blood cell production and growth.*

As red blood cells (RBCs) develop, grow and respond to disease pathophysiology they exhibit a variety of changes. Over their lifespan, RBCs typically exhibit significant loss of both volume and hemoglobin mass, due to processes such as vesiculation. While these processes have been well studied in experimental models (e.g mice, etc.), due to the difficulties of in vivo measurement of blood cells, patient-specific estimates of volume and hemoglobin loss at the cellular level are challenging to obtain.

To address this issue, we propose use of mass balance equations to estimate cell clearance, and volume and hemoglobin loss. This model is applied using both routine clinical measurements such as the complete blood count, and single-cell flow cytometry measurements from which the blood count is derived.

We apply this mass balance system to analysis of a group of N = 28 healthy subjects who all donated blood, and 3 weeks later underwent either autotransfusion of this blood (N = 14) or had a placebo saline injection (N = 14). This analysis highlights both qualitative and quantitative differences in blood production between the two groups at various points throughout the study. (Received September 14, 2020)

1163-92-951 **Kathleen M Storey*** (`storeyk@umich.edu`) and **Trachette L Jackson**. *A cellular automaton model of the spatial and immune-related dynamics involved in a combination of oncolytic viral therapy and anti-PD-1 immunotherapy.*

Oncolytic viral therapies and immunotherapies are of growing interest to cancer researchers and clinicians, due to their selectivity for tumor cells over healthy cells and their immunostimulatory properties. Uncertainty remains regarding the circumstances under which the immune system effectively assists in eliminating tumor cells. We develop a three-dimensional cellular automaton (CA) model of a lethal brain tumor, glioblastoma, undergoing treatment with a combination of an oncolytic Herpes Simplex Virus and an anti-PD-1 immunotherapy. We extend our previous work, which models this combination therapy using an ordinary differential equation model, to the spatially explicit CA model, in order to consider the spatial effects of the treatment. We use a mechanistic approach to model the interactions between distinct populations of immune cells, incorporating both innate and adaptive immune responses to oncolytic viral therapy, and including a mechanism of adaptive immune suppression via the PD-1/PD-L1 checkpoint pathway. We discuss the role of several significant parameters involved in the innate and adaptive immune response to both treatment modalities. Additionally, we utilize our model to determine optimal viral dosing, in both the temporal and spatial contexts. (Received September 14, 2020)

1163-92-954 **Sofya Zaytseva***, szaytseva@uga.edu, and **Romuald Lipcius**, **Leah B Shaw** and **Donglai Gong**. *Image Analysis Approach to Understanding Spatial Patterns in Intertidal Oyster Reefs.* Preliminary report.

This talk focuses on pattern formation in intertidal oyster reefs. Despite being a resilient species, the Eastern oyster population has plummeted over the last century, making reef restoration of critical importance. While various aspects of reef development have been studied in the past, the role of flow in oyster reef development remains not well understood. Using drone imagery of an extensive intertidal reef network, we develop an image classification method to delineate the individual oyster reefs and extract useful information regarding their geometry. We then undertake a machine learning approach to investigate different reef configurations (string reefs (perpendicular to flow), fringing reefs (parallel to flow) and patch reefs (no particular orientation)) and explore the interplay between reef geometry, flow conditions and bathymetry and their role in reef persistence. This becomes particularly important for reef restoration and can help determine suitable locations and optimal configurations for the construction of artificial reefs. (Received September 14, 2020)

1163-92-962 **Michael C. Reed*** (`reed@math.duke.edu`), 120 Science Drive, Campus box 90320, Durham, NC 27708-0320. *Sex differences in metabolism.*

James Donaldson's deep and booming laughter, filled with intelligence, good will, humor, and love of life has reverberated in my head since we became friends in the 1970s. I will comment on his many contributions to mathematics, to Howard University, and to humanity. And then, in his honor, I will discuss recent research that I believe he would like. We do not think of the liver as a sex organ, but men and women in the child-bearing years have very different livers. Many enzymes in one-carbon metabolism are upregulated or downregulated by the sex hormones and, as a consequence, concentrations and biochemical reaction velocities are different in men and women. Mathematical modeling has given new understanding of these phenomena, and the underlying

evolutionary reasons. For example, we now understand why homocysteine, the major biochemical marker for cardio-vascular disease, is lower in women. Thus, investigation of sex differences in metabolism is important for precision medicine.   (Received September 14, 2020)

1163-92-967   **Lauren A Sugden*** (`sugden1@duq.edu`). *Mining genomic data for patterns of adaptive mutations, genes, and pathways.* Preliminary report.

Interpreting the signatures of human evolution in modern-day genomic data is a persistent challenge that requires ever more sophisticated mathematical approaches. I will describe here a framework for detecting patterns of adaptation at the single mutation level, and up to the level of genes and gene pathways. I will first describe a probabilistic model that leverages the correlated patterns left by a mutation spreading rapidly through a population, as well as a hidden Markov model that can improve sensitivity and provide measures of uncertainty. I will discuss recent efforts to build on this model to incorporate patterns of adaptation on larger genomic scales, in order to assess the adaptive force experienced by a gene or pathway as a functional unit. I will demonstrate the ability of the initial probabilistic model to identify well-known adaptive mutations in well-studied populations, as well as its usefulness in proposing plausible targets of selection in the ‡Khomani San hunter gatherer population of southern Africa, an example of a historically underrepresented population in genomic studies. I will then share some preliminary data concerning the hidden Markov model integration of multiple scales, and the benefits derived from this approach.   (Received September 14, 2020)

1163-92-981   **W. Christopher Strickland*** (`cstric12@utk.edu`), 227 Ayres Hall, Knoxville, TN 37996, and **Owen Queen**, **Leigh Pearcy**, **Tricia Phillips** and **Suzanne Lenhart**. *Using agent-based models to explore network dynamics in addiction epidemiology.* Preliminary report.

Opioid addiction has become a national health crisis in recent years, with involvement in 66% of all drug overdose deaths in 2016 and high economic costs. In contrast to the dynamics of a classic disease or illicit drug epidemic, opioid addiction has its roots in prescription medication – a fact which greatly increases the exposed population and mathematically suggests non-contact based routes of addiction. In this talk, I will present recent work on epidemic models for opioid addiction and treatment, including dynamics involving heroin and fentanyl and projections based on data from the state of Tennessee. I will then describe an undergraduate project that utilizes an agent-based approach to examine the well-mixedness assumption of these models in the presence of network structure.   (Received September 14, 2020)

1163-92-997   **John T. Nardini*** (`jtnardin@ncsu.edu`). *TDA discriminates parameters in models of angiogenesis.*

Spatial networks naturally arise in many phenomena from biology, physics, and political science. Methods from algebraic topology are gaining popularity for inferring the properties of networks that arise over many scales, yet determining which methods are appropriate for certain applications can be challenging. We focus on the analysis of networks that arise from tumor-induced blood vessel formation, where the network structure critically determines its function in both healthy and abnormal biological tissue. A simple agent-based model of this process shows that the network morphology changes based on the model's underlying parameters. We propose two novel approaches to analyze the topology of these simulated networks and demonstrate how they can be combined with methods from data science to perform a global sensitivity analysis for the model. We find that one method, which we call a plane sweeping approach, can perform a sensitivity analysis that reliably separates parameter space into regions that produce similar network structures. We propose that this methodology can be used in combination with experimental images to infer how such networks developed.   (Received September 14, 2020)

1163-92-1005   **Shweta Bansal***, shweta.bansal@georgetown.edu. *Infectious disease surveillance and modeling through spatial "big data".* Preliminary report.

During one of epidemiology's formative moments, John Snow mapped London households with cholera and succeeded in highlighting the risk of disease associated with the Broad Street pump. Since then, spatial investigations have played a critical role in improving our understanding of the associations between risks and disease outcomes. Modern electronic resources allow us to carry out spatial epidemiology studies by increasing accessibility to populations over space and time, and by providing digital data on health behaviors and health outcomes at unprecedented breadth and depth. In this talk, I will discuss case studies where we have applied an ecological approach and spatial big data modeling to inform disease surveillance, disease control, and public health policy.   (Received September 14, 2020)

1163-92-1021   **Hayriye Gulbudak**, **Paul L. Salceanu** and **Gail S.K. Wolkowicz***
(`wolkowic@mcmaster.ca`), Department of Mathematics and Statistics, 1280 Main Street
West, Hamilton, Ontario L8S 4K1, Canada. *A delay model for persistent viral infections in
replicating cells.*
Persistently infecting viruses remain within infected cells for a prolonged period of time without killing the cells
and can reproduce via budding virus particles or passing on to daughter cells after division. The ability for
populations of infected cells to be long-lived and replicate viral progeny through cell division may be critical for
virus survival in examples such as HIV latent reservoirs, tumor oncolytic virotherapy, and non-virulent phages in
microbial hosts. A model for persistent viral infection within a replicating cell population, time delay and eclipse
stage prior to infected cell replication are considered. Reproduction numbers providing existence and stability
of the equilibria of the system, along with identifying the emergence of several bifurcations, including backward,
Hopf, and Bogdanov-Takens bifurcations are discussed. We investigate the possibility of long-term survival
of the infection (represented by chronically infected cells and free virus) in the cell population by utilizing the
mathematical concept of robust uniform persistence. Using the Matlab toolbox, DDE-Biftool, with the estimated
parameter values for phage-microbe systems, we show regimes of complex bifurcation dynamics and address how
considering delay can affect outcomes.   (Received September 14, 2020)

1163-92-1027   **Christina Edholm*** (`cedholm@scrippscollege.edu`). *A Mathematical Model of Contact
Tracing During the 2014-2016 West African Ebola Outbreak.*
The 2014-2016 West African outbreak of Ebola Virus Disease (EVD) was the largest and most deadly to date.
Contact tracing, following up those who may have been infected through contact with an infected individual to
prevent secondary spread, plays a vital role in controlling such outbreaks. However, there were many complica-
tions and challenges to contact tracing efforts during the 2014-2016 outbreak. We present a system of ordinary
differential equations to model contact tracing in Sierra Leone during the outbreak. Using data on cumulative
cases and deaths we estimate most of the parameters in our model. We include the novel features of counting
the total number of people being traced and tying this directly to the number of tracers doing this work. Some
parameters change over time as a result of changing behaviors in the population. We explore the role contact
tracing played in eventually ending the outbreak and examine the potential impact of improved contact tracing
on the death toll.   (Received September 14, 2020)

1163-92-1035   **Evan Milliken*** (`evan.milliken@louisville.edu`), Department of Mathematics, 328
Natural Sciences Bldg, University of Louisville, Louisville, KY 40292. *Deterministic and
stochastic models COVID-19 with vaccination.* Preliminary report.
SARS-CoV-2, the virus which causes COVID-19, rapidly spread around the world during 2020, resulting in a
global pandemic. An unprecedented global effort has been made to develop a vaccine to control the spread of
this virus and the resulting deadly disease. Recently, studies indicating rapid declines in antibody levels and
controversial reports of reinfection have led to speculation of waning immunity to the disease and, potentially,
to vaccines in development. Existing research on deterministic mathematical models which include waning
immunity and vaccination have variously indicated the possibility of backward bifurcation or the rise of large-
scale oscillations. In this talk, deterministic and stochastic models of the population-level spread of SARS-CoV-
2 are presented and analyzed. Results are discussed through the lens of implications for vaccination policy.
(Received September 14, 2020)

1163-92-1043   **Leah B. Shaw*** (`lbshaw@wm.edu`). *Extinction in a stage-structured population with
fluctuating dispersal rates.*
Adults of reef-building species such as oysters are non-motile. New individuals arise in the population when
larvae, which are dispersed by water currents, settle on an existing reef. We develop a stage-structured stochastic
differential equation model for oyster populations. Due to positive feedback interactions between oysters, we
include an Allee effect, in which populations above a threshold can grow but below the threshold approach
extinction. Larval transport depends on rainfall, so we include stochastic fluctuations in larval availability due
to variable rainfall. We extend to a metapopulation model in which reefs are coupled by dispersal of larvae.
Self-replenishment of a reef by its own larvae can also occur. We study the risk of population extinction in the
model. The roles of life history strategy and fluctuating connectivity are explored.   (Received September 14,
2020)

1163-92-1109   **Nicole Pagane***, npagane@rockefeller.edu, and **Devany West**, **Quinn MacPherson**, **Bruno Beltran**, **Andrew J. Spakowitz** and **Viviana I. Risca**. *Coarse-graining DNA mechanics to study mesoscale chromatin geometries and interdigitation.*
DNA behaves as a semi-flexible polymer, well described by the worm-like chain model. In order to study how the mechanical properties of DNA interact with nucleosome geometry in chromatin, we previously implemented a discretized stretchable, shearable worm-like chain model in a coarse-grained Monte Carlo simulation codebase called wlcsim. To study the mesoscale structure and fluctuations of chromatin—the stiff sub-kilobase regime with just a handful of nucleosomes—we now incorporate explicit twisting rigidity, excluded volume, and inter-nucleosome potential terms into the wlcsim code. We simulate short single and multiple chromatin fibers to assess how these pairwise repulsive and attractive terms modulate fiber geometries, compaction, and phase separation. We then characterize features of the single-fiber structure space and the multiple-fiber phase space. Lastly, we generate predicted contact matrices that enable comparison of simulated chromatin structure ensembles with experiments in cultured cells.   (Received September 14, 2020)

1163-92-1144   **Yeona Kang*** (`yeona.kang@howard.edu`), 204 Academic Service Building B, Washington, DC 20059, and **Keith Jamison**, **Rayan Lowder**, **Nicholas D Schiff**, **Amy Kuceyeski** and **Sudhin A Shah**. *Alteration of neuronal integrity underlying executive attention impairments following traumatic brain injury: PET study utilizing [11C] Flumazenil.* Preliminary report.
Chronic deficiencies in executive attention have been demonstrated after Traumatic Brain Injury (TBI). Very little is known regarding the biology and mechanism underlying this impairment. There is a critical need for quantitative and integrative approaches that can identify and measure the neuronal damage. This study was measured functional neuronal integrity within the anterior forebrain mesocircuit with [11C] flumazenil (FMZ) PET and evaluate its relationship to impaired executive attention. 8 TBI patients and 20 heathy control (HC) had a dynamic FMZ-PET imaging. 16 anterior mesocircuit regions were considered as cortical regions of interest. FMZ-Binding was calculated by the simplified reference tissue model with the pons as a reference. We found there is no significant differences between the populations for volume fraction and cortical thickness but there were differences in the FMZ-binding. Individual scores of attention were predicted (R2=0.55 in executive attention and R2 =0.77 in mean reaction time). Therefore, a lower than normal FMZ-binding was present in subjects of TBI, indicating loss of neuronal integrity. These results demonstrate the feasibility of detecting altered neuronal integrity in post-acute TBI using a novel molecular imaging tool of FMZ-PET.   (Received September 14, 2020)

1163-92-1148   **Laura S Storch*** (`laura.storch@oregonstate.edu`), Corvallis, OR 97331, and **Sarah L Day**. *A topological data analysis approach to understanding critical transitions in spatially explicit populations.*
Understanding and predicting critical transitions in spatially extended populations is challenging, both due to the high dimensionality of the systems and complexity of their dynamics. Here, we track dynamical changes in a spatially extended population by observing and quantifying changes in the population distribution patterns through a critical transition. In these studies, the critical transition is a system parameter shift that leads to a global extinction event. We use a simple density-dependent coupled patch model with Ricker map growth on a 2D lattice to create population distribution patterns. Using topological data analysis to quantify the spatial patterns, we calculate Betti numbers, which count certain topological features in a topological space. We find that the change in Betti numbers (corresponding with the change in spatial patterns) en route to a critical transition depends on the time scale of the changing system parameter driving said critical transition. We explore the relationship between changing Betti numbers and the time scale of the parameter drift. We hope to use this technique on spatially explicit ecological data sets and/or GIS/satellite imaging time series.   (Received September 14, 2020)

1163-92-1229   **Eric Marland*** (`marlandes@appstate.edu`) and **Gregg Marland** (`marlandg@appstate.edu`). *Dynamics of Carbon Storage in Wood Products.*
When forests are harvested all of the carbon removed from the site is not instantly oxidized and released to the atmosphere as CO2. While some of the C may be released in the year of harvest, some may be sequestered in durable wood products for years, even centuries. In estimating CO2 emissions, we want to recognize that while some products are harvested but not oxidized there also is some CO2 released from oxidation of products harvested in previous years. Of interest is the long term trend in the overall carbon stock from harvested wood products and the relationship of that pool with forest carbon stocks at different time and spatial scales. In this presentation we will discuss the current stock of harvested wood products in the US, how it is changing, and how

a forest manager can estimate the actual net, effective discharge of CO2 from a given forest harvest.   (Received September 15, 2020)

1163-92-1239   **Jan Poleszczuk***, Ks. Trojdena 4 street, 02109 Warsaw, Poland. *Computational modeling of pulse wave propagation: how mathematical model calibrated with pulse wave recording can be used to assess the state of patient's cardiovascular system.*

Cardiovascular (CV) diseases are the leading cause of death worldwide. Therefore, there is a constant demand for more accurate and non-invasive methods for cardiovascular system state assessment. Therefore, we focused on developing a mathematical model which, after calibration with patient-specific data, would provide new personalized information about CV system state.

To this extent, we model the blood transport in a bifurcating binary tree of fifty-five larger systemic arteries in which individual vessels are axisymmetric elastic cylinders tapering along their length. We describe spatiotemporal changes in the cross-sectional area of the artery (equivalently blood pressure) and blood flow using an 1D approach.

Proposed model was confronted with the actual recordings of the blood pressures in the peripheral arteries collected in a group of healthy individuals and hemodialysis patients. We show that, after parameter estimation procedure, the model is able to provide new patient-specific insights into CV system state that are unattainable with existing non-invasive methods.   (Received September 15, 2020)

1163-92-1270   **Margaret A Grogan*** (`mgrogan1@vols.utk.edu`) and **Olivia Prosper**. *Exploring Reproduction Numbers of Two Malaria Models.* Preliminary report.

Reproduction numbers are the go-to threshold to calculate for infectious disease models in order to determine if an outbreak will die out or not. In my exploration and derivation of two Malaria models, I have dealt with trying to calculate these thresholds and think about them not only as algebraic groupings of parameters but as biologically interpretable expressions. In this talk, I will describe the processes and methods I used to calculate reproduction numbers for two of my current working models for Malaria, analyze them with a biological perspective, and present some interesting results and plots   (Received September 15, 2020)

1163-92-1274   **Yanyu Xiao*** (`xiaoyu@ucmail.uc.edu`), POBox 210025, Cincinnati, OH 45219. *Investigations of the impact of contact patterns on disease transmissions.*

Contact rates play an important role in modeling the transmissions of infectious diseases. Here, we investigate the contact patterns at different locations and examine the different effects on the disease dynamics. A study case will be presented for COVID-19 transmission dynamics.   (Received September 15, 2020)

1163-92-1295   **Abba Gumel*** (`agumel@asu.edu`), School of Mathematical and Statistical Scienc, Tempe, AZ 85287. *Mathematics of the Dynamics and Control of the COVID-19 Pandemic.*

The novel coronavirus that emerged in December of 2019 (COVID-19), which started as an outbreak of pneumonia of unknown cause in the city of Wuhan, has become the most important public health and socio-economic challenge humans have faced since the 1918 Spanish flu pandemic. Within weeks of emergence, the highly-transmissible and deadly COVID-19 pandemic spread to almost every part of the world, so far accounting for nearly 30 million confirmed cases and over 900,000 deaths, in addition to incurring severe economic burden globally. In this talk, I will discuss our work on modeling the spread and control of COVID-19. I hope to convince you that, despite the enormous public health and socio-economic devastation it exudes globally, COVID-19, like its coronavirus cousins (SARS and MERS), is a respiratory disease that is controllable using basic public health interventions.   (Received September 15, 2020)

1163-92-1305   **Sebastian J Schreiber*** (`sschreiber@ucdavis.edu`), Department of Evolution and Ecology, One Shields Avenue, University of California, Davis, Davis, CA 95616. *Species coexistence in a temporally autocorrelated world.*

All species experience temporal fluctuations in environmental conditions e.g. temperature or mortality risk. These fluctuations often are autocorrelated e.g. warmer years tending to be followed by warmer years. How these autocorrelations influence species coexistence is, largely, an open problem. Recently, Benaim and Schreiber (J. Math. Bio. (2019) 79:393) developed new mathematical results to characterize coexistence and extinction for stochastic, multispecies models with temporal autocorrelations. These characterizations rely on Lyapunov exponents at stationary distributions supporting a subset of species. Applying these methods to classical ecological modules of exploitative competition and apparent competition, I determine how autocorrelated temporal fluctuations alter ecological outcomes. For example, if survivorship of competing species fluctuate, then negative autocorrelations promote coexistence while positive autocorrelations lead to stochastic priority effects. In contrast, positively autocorrelated fluctuations in attack rates of a shared predator can mediate coexistence, while

negatively autocorrelated fluctuations generate stochastic priority effects. These results highlight the importance of temporal autocorrelations in structuring ecological communities.   (Received September 15, 2020)

1163-92-1307   **Nicholas Mattia Marazzi***, marazzin@mail.missouri.edu, and **Giovanna Guidoboni**, **Riccardo Sacco**, **Josh Fraser**, **Kannappan Palaniappan** and **Virginia Huxley**. *Fluid and proteins in microvascular networks: importance of heterogeneity in geometrical and biophysical properties.*

The main goal of this study is to illustrate how the estimate of the fluid exchange between microvasculature and surrounding tissue is impacted by assuming heterogenous or homogeneous network properties. To this purpose, the microvascular fluid exchange is studied in three different configurations: homogeneous (all vessels in the network are identical with respect to dimensions and biophysical properties), class-uniform (class-specific dimensions and biophysical properties) and heterogeneous scenario (class-specific biophysical properties and vessel-specific geometrical properties). The assumption of homogeneous distribution of geometrical and biophysical network leads to an overestimate of $J_{f,tot}$ of 317   (Received September 15, 2020)

1163-92-1329   **Benjamin A Levy*** (`blevy1@fitchburgstate.edu`), 160 Pearl St, Fitchburg, MA 01420. *Modeling the Effect of HIV/AIDS Stigma on HIV Infection Dynamics in Kenya.*

Stigma towards people living with HIV/AIDS (PLWHA) has impeded the response to the disease across the world. Widespread stigma leads to poor adherence to preventative measures while also causing PLWHA to avoid testing and care, delaying important treatment. Stigma-induced behavior therefore lowers treatment rates in a community while increasing transmission rates and death rates. Levels of HIV/AIDS-related stigma are particularly high in sub-Saharan Africa, which contributed to a surge in cases in Kenya during the late 20th century. Since the early 21st century the United Nations and governments around the world have worked to eliminate stigma from society. Resulting public health education campaigns have improved the perception of PLWHA over time, but HIV/AIDS remains a significant problem, particularly in Kenya. We formulate a system of ordinary differential equations to model the effect of stigma on the spread of HIV in Kenya. We measure how stigma is changing over time using survey data, estimate parameters for our model using time-series data, and consider the impact of a changing stigma level on disease dynamics. Our results compare model output from various hypothetical scenarios and quantify the impact of HIV/AIDS-related stigma in Kenya.   (Received September 15, 2020)

1163-92-1338   **R J Braun*** (`rjbraun@udel.edu`), **L J Cummings** and **T A Driscoll**. *Two-layer models for tear film dynamics.*

After each blink, the tear film on the ocular surface establishes a multilayer fluid film that helps protect the ocular surface and promote good vision. Tear breakup (TBU) is a mode of failure of the tear film where the aqueous layer thins dramatically, and it is an etiological factor in dry eye disease, which affects millions. Several aspects of TBU dynamics and properties can be understood well from single layer models of the tear film   (Received September 15, 2020)

1163-92-1340   **René A. Salinas*** (`salinasra@appstate.edu`), Department of Mathematical Sciences, 342 Walker Hall, 121 Bodenheimer Drive, Boone, NC 28608. *Individual-Based Models in Wildlife Management: How we got here and where are we going.* Preliminary report.

Individual-based models (IBMs) have been used in biology for over 40 years. As demographic and spatial data became more standard, the utilization of IBMs increased as a preferred tool in modeling populations. In this talk, I will present an historical overview on how advances in data acquisition increased the utility of IBMs, discuss some recent work on feral hog management, and use recent articles to suggest how advances in data science will increase the power of IBMs in understanding the complexity of populations.   (Received September 15, 2020)

1163-92-1342   **Andreas Buttenschoen*** (`abuttens@math.ubc.ca`), 1984 Mathematics Road, Vancouver, BC V6T1Z2. *A 3D cell-based model of cell migration in the extracellular matrix, facilitated by filopodia, degradation, and cell deformation.*

Cell migration in the extracellular matrix (ECM) or ECM-like environments is driven by cells attaching filopodia to ECM fibers and pulling on them. At the same time, however, high densities of the ECM act as a mechanical barrier for cell migration. To migrate effectively, cells maneuver through the ECM, finding holes through which they can squeeze, or mechanically adapt the elastic ECM by deformation or degradation. The combination of these mechanisms results in cells that move optimal in medium dense ECM environments. Here we ask whether these three space negotiation mechanisms in combination with migration via pulling are sufficient to explain larger scale observations in cell migration, namely directed migration in microtrack assays, single cell

ECM invasion and multicellular coordinated migration. To answer these questions, we built a computational cell-based model of cell migration using cells that can adapt their shape, mechanically interact with an elastic ECM, and degrade the ECM.

Joint work with Paul van Liedekerke, Dirk Drasdo, and Margriet Palm.   (Received September 15, 2020)

1163-92-1362   **Laura F. Strube*** (`lfstrube@vt.edu`), lfstrube@vt.edu, and **Maya Walton** and **Lauren M. Childs**, lchilds@vt.edu. *The role of repeat infection in the dynamics of a simple model of waning and boosting immunity.*

Some infectious diseases produce lifelong immunity while others only produce temporary immunity. In the case of short-lived immunity, the level of protection wanes over time and may be boosted upon re-exposure, via infection or vaccination. Previous work developed a simple model capturing waning and boosting immunity, known as the Susceptible-Infectious-Recovered-Waned-Susceptible (SIRWS) model, which exhibits rich dynamical behavior including supercritical and subcritical Hopf bifurcations among other structures. Here, we extend the bifurcation analyses of the SIRWS model to examine the influence of all parameters on these bifurcation structures. We show that the bistable region, involving both a stable fixed point and a stable limit cycle, exists only for a small region of biologically realistic parameter space. Furthermore, we contrast the SIRWS model with a modified version, where immune boosting depends on the occurrence of a secondary infection. Analysis of this extended model shows that oscillations and bistability, as found in the SIRWS model, depend on strong assumptions about infectivity and recovery rate of secondary infection. Understanding the dynamics of models of waning and boosting immunity is important for accurately assessing epidemiological data.   (Received September 15, 2020)

1163-92-1375   **Brenton LeMesurier*** (`lemesurierb@cofc.edu`) and **Alexander Kasman** (`kasmana@cofc.edu`). *Mathematical modeling of DNA transcription bubbles.* Preliminary report.

In 1990, Englander et al proposed the sine-Gordon equation (sGE) as a basic mathematical model of the DNA transcription bubble. sGE is an integrable PDE having exact solutions in the form of kink solitons. However, the derivation of this continuous model relies on many dubious approximations and assumptions. In particular, it assumes uniformity of the bases along the DNA strands and symmetry of the two helices.

Numerous authors have proposed and studied refinements over the intervening decades. This talk describes some of those modeling efforts, with an emphasis on the effects of symmetry-breaking and the implications of the sequence-dependent dynamics. The good news is that some useful core aspects of the sine-Gordon approximation persist. It still has coherent kink solutions whose dynamics are of possible relevance to DNA transcription bubbles, and shows some interesting effects of codon non-uniformity.   (Received September 15, 2020)

1163-92-1382   **Asma Alshehri**, **John Ford** and **Rachel Leander***, Rachel.Leander@mtsu.edu. *The impact of maturation time distributions on the structure and growth of cellular populations.*

Here we study how the structure and growth of a cellular population vary with the distribution of maturation times from each stage. We consider two cell cycle stages. The first represents early G1. The second includes late G1, S, G2, and mitosis. Passage between the two reflects passage of an important cell cycle checkpoint known as the restriction point. We model the population as a system of partial differential equations. After establishing the existence of solutions, we characterize the maturation rates and derive the steady-state age and stage distributions as well as the asymptotic growth rates for models with exponential and inverse Gaussian maturation time distributions. We find that the stable age and stage distributions, transient dynamics, and asymptotic growth rates are substantially different for these two maturation models. We conclude that researchers modeling cellular populations should take care when choosing a maturation time distribution, as the population growth rate and stage structure can be heavily impacted by this choice. Furthermore, differences in the models' transient dynamics constitute testable predictions that can help further our understanding of the fundamental process of cellular proliferation.   (Received September 15, 2020)

1163-92-1383   **Omar Saucedo*** (`osaucedo@vt.edu`). *Understanding how structural network change impacts infectious diseases.* Preliminary report.

Throughout our daily lives, we establish connections with other individuals either by visiting a grocery store, attending a local social gathering, or meeting with family. We can view the interactions as a person's general social network. Certain events such as a large-scale natural disaster can disrupt these networks by either reducing the number of connections or making certain connections weaker. As observed in the recent global pandemic, the localities that enacted strong health measures with regard to minimizing social interactions helped reduce

the propagation of the disease. In this talk, we will examine how changes in the network structure can impact the spread of an infectious disease.   (Received September 15, 2020)

1163-92-1421   **Kamrine Poels*** (`kpoels@g.harvard.edu`), **Adam J Schoenfeld**, **Alex Makhnin**, **Yosef Tobi**, **Yuli Wang**, **Aaron Hata**, **Scott L Weinrich**, **Helena A Yu** and **Franziska Michor**. *Identification of optimal dosing schedules of dacomitinib and osimertinib for a phase I/II trial in advanced EGFR-mutant non-small cell lung cancer.*

Despite the clinical success of the third-generation EGFR inhibitor osimertinib as first-line treatment of EGFR-mutant non-small cell lung cancer (NSCLC), resistance predictably arises, and alternative therapies are needed. Dacomitinib, a pan-HER inhibitor, is approved for first-line treatment and results in different acquired EGFR mutations that mediate on-target resistance. A combination of osimertinib and dacomitinib could induce more durable responses by preventing on-target acquired EGFR mutations. We present an integrated computational modeling and experimental approach to identify an optimal dosing schedule for osimertinib and dacomitinib combination therapy. We developed a predictive modeling platform that encompasses tumor heterogeneity and inter-subject pharmacokinetic variability to predict tumor evolution under different dosing schedules, parameterized using in vitro dose-response data. This model was validated using cell line data and used to identify an optimal combination dosing schedule. Our dosing schedule was subsequently confirmed tolerable in an ongoing phase I clinical trial at MSKCC, with some dose modifications, demonstrating that our rational modeling approach can be used to identify appropriate dosing for combination therapy in the clinical setting.   (Received September 15, 2020)

1163-92-1425   **Libin Rong*** (`libinrong@ufl.edu`), Department of Mathematics, University of Florida, Gainesville, FL 32611. *Dynamics of a new HIV model with the activation status of infected cells.* Preliminary report.

The activation status can dictate the fate of an HIV-infected CD4+ T cell. Infected cells with a low level of activation remain latent and do not produce virus, while cells with a higher level of activation are more productive and thus likely to transfer more virions to uninfected cells. How the activation status of infected cells affects HIV dynamics under antiretroviral therapy remains unclear. We develop a new mathematical model that structures the population of infected cells continuously according to their activation status. The effectiveness of antiretroviral drugs in blocking cell-to-cell viral transmission is assumed to decrease as the level of activation of infected cells increases because the more virions transferred from infected to uninfected cells, the less effectively the treatment is able to inhibit the transmission. The basic reproduction number of the model is shown to determine the existence and stability of the equilibria. This study also provides a new modeling framework to study the observations, such as the low viral load persistence, extremely slow decay of latently infected cells and transient viral load measurements above the detection limit, in HIV-infected patients during suppressive antiretroviral therapy.   (Received September 15, 2020)

1163-92-1427   **Alexis L. White*** (`alexis.white@ufl.edu`), University of Florida, Gainesville, FL 32611, and **Holly D. Gaff** (`hgaff@odu.edu`), Dept of Biological Sciences, Norfolk, VA 23529. *Simulation of an integrated tick management program with a tick-killing robot and guinea fowl.*

Ticks are vectors of disease-causing pathogens that affect humans, wildlife, and domestic animals. Effective control measures are needed to reduce the risk of encountering ticks, and thus reduce risk of tick-borne disease. Interated tick management (ITM) has been used to implement multiple tick control methods at once to increase efficacy but have not been well studied because of the complexities of tick and host ecology. Using the data from a tick surveillance study, along with the results from tick control field studies, an agent based model was developed to explore how integrated tick management tools interact to increase efficacy. Within the model we explore an ITM program within a backyard that uses three forms of control: (1) TickBot, a tick-killing robot that lures ticks to a treated area (2) guinea fowl as a biological control, but are also hosts to juvenile life stages of ticks (3) lawn mowing to decrease favorable habitat of ticks. Results from the model reveal the complex dynamics between control strategies and emphasize the need for further field studies to better under these forms of tick control and improve model parameters.   (Received September 15, 2020)

1163-92-1445   **Sherry E Scott*** (`sscott@msri.org`), **K. Barley**, **K. Cook** and **A. Gumel**. *Which infectious classes are the main drivers for COVID -19?*

There is evidence that a significant portion of the infected cases in the transmission of COVID-19 come from the asymptomatic and presymptomatic cases. We design a mathematical model to evaluate the contribution of asymptomatic and presymptomatic transmission using data from the states of Arizona, Florida, New York and

Washington. More specifically we address the question of how many new cases are generated by presymptomatic and asymptomatic infectious individuals and seek to use this information to guide and determine postlockdown strategies.   (Received September 15, 2020)

1163-92-1465   **Mary Silber*** (`msilber@uchicago.edu`), Department of Statistics, University of Chicago, Chicago, IL 60637. *A fast-slow pattern formation model for investigating large-scale vegetation bands in drylands.*

From infiltration of water into the soil during rainstorms to seasonal plant growth and death, eco-hydrological processes relevant to the formation of banded vegetation patterns in drylands occur across multiple timescales. A new fast-slow switching model, developed to capture key processes on appropriate timescales, is introduced in the form of reaction-advection-diffusion equations. The fast system captures hydrological processes that occur on timescales associated with major rainstorms. The slow system acts between rain events, on a timescale that evolves vegetation and soil moisture over the dry seasons. We explore the fast-slow switching model through numerical simulation on a one-dimensional hillslope, and find agreement with certain observations about the pattern formation phenomenon, including band spacing and upslope colonization rates. This fast-slow model framework introduces a tool for investigating the possible impact of changes to frequency and intensity of rain events in dryland ecosystems, as well as stochasticity of rain inputs. (This talk is based in part on a paper coauthored by P Gandhi, S Bonetti, S Iams, and A Porporato, that appeared in Physica D in 2020.)   (Received September 15, 2020)

1163-92-1466   **Xi Huo***, Ungar 515, 1365 Memorial Dr, Coral Gables, FL 33146-2508. *Antimicrobial de-escalation: How mathematical models can infer the design of clinical studies.*

Antimicrobial de-escalation is a highly recommended and widely practiced drug use strategy in intensive care units. It is believed to play important roles in reducing the development and transmission of antibiotic resistance, as well as reducing mortality and inappropriate empiric therapies. However, such benefits were not uniformly observed in clinical studies, making it hard to conclude the benefits and trade-offs of such a strategy. In this talk, we will present results from mathematical models, in terms of individual-based simulations and long-term behavior analysis, to infer the expected outcomes of clinical studies that compare de-escalation with traditional therapy. We believe that modeling could assist the design of many clinical studies to help researchers to obtain reasonable expectations of the outcomes, plan ahead analytical methods, and adjust data collection methods. (Received September 15, 2020)

1163-92-1486   **Rachel R. Harman*** (`rachel_harman@outlook.com`) and **James T. Cronin**. *Empirically assessing density-dependent emigration within fragmented environments using insects.*

Habitat fragmentation creates smaller habitable patches often separated by a more hostile matrix. In this landscape, dispersal is a fundamental process that generates a metapopulation network and range expansion. An individual's decision to emigrate from its habitat may be influenced by the conspecific density, which, in small patches, may affect population persistence. Empirical studies have often ignored nonlinear forms of density-dependent emigration (DDE) by utilizing methods that include few densities, a small range of densities, or statistics that preclude a nonlinear term. Here, we assessed the emigration of insects, including Ischnodemus falicus, a gregarious marsh bug, using mark-release-recapture techniques. Forty releases that ranged from 3 to 180 individuals (twice the highest density observed in the field) were conducted. The resulting DDE form was non-linear negative, suggesting a u-shaped response curve that would not have been observed with a smaller density range; however, highly dense populations may be created as a consequence of habitat fragmentation. Although it can be difficult, using empirical methods that allow for different forms of DDE to be quantified will improve conservation efforts by better identifying populations at risk of extinction.   (Received September 15, 2020)

1163-92-1515   **James Webber** (`james.webber@tufts.edu`), 161 College Ave., Medford, MA 02155, **Eric Todd Quinto*** (`todd.quinto@tufts.edu`), 503 Boston Ave., Medford, MA 02155, and **Eric Miller** (`eric.miller@tufts.edu`), 161 College Ave., Medford, MA 02155. *Microlocal Analysis of Joint Reconstruction in X-ray+Compton Tomography.*

We present a new joint reconstruction method for the simultaneous reconstruction of attenuation coefficient and electron density from X-ray transmission and Compton-scattered backscattered data. We use microlocal analysis to show that the Compton and X-ray CT data set are complementary–this ensures that most wavefront directions of the object are visible from the combined data. We use a Lambda tomography penalty term in the regularization procedure of the algorithm to integrate the two data sets together. We evaluate our reconstruction method on the "parallel line segment" acquisition geometry of [arXiv:1907.00418, 2019] which is motivated by

a specific architecture for airport security screening. We first present a novel microlocal analysis of this data acquisition geometry which explains the nature of image artefacts when the attenuation coefficient and electron density are reconstructed separately. We introduce a new joint reconstruction scheme for low effective atomic number imaging characterized by a regularization strategy whose structure is derived from lambda tomography principles and motivated directly by the microlocal analysis results. Finally, we show the effectiveness of our method in combating the noise and image artefacts on simulated phantoms.   (Received September 15, 2020)

1163-92-1519   **Natalie Petruzelli*** (ngp01978@sjfc.edu) and **Keegan Kresge**. *Analyzing Epidemic Thresholds on Dynamic Network Structures.*
COVID-19 epidemics in parts of the U.S. have shown unexpected shifts from exponential to linear growth in the number of daily new cases. We explore a network-based epidemic model that interpolates between lattice-like and configuration model networks while keeping the degree distribution and reproduction number (R_0) constant to show these dynamics. This model gives nodes locations and connects them to their nearest neighbors, rearranging a proportion p of the edges in a configuration model subnetwork. As p increases, we observe a shift from linear to exponential growth. Real human contact networks have many local interactions and fewer long-distance ones, so social distancing affects the effective reproduction number R_t and the proportion of long-distance connections. While the impact of changes in R_t is understood, less is known about the effect of subtle changes in network structure. Our analysis finds that the threshold between dynamics occurs with a low percentage of reconfigured edges. Furthermore, the total infections in an epidemic substantially increase around the threshold even with a constant R_0. This study reveals that enacting and relaxing social distancing restrictions has more complex and dramatic effects on epidemic dynamics than previously thought.   (Received September 15, 2020)

1163-92-1536   **Rebecca A Everett***, reverett@haverford.edu, and **Alexis Kuerbis**, **Allison Lewis**, **Jing Li** and **Angela Peace**. *Data-Driven Mathematical Modeling of the Dual Process Framework of Addiction Among Problem Drinkers.*
Alcohol use disorder (AUD) is a costly, heterogeneous disorder that comprises a continuum of symptoms and severity of associated problems. Problem drinkers make up about half of all individuals with AUD, and tend to be characterized as having mild to moderate levels of AUD severity and higher psychosocial functioning compared to those with severe AUD. We are interested in identifying mechanisms of change in drinking behavior among problem drinkers. We apply a discrete dynamical system model to clinical data using mixed effects parameterization in order to identify both population and individual level parameters.   (Received September 15, 2020)

1163-92-1574   **Yafei Wang*** (wangyafe@iu.edu), 700 N Woodlawn Ave, Bloomington, IN 47408, and **Heber Rocha**, **Randy Heiland** and **Paul Macklin**. *Simulation of drug-loaded nanoparticles on cancer treatment: an agent-based modelling approach.* Preliminary report.
Cancer is a complex systems problem that involves tumor cells and their microenvironment. The application of engineered nanomaterials in medicine for therapy or diagnosis and nanomedicine-biological systems interaction is becoming significant. The mathematical modelling on cancer treatment with nanoparticles (NPs) can be used to explore NPs, tissue, cell, and drug response parameters for a growing tumor, and see for themselves how things could play out for their design choices.

In this work, we propose an agent-based model (with PhysiCell) to investigate the therapeutic designs of cancer treatment with NPs/drug, where NPs internalization, drug release, NPs/drug pass in inheritance and drug effects on tumor cells are explored. Our simulation studies show that drug-loaded nanoparticles have some allowed promising new options for cancer therapy, and the point to the power of using large-scale model exploration to tune and improve therapy. In particular, we introduce a novel tracking of nanoparticle populations in each individual cell, allowing better modelling of drug release by internalized nanoparticles, and also providing the capability of modelling the nanotherapy of tumor in multi-generation and long-term therapeutic implications. (Received September 15, 2020)

1163-92-1575   **Cameron Browne*** (cambrowne@louisiana.edu), **Hayriye Gulbudak** and **Joshua Macdonald**. *Differential impacts of contact tracing and lockdowns on outbreak size in COVID-19 model applied to China.*
Rapid growth of the COVID-19 epidemic in China induced extensive efforts of contact tracing and social-distancing/lockdowns. We construct a novel infectious disease model incorporating these distinct quarantine measures (contact tracing and self-quarantine) as reactionary interventions dependent on current infection levels. Derivation of the final outbreak size leads to a simple inverse proportionality relationship with self-quarantine

rate, revealing a fundamental principle of exponentially increasing cumulative cases when delaying mass quarantine or lockdown measures beyond a critical time period. In contrast, contact tracing results in a proportional reduction in reproduction number, flattening the epidemic curve but only having sizable impact on final size when a large proportion of contacts are traced. We fit the mathematical model to data from China on reported cases and quarantined contacts, finding that lockdowns had an overwhelming influence on compressing the outbreak, whereas contact tracing played a role in reducing peak number of infected. Sensitivity analysis under different re-opening scenarios illustrate the differential effects that responsive contact tracing and lockdowns can have on subsequent outbreaks.   (Received September 15, 2020)

1163-92-1580   **Laura Staugler*** (lstaugler@wpi.edu) and **Andrea Arnold** (anarnold@wpi.edu). *Using Ensemble Kalman Filter to Estimate Stochastic Current in the FitzHugh-Nagumo Model.*
The FitzHugh-Nagumo model is widely used for understanding the dynamics of a single neuron. While it may be possible to experimentally measure the voltage, the stochastic input current may be unmeasurable. The aim of this work is to estimate stochastic currents given noisy voltage data for a single neuron. We utilize ensemble Kalman filtering with parameter tracking to estimate the applied current and associated FitzHugh-Nagumo model states. Results are demonstrated using time-varying stochastic currents and currents that switch between deterministic and stochastic.   (Received September 15, 2020)

1163-92-1641   **Kevin Flores*** (kbflores@ncsu.edu), Department of Mathematics, North Carolina State University, Raleigh, NC 27695. *Equation learning for partial differential equation models of biological transport.*
The use of partial differential equation (PDE) models for biological transport has seen widespread use in describing ecological, biomedical, and developmental processes. In this talk we review recent progress in learning PDE models directly from spatiotemporal data, focusing on specific challenges encountered in biology such as high noise levels and limited time samples. The main application we will discuss is learning models that describe collective cell migration from time-lapse microscopy data. Specifically, we used equation learning to derive a new PDE with a structure similar to the nonlinear Fisher-KPP equation, and that is also more accurate than previously used models for describing cell migration data.   (Received September 15, 2020)

1163-92-1654   **Suzanne Sindi***, 5200 North Lake Road, Merced, CA 95340. *Modeling and Parameter Inference in Biological Systems.*
Biological systems are inherently complex, and mathematical modeling is a natural tool for gaining intuition and generating novel hypotheses. However, a critical challenge in biological systems is their uncertainty. Uncertainty can take place in many forms, and this talk will explore two of them.

First, even if we have knowledge of mechanisms in biology, we often do not know critical parameters in how critical components interact. In this talk I will describe efforts to use Bayesian parameter inference and model selection to discover the correct model and its corresponding values.

Second, in the analysis of mathematical models we often focus on asymptotic behavior. However, true biological systems operate on finite time scales. In this work we are interested in studying a problem in cell biology where a critical event must take place before the cell divides. We frame this problem as calculating the first arrival times of a stochastic process.   (Received September 16, 2020)

1163-92-1655   **Suzanne Sindi***, 5200 North Lake Road, Merced, CA 95343, **Mikahl Banwarth-Kuhn**, 5200 North Lake Road, Merced, CA 95343, and **Jordan Collignon**, 5200 North Lake Road, Merced, CA 95434. *Multiscale Modeling of Prion Aggregate Dynamics in Yeast.*
Prion proteins are responsible for a variety of neurodegenerative diseases in mammals such as Creutzfeldt-Jakob disease in humans and "mad-cow" disease in cattle. While these diseases are fatal to mammals, a host of harmless phenotypes have been associated with prion proteins in S. cerevisiae, making yeast an ideal model organism for prion diseases. Most mathematical approaches to modeling prion dynamics have focused on either the protein dynamics in isolation, absent from a changing cellular environment, or modeling prion dynamics in a population of cells by considering the "average" behavior. However, such models have been unable to recapitulate in vivo properties of yeast prion strains.

In this talk, I will show some results from recent individual based simulations where we study how the organization of a yeast population depends on the division and growth properties of the colonies. Each individual cell has their own configuration of prion aggregates and we study how the population level phenotypes are a natural consequence of the interplay between the cell cycle, budding cell division and aggregate dynamics.

We quantify how common experimentally observed outcomes depend on population heterogeneity. (Received September 16, 2020)

1163-92-1665   **Edward T Dougherty*** (`edougherty@rwu.edu`), Roger Williams University, Bristol, RI 02809, Bristol, RI 02809, and **Kaia R. Lindberg**. *Electrotherapy Impact on Neuronal Electrophysiology.*
Electrotherapies continue to mitigating the debilitating symptoms of numerous brain disorders, yet the underlying cellular effects of these treatments are not comprehensively understood. Using a mathematical model that couples the Poisson-Nernst-Planck system of partial differential equations and Hodgkin-Huxley based ordinary differential equations, the effects of deep brain stimulation on neuronal electrophysiology are investigated. Our results demonstrate that deep brain stimulation elevates transmembrane potential to facilitate action potential firing, and in addition, yields an influx of calcium which is known to be critical in the secretion of neurotransmitters for proper, healthy cellular functioning. Finally, comparisons to other modes of electrotherapy highlight differences in cellular-level impacts among these different treatment forms. (Received September 16, 2020)

1163-92-1670   **Robert Gatenby*** (`robert.gatenby@moffitt.org`), 12902 Magnolia Dr, Tampa, FL. *Ecology and evolution in control and cure of metastatic cancers.*
Clinical oncology investigations have largely focused on new drug discovery. However, most metastatic cancers remain fatal because even highly effective treatments usually fail due to evolution of resistance. Arguably, Darwinian dynamics are thus the proximate cause of death in many cancer patients.

From an evolutionary perspective, the conventional "maximum tolerated drug dosage" strategy is often suboptimal because it imposes maximal selection pressure for resistance while eliminating the treatment-sensitive populations, which are potential competitors.

An alternative approach seeks to prolong response and tumor control by exploiting evolutionary principles through modulating the treatment schedule to suppress proliferation of resistant population. Studies have found this approach can be successful.

More recent work has proposed evolutionary dynamics can be used to cure currently fatal cancers by strategic sequencing of available agents. This is based on a paradigm that curing cancer is equivalent to an Anthropocene extinction and can be achieved through a series of frequently small eco-evolutionary perturbations. Examples include treatments for pediatric Acute Lymphocytic Leukemia. Clinical trials using this extinction strategy" are now underway. (Received September 16, 2020)

1163-92-1682   **Oyita Udiani*** (`udianio@vcu.edu`), 1015 Floyd Avenue, Richmond, VA 23284. *Adaptive specialization in multi-species predator-prey systems.*
Competitive interactions among species and natural enemies determine both species diversity in ecological communities and phenotypic diversity of those species. For example, in bacterial communities, the cast of characters includes opportunistic predators (protists, predatory bacteria and phages) that differ in prey range (generalists vs. specialists) and the capacity to adapt in changing environments. In this talk, I will introduce a dynamical systems framework for the study of multi-species consumer-resource interactions within an ecosystem. I will discuss preliminary insights from this framework that can help uncover how selective trade-offs experienced by consumers may constrain resource use patterns within a community. (Received September 16, 2020)

1163-92-1689   **Calistus Ngonghala***. *The impact of non-pharmaceutical interventions on COVID-19.*
The novel coronavirus (COVID-19), which emerged from China in December 2019 continues to cause devastating public health and socio-economic impact in many parts of the world. In the absence of a safe and effective treatment or vaccine against the virus, non-pharmaceutical interventions (NPIs) are the main tools for controlling and mitigating the burden of the pandemic. We develop and use mathematical models to assess the impact of NPIs such as social distancing, lockdown measures, face-mask use in public, case detection, etc., on the burden of the pandemic in some US states and the entire US. Our results show that pre-symptomatic and asymptomatic humans are the main drivers of the pandemic in the US, thereby emphasizing the need for random testing and contact tracing. Also, our results indicate that a) early lifting of strict lockdown measures could trig- ger a devastating second COVID-19 wave, while extending the lockdown period by a short time, e.g., two weeks, would have suppressed the devastating post-lockdown resurgence in the US; b) wide-scale use of face masks in public could halt the post-lockdown resurgence of COVID-19 in the US; and c) implementing lockdown and mask-use measures two weeks earlier would have prevented the pandemic from taking off significantly in the US. Thus, the study suggests that, like other past Coronavirus outbreaks, COVID-19 might be controllable using basic non-pharmaceutical interventions. (Received September 21, 2020)

93 ► *Systems theory; control*

1163-93-42   **Michael Malisoff*** (`malisoff@lsu.edu`), Department of Mathematics, 303 Lockett Hall,
Field House Drive, Louisiana State University, Baton Rouge, LA 70803-4918. *Feedback
Stabilization of Control Systems Arising in the Visual Landing of Aircraft.*
We discuss a visual landing problem in which an aircraft must align with a runway using a body fixed camera.
Such problems arise when emergencies call for civilian or military aircraft to land on unequipped runways and
when there may be GPS loss. We help address this problem using a nonlinear control systems approach, by
providing new output feedback designs with imprecise output measurements where the outputs can also contain
delays and sampling. This research is collaborative with Laurent Burlion from the Rutgers University Department
of Mechanical and Aerospace Engineering and with Frederic Mazenc from INRIA in France. No prerequisite
background in feedback control will be needed to understand and appreciate this talk.   (Received July 27, 2020)

1163-93-382   **Wei Kang*** (`wkang@nps.edu`), Department of Applied Mathematics, Naval Postgraduate
School, Monterey, CA 93943. *On the Observability of Adversarial Swarm Dynamics.*
In this talk, I will introduce a quantitative measure of partial observability for dynamical systems. For networked
systems such as swarms of unmanned vehicles, the concept and theory of partial observability are developed using
a computational approach. The new approach solves some problems for which traditional observability analysis
and estimation methods, such as Kalman filters, fail to address. The theory and numerical algorithms are
illustrated using an example of detecting swarm cooperating strategy.   (Received September 04, 2020)

1163-93-518   **Yang Shen** and **Bin Zou*** (`bin.zou@uconn.edu`). *Mean-Variance Portfolio Selection in
Contagious Markets.*
We consider a mean-variance portfolio selection problem in a financial market with contagion risk. The risky
assets follow a jump-diffusion model, in which jumps are driven by a multivariate Hawkes process with mutual
excitation effect. The mutual excitation feature of the Hawkes process captures the contagion risk in the sense
that each price jump of an asset increases the likelihood of future jumps not only in the same asset but also
in other assets. We apply the stochastic maximum principle and linear-quadratic control technique to solve the
problem and obtain the efficient strategy and efficient frontier in semi-explicit forms, subject to a non-linear
partial differential equation.   (Received September 08, 2020)

1163-93-977   **Hemant A. Patil*** (`hemant_patil@daiict.ac.in`), Room No. 4103, Faculty Block-4, Near
Indroda Circle, DA-IICT, Gandhinagar, Gujarat 382007, India. *Modeling Spectrum Decay
in Hilbert Space for Replay Spoof Detection.* Preliminary report.
Automatic Speaker Verification (ASV) refers to verifying the claimed identity of a speaker with the help of
machines. We would like an ASV system to be robust under various adverse conditions, such as microphone and
transmission channel, and acoustic noise. Such robustness makes an ASV system to be vulnerable to various
spoofing attacks as it tries to nullify these effects and thereby making spoofed speech more close to the natural
speech of genuine speakers. Hence, there is a need to develop countermeasures for various spoofing attacks. This
talk will concentrate on developing Spoofed Speech Detection (SSD) system for replay spoof, which is very easy
to mount but highly challenging to detect. Various technical challenges associated with replay SSD problem
will be analyzed along with relevant mathematical modeling of replay. Since the replay spoof is known to be
affected by bandpass frequency response characteristics of transmission channel and acoustic room, we model its
spectrum decay in Hilbert space formulation, in particular, using Lipchitz global vs. local regularity to measure
the spectrum decay using Lipchitz exponent. Finally, talk will discss Linear Prediction (LP)-based spectrum
whitening method to model spectrum decay.   (Received September 14, 2020)

1163-93-1657   **Saroj Aryal*** (`saryal@georgian.edu`), 900 Lakewood Ave, Lakewood, NJ 08701, and
Sarita Nemani (`snemani@georgian.edu`), 900 Lakewood Ave, Lakewood, NJ 08701. *On
Hurtwiz stability of Composite Polynomials.* Preliminary report.
A conjecture on robustness of stable transfer polynomials has been solved in negative. In this work, we formulate
a characterization of stable transfer polynomials, which will be used to identify a class of polynomials satisfying
the conjecture. Further results will be derived as variations of the conjecture.   (Received September 16, 2020)

94 ▶ *Information and communication, circuits*

1163-94-282   **Pantelimon Stanica*** (pstanica@nps.edu). *A multiplicative differential uniformity of block ciphers' Sboxes.*

Inspired by a practical attack on block ciphers based upon a modification of the differential attack in [N. Borisov, M. Chew, R. Johnson, D. Wagner, Multiplicative Differentials, Fast Software Encryption, FSE 2002], we defined recently (2019) a new (output) multiplicative differential, and the corresponding differential uniformity. Quite a few papers have appeared within months on this new notion that is interesting from both a mathematical and cryptography perspective. Here, we go through some of the results that were developed in the past year connected to this concept. For example, we find the *c*-differential uniformity of the inverse function (as used in the Advanced Encryption Standard), as well as some other (almost) perfect nonlinear functions, in both even and odd dimensional binary finite fields, just to mention a few such results. The proof methods are number theoretical in nature. Also, the *c*-differential uniformity of some real-life ciphers' Sboxes will be displayed. (Received August 31, 2020)

1163-94-330   **Nael Rahman** and **Vladimir Shpilrain*** (shpilrain@yahoo.com). *MAKE: a Matrix Action Key Exchange.*

We offer a public key exchange protocol based on a semidirect product of two cyclic (semi)groups of matrices over Z_p. One of the (semi)groups is additive, the other one multiplicative. This allows us to take advantage of both operations on matrices to diffuse information. We note that in our protocol, no power of any matrix or of any element of Z_p is ever exposed, so all standard attacks on Diffie-Hellman-like protocols (including Shor's quantum algorithm attack) are not applicable.   (Received September 02, 2020)

1163-94-385   **Thor Martinsen** and **Vikram Kanth*** (vkkanth@nps.edu), Naval Postgraduate School, Monterey, CA. *The Self-Shrinking Conflation Generator: A New Pseudorandom Bit Generator.* Preliminary report.

The backbone of many cyber security applications and algorithms requires random numbers. Due to the fact that true randomness is hard to capture and use, pseudo-random number generators (PRNG) are used to approximate it. One of the most commonly used pseudo-random generators is the Linear Feedback Shift Register (LFSR), which is fast, computationally inexpensive, and has excellent statistical properties. LFSRs have many weaknesses, some of which were addressed by decimation-based sequence generators like the self-shrinking generator (SSG). The SSG is also vulnerable to attack. We propose an improvement to the SSG called the self-shrinking conflation generator (SSCG). Our approach uses the observation that what is discarded during the self-shrinking process is from a cryptographic perspective, just as good as what is kept. By combining the normally discarded bits with those that are retained, we create a modified bitstream with improved characteristics. We provide some mathematical security analysis associated with this approach, apply the NIST statistical test suite to several different bitstreams created using LFSRs, and compare our results to that of the SSG.   (Received September 04, 2020)

1163-94-441   **María Isabel González Vasco**, **Ángel Pérez del Pozo**, **Rainer Steinwandt*** (rsteinwa@fau.edu) and **Adriana Suárez Corona**. *Password-Authenticated Key Establishment in the Advent of Scalable Quantum Computing.*

To address the impact of large-scale quantum computing, new designs for cryptographic key establishment protocols now commonly integrate tools from post-quantum cryptography. A standard approach is to rely entirely on cryptographic tools, e.g., post-quantum signatures, for which no efficient quantum cryptanalytic attacks are known.

Available quantum resource estimates to implement, e.g., Shor's algorithm, suggest that quantum cryptanalytic attacks are not an imminent threat yet, and this talk focuses on a quantum future scenario. Here, an adversary is restricted to classical computation during (today's) protocol execution, but can leverage quantum computing after the protocol execution has ended (in the future). Such a security model opens up the possibility to temporarily rely on established hardness assumptions, even if in the long-term a quantum cryptanalytic attack becomes feasible. The talk shows how password-authenticated (group) key establishment can be realized efficiently in such a setting.

1163-94-841   **Akram Aldroubi*** (`akram.aldroubi@vanderbilt.edu`), **Shiying Li**
                   (`sl8jx@virginia.edu`) and **Gustavo Rohde** (`gr2z@virginia.edu`). *Partitioning signal
                   classes using transport transforms for data analysis and machine learning.*
A relatively new set of transport-based transforms (CDT, R-CDT, LOT) have shown their strength and great
potential in various image and data processing tasks such as parametric signal estimation, classification, cancer
detection among many others. In this talk we give an overview of the transport transforms and their relations
to transport theory. We then provide conditions under which classes of signals that are created by algebraic
generative models are transformed into convex sets by the transport transforms. Such convexification of the
classes simplify the classification and other data analysis and processing problems when viewed in the transform
domain.   (Received September 13, 2020)

1163-94-1069   **Lubjana Beshaj*** (`lubjanabeshaj@gmail.com`), Highland Falls, NY 10928, and **Gaurav
                   Tyagi**. *Challenges of using neural networks for encryption.*
The artificial neural networks domain is thriving in current times, mainly because ANN are useful in learning
to distinguish patterns that are sometimes even impossible for humans. The most common characteristic of
ANN, described in many literary papers, is "They work as Black Box". Which is true because ANN learn
iteratively from the training data and final state is not reproducible by using any explicit formulation. To use
ANN for encryption is a valid choice for a multiple of reasons, but in this paper, we are analyzing challenges and
inconsistencies which needed to be addressed to use ANN for encryption.   (Received September 14, 2020)

1163-94-1510   **David Jao*** (`djao@uwaterloo.ca`), 200 University Ave. W, Waterloo, Ontario N2L3G1,
                   Canada. *Implementation of isogeny-based cryptography.* Preliminary report.
The past few years have seen dramatic progress in state of the art algorithms for implementing and evaluating
isogenies in the context of public-key cryptography. In this talk we survey recent improvements in the asymp-
totic complexity of isogeny evaluation and present detailed case studies of optimized performance benchmarks.
(Received September 15, 2020)

97 ▶ *Mathematics education*

1163-97-100   **Joshua D Chesler*** (`josh.chesler@csulb.edu`) and **Jeffrey Pair**
                   (`jeffrey.pair@csulb.edu`). *Departmental partnerships and programmatic changes in
                   support of future high school teachers.*
At a university with a large population of mathematics majors on a secondary teach track, we developed spe-
cialized courses for these students by forming partnerships between mathematics educators and mathematicians.
By co-developing and co-teaching initial iterations of the courses, we have embraced and amplified the idea that
all department members have a role in preparing mathematics teachers. So far, courses in Abstract Algebra
and Mathematical Modeling have been developed through this process. We will discuss the motivation for and
design of our courses, student feedback, programmatic changes and constraints, and ongoing plans.   (Received
August 14, 2020)

1163-97-121   **Nicholas H Wasserman***, 525 W 120th St, Box 210-M, New York, NY 10027.
                   *Pedagogical Mathematical Practices as a way to Develop Pedagogy from Mathematics
                   Coursework.*
One of the challenges of mathematics courses in relation to developing pre- or in-service teachers' mathematical
knowledge for teaching (MKT) is balancing pedagogical aims with mathematical ones. The primary aim of
mathematics coursework is mathematics; but the development of MKT necessitates some degree of conversing
about pedagogy. Wasserman et al. (2017; 2019) developed a novel instructional model to design materials for
a real analysis course, and have previously discussed how the classroom teaching situations in these modules
supported the mathematical aims of a real analysis course. In this talk, I discuss further the nature and
development of the pedagogical aims - characterized as Pedagogical Mathematical Practices (PMPs). In a
manner akin to Shulman's (1986) notion of pedagogical content knowledge (PCK) as the intersection of domains
of knowledge, PMPs would be the intersection of domains of practice. They describe where mathematical practice
meaningfully intersects with mathematics teaching practice; they allow for conversing about pedagogy in ways
that also are mathematical. I share examples of these PMPs, their use within module materials, and results from
a study following secondary teachers into their classrooms to explore the impact on classroom teaching practice.
(Received August 18, 2020)

1163-97-164   **Jenna Tague*** (`jenna.tague@cloviscollege.edu`). *Structures to Support Students in an Undergraduate Mathematics Program.* Preliminary report.

Increasing diversity across Science, Technology, Engineering, and Mathematics (STEM) fields is an economic and social imperative. However, the question of how to support underrepresented students through the four or more years required to complete a STEM majors is unanswered (Karp, 2011). The purpose of this proposal is to report on two years of a 5-year longitudinal study, which examines structures put in place to provide support to low-income, underrepresented students in an undergraduate mathematics major. In particular, I describe the supports we designed, why they were chosen, as well as qualitative themes that help to assess those support structures.   (Received August 24, 2020)

1163-97-267   **Tracy Weyand*** (`weyand@rose-hulman.edu`). *Assigning versus integrating: How a skeptic became a believer.*

During the summer of 2018, I attended SIMIODE's MINDE workshop with the goal of finding more projects that I could assign my students to provide them with the opportunity to apply what they learned in class. When I left the workshop, I was still a skeptic of both the modeling-first approach and the ability to fit this in with the required curriculum. Even though I assigned new projects that fall, it didn't appear to have a positive impact on my students. I slowly started working towards "integrating" modeling rather than "assigning" modeling, as was introduced to me in the workshop. That was when I noticed a difference from my students, and that is when this skeptic became a believer.

I will describe how I am slowly transitioning my course(s) to incorporate more modeling without overburdening myself and while keeping to a common course schedule/curriculum. I'll explain several ways I have found, adapted, and integrated modeling into my course(s), and how this process has led to the publication of some of my modeling scenarios by SIMIODE. I will also note similarities and differences I have noticed from my students throughout this process.   (Received August 31, 2020)

1163-97-270   **Tracy Weyand*** (`weyand@rose-hulman.edu`). *Integrating Modeling into Differential Equations Courses.*

I'm certain that you too have experienced it; just because a student can use a certain solution (or analysis) technique for a differential equation does not mean that he/she can successfully apply that same technique to a physical situation. Many scientists and engineers will be doing exactly this, so how do we make sure students walk out of our class prepared?

I will describe how I am slowly transitioning my differential equations course(s) to incorporate more modeling and applications without overburdening myself and while keeping to a common course schedule and curriculum. I'll describe several ways I have found, adapted, and integrate modeling in my course(s). This will include my work with the SIMIODE (Systemic Initiative for Modeling Investigations & Opportunities with Differential Equations) as both a user and author of their modeling scenarios. I will also note similarities and differences that I have noticed about my students throughout this process.   (Received August 31, 2020)

1163-97-278   **Michael Alexander Radin*** (`michael.a.radin@gmail.com`), Rochester Institute of Technology, College of Science, School of Mathematical Sciences, Rochester, NY 14623. *New journey of discoveries in online education; opportunities and innovations.*

Due to corona virus crisis we were forced to switch to teaching online after teaching in the traditional face to face classroom atmosphere. We may view this as a negative experience that takes us out of our comfort zone or we may view this a new world of discoveries that can lead to new opportunities and innovations. Our first aim will be to discuss the advantages and disadvantages of teaching online versus traditional face to face classroom atmosphere. Our second aim will be to share new teaching practices in the digital atmosphere. How do we inspire students to learn in the digital atmosphere? Do students perform better in comparison to the traditional classroom atmosphere?   (Received August 31, 2020)

1163-97-320   **Mara P Markinson*** (`mara.markinson@qc.cuny.edu`). *Improving Prospective and Novice Secondary Mathematics Teachers' MKT Through a College-Level Course in Euclidean Geometry.* Preliminary report.

The session will focus on the development of a college-level course in Euclidean Geometry for prospective and novice secondary mathematics teachers. Emphasis will be on how the course's syllabus, activities, requirements, and structure fosters the growth of MKT in prospective and novice secondary teachers, specifically in Euclidean geometry and its teaching.   (Received September 02, 2020)

1163-97-375 **Brynja R Kohler*** (`brynja.kohler@usu.edu`), Dept of Mathematics and Statistics, Logan, UT 84322, and **William S Tidwell**. *Mathematical Modeling Curriculum Material for Secondary Teacher Education.*

Several years ago, the Mathematics Teacher Education Partnership formed a research action cluster with the aim of developing pre-service mathematics teachers' mathematical knowledge for teaching (MKT) in various content courses. This research action cluster, called the Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools [MODULE(S2)], is now piloting curriculum materials in college geometry, abstract algebra, statistics, and mathematical modeling with faculty across the country. The mathematical modeling course engages pre-service teachers in mathematical modeling activities, metacognition on the process of modeling, and reflection on best practices for implementing this type of learning in secondary schools. In this presentation, we'll introduce the mathematical modeling curriculum materials, describe the themes and key ideas from each module, the structure and flow of lessons, features of materials for instructors and students, and share activities that focus on developing MKT specific to mathematical modeling including "Fighting Floods with Sandbags" from Module 1, "Modeling Memorization" from Module 2, and "Area of Sioux Reservation Land" from Module 3. (Received September 04, 2020)

1163-97-383 **Mary Lynn Reed*** (`mlrsma@rit.edu`), 2334 Gosnell Hall, 85 Lomb Memorial Drive, Rochester, NY 14623. *Emphasizing Communication Skills and Career Perspectives in a Senior Capstone Course in Mathematics.*

After spending more than twenty years working as a mathematician and technical leader in government and industry, and overseeing the entire mathematics hiring program at one of the largest employers of mathematicians in the world (the National Security Agency), the author brought a unique "employer-oriented" perspective to the Senior Capstone Course in Mathematics at the Rochester Institute of Technology this past Spring. In this talk we describe how this course was designed and implemented to emphasize knowledge and skills that would be useful to graduating math majors, with a particular focus on written and verbal communication of technical information for a variety of audiences. Assignments included several different styles of oral presentation (including "dry runs" with peer feedback) and the writing of an Executive Summary document. The centerpiece of the course was a Capstone project, independently investigated and developed by students based on their interests. Regular class meetings offered the opportunity for discussion and reflection on topics of relevance to mathematically-focused careers. A focal point for weekly class discussions was Cathy O'Neil's popular book {*Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy*}. *(Received September 04, 2020)*

1163-97-390 **David Hartenstine*** (`hartend@wwu.edu`), **Perry Fizzano**, **Joseph A. Brobst**, **Joanna K. Garner** and **James Lamar Foster**. *Peer and Recent Alumni Mentoring for Math and Computer Science Students in an NSF S-STEM Program.*

The CS/M Scholars Program at Western Washington University (WWU) is an NSF Track 2 S-STEM project. The grant supports students (CS/M Scholars) majoring in math or computer science. A hallmark element of the program is a near-peer mentoring program. Each first- and second-year CS/M Scholar is paired with a junior or senior in the program. The junior and senior CS/M Scholars are each mentored by an Early Career Professional Mentor (ECPM), a recent WWU math or CS alum. Six times during the school year, mentors and mentees engage in short conversations on a provided theme. Summaries written by mentees have provided research data and feedback for program improvement. Preliminary research findings indicate that mentoring relationships provide safe spaces for students' identity exploration and development of commitments towards career paths and life goals. Each of the cases analyzed included articulation and exploration of students' beliefs, self-perceptions, goals, and action possibilities, suggesting that meaningful interventions to promote student retention in mathematics and CS can be implemented under time and resource constraints. Topics discussed will also include: recruitment of ECPMs, management, feedback from students and ECPMs, challenges faced, and plans for the future. (Received September 04, 2020)

1163-97-392 **Derege H Mussa*** (`dxm146130@utdallas.edu`), University of Texas at Dallas, Department of Mathematical Sciences, Richardson, TX 75080, and **Jigarkumar S Patel** (`jsp061000@utdallas.edu`), University of Texas at Dallas, Department of Mathematical Sciences, Richardson, TX 75080. *Case Study in Effective Calculus Course; Specifically in Applied Calculus II course In the University of Texas at Dallas.* Preliminary report.

Calculus course plays vital role in the study of undergraduate mathematics however understanding of the concept is not being met in most calculus courses. Hence there is a need to take special care in determining an effective method of teaching students in these courses. If we want them to gain a conceptual understanding that goes

beyond performing routine calculation. The major focus is students' conceptual understanding of the course, in particular, in determining how we can teach the course and improve the teaching an learning process so that the student will have more internally consistent concept image. Applied calculus II course is one of the sequence courses in UTD that gives students a substantial mathematical concept and skill set to have a profound impact on their attitude and success towards their advanced courses. The purpose this paper is to investigate student understanding of the course, identifying factors which affect students and improve their understanding of the course. The paper discuss new results including identifying the topics that has impact to the course and assessing student understanding of the course with possible recommendation   (Received September 04, 2020)

1163-97-396   **Cynthia O Anhalt*** (`canhalt@math.arizona.edu`), Department of Mathematics, 617 N. Santa Rita Ave., Tucson, AZ 85721, and **Ricardo Cortez**, Mathematics Department, 6823 St. Charles Ave., New Orleans, LA 70118. *Examining MKT in Mathematical Modeling through PSTs' Participation in Simulations of Practice.* Preliminary report.

Research in prospective teachers' MKT development specific to mathematical modeling is gaining momentum. Curriculum across K-12 calls for modeling, and teacher preparation needs to be inclusive of modeling in both content and pedagogy courses. The [MODULE(S)2] project has developed a modeling curriculum for teacher preparation that includes simulations of practice (SoP) in which PSTs solve a modeling problem, and then enter a simulated teaching space and record their thinking by videotape or in writing. These SoP experiences aim to reveal PSTs' thinking on student-developed models and planning discussions on the mathematics and contextualization of student results. We present an SoP analysis on the decreasing area of the Sioux Reservation land modeling task, which explores different methods for finding the area of land in connection to the injustice deeply rooted in the treatment of indigenous people in the U.S. Preliminary results indicate that PSTs do not consider inclusion of critical discussions in relation to the problem context, unless asked explicitly. This leads to questions about critical social issues as problem contexts, and explicit attention to teacher pedagogical preparation in structuring discussions around the contextualization of the mathematical results.   (Received September 07, 2020)

1163-97-426   **Iordanka N Panayotova*** (`iordanka.panayotova@cnu.edu`), 1 Avenue of the Arts, Department of Mathematics, Newport News, VA 23606, and **Maila Hallare**. *Inquiry-Based Learning: Mathematical Model of Three Fish Population Dynamics in the Chesapeake Bay.* Preliminary report.

We present an inquiry-based project that uses differential equations to study the relationships between three fish species in the Chesapeake Bay and the ecological issues associated with them. Striped bass are a commercial and recreational catch, and they are a key predator in the food web that preys on menhaden. Menhaden, often called "the most important fish in the sea," are a critical link in the food chain but are severely overfished. The blue catfish,an invasive fish, competes with striped bass to prey on menhaden and may cause severe ecological harm in the region. We analyze biological interactions (two-predator competition on one prey), ecological concerns (overfishing and invasion), and the policy approach (harvesting) using a series of scenarios based on some simplifying assumptions to demonstrate the application of theoretical concepts to actual fisheries in the Bay. The project is an excellent instructional tool as it involves elementary skills such as finding the roots of polynomial equations, computing eigenvalues and eigenvectors, and advanced topics in differential equations and dynamical systems such as Routh-Hurwitz criteria and the Center Manifold Theorem. Numerical simulations using MATLAB are used to confirm theoretical results and visualize time-evolutions.   (Received September 06, 2020)

1163-97-444   **Keith E. Hubbard*** (`hubbardke@sfasu.edu`), Box 13040 SFA Station, Nacogdoches, TX 75962. *Early engagement of STEM Majors into a Community of STEM Majors.*

Our S-STEM project, SMART Texas, sought to engage Scholars as soon as they arrived on campus with a Residential Learning Community, a Student Success course, personalized faculty mentoring, and STEM fieldtrips. Some missteps notwithstanding, we achieved 3 times the graduation rate of a paired longitudinal comparison group, along with superior engagement on STEM activities and undergraduate research. We will share our findings and lessons learned.   (Received September 07, 2020)

1163-97-457   **Jessie C Hamm*** (`hammj@winthrop.edu`), Rock Hill, SC 29732. *Getting Started in Outreach: The Dos and Don'ts for Success.*

Getting started in outreach can seem overwhelming. There's funding, planning, recruiting, and so much more. I was fortunate enough to learn from some amazing mentors during graduate school and bring those skills to my own institution. Before my arrival, our institution had little to no math outreach but I have hosted multiple

events each of the 6 years I have been there! I have learned a lot about building a successful outreach program and would love to share some dos and don'ts to success for those new to outreach!   (Received September 07, 2020)

1163-97-460   **Cody L Patterson*** (`codypatterson@txstate.edu`), Department of Mathematics, Texas State University, 601 University Drive, San Marcos, TX 78666. *Looking inside the black box: Thinking about procedures for solving equations in a capstone course for secondary teachers.*

I will share some tasks that I have used in a capstone course for preservice secondary teachers to gain insight about how prospective teachers think about procedures commonly used to solve equations and systems. One goal of this part of the course is to help teachers better understand the conceptual underpinnings of these procedures, and equip them to explain structural features that often prove surprising or confusing for students, such as "setting each factor equal to zero" when solving a quadratic equation, "extraneous solutions" when solving a radical equation, and special cases for systems of linear equations. I will discuss some challenges that regularly occur when we delve into this topic, and share some new research that may provide insight into these challenges. (Received September 07, 2020)

1163-97-501   **Rebecca McGraw***, 1700 N Indigo Dr, Tucson, AZ 85745. *Aspects of covariational reasoning and related thinking in the work of pre-service middle/high school teachers.*

Reasoning about functions and function relationships across representations and contexts remains central to both high school and college mathematics. Researchers have found that difficulties with particular aspects of this content, namely challenges with constructing and interpreting graphs, can create obstacles to success across STEM fields (Oehrtman, Carlson, & Thompson, 2008). In this talk, I will focus on the work of several undergraduate students who were preparing to become middle/high school teachers at the time of the study. These students completed multiple problems that were designed specifically to elicit thinking about key ideas (e.g., how changes in speed of a variable in motion are reflected in the shape of the graph). Based on my analysis of this data, I will discuss relationships between future teachers' mathematical thinking and aspects of their emerging MKT in this content area. I will also suggest avenues for further research as indicated by my own work and the current state of MKT research in this subarea.   (Received September 08, 2020)

1163-97-520   **Brandi Henry*** (`brandi.henry@temple.edu`) and **Rose Kaplan-Kelly** (`rose.kaplan-kelly@temple.edu`). *Engaging Middle Schoolers in Mathematics: Pre- and Post-Spring 2020.*

We will discuss the background of two mathematical outreach programs we have recently co-organized at Temple University for middle school students. In addition to an overview of program goals, intended audiences, and logistical structure for each program, we will also highlight: applying feedback to grow a program, making programs accessible, adapting to a virtual format, and our perspective as graduate students.

(1) Sonia Kovalevsky Mathematics Day: Temple University's edition of this national program is a day-long event held each year to encourage middle school girls in mathematics by introducing them to fun topics and mentorship from women in mathematics.

(2) Temple University's Summer Mathematics Interactive Learning Experience (TU SMILE): A week-long virtual program designed to expose students to mathematical concepts outside of the traditional middle school curriculum and foster interest in mathematics.

(Received September 08, 2020)

1163-97-525   **Andrew M. Ross*** (`andrew.ross@emich.edu`), **Stephanie Casey** (`scasey1@emich.edu`) and **Melody Wilson** (`melody@umich.edu`). *Developing Statistical Knowledge for Teaching with an emphasis on Equity Literacy.*

This presentation will showcase our work to create curriculum materials that develop teachers' Statistical Knowledge for Teaching as part of the MODULE(S2) project. We will discuss the process we used to select the content for the course, involving the Common Core State Standards and the Association of Mathematics Teachers Educators (AMTE) Standards for Preparing Teachers of Mathematics (with a focus on their equity literacy standards). We will also discuss how the materials integrate development of statistical knowledge along with pedagogical practices. Attendees will have an opportunity to engage with activities from the materials that illustrate how we develop statistical knowledge for teaching with an emphasis on equity literacy. Results of a study on the development of teachers' statistical knowledge for teaching, including equity literacy, will also be shared.   (Received September 08, 2020)

1163-97-537   **Elizabeth A. Burroughs*** (`burroughs@montana.edu`). *Five types of connections between undergraduate mathematics and teaching secondary mathematics.*

Robust preparation of future secondary mathematics teachers requires attention to the acquisition of mathematical knowledge for teaching. In the last decade, researchers have focused on the ways that the study of advanced mathematics as part of teacher preparation can promote prospective teachers' mathematical knowledge for teaching. The META Math project operationalizes mathematical knowledge for teaching to support mathematicians in addressing connections to teaching in their instruction. We identify five types of connections between undergraduate mathematics and teaching secondary mathematics: content knowledge; explaining mathematical content; looking back and looking forward; school student thinking; and guiding school students' understanding. We focus on the role of hypothetical human beings as a key feature of tasks included in mathematics content courses that promote mathematical knowledge for teaching.   (Received September 08, 2020)

1163-97-542   **Gowri Meda*** (`medag@oes.edu`), 6300 SW Nicol Road, Portland, OR 97223. *High school students loving the proofs of the laws of exponents (for the special case of positive integer exponents): Why this is an important goal and how to accomplish it!* Preliminary report.

How often do math professors in courses for freshmen happily share that *almost all* their students are not only able to successfully recall the laws of exponents but they can also provide the precise proofs? *Not very often* is my guess, based on my 19 years of college level teaching. Instead, we hear college professors complain about students' mistakes on all levels, especially in calculus classes. However, it is possible to send our high school students to college with a strong conceptual understanding of mathematics, including the laws of exponents. My experiences teaching 9th and 10th graders, at an independent school over the last four years, inform me that these young minds are fully capable of exploring the world of mathematics with precise definitions, making conjectures, and understanding and learning rigorous proofs. In this talk, I will discuss how to engage 9th and 10th graders with the proofs of the laws of exponents (for the special case of positive integer exponents) in a way that they will love to present the proofs at any time (no preparation required!). Based on the work of Hung-Hsi Wu, I will also discuss how to use the idea of interpolation of functions to introduce the important exponential function.   (Received September 08, 2020)

1163-97-601   **Thomas Haslwanter*** (`thomas.haslwanter@fh-ooe.at`). *Bringing quaternions to the people - a 177 year challenge.*

In my research and teaching, I have been facing a constant challenge to explain quaternions to my students and research colleagues, who typically have a background in engineering and not in mathematics.

Over the last few years I have come up with a set of simple rules and tools that I have been using to make quaternions more "palatable" to users, and to allow them to work with rotations, even if they don't understand all the mathematical details. While those rules and tools don't provide any new insight to quaternions, I believe that my experiences may be helpful to other "quaterniologists" for teaching and presentations. Since most students have a very hard time visualizing rotations in rotated coordinate systems, those rules try to keep each individual step as simple as possible, and allow the user to find a correct solution algorithm even if they cannot visualize the bigger picture.

To facilitate the development of real world applications, I have also created a Python package, called "scikit-kinematics", which should allow users to get going with quaternions as quickly as possible, without getting bogged down with numerical implementation problems.   (Received September 10, 2020)

1163-97-614   **Erika L. Ward*** (`eward1@ju.edu`). *Metric Spaces Sooner: Context in Undergraduate Analysis Courses.*

A standard one-semester undergraduate real analysis course revisits calculus and develops the rigorous mathematical underpinning for those ideas. As an undergraduate student, I found this incredibly frustrating: I was already convinced of the truth of these ideas. Now, as an analyst who teaches undergraduate real analysis, I have seen that frustration reflected back by my students. Introducing the broader context – wanting to understand how the calculus they know operates so that we can try to do calculus in other contexts – provides the grounding and motivation that I and my students often found to be missing. I will discuss that motivation, and share examples of how I incorporate this into the course, and student reactions and outcomes.   (Received September 10, 2020)

1163-97-622   **Tamas Wiandt*** (`tiwsma@rit.edu`), 85 Lomb Memorial Dr, Rochester, NY 14623. *Mathematics in RIT's First Class Academy.*

'Summer melt' was a common problem for colleges even before COVID-19. This year there were very serious concerns that the usual melt might turn into a flood as college plans were reevaluated by seniors. Rochester

Institute of Technology tried to counter this effect by introducing First Class Academy, an online, interactive summer program offering free college credit through non-conventional courses. One of these was a mathematics course designed to reach out to majors and non-majors alike. I will talk about the structure, the topics, and the challenges of this course and the feedback the students provided.   (Received September 10, 2020)

1163-97-690   **Bernard Brooks*** (`bpbsma@rit.edu`). *Designing a Course Video Module for Dual Use.*
Instructors for university courses are, as of the writing of this abstract, not certain if their classes can and will be delivered in the standard in-person lecture format. We are being asked to be prepared to move our material online rapidly if need be and to be able to deliver our material flexibly to quarantined students. Delivering mathematics lectures via video is much more time consuming and takes more effort than preparing a standard in-person lecture. But by creating the online material with the thought of flexibility and applicability to supplement traditional in-person lecturing an instructor can recoup some of the tremendous time investment required to produce quality online material. In this talk we will outline some best practices that will allow the online material to be reused and repurposed throughout the academic's career.   (Received September 11, 2020)

1163-97-792   **Christina Eubanks-Turner*** (`ceturner@lmu.edu`), 1 LMU Drive, University Hall, Suite 2714, Los Angeles, CA 90278. *Emphasizing Mathematical Knowledge for Teaching in a Graduate-Level Introduction to Proofs Course for High School Teachers.* Preliminary report.
In this presentation, I discuss the features and materials of an introduction to proofs for high school course designed to help enhance teachers' mathematical knowledge for teachers. Additionally, I will discuss the history and impetus for the course design some of the major learnign goals and give details about the course vision. (Received September 12, 2020)

1163-97-810   **Nathaniel S Barlow*** (`nsbsma@rit.edu`). *Using 3D Prints in differential equations and complex variables courses.*
I will discuss the incorporation of custom made 3D prints in math courses, with specific examples of their use in differential equations, boundary value problems, and complex analysis. Normally, this would be a hands-on talk. I will still bring my big box of math prints to show (over the screen), but will also discuss no-touch replacements for these learning tools, including augmented reality.   (Received September 13, 2020)

1163-97-825   **Evelina G Lapierre*** (`elapierre@jwu.edu`). *Leonardo da Vinci and Integrative Learning.*
Integrative learning is defined as the ability to connect, synthesize and relevantly apply concepts between two or more disciplines. Many universities are requiring integrative learning courses as part of their general education programs in order to promote the importance of adaptive thinking. In the Fall of 2017, I debuted a course that integrates math, art and culture while examining Leonardo da Vinci's life, art and work as a military engineer. After some growing pains, it has become a popular course that runs every term and is also offered in the summer as a study abroad experience in Florence, Italy. I will share my course syllabus and a few assignments. (Received September 13, 2020)

1163-97-843   **Mark H Goadrich*** (`goadrich@hendrix.edu`), Department of Mathematics / Computer Science, 1600 Washington Avenue, Conway, AR 72032. *Incorporating Successful MCM Strategies Into Curricular Learning Outcomes.*
In our Scientific Computing course at Hendrix College, students learn the techniques of data visualization, clustering, dynamical systems modeling through both differential equation approximation and agent-based modeling, Monte Carlo simulations, and global search algorithms. For the latter half of the course, students work in teams on a final project, where they choose a problem from a previous MCM, develop and test models supported with multiple figures and graphs, and write an extensive paper on their findings following the expected style of MCM solutions. Additionally, through participating in the MCM and writing a reflection paper on their experience, our students can earn one of their three required engaged learning credits. This tight integration of modeling techniques into our curriculum has proven successful, both in recruiting students to work on the MCM each year, and preparing them for producing MCM entries with the expected rigor and structure of Meritorious solutions. (Received September 13, 2020)

1163-97-845   **Gwen Gilinger*** (`ggilinger@ccbcmd.edu`), **Malissa Rivera**, **Sonja Schmitz**, **Vell Lyles** and **Michael Venn**. *Accelerated Math Courses as a Means of Recruitment, Retention and Cohort Building in an NSF S-STEM Grant.* Preliminary report.
A major impediment to retention of students in STEM fields (especially women and minorities) is the successful completion of the necessary math courses for their major. In community colleges, a significant number of

individuals require remedial math and can take several years to reach Calculus I, a necessary course for most STEM majors. In 2019 The National Science Foundation (NSF) S-STEM grant "Mathematics Acceleration for STEM Students" (MASS) was awarded to The Community College of Baltimore County (CCBC) with the goal of improving retention and success of students in STEM fields by accelerating them through the required math courses to or through Calculus I in a year. Students who receive the scholarship become part of a larger organization known as STEM Core. STEM Core was developed by Growth Sector, a California based non-profit devoted to opening STEM careers to students from low-income backgrounds and one of the major components of STEM Core is the mathematics acceleration. In this talk I will focus on the challenges we have faced and the opportunities we have found in recruiting students for this grant, the use of accelerated math classes as a means of cohort building and what the data say so far in terms of retention of our STEM students.   (Received September 13, 2020)

1163-97-871   **Rachel Schwell*** (`schwellrac@ccsu.edu`). *Creating the Student You Want: Modeling Meta Goals through Inquiry in Proof-Based Courses.*
Instructors of proof-based courses tend to teach the way we were taught: through watching an expert efficiently present previously-worked-out solutions. Yet how many of us simultaneously lament the unrealistic and misguided expectations many of our students seem to hold in the proving and problem-solving process? There are many aspects of successfully doing mathematics that many of us who chose to pursue higher-level math did not need to be explicitly taught; for example carefully parsing definitions, "playing around" with examples and non-examples, and most importantly, the understanding that the proving process is not linear, not algorithmic, and not fast. I venture that many if not most of our students will continue to miss these important aspects, which we will call "meta-goals", until our in-class approach itself models them. Together we will discuss and explore the meta-goal benefits of a more hands-off and student-led class structure in proof-based courses. For those looking for more detailed and personal guidance on implementing this type of approach, we will also discuss a national and regional NSF-funded community developed over the past two years aimed at expanding and supporting the use of inquiry-based approaches.   (Received September 14, 2020)

1163-97-888   **Radu V Balan*** (`rvbalan@umd.edu`), 4176 Campus Drive, William E. Kirwan Hall, William E. Kirwan Hall, College Park, MD 20742. *Mathematical Modeling Activities at the University of Maryland.*
Mathematical Modeling is a ubiquitous activity that transcends the boundaries of any one single field. Due to this versatility, mathematical modeling does not have a standard textbook that can be adopted in the teaching process. In this talk I shall present mathematical modeling activities at the University of Maryland, including how I got involved in the MCM/ICM competitions a few years ago. All these accomplishments are due to the extraordinary talent and determination of those undergraduate students who took part in these competitions. (Received September 13, 2020)

1163-97-904   **Amanda Beecher***, abeecher@ramapo.edu. *ICM Director's Overview of the Interdisciplinary Contest in Modeling: What Advisors Need to Know.*
The Interdisciplinary Contest in Modeling (ICM) is run parallel to the Mathematical Contest in Modeling (MCM) with the same format. In 2020, students formed over 7200 teams to address one of three open-ended problems (D-Operations Research/Network Science, E-Environmental Science, F-Policy). The interdisciplinarity of this contest provides a mechanism to bring students and colleagues from many other disciplines together as authentic collaborative groups to solve real world problems. We discuss all aspects of the contest, past problems, judging, and results to provide   (Received September 14, 2020)

1163-97-920   **Ann Brett*** (`ann.brett@jwu.edu`), Providence, RI 02903. *Creative Teaching Methods Used in the Integrative Learning Course "Explorations in Symmetry".* Preliminary report.
The Integrative Learning Course course "Explorations in Symmetry" introduces the student to the basic concept of symmetry and its important role as a unifying agent in the understanding of mathematics, nature, art, architecture and music. Topics covered in the course include an introduction to group theory, the mathematical language of symmetry, transformations, general symmetry principles, and applications. In this talk, we discuss some strategies used to teach the highly mathematical concepts of symmetry to general education students, and some successes students in the class have had applying the course content to their own careers and interests. (Received September 14, 2020)

1163-97-935   **Amanda Tucker*** (`amanda.tucker@rochester.edu`), 96 Dartmouth St, Rochester, NY
14607. *Lessons learned while directing GirlsGetMath@Rochester.* Preliminary report.

I will discuss lessons learned while starting and directing GirlsGetMath@Rochester, a program in applied and computational mathematics for high school aged girls interested in mathematics at the University of Rochester. This is the first sister program of GirlsGetMath@ICERM.   (Received September 14, 2020)

1163-97-955   **Triet M. Pham*** (`tmp140@math.rutgers.edu`), Rutgers University, Hill Center - Busch
campus, 110 Frelinghuysen Rd, Piscataway, NJ 08854. *Advising Master students in
financial competitions with modeling components.*

At the Rutgers Master in Math Finance program, our students regularly participate in competitions organized by the practitioners in the finance industry. The International Association for Quantitative Finance competition is very similar to the Interdisciplinary Contest in Modeling (ICM) where the students have to submit a 10 page research paper on a particular financial topic. The other competitions, such as the the PRMIA Risk Management Challenge and the CME Group University Trading Challenge all have modeling components behind their common goal of successfully managing some portfolios. In this talk, I will describe how these competitions fit into our Master program, both in terms of the curriculum and the practical training for the students while still on campus. I want to emphasize the gap that I have observed from the students applying their classroom knowledge to solve a "real world" problem; the challenges they face in selecting the right technical tools, in not over-complicating the problem and in communicating with their team mates. Finally I will relate this to my experience in advising two undergraduate students in the recent ICM competition, with emphasis on the difference between advising Master level and advanced undergraduate students.   (Received September 14, 2020)

1163-97-987   **Wesley Hamilton*** (`wham@live.unc.edu`). *Organizing and running COMAP preparatory
workshops: one department's story.*

In December 2019, I decided to start a departmental initiative to prepare undergraduates for the 2020 COMAP MCM/ICM. In January 2020, I organized and co-ran twice weekly workshops on both mathematical and computational topics, as well as guided readings of previous competitions' winning solutions. In February/March a total of 7 teams from my school competed: all were successful, two received honorable mention, and one was a meritorious winner. In this talk, I'll talk about my department's journey, including soliciting help from my fellow graduate students, what resources we found most helpful, and what modifications we'll incorporate for future iterations, including workshops run during the Fall and Winter before the 2021 COMAP competition. (Received September 14, 2020)

1163-97-1000   **Steven B Horton*** (`mathlete005@gmail.com`) and **William C Bauldry**
(`bauldrywc@appstate.edu`). *MCM Director's Overview of the Mathematical Contest in
Modeling: What Advisors Need to Know.*

The Mathematical Contest in Modeling (MCM) is an undergraduate level contest occurring annually in February. In 2020, students formed over 13,700 teams to address one of three open-ended problems (A-Continuous, B-Discrete, C-Data Insights) during the 100-hour weekend contest period. During the contest students build and analyze a model to address the problem, and then write a 25-page paper to showcase their work. We will discuss the MCM in general and will examine some sample problems to highlight characteristics of a competitive paper. We will also provide insights to advisors on how to put together a team, prepare for the contest, and be successful. (Received September 14, 2020)

1163-97-1012   **Eddy Kwessi*** (`ekwessi@trinity.edu`), 1 Trinity Place, San Antonio, TX 78212. *A
Trinity University Perspective on Mathematical Competition in Modeling.*

The Consortium for in Mathematics and its Applications (COMAP) organizes a varieties of competitions every year. I became involved as newly hired faculty member at Trinity University with the selection and preparation of students who will take part in the Mathematical Competition in Modeling (MCM) in 2011. Nearly every year since then, I have been part of this competition as student advisor. Our teams have had relatively great success throughout the years and that probably speaks volume to our selection and preparation processes. In this talk, I will share our unique perspective from selection to competition, the lessons learned, and the advice I would give to any faculty member or student willing to embrace this challenging but worthwhile endeavor.   (Received September 14, 2020)

1163-97-1014   **Shuai Jiang***, 182 Goerge St, Providence, RI 02912, and **Rebecca Santorella**, 182
George St, Providence, RI 02912. *Localizing the COMAP Competitions.*

COMAP's Mathematical Contest in Modelling (MCM/ICM) is a unique challenge for undergraduates in which a group of three are tasked with producing a short academic paper in the span of four days. The contest differs

in structure from many classical math competitions in both the structure, rules, output, and grading criteria. In particular, undergraduates are required to produce an academic paper, which many of them have little to no experience with prior to the competition. We find that creating our own mini contest prior to MCM/ICM allows students to understand the expectations of the competition and receive detailed feedback on their writing and modelling approach. Teams that perform well in our local competition generally excel at MCM/ICM.   (Received September 14, 2020)

1163-97-1031   **David S. Ross***, dsrsma@rit.edu, and **Nathaniel Barlow**. *The logical methodology of mathematical modeling.*
The curriculum of RIT's PhD program in mathematical model trains students to be applied mathematicians, experts in mathematics who use mathematical analysis, computational methods, and logic to solve application field problems. In this talk we will explain how, and why, mathematics proper is secondary to logical methodology in the enterprise of modeling. We will discuss how we teach the logical methodology of modeling in our graduate classes. We will discuss practical examples that we use in the curriculum, and examples of industrial collaborations and how we use them to educate modelers. These will include problems related to immuno-assay chemistry, micro-energy harvesting, cancer treatment, sales projection, cardiopulmonary disease, and contact lens mechanics that have come to us from various industrial firms.   (Received September 14, 2020)

1163-97-1032   **Jason Douma*** (jason.douma@usiouxfalls.edu), 1101 W 22nd St, Sioux Falls, SD 57105. *Math Modeling Hub: a Space for Community, Collaboration, and Resource Sharing.* Preliminary report.
The Math Modeling Hub (MMHub) serves as a community of practice for math modeling educators, as well as a repository of free open educational resources for math modeling. These resources include materials for classroom use, professional development for educators, and "sandbox" areas where educators can collaboratively develop, edit, and test instructional materials. The MMHub also sustains a community of practice by hosting forums, webcasts, workshops, and mentoring networks. By design, the MMHub includes educators from all grade levels (pre-K through college/university) as a way to promote continuity and quality in the modeling experiences students encounter throughout their journey. There is much work yet to be done in building the Math Modeling Hub community and growing its repository. The rate at which it achieves its aims–and the impact it has along the way–will depend on the contributions of enthusiastic colleagues dedicated to supporting math modeling education across all grade levels.   (Received September 14, 2020)

1163-97-1140   **Kathleen G Snook*** (kgsnook@gmail.com), 471 Old Rd to 9 Acre Corner, Concord, MA 01742. *An Overview of COMAP and its Modeling Contests.*
Founded in 1980 with a simple mission, "to increase the presence of mathematical modeling and applications in our nation's classrooms," the Consortium for Mathematics and Its Applications (COMAP) has produced modules, journals, newsletters, textbooks, videos, and national reports in support of mathematics teaching and learning. Since 1985, COMAP has also annually held its international Mathematical Contest in Modeling (MCM), and in 1999 added its Interdisciplinary Contest in Modeling (ICM). Now, celebrating its 40th year, COMAP continues to support the integration of mathematical modeling into the curriculum.   During this introductory portion of this Special Session, we will overview COMAP's curricular materials and its international contests.   (Received September 14, 2020)

1163-97-1249   **Darlene M Olsen*** (dolsen1@norwich.edu), 158 Harmon Drive, Northfield, VT 05663, and **Christine Latulippe** and **Joe Latulippe**. *Promoting Success of Undergraduate STEM Students Through Scholarships, Mentoring and Curricular Improvements in First-year Mathematics Courses.* Preliminary report.
Norwich University, a private military college that serves both civilian and Corps of Cadets students, secured a NSF S-STEM award to develop a program to attract and retain highly talented, low-income students who are pursuing baccalaureate degrees in biochemistry, biology, chemistry, mathematics, neuroscience, and physics. Norwich recognizes that students who enter college with less experience in mathematics are less likely to graduate with a degree in a STEM discipline. With that in mind, the research aims of the program are to: 1) measure the benefits of corequisite implementation of precalculus and calculus to help students complete the required calculus sequence by the end of their first year; 2) implement and assess a leadership training program to improve the communication and leadership skills of peer tutors in mathematics courses; and 3) measure the effect of improved academic, financial, and career support for the students across their academic career at Norwich. The two topics of this presentation are: 1) the lessons learned in the process of securing and implementing the NSF S-STEM

award; and 2) the content, delivery, and outcomes of the first semester offering of the corequisite implementation of precalculus and calculus.   (Received September 15, 2020)

1163-97-1348   **Peter Haine*** (`phaine@mit.edu`), 122 Otis St, #2, Cambridge, MA 02141. *PRIMES Circle at MIT.*

The PRIMES Circle program at MIT is a reading program aimed at increase diversity in the mathematical community by helping students from underrepresented groups to develop their interest in mathematics and to set them on a path toward pursuing a math-based major in college. For four months, late January through mid-May, students work with MIT professors and undergraduates to study mathematics and to solve challenging math problems beyond high school curriculum. Students focus on a specific area of mathematics, read literature in that area, and practice problem solving, expository writing, and presentation skills.

In this talk, we'll give an discuss what the PRIMES Circle program entails and some best practices in training undergraduates to be caring and effective mentors.   (Received September 15, 2020)

1163-97-1349   **Francesca Bernardi*** (`fbernardi@wpi.edu`) and **Katrina Morgan** (`katrina.morgan@northwestern.edu`). *Merging Mathematics and Media to Increase Representation and Build Community.*

Students from underrepresented groups face numerous barriers of entry in STEM. Fostering a community of young people who are interested in Mathematics, but do not necessarily see themselves as potential Mathematicians, can help them feel welcome in the field and build their confidence. Outreach programs aligning STEM communication with Mathematics can be a point of entry for such students, who often find these events to be less intimidating than those focused exclusively on technical activities.

Incorporating communication aspects in outreach serves several purposes: It encourages the participation of non-traditional STEM students by building a community of young people passionate about Mathematics and communicating it to the public; it helps educate these students to become better Mathematicians, teaching them communication skills as well as challenging Math topics; and it shows them the importance of focusing on both aspects.

We report experiences from Girls Talk Math, a free Math and media camp for female and gender non-conforming high school students founded in 2016 at the University of North Carolina at Chapel Hill. Girls Talk Math is now hosted on four major US campuses and in 2020 was held virtually as a joint effort with campers from all four institutions.   (Received September 15, 2020)

1163-97-1365   **Shelby Stanhope*** (`shelby.stanhope@afacademy.af.edu`). *Supporting Spatial Conceptualization in Multivariable Calculus Using 3D Printed Surfaces and Experiential Learning Field Trips.*

A unique transition occurs as students enter multivariable calculus. Up to this point, students have spent their mathematical careers becoming experts in the two-dimensional xy-plane. Adding another dimensions allows us to explore this 3D world we live in, but the transition to three dimensional mathematical thinking does not come easily to many students. To support students' spatial understanding of concepts in the course, we introduced tactile manipulatives and experiential learning field trips. In this presentation, I will discuss the use 3D printed surfaces in the classroom. Students can touch, write on, and rotate the surfaces as they work with a classmate to build their understanding of new concepts through a series of guided activities. Additionally, I will discuss two experiential learning activities, which provide the opportunity for students to see math in action, bringing together concepts they have seen, but can now feel and experience themselves. One activity requires a sloped surface (like a small hillside or graded slope of grass) and the other can be accomplished remotely (via video field trip to a wind tunnel). Student testimonies reveal that they often think back to the field trip activities to discuss concepts and to explain material to other students.   (Received September 15, 2020)

1163-97-1370   **Ben Galluzzo*** (`bgalluzz@clarkson.edu`), Clarkson University, 8 Clarkson Avenue, 5815, Potsdam, NY 13699. *Math Modeling with Few to No Prerequisites.*

Many college students are aware that mathematics is important in the study of sciences and engineering (if for no other reason than that those majors require so many math courses). However, few know how mathematical models are developed, and even fewer have experience solving problems independent of a mathematical method prescribed by their current course content. In this talk we'll discuss a stand-alone modeling course designed to serve a broad audience. In particular, we'll share the course structure that emphasizes engagement in the modeling process and provides students with a framework for modeling in future academic (and career) endeavors. (Received September 15, 2020)

1163-97-1397   **Daniel J Teague*** (`teague@ncssm.edu`), 105 Gallagher Ct, Morrisville, NC 27560-7762. *Moving from Remembering to Thinking: Mathematical Modeling in High School.*

The mathematics program at the North Carolina School of Science and Mathematics has a modeling focus, and we offer an elective course in Mathematical Modeling. The goal of the modeling course isn't to learn new mathematics (although they do), or to study the classic models that others have built, but to learn how to use creatively and profitably whatever mathematics the student knows. We have seen that mathematical modeling in all classes engages and energizes high school students in ways that "standard mathematics" doesn't and can't. Modeling opens students to the creative aspects of mathematics and places the emphasis and focus on thinking rather than remembering. Modeling is slow, so mathematics is no longer a speed race. Mathematical modeling becomes an adventure in merging our mathematical knowledge with our understanding of how the world works; with each informing and strengthening the other. Examples from NCSSM's curriculum and students will illustrate what is possible.   (Received September 15, 2020)

1163-97-1400   **Sherry Kasper*** (`skasper@leeuniversity.edu`), **Sarah Schlosser**, **Anthony Minutolo** and **Blayne Carroll**. *Lee University S-STEM: The integration of Math and Science Scholars.* Preliminary report.

The integrated Math and Science Scholars (iMASS) program is an NSF funded S-STEM grant that began in the spring of 2015. This program seeks to introduce students to STEM industry and research opportunities in the region, create a learning community, and increase retention in our academically-talented, financially-eligible students, many of whom are minority or first-generation. In addition, we strive to create interdisciplinary conversations between math and science students. To facilitate these cross-disciplinary actions, the students are cohorted in courses such as Experimental Statistics and Mathematical Modeling. This presentation will examine key activities used to foster interdisciplinary collaboration and their impact on students. We also will discuss lessons learned regarding fostering learning communities that prepare students from a variety of STEM fields to work collaboratively on future problems.   (Received September 15, 2020)

1163-97-1415   **Daniel M Harris***, 184 Hope St, Box D, Providence, RI 02912. *Building New Opportunities: Integration of Outreach, Education and Research.*

In this talk, I will describe a synergistic approach through which my education and research efforts have directly translated into new outreach activities. In particular, I will showcase examples where novel and accessible physical demonstrations are developed by students and then utilized for activities within outreach programs including Girls Talk Math (UNC Chapel Hill), Girls Get Math (ICERM), STEM Day (Brown), and others. The rapid expansion of makerspaces across the country and the growing open hardware movement have facilitated a new paradigm where such hands-on devices and demonstrations can be shared broadly and readily recreated in a rapid and cost-effective manner. While ultimately serving to enhance the breadth of activities available to outreach programs, the framework presented also has the potential to increase visibility and provide additional context for student's academic work. Future outlook and other perspectives resulting from this experience will be shared.   (Received September 15, 2020)

1163-97-1462   **Kayla Blyman*** (`profkaylablyman@gmail.com`). *Painting with Robots, Playing with Bubbles, and Introducing Mathematical Modeling - A Virtual Potpourri of Ideas for Outreach from My Outreach Library.*

If you go looking for them, outreach opportunities are everywhere. (They will even come looking for you sometimes!) But not all outreach opportunities are built the same. They each bring with them a unique environment, a unique timeline, and unique participants. Those participants will be at different stages in their education and will come with a variety of levels of willingness to engage depending on the opportunity. Consequently, outreach can be quite intimidating.

One of the ways to begin making engaging in outreach opportunities less intimidating (and less labor intensive) is to start building a library of outreach activities. As you come up with ideas at random times you can add the draft of your idea to your library and then carve out a little time to expand (or condense) the idea and add details. Each activity will be appropriate for some situations, but not all. The more entries you have in your library the more likely you are to be able to take an idea "off the shelf" and implement it. This will allow you to experience all the joys of outreach with minimal new effort behind the scenes.

In this talk I will share with you my outreach library entry technique as well as some of my favorite entries to date.   (Received September 15, 2020)

1163-97-1565   **Jose Contreras*** (`jncontrerasf@bsu.edu`). *Infinity: A Historical and Cultural Approach.* Preliminary report.

The idea of infinity has been present not only in mathematics, but also in other human affairs such as religion and art. Many scholars have examined the idea of infinity, from the ancient Greek philosopher Aristotle to Newton, to Weierstrass, and to Cantor, the conqueror of infinity.

Today, infinity permeates not only mathematics but also the arts and sciences. In this presentation, I will illustrate how I approach the history of infinity, particularly mathematical infinity, in my history of mathematics course. In the presentation, like in class, I will discuss notable people and cultures' contributions to and conceptions about infinity and how people with a diversity of backgrounds (e.g., ethnicity, economic status, national origin, gender, age, religious viewpoints) have used the idea of infinity on their work or have made contributions to the development of the concept of infinity. I will also discuss how Cantor's religious background and viewpoints gave him the courage to persist on investigating the mathematical infinity as a number in spite of the tremendous oppositions to his revolutionary ideas.   (Received September 15, 2020)

1163-97-1576   **Ranthony A.C. Edmonds*** (`edmonds.110@osu.edu`) and **John H. Johnson, Jr.**. *Hidden Figures, Service-Learning, and Mathematical Community.*

In this talk we highlight the first service-learning course offered by the Department of Mathematics at The Ohio State University, Intersections of Mathematics and Society: Hidden Figures. In this course, students interact with local "Hidden Figures," who are STEM leaders in the Columbus Metro Area from underrepresented backgrounds, and participate in service-learning through the development of original STEM programming related to course content. We focus our discussion on the development of the service-learning and community engagement components of the course, and lessons learned along the way.   (Received September 15, 2020)

1163-97-1601   **James J Madden*** (`mmmad@lsu.edu`), Dept. Math., 301 Lockett Hall, Louisiana State University, Baton Rouge, LA 70803. *Using Historical Sources to Understand the Contemporary K-12 Mathematics Curriculum.* Preliminary report.

The Mathematics Department at Louisiana State University offers a variety of graduate-level courses for secondary mathematics teachers that can be taken to meet the requirements of the LSU Masters of Natural Sciences degree program or the LSU Graduate Certificate in Mathematics for Advanced Secondary Instruction. Over the years, we have experimented with many different instructional designs. One of the most engaging things we have tried is reading and discussing original historical sources (such as Fibonacci, Viète, Descartes, Oughtred) or transcriptions of these sources in modern language. Direct connections to the contemporary K-12 curriculum are often easy to make. At the same time the historical sources frequently lead to novel perspectives on familiar topics, and they support teachers in anticipating and understanding the ways that students may interpret what they are learning. We will distribute some classroom materials that we have have had success in using. (Received September 15, 2020)

1163-97-1606   **Jose N Contreras*** (`jncontrerasf@bsu.edu`). *Developing Mathematical Knowledge for Teaching Geometry within Dynamic Geometry Environments.*

In this presentation, I will describe an inquiry-based geometry course that I developed to engage my prospective and practicing secondary mathematics teachers in searching for patterns and in posing and solving geometry problems within the support of Dynamic Geometry Software (DGS). The course is based mainly on three types of activities.

The first type of activities guides students to discover geometric patterns with the aid of DGS and then make and test conjectures. An example of this type of activities involves examining the Pythagorean configuration to discover additional patterns including relationship among areas of polygons.

The second type of activities includes problems that can be represented and modeled with DGS. An example of this type of activities involve investigating the treasure problem.

The third type of activities includes geometric problems whose attributes can be modified to pose additional interesting mathematical problems. An example of this type of activities involve the Varignon theorem. Students use DGS to formulate conjectures about the solutions to the problems posed.

Learning how to use DGS and crafting proofs are two pedagogical aspects of mathematical knowledge for teaching that my students further develop by completing the course.   (Received September 15, 2020)

1163-97-1660   **Perla L Myers*** (`pmyers@sandiego.edu`), 2011 Garfield Rd, San Diego, CA 92110, and **Odesma Dalrymple** (`odesma@sandiego.edu`), **Amanda Ruiz** (`alruiz@sandiego.edu`), **Jane Friedman** (`janef@sandiego.edu`), **Drew Talley** (`dtalley@sandiego.edu`) and **Judith Liu** (`liuusd@sandiego.edu`). *Project AnchorSTEM: Engaging with Community for Student Success in Mathematics, Engineering and Science.* Preliminary report.

AnchorSTEM is a new S-STEM project that focuses on University of San Diego's role of Anchor Institution by building on the structures from past S-STEM projects, while sustaining and strengthening partnerships with pre-college serving community organizations to establish a reciprocal, ongoing mechanism to support the retention, graduation and success in Mathematics, Engineering and Science of local "Rising Stars" (students with financial need, especially those from underrepresented groups in STEM). Through this project, we are developing, implementing and evaluating a collaborative program to recruit and advance a diverse group of STEM majors by placing the community front and center. We recognize that (a) where students come from and the assets they bring are important, (b) it takes the whole community working together to advance each member, and (c) retaining and strengthening student ties with community and family helps them succeed in college. In this session, we will share lessons learned in previous projects and how they shaped the development of this new project, as well as our first insights from work on Project AnchorSTEM. This project is funded by NSF's Scholarships in Science, Technology, Engineering, and Mathematics program.   (Received September 16, 2020)

1163-97-1683   **Maya Klaib*** (`maya_klaib@redlands.edu`) and **Maxwell Auerbach** (`maxwell.auerbach@emory.edu`). *Strategies for Creating a Student Driven Environment for the COMAP Modeling Contests, from Students' Perspectives.*

Some capable students may be intimidated by spending a weekend participating in a mathematics competition, while others would dream of nothing else. As two students on the opposite ends of this spectrum who worked together to form a team, we understand the difficulty in creating a student driven environment where each person can be part of a productive and successful team. In this talk, we will provide invaluable insight into successful actions faculty can take to help create a student driven environment and to assist and advise their teams. We will also discuss how students can be led to form their own teams, and how to help motivate students.   (Received September 16, 2020)

Table of Contents for MAA Abstracts of Invited Addresses, Presentations by Teaching Award Winners, SIGMAA Guest Lecturers, and Session Papers

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MAA Invited Addresses, Presentations by Teaching Award Winners, and SIGMAA Guest Lecturers

1163-A0-1698   **David Austin***. *Stories my students taught me.*
We all carry stories of our mathematical journey and how it's molded our commitment to serving our students and to ensuring that they feel they belong in our community. But it was when I really started listening to my students' stories that this profession became profoundly meaningful. I'd like to share some of how my best teachers, my students, have influenced the way that I think about my work with them and how their voices help me feel that I'm better serving the discipline we've all devoted ourselves to.   (Received November 5, 2020)

1163-A0-1699   **Elaine A. Kasimatis***. *What are the right questions?*
If "wisdom begins in wonder", as Socrates asserted, and if one of the measures of effective teaching is how well it fosters understanding, then how do we kindle wonder? For all the hours and creative energy we spend on planning lessons, which aspects of our efforts actually translate into genuinely stirring our students to wonder, to be curious learners? Artist and educator Josef Albers maintained that, "Good teaching is more a giving of right questions than a giving of right answers." And writer Madeleine L'Engle shared her wish that "we worried more about asking the right questions instead of being so hung up on finding answers." So, what are the right questions? In this talk we will consider how crafting and refining questions can be used to guide students to pose conjectures and construct their own understanding as they engage in the mathematical process.   (Received November 5, 2020)

1163-A0-1700   **Dave Kung***. *I. Uncommon Inspirations for Teaching, II. The Price of Equity in Mathematics*
I. Teaching is difficult work. We can better meet the challenge of helping students explore, struggle with, and ultimately understand mathematics by finding inspiration in unlikely places. I've learned invaluable lessons by thinking about Circ du Soleil performers, violin teachers, my daughter's pool play, and even television dramas. Where will you find inspiration for your teaching? II. The mathematics community has a dismal record when it comes to issues of racial and gender equity, from our introductory math classes, to our graduate programs, to faculty hiring and retention, to awards like this one. What would it cost people like me to #DisruptMath and embrace a more equitable vision? Am I willing to do what needs to be done? Would you be willing to join me? (Received November 5, 2020)

Inquiry-Based Learning and Teaching

1163-A1-67   **Blain Patterson*** (`pattersonba@vmi.edu`) and **Sarah Patterson**. *Learning Programming through Tactivities.* Preliminary report.
In this talk, we will discuss how to incorporate tactile activities or "tactivities" into an introduction to programming course. The difficulties students face with programming is often associated with logic and critical thinking, rather than the use of a particular language. Therefore, these tactivities are agnostic of technology and instead focus on conceptual understanding rather than syntax. A few examples of tactivities will be provided as well as insights into how this may play out in your classroom.   (Received August 05, 2020)

1163-A1-257   **Thomas Pawlaschyk** and **Sven-Ake Wegner***, University of Hamburg, Germany. *Homework tasks that engage students in conjecturing, refuting and proving.*
In the talk we report on an implementation of discovery-oriented problems in courses on Real Analysis and Differential Equations. We explain a type of task-design that gives students the opportunity to conjecture, refute and prove. What is new in our approach is that the complexity in our problems is limited and thus the tasks can also be used in homework assignments. In addition to several concrete examples we also present feedback and assessment outcomes of our students. The presentation is based on [PW20].

Reference

[PW20] T. Pawlaschyk and S.-A. Wegner, Engaging students in conjecturing through homework in Real Analysis and Differential Equations. Int. J. Math. Educ. Sci. Technol. **51**, no. 3 (2020), 439–450, DOI:10.1080/0020739X.2019.1656832.
(Received September 04, 2020)

1163-A1-371   **Christine von Renesse*** (cvonrenesse@westfield.ma.edu), **Volker Ecke** and **Patrick Rault**. *Grassroots Community Organizing: You can do it!*
Every semester we learn from our experiences and grow as teachers, and lately this has been exemplified by the movement of classes to hybrid and online environments and the raising awareness of racial inequities. In this interactive session, participants will ponder a teaching struggle or question and how they would best facilitate a faculty event to jointly work on ideas and develop solutions. You don't have to be an expert on anything to be able to facilitate an effective, fun, and engaging event for faculty. We will help participants identify places to self-organize grassroots-style events, and provide a window into the workings of regional IBL communities of mathematics educators (www.iblcommunities.org).   (Received September 04, 2020)

1163-A1-488   **Lisa Hilken*** (hilken@math.uni-tuebingen.de), , Germany, and **Carla Cederbaum** (cederbaum@math.uni-tuebingen.de), , Germany. *Hands-on Differential Geometry.*
In the course "Hands-on Differential Geometry", pre-service teachers develop several definitions of Elementary Differential Geometry by themselves. Topics are among others the curvature of plane and space curves and surfaces. The course consists of four cycles. At the beginning of each cycle, every group chooses one of the given problems to work on. The students then have the possibility to gain some intuition about their mathematical problem by using hands-on materials. Afterwards, they try to solve their problem in a mathematically rigorous way. At the end of each cycle, every group presents their results to the whole group. For example, the students are asked to develop a notion and definition for the curvature of plane curves. They may ride a bike or walk along chalk-drawn curves to get a feeling for the relation between the radius and the curvature of a circle or the radius of a circle and the angle of the handlebars to the bike. The course aimed at strengthening availing beliefs about mathematics and the self-efficacy. We will describe the seminar including some of the mathematical problems we posed, give an idea of the quality of the solutions found by the students, and share insights from the accompanying research.   (Received September 08, 2020)

1163-A1-530   **Monica D. Morales-Hernandez***, 413 Science Building, One South Avenue, Adelphi University, Garden City, NY 11530-0701, and **Sherli Koshy Chenthittayil** and **Nikeetha Farfan D'Souza**. *Biographical essays as an effective tool to engage International students in American College Mathematics.* Preliminary report.
Diversity and Inclusion are two words that have become very popular in the last years. As mathematicians we often listen, write, read, and implement policies to diversify our classrooms, but what happens when our classrooms are already diverse? How do we actively practice inclusion? How do we modify and improve our teaching techniques and the way we communicate math to a group of students whose main characteristics are that they are diverse, they have limited math experience, and also have limited English skills? How do we modify the math curriculum so that the students can successfully complete the class? How do we actively practice inclusion in an ESL College Math classroom?. In this project we study whether historical exposure improves engagement of international students in math classes. We make use of biographies of mathematicians as a tool to teach mathematical concepts as well as a tool to cultivate inclusion and exemplify the not so visible diversity of the field of mathematics.   (Received September 08, 2020)

1163-A1-594   **Su Liang***, One UTSA Circle, University of Texas, Department of Mathematic, San Antonio, TX 78249. *Visualizing Connected Mathematics through Problem Solving.*
Inquiry-based Learning (IBL) provides opportunities for students to understand mathematical ideas by actively engaging in the process of doing mathematics. Meaningful class activities are necessary to ensure teaching/learning effectiveness in IBL classrooms. In the session, the author is going to share a class activity that helps preservice mathematics teachers see how mathematical ideas are systematically connected across grade levels from different perspectives such as arithmetic, algebra, and calculus. This activity will demonstrate how a class task can facilitate a meaningful inquiry by engaging students in doing meaningful mathematics.   (Received September 10, 2020)

1163-A1-617 **Hyman Bass*** (`hybass@umich.edu`), 3281 Wylie Rd., Dexter, MI 48130. *Group Theory on the Number Line.* Preliminary report.

I propose a way that (concrete) group theory can illuminate some basic ideas of school math, through studying additive and multiplicative groups of real numbers. The key is Division with Remainder: (DwR) For a and b > 0 in R, a = qb + r with q in Z (integers) and $0 \leq r < b$. Let A be a subset of R and a in A. Call a isolated in A if, for some r > 0, (a-r, a+r) contains no element $\neq$ a from A. Call A discrete if all of its elements are isolated, and uniformly discrete if some r > 0 works for all a in A. Call A an additive group if A is closed under + and −. Let A be a real additive group. The following follow easily from (DwR). (I) If 0 is isolated in A then A is uniformly discrete. (II) A is either discrete or dense in R. (III) If A is discrete then A = Za for a unique a $\geq$ 0. (IV) Za + Zb is discrete iff a and b are commensurable. In this case, d = gcd(a, b) $\geq$ 0 and m = lcm(a, b) $\geq$ 0 are defined by, Za + Zb = Zd and Za $\cap$ Zb = Zm. Using the continuous group isomorphism exp: R —> (0, ∞), one can similarly describe discrete multiplicative subgroups of R. Studying the additive and multiplicative structures of C and of Z/Zm is more complex.. (Received September 10, 2020)

1163-A1-873 **Shubhangi Sadanand Stalder*** (`stalder@uwm.edu`), 1500 N. University Drive, Waukesha, WI 53188. *Cultivating an Intuitive Sense of Arithmetic of Functions.*

Time is often tight when teaching College Algebra and Pre-Calculus courses. In this talk, I will present an efficient approach in how you can use inquiry-based learning strategies by choosing concrete examples of functions written in set, tabular, or graphical representation to find the sum, difference, product, quotient, and composition of functions. Transformation of functions then comes for free as a happy byproduct. One benefit of this kind of inquiry-based learning is that students learn about the arithmetic and transformation of functions simultaneously and can develop a deeper or intuitive understanding of the concepts instead of just performing the tasks robotically or as part of rote memorization. Cultivating this intuitive sense can also generate curiosity. Students may want to ask how they can apply their knowledge to graphs of other functions like $y = \ln{(x-1)}(x+2), y = (e^x - 1)(e^x + 2)$, or $y = \frac{1}{(e^x - 1)}$, and so on. Students can then be allowed or encouraged to verify their intuitive answers using graphing utilities. This kind of training can also be helpful for later Calculus courses. (Received September 13, 2020)

1163-A1-1016 **Carol Ann Downes*** (`downes@hendrix.edu`). *Discovering L'Hospital's Rule In A Calculus Lab.* Preliminary report.

Within a 50 minute calculus lab activity, students discover L'Hospital's Rule for the 0/0 and ∞/∞ indeterminant forms by working through a scaffolded activity in small groups and using open-sourced technology. First, students analyze a limit with form 0/0 by exploring the linear approximations of each function in the ratio, and then, students explore a limit with form ∞/∞ by analyzing the rates of change of each function in the ratio. Essentially, students learn the why and how behind L'Hospital's Rule for these forms by discovering the overarching structure of the proof as explored through examples. Once they have discovered the rule, students synthesize what they learned in a lab essay assignment graded not just on mathematical correctness, but also on how well the student guides the reader through learning L'Hospital's Rule. By introducing L'Hospital's Rule in this way, we engage and challenge students on the foundational concepts even if they have seen the mechanics of the rule in high school. (Received September 14, 2020)

1163-A1-1039 **Kelly Gomez Johnson*** (`kgomezjohnson@unomaha.edu`), University of Nebraska at Omaha, Roskens Hall 406N, 6005 Dodge Street, Omaha, NE 68182, **Paula Jakopovic** (`paulajakopovic@unomaha.edu`), University of Nebraska at Omaha, Roskens Hall 308, 6005 Dodge Street, Omaha, NE 68182, and **Patrick X. Rault** (`prault@unomaha.edu`), University of Nebraska at Omaha, Durham Science Center, 6005 Dodge Street, Omaha, NE 68182. *Developing a Network of Regional Inquiry-Based Learning (IBL) Communities: Preliminary Findings & Key Learnings on a Grassroots Initiative.* Preliminary report.

Finding and creating communities of like-minded colleagues is easier said than done. Instructional and philosophical change, around the practice of inquiry, is complex and therefore, having access to quality resources, expertise and a supportive network can make a big difference. Especially during a global pandemic and sociopolitical unrest, leveraging existing and emerging networks of faculty working jointly on ideas and developing solutions can be advantageous. During this session, participants will hear about the inaugural year of a federally funded network of regional inquiry-based learning communities. Topics covered will include the network's mission and goals, key infrastructure and implementation elements, and preliminary findings pertaining to participants' engagement in their regional inquiry-based learning community and the broader network (adapted from a value framework developed by Wenger, Traynor, & DeLaat, 2014). Additionally, we will share insights on how regional

communities address important issues in mathematics teaching and learning in terms of access, inclusivity, idea exchange, and innovation during times of change (www.iblcommunities.org).   (Received September 14, 2020)

1163-A1-1196   **Jessica Oehrlein*** (`jessica.oehrlein@columbia.edu`). *Mathematics Research-Like Experience in a Discrete Structures Course for Computer Science Students.*

In a graph theory and algorithms course for computer science first years and sophomores, one option for a course project was a guided research-like experience in graph theory. Students explored properties of a family of graphs by building examples, proving or providing counterexamples to given statements, developing their own conjectures, and extending to a broader set of graph families. I will describe the project, what went well, what feedback students gave, and what I would change in the future.   (Received September 15, 2020)

1163-A1-1252   **Jonathan P Keiter*** (`jkeiter@esu.edu`). *Using 3D Slices to Explore Partial Derivatives and Directional Derivatives.* Preliminary report.

I will describe two projects that have the students working with 3D manipulatives to explore the concepts of partial derivatives and directional derivatives. By working with interlocking wood sheets that form a surface on the top, the students have a deeper understanding of rates of change for functions of two variables. They see how the slopes in the x- and y-directions are independent of each other. Students working in groups will each have a surface created by interlocking sheets. They will investigate the surface and then remove two perpendicular slices (cross sections of the function) that meet at one point on the surface. They will then trace the curves created at that point on the two slices and estimate the slope. In a second project, the students learn how the slope of a surface changes depending on the direction and point chosen. On a point that is identified, the students will use a ruler to measure the height changes when they move to the eight corners that surround the point. They will use those measurements to estimate the slope in those eight directions.   (Received September 15, 2020)

1163-A1-1385   **Caleb L Adams***, PO Box 6942, Radford, VA 24142. *Enhancing Student Engagement in Online Discussions.* Preliminary report.

Today's college student is a member of the generation where the internet has been commonplace throughout their entire life. With more courses becoming available online, instructors can find generating a positive, rewarding experience as one goal they may work towards to ensure their students' satisfaction of being members of the online course. One item that has been noted to improve the students experience is high engagement practices in the online course. In this presentation, several important factors will be highlighted providing instructors insight to tactics that can be implemented to enhance student engagement in their online courses.   (Received September 15, 2020)

1163-A1-1540   **Jose N Contreras*** (`jncontrerasf@bsu.edu`), 3409 W Wheatfield Ln, Muncie, IN 47304. *Using GeoGebra and a Problem-Posing Approach to Create an Inquiry-Based Learning Environment for Secondary Mathematics Teachers.*

In this presentation, I will illustrate how I guide my prospective secondary mathematics teachers to discover theorems related to the power of a point using a problem-posing approach supported by GeoGebra (GG). In this investigation, the students discover and unify the four theorems associated with the power of a point: the secant-secant theorem, the secant-tangent theorem, the tangent-tangent theorem, and the chord-chord theorem. In this investigation, the students are also engaged not only in making conjectures, but also in developing proofs to prove each of the four theorems.

As teacher educators, we need to design learning tasks for practicing and future teachers that deepen their understanding of the content they teach or are likely to teach. Having a profound understanding of a mathematical idea involves seeing the connectedness of mathematical ideas. By discovering and unifying the power-of-a-point theorems and proofs, my students experience what it means to understand deeply a mathematical theorem. GG was an instrumental pedagogical tool that facilitated and supported the investigation in three main ways: as management tool, motivational tool, and cognitive tool.   (Received September 15, 2020)

1163-A1-1570   **Brooke Brown*** (`brown.brooke.m@gmail.com`), **Howard Bartels** and **Allyson Hallman-Thrasher**. *Who is successful in IBL Calculus and how does IBL support student success?*

Students in non-mathematics-intensive majors are often expected to struggle in calculus due to poor background knowledge, dislike of mathematics, or the lack of a need to be prepared for follow-up mathematics courses. We will discuss the performance and gains in affective domains of non-mathematics-intensive majors enrolled in an inquiry-based learning (IBL) Calculus I course. We will share what features of the IBL structure students with non-mathematics-intensive majors reported as being beneficial.   (Received September 15, 2020)

1163-A1-1572   **Kayla Heacock** and **Kelly Bubp*** (`kmbubp@frostburg.edu`). *Student Buy-In and Affective Gains in IBL Calculus.*

Student buy-in is essential to the success of inquiry-based learning courses. The creation of a supportive and productive learning environment can lead to gains in both learning and affective domains. We will discuss assignments designed to foster student buy-in through introducing students to learning through effort, collaboration, struggle, and from mistakes. Additionally, we will share data on the impact of these assignments on students' reported gains in related affective domains such as persistence, confidence, and positive attitude toward mathematics.   (Received September 15, 2020)

1163-A1-1587   **Tim Archie***, 580 UCB, Boulder, CO 80309-0580, and **Devan Daly** and **Sandra Laursen**. *How much is enough professional development? Outcomes of short and extended workshops on inquiry-based learning in college mathematics.*

Teaching-related professional development (TPD) supports instructors' uptake of IBL and similar student-centered teaching approaches. Working with the NSF-supported PRODUCT project of the Academy of Inquiry Based Learning, we studied two forms of TPD: four-day intensive workshops (IWS) for instructors ready for a deep dive on IBL and shorter traveling workshops (TWS) that introduce IBL methods and encourage instructors to learn more. Here we compare audiences and outcomes for the IWS and TWS to consider their affordances and limitations in fostering uptake of IBL. Both types of workshops increased participants' IBL capacity, as measured by their knowledge, skills, motivation to use IBL, and belief in its effectiveness. For the IWS, IBL capacity increased from pre- to post-workshop, and 94% of participants reported implementing IBL practices. Regression analyses showed that IBL capacity is strongly related to IBL uptake. Thus these workshops were effective in building IBL capacity and in turn promoting implementation. TWS data show that a majority of respondents reported gains in IBL capacity measures, especially growth in IBL knowledge and motivation. We will offer design suggestions for short and extended workshops that help them to reach relevant and achievable objectives.   (Received September 15, 2020)

1163-A1-1589   **Amy Ksir*** (`ksir@usna.edu`), **Nina White**, **Jane R. Cushman**, **Christine von Renesse** and **Patrick X. Rault**. *Starting and Running a Community for Mathematics Inquiry in Teaching.*

A Community for Math Inquiry in Teaching (ComMIT) is a professional learning community of college math instructors who support each other in the practice of teaching using inquiry. Each community provides professional development, collaboration opportunities, and mentoring, and fosters a welcoming environment for faculty as we continue to develop our practices of facilitating equitable student collaboration, engaging students in rich mathematical tasks, using student thinking to drive course decisions, and valuing all students as they bring their whole, authentic selves into the classroom.

The presenters of this session come from the leadership teams of five different such regional communities. In this talk, we will share a toolkit we have been developing for others who are interested in starting a community of their own. This toolkit includes case studies of events and activities, leadership structures, communication strategies, and other nuts and bolts of starting and sustaining an inclusive community. The toolkit also includes a set of guiding questions for new community leaders; in this session we will engage with some of those questions as participants reflect on their regional needs, existing networks, and opportunities.   (Received September 15, 2020)

Incorporating Realistic Applications of Mathematics Through Interdisciplinary Collaborations

1163-A5-221   **Deborah C. Arangno*** (`darangno@hcc-nd.edu`). *An Application of Mathematical Structures to a Philosophical Debate.*

: In contemporary metaethics, realist-antirealist debates centering on the ontology of moral properties continue to generate significant and exciting research. We have explored the use of methods from Graph Theory to clarify a debated position of Meta-Ethics, previously encumbered by intrinsic vagueness and ambiguity. We employ rigorous mathematical formalism to symbolize, parse, and thus disambiguate, particular philosophical questions regarding ethical ontological materialism of the reductivist variety. In our research, we seek to revisit the once vexed question regarding the multiple-realizability of moral properties by employing the mathematical machinery of hypergraphs. The utilization of hypergraphs offers explanatory flexibility to the hypothesis that

there are, potentially, an infinite number of unrelated subvening facts (viz., the subvening sets of natural non-moral properties) which have the possibility of coming together as subsets (hyperedges) to form supervening moral properties. We hope our research will be of interest to both the Analytic Philosophy and the applied Mathematics communities, and will help facilitate discussions of metaethics, particularly pertaining to ethical naturalism. We believe there is much research yet to be done. (Received August 28, 2020)

1163-A5-750   **Briana Lynne Edwards*** (`bedwards2016@my.fit.edu`) and **Vladislav Bukshtynov** (`vbukshtynov@fit.edu`). *Multi-Sample PCA Applications for Reconstructing Structure of Biological Tissues.*
Electrical Impedance Tomography (EIT) is an evolving medical imaging technique with particularly promising applications for cancer detection. EIT-based screening is an attractive alternative to current detection methods because it is non-invasive and cost-effective. However, the underlying formula is a highly ill-posed nonlinear inverse problem requiring advanced computational algorithms to produce results that are competitive with existing screening methods. This work aims to increase the reliability and specificity of the computational framework to solve the inverse EIT problem by integrating multiple datasets into the principal component analysis (PCA) re-parameterization via multiple factor(ial) analysis (MFA). These datasets are represented by synthetically generated groups of samples, or solution images, related by their topology. An efficient solution space parameterization is facilitated by the multi-table PCA transformation constructed from properly combined sample groups. The efficacy of the PCA/MFA is maximized by setting the sample group weights and the number of PCA components as optimization parameters. Our first computational results obtained for multiple 2D synthetic models will be presented along with a discussion on future applications to 3D clinical data. (Received September 12, 2020)

1163-A5-973   **R E Bergee*** (`bergeere@vcu.edu`), **E L Boone**, **R A Ghanam** and **B Stewart-Koster**. *Sequential Optimal Designs for Estimating Population Dynamics.*
Studying population dynamics can assist environmental managers in making better decisions. Traditionally, the sampling of species has been recorded on a regular time frequency. However, sampling can be an expensive process due to financial and physical constraints. Limiting sample size makes it challenging to accurately estimate the population dynamics. Thus, a new and novel approach is proposed to collecting samples based on the dynamics associated with populations of interest. The Lotka-Volterra differential equations are employed to simulate species composition. The goal of this research is to develop a methodology that learns about dynamical systems in a sequential manner and determines an optimal sampling regime for ecologists. (Received September 14, 2020)

1163-A5-1054   **Theresa A. Jorgensen*** (`jorgensen@uta.edu`), **Elizabeth M. Griffith**, **W. Ashley Griffith**, **Brittan Wogsland**, **Lindsey Hernandez** and **J. Lowe**. *Integrating Geoscience Contexts into Precalculus.*
Motivated by the goals of 1) improving the success of intended STEM majors in first year mathematics courses and 2) introducing first year students to the geosciences to increase the number and diversity of students who choose a geoscience major, our team of mathematicians and geoscientists are modifying the curriculum of Precalculus by integrating current geoscience research. We successfully piloted this collaboration in College Algebra and are extending it to Precalculus. For each Precal unit our geoscientists chose a corresponding geoscience theme and a scientist who does research related to that theme. We have developed videos featuring each geoscientist describing how the mathematical skill the students are learning is essential for her research. A portion of student homework exercises and problem-solving labs are cast in the context of that geoscience research. We present pre- and post-course survey results that assess student interest and self-efficacy in science and math as well as career perceptions of geoscience and related sciences. We highlight curricular examples from implementation of the geoscience themes, in particular volcanology aligned with a unit on functions, and planetary geoscience aligned with a unit on unit circles, trig functions, and right triangles. (Received September 14, 2020)

1163-A5-1145   **Girija S Nair-Hart***, nairhaga@ucmail.uc.edu, OH. *Fostering Increased Sense of Belonging in a Calculus Class through an Inter-disciplinary Project in the Context.* Preliminary report.
The purpose of this project was to provide students a more fulfilling experience in their Calculus 1 class through a hands-on project that highlights applications of Calculus in relatable real-life contexts. Biology, Physics, and Chemistry faculty participated in this project that examined blood flow in the smaller artery in arterial bifurcation. Based on a text book exercise, informed by Poiseuille's Law, the Navier-Stokes equations, and the concept of optimization, students examined the relationship among the angles and radii of bifurcating

blood vessels. They calculated the branching angle of the smaller vessel that minimized friction along the branching path. In groups students constructed various models of vascular branching describing fluid flow through blood vessels and tested their best models. Students stated that this hands-on project helped them realize the importance of calculus and its connections to other disciplines in ways that they have never imagined before. They also felt a sense of belonging in the class and established a stronger connection with their peers. The presentation will detail the project, results of successful collaboration, and its impact on students' experiences. Insights gained for project modification will also be presented.   (Received September 14, 2020)

1163-A5-1352	**Lindsay C Good*** (`lgood2@pacollege.edu`), 850 Greenfield Road, Lancaster, PA 17601, and **Daniel L Ozimek** (`dozimek2@pacollege.edu`). *Integrating Quantitative Reasoning and Clinical Judgment: An On-going Collaboration Between Mathematics and Nursing Faculty at a Health Sciences College.*

Since 2017, a multidisciplinary task force, supported by the Mathematical Association of America, the Charles A. Dana Center at the University of Texas at Austin, and Quality and Safety Education for Nurses (QSEN), has been working to strengthen partnerships between the mathematics and nursing communities in order to improve quantitative education practices in nursing programs. Consistent with recently published recommendations from this task force, mathematics and nursing faculty at Pennsylvania College of Health Sciences have collaborated to create authentic dosage calculation activities designed to integrate quantitative reasoning with clinical judgment processes. This presentation will include a brief history of this on-going collaborative effort, lessons learned that can influence future collaborations, and examples of co-constructed dosage tasks. Presenters will also share a tentative framework that captures how the collaborative project has influenced mathematics faculty members' approaches to teaching dosage calculations.   (Received September 15, 2020)

1163-A5-1363	**Pavel Belik***, belik@augsburg.edu, and **Joan Kunz**. *Incorporating Chemistry into a Calculus Lab.*

As part of the national SUMMIT-P project, faculty from Mathematics, Economics, and Chemistry at Augsburg University renovated the calculus sequence to meet the needs of the partner disciplines. As one of the outcomes, weekly labs in Calculus I and II now feature partner discipline contexts/questions and are placed later to make it less obvious what calculus ideas and tools are needed in the hopes of improving transference. We will present one such lab that connects titration, a procedure taught in our General Chemistry II class, to calculus concepts needed to identify points of significance taking place during the experiment. This lab was co-developed by the Mathematics and Chemistry faculty and the work was supported by the National Science Foundation under NSF award number 1625142. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. (Received September 15, 2020)

1163-A5-1414	**John C Merkel*** (`jmerkel@oglethorpe.edu`) and **Robert Dougherty-Bliss**. *A Compartmental Analysis Model of Persistent Environmental Toxins.* Preliminary report.

The presenters are an instructor-student pair who worked with Dr. Clem Welsh of the Centers for Disease Control and Prevention on a project modeling the propogation of polychlorinated biphenyls through the human body. Dougherty-Bliss, a graduate student who completed this project during his undergraduate studies, will discuss the research and its influence on his mathematical career. Merkel will discuss the subsequent development of differential equations course materials based on this project.   (Received September 15, 2020)

1163-A5-1418	**Mike May*** (`mike.may@slu.edu`). *Variables and Parameters in Math for Business.* Preliminary report.

At Saint Louis University, we have an ongoing seminar to discuss how to make math education more effective fro business students. A topic consistently brought up by the business faculty is the need for students to distinguish variables and parameters in multivariable functions. Traditionally, introductory math courses work in functions of one variable and do this by designating all but one of the variables as parameters. In the applied setting, the variable moves along the curve and parameters move the curve.

The collaboration led to modification in both math and business courses. For business courses we produced visualization tools for examples of using parameters and variables in introductory courses in accounting, economics and finance. For math courses, this has led to an adjustment in a business flavored college algebra course where the students work with the multivariable functions concerning loans, but then have to answer questions where the changing context changes which quantities are parameters and which are variables.   (Received September 15, 2020)

1163-A5-1460   **Janet M Shiver*** (`janet.shiver@cwu.edu`), 400 E. University Dr, Mathematics Department, Ellensburg, WA 98926, **Ian Quitadamo** (`ian.quitadamo@cwu.edu`), 400 E. University Dr., Biology Department, Ellensburg, WA 98926, and **Tim Sorey** (`tim.sorey@cwu.edu`), 400 E. University Dr., Chemistry Department, Ellensburg, WA 98926. *Integrating Mathematics, Science, Engineering and Technology to Prepare Secondary Teachers.*

Designing a course to prepare secondary mathematics and science teachers that truly integrates STEM was both the goal of our team (chemist, biologist and mathematician) and a monumental challenge. Throughout the design and revision process we found ourselves struggling to develop authentic problems, transform closed mindsets, and balance varying abilities in mathematics, science and engineering. Once we began understanding the similarities in scientific inquiry, mathematical problem solving and the engineering practices, we were able to design a well-developed course that allows scientists to see the value in using mathematics to make better decisions, mathematicians the value of true scientific inquiry, and engineers the value of scientific exploration combined with mathematical precision to provide the best results to problems. This paper takes a look at the challenges and triumphs associated with developing and executing a truly integrated STEM course.   (Received September 15, 2020)

1163-A5-1585   **Gregory V. Bard*** (`bardg@uwstout.edu`), Dept of Math Stat & Comp Sci, Jarvis Hall Science Wing, University of Wisconsin—Stout, Menomonie, WI 54751. *Realistic Yet Comprehensible Models of Applied Problems using Monte-Carlo Simulations in Discrete Mathematics.* Preliminary report.

As enthusiasm about data-science spreads among the student population, it is convenient for the topic of Monte-Carlo Simulations to be injected into courses that undergraduates take in the first two years. Not only are Monte-Carlo Simulations a good tool for modeling complex phenomena, they are also a necessary prerequisite for the relatively new statistical technique of "bootstrapping."

In this talk, the presenter will discuss three examples of realistic and relevant (but comprehensible), applications of Monte-Carlo simulation, for three minutes each. These are taken from a course in discrete mathematics, aimed at computer-science, cybersecurity, and computer-engineering majors in their third semester. They are for (1) the introduction to polling error in political science, including a first glimpse at confidence intervals; (2) for the famous observation about Archibald Wald regarding armor plating on allied bombers in WWII; (3) for the unfair impact of attendance policies of a famous grocery store on employees of differing socio-economic classes. These are demonstrated using the computer-algebra system Sage. While political science, aeronautical engineering and sociology are distant from the computing disciplines, students have reacted favorably to these extended examples.   (Received September 15, 2020)

Harmonic Analysis and Applications to Complex Analysis and Partial Differential Equations

1163-AD-113   **Yury Grabovsky** and **Narek Hovsepyan*** (`narek.hovsepyan@temple.edu`). *Optimal error estimates for analytic continuation from a curve with imprecise data.*

Analytic functions in a domain Ω are uniquely determined by their values on any curve Γ lying in the interior (or on the boundary) of Ω. We are interested in a sharp quantitative version of this statement. Given f analytic and of order one in Ω, assume that it is small on Γ (say, of order ϵ), how large can f be at a point z away from the curve? When the sizes of f are measured in Hilbert space norms we give a sharp bound on $|f(z)|$ in terms of a linear integral equation of Fredholm type. We show that the bound behaves like a power law ϵ^γ for some $\gamma = \gamma(z) \in (0, 1)$. In special geometries (such as the annulus, ellipse or upper half-plane) the solution of the integral equation and the corresponding exponent can be found explicitly.   (Received August 16, 2020)

1163-AD-798   **Jeongsu Kyeong*** (`tuj64025@temple.edu`), 4040 Presidential Blvd, Philadelphia, PA 19131. *Singular Integral Operators for Elliptic Boundary Value Problems.*

The plan is to discuss the employment of singular integral operators in the study of second order elliptic partial differential equations in Lipschitz domains. More specifically, the presentation will be centered around the relationship between the geometry of the domain and the availability of a Fredholm theory for the boundary to boundary singular integral operators associated with the given PDE. The talk will be accessible to graduate students who have completed a first year graduate course in measure theory.   (Received September 13, 2020)

1163-AD-849   **Dorina Mitrea*** (`dorina_mitrea@baylor.edu`), **Irina Mitrea** and **Marius Mitrea**. *A Sharp Divergence Theorem and Applications to Complex Analysis.*

In this talk I will discuss a version of the Divergence Theorem for vector fields defined in rough domains satisfying minimal conditions. As applications of this theorem I will prove a very general version of the Cauchy-Pompeiu representation formula and of the classical Residue Theorem in complex analysis. (Received September 13, 2020)

1163-AD-915   **Azita Mayeli*** (`amayeli@gc.cuny.edu`), New York City, NY. *An interplay between Gabor bases and Fuglede Conjecture.* Preliminary report.

The Fuglede Conjecture asserts that a bounded and measurable domain Ω in $\mathbb{R}^d$ tiles the entire space by countable many copies of its translations if and only if the Hilbert space $L^2(\Omega)$ has an orthogonal basis of exponentials. The conjecture has been disproved for dimension $d \geq 5$ by Tao (2003) and later for $d \geq 3$ by other mathematicians. However, the conjecture holds true for special cases in all dimensions. In this talk, we show how the study of the Gabor bases problem can be related to the study of Fuglede Conjecture in general. More precisely, we assert that for a characteristic function $g := \chi_\Omega$ of a set Ω, the function g generates a Gabor bases for $L^2(\mathbb{R}^d)$ with respect to a countable Gabor spectrum if and only if the Fuglede Conjecture holds ture for Ω. We term our assertion *Fuglede-Gabor Problem* and prove that it is true for special cases of Gabor spectrums. This is a joint work with Chun-Kit Lai of SFSU. (Received September 14, 2020)

1163-AD-1041   **Ellen Urheim*** (`urheim@math.upenn.edu`). *Geometrically stable oscillatory integral operators.*

One group of objects studied in harmonic analysis are oscillatory integrals, an example of which is the Fourier transform. An oscillatory integral operator usually takes an input function, multiplies by a cutoff function and a function oscillating at a rate given by a parameter λ, and then integrates. In general, we expect that the more oscillation we have (the bigger λ is), the smaller the output should be, due to cancellation happening in the integral. We often seek to quantify this relationship via a decay rate that depends on λ. In this talk, we will discuss some particular examples of oscillatory integral operators, including examples from recent work with P. T. Gressman of operators for which the decay rates do not change when we make small enough modifications to the oscillatory function and the cutoff function. (Received September 14, 2020)

1163-AD-1242   **Shari Moskow***, 321 Llandrillo Rd, Bala Cynwyd, PA 19004. *Inverse Problems: Determining the Equation from the Solution.*

Mathematicians and scientists are often looking for the solution to a known partial differential equation. However, sometimes rather than solving the equation, we are working backwards from data to find the equation itself. This kind of inverse thinking shows up in areas like medical imaging, remote sensing, nondestructive testing, and many other areas of science. Specialists may be using this approach to find a tumor in a breast or to locate oil in a geological reservoir. However, the smallest miscalculations in the acquired data can lead to large errors in defining the equation's parameters. We will discuss the challenges facing scientists and mathematicians in solving these "inverse problems." (Received September 15, 2020)

Supporting Mathematics in the Developing World

1163-AE-521   **Ingrid Daubechies***, Mathematics Department, Duke University, 120 Science Drive, Campus Box 90320, Durham, NC 27708-0320. *Getting Involved in Supporting Mathematics in Developing Countries through the International Mathematical Union.*

The International Mathematical Union (IMU) supports mathematics in developing countries through the Commission for Developing Countries (CDC). The CDC's programs include grants, lecturing, scholarships, special programs and travel support to the International Congress of Mathematicians. Of particular interest to the MAA/AMS communities are the lecturing and scholarship programs where mathematicians can volunteer to teach in developing countries, financially support students, or both. Started in 2008, the Volunteer Lecturer Program (VLP) offers universities in the developing world lecturers for intensive 3-4 week courses in mathematics at the advanced undergraduate or master's level. Volunteers have taught courses in 21 countries in Africa, Asia and Latin America. Started in 2017, the Graduate Research Assistantships in Developing Countries Program (GRAID) is for research assistantships to graduate (MSc or PhD) students of research groups working in a developing country where $3,500 is sufficient to fully fund a graduate student. Using funds donated through Friends of the IMU (FIMU), GRAID has funded research groups in 8 countries in Africa and Asia. This talk

will give an overview of the programs, sample cases and opportunities to get involved.   (Received September 08, 2020)

1163-AE-762   **R Sanchez-Lamoneda*** (`lamonedar@gmail.com`), Calle 58-A. #37-94, Bogota, 111321, Colombia. *The impact of mathematical olympiads on the mathematics community of Venezuela.*

Mathematical olympiads are a very effective activity to promote mathematics among young students, teachers and the school community. They are also a powerful tool to discover mathematically gifted young people. The Venezuelan mathematical olympiads started in 1976, a year after the first graduate program in mathematics started in the country. Mathematical studies are relatively new in Venezuela. Undergraduate studies in mathematics began at the end of the fifties of the last century. I will explain how mathematical olympiads in Venezuela made it possible for young students to be aware of mathematics as a possible career. Mathematical olympiads give students the opportunity to discover their mathematical talent and show them a possible path to mathematics when they go to the university.   (Received September 12, 2020)

1163-AE-1015   **Miranda I Teboh-Ewungkem*** (`mit703@lehigh.edu`), Department of Mathematics, Lehigh University, Bethlehem, PA 18015. *Supporting Mathematics in Developing Countries: Case of Collaboration with Dr Gideon Ngwa and the University of Buea, Cameroon.*

Many African diaspora mathematicians with basic math training from African universities are eager to engage with the institutions that trained them. They need support to make those efforts meaningful. Some practicing mathematicians working in developing countries but trained in developed nations need encouragement to continue to stay in their home countries to promote the development of mathematics there. Women and others in mathematics in developing countries need role models, mentors and active collaboration. In this talk, I present my collaborative efforts with African institutions, starting with my sustained collaboration with Dr. Ngwa, Dept. of Mathematics, Univ. of Buea, Cameroon, a mathematician trained in Oxford. Next, I share my engagements with the Math Dept., Univ. of Buea, through teaching, supervision and in organizing internationally attended mathematics meetings, as well as in training students, engaging them in research and, in mentoring junior faculty. I will also highlight my role in examining students' work from other African universities, discussing successes/challenges encountered, the roles ADMP, GRAID, and the NSF have played, via grants, and continue to play in enabling the successes of the aforementioned collaborations. I will conclude with future plans.   (Received September 14, 2020)

1163-AE-1283   **Helene R Tyler*** (`helene.tyler@manhattan.edu`), Department of Mathematics, Manhattan College, 4513 Manhattan College Parkway, Riverdale, NY 10471. *Reflections on Mathematical Development in Southeast Asia.*

Nearly 11 years after my first trip to Cambodia, there has been tremendous growth in the local mathematical community. Seeds of mathematics planted in Phnom Penh continue to flourish in the city and in the provinces. I will share stories and updates from former students and fellow educators, and reflect on an evolving notion of how we measure success.   (Received September 15, 2020)

1163-AE-1367   **Sofiat Olaosebikan*** (`sofiat.olaosebikan@glasgow.ac.uk`). *A story on how my journey through AIMS led me to becoming an academic in the UK.*

Growing up, I was passionate about mathematical problem solving. This led to my decision to study Mathematics for my first degree at the University of Ibadan, Nigeria. Along the line, I also realised my passion for computing because my father bought me my first computer at age 15. Throughout my undergraduate studies, I thought hard about how to combine my maths and computing skills for solving real-world problems. I searched the internet, I was overwhelmed, and worst of all, I had no one to point me in the right direction.

In 2015, I received a Masters scholarship to study at the African Institute for Mathematical Sciences (AIMS). During my time at AIMS, I was introduced to Python programming, graph theory, algorithms, and other interesting subjects in mathematical sciences. I felt empowered!

In this talk, I am going to reflect on my experience as a student at AIMS. To be specific, I will focus on how learning from the visiting international academics helped me identify my mathematical strengths, and how working with them gave me the confidence to further develop my passion for research. I will also briefly talk about the Programming Workshop for Scientists in Africa (PWSAfrica) – an initiative I started in 2018 to empower young scientists with computer programming skills.   (Received September 15, 2020)

1163-AE-1442   **Graeme Fairweather*** (`graeme.fairweather@gmail.com`), 45 South Joyce Street, Golden, CO 80401. *Mathematics – and Angkor Wat, Bagan and Luang Prabang.* Preliminary report.

What is the connection between these three UNESCO World Heritage Sites and mathematics/mathematics education? Well they are places visited by participants in a series of annual conferences held in S.E. Asia. This series, entitled "International Conference on Mathematics and Mathematics Education in Developing Countries", was established in 2008 in Cambodia and, for five years, was held in Phnom Penh. In 2013, the venue was changed to Myanmar where it was hosted for five years by Mandalay University and Yangon University. The National University of Laos in Vientiane, Laos, hosted the 11th and 12th conferences. It was planned to hold the 13th conference in 2020 at Souphanouvong University in Luang Prabang, a picturesque town north of Vientiane, but it has been postponed because of the covid-19 pandemic. Since 2013, pre-conference workshops have been held for three groups, high school teachers, undergraduates, and graduate students/faculty. These have been directed by distinguished international faculty and have been well-received, attracting in total over 100 participants each year. In this presentation, features of the workshops and conferences are described, and positive outcomes of these events as well as continuing challenges facing the organizers discussed.   (Received September 15, 2020)

Enlightening Mathematical Proofs from Geometry, Calculus, Linear Algebra, Probability or Combinatorics

1163-AF-233   **William Dunham*** (`bdunham@brynmawr.edu`). *J. J. Sylvester and Odd Perfect Numbers.*
"Whether there are any odd perfect numbers," wrote Euler, "is a most difficult question." His statement was prophetic, for the existence of such numbers remains unresolved to this day. However, mathematicians have established necessary conditions for odd perfect numbers (should any exist). In this talk we prove one of these: an odd perfect number must have at least three different prime factors. The proof, from J. J. Sylvester in 1888, is as elementary as it is elegant.   (Received August 29, 2020)

1163-AF-243   **Dan Kalman*** (`kalman@american.edu`). *Optimization as a Proof Technique.*
The usual goal of optimization is to answer a question about possible extreme cases for a phenomenon or context. But sometimes optimization provides a means for proving theorems. As a simple example, to prove the validity of an inequality of the form $f(x, y, \ldots) \geq 0$ in a domain Ω, it suffices to minimize f over Ω and show that the minimum value is nonnegative. In this talk we will consider several examples where optimization elegantly proves results that do not at first glance appear to have anything to do with optimization. In some of these examples the objective function is very simple so that the key feature of the optimization concerns the geometry of the domain of optimization.   (Received August 30, 2020)

1163-AF-248   **R. J. Swift*** (`rjswift@cpp.edu`), Department of Mathematics and Statistics, Cal Poly Pomona, Pomona, CA 91768. *Solids of Revolution and the Herschel-Maxwell Theorem.*
A characterization of a continuous nonnegative function f that is radially symmetric is obtained. The motivation of the problem arises from an elementary consideration of solids of revolution with the Herschel-Maxwell Theorem for the multivariate normal distribution the consequence.   (Received August 30, 2020)

1163-AF-285   **Francis Edward Su*** (`su@math.hmc.edu`), 301 Platt Blvd, Claremont, CA 91106. *Five different proofs of Sperner's Lemma.*
One way to measure the significance of a mathematical result is by considering the other things that are equivalent to it. Sperner's lemma (1928) is the combinatorial equivalent of the Brouwer fixed point theorem in topology, an important theorem by any measure. Another way is to measure a result's significance is to look at how many different proofs it admits. I'll describe at least five delightful proofs of Sperner's lemma that reveal different aspects of the allure of this combinatorial gem.   (Received September 01, 2020)

1163-AF-297   **Mits Kobayashi*** (`mitsuo.kobayashi@dartmouth.edu`). *Visualizing Infinite Series.*
Historically, infinite series had a geometric interpretation, although modern practice is to work with them completely algebraically. We briefly give some examples of classical "geometric" series as well as a more recent attempt by Viggo Brun to give a geometric interpretation to a series for $\pi/4$. Although Brun's method is ad hoc, we show that the same result and others can be found using standard results in single-variable calculus. We demonstrate the wider applicability of this method by giving a geometric interpretation to a series involving the Euler-Mascheroni constant.   (Received September 01, 2020)

1163-AF-304   **Alan Krinik*** (`ackrinik@cpp.edu`), **Jeremy J Lin**, **Thuy Vu Dieu Lu**, **Mark Dela**, **David Perez**, **David Beecher** and **Weizhong Wong**. *Generalized Ballot Box Problem Using Linear Algebra.*

Suppose that in an election between two candidates A and B that A wins by k votes. The classical ballot box problem asks what is the probability that candidate A is never behind candidate B during the counting of n ballots. If we assume that each voter cast a vote for either A or B with probability 1/2, then there is an elegant, well-known, combinatorial solution to this problem.

We generalize this ballot box problem by assuming that the probabilities of voting for A or B correspond to one-step transition probabilities of certain birth-death chains. This allows for non-uniform (or varying) probabilities and also allows for the probability of abstention from voting. Under these circumstances, we present a method to solve this generalized ballot box problem in terms of powers of certain matrices.

This linear algebraic approach leads to studying formulas, that scale up, for powers of certain types of nxn tridiagonal matrices in terms of their known eigenvalues. This work also produces explicit expressions for the transient probabilities of various finite birth-death chains or processes.   (Received September 01, 2020)

What does an Introduction to Data Science course look like?

1163-AH-698   **Mine Cetinkaya-Rundel***, University of Edinburgh, 2257 JCMB, Edinburgh, United Kingdom. *Introductory data science, a fresh look.*

Modern statistics is fundamentally a computational discipline, but too often this fact is not reflected in our statistics curricula. With the rise of data science, it has become increasingly clear that students want, expect, and need explicit training in this area of the discipline. Additionally, recent curricular guidelines clearly state that working with data requires extensive computing skills and that statistics students should be fluent in accessing, manipulating, analyzing, and modeling with professional statistical analysis software. In this talk, we will describe a fresh approach to teaching data science at the introductory level, introduce the design philosophy behind the curriculum, and give examples from course materials as well as from student projects. We will also discuss new directions in assessment and tooling as we scale up the course and move it online.   (Received September 11, 2020)

1163-AH-999   **Karl RB Schmitt*** (`krbschmitt@gmail.com`). *Client-Based Projects and Transformative Failure as an Introduction to Data Science.*

Data science is a field that is executed 'in-context', that is, in a domain on specific needs or questions. At Valparaiso University, students experience this throughout their degree starting with semester-long client-consulting projects in our "Introduction to Data Science" course. This sets the stage for later learning, providing an exciting engagement with the field, but also leading students to experience transformative failures which improve their data acumen. In this talk, I will provide an overview of the data science topics covered, but more importantly, I will talk about the *other* skills that are included in this course. After teaching it multiple times, the format was evolved to provide more significant support to the students during their projects. This shift was important for later success in the major and also led to significant student success in jobs, research, and other venues. (Received September 14, 2020)

1163-AH-1241   **Donghui Yan***, 285 Old Westport Road, Dartmouth, MA 02747. *A first course in Data Science.*

Data science is a discipline that provides principles, methodology, and guidelines for the analysis of data for tools, values, or insights. Driven by a huge workforce demand, many academic institutions have started to offer degrees in data science, with many at the graduate, and a few at the undergraduate level. Quite a few articles have been published that deal with graduate data science courses, much less so dealing with undergraduate ones. Our discussion will focus on undergraduate course structure and function, and specifically, a first course in data science. Our design of this course centers around a concept called the data science life cycle. That is, we view tasks or steps in the practice of data science as forming a process, consisting of states that indicate how it comes into life, how different tasks in data science depend on or interact with others until the birth of a data product or a conclusion. Naturally, different pieces of the data science life cycle then form individual parts of the course. Details of each piece are filled up by concepts, techniques, or skills that are popular in industry. Consequently, the design of our course is both "principled" and practical.   (Received September 15, 2020)

1163-AH-1490 **Tiffany A Timbers*** (`tiffany.timbers@stat.ubc.ca`), Earth Sciences Building, Rm 3152, 2207 Main Mall, Vancouver, BC V6T 1Z4, Canada. *Introduction to Data Science at the University of British Columbia - an accessible course with an emphasis on reproducible workflows.*

DSCI 100, the Introduction to Data Science course at the University of British Columbia in Vancouver, BC, was first launched in January 2019. It was designed as a first experience for students to gain skills in the areas of assembling, analyzing, and interpreting data. The high level course outcome that we aim for is that by the end of the course, students will be able to implement a data science workflow, by reading data from a local or remote file, "wrangling" (managing) the data intelligently, and creating tables and/or figures that convey a justifiable story based on the data. The course also introduces students to the landscape of statistical questions, and focuses on teaching intuitive and established supervised and unsupervised methods to answer predictive and exploratory questions. Examples include k-nearest neighbour classification, k-nearest neighbour regression, linear regression and k-means clustering. Rich problems with real data sets are used throughout the course. Statistical inference for point estimates is introduced at the end of the course to motivate follow-on statistical courses where inference and generative models are explored in more detail. (Received September 15, 2020)

1163-AH-1537 **Mark Daniel Ward*** (`mdw@purdue.edu`), 1301 Third Street, West Lafayette, IN 47906. *Creating Data Sciences Courses that are Accessible for All Undergraduate and Graduate Students.*

At Purdue University, in The Data Mine, we have created more than 20 new Learning Communities and have started 23 new Corporate Partnerships in the last two years. We also have a new series of Data Science courses that are designed to be accessible for all students; no prerequisite is needed. Nonetheless, the courses cover several popular tools for students who are new to the Data Sciences, including R, Python, SQL, UNIX, etc. In this talk, we will reflect briefly on the reproducibility of this model, including the opportunities and challenges inherent in building a campus-wide program of this nature. (Received September 15, 2020)

Coding Theory and Geometry

1163-AI-290 **Zilin Jiang**, **Jonathan Tidor**, **Yuan Yao**, **Shengtong Zhang** and **Yufei Zhao*** (`yufeiz@mit.edu`). *Equiangular lines and spectral graph theory.*

Equiangular lines are configurations of lines all passing through the origin and pairwise making the same angle. What is the maximum number of such equiangular lines in a given dimension? I will tell you what we know about the problem, some recent developments, and why it is connected to spectral graph theory. (Received September 01, 2020)

1163-AI-928 **Zeev Dvir*** (`zdvir@princeton.edu`). *Locally Correctable Codes and the Sylvester-Gallai theorem.*

The famous Sylvester-Gallai theorem states that, any finite set of points in the real plane, not all on a single line, defines at least one 'ordinary' line (a line passing through only two points). In recent years, quantitative/robust variants of this theorem turned out to have a surprising applications in coding theory. In particular, they are closely related to codes with 'local' decoding procedures. In this talk I will explain this connection and survey some of the known results and open problems. (Received September 14, 2020)

1163-AI-1084 **Beth Malmskog*** (`bmalmskog@coloradocollege.edu`), **Kathryn Haymaker** and **Gretchen Matthews**. *Locally Recoverable Codes with Many Recovery Sets from Number Theory and Geometry.*

Error correcting codes are systems for incorporating redundancy into stored or transmitted data, so that errors can be identified and even corrected. A good error correcting code is efficient and can correct many errors relative to its efficiency. These codes are ubiquitous in the digital age, and many excellent codes arise from algebraic constructions. The increasing importance of cloud computing and storage has created a need for codes that protect against server failure in large computing facilities. One way of approaching this problem is to ask for local recovery. An error correcting code is said to be locally recoverable if any symbol in a code word can be recovered by accessing a subset of the other symbols. This subset is known as the helper or recovery set for the given symbol. It may be desirable to have many disjoint recovery sets for each symbol, in case of multiple server failures or to provide many options for recovery. This talk describes how geometry and number theory can be powerful tools for creating such codes, including a construction using fiber products of curves to generate arbitrarily many recovery sets. (Received September 14, 2020)

1163-AI-1177   **Gretchen Matthews*** (`gmatthews@vt.edu`), Department of Mathematics, Virginia Tech, Blacksburg, VA 24061. *Toward classifying multipoint codes.*

Algebraic geometry codes are defined by specifying divisors G and D on a curve X over a finite field; codewords arise as functions specified by G are evaluated at points in the support at D. The resulting code is called a one-point code if the support of G has cardinality one and a multipoint code otherwise. Algebraic geometry codes have been shown to have remarkable properties in terms of classical error correction, and they may be studied by considering attributes of the curve and divisors. Closely related are Weierstrass semigroups which govern the dimension of the code. In this talk, we consider to what extent curve automorphisms and Weierstrass semigroups influence the classification of multipoint Hermitian codes, meaning those defined on the Hermitian curve. (Received September 14, 2020)

1163-AI-1443   **Christine A Kelley*** (`ckelley2@unl.edu`) and **Michelle Haver**. *Applications of finite geometries in coding theory.*

Codes based on finite geometries have a rich structure that lends them to many applications. One notable class is the class of finite geometry low-density parity-check (FG-LDPC) codes, in which the incidence matrix of the geometry may be used as the parity-check matrix of the code. In this talk we highlight some of the important known results of these finite geometry codes. We will then present some new results on codes obtained by lifts of finite geometry incidence graphs, and how the resulting codes can be used in a variety of applications, including distributed storage. (Received September 15, 2020)

1163-AI-1672   **Shubhangi Saraf***, shubhangi.saraf@gmail.com. *Some recent results on high rate local codes.*

In this talk we will give a high level survey on some recent results about high rate locally testable and locally decodable codes. I will discuss the construction of high rate locally testable and locally decodable codes with subpolynomially small query complexity. Time permitting I will also mention constructions of locally testable and locally decodable codes with rate and distance approaching the Gilbert-Varshamov bound (which is the best rate-distance tradeoff known for general binary error-correcting codes). Based on joint works with Swastik Kopparty, Or Meir, Rafael de Oliveira, Noga Ron-Zewi, Mary Wootters, Rafael de Oliveira, Sivakanth Gopi (Received September 16, 2020)

Promoting Women in Mathematics

1163-B1-284   **Carla Cederbaum*** (`cederbaum@math.uni-tuebingen.de`), Mathematics Department, Auf der Morgenstelle 10, 72070 Tuebingen, Germany. *Math Mentoring: experiences with training the mentors.*

Many higher education institutions, mathematics departments, etc. offer or plan to put in place mentoring programs for women and/or minority mathematicians at various stages of their careers. Of course, one key point in starting and running a successful mentoring program is the selection of suitable mentors. Another key point identified by social science research on the success of mentoring programs is to provide adequate training for the selected mentors. I will report on my experiences with establishing, running, and visiting mentoring programs for women mathematicians of all career stages — undergraduate student through full professor — at various institutions both in the US and in Germany.

In particular, I will present the components of a mentoring training developed in collaboration with the mathematics, mentoring, and mathematics education scholars Benjamin Cooke and Sarah Schott from Duke University as well as with the gender and diversity scholar Katrin Gra{ß} from T{'u}bingen University, Germany. The training materials will be made freely available under a Creative Commons license after the presentation. (Received September 01, 2020)

1163-B1-302   **Monica D Morales-Hernandez*** (`mmoraleshernandez@adelphi.edu`), One South Av, Garden City, NY, and **Sherli Koshy Chenthittayil**, **Josh P. Hiller** (`johiller@adelphi.edu`) and **Peter Maceli** (`pmaceli@ithaca.edu`). *Are Women in Computational Mathematics a myth?* Preliminary report.

The lack of female representation in academic positions in STEM fields such as Mathematics is widely known and it inherently reduces the visibility of research conducted by female academics compared to their male peers.

In this work, we quantify female representation among publications in 10 volumes of the SIAM Journal on Scientific Computing.

We identify possible relationships between the lack of female representation in authorships and the female representation on Editorial Boards. We apply different statistical procedures to identify possible causes of this gender gap

We study whether gender bias in the review processes is a cause in the disparity in the number of publications where female researchers are involved. We apply the same statistical procedures to determine whether geographical location influences the representation of female mathematicians in authorships.   (Received September 01, 2020)

1163-B1-374   **Joanna G. Jauchen*** (`jjauchen@gmu.edu`). *Activist Faculty Service: Framing the Work of Women Faculty in Gender-Based Initiatives.*
In the literature on initiatives to increase women's participation in STEM, many studies focus on impact to participants. The service of women faculty leading these initiatives has not been extensively examined. This type of faculty service is similar to race-based service which has been conceptualized both as a cultural tax levied by universities and as a form of critical agency that faculty undertake (Baez, 2000; Padilla, 1994). In this talk, I describe how I have started to see the service of women faculty in gender-based initiatives through this lens of critical agency, as a form of institutional activism. Through an intersectional feminist critical lens, I will present a review of institutionalized forms of activism, including Katzenstein's (1990; 1998) work on how institutions constrain activist work, Meyerson's (2001) framework on types of institutional activism, and Kezar and Lester's (2011) work on grassroots leadership in universities. From that review, I pose a series of reflective questions for participants to consider through this institutional activist lens. My aim is to provide faculty new ways to contextualize their own service and provide a sense of effective practices that can be leveraged in gender-based work.   (Received September 09, 2020)

1163-B1-413   **Sun-Yung Alice Chang** (`chang@math.princeton.edu`), Department of Mathematics, Princeton University, Fine Hall, Washington Road, Princeton, NJ 08544-1000, **Dusa McDuff** (`dusa@math.columbia.edu`), Department of Mathematics, Barnard College, Columbia University, New York, NY 10027, **Margaret A. Readdy*** (`margaret.readdy@uky.edu`), Department of Mathematics, University of Kentucky, Lexington, KY 40506, and **Karen Uhlenbeck** (`uhlen@math.utexas.edu`), One Einstein Road, Princeton, NJ 08540. *The Women and Mathematics Program at the Institute for Advanced Study.*
The Women and Mathematics (WAM) program at the Institute for Advanced Study in Princeton has been in existence since 1994. It has developed into a one week mathematical and mentoring workshop composed of approximately 60 undergraduate, graduate and postdoctoral participants to learn a current topic in mathematics. The two weeklong Uhlenbeck and Terng lectures, coupled with the teaching assistant-led problem sessions, and an intergenerational group of individuals from the WAM committee, industry, academics and government, round out the program. We will discuss the WAM program, including its recruitment efforts, learning and mentoring components, new initiatives, and some of the challenges an e-program presents.   (Received September 05, 2020)

1163-B1-455   **Sophia T. Santillan** and **Victoria Akin*** (`toriakin@math.duke.edu`), 120 Science Drive, 117 Physics Building, Campus Box 90320, Durham, NC 27708, and **Lauren Valentino**. *An Engineer, a Mathematician, and a Sociologist (all Women) Walk into a ~~Bar~~ Classroom...*
Middle school is a critical point in adolescent development during which stereotypes and cultural norms can dampen many girls' interest in math and STEM. Factors affecting this trend include a lack of female STEM role models, and math-specific beliefs, mindsets, and values. Duke Girls Exploring Math (DukeGEM), created by an interdisciplinary research team, is a program designed to address the gender gap in math by hosting innovative weekend and summer workshops for local middle school girls. These workshops are unique in that they combine collaborative mathematics problem solving with interactive discussions of the socio-cultural factors that affect STEM persistence. DukeGEM employs a social identity approach to nurture STEM-compatible beliefs alongside a rich problem-based mathematics curriculum to bridge the gender gap in math and STEM. Each spring semester workshop includes a hands-on math puzzle to be tackled in groups, and a gender-and-STEM activity/discussion that focuses on improving participants' math identity. Summer workshops also include visits to female-led Duke STEM labs. Early data demonstrated increases in both math performance and average math confidence. Participants also showed an increased belief that innate sex differences do not cause the gender gap in STEM. (Received September 07, 2020)

1163-B1-497   **Jenna P. Carpenter*** (`carpenter@campbell.edu`), Po Box 115, Buies Creek, NC 27506. *Building a More Diverse Faculty: Research-Based Strategies That Work.*
STEM disciplines in general, including the mathematical sciences and engineering, have long struggled with attracting and retaining a robustly diverse community of faculty, including women. Even when underrepresented faculty are hired, they tend to be congregated at lower levels, such as non-tenure track or assistant professor tenure-track positions, versus the full professor, administrative and other prestige positions. Research, however, has identified a number of practical strategies that work to diversify the faculty. At Campbell University, we have used these best practices to create a School of Engineering whose full-time faculty has 100% women administrators, a mechanical engineering faculty that is 80% women, and an overall faculty that is 62% women. This talk will review these strategies and how to implement them for success. (Received September 08, 2020)

1163-B1-499   **Wesley Hamilton*** (`wham@live.unc.edu`). *The Workshop for Inclusive Teaching Practices: organization and impact.*
In spring 2019, I co-organized the Workshop for Inclusive Teaching Practices (WITP) at UNC-CH. The purpose of this workshop was to connect graduate students and faculty keen on incorporating more inclusive teaching practices into their classrooms, and consisted of invited talks, a panel discussion, and small/large group discussions; as an example, one of the discussions was centered around the "Avoiding Gender Bias in Reference Writing" handout from the University of Arizona Commission on the Status of Women with sample letters of recommendation to analyze. A second iteration of the workshop was organized for the spring 2020 semester with a theme of mentorship, before COVID-19 forced us to reschedule everything into an online format during fall 2020. In this talk, I'll discuss how my collaborators (Katri Morgan (UNC-CH) and Paul Kruse (UNC-CH) in 2019, and Suzanne Crifo (Duke) and Katherine Harris (NCSU) in 2020) and I organized and ran the WITP, with specific tips and suggestions for others interested in organizing similar workshops. (Received September 08, 2020)

1163-B1-500   **Lyn McQuaid*** (`mcquaid@kutztown.edu`). *Endeavors to Attract Women to Mathematics: The Obvious and The Covert.*
I have tried many different ways to encourage women of all ages to study mathematics. In this talk, I will start with some of the more obvious attempts. Such attempts include starting a club to support women majoring and minoring in mathematics, holding an annual High School Mathematics Day for Girls, and creating a general education course entitled Women in Mathematics. I will also disclose the many covert methods I have employed such as using examples of women mathematicians every chance I could in every class I taught and endeavoring to have myself or one of my female colleagues appear at outreach events and in photos promoting the Department of Mathematics at my university. (Received September 08, 2020)

1163-B1-505   **Tim McEldowney*** (`tmcel001@ucr.edu`), WV 26505. *Building the Bridge: Creating a new pathway for Underrepresented Minority Women into Calculus.* Preliminary report.
Faculty in the Department of Mathematics at West Virginia University (WVU), an institution where only 7.8% of students identify as African-American or Hispanic, implemented an Emerging Scholars Program (ESP) in 2009 to support underrepresented minority students in their journey through Calculus. ESP at WVU has been successful at retaining and supporting URM students over the decade. For example, URM students who go through ESP Calculus have a significantly higher STEM graduation rate than non-ESP URM students. However, women are underrepresented in the program (30.6%) and in Fall of 2019 we ended up with an ESP Calculus I class that was all men. Looking through institutional data we found that women are underrepresented in Calculus I at WVU even more than the national average (34% vs. 45%), yet are better represented in WVU's lower level mathematics courses like College Algebra (47%). To address this underrepresentation of women in ESP we created a Summer Bridge to get underrepresented minority women from College Algebra and Trigonometry to prepare them for entering ESP Calculus I in the fall semester. In this talk we will discuss the data that led to the creation of the Bridge, how we designed it, and challenges the initial Summer Bridge faced, like a global pandemic. (Received September 08, 2020)

1163-B1-524   **Matthew K Voigt*** (`mkvoigt@gmail.com`), 320 Falls St, 502, Greenville, SC 29601. *Resources to Support Queer-Spectrum STEM students in Undergraduate Mathematics Programs.*
There is limited research examining the types and features of identity resources leveraged by queer-spectrum STEM students in undergraduate mathematics programs. In the follow paper presentation, I will share research that was conducted with 17 queer-spectrum students at four universities across the United States. Drawing on thematic analysis and identity resource constructs from Nasir (2011), I identified material, relational, and

ideational resources that Queer-spectrum students identified as either contributing to or hindering their experiences in STEM. Three cross-cutting findings were identified among the 22 identified identity resources. First, Queer-spectrum students' participation was fostered through the creation of smaller "safe spaces" and relationships in STEM. Second, Queer-spectrum student's sense of belonging was supported through resources that fostered academic and social integration (e.g, oSTEM). Third, the lack of positive ideational resources (e.g., view of STEM as white straight male centric) contributed to a lessened sense of both belonging and perceived ability. (Received September 08, 2020)

1163-B1-536 **Guadalupe I Lozano*** (`guada@math.arizona.edu`), **Marla A Franco** and **Vignesh Subbian**. *Equity in STEM at Hispanic Serving Institutions: Adaptive Case Studies as Catalysts for Change.* Preliminary report.

In 2017, the University of Arizona organized and carried out the first of 11 national conferences intended to inform the design of NSF's inaugural Hispanic Serving Institutions (HSI) program. The conference led to consensus recommendations for transforming STEM education at HSIs (Lozano, Franco, Subbian, 2018). In this talk, we discuss how three Adaptive Case Studies, construed as Broader Impacts of our grant, served to move from recommendation to actionable change in two intersectional areas: equity and assessment, and equity and access. These areas were identified as being underdeveloped in higher ed spaces relative to others, yet central and critical for brokering lasting systemic change at HSIs. (Received September 08, 2020)

1163-B1-641 **Carolyn Hamilton** and **Violeta Vasilevska*** (`violeta.vasilevska@uvu.edu`), Orem, UT 84058. *Engaging High School and Undergraduate Female Students in Extracurricular Math Activities.* Preliminary report.

In this talk, the structure of the Utah Valley University outreach program Math Girls Rock! will be discussed. Since 2011, this program has engaged hundreds of high school and undergraduate female students through hands-on extracurricular math activities and encouraged them to continue their education. Math Girls Rock! is designed as a year-long Math Club for Young Women, offering weekly meetings with the undergraduate female students and a series of after-school meetings (2 times per semester) for the high-school girls. The program offers mentoring on two levels: the project directors mentor the undergraduate students who then mentor the high school girls through the designed hands-on projects. It engages both student groups in various fun and interesting math projects that show that math is exciting, beautiful, and applicable to various other fields. In addition, the projects are designed to spark student curiosity, to stimulate student inquiry, to get them to get "their hands dirty" with math, and to inspire them to want to learn more. Moreover, the pre- and post-survey data on how learning math concepts in these types of informal settings affected the participants' attitudes toward mathematics will be presented. (Received September 10, 2020)

1163-B1-739 **Julianne Vega*** (`jvega30@kennesaw.edu`). *Girls and Non-Binary Students in STEM Magazine.*

In Summer 2020, I learned about the work of two young, aspiring women scientists Avani Ahuja (Georgetown Country Day School '22) and Layla Dawit (Sidwell Friends '22) and since then I have been mentoring these women as they continue to expand efforts. These two young women have been organizing, editing, and producing a Girls and Non-Binary Students in STEM magazine which consists of STEM articles written for and by high school girls and non-binary students and interviews with women scientists. In this talk, I will highlight their motivation behind the magazine, how they are beginning to expand their work, and how we as mathematicians can encourage more young leaders, like these two, to flourish. (Received September 12, 2020)

1163-B1-991 **Jacqueline M Dewar*** (`jdewar@lmu.edu`), Loyola Marymount University, Mathematics Department, UH 2700, 1 LMU Drive, Los Angeles, CA 90045. *Encouraging Women in Mathematics Through an Interdisciplinary Course.*

This talk describes (and provides resources for) an interdisciplinary course grounded in the biographies and work of ten women mathematicians (Hypatia to Emmy Noether). The course aimed to engage students in mathematical topics related to the women's mathematical work and to address gender equity in mathematics education and mathematics-related careers in the US. Many of the themes that emerge from examining the lives and work of these historical figures remain relevant today. The course encouraged students, some of whom were future K–12 teachers, to adopt a more expert view of mathematics as a study of patterns, rather than of numbers. The students also encountered 21st century role models for women doing mathematics and the future teachers gained knowledge of equitable classroom practices and resolve to incorporate these into their teaching. A follow-up study of those who became teachers revealed implications for undergraduate teaching practice and teacher preparation programs that will be shared. (Received September 14, 2020)

1163-B1-1337   **James Morrow*** (`jmorrow@mtholyoke.edu`) and **Charlene Morrow**
                   (`cmorrow@mtholyoke.edu`). *Empowering Young Women to Love Mathematics.*
We will present a set of principles and practices designed to empower young women in mathematics. These ideas are based on research about making the learning environment responsive to preferred ways that women learn and strategies that help them feel supported as they encounter mathematics and build understanding.

Over a period of 30 years, the authors have applied and refined this research and put it into practice in the realm of mathematics classes at both the secondary and post-secondary levels. We will describe how we apply our methods to learning, problem solving, problem posing, and appreciating mathematics.

We will describe how to help students become active learners and independent, but not isolated, learners by explaining ways of listening to, responding to, and guiding them, and how to introduce them to a broader picture of the world of mathematics. We will also describe some outcomes for students who have experienced such methods, including how our methods have helped students develop problem-solving strategies and increased their ability to pose problems. Finally, we will give examples of how such methods have changed the teaching practices of several teachers who have worked with us.   (Received September 15, 2020)

1163-B1-1391   **Katrina Morgan*** (`katrina.morgan@northwestern.edu`) and **Francesca Bernardi**.
                   Addressing Barriers Through Outreach at the Girls Talk Math Camp.
Young women entering mathematics face several known barriers to success. These include lack of encouragement, a narrow picture of what it means to be a "math person" and not seeing themselves represented amongst mathematicians. The Girls Talk Math camp was founded in 2016 by the authors with the goal of addressing such barriers. Girls Talk Math is a free day camp for girls and gender nonconforming students in high school. The program's core curriculum has campers working in groups on four activities: Completing one of several advanced problem sets; researching a female mathematician; writing a blog post about the math they learn; and writing and recording a podcast about their female mathematician. Each component was designed to encourage curiosity and give campers agency over their learning. This talk will discuss the rationale behind the structure of the program and the role of outreach in promoting diversity in mathematics.   (Received September 15, 2020)

1163-B1-1527   **Suzanne E. Crifo*** (`suzanne.crifo@duke.edu`), 4 East Campus Union Drive, Box 90694,
                   Duke University, Durham, NC 27708, and **Kimberly Bethea**
                   (`kimberly.bethea@duke.edu`). *STEM Advancement through Group Engagement (SAGE)
                   Learning Communities: Developing Student Leaders.* Preliminary report.
The Stem Advancement through Group Engagement (SAGE) Program at Duke University is an opportunity for undergraduate students in Biology, Chemistry, and Math to work in a small group setting in weekly study sessions. These learning communities are led by peer facilitators, who engage in weekly professional development. For the first semester in which SAGE-Math is offered, all peer facilitators happen to be female-identifying. Their professional development spans a variety of areas, including: devising and implementing a session plan; diversity, equity, and inclusion training; observing and providing feedback to peers; active learning techniques; and growth mindset, metacognition, and self-regulated learning. Ideally, the professional expectations and development will boost facilitators' confidence both in math and as a peer leader.   (Received September 15, 2020)

1163-B1-1552   **Daniel Collister*** (`dcoll010@ucr.edu`), **Savanna Gee** (`sgee004@ucr.edu`), **Po-Ning
                   Chen** (`poningc@ucr.edu`) and **Elizabeth Dyer** (`elizabeth.dyer@mtsu.edu`). *Remote
                   Advanced Mathematics Program 2020 - RAMPing Up Representation in Mathematics:
                   Program evolution in a pandemic.*
The lack of diversity observed in our mathematics department has seemed related to a sense of disconnection from the field and isolation felt by students from underrepresented communities. One response that the UCR Mathematics Department has employed over the last three years has been a summer bridge program where students hone upper division math skills while connecting with colleagues and professionals in and beyond the department.

When COVID forced "AMP" to become RAMP: Remote Advanced Mathematics Program, the 2020 instructors and organizers took the opportunity to revisit the heart of the program and leverage new pedagogical approaches in the online format. The revised practices resulted not only in students deepening their understanding of mathematical proofs, but also fostered a sense of belonging in the department and field. In this talk we share about the asynchronous teaching strategies, differentiated instructional approaches, and journaling activities that we saw engage and empower students from diverse backgrounds even in light of the social distancing practices of the pandemic.   (Received September 15, 2020)

1163-B1-1591   **Kaitlin Hill*** (`hillk@wfu.edu`) and **Kathryn McCormick** (`kathryn.mccormick@csulb.edu`). *Learning with GLEAM: A Saturday enrichment program for middle school girls.*

Girls Learning and Exploring Applications of Mathematics (GLEAM) was a Saturday enrichment program for middle school girls in high-diversity schools in Minneapolis/St. Paul, Minnesota, that ran in AY 2018-'19 and '19-'20. We recruited middle school teachers and undergraduate teaching assistants to work with the program, and we worked with these teachers to help recruit students from their school. Our outreach model prioritized the role of partner teachers as essential facilitators in reaching out to their students and providing a trusted foundation for learning new and fun topics. In this talk we will discuss program logistics as well as challenges and victories we encountered along the way.   (Received September 15, 2020)

1163-B1-1599   **Andrea Arnold*** (`anarnold@wpi.edu`). *A Virtual Sonia Kovalevsky Day Outreach Event for Middle School Girls.*

Sonia Kovalevsky Day (SK Day) outreach events are held nationally by local chapters of the Association for Women in Mathematics (AWM), with the goal of encouraging young women to continue their study of mathematics. In this talk, I will describe an online version of SK Day that was held remotely in Spring 2020 by the AWM Student Chapter at Worcester Polytechnic Institute. In particular, I will discuss how this virtual event successfully incorporated interactive online math activities to engage middle school girls from four different states across New England.   (Received September 15, 2020)

1163-B1-1609   **Tanya Moore*** (`tanya@intersectinglines.us`). *A Storied Perspective: What the Lives and Careers of Women in Math Can Teach us About Broadening Participation.* Preliminary report.

Women have been active participants in the mathematical sciences throughout history, yet chronic underrepresentation of women in the field persists and the contributions of women are not largely known. As questions arise about how best to increase recruitment and retention of women in mathematics, examining the lives of women mathematicians can be instructive. The author explores how we might use biographies to inspire, motivate and change the narrative around who mathematicians are and the work that they do. This presentation focuses specifically on three African American women mathematicians, Evelyn Boyd Granville, Dorothy Hoover and Gladys B. West, and shares how their lives can be used as road maps for efforts focused on broadening participation in math.   (Received September 15, 2020)

1163-B1-1635   **Adeli Hutton***, adeli.hutton@wustl.edu. *Factors Driving Women to Choose Math and Persist: A Review of the Research.*

Mathematics has the lowest retention rate of any college major and is lower yet for women. In a large-scale controlled study, Ellis et al. found that women are 1.5 times more likely than men to leave STEM majors after taking Calculus 1, although not due to a lack of mathematical ability. Despite this high attrition being a well-known problem, conclusive evidence determining the factors that lead women to persist in math rather than drop out, especially in more advanced levels, has not yet been well established. This talk will summarize the results of a literature review studying factors that drive women to pursue math degree programs and persist in them. Specifically, it will introduce tactics shown to be promising in recruitment and retention and also pedagogical methods proved efficacious in retaining women in STEM degrees. This talk will then summarize plans for further research based on the conclusions of the literature review. The speaker will also discuss how her own experiences intersect with the research results from both the perspectives of being a math post-baccalaureate student in a bridge program and a volunteer with several outreach and community engagement projects that aim to inspire, encourage, and support high school and undergraduate women interested in math.   (Received September 15, 2020)

Mathematics and the Life Sciences: Initiatives, Programs, Curricula

1163-D1-666   **Alessandra Pantano*** (`apantano@uci.edu`), **Lora D Weiss** (`weisslo@gvsu.edu`) and **Nicole Fider** (`nfider@math.arizona.edu`). *Collaborative research experiences in Mathematics and the Life Sciences for high school and college students.* Preliminary report.

Supported by the NSF-Simons Center for Multiscale Cell Fate Research at UCI, MathBioU and Math ExpLR are nonresidential interdisciplinary summer programs that bring together talented college and pre-college students with UCI faculty from the Departments of Mathematics, Developmental and Cell Biology, Chemical and

Biomedical Engineering, Physics and Astronomy and the UCI STEM Cell Research Center, for six intense weeks of research in mathematical biology.

A distinctive feature of this summer research program is its multigenerational nature. High school and undergraduate students work together to learn Matlab, LaTeX and mathematical modeling, and they perform high-quality research in mixed teams under faculty supervision. This collaboration leverages the unique abilities and motivation of the different age-groups and allows for cross-mentoring.

Our 3-year program evaluation has revealed very positive impact on all students involved. In this talk, we will share program results and we will discuss the challenges and successes involved in moving the program online in Summer 2020. Moreover, we will present some ideas regarding the creation of an "Intro to Research in Mathematical Biology" course that builds upon this summer program experience.   (Received September 11, 2020)

1163-D1-802   **Vinodh Kumar Chellamuthu***, 225 South University Avenue, Saint George, UT , and **Noelle West**. *Modeling the Effects of Passive Immunity in Birds for the Disease Dynamics of West Nile Virus.*

West Nile Virus (WNV) is a mosquito-borne virus that circulates among birds but also affects humans. Migrating birds carry these viruses from one place to another each year. WNV has spread rapidly across the continental United States resulting in numerous human infections and deaths. Several studies suggest that larval mosquito control measures should be taken as early as possible in a season to control the mosquito population size. Also, adult mosquito control measures are necessary to prevent the transmission of WNV from mosquitoes to birds and humans. To better understand the effective strategy for controlling affected larvae mosquito population, we have developed a mathematical model using a system of first order differential equations to investigate the transmission dynamics of WNV in a mosquito-bird-human community. We also incorporated vertical transmission in mosquitoes and passive immunity in birds to more accurately simulate the spread of the disease.   (Received September 13, 2020)

1163-D1-1361   **Meredith L Greer***, 205 Hathorn Hall, Bates College, Lewiston, ME 04240. *Teaching Mathematical Epidemiology in the Time of COVID-19.* Preliminary report.

Models for mathematical epidemiology, such as SIR (Susceptible-Infectious-Removed), have been used as examples in undergraduate mathematics courses such as calculus and differential equations. Some undergraduate biology curricula discuss these models as well. Network models of disease spread appear in math, biology, and computer science curricula. For several years, such models have been examples of the ways math, biology, and computer science could intersect. Though clearly useful, they were not a central focus of daily news.

Now, due to COVID-19, every student and instructor has likely heard extensively about using models and data to track and understand epidemics. Political interpretations abound. Instructors no longer need to explain why these models are studied. Students may have strong opinions about which models to study or how models should be used (or not used) for the development of public policy.

This talk develops strategies for teaching epidemiological models in and after 2020. Topics may include: communication between students and instructors; selecting course topics and activities; managing mathematical or scientific discussions involving people with conflicting opinions; building from classroom topics toward civic engagement; and more.   (Received September 15, 2020)

Combining Technological Tools and Innovative Practices to Improve Student Learning Outcomes

1163-D5-72   **Leslie B. Jones*** (`lbjones@ut.edu`), The University of Tampa, 401 W Kennedy Blvd, Tampa, FL 33606, and **Tim Smith**. *Introductory Computer Programming Courses in Mathematics Curriculum.*

One of the disciplines most commonly integrated into mathematics curriculum is computer science. We present the results of surveys and curricular research on introductory computer programming courses that are required or recommended for mathematics degrees at U.S. college and universities with undergraduate populations between 5,000 and 20,000 students. Our findings shed light on the prevalence of introductory computer programming courses in mathematics curriculum, who teaches these courses, which programming languages are most popular and what drives the programming language decision. The results of our study highlight the importance of the active involvement of mathematics faculty in setting learning outcomes in introductory programming courses, capitalizing on skills and concepts learned in introductory programming courses by incorporating them into

subsequent mathematics courses, and determining programming languages in use. (Received September 04, 2020)

1163-D5-305 **Younhee Lee*** (`leey6@southernct.edu`) and **Fernanda Bonafini** (`fcb5100@psu.edu`). *Pre-service teachers' use of technology: Exploring opportunities to amplify and reorganize students' thinking through the creation of video lessons.*
The demand for teachers to effectively promote remote learning has never been higher due to the outbreak of the COVID-19 pandemic. This report will present findings of a qualitative study investigating prospective teachers' use of mathematical action technology as they created video lessons on iPads. The research design involved implementing an iPad project in a Technology for Secondary Mathematics Education course with two different cohorts of students. Data comprises their project outcomes with a follow-up survey, and it was analyzed using TPACK framework. In this talk, we focus on participants' use of technological knowledge, in particular, through the categorization of cognitive technologies as 'Reorganizer' and 'Amplifier' (Pea, 1985, 1987). According to Pea, using technology as Reorganizer provides learners the opportunity to alter and expand the way of thinking in their approaches to solving the given problem. On the other hand, using technology as Amplifier involves delegating the tool to handle tedious work such as computations and graphing efficiently. The report will discuss how most participants remained at using technology as Amplifier and how to guide prospective teachers to implement technology as Reorganizer based on empirical data. (Received September 04, 2020)

1163-D5-339 **Tanvir Prince*** (`tprince@hostos.cuny.edu`), NY. *Teaching Mathematics in a Community College During COVID-19 - Importance of Text Message Communication.*
It is more important than any other time to be able to communicate with students in real time outside the class time to motivate them and encourage them to do homeworks and submit assessment on time. We need to communicate not by email but something faster and quicker. No doubt, the answer is "text message". But how can a faculty establish text message communication with students without compromising his/her phone number? The answer is on "Remind". I find it very reliable, fast and most importantly safe. Some features include:

*You can control if students can message each other or not *The messages from students go to the app instead of your phone's text message system. This way you separate personal text messages with the student's text messages. *Your own phone number is hidden *Enroll students in the beginning of the semester is very effortless, easy and quick *Individual accounts are completely free and you can sign in with your google account.

I will talk about this in detail and after the presentation the audience will have a clear understanding about various features of this "remind" software and will be able to use it in their classes. (Received September 09, 2020)

1163-D5-523 **Yaomingxin Lu*** (`luyaomingxin@gmail.com`), 1903 W Michigan Ave, Kalamazoo, MI 49008-5248. *Developing Undergraduate Students' Understanding of Variability through the Use of Dynamical Statistical Software.*
A major component of statistical thinking involves understanding and describing variability in data (Moore, 1990). Advances in technology allow for the development of tasks that can engage students more readily in data analysis. In particular, dynamic statistics software has made it easier for the learners to move from learning about descriptive statistics procedures such as calculating the standard deviation, to a more conceptual understanding of variability. This presentation will discuss the teaching innovation of implementing dynamical statistical software, TinkerPlots, in a statistics content course for education majors. Two cases of student pairs working on an exploration task using TinkerPlots will be discussed. In both cases, the use of TinkerPlots had dynamically engaged the students with the data while thinking about spread with the use of different plots and tools. Thus, this tool helped their understanding of different aspects of variability move forward. My findings support the notion that the ways students visualize concepts directly impact the ways in which they understand such concepts. It is essential to continue to investigate what sense students make of these visualizations intended to support students. (Received September 08, 2020)

1163-D5-686 **Peri Shereen*** (`pshereen@csumb.edu`) and **Lipika Deka**. *Tech tools to promote classroom community and improve student learning in upper division courses.*
Reading apprenticeship (RA) is a pedagogical framework which understands that metacognitive conversations occur when the "personal", "social", "knowledge-building", and "cognitive" dimensions to learning are being tapped into at the same time. The authors recently blended in RA to an active learning model in abstract algebra and real analysis courses. These implementations leaned heavily on building the personal and social dimensions of learning through student interaction, presentations, and more. Due to COVID-19 our model was disrupted. Rather than regressing, the authors discovered new technological tools, which could be adapted to a

remote learning environment and that continued to build the social and personal dimensions, while maintaining an active learning model. We will present two tools in particular. The first is Perusall an online platform where students read text, watch videos all while annotating and commenting publicly to their classroom community. In addition, we will discuss how discussion boards can be used as an effective space for classroom presentations. We will present our anecdotal evidence to how adapting these new technological tools built the social and personal dimensions of learning.   (Received September 11, 2020)

1163-D5-837   **Jennifer Clinkenbeard*** (`jclinkenbeard@csumb.edu`), **Alison Lynch** (`alynch@csumb.edu`) and **Sandra Zuniga Ruiz** (`szunigaruiz@csumb.edu`). *Mathematics Faculty Professional Development and Student Experiences in Distance Learning.* Preliminary report.

In the shift to distance learning, mathematics faculty needed to quickly build capacity for innovative practices to continue to provide high-quality mathematics instruction. The ubiquity and variety of technological tools available, coupled with the need to adapt course materials so that students could equitably and meaningfully participate from a wide variety of home situations, locations, and access to technology, meant that many faculty members felt underprepared and overwhelmed. To help address these issues, CSUMB mathematics and statistics faculty adopted a peer-instruction approach, where faculty members designed and facilitated modular professional development (PD) sessions for one another. We discuss the design and implementation of this PD series, with a focus on how faculty members chose which tools to implement into their teaching, and for what purposes. We present results from an informal faculty survey including how faculty adapted their courses, as well as how they envision their courses when they return to face-to-face instruction. Finally, we compare results from a student experiences survey in general education mathematics during distance learning and face-to-face instruction during the previous year.   (Received September 13, 2020)

1163-D5-1052   **Adam F Childers** (`childers@roanoke.edu`), VA , and **David G Taylor*** (`taylor@roanoke.edu`). *Classroom Stats: The Classy, Real, and Simple Solution to Modernize and Increase Engagement in Statistics Classrooms.*

Randomized comparative experiments are a foundational part the statistics education and are one of the most practical and useful ways to find causal relationships. Teaching these methods is done in every introductory statistics course but conducting them in the classroom is challenging. Efficiently randomizing the class, collecting the data, and then analyzing it is not trivial and can be time consuming. In this talk we will show how to use the free data collection and analysis platform Classroom Stats to conduct and analyze randomized comparative experiments, live in class, in a matter of minutes. Combining two technologies – a web application instructors use to design the experiment and analyze the data along with a mobile application students use to enter data – Classroom Stats makes conducing randomized experiments easy. The speed and efficiency of conducting these experiments gives students multiple repetitions with the entire statistical process and uses student data which have both been shown in the statistics education literature to promote skill retention.   (Received September 14, 2020)

1163-D5-1062   **Younggon Bae*** (`younggon.bae@utrgv.edu`), **Zareen G. Rahman** (`rahmanzx@jmu.edu`) and **V. Rani Satyam** (`vrsatyam@vcu.edu`). *Virtual Group Work on a Shared Dynamic Geometry Task in an Online Synchronous Classroom Environment.* Preliminary report.

The goal of this study is to explore affordances and limitations of virtual group work on shared dynamic geometry activities via Zoom in support of preservice secondary mathematics teachers' active engagement with mathematics, technology, and group discourse in online synchronous classroom environments. In this study, three mathematics education professors collectively designed an instructional task to engage students with Euclidean transformations in a Desmos applet, followed by group discussion and presentation. The task design includes a technology tutorial for students and creation of group norms for productive collaboration in a virtual space. Groups in breakout sessions work on the task collaboratively thru screen-share in Zoom, exploring the different transformations and creating a Google presentation to report how they characterize and compare each rigid motion. We collect recordings of group discussion, presentation, and reflection on their virtual group work. In this presentation, we will share with the audience what we learned from this experience about students' challenges with engaging in this virtual collaboration on mathematical tasks and the instructor's role in task design and enactment to support students.   (Received September 14, 2020)

1163-D5-1080   **Cathy S Liebars*** (`liebars@tcnj.edu`). *Math homework videos on Flipgrid.* Preliminary report.

Flipgrid is a simple, free, and accessible video discussion experience for educators and their learners. The presenter will describe the use of Flipgrid for presentation of homework problems in a capstone-like course for preservice teachers called Topics in Secondary Mathematics from an Advanced Viewpoint. Homework is assigned regularly and each student in the class chooses a homework problem that they will present by making a Flipgrid video. They do this a total of five times over the course of the semester. This determines their homework grade. Flipgrid allows for video responses so other students can view the homework videos and make a video comment or ask a question. This experience, including an example and the rubric used for evaluation, will be shared and discussed. (Received September 14, 2020)

1163-D5-1118   **Jessie A Hamm*** (`hammj@winthrop.edu`), Rock Hill, SC 29732. *Teaching an Inquiry Based Proofs Course in a Virtual Setting.*

When we think of inquiry based learning classes a virtual environment doesn't typically come to mind. In fact, it seems like one of the worst settings for such a course structure. This fall I am teaching my first ever fully synchronous virtual courses. One of these is an Introduction to Proofs course in which I am using IBL and mastery based grading (for the first time!). Though I was nervous about the course I can honestly say it has been such a pleasant surprise. I have learned how to effectively use many technological tools in this virtual IBL setting! Some of these include breakout rooms, Jamboards, groupme, and note-taking apps on my ipad. In this talk I will share some of my experiences with these tools, both the successes and the (productive) failures. (Received September 14, 2020)

1163-D5-1137   **Adam Case*** (`adam.case@drake.edu`), Drake University, Collier-Scripps Hall, 3rd Floor, 2702 Forest Avenue, Des Moines, IA 50311. *Using Computer Game Software to Teach Creative Problem Solving in a First-Year Seminar.* Preliminary report.

In this talk, I discuss an experimental first-year seminar that uses computer game software to teach creative problem solving to a class of twenty first-year college students. The software used is a puzzle game (which can be played cooperatively by two students on a single Apple iPad) where the rules to each puzzle are never explicitly stated. Students, therefore, must derive the rules themselves and master them. As students make progress in the game, they build up a significant body of knowledge regarding the rules to the puzzles and various strategies for solving them. I also describe an initial quantitative study in which I explored how students' attitudes toward their own creative problem solving abilities changed by the end of the semester. A reflection on the results of the study will be discussed, along with ideas for improving the course. (Received September 14, 2020)

1163-D5-1138   **Daniel R Jordan*** (`djordan@colum.edu`), Department of Science and Mathematics, Columbia College Chicago, 600 S Michigan Ave, Chicago, IL 60605-1996. *Using Miro in Remote Flipped Classes.*

Miro, as described on its website, "is a collaborative online whiteboard platform designed for remote and distributed teams." While geared primarily for use by industry, Miro's unbounded whiteboard is both easy to use and rich in features, including free-hand drawing, drag-and-drop incorporation of files, and basic team management. In this presentation, I will demonstrate Miro and describe how I used it to teach a flipped Linear Algebra course. (Received September 14, 2020)

1163-D5-1179   **Hema Gopalakrishnan*** (`gopalakrishnanh@sacredheart.edu`), 5151 Park Avenue, Fairfield, CT 06825. *Active Learning during Online Instruction.*

For the fall semester of 2020, the math department at Sacred Heart University adopted a hybrid model for its lower division courses. Students attended classes synchronously online and in-person. In this talk I will share my experience of using digital applications such as the Google Jamboard in teaching second semester calculus, and how these applications helped to create active learning opportunities, and encouraged student interaction and engagement in the classroom. (Received September 15, 2020)

1163-D5-1257   **Darryl Jim Chamberlain Jr.*** (`dchamberlain31@ufl.edu`) and **Russell Jeter**. *Automated AF: Leveraging augmented intelligence to provide automated, actionable feedback.* Preliminary report.

Feedback students can use to advance their learning, known as actionable feedback, is critical for student development but difficult to provide. We propose a method to utilize augmented intelligence to provide automated, actionable feedback. This method focuses on exatithow a student answers a question by providing a connection between exatitwhat a student answers and the mathematical conception(s) that would lead to such an answer. Results from implementing the method in a 300 student College Algebra course (with a coordinator and 1

teaching assistant) will be presented to showcase the benefits and pitfalls of automated, actionable feedback. (Received September 15, 2020)

1163-D5-1310 **Monica M VanDieren*** (`vandieren@rmu.edu`). *Integrating WeBWorK, CalcPLot3D, and Flipgrid in a mostly asynchronous online multivariable calculus class.* Preliminary report. When face-2-face classes were canceled and my multivariable calculus abruptly moved online, I turned to WeB-WorK. The platform, along with its extensive Open Problems Library (OPL), provided enough utility to not only serve as a summative assessment tool, but also as a means to deliver asynchronous content and formative assessments to my students. By integrating CalcPlot3D and Flipgrid (a video-based discussion board) into WeBWorK problems, I was able to deliver an asynchronous course completely within the WeBWorK platform. In this talk I will discuss how I leveraged the OPL and small changes to the essay prompt, solution subroutine, and iframe embeddings to create lectures with conceptual understanding exit tickets, often involving interactive CalcPlot3D graphics. I'll discuss how I used the essay prompt in WeBWorK to integrate Flipgrid as not only a summative assessment tool but also as a way to build community among the students with virtual study groups. Through weekly check-ins with students in WeBWorK, I also monitored student progress on meta-cognitive learning outcomes, which were of increasing importance as the students adapted to a new learning environment. (Received September 15, 2020)

1163-D5-1350 **Shelby Stanhope*** (`shelby.stanhope@afacademy.af.edu`). *Supporting Spatial Conceptualization in Multivariable Calculus Using 3D Printed Surfaces, the Visualization Applet CalcPlot3D, and Experiential Learning Activities.*
A unique transition occurs as students enter multivariable calculus. Up to this point, students have spent their mathematical careers becoming experts in the two-dimensional xy-plane. Adding another dimensions allows us to explore this 3D world we live in, but the transition to three dimensional mathematical thinking does not come easily to many students. To best support students' spatial understanding of concepts in the course, we should provide interactive computer visualizations, tactile manipulatives, and experiential learning opportunities. In this presentation, I will discuss the use 3D printed surfaces in the classroom. Students can touch, write on, and rotate the surfaces as they work with a classmate to build their understanding of new concepts. Additionally, the free applet CalcPlot3D can be used to provide interactive computer visualizations. The program requires no coding and is extremely accessible to students. I will present demonstrations that instructors can use to illuminate concepts and visualizations that students can easily create themselves. Finally, I will discuss two experiential learning activities, which provide the opportunity for students to see math in action, bringing together concepts they have seen, but can now feel and experience themselves. (Received September 15, 2020)

1163-D5-1366 **Molly M Sutter*** (`molly.sutter@wsu.edu`) and **William L Hall**. *Math in the Time of COVID-19: Survey Results from Mathematics Instructors on Their Experiences in Spring 2020.*
COVID-19 has brought with it new challenges and pedagogical strategies in teaching. In spring 2020, our university moved instruction online. A survey was distributed to all instructors teaching in spring 2020. We share results of responses from the subset of survey respondents from mathematics and statistics instructors about this sudden shift and their experiences during this time. Mathematics instructors reported developing new modeling activities involving COVID-19 and created opportunities for mathematical discussion using various techniques and tools. Still instructors faced challenges with lack of technology available and slow or no internet access for students and themselves. New commitments and responsibilities arose, creating responsibilities beyond academics for students and teachers. One theme we identified in how instructors addressed these challenges was the practice of humanizing education, including connecting with students on a personal level, adjusting course requirements, placing value on students' experiences, and promoting good mental health all around. This was particularly important as isolation created a new hurdle for everyone. In doing so, instructors bridged the gap between teacher and student, placing value on the experiences we all are going through. (Received September 15, 2020)

1163-D5-1386 **Matthew C Mauntel*** (`mmauntel@fsu.edu`), 209J Stone Building, 1114 W Call St, Stone Building, Tallahassee, FL 32306. *Chase that Rabbit! Student Structuring of Space in Linear Algebra through Digital Game-based Learning.* Preliminary report.
There is an increased need for distance learning in our society due to the recent pandemic. One possible avenue is to pursue digital game-based learning. The game Vector Unknown was designed using principles from the Realistic Mathematics Education (RME) to connect student's algebraic and geometric understandings of vector equations in linear algebra. The goal of the game consists of guiding an avatar to a goal location by selecting

appropriate vectors and scaling them. This research utilizes the game as realistic basis for student understanding of 2D space structured by linear combinations, which may provide crucial intuitions between concepts such as span, linear independence, and basis. Participants were chosen from a large Southern university with a variety of gaming and linear algebra experience. Participants played and analyzed Vector Unknown to understand its mathematical design. The participants were then asked to design their own version of 3D version of the game to see how their reasoning about linear combinations in 2D space extends to higher dimensions.   (Received September 15, 2020)

1163-D5-1392   **Matthew Prudente***, 400 Saint Bernardine St., Reading, PA 19607. *We're Talking About Practice: A Mastery-Based Grading Approach to Precalculus.* Preliminary report.

Mastery-Based Grading (MBG) is a learning modality that structures course material to emphasize time, flexibility, and student responsibility. It is a way for students to take ownership of their learning. Traditional number or letter grades are often replaced with "badge" grades representing mastery of a topic, progression towards mastery, or improvement needed. Final grades are calculated from a cumulative total of these badge grades. For a Precalculus class, certain adaptations were made to motivate students to be responsible for their learning and demonstrate preparedness before entering Calculus I.   (Received September 15, 2020)

1163-D5-1489   **Suzanne Dorée*** (`doree@augsburg.edu`), **Tracy Bibelnieks** and **Jody Sorensen**. *Learning New Tricks: Leveraging Google Apps to Teach Inquiry-Based Discrete Mathematics.*

Like many of our colleagues nationally, in Fall 2020 we converted our Discrete Mathematics courses to online delivery. To maintain a similar look-and-feel with our in-person classes, we opted to teach synchronously over Zoom. We used a combination of Google Apps, especially Jamboards and Slides, integrated with our LMS (Moodle) to provide environments for students to actively collaborate in Zoom Breakout Rooms during class. We used Google Drive to share course materials across three similar courses at two different institutions – Discrete Math Structures (intro to proofs level for math majors), Discrete Math for Computing (lower level, for MIS and CS majors), and a larger enrollment Discrete Math (for math and CS). We used Google appointments to schedule office hours and study groups used Google Hangouts to meet. One advantage of these products was that they were immediately available free of charge to our students and compatible across platforms, and with Chromebooks. In this presentation we share some lessons we learned that saved us time, some challenges that remain, and feedback from students on their experiences.   (Received September 15, 2020)

1163-D5-1555   **Carly Briggs*** (`carlybriggs@bennington.edu`). *Using Technology to Adapt Student Presentations for Remote Learning.*

It can be challenging and time consuming to provide students with opportunities to communicate their reasoning about mathematical concepts both verbally and in writing. One efficient way is through student presentations. In my courses, students are responsible for presenting a proof or solution and for giving feedback to each presenter. Using best practices from other disciplines, I formalized this presentation and feedback process for in-person, synchronous courses. I recently adapted this process as an asynchronous component for a remote course using video recordings and the messaging platform Slack. In this talk, I will share my methods, compare the benefits and pitfalls of both the in-person and remote processes, and provide samples from a recent Linear Algebra course. (Received September 15, 2020)

1163-D5-1561   **Katherine Walsh Hall***. *Creating and Using GeoGebra 3D Models in Multivariable Calculus.* Preliminary report.

In multivariable calculus, many students struggle to visualize many of the concepts because they have a hard time picturing surfaces and solids in $\mathbb{R}^3$. In past semesters, we have given students a chance to see 3D printed models and use Play-Doh to help visualize such concepts. With the move to online courses, we have been relying more heavily on virtual 3D graph models, many of which are developed using GeoGebra.

    There are several ways students might see these - watching them be used during lecture, interacting with pre-built demos on their own or building their own demos. In this talk, we see several examples of such models and discuss how we can analyze student learning outcomes in these three different methods and what we found through a preliminary study during Fall 2020. This work will help determine the best use cases for these models in future semesters. We will also talk about what we learned as best practices for sharing such models and having students gain comfort with the application so they can create their own models.   (Received September 15, 2020)

The EDGE (Enhancing Diversity in Graduate Education) program: Pure and Applied talks by Women Math Warriors

1163-F1-679   **Samantha K Fairchild***, skayf@uw.edu. *Counting social interactions for discrete subsets of the plane.*

Given a discrete subset V in the plane, how many points would you expect there to be in a ball of radius 100? What if the radius is 10,000? When V arises as orbits of non-uniform lattice subgroups of SL(2,R), we can understand asymptotic growth rate with error terms of the number of points in V for a broad family of sets. A crucial aspect of these arguments and similar arguments is understanding how to count pairs of saddle connections with certain properties determining the interactions between them, like having a fixed determinant or having another point in V nearby. We will focus on a concrete case used to state the theorem and highlight the proof strategy.   (Received September 11, 2020)

1163-F1-926   **Raegan Higgins*** (`raegan.higgins@ttu.edu`), Texas Tech University, Department of Mathematics & Statistics, Lubbock, TX 79402-1042, **Casey J Mills** (`casey.j.mills@ttu.edu`), Texas Tech University, Department of Mathematics & Statistics, Box 41042, Lubbock, TX 79409-1042, and **Angela Peace** (`a.peace@ttu.edu`), Texas Tech University, Department of Mathematics & Statistics, Box 41042, Lubbock, TX 79409. *Modeling Intermittent Hormone Therapy for Prostate Cancer using Dynamic Equations on Time Scales.*

Prostate cancer is often treated by intermittent androgen deprivation therapy which requires patients to shift between periods of androgen suppression treatment and no treatment. Prostate-specific antigen levels are used to track changes in tumor volume of prostate cancer patients undergoing this therapy. Traditionally, continuous ordinary differential equations are used to estimate prostate-specific antigen levels. In this presentation, we will use dynamic equations to estimate these levels and construct a novel time scale model to account for both continuous and discrete time simultaneously. This accounts for pauses between treatment cycles during intermittent androgen deprivation therapy. We compare our model to data sets of prostate-specific antigen levels to determine any similarity between on treatment intervals and those in our time scale.   (Received September 14, 2020)

1163-F1-1007   **Michelle Craddock Guinn*** (`michelle.guinn@belmont.edu`), 1900 Belmont Blvd, Janet Ayers Academic Center 4046, Nashville, TN 37212, and **Brad Schleben**. *Experiencing Mathematics Abroad.*

Study abroad experiences have been recognized as providing students with valuable opportunities to work with individuals and groups different from themselves, to incorporate diverse viewpoints into their work and to engage in meaningful experiences outside their culture. This article focuses on one of the mathematics courses in Belmont University's study abroad program that was designed to synthesize course content and authentic learning experiences in order to address the diverse set of student learning outcomes. While improved student engagement in cultural understanding and the promotion of intellectual diversity took on a central role in the course design and assessment, a secondary goal was an improved student perception of mathematics and its application. We examine the course in action by looking at three example assignments, followed by their connections to program experiences, and how these things coordinate to meet the student learning outcomes. (Received September 14, 2020)

1163-F1-1117   **Laurel A Ohm*** (`laurel.ohm@nyu.edu`). *Accuracy of slender body theory in approximating force exerted by thin fiber on viscous fluid.*

We consider the accuracy of slender body theory in approximating the force exerted by a thin fiber on the surrounding viscous fluid when the fiber velocity is prescribed. We term this the slender body inverse problem, as it is known that slender body theory converges to a well-posed PDE solution when the force is prescribed and the fiber velocity is unknown. From a PDE perspective, the slender body inverse problem is simply the Dirichlet problem for the Stokes equations, but from an approximation perspective, nonlocal slender body theory exhibits high wavenumber instabilities which complicate analysis. Here we consider two methods for regularizing the slender body approximation: spectral truncation and the δ-regularization of Tornberg and Shelley (2004). For a straight, periodic fiber with constant radius $\epsilon > 0$, we explicitly calculate the spectrum of the operator mapping fiber velocity to force for both the PDE and the approximations. For both the truncated and δ-regularized approximations, we obtain convergence results to the PDE solution as $\epsilon \to 0$. Moreover, we determine the

dependence of the δ-regularized error estimate on the regularization parameter δ. (Received September 14, 2020)

1163-F1-1142   **Nancy Scherich** and **Sherilyn Tamagawa*** (`shtamagawa@davidson.edu`). *Niebrzydowski Algebras and Trivalent Spatial Graphs.* Preliminary report.

In this talk, we will introduce virtual Niebrzydowski algebras, algebraic structures with two ternary operations and a partially defined multiplication, whose axioms are motivated by the Reidemeister moves for Y-oriented virtual trivalent spatial graphs and handlebody-links. We will also show examples of the counting invariant on some trivalent spatial graphs and classify the small virtual Niebrzydowski algebras. (Received September 14, 2020)

1163-F1-1154   **Tai-Danae Bradley*** (`tai.danae@math3ma.com`). *Exploring Language with Linear Algebra.*

The linear algebra of quantum mechanics provides a good starting point for investigating mathematical structure found in natural language. In this talk, I'll give a brief tour of these ideas, showing how a probability distribution on a finite set of sequences can be modeled by a rank 1 density operator, how to assign reduced density operators to expressions in language, and how this assignment fits nicely within the framework of category theory. (Received September 14, 2020)

1163-F1-1288   **Carolyn Otto*** (`ottoa@uwec.edu`) and **Sydney Dame** (`damesg7254@uwec.edu`).
    Topological Theory of Virtual Knotting in Protein Folding. Preliminary report.

Proteins are known to fold and form knots that can be studied using knot theory. We aim to devise an updated method of protein folding by reviewing the model proposed in Flapan et. al (2018) by incorporating virtual knotting. We will be looking at identifying nontrivial virtual knots that can be found using the modified Flapan theory. (Received September 15, 2020)

1163-F1-1422   **Kelly Ruth Buch*** (`kbuch@vols.utk.edu`), 227 Ayres Hall, 1403 Circle Drive, Knoxville,
    TN 37996-1320. *Basal Sprout Production in a Fatal Vector-Borne Tree Disease: Do Sprouts Help or Hurt?*

Many tree species respond to disturbances by producing shoots, called basal sprouts, from the base and root system of a dying tree, and these sprouts can help maintain the population after the disturbance. In the case of many fatal tree diseases, the production of basal sprouts can be a key contributor to population resurgence post-epidemic. However in the case of fatal vector-borne tree diseases, the production of basal sprouts by infected host trees may worsen the epidemic and lead to local host extinction rather than maintenance of the population. We consider as a case study Laurel Wilt, which is a fatal fungal tree disease vectored by an invasive beetle. After a susceptible tree becomes infected and dies from Laurel Wilt, the dead tree provides suitable host material for vector reproduction. Using a stage structured SI ODE model, we explore the effect of basal sprout production on the populations of both host and vector. We interpret our results to provide insight on the circumstances under which the production of basal sprouts within a fatal vector-borne tree disease system leads to an endemic disease state, a disease-free host population, or the local extinction of both host and vector. (Received September 15, 2020)

1163-F1-1423   **Betsy Camano***, bcamano.student@manhattan.edu. *The Art of Precision and Clarity in Mathematical Thinking.* Preliminary report.

I would like to converse about the positive rippling effect of mathematical thinking in any field and in life. (Received September 15, 2020)

1163-F1-1470   **Casey L. Johnson*** (`casey.johnson@cgu.edu`), 504 1/2 Begonia Ave, Corona Del Mar,
    CA 92625. *Can you hear the shape of a Stieltjes star graph?*

How much information do the frequencies of small vibrations for star graph configurations of connected Stieltjes strings give us? The spectrum of a single Stieltjes string, a thread bearing a finite number of point masses, is uniquely determined by the number and size of the masses. In 2002, F.R. Gantmakher and M.G. Krein solved the inverse problem which identified the location and mass of each bead given just the spectrum corresponding to Dirichlet boundary conditions and the spectrum corresponding to Neumann boundary conditions. Joining multiple Stieltjes strings of various lengths together to form a star graph shape has fascinating implications on the spectrum of the graph. For these new star graphs, is the spectrum still uniquely determined? Can we determine the configuration of the star graph from the spectrum of the graph? (Received September 15, 2020)

1163-F1-1477   **Genesis J. Islas*** (`gjislas@asu.edu`). *Node generation for radial basis function partition of unity method for computations on surfaces.* Preliminary report.

Radial Basis Functions (RBFs) are a popular meshfree method for solving partial differential equations. RBF approximations are highly accurate and can exhibit exponential convergence. A partition of unity method can be used to improve computational cost and to prevent excessively large condition numbers. The domain decomposition also makes the problem highly parallelizable. In this talk, we present a node generating algorithm for the radial basis function partition of unity method on surfaces. The stability and convergence analysis of the associated PDE solver will be presented. (Received September 15, 2020)

1163-F1-1531   **Orsola Capovilla-Searle*** (`ocapovilla@gmail.com`), Duamrh, NC 27701. *Obstructions of exact Lagrangian cobordisms of Legendrian links.* Preliminary report.

Topological links are smooth embeddings of disjoint copies of circle into the 3-sphere, are considered equivalent up to smooth isotopy. Legendrian links Λ in $(\mathbf{R}^3, ker(dz - ydx))$ are topological links that satisfy the additional geometric condition that their tangent vector lies on the contact structure, that is $T_p\Lambda \subset ker(dz - ydx)$ for any $p \in \Lambda$. In the category of Legendrian links exact Lagrangian cobordisms correspond to morphisms, and many Legendrian invariants are functorial in this category. We have new obstructions to the existence of embedded and immersed exact Lagrangian cobordisms. This is joint work with Legout, Limouzineau, Murphy, Pan, and Traynor. (Received September 15, 2020)

1163-F1-1612   **Dorian Smith*** (`smi01055@umn.edu`), 2089 Carter Ave, Saint Paul, MN 55108. *The Radio Number of Banana Tree Graphs.*

Let G be a connected graph. For any two vertices u and v, let $d(u,v)$ denote the distance between u and v in G. The diameter of G is the maximum distance between any pair u, v and is denoted by $\operatorname{diam}(G)$. The radio k-labeling for G is a function $f : V(G) \to \{0, 1, 2, 3, \ldots, k\}$ such that $\|f(u) - f(v)\| \geq \operatorname{diam}(G) - d(u,v) + 1$. The radio number of a graph G, denoted by $rn(G)$, of a graph G is the smallest such k such that G has a radio k-labeling. The radio number of various trees are known. Our tree of interest is the banana tree. The following definitions are needed to define a banana tree. A leaf is a vertex with only one neighbor. A m-star is defined as a tree with m leaves and one apex. We define the (n, m)-banana tree denoted, $B(n, m)$, to be the tree obtained by joining one leaf of each of n copies of a $(m - 1)$- star to a single root. The radio number of a banana tree is known. We study the properties of the radio k-labeling of new trees formed by adjoining vertices and edges to a banana tree. (Received September 15, 2020)

1163-F1-1631   **McCleary Philbin*** (`philb036@umn.edu`). *A spectral sequence to compute algebraic de Rham cohomology.*

Algebraic de Rham cohomology is computed from the hypercohomology of a complex, so we may also compute it using a spectral sequence. In this talk, I will go through preliminaries needed in order to define algebraic de Rham cohomology, then I will discuss some things we know about the associated Hodge de Rham spectral sequence and pose some questions we'd like to answer. (Received September 15, 2020)

Mathematics and Sports

1163-F5-294   **Filippo Posta*** (`filippo.posta@estrellamountain.edu`), Phoenix, AZ. *Football match forecasting project as mean to foster research opportunities during first two years of college.*

A common issue with STEM students is that they are capable of repeating what they already know, but they have a hard time extrapolating their knowledge to practical problems. Sports analytics provides a good umbrella of possibilities to apply statistical and mathematical skills to problems that are of interest to the students. This talk summarizes the results and experiences of creating mathematical models that are solely based on the expected goal statistics obtained from a publicly available database. We created three probabilistic models based on expected goals statistic and compared them with two well-established probabilistic models using binomial deviance, squared error, and profitability in the betting market as evaluation metrics. Our best model adjusted the expected goal statistics for home-field advantage and outperformed the two probabilistic models used as gold standard. Two of our models were profitable under certain betting conditions. Our approach provided a framework amenable to students with basic statistical knowledge without sacrificing the potential impact of the findings. (Received September 01, 2020)

1163-F5-376 **Diana S Cheng*** (`dcheng@towson.edu`) and **John B Gonzalez**. *Figure skating scores reveal a cold hard truth: Relationships between artistic and technical scores in the 2018-19 season.*

After scoring transparency controversies were embarrassingly apparent at the 2002 Winter Olympic Games, the International Skating Union developed a numeric scoring system called the International Judging System to rate figure skating performances. Internationally competitive skaters and their coaches now formulate practices and performances with the intention of maximizing their earned scores. We examined scoring data from over 18,000 performances in 120 competitions during the last full season of figure skating, which was 2018-19. Our interpretations of this data show that imbalances within skating performances now exist due to skaters' maximizing their scores based on the judging system; and that the artistic (Program Component) and technical (Total Element) scores are highly correlated and directly related, even though this may not have been the intention when the scoring system was initially designed. (Received September 15, 2020)

1163-F5-839 **Tim Zeitvogel*** (`tim.zeitvogel@pepperdine.edu`) and **Timothy A Lucas** (`timothy.lucas@pepperdine.edu`). *Modeling the Influence of In-Match Dynamics on Tennis Outcomes.* Preliminary report.

Tennis is different from many other sports because the goal is to achieve enough points to win the match rather than to have more points than your opponent when the clock runs out. Our research attempts to understand the impact of the unique scoring system of tennis by analyzing the influence of potential in-match factors such as momentum and the significance of points. We created an in-match stochastic model for the overall win probability based on the service and return ability of individual players. We then defined a regression model for the service point win probability based on average service win probability, pre-match player estimations, the outcome of the previous point, and the relative significance of the point. This allowed us to compare the quality of commonly used metrics for player estimation and ranking. Our analysis of the regression model includes best-of-three and best-of-five tournaments on three surfaces. In order to measure the mental toughness of players, we defined a pressure index that compares the expected service and return game win probabilities with the actual outcomes. Based on our results, we created a dynamic in-match forecast model for match win probability, which incorporates pre-match estimations, the pressure index, and in-match trends. (Received September 13, 2020)

1163-F5-1244 **Amanda Harsy*** (`harsyram@lewisu.edu`), IL, and **Megan Vesta**, **Sheila Lesiak** and **Maria Rodriguez Del Corral**. *A Markov Chain Model For Predicting College Baseball.*

Ranking sports teams can be a challenging task, and using straight win percentage can be misleading at times. Among the many mathematically inspired sports ranking systems, linear algebra methods are among the most elegant and simple. In this research, we focus on applying a Markov chain method to predict the future results of NCAA Division 1 College Baseball. In particular, we investigate whether win streaks can help predict the final standings for college baseball. (Received September 15, 2020)

1163-F5-1566 **Cameron C Cook*** (`ccook54@vols.utk.edu`), Knoxville, TN 37919. *Fuel Allocation and Nutrition Management for a Runner Competing in a Race.* Preliminary report.

Nutrition is an integral part of successfully running long distance races such as a marathon and needs to be included in models of running strategies. We formulate a system of ordinary differential equations to represent the velocity, energy, and nutrition for a runner competing in a long-distance race. The energy compartment represents the energy available in the runner's muscles. The food consumed during the race is a source term for the nutrition differential equation. With our model, we are investigating strategies to manage the nutrition and force (source in velocity differential equation), to minimize the running time in a fixed distance race. (Received September 15, 2020)

1163-F5-1602 **John Edward Vesterman*** (`vestermanje21@mail.vmi.edu`), 450 Burma Rd., VMI Box 81756, Lexington, VA 24450, and **John David**. *Model Analysis Techniques Applied to Shots in Roanoke College Basketball Games.*

In NCAA basketball, it is not always obvious which factors make a shot good: court position, what type of shot, if the shooter was closely guarded, etc. In this talk, we will be using data of every shot taken in a Roanoke College Men's and Women's Basketball home game from 2017-2019 collected by the Stat Crew. By applying multiple mathematical techniques such as Decision Forests, Partial Dependency Plots and Shapley Values, I will evaluate the importance of each aspect of a shot and game performance by comparing points scored vs. expected points scored as given by our model. This approach will allow teams to decide which shots to take and not to take, by understanding the points gained or lost on specific shots and which factors of the shot contributed to this. (Received September 15, 2020)

1163-F5-1607   **Jordan E Coomes*** (`jcoomes3@radford.edu`), Radford, VA 24142, and **Caleb L Adams** (`cadams5@radford.edu`), Radford, VA 24142. *Effects on the Outcome of Points Following a Timeout in Volleyball - a Statistical Sport Analysis.* Preliminary report.

Applying statistical analysis, the aim of this study is to determine the effectiveness of timeouts within the game of volleyball. Athletic competitions are full of variables – for instance, coaching strategies or the effect of momentum on a player's mentality can affect how an athlete performs in competition. The analysis conducted is an examination of whether timeouts create a true positive impact on subsequent points following a timeout taken in a Division I volleyball match. Results presented have the potential to influence timeout strategy in the future of collegiate volleyball. Additionally presented is an analysis of when timeouts are called and generally how long it takes the timeout to become effective.   (Received September 15, 2020)

1163-F5-1650   **Mark Kozek*** (`mkozek@whittier.edu`), Department of Mathematics & Computer Science, Whittier College, Whittier, CA 90608-0634. *Soccermetrics: An Interdisciplinary Course on Soccer Analytics.* Preliminary report.

We reflect on our experience creating and teaching *Soccermetrics*, an interdisciplinary course (aimed at the general undergraduate population) that focuses on quantitative techniques from mathematics or finance that are used in soccer analytics.

We begin the course with an overview of elementary statistics, spreadsheet computing, and matrix arithmetic. Depending on student preparation/interest, we may include concepts from linear algebra, game theory, or computer programming. We evaluate data from soccer matches to develop metrics, create team rankings, predict match outcomes, analyze penalty kick shoot-outs, valuate players, discover fallacies in common soccer "facts", etc. We also survey the economics of operating a soccer team, of building a soccer stadium, and of hosting a World Cup.   (Received September 16, 2020)

Mathematical Experiences and Projects in Business, Industry, and Government (BIG)

1163-I1-437   **Mihhail Berezovski*** (`mihhail.berezovski@gmail.com`). *Bringing business, industry, and government into undergraduate research.*

In this talk, we will discuss the challenges of bringing collaboration with real business, industry, or government (BIG) into the undergraduate research. We highlight differences and challenges compare to traditional undergraduate research. We will discuss the possible ways to response to COVID19 situation and share ideas for successfully designing and mentoring such projects. The steps to build successful collaboration with BIG will also be presented.   (Received September 06, 2020)

1163-I1-1147   **Scott Mansfield Sanders*** (`sanderssm@etsu.edu`), Johnson City, TN, and **Abderrahim Elallam** and **Benjamin Williams**. *Mathematical Assessment of Ballad Health's Diabetes Management Program.* Preliminary report.

Ballad Health, the primary healthcare provider for Northeast TN, has been developing a comprehensive Diabetes Management Program since 2018. Participants receive individualized coaching from Pharmacists, Nutritionists, Wellness Coaches, and other experts with the goal of reducing healthcare spending and improving overall patient outcomes. Healthcare costs are covered by Ballad, and participants provide a range of data to Ballad on an annual basis including various biometrics, lab data, and health risk assessments. Data from more than 600 participants will be used to assess the effectiveness of Ballad's program in helping patients get their Diabetes under control. We determine whether the program has resulted in significant improvements in biometrics, lab readings, or overall healthcare spending. Furthermore, we seek patterns within age groups and urban versus rural settings. We develop a ratings system to evaluate the progress of participants and provide insights into how other factors like emotional health, chronic conditions, and modifiable behaviors affect outcomes. We accomplish these goals using statistical analysis, math modeling, and basic machine learning techniques and inform Ballad as to how their coaching program has been most and least effective.   (Received September 14, 2020)

1163-I1-1169   **Debbie Yuster*** (`dyuster@ramapo.edu`), Ramapo College of New Jersey, 505 Ramapo Valley Rd., Mahwah, NJ 07430. *Stopping terrorists, saving the environment, and keeping network traffic flowing: Undergraduate research using industry-supplied problems.*

With funding and training from the MAA's PIC Math program, I taught an upper-division Math elective in Spring 2017. Students were divided into three groups, each of which researched a problem of interest to a partner company, using math and coding. I will give an overview of forming the corporate partnerships and running

the course, and will discuss the three research partners and projects: 1) A financial company that must check account holder names against a terrorist watch list, who asked students to evaluate the performance of their name-matching algorithm, 2) A company whose ships lay undersea communications cables, who asked students to create a model that estimates their fleet's toxic emissions, and 3) A digital forensics company, who asked students to analyze a control method used to avoid computer network congestion. (Received September 14, 2020)

1163-I1-1198   **Hanif Heidari*** (`hanif.heidari@gmail.com`), Department of Applied Mathematics, Damghan University, Damghan, Semnan, Iran. *Predicting the time series with big or small data.*

Prediction the time series is a hot topic in recent years since it have many applications in science and engineering. Some statistical and artificial intelligence methods are improved to predict the time series. But, there does not exist any methods that works on every time series. Researchers and analyst have to test some methods to find the best way for the prediction. In this presentation, the time series are classified to big and small data. The methods are classified to statistical and artificial intelligence methods. Some suitable methods for each time series class will be introduced. The methods are applied on some business and industrial problems. The results show that the proposed classification between the types of the problems and methods leads to more accurate results. At the end we will give some prospective points for the future work of this research area. (Received September 15, 2020)

1163-I1-1399   **Aaron B Luttman*** (`aaron.luttman@pnnl.gov`). *Nuclear Fusion – Government and Industry Research Opportunities for Mathematicians.*

Despite the joke that nuclear fusion "will always be 10 years away," devices capable of initiating nuclear fusion in a controlled environment have actually been around since at least the 1960's. These devices take in more energy than they put out – hence not solving the world's energy problems – but they have important uses, especially as a mechanism for generating a large number of neutrons in a very short time duration. In this work, we will present some recent research on fusion devices that is specifically focused on controlling the neutron production, along with some scientific applications in industry and government. Controlling production requires that one understand the phenomena leading to the fusion reaction, which is partially done using computational models and partially determined experimentally, with new mathematical models and computational algorithms required to extract the information of interest from measurements. In addition to results from a US Department of Energy (DOE) pulsed neutron source, we will discuss additional projects and lines of inquiry that are ripe for mathematicians interested in joining the DOE research enterprise. (Received September 15, 2020)

1163-I1-1544   **Debra Gladden*** (`dgladden@leeuniversity.edu`), 1643 North Ocoee St, Cleveland, TN 37311. *Modeling the Degradation of Lithium Ion Batteries.* Preliminary report.

Lithium Ion batteries continue to be the industry standard in rechargeable batteries due to their high energy transfer and low cost. However, occasionally, these batteries fail to function well and even explode. In order to understand and predict this volatile behavior, we model a healthy battery. We analyze data from a physical battery model at Oak Ridge National Labs under ideal circumstances and build in noise to the ideal model. (Received September 15, 2020)

The Philosophy of Mathematics, in Memory of Reuben Hersh

1163-I5-1156   **Rachel Rupnow*** (`rrupnow@niu.edu`) and **Eric Johnson** (`ejohnson13@niu.edu`). *Algebraists' Metaphors for Sameness: Philosophies, Variety, and Commonality.*

In order to understand how mathematicians relate mathematical concepts to sameness, surveys were sent to algebraists throughout the United States. The relevant portion of the data set for this talk focuses on open-ended responses to three questions on the meaning of sameness in math, the meaning of sameness in abstract algebra, and similarities or differences between sameness in algebra and other branches of math. Responses from 197 participants were analyzed using conceptual metaphors (Lakoff, G., & Nunez, R. (1997). The metaphorical structure of mathematics: Sketching out cognitive foundations for a mind-based mathematics. In Lyn D. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 21-89). Mahwah, NJ: Erlbaum.). Conceptual metaphors connect a target domain, in this case sameness, to a source domain that provides another way of reasoning (e.g., Sameness is a concept shown by structure-preservation.). Algebraists' metaphors grouped

into clusters including philosophical stances toward sameness, discipline-based instantiations of sameness, and informal language for sameness. Implications for discussing sameness-based conceptual connections across courses are discussed in light of the connections made by some of the mathematicians.   (Received September 14, 2020)

1163-I5-1276   **Thomas Drucker*** (`druckert@uww.edu`). *Why Is There a Question About Why There Is Philosophy of Mathematics At All?* Preliminary report.

Ian Hacking has observed that philosophizing about mathematics is haunted by Plato's ghost. Reuben Hersh was part of a movement to lay Plato's ghost in understanding how mathematics works. His approach was based on observing mathematical practice and eliminating some of the traditional problems raised in a philosophical setting. This talk suggests that, while Hersh's approach has some distinctive features, it's not as though it suffices to lay Platonic issues entirely at rest.   (Received September 15, 2020)

The Teaching and Learning of Undergraduate Ordinary Differential Equations, in memory of William E. Boyce

1163-J5-469   **Krista L Lucas** and **Timothy A Lucas*** (`timothy.lucas@pepperdine.edu`). *Using Mobile Apps to Enhance Learning in Differential Equations.*

It is increasingly important for mathematics students to engage in active learning along with discussion of material with their peers. A key for these students to understand mathematical models that incorporate differential equations is visualizing slopefields, phase planes and solutions. *Slopes* is a mobile application with an intuitive interface, designed to visualize solutions to differential equations and support active learning in the classroom. *Slopes* is currently available for iPad, iPhone and Android phones, which are highly portable and feature larger touch screens that allow students to view and manipulate content easily. To study the possible benefits of the app, we implemented group activities using *Slopes* into an ordinary differential equations class, conducted observations and focus groups, and examined final poster projects on modeling topics. We found that students used *Slopes* to visualize solutions, aid in discussion and cooperation, and demonstrate understanding of differential equations concepts.   (Received September 07, 2020)

1163-J5-484   **Viktoria Savatorova*** (`viktoria.savatorova@ccsu.edu`), 1615 Stanley street, New Britain, CT 06053. *Tools of the trade or how to encourage student's engagement in remote ODE class.*

The main feature of this course is that every chapter starts with a "story". The purpose of the "story" is to set the stage: we make assumptions, derive equations to solve, and formulate questions to answer. Based on equations we select a method to learn. We assemble our toolbox starting with linear equations and analytical methods of their solutions. We master the method and test solutions asking and answering "what if" questions. Then we start relaxing our assumptions. Equations become more complicated, and new tools are needed. We learn how software can help to visualize solutions and how to interpret vector fields and phase portraits. We move from linear to nonlinear problems and introduce numerical methods as our new tools. We master numerical methods and test them using original exact analytical solution as a limiting case. As we move forward, we assemble the toolbox of analytical and numerical methods we can use to solve differential equations and systems of differential equations. The class work is split into several projects. Students work in groups submitting and presenting their results on the different stages of each project. They are encouraged to communicate with their peers and instructor providing a feedback on work of others and supplementing it.   (Received September 08, 2020)

1163-J5-860   **Deborah Hughes Hallett***, dhh@math.arizona.edu, and **Dan Flath**, dflath@gmail.com. *Unlocking the Differential Equations in Covid-19 data.* Preliminary report.

The first step in modeling is often the hardest: Identifying the functional form of the model. Covid-19 infection data provide an important setting in which to do this, allowing for estimates of parameters of public importance. Using Covid-19 data from around the world, this talk will give examples of the identification of exponential, logistic, and Gompertz growth. As can be done in Calculus II or Introductory Differential Equations, we estimate the relative growth rate from the data and study its behavior to identify possible models.   (Received September 13, 2020)

1163-J5-864   **Ayse Sahin***, ayse.sahin@wright.edu, **Deborah Hughes Hallett**, dhh@math.arizona.edu, and **Rob Indik**, iindik@math.arizona.edu. *Systems of Differential Equations with Covid-19 Data: From Classroom to REU.* Preliminary report.

The accessibility of up to date data for the spread of Covid-19 allows undergraduates at all levels to engage with systems of differential equations. Students with little technical background can successfully build and interpret an SIR model for the spread of Covid-19. Using software, they can model the impact of social distancing, drug treatments, vaccination, and see under what circumstances the disease dies out, the population reaches herd immunity, or has a second wave.

Advanced undergraduate summer research students at Wright State University were able to access and make progress on questions motivated by current research. Motivated by recent work of Lega, they investigated the appropriate modification of the SIR model that would result in logistic growth of the cumulative number of infections.   (Received September 13, 2020)

1163-J5-881   **Itai Seggev*** (`is+research@cs.hmc.edu`), Wolfram Research, Inc, 100 Trade Center Dr, Champaign, IL 61820. *Making Mathematica Work for You in Teaching DEs.*

Mathematica has long been recognized as a powerful system for doing mathematics, including symbolic and numerical differential equations. Perhaps because it has been doing this for so long, it also has a reputation for being difficult to use, especially for differential equations. In this talk I will survey a variety of new and not-so-new ways in which Mathematica has been made more friendly to the differential equations learner, suggest ways to make your own dedicated commands for your class, and generally argue that Mathematica is the right tool for the differential equations classroom.   (Received September 13, 2020)

1163-J5-1047   **Samer Habre*** (`shabre@lau.edu.lb`) and **Beverly Henderson West** (`bhw2@cornell.edu`). *Presenting CODEE's 2020 Special Issue, "Engaging Learners: Differential Equations in Today's World".*

"Engaging Learners: Differential Equations in Today's World" is the theme for the 2020 special issue of the CODEE Journal (codee.org). Our online journal has international impact; more than 2/3 of the downloads are outside the United States, and include more than 70% of the 195 countries listed on internet sources. So it is fitting that we open the special issue with an article on "Internationalizing the Calculus Curriculum"; its source at an historically black university emphasizes that our CODEE consortium profits much from diversity. You will find a variety of other creative papers in this publication. Two address ecological dynamics, with one project on three-species fisheries in Chesapeake Bay, and another on whale population management (which also suggests several variations to the usual modeling process). Two articles address epidemics – one surveys historical models, leading to a focus on Ebola; another details how students volunteered to mount a research project on Covid-19, precisely when the pandemic was evolving in Spring 2020. Our presentation will give further details on these and other contributions that "engage learners".   (Received September 14, 2020)

1163-J5-1074   **James S Sochacki*** (`sochacjs@jmu.edu`), sochacjs@jmu.edu. *Exploring Differential Equations with Exponents and Delays.*

I learned differential equations from the 3rd edition of Differential Equations and Boundary Value Problems by Boyce & DiPrima and an energetic professor in 1977. From that point on, I knew I would study differential equations and modeling. Three problems in this book caught my attention as a student: (1) $x' = x^{\frac{1}{3}}$ (2) $x' = x^2$ and (3) the rocket problem. In this talk, I will discuss the generic differential equation: $x' = x^r$ and a rocket chasing a planet (or a dog chasing a rabbit). For the former, I will discuss properties for real numbers r and questions of inquiry that I give my students related to this problem and the latter will be the discovery that two students made for an NSF-REU summer. There will be computer generated figures and animations.   (Received September 14, 2020)

1163-J5-1404   **Francesca Bernardi*** (`fbernardi@wpi.edu`) and **Manuchehr Aminian** (`maminian@cpp.edu`). *Epidemiology and the SIR model: Historical Context to Modern Applications.*

Learning how ODEs are commonly used to model real-world problems can be a gateway into applied mathematics for undergraduate students. We present a project to teach students about mathematical epidemiology utilizing historical documents and primary sources, as well as data and articles from recent events. Students begin by reading excerpts of an early 1900s report discussing the Indian plague epidemic and the seminal 1927 Kermack and McKendrick paper introducing the SIR model. Students then fit SIR models to available compiled data sets from the Ebola outbreak of 2014-2016 in West Africa. Additionally, students explore how the outbreak is affected by context and culture, such as local attitudes towards government health recommendations. Throughout this

project, participants explore mathematical, historical, and sociological aspects of the SIR model and approach data analysis and interpretation. Based on their work, students form opinions on public health decisions and related consequences. This curriculum has been assigned (in different versions) as part of a class syllabus, as an undergraduate research project, and as an extra credit assignment at multiple universities in the United States. Feedback from students has been encouraging. (Received September 15, 2020)

1163-J5-1545   **Michael C. Barg*** (`mbarg@niagara.edu`). *Forming a Collaborative Research Group with an Introductory Differential Equations Class.* Preliminary report.

Amidst the educational upheaval that was the latter part of the Spring 2020 semester, I revised my traditional introductory differential equations project in such a way as to harness the energy of the ever-evolving COVID-19 pandemic. I formed a research group with my students in order to explore variations on the standard SIR model. There were some tasks that all group members completed, while other responsibilities were delegated to individuals who reported their findings to the larger group. A regular Wednesday evening meeting served as a requirement that all research group members had to complete. My students embraced this opportunity to work together. We collected data, estimated parameters in our models, and computed numerical solutions to SIR-based systems of differential equations. In this time of great uncertainty, we found comfort in the project and sought ways to understand our models and their relationships to a real-life developing situation. This session will include an account of the transition to the collaborative project, some details of our work, and commentary from students. While I hope that such a devastating scenario does not present itself again, the situation provided an opportunity of a lifetime for my students. This is our story. (Received September 15, 2020)

1163-J5-1656   **Douglas B Meade*** (`meade@math.sc.edu`), Department of Mathematics, University of South Carolina, Columbia, SC 29205. *Some Thoughts About Series Solutions in the 21st Century.*

The classical series methods for ordinary differential equations have a beauty and appeal on their own, many modern classes do not have enough time to fully develop and apply these techniques.

In many practical settings it suffices to know the first few terms of the series expansion. We will explore different ways to obtain these approximate solutions, including Picard iterations and repeated differentiation.

A common theme of these methods is that they are iterative, and each step is easily implemented with symbolic computation tools. This makes these methods particularly well-suited to courses taught in an online or hybrid format. The examples discussed will include both theoretical and practical applications. (Received September 16, 2020)

Recreational Mathematics: Puzzles, Card Tricks, Games, and Gambling

1163-L5-259   **Doug Chatham*** (`d.chatham@moreheadstate.edu`), Department of Mathematics, Morehead State University, Morehead, KY 40351. *Social Distancing on the Chessboard.* Preliminary report.

On an $n \times n$ chessboard, two squares are at most two queen moves (or rook moves) from each other. We ask how many blocking pieces ("pawns") need to be placed on the board to increase the maximum possible distance d ("diameter") from 2 to some other desired number. We produce an algorithm to answer the question for given values of n and d. We show that, for $n \geq 3$, to increase the diameter for queens and rooks to 3, one pawn is needed. Also, for $n \geq 4$, to increase the diameter to 4, three pawns are needed for the queens and two pawns for the rooks. (Received August 31, 2020)

1163-L5-464   **Jathan Austin*** (`jwaustin@salisbury.edu`), **Brian Kronenthal**, **Susanna Molitoris Miller** and **Jonathon Miller**. *Counting Socially-Distanced Catan Configurations.*

Constructing a board for the game Catan includes a requirement that no red number tiles are adjacent to one another in a hexagonal grid formation. Previous countings of the number of boards omitted this restriction. This paper revisits the number of possible boards incorporating this constraint. (Received September 07, 2020)

1163-L5-563   **Anne Quinn*** (`quinna@edinboro.edu`). *Transformations and Simulations on the Game of Set.*

The Game of Set is a popular and addictive game which can be enjoyed by people of all ages. Although the talk will begin with a brief introduction for beginners (see my article at setgame website), it will mainly focus on new insights for experienced players. Matrix transformations will be applied to see the effect on theorems about

types of sets and about minimum, average, and maximum number of sets. Simulation and modular arithmetic will be also be utilized to study results for dice versions of the Set game. Differences between the two types of Set games will be highlighted.   (Received September 09, 2020)

1163-L5-575   **Anduriel Widmark*** (`anduriel@andurielstudios.com`), Denver, CO. *Creative Design Using Polystix- Homogenous Cylinder Rod Packings.*

Exploring the various ways cylinders can be symmetrically arranged to fill space with physical models is a fun and tactile way to learn about symmetry while also making a beautiful sculpture. In this paper, groups of non-intersecting congruent cylinder packings, related through cubic symmetry, are defined and classified. The basic techniques and materials required to reproduce these polystix models are outlined. Several examples of different arrangements are presented to highlight the range of geometric concepts that polystix can illustrate. In addition to being a great hands-on learning project, building polystix is also a practical system for creative and artistic design.   (Received September 09, 2020)

1163-L5-600   **Joshua Harrington**, **Kedar Karhadkar**, **Madeline Kohutka**, **Tessa Stevens** and **Tony W. H. Wong*** (`wong@kutztown.edu`), Department of Mathematics, Kutztown University of Pennsylvania, 15200 Kutztown Road, Kutztown, PA 19530. *Two dependent versions of probabilistic chip-collecting games.*

Alice and Bob take turns to collect chips in the following manner. In each turn, Alice tosses a fair coin, which decides whether she collects a or b chips, where a and b are positive integers. If Alice collects a chips, then Bob collects b chips, and vice versa. We consider two variants of game play that have different rules in determining the winner. Namely, the winner of Game 1 is the first player to collect at least n chips, while the winner of Game 2 is the first player to collect a positive number of chips congruent to 0 modulo n. We fully determine the formula for the winning probabilities of each player in Game 1, and determine the best and worst case scenarios in terms of winning probabilities in Game 2.   (Received September 10, 2020)

1163-L5-675   **Kenneth Levasseur*** (`kenneth_levasseur@uml.edu`), Mathematical Sciences, 1 University Ave., Lowell, MA 01852. *"Pass the Buck" on a Complete Binary Tree.*

The Stochastic Abacus is employed to compute winning probabilities at each level of the game "Pass the Buck" on a complete binary tree with the starting vertex being the root of the tree. The derivation is also generalized to play on complete l-ary trees.   (Received September 11, 2020)

1163-L5-736   **Daniel M. Kane** (`dakane@ucsd.edu`) and **Scott Duke Kominers*** (`kominers@fas.harvard.edu`). *Prisoners, Rooms, and Lightswitches.*

We introduce a new variant of the classic prisoners and light switches puzzle: A warden leads his n prisoners in and out of r rooms, one at a time, in some order, with each prisoner eventually visiting every room an arbitrarily large number of times. The rooms are indistinguishable, except that each one has s light switches; the prisoners win their freedom if at some point a prisoner can correctly declare that each prisoner has been in every room at least once. *What is the minimum number of switches per room, s, such that the prisoners can manage this?* We show that if the prisoners do not know the switches' starting configuration, then they have no chance of escape—but if the prisoners do know the starting configuration, then the minimum sufficient s is surprisingly small. The analysis gives rise to a number of puzzling open questions, as well.   (Received September 12, 2020)

1163-L5-875   **Rachel A Perrier*** (`rachel.perrier@wsu.edu`) and **Jessica M Dickson** (`jmdickson@wsu.edu`). *Dots-and-Polygons.*

Dots-and-Boxes is a popular children's game whose winning strategies have been studied by Berlekamp, Conway, Guy, and others. We consider two variations, Dots-and-Triangles and Dots-and-Polygons, both of which utilize the same lattice game board structure as Dots-and-Boxes. The nature of these variations along with this lattice structure lends itself to applying Pick's theorem to calculate claimed area. Several strategies similar to those studied in Dots-and-Boxes are used to analyze these new variations.   (Received September 13, 2020)

1163-L5-980   **Edmund A. Lamagna*** (`eal@cs.uri.edu`), University of Rhode Island, Kingston, RI 02881. *Frogs + Puzzles = Algorithmic Thinking.*

Recreational mathematics provides a rich source of interesting, entertaining problems that can be used in the classroom to develop mathematical thinking and problem-solving skills. Several versions of peg solitaire in which frogs perform various feats, such as exchanging positions, are considered. While some of the puzzles are well known, the mathematics underlying their solutions is not. The problems offer a vehicle for learning about algorithms in an "unplugged" way, without computer programming. The puzzles considered involve significant mathematical content and provide a springboard into such topics as the analysis of algorithms, lower bounds and

optimality, parity, and the generalization of solutions. The puzzles also promote the use of important problem-solving paradigms such starting with small cases and observing patterns to develop intuition about a problem. (Received September 14, 2020)

1163-L5-1095   **Tomas Guardia***, 502 East Boone Avenue, MSC 2516, Gonzaga University, Spokane, WA 99528. *The Pythagorean Games Metromachia and Rithmomachia.* Preliminary report.
There is evidence of number theory in Rithmomachia (e.g., Fibonacci, Pell, and Jacosthal Numbers). We have made progress in linking the infinite board of Rithmomachia to infinity with these numbers. Metromachia is a geometric game based on the same principle of Rithmomachia. We expect to find the connection of Greek Number Theory with Metromachia in the fashion of the previous work with Rithmomachia   (Received September 14, 2020)

1163-L5-1203   **Eric Sundberg***, 1600 Campus Road, Los Angeles, CA 90041, and **Ramin Naimi**. *Tic-Tac-Toe on the Boolean Hypercube.* Preliminary report.
We introduce a tic-tac-toe-style game on the vertices of the n-dimensional Boolean hypercube $\{0,1\}^n$ where the winning sets are the sets of vertices of k-dimensional subcubes. We also present pairing strategies which allow the second player to force at least a draw.   (Received September 15, 2020)

1163-L5-1323   **Andrea M. Potylycki*** (andrea.potylycki@alvernia.edu), 400 St Bernardine Street, Reading, PA 19607. *A Two-Player Pebbling Game.* Preliminary report.
Given a graph G with pebbles on the vertices, we define a pebbling move as removing two pebbles from a vertex and placing one pebble on its neighbor. The pebbling number, $\pi(G)$, of G is the least number of pebbles needed so that any arrangement of the $\pi(G)$ pebbles can reach any goal vertex through a sequence of pebbling moves. We define a new two-person pebbling game, called Two-Player Graph Pebbling with players Mover and Defender. The value $\eta(G)$ is defined as the minimum number of pebbles such that given every configuration of the $\eta(G)$ pebbles and every root vertex, r, Mover has a winning strategy. We determine winning strategies for Mover on cycles and on joins of certain graphs.   (Received September 15, 2020)

1163-L5-1417   **Andrew Sward** (andrewsward@augustana.edu), 1520 32nd St., Rock Island, IL 61201, **Duc Pham*** (ducmpham17@augustana.edu), 3206 8th Ave, Rock Island, IL 61201, and **April Tran** (chitran17@augustana.edu), 628 42nd St, Rock Island, IL 61201. *Cyclical Tetris.* Preliminary report.
Cyclical Tetris is a way of playing Tetris with a repeatable pattern of pieces. Given such a pattern of pieces, is it possible to play that pattern forever? We investigate all patterns of length 2 and 3 using a genetic algorithm AI to play the patterns and identify patterns that may not be infinitely playable. We examine some such patterns rigorously to show that they are in fact unplayable on certain board widths. In particular we show the SZ pattern is impossible to play on any boards of width congruent to 2 (mod 4).   (Received September 15, 2020)

1163-L5-1645   **Michael Czekanski*** (mczekanski1@gmail.com), 14 Old Chapel Road, Middlebury, VT 05753, and **Alex Lyford**, 14 Old Chapel Road, Middlebury, VT 05753. *Probabilistic Decision Making—Looking Beyond Expected Value.*
Life is full of decisions. Whether it's decisions about catching the next train, beating traffic, or playing board games, we constantly engage with probabilistic decision making. Students in introductory statistics courses often learn that we should maximize the expected value of our decisions, which is a good strategy in the long run, but not necessarily the best decision given the circumstances. In this talk, we examine the efficacy of this rule and others in the board game Camel Up. By implementing a series of intelligent agents, we explore how these rules perform and how they work against each other. Furthermore, we look at how these rules perform in comparison to the intuition that students gain in an introductory statistics course. We seek to determine if there is an optimal rule for playing Camel Up, and if so, is it easy to remember? Computers can create complex strategies for games that often beat the best humans. We investigate if a simple, easy-to-learn decision-making strategy based on more than just expected value can perform similarly to that of complex algorithms.   (Received September 15, 2020)

Data-Driven Modeling Projects to Motivate Active Learning and Engagement

1163-M1-856 **Deborah Hughes Hallett*** (`dhh@math.arizona.edu`), dhh@math.arizona.edu, and **Enrique Acosta**, enriqueacostajaramillo@gmail.com. *Why Model? Why Plot? Spreadsheets and Covid-19 Data in the Classroom.* Preliminary report.
Covid-19 has (unfortunately) put tons of data and important questions right in front of us. Spreadsheets put the power in students hand to investigate these questions, and sometimes answer them. Using a spreadsheet, students can see for themselves why published data is usually smoothed into 7-day averages. They can investigate the spread in different regions and see the impact of changing directives—for example, lock downs and social distancing. They can make predictions from a scatterplot. (Received September 14, 2020)

1163-M1-1165 **Michael A Karls*** (`mkarls@bsu.edu`), Ball State University, Muncie, IN 47306. *A Two-Dimensional Groundwater Modeling Project.* Preliminary report.
The purpose of this project is to derive and investigate a mathematical model for two-dimensional flow of groundwater. This model consists of a partial differential equation known as the two-dimensional groundwater flow equation as well as appropriate boundary conditions and initial data. After finding analytical and numerical solutions to the resulting initial value – boundary value problem, we will compare our models via an example implemented in Mathematica. This project came about as a result of an Undergraduate Honors Thesis project at Ball State University. (Received September 14, 2020)

1163-M1-1204 **Rodica Cazacu*** (`rodica.cazacu@gcsu.edu`). *Project-based assessment using real data.*
Each semester, when I start teaching my Introduction to Mathematical Modeling course, I can't stop thinking about how my new students will see the world around them after taking my class! Since I took over and redesigned this course, I was looking for ways to let my students express themselves while applying what they learn in my class. And how else they could do that better than working on their own projects? This presentation will show how I started using projects in large classes of non-science majors, instructing my students and guiding their work, while allowing them to leave their mark in the process. From small projects to a final project that replaces the final exam, my students will review all key concepts working with data they collect or find interesting and useful, using mathematics to interpret and analyze their findings. (Received September 15, 2020)

1163-M1-1222 **Becky Sanft*** (`bsanft@unca.edu`) and **Anne Walter** (`waltera@stolaf.edu`). *Exploring Mathematical Modeling in Biology Through Case Studies and Experimental Activities.*
Exploring Mathematical Modeling in Biology Through Case Studies and Experimental Activities, written collaboratively by a mathematician and biologist, provides supporting materials for a course taken simultaneously by students majoring in the mathematical sciences and those in the life sciences. The text is designed to actively engage students in the process of modeling through a collection of case studies and wet labs connecting mathematical models to real data. In this talk we will provide an overview of the book and present a case study on immunotherapy in prostate cancer. Students formulate a system of differential equations that describe the interactions among tumor vaccine cells, immune response, and prostate cancer cells, and data from a prostate-specific antigen (PSA) test are used to estimate model parameters. The model is used to test vaccination schedules, and sensitivity analysis is used to identify model parameters that, if manipulated, are likely to make the greatest difference in cancer patient survival time. Through this case and the other case studies and labs, the reader will see the utility of models for understanding complex systems, making predictions, testing control strategies, and identifying further questions. (Received September 15, 2020)

1163-M1-1285 **Emilie Hancock*** (`emilie.hancock@cwu.edu`), emilie.hancock@cwu.edu, and **Reilly Hennessey**. *Pulling Ladders from Walls: Bringing Modeling to Life in Introductory Calculus.*
Modeling applications in introductory calculus textbooks can feel static and formulaic. The problems and variables are already defined and assumptions have been made. A falling ladder is briefly introduced before students create an equation, take some derivatives, find the missing rate, and then move on to expanding balloons. In an attempt to provide Calculus I students with a more authentic mathematical modeling experience, we designed a short, inquiry-based project for students to explore calculus-based models involving related rates. Over three days, students pulled 2x4 boards away from walls to collect time and distance data, used the data to generate relevant equations, and assessed model assumptions and limitations as they compared the textbook model with the real-world data. In this talk, we describe how we combined lab materials from Project CLEAR

Calculus with Buck Institute's essential elements of project-based instruction to highlight the mathematical modeling process in Calculus I. We also share student work and our reflections on implementation.   (Received September 15, 2020)

1163-M1-1332   **Greg Hartman*** (`hartmangn@vmi.edu`) and **Katherine Crowley** (`crowleykd@vmi.edu`).
Learning modeling using non-STEM data: sample projects from Math That Matters, an inquiry-based introduction to mathematical modeling for students without calculus.
Virginia Military Institute's Math That Matters courses are a two-semester modeling sequence designed for students whose major does not require calculus. The sequence teaches modeling through active learning projects using scenarios and data sets provided by the departments the courses service. Three sample projects are given. In one, students analyze voter characteristics using a large and messy data set provided by the Department of International Studies, requiring pivot tables in Excel to prepare a hypothesis test. In another, students design an art exhibit with 40 paintings and sculptures, determining art placement and visitor traffic flow using scale models. Finally, we highlight one student project from the second semester, where students work in pairs on a semester-length modeling project of their own design. We describe how and why we designed this course and how it can be tailored to use data from your institution.   (Received September 15, 2020)

1163-M1-1374   **Olivia M Carducci*** (`ocarducci@seu.edu`). *A Partnership Between Math and Career Services.*
Last year the math department developed a new course for incoming freshmen math majors. Mathematical Investigations is designed to introduce new math majors to college level mathematics and each other. A major goal of the course is to provide answers to the question "What can you do with a math major?" Of course, we partnered with Career Services to provide information on employment and the services they provide to help students find internships and jobs.

More significantly, this course has partnered with Career Services to take the data from their graduating senior survey on future plans of graduating students and provide a one-page summary of the results. The professor will offer a module on data visualization and then turn the students loose on the data from Career Services. Rather than tell the students that they will be qualified to work with data; students will gain experience in working with data.   (Received September 15, 2020)

1163-M1-1509   **Patrice Tiffany***, 63 Mckenna Street, Blauvelt, NY 10913. *Modeling Ebola and Covid Data Driven Models in Differential Equations on a Remote Platform.*
Mathematical modeling is a critical tool in our world. Mathematical modeling is also a valuable pedagogical tool. The differential equations course, by its nature lends itself to be a modeling rich course. Must we abandon this modeling approach when we transfer to a remote platform? I do not believe so. We cannot transform the course by means of just a document camera. However, the course can be transformed to a viable remote experience. The new course may be more structured. The group work may be mirrored in break out sessions and the students' work may be monitored more closely. This modeling unit introduces the separation of variables technique through group work, modeling the spread of Ebola in western Africa. From there, the students go on to individual projects where they model the spread of COVID in individual states in the U.S. I will show how an in-class data driven modeling unit was transformed onto a remote platform.   (Received September 15, 2020)

1163-M1-1551   **Corban Harwood*** (`rharwood@georgefox.edu`). *Introducing Differential Equations through a Simulation, Data Analysis, and Modeling Project.*
Visually analyzing data is a key modeling skill that can actively engage students across the curriculum, from making content tangible to guiding further analysis. In this talk, we will share an introductory project in differential equations where students investigate the impact of heightened hygiene and decreased interactions on the spread of an infectious disease. Students first focus on the common cold to bring their own experience to bear, and then extend the analysis and conclusions to COVID-19. Students generate and analyze their own data, engage with each other and the content as they argue and reason through the modeling steps. Also, we will share modifications of this project for remote teaching where students perform the group simulation over Zoom, analyze that data through a collaborative spreadsheet, and record a presentation of their results.   (Received September 15, 2020)

Mathematics and Music

1163-M5-749 **Anil Venkatesh*** (avenkatesh@adelphi.edu) and **Viren Sachdev**. *Internal Symmetries in Musical 12-Tone Rows.*

In music, a 12-tone row is any of the 12! possible orderings of notes in the Western chromatic scale. The musical notes of a 12-tone composition must always arise in the same order, cycling repeatedly through a predetermined "row" of twelve notes. The repetitive structure of 12-tone music lends itself to mathematical study. In 2003, Hunter and von Hippel investigated symmetry in 12-tone rows, using group theory to enumerate equivalence classes of rows under a group of music-theoretic symmetries. They found that highly symmetric rows constitute just 0.13% of the 12! possibilities, and yet these rows arise in 10% of actual compositions. In a previous talk, we conjectured that the remaining 90% of 12-tone compositions, while not entirely self-symmetric, might contain shorter repetitions and symmetries that were intuitively attractive to the composers. In this talk, we introduce a way to measure the occurrence of short repetitions and symmetries that go undetected in the analysis of Hunter and von Hippel. We present a new hierarchy of symmetry for 12-tone rows and show that composers favor symmetric substructures in their work. (Received September 12, 2020)

1163-M5-905 **Maria C Mannone*** (maria.mannone@unive.it), University of Palermo, Ca' Foscari University, via Archirafi 34 (Palermo), Ca' Bottacin, Calle Crosera, 3911 (Venice), 90123 Palermo/Venice, Italy. *Categories, Color Gestures, and the "Souvenir Theorem": using Math to navigate the Complexity of Music and the Visual Arts.*

Arts, including music, seem to be far away from mathematical rigor. However, some mathematical constructions can well describe artistic forms and features and image/music translations. With the language of categories, in fact, we can describe forms as morphisms connecting points in space; form transformations, as natural transformations; visual-to-music form and structure mappings, as functors (mappings between categories). Effective translations verify gestural similarity: 'similar' visual sketches and articulations/melodic profiles appear as being generated by the same gesture. We can define color gestures as mappings between points in the RGB space, and timbre gestures as mappings between timbres. Timbre-color correspondence requires equivalence classes and quotient categories. A timbre is mapped onto a color-class of colors and vice versa, satisfying a perceptive analogy between color and timbre bands (e.g., tension/relaxation feeling). We also present here the Souvenir Theorem (ST) and the Art Conjecture (AC). According to AC, the same idea can be identifiable and well-defined through its embodiments in similar artworks from different media. ST, with practical implications, is a condition of minimal recognizability of a form, allowing the creation of musical souvenirs. (Received September 15, 2020)

1163-M5-929 **Federico Favali*** (federicofavali@gmail.com), Independent researcher, *The Inner Geometry of the Prelude op.74 no.1 by Skrjabin.*

The Preludes for piano by Skrjabin have been analyzed in depth by many musicologists throughout the past decades. Despite the studies already written, this paper aims to analyze one of them according to a new analytical approach. The piece taken into consideration is the Prelude op.74 no.1. The starting point will be a geometrical view of the harmony. In the large field of studies on the relationship between mathematics and music, the aspect of the geometric representation of chords has already been investigated. For example, the studies of Dmitri Tymoczko are fundamental in this sense. This study builds on what has been done so far and wants to show how certain geometric figures derive from the analysis of chords in the Prelude. In other words the harmonic chain of the piece will be described geometrically, using hexagons. These figures will be represented graphically. Thus, the analysis of the piece will be a graphic analysis on how these figures change. The comparison among the harmonies of the composition will by made comparing the hexagons: their characteristics will reflect the characteristics of the chords. Such a type of study is the first step toward a detailed and accurate theory of harmony explained geometrically. (Received September 14, 2020)

1163-M5-1211 **Elijah Jarreau Bergevin*** (elijah.bergevin@cwu.edu), 1201 E Helena Ave, Unit 13302, Ellensburg, WA 98926, and **Sooie-Hoe Loke** (sooie-hoe.loke@cwu.edu), 400 E University Way, Samuelson Hall 208, Ellensburg, WA 98926. *Estimating Jazz Standard Key and Form with Hidden Markov and Hidden Semi-Markov Models post EM Optimization.*

With the rise of the digital age there has come an ever-growing supply of art in need of methods for analysis and cataloging. This project addresses music in notated form. Hidden Markov model (HMM) and hidden semi-Markov model (HSMM) are utilized to interpret the structures of American jazz standard repertoire. Ten charts with key modulation and ten with bisectional form are digitally converted for purpose of model optimization and model evaluation, individually. Two separate relationships are explored. First, the 24 major and minor keys of the Western musical canon are designated as hidden states. Pairs of consecutive chord symbols represent

observable emissions from those keys. The initial, emission, and transition parameters of HMMs—shaped by previous work on tonal relations—are then optimized. Second, the bisectional nature of a large body of jazz standards determines two hidden states. Melodic pitch is used as an observable indicator. HMM parameters and HSMM parameters (including duration) are semi-uniformly initiated, and optimized. A discussion of results and implications for digital cataloging/retrieval as well as for composition software is included.   (Received September 15, 2020)

1163-M5-1379   **Landon H. Buell*** (`lhb1007@wildcats.unh.edu`), Durham, NH 03824, and **Kevin M.**
                    Short (`kevin.short@unh.edu`), Dept. of Mathematics & Statistics, University of New
                    Hampshire, Durham, NH 03824. *Musical Instrument Classification Using a Hybrid Neural*
                    Network.

Classifying audio signals with machine learning has become an important topic of research in the past few years. Models often involve the input of a 2-D spectrogram or 1-D feature vector into a unimodal network such as a Convolutional Neural Network (CNN) or Multilayer Perceptron (MLP). In this study, we explore automatic classification of musical instruments using a new hybrid neural-network architecture that combines the CNN and MLP models and provides superior performance over models that rely solely on one or the other. This hybrid network uses two branches, one being a CNN to process an image-like 2-D spectrogram, and the other being an MLP to process a 1-D feature vector. Within the model, a hidden layer combines activations from the two branches by concatenating them into a single 1-D dense layer, thus any predictions are a product of both branches. We describe in detail the creation of the spectrogram and features, as well as how they influence the chosen network architecture. We finish with a practical demonstration that uses this classifier model to match waveforms from a chaotic music synthesizer to real-world musical instruments. Training data is from studio recordings of the Philharmonia Symphony Orchestra and University of Iowa's Electronic Music Studios. (Received September 15, 2020)

1163-M5-1441   **Gareth E Roberts*** (`groberts@holycross.edu`), 1 College Street, Dept. of Mathematics
                    and Computer Science, College of the Holy Cross, Worcester, MA 01610. *Bartók,*
                    Fibonacci, and the Golden Ratio: Fact or Fiction?

In 1955, Hungarian music theorist Ernő Lendvai published research claiming the existence of the Fibonacci numbers and the golden ratio in several works of the famous Hungarian composer Béla Bartók. Lendvai's discoveries and arguments were ground-breaking, leading to a flurry of claims that Bartók and other well-known composers (e.g., Debussy) were consciously incorporating sacred mathematical proportions into their music. Unfortunately, Lendvai made some crucial errors in his analysis, cherry-picked favorable data, and made some questionable assumptions. Following the excellent analysis of Roy Howat, we will discuss the evidence for and against the purported use of the Fibonacci numbers and golden mean in Bartók's music, focusing our attention on his ethereal Music for Strings, Percussion and Celesta.   (Received September 15, 2020)

1163-M5-1684   **Larry G Blaine*** (`lblaine@plymouth.edu`), PO Box 605, Plymouth, NH 03264. *The*
                    Hexachord Theorem and Some of its Consequences.

The Hexachord Theorem is perhaps the foremost example of a non-trivial mathematical result whose origins lie purely in the theory of musical composition. It says, roughly, that if the twelve notes of a "chromatic scale" (Think of any twelve consecutive notes on a piano keyboard) are partitioned into two sets of six notes each, the interval relations among the six notes are the same for both sets. In this talk, we will explore the history of the theorem, outline the methods of proof, discuss its relevance to music,and examine some generalizations. (Received September 16, 2020)

Teaching Math Modeling and Sustainability

1163-N1-345   **Kaitlyn A Perry*** (`k.perry@wingate.edu`). *Energy Conservation in the Classroom.*
Wingate University received an Energy Conservation Grant from the Jessie Ball DuPont Educational Fund to help transform the campus into a more sustainable one; however, energy conservation happens behind walls and therefore, it is often invisible and underappreciated. Instead of only changing the buildings, we decided to bring the ideas of sustainability into the curriculum to educate our students about energy conservation on campus and let them use their math skills to support what they have learned. In this talk, we will go over the ways to use data from energy savings to introduce sustainability in a variety of introductory math courses.   (Received September 09, 2020)

1163-N1-858   **Deborah Hughes Hallett***, dhh@math.arizona.edu, and **Tom Tucker**, Ttucker@mail.colgate.edu. *Sea Level Rise and its Costs.* Preliminary report.
The impact of climate change is evident in the past summer's fires and hurricanes and the relentless rise of the sea. But how many of us appreciate the enormous costs that sea level rise is expected to pose? An understanding of possible future costs and their present values enables our students to prepare for a future in which these costs occur. This talk suggests data that can be used to develop the concept of present value in Precalculus and Calculus II in the context of sea level rise and its estimated impact.   (Received September 13, 2020)

1163-N1-1114   **Victor J. Donnay*** (`vdonnay@brynmawr.edu`), Math Dept, Bryn Mawr College, 101 N. Merion Ave, Bryn Mawr, PA 19010. *Using Renewable Energy Data to Introduce Integration.*
Many of our campuses are making investments in solar or wind energy systems. The data from these systems is often displayed in real time via the internet. We will show how this power data can be used to motivate the introduction of integration and lead in a natural way to numerical integration and Riemann sums. This example is one of many arising in a course on Math Modeling and Sustainability in which fundamental mathematical concepts are taught in an engaging way by linking to issues of sustainability.   (Received September 14, 2020)

2010 MATHEMATICS SUBJECT CLASSIFICATION

Compiled in the Editorial Offices of MATHEMATICAL REVIEWS and ZENTRALBLATT MATH

- **00** General
- **01** History and biography
- **03** Mathematical logic and foundations
- **05** Combinatorics
- **06** Order, lattices, ordered algebraic structures
- **08** General algebraic systems
- **11** Number theory
- **12** Field theory and polynomials
- **13** Commutative rings and algebras
- **14** Algebraic geometry
- **15** Linear and multilinear algebra; matrix theory
- **16** Associative rings and algebras
- **17** Nonassociative rings and algebras
- **18** Category theory; homological algebra
- **19** *K*-theory
- **20** Group theory and generalizations
- **22** Topological groups, Lie groups
- **26** Real functions
- **28** Measure and integration
- **30** Functions of a complex variable
- **31** Potential theory
- **32** Several complex variables and analytic spaces
- **33** Special functions
- **34** Ordinary differential equations
- **35** Partial differential equations
- **37** Dynamical systems and ergodic theory
- **39** Difference and functional equations
- **40** Sequences, series, summability
- **41** Approximations and expansions
- **42** Fourier analysis
- **43** Abstract harmonic analysis
- **44** Integral transforms, operational calculus
- **45** Integral equations
- **46** Functional analysis
- **47** Operator theory
- **49** Calculus of variations and optimal control; optimization
- **51** Geometry
- **52** Convex and discrete geometry
- **53** Differential geometry
- **54** General topology
- **55** Algebraic topology
- **57** Manifolds and cell complexes
- **58** Global analysis, analysis on manifolds
- **60** Probability theory and stochastic processes
- **62** Statistics
- **65** Numerical analysis
- **68** Computer science
- **70** Mechanics of particles and systems
- **74** Mechanics of deformable solids
- **76** Fluid mechanics
- **78** Optics, electromagnetic theory
- **80** Classical thermodynamics, heat transfer
- **81** Quantum theory
- **82** Statistical mechanics, structure of matter
- **83** Relativity and gravitational theory
- **85** Astronomy and astrophysics
- **86** Geophysics
- **90** Operations research, mathematical programming
- **91** Game theory, economics, social and behavioral sciences
- **92** Biology and other natural sciences
- **93** Systems theory; control
- **94** Information and communication, circuits
- **97** Mathematics education

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