Abstracts
of Papers Presented to the American Mathematical Society

Volume 43, Number 1, Issue 207 Winter 2022

Abstracts for

ISSN 2689-4831 (online) Pages 1–462

American Mathematical Society

SUBMISSION INFORMATION

ABSTRACTS PUBLISHED IN THIS JOURNAL are those submitted by authors who intend to present them at AMS meetings. The abstracts for each meeting are sorted by the two-digit 2020 Mathematics Subject Classification chosen by the author and appear in the order received. See the 2020 Mathematics Subject Classification at

mathscinet.ams.org/msc/msc2020.html.

The expanded classification system, beyond classification 97 of the 2020 Mathematics Subject Classification, for sessions and talks at the Joint Mathematics Meetings is available at

https://www.ams.org/meetings/national/session-classifications.

THE SUITABILITY OF A PAPER for presentation to the Society is judged from the abstract as outlined in Article X, Section 6 of the bylaws:

> Papers intended for presentation at any meeting of the Society shall be passed upon in advance by a program committee appointed by or under the authority of the Council; and only such papers shall be presented as shall have been approved by such committee. Papers in a form unsuitable for publication, if accepted for presentation, shall be referred to on the program as preliminary communications or reports.

In addition, the editors of the *Abstracts* have adopted the following policy: In order to be accepted for publication, an abstract must have mathematical content. It should not contain libelous, defamatory, or tasteless remarks, commercial promotions, nor political or religious arguments. The AMS assumes no responsibility for the content or inappropriate remarks in any published abstract.

GENERAL INFORMATION ON ABSTRACTS is found at http://www.ams.org/journals/abs.

ABSTRACTS ARE PUBLISHED ELECTRONICALLY from copy submitted by the author. Web submission is the required electronic format. To access the interactive template, visit the *Abstracts* submission page on the AMS website at

http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Step-by-step submission instructions are included in the template. No knowledge of LaTeX is necessary; however, any mathematical displays or accent marks in text or in names must be coded in LaTeX. Requests for general information concerning abstracts may be sent to abs-coord@ams.org.

THE ABSTRACT RECEIPT DEADLINES FOR ALL MEETINGS will be strictly enforced. Late papers cannot be accommodated. When all talks have been scheduled for a given meeting, the attendant abstracts will be available for viewing on the AMS website through the program display for that meeting.

NOTATIONS IN THIS JOURNAL are the following:

* Indicates who will present the paper at the meeting.

SUBSCRIPTION INFORMATION

CHANGES OF ADDRESS AND ALL GENERAL CORRESPONDENCE should be sent to Member and Customer Services, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2213 USA (email: amsmem@ams.org or fax: 401-455-4026); dues payments and orders for AMS publications should be addressed to the Society at P.O. Box 845904, Boston, MA 02284-5904 USA; all correspondence relating to advance registration for meetings should be addressed to the Society, 201 Charles Street, Providence, RI 02904-2213 USA.

Abstracts of Papers Presented to the American Mathematical Society (ISSN 2689-4831 (online)) is published electronically four times a year by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2213. The subscription price for Volume 43 (2022) is US$212 list, US$169.60 institutional member, US$127.20 individual member. Subscription renewals are subject to late fees. See www.ams.org/customers/macs-faq.html#journal for more information.

PAPERS PRESENTED AT MEETINGS

THIS CALENDAR lists meetings of the Society which have been approved by the Council at which papers may be presented. Programs of Annual Meetings appear in the *Notices* and on the AMS website; programs for sectional meetings appear on the AMS Web pages in the Meetings & Conferences section, and are electronically archived in the *Notices* section on the AMS website.

MEETING #	DATE	PLACE	ABSTRACT DEADLINE	ABSTRACT ISSUE
1175	March 11–13, 2022	Charlottesville, VA, CANCELED	EXPIRED	Vol. 43, No. 2
1176	March 19–20, 2022	Virtual Meeting	EXPIRED	Vol. 43, No. 2
1177	March 26–27, 2022	Virtual Meeting	EXPIRED	Vol. 43, No. 2
1178	May 14–15, 2022	Virtual Meeting	March 15	Vol. 43, No. 3
1179	September 17–18, 2022	El Paso, TX	July 26	Vol. 43, No. 3
1180	October 1–2, 2022	Amherst, MA	August 16	Vol. 43, No. 4
1181	October 15–16, 2022	Chattanooga, TN	August 23	Vol. 43, No. 4
1182	October 22–23, 2022	Salt Lake City, UT	August 30	Vol. 43, No. 4

Virtual Joint Mathematics Meetings, April 6–9, 2022

Abstracts of the 1174th Meeting.

00 ► General and overarching topics; collections

1174-00-5207 **Daniel Krashen*** (daniel.krashen@gmail.com), Rutgers University. *Field patching and algebraic structures*

In this talk we will explore field arithmetic as seen through the lens of algebraic structures such as quadratic forms and division algebras. In particular, I will describe some open problems and how the technique of field patching, first introduced by David Harbater and Julia Hartmann, has given some new tools for progress and insights. (Received November 18, 2021)

1174-00-5209 **Karen E. Smith*** (kesmith@umich.edu), University of Michigan. *Resolutions of Singularities and Rational Singularities*

In these three talks, I hope to share some of the beauty of an ancient field of mathematics called Algebraic Geometry, and some of the excitement of modern techniques used to investigate it. The talks do build on each other, but each can stand alone as well.

Algebraic geometry is the study of algebraic varieties, or geometric shapes described by polynomial equations. You already know many examples, such as the circle, whose polynomial equation is x2 + y2 = 1; or a sphere. Algebraic varieties are ubiquitous throughout mathematics and its applications to science and engineering. Not only do they naturally arise in important contexts—the set of all rigid transformations of space, for example, can be given the structure of an algebraic variety—but often complicated behavior can be described (or approximated) by polynomials. Because polynomials are relatively easy to manipulate by hand or by machine, algebraic geometry is a tool for scientists, engineers and even artists, as well as a rich source of examples throughout mathematics. Of course, algebraic geometry is also beautiful theoretical subject in its own right, and it is from this perspective that the talks approach the subject. (Received November 16, 2021)

1174-00-5217 **Monica Jackson*** (monica@american.edu), American University. *Spatial Data Analysis for Public Health Data.*

Spatial data analysis concerns data that are correlated by location, and relies upon the assumption that objects closer together in space (e.g. geographical location) will most likely have similar responses. This talk provides an overview of graphical and quantitative methods I developed for the analysis of spatial data. Emphasis is on lattice data (also known as areal data or aggregated data) however modeling of geostatistical data and point patterns will be discussed. I will apply these methods to public health data with applications to cancer trends, maternal mortality in the Dominican republic, and COVID-19 disease surveillance. (Received November 15, 2021)

1174-00-5229 **Anna Gilbert*** (anna.gilbert@yale.edu), Yale University. *Metric representations: Algorithms and Geometry*

Given a set of distances amongst points, determining what metric representation is most "consistent" with the input distances or the metric that best captures the relevant geometric features of the data is a key step in many machine learning algorithms. In this talk, we discuss a number of variants of this problem, from convex optimization problems with metric constraints to sparse metric repair. (Received November 18, 2021)

1174-00-7120 **Catherine Faith Brennan*** (catherine.brennan@colorado.edu), University of Colorado, Boulder. *Exploring the Potential for Gerrymandering Within Single And Multi Member Legislative Redistricting Plans* Preliminary report.

This project explores ways of quantifying what constitutes a gerrymandered electoral districting plan, and compares the potential for gerrymandering between single member and multi member legislative districts. I attempt to illustrate the benefits of using a ranked choice voting system within multimember districts as opposed to a single member district winner-take-all election system. Using Markov Chain Monte Carlo methods, I have created large random samples of districting plans for the Colorado State Senate election for both Single Member Districting Plans and a simulation of Multimember Districting plans using the Single Transferable Vote (STV) ranked choice voting method. The project uses Ensemble Analysis to analyze the distribution of districting plans and identify outliers. In addition, an analysis of competing metrics of fairness between the two types of simulated plans is conducted. (Received September 12, 2021)

1174-00-7211 **Susan Happersett*** (fibonaccisusan@icloud.com), Artist. *Hyperbolic Paper Sculptures*

In my quest to find new ways to incorporate my mathematically generated drawings into sculptures, I have developed a number of paper hyperbolic forms.

My sculpting process connecting basic geometric shapes is quite straightforward. Each hyperbolic form consists of a set of identical circles, squares or other polygons with patterning on both sides. Limiting sculptures to multiples of one particular geometric shape makes them well-suited for transforming digital prints into 3-D art. Following some basic rules of symmetry, artists can make their own unique sculptures covering the surfaces of the hyperbolic forms with their own art work.

This presentation will demonstrate how to build these hyperbolic forms as well as discuss which symmetries work best on each.I am excited to see where other artists take these forms. (Received September 17, 2021)

1174-00-7234 **David A Reimann*** (dreimann@albion.edu), Albion College. *Visual Representations of Natural Numbers using Geometric Patterns*

Natural numbers can be visually represented by a geometric arrangement of simple visual motifs. This representation is not unique because any partition of an integer n can generate at least one geometric pattern. Thus the number of partitions of n is a lower bound on the number of geometric patterns. For example, there are 17977 partitions for the number 36; it is both a square number (6^2) and a triangular number ($1+2+3+4+5+6+7+8$). Aesthetic considerations often favor patterns with some degree of symmetry, such as patterns that fix a single point or wallpaper patterns. A series of geometric designs for the numbers 1–100 were created to visually highlight some properties of each number. The designs use a variety of motifs and arrangements to provide a diverse yet cohesive collection. One application of these patterns is as a teaching tool for helping students recognize and generalize patterns and sequences. (Received September 13, 2021)

1174-00-7576 **Paul Goethals** (paul.goethals@westpoint.edu), United States Military Academy, **Hannah Lynne Ball*** (hannah.ball@westpoint.edu), United States Military Academy, and **Vic Trujillo** (victor.trujillo@westpoint.edu), United States Military Academy. *Modeling High Energy Laser Propagation through Rain*

High energy lasers have been a subject of interest for the US military for several decades, resulting in the employment of several systems of high energy lasers in use for the Army. Lasers pose a solution to many tactical and strategic problems involving engagements with physical targets and so their interaction with various materials is an important topic of study. Previous research has considered the propagation of high energy lasers through clouds, water, and how individual atmospheric conditions impact beam propagation. This research considers the propagation of high energy laser beams through complex atmospheric conditions—specifically, propagation through rain. Factors considered include rain, temperature, wind, changing electromagnetic fields, and atmospheric particulates to develop a comprehensive model. The underpinning mathematics are based on a combination of Maxwell's equations, the Mie solution, and paraxial approximations to Maxwell's equations as well as previously developed modeling solutions for individual conditions. Experiments using lasers are performed as a basis for the analysis of beam characteristics and their effects. (Received September 15, 2021)

1174-00-7772 **Luke K Stoner*** (luke.stoner@westpoint.edu), United States Military Academy, and **Connor Hirsch** (connor.hirsch@d3systems.com), D3 Systems. *Evaluating Twitter as a Measure for Public Opinion Using Statistical Analysis*

As the environment for modern conflict continues to evolve and become more urbanized, the ability to collect accurate, timely public opinion data will become increasingly important to the success of international security operations. While public opinion data is still largely collected through traditional polling, Twitter data scraping has emerged as a popular alternative to the conventional methodology. This research examines the effectiveness of techniques such as Twitter data scraping as a potential replacement for in-person or phone survey collection. A traditional survey of a country located in the increasingly contested and strategic Horn of Africa is performed, focusing on pressing topics such as COVID-19, great-power competition, election violence, and terrorism. Simultaneously, data from Twitter media local to the country is analyzed to assess the accuracy and effectiveness of data scraping in assessing public opinion. Both descriptive and inferential statistical analyses are performed to generate insights and evaluate the predictive capability of the social media platform. (Received September 16, 2021)

1174-00-8225 **Randall E Cone*** (recone@salisbury.edu), Salisbury University. *Freewill, Mathematics, and Sophocles*

Few plays have generated more speculation about Fate and Freewill than the Oedipus plays by Sophocles. In this presentation, we discuss the structure and nature of these plays from mathematical and data analytic perspectives. In addition, we present novel visualizations of the plays, focusing on the application of mathematical techniques to the associated graphic designs. (Received September 18, 2021)

1174-00-8992 **Sage Stanish*** (sagestanish@posteo.net), College of William & Mary, **Sarah Day** (sldayx@wm.edu), College of William and Mary, **Saskia Mordijck** (smordijck@wm.edu), College of William & Mary, and **Benjamin Dudson** (benjamin.dudson@york.ac.uk), Lawrence Livermore National Laboratory, Livermore, California 94550, York Plasma Institute, Department of Physics, University of York, York YO10 4LZ, UK. *Topological Data Analysis and its Application to Drift Wave Turbulence* Preliminary report.

Anomalous transport is a critical phenomenon in plasma physics and predicting its behavior will aid in the design of future fusion devices. Anomalous transport is the result of drift-wave turbulence that naturally occurs in magnetized plasmas. Here we use Topological Data Analysis (TDA) to study the Hasegawa Wakatani (H-W) model, which captures key features of drift-wave turbulence. TDA has been made possible by recent advances in computing and theoretical mathematics and computes the topology of arbitrary data. The goal is to identify topological structures in the H-W model and link them to the physical mechanisms behind drift-wave turbulence. Previous work applying TDA to classical fluid mechanics such as Kolmogorov Flow and Rayleigh-Bénard Convection has been promising. The authors were able to identify periodicity in the flow patterns and classify all states of the systems uniquely using TDA metrics. In this poster we will apply similar techniques to simulations of the H-W model. We look to identify predictors of change in the turbulent transport characteristics and connect these to the traditional descriptions of turbulent flow.

Funded by: Charles Center Research Grant, DE-SC0007880, Army Research Office Grant Number W911NF-18-1-0306 (Received September 21, 2021)

1174-00-9299 **Martin Levin*** (mdlevin_public@msn.com), retired. *Contemplation on the Platonic Solids*

Euclid proved that there exists a regular dodecahedron by circumscribing one around a cube. In this very well known construction, each of the 12 edges of the cubes lies (as a diagonal) on one of the 12 faces of the dodecahedron. What seems to be new—as far as I can tell—is that, if we regard the faces of the dodecahedron as extended into space, then there is exactly one other way to place the cube so that each of its edges lies on one face of a dodecahedron. I show this in a geometric sculpture (in stainless steel and copper) in the art exhibit here. When a beautiful geometric relationship is depicted with fine craftsmanship, then one has, in one piece, both a geometric model and a beautiful sculpture that is a joy to ponder. This model is just one example. By aligning two different Platonic solids so that some order three axes coincide, and varying the relative sizes of the two, one finds many interesting incidence relations between the two polyhedra. Moreover, such models provide a very visual and accessible way to introduce concepts of group theory, projective geometry, and topology. (Received September 20, 2021)

1174-00-9844 **Elaine Spiller*** (elaine.spiller@marquette.edu), Marquette University. *Uncertainty quantification of coupled multi-physics systems*

It is of great interest to construct surrogates or emulators for computationally intensive models with high dimensional input and output spaces. A different but related topic of interest is uncertainty quantification of coupled models—that is, when a functional of the output from one model serves as part of the inputs for a second model. In particular this kind of coupling arises in multi-scale and multi-physics computer models, e.g. when specialized computer codes are used to simulate different parts of the physics of a highly complex mathematical model. We will describe the construction of linked Gaussian process emulators of fluid flow coupled to the mechanics of a deformable porous material where specialized codes operate on the fluid and solids portions independently, and are coupled by an exchange of high-dimensional output data from each individual model. (Received September 20, 2021)

1174-00-10381 **Grant Sanderson*** (grant@3blue1brown.com), 3blue1brown. *When do programatic visualizations actually help in teaching math?*

The successful efforts to popularize math often involve evocative visualizations, and in many cases the most stunning diagrams are those generated programatically. A natural question to ask is when these visualizations help to genuinely teach the math, and when they're merely eye candy. This talk aims to explore this question through several examples, primarily in the context of math popularization on YouTube. (Received September 21, 2021)

1174-00-10484 **Frank A Farris*** (ffarris@scu.edu), Santa Clara University. *Coloring Polar Zonohedra* Preliminary report.

In his talk at Bridges 2021, George Hart gave an accessible introduction to polar zonohedral, a rotationally-symmetric class of polyhedra whose faces are all rhombi. Implementing his formulas in Grasshopper and Rhino, I have explored ways to color these appealing shapes. I will review the mathematics and describe a few of the endless possibilities. (Received September 21, 2021)

1174-00-10508 **Samit Bhattacharyya** (samit.b@snu.edu.in), Shiv Nadar University. *Assessing the impact of human choice to self-isolate on the COVID-19 pandemic through a coupled epidemiological-dynamic game framework*

Self-quarantine and social-distancing were two major control measures against COVID-19 before vaccines were authorized for emergency use in many countries. However, public perception and human response to these measures hindered their success in many countries. This resulted in huge outbreaks with devastating effects on humans, health systems and the economies of these countries. To unravel the impact of this behavioral response to social-isolation (i.e., self-quarantine or social-distancing) on the burden of the COVID-19 pandemic, a model framework that integrates COVID-19 transmission dynamics with a multi-strategy evolutionary game approach of individual decision-making is proposed and analyzed. The framework is used to characterize the evolution of human choices in social-isolation as the disease progresses and public health control measures such as mandatory lockdowns are implemented. Analysis of the model illustrates that social-distancing plays a major role in reducing the burden of the disease compared to self-quarantine. Parameter estimation using COVID-19 incidence data, as well as different lockdown data sets from India, and scenario analysis involving a combination of Voluntary-Mandatory implementation of self-quarantine and social-distancing shows that the effectiveness of this approach depends on the type of isolation, and the time and period of implementation of the selected isolation measure during the outbreak. (Received September 21, 2021)

1174-00-10597 **Po-Shen Loh*** (ploh@cmu.edu), Carnegie Mellon University. *Taking Math to YouTube, Social Media, and Public Parks*

For several years, the speaker has been experimenting with unusual methods of communicating math to general audiences. In this informal and interactive talk, he will share about his experiences building reality-TV-style video collaborations with Instagram influencers, co-creating YouTube videos totaling over 10 million views, running a live Q&A YouTube show answering math questions during the COVID-19 pandemic, and running a 2-month outdoor speaking tour in public parks throughout 40 cities across America. (Received September 21, 2021)

1174-00-10724 **Lauren Gala*** (milaur@upenn.edu), University of Pennsylvania. *LaTeX Collaborations: Expanding Graduate Student Support* Preliminary report.

This talk presents two case studies of cross-campus collaborations with a librarian to expand LaTeX services for graduate students while capitalizing on resources licensed by the Penn Libraries like MediaSpace and Overleaf. The first case study centers on a LaTeX online training course created for the Graduate Student Center. I will explain the process for determining learning objectives, designing practice exercises, and producing video lessons. Next, I will discuss the project to code a LaTeX Ph.D. dissertation template for the Graduate Division of the School of Arts and Sciences and review the project scope, roles, and expectations. In addition to describing the partnership with mathematics faculty to develop the rollout and a sustainable support model, I will report on the status of the template project. (Received September 21, 2021)

1174-00-10741 **Douglas James Dunham*** (ddunham@d.umn.edu), University of Minnesota Duluth, and **Lisa Shier** (kwajshier@yahoo.com), University of Maryland Global Campus. *An Escher-inspired fish pattern on the 6,6—3 polyhedron.* Preliminary report.

We describe a fish pattern on part of the regular triply periodic polyhedron {6,6—3} in the style of M.C. Escher. We previously designed a fish pattern on the {4,6—4} polyhedron in which the fish switched directions along backbone lines. The motivation for the new pattern was to fix that problem. In 1926 H.S.M. Coxeter and John Flinders Petrie proved that there were exactly three regular triply periodic polyhedra: {4,6—4}, {6,4—4}, and {6,6—3}. In general, the {p,q—r} polyhedron has regular p-sided polygonal faces meeting q at a vertex, with regular r-sided polygonal holes. We convinced ourselves that we could not design a pattern with fish swimming in consistent directions along backbone lines on either the {4,6—4} or the {6,4—4} polyhedron, hence our use of the {6,6—3} polyhedron.

As with our previous {4,6—4} polyhedron, the fish along backbone lines on the {6,6—3} polyhedron are all the same color and swim in the same direction. There are six families of parallel backbone lines embedded in the {6,6—3}, each parallel to one of the pairs of face diagonals of a cube. (Received September 21, 2021)

1174-00-10976 **Joachim Mueller-Theys*** (Mueller-Theys@gmx.de), independent researcher. *On the Relations Induced by Partitions*

The equivalence relation induced by a partition $\mathcal{P}$ of M in standard manner may be understood as *(alternative) similarity* $a \sim_{\mathcal{P}} b$ defined by $\exists P \in \mathcal{P}\ a \sim_P b$, whereby $a \sim_P b$:iff $P(a)$ & $P(b)$ ($a, b \in M$, $P(a)$:iff $a \in P$). We found out that $\sim_{\mathcal{P}}$ then coincides with *(universal) equality* $\equiv_{\mathcal{P}}$, $a \equiv_{\mathcal{P}} b$ defined by $\forall P \in \mathcal{P}\ a \equiv_P b$, whereby $a \equiv_P b$:iff $P(a) \Leftrightarrow P(b)$. In addition to this new characterization, the $\mathcal{P}$-equalities subsume Leibniz's indiscernibility ($\mathcal{P} := \{P\colon P \subseteq M\}$), then coinciding with identity =, and $\equiv_{P/\mathcal{P}}$ make possible mathematical analysis and characterizations of statements like “all men are equal”. Many details may be found in the ASL-abstract. (Received September 21, 2021)

1174-00-10993 **Rachel Dennis*** (rdennis6@u.rochester.edu), University of Rochester. *An Analysis of Novel Data Acquisition Methods in Compton Tomography*

I introduce two methods for image reconstruction using Compton Tomography. Each method of data collection involves integrating over circular arcs passing through an object and using a filtered backprojection to reconstruct an image of the object. I model each method using a characteristic function to represent the object being imaged and analyze the artifacts which show up in the reconstruction. I discuss methods of reducing the artifacts to generate a more accurate image and evaluate the effectiveness of each method. (Received September 21, 2021)

1174-00-11242 **Darrah P Chavey*** (chavey@beloit.edu), Beloit College. *Symmetric Designs of Mirror Curves Inspired by African Sona*

The Cokwe (Tchokwe, Chokwe) people of Southwest Africa developed a drawing technique that creates curves within a grid of dots with “mirror” edges, combined with internal walls, where the curves bounce, following tightly constrained rules. These drawings are closely related to Celtic knots and Indian Kolam designs, but are even more mathematical than those traditions. The Cokwe tradition includes a strong preference for monolineal circuits (Eulerian circuits) and for symmetric drawings. Inspired by these designs, we were able to construct symmetric monolineal Sona drawings for essentially any size grid, where the grid itself had central, strip, or wallpaper symmetry, while using any of the rectangular symmetry groups (central, strip, or wallpaper). These results create dense infinite families of designs which we believe would meet the aesthetic criteria of the classic Cokwe artists. We discuss the ways we have used the search for such patterns as creative exercises in an ethnomathematics classroom. (Received September 22, 2021)

1174-00-11761 **Habibolla Latifizadeh*** (hl0034@mix.wvu.edu), School of Mathematical and Data Sciences, West Virginia University, **David Klinke II** (David.Klinke@mail.wvu.edu), WVU Cancer Institute, West Virginia University, **Audry Fernandez**, WVU Cancer Institute, West Virginia University, **Wentao Deng** (wdeng@ufl.edu), West Virginia University, and **Anika C. Pirkey** (anika.pirkey@mail.wvu.edu), West Virginia University. *Data-driven learning how oncogenic gene expression locally alters heterocellular networks.* Preliminary report.

In a healthy body, tissue repair and healing occur through a coordinated process of cellular proliferation, programmed cell death, migration, and immune surveillance. These biological processes are executed by different cell types, whose function is regulated by genes. In cancer, alteration in particular genes, called oncogenes, change how malignant cells carry out these processes. In humans, how these oncogenes then effect change in the composition and function of the variety of other cell types within a special tissue is unclear. Identifying how these networks of communicating cells are altered in disease is central for the rational design of new drugs. While the number of samples is large in the data for bulk (mixture of cells) gene expression, inferring cellular level networks using all genes as unique features is computationally intractable. Using gene signatures that uniquely define cell subsets as priors, digital cytometry can be used to deconvolute the mixture by projecting the transcriptomic space onto a smaller number of features that estimate the prevalence of stromal and immune cell types and the differentiation state of malignant cells present within the sampled tumor. Some features are normally distributed which is consistent with the assumptions that underpin conventional network inference algorithms. Following the Gaussian distribution of the features, causal Bayesian Networks can be potentially used as a mechanistic and probabilistic graphical framework that elucidate a directed graph-based representation not only to formulate uncertainty in the cellular networks, but to capture a deeper knowledge about conditional linear dependencies and causal influences among gene expression associated with oncogenesis and stromal and immune cell subsets. Therefore, we posit that combining digital cytometry with Bayesian Network inference can potentially create a tractable computational problem to infer how cell level networks are changed in disease state. (Received October 17, 2021)

1174-00-12170 **Megan J Chambers*** (mjchambe@ncsu.edu), North Carolina State University. *Approximate Bayesian computation for generating a three-dimensional structured tree model of the pulmonary arterial network.* Preliminary report.

The pulmonary arteries form a rapidly branching network that serves to perfuse the lungs and oxygenate blood. Pressure and flow in the main pulmonary artery can be determined by solving a multiscale 1D fluid dynamics model, where large arteries' dimensions are explicitly defined and small vessels are represented by self-similar structured trees. These trees are generated from parameters such as minimum vessel radius, radius scaling factors, vessel length-to-radius ratio, and branching angles. In this study, we aim to optimize the formation of the structured trees so that they closely resemble pulmonary arterial networks in CT scans. This is accomplished via approximate Bayesian computation, whereby the structured trees' parameters are extracted from distributions of observed parameter values, and summary statistics of the generated trees are compared to those from the data. Optimizing the creation of these trees would allow us to predict perfusion in lungs of patients with and without pulmonary hypertension, which has long been associated with changes in the arterial morphology. Additionally, these methods could be extended to characterize other physiological networks and inform fluid dynamics models in other organs. (Received November 3, 2021)

01 ► History and biography

1174-01-5214 **Edray Goins*** (ehgoins@mac.com), Pomona College. *Addressing Anti-Black Racism in Our Departments.*

In April 2021, the PBS Newshour ran a story with the headline "Even as colleges pledge to improve, share of engineering graduates who are Black declines". Indeed, there is a dearth of Black students in our mathematics classrooms. A 2018 study by the Pew Research Center found that Black students earned just 7 percent of STEM bachelor's degrees. Unfortunately, this is an issue for our faculty as well. A 2017 report in Inside Higher Ed states that there has been an increase over time in the diversity of senior and junior faculty members in the STEM fields—except black faculty. A New York Times article, titled "For a Black Mathematician, What It's Like to Be the 'Only One' ", quoted that there are just a dozen black mathematicians among nearly 2,000 tenured faculty members in the nation's top 50 math departments.

What can we as faculty members do to make our mathematics departments more welcoming and diverse for Black students and faculty alike? These are daunting problems, and many with an interest in presenting solutions do not even have tenure! In this interactive presentation, we present some practices that even tenure-track faculty can engage in to showcase how #BlackLivesMatter—from increasing the number of pathways for majors, to building community by conducting research with students, and having hard conversations within hiring committees. (Received November 15, 2021)

1174-01-5535 **Lawrence Arthur D'Antonio*** (ldant@ramapo.edu), Ramapo College. *Mr. Newton's Approbation: John Flamsteed and Isaac Newton* Preliminary report.

In 1675, Flamsteed became the first Astronomer Royal of England. He is perhaps most famous for his star catalog *Historia Coelestis Britannica*. Newton used (and sometimes abused) Flamsteed to obtain observations of the moon, which Newton needed for his lunar theory. In this talk we will examine their complicated relationship. Flamsteed sought Newton's approbation until it became clear that it would never be forthcoming. (Received August 21, 2021)

1174-01-5583 **Shoo Seto** (shoseto@fullerton.edu), California State University, Fullerton, and **Bogdan D. Suceava*** (bsuceava@fullerton.edu), California State University Fullerton. *Kentaro Yano (1912-1993) and Tadashi Nagano (1930-2017) : Two Geometers and Their Works*

Kentaro Yano is today remembered for his contributions on geometric structures on differential manifolds, mostly published after 1947. His first doctoral thesis was written under Elie Cartan's coordination before 1938 and inspired a lifelong direction of research including differential geometry on complex and almost complex spaces, or integral formulas in Riemannian geometry, among other topics. Besides an impressive body of mathematical work, Kentaro Yano was interested in writing for large audiences, and we will describe his interest and his books written in Japanese. Tadashi Nagano defended his doctoral dissertation under Kentaro Yano's supervision at University of Tokyo in 1959 and became a very influential and respected geometer. His body of work includes 10 papers written with Shoshichi Kobayashi. We will present his life and work, his recognitions and his lasting influence. Our presentation concludes with an overview of the volume *Differential Geometry and Global Analysis. In Honor of Tadashi Nagano*, edited by B.-Y. Chen, N.D. Brubaker, T. Sakai, B.D. Suceavă, M. Sumi Tanaka, H. Tamaru, and M.B. Vajiac, to appear as Vol. 777 in the *Contemporary Mathematics* series of the American Mathematical Society (expected to be released in 2022). (Received August 23, 2021)

1174-01-6938 **Davide Rizza*** (d.rizza@uea.ac.uk), University of East Anglia. *The origins of model theory: reflections on Mal'tsev's contribution* Preliminary report.

Model theory is often regarded as a natural development Tarski's formal semantics, as presented in his celebrated 1933 paper.

However seemingly plausible, this account of the birth of model theory is immediately at variance with some significant facts: key contributions to the subject predate Tarskian formal semantics and even some fundamental results of Tarski's (e.g. his work on real closed fields) did not depend on his 1933 paper.

We propose to look at the early history of model theory along different lines. In particular, we suggest that this area of research grew out of several attempts at articulating general methods for the study of mathematical problems in terms of the duality between linguistic conditions and contexts of enquiry in which these conditions subsist.

We articulate this suggestion by revisiting the classic contributions by A. I. Mal'tsev on compactness and local theorems in group theory. (Received September 11, 2021)

1174-01-7200 **Glen R. Van Brummelen*** (glen.vanbrummelen@twu.ca), Trinity Western University. *The Tangled Tale of the Tangent*

The tangent function in trigonometry has a long and complicated history: depending on how one defines its invention or discovery, one might choose a birth date anywhere between antiquity and the 18th century. We shall explore the emergence of the tangent in several cultures (especially Greece, Islam, China, and the West) when something like the tangent emerged, discussing shades of meaning in the identification of moments of discovery. We conclude with a new assertion of where the journey began that led to the tangent function we use today, within mathematical astrology in Italy in the 15th century. (Received September 13, 2021)

1174-01-7320 **Abe Edwards*** (aedwards@msu.edu), Michigan State University. *(Not) Green's Theorem: The Curious History of a Theorem and its Pedagogical Implications*

In 1828 the English mathematician George Green (1793 – 1841) published *"An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism"*. In this *Essay*, an unknown miller from Nottinghamshire established the first mathematical theories of electricity and magnetism, introduced potential functions, and laid the foundation for later work by Maxwell, Thompson, and others. One thing Green did *not* do in his 1828 essay was discuss the theorem that now bears his name. In this talk, we explore the treatment of Green's Theorem in several 19th century manuscripts including *"On Integrals that Extend Over All of the Points of a Closed Curve"* (Cauchy, 1846) and *"Foundations for a General Theory of Functions of a Complex Variable"* (Riemann, 1851). In addition to being a fascinating story of how mathematical truth is established over time, the evolution of Green's Theorem provides compelling reasons for instructors to incorporate primary source texts in the undergraduate mathematics classroom. (Received September 14, 2021)

1174-01-7555 **Maria R Zack*** (mzack@pointloma.edu), Point Loma Nazarene University. *Barbaro and Vitruvius: A Mathematician's Translation of an Architectural Text* Preliminary report.

Marcus Vitruvius Pollio, known as Vitruvius, was a first century BC Roman architect and engineer best known for his work *De architectura*. This multi-volume text discusses standard architectural forms in the ancient world. Vitruvius' work was "rediscovered" in the sixteenth century and was one of the books that fed the interest of symmetry and classical forms in European architecture, particularly in the work of architects such as Andrea Palladio. The cleric, architect, and student of mathematics, Daniel Barbaro translated *De architectura* into Italian in 1567. Vitruvius' original text contained almost no calculations, however Barbaro's commentary on *De architectura* contains a variety of mathematical topics. This talk will look at some of the mathematics in Barbaro's version of Vitruvius. (Received September 15, 2021)

1174-01-7556 **Amy Ackerberg-Hastings*** (aackerbe@verizon.net), MAA Convergence. *HoM Toolbox: Historiography and Methodology for Mathematicians* Preliminary report.

How and where do mathematicians learn to research and write history? How easily can they find out about current methods and discussions by historians of mathematics and science? *MAA Convergence* is helping address both questions with a new article series that will give an overview of professional practice in history and provide examples of how historians of mathematics have applied specific methods and theories of historical interpretation. The initial installment considers:

(1) What is history? Why should readers want to research and write it well?
(2) How do we know about the past?
(3) How do we create history based on what we know about the past?
(4) What is the history of the history of mathematics?
(5) How can readers articulate their own philosophies of the history of mathematics?

This talk will focus on the fifth activity as an exercise that not only encourages researchers, educators, and students to consume quality scholarship in the history of mathematics but also calls them to join the effort to produce original research in this field. I will end the presentation by asking for audience feedback on what mathematicians new to the endeavor need to know to research and write the history of mathematics. (Received September 15, 2021)

1174-01-7575 **Brigitte Stenhouse*** (bstenhouse17@gmail.com), University of Oxford. *Choosing to publish a 'popular' work: Three books by Mary Somerville advocating for analytical mathematics in 1830s Britain*

In her 1831 book *Mechanism of the Heavens*, Mary Somerville actively promoted the adoption of analytical methods both to mathematicians and to those natural philosophers who were not mathematically literate. She argued that a true understanding of physical astronomy — the motions and shapes of the planets, moons, and comets — could only be reached by those who understood mathematical analysis. However, although she continued to publish articles and books on the physical sciences, none of Somerville's subsequent works spoke directly to mathematicians nor did they prioritise mathematical questions. This is somewhat surprising considering that Somerville had explicitly depicted analytical mathematics as a fertile ground waiting to be farmed.

In this talk I will give an overview of two book-length manuscripts first written by Somerville in the 1830s — one a volume on the form and rotation of planets, the other an analytical work on curves and surfaces. Somerville evidently spent large amounts of time on these technical manuscripts, which include hundreds of hand-drawn diagrams, but in 1834 they were left by the wayside and she instead brought a survey of recent scientific advancements to press in which all the mathematical formulae had been removed. By considering the market for mathematical books at the time, alongside Somerville's place in the scientific community as a self-educated woman, I will demonstrate why these manuscripts remained unfinished and unpublished. (Received September 15, 2021)

1174-01-7609 **Sarah Glaz*** (sarah.glaz@uconn.edu), The University of Connecticut. *Enheduanna—Princess, Priestess, Poet and Mathematician*

This talk is based on the paper by the same title, which appeared in The Mathematical Intelligencer, Summer 2020 issue. The paper brings together a variety of poems and poem fragments written in ancient Mesopotamia that allow us to understand the origins of mathematics in its historical, social and cultural context. Enheduanna, Mesopotamian high priestess of the moon god, Nanna, and daughter of King Sargon, is the first recorded author in history. The talk is organized around my translations of four of Enheduanna's temple hymns, each one of which highlights a different aspect of early mathematics. Among these, Temple Hymn 42, stands out as one of the oldest historical sources which holds a mirror to what mathematics meant for the people of that time period, and also depicts the mathematicians of the day and suggests what their job was. The poetry of ancient Mesopotamia portrays mathematics as an important activity, one worth mentioning, perhaps even bragging about, in a religious hymn—an activity that was not separate from hymn writing or from spiritual duties and observances. The paper considers the possibility that Enheduanna herself might had been a mathematician, and concludes with my poem "Enheduanna"—a tribute to this extraordinary woman. (Received September 15, 2021)

1174-01-7627 **Karen H Parshall*** (khp3k@virginia.edu), University of Virginia. *Topology in 1930s America: A Tale of Two "Camps"*

By the 1930s, two rival "camps" of topology had evolved in the United States: point-set topology animated by Robert L. Moore at the University of Texas and combinatorial (or algebraic) topology fostered by Oswald Veblen and, especially, Solomon Lefschetz at Princeton. This talk will sketch the contours of the American topological landscape in the decade before the outbreak of World War II and highlight the differences and divisions between the adherents of the two approaches as they jockeyed for influence within the American mathematical research community. (Received September 15, 2021)

1174-01-7697 **Brenda Davison*** (bdavison@sfu.ca), SFU. *Poincaré, Celestial Mechanics and Divergent Series* Preliminary report.

In the early 1890s, Henri Poincaré (1854-1912) published a three volume work titled "New Methods of Celestial Mechanics". In part, this work reconciled the ad hoc asymptotic expansions used by astronomers with the more rigorous and different understanding of convergence of the pure mathematician. This talk will examine what Poincaré contributed to the theory of asymptotic expansions in the "New Methods" and in the few papers that he published on this topic just before "New Methods". I will also comment on how this work helped to establish asymptotic expansions as an important mathematical tool. (Received September 15, 2021)

1174-01-8888 **Thomas L Drucker*** (druckert@uww.edu), University of Wisconsin–Whitewater. *John Horton Conway as Historian* Preliminary report.

John H. Conway was one of the most creative mathematicians of the second half of the twentieth century. His death during the pandemic was a loss to the mathematical community, even if his ill health had recently limited his ability to take part in its activities. Fortunately, he was able to see the publication of Siobhan Roberts's biography of him (Genius at Play, 2015), a tribute both to his accomplishments and his distinctive personality. This talk will present a view of Conway's attitude toward history and some of the strong feelings he had about issues from Greek mathematics forward. It is based on conversations, correspondence, and published work of Conway. (Received September 20, 2021)

1174-01-9135 **Colin B McKinney*** (mckinnec@wabash.edu), Wabash College. *Distilling the Canon: Curating a Sourcebook of Greek Mathematics* Preliminary report.

I've been working jointly with Victor Katz and Clemency Montelle on a Sourcebook of Greek Mathematics, to join the series of two other sourcebooks edited by Victor. This talk will pull back the curtain a bit on how we've set about to curate and distill nearly a thousand years of mathematics into a single volume. What to include, what not to include? How technical should material be? How have we worked with a variety of sources, some in English, others in older languages? How have we handled diagrams and labeling? Using examples from our work, I'll try to shed light on this monumental task. (Received September 20, 2021)

1174-01-9386 **Edray Herber Goins*** (ehgoins@mac.com), Pomona College, and **Robin Todd Wilson** (robinwilson@cpp.edu), California State Polytechnic University, Pomona. *MADDER: Mathematicians of the African Diaspora Database's Ensemble of Researchers* Preliminary report.

In 1997, Scott Williams (SUNY Buffalo) founded the website "Mathematicians of the African Diaspora," which has since become widely known as the MAD Pages. Williams built the site over the course of 11 years, creating over 1,000 pages by himself as a personal labor of love. The site features more than 700 African Americans in mathematics, computer science, and physics as a way to showcase the intellectual prowess of those from the Diaspora.

Soon after Williams retired in 2008, Edray Goins (Pomona College), Donald King (Northeastern University), Asamoah Nkwanta (Morgan State University), and Weaver (Varsity Software) have been working since 2015 to update the Pages. Edray Goins and Robin Wilson (Cal Poly Pomona) led a research group of 13 undergraduates during the 2021-22 academic year to write more biographies for the new MAD Pages.

In this talk, we discuss the results from MADDER (Mathematicians of the African Diaspora Database's Ensemble of Researchers), recalling some stories of the various biographies of previously unknown African American mathematical scientists, and reflecting on some of the challenges of running a math history REU. This project is funded by CURM. (Received September 20, 2021)

1174-01-10311 **Julia Tomasson*** (jct2182@columbia.edu), Columbia University. *The 'Quaestio de certitudine mathemaricarum,' Ibn Rushd and Islamic Mathematics* Preliminary report.

The so-called 'Quaestio de certitudine mathematicarum' debate of the sixteenth and seventeenth centuries is a rich site of inquiry for those interested in the early modern exact sciences, the history of mathematical proof, and the cultural history of mathematics more broadly. In the Quaestio, mathematicians—mostly associated with the University of Padua—argued whether the discipline of mathematics could claim that mathematical proofs are 'certain' in an Aristotelian logical framework, and if not, what types of proofs ought mathematicians to use to ensure certainty. While this debate is framed as a reckoning with Aristotle, the first citation in the first sentence of the first tract of the debate (Piccolomini, 1547) was not to Aristotle but to the Muslim polymath Averroes (Latin name for Ibn Rushd, 1126-1198 CE). Moreover, much of the debate is a discussion of Latin commentaries on Averroes' interpretations of Aristotle. Is this just a philosophical or rhetorical connection? Many Ottoman scholars were training at the University of Padua at this time. How did this effect the culture and practice of mathematical learning, teaching, and research? Were there similar debates happening in Arabic commentaries of Averroes work on mathematics and philosophy of mathematics that we might include in a more robust history of the Quaestio? (Received September 21, 2021)

1174-01-10334 **Cain Edie-Michell*** (cain.edie-michell@vanderbilt.edu), UC San Diego, and **Masaki Izumi** (izumi@math.kyoto-u.ac.jp), Kyoto. *Classification results for quadratic fusion categories*

Quadratic fusion categories are fusion categories whose simple objects are a finite group, and a single orbit of non-invertibles under the group action. These categories provide the largest class of *sporadic* fusion categories currently known to the community. I will describe some recent progress in the classification of quadratic fusion categories. (Received September 21, 2021)

1174-01-10363 **Moira Chas*** (moira.chas@stonybrook.edu), Stony Brook University. *What do Alicia Boole Stott, Ancient Indian Poets and Coloring Maps have in common?* Preliminary report.

Alicia Boole Stott was a Victorian housewife who developed a deep grasp of four dimensional geometry and managed to express it with marvelous drawings and models.

Ancient Indian Poets counting the rhythms of verses found sequences of integers which are very familiar to nature, to art and to mathematicians.

The history of coloring maps is intricate, colorful and occasionally confusing but it will be hopefully clarified during this talk with the help of crocheted mathematical art.

These are just three instances of the enriching dialog of mathematics and art. (Received September 21, 2021)

1174-01-10456 **Dwight Anderson Williams*** (dwight@mathdwight.com), MathDwight. *The ABCs of Pandemic Postdoc-ing*

From algebras to Zoom, we take a moment to review some local and global events in recent history, to shout out a network of people that have built up my community (and maybe yours, too!), and to ask some questions without ready-made answers. No, this talk is not to teach anyone on how to be a successful postdoc for any amount of time, especially during a pandemic. This talk is for therapy with the hope of connecting with good people more than six feet away in 20 minutes or less. (Received September 21, 2021)

1174-01-11045 **Brittany Carlson*** (bcarl005@ucr.edu), University of California, Riverside. *Villainizing the Woman Mathematician in Nineteenth-Century Literature*

The nineteenth-century ushered in a wave of villainizing mathematics and mathematicians. While popular printed media villainized a few masculine figures for their superior mathematical abilities, such as the infamous Professor Moriarty, the overwhelming majority of villainized nineteenth-century mathematicians were women. In this talk, I will survey various literary sources that defile the reputation of women mathematicians. For example, Charles Dickens's Hard Times, Charles Kingsley's Hypatia, and George Bernard Shaw's Mrs. Warren's Profession portray successful women mathematicians as cold and calculating, incapable of feeling love and acquiring true happiness when, in reality, the only thing these women are incapable of is adhering to nineteenth-century gender norms. Studying this literary archive exposes many anxieties about women mathematicians. Acknowledging and understanding these anxieties about the historical woman mathematician is vital for understanding why women were barred from mathematical study and are still minoritized in the field today. (Received September 21, 2021)

1174-01-11228 **Daniel J Fairbanks*** (daniel.fairbanks@uvu.edu), 800 W University Parkway, Department of Biology, Utah Valley University. *Gregor Mendel's experiments: Does smpling without replacement and the pollen-tetrad model of DNA segregation explain the bias?*

The data of Gregor Mendel founded the science of genetics. He described the units of heredity as "formative elements" ("bildungsfähigen Elemente"), now known as genes, of a material nature ("materiellen Beschaffenheit") now known as DNA. He determined, based on observed mathematical ratios, that these formative elements "reciprocally segregate themselves" during formation of the reproductive cells, a process now known as meiosis. This segregation produces predictable ratios in inheritance, reported by Mendel. His results, however, have been criticized as early as 1902, and especially by Sir Ronald Fisher in 1936, for their consistent bias toward expectation. Some have gone so far as to accuse Mendel of deliberate fraud. Others have have defended Mendel, claiming that sampling without replacement of DNA molecules during fertilization explains the bias. The pea plant provides an ideal situation for testing this model experimentally. This presentation examines the results of this experimental testing and their implications for sampling without replacement in the pollen-tetrad model. These results indicate that the pollen-tetrad model and sampling without replacement fail to explain the bias in Mendel's experiments. (Received September 22, 2021)

1174-01-11753 **Emelie A. Kenney*** (kenney@siena.edu), Siena College. *In their own right: Polish women mathematicians whose husbands have enjoyed more recognition* Preliminary report.

Often, we are familiar with the mathematical work of male relatives of women mathematicians, but less so with the women themselves. In this paper, we focus on some achievements of several Polish women whose careers as mathematicians spanned a period in 20th century history that created great difficulties for them and for their more famous spouses. Any list of such women must include Stanislawa Nikodym, Maria Matuszewska, Zofia Mikolajska, and Janina Hosiasson-Lindenbaum. (Received October 7, 2021)

03 ► Mathematical logic and foundations

1174-03-5589 **James Walsh*** (jameswalsh@cornell.edu), Cornell University. *On the hierarchy of natural theories*

Gödel famously showed that there is no universal axiomatic theory for the development of arithmetic. Instead, we are left with a vast array of ever stronger competing axiomatic theories. Perhaps the main tool for comparing these axiomatic theories is by measuring their relative consistency strength. It is a well-known empirical phenomenon that it is always possible to compare *natural* axiomatic theories by relative consistency strength, even though axiomatic theories are not generally comparable according to consistency strength. Why are the natural axiomatic theories linearly ordered by consistency strength? Without a precise mathematical definition of naturalness, it is unclear how to study this question mathematically. I will discuss some strategies for addressing this problem that have been developed recently. (Received August 23, 2021)

1174-03-7103 **Aaron William Anderson*** (aaronanderson@math.ucla.edu), University of California, Los Angeles. *Combinatorial Bounds in Distal Structures*

We provide polynomial upper bounds for the minimal sizes of distal cell decompositions in several kinds of distal structures, particularly weakly o-minimal and P-minimal structures. The bound in general weakly o-minimal structures generalizes the vertical cell decomposition for semialgebraic sets, and the bounds for vector spaces

in both o-minimal and p-adic cases are tight. As an application, we prove some sum-product bounds for distal fields, and bound the number of distinct distances determined by n points in $\mathbb{Q}_p^2$. (Received September 12, 2021)

1174-03-7219 **Roland Walker*** (rwalke20@uic.edu), University of Illinois at Chicago. *Distality Rank*
Building on Pierre Simon's notion of distality, we introduce distality rank as a property of first-order theories and give examples for each rank m such that $1 \leq m \leq \omega$. For NIP theories, we show that distality rank is invariant under base change. We also define a generalization of type orthogonality called m-determinacy and show that theories of distality rank m require certain products to be m-determined. Furthermore, for NIP theories, this behavior characterizes m-distality. If we narrow the scope to stable theories, we observe that m-distality can be characterized by the maximum cycle size found in the forking "geometry," so it coincides with $(m-1)$-triviality. On a broader scale, we see that m-distality is a strengthening of Saharon Shelah's notion of m-dependence. (Received September 13, 2021)

1174-03-7242 **Nicholas Ramsey** (nickramsey@math.ucla.edu), University of California, Los Angeles, and **Alex Kruckman*** (akruckman@wesleyan.edu), Wesleyan University. *Kim's Lemmas and Tree Properties* Preliminary report.
One of the most important steps in the development of simplicity theory by Kim and Pillay in the 1990s was a result now known as Kim's Lemma: In a simple theory, if a formula divides, then this dividing is witnessed by every Morley sequence in the appropriate type. More recently, variants on Kim's Lemma have been shown (by Chernikov, Kaplan, and Ramsey) to follow from, and in fact characterize, two generalizations of simplicity in different directions: the combinatorial dividing lines NTP2 and NSOP1. After surveying the Kim's Lemmas of the past, I will speculate about a new variant of Kim's Lemma, and a corresponding new tree property, which generalizes both TP2 and SOP1. This is joint speculation with Nick Ramsey. (Received September 23, 2021)

1174-03-7641 **Diego Antonio Rojas*** (darojas@iastate.edu), Iowa State University. *Effective vague convergence of measures on the real line* Preliminary report.
Recently, McNicholl and Rojas developed a framework to study the effective theory of weak convergence of measures on $\mathbb{R}$. In this talk, we introduce a similar framework to study the effective theory of vague convergence of measures on $\mathbb{R}$. In particular, we define two effective notions of vague convergence of measures in $\mathbb{R}$ and show that they are equivalent. However, unlike effective weak convergence, we show that an effectively vaguely convergence sequence need not have a computable limit. Nevertheless, we show that for a computable sequence $\{\mu_n\}_{n\in\mathbb{N}}$ of measures in $\mathbb{R}$, effective weak and vague convergence of measures coincide whenever $\{\mu_n(\mathbb{R})\}_{n\in\mathbb{N}}$ has a computable modulus of convergence. (Received September 17, 2021)

1174-03-7919 **Neer Bhardwaj*** (nbhard4@illinois.edu), University of Illinois at Urbana-Champaign. *On the Pila-Wilkie theorem*
I'll give an account of the Pila-Wilkie counting theorem and some of its extensions and generalizations. I'll talk in some detail about our approach where we use semialgebraic cell decomposition to simplify part of the original proof. Only very basic knowledge of o-minimality will be assumed; this is joint work with Lou van den Dries. (Received September 16, 2021)

1174-03-7972 **Ruggero Ferro** (ruggero.ferro@univr.it), Retired from University of Verona, Italy, and **Alejandro Javier Cuneo*** (Alejandro.cuneo@unc.edu.ar), University of Cordoba, Argentina. *An unorthodox Philosophy of Mathematics*
The classroom experience in teaching mathematics is a source about how we know mathematics, and the adequateness to teaching could be a useful criterion in choosing among different foundations.

The development of student's knowledge puzzles us about the idea of mathematics either as a set of inborn notions or of notions taken from an unreal world existing somewhere. The empirical attitude is closer to the way of knowing, but its traditional presentation has difficulties in justifying non-concrete notions. These notions are part of an evolving model of reality created by humans, clashing with a static, preexistent, not experienceable world. Such models of reality become more complex to face the complication of reality, yielding to the impossibility of grasping at once all its features, that will have to be discovered by future scholars.

We are proposing to upgrade the traditional empiricism by considering not only the usual five senses but also a few well determined internal senses. These, in addition to memory and to specific mental operations producing further perceptions, are adequate to construct mathematical notions.

Cfr. A.J. Cuneo, R. Ferro: From the Classroom: Towards A New Philosophy of Mathematics in MAA Notes 86 Using the Philosophy of Mathematics in Teaching Mathematics. (Received September 20, 2021)

1174-03-8058 **Michael A Shulman*** (shulman@sandiego.edu), University of San Diego. *Complementary foundations for mathematics: when do we choose?*

If one foundation for mathematics were clearly the most true or useful, there would be no debate. Instead, each foundation is well-suited for some purposes and ill-suited for others, as when we compare quantum mechanics and general relativity, or C++ and Python. Thus the question is not how to choose a foundation once and for all, but when to choose one foundation and when to choose another; and we can only answer this with examples. For instance, ZF set theory is good at studying well-foundedness; categorical set theory is good at relating different mathematical universes; constructive mathematics is good at continuity and computability; and homotopy type theory is good at invariant higher structures. (Received September 17, 2021)

1174-03-8127 **Alexi Block Gorman*** (atb2@illinois.edu), The Fields Institute. *Characterizing Tameness and Definability from k-regular Subsets of* $\mathbb{R}$ Preliminary report.

Büchi automata are the natural analogue of finite automata in the context of infinite strings (indexed by the natural numbers) on a finite alphabet. We say a subset X of the reals is k-regular if there is a Büchi automaton that accepts (one of) the base-k representations of every element in X, and rejects the base-k representations of each element in its complement. These sets often exhibit fractal-like behavior—e.g., the Cantor set is 3-regular. There are remarkable connections in logic to Büchi automata, particularly in model theory. In this talk, I will give a characterization of when the expansion of the real ordered additive group by a predicate for a closed k-regular subset of [0,1] is model-theoretically tame (d-minimal, NIP, NTP2). Moreover, I will discuss how this coincides with geometric tameness, namely trivial fractal dimension. This will include a discussion of how the properties of definable sets vary depending on the properties of the Büchi automaton that recognizes the predicate subset. (Received September 17, 2021)

1174-03-8656 **Daniel Meskill** (dmeskill@unc.edu), University of North Carolina - Chapel Hill, **Marie Neubrander*** (mneubrander@crimson.ua.edu), University of Alabama, **Yiyao Zhang** (zhan3206@gmail.com), Purdue University, and **Anh Doan** (doana@hollins.edu), Hollins University. *Detecting Small Multi-Set Differences Efficiently for Ads Data Privacy* Preliminary report.

Advertisers in internet advertising access data about their ad campaigns across devices through secure, privacy-centric environments. To further improve the security protections for these environments, constraints against SQL queries involving multiple sets, or multi-sets, are in place. The purpose of this project is to develop efficient strategies and data structures to better detect multi-set overlaps and differences in order to better protect user privacy when evaluating multi-set queries. In this session, we will show our investigation into algorithms for detecting 3+ set differences. We developed two algorithms: the maximum-ID method and the reduced row echelon form (RREF) linear algebra-based method. The maximum-ID method uses element frequencies, or the number of times an element appears in a set, as a means to detect multi-set differences. The RREF method formulates queries and sets operations as vectors and matrices and converts the detection into a problem of matrix operations. For each method, we put forth theoretical analyses demonstrating privacy violations always caught coupled with experimental results highlighting additional potential use cases. (Received September 19, 2021)

1174-03-8739 **Jonathan Lenchner*** (jon.lenchner@gmail.com), IBM T.J. Watson Research Center. *Computational Complexity through the Informational Lens: A Glimpse into Descriptive Complexity via a Game that Characterizes the Number of Quantifiers Needed to Express a Property in First Order Logic*

In this talk we will go through a few of the highlights of descriptive complexity, a field that looks at computational complexity through the informational lens. The talk will begin with a discussion of Fagin's Theorem, a famous result that kicked off the field in 1974 by showing that NP can be characterized by the type of logical formulas needed to describe the associated problems and without saying a word about time or even space complexity. The talk will then focus on descriptive and computational complexity results that have been obtained using different sorts of combinatorial games, including Ehrenfeucht-Fraïssé games, Fagin games, Ajtai-Fagin games and most recently, a multi-structural game that my colleagues and I have rediscovered. The games were originally conceived of by Neil Immerman back in 1981. The most recent work is joint with Ron Fagin, Ken Regan, Nikhil Vyas and Ryan Williams. (Received September 19, 2021)

1174-03-8887 **Alex Mennen*** (alexmennen@math.ucla.edu), UCLA. *Semi-equationality: a unification of equational and distal theories*

I will introduce two closely-related properties, which we've been calling "semi-equational" and "strongly semi-equational", which can be thought of as attempts to complete the analogy "stable : equational :: NIP : ?", or as one-sided versions of distality. The relationship between these and other model-theoretic properties will be discussed, and I will give some examples of (strongly) semi-equational theories, and describe some techniques for showing that theories are not (strongly) semi-equational. The results in this talk are joint work with Artem Chernikov. (Received September 20, 2021)

1174-03-9110 **Toby Meadows*** (meadowst@uci.edu), University of California, Irvine. *Foundations & Interpretability*

In this paper, I will discuss how interpretability might be used to resolve a number of debates between competing foundational systems. For example, it is well-known that the set theory ZFC is, roughly speaking, bi-interpretable with category theory $ETCS$ augmented with an axiom corresponding to Replacement. I suggest that the philosophical significance of this result depends greatly upon what we expect from a Foundation of Mathematics. Indeed results like these can be useful tools for uncovering those expectations. Making use of interpretability considerations, I will then present and discuss a somewhat standard argument for the claim that: if we accept that foundations should not hinder mathematical practice, then the large cardinal hierarchy provides the natural way of satisfying this demand. (Received September 20, 2021)

1174-03-9193 **Robert M. Anderson*** (robert.anderson@berkeley.edu), UC Berkeley, **M. Ali Khan** (akhan@jhu.edu), Johns Hopkins University, **Haosui Duanmu** (duanmuhaosui@berkeley.edu), Harbin Institute of Technology, UC Berkeley, and **Metin Uyanik** (m.uyanik@uq.edu.au), University of Queensland. *Extended Solutions of Economic Models with a Measure Space of Agents* Preliminary report.

Models with a measure space Ω of agents are ubiquitous in economic theory. We consider measure space models with widespread externalities, in which individual agents' preferences depend continuously on $L^1_+(\Omega)$ endowed with the norm topology. Equilibria may fail to exist in Ω because there are not "enough" measurable sets. We adapt a new nonstandard analysis technique developed for Markov Processes by Duanmu. Duanmu's technique is a powerful tool for extending known results for finite models to infinite models, including measure space models. We first show that equilibria exist in ${}^*L^1_+(\hat{\Omega})$, where $\hat{\Omega}$ is a hyperfinite partition of ${}^*\Omega$. The standard parts of such equilibrium allocations are in $L^1_+(\bar{\Omega})$, where $\bar{\Omega}$ is the Loeb space generated by $\hat{\Omega}$, and as such are outside the domain of the preferences in the original model. We extend the preferences to $L^1_+(\bar{\Omega})$ by taking the standard parts (in the usual topology on preferences) of the preferences on ${}^*L^1_+(\hat{\Omega})$. To our knowledge, this is the first example of such an extension of preferences to allocations on an expanded measure space. Under the stronger assumption that preferences are continuous in the weak topology generated by $L^\infty(\Omega)$ on $L^1(\Omega)$, the equilibria can be pushed down to equilibria in the original space $L^1_+(\Omega)$. (Received September 20, 2021)

1174-03-9273 **Mauro Di Nasso*** (mauro.di.nasso@unipi.it), University of Pisa, Italy. *Nonstandard natural numbers in arithmetic Ramsey Theory and topological dynamics* Preliminary report.

The use of hypernatural numbers ${}^*\mathbb{N}$ in nonstandard analysis has recently found several applications in arithmetic Ramsey theory. The basic observation is that every infinite number $\nu \in {}^*\mathbb{N}$ corresponds to an ultrafilter on $\mathbb{N}$, and the algebra of ultrafilters is a really powerful tool in this field.

Furthermore, hypernatural numbers are endowed with a natural compact topology, i.e. the S-topology, and one can apply the methods of topological dynamics on ${}^*\mathbb{N}$ considering the shift operator $S : \xi \mapsto \xi + 1$. This very peculiar dynamic has interesting characteristics.

In this talk I will survey some of the recent applications, with the aim of showing that the methods of nonstandard analysis can be used successfully in this area of combinatorics. In particular, I will present a new result in the style of Hindman's Theorem, on the existence of infinite monochromatic configurations in any finite coloring $\mathbb{N} = C_1 \cup \ldots \cup C_r$ of the natural numbers. A typical example is the following monochromatic pattern: a, b, c, $\ldots$, $a+b+ab$, $a+c+ac$, $b+c+bc$, $\ldots$, $a+b+c+ab+ac+bc+abc$, $\ldots$. (Received September 20, 2021)

1174-03-9413 **Wilfried Sieg*** (ws15@andrew.cmu.edu), Carnegie Mellon University. *Methodological Frames: Mathematical structuralism and proof theory*

The juxtaposition of mathematical structuralism (as practiced by Dedekind in the 1870s and 1880s) and proof theory (as articulated by Hilbert in the 1920s and Bernays in the 1930s) indicates a programmatic goal. I want to turn our attention from choosing between different "foundations" to focusing sharply on two central tasks in the philosophy of mathematics; namely, to understand the role of abstract structures in mathematical practice and the function of accessibility notions in methodological frames. Proof theory is playing a mediating role in such investigations. (Received September 20, 2021)

1174-03-9576 **David J. Webb*** (dwebb@math.hawaii.edu), University of Hawaii at Manoa. *Reducibilities between* MLR *and* Either(MLR) Preliminary report.

We investigate which reducibility notions suffice to output a (Martin-Löf) random real given a pair of input oracles, an unknown member of which is itself random. We demonstrate that truth-table reducibility suffices, showing that the classes of Kolmogorov-Loveland random reals and Martin-Löf random reals are truth-table Medvedev equivalent, answering a question of Miyabe. We also investigate whether even stronger reducibilities can be used, showing that positive, linear, and bounded truth-table reductions can fail to output randomness given such oracles. (Received September 20, 2021)

1174-03-9588 **John R Burke*** (jburke@ric.edu), Rhode Island College. *Strict Finite Foundations of Mathematics*

Strict Finitism is a minority view in the philosophy of mathematics. In this talk, we will give a historical overview of the development of the strict finite philosophical stance. We will review some proposals for strict finite arithmetic and strict finite logic and discuss its viability as a foundation for mathematics. We will then turn our attention to finitism in geometry and announce new research proposing a strict finite foundation for geometric constructions which applies classical logic. Topics which will also be touched on are fictionalism, operationalism, and automated theorem provers. (Received September 20, 2021)

1174-03-9633 **David A. Ross*** (ross@math.hawaii.edu), Department of Mathematics, University of Hawaii, Honolulu, HI 96822. *Asymptotic fixed points and the nonstandard hull of a metric space*

An important result about asymptotic fixed points on a metric space can be obtained in an unexpected (but natural) way by applying a classical result about fixed points to a suitable nonstandard hull. (Received September 20, 2021)

1174-03-9748 **Lavinia Ciungu*** (lciungu@sfc.edu), St Francis College. *Valued Quantum B-algebras* Preliminary report.

Rump and Yang introduced the concept of quantum B-algebras, and proved that the quantum B-algebras can provide a unified semantic for non-commutative algebraic logic. Almost all implicational algebras studied so far (pseudo-effect algebras, residuated lattices, pseudo MV/BL/MTL-algebras, bounded non-commutative R'-monoids, pseudo-hoops, pseudo BCK/BCI-algebras) are quantum B-algebras. In this presentation, we de ne and study the pseudo-valuations on unital quantum B-algebras. The positive, strong and commutative pseudo-valuations are defied, and it is proved that the kernel of a positive strong and commutative pseudo-valuation is a commutative filter. If a unital quantum B-algebra X is integral, we prove that all pseudo-valuations of X are commutative. We show that there is a correspondence between the positive strong pseudo-valuations and the filters of a unital quantum B-algebra. Special results are proved for particular classes of quantum B-algebras, such as involutive quantum B-algebras and quantum B-algebras with product. An extension theorem is proved for the pseudo-valuations defined on the subalgebra of regular elements of a pointed quantum B-algebra. Finally, we de ne the notion of perfect pseudo-valuations on quantum B-algebras with product and we investigate certain of their properties. (Received September 20, 2021)

1174-03-9810 **James Hanson*** (jhanson9@umd.edu), University of Maryland, College Park. *Generic stability and randomizations*

I will discuss some current research on the relationship between generic stability of a type in a randomization and properties of the corresponding type or measure in the original theory. (Received September 20, 2021)

1174-03-10010 **Aniruddha Ghosh** (aghosh23@jhu.edu), The Johns Hopkins University. *Existence of Berk-Nash Equilibrium via Nonstandard Analysis*

In an economic dynamic optimization problem, the agent faces a Markov decision process, where a transition probability function determines the evolution of a state variable as a function of the previous state and the agent's action. Esponda and Pouzo introduced an equilibrium concept, known as the Berk-Nash equilibrium, to describe the steady states of the agent's learning dynamics when the agent is a Bayesian learner with a misspecified model. They establish the existence of the Berk-Nash equilibrium for regular Markov decision processes with finite action and state spaces. We extend their results to regular Markov decision processes with compact action and state spaces using nonstandard analysis. (Received September 21, 2021)

1174-03-10116 **Omer Ben-Neria** (omer.bn@mail.huji.ac.il), Einstein Institute of Mathematics, Jerusalem, and **Thomas D Gilton*** (tdg25@pitt.edu), University of Pittsburgh. *Club Stationary Reflection and the Special Aronszajn Tree Property*

A fruitful line of research in set theory investigates the tension between compactness and incompactness principles, in particular, when principles in these categories are in fact jointly consistent. In a recent result with Omer Ben-Neria, we have established such a joint consistency result, showing that Club Stationary Reflection is consistent with the Special Aronszajn Tree Property on the cardinal ω_2.

The tension between these two principles shows up in the very different properties of our posets which we must maintain throughout the course of our construction. We first introduce the idea of an $\mathcal{F}_{\mathrm{WC}}$-*Strongly Proper* poset ($\mathcal{F}_{\mathrm{WC}}$ is the weakly compact filter). These posets use systems of continuous residue functions to witness strong genericity. We then show how to specialize trees on ω_2 following such a forcing, generalizing the work of Laver and Shelah. We also show that the composition of Levy collapsing a weakly compact followed by our club adding is in this class. Additionally, we develop new ideas for preserving Aronszajn trees and for stationary sets which do not make use of the usual closure assumptions. For instance, we show that our club adding posets don't add branches to Aronszajn trees of interest and that quotients of the specializing forcing preserve stationary sets of countable cofinality. In this talk we will survey these posets and sketch our proof of specializing, as well as proofs our preservation theorems. (Received September 21, 2021)

1174-03-10426 **Kyle Gannon*** (gannon@math.ucla.edu), UCLA. *Convoluted Dynamics*

This talk is about NIP groups and the dynamical systems associated with definable convolution. If T is an NIP theory expanding a group, $\mathcal{G}$ is a monster model of T and G is a small elementary submodel of $\mathcal{G}$, then both the space of global automorphism invariant measures over G as well as the space of global finitely satisfiable measures over G form left continuous compact Hausdorff semigroups under definable convolution. This allows one to study these objects through the lens of Ellis theory. We discuss how to find and describe the minimal ideals of these semigroups.

This is joint work with Artem Chernikov. (Received September 21, 2021)

1174-03-10604 **Daniel C Bowerman*** (dcbhz9@mst.edu), Missouri University of Science and Technology. *Nonstandard Methods Applied to Loops* Preliminary report.

The Abelian Inner Mapping (AIM) Conjecture is an ongoing problem in loop theory that has been worked on by mathematicians for over a decade. In this paper, we look at how we can apply ultraproducts and nonstandard methods to find cases when infinite loops satisfy the AIM conjecture and cases where we can generate infinite loops that satisfy the AIM conjecture. (Received September 21, 2021)

1174-03-10661 **Caroline Terry*** (terry.376@osu.edu), The Ohio State University. *Problems in model theory and combinatorics*

In this talk we discuss some recent progress and open problems in model theoretic combinatorics. (Received September 21, 2021)

1174-03-10676 **Gabriel Conant*** (conant.38@osu.edu), The Ohio State University. *Tame regularity and continuous model theory*

I will discuss recent research on continuous analogs of stable regularity. This is will include joint work with Nicolas Chavarria Gomez and Anand Pillay. (Received September 21, 2021)

1174-03-10699 **Anand Pillay*** (apillay@nd.edu), University of Notre Dame. *On stability of abelian structures with a homomorphism to a compact Hausdorff group*

This is joint work with Nicolas Chavarria Gomez.

We study abelian structures with a homomorphism to a compact Hausdorff group T as structures in a certain version of continuous logic. We prove an analogue of the well-known quantifier elimination down to Boolean combinations of positive primitive formulas in abelian structures. We conclude stability. If we have time we will discuss the relevant version of continuous logic and the translation to other versions. (Received September 21, 2021)

1174-03-11249 **Emil Daniel Schwab** (eschwab@utep.edu), Department of Mathematical Sciences, University of Texas at El Paso, and **Gabriela Schwab** (gabrielas@epcc.edu), Department of Mathematics, El Paso Community College. *On the relation between Łukasiewicz fuzzy systems and neural networks with ReLU activation* Preliminary report.

Explainability of neural networks and deep learning architectures, has gained interest recently, as these are used in several safety critical applications. Neural networks with ReLU activation function, (i.e., $ReLU(x) = \max\{x, 0\}$, for $x \in \mathbb{R}$) have shown to be very powerful tools, because they allow implementation of efficient learning algorithms, however, their explainability is traded for their efficiency. Łukasiewicz fuzzy systems are defined using the Łukasiewicz implication and t-norm, and they can be used to approximate continuous functions using implicative fuzzy rules, which are easily interpretable in the framework of fuzzy logic and fuzzy set theory. In this paper we study the relation between Łukasiewicz fuzzy systems and neural networks with ReLU activation, improving in this way the explainability of neural networks. (Received September 22, 2021)

1174-03-12196 **Nicolas Fillion*** (nfillion@sfu.ca), Simon Fraser University. *Trust but Verify: What Can We Know About the Reliability of a Computer-Generated Result?*

Since the Second World War, science has become increasingly reliant on the use of computers to perform mathematical work. Today, computers have justifiably become a trusted ally of scientists and mathematicians. At the same time, there is a panoply of cases in which computers generate demonstrably incorrect results; and there is currently no reason to expect that this situation will change. This prompts the careful user to verify computer-generated results, but it is clear that we are often not in a position to review the work of computers as we would traditionally review a putative derivation or calculation. In this sense, computational processes are epistemically opaque.

Since Humphreys introduced the phrase 'epistemic opacity' in the philosophical literature in 2004, the concept of opacity has been developed along different lines; furthermore, many incompatible claims have been advanced—be they about what opacity is or about whether we should worry about it—leaving this field of the philosophy of computing in a state of confusion. In this paper, we propose a framework that disentangles three core questions (1. What kinds of epistemic opacity are there in scientific computing? 2. Should we worry about epistemic opacity? 3. Should we seek greater transparency whenever possible?) and systematically survey how their answers inter-relate. (Received November 15, 2021)

05 ► Combinatorics

1174-05-5424 **Karla Rubi** (krubi1@toromail.csudh.edu), CSU-Dominguez-Hills, **Luis Gomez*** (lag018@uark.edu), University of Arkansas, and **Jorden Terrazas** (jordenpterrazas@gmail.com), Southern Methodist University. *Failed Zero Forcing of Graphs*

Given a graph G, the zero-forcing number of G, $Z(G)$, is the smallest cardinality of any set S of vertices on which repeated applications of the forcing rule results in all vertices being in S. The forcing rule is: if a vertex v is in S, and exactly one neighbor u of v is not in S, then u is added to S in the next iteration. Zero-forcing numbers have attracted great interest over the past 15 years and have been well studied. In this paper we investigate the largest size of a set S that does not force all of the vertices in a graph to be in S. This quantity is known as the failed zero-forcing number of a graphs and will be denoted by $F(G)$, and has received attention in recent years. We present new results involving this parameter. In particular, we completely characterize all graphs G where $F(G) = 2$, solving a problem posed in 2015 by Fetcie, Jacob, and Saavedra. We also determine failed zero-forcing numbers for several families of graphs including circulant graphs, nearly regular graphs, and graphs with pendant paths. (Received August 20, 2021)

1174-05-5425 **Emma Miller*** (millere08@moravian.edu), Moravian College, and **Andrew Shannon** (a.shannon9@gmail.com), Pomona College. *On the Characterization of Rank 6 Graphs with Triangles*

The *rank* of a graph is defined to be the rank of its 01-adjacency matrix. All graphs with ranks up to 5 are completely characterized by providing finitely many reduced graphs of the desired rank which can then be expanded via multiplication of vertices. Moreover, triangle free graphs with rank 6 are completely characterized.

In this talk, we discuss a computational method adapted from Akbari, Cameron, and Khosrovshahi that constructs all reduced graphs with triangles with a given rank. We then introduce a technique pertaining to neighborhoods that allows for certain edges to be deleted from a graph while maintaining its rank. We discuss how this reduction provides a new way of characterizing graphs of a given rank. (Received August 20, 2021)

1174-05-5430 **Adam Eshel*** (aeshel@u.rochester.edu), University of Rochester, and **Ellen Karolina Persson** (nellepersson@berkeley.edu), University of California, Berkeley. *Community Structure of Cosponsorship Networks in the US Congress 2003-2021*

We study the legislative cosponsorship networks for the ten most recent US Congresses 108-117 (2003-2021). Three different edge weighting schemes for legislative cosponsorship networks have been proposed in the literature and this paper provides a systematic comparison of the network analysis for each scheme. (Received August 20, 2021)

1174-05-5437 **William Nettles*** (nettlewr@dukes.jmu.edu), James Madison University, and **Riley Stephens** (Riley.Stephens@UTDallas.edu), University of Texas at Dallas. *Efficient (j,k)-Domination on Chrysalises*

Rubalcaba and Slater define an *efficient* (j,k)*-dominating function* on graph G as a function $f : V(X) \to \{0,\ldots,j\}$ so that for each $v \in V(X)$, $f(N[v]) = k$, where $N[v]$ is the closed neighbourhood of v (Robert R. Rubalcaba and Peter J. Slater. Efficient (j,k)-domination. *Discuss. Math. Graph Theory*, 27(3):409-423, 2007). For regular graph G the set of efficient dominating functions is closely related to the (-1)-eigenspace of G.

A 3*-regular caterpillar* is a tree obtained from a path by adding a pendant vertex to every vertex of degree 2. A *chrysalis* is a 3-regular graph constructed by adding a cycle through the leaves of a 3-regular caterpillar. We characterize the planar chrysalises that admit efficient dominating functions, as well as the values j and k for which an efficient (j,k)-dominating function can be constructed. Towards extending our characterization to all chrysalises we characterize efficient domination on a class of non-planar chrysalises.

Chrysalises can also be generalized to r-regular graphs for $r \geq 4$. An r*-chrysalis* is the graph obtained from $G_{l,r} = P_l \Box K_{r-1}$ by adding vertices x_1, x_l, and all of the edges between x_i and the ith clique G, as well as the edge x_1x_l. We characterize which of these graphs admit an efficient dominating function, and show how any efficient dominating function can be constructed via a basis for the (-1)-eigenspace. (Received August 20, 2021)

1174-05-5475 **Christopher Soto*** (christopher.soto32@qmail.cuny.edu), Queens College, City University of New York, **Parneet Gill** (pgill023@mail.fresnostate.edu), California State University, Fresno, and **Pamela Vargas** (pvargas@smith.edu), Smith College. *Preferential and k-Zone Parking Functions*

Parking functions are vectors that describe the parking preferences of n cars that enter a one-way street containing n parking spots numbered 1 through n. A list of each car's preferences is also compiled into vectors in which we denote as $(a_1,\ldots,a_n)$, such that a_i is the parking preference for car i. The classical parking rule allows cars to enter the street one at a time going to their preferred parking spot and parking, if that space is unoccupied. If it is occupied, they then proceed down the one-way street and park in the first available parking spot. If all cars can park, we say the vector $(a_1,\ldots,a_n)$ is a parking function.

In our research, we introduce new variants of parking function rules with backward movement called k-Zone, preferential, and inverse preferential functions. We study the relationship between k-Zone parking functions and k-Naples parking functions and count the number of parking functions under these new parking rules which allow cars that find their preferred spot occupied to back up a certain parameter. One of our main results establishes that the set of non-increasing preference vectors are k-Naples if and only if they are k-Zone. For one of our findings we provide a table of values enumerating these new combinatorial objects in which we discover a unique relationship to the order of the alternating group A_{n+1}, numbers of Hamiltonian cycles on the complete graph, K_n, and the number of necklaces with n distinct beads for $n!$ bead permutations. (Received September 4, 2021)

1174-05-5477 **Dylan Alvarenga*** (dalvarenga@cpp.edu), Cal Poly Pomona, **Camelle Audrey Tieu** (tieuca@uci.edu), UC Irvine, and **Yasmin Agullion** (yaguill1@swarthmore.edu), Swarthmore College. *The Bijection between Parking Functions and the Tower of Hanoi*

Parking functions with "bumps" have a total of d spaces between the desired parking spots of n cars and the spots in which they actually park. The set of parking functions of length n with exactly one bump is equinumerous to the set of distinct states in the famous combinatorial puzzle, the Tower of Hanoi, with $n+1$ disks and $n+1$ pegs. In our research, we created two functions that illustrate an explicit bijection between the two sets. (Received September 10, 2021)

1174-05-5555 **Emily J King*** (emily.king@colostate.edu), Colorado State University. *The Combinatorics of Equiangular Tight Frame Substructures*

Given a frame, it is often of interest to ask if a subset of frame vectors also have certain frame properties. The answer to this question has implications for the robustness of the frame representation to erasures and appropriateness of the frame in compressed sensing applications and such subsets sometimes yield optimal subspace configurations. In this talk two combinatorial objects which encode information about equiangular tight frame substructures will be presented: binders and paired difference sets. Connections to other combinatorial objects will also be touched on. (Received August 22, 2021)

1174-05-5588 **Pamela Estephania Harris*** (peh2@williams.edu), Williams College, **Carolina Benedetti** (c.benedetti@uniandes.edu.co), Universidad de los Andes, **Christopher R. H. Hanusa** (christopher.hanusa@qc.cuny.edu), Queens College, **Alejandro H Morales** (ahmorales@math.umass.edu), University of Massachusetts, Amherst, and **Anthony Simpson** (alsimpson1996@gmail.com), Williams College. *Kostant's partition function and magic multiplex juggling sequences*

Kostant's partition function is a vector partition function that counts the number of ways one can express a weight of a Lie algebra $\mathfrak{g}$ as a nonnegative integral linear combination of the positive roots of g. Multiplex juggling sequences are generalizations of juggling sequences that specify an initial and terminal configuration of balls and allow for multiple balls at any particular discrete height. Magic multiplex juggling sequences generalize further to include magic balls, which cancel with standard balls when they meet at the same height. In this talk, we present a combinatorial equivalence between positive roots of a Lie algebra and throws during a juggling sequence. This provides a juggling framework to calculate Kostant's partition functions, and a partition function framework to compute the number of juggling sequences. (Received August 23, 2021)

1174-05-5608 **Andrew R Tawfeek*** (atawfeek@uw.edu), University of Washington, and **Ivan Contreras** (icontreraspalacios@amherst.edu), Amherst College. *On discrete gradient vector fields and Laplacians of simplicial complexes*

Discrete Morse theory, a cell complex-analog to smooth Morse theory allowing homotopic tools in the discrete realm, has been developed over the past few decades since its original formulation by Robin Forman in 1998. In particular, discrete gradient vector fields on simplicial complexes capture important topological features of the structure. We prove that the characteristic polynomials of the Laplacian matrices of a simplicial complex are generating functions for discrete gradient vector fields when the complex is a triangulation of an orientable manifold. Furthermore, we provide a full characterization of the correspondence between rooted forests in higher dimensions and discrete gradient vector fields. (Received August 23, 2021)

1174-05-5674 **Satyan L. Devadoss** (devadoss@sandiego.edu), University of San Diego, and **Daoji Huang*** (daoji_huang@brown.edu), ICERM. *Discrete Spaces of Phylogenetic Structures* Preliminary report.

Phylogenetic inference of biological species often involves comparing labeled metric trees, performing discrete operations to change one tree into another. Billera, Holmes, and Vogtmann (2001) created a "phylogenetic tree space" T_n with additive edge lengths, conveying the distance between trees based on NNI (*nearest neighbor interchange*) operations. However, the SPR (*subtree prune and regraft*) operation is sometimes more relevant to biology, especially due to the nature of horizontal gene transfer. The "phylogenetic orange space" E_n, introduced by Kim (2000) endowed with multiplicative edge weights, allows trees with infinite edge lengths that captures the relations based on SPR operations.

We construct and study a folding map from the compactified tree space T_n to orange space E_n, providing a topological connection between NNI and SPR moves. This allows the introduction and fully characterization of an intermediate compactified space, dubbed the "discrete path space", as an effort to better understand the boundary combinatorics of these spaces. (Received August 24, 2021)

1174-05-5676 **Elizabeth Gross** (egross@hawaii.edu), University of Hawai'I At Manoa, and **Nicole Yamzon*** (nyamzon2@illinois.edu), University of Illinois. *Binomial ideals of domino tilings*

In this talk, we consider the set of all domino tilings of a cubiculated region. The primary question we explore is: How can we move from one tiling to another? Tiling spaces can be viewed as spaces of subgraphs of a fixed graph with a fixed degree sequence. Moves to connect such spaces have been explored in algebraic statistics. Thus, we approach this question from an applied algebra viewpoint, making new connections between domino tilings, algebraic statistics, and toric algebra. Using results from toric ideals of graphs, we are able to describe moves that connect the tiling space of a given cubiculated region of any dimension. This is done by studying binomials that arise from two distinct domino tilings of the same region. Additionally, we introduce tiling ideals and flip ideals and use these ideals to restate what it means for a tiling space to be flip connected. Finally, we show that if R is a 2-dimensional simply connected cubiculated region, any binomial arising from two distinct tilings of R can be written in terms of quadratic binomials. As a corollary to our main result, we obtain an alternative proof to the fact that the set of domino tilings of a 2-dimensional simply connected region is connected by flips. This is joint work with Elizabeth Gross. (Received August 24, 2021)

1174-05-5706 **Sergi Elizalde** (sergi.elizalde@dartmouth.edu), Dartmouth College, and **Benjamin Adenbaum*** (benjamin.m.adenbaum.gr@dartmouth.edu), Dartmouth College. *Rowmotion on 321-avoiding Permutations* Preliminary report.

We give a natural definition of *rowmotion* for 321-avoiding permutations, by translating, through bijections involving Dyck paths and the Lalanne–Kreweras involution, the analogous notion for antichains of the positive root poset of the A_{n-1} root system. We prove that some permutation statistics, such as the number of fixed points, are homomesic under rowmotion, meaning that they have a constant average over its orbits. Our proofs use a combination of tools from Dynamical Algebraic Combinatorics and structural properties of pattern-avoiding permutations. (Received August 31, 2021)

1174-05-5741 **Brendan Rooney*** (brsma@rit.edu), Rochester Institute of Technology. *Regular Graphs with $q = 2$*

Given a graph G on n vertices, $\mathcal{S}(G)$ is the set of symmetric $n \times n$ matrices with the same off-diagonal zero pattern as the adjacency matrix $A(G)$. We say that a connected graph G has $q = 2$ if there is a matrix $M \in \mathcal{S}(G)$ with exactly 2 distinct eigenvalues. Several classes of graphs with $q = 2$ have been identified (e.g., complete graphs, hypercubes, and complete bipartite graphs $K_{n,n}$). However, we are still far from a full characterization of graphs with $q = 2$.

In this talk, we give a characterization of the k-regular graphs with $q = 2$ for $k \leq 4$. Our characterization leads to infinite families of k-regular graphs with $q = 2$ for all $k \geq 5$. We will also discuss connections to previous work on combinatorially orthogonal matrices, and quadrangular graphs. (Received August 27, 2021)

1174-05-5764 **Beth Anne Castellano*** (castelle@lafayette.edu), Lafayette College, and **Marcella Manivel** (marcella11565@gmail.com), University of Minnesota, Carleton College. *A Graph Theoretic Approach to Regularity of Toric Ideals* Preliminary report.

One measure of the complexity of an ideal is its *regularity*, which may be difficult to compute with algebraic tools. To study the regularity of certain *toric ideals* (prime ideals generated by differences of monomials) with quadratic initial ideals, we instead make use of correspondences between the initial ideals and graphs. Moreover, a key theorem by Khosh-Ahang and Moradi gives a method of computing the regularity of a monomial ideal using only the induced matching number of a particular graph associated to it. We find that we can apply this theorem when these graphs are *down-left graphs*. This research was conducted as part of the 2021 Hobart and William Smith Colleges Mathematics REU, supported by the National Science Foundation under grant no. DMS 1757616. (Received September 10, 2021)

1174-05-5895 **Karel Devriendt*** (devriendt@maths.ox.ac.uk), University of Oxford, and **Renaud Lambiotte** (renaud.lambiotte@maths.ox.ac.uk), University of Oxford. *Resistance curvature - a new discrete curvature on graphs* Preliminary report.

The effective resistance is an important property of graphs with a well-developed theory and many applications in computer science, engineering and network science. We formulate a new application of effective resistances in the context of discrete curvature: we propose the *resistance curvatures*, p defined on the vertices and κ on the edges of a graph as discrete curvatures. Both curvatures follow from a simple, local definition on the graph as

$$p_i = 1 - \frac{1}{2} \sum_{k:k\sim i} c_{ik}\omega_{ik} \quad \text{and} \quad \kappa_{ij} = \frac{2(p_i + p_j)}{\omega_{ij}}$$

for vertices i and edges ij, where $k \sim i$ are neighbours of i, c_{ij} the weight of an edge and ω_{ij} the effective resistance between its end-points. From properties of the effective resistance, several bounds on p and κ immediately follow, and we find connections to the graph topology such as $\sum_i p_i = \#$ connected components, and a result of Miroslav Fiedler which states that if p is positive for all vertices, then removing any k vertices can disconnect the graph in at most $k + 1$ components. To support the interpretation as curvatures, we furthermore find that p and κ reproduce several properties of differential curvature in the context of graph: in random geometric graphs we find convergence of p to the underlying curvature, κ can be bounded from both sides by well-established notions of discrete curvature and there is a natural associated discrete Ricci flow. (Received September 13, 2021)

1174-05-5901 **Jakeyl Johnson*** (jjohnson@msri.org), Partner, and **Daniel Quinonez** (dquinonez@msri.org), Partner. *Unit Interval Rational Parking Functions* Preliminary report.

A *unit interval rational parking function* is a parking function with m cars and n spots, where $m \leq n$, and each car can park in their preferred spot or the next. This concept unifies two existing generalizations of classical parking functions: rational parking functions, with m cars and n spots, and unit interval parking functions, where each car can park in their preferred spot or the next. A *preferential arrangement* on the set $[m] = \{1, \ldots, m\}$ is a ranking of the elements with ties allowed. Separating the blocks with bars creates a *barred preferential arrangement*. In this paper, we show a bijection between the set of unit interval rational parking functions with m cars and $\ell = n - m$ additional spots, and the set of barred preferential arrangements on $[m]$ with ℓ bars. This bijection yields the use of the recursive formula by Ahlbach, Usatine, and Pippenger for counting the number of unit interval rational parking functions. (Received August 31, 2021)

1174-05-5912 **Alexander Nicholas Sietsema** (sietsem6@msu.edu), Michigan State University, **James Schmidt** (schmi710@msu.edu), Michigan State University, **Bruce Eli Sagan*** (sagan@math.msu.edu), Michigan State University, **Rachel Domagalski** (domagal9@msu.edu), Michigan State University, **Jinting Liang** (liangj26@msu.edu), Michigan State University, and **Quinn Minnich** (minnichq@msu.edu), Michigan State University. *Cyclic shuffle compatibility*

Define a (linear) permutation to be any sequence of distinct positive integers. Given two permutations π and σ on disjoint underlying sets, we denote by $S(\pi, \sigma)$ the set of shuffles of π and σ. A statistic is a function St whose domain is the set of permutations. A statistic is shuffle compatible if the distribution of St on $S(\pi, \sigma)$ depends only on $\mathrm{St}(\pi)$ and $\mathrm{St}(\sigma)$ and their lengths rather than on the individual permutations themselves. This notion is implicit in the work of Stanley in his theory of P-partitions. The definition was explicitly given by Gessel and Zhuang who proved that various permutation statistics were shuffle compatible. Shuffle compatibility is also important in expressing the product of two fundamental quasisymmetric functions. Recently Adin, Gessel, Reiner, and Roichman defined a cyclic version of the algebra of quasisymmetric functions where a cyclic analogue of shuffle compatibility comes into play. We show how linear shuffle compatibility results can be lifted to the cyclic realm. (Received August 31, 2021)

1174-05-5945 **Aleyah Dawkins*** (adawkin@gmu.edu), George Mason University. *Cheeger constant of Cartesian products of Cayley graphs arising from Generalized Dihedral Groups* Preliminary report.

Let n, x be positive integers satisfying $1 < x < n$. Let $H_{n,x}$ be a group of the form $\langle a, b | a^n = b^2 = (ba)^x = 1\rangle$. $H_{n,x}$ will be referred to as the generalized dihedral groups. It is possible to associate a cubic Cayley graph to each such group. Bounds on the Cheeger constant, $h(G)$, of these graph is known. We consider the problem of finding the Cheeger constant, $h(G \,\square\, G)$, of the Cartesian product of these graphs and comparing it to the known bounds on $h(G)$. We also obtain some bounds on Cartesian products of regular graphs. (Received September 1, 2021)

1174-05-5997 **Julianne Vega** (jvega30@kennesaw.edu), Kennesaw State University, **Cameron Lowe*** (clowe35@students.kennesaw.edu), Kennesaw State University, **Heather Willis** (heatherrw258038@gmail.com), OSC a Breckenridge Company, **Justin McClung** (jmcclun4@students.kennesaw.edu), Kennesaw State University, **Robert Shipp** (rshipp2@students.kennesaw.edu), Kennesaw State University, and **Andrés R. Vindas-Meléndez** (andres.vindas@berkeley.edu), MSRI & UC Berkeley. *t-Stack Sorting Subpolytopes of the Permutahedron* Preliminary report.

In 1968, Knuth introduced the stack sorting algorithm which attempts to chronologically sort an inputted sequence, in our case a permutation. Using the stack sorting algorithm, we traverse the vertices and edges of the permutahedron. The n-permutahedron is the (n-1)-dimensional polytope generated by the convex hull of permutations of the first n natural numbers. We consider subpolytopes of the permutahedron arising from the convex hull of sequences generated by iterations of the stack sorting algorithm. For a particular family of subpolytopes, we determine their dimension and prove that they are simplices. We conjecture that this family of simplices has Ehrhart polynomials with coefficients in Pascal's triangle. (Received September 2, 2021)

1174-05-6016 **Alexander Moll*** (alexander.moll@umb.edu), University of Massachusetts Boston. *Weighted enumeration of ribbon paths via Jack polynomials*

We introduce a generalization of Motzkin paths that we call "ribbon paths." For a particular choice of weights, we express weighted sums of ribbon paths in terms of Jack polynomials. As a consequence, our weighted sums are equal to weighted sums of certain ribbon graphs on non-oriented real surfaces. Our results are new even for ribbon paths without "slides," in which case the weighted sums are expressed through Schur polynomials and the corresponding real surfaces are oriented. (Received September 2, 2021)

1174-05-6144 **Steven Robert Edwards*** (sedwar77@kennesaw.edu), Kennesaw State University, and **William Griffiths** (wgriff17@kennesaw.edu), Kennesaw State University. *Generalized Delannoy Numbers and Delannoy Constructions* Preliminary report.

The Delannoy numbers, D(m,n), count the number of Queen's Walks, which are lattice paths from the origin to (m,n), where the allowed steps are up, right, and diagonal. The Generalized Delannoy numbers count such paths with various restrictions. All Delannoy numbers satisfy the same recursion, and many relations among these numbers have been discovered, including some which have been proved using path interpretations. The Generalized Delannoy numbers can be created from the Delannoy numbers via the Delannoy Construction, which creation is the most fundamental example of such. The Delannoy Construction is a method of producing infinite collections of recursive arrays from a "seed" which produces a structured array, where the structure consists of canonical functions which create the structure. (Received September 16, 2021)

1174-05-6192 **Wasin So*** (wasin.so@sjsu.edu), San Jose State University. *How Many Non-isospectral Integral Circulant Graphs Are There?* Preliminary report.

The answer to the question in the title is contained in the following conjecture from 2005: There are exactly $2^{\tau(n)-1}$ non-isospectral integral circulant graphs of order n,
where $\tau(n)$ is the number of divisors of n. In this note, we review some background about this conjecture, which is still open. Moreover, we affirm this conjecture for some special cases of n, namely, $n = p^k, pq^k$, or p^2q with primes $2 \leq p < q$ and integer $k \geq 1$; or $n = pqr$ with primes $2 < p < q < r$. A common technique used in the proofs of these different cases is the notion of super sequence, a positive sequence in which each term is greater than the partial sum of all previous terms. (Received September 6, 2021)

1174-05-6205 **Lina Liu** (lliu354@wisc.edu), University of Wisconsin-Madison, **Aalliyah Celestine*** (acelest6@xula.edu), Xavier University of Louisiana, and **Jacob van der Leeuw** (jvanderleeuw@email.arizona.edu), University of Arizona. *Enumerating k-Typed Parking Functions and Their Properties*

Motivated by open problems presented in Mark Dukes' recent paper on parking functions in 2021, we explore a new variant called k-typed parking functions (k-TPFs). These are tuples describing the parking of M cars on a street with M parking spots. Each car is assigned one of k ordered types, with m_i denoting the number of cars of type i; hence, $M = m_1 + m_2 + \cdots + m_k$. We count the total number of exact k-TPFs of length M, and remark that when $k = 2$, $m_1 = n$, and $m_2 = n - 1$, our result recovers the enumeration of classical parking functions of order n. We also establish that every exact k-TPF corresponds to a unique parking configuration C. We denote the set of all permutations of the preference lists P_i in $\alpha = (m_1; P_1, P_2, \ldots, P_k)$ as fam(α). Lastly, we prove each fam(α) is a disjoint subset of all k-TPFs of length M and derive a formula for counting the number of elements in each fam(α). (Received September 6, 2021)

1174-05-6216 **Shane Chern*** (chenxiaohang92@gmail.com), Dalhousie University, and **George Beck** (george.beck@gmail.com), Dalhousie University. *Reciprocity between partitions and compositions*

In this talk, I will discuss an extension of the work of Andrews, Beck and Hopkins by considering partitions and compositions with bounded gaps between each pair of consecutive parts. I will show that both their generating functions and two matrices determined by them satisfy certain reciprocal relations. This is joint work with George Beck. (Received September 7, 2021)

1174-05-6330 **Shuxing Li*** (shuxingl@sfu.ca), Simon Fraser University. *Packings of Partial Difference Sets*

As the underlying configuration behind many elegant finite structures, partial difference sets have been intensively studied in design theory, finite geometry, coding theory, and graph theory. Over the past three decades, there have been numerous constructions of partial difference sets in abelian groups with high exponent, accompanied by numerous very different and delicate techniques. Surprisingly, we manage to unify and extend a great many previous constructions in a common framework, using only elementary methods. The key insight is that, instead of focusing on one single partial difference set, we consider a packing of partial difference sets, namely, a collection of disjoint partial difference sets in a finite abelian group. This conceptual shift leads to a recursive lifting construction of packings in abelian groups of increasing exponent.

This is joint work with Jonathan Jedwab. (Received September 8, 2021)

1174-05-6356 **Darren A. Narayan*** (dansma@rit.edu), Rochester Institute of Technology. *Real World Applications of Graph Theory*

The presentation will feature examples of concrete real world applications of graph theory. These include: (i) analysis of functional magnetic resonance imaging (MRI) data using betweenness centrality (Rochester Center for Brain Imaging), (ii) analysis of fiber-optic networks using betweenness centrality (National LambdaRail), (iii) application of graph metrics to analyze the efficiency of the Atlanta Metro (MARTA) network, and (iv) Using graph theory to analyze anatomical connectivity of the human brain. (Received September 8, 2021)

1174-05-6360 **Leslie Hogben*** (hogben@aimath.org), Iowa State University & American Institute of Mathematics. *Extreme values of parameters related to zero forcing and the inverse eigenvalue problem of a graph*

Zero forcing is an iterative process that repeatedly applies a rule to fill vertices of a graph. The zero forcing number is the minimum number of initially filled vertices that are needed to fill all vertices through the process, and is denoted by $Z(G)$. The maximum multiplicity of an eigenvalue (or maximum nullity) among matrices having off-diagonal nonzero pattern described by the edges of the graph G is denoted by $M(G)$. It is known that $M(G) \leq Z(G)$, and this relationship is one of the origins of zero forcing (and the source of its name). Several variants of zero forcing number and maximum nullity, such as positive semidefinite zero forcing number and maximum nullity, have also been studied. For each variant of zero forcing, other parameters of interest include the propagation time (number of rounds needed to fill all vertices) and throttling number (which minimizes a combination of resources used to accomplish a task and time needed to accomplish it). This talk will survey results on extreme values of various zero forcing numbers, propagation times, throttling numbers, and maximum nullities. (Received September 8, 2021)

1174-05-6371 **Kiril Atanasov Bangachev*** (kirilb@princeton.edu), Princeton University. *On the Asymmetric Generalizations of Two Extremal Questions on Friends-and-Strangers Graphs*

For two graphs X and Y with vertex sets $V(X)$ and $V(Y)$ of the same cardinality n, the friends-and-strangers graph $\mathsf{FS}(X,Y)$ was recently defined by Defant and Kravitz. The vertices of $\mathsf{FS}(X,Y)$ are the bijections from $V(X)$ to $V(Y)$, and two bijections σ and τ are adjacent if they agree everywhere except at two vertices $a, b \in V(X)$ such that a and b are adjacent in X and $\sigma(a)$ and $\sigma(b)$ are adjacent in Y. We show that if X and Y have minimum degrees $\delta(X)$ and $\delta(Y)$ that satisfy $\delta(X) > n/2$, $\delta(Y) > n/2$, and $2\min(\delta(X), \delta(Y)) + 3\max(\delta(X), \delta(Y)) \geq 3n$, then $\mathsf{FS}(X,Y)$ is connected. As a corollary, we settle a recent conjecture by Alon, Defant, and Kravitz. We further show that if X and Y are edge-subgraphs of $K_{r,r}$ satisfying $\delta(X) + \delta(Y) \geq 3r/2 + 1$, then the graph $\mathsf{FS}(X,Y)$ has exactly two connected components. As a corollary, we provide an almost complete answer to another question by Alon, Defant, and Kravitz. (Received September 8, 2021)

1174-05-6375 **Matthew Prudente*** (matthew.prudente@alvernia.edu), Alvernia University. *On Two-Player Graph Pebbling*

Graph pebbling can be extended to a two-player game on a graph G, called *Two-Player Graph Pebbling*, with players Mover and Defender. The players each use pebbling moves, the act of removing two pebbles from one vertex and placing one of the pebbles on an adjacent vertex, to win. Mover wins if they can place a pebble on a specified vertex. Defender wins if the specified vertex is pebble-free and there are no more pebbling moves on the vertices of G. The *Two-Player Pebbling Number* of a graph G, $\eta(G)$, is the minimum m such that for every arrangement of m pebbles and for any specified vertex, Mover can win. In this talk, we specify the winning player for paths, cycles, and the join of certain graphs. (Received September 8, 2021)

1174-05-6402 **Russell Lyons*** (rdlyons@indiana.edu), Indiana University, and **Graham White** (grahamwhite@alumni.stanford.edu), none. *Monotonicity is special for continuous-time random walks on groups*

Consider continuous-time random walks on Cayley graphs where the rate assigned to each edge depends only on the corresponding generator. We show that the limiting speed is monotone increasing in the rates for infinite Cayley graphs that arise from Coxeter systems, but not for all Cayley graphs. On finite Cayley graphs, we show that the distance to stationarity is monotone decreasing in the rates for Coxeter systems and for abelian groups, but not for all Cayley graphs. We also find several examples of surprising behaviour in the dependence of the distance to stationarity on the rates. (Received September 12, 2021)

1174-05-6440 **Gary R.W. Greaves*** (gary@ntu.edu.sg), Nanyang Technological University. *Hermitian matrices of roots of unity and their characteristic polynomials*

Many known configurations of Equiangular Tight Frames correspond to Hermitian matrices of roots of unity. Motivated by the pursuit of necessary conditions for their existence, we consider spectral properties of such matrices. Our main results are conjecturally sharp upper bounds on the number of residue classes of the characteristic polynomial of a Hermitian matrix of roots of unity modulo ideals generated by powers of $(1-\zeta)$, where ζ is a root of unity. During this talk, I will present a generalisation of a classical result of Harary and Schwenk about a relation for traces of powers of a graph-adjacency matrix, which is a crucial ingredient for the proofs of our main results. (Received September 9, 2021)

1174-05-6451 **Jean Joseph*** (jsjean1@hotmail.com), West Palm Beach, FL. *WHEN DOES THERE EXIST AN EMBEDDING BETWEEN ANY TWO GENERALIZED HYPERGRAPHS?* Preliminary report.

Let X be any set with an infinite sequence of relations $\rho_1, \rho_2, \rho_3, \ldots$ with the arity of each relation being equal to its index; for instance, ρ_2 is a binary relation while ρ_3 is a ternary relation. For the sake of this talk, we call this structure a **generalized hypergraph**. Given any two generalized hypergraphs, we give conditions for the existence of an embedding from one to the other. (Received September 9, 2021)

1174-05-6455 **Esther Banaian** (banai003@umn.edu), University of Minnesota, **Zhichun Zhang*** (zzhang3@swarthmore.edu), Swarthmore College, and **Amy Tao** (atao3@wellesley.edu), Wellesley College. *Friezes from Dissection* Preliminary report.

A frieze on a polygon is a map from the diagonals of the polygon to an integral domain which respects the Ptolemy relation. Motivated by Holm and Jørgensen's study of friezes from gluing together regular sub-polygons, we study friezes over the integral domain $\mathbb{Z}[\sqrt{2}]$ and $\mathbb{Z}[\sqrt{3}]$. The rings $\mathbb{Z}[\sqrt{2}]$ and $\mathbb{Z}[\sqrt{3}]$ correspond to dissecting a polygon into triangles and squares and into triangles and (regular) hexagons, respectively. This was a generalization of work by Conway and Coxeter, who found that friezes over positive integers on an n-gon are in bijection with triangulations of an n-gon. In particular, we are interested in *unitary* friezes on a polygon. A frieze is unitary if there exists a triangulation of the underlying polygon such that every arc in the triangulation has a weight in the frieze that is a unit in the respective integral domain. Conway and Coxeter found that all friezes on a polygon over positive integers are unitary. We explore the characterization of unitary friezes on a polygon over $\mathbb{Z}[\sqrt{2}]$ and $\mathbb{Z}[\sqrt{3}]$. While in $\mathbb{Z}[\sqrt{2}]$ we find a whole family of dissections involving squares and triangles that give rise to a unitary frieze, we claim that in $\mathbb{Z}[\sqrt{3}]$ the only unitary friezes come from triangulations - i.e. we cannot use a hexagon in the dissection. We also show that the family of dissections giving unitary friezes in $\mathbb{Z}[\sqrt{2}]$ has connections to well-known, infinite continued fractions. (Received September 21, 2021)

1174-05-6846 **Andrew Berget*** (andrew.berget@wwu.edu), Western Washington University. *Schur classes of matroids* Preliminary report.

Schur classes of matroids are combinatorial analogues of intersections of Schubert varieties with torus orbits in a Grassmannian. In this talk, I will explicate a series of positivity conjectures for matroids, one of which states that the degrees of Schur classes of matroids are non-negative integers. I will present ongoing work to resolve the latter conjecture, joint with Eur, Fink and Spink. (Received September 9, 2021)

1174-05-6899 **Emelie J Curl*** (curlej@hollins.edu), Hollins University. *The Zero Forcing Number of Graph Complements* Preliminary report.

Zero forcing is a basic iterative propagation game implemented on graphs. This graph parameter is often studied in the context of other graph parameters such as minimum rank, maximum nullity, and the path cover number to name a few. However, by examining solely the zero forcing number, a natural question is: How can we (nicely) compute the zero forcing number of the complement of a connected graph? Simultaneously, what tools and strategies can we develop for computing (or at least bounding) the zero forcing number of the complement of a graph constrained by considering certain classes of graphs, such as: bipartite, unicyclic, and other families containing more than one cycle. This talk will focus on presenting some results related to answering these questions. (Received September 10, 2021)

1174-05-6966 **Brandon Lee*** (blee006261@gmail.com), University of Texas at El Paso. *Laplacian Eigenvalues of Bipartite Kneser-Like Graphs* Preliminary report.

The bipartite Kneser-like graph $G(a,b)$ is constructed as follows, take for any integers $a > b > 1$, let $n = a+b+1$, then let $\mathcal{A}$ be a set of all a-sized subsets of $\{1,\ldots,n\}$. Similarly let $\mathcal{B}$ be the set of all b-sized subsets of $\{1,\ldots,n\}$. Draw an edge between a vertex in $\mathcal{A}$ and a vertex in $\mathcal{B}$ if their intersection is disjoint. We conjecture that the eigenvalues of the Laplacian matrix of this graph, denoted $L(G(a,b))$, are all non-negative integers.

We further conjecture a more precise formula for the multiplicity of any eigenvalue and a general formation of the eigenvectors for the b, $b-1$, $a+2$, and $a+3$ eigenvalues for $L(G(a,b))$. Thus far, we have proved that: every eigenvector has a related pair with a similar eigenvalue and eigenvector, which implies the sequence of multiplicities is symmetric; that all the eigenvalues of $L(G(a, 2))$ are non-negative integers; and established a transformation to a smaller matrix. (Received September 10, 2021)

1174-05-6972 **Cynthia Marie Rivera Sánchez** (cynthia.rivera15@upr.edu), University of Puerto Rico - Río Piedras Campus, **Lauren June Quesada** (ljqa2018@mymail.pomona.edu), Pomona College, and **Aaron Ortiz*** (aortiz61@miners.utep.edu), The University of Texas at El Paso. *The Defective Parking Space and Standard Young Tableaux* Preliminary report.

Parking functions and their enumerative properties have been studied in depth in the field of combinatorics and in computer science for their pertinence to hashing functions. A well-known result is the Catalan parameterization of the number of orbits of the standard parking space, the set of parking functions under the action of the symmetric group. We introduce d-defective parking spaces, showing defect is an invariant with regards to permutations. We also establish a connection and conjecture a bijection between the orbits of d-defective parking space and standard Young tableaux of shape $(n+d, n-d-1)$. Additionally, we provide a recursive formula for the number of orbits decomposing the d-defective parking space and establish an interesting link to Catalan convolutions. (Received September 10, 2021)

1174-05-6974 **Lindsey-Kay Lauderdale*** (llauderdale@towson.edu), Towson University. *Graovac-Pisanski's distance number for quaternion graphs* Preliminary report.

The Graovac-Pisanski's distance number of a graph G measures the average distance each vertex is displaced under the elements of the automorphism group of G. In this talk, we will discuss the Graovac-Pisanski's distance number for graphs whose automorphism groups are isomorphic to a quaternion group. (Received September 10, 2021)

1174-05-6983 **Molena Nguyen*** (molena.thy.nguyen@gmail.com), North Carolina State University. *Take-away Impartial Combinatorial Game on Different Geometric and Discrete Structures* Preliminary report.

Following from the winning strategy for a Take-Away Impartial Combinatorial Game on only Oddly Uniform or only Evenly Uniform Hypergraphs in the Ph.D. Dissertation of Dr. Kristen Barnard (an Assistant Professor of Mathematics at Berea College), Molena Nguyen found the new winning strategy for a Take-Away Game on neither Oddly nor Evenly Uniform Hypergraphs during her Undergraduate Independent Research opportunity. However, those neither Oddly nor Evenly Uniform Hypergraphs must meet the given special requirements. In a Take-Away Game on hypergraphs, two players take turns to remove the vertices and the hyperedges of a hypergraph. In each turn, a player must remove either only one vertex or only one hyperedge. When a player chooses to remove one vertex, all of the hyperedges that contain the chosen vertex are also removed. When a player chooses to remove one hyperedge, only that one chosen hyperedge is removed. Whoever removes the last vertex wins the game. All of the new theorems in this research paper are in agreement with the previous theorems in Dr. Kristen Barnard's Ph.D. Dissertation. (Received September 11, 2021)

1174-05-6996 **Polona Oblak** (polona.oblak@fri.uni-lj.si), University of Ljubljana, and **Helena Šmigoc** (helena.smigoc@ucd.ie), University College Dublin. *On the inverse eigenvalue problem for block graphs*

The inverse eigenvalue problem of a graph is asking whether a given spectrum can be realized by some matrix whose zero-nonzero pattern is prescribed by the adjacency of the graph. The strong spectral property is a matrix condition that allows one to perturb the matrix without changing its spectrum, and it is powerful in creating matrices with the same spectrum and more nonzero entries. In this talk, we will introduce some techniques for matrices without the strong spectral property and provide partial solutions to the inverse eigenvalue problem for block graphs. (Received September 11, 2021)

1174-05-7003 **Andrew V. Sills*** (asills@georgiasouthern.edu), Georgia Southern University. *An application of integer partitions to nonparametric statistics*

We examine how partition theory plays a natural role in the Wilcoxon rank sum test. In 1945, when Wilcoxon introduced his rank sum test, he addressed the issue of how to assign ranks in cases where ties exist in the collected data, but did not discuss distributional results. In 1947, Mann and Whitney derived the distribution, including several moments, for a statistic essentially equivalent to Wilcoxon's rank sum, but avoided the complications that arise with ties by assuming continuous measurements. In practice, it seems that modern statistical software avoids dealing with the exact distributions when ties exist by using a normal approximation, even in the case of small sample sizes where such an approximation may not be justified. In this talk, we will examine how to use a slight generalization of ideas from the theory of integer partitions to derive exact distributions for the Wilcoxon rank-sum statistic when ties appear in the data. These ideas can be implemented easily in a CAS such as Mathematica or Maple, and with additional care and effort, in a lower level language like C. (Received September 11, 2021)

1174-05-7028 **Jiayuan Wang*** (j453w588@gwu.edu), George Washington University. *Fully commutative elements in complex reflection groups* Preliminary report.

Fully commutative elements in types B and D are completely characterized and counted by Stembridge. Recently, Feinberg-Kim-Lee-Oh have extended the study of fully commutative elements from Coxeter groups to the complex setting, giving an enumeration of such elements in $G(m,1,n)$. In this note, we prove a connection between fully commutative elements in B_n and in $G(m,1,n)$, which allows us to characterize fully commutative elements in $G(m,1,n)$ by pattern avoidance. Furthermore, we present a counting formula for such elements in $G(m,1,n)$. (Received September 11, 2021)

1174-05-7115 **William Verreault*** (william.verreault.2@ulaval.ca), Université Laval. *MacMahon Partition Analysis: a discrete approach to broken stick problems*

I will present a discrete approach to solve problems on forming polygons from broken sticks, which is akin to counting polygons with sides of integer length subject to certain Diophantine inequalities. Namely, we use MacMahon's Partition Analysis to obtain a generating function for the size of the set of segments of a broken stick subject to these inequalities. In particular, we use this approach to show that for $n \geq k \geq 3$, the probability that a k-gon cannot be formed from a stick broken into n parts is given by $n!$ over a product of linear combinations of partial sums of generalized Fibonacci numbers, a problem which proved to be very hard to generalize in the past. (Received September 16, 2021)

1174-05-7125 **Jeremiah D Bartz*** (jeremiah.bartz@und.edu), University of North Dakota. *A Ternary Tree of Triangular Triples*

A common saying is that good things come in threes. In 1934, B. Berggren may have agreed when unearthing three matrices which generate a ternary tree of all primitive Pythagorean triples from the initial triple $(3,4,5)$. We show that a similar structure exists when replacing primitive triples with triangular triples. The construction of both ternary trees are intertwined, involving Berggren's three matrices and a third ternary tree. Perhaps good things really do come in threes! (Received September 12, 2021)

1174-05-7127 **Jessica De Silva*** (jdesilva1@csustan.edu), California State University, Stanislaus, **Adam B Dionne** (abd2@williams.edu), Williams College, and **Aidan Dunkelberg** (aidandunkelberg@bellsouth.net), Williams College. *Very well-covered graphs with the Erdös-Ko-Rado Property*

The Erdös-Ko-Rado (EKR) theorem states that the maximum size of an intersecting family of r-element subsets of $[n]$ can be attained by taking all subsets containing some fixed element. This classical result in extremal combinatorics has been rephrased in the language of graph theory as an extremal property regarding independent sets of a graph. A family of independent r-sets of a graph G is an r-star if every set in the family contains some fixed vertex v. A graph is r-EKR if the maximum size of an intersecting family of independent r-sets can be attained by an r-star. In this context, the classical EKR theorem determines the values of r for which the empty graph satisfies the r-EKR property. This talk will motivate and present early findings in the search for very well-covered graphs with the r-EKR property. (Received September 12, 2021)

1174-05-7146 **Andrew Vince*** (avince@ufl.edu), University of Florida. *The Average Size of a Connected Vertex Set of a Graph*

Although connectivity is a basic concept in graph theory, the enumeration of connected subgraphs of a graph has only recently received attention. The topic of this talk is the average order of a connected induced subgraph of a graph. This generalizes, to graphs in general, the average order of a subtree of a tree, a topic initiated by R. Jamison in 1983. (Received September 13, 2021)

1174-05-7170 **Mark Kempton*** (mkempton@mathematics.byu.edu), Brigham Young University. *A 2-separation formula for effective resistance*

A graph has a k-separation if the removal of some k vertices disconnects the graph. Computing effective resistance in a graph with a 1-separation is very straightforward. We present a formula for computing effective resistance in a graph with a 2-separation. This takes advantage of a beautiful combinatorial characterization of effective resistance using the enumeration of spanning trees and forests in the graph. (Received September 13, 2021)

1174-05-7185 **Joshua Carlson*** (joshua.carlson@drake.edu), Drake University, and **John Petrucci** (jmp10@williams.edu), Williams College. *Hopping Forcing on Graphs*

Zero forcing is a combinatorial game played on graphs that models the spread of information by using a color change rule to change the color of vertices to blue. In a game where all vertices eventually become blue, the number of vertices that are colored blue initially bounds the maximum nullity of a family of matrices associated with the graph on which that game is played. Many variants of zero forcing have been shown to have connections to numerous important graph parameters. One variant, called Z-floor forcing, is equivalent to the proper path-width of a graph. Z-floor forcing combines the standard color change rule of zero forcing with another rule called *hopping*. This talk will introduce a new variant called *hopping forcing* which uses the hopping color change rule alone and has its own connections to interesting graph parameters. (Received September 13, 2021)

1174-05-7274 **Ben Small*** (bentsm@gmail.com), -. *A proof of a tight upper bound on PSD propagation time (and other recent results)* Preliminary report.

Recent research has lead to significant developments regarding positive semidefinite (PSD) zero forcing and propagation time. This talk will present (with proof) a tight upper bound on the PSD propagation time of a graph and some implications of that bound. It will also discuss some of the other recent results on the topic. (Received September 18, 2021)

1174-05-7282 **Mohsen Aliabadi*** (aliabadi@iastae.edu), Iowa State University. *Recent developments in the theory of matchings* Preliminary report.

In this talk, we introduce the notions of matching matrices in groups and vector spaces, which lead to some necessary conditions for existence of acyclic matching in abelian groups and its linear analogue. We also discuss the linear matching property in field extensions to find a dimension criterion for linear matchable bases. We provide an upper bound for the dimension of primitive subspaces in a separable field extension. This may be considered as an analog of Artin's theorem on intermediate subfields of a given simple field extension. We finally provide an application of matchings about the fundamental theorem of algebra. Our tools mix combinatorics and linear algebra. (Received September 13, 2021)

1174-05-7288 **Eric R Dolores-Cuenca*** (eric.rubiel@u.northwestern.edu), Yonsei University, and **Jose Antonio Arciniega-Nevarez** (fenix.dgo@gmail.com), Universidad de Guanajuato. *Power series representing posets*

Consider the category of finite posets. We prove that two geometrically inspired operations on this category are transported to labeling series. Posets generated by those operations are named Wixárika posets. We provide an algorithm to determine if a power series is associated to a Wixárika poset and we compute the number of labeling functions for such posets. Consequently, we prove new identities of binomial coefficients. (Received September 14, 2021)

1174-05-7291 **Maria C. Mannone*** (maria.mannone@unive.it), Department of Mathematics and Computer Science, University of Palermo; European Centre for Living Technology, Ca' Foscari University of Venice. *CubeHarmonic: when the Rubik's cube becomes a musical instrument* Preliminary report.

It is said that the Rubik's cube had first been proposed by Ernö Rubik as a pedagogical tool, to help students visualize and manipulate rotations and permutations in Group Theory. The Rubik's cube presents different colors to distinguish its faces and help its solving. In the main version of the cube, each face contains nine squares (facets). Associating a note of a musical chord with each square, each face presents a 9-note chord. Mixing the cube, we mix the chords. It is the core idea of the CubeHarmonic, a math-inspired musical instrument. The rotational symmetries allow us to make cyclic progressions of the chords: after a precise sequence of moves, we return to the same chords. Therefore, it is possible to study the chord-preserving symmetries. Locally, on the CubeHarmonic we can recreate portions of tonnetz, a lattice used to study symmetries of chords in Western musical theory. Recently, the idea of CubeHarmonic has been extended to four dimensions, using computational resources and augmented reality. The hyper-Rubik's cube, tridimensionally developed in eight Rubik's cubes mutually connected, adds degrees of freedom to the musical realizations. It allows us to "listen to the sound of multiple dimensions." We present some prototypes of CubeHarmonic and HyperCubeHarmonic developed in collaboration with researchers of the RIEC of Tohoku University and Toyo University in Japan. (Received September 14, 2021)

1174-05-7316 **Tien Chih** (tien.chih@msubillings.edu), Montana State University Billings. *Report on CURM project: Homotopy of Graphs*

We will discuss the 2020-2021 CURM funded undergraduate research project on Homotopy of Graphs, joint between MSU Billings and Fort Lewis College. Students from both campuses worked on questions in ×-homotopy of grpahs, building on results by Chih & Scull, studying 'spider webs' of graphs induced by patterns in homotopy of graph morphisms. MSUB students investigated automorphisms of graphs, and FLC students enumerated and studied the structure of certain classes of spider web graphs. Mathematically the project was a great success, and some of our students are currently writing up some of their results for publication.

When we proposed this project, we planned for more face to face collaboration. Because of COVID our students did not wind up with much direct interaction. We did share code and results back and forth, and MSUB students wound up citing FLC result in their work. We, the team leaders, were in much closer contact throughout, with weekly zoom check-ins and many many emails.

In this talk, we will share some of the results of our students and talk about the collaborative process under the constraints of the last year. (Received September 14, 2021)

1174-05-7319 **Jonathan Cutler*** (jonathan.cutler@montclair.edu), Montclair State University, and **Deepak Bal** (bald@montclair.edu), Montclair State University. *Enumerative Nordhaus-Gaddum inequalities*

Nordhaus and Gaddum proved the following inequalities that give upper and lower bounds on the sum and product of the chromatic number of a graph and its complement.

$$2\sqrt{n} \leq \chi(G) + \chi(\overline{G}) \leq n+1 \quad \text{and} \quad n \leq \chi(G)\chi(\overline{G}) \leq \frac{(n+1)^2}{4}.$$

Inspired by these results, Nordhaus-Gaddum inequalities have been studied for many other graph invariants. Recently, Wagner gave a lower bound on the sum of the number of dominating sets in a graph on its complement. In this talk, we discuss some related results and their connections to well-known areas of study in graph theory. (Received September 14, 2021)

1174-05-7343 **Jake Levinson** (jake_levinson@sfu.ca), Simon Fraser University, and **Maria Monks Gillespie*** (maria.gillespie@colostate.edu), Colorado State University. *Schubert curves and other intersections in Grassmannians*

We will present a visual introduction to Schubert calculus on the Grassmannian and the construction of Schubert curves via flags osculating the real rational normal curve. We will end with a visual demonstration of our combinatorial algorithm on Young tableaux that computes a monodromy operator on Schubert curves. (Received September 14, 2021)

1174-05-7369 **Francois G. Meyer*** (fmeyer@colorado.edu), University of Colorado at Boulder. *Multiscale Analysis of the Pattern of Connectivity in Dynamic Community Networks*

The main contribution of this work is a detailed analysis of changes in the pattern of connectivity in a dynamic community graph. This model is formed by adding new vertices, and randomly attaching them to the existing nodes. The goal of the work is to detect the time at which the graph dynamics switches from a normal evolution – where balanced communities grow at the same rate – to an abnormal behavior – where communities start merging.

In order to circumvent the problem of decomposing each graph into communities, we use a metric to quantify changes in the graph topology as a function of time. The detection of anomalies becomes one of testing the hypothesis that the graph is undergoing a significant structural change.

The theoretical analysis of the statistical test is validated with several experiments on real dynamic networks that confirm that the resistance perturbation provides a very sensitive statistic to detect changes in dynamic networks. (Received September 14, 2021)

1174-05-7410 **Xufei Liu*** (xufeiliu2000@gmail.com), Georgia Institute of Technology, **Eugene Fiorini** (efiorini@dimacs.rutgers.edu), Rutgers University, **Jared Glassband** (jeg328@cornell.edu), Cornell University, **Garrison Lee Koch** (kochg02@moravian.edu), Moravian University, **Sophia Marie Lebiere** (sophialebiere@gmail.com), Tufts University, **Evan Sabini** (esabini@villanova.edu), Villanova University, **Nathan Shank** (shankn@moravian.edu), Moravian University, and **Andrew Woldar** (andrew.woldar@villanova.edu), Villanova University. *On the Girth of Assignment Graphs Generated from Digraphs*

Graph pebbling is a mathematical game played on a graph G with no loops or multiple edges. A standard pebbling move consists of removing two pebbles from a vertex and adding one pebble to an adjacent vertex. An assignment graph is a Hasse diagram derived from each sequence of possible pebbling moves. This project focuses on assignment graphs generated from digraphs with no bidirectional edges. We analyze what properties yield certain girths. In particular, we demonstrate the connection between the girth of assignment graphs and the number of permutations on $[n]$ containing no strong fixed points and no small descents via inclusion-exclusion. Our results are motivated by a problem concerning assignment graphs of prescribed girth. (Received September 14, 2021)

1174-05-7452 **Adam Widtsoe Knudson*** (adamwidtsoe16@gmail.com), Brigham Young University, and **J. Nolan Faught** (faught3@gmail.com), Brigham Young University. *Kemeny's constant in graphs with a 1-separation*

Kemeny's constant is a graph invariant useful for measuring connectivity of a graph. We will present a formula for computing Kemeny's constant of a graph with 1-separations and some of it's applications. (Received September 14, 2021)

1174-05-7457 **Michael Javier Rivera*** (michael.rivera34@upr.edu), University of Puerto Rico Mayaguez, **Naima Nader** (nnader@vassar.edu), Vassar College, and **Erika King** (eking@hws.edu), Hobart and Williams Colleges. *Induced Matching Game on Graphs*

A *matching* is an independent set of edges in a graph G. By independent we mean that no two edges in the set are adjacent. An *induced matching* is a matching with an additional property that no two of its edges are joined by an edge in G. An induced matching M in a graph G is *maximal* if no other induced matching in G contains M. We propose a game in which two players alternate choosing edges on a graph while maintaining an induced matching. They continue until the matching is maximal and the last player to go wins. We will describe winning strategies for this game and prove results on several families of graphs. In addition, we will talk about some new families of graphs where the winner is pre-determined. These graphs are called well-indumatchable or parity graphs. This material is based upon work supported by the National Science Foundation under grant no. DMS 1757616. (Received September 14, 2021)

1174-05-7466 **Ayush Garg*** (ayushg.iitd@gmail.com), IIT Delhi, **Cat Raanes** (17craanes@castilleja.org), University of British Columbia, and **Michelle Olson** (olson_m_e@csu.fullerton.edu), California State University, Fullerton. *Convex Geometries and their representation by circles on the plane*

A convex geometry is a closure system satisfying the anti-exchange property. We documented all convex geometries on 4- and 5-element base sets with respect to their representation by circles on the plane, with respect to the convex hull closure operator. All 34 non-isomorphic geometries on a 4-element set can be represented by circles, and of 672 known geometries on a 5-element set, we made representations of 623. Of the 49 remaining geometries on a 5-element set, one was already shown not to be representable due to the Weak Carousel property, as articulated by Adaricheva and Bolat (Discrete Mathematics, 2019). We discovered novel geometrical properties, namely the Triangle Property, the Opposite Property, the Triple Q property and the Nested Triangles Property, using which we show that 40 more of these convex geometries cannot be represented by circles on the plane. All properties are based on one circle being in the convex hall of two or three other circles, denoted by implications. The opposite property states that If $bcd \to e$ is a tight implication and $ab \to e$, $ac \to e$, and $ad \to e$, then $a \to e$. We also expand closure systems to systems with several unary predicates (colours) and apply this approach to representing convex geometries by coloured circles. Finally, we study representations of convex geometries with shapes other than circles or points and show that all 49 "impossible" geometries on 5-element set are representable by ellipses. (Received September 17, 2021)

1174-05-7468 **Stephen G. Hartke** (stephen.hartke@ucdenver.edu), University of Colorado Denver, and **Eric Culver*** (eric.culver@ucdenver.edu), University of Colorado Denver. *Relationship between Correspondence Coloring and Alon-Tarsi Number of a Graph* Preliminary report.

Correspondence coloring is a generalization of list coloring in which the colors appear differently to different vertices. The Alon-Tarsi number of a graph is a parameter derived from the Combinatorial Nullstellensatz which upper bounds most coloring parameters of a graph. However, it fails to be an upper bound on the correspondence chromatic number, and in fact, the exact relationship is not well-understood. We investigate this relationship in more detail, seeing when the correspondence chromatic number is larger, and when the Alon-Tarsi number is larger, and how big of a gap there can be. (Received September 14, 2021)

1174-05-7476 **Dorian Smith*** (smi01055@umn.edu), University of Minnesota Twin Cities. *Sandpile groups for cones over certain trees* Preliminary report.

Let $G = (V, E)$ be a loopless undirected, connected graph. The sandpile group $K(G)$ is the finite abelian group isomorphic to the cokernel of the reduced graph Laplacian of G. Here we prove the exact group structure of $K(G)$ for two types of graphs G: a path with a cone vertex attached, commonly referred to as an n-fan, and that of a star with a cone vertex attached. The motivation is that we hope that these two families will be extreme cases, giving bounds on the structure of the sandpile groups for all other cones over trees on a fixed number of vertices. (Received September 14, 2021)

1174-05-7524 **Nicholas W Landry*** (nicholas.landry@colorado.edu), University of Colorado Boulder, and **Juan G Restrepo** (juanga@colorado.edu), University of Colorado Boulder. *Hypergraph dynamics: assortativity and the expansion eigenvalue*

The largest eigenvalue of the matrix describing a network's contact structure is often important in predicting the behavior of dynamical processes. We extend this notion to hypergraphs and motivate the importance of an analogous eigenvalue, the *expansion eigenvalue*, for hypergraph dynamical processes. Using a mean-field approach, we derive an approximation to the expansion eigenvalue and its associated eigenvector in terms of the degree sequence for uncorrelated hypergraphs. We introduce a generative model for hypergraphs that includes degree assortativity, and use a perturbation approach to derive an approximation to the expansion eigenvalue and its corresponding eigenvector for assortative hypergraphs. We validate our results with both synthetic and empirical datasets. We define the *dynamical assortativity*, a dynamically sensible definition of assortativity for uniform hypergraphs, and describe how reducing the dynamical assortativity of hypergraphs through preferential rewiring can extinguish epidemics. (Received September 15, 2021)

1174-05-7535 **Lawrence Hueston Harper*** (larryharper888@gmail.com), University of California-Riverside. *The Morphismology of Combinatorics Meets Distributive Lattices*

The Morphismology of Combinatorics Meets Distributive Lattices

Morphismology is the approach to mathematics touted by Saunders MacLane. At the end of Chaper 1 of his classic monograph, Categories for the Working Mathematician (2nd. Ed.), he writes, "Category theory asks of every mathematical object, 'What are the morphisms?' ". In the ensuing chapters he shows how those morphisms, and the structure of the resulting category, can be used as tools to study the object and solve related problems. For fifty years, applying MacLane's dictum to combinatorial problems has been a large part of this author's work, leading to connections with all parts of mathematics. In particular we encountered distributive lattices repeatedly. In this lecture we will give an overview of two of those encounters: Answering a fundamental question about Steiner operations (morphisms for isoperimetric problems on graphs): How to efficiently generate the range of a Steiner operation? And suggesting new variants of Sperner's theorem (The largest antichain on the Boolean lattice with n generators is the largest (middle) rank): Is the free distributive lattice on n generators Sperner? (Received September 14, 2021)

1174-05-7558 **Christopher Oneill*** (musicman3320@gmail.com), San Diego State University, **Tara Gomes** (gomes072@umn.edu), University of Minnesota, and **Eduardo Torres Davila** (torre680@umn.edu), University of Minnesota. *Numerical semigroups, minimal presentations, and posets*

A *numerical semigroup* is a subset S of the natural numbers that is closed under addition. One of the primary attributes of interest in commutative algebra are the relations (or trades) between the generators of S; any particular choice of minimal trades is called a *minimal presentation* of S (this is equivalent to choosing a minimal binomial generating set for the defining toric ideal of S). In this talk, we present a method of constructing a minimal presentation of S from a portion of its divisibility poset. Time permitting, we will explore connections to polyhedral geometry.

No familiarity with numerical semigroups will be assumed for this talk. (Received September 15, 2021)

1174-05-7586 **Eric S. Egge*** (eegge@carleton.edu), Carleton College. *Pattern-Avoiding Fishburn Permutations and Ascent Sequences* Preliminary report.

A Fishburn permutation is a permutation which avoids the bivincular pattern $(231, \{1\}, \{1\})$. That is, it has no subsequence order isomorphic to 231 in which the first two entries of the subsequence are adjacent and the two smallest entries differ by one. These permutations are named for the fact that they are counted by the Fishburn numbers $\xi(n)$, which satisfy

$$\sum_{n=0}^{\infty} \xi(n) q^n = 1 + \sum_{n=1}^{\infty} \prod_{j=1}^{n} (1 - (1-q)^j).$$

An ascent sequence is a sequence $a_1, \ldots, a_n$ of nonnegative integers such that $a_1 = 0$ and for each $j \geq 2$, the entry a_j is at most one more than the number of ascents in $a_1, \ldots, a_{j-1}$. In this talk we will discuss pattern-avoiding Fishburn permutations, building on work of Gil and Weiner. We will also describe connections between pattern-avoiding Fishburn permutations and pattern-avoiding ascent sequences. (Received September 15, 2021)

1174-05-7608 **Nathaniel Veimau** (nveimau1@swarthmore.edu), Swarthmore College. *The Expected Number of Distinct Patterns in a Random Permutation* Preliminary report.

Let π be a uniformly chosen random permutation on $[n]$. Analyzing the interaction between the numbers in two overlapping sets of k-positions, denoted π_1 and π_2, we have found the probability that the sets are order isomorphic. This probability comes from a critical lemma which proves that, in order for two sets to be order isomorphic, their overlap entries must also be order isomorphic. Once we have established the probability that π_1 is order isomorphic to π_2, we will use this to find the expected value of the number of distinct permutations in π by defining a function of the overlap. This function is based on a second lemma where we show that the number of repeated patterns is less than or equal to the number of pairs of order isomorphic patterns, which helps us to derive an upper bound on the required expectation. We will describe our significant progress in showing that the expected number of distinct permutations is $2^n(1 - o(1))$. (Received September 16, 2021)

1174-05-7612 **Isabel Byrne*** (isabeltb@vt.edu), Virginia Tech, and **Veronica D Borras-Serrano** (veronica.borras@upr.edu), University of Puerto Rico, Mayaguez. *The Expected Number of Distinct Patterns in Random Permutations* Preliminary report.

Let π be a uniformly chosen random permutation on $[n]$. Analyzing the interaction between the numbers in two overlapping sets of k-positions, denoted π_1 and π_2, we have found the probability that the sets are order isomorphic. This probability comes from a critical lemma which proves that, in order for two sets to be order isomorphic, their overlap entries must also be order isomorphic. Once we have established the probability that π_1 is order isomorphic to π_2, we will use this to find the expected value of the number of distinct permutations in π by defining a function of the overlap. This function is based on a second lemma where we show that the number of repeated patterns is less than or equal to the number of pairs of order isomorphic patterns, which helps us to derive an upper bound on the required expectation. We will describe our significant progress in showing that the expected number of distinct permutations is $2^n(1 - o(1))$. (Received September 16, 2021)

1174-05-7682 **Jiaxi Nie*** (jin019@ucsd.edu), University of California San Diego, and **Jacques Verstraëte** (jbaverstraete@gmail.com), University of California San Diego. *Randomized greedy algorithm for independent sets in regular uniform hypergraphs with large girth*

In this paper, we consider a randomized greedy algorithm for independent sets in r-uniform d-regular hypergraphs G on n vertices with girth g. By analyzing the expected size of the independent sets generated by this algorithm, we show that $\alpha(G) \geq (f(d,r) - \epsilon(g,d,r))n$, where $\epsilon(g,d,r)$ converges to 0 as $g \to \infty$ for fixed d and r, and $f(d,r)$ is determined by a differential equation. This extends earlier results of Gamarnik and Goldberg for graphs. We also prove that when applying this algorithm to uniform linear hypergraphs with bounded degree, the size of the independent sets generated by this algorithm concentrate around the mean asymptotically almost surely. (Received September 15, 2021)

1174-05-7683 **Miles Mena*** (milestmena@lewisu.edu), Lewis University, **Zhanar Berikkyzy** (zberikkyzy@fairfield.edu), Fairfield University, **Liyang Zhang** (lzhang2@fairfield.edu), Fairfield University, **Isabel Trindade** (isabel.trindade@yale.edu), Yale University, **Jonathan Figuero-Reyes** (jonathanfigueroa@gmail.com), Baruch College, and **Natalie Jean-Michel** (nfj1@williams.edu), Williams College. *Spanning Forest Probabilities on Path Graphs and Other Graph Families* Preliminary report.

From a simple graph, a forest-building process can be applied to create a new graph from the spanning forest on the original graph. The process is summarized as follows: remove all the edges from the graph and enumerate them in any fashion, then, in order, add the edges if and only if the edge can be drawn from a vertex that has not already been visited by an edge.

This talk provides probabilities, among other things, of k-component graphs arising after the forest building process on a collection of graphs including path graphs. A generalization of these probabilities is given and we establish a connection to the peaks and valleys problem in a set of integers. (Received September 15, 2021)

1174-05-7715 **Tomás Aguilar-Fraga*** (tomas.aguilar.fraga@gmail.com), Harvey Mudd College, **Kobe Amir Lawson-Chavanu** (kobelawson@gmail.com), Morehouse College, and **Dirk Antony Tolson** (tolsond@sonoma.edu), Sonoma State University. *Counting ℓ-Interval Parking Functions: Preliminary Report* Preliminary report.

Parking functions of length n, that is, vectors representing the number of ways n cars can park on a one way street, are a well-studied combinatorial object. One variation on them is to consider each car as parking, at most, a fixed interval ℓ away from their preference. We call these ℓ-interval parking functions. Scholar Kimberly P. Hadaway has shown that, when $\ell = 1$, these functions are in bijection with the Fubini rankings of the same length. In this talk, we expand upon her work to present a generalised recursive formula for when ℓ is any natural number. Additionally, we present formulae for the number of nondecreasing ℓ-interval parking functions, while also finding new and interesting connections to objects such as the Fubini rankings, Dyck paths, and the Fibonacci numbers. (Received September 15, 2021)

1174-05-7786 **Tony Wing Hong Wong** (wong@kutztown.edu), Kutztown University of Pennsylvania, Kutztown University, **Kayla Barker*** (kaylaabarker5@gmail.com), Stockton University, **Mia DeStefano** (mdestefano@vassar.edu), Vassar College, **Michael Gohn** (mg7785@desales.edu), DeSales University, **Joseph Miller** (miller1374@uwlax.edu), University of Wisconsin-La Crosse, and **Jacob Roeder** (jwroeder19@my.trine.edu), Trine University. *Generalized Two-player Pebbling Games on Simple Graphs*

Graph pebbling is a mathematical game played on an undirected graph with no loops or multiple edges. An (a, b) graph pebbling game involves two players who alternate making moves in the following manner: removing a pebbles from a vertex and adding b pebbles to an adjacent vertex, with $a > b$. The first player having no available move loses the game. We determine some conditions for which the $(k + 1, k)$ graph pebbling game played on the complete graph K_n results in a P- or N-game. We also analyze the standard $(2, 1)$ graph pebbling game on other simple graphs G. (Received September 16, 2021)

1174-05-7800 **Michael Tait*** (michael.tait@villanova.edu), Villanova University. *Extremal questions re the minimum number of distinct eigenvalues of a graph*

Given a graph G, $q(G)$ is the minimum number of eigenvalues of a matrix whose zero/nonzero pattern corresponds to the edges of G. Any nonempty graph has $q(G) \geq 2$ and in this talk we discuss extremal questions about which graphs may attain $q(G) = 2$. This is joint work with the "AIM q-group" (Received September 16, 2021)

1174-05-7834 **Ethan Partida*** (ethanpartida@msn.com), University of Minnesota Twin Cities, **Russell Barnes** (rjbarnes@hmc.edu), Harvey Mudd College, **Fran Herr** (herrf@uw.edu), University of Washington, and **Cece Henderson** (ch3@wellesley.edu), Wellesley College. *Simon's conjecture and extending shellings of simplicial complexes* Preliminary report.

A pure simplicial complex is said to be 'shellable' if there exists an ordering of its facets with particular intersection criteria. A shellable d-dimensional complex is 'shelling completable' if it can be extended to a shelling of the d-skeleton of a simplex. Simon's conjecture states that any shellable complex is shelling completable, and work from a previous REU established this for the special case of vertex-decomposable (VD) complexes.

To describe combinatorial obstructions of 'stuck' shellings, we prove that certain classes of shellable complexes can always be extended by a predetermined number of facets. We also describe combinatorial and topological obstructions to 'stuck' shellings. Generalizing flag simplicial complexes, we examine the role that missing faces play in completions of shellings. Lastly, we show that extending shellings of VD subcomplexes to arbitrary VD complexes is not always possible. (Received September 17, 2021)

1174-05-7837 **Alice Oveson*** (alice.oveson@gmail.com), Brigham Young University. *Comparison of PageRank Variations* Preliminary report.

I will survey two variations on the classical PageRank algorithm, namely Non-Backtracking PageRank and μ-PageRank and give information on how the three algorithms compare in computation time and final result. In recent work, we have proved that PageRank and Non-Backtracking PageRank give the same result when applied to a bipartite biregular graph. I will analyze the prospect of clustering with the various algorithms and discuss the strengths and drawbacks of each. (Received September 16, 2021)

1174-05-7838 **Eliana Tolosa Villarreal*** (etolosavillarreal@mail.sfsu.edu), San Francisco State University, Università degli studi di Genova. *Positive signed sum systems and odd-lecture hall polytopes* Preliminary report.

Given a vector $\mathbf{x}$ of real numbers, we are interested in finding out which additions and subtractions of some of its entries are positive. The family of such subsets with positive sum is called the *positive signed sum system*

$\mathcal{Q}^+(\mathbf{x})$. We endow the positive signed sum system with the structure of a simplicial complex $\Delta(\mathcal{Q}^+(\mathbf{x}))$. We study the f-vector and h-vector of the this complex, which are invariant under the choice of $\mathbf{x}$, uncovering an unexpected relation with the theory of s-lecture hall polytopes. (Received September 16, 2021)

1174-05-7858 **Brian Kronenthal** (kronenthal@kutztown.edu), Kutztown University of Pennsylvania, Kutztown University, **Mason Nakamura** (Mason.Nakamura1@marist.edu), Marist College, and **Kevin Hua*** (kehxw@outlook.com), Pomona College. *Induced-Saturation with Star Graphs*

Using the definition of Tennenhouse, a graph G is *induced H-saturated* if there exists no induced subgraph H in G, but for every edge $e \in \overline{G}$, $G + e$ has an induced subgraph H. Tennenhouse showed the existence of induced $K_{1,3}$-saturated graphs for $n \geq 12$. Inspired by his results for this star graph $K_{1,3}$, we show there exist induced $K_{1,3}$-saturated graphs on n vertices if and only if $n \geq 8$. Additionally, we construct arbitrarily large induced $K_{1,m}$-saturated graphs for $4 \leq m$ by adopting techniques from Behrens, Erbes, Santana, Yager, and Yeager. Finally, for the double star $D_{2,2}$, we show via a constructive proof that there exists an induced $D_{2,2}$-saturated graph on n vertices if and only if $n \geq 12$. (Received September 17, 2021)

1174-05-7864 **Margherita Maria Ferrari** (mmferrari@usf.edu), University of South Florida, **Nataša Jonoska** (jonoska@mail.usf.edu), University of South Florida, **Devon Conant** (devonconant@usf.edu), University of South Florida, and **Lina Fajardo Gomez*** (fajardogomez@usf.edu), University of South Florida. *DNA Segment Arrangements and Delannoy Numbers* Preliminary report.

We use combinatorial techniques to study gene rearrangements. During reproduction, the transcriptionally active macronucleus in certain species of ciliates is disintegrated and regenerated from a germline micronucleus where each gene is fragmented and scrambled. Fragments may be out of order or reversed, separated by "junk" DNA segments which may be excised as circular molecules. The alignment of short repeat sequences, called pointers, at the endpoints of gene fragments guides the recombination process. Analysis of the sequences cyclic molecules indicates that fragments of one or two genes are involved in cyclization. We propose a combinatorial model to describe each linear arrangement of gene fragments neighboring circular molecule junctions as a sequence of pointers, and we call these sequences legal strings. We describe a bijection between legal strings and lattice paths from $(0, 0)$ to (m, n) using only steps $(1, 0)$, $(0, 1)$ and $(1, 1)$, which are counted with the square array of Delannoy numbers. (Received September 16, 2021)

1174-05-7868 **Austin Schaibley*** (austinbschaibley@lewisu.edu), Lewis University, and **Chiara Hurd** (ChiaraMHurd@lewisu.edu), Lewis University. *Graph theoretical design strategies for modeling web graphs using self-assembling DNA* Preliminary report.

Self-assembly is a term used to describe the process of a collection of components combining to form an organized structure without external direction. The unique properties of double-stranded DNA molecules make DNA a valuable structural material with which to form nanostructures, and the field of DNA nanotechnology is largely based on this premise. By modeling nanostructures with discrete graphs, efficient DNA self-assembly becomes a mathematical puzzle. These nanostructures have wide-ranging applications, such as containers for the transport and release of nano-cargos, templates for the controlled growth of nano-objects, and in drug-delivery methods. This research project centers around the exploration of the graph theoretical and combinatorial properties of DNA self-assembly of web graphs in three different laboratory conditions. (Received September 16, 2021)

1174-05-7872 **Jacob David** (jdavid@exeter.edu), Phillips Exeter Academy, **Christopher Wu*** (cw86459@eanesisd.net), Westlake High School, **Suho Oh** (suhooh@txstate.edu), Texas State University, and **Pierce Lai** (pierce.lai37@gmail.com), Massachusetts Institute of Technology. *Triconed Graphs, weighted forests, and h-vectors of matroid complexes*

A well-known conjecture of Stanley is that the h-vector of a matroid is a pure $\mathcal{O}$-sequence. There have been numerous papers with partial progress on this conjecture, but it is still wide open. In particular, for graphic matroids coming from taking the spanning trees of a graph as bases, the conjecture is mostly unsolved. In graph theory, a set of vertices is called dominating if every other vertex is adjacent to some vertex inside the chosen set. Kook proved Stanley's conjecture for coned graphs, which is the class of graphs that are dominated by a single vertex. Cranford et al extended that result to biconed graphs, which is the class of graphs dominated by a single edge. In this paper we extend that result to triconed graphs, the class of graphs dominated by a path of length 2. (Received September 16, 2021)

1174-05-7899 **Thomas Meyer*** (tmeyer23@amherst.edu), Amherst College, **Ethan Spingarn** (espingarn23@amherst.edu), Amherst College, **Sabrina Mi** (scmi@uchicago.edu), University of Chicago, and **Beata Casiday** (beata.casiday@yale.edu), Yale University. *Spinors and Graph Theory* Preliminary report.

The evolution of quantum mechanical systems can be described by the Schrödinger equation, and the Dirac equation for spinors (particles with spin). In this presentation, we study the behavior of spinors on a graph, as predicted by the Dirac equations, using analytic and algebraic methods. Analytically, we investigate the evolution of quantum states using graph-theoretic versions of the Schrödinger and Dirac equations, and we study discrete models of the Laplacian and Dirac operators. Algebraically, we study spinors as linear representations of the spin group using graph Clifford algebras. We also give gluing results on the centers of graph Clifford algebras, and on Dirac operators associated to lattice graphs.

This research was conducted as part of Yale's SUMRY REU 2021. (Received September 16, 2021)

1174-05-7926 **H. Tracy Hall*** (h.tracy@gmail.com), Hall Labs, LLC. *The inverse eigenvalue problem for Hermitian-labeled multigraphs* Preliminary report.

The inverse eigenvalue problem for a multigraph G asks which spectra are possible for a real symmetric matrix with pattern specified by G. (Multiple parallel edges mean that either zero or nonzero is a valid matrix entry.) Eigenvalue multiplicity lists give rise to an important sub-question, as does maximum nullity. Each of these questions has a "strong" variant, requiring well-behaved realizing matrices, for which the answers are monotone with respect to the graph minor ordering. As a consequence, via the Robertson-Seymour Theorem, for certain spectral questions the set of achieving graphs is completely specified by some finite list of minor-minimal examples.

Recent work extends these questions and their strong variants to sign-labeled multigraphs, for which finite lists of minor-minimal examples are still expected: Geelen, Gerards, and Whittle claim an extension of the Graph Minor Theorem to group-labeled multigraphs (in which edges are oriented and labeled by group elements) for finite abelian groups such as $\mathbb{Z}/2\mathbb{Z}$.

A further extension of these questions and their strong variants appears to possible for the class of Hermitian matrices. In this case, the graph should independently specify the quality (zero, strictly positive, strictly negative, or undetermined) of both the real and imaginary components for each off-diagonal entry. This can be accomplished using a multigraph whose edges are oriented and labeled by elements of the group $\mathbb{Z}/4\mathbb{Z}$. (Received September 16, 2021)

1174-05-7948 **Ram Krishna Goel*** (ram.krishna.goel@gmail.com), Krishna Homeschool, MIT PRIMES-USA program. *Products of reflections in smooth Bruhat intervals* Preliminary report.

A permutation is called smooth if the corresponding Schubert variety is smooth. Gilboa and Lapid prove that in the symmetric group, multiplying the reflections below a smooth element w in Bruhat order in a "compatible order" yields back the element w. In this project we investigate generalizations of this result to other finite Weyl groups and study finer properties of this decomposition of w. (Received September 17, 2021)

1174-05-7969 **Viktoriya Bardenova*** (vlbardenova6187@eagle.fgcu.edu), Florida Gulf Coast University, **Erik A Insko** (einsko@fgcu.edu), Florida Gulf Coast University, and **Vincent Ciarcia** (vciarcia@fgcu.edu), Florida Gulf Coast University. *Markov Models for the Tipsy Cop and Robber Game on Graphs*

In this paper we analyze and model three open problems posed by Harris, Insko, Prieto-Langarica, Stoisavljevic, and Sullivan in 2020 concerning the tipsy cop and robber game on graphs. The three different scenarios we model account for different biological scenarios. The first scenario is when the cop and robber have a consistent tipsiness level though the duration of the game; the second is when the cop and robber sober up as a function of time; the third is when the cop and robber sober up as a function of the distance between them. Using Markov chains to model each scenario we calculate the probability of a game persisting through M rounds of the game and the expected game length given different starting positions and tipsiness levels for the cop and robber. (Received September 17, 2021)

1174-05-7976 **Sunrose Thapa Shrestha** (sunrose.shrestha@gmail.com), Wesleyan University, **Jane Wang** (janewang03@gmail.com), Indiana University Bloomington, **Michael Kielstra*** (pmkielstra@college.harvard.edu), Harvard University, **Cameron Thomas** (Cameron.dthomas@morehouse.edu), Morehouse College, **Chenyang Sun** (cs19@williams.edu), Williams College, **Diana Davis** (dianajdavis@gmail.com), Phillips Exeter, and **Samuel Lelièvre** (samuel.lelievre@gmail.com), Université Paris-Sud, Université Paris-Saclay. *Towards the Language Complexity of the Regular Hexagon* Preliminary report.

Recently, the study of polygonal billiards – that is, the study of paths taken by idealized balls rolling in straight lines without friction on tables shaped like regular polygons, bouncing off the sides at the angles one would expect – has been the subject of some interest. We consider, in particular, methods to study billiard trajectories via the concept of language complexity, a function of n which gives the number of words of length n in some language. It is common to analyze the complexity of billiard trajectories by viewing them as words expressed in the labels of the sides of the table, with letters in a word corresponding to bounces in a trajectory, and analyzing the complexity of the language formed in the process. We present work towards a proof of a conjecture that the language complexity of a regular hexagonal billiard table is, asymptotically, $\frac{621}{32\pi^2}n^3$. This involves calculating the asymptotic density of lattice points on the hexagonal grid and making progress towards finding the number of trajectories on the grid bounded by some given length. (Received September 17, 2021)

1174-05-8031 **Jeremy L. Martin** (jlmartin@ku.edu), University of Kansas, **Mei Yin** (mei.yin@du.edu), University of Denver, **Derek Hanely** (derek.hanely@uky.edu), University of Kentucky, **Daniel A McGinnis** (dam1@iastate.edu), Iowa State University, **Andrés R. Vindas Meléndez*** (avindas@msri.org), MSRI & UC Berkeley, **Dane Miyata** (dmiyata@uoregon.edu), University of Oregon, and **George Nasr** (george.nasr@huskers.unl.edu), University of Oregon. *Ehrhart Theory of Panhandle & Paving Matroids* Preliminary report.

Ehrhart theory is a topic in geometric combinatorics which involves the enumeration of lattice points in integral dilates of polytopes. We introduce pan-handle matroids, which can be understood as a certain lattice-path matroid. We provide a formula for the Ehrhart polynomial of the matroid polytope of pan-handle matroids. This Ehrhart polynomial allows us to obtain a formula for matroid polytopes of paving matroids. (Received September 17, 2021)

1174-05-8041 **Emily J Evans** (ejevans@mathematics.byu.edu), Brigham Young University, **Amanda E. Francis** (aef@ams.org), Mathematical Reviews, American Mathematical Society, and **Rebecca Jones*** (rebeccajones@mathematics.byu.edu), Brigham Young University. *A comparison of resistance distance and Katz distance*

Although there are a large number of proposed techniques for link prediction and many papers that look at large scale effectiveness of differing techniques, little has been done to compare precise link ordering between two different methods. In this talk we will compare two such methods, resistance distance and Katz distance. We will compare these two techniques on several families of graphs and provide some closed exact formulas for Katz distance on these graphs. (Received September 17, 2021)

1174-05-8066 **Bryan A Curtis*** (bcurtis1@iastate.edu), Iowa State University. *Strong Nullity Interlacing Property* Preliminary report.

Let $S(G)$ denote the set of real symmetric matrices with graph G. For a matrix $A \in S(G)$ write $A(n)$ to denote the principal submatrix of A obtained by deleting vertex n. The strong nullity interlacing property (SNIP) is a new linear algebraic tool that is useful for simultaneously studying the nullity of matrices $A \in S(G)$ and of $A(n)$. This talk will introduce the SNIP and investigate its various applications and properties. These concepts are related to and motivated by the strong Arnold property and the λ, μ-problem. (Received September 17, 2021)

1174-05-8079 **Ryan Christopher Lynch** (rlynch6@nd.edu), University of Notre Dame, and **Natalie Robin Dodson*** (ndodson@middlebury.edu), Middlebury College. *Universal Cycles* Preliminary report.

A universal cycle (U-cycle) is a cyclic listing of all the elements of a combinatorial structure where each object appears exactly once. We prove that a U-cycle exists for the class of k-Naples parking functions. We then apply the related concept of a graph universal cycle (GU-cycle) to combinatorial objects that cannot be U-cycled, and ultimately demonstrate that Slow Rising Permutations can be GU-cycled. We research the possibility of applying GU-cycles to finite groups. Furthermore, we conjecture that alternating permutations are GU-cyclable and demonstrate progress towards this claim. (Received September 17, 2021)

1174-05-8089 **Dashleen Gonzalez** (dashleen.gonzalez@upr.edu), University of Puerto Rico, Ponce. *No Three in a* θ Preliminary report.

The no-three-in-a-line problem asks for the maximum number of points that can be placed on an $n \times n$ grid such that no three of them lie in a line. It has remained unsolved for over 100 years, even though it has an easily proven upper bound of $2n$. Inspired by this problem, we propose an extension similar to the one studied by Gossell and Johnson: How many points can be chosen on an $n \times n$ grid such that no three of them form an angle of θ? We classify the angles that yield interesting problems and focus on angles that appear in surprising configurations on the grid. We prove upper and lower bounds for specific angles and discuss the geometric properties of the grid. (Received September 17, 2021)

1174-05-8102 **Bonnie C. Jacob*** (bcjntm@rit.edu), Rochester Institute of Technology. *Toward a universal approach for the zero forcing span of a graph*

The zero forcing number of a graph, $\mathrm{Z}(G)$, has become a well-studied problem related to the inverse eigenvalue problem on graphs. Of course, standard zero forcing is just one example of a class of zero-forcing-style problems on graphs that consider a set of related questions: if we start with a subset of the vertex set colored blue, and the rest white, what is the minimum number of blue vertices needed at the beginning of the process in order to turn all vertices blue under repeated applications of a color change rule that is specific to the problem? For each of these problems, we can ask the following related question, which we pose here in the context of $\mathrm{Z}(G)$: Given a graph G, what is the minimum value k such that ANY set of vertices of cardinality $\mathrm{Z}(G) + k$ is a zero forcing set? We call this concept the *zero forcing span* of a graph. In this talk, we discuss some properties of zero forcing spans in the context of various zero-forcing-style problems, and also comment on what we can say about zero-forcing-style spans in general. (Received September 17, 2021)

1174-05-8115 **Mary Flagg*** (flaggm@stthom.edu), University of St. Thomas, **Daniela Ferrero** (dferrero@txstate.edu), Texas State University, **Chassidy Bozeman** (cbozeman@mtholyoke.edu), Mt. Holyoke College, **Beth Morrison Bjorkman** (bjorkman.beth@gmail.com), Air Force Research Laboratory, **Cheryl Grood** (cgrood1@swarthmore.edu), Swarthmore College, and **Carolyn Reinhart** (carolynreinhart196@gmail.com), Swarthmore College. *TAR Reconfiguration for Power Domination*

Reconfiguration studies the connections between feasible sets for a particular graph problem. Power domination is a graph coloring game inspired by the challenge of efficiently monitoring a power network. A power dominating set is a subset of the vertices of the graph which will observe all of the graph according to the color change rule based on the physics. Given a graph G, the TAR reconfiguration graph of G is a graph with power dominating sets as vertices and two vertices are connected by an edge if one set may be obtained from the other by adding or removing one vertex. We study the properties of the TAR reconfiguration graph of G and its relation to the properties of G. (Received September 18, 2021)

1174-05-8136 **Junxuan Shen*** (jshen@caltech.edu), California Institute of Technology. *The Structural Incidence Problem for Cartesian Products* Preliminary report.

We prove new structural results for point-line incidences. An incidence is a pair of one point and one line, where the point is on the line. The Szemerédi-Trotter theorem states that n points and n lines form $O(n^{4/3})$ incidences. This bound has been used to obtain many results in combinatorics, number theory, harmonic analysis, and more. While the Szemerédi-Trotter bound has been known for several decades, the structural problem remains wide-open. This problem asks to characterize the point-line configurations with $\Theta(n^{4/3})$ incidences.

We completely characterize the case where the point set is a lattice.

Theorem. Let $1/3 < \alpha < 2/3$. Let $\mathcal{P}$ be a lattice of size $n^{\alpha} \times n^{1-\alpha}$ and $\mathcal{L}$ be a set of n lines, such that $\mathcal{P}$ and $\mathcal{L}$ form $\Theta(n^{4/3})$ incidences. Then $\mathcal{L}$ contains $\Omega(n^{1/3}/\log n)$ families of $\Theta(n^{2/3})$ parallel lines. The y-intercepts of each family of parallel lines form an arithmetic progression.

When $\alpha < 1/3$ or $\alpha > 2/3$, it is impossible to have $\Theta(n^{4/3})$ incidences. We also obtain similar results where $\mathcal{P} = A \times B$ where only one of A, B is an arithmetic progression. That is, in the case where only one axis of the Cartesian product $\mathcal{P}$ behaves like a lattice. (Received September 18, 2021)

1174-05-8167 **Sophia Rai Benjamin** (sophia.r.benjamin@gmail.com), North Carolina School of Science and Mathematics, **Quinn B Perian** (quinn.perian@outlook.com), Stanford Online High School, and **Arushi Mantri*** (arushi.mantri@gmail.com), Jesuit High School Portland. *On the Wasserstein Distance Between k-Step Probability Measures on Finite Graphs* Preliminary report.

We consider random walks X, Y on a finite graph G with respective lazinesses $\alpha, \beta \in [0, 1]$. Let μ_k and ν_k be the k-step transition probability measures of X and Y. In this paper, we study the Wasserstein distance between μ_k and ν_k for general k. We consider the sequence formed by the Wasserstein distance at odd values of k and the sequence formed by the Wasserstein distance at even values of k. We first establish that these sequences always converge, and then we characterize the possible values for the sequences to converge to. We further show that each of these sequences is either eventually constant or converges at an exponential rate. By analyzing the cases of different convergence values separately, we are able to partially characterize when the Wasserstein distance is constant for sufficiently large k. (Received September 17, 2021)

1174-05-8211 **Dermot McCarthy*** (Dermot.McCarthy@ttu.edu), Texas Tech University. *Generalized paley graphs, multicolor diagonal Ramsey numbers and modular forms.*

In this talk, we introduce generalized Paley graphs and study their complete subgraphs.

The generalized Paley graph of order q, $G_k(q)$, is the graph with vertex set $\mathbb{F}_q$, where q is a prime power such that $q \equiv 1 \pmod{k}$ if q is even, or, $q \equiv 1 \pmod{2k}$ if q is odd, and, ab is an edge if and only if $a - b$ is a k-th power residue.

In particular, we provide a formula, in terms of finite field hypergeometric functions, for the number of complete subgraphs of order four contained in $G_k(q)$, $\mathcal{K}_4(G_k(q))$, which holds for all k. This generalizes the results of Evans, Pulham and Sheehan on the original (k=2) Paley graph. We also provide a formula, in terms of Jacobi sums, for the number of complete subgraphs of order three contained in $G_k(q)$, $\mathcal{K}_3(G_k(q))$. In both cases, we give explicit determinations of these formulae for small k.

We show that the number of complete subgraphs of generalized Paley graphs can be used to yield lower bounds for multicolor diagonal Ramsey numbers. Using the formulae described above, we state explicitly lower bounds for the multicolor diagonal Ramsey numbers $R_k(4) = R(4, 4, \cdots, 4)$ and $R_k(3) = R(3, 3, \cdots, 3)$ for small k and compare to known bounds.

We also examine the relationship between both $\mathcal{K}_4(G_k(q))$ and $\mathcal{K}_3(G_k(q))$, when q is prime, and Fourier coefficients of modular forms. (Received September 18, 2021)

1174-05-8230 **Walter Bridges*** (wbridg6@lsu.edu), University of Cologne. *Product-sum identities from restricted plane partitions* Preliminary report.

A cylindric partition is a sort of restricted plane partition that can be thought of as wrapping around the surface of a cylinder. A *profile* describes their shape, and Borodin proved that for every profile, the generating function is a (modular) infinite product. Recently, Corteel and Welsh proved systems of recurrences for cylindric partitions of all profiles and found sum generating functions that solve these in several cases, thus establishing new connections between product-sum identities and an important combinatorial object.

We discuss an extension of Corteel–Welsh's work to symmetric cylindric and double skew shifted plane partitions, which, thanks to recent work of Han and Xiong, also come with infinite product generating functions. This is joint work with Ali Uncu. (Received September 18, 2021)

1174-05-8276 **Aram Bingham*** (aram@matmor.unam.mx), Centro de Ciencias Matemáticas, Universidad Nacional Autónoma de México. *Bijections for posets of clans*

Borel subgroup orbits of classical symmetric spaces are parametrized by families of signed involutions called *clans*, which provide a combinatorial model for studying questions related to Schubert calculus on symmetric spaces. For symmetric spaces of Hermitian type, clans are grouped into "sects" corresponding to Schubert cells of an associated Grassmannian variety, yielding a cell decomposition of the symmetric space and facilitating a combinatorial description of the closure (Bruhat) order on the orbits. This decomposition reveals coincidences of clans in the largest sect with other well-studied posets of matrix Schubert varieties. We further describe explicit bijections between clans for Hermitian type symmetric spaces and several other combinatorial families of objects, including certain rook placements, set partitions, and weighted Delannoy paths. (Based on joint work with Mahir Can and Özlem Uğurlu.) (Received September 18, 2021)

1174-05-8293 **Andrew Hardt*** (hardt040@umn.edu), University of Minnesota. *Lattice Models, Hamiltonian Operators, and Symmetric Functions*

We will explore the question of which functions are expressible both in terms of the partition function of a solvable lattice model and as the time-evolution of a Hamiltonian operator. Both of these combinatorial objects arise from representations of quantum groups, but in different ways.

In the case of six-vertex model, this intersection turns out to be at the physically motivated *free fermion point*. Using Hamiltonians allows us to compute that the partition function of the free fermionic lattice model is always a supersymmetric Schur function.

In a q-deformation where we introduce *charge* into the model, a similar result holds, and the partition function is a super-LLT polynomial. It turns out that lattice models in number theory originally constructed to represent metaplectic Whittaker functions are also ideally suited to have Hamiltonian operators. (Received September 18, 2021)

1174-05-8303 **Sam Spiro*** (sspiro@ucsd.edu), University of California, San Diego, and **Xiaoyu He** (alkjash@stanford.edu), Princeton. *Maximal Independent Sets in Clique-free Graphs*

An independent set I of a graph G is said to be a maximal independent set (MIS) if it is maximal with respect to set inclusion. Nielsen proved that the maximum number of MIS's of size k in an n-vertex graph is asymptotic to $(n/k)^k$, with the extremal construction being a disjoint union of k cliques with sizes as close to n/k as possible. In this talk we study how many MIS's of size k an n-vertex graph G can have if G does not contain a clique K_t. We prove for all fixed k and t that there exist such graphs with $n^{\lfloor \frac{(t-2)k}{t-1} \rfloor - o(1)}$ MIS's of size k by utilizing recent work of Gowers and B. Janzer on a generalization of the Ruzsa-Szemerédi problem. We prove that this bound is essentially best possible for triangle-free graphs when $k \leq 4$. This is joint work with Xiaoyu He and Jiaxi Nie. (Received September 18, 2021)

1174-05-8304 **Justin M. Troyka*** (jutroyka@davidson.edu), Davidson College, and **Yan Zhuang** (yazhuang@davidson.edu), Davidson College. *Connections between permutation clusters and generalized Stirling permutations*

Permutation clusters are studied in the context of consecutive patterns in permutations, in the cluster method of Elizalde and Noy (2012) which counts the permutations avoiding a given consecutive pattern. We count $2134\ldots(m+1)$-clusters-clusters according to inverse descent number and according to inverse peak number, and we find connections to the m-Stirling permutations and mth-order Eulerian polynomials of Gessel (1978) and the (1/2)-Eulerian polynomials of Savage and Viswanathan (2012).

From these results, we count the permutations avoiding the consecutive pattern $2134\ldots(m+1)$. This is made possible using the second author's new analog of the cluster method that applies to the Malvenuto–Reutenauer algebra of permutations, an algebra which generalizes the symmetric functions, quasisymmetric functions, and noncommutative symmetric functions. Applying various homomorphisms to this new cluster method recovers Elizalde and Noy's cluster method and Elizalde's q-analog which counts according to inversions, and it also yields several other new permutation cluster methods refined by various permutation statistics. The ones refined by inverse descent number and inverse peak number allow us to count the pattern-avoiding permutations according to these statistics, directly from our enumeration of clusters. (Received September 18, 2021)

1174-05-8312 **Jagdeep Singh*** (jsing29@lsu.edu), Louisiana State University, and **James Oxley** (oxley@math.lsu.edu), Louisiana State University. *Generalizing Cographs to 2-cographs*

The class of cographs or complement-reducible graphs is the class of graphs that can be generated from K_1 using the operations of disjoint union and complementation. In this talk, we consider 2-cographs, a natural generalization of the well-known class of cographs. We show that, as with cographs, 2-cographs can be recursively defined. However, unlike cographs, 2-cographs are closed under induced minors. We consider the class of non-2-cographs for which every proper induced minor is a 2-cograph and show that this class is infinite. Our main result finds the finitely many members of this class whose complements are also induced-minor-minimal non-2-cographs. This is joint work with James Oxley. (Received September 18, 2021)

1174-05-8359 **Andrew Reimer-Berg*** (Andrew.Reimer-Berg@colostate.edu), Colorado State University. *A combinatorial proof of a geometric enumeration via generalized RSK* Preliminary report.

We present a new combinatorial proof of an identity that appeared in a recent paper by Farkas and Lian. Their proof was geometric, and arose in the study of linear series on curves. The theory of Schubert calculus leads to a natural combinatorial interpretation of the identity in terms of Young tableaux, and our proof generalizes the well-known RSK bijection on words. (Received September 18, 2021)

1174-05-8365 **Sarah Allred*** (sallre4@lsu.edu), Louisiana State University, **Bogdan Oporowski** (bogdan@math.lsu.edu), Louisiana State University, and **Guoli Ding** (ding@math.lsu.edu), Louisiana State University. *Unavoidable Induced Subgraphs of Large 2-connected Graphs*

It is well known that, for every positive integer r, every sufficiently large connected graph contains an induced subgraph isomorphic to one of K_r, $K_{1,r}$, and P_r. We prove an analogous result for 2-connected graphs. In particular, we show that every sufficiently large 2-connected graph contains an induced subgraph isomorphic to one of K_r, a subdivision of $K_{2,r}$ with possibly an edge joining the two vertices of degree r, and a graph that has a well-described ladder structure. (Received September 18, 2021)

1174-05-8382 **Andrew Li Zhang*** (andrewzhang05719@gmail.com), Wayzata High School, **Hua Wang** (hwang@georgiasouthern.edu), Georgia Southern University, **Jenny Wei** (jenwwwww@gmail.com), Allen High School, and **Raymond Wang** (raymond.m.wang@gmail.com), Westwood High School. *A flow network model to track Covid infection through time*

In this work, we modeled the spread of Coronavirus (COVID-19) in New England by representing the region as a flow network with multiple time periods, and then applying the Ford-Fulkerson algorithm with various improvements. We developed a systematic way to evolve the flow network between time periods by introducing the evolution node, which increases or decreases the flow in between time periods based on the factors of new vaccinations, new cases, and a travel metric. To accommodate these adjustments, the Ford-Fulkerson algorithm is modified to handle both minimum and maximum bounds on edge capacities, as well as multiple sources and sinks of flow. Through recursive applications of the Ford-Fulkerson algorithm and the analysis of the maximum flow and minimum cut, we used this model to identify and analyze critical land borders between the New England states and simulate the effects of potential quarantine, close-down, or prioritized vaccination. Our model and results can be beneficial to infectious disease control and prevention in general. This work was supervised by Professor Hua Wang from Georgia Southern University. (Received September 19, 2021)

1174-05-8451 **Toufik Mansour** (tmansour@univ.haifa.ac.il), University of Haifa, **Diego Villamizar*** (dvillami@tulane.edu), Aalto university, **Rigoberto Flórez** (rflorez1@citadel.edu), The Citadel, **Jose Luis Ramirez** (jolura1@gmail.com), Universidad Nacional de Colombia, and **Fabio A. Velandia** (fvelandias@unal.edu.co), Universidad Nacional de Colombia. *Restricted Dyck paths*

In this talk we will discuss a subfamily of classic lattice paths, the Dyck paths, called restricted $d-$Dyck paths. These are paths in the first quadrant of the $xy-$plane that start at the origin and end on the $x-$axis consisting of north-east and south-east steps. The restriction is that the difference in between the height of two consecutive valleys (local minima) is at least d (rightmost valley minus leftmost valley). We will discuss some statistics on the paths such as the total area and the number of peaks (local maxima). Connections to non-crossing partitions and polyominoes will be given. (Received September 19, 2021)

1174-05-8452 **Caleb J Fong*** (calebfjx@gmail.com), University of St Andrews, and **Kai Zheng** (kzzheng@mit.edu), Massachusetts Institute of Technology. *Highly Revstack-Sorted Images of S_n* Preliminary report.

The study of West's stack-sorting map s has in recent years demonstrated connections between various fields of mathematics such as free probability theory and convex geometry. An interesting variant of West's map s is the *reverse-stack* (revstack) sorting map $\mathcal{T}$, which takes an input permutation π, reverses it, and then passes it through s. Interest in the map $\mathcal{T}$ goes beyond the surface-level novelty of its combinatorial results. As precisely stated in Steingrímsson's Sorting Conjecture, it is widely believed that on average – permutations in S_n sort to the identity under fewer iterations of $\mathcal{T}$ than iterations of s.

In this talk, we will use a sequence of structural lemmas to show that for k sufficiently large compared to n, the image sets $\mathcal{T}^k(S_n)$ are enumerated by a simple sequence. This echoes a result due to Defant in 2020, in which a similar enumeration is obtained for the map s. The enumeration that we prove involves showing that for these large values of k, every permutation in $\mathcal{T}^k(S_n)$ has at most one descent (i.e., an index i in the permutation $\pi = \pi_1 \cdots \pi_n$ for which $\pi_i > \pi_{i+1}$). This then leads naturally into some ideas for further work, including a proposed characterization of the maximum number of descents of a permutation in $\mathcal{T}^k(S_n)$, for *all* values of k.

This is based on work done at the University of Minnesota Duluth REU, with the support of the NSF and the NSA. (Received September 19, 2021)

1174-05-8510 **Seamus Albion*** (seamus.albion@univie.ac.at), Universität Wien. *2-core Littlewood identities*

In his 1940 textbook on group characters, Littlewood wrote down two summation formulae for Schur functions where the sum is restricted to partitions whose Young diagram contains only columns or rows of even length. Now called Littlewood identities, these formulae and their generalisations have found many applications in combinatorics, representation theory and hypergeometric series. Recently Lee, Rains and Warnaar conjectured a pair of novel Littlewood-type identities for Macdonald polynomials where the sum is over partitions with empty 2-core. I will explain how to resolve their conjectures in the Schur function case, and discuss some further fascinating conjectures involving partitions with empty 2-core. (Received September 19, 2021)

1174-05-8511 **Andrés R. Vindas Meléndez** (avindas@msri.org), MSRI, **Andrés Ramos Rodríguez** (ramosandres443@gmail.com), Universidad de Puerto Rico, Rio Piedras, **Christo Meriwether Keller** (cmkeller@umass.edu), Umass Amherst, **Eunice Sukarto** (eunicesukarto@berkeley.edu), University of California, Berkeley, **Laura Colmenarejo** (lcolmen@ncsu.edu), University of Massachusetts Amherst, and **Zakiya Jones** (zakiyacmjones@gmail.com), Pomona College. *Counting k-Naples Parking Functions Through Permutations and the k-Naples Area Statistic*

The k-Naples parking functions of length n (a generalization of parking functions) are defined by requiring that a car which finds its preferred spot occupied must first back up a spot at a time (up to k spots) before proceeding down the street. Note that the well-studied parking functions are the specialization of k to 0. For fixed $0 \leq k \leq n-1$, we define a function φ_k which maps a k-Naples parking function to the permutation denoting the order in which its cars park. In this talk we provide a result which establishes that summing the sizes of the fibers of φ_k gives a new formula for the number of k-Naples parking functions as a sum over the permutations of length n. This formula for enumerating k-Naples parking functions is not recursive, in contrast to the previously known. Moreover, it can be expressed as the product of the lengths of particular subsequences of permutations, and its specialization to $k=0$ gives another way to describe the number $PF_n = (n+1)^{n-1}$ of parking functions of length n. Lastly, we will relate the q-analog of our formula to a new statistic that we denote $\texttt{area}_k$ and call the *k-Naples area statistic*, the specialization of which to $k=0$ gives the usual `area` statistic on parking functions. (Received September 19, 2021)

1174-05-8512 **Lon Mitchell*** (lonmitchell@usf.edu), University of South Florida, and **Sivaram K. Narayan** (sivaram.narayan@cmich.edu), Central Michigan University. *On minimum semidefinite rank and zero-forcing related parameters for signed graphs.*

We present some recent results on minimum semidefinite rank of signed graphs and their implications for zero-forcing related parameters. (Received September 19, 2021)

1174-05-8587 **Rachana Madhukara*** (rachanam@mit.edu), MIT. *Adjacency and Broadcast Dimension of Grid and Directed Graphs*

A function $f : V(G) \to \mathbb{Z}_{\geq 0}$ is a *resolving broadcast* of a simple undirected graph G if, for any distinct $x, y \in V(G)$, there exists a vertex $z \in V(G)$ with $f(z) > 0$ such that $\min\{d(x,z), f(z)+1\} \neq \min\{d(y,z), f(z)+1\}$. The *broadcast dimension* $\mathrm{bdim}(G)$ of G is the minimum of $\sum_{v \in V(G)} f(v)$ over all resolving broadcasts f of G. Similarly, the *adjacency dimension* $\mathrm{adim}(G)$ of G is the minimum of $\sum_{v \in V(G)} f(v)$ over all resolving broadcasts f of G where f only takes values in $\{0,1\}$. Note that these parameters are defined analogously for directed graphs by considering directed distances.

We partially resolve a question of Zhang by calculating the adjacency dimension of certain Cartesian products of path graphs, namely $\mathrm{adim}(P_2 \square P_n)$ and $\mathrm{adim}(P_3 \square P_n)$ for all n. Additionally, we study the behavior of adjacency and broadcast dimension on directed graphs. First, we explicitly calculate the adjacency dimension of a directed complete k-ary tree, where every edge is directed towards the leaves. Next, we prove that $\mathrm{adim}(\vec{G}) = \mathrm{bdim}(\vec{G})$, where $\vec{G}$ is any orientation of G. Furthermore, we show that $\mathrm{bdim}(G)/\mathrm{bdim}(\vec{G})$ is unbounded and also not uniformly bounded away from 0. (Received September 21, 2021)

1174-05-8592 **Stoyan Dimitrov*** (sdimit6@uic.edu), University of Illinois At Chicago, and **Niraj Khare** (nkhare@cmu.edu), Carnegie Mellon University in Qatar. *Moments of permutation statistics and central limit theorems*

We show that if a permutation statistic can be written as a linear combination of bivincular patterns, then its moments can be expressed as a linear combination of factorials with constant coefficients. This generalizes a result of Zeilberger. We use an approach of Chern, Diaconis, Kane and Rhoades, previously applied on set partitions and matchings. In addition, we give a new proof of the central limit theorem (CLT) for the number of occurrences of classical patterns, which uses a lemma of Burstein and Hästö. We give a simple interpretation

of this lemma and an analogous lemma that would imply the CLT for the number of occurrences of any vincular pattern. Furthermore, we obtain explicit formulas for the moments of the descents and the minimal descents statistics. The latter is used to give a new direct proof of the fact that we do not necessarily have asymptotic normality of the number of pattern occurrences in the case of bivincular patterns. (Received September 19, 2021)

1174-05-8613 **Joshua D. Laison*** (jlaison@willamette.edu), Willamette University, **Kyle Salois** (kylesalois@gmail.com), Colorado State University, **Caroline Daugherty** (daughertyc@kenyon.edu), MIT, and **Rebecca Robinson** (rebrobin@umflint.edu), University of Colorado Denver. *Intersection graphs of convex sub-polygons of a polygon, a story in pictures* Preliminary report.

We introduce intersection graphs of maximal convex sub-polygons of polygons with sides parallel to a regular $2k$-gon. If P is a polygon with sides parallel to a regular $2k$-gon, let S be the set of all maximal convex polygons contained in P. Form the intersection graph G with a vertex for each polygon in S, and an edge between vertices if their corresponding polygons intersect in their interiors. We say that G is a k-MSP (maximal sub-polygon) graph.

We find representations of familiar families of graphs (trees, cycles) and find separating examples of graphs which are k-MSP graphs and not j-MSP graphs for $k \neq j$. As a presentation in the Presenting Research Mathematics through Visual Storytelling session, these proofs will be presented without words. (Received September 19, 2021)

1174-05-8637 **Isabella Quan*** (isabella.quan6@gmail.com), Westlake High School. *On Snowflakes and Pizza: Graph Theoretic Properties of 2D Steiner Solutions*

Steiner's plane cutting problem asks for configurations of lines in the plane that create the maximum number of regions (termed S-solutions). Following the work of Baril and Santos, we associate to every S-solution a graph G in the plane and establish graph theoretic properties common to these graphs; for instance, all S-solutions are bipartite, with a minimum degree of 2. We also introduce a new technique for analyzing and constructing S-solutions, called the "snowflake transformation". This allows us to construct families of S-solutions whose graphs have small vertex degree. This technique also yields a new method of constructing traceable S solutions for all n. Finally, we use the snowflake transformation method to construct S solutions with n lines whose graphs have large maximum independent sets. We hope to extend our methods to study graph-theoretic properties of hyperplane arrangements in higher dimensions. We also hope to develop tools to determine sufficient conditions for a planar graph to correspond to an S-solution. (Received September 19, 2021)

1174-05-8728 **Olivia McGough*** (mcgougho@reed.edu), Reed College, **Gopal Goel** (gopal.krishna.goel@gmail.com), Massachusetts Institute of Technology, and **David Perkinson** (davidp@reed.edu), Reed College (professor). *Counting weighted maximal chains in the circular Bruhat order*

The *Bruhat order* is a partial ordering on S_n, graded by the number of inversions. Its cover relations have the form $\pi s_{ij} \lessdot \pi$ where $s_{ij} := (i,j)$ is a transposition such that $\ell(\pi) = \ell(\pi s_{ij}) + 1$. Define the *weight* of a covering $\pi s_{ij} \lessdot \pi$ with $i < j$ to be $\alpha_i + \alpha_{i+1} + \ldots \alpha_{j-1}$, and the weight of a maximal chain to be the product of the weights of its cover relations. Stembridge shows that the sum of the weights of the maximal chains is

$$\frac{\binom{n}{2}!}{1^{n-1}2^{n-2}\ldots(n-1)^1} \prod_{1\leq i<j\leq n} \alpha_i + \alpha_{i+1} + \ldots \alpha_{j-1}$$

The totally nonnegative Grassmannian $\mathrm{Gr}(k,n)_{\geq 0}$ is the subset of the real Grassmannian $\mathrm{Gr}(k,n)$ consisting of points with all nonnegative Plücker coordinates. The circular Bruhat order is a poset isomorphic to the face poset of Postnikov's positroid cell decomposition of $\mathrm{Gr}_{\geq 0}(k,n)$. We define "circular" analogues of Stembridge's weights and coverings, and provide a closed formula for the sum of its weighted maximal chains:

$$f(k,n)(\alpha_1 + \ldots \alpha_n)^{k(n-k)}$$

where $f(k,n)$ is the number of Young tableaux for the $k \times (n-k)$ rectangle. (Received September 19, 2021)

1174-05-8731 **Isabella M Sholtes*** (bellamarie2700@gmail.com), Washington & Jefferson College, and **Robert Muth** (rmuth@washjeff.edu), Washington & Jefferson College. *Cuspidal ribbon tableaux in affine type A*

For any convex preorder on the set of positive roots of affine type A, we construct associated combinatorial objects called cuspidal ribbons. We further show that every skew shape has a unique ordered tiling by these cuspidal ribbons. This combinatorial data is applicable to other areas of algebra and geometry, such as the representation theory of Specht modules. (Received September 19, 2021)

1174-05-8742 **Yifan Zhang*** (yifan12@illinois.edu), University of Illinois at Urbana-Champaign, **Wei Wang** (weiw10@illinois.edu), University of Illinois at Urbana-Champaign, and **Hsin-Hui Judy Chiang** (hsinhui2@illinois.edu), University of Illinois at Urbana-Champaign. *Subsums of Random Numbers* Preliminary report.

If you pick n random numbers in $[0,1]$, what is the probability that their sum also falls into the interval $[0,1]$? The answer turns out to be $1/n!$, which can be seen using symmetry arguments or multidimensional integrals. In our research, we consider more generally the probability, $p(n,k)$, that, given n random numbers in $[0,1]$, there are k of these numbers whose sum falls into the interval $[0,1]$.

We prove the explicit formula

$$p(n,k) = 1 + \frac{1}{(k-1)!}\sum_{i=1}^{k-1}(-1)^{k-i}\cdot\binom{k-1}{i}\cdot i^n$$

We also show that $p(n,k)$ satisfies the recurrence

$$p(n,k) = \frac{1}{k}\cdot p(n-1,k-1) + \frac{k-1}{k}\cdot p(n-1,k)$$

and has generating function

$$\sum_{n=k}^{\infty} p(n,k)x^k = \frac{x^k}{k!\cdot(1-x)(1-\frac{1}{2}x)\cdots(1-\frac{k-1}{k}x)}.$$

These formulas are analogous to formulas for Stirling numbers of the second kind, $S(n,k)$, which count the number of ways to partition an n-element set into k nonempty subsets. (Received September 19, 2021)

1174-05-8779 **Charles D Burnette*** (cburnet2@xula.edu), Xavier University of Louisiana. *Involution factorizations of Ewens random permutations*

An involution is a bijection that is its own inverse. Given a permutation σ of $[n]$, let $\mathsf{invol}(\sigma)$ denote the number of ways to express σ as a composition of two involutions of $[n]$. The statistic invol is asymptotically lognormal when the symmetric groups $\mathfrak{S}_n$ are each equipped with Ewens Sampling Formula probability measures of some fixed positive parameter θ. In this talk, we strengthen and generalize previously determined results on the limiting distribution of $\log(\mathsf{invol})$ for uniform random permutations, i.e. the specific case of $\theta = 1$. We also investigate the first two moments of invol itself. (Received September 19, 2021)

1174-05-8784 **Shanise Walker*** (walkersg@uwec.edu), University of Wisconsin Eau Claire. *The Game of Cycles on Cactus Graphs*

The Game of Cycles is a two-player impartial mathematical game, introduced by Francis Su in 2020 in his book *Mathematics for Human Flourishing.* The game is played on simple planar graphs in which players take turns marking edges using a sink-source rule. The game of cycles has been explored on graphs with certain types of symmetry where a mirror-reverse strategy determined which player could win the game. In this talk, we analyze the game of cycles for specific types of cactus graphs using a modified version of the mirror-reverse strategy. (Received September 19, 2021)

1174-05-8829 **Soichi Okada*** (okada@math.nagoya-u.ac.jp), Nagoya University. *Enumeration of shifted plane partitions of double staircase shape via intermediate symplectic characters*

S. Hopkins conjectured a simple product formula for the number of shifted plane partitions of double staircase shape with bounded entries, where a double staircase is a shifted shape obtained by pasting two shifted staircases back to back. The conjecture was then proved by Hopkins and Lai using the theory of lozenge tilings of the triangular lattice. In this talk, we provide an alternate proof of Hopkins' conjecture by using Proctor's intermediate symplectic characters. In fact, we establish a factorization formula for a certain summation of intermediate symplectic characters, and deduce Hopkins' conjecture by specializing the variables to 1. This approach enables us to find q-analogues, which can be considered as generalizations of the MacMahon and Bender–Knuth ex-conjectures on symmetric plane partitions. (Received September 20, 2021)

1174-05-8912 **Christian Gaetz*** (crgaetz@gmail.com), Harvard University. *On some generalizations of weak Bruhat order* Preliminary report.

I will discuss the combinatorics of a new family of lattices which appear in the theory of total positivity. These lattices are closely related to the weak Bruhat order on a finite Coxeter group, a very well-studied class of semidistributive lattices. (Received September 20, 2021)

1174-05-8991 **Jason O'Neill*** (jmoneill@ucsd.edu), UC San Diego. *Towards supersaturation for oddtown and eventown* Preliminary report.

Given a collection $\mathcal{A}$ of subsets of an n element set, let $\mathrm{op}(\mathcal{A})$ denote the number of distinct pairs $A, B \in \mathcal{A}$ for which $|A \cap B|$ is odd. Using linear algebra arguments, we prove that for any collection $\mathcal{A}$ of $2^{\lfloor n/2 \rfloor} + 1$ even-sized subsets of an n element set that $\mathrm{op}(\mathcal{A}) \geq 2^{\lfloor n/2 \rfloor - 1}$. We also prove that for any collection $\mathcal{A}$ of $n+1$ odd-sized subsets of an n element set that $\mathrm{op}(\mathcal{A}) \geq 3$, and that both of these results are best possible. We also consider the problem for larger collections of odd-sized and even-sized sets respectively. (Received September 20, 2021)

1174-05-9003 **Anton Dochtermann** (dochtermann@txstate.edu), Texas State University, **Ayah Almousa*** (aka66@cornell.edu), University of Minnesota - Twin Cities, and **Benjamin Smith** (benjamin.smith-3@manchester.ac.uk), University of Manchester. *Triangulations of Root Polytopes and Polarizations* Preliminary report.

We show that every triangulation of a root polytope gives rise to a polarization of a so-called "restricted power" of the graded maximal ideal in a polynomial ring and discuss the algebraic and combinatorial implications of this fact. (Received September 20, 2021)

1174-05-9004 **Stephen Melczer*** (smelczer@uwaterloo.ca), University of Waterloo. *New Applications of Analytic Combinatorics*

We discuss new applications of analytic combinatorics, including problems in information theory, biology, and algebraic statistics. (Received September 20, 2021)

1174-05-9009 **Tiadora Ruza*** (tvruza@uwaterloo.ca), University of Waterloo. *Limit Theorems for Permutations with Restricted Cycles: An Experimental Journey*

Analytic combinatorics in several variables is a field of study focused on the derivation of limit behaviours of multivariate sequences. This talk will walk through proving a local central limit theorem for a family of permutations with restricted cycles, using techniques from the theory of analytic combinatorics in several variables. The focus of the talk will be not only on general methods for proving a local central limit theorem, but also on the benefits of experimentation with mathematical software to provide insight on how to apply the tools provided by analytic combinatorics in several variables. As a key part of the proof, we compute a symbolic determinant using a powerful (and somewhat general) method involving LU-factorization. (Received September 20, 2021)

1174-05-9029 **William Jonathan Keith*** (wjkeith@mtu.edu), Michigan Technological University, and **Wei-Lun Tsai** (wt8zj@virginia.edu), University of Virginia. *Open Problems in Combinatorics and Modular Forms, Part II*

The second part of this extended discussion. We will present some widely interesting open problems in the fields of combinatorics and modular forms at this special session. We will also give a brief necessary background regarding these open problems. Our main objective for this presentation is to help bridge the researchers from these two fields. Both W. Keith and W.-L. Tsai will present during this talk. A portion of the time will be devoted to audience discussion and we are eager to hear your input. (Received September 20, 2021)

1174-05-9030 **Isaac Reiter** (ireit426@live.kutztown.edu), Polymath Jr. REU, **Nicholas Tong Chiem*** (nick_chiem@yahoo.com), Polymath Jr. REU, **Blake Bates** (bates326@umn.edu), Polymath Jr. REU, **Pablo Blanco** (pablancoh@aol.com), Polymath Jr. REU, **Sarvagya Jain** (sarvagyajain@iisc.ac.in), Polymath Jr. REU, **Kevin Zhou** (kevinazhou150@gmail.com), Polymath Jr. REU, **Hanna Mikulás** (hanna@minerva.kgi.edu), Polymath Jr. REU, **Risa Fines** (finesr@carleton.edu), Polymath Jr. REU, and **Maja Lie** (maja.alzy.lie@gmail.com), Polymath Jr. REU. *Preliminary Report: Anti-Ramsey Multiplicities* Preliminary report.

There has been an extensive amount of research done on Ramsey theory, but not nearly as much on anti-Ramsey theory. In this project we study the maximum number of rainbow copies of a given graph H, asymptotically within an edge-colored K_n. A graph H is r-rainbow-common if the maximum proportion of rainbow copies of H in K_n is achieved through a random edge-coloring with r colors, and we call a graph r-rainbow-uncommon if H is not r-rainbow-common. Not much is known about rainbow-commonality of graphs. De Silva et al recently investigated the problem for complete graphs, stars, and disjoint unions of stars. They conjectured that the cycle graph C_k is k-rainbow uncommon. We verify this conjecture for $k = 4, 5, 6$ with specific constructions. De Silva et al also conjectured that if H is r-rainbow-uncommon, then H is $(r+1)$-rainbow-uncommon. We prove that this conjecture holds for K_3 and K_4. This is joint work with the Polymath Jr group on anti-Ramsey multiplicities. (Received September 20, 2021)

1174-05-9038 **Edo Biluar*** (`biluared@grinnell.edu`), Grinnell College. *Counting Unit Simplices in* $\mathbb{R}^d$

A *unit k-simplex* is a set of k points in $\mathbb{R}^d$ with pairwise unit distances between them. An interesting problem in Discrete Geometry asks for the maximum possible number of unit k-simplices that can be determined by a set of n points in $\mathbb{R}^d$. For $d \geq 4$ and $1 \leq k \leq d/2$, it is not very hard to show that the answer to this question is of the order n^k. However, for $d/2 < k < d+1$ the problem is wide open for most cases. Erdős and Purdy conjectured that for that setting the answer is of the order $n^{d/2}$. The conjecture is motivated by constructions that match this bound. We make progress towards this conjecture by confirming it for the case of $k = d/2 + c$ for any constant c and sufficiently large $d = d(c)$. Our proof builds on the work of Agarwal and Sharir; using classical cutting techniques one can reduce the problem to solving a large family of geometrically defined linear optimisation problems. By a careful analysis we solve the corresponding linear optimisations to achieve our result. (Received September 20, 2021)

1174-05-9068 **Shira Zerbib*** (`zerbib@iastate.edu`), Iowa State University. *A generalized Caccetta-Haggkvist conjecture* Preliminary report.

Generalizing the Caccetta-Haggkvist (CH) conjecture, we conjecture that every digraph G has a directed cycle of size at most $\lceil \sum_{v \in V(G)} \frac{1}{deg^+(v)} \rceil$. We prove this when $deg^+(v) \leq 2$ for all v. We also address a rainbow generalization of the conjecture: if $F_1, \ldots, F_n \subseteq E(K_n)$, then they have a rainbow cycle of size $\lceil \sum_{i=1}^{n} \frac{1}{|F_i|} \rceil$ or less. ("Rainbow" means choosing a distinct representative for some of the sets. The CH conjecture is the case in which the F_i's are the out-stars in the directed graph.) We prove this conjecture when $|F_i| \leq 2$ for all i. Joint with Ron Aharoni, Eli Berger and Maria Chudnovsky. (Received September 20, 2021)

1174-05-9094 **Amanda Burcroff** (`agburcroff@gmail.com`), Durham University, and **Grace O'Brien*** (`graceob@umich.edu`), University of Michigan-Ann Arbor. *Unimodality and monotonic portions of certain domination polynomials* Preliminary report.

Given a simple graph G on n vertices, a subset of vertices $U \subseteq V(G)$ is *dominating* if every vertex of $V(G)$ is either in U or adjacent to a vertex of U. The *domination polynomial* of G is the generating function whose coefficients are the number of dominating sets of a given size. We examine the domination polynomials of many well-known families of graphs. In particular, we show unimodality for spiders with at most 400 legs (of arbitrary length), lollipop graphs, arbitrary direct products of complete graphs, and Cartesian products of two complete graphs. Additionally, we prove that for every graph, a portion of the coefficients are non-increasing, where the size of the portion depends on the upper domination number, and in certain cases this is sufficient to prove unimodality. Finally, we study graphs with m *universal vertices*, i.e., vertices adjacent to every other vertex, and show that the last $(\frac{1}{2} - \frac{1}{2^{m+1}})n$ coefficients of their domination polynomial are non-increasing. (Received September 20, 2021)

1174-05-9133 **Samantha Miller-Brown*** (`sam413@lehigh.edu`), Lehigh University. *Specht Modules as 0-Hecke Algebra Modules* Preliminary report.

The well studied 0-Hecke algebra is a deformation of the symmetric group, $\mathfrak{S}_n$. While the representation theory of the symmetric group is encoded via the Frobenius characteristic map using symmetric functions, there is an analogous map which encodes representations of the 0-Hecke algebra using quasisymmetric functions. We call a 0-Hecke algebra module *compatible* with an $\mathfrak{S}_n$ module if they have related actions and the same respective Frobenius images. We focus our attention now on the Specht modules, S^λ. Using polytabloids indexed by standard Young tableaux as a basis for S^λ, one can define an $\mathfrak{S}_n$ action such that the Frobenius image is precisely the schur function s_λ. In this talk, we aim to show that by deforming the commonly defined $\mathfrak{S}_n$ action on the standard polytabloids, we obtain a well-defined, compatible 0-Hecke algebra module. In particular, we will show that we are able to produce a well-defined 0-Hecke algebra action on the Specht modules with quasisymmetric Frobenius image s_λ. (Received September 20, 2021)

1174-05-9183 **Elaine Wong*** (`elaine.wong@ricam.oeaw.ac.at`), Austrian Academy of Sciences, RICAM, **Benjamin Hackl** (`benjamin.hackl@aau.at`), Alpen-Adria-Universität Klagenfurt, and **Jesse Selover** (`jselover@umass.edu`), UMass Amherst. *Asymptotics for Diagonal Coefficients of Multivariate Rational Functions in SageMath* Preliminary report.

In this talk, we present a SageMath implementation of algorithms from a recent paper of Melczer and Salvy that enable the computation of dominant asymptotics for the diagonal coefficients of multivariate rational functions. These asymptotics rely on the existence and computation of minimal critical points of the rational function. In particular, we describe a symbolic-numeric approach to achieve this: the symbolic side comes from computing zeroes of a certain polynomial system via the Kronecker representation (giving us the critical points) and the numeric side comes from minimality testing via the refinement of precision bounds. We showcase a few examples in the combinatorial case and report a bit on ongoing/future work. (Received September 20, 2021)

1174-05-9212 **Robin A Pemantle*** (`pemantle@math.upenn.edu`), University of Pennsylvania. *ACSV: recent improvements* Preliminary report.

I will discuss some of the most recent improvements to ACSV that will appear in the forthcoming second edition of "Analytic Combinatorics in Several Variables" (with Mark Wilson and Steve Melczer). (Received September 20, 2021)

1174-05-9221 **Stefan Trandafir*** (`strandaf@sfu.ca`), Simon Fraser University, and **Adam Afandi** (`adamafandi46@gmail.com`), University of Munster. *Littlewood-Richardson coefficients from the vector partition perspective* Preliminary report.

The Littlewood-Richardson (LR) coefficients $c^{\nu}_{\lambda,\mu}$ are the structure constants for the decomposition of the tensor product of irreducible representations V_λ and V_μ of $GL_k(\mathbb{C})$ into a direct sum of irreducible representations. Using the hive interpretation of LR coefficients introduced by Berenstein and Zelevinsky, Rassart showed that the LR coefficients associated to GL_k can be represented using a vector partition function. From this form, he showed that there exists a piecewise-polynomial p_k such that $c^{\nu}_{\lambda,\mu} = p_k(\lambda, \mu, \nu)$ for all λ, μ, ν with $l(\lambda), l(\mu), l(\nu) \leq k$. More recently, Briand and Rosas have utilized this perspective to prove that there are 144 linear symmetries of the LR coefficients associated to SL_3.

We have found a novel determinant formula for the LR coefficients associated to GL_k, and also found a geometric interpretation for a known stabilization result. Additionally, we have computed the pieces of polynomiality for the GL_4 case, and formulae for several of the pieces (for which the polynomial is a binomial coefficient). We have used this computation to prove that there are 12 symmetries for the LR coefficients associated to SL_k for $k = 4, 5, 6, 7$. In subsequent work we hope to generalize the determinant formula and describe the pieces for which the piecewise-polynomial is a binomial coefficient for any k. (Received September 20, 2021)

1174-05-9226 **Hein Van Der Holst** (`hvanderholst@gsu.edu`), Georgia State University, **Marina Arav*** (`marav@gsu.edu`), Georgia State University, and **Fredrick Scott Dahlgren** (`fdahlgren1@student.gsu.edu`), Georgia State University. *Signed graphs with maximum nullity two.*

A signed graph is a pair (G, Σ), where $G = (V, E)$ is a graph (in which parallel edges are permitted, but loops are not) with $V = \{1, \ldots, n\}$ and $\Sigma \subseteq E$. The edges in Σ are called odd and the other edges of E even. By $S(G, \Sigma)$ we denote the set of all symmetric $n \times n$ matrices $A = [a_{i,j}]$ with $a_{i,j} < 0$ if i and j are adjacent and connect by only even edges, $a_{i,j} > 0$ if i and j are adjacent and connected by only odd edges, $a_{i,j} \in \mathbb{R}$ if i and j are connected by both even and odd edges, $a_{i,j} = 0$ if $i \neq j$ and i and j are non-adjacent, and $a_{i,i} \in \mathbb{R}$ for all vertices i.

Arav, Hall, Li, and van der Holst characterized the 2-connected signed graphs (G, Σ) with $M(G, \Sigma) = 2$. In this talk, we give a complete characterization of the signed graphs (G, Σ) with $M(G, \Sigma) \leq 2$. (Received September 20, 2021)

1174-05-9283 **Ralihe Raul Villagran Olivas*** (`ralihevillagran@gmail.com`), Cinvestav-IPN. *On graphs with real algebraic co-rank at most* 3 Preliminary report.

Recently, there have been found relations between the algebraic co-rank of a graph and the zero-forcing number along with the minimum rank. In this talk we continue on this direction by exploring the graphs with real algebraic co-rank at most 3, aiming to shed light on the conjecture that the real minimum rank is bounded from above by the real algebraic co-rank. (Received September 20, 2021)

1174-05-9290 **Kyle Salois*** (`kyle.salois@colostate.edu`), Colorado State University. *Inequality of a Class of Near-Ribbon Skew Schur Q-functions*

While equality of skew Schur functions is well understood, the problem of determining when two skew Schur Q-functions are equal is still largely open. It has been studied in the case of ribbon shapes in 2008 by Barekat and van Willigenburg. We approach the problem for *near-ribbon* shapes, formed by adding one box to a ribbon skew shape; in particular, *frayed ribbons*, which are the near-ribbons whose shifted skew shape is not an ordinary skew shape. We conjecture with evidence that all Schur Q-functions for frayed ribbon shapes are distinct up to antipodal reflection. We prove this conjecture for several infinite families of frayed ribbons, using a new approach via the "lattice walks" version of the shifted Littlewood-Richardson rule discovered in 2018 by Gillespie, Levinson, and Purbhoo. (Received September 20, 2021)

1174-05-9312 **Prasad Senesi*** (senesi@cua.edu), The Catholic University of America. *Solid waste collection and work: a variation on a standard routing optimization problem* Preliminary report.

Route optimization is a common problem in business and industry. Our research team investigated the problem of optimizing the routes taken by trash trucks in a local county. Whereas the problem of finding a route of minimal distance is familiar, we considered the more challenging problem of optimizing a route with respect to the work done by a truck, which is a function not only of distance but also of accrued weight as more trash sites are visited. This variation on the standard route optimization problem posed some remarkable challenges for us. In this report on our research project, we will discuss data collection, graph-theoretic modeling of the problem, strategies, algorithms, and future directions. (Received September 20, 2021)

1174-05-9324 **Dashleen Gonzalez*** (dashleen.gonzalez@upr.edu), University of Puerto Rico, Mayaguez. *DNA Coding Theory* Preliminary report.

Next generation sequencing methods determine the genomic sequences in several samples simultaneously. It is now possible to sequence much faster than before. In order to distinguish the samples sequenced simultaneously one must use a DNA Barcode. In this work we use DNA barcodes derived from BCH codes and other Algebraic Codes. There are previous constructions using BCH Codes, Hamming Codes, Golay codes or LDPC codes. In this work we construct DNA barcodes as a subcode of quaternary BCH codes. We take into consideration the restrictions from the sequencing technology. We present our results on the information rate of the resulting DNA barcodes. (Received September 20, 2021)

1174-05-9325 **James H Lin*** (jameslin@mit.edu), MIT. *Rowmotion on m-Tamari and BiCambrian Lattices*

Rowmotion is a well-studied operator on the distributive lattice of order ideals of a poset. In 2016, Barnard generalized the definition of rowmotion to semidistributive lattices, and later in 2018, Thomas and Williams generalized the definition of rowmotion to trim lattices. We partially resolve a conjecture by Thomas and Williams that the order of rowmotion on the (trim) (a, b)-Tamari lattice is $a + b - 1$ – namely, we describe and prove the orbit structure in the case when $b \equiv 1 \pmod{a}$. Furthermore, we additionally prove that the down-degree statistic on this special case of Tamari lattices is homomesic for rowmotion. Finally, we settle a conjecture by Thomas and Williams on the orbit structure of the (semidistributive) biCambrian lattice with respect to a bipartite Coxeter element. (Received September 20, 2021)

1174-05-9330 **James Edward Garrison*** (jamesgarrison99@gmail.com), Hampden-Sydney College. *Symmetry Parameters in Kneser Graphs* Preliminary report.

A graph G is said to be *d-distinguishable* if there exists a not-necessarily-proper coloring with d colors so that only the trivial automorphism preserves the color classes. The *paint cost of d-distinguishing*, denoted $\rho^d(G)$, is defined as the minimum size of the complement of a color class over all d-distinguishing colorings for G. Our work also utilizes *determining sets* of G, a set of vertices S of G such that every automorphism of G is uniquely determined by its action on S. The *determining number* of a graph is the size of a smallest determining set. We investigate the paint cost of d-distinguishing families of Kneser Graphs using the determining number of those families. Our preliminary findings include the determining number for $K_{n:2}$, which we plan to use to find $\rho^d(K_{n:2})$. (Received September 20, 2021)

1174-05-9333 **Kevin Long*** (kevinlong@gwu.edu), George Washington University, and **Joseph Bonin** (jbonin@gwu.edu), George Washington University. *The free m-cone of a matroid and its $\mathcal{G}$-invariant* Preliminary report.

The $\mathcal{G}$-invariant is a matroid invariant introduced by Derksen which generalizes the Tutte polynomial. Derksen and Fink later proved that $\mathcal{G}$ is the universal valuative invariant for matroid base polytopes. Bonin and Kung defined another invariant, the catenary data, and proved that it was equivalent to $\mathcal{G}$. A third matroid invariant of interest is the configuration, introduced by Eberhardt, which determines the $\mathcal{G}$-invariant for a matroid with no coloops. Examples of matroids with the same $\mathcal{G}$-invariant and different configurations include Dowling matroids of rank at least 4 based on groups of the same order. However, the smallest of these examples have 28 elements, which make them unwieldy, and not many other examples are known. We give a matroid construction, called the free cone, based on any pair of nonisomorphic matroids with the same $\mathcal{G}$-invariant, which produces a pair of matroids with the same $\mathcal{G}$-invariant and different configurations. The free cone provides smaller examples that are easier to work with. In this talk, we will give an overview of these matroid invariants. We will also define the free cone, and outline how its structure gives the desired result using the catenary data. (Received September 20, 2021)

1174-05-9359 **Rebecca Nicole Smith*** (rnsmith@brockport.edu), SUNY Brockport, and **Lara K Pudwell** (lkp42@yahoo.com), Valparaiso University. *Shuffle sorting permutations* Preliminary report.

We consider the problem of sorting permutations using four different shuffle algorithms. Our results include some characterizations and enumerations of the sortable permutations using each algorithm, comparisons between certain pairs of the algorithms, and some results about the worst case scenario for one of the algorithms. (Received September 20, 2021)

1174-05-9374 **Cory Palmer** (cory.palmer@umontana.edu), University of Montana, **Daniel Johnston** (djohnst1@skidmore.edu), Skidmore College, and **P. Mark Kayll*** (mark.kayll@umontana.edu), University of Montana. *Restricted matchings in Turán graphs* Preliminary report.

In *Algebraic Combinatorics*, Godsil remarks on derangements that "counting them is a traditional preoccupation of Combinatorics texts." So perhaps we've all seen enough of derangements! But perhaps not ... attend this talk and decide for yourself. (Joint work with Dan Johnston and Cory Palmer.) (Received September 20, 2021)

1174-05-9383 **Kassie Archer*** (karcher@uttyler.edu), University of Texas at Tyler, and **Christina Graves** (cgraves@uttyler.edu), University of Texas at Tyler. *A new statistic on Dyck paths* Preliminary report.

We investigate a new statistic on Dyck paths recently introduced in a paper in order to enumerate 321-avoiding permutations composed only of 3-cycles. This statistic is a product of binomial coefficients that depend on the lengths of ascents and descents in the Dyck path. We'll talk about the relationship between this statistic and 321-avoiding permutations composed of only 3-cycles, and also mention some other nice properties of this statistic. The proofs presented will use bijections with Motzkin paths and generalized Dyck paths. (Received September 20, 2021)

1174-05-9396 **Megan A. Martinez*** (mmartinez@ithaca.edu), Ithaca College. *Teaching Old Bijections to do New Tricks*

Throughout the years, several bijections between permutations and inversion sequences have been developed. However, not as much has been done to investigate how these bijections map pattern-avoiding permutation classes to pattern-avoiding inversion sequence classes. In this talk, we will discuss the surprising (and nice!) interplay between some of these bijections that map permutation to inversion sequences, and what pattern-avoiding results can be mined from them. (Received September 20, 2021)

1174-05-9424 **Jd Nir*** (jd.nir@umanitoba.ca), University of Manitoba, and **Kyle Murphy** (kyle.murphy@dsu.edu), Dakota State University. *Paths of Length Three are K_{r+1}-Turán-Good*

The generalized Turán problem $\mathrm{ex}(n, T, F)$ is to determine the maximal number of copies of a graph T that can exist in an F-free graph on n vertices. Recently, Gerbner and Palmer noted that the solution to the generalized Turán problem is often the original Turán graph. They gave the name "F-Turán-good" to graphs T for which, for large enough n, the solution to the generalized Turán problem is realized by a Turán graph. They prove that the path graph on two edges, P_2, is K_{r+1}-Turán-good for all $r \geq 3$, but they conjecture that the same result should hold for all P_ℓ. In this paper, using arguments based in flag algebras, we prove that the path on three edges, P_3, is also K_{r+1}-Turán-good for all $r \geq 3$. (Received September 20, 2021)

1174-05-9425 **John F Burkhart** (jfb84@nau.edu), Northern Arizona University, and **Alexander P Stewart*** (aps293@nau.edu), Northern Arizona University. *Characterizing the structure of reversals acting on signed* Preliminary report.

A reversal acting on a signed permutation π reverses the order of elements in consecutive positions and changes their signs. The reversal distance of a signed permutation π is equal to the minimal number of reversals needed to transform π into the identity permutation. Each signed permutation can be encoded in a so-called a breakpoint diagram that illuminates the underlying structure of the corresponding signed permutation. In particular, Hanenhalli and Pevzner provide a formula for easily calculating the reversal distance of a signed permutation using its breakpoint diagram. In this talk, we will characterize special classes of signed permutations by describing the structure of their corresponding breakpoint diagrams. We will also summarize connections to genome rearrangement problems. (Received September 21, 2021)

1174-05-9427 **Katie Anders*** (kanders@uttyler.edu), University of Texas at Tyler. *The Universal Difference Property for generalized graph splines over principal ideal domains* Preliminary report.

We study the generalized graph splines introduced by Gilbert, Tymoczko, and Viel and focus on an attribute known as the Universal Difference Property (UDP). Anders, Arreola, Asencio, Ireland, and Smith showed that UDP is satisfied for certain families of edge labeled graphs, including paths, trees, and cycles, and used Prüfer domains to show that not every edge labeled graph satisfies UDP. Altinok and Sarıoğlan showed that UDP is satisfied for any edge labeled graph over a principal ideal domain. (Received September 20, 2021)

1174-05-9454 **Keith Leung** (lleungyukchi@outlook.com), Columbia University. *Colored Gelfand-Tsetlin Patterns and Symmetric Lattice Models* Preliminary report.

In the search for a bijective version of the proof presented in Weyl Group Multiple Dirichlet Series by Brubaker, Bump, and Friedberg, which states that two definitions of the p-parts of a multiple Dirichlet series given a Type A root system are equal, our group, mentored by Ben Brubaker himself, tackled defining new combinatorial objects: colored symmetric lattice models and their corresponding colored Gelfand-Tsetlin patterns. In this talk, we will discuss the main problem, which is motivated by representation theory and number theory, how our definitions tackle this problem, and our conjecture towards a bijective proof. In particular, we conjecture a generalization of the Schützenberger involution to these new classes of combinatorial objects. (Received September 20, 2021)

1174-05-9465 **Jonny Quezada** (jonnyquezada@unomaha.edu), University of Nebraska At Omaha, **William Chettleburgh** (chettleburghw@gmail.com), Michigan State University, **Naftoli Kolodny*** (nzk13@yahoo.com), Suny Binghamton, and **Luke Barbarita** (lukebarbarita@yahoo.com), University of California Irvine. *Enumerating Polyominoes on the Torus and Other Finite Surfaces*

Since their popularization in the mid-20th century, significant attention has been directed to the counting of polyominoes on an infinite plane. A polyomino is a shape consisting of unit squares that are connected by their edges. In our research, we pursued the less examined problem of counting polyominoes on finite surfaces with different dimensions, such as the torus and Möbius strip. Specifically, we examined the fixed polyominoes, where different orientations of a polyomino are considered distinct. We first performed by-hand enumeration for surfaces of smaller dimensions to learn more about the properties of polyominoes on each surface. Utilizing existing algorithms as a foundation, we then employed computational methods (such as recursive growth and transition matrices) to enumerate and count polyominoes for larger cases. We were able to count up to the 19-ominoes on the 6×6 torus, cylinder, and finite grid. Additionally, we proved results regarding special classes of polyominoes. This presentation will summarize the results of the CC-REU NSF summer REU experience (DMS-2050692) where these questions were explored. (Received September 20, 2021)

1174-05-9549 **Yuki Takahashi*** (takahash@grinnell.edu), Grinnell College. *Lifting methods in mass partition problems*

Many results in mass partitions are proved by lifting $\mathbb{R}^d$ to a higher-dimensional space and dividing the higher-dimensional space into pieces. We extend such methods to use lifting arguments to polyhedral surfaces. Among other results, we prove the existence of equipartitions of $d+1$ measures in $\mathbb{R}^d$ by parallel hyperplanes and of $d+2$ measures in $\mathbb{R}^d$ by concentric spheres.

For measures whose supports are sufficiently well separated, we prove results where one can cut a fixed (possibly different) fraction of each measure either by parallel hyperplanes, concentric spheres, convex polyhedral surfaces of few facets, or convex polytopes with few vertices. (Received September 20, 2021)

1174-05-9556 **Opel Jones*** (ojones@towson.edu), Towson University. *Enumeration of Dumont permutations avoiding certain four-letter patterns*

In 1974, Dumont proved two types of permutations are counted by the same sequence. The first type is a permutation wherein each entry must be immediately followed by a smaller entry, and each odd entry must be immediately followed by a larger entry, or ends the permutation (i.e., the last entry must be odd). The second type is a permutation wherein each entry at an even position is a deficiency, and each entry at an odd position is a fixed point or an excedance. These are now known as Dumont permutations of the first and second kinds. In 2010, Burstein, Josuat-Vergès, and Stromquist proved that there are two more types of permutations which are also counted by that same sequence, now known as Dumont permutations of the third and fourth kinds. In this talk we will discuss enumerations of Dumont permutations of the fourth kind with certain restrictions, that is avoiding certain patterns or with minimal occurrences of certain patterns. We will also briefly discuss their proofs which involve methods using induction, block decomposition, Dyck paths, and generating functions. We will conclude with a conjecture that two four-letter patterns are Wilf-equivalent on Dumont permutations of the first kind. (Received September 20, 2021)

1174-05-9572 **Dawei Shen*** (shen.dawei@wustl.edu), Washington University In St. Louis, **Frank Lu** (franklu@princeton.edu), Princeton University, and **Siki Wang** (swang21@cmc.edu), Claremont McKenna College. *%-Immanants and Kazhdan-Lusztig Immanants*

In this paper, we investigate the relationship between Kazhdan-Lusztig immanants, which were introduced in Rhoades-Skandera (2006), and %-immanants, which were introduced in Chepuri-Sherman-Bennett (2021). As noted by Skandera (2008), Kazhdan-Lusztig immanants are important dual canonical basis elements of the coordinate ring of $GL_n(\mathbb{C})$. Our main result is a classification of when a 321-avoiding Kazhdan-Lusztig immanant can be written as a linear combination of %-immanants. This result uses a formula in Rhoades-Skandera (2005) to compute 321-avoiding Kazhdan-Lusztig immanants. We also partially extend our classification to general Kazhdan-Lusztig immanants: we obtain a necessary condition for a Kazhdan-Lusztig immanant to be a linear combination of %-immanants and a classification for a Kazhdan-Lusztig immanant to be written as a sum of at most two %-immanants. Finally, we conjecture an explicit formula for computing the Kazhdan-Lusztig immanants coming from a 1324-, 32154-, 21543-avoiding permutation, and using this conjectural formula, we derive expressions for 1324-, 24153-, 31524-, 32154-, 21543-, 231564-, 312645-, 426153-avoiding Kazhdan-Lusztig immanants as a sum of %-immanants. (Received September 20, 2021)

1174-05-9573 **Jonathan Jay Fang*** (jjfang@brandeis.edu), Brandeis University, and **Zachary R Hamaker** (zhamaker@ufl.edu), University of Florida. *On pattern avoidance in matchings and involutions* Preliminary report.

We study the relationship between two notions of pattern avoidance for involutions in the symmetric group and their restriction to fixed-point-free involutions. We introduce the notion of involution containment, and show that pattern avoidance for involutions can always be expressed in terms of involution containment through an effective algorithm. We also give partial results characterizing classes of involutions where the converse holds. Additionally, we describe enumerative results for involution classes avoiding involutions of length 3. (Received September 21, 2021)

1174-05-9622 **Ryan Jeong*** (rsjeong@sas.upenn.edu), University of Pennsylvania. *On Structural Aspects of Friends-And-Strangers Graphs*

Given two graphs X and Y on the same number of vertices, the friends-and-strangers graph $\mathsf{FS}(X,Y)$ has as its vertices all $n!$ bijections from $V(X)$ to $V(Y)$, with bijections σ,τ adjacent if and only if they differ on two adjacent elements of $V(X)$ whose images are adjacent in Y. We investigate a number of questions concerning structural properties of these graphs. Specifically, we study necessary and sufficient conditions for $\mathsf{FS}(X,Y)$ to be connected for all graphs X from some set: we prove that $\mathsf{FS}(X,Y)$ is connected for all biconnected X if and only if $\overline{Y}$ is a forest with trees of jointly coprime size, resolving a conjecture of Defant and Kravitz. We initiate and make significant progress toward determining the girth of $\mathsf{FS}(X,\mathrm{Star}_n)$ for connected graphs X. Finally, we study the diameters of friends-and-strangers graphs: we show there exist infinitely many values of n for which there are n-vertex graphs X and Y with the diameter of a component of $\mathsf{FS}(X,Y)$ at least $n^{(\log n)/(\log\log n)}$, so diameters of connected components of friends-and-strangers graphs fail to be polynomially bounded in the size of X and Y; this resolves a question posed by Alon, Defant, and Kravitz. We also show that any component of $\mathsf{FS}(\mathrm{Path}_n,Y)$ has diameter at most $|E(Y)|$, and the diameter of $\mathsf{FS}(\mathrm{Cycle}_n,Y)$ is $O(n^3)$ whenever $\mathsf{FS}(\mathrm{Cycle}_n,Y)$ is connected. (Received September 20, 2021)

1174-05-9657 **Shijing Chen*** (shijing.chen@pepperdine.edu), Pepperdine University. *Topology of Hamiltonian cycles in regular triangulations of the torus* Preliminary report.

Let G be a graph embedded on a surface S. Each cycle in G is an element of a homotopy class of simple closed curves on S. We consider the distribution of Hamiltonian cycles among homotopy classes of curves in the case where S is a torus and G is the edge graph of a regular triangulation of S. (Received September 20, 2021)

1174-05-9665 **David Narváez*** (david.narvaez@rochester.edu), University of Rochester. *Beyond the Resolution of Keller's Conjecture*

In 2019 in joint work with Brakensiek, Heule, and Mackey, we solved the last open case of Keller's conjecture: we found that in every tiling of the 7-dimensional space using identical cubes there are two cubes that share a face. Our proof is based on previous work by Kisielewicz and Łysakowska who cornered the conjecture in dimension 7 to the problem of finding large cliques in three graphs known as Keller graphs. We encoded the clique-finding problem as a Boolean formula and made extensive use of state-of-the-art techniques in automated reasoning, in particular verified satisfiability solving and symmetry breaking, to decide the satisfiability of this formula. The

first part of this talk provides an overview of the proof, including the background and the connection between Keller graphs and Keller's conjecture, the encoding of the clique-finding problem as a Boolean formula, the symmetries this formula exhibits, and the verification of our solution. The second part of this talk focuses on future work and related questions whose answers would let us greatly simplify the proof approach, and thus achieve a better understanding of Keller's conjecture. (Received September 20, 2021)

1174-05-9666 **Jacob Matherne** (acobm@math.uni-bonn.de), University of Bonn. *On the Newton Polytopes of Chromatic Symmetric Functions*

Chromatic symmetric functions of $(3+1)$-free incomparability graphs (in particular, incomparability graphs of unit interval orders) are well-studied symmetric functions in algebraic combinatorics, generalizing the chromatic polynomial. Motivated by the Stanley-Stembridge conjecture that the chromatic symmetric function of a $(3+1)$-free incomparability graph is e-positive, we show that the allowable coloring weights for such a graph are the lattice points of a permutahedron P_λ, and give a formula for the dominant weight lambda. We also give conjectures about stability and the Lorentzian property of these symmetric functions. (Received September 20, 2021)

1174-05-9671 **Kelsey Knoblock*** (knobl2kl@cmich.edu), Central Michigan University, **Robert W Bell** (bellro@msu.edu), Michigan State University, **Cassidy George** (ctgeorge@dons.usfca.edu), University of San Francisco, **Emme McMullen** (emcmullen@g.hmc.edu), Harvey Mudd College, and **Zach Stewart** (stewa719@msu.edu), Michigan State University. *Tandem Cops and Robbers and Outerplanar Graphs* Preliminary report.

Cops and Robbers is a pursuit and evasion game played on a finite discrete graph. The cop player wins by moving one of their pawns to the same vertex as the opponent's robber pawn. The players take turns moving their pawns, with the cop player always moving first. It is a complete information game, hence both players can see the entire graph and each other's positions at all times. The cop number of a graph G, denoted $c(G)$, is the minimum number of cops that are needed to ensure the cop player can win the game, assuming the robber always plays optimally. No intrinsic characterization for $c(G) = 2$ graphs is known. Tandem cops, a version of the game created by Clarke and Nowakowski, was created to work toward a characterization of cop number two graphs. Tandem cops are two cops that must remain adjacent before and after each move, hence graphs on which tandem cops can win are a subset of cop number two graphs. We investigate when outerplanar graphs are tandem-win. We present a theorem that a finite connected outerplanar graph G is tandem-win if and only if every k-cycle in G has a chord if $k \geq 5$. In addition, we explore the variation Cops and Robbers on oriented graphs. On these graphs, the pawns can only be moved in one direction on an edge, simulating one-way streets. No characterization for $c(G) = 1$ oriented graphs is known. We share some examples that illustrate the complexity of Cops and Robbers on oriented graphs. (Received September 20, 2021)

1174-05-9699 **Theo McKenzie*** (mckenzie@math.berkeley.edu), University of California, Berkeley. *Many Nodal Domains in Random Regular Graphs*

Nodal domains are the connected components of a graph when vertices are partitioned according to the sign of an eigenvector. For more than 100 years, they have been used to discern the structure of eigenvectors of operators in both continuous and discrete space.

We prove that with high probability, a randomly sampled d-regular graph is such that all eigenvectors of very negative eigenvalues of the adjacency matrix have an almost-linear number of nodal domains.

This confirms simulations of Dekel, Lee, and Linial and reinforces the random wave conjecture on graphs. (Received September 20, 2021)

1174-05-9700 **Grace Mulry** (gracemulry@utexas.edu), University of Texas at Austin, **Kyla Shappell** (Kyla.m.shappell@email.shc.edu), Spring Hill College, and **Kathleen Ryan** (Kathleen.Ryan@desales.edu), DeSales University. *The Bipartite Graph Reduction Game*

A *game-labeling* of a bipartite graph is one in which the vertices have non-negative labels and the label sums of the partite sets are equal. A *reduction across an edge* is one in which the endpoints are both reduced by the same integral amount. We introduce a single player game on a game-labeled bipartite graph G where each move is a reduction across an edge and no move produces a negative label. The goal of the game is to reduce all labels in G to 0, and if the player succeeds in doing so, the player wins the game. In this presentation, we present a necessary and sufficient sum condition for detecting the solvability of the game. We also demonstrate the connection between this game and Double Choco Puzzles. (Received September 20, 2021)

1174-05-9736 **Justin K Wisby*** (justinwisby@gmail.com), Florida International University, and **Walter Carballosa Torres** (wcarball@fiu.edu), Florida International University. *Bounds for total k-domination of Cartesian product of complete graphs.* Preliminary report.

Let $G = (V, E)$ be a finite undirected graph. A set S of vertices in V is said to be total k-dominating if every vertex in V is adjacent to at least k vertices in S. The total k-domination number, $\gamma_{kt}(G)$, is the minimum cardinality of a total k-dominating set in G. In this work we study the total k-domination number of Cartesian product of two complete graphs which is a natural lower bound of the total k-domination number of Cartesian product of two graphs. In particular, we obtain upper and lower bounds for every product of complete graphs. In addition, we obtain the exact values of the 2-total domination number of $K_n \square K_m$ for all $n, m \geq 2$. (Received September 20, 2021)

1174-05-9753 **Torin Greenwood*** (torin.greenwood@ndsu.edu), North Dakota State University, and **Mark Wilson** (markwilson@umass.edu), University of Massachusetts Amherst. *Asymptotics from higher-dimensional algebraic generating functions* Preliminary report.

Although the asymptotics of coefficients from multivariate rational generating functions and univariate algebraic generating functions are well-studied, a systemized approach for analyzing multivariate algebraic generating functions has not yet emerged. In 1996, Safonov published an algorithm that encodes the coefficients of a d-dimensional algebraic generating function within a (d + 1)-dimensional rational generating function. Tools from the study of analytic combinatorics in several variables can then be applied to extract asymptotics from this rational function. Algebraic generating functions arise in many contexts, including trees, Catalan objects, RNA secondary structures, and lattice paths in confined spaces. We explore applying Safonov's algorithm to several examples. (Received September 20, 2021)

1174-05-9787 **Thomas M. McKenzie*** (mckenzie@gonzaga.edu), Gonzaga University, **Nick Linthacum** (nlinthacum@zagmail.gonzaga.edu), Gonzaga University, and **Lin Ai Tan** (ltan2@zagmail.gonzaga.edu), Gonzaga University. *Deques on a Torus* Preliminary report.

To construct a one-page torus book embedding of a graph we arrange the vertices on a circle along the equator of the torus and then place the edges on the surface of the torus so that no two edges cross. This definition can be extended to an n-page torus book by maintaining the same circular vertex ordering on each page. We give an application of n-page tours books to computer science, showing that they correspond to torus deques. Optimal embeddings of certain graphs on torus books and applications to delivery systems are considered. (Received September 20, 2021)

1174-05-9789 **Jurgen Kritschgau*** (kritschgauj@gmail.com), Carnegie Mellon University. *Updates on Forbidden Induced Subgraphs and Throttling* Preliminary report.

Throttling seeks to minimize the sum of the size of a zero forcing set (standard or otherwise) and its propagation time. This talk will focus on the use of forbidden induced subgraphs in throttling in general, with an emphasis on the similarity between standard, PSD, and skew throttling. Connections to weighted throttling and product throttling will also be drawn. (Received September 20, 2021)

1174-05-9802 **Craig Erickson** (cerickson30@hamline.edu), Hamline University, **Kevin Grace*** (kevin.m.grace@vanderbilt.edu), Vanderbilt University, and **Alathea Jensen** (jensena@susqu.edu), Susquehanna University. *A combinatorial bound on the number of distinct eigenvalues of a graph*

The smallest possible number of distinct eigenvalues of a graph G, denoted $q(G)$, has a combinatorial bound in terms of unique shortest paths in the graph; $q(G)$ is bounded below by k, where k is the number of vertices of a unique shortest path joining any pair of vertices in G. Thus, if n is the number of vertices of G, then $n - q(G)$ is bounded above by the size of the complement (with respect to the vertex set of G) of the vertex set of the longest unique shortest path joining any pair of vertices of G. In this talk, we present some results on the minor-monotone floor of $n - k$, which is the minimum of $n - k$ among all graphs of which G is a minor. (Received September 20, 2021)

1174-05-9808 **Shannon R Overbay** (overbay@gonzaga.edu), Gonzaga University. *Cylinder, Torus, and Möbius Book Embeddings of Graphs* Preliminary report.

A standard book embedding of a graph is a layout of the vertices of the graph on a line in three space. The edges of the graph are then placed on half planes (pages) extending from the line in such a way that no two edges cross each other or the spine. We extend this standard definition of a book embedding of a graph to a cylindrical book, a torus book, and a Möbius book. Edge bounds are given for all of these books. Optimal embeddings of complete graphs are then considered. (Received September 20, 2021)

1174-05-9812 **Louis A Deaett** (louis.deaett@quinnipiac.edu), Quinnipiac University. *On the strong maximum nullity of a connected bipartite graph*

For a connected bipartite graph G with bipartition U, W and —U—=—W—, we denote by $Q(G)$, the set of all $U \times W$ matrices $M = [m_{i,j}]$ with $m_{i,j} \neq 0$ if and only if ij is an edge of G, and we denote by $N(G)$, the set of all $U \times W$ matrices $X = [x_{i,j}]$ with $x_{i,j} = 0$ if ij is an edge of G. A matrix $M \in Q(G)$ has the ASAP if there is no nonzero matrix $X \in N(G)$ such that $X^T M = 0$ and $MX^T = 0$. There exists a matrix $M \in Q(G)$ satisfying the ASAP if and only if G has a perfect matching. For a connected bipartite graph G having a perfect matching, denote by $\zeta(G)$ the maximum nullity of any $M \in Q(G)$ satisfying the ASAP.

In this talk, we discuss the class of connected bipartite graphs G with $\zeta(G) = 0$ and the class of connected bipartite graph G with $\zeta(G) \leq 1$. We do this in terms of forbidden substructures (minors). We also discuss the signed version. (Received September 20, 2021)

1174-05-9915 **Enrico Celestino Colon*** (ecolon@mit.edu), Massachusetts Institute of Technology. *Split-multiplicity-free Flagged Schur Polynomials*

Quiver coefficients come from the study of a general kind of degeneracy locus associated to an oriented quiver of type A. They can be obtained by expanding the Schubert polynomials into the split-Schur polynomials, and possess very rich combinatorial structures. In this paper, we investigate the problem of determining which Schubert polynomials are split-multiplicity-free by looking at two meaningful special cases: the flagged Schur polynomials, which are Schubert polynomials of vexillary permutations, and the Stanley symmetric polynomials, which are stable limits of Schubert polynomials. Specifically, we present a necessary and sufficient condition on a shape λ for the flagged Schur polynomials s_λ^b to be split-multiplicity-free, given a generic flag. We also discuss progress on the Stanley symmetric polynomials via Rothe diagram. (Received September 21, 2021)

1174-05-9955 **Daryl R. DeFord** (daryl.deford@wsu.edu), Washington State University, and **Elliot Kimsey*** (elliot.kimsey@wsu.edu), Washington State University. *Analysis of Population Imbalances on Washington State Dual Graphs*

The political redistricting problem can be analyzed by studying the partitions of a dual graph associated to geographic units in a given state. A partition is said to be permissible if it satisfies a set of constraints, such as the connectedness of the induced subgraphs of its components. Another common constraint is population balance, which requires bounding weighted sums across the nodes of each component. In this talk we will describe recent work comparing population totals and projections on several dual graphs representing Washington State, motivated by measuring the impacts of within-cycle malapportionment, the deviations between district populations caused by shifts during the census cycle. We will discuss several approaches and metrics for characterizing these imbalances, as well as related Markov chain sampling results. (Received September 21, 2021)

1174-05-9975 **Saad I El-Zanati*** (saad.elzanati@gmail.com), Illinois State University. *Research Experiences for Pre-service and In-service Secondary Math Teachers*

The Illinois State University Research Experiences for Undergraduates and Teachers (REU/RET) is an eight-week summer research experience for 8 (or more) pre-service and 4 in-service secondary mathematics teachers. The REU has been held annually since 2007. Participants explore research topics in discrete mathematics with emphasis on experimentation, formulation of researchable questions, careful reasoning and justification, and clear, precise reporting. An added emphasis is on the development and implementation, by the REU participants, of a one-week Mathematics Research Camp for 12–18 high school students from the Chicago Public Schools district. The main objective of the project is to provide research opportunities for the participants, helping them to become highly qualified mathematics teachers who can meet the national demands for increased student proficiency and adapt to the changing needs of our technological society. The desired project outcomes include: (1) the development of "*teacher-scholars*"–teachers who experience scholarship in mathematics in a setting that emphasizes the interconnections among theory, procedures, and applications and who develop the habits of mind of a mathematical thinker and (2) research publications authored jointly by participants and faculty.

In this presentation, we give an overview of the experience and discuss how aspects of it can implemented at other institutions. (Received September 21, 2021)

1174-05-10000 **Tom Roby*** (tom.roby@uconn.edu), UConn. *Rowmotion in products of two chains*

The *rowmotion* operator on the order ideals (or antichains) of a finite poset is of long-standing interest, and there has recently been a flurry of research activity around understanding its dynamical and other properties. Questions of periodicity, orbit sizes, and *homomesy* (finding statistics that have the same average over all orbits) abound. There are also connections with representation theory that are still only partially understood. This talk will give an introductory overview of these ideas for the special (nontrivial) case where the poset is a product of two chains. (Received September 21, 2021)

1174-05-10008 **Diego Gonzalez-Moreno** (dgonzalez@cua.uam.mx), UAM-Cuajimalpa, **Mika Olsen*** (olsen@cua.uam.mx), UAM-Cuajimalpa, **Julián Alberto Fresán-Figueroa** (jfresan@cua.uam.mx), UAM-Cuajimalpa, and **Camino Balbuena** (m.camino.balbuena@gmail.com), Retired professor. *Colorings in Moore graphs*

Coloring in graphs is one of the most studied topics both for theoretical interest and for the amount of applications they have, and vertex coloring as well as edge colorings are studied. A vertex coloring of a graph induces a partition of the vertex-set into chromatic classes. For the classic coloring, each chromatic class induces an independent set. This coloring can be generalized by changing the condition on the chromatic classes. The generalizations may be defined due to theoretical interest or due to some particular application, or both. In this talk I am going to consider two colorings arising from problems. The packing chromatic number has applications in radio frequency assignment. A vertex coloring of a graph is a packing coloring if the colors are natural numbers and two vertices of the color i are at a distance at least i+1. The packing chromatic number is the minimum number of colors in a packing color. An edge coloring of a graph is rainbow if there is a rainbow path between any pair of vertices, that is, a path where no two edges have the same color. The rainbow connectivity is defined as the minimum number of colors of a rainbow coloring and has applications in cybersecurity. I present results for rainbow connectivity and for the packing chromatic number of Moore-graphs. A Moore graph is a (k,g)-cage (a k-regular graph, with girth g) whose order reaches the Moore bound. Moore-graphs with girth 6, 8, or 12 have structural properties inherited from finite geometries that are very useful. (Received September 21, 2021)

1174-05-10022 **Mario Sanchez*** (mariomtfg@gmail.com), Cornell University. *Spectrahedral Lifts of Generalized Permutahedra* Preliminary report.

Spectrahedra are convex objects given by the feasible region of a semidefinite program. Like their linear programming counterpart - polytopes - these objects have many interesting combinatorial properties. Many combinatorial objects such as graphs, posets, and matroids can be studied through a family of polytopes called generalized permutahedra. In this talk we present work-in-progress on a spectrahedral analogue of generalized permutahedra. This allows us to associate to each of the aforementioned combinatorial objects a spectrahedron through the theory of submodular functions. (Received September 21, 2021)

1174-05-10074 **Dony Varghese*** (dvarghes@vols.utk.edu), The University of Tennessee, Knoxville. *Characteristic Sets of Matroids* Preliminary report.

A matroid is a combinatorial structure on a finite set which generalize the concept of linear independence of vectors in a vector space. The linear characteristic set of a matroid is either a finite or a cofinite set. But not much is known about the possible algebraic characteristic sets. We look at other possible algebraic characteristic sets and talk about a different type of characteristic set. (Received September 21, 2021)

1174-05-10091 **Katherine Moore*** (kmoore@amherst.edu), Amherst College. *Communities in Data: Partitioned Local Depth*

Concepts of conflict and alignment give rise to a perspective from which to consider community structure in distance-based data. In particular, a simple socially-inspired approach to localization gives rise to a measure of local depth which can then be partitioned to describe cohesion between points. Leveraging a natural threshold distinguishing strong and weak ties, the connected components of the associated networks provide a means to reveal any (community) clusters within the data. Throughout, no additional user inputs, distributional assumptions nor optimization criteria are employed. Since the perspective only leverages distance comparisons of the form "is x more similar to y or to z?", the sense of community structure revealed here can be particularly useful in settings in which local density varies or numeric distances are uncertain. A few examples to high-dimensional data and that with varying density are provided, together with applications to linguistic, genetic and cultural values data. (Received September 21, 2021)

1174-05-10109 **Sherry Shen** (ys564@cam.ac.uk), University of Cambridge, **Lucy Wickham*** (lucygwickham@gmail.com), The College of Wooster, **Evan Knowles** (knowleew202@potsdam.edu), SUNY Potsdam, **Uriel Alejandro Nolasco Hernández** (no379316@uaeh.edu.mx), Universidad Autònomo del Estado de Hidalgo, Mexico, and **Luis Angel Topete Galvàn** (luisatg96@gmail.com), Universidad Autònomo del Estado de Hidalgo, Mexico. *Links in Projectively Embedded Graphs*

A graph G is *nonseparating projective planar* if G has a projective planar embedding without a nonsplit link. Nonseparating projective planar graphs are closed under taking minors and are a superclass of projective outerplanar graphs. We partially characterize the minor-minimal (simplest) separating projective planar graphs by proving that given a minor-minimal nonouter-projective-planar graph G, either G is minor-minimal separating projective planar or $G\dot{\cup}K_1$ is minor-minimal weakly separating projective planar, a necessary condition for G to be separating projective planar.

One way to generalize separating projective planar graphs is to consider *type I 3-links* consisting of two cycles and a pair of vertices. A graph is *intrinsically projective planar type I 3-linked (IPPI3L)* if its every projective planar embedding contains a nonsplit type I 3-link. We partially characterize minor-minimal IPPI3L graphs by classifying all minor-minimal IPPI3L graphs with three or more components, and finding many others with fewer components.

A graph is *intrinsically projectively linked (IPL)* if its every embedding in projective space contains a nonsplit link. Some minor-minimal IPL graphs have been found previously. We determine that no minor-minimal IPL graphs on 16 edges exists and identify new minor-minimal IPL graphs by applying $\Delta - Y$ exchanges to $K_7 - 2e$. (Received September 21, 2021)

1174-05-10133 **Esther Dawn Conrad** (edconrad@iastate.edu), Iowa State University. *PMU-defect-robust power domination* Preliminary report.

The power domination problem finds the placement of the minimum number of sensors called phasor measurement units (PMUs) needed to monitor an electric power network. We consider the minimum number of sensors and appropriate placement to ensure monitoring when k sensors are allowed to fail. That is, what is the minimum multiset of the vertices, S, such that for every $F \subseteq S$ with $|F| = k$, $S \setminus F$ is a power domination set. Such a set is called a k-robust power domination set. This generalizes the work done by Pai, Chang, and Wang in 2010 on vertex-fault-tolerant power domination, which did not allow for multiple sensors to be placed at the same vertex. We provide general bounds and determine the k-robust power domination number of some graph families. (Received September 21, 2021)

1174-05-10161 **Nathan Lemons** (nlemons@lanl.gov), Los Alamos National Laboratory, and **Anastasia Halfpap*** (anastasia.halfpap@umontana.edu), University of Montana. *Minimum positive co-degree problems for 3-graphs* Preliminary report.

Let $\mathcal{H}$ be a non-empty, r-uniform hypergraph. The *minimum positive co-degree* of $\mathcal{H}$, denoted δ^+_{r-1}, is the maximum k such that if S is an $(r-1)$-set contained in a hyperedge of $\mathcal{H}$, then S is contained in at least k hyperedges of $\mathcal{H}$. In this talk, we focus on 3-uniform graphs, and seek to maximize δ^+_2 subject to the forbidding of some sub-hypergraph $\mathcal{F}$. In addition to a survey of specific new results, we will highlight some differences between the problem of maximizing δ^+_{r-1} and the problem of maximizing the standard minimum co-degree. (Received September 21, 2021)

1174-05-10192 **Everett Sullivan*** (everettsu@gmail.com), Virginia Tech, and **Alex Klotz** (Alex.Klotz@csulb.edu), California State University, Long Beach. *Exactly-Solvable Self-Trapping Lattice Walks*

A Growing Self-Avoiding Walk (GSAW) is a random walk on a graph, where it is not allowed to revisit any vertex. Additionally the walk can be modified to be more likely to visit a vertex that is near vertices that have already been visited, a type of weighting known as nearest-neighbor attraction. These walks are used to model the statistics of polymer chains in poor solvents. A active area of research is understanding how long such a walk will be before it terminates, also known as the mean trapping length. We consider such paths of lattices that are two to three vertices wide. By creating a finite state automata that can uniquely describe all such paths, a generating function can be created that allows for exact computation of mean trapping length, the asymptotic behavior of the trapping probability distribution, and the effect of nearest-neighbor attraction. (Received September 21, 2021)

1174-05-10214 **Neal Bushaw*** (nobushaw@vcu.edu), Virginia Commonwealth University. *Bridging the Gap Between Monochrome and Rainbow* Preliminary report.

The forbidden subgraph problem is among the classics in its area – how many edges can an F-free graph have? Asking for the maximum here recovers the extremal number, while asking for the minimum gives the saturation number. Recent work by numerous authors has explored a coloring variation on this problem – how many edges can there be in a properly edge-colored graph avoiding a rainbow copy of F (where rainbow means each of its edges receives its own color)? Again, one can ask for a maximum or minimum here, and one discovers the rainbow extremal and rainbow saturation numbers. In this talk, we discuss the recently introduced k-unique saturation, allowing us to create a spectrum of extremal functions (and saturation functions) covering the space between traditional and rainbow. This is joint work with Vic Bednar and Moheng Zhang. (Received September 21, 2021)

1174-05-10221 **Robert McCloskey*** (robert.y.mccloskey@gmail.com), Lehigh University. *Generalizing Transition Matrices of Symmetric Function Bases to QSym and NSym*

Transition matrices between well-known bases of symmetric functions often encode useful combinatorial data, as shown by work of Eğecioğlu, Remmel, and MacDonald. The quasisymmetric functions, $QSym$, and noncommutative symmetric functions, $NSym$, are dual spaces both generalizing the symmetric functions; each have several bases whose transition matrices imitate those from the classical cases. In this talk, we will review some of the symmetric function change-of-basis results, define their generalizations in the $QSym$ and $NSym$ cases, and explore what representation theoretic data about representations of the 0-Hecke algebra these can encode. (Received September 21, 2021)

1174-05-10228 **Norman Fox** (foxb@apsu.edu), AUSTIN PEAY STATE UNIVERSITY, and **Holly M Abrams*** (abrams.holly@yahoo.com), AUSTIN PEAY STATE UNIVERSITY. *An Exploration of Odd Prime Graph Labelings*

An odd prime labeling is a variation of a prime labeling in which the vertices of a graph of order n are labeled with the distinct odd integers 1 to $2n-1$ so that the labels of adjacent vertices are relatively prime. This paper investigates many different classes of graphs including stacked prisms, binary trees and caterpillars, and uses various methods to construct odd prime labelings. (Received September 21, 2021)

1174-05-10235 **Boris Brimkov*** (boris.brimkov@sru.edu), Slippery Rock University. *On minimal zero forcing sets* Preliminary report.

In this talk, we discuss minimal zero forcing sets of graphs. In particular, we explore the relation between the maximum size of a minimal zero forcing set and the zero forcing number; we also explore the number of distinct minimal zero forcing sets in various families of graphs. (Received September 21, 2021)

1174-05-10257 **Rodrigo Reynaldo Rios** (rodrigoreyrios@gmail.com), Florida Atlantic University. *Using Graph Coloring to Measure Network Reliability* Preliminary report.

Given a simple graph G, otherwise referred to as a network, one can assign a measure of reliability as the number of deletions required to force a network into a state of failure defined by some condition(s). Network reliability has previously been studied extensively on failure states defined by graph properties such as diameter and order. However, we analyze networks on chromatic number and chromatic index under vertex and edge removals as forms of network reliability parameters in order to find the most and least reliable graphs in $G(n,m)$, the set of Graphs with n vertices and m edges. (Received September 21, 2021)

1174-05-10329 **John Jasper*** (john.jasper@sdstate.edu), South Dakota State University. *Toward a Taxonomy of Real Equiangular Tight Frames* Preliminary report.

In the search for new equiangular tight frames (ETFs), a fruitful approach has been to identify combinatorial "mutations" of existing algebraic constructions. This program has delivered a few of general "species" of ETFs such as Steiner ETFs and Tremain ETFs. As the study of ETFs matures, we find that "new" constructions are secretly members of these species. These realizations frequently lead to new constructions of ETFs and of strongly regular graphs. (Received September 21, 2021)

1174-05-10350 **Michael Cowen*** (mcowen@g.clemson.edu), Clemson University. *Power Edge Ideals*

Every electrical power system can be modeled by a graph G whose vertices represent buses and whose edges represent power lines. A *phasor measurement unit* (PMU) is a device that can be placed at a bus to observe the voltage at that bus as well as the current and its phase through all incident power lines. The problem of monitoring the entire electric power system using the fewest number of PMUs is closely related to the well-known vertex covering and dominating set problems in graph theory.

In this talk, we will give an overview of the PMU placement problem and its connections to commutative ring theory. We will define the *power edge ideal* I_G^P of a graph G with n vertices in a polynomial ring $R = k[X_1, \ldots, X_n]$ and we will describe some algebraic properties of the quotient R/I_G^P. (Received September 21, 2021)

1174-05-10380 **Houston Schuerger*** (houstonschuerger@my.unt.edu), Trinity College. *Zero Forcing and Parallel Increasing Path Covers* Preliminary report.

Zero Forcing and Parallel Increasing Path Covers

Houston Schuerger, Trinity College, Houston.Schuerger@trincoll.edu

We introduce a new, but equivalent, way of considering zero forcing, which we will refer to as parallel increasing path covers. We first show that this notion of parallel increasing path covers is equivalent to that of zero forcing. The result of this equivalence is that each vertex of a chain set can be associated with an element of a partition which encodes the zero forcing process. From the information contained within, we are able to provide new proofs of familiar results, produce new results concerning propagation time and positive semidefinite propagation time of graphs, and provide ways to construct graphs with certain zero forcing sets. (Received September 21, 2021)

1174-05-10411 **On Ki Luo*** (astridxlok@gmail.com), Research Science Institute, Pui Ching MIddle School. *The Johnson-Leader-Russell Question on Square Posets*

We study the problem of finding the maximum number of maximal chains in a given size-k subset of a square poset $[n] \times [n]$. This was proposed by Johnson, Leader, and Russell but not yet solved. Kittipassorn had given a conjectural solution to the problem. We verify Kittipassorn's conjecture for $0 \leq k \leq 3n - 2$ and solve a variant problem for the case $3n - 1 \leq k \leq 4n - 4$, which also supports the conjecture. For general k, we find that the optimal configuration is given by a 1-Lipschitz function. We also generalize the problem to rectangle posets and give a solution to one particular poset. (Received September 21, 2021)

1174-05-10434 **Helen Jenne** (helen.jenne@gmail.com), Unaffiliated, **Benjamin J Young*** (bjy@uoregon.edu), University of Oregon, and **Gautam Webb** (gwebb@uoregon.edu), University of Oregon. *The combinatorial PT-DT correspondence*

We show that two generating functions from algebraic geometry (the combinatorial Calabi-Yau topological vertices in DT theory and PT theory) are equal, up to a constant factor of Macmahon's generating function for plane partitions.

There's no algebraic geometry in this talk - the generating functions in question are combinatorial, and count plane-partition-like objects. We make use of correspondences (both folklore and novel) between the these objects and various instances of the dimer and double-dimer models, from statistical mechanics; our main tool is the "condensation identity" in these models, due respectively to Kuo and Jenne (closely related to Dodgson's condensation identity, and the Desnanot-Jacobi identity, in the theory of determinants).

Indeed, there are also no slides in this talk, and I won't even use a computer, strictly speaking. Plane partitions have a natural interpretation in terms of stacks of unit cubes, so I'll be able to give the entire talk using the video game Minecraft, running on a Nintendo Switch. (Received September 21, 2021)

1174-05-10502 **Gordon Rojas Kirby** (girkirby@gmail.com), Arizona State University, and **Nicolle Gonzalez*** (Nicolle@math.ucla.edu), UCLA. *Eulerian Numbers and the Tutte Polynomial* Preliminary report.

The Tutte polynomial is a two variable polynomial graph invariant. Due to its computational complexity it is often specialized at certain values which give us information about the graph such as the number of spanning trees or types of arrangements. In the case of the complete graph, one particular value T(1,-1) yields the so called Eulerian numbers, which count the number of up-down permutations. This fact was proven by Kuznetsov, Pak, and Postnikov using algebraic recursions and analytically by Ardila. In these joint talks, we will discuss ongoing investigations on how to understand the connection between T(1,-1) and the Eulerian numbers combinatorially and geometrically. (Received September 21, 2021)

1174-05-10522 **Marie Meyer** (mmeyer2@lewisu.edu), Lewis University, and **Paige Allen*** (paigeallen@lewisu.edu), Lewis University. *Laplacian Simplices Associated to Graphs*

This presentation introduces properties of the Laplacian simplex, which is constructed from a finite graph by taking the convex hull of the columns of its Laplacian matrix. For a graph with n vertices, the resulting polytope is an n-1 dimensional simplex. Recently, there is a heightened interest in studying polytopes associated to graphs.

It is of particular interest to explore how the geometric properties of the polytope relate to the discrete properties of the graph. For instance, the volume of a Laplacian simplex is related to the spanning trees of its underlying graph. We analyze the fundamental parallelepiped associated to a Laplacian simplex in order to describe a bijection between its lattice points and the labeled rooted spanning trees of the underlying graph. Finally, we demonstrate graph constructions to present new results regarding the form of lattice points in the fundamental parallelepiped for different families of graphs. (Received September 21, 2021)

1174-05-10525 **Jozsef Balogh** (jobal@illinois.edu), Univerisity of Illinois at Urbana-Champaign. *Maximum size intersecting families of bounded minimum positive co-degree*

Let $\mathcal{H}$ be an r-uniform hypergraph. The *minimum positive co-degree* of $\mathcal{H}$ is the minimum k such that if S is an $(r-1)$-set contained in a hyperedge of $\mathcal{H}$, then S is contained in at least k hyperedges of $\mathcal{H}$. For $r \geq k$ fixed and n sufficiently large, we determine the maximum possible size of an intersecting r-uniform n-vertex hypergraph with minimum positive co-degree at least k and characterize the unique hypergraph attaining this maximum. This generalizes the Erdős-Ko-Rado theorem which corresponds to the case $k = 1$. Our proof is based on the delta-system method. (Received September 21, 2021)

1174-05-10528 **Grigoriy Blekherman** (greg@math.gatech.edu), Georgia Tech, **Annie Raymond*** (raymond@math.umass.edu), University of Massachusetts, Amherst, **Mohit Singh** (mohit.singh@isye.gatech.edu), Georgia Tech, and **Rekha Thomas** (rrthomas@uw.edu), University of Washington. *Tropicalization of graph profiles*

The number of homomorphisms from a graph H to a graph G, denoted by $\hom(H;G)$, is the number of maps from $V(H)$ to $V(G)$ that yield a graph homomorphism, i.e., that map every edge of H to an edge of G. Given a fixed collection of finite simple graphs $\{H_1,\ldots,H_s\}$, the graph profile is the set of all vectors $(\hom(H_1;G),\ldots,\hom(H_s;G))$ as G varies over all graphs. Graph profiles essentially allow us to understand all polynomial inequalities in homomorphism numbers that are valid on all graphs. Profiles can be extremely complicated; for instance the full profile of any triple of connected graphs is not known. To simplify these objects, we introduce their tropicalization which we show is a closed convex cone that still captures interesting combinatorial information. We explicitly compute these tropicalizations for some sets of graphs, and relate the results to some questions in extremal graph theory. (Received September 21, 2021)

1174-05-10537 **Francis Edward Su*** (su@math.hmc.edu), Harvey Mudd College, and **Frédéric Meunier** (frederic.meunier@enpc.fr), École Nationale des Ponts et Chaussées. *Mulitlabeled versions of Sperner's Lemma and a Survivor-Style Cake-Cutting Theorem*

We prove a multi-labeled version of Sperner's Lemma, which as an interesting by-product provides new proofs of existing fair division theorems as well as dual theorems that were not known before. In particular we show a new "Survivor-style" cake-cutting theorem: For N players, there exists a division of cake into $N-1$ pieces today such that, no matter which player is voted off the island tonight, all remaining players tomorrow can choose pieces of cake in an envy-free way. (Received September 21, 2021)

1174-05-10549 **Alexandr Kostochka** (kostochk@illinois.edu), University of Illinois At Urbana-Champaign, **Ruth Luo*** (ruluo@ucsd.edu), University of California, San Diego, and **Grace McCourt** (mccourt4@illinois.edu), University of Illinois at Urbana-Champaign. *Dirac conditions for hamiltonian hypergraphs*

Dirac proved that any n-vertex graph with minimum degree at least $n/2$ contains a hamiltonian cycle. We prove an analogue for uniform hypergraphs. That is, we give exact bounds for the minimum degree of r-uniform hypergraphs that forces the existence of hamiltonian Berge cycles. In particular, the bounds are different when r is small or large compared to the number of vertices. (Received September 21, 2021)

1174-05-10551 **Puck Rombach** (puck.rombach@uvm.edu), University of Vermont. *On rainbow Turán numbers of paths and other trees*

For a fixed graph F, we would like to determine the maximum number of edges in a properly edge-colored graph on n vertices which does not contain a rainbow copy of F, that is, a copy of F all of whose edges receive a different color. This maximum, denoted by $ex^*(n,F)$, is the *rainbow Turán number* of F. We show that $ex^*(n,P_k) \geq \frac{k}{2}n + O(1)$ where P_k is a path on $k \geq 3$ edges and similar bounds for other trees using similar techniques. (Received September 21, 2021)

1174-05-10580 **Anna Kirkpatrick*** (kirkpatrickae@gmail.com), SAS Institute, **Chidozie Onyeze** (chidozieonyeze@gatech.edu), Microsoft, **Prasad Tetali** (tetali@math.gatech.edu), Carnegie Mellon University, **Cassie Mitchell** (cassie.mitchell@bme.gatech.edu), Georgia Institute of Technology, **David Kartchner** (david.kartchner@gatech.edu), Georgia Institute of Technology, **Stephen Allegri** (saallegri@gatech.edu), Georgia Institute of Technology, **Davi Nakajima An** (dna@gatech.edu), Georgia Institute of Technology, and **Evie Sandeep Ira Davalbhakta** (edavalbhakta3@gatech.edu), Georgia Institute of Technology. *Exploring Optimizations to HeteSim for Computing Relatedness in Heterogeneous Information Networks*

Heterogeneous information networks, or knowledge graphs, are valuable tools for analyzing insights from the vast number of papers published in the biomedical sciences. Informally, a heterogeneous information network is a directed graph in which each node corresponds to a biomedical concept and each directed edge encodes a relationship between concepts. SemNet is a heterogeneous information network with approximately 300,000 nodes and 20,000,000 edges built from the abstracts of papers in the PubMed database. This work builds on the SemNet codebase with the goal of reducing algorithm runtime. We focus specifically on the similarity score HeteSim, which has been successfully used in multiple biomedical heterogeneous information networks. However, HeteSim can require prohibitive runtime. Performance improvements to HeteSim computation algorithms based on data structure changes and randomized approximation algorithms will be presented. Both theoretical bounds on algorithm performance and empirical timing results will be included. (Received September 21, 2021)

1174-05-10625 **Hunter Loftis** (iloftis@brynmawr.edu), Bryn Mawr College, **Sean James Bezos O'Connor*** (soconn19@jhu.edu), Johns Hopkins University, and **Josue Ramirez** (ramirezantonio.j@northeastern.edu), Northeastern University. *Cycle and Path Decompositions of Even Hypercubes* Preliminary report.

A graph G decomposes a graph H if the edge set of G can be partitioned into edge-disjoint subgraphs isomorphic to H. We consider decompositions of the n-dimensional hypercube Q_n into paths and cycles. A conjecture by Erde (2014) asserts that if n is even, then a path of length ℓ decomposes Q_n if the necessary conditions that $\ell < 2^n$ and ℓ divides the number of edges of Q_n hold. Offner et al. (2021) proved, among other results, that if n is the sum of at most two powers of 2, then the cycle with the largest length divisible by n while still satisfying the necessary conditions provided by Erde decomposes Q_n. We have improved this result to the case of n being the sum of at most three powers of 2. Corollaries of this result strengthen many other results of Offner et al. (Received September 21, 2021)

1174-05-10672 **Emina Soljanin*** (emina.soljanin@rutgers.edu), Rutgers University. *Codes, Graphs, and Hyperplanes in Data Access Service* Preliminary report.

Distributed computing systems strive to maximize the number of concurrent data access requests they can support with fixed resources. Replicating data objects according to their relative popularity and access volume helps achieve this goal. However, these quantities are often unpredictable. In emerging applications such as edge computing, even the expected numbers of users and their data interests extensively fluctuate, and data storage schemes should support such dynamics. Erasure-coding has emerged as an efficient and robust form of redundant storage. In erasure-coded models, data objects are elements of a finite field. Each node in the system stores one or more linear combinations of data objects. This talk asks 1) which data access rates an erasure-coded system can support and 2) which codes can support a specified region of access rates. We will address these questions by formulating them as some known and some new combinatorial optimization problems on graphs. We will explain connections with batch codes. This talk will also describe how, instead of a combinatorial, one can adopt a geometric approach to the problem. (Received September 21, 2021)

1174-05-10693 **Joseph Rennie** (rennie2@illinois.edu), University of Illinois at Urbana-Champaign, **Gordon Kirby** (gkirby@math.ucsb.edu), University of California, Santa Barbara, **David Melendez** (davidmelendez@knights.ucf.edu), University of Central Florida, and **Lucy Martinez*** (lm1154@math.rutgers.edu), Rutgers University. *Closed formulas for Kostant's Partition Function for $\mathfrak{sl}_4(\mathbb{C})$*

Kostant's partition function is a vector partition function which counts the number of ways to express a weight of a Lie algebra, $\mathfrak{g}$, as a nonnegative integral sum of the positive roots of $\mathfrak{g}$. In this presentation, we extend previous work on the value of Kostant's partition function in the Lie algebra of type A_3 to the q-analog of Kostant's partition function. These results, when evaluated at $q = 1$, recover formulas of De Loera and Sturmfels. (Received September 21, 2021)

1174-05-10701 **Alexander Clifton*** (aclift2@emory.edu), Emory University. *k-Covers of Triangular Grids* Preliminary report.

Alon and Füredi determined the minimum number of affine hyperplanes needed to cover all but one point of an n-dimensional rectangular grid. It is natural to extend this question to higher covering multiplicities as well as to triangular grids. In the case of two dimensions, we let T_n denote a triangular lattice with n points along a side. We discuss the smallest number of lines needed to cover all the points of T_n at least k times each. In particular, we show a surprising connection between this problem and the linear relaxation of the problem of covering all the points of T_k once each.

Joint work with Abdul Basit and Paul Horn. (Received September 21, 2021)

1174-05-10706 **Dheer Noal Desai*** (dheernsd@udel.edu), University of Delaware. *Spectral Turán Problems*

The odd wheel W_{2k+1} is the graph formed by joining a vertex to a cycle of length $2k$. The (k,r)-fan is the graph consisting of k copies of the complete graph K_r which intersect in a single vertex, and is denoted by $F_{k,r}$. In this talk, we will investigate the largest value of the spectral radius of the adjacency matrix among all n-vertex graphs that do not contain some subgraph. We will compare the structures of the Turán-extremal and spectral extremal graphs for some examples. We show that for small odd wheels and all fans, the spectral extremal graphs are among the Turán-extremal graphs on n vertices, but for larger wheels the family of spectral extremal graphs and the family of Turán-extremal graphs are disjoint.

We will give an overview of similar results and describe a method that may help us find new ones. This is joint work with Sebastian Cioabă (University of Delaware), Liying Kang (Shanghai University), Yongtao Li (Hunan University), Zhenyu Ni (Shanghai University), Michael Tait (Villanova University) and Jing Wang (Shanghai University). (Received September 21, 2021)

1174-05-10729 **Harris Spencer Johnson** (harris.johnson@morehouse.edu), Morehouse College, **Jai James*** (jj8327a@american.edu), American University, **Jordan Turner** (jordan.turner@morehouse.edu), Morehouse College, and **Thea Nicholson** (thea1.nicholson@gmail.com), Xavier University. *Riordan Array Representations of Catastrophe Paths*

In this project we examine several specific sets of integer lattice paths consisting of up steps (1,1), level steps of either length (1,0) or (2,0), and down steps of various lengths. Many of these lattice paths will generate significant sequences, notably the Catalan, Schröder and Motzkin Numbers. We also show how generating functions can be obtained using these integer lattice paths that will result in the aforementioned sequences. This project will also prove the existence of Riordan Array representations of the notable paths. Lastly, this project considers Catastrophe paths and examines how one set of integer lattice paths consisting of up steps of length $(1,1)$ and down steps of arbitrary lengths $(1,-1)$, $(1,-2)$, $(1,-3)$, $(1,-4)$, $\ldots$ is a binary shift "between" the Catalan Path and Motzkin Path. There are also results considering a potential bijection between a 'faux' catastrophe path and rooted trees. (Received September 21, 2021)

1174-05-10749 **Nachama Stern*** (snstern@smith.edu), Smith College, and **Issa Susa** (isusa@smith.edu), Smith College. *Firefighter games on graphs*

The Firefighter Problem is described in the following manner: Imagine that, at time 0, a fire breaks out at a vertex of a graph G. At each subsequent time, one can place the firefighter on a vertex to "defend" a vertex of G from the fire and then the fire spreads from each "burning" vertex to all of its undefended adjacent vertices. Once a vertex is burning or defended, it remains so from then on-wards. The process terminates when the fire can no longer spread. On a finite graph G, when the process terminates there are three possible statuses for each vertex. This presentation will discuss strategies for playing the firefighter game on different graph types. (Received September 21, 2021)

1174-05-10750 **Eric G Ramos*** (e.ramos@bowdoin.edu), Bowdoin College. *The Graph Minor Theorem in Topological Combinatorics*

We study a variety of natural constructions from topological combinatorics, including matching complexes as well as other graph complexes, from the perspective of the graph minor category of Miyata, Proudfoot, and Ramos. We prove that these complexes must have universally bounded torsion in their homology across all graphs. One may think of these results as arising from an algebraic version of the graph minor theorem of Robertson and Seymour. (Received September 21, 2021)

1174-05-10761 **Andrew Beveridge*** (abeverid@macalester.edu), Macalester College, and **Ian Calaway** (icalaway@stanford.edu), Stanford University. *Approval Ballot Triangles and Magog Triangles*

Bertrand's Ballot Problem enumerates the number of ways to count ballots so that candidate 1 never trails candidate 2. We generalize this problem by considering an approval ballot election between n candidates. In an approval ballot election, each voter endorses a subset of candidates, rather than voting for just one person. The general approval ballot problem becomes: how many ways can the ballots be counted so that candidate k never trails candidate $k+1$?

This formulation yields a family of binary triangular arrays, called approval ballot triangles (ABTs), that are in bijection with totally symmetric self-complementary plane partitions. We show that ABTs unify three different TSSCPP families: magog triangles, kagog triangles, and (the newly discovered) lagog triangles. We then further the connection between TSSCPPs and ballot problems by giving a decomposition of a strict-sense ballot into a list of sequentially compatible ABTs. (Received September 21, 2021)

1174-05-10785 **Adam Blumenthal*** (blumenam@westminster.edu), Westminster College. *Independent Domination in Large Regular Graphs* Preliminary report.

A set of vertices in a graph G is said to be an independent dominating set if it is both independent and dominating. The size of a smallest independent dominating set in G is called the independent domination number, denoted $i(G)$. In 1964, Rosenfeld proved that for all regular graphs, $i(G)/n \leq \frac{1}{2}$. Bounds on $i(G)/n$ have been studied for regular graphs with a particular degree in some small cases. We will discuss some bounds on $i(G)/n$, particularly for connected graphs of large degree. (Received September 21, 2021)

1174-05-10787 **William Chang*** (chan087@usc.edu), university of southern california. *Asymptotical bounds on maximal cliques in K_t-minor free graphs* Preliminary report.

In this poster, we study bounds for maximal cliques of fixed sizes in K_t-minor free graphs. We apply the container's method to obtain our results, which allow us to restrict our attention to dense substructures of the original graph. We break the size of the maximal cliques into the middle and high ranges and develop bounds for each. In the upper range we have at most $n\left(\frac{2t+1-k}{k}\right)^k 2^{o(t)}$ cliques, while for the middle ranges we have a bound of $n2^{2t-2k+1+o(t)}$. We also show that, more generally, that the total number of maximal cliques is $2^{\frac{1}{3}(2t-4)}$. (Received September 21, 2021)

1174-05-10807 **Peter Kagey*** (kagey@usc.edu), University of Southern California. *The expected value of letters in permutations with a given number of k-cycles.*

In this talk, we discuss permutations $\pi \in S_n$ with exactly m transpositions. In particular, we are interested in the expected value of the first letter of a permutation, $\pi(1)$, when such permutations are chosen uniformly at random. When n is even, this expected value is approximated closely by $(n+1)/2$, with an error term that is inversely related to the number isometries of the $(n/2-m)$-dimensional hypercube that move every face.

Furthermore this construction generalizes to allow us to compute the expected value of $\pi(i)$, the i-th letter of a permutation $\pi \in S_n$ with exactly m k-cycles where $k \mid n$. In this case, the expected value has an error term which is instead inversely related to the number derangements of the generalized symmetric group $S(k, n/k-m)$.

In the case that $k \nmid n$, the expected value of $\pi(i)$ is precisely $(n+1)/2$. Indirectly, this suggests the existence of a reversible algorithm to insert a letter into a permutation which preserves the number of k-cycles. In the talk, we will explicitly construct such an bijection. (Received September 21, 2021)

1174-05-10837 **Derek Young*** (dyoung@mtholyoke.edu), Mount Holyoke College. *Minimum Rank and Zero Forcing Parameters for Cobipartite Graphs*

The minimum rank problem for a graph is the problem of determining the minimum rank over the set of symmetric matrices described by the graph. The minimum rank of a graph has been studied by using zero-nonzero patterns. In (Barioli et al., 2009), it was shown that one can convert the minimum rank problem for cobipartite graphs to the minimum rank problem for a zero-nonzero pattern constructed from the cobipartite graph. In this talk, a connection between when the triangle number of a zero-nonzero pattern is the same as the minimum rank of the zero-nonzero pattern and when the maximum nullity and zero forcing number are the same for a cobipartite graph is discussed. We also mention the relationship between a variety of different zero forcing parameters for cobipartite graphs.

References

[1] F. Barioli, S. M. Fallat, H. T. Hall, D. Hershkowitz, L. Hogben, H. van der Holst, and B. Shader. On the minimum rank of not necessarily symmetric matrices: a preliminary study. *Electron. J. Linear Algebra*, 18:126–145, 2009.

(Received September 21, 2021)

1174-05-10851 **Anuj Sakarda** (yahoosakarda@gmail.com), MIT PRIMES, **Jerry Tan*** (jerryt4444go@gmail.com), MIT PRIMES, **Armaan G Tipirneni** (22tipirneni@lexingtonma.org), MIT PRIMES, and **Feng Gui** (fenggui@mit.edu), MIT. *On the Distance Spectra of Extended Double Stars*

The distance matrix of a connected graph is defined as the matrix in which the entries are the pairwise distances between vertices. The distance spectrum of a graph is the set of eigenvalues of its distance matrix. A graph is said to be determined by its distance spectrum if there does not exist a non-isomorphic graph with the same spectrum. The question of which graphs are determined by their spectrum has been raised in the past, but it remains largely unresolved. Extended double stars are comprised of two stars whose centers are connected to a common vertex. We will prove that extended double stars are determined by their distance spectra. (Received September 21, 2021)

1174-05-10874 **Viveck Cadambe** (viveck@psu.edu), Pennsylvania State University, and **Srikanth Manjunatha** (sxb5973@psu.edu), Pennsylvania State University. *An Extreme Point Approach to Subset Selection*

Given a training data set, we develop a method to obtain a representative subset of training data meant for classification. Our focus is on subsets that are particularly suitable for training linear classification models. Our method is to reduce data sets in a manner that essentially preserves the convex hulls of the various classes of the data set. We develop scalable randomized algorithms for convex hull approximation, and obtain rigorous guarantees for models trained on subsets of linearly separable data sets. We empirically demonstrate the effectiveness of our approach for both training and data augmentation on the MNIST data set. (Received September 21, 2021)

1174-05-10884 **Emarie De La Nuez*** (edelanuez@smith.edu), Smith College, **Raina Okonogi-Neth** (rokonogineth@smith.edu), Smith College, and **Annabel Yim** (ayim@smith.edu), Smith College. *Using Mathematical Modeling to Explore the Genealogy of Written Text*

In this project, we explored the ways that network theory could be applied to understand thematic connections between works of literature. We used the text Much Ado About Nothing by William Shakespeare as our root piece of literature. (Received September 21, 2021)

1174-05-10885 **Samuel Thomas Lowery*** (stl1006@sru.edu), Slippery Rock University. *Coloring Intersection Points of Line Segments* Preliminary report.

We explore the problem of coloring the intersection points of a set of line segments, so that no points on the same segment have the same color. Through a combination of graph theoretic, geometric, and computational approaches, we obtain novel results about the minimum required number of colors for such a coloring for different arrangements of segments. (Received September 21, 2021)

1174-05-10886 **Jake Ruotolo*** (jakeruotolo@knights.ucf.edu), University of Central Florida, **Kevin Wang** (wangkevi@grinnell.edu), Grinnell College, and **Fan Wei** (fanw@princeton.edu), Princeton University. *A Sharp Bound on the number of Independent Transversals*

Let G be a graph and $V_1 \cup V_2 \cup \ldots \cup V_k$ partition $V(G)$. Let $d_{i,j}$ denote the edge density of the pair (V_i, V_j). An independent transversal is an independent set of G with exactly one vertex in each V_i. In this paper, we prove an asymptotically sharp upper bound for the maximum number of independent transversals given the $d_{i,j}$'s. Bounding the number of independent transversals is a common theme in extremal combinatorics. It often appears in inducibility type results for many combinatorial structures such as permutations and graphs. This paper combines ideas and techniques from various subfields of mathematics: optimization theory plays a key role in the proof of our main result and we discuss some stability results which have an analytic flavor. (Received September 21, 2021)

1174-05-10926 **Sally Cockburn** (scockbur@hamilton.edu), Hamilton College, **Debra L Boutin** (dboutin@hamilton.edu), Hamilton College, **Katherine Perry*** (kperry@soka.edu), Soka University of America, **Sarah Loeb** (sloeb@hsc.edu), Hampton Sydney College, and **Lauren Keough** (keoulaur@gvsu.edu), Grand Valley State University. *Breaking Symmetries: Determining and Distinguishing Mycielskian Graphs* Preliminary report.

Symmetry in a graph G can be measured by investigating possible automorphisms of G. One way to do this is to color the vertices of G in such a way that only the trivial automorphism can preserve the color classes. If such a coloring exists with d colors, G is said to be *d-distinguishable*. The smallest d for which G is d-distinguishable is its *distinguishing number*. In this talk we'll investigate this parameter in the setting of simple graphs achieved

by applying the traditional Mycielskian and generalized Mycielskian constructions. The traditional Mycielskian construction was introduced by Mycielski in 1955 to prove that there exist triangle-free graphs with arbitrarily large chromatic number. (Received September 21, 2021)

1174-05-10969 **Nicholas C Radley*** (nicholascradley@gmail.com), La Salle University. *Partitioning, Propagation Time, and Determination of End States for Multi-Color Forcing in Graphs* Preliminary report.

A recent paper introduced a generalization of zero forcing on graphs to include more than two colors. In this paper, we introduce a new method for examining the process of multi-color forcing with propagation that includes a variation on partitioning and extends the notion of color contraction. We identify some interesting properties of this extension of color contraction, and from these we improve the upper bound on time to termination for the forcing process for small numbers of colors and specific families of graphs. Finally, we present results regarding determination of the end state for some graphs given their initial colorings. (Received September 21, 2021)

1174-05-10981 **Maura Suzanne Hegarty*** (mshegarty@umass.edu), University of Massachusetts, Amherst, and **William Dugan** (wdugan@umass.edu), University of Massachusetts, Amherst. *The Pitman-Stanley Polytope and Flow Polytopes* Preliminary report.

In 1999, Pitman and Stanley introduced the Pitman-Stanley polytope along with its volume formula and lattice point formula. The Pitman-Stanley polytope is well studied due to its connections to probability, parking functions, the generalized permutahedra, and flow polytopes. Its lattice points correspond to plane partitions with entries 0 and 1. Pitman and Stanley remarked that their polytope can be generalized so that lattice points correspond to plane partitions with entries $0, 1, ..., m$. This generalization has since been untouched. We study this generalization, its lattice points, and its volume. We also realize the generalized Pitman-Stanley Polytope as a flow polytope which opens the door to new tools for studying this polytope. (Received September 21, 2021)

1174-05-11024 **William J. Martin*** (martin@wpi.edu), Worcester Polytechnic Institute, and **Ethan Washock** (ewashock@wpi.edu), Worcester Polytechnic Institute. *The ideal of polynomials vanishing on an equiangular tight frame* Preliminary report.

One approach in the search for sets of equiangular lines in $\mathbb{R}^d$ or $\mathbb{C}^d$ is to choose a set of polynomials in d variables whose zero set is a set of lines with the desired property. With this in mind, we approach the opposite question and ask, for known sets of equiangular lines, what is the simplest description of the homogeneous ideal of polynomials that vanish on those lines? In this talk, we focus on the case of the Paley ETFs, the Paley graphs arising from finite fields $\mathbb{F}_q$, $q \equiv 1 \pmod 4$, and other strongly regular graphs with Paley-like parameters. (Received September 21, 2021)

1174-05-11035 **Patricio Gallardo*** (pgallard@ucr.edu), UC Riverside. *On the geometry and classification of flow polytopes* Preliminary report.

Given an acyclic directed graph, if one considers all nonnegative flows on the edges subject to a boundary condition at the vertices, then one arrives at a polytope parametrizing all such flows. Such a setting determines a classification problem because there is only a finite number of polytopes for a given graph. Unfortunately, there is no classification of such flow polytopes for most graphs. We will describe computational tools developed with M. Barker (WashU), D. Mckenzie (UCLA), and ongoing work with A. Goodwin and J. Orellana-Hernandez at UCR to study families of polytopes of dimensions larger than three. We will also discuss connections to algebraic geometry. (Received September 21, 2021)

1174-05-11096 **Nicolle Gonzalez** (nicolle@math.ucla.edu), University of California Los Angeles. *Alternating permutations and the Tutte polynomial*

The Tutte polynomial is a two variable polynomial graph invariant. Due to its computational complexity it is often specialized at specific values which give us information about the graph such as the number of spanning trees or types of arrangements. In the case of the complete graph, one particular value $T(1, -1)$ yields the so called Eulerian numbers, which count the number of up-down permutations. This fact was proven by Kuznetsov, Pak, and Postnikov using algebraic recursions and analytically by Ardila. In these joint talks, we will discuss ongoing investigations on how to understand the connection between $T(1, -1)$ and the Eulerian numbers combinatorially and geometrically. (Received September 21, 2021)

1174-05-11138 **Matt Lehman*** (matthew.lehman@umb.edu), University of Massachusetts Boston. *Counting Rhythms: Subsets and Explicit Bijections*

A *rhythm* is a string used to model musical rhythms. It's comprised of an alternating set of two binary strings: zeros and ones (nodes), and dots and dashes (rhodes), in node/rhode pairs, with a restriction. We define some special rhythm types: *definite*, *cyclic* and *acyclic*, count those using Fibonacci and Lucas numbers, and use those results to count general rhythms. When counts match we provide corresponding bijections, e.g., between the definite acyclic rhythms of length n and all acyclic rhythms of length $(n-1)$. (Received September 21, 2021)

1174-05-11199 **Letong Hong*** (clhong@mit.edu), Massachusetts Institute of Technology. *The Pop operator on m-Tamari lattices* Preliminary report.

Motivated by the pop-stack-sorting map on the symmetric groups, Defant defined an operator $\mathsf{Pop}_M : M \to M$ for each complete meet-semilattice M by

$$\mathsf{Pop}_M(x) = \bigwedge(\{y \in M : y \lessdot x\} \cup \{x\}).$$

We study the dynamics of $\mathsf{Pop}_{\mathrm{Tam}_n(m)}$, where $\mathrm{Tam}_n(m)$ is the nth m-Tamari lattice. An element x is t-Pop-sortable if $\mathsf{Pop}^t(x) = \hat{0}$, the minimal element of the lattice, and we let $h_t(m,n)$ denote the number of t-Pop-sortable elements in $\mathrm{Tam}_n(m)$. We answer a conjecture by Defant that the generating function $\sum_{n\geq 1} h_t(m,n)z^n$ is rational. We also enumerate the number of 3-Pop-sortable elements. Moreover, in the case of the Tamari lattice of Dyck paths (i.e. $m = 1$), we give explicit form of the generating function. (Received September 21, 2021)

1174-05-11219 **Sohyeon Jung*** (Sjung@smith.edu), Smith College, **Veronica Lang** (Vlang@smith.edu), Smith College, **Risa Vandergrift** (Rvandergrift@smith.edu), Smith College, **Emily Hafken** (Ehafken@smith.edu), Smith College, and **Aviv Bernstein Livne** (Abernsteinlivne@smith.edu), Smith College. *Springer fibers and webs*

A flag is a sequence of nested subspaces in a fixed n-dimensional vector space, namely the choice of a line contained in a plane contained in a 3-dimensional space and so on. Given a matrix X, the Springer fiber of X is the collection of "eigenflags" for X, namely the flags for which X preserves each nested subspace.

In this talk, we study the combinatorics and geometry of a particular family of Springer fibers that arise in combinatorics, representation theory, and knot theory. We give some results about how to partition these Springer fibers into cells that are encoded by a kind of combinatorial knot-theoretic graph called a web. (Received September 21, 2021)

1174-05-11234 **Sean English** (senglish@illinois.edu), University of Illinois Urbana-Champaign, **Emily Heath*** (eheath@iastate.edu), Iowa State University, **József Balogh** (jobal@illinois.edu), University of Illinois Urbana-Champaign, and **Robert Krueger** (rak@illinois.edu), University of Illinois Urbana-Champaign. *Lower bounds on the Erdős-Gyárfás problem*

Given positive integers p and q, a (p,q)-coloring of the complete graph K_n is an edge-coloring in which every p-clique receives at least q colors. Erdős and Shelah posed the question of determining $f(n,p,q)$, the minimum number of colors needed for a (p,q)-coloring of K_n. Recently, Pohoata and Sheffer introduced the color energy technique to find lower bounds on this function. In this talk, we will present new lower bounds for several families of (p,q) which we obtain by further developing their method. (Received September 21, 2021)

1174-05-11278 **Jonathan Figueroa Reyes*** (jonathanfigueroa2001@gmail.com), Baruch College. *Spanning tree building process on families of graphs* Preliminary report.

Given a simple graph, we can use the forest-building process to identity a spanning forest, as follows: we sort the edges of the graph in some order, then, moving in the order of the list, we keep each edge only if it is incident to a vertex that is not already incident to any previous edge, which will result in a spanning forest. To identify all possible spanning forests, we can repeat this process with every possible permutation of the edges. Therefore, the probability of obtaining a spanning tree (a single-component spanning forest) can be calculated as a fraction of the number of single-component spanning forests divided by the total number of spanning forests.

In this talk, we derive the probability of obtaining a spanning tree for graphs of various shapes and arbitrary sizes. (Received September 22, 2021)

1174-05-11282 **Xiaofeng Gu** (xgu@westga.edu), University of West Georgia, **Sulin Song*** (ss0148@mix.wvu.edu), West Virginia University, and **Hong-Jian Lai** (hjlai@math.wvu.edu), West Virginia University. *On hamiltonian line graphs of hypergraphs*

A graph is **supereulerian** if it has a spanning eulerian subgraph. Harary and Nash-Williams in 1968 proved that the line graph of a graph G is hamiltonian if and only if G has a dominating eulerian subgraph, Jaeger in 1979 showed that every 4-edge-connected graph is supereulerian, and Catlin in 1989 proved that every graph with two edge-disjoint spanning trees is a contractible configuration for supereulerianicity. Utilizing the notion of partition-connectedness of hypergraphs introduced by Frank, Király and Kriesell in 2003, we generalize the above-mentioned results of Harary and Nash-Williams, of Jaeger and of Catlin to hypergraphs by characterizing hypergraphs whose line graphs are hamiltonian, and showing that every 2-partition-connected hypergraph is a contractible configuration for supereulerianicity. (Received September 22, 2021)

1174-05-11289 **Steve Butler** (butler@iastate.edu), Iowa State University, **Ayomikun Adeniran*** (aadenira@colby.edu), Colby College, **Cyrus Hettle** (cyrushettle@gmail.com), Georgia Institute of Technology, and **Hayan Nam** (hnam@iastate.edu), Iowa State University. *Parking completions*

Parking functions are a well-known generalization of permutations. One interesting generalization of these combinatorial objects are *parking completions*. Given a strictly increasing sequence $\mathbf{t} = (t_1, \ldots, t_m)$ with entries from $[n]$, we can think of $\mathbf{t}$ as a list of spots already taken in a street with n parking spots, and $\mathbf{c} = [n] \setminus \mathbf{t}$ as a list of parking preferences where the i-th car attempts to park in the c_i-th spot and if not available then proceeds up the street to find the next available spot, if any. A parking completion corresponds to a set of preferences $\mathbf{c}$ where all cars park. In this talk, we will explore how parking completions are related to restricted lattice paths. We will also present results for both the ordered and unordered variations of the problem by use of a pair of operations (termed **Join** and **Split**). A nice consequence of our results is a new volume formula for most Pitman-Stanley polytopes. (Received September 28, 2021)

1174-05-11300 **Michael Ren*** (mren36@mit.edu), Massachusetts Institute of Technology. *Wilf Equivalences and Stanley-Wilf Limits for Patterns in Rooted Labeled Forests*

Building off recent work of Garg and Peng, we continue the investigation into classical and consecutive pattern avoidance in rooted forests, resolving some of their conjectures and questions and proving generalizations whenever possible. Through extensions of the forest Simion-Schmidt bijection introduced by Anders and Archer, we demonstrate a new family of forest-Wilf equivalences, completing the classification of forest-Wilf equivalence classes for sets consisting of a pattern of length 3 and a pattern of length at most 5. We also find a new family of nontrivial c-forest-Wilf equivalences between single patterns using the forest analogue of the Goulden-Jackson cluster method, showing that a $(1-o(1))^n$-fraction of patterns of length n satisfy a nontrivial c-forest-Wilf equivalence and that there are c-forest-Wilf equivalence classes of patterns of length n of exponential size. Finally, we prove a forest analogue of the Stanley-Wilf conjecture for avoiding a single pattern as well as certain other sets of patterns. Our techniques are analytic, easily generalizing to different types of pattern avoidance and allowing for computations of convergent lower bounds of the forest Stanley-Wilf limit in the cases covered by our result. (Received September 30, 2021)

1174-05-11777 **Kevin Iga*** (kiga@pepperdine.edu), Pepperdine University. *Adinkras: A collaboration across subjects, fields, and backgrounds.*

Abstract body/text including any latex coding if necessary. In the summer of 2019, ADJOINT at MSRI sponsored a collaboration between two Black male physicists (Jim Gates, Brown University and Vincent Rodgers, University of Iowa), a White female combinatorist (Carly Klivans, Brown University), and an Asian American male 4-manifold topologist (Kevin Iga, Pepperdine University). One of the subjects involved Adinkras, which are graphs with colored edges satisfying various properties, developed in 2004 by Jim Gates and Michael Faux to study the mathematics of supersymmetric theories in particle physics. In this presentation I will explain what Adinkras are, what they are good for, and some of the interesting interactions with other areas of mathematics. I will provide an update on the status of these collaborations. (Received October 20, 2021)

1174-05-12179 **Katelynn Kochalski*** (kochalski@geneseo.edu), SUNY Geneseo. *Modified Game of Best Choice*

In the game of best choice, a list of candidates is interviewed and after each interview the committee must decide whether to irrevocably pass or accept the candidate. When making this decision, the committee can compare the applicant to previous applicants but has no other information. The game is won if the best candidate is hired and it is known that to optimize this probability, one should wait a designated length of time and then hire the next best candidate they see. Candidates are then incentivized to have their interview as soon after this calibration period as possible. We study a variant of this problem which helps mitigate this feature by forbidding some candidate orders. We seek strategies where candidates have no incentive to change their interview time. In our work we completely characterize all these strategy indifferent interview orders. This talk will give that characterization and the associated probabilities of winning that can occur. (Received November 9, 2021)

1174-05-12240 **Anup Rao*** (anuprao@cs.washington.edu), University of Washington. *Sunflowers: from soil to oil*

A sunflower is a collection of sets whose pairwise intersections are all the same. Erdos and Rado showed that any large family of sets of size k must contain a large sunflower, and made a conjecture about the dependence of the size of sunflower on the size of the family of sets. Very recently, Alweiss, Lovett, Wu and Zhang made significant progress towards proving their conjecture. I discuss the key ideas involved in this line of work, and show how this problem is connected to a diverse array of applications in mathematics and computer science. (Received December 5, 2021)

06 ► Order, lattices, ordered algebraic structures

1174-06-5918 **Bernd S. W Schroder*** (Bernd.Schroeder@usm.edu), The University of Southern Mississippi. *The Automorphism Conjecture for Ordered Sets of Width 10 or Less* Preliminary report.

The automorphism conjecture for ordered sets is a combinatorial conjecture which states that the ratio of the number of automorphisms to the number of endomorphisms goes to zero as the size of the underlying ordered set goes to infinity. This talk will outline a method to determine, for a given width, the indecomposable ordered sets for which the number of automorphisms is maximal. This result is then used to prove the automorphism conjecture for ordered sets of width less than or equal to 10. Perhaps more importantly, the requisite insights into the structure of the automorphism group of an ordered set could be the key to resolving the automorphism conjecture in general. (Received August 31, 2021)

1174-06-7798 **Milan Haiman*** (milan.haiman@gmail.com), MIT. *The Dimension of Divisibility Orders and Multiset Posets*

The Dushnik-Miller dimension of a poset P is the least d for which P can be embedded into a product of d chains. Lewis and Souza showed that the dimension of the divisibility order on the interval of integers $[N/\kappa, N]$ is bounded above by $\kappa(\log\kappa)^{1+o(1)}$ and below by $\Omega((\log\kappa/\log\log\kappa)^2)$. We improve the upper bound to $O((\log\kappa)^3/(\log\log\kappa)^2)$. We also prove a more general result on posets of multisets ordered by inclusion. (Received September 16, 2021)

1174-06-8176 **Madelyn Kisner*** (madelynkisner1614@gmail.com), W&J, and **Abigail Ciasullo** (ciasulloaj@washjeff.edu), W&J. *Searching for Ideals in Hypercube Posets* Preliminary report.

We consider the problem of searching for an unknown ideal μ in the hypercube poset $\beta(n)$. Elements of $\beta(n)$ may be sequentially queried for membership in μ, with at most k positive queries permitted. We provide search strategies that identify μ in a minimal number of queries in the cases $k = 1, k \geq n$, and provide a close-to-optimal strategy that yields a strong bound on queries in the case $k = 2$. (Received September 17, 2021)

1174-06-8519 **Robert W. Muth** (rmuth@washjeff.edu), Washington & Jefferson College, **Vikram G Singh*** (singhvg@washjeff.edu), Washington & Jefferson College, and **Derric Denniston** (dennistond@washjeff.edu), Washington & Jefferson College. *The Configuration Space of a Robotic Arm Over a Graph*

We investigate the configuration space S associated with the movement of a robotic arm on a grid over an underlying graph G. We study an associated PIP (poset with inconsistent pairs) IP consisting of indexed paths on G. This PIP acts as a combinatorial model for the robotic arm, and we use IP to show that the space S is a CAT(0) cubical complex, generalizing work of Ardila, Bastidas, Ceballos, and Guo. This establishes that geodesics exist within the configuration space, and yields explicit algorithms for moving the robotic arm between different configurations in an optimal fashion. We also give a tight bound on the diameter of the robotic arm transition graph—the maximal number of moves necessary to change from one configuration to another—and compute this diameter for a large family of graphs G. (Received September 19, 2021)

1174-06-8614 **Rebecca Patrias** (patr0028@stthomas.edu), University of St Thomas, and **Oliver Pechenik*** (oliver.pechenik@uwaterloo.ca), University of Waterloo. *Dynamics of plane partitions*

Consider a plane partition P as an order ideal in the product $[a]\times[b]\times[c]$ of three chain posets. The combinatorial rowmotion operator sends P to the plane partition generated by the minimal elements of its complement. We show that rowmotion exhibits a strong form of resonance with frequency $a+b+c-1$, in the sense that each orbit size shares a prime divisor with $a+b+c-1$. This confirms a 1995 conjecture of Peter Cameron and Dmitri Fon-Der-Flaass. I'll also discuss more recent work on other posets. (Received September 19, 2021)

1174-06-8918 **Robert W Ghrist** (ghrist@math.upenn.edu), University of Pennsylvania, and **Gregory Henselman-Petrusek*** (gh10@princeton.edu), University of Oxford. *How to analyze a persistence module of abelian groups into simple parts, with applications* Preliminary report.

The breakdown of persistence modules into simple parts or "bars" is basic to topological data analysis. However, such breakdowns exist only for persistence modules derived from homology with field coefficients. This creates a natural gap between persistence and the wider field of homological algebra.

Recently, Patel narrowed this gap by defining barcodes for tame persistence modules in categories outside Vect. However, in a departure from the linear regime, these barcodes do not accompany an algebraic decomposition of the module itself – there is no notion of a "cycle representative" for one of these bars.

This work supplies the missing link. We introduce a new decomposition scheme that extends the notion of "factorization into bars" from the linear regime to any abelian category (as well as many non-abelian categories). This generalization has diverse applications in theory, computation, and visualization. This is joint work with Robert Ghrist. (Received September 20, 2021)

1174-06-9964 **Sandor Radeleczki*** (matradi@uni-miskolc.hu), Mathematical Institute, University of Miskolc. *Atoms and coatoms in the lattice of residuated maps* Preliminary report.

Abstract: We describe the coatoms of the lattice $\mathrm{Res}(L)$ formed by the residuated maps of a completely join-and meet-dense complete lattice, by using the so-called ordered relations. As a generalization of meet-distributive lattices, a lattice class $\mathcal{K}$ related strongly with representation problems is introduced. We show that for any lattice $L \in \mathcal{K}$ with finite height, the atoms of $\mathrm{Res}(L)$ are two-valued weakly regular maps. For any lattice L of finite height we characterize those weakly regular maps $f\colon L \to L$ which are atoms in $\mathrm{Res}(L)$. (Received September 21, 2021)

1174-06-10198 **Karell Bertet*** (kbertet@univ-lr.fr), La Rochelle University, and **Christophe Demko** (cdemko@univ-lr.fr), La Rochelle University. *On the links between NextPriorityConcept algorithm and generalized convex hulls.* Preliminary report.

Formal Concept Analysis is a mathematical formalism offering many methods that can be used in various fields of computer science. FCA highlights the structure of lattice or "concept lattice", defined for binary or categorical data, where a concept is a pair composed of a maximal subset of objects together with their shared data. The whole set of concepts is naturally organized in a hierarchical graph, called the concept lattice. The fundamental theorem in FCA establishes a bijection between lattices and "reduced" binary datasets [BM70] via concept lattice generation, called the "Galois" connection. A nice result establishes that the composition of the two operators of a Galois connection is a closure operator. The notion of closure operator is central in lattice theory. From the original binary formalism of FCA, different extensions to non-binary data have been proposed [FR00] [GK01], by establishing that the Galois connection between a data space and a description space is maintained, as long as the description space verifies the lattice property. We recently introduced in [DBFVK20] a new algorithm, NextPriorityConcept, that is capable of generating concepts from complex and heterogeneous data with a generic description to provide predicates describing a subgroup of objects. We have observed that the generic use of predicates describing subgroups correspond to generalized convex hulls. In this talk, we will present some links between our algorithm and the theory of convex structures. (Received September 21, 2021)

1174-06-10402 **Kira Adaricheva*** (kira.adaricheva@hofstra.edu), Hofstra University, USA, **J. B. Nation** (jb@math.hawaii.edu), University of Hawaii, USA, **Jennifer Hyndman** (hyndman@unbc.ca), University of Northern British Columbia, Canada, and **Joy Nishida** (jnishida@hawaii.edu), University of Hawaii, USA. *A primer on quasivariety lattices*

The goal of the talk to announce a monograph to be published by Canadian Mathematical Society/Springer at the end of 2021.

An old problem of Birkhoff and Mal'cev asks for a characterization of lattices of subquasivarieties. It is well known that these lattices are join-semidistrbutive.

There are two parts to the question: (1) describe properties that hold in subquasivariety lattices; (2) represent lattices satisfying some given properties as subquasivariety lattices.

Progress up until 1998 is recorded in V. Gorbunov's book (Plenum, NY 1998). But more recent results have cast the problem in a different light.

The first part of the monograph develops the basic theory of subquasivariety lattices in the very general context. The main result represents the lattice of subquasivarieties as the lattice of H-closed algebraic subsets of an algebraic lattice with a monoid H of operators. One can use the representation to find restrictions on the equational closure operator on a lattice of subquasivarieties, strengthening earlier results. The second part of the book discusses some of these restrictions.

The third part shows how to represent some pairs consisting of a finite lattice and an equaclosure operator on it as a lattice of subquasivarieties and its natural equational operator. A fourth part discusses representations of dually algebraic distributive lattices.

A companion monograph by J.B. Nation and J. Hyndman (Canadian Mathematical Society/Springer, 2018) focuses on locally finite quasivarieties. (Received September 21, 2021)

1174-06-10630 **Csaba Biro*** (`csaba.biro@louisville.edu`), University of Louisville, and **Sida Wan** (`sida.wan@louisville.edu`), University of Louisville. *Ramsey theoretical results on product of chains* Preliminary report.

Let $[n]^k$ be the poset of k-tuples $(a_1, \dots, a_k)$ with $1 \leq a_i \leq n$ for each i, partially ordered coordinatewise. One can view $[n]^k$ as the product of k chains, each of size n. A classic theorem, called the Product Ramsey Theorem states that for all m, l, r, k there exists n, such that every r-coloring of the $[m]^k$ subgrids of $[n]^k$, there is a monochromatic subgrid $[l]^k$. In this research, we consider various applications and variations of the Product Ramsey Theorem. (Received September 21, 2021)

1174-06-10668 **Karen L Collins*** (`kcollins@wesleyan.edu`), Wesleyan University. *The Distinguishing Number of a Poset*

The distinguishing number of poset P, denoted by $D(P)$, is the minimum number of colors needed to color the points of P so that the only automorphism of P that preserves colors is the identity. In this talk we discuss results about $D(P)$ and a related parameter applied to distributive lattices. (Received September 21, 2021)

1174-06-10767 **Lachesis W Cavin*** (`twczqd@umsystem.edu`), missouri university of science and technology. *Representing Lattices of Height 2 as Congruence Lattices of Flag-transitive G-s* Preliminary report.

A flag-transitive G-set is formed by the transitive action of an automorphism group on the flag set of a block design. Consider a finite lattice L of height 2 which has not been proven representable as a congruence lattice of a finite algebra. If L is representable as a congruence lattice of a finite algebra, then it is representable as the congruence lattice of a flag-transitive G-set. (Received September 21, 2021)

1174-06-10821 **Jonathan David Farley*** (`lattice.theory@gmail.com`), Morgan State University. *Semi-distributivité, égalité, and Stanley: An Unpublished Question of Björner from 1997*

Modern lattice theory, the abstract study of order and hierarchy, was born at Harvard in the 1930's, a creation of my undergraduate thesis advisor Professor Garrett Birkhoff. (Technically, Barry Mazur was my undergraduate thesis advisor, since Professor Birkhoff had retired.) His colleague Gian-Carlo Rota wrote, citing a prediction of I. M. Gelfand, that "lattice theory will play a leading role in the mathematics of the twenty-first century".

Using the g-Theorem on polytopes, Bjorner proved a result about how the number of totally ordered subsets of a finite distributive lattice grows as the subsets increase in size. He then asked in 1997 if that result could be proven combinatorially.

At "the other end of the galaxy," one finds Priestley duality for distributive lattices, a way of understanding distributive lattice-ordered algebraic structures by means of topology.

One day, on an airplane crossing the Atlantic, I saw these two notions collide.

Let P be an n-element, naturally labeled poset. For $k \in \mathbb{N}$, let h_k be the number of linear extensions of P with exactly k descents. I prove combinatorially what Björner proved with the g-Theorem, that for $k \leq \frac{n-1}{2}$, $h_{n-1-k} \leq h_k$.

There are connections with semidistributive lattices and Stanley's P-partitions.

(I had originally planned to speak on these results at the 2014 AMS meeting, but instead I flew to Turkey where I met someone who later claimed she was connected to the Russian mafia. But that's quite a different tale. . .) (Received September 21, 2021)

1174-06-10949 **Radu Negulescu*** (`radu.negulescu.ca@gmail.com`), -. *A Taxonomy of Lock Faults in Concurrent Systems* Preliminary report.

Concurrent systems are practically everywhere: digital circuits, threaded software, and discrete event human workflows are ubiquitious examples of such systems. We propose a characterization of three main types of faults of concurrent systems (deadlock, livelock, and starvation) using Boolean lattices. We show our characterization is satisfactory in that it matches empirical examples of lock faults and it is consistent with known properties of operations on concurrent systems. (Received September 21, 2021)

1174-06-11034 **Gary Gordon*** (gordong@lafayette.edu), Lafayette College. *Families of linear relations among subsets of posets*

Brylawski proved the coefficients of the Tutte polynomial of a matroid satisfy a set of linear relations. We extend these relations to a generalization of the Tutte polynomial that includes antimatroids. In this talk, we concentrate on applications to posets, which are examples of antimatroids in several different ways. In each case, new linear identities involving 'convex' subsets of posets are derived. (Received September 21, 2021)

1174-06-11226 **Sara Chari*** (schari0301@gmail.com), Bates College, and **Deveena Banerjee** (deveena.r.banerjee@vanderbilt.edu), Vanderbilt University. *Higher Dimensional Origami Constructions* Preliminary report.

When performing origami folds on a piece of paper, we may view the paper as the complex plane and the folds as lines in the plane. We start with two seed points and make a fold through each, generating a new intersection point, and by iterating this process for each pair of points formed, we generate a subset of the complex plane. We extend previously known results about the algebraic and geometric structure of these sets to higher dimensions. In the case when the set obtained is a lattice, we explore the relationship between the set of angles and the generators of the lattice and determine how introducing a new angle alters the lattice. (Received September 21, 2021)

08 ► General algebraic systems

1174-08-5799 **Carlos Améndola** (carlos.amendola@tum.de), Technical University of Munich. *Estimating Gaussian mixtures using sparse polynomial moment systems*

The method of moments is a statistical technique for density estimation that solves a system of moment equations to estimate the parameters of an unknown distribution. A fundamental question critical to understanding identifiability asks how many moment equations are needed to get finitely many solutions and how many solutions there are. We answer this question for classes of Gaussian mixture models using the tools of polyhedral geometry. Using these results, we present a homotopy method to perform parameter recovery, and therefore density estimation, for high dimensional Gaussian mixture models. The number of paths tracked in our method scales linearly in the dimension. (Received August 27, 2021)

1174-08-10994 **Charlotte Aten*** (caten2@ur.rochester.edu), University of Rochester. *Distributive lattices in rock-paper-scissors*

I will describe, primarily by way of examples, a process for manufacturing a generalized rock-paper-scissors game from a finite group $\mathbf{G}$ which I first introduced in my 2020 paper "Multiplayer rock-paper-scissors", which appeared in Algebra Universalis. These games may be viewed as algebras $\mathbf{A} = (A, f)$ with a single basic n-ary operation f, where n is the number of players and A is the set of items from which a player may choose. The congruence lattices of these *regular rock-paper-scissors magmas* are strongly related to certain "convex" subgroups of $\mathbf{G}$ in that there is a poset $\mathbf{P}$ associated with the cosets of such subgroups and the congruence lattice $\mathbf{Con}(\mathbf{A})$ of $\mathbf{A}$ is the lattice of maximal antichains of $\mathbf{P}$, which by a result of Dilworth is always distributive. Time permitting I will go on to address that these congruence-distributive finite regular rock-paper-scissors magmas generate the varieties generated by hypertournament algebras and how the congruence-distributivity of these generating algebras may be brought to bear on the problem of understanding the structure of these varieties of interest. (Received September 21, 2021)

11 ► Number theory

1174-11-5693 **Robert D Hough*** (robert.hough@stonybrook.edu), SUNY Stony Brook. *Subconvexity of the Shintani zeta function*

A subconvex estimate is made for the Shintani zeta function enumerating class numbers of binary cubic forms. The method also proves subconvexity for its Maass form twisted version. The method is general, and may have further applications to subconvex estimates for zeta functions of prehomogeneous vector spaces, which is novel due to the high degree. Joint work with Eun Hye Lee. (Received August 25, 2021)

1174-11-5736 **George E Andrews*** (gea1@psu.edu), Pennsylvania State University, Pennsylvania State University, University Park, PA. *Schmidt Type partitions and modular forms*

This work is joint with Peter Paule. In 1999, Frank Schmidt noted that the number of partitions of integers into distinct parts in which the first, third, fifth, etc. summands add to n is equal to p(n), the number of ordinary partitions of n. By invoking MacMahon's theory of Partition Analysis, we provide a context for this result which leads directly to many other theorems of this nature. (Received August 26, 2021)

1174-11-5809 **Eduardo G Aponte*** (edugio56@yahoo.com), UNEXPO, Caracas. Venezuela.. *Non-consecutive partial sums and the geometric series* Preliminary report.

In this work we consider the partial sums of the geometric series whose terms are exponential functions. From them, formulas for sums and series of real numbers raised to the power m are obtained.

We start from partial sums of the geometric series by means of which partial sums of non-consecutive real numbers are generated raised to integer powers m, with $m \geq 0$. This is achieved by differentiating m times the partial sums of the geometric series. Additionally, we consider the case of alternation of signs in the geometric series and doing the same differentiation process we obtain similar results. On the other hand, introducing a parameter $b > 0$ and following the same argument, additional results are found. For all the cases considered in the work, recurrence relations are obtained, which allow building all the sums using only the first term. These formulas are explicit and more general to the ones obtained by F. T. Howard in his 1994's article.

Finally, integrating the partial sums of the geometric series with exponential terms that generate non-consecutive sums and non-consecutive alternating sums, we obtain a set of partial sums of series in integral form, with $m \in \mathbb{C}$ and for particular parameter values, the ζ Riemann function. (Received August 28, 2021)

1174-11-5813 **Ken Ono*** (ken.ono691@gmail.com), University of Virginia, **Ian Wagner** (iwagner@vanderbilt.edu), Vanderbilt University, and **Kathrin Bringmann** (kbringma@math.uni-koeln.de), University of Cologne. *Modular q-series arising from partitions*

Generating functions for various divisor functions offer examples of classical modular forms. What can be said about such functions which are not modular? Here we establish a framework of such q-series which arise from the Bloch-Okounkov bracket operators of weighted t-hooklength partition functions. These q-series give rise to families of mock modular forms and quantum modular forms. (Received August 28, 2021)

1174-11-5966 **Wen-Ching Winnie Li** (wli@math.psu.edu), The Pennsylvania State University, and **Fang-Ting Tu*** (tu@math.lsu.edu), Louisiana State University. *A Whipple ${}_7F_6$ formula revisited*

A well-known formula of Whipple relates certain hypergeometric values ${}_7F_6(1)$ and ${}_4F_3(1)$. In this talk we revisit this relation from the viewpoint of the underlying hypergeometric data HD, to which there are also associated hypergeometric character sums and Galois representations. We explain a special structure behind Whipple's formula when the hypergeometric data HD are primitive and defined over $\mathbb{Q}$. In this case, by the work of Katz, Beukers, Cohen, and Mellit, there are compatible families of ℓ-adic representations of the absolute Galois group of $\mathbb{Q}$ attached to HD. For specialized choices of HD, these Galois representations are shown to be decomposable and automorphic. As a consequence, the values of the corresponding hypergeometric character sums can be explicitly expressed in terms of Fourier coefficients of certain modular forms. We further relate the hypergeometric values ${}_7F_6(1)$ in Whipple's formula to the periods of modular forms occurred. (Received September 1, 2021)

1174-11-5992 **Ryan Kenichi Tamura** (rtamura1@berkeley.edu), University of California, Berkeley, **Adriana L Duncan** (addielouduncan@yahoo.com), University of Texas at Austin, **Simran Khunger** (skhunger@andrew.cmu.edu), Carnegie Mellon University, and **Holly Swisher*** (swisherh@math.oregonstate.edu), Oregon State University. *Generalizations of the Alder-Andrews Theorem in Partition Theory*

Partitions with parts differing by at least d, i.e. d-distinct partitions, arise in a famous identity of Euler and the first Rogers-Ramanujan identity which state that the number of 1-distinct or 2-distinct partitions of n is equal to the number of partitions of n into parts that are ± 1 modulo 4 or 5, respectively. Alder showed that for $d \geq 3$ no identity of such a type exists, and conjectured what is now the Alder-Andrews Theorem, that there are at least as many d-distinct partitions of n as partitions of n into parts that are ± 1 modulo $d + 3$. This was proved partially by Andrews in 1971, Yee in 2008, and was fully resolved by Alfes, Jameson and Lemke Oliver in 2011.

In 2020, Kang and Park constructed an extension of Alder's conjecture which relates to the second Rogers-Ramanujan identity. Namely, that there are at least as many d-distinct partitions of n with parts ≥ 2 as

partitions of n into parts that are ± 2 modulo $d+3$, excluding the part $d+1$. Kang and Park proved this when $d = 2r-2$ and n is even.

Here we discuss our recent proof of Kang and Park's conjecture for all $d \geq 62$. Additionally, we present a generalized infinite family of conjectures and discuss proofs of these for infinite classes of n and d. We conclude with a discussion of remaining open questions.

This work is joint with Adriana Duncan, Simran Khunger, and Ryan Tamura. It was initiated during the 2020 Oregon State University summer REU program, funded by NSF grant DMS-1757995 and Oregon State. (Received September 2, 2021)

1174-11-5998 **Jakob Streipel*** (jakob.streipel@wsu.edu), Washington State University. *Twisted moments of* GL(3) × GL(2) *L-functions* Preliminary report.

We compute an asymptotic formula for the twisted moment of GL(3) × GL(2) L-functions and their derivatives, and use this to prove that symmetric square lifts of GL(2) Maass forms are uniquely determined by the central values of the derivatives of GL(3) × GL(2) L-functions. (Received September 2, 2021)

1174-11-6021 **Tyler Cleveland Russell*** (trussell12@patriots.uttyler.edu), The University of Texas at Tyler. *Polynomials Associated to Integer Partitions*

Integer partitions express the different ways that a positive integer may be written as a sum of other positive integers. Here we explore the analytic properties of a polynomial $f_\lambda(x)$ that we call the partition polynomial for the partition λ, with the hope of learning new properties of partitions. We prove a recursive formula for the derivatives of $f_\lambda(x)$ involving Stirling numbers of the second kind, show that the set of integrals from 0 to 1 of a normalized version of $f_\lambda(x)$ is dense in $[0, 1/2]$, pose a few open questions, and formulate a conjecture relating the integral to the length of the partition. We also provide specific examples throughout to support our speculation that an in-depth analysis of partition polynomials could further strengthen our understanding of partitions. (Received September 2, 2021)

1174-11-6023 **Hari Ramakrishnan Iyer*** (hiyer@college.harvard.edu), Harvard College, **Sanford Lu** (sl7895@nyu.edu), New York University, and **Dang Dang** (dang.dang@stonybrook.edu), Stony Brook University. *One-level density of a family of L-functions over function fields* Preliminary report.

Katz and Sarnak conjectured a correspondence between n-level density statistics of zeroes of families of L-functions and the eigenvalues of random matrix ensembles. The particular ensemble depends on the symmetry type of the family, which is a classical compact group. We build upon previous work by Waxman, which has shown that L-functions associated with Hecke characters on Gaussian integers $\mathbb{Z}[i]$ have zeroes which are modeled by the eigenvalues of symplectic matrices. We consider analogous L-functions associated with "super-even" characters in the function field setting. Though these characters have been studied using random matrix theory as $q \to \infty$ (for $\mathbb{F}_q[t]$), we instead consider the limit where the degree K of the modulus of the Dirichlet character is large; note that this is equivalent to the large conductor limit, since $K \log q$ is proportional to the average logarithmic conductor of super-even L-functions at $s = 1/2$. We compute the limiting one-level density for this family of L-functions and we show that it matches a symplectic distribution for a class of test functions f whose Fourier transform $\hat{f}$ is compactly supported in $(-1, 1)$. We directly calculate the main term and a lower order term for the one-level density. In addition, we apply the L-functions Ratios Conjecture to compute the one-level density, and we show agreement with the unconditional result for restricted support to arbitrarily lower order $O(K^{-a})$ for all $a > 1$. (Received September 3, 2021)

1174-11-6028 **Chad Awtrey*** (chad.awtrey@gmail.com), Samford University. *Totally ramified p-adic fields of degree $2p$*

Let p be a prime number and $\mathbb{Q}_p$ the field of p-adic numbers. For any positive integer n, there are only finitely many extensions of $\mathbb{Q}_p$ of degree n up to isomorphism. Consequently, an important area of research is concerned with characterizing these classes by determining a defining polynomial for each extension along with ramification information (e.g., the Galois group of the normal closure). Such information is known in some cases; for example, when $p \nmid n$, $p = n$, or $n < 16$. In this talk, we give a characterization of extensions in the case $n = 2p$, which extends and generalizes previous work. In particular, we determine the following for each isomorphism class of totally ramified extensions: its automorphism group, subfields, an Eisenstein polynomial defining the extension, and the structure of the polynomial's Galois group. More concretely, the Galois group is a semidirect product of the form $C_p^s \rtimes (C_e \rtimes C_f)$ for some $s \in \{1, 2\}$, and we determine e, f, and s symbolically in terms of the polynomial's coefficients. For $p \leq 23$, we identify the Galois group up to conjugacy as a transitive subgroup of S_{2p}. (Received September 3, 2021)

1174-11-6060 **Dongxi Ye** (yedx3@mail.sysu.edu.cn), Sun Yat-sen University, and **Nathaniel Mayes** (nathaniel.mayes01@utrgv.edu), University of Texas Rio Grande Valley. *Ramanujan Type Congruences for Products of Higher Level*

In 1919, Ramanujan formulated congruences modulo powers of $5, 7$, and 11 for the classical partition function. These were later extended to other partition-theoretic generating functions, including those for p-cores. In this talk, we will discuss a class of Klein form quotients that satisfy similar Ramanujan type congruences for powers of p. Congruences satisfied by coefficients of generating functions for p-cores appear as special cases. In this talk, the focus will be on primes $p \geq 13$. We demonstrate how a permutative action of $\Gamma_0(p)$ on each generating function implies an algebraic symmetry in which quotients satisfying Ramanujan type congruences are divided into equivalence classes. Classes of products arise that satisfy Ramanujan type congruences for all sufficiently large primes.

As time permits, a sieving process will be described through a decomposition of the space of modular forms of weight 1 for $\Gamma_1(p)$ as a direct sum of subspaces of modular forms for $\Gamma(p)$ of the form $q^{r/p}\mathbb{Z}[[q]]$ for each congruence class r modulo p. Since the relevant bases elements generate the graded algebra of modular forms for these congruence groups, the weight one decompositions determine series dissections for modular forms of level p that lead to additional classes of congruences. A number of novel byproducts are obtained from dissection identities. The techniques constitute a foundation for the study of congruences for modular forms of any weight and level. (Received September 4, 2021)

1174-11-6070 **Amanda Folsom*** (afolsom@amherst.edu), Amherst College, **Anna M Dietrich** (adietrich22@amherst.edu), Amherst College, **Keane Ng** (kng23@amherst.edu), Amherst College, **Chloe Stewart** (cstewart22@amherst.edu), Amherst College, and **Shixiong Xu** (sxu23@amherst.edu), Amherst College. *Overpartition ranks and quantum modular forms*

For each $d \in \mathbb{N}$, we establish an infinite family of weight $1/2$ quantum modular forms from the overpartition M_d-rank generating function. Infinite quantum families from both the Dyson rank overpartition generating function and the overpartition M_2-rank generating function, appear as special cases of our work. As a corollary, we obtain explicit closed expressions which may be used to evaluate Eichler integrals of certain weight $3/2$ theta functions. (Received September 6, 2021)

1174-11-6208 **Noah Solomon** (nsolomon22@amherst.edu), Amherst College, and **Elizabeth Pratt** (epratt22@amherst.edu), Amherst College. *Quantum Jacobi forms and sums of tails identities*

Sums of tails identities feature sums of differences between infinite products and their nth partial products. Some seemingly curious sums of tails identities appearing in Ramanujan's Lost Notebook were later shown to be connected to a Hecke L-function, and even more recently, other sums of tails identities have been related to quantum modular forms. In this talk, we will discuss results on the interconnected topics of quantum Jacobi forms, sums of tails identities, and generating functions for values of certain L-series. (Received September 6, 2021)

1174-11-6247 **Jingbo Liu*** (jliu@tamusa.edu), Texas A&M University-San Antonio. *On a Waring's problem for Hermitian lattices*

Assume E is an imaginary quadratic field and $\mathcal{O}$ is its ring of integers. For each positive integer m, let I_m be the free Hermitian lattice of rank m over $\mathcal{O}$ having an orthonormal basis. For each positive integer n, let $\mathfrak{S}_{\mathcal{O}}(n)$ be the set of all Hermitian lattices of rank n over $\mathcal{O}$ that can be represented by some I_m. Denote by $g_{\mathcal{O}}(n)$ the smallest positive integer g such that each Hermitian lattice in $\mathfrak{S}_{\mathcal{O}}(n)$ can be represented by I_g. In this talk, we shall provide an explicit upper bound for $g_{\mathcal{O}}(n)$ for all imaginary quadratic fields E and positive integers n. (Received September 7, 2021)

1174-11-6252 **Eun Hye Lee*** (EunHye.Lee@stonybrook.edu), Stony Brook University. *Subconvexity of Shintani Zeta Functions* Preliminary report.

In this talk, we introduce and prove a subconvex estimate for the Shintani ζ function enumerating class numbers of binary cubic forms. (Received September 7, 2021)

1174-11-6322 **Atul Dixit*** (adixit@iitgn.ac.in), IIT Gandhinagar, and **Ankush Goswami** (ankushgoswami3@gmail.com), IIT Gandhinagar. *Title: A closed-form evaluation of a bivariate generating function associated with overpartition pairs* Preliminary report.

This work originated from our quest to obtain an spt-type function starting with the quintuple product identity. It enabled us to obtain a new identity for a bivariate generating series, giving its closed-form evaluation. The coefficients of this series are connected with a function counting certain overpartition pairs recently introduced by

Bringmann, Lovejoy and Osburn. More so, our identity enables us to obtain a closed-form evaluation of a double series using Chebyshev polynomials of the second kind, thereby resulting in an analogue of Euler's pentagonal number theorem. We also show that the aforementioned identity connects a double series with a weight 7/2 theta series using a result of Shimura. This is an ongoing joint work with Ankush Goswami. (Received September 7, 2021)

1174-11-6333 **Rahul Kumar*** (rks0999@gmail.com), IIT Gandhinagar, IBS-CGP POSTECH, South Korea. *Extended higher Herglotz functions*

In 1975, Don Zagier obtained a new version of the Kronecker limit formula for a real quadratic field which involved an interesting function $F(x)$ which is now known as the *Herglotz function*. As demonstrated by Zagier, and very recently by Radchenko and Zagier, $F(x)$ satisfies beautiful properties which are of interest in both algebraic number theory as well as in analytic number theory. In this paper, we study $\mathcal{F}_{k,N}(x)$, an extension of the Herglotz function which also subsumes *higher Herglotz function* of Vlasenko and Zagier. We call it the *extended higher Herglotz function*. It is intimately connected with a certain generalized Lambert series. We derive two different kinds of functional equations satisfied by $\mathcal{F}_{k,N}(x)$. Radchenko and Zagier gave a beautiful relation between the integral $\int_0^1 \frac{\log(1+t^x)}{1+t}\,dt$ and $F(x)$ and used it to evaluate this integral at various rational as well as irrational arguments. We obtain a relation between $\mathcal{F}_{k,N}(x)$ and a generalization of the above integral involving polylogarithm. The asymptotic expansions of $\mathcal{F}_{k,N}(x)$ and some generalized Lambert series are also obtained along with other supplementary results.This is joint work with Atul Dixit and Rajat Gupta. (Received September 8, 2021)

1174-11-6334 **Rajat Gupta*** (rajat_gupta@iitgn.ac.in), IIT Gandhinagar. *Koshliakov zeta functions and Modular Relations*

Nikolai Sergeevich Koshliakov was an outstanding Russian mathematician who made phenomenal contributions to number theory and differential equations. In the aftermath of World War II, he was among the many scientists arrested on fabricated charges and incarcerated. Under extreme hardships while still in prison, Koshliakov (under a different name, 'N. S. Sergeev') wrote two manuscripts out of which one was lost. Fortunately, the second one was published in 1949, although, to the best of our knowledge, no one studied it until the last year when Prof. Atul Dixit and I started examining it in detail. This manuscript contains a complete theory of two interesting generalizations of the Riemann zeta function having their genesis in heat conduction and is truly a masterpiece! In this talk, we will discuss some of the contents of this manuscript and then give some new results (modular relations) that we have obtained in this theory. This is joint work with Prof. Atul Dixit. (Received September 8, 2021)

1174-11-6335 **Spencer Hamblen*** (shamblen@mcdaniel.edu), McDaniel College. *Waring's Problem in Quaternion Rings*

Quaternions are often used to demonstrate the proof of Lagrange's Four Square Theorem. The generalization of this theorem to higher powers is Waring's Problem, which itself can be generalized to any ring R: what is the minimum number of k-th powers in R, (the Waring number $g_R(k)$) necessary to represent every element in R that can be represented as the sum of k-th powers?

Paul Pollack recently proved the existence of the Waring number over integral quaternions for any power k. We investigate Waring numbers in $LQ_{a,b}$, the set of elements with integer coefficients in the quaternion algebra with $i^2 = -a$ and $j^2 = -b$. We calculate $g_{LQ_{a,b}}(2)$ for many values of a and b, and give general bounds for $g_{LQ_{a,b}}(k)$ when $k = 2$, 3, and 4. (Received September 8, 2021)

1174-11-6341 **Soumyarup Banerjee*** (soumya.tatan@gmail.com), Indian Institute of Technology Gandhinagar, India. *Finiteness theorems for universal sums of squares of almost primes*

The Conway–Schneeberger Fifteen theorem states that a given positive definite integral quadratic form is *universal* (i.e., represents every positive integer with integer inputs) if and only if it represents the integers up to 15. This theorem is sometimes known as "Finiteness theorem" as it reduces an infinite check to finite one. In this talk, I would like to present my recent work along with Ben Kane where I have investigated quadratic forms which are universal when restricted to almost prime inputs and have established finiteness theorems akin to the Conway–Schneeberger 15 theorem. (Received September 8, 2021)

1174-11-6372 **Bella Tobin** (bella.tobin@okstate.edu), Oklahoma State University, **Angelica Babei** (babeiangelica@gmail.com), McMaster University, **Lea Beneish** (leabeneish@berkeley.edu), University of California, Berkeley, and **Manami Roy** (mroy17@fordham.edu), Fordham University. *Generalized Ramanujan-Sato series arising from modular forms II*

Motivated by work of Chan, Chan, and Liu, we obtain a new general theorem yielding corollaries that produce generalized Ramanujan-Sato series for $1/\pi$. We use these corollaries to construct explicit examples arising from modular forms on arithmetic triangle groups. This work is joint with Angelica Babei, Lea Beneish, Manami Roy, Bella Tobin, and Fang-Ting Tu. It was initiated as part of the Women in Numbers 5 workshop. (Received September 8, 2021)

1174-11-6413 **Shashank Kanade*** (shashank.kanade@du.edu), University of Denver, **Antun Milas** (antun.milas@gmail.com), SUNY at Albany, and **Matthew C Russell** (russell2@math.rutgers.edu), Rutgers University. *Nahm-type sums with double poles*

Nahm sums are q-hypergeometric series with a specific form: $\sum_{\mathbf{n}\geq \mathbf{0}} \frac{q^{\mathbf{n}^t\cdot \mathrm{C}\cdot \mathbf{n}+\mathbf{n}^t\cdot \mathrm{B}}}{(q)_{n_1}(q)_{n_2}\cdots(q)_{n_r}}$. In a recent work, Milas and Jennings-Shaffer studied versions of such series where each Pochhammer in the denominator of the summands appears with a power higher than 1. They proved many new identities involving such series. A wide variety of such identities also arose in the work of Córdova, Gaiotto and Shao on Schur's indices of certain physical theories. In this talk, I will present such Nahm-type sum representations involving double poles for Gordon-Andrews and Andrews-Bressoud series and also for Rogers' false theta functions. This is a joint work with A. Milas and M. C. Russell. (Received September 8, 2021)

1174-11-6453 **Josh Carlson** (joshua.carlson@drake.edu), Williams College, and **Eva Goedhart** (eva.goedhart@williams.edu), Williams College. *Sequences of consecutive factoradic happy numbers*

Given a positive integer n, the factorial base representation of n is given by $n=\sum_{i=1}^{k} a_i\cdot i!$, where $a_k\neq 0$ and $0\leq a_i\leq i$ for all $1\leq i\leq k$. For $e\geq 1$, we define $S_{e,!}:\mathbb{Z}_{\geq 0}\to\mathbb{Z}_{\geq 0}$ by $S_{e,!}(0)=0$ and $S_{e,!}(n)=\sum_{i=0}^{n} a_i^e$, for $n\neq 0$. For $\ell\geq 0$, we let $S_{e,!}^{\ell}(n)$ denote the ℓ-th iteration of $S_{e,!}$, while $S_{e,!}^{0}(n)=n$. If $p\in\mathbb{Z}^+$ satisfies $S_{e,!}(p)=p$, then we say that p is an e-power factoradic fixed point of $S_{e,!}$. Moreover, given $x\in\mathbb{Z}^+$, if p is an e-power factoradic fixed point and if there exists $\ell\in\mathbb{Z}_{\geq 0}$ such that $S_{e,!}^{\ell}(x)=p$, then we say that x is an e-power factoradic p-happy number. Note an integer n is said to be an e-power factoradic happy number if it is an e-power factoradic 1-happy number. In this paper, we prove that all positive integers are 1-power factoradic happy and, for $2\leq e\leq 4$, we prove the existence of arbitrarily long sequences of e-power factoradic p-happy numbers. A curious result establishes that for any $e\geq 2$ the e-power factoradic fixed points of $S_{e,!}$ that are greater than 1, always appear in sets of consecutive pairs. Our last contribution, provides the smallest sequences of m consecutive e-power factoradic happy numbers for $2\leq e\leq 5$, for some values of m. (Received September 10, 2021)

1174-11-6466 **Shivajee Gupta*** (shvajee9137@gmail.com), IIT Gandhinagar. *A modular relation involving non-trivial zeros of the Dedekind zeta function, and the Generalized Riemann Hypothesis* Preliminary report.

We give a number field analogue of a result of Ramanujan, Hardy and Littlewood, thereby obtaining a modular relation involving the non-trivial zeros of the Dedekind zeta function. We also provide a Riesz-type criterion for the Generalized Riemann Hypothesis for $\zeta_{\mathbb{K}}(s)$. New elegant transformations are obtained when $\mathbb{K}$ is a quadratic extension, one of which involves the modified Bessel function of the second kind. (Received September 13, 2021)

1174-11-6887 **John M Voight*** (jvoight@gmail.com), Dartmouth, **Xavier Roulleau** (Xavier.Roulleau@univ-amu.fr), Angers University, and **Donald Cartwright** (donald.cartwright@gmail.com), University of Sydney. *A refinement of Bezout's Lemma and elements of order 3 in some rational quaternion algebras*

Given coprime $a,b\in\mathbb{Z}$, there exist $x,y\in\mathbb{Z}$ such that $ax-by=1$ by Bézout's lemma. We prove that, after possibly interchanging a,b, that we may choose x,y to be norms from the Eisenstein integers. We do this via the study of local embeddings of quadratic rings into certain orders in rational quaternion algebras. (Received September 10, 2021)

1174-11-6890 **Gonzalo Tornaria** (tornaria@cmat.edu.uy), Universidad de la Republica, and **Jeffery Hein** (sandamnit@gmail.com), Independent. *Computing classical modular forms as orthogonal modular forms*

Birch gave an extremely efficient algorithm to compute a certain subspace of classical modular forms using the Hecke action on classes of ternary quadratic forms. We extend this method to compute all forms of non-square level using the spinor norm, and we exhibit an implementation that is very fast in practice. (Received September 10, 2021)

1174-11-6897 **Cristina Maria Ballantine*** (cballant@holycross.edu), College of the Holy Cross. *Almost Partition Identities*

An almost partition identity is an identity for partition numbers that is true asymptotically 100% of the time and fails infinitely often. We prove a new kind of almost partition identity, namely the number of parts in all self-conjugate partitions of n is almost always equal to the number of partitions of n in which no odd part is repeated and there is exactly one even part (possibly repeated). Not only does the identity fail infinitely often, but also the error grows without bound. In addition, we prove several identities involving the number of parts in restricted partitions. (Received September 10, 2021)

1174-11-7010 **Christopher Jennings-Shaffer*** (chrisjenningsshaffer@gmail.com), University of Denver. *Some Rogers-Ramanujan Type Identities with False Theta Functions*

The Rogers-Ramanujan identities are two classical identities relating certain basic hypergeometric series with infinite products. Originally due to Rogers and rediscovered by Ramanujan, these identities have found their way into subjects including number theory, combinatorics, vertex operator algebras, statistical mechanics, and more. In every derivation of these identities, the infinite product comes from applying the Jacobi triple product identity to a classical theta function. Theta functions are well studied objects and of great importance due to their modular properties, and in simplest terms may be viewed as a sum over the integers of a variable raised to a quadratic term. False theta functions arise from a change in the sign of certain summands, and are not modular. While nearly as old as theta functions, false theta functions have attracted attention recently as they are appear almost as often as theta functions, but their properties are not well studied.

We review the history of classical Rogers-Ramanujan identities, theta functions, and false theta functions. We then give a few new Rogers-Ramanujan type identities where the "product side" is not a product, but a false theta function. These false theta functions arise as the characters of irreducible modules for the N=1 super-singlet superalgebra. The key ingredient in proving these new identities is Bailey's lemma. (Received September 11, 2021)

1174-11-7108 **Amanda Welch** (arwelch@eiu.edu), Eastern Illinois University. *Beck-type companion identities for Franklin's identity*

The original Beck conjecture, now a theorem due to Andrews, states that the difference in the number of parts in all partitions into odd parts and the number of parts in all strict partitions is equal to the number of partitions whose set of even parts has one element, and also to the number of partitions with exactly one part repeated. This is a companion identity to Euler's identity. The theorem has been generalized by Yang to a companion identity to Glaisher's identity. Franklin generalized Glaisher's identity, and in this article, we provide a Beck-type companion identity for Franklin's identity and prove it via a modular refinement. Andrews' and Yang's respective theorems fit naturally into this very general frame. (Received September 12, 2021)

1174-11-7122 **Ralf Schmidt*** (ralf.schmidt@unt.edu), University of North Texas, **Jennifer Johnson-Leung** (jenfns@uidaho.edu), University of Idaho, and **Brooks K. Roberts** (brooksr@uidaho.edu), University of Idaho. *Computing bad Euler factors for paramodular forms*

Paramodular forms are Siegel modular forms of degree 2 with respect to the paramodular group of some level N. They admit a theory of old- and newforms similar to the well-known Atkin-Lehner theory for elliptic modular forms. To every cuspidal paramodular newform f there is attached a spin (degree-4) L-function $L(s,f)$, which admits an analytic continuation and functional equation.

For a prime p not dividing the level N the local Euler factors $L_p(s,f)$ can be determined from the Fourier expansion of f, provided enough Fourier coefficients are known. In this talk we will explain how to also obtain the local Euler factors $L_p(s,f)$ from the Fourier coefficients for primes p dividing N, in a way that can easily be programmed into a computer.

Our method is based on the theory of stable Klingen vectors for irreducible, admissible representations of GSp$(4, F)$ over a p-adic field F. This is joint work with Jennifer Johnson-Leung and Brooks Roberts. (Received September 12, 2021)

1174-11-7159 **Mircea Merca*** (mircea.merca@profinfo.edu.ro), University of Craiova, Romania. *On a function involving the cubic partitions* Preliminary report.

The partitions in which even parts come in two colours are known as cubic partitions. We introduce and investigate the cubic partition function $A(n)$ which is defined as the difference between the number of cubic partitions of n into an even numbers of parts and the number of cubic partitions of n into an odd numbers of parts. We present a collection of identities relating $A(n)$ and provide analytic proofs. Among these identities we remark two Ramanujan-like congruences: for all $n \geqslant 0$, $A(9n+5) \equiv 0 \pmod 3$ and $A(27n+26) \equiv 0 \pmod 3$. Combinatorial interpretations for $A(n)$ are given in terms of the ordinary partitions into parts congruent to $\pm 1, 4 \pmod 8$ when n is even and congruent to $\pm 3, 4 \pmod 8$ when n is odd. In this context, some infinite families of linear inequalities involving $A(n)$ are proposed as open problems. (Received September 13, 2021)

1174-11-7196 **Pietro Mercuri*** (mercuri.ptr@gmail.com), Università Sapienza di Roma. *Jagged partitions and Rogers-Ramanujan type identities*

We give a definition of k-jagged partitions, where k is a positive integer, and we explain how to use them to find analytical identities of Rogers-Ramanujan type for some given partition identities. In particular we focus on a bijection given by Alladi, Andrews and Gordon. (Received September 13, 2021)

1174-11-7197 **Martin H Weissman*** (weissman.marty@gmail.com), University of California, Santa Cruz. *Visualizing Primes*

We study prime numbers through a mix of experiment, analysis, and proof. Landmark theorems and conjectures capture the distribution of prime numbers. But these results are difficult for outsiders to understand, being couched in subtle asymptotics, notations with big O's and little o's, error terms with dependent constants, etc. In this talk, I will convey stories about prime numbers, using tools of data visualization rather than formulas throughout. (Received September 13, 2021)

1174-11-7270 **Madeline Locus Dawsey*** (mdawsey@uttyler.edu), University of Texas At Tyler, **Robert Peter Schneider** (robert.schneider@uga.edu), University of Georgia, and **Matthew Ronald Just** (justmatt@uga.edu), Emory University. *A new partition statistic and applications* Preliminary report.

Partition theory has traditionally revolved around the size statistic for partitions and the partition function $p(n)$, which counts the number of partitions of size n. In recent work with Just, Schneider, and Sharp, I have explored a new perspective on partitions that focuses more on multiplicative aspects of partitions than their classical additive structure. We define a new, multiplicative partition statistic called the "supernorm" which shifts the focus of study from partitions themselves to prime factorizations of natural numbers based on the parts of partitions. We give several applications of the supernorm to partition bijections, density theorems, and Hurwitz class numbers. (Received September 13, 2021)

1174-11-7283 **Marc Chamberland*** (chamberl@grinnell.edu), Grinnell College. *Asymptotics of Sums over Primes and the Riemann Zeta Function* Preliminary report.

The classical result of Mertens, enhanced by the Prime Number Theorem, states that

$$\sum_{p \leq n} \frac{\log p}{p} = \log(n) + C + o(1)$$

as $n \to \infty$, where the sum is over the primes p and the constant C can be specified. In this talk, we generalize this formula to expressions of the form

$$\sum_{p \leq n} \frac{(\log p)^k}{p^s}$$

where k is a non-negative integer and s is real. For the case when $s < 1$, the formulas are conditioned on information concerning the zeros of the Riemann zeta function. (Received September 14, 2021)

1174-11-7284 **Frank G. Garvan** (fgarvan@ufl.edu), University of Florida, and **John W Streese*** (jstreese@ufl.edu), Univ of Florida. *Rank and Crank partition congruences and quasi-modular forms.* Preliminary report.

We consider problems of congruences for the rank and crank of partitions, using rank and crank moments. We briefly cover background material on modular forms and then improve results of Bringmann, Garvan, and Mahlburg by reducing the bound on the weights of the Rank moments modulo ℓ, for $\ell > 3$. We further explore what relationships can be drawn between modular function weights and these rank moment congruences. (Received September 13, 2021)

1174-11-7307 **Robert Osburn** (robert.osburn@ucd.ie), University College Dublin, Ireland. *Quantum modularity of partial theta series with periodic coefficients*

Theta series first appeared in Euler's work on partitions, but was systematically studied later by Jacobi. In his Lost Notebook, Ramanujan wrote down many identities (without proof) involving the so-called partial theta series. Unlike the theta series which are modular forms, the theory of partial theta series is not well understood. In this talk, I will consider a family of partial theta series and show their "quantum modular" behaviour. This is based on my recent joint work with Robert Osburn (UCD). The talk should be accessible to graduate and advanced undergraduate students. (Received September 14, 2021)

1174-11-7517 **Andreas Weingartner*** (weingartner@suu.edu), Southern Utah University. *Somewhat smooth numbers in short intervals*

We use exponent pairs to establish the existence of many x^a-smooth numbers in short intervals $[x-x^b,x]$, when $a>1/2$. In particular, $b=1-a-a(1-a)^3$ is admissible. Assuming the exponent-pairs conjecture, one can take $b=(1-a)/2+\epsilon$. As an application, we show that $[x-x^{0.4872},x]$ contains many practical numbers when x is large. (Received September 14, 2021)

1174-11-7620 **Joshua Harrington*** (jsharrin@cedarcrest.edu), Cedar Crest College, and **Andrew Vincent** (andrew@centrodesolutions.com), Centrode Solutions. *Super totient and exceptional totient numbers*

Let n be a positive integer, let $R(n)=\{a : 1\le a\le n \text{ and } \gcd(a,n)=1\}$, and let $R_e(n)=\{a : 1\le a\le n \text{ and } \gcd(a,n)=\gcd(a-1,n)=1\}$. We call n a *super totient number if $R(n)$ can be partitioned into two sets of equal sum and we call n an exceptional totient number of $R_e(n)$ can be partitioned into two sets of equal sum. In this talk, we provide a full classification of super totient numbers and exceptional totient numbers. We further explore when $R(n)$ and $R_e(n)$ can be partitioned into k sets of equal sum.*

This talk discusses work recently conducted with undergraduate students at Cedar Crest College. *(Received September 15, 2021)*

1174-11-7654 **Maria De Los Angeles Chara** (charamaria@gmail.com), Universidad Nacional del Litoral, and **Ernest Guico** (ernest.guico@gmail.com), Independent. *Our current state of affairs: Locally Recoverable Codes*

An error correcting code is a way of encoding information for storage or transmission while incorporating redundancy so that if any information is lost, it can be recovered from what remains. Cloud storage applications motivated the invention of Locally Recoverable Codes (LRC), where any codeword symbol can be recovered from a small collection of other symbols in the codeword. This talk will introduce a class of LRCs built from maps between curves over finite fields and give examples of these in action. This talk is meant to be a dialogue and is meant to invite number theorists to start thinking about LRCs. (Received September 15, 2021)

1174-11-7658 **Trevor Hyde** (tghyde@uchicago.edu), University of Chicago, **Zoë Bell** (zbell@g.hmc.edu), Harvey Mudd College, **Jasmine Camero** (jasmine.camero@emory.edu), Emory University, **Fiona Lu** (clu@csusm.edu), California State University, San Marcos, **Rebecca Miller** (r.lauren.miller@slu.edu), Saint Louis University, and **Eric Zhu** (eric_zhu1@brown.edu), Georgia Institute of Technology, Brown University. *Proportion of Periodic Points for Lattès maps over Finite Fields*

A Lattès map can be created from looking at the multiplication-by-d map of an elliptic curve E defined over a finite field $\mathbb{F}_q$. This talk will focus on discussing the proportion of periodic points for a Lattès map in $\mathbb{P}^1(\mathbb{F}_q)$ and the techniques used to find it. In particular, this talk will focus on supersingular elliptic curves and discuss the nice properties these elliptic curves have over $\mathbb{F}_q$ that allow us to find formulas for the proportions of periodic points for the associated Lattès. Please feel invited to ask questions if something doesn't make sense. (Received September 15, 2021)

1174-11-7694 **Alex Iosevich*** (iosevich@math.rochester.edu), University of Rochester. *Vapnik-Chervonenkis dimension and point configurations in vector spaces over finite fields*

Let X be a set and $\mathcal{H}$ a collection of functions from X to $\{0,1\}$. We say that $\mathcal{H}$ shatters a finite set $C\subset X$ if the restriction of $\mathcal{H}$ yields every possible function from C to $\{0,1\}$. The VC-dimension of $\mathcal{H}$ is the largest number d such that there exists a set of size d shattered by $\mathcal{H}$, and no set of size $d+1$ is shattered by $\mathcal{H}$. Vapnik and Chervonenkis introduced this idea in the early 70s in the context of learning theory, and this idea has also had a significant impact on other areas of mathematics. In this paper we study the VC-dimension of a class of functions $\mathcal{H}$ defined on $\mathbb{F}_q^d$, the d-dimensional vector space over the finite field with q elements. Define

$$\mathcal{H}_t^d = \{h_y(x) : y \in \mathbb{F}_q^d\},$$

where for $x \in \mathbb{F}_q^d$, $h_y(x) = 1$ if $||x-y|| = t$, and 0 otherwise, where here, and throughout, $||x|| = x_1^2+x_2^2+\cdots+x_d^2$. Here $t \in \mathbb{F}_q$, $t \neq 0$. Define $\mathcal{H}_t^d(E)$ the same way with respect to $E \subset \mathbb{F}_q^d$. The learning task here is to find a sphere of radius t centered at some point $y \in E$ unknown to the learner. The learning process consists of taking random samples of elements of E of sufficiently large size.

We are going to prove that when $d = 2$, and $|E| \geq Cq^{\frac{15}{8}}$, the VC-dimension of $\mathcal{H}_t^2(E)$ is equal to 3. (Received September 15, 2021)

1174-11-7724 **Shreejit Bandyopadhyay*** (sxb437@psu.edu), Pennsylvania State University. *Andrews-Beck type Congruences Related to the Crank of a Partition* Preliminary report.

George Beck recently made certain conjectures related to the ranks and cranks of partitions. The conjectures related to the rank were proved by Andrews by using a suitable generating function and subsequently invoking results due to Atkin and Swinnerton-Dyer. In this talk, we revisit the conjectures on the crank by decomposing the relevant crank generating function. We further explore connections with certain identities involving generalized Lambert series and Apple-Lerch sums. (Received September 15, 2021)

1174-11-7767 **Aidan Kelley*** (aidankelley@wustl.edu), Washington University in St. Louis. *Gauss-like Exponential Sums from Whitaker Coefficients* Preliminary report.

We study multi-variate exponential sums that arise in Brubaker and Friedberg's study of Whitaker coefficients for maximal parabolic Eisenstein series on metaplectic covers of split reductive groups. Specifically, we study the case of the maximal parabolic coming from removing the second root in the Dynkin diagram A_n of GL_{n+1}. We show under lax conditions that these sums factor as a product of quadratic Gauss sums. In the small remaining conditions, we compute their values explicitly. In the $n = 5$ case, we show how our results relate to a conjecture by Chinta about the equivalence of two Dirichlet series associated to A_5. (Received September 21, 2021)

1174-11-7813 **Amita Malik*** (amita.malik@aimath.org), Max Planck Institute, Bonn. *Partitions into primes in arithmetic progressions*

In this talk, we discuss the asymptotic behavior of the number of ways to write a given positive integer as a sum of primes concerning a Chebotarev condition. In special cases, this reduces to the study of partitions into primes in arithmetic progressions. While this study for ordinary partitions goes back to Hardy and Ramanujan, partitions into primes were recently re-visited by Vaughan. Our error term is sharp and improves on previous known estimates in the special case of primes as parts of the partition. As an application, monotonicity of this partition function is established explicitly via an asymptotic formula in connection to a result of Bateman and Erdős. (Received September 16, 2021)

1174-11-7824 **Jim L. Brown*** (jimlb@oxy.edu), Occidental College. *Eigenform products of Siegel modular forms with nontrivial level*

Given two eigenforms, it is a natural question to ask if the product of the eigenforms is again an eigenform. In the case of elliptic modular forms this was answered in the full level case by Duke and Ghate and in the general level case by Johnson. In this talk we will discuss this problem for genus two Siegel modular forms in the full level and general congruence level cases. This is work that was conducted with two separate groups of REU students. (Received September 16, 2021)

1174-11-7826 **Alia Hamieh*** (alia.hamieh@unbc.ca), University of Northern British Columbia, and **Nathan Ng** (nathan.ng@uleth.ca), University of Lethbridge. *Mean Values of Long Dirichlet Polynomials with Higher Divisor Coefficients* Preliminary report.

In this talk, I report on joint work with Nathan Ng. Assuming a conjectural formula for a certain family of additive divisor sums, we establish an asymptotic formula for mean values of long Dirichlet polynomials with higher order shifted divisor functions as coefficients. This proves a conjecture of Coney and Keating under the assumption of an additive divisor conjecture. As a consequence, we prove a special case of a conjecture of Conrey and Gonek where the additive divisor conjecture is known. (Received September 16, 2021)

1174-11-7851 **Dan Fretwell** (daniel.fretwell@bristol.ac.uk), University of Bristol, **Eran Assaf*** (assaferan@gmail.com), Dartmouth, **Adam Logan** (adam.m.logan@gmail.com), Tutte Institute for Mathematics and Computation, Carleton University, Tutte Institute for Mathematics and Computation, Carleton University, **Colin Ingalls** (colin.ingalls@gmail.com), Carleton University, and **Spencer Secord** (spencer.e.secord@gmail.com), Carleton University. *Quaternary Quadratic Forms, Theta Lifts and Siegel modular forms* Preliminary report.

Orthogonal modular forms are algebraic modular forms arising from lattices in quadratic spaces. We study this space for anisotropic quaternary quadratic spaces over a totally real number field F. By studying the image of the even Clifford functor, which is a faithful functor to the category of orders in quaternion algebras, it is possible to define a half-Clifford functor, and characterize its image. We show that this functor preserves the Hecke-module structure between these two categories, and by applying Jacquet-Langlands correspondence, we realize it as the subspace of Siegel modular forms which are theta lifts of classical and Hilbert modular forms. We conclude with applications to non-vanishing results of various theta lifts. (Received September 17, 2021)

1174-11-7856 **Andrew V. Sutherland** (drew@math.mit.edu), MIT, **Andrew R Booker** (andrew.booker@bristol.ac.uk), University of Bristol, **Jeroen Sijsling** (jeroen.sijsling@uni-ulm.de), Universitat Ulm, and **Dan Yasaki** (d_yasaki@uncg.edu), University of North Carolina, Greensboro. *Sato-Tate groups and modularity for atypical abelian surfaces*

We discuss in detail what it means for an abelian surface A over a number field to be modular, organizing conjectures and theorems that associate to A a modular form with matching L-function. The explicit description of this modular form depends on the real Galois endomorphism type of A, or equivalently on its Sato–Tate group. For A defined over the rational numbers, this description can involve classical, Bianchi, or Hilbert modular forms; and for each possibility, we provide a genus 2 curve with small conductor from which it arises. (Received September 16, 2021)

1174-11-7905 **Andrew G. Earnest*** (aearnest@siu.edu), Southern Illinois University Carbondale. *Integers represented by ternary quadratic forms*

This talk will focus on the classical problem of representing integers by integral ternary quadratic forms. The emphasis here will be on positive definite forms, although some of the results to be described apply also to the case of indefinite forms. Some recent results and open problems will be presented, and the status of several long-standing conjectures will be reviewed. (Received September 16, 2021)

1174-11-7993 **Christina Roehrig*** (croehrig@math.uni-koeln.de), University of Cologne. *Siegel theta series for quadratic forms of signature* $(m-1,1)$ Preliminary report.

Studying theta series for indefinite quadratic forms provides numerous examples of elliptic and Siegel modular forms. However, these examples are in general non-holomorphic. In the special case where we consider quadratic forms of signature $(m-1,1)$, we can follow a result by Livinsky in order to generalize the construction of elliptic modular forms by Kudla and Millson to higher genus $n \in \mathbb{N}$. In this talk, we outline this particular construction of non-holomorphic modular Siegel theta series and describe the holomorphic part of these series. This proves to be an analogue to Zwegers' result on elliptic theta series. (Received September 17, 2021)

1174-11-8001 **Bianca Thompson** (bthompson@westminstercollege.edu), Westminster College, **Beth Malmskog** (bmalmskog@coloradocollege.edu), Colorado College, and **Mckenzie West*** (westmr@uwec.edu), University of Wisconsin-Eau Claire. *Using Curves to Store Information* Preliminary report.

Error correcting codes are used to store information efficiently while still allowing for recovery in the case of partial loss. Recently work has begun to construct codes using the algebraic geometric properties of curves over finite fields. In this talk, we'll introduce the general construction of codes from curves and provide a few explicit examples of locally recoverable codes created using maps between curves over finite fields. This is a preliminary report of work started at the inaugural Rethinking Number Theory workshop. (Received September 17, 2021)

1174-11-8048 **Juanita Duque-Rosero*** (Juanita.duque.rosero.gr@dartmouth.edu), Dartmouth College. *Enumerating triangular modular curves of small genus*

Triangular modular curves are a generalization of modular curves that arise from quotients of the upper half-plane by congruence subgroups of hyperbolic triangle groups. These curves arise from Belyi maps with monodromy $\mathrm{PGL}_2(\mathbb{F}_q)$ or $\mathrm{PSL}_2(\mathbb{F}_q)$. In this talk, we will present a computational approach to enumerate all triangular modular curves of genus 0, 1, and 2. (Received September 17, 2021)

1174-11-8061 **Ali Kemal Uncu*** (aku21@bath.ac.uk), Austrian Academy of Sciences (RICAM), University of Bath, and **Wadim Zudilin** (w.zudilin@math.ru.nl), Radboud University. *Reflecting (on) the modulo 9 Kanade-Russell (conjectural) identities*

We examine complexity and versatility of five modulo 9 Kanade–Russell identities through their finite (aka polynomial) versions and images under the $q \mapsto 1/q$ reflection. (Received September 17, 2021)

1174-11-8134 **Benjamin Martin Baily** (bmb2@williams.edu), Williams College, **Justine Dell** (jdell@haverford.edu), Haverford College, **Alicia G. Smith Reina** (ags6@williams.edu), William College, **Yingzi Yang** (yyingzi@umich.edu), University of Michigan, **Rene Isaac Mijares** (RIM1@WILLIAMS.EDU), Williams College, and **Irfan Durmic** (id5@williams.edu), Williams College. *The Bergman Game*

Every positive integer may be written uniquely as a base-φ decomposition–that is a sum of powers of φ, the golden mean, which includes no two adjacent powers. Guided by earlier work on a two-player game which produces the Zeckendorf Decomposition of an integer, we define a related game played on an infinite tuple of non-negative integers which decomposes a positive integer into its base-φ expansion. We call this game the Bergman Game. We prove that the longest possible Bergman game on an initial state S with n summands terminates in $\Theta(n^2)$ time, and we also prove that the shortest possible Bergman game on an initial state terminates in $\Theta(n)$ time. We also show a linear bound on the maximum length of the tuple used throughout the game. (Received September 17, 2021)

1174-11-8179 **Gaurav Bhatnagar*** (bhatnagarg@gmail.com), Ashoka University. *An elementary proof and extensions of Ramanujan's congruences $p(5n+4) \equiv 0$ and $\tau(5n+5) \equiv 0$ (mod 5).*

We give an elementary proof of Ramanujan's famous congruences $p(5n+4) \equiv 0$ (mod 5) and $\tau(5n+5) \equiv 0$ (mod 5). The proof requires no more than what Euler and Jacobi knew. The proof extends to embed the congruences into 4 infinite families of congruences for rational powers of the eta function. Many congruences of this nature have been found recently by Chan and Wang (2019); seven of their assertions are covered in our list. The same classical techniques give some further curious congruences for the sum of divisors function. These congruences follow from recurrence relations of the kind Euler gave for the partition function.

This is joint work with Hartosh Singh Bal. (Received September 17, 2021)

1174-11-8185 **Chi-Yun Hsu*** (cyhsu@math.ucla.edu), University of California, Los Angeles, UCLA, **Boya Wen** (bwen25@wisc.edu), University of Wisconsin-Madison, **Isabella Negrini** (isabella.negrini@mail.mcgill.ca), McGill University, and **Hannah Burson** (hburson@umn.edu), University of Minnesota. *Beck type identities related to certain mock theta functions*

Euler's partition identity states that the number of partitions of a positive integer n into odd positive integers equals the number of partitions of n into distinct positive integers. Recently, Beck conjectured and Andrews proved the following related identity: the difference between the number of parts in all partitions of n into odd parts and the number of parts in all partitions of n into distinct parts is the number of partitions of n with exactly one part repeated. There are many partition identities arising from modular forms, related to which one can ask for a Beck-type identity. In a joint work with Ballantine, Burson, Folsom, Negrini, and Wen, we establish Beck-type identities arising from the mock theta functions $\omega(q)$, $\mu(q)$ and $\phi(q)$. This work is part of the Women in Number 5 program. (Received September 18, 2021)

1174-11-8204 **Jonathan Love*** (jon.love@mcgill.ca), CRM Montreal. *Computing cusp forms over function fields* Preliminary report.

There is a vast literature and set of computational tools available for modular forms over number fields, but the function field case is comparatively less well understood, and far fewer examples have been generated. In this talk, I will summarize an algorithm that can be used to compute the space of everywhere unramified cusp forms over the function field of a curve X over $\mathbb{F}_p$. We will focus on one of the key ingredients in this algorithm, which is a method to test for isomorphism between rank 2 vector bundles on X. (Received September 18, 2021)

1174-11-8213 **Monika Yadav*** (monikay@iiitd.ac.in), Indraprastha Institute of Information Technology Delhi, New Delhi, India. *Mass formulae for Euclidean self-orthogonal and self-dual codes over finite commutative chain rings*

Self-orthogonal and self-dual codes constitute the two most important and well-studied classes of linear codes having rich algebraic structures. These two classes of linear codes have nice connections with the theory of modular forms, unimodular lattices and the theory of designs. In 1990's, Calderbank and Nechaev viewed many binary non-linear codes as Gray images of linear codes over the ring $\mathbb{Z}_4$ of integers modulo 4. The problem of counting all self-orthogonal and self-dual codes over finite commutative chain rings has attracted a lot of attention. In this talk, we will obtain explicit mass formulae for all Euclidean self-orthogonal and self-dual codes of an arbitrary length over finite commutative chain rings of odd characteristic. We will also derive necessary and sufficient conditions for a linear code of an arbitrary length over a finite commutative chain ring to be Euclidean self-orthogonal and to be Euclidean self-dual. We will also provide a recursive method to construct a Euclidean self-orthogonal (resp. self-dual) code of the type $\{k_1, k_2, \cdots, k_e\}$ and length n over a finite commutative chain ring with the nilpotency index e from a Euclidean self-orthogonal (resp. self-dual) code of the type $\{k_1 + k_2, k_3, \cdots, k_{e-1}\}$ and of the same length n over a finite commutative chain ring with the nilpotency index $e - 2$ and vice versa, where $e \geq 4$ is an integer and k_1, k_2, $\cdots$, k_e are non-negative integers satisfying $k_1 + k_2 + \cdots + k_e \leq n$. (Received September 18, 2021)

1174-11-8214 **Sandeep Sharma*** (sandeeps@iiitd.ac.in), Indraprastha Institute of Information Technology Delhi, New Delhi, India. *Multi-twisted additive codes over finite fields*

Additive codes over the finite field $\mathbb{F}_4$ were introduced and studied by Calderbank et al. (1998) as a natural generalization of linear codes. Later, Rains (1999) and Bierbrauer and Edel (2000) defined and studied additive codes over arbitrary finite fields. These codes not only constitute an important family of error-correcting codes, but are also useful in the construction of quantum error-correcting codes. In this talk, we will introduce a new class of additive codes over finite fields, viz. multi-twisted (MT) additive codes. We will study their algebraic structures by writing a canonical form decomposition for these codes and provide an enumeration formula for these codes. By placing ordinary, Hermitian and $*$ trace bilinear forms, we will further study their dual codes and derive necessary and sufficient conditions under which a MT additive code is self-dual and self-orthogonal. We will also derive a necessary and sufficient condition for the existence of a self-dual MT additive code over a finite field, and provide explicit enumeration formulae for self-dual and self-orthogonal MT additive codes over finite fields with respect to the aforementioned trace bilinear forms. We will also identify and list several good codes within the family of MT additive codes over finite fields. We will also discuss some interesting open problems in this direction. (Received September 18, 2021)

1174-11-8269 **Lisa Kaylor*** (kaylorlm@alma.edu), Alma College. *Quaternary Lattices of Discriminant 4p* Preliminary report.

Let L be a positive definite even quaternary integral lattice. It was shown by Hsia and Hung that the degree 2 theta series corresponding to the classes of such lattices with discriminant $p \equiv 1 \bmod 4$ and minimum 2 are linearly independent. We consider these lattices with $dL = 4p$ where $p > 13$ is a prime congruent to 3 mod 4. There are two genera of lattices in this case, which are considered separately. We show that the degree 2 theta series of the classes with nontrivial orthogonal group are linearly independent within each genus. (Received September 18, 2021)

1174-11-8320 **Padmavathi Srinivasan*** (padma.0289@gmail.com), University of Georgia. *Grothendieck's section conjecture at the boundary of moduli space*

Grothendieck's section conjecture predicts that rational points on curves over arithmetically interesting fields (e.g. number fields) are in bijection with purely group-theoretic data (viz. sections to an exact sequence) constructed from the fundamental group of the curve. The difficulty in proving this conjecture is due to the enormous size of the groups in question, so examples where this conjecture holds are sparse. In this talk, I will describe joint work with Wanlin Li, Daniel Litt and Nick Salter, where we give a new way of showing many examples where this conjecture holds exist, by showing no sections exist, and hence no rational points. The key idea is a degeneration argument—we study group-theoretic obstructions to having sections in families of smooth curves, and study the behaviour of these obstructions close to a point where the curves degenerate and acquire singularities. (Received September 18, 2021)

1174-11-8412 **Shaoyun Yi** (shaoyun@mailbox.sc.edu), University of South Carolina. *Dimensions for the spaces of Siegel cusp forms of Klingen level 4*

Many mathematicians have studied dimension and codimension formulas for the spaces of Siegel cusp forms of degree 2. The dimensions of the spaces of Siegel cusp forms of non-squarefree levels are mostly not available in the literature. This talk will present new dimension formulas of Siegel cusp forms of degree 2, weight k, and level 4 for two congruence subgroups. Our method relies on counting a particular set of cuspidal automorphic representations of GSp(4) and exploring its connection to dimensions of spaces of Siegel cusp forms of degree 2. This work is joint with Ralf Schmidt and Shaoyun Yi. (Received September 18, 2021)

1174-11-8416 **Alexander J Barrios** (abarrios@carleton.edu), Carleton College. *Tamagawa numbers for rational elliptic curves with non-trivial torsion*

Suppose $E/\mathbb{Q}$ be an elliptic curve with a non-trivial torsion point, and c is the global Tamagawa number of E. Motivated by the work of Lorenzini, we study the quotient $\frac{c}{|E(\mathbb{Q})_{\text{tors}}|}$ which appears as a factor in the leading term of the L-function of $E/\mathbb{Q}$ in the conjecture of Birch and Swinnerton-Dyer. Lorenzini showed that there are finitely many elliptic curves with a torsion point of order at least 4 and global Tamagawa number 1. In joint work with Alexander Barrios, we found all rational elliptic curves with a 2-torsion and 3-torsion point, which have a global Tamagawa number equal to 1. With Lorenzini's work, this result classifies all the rational elliptic curves with global Tamagawa number equal to 1 and a non-trivial torsion point. We obtain our results by considering a parametrized families of elliptic curves with a non-trivial torsion subgroup and computing the local Tamagawa numbers in terms of the parameters. (Received September 18, 2021)

1174-11-8425 **Aaron J Pollack** (apollack@ucsd.edu), University of California, San Diego, **Fatma Çiçek** (fcicek@iitgn.ac.in), Indian Institute of Technology Gandhinagar, **Giuliana Davidoff** (gdavidof@mtholyoke.edu), Mount Holyoke College, **Sarah Dijols** (sarah.dijols@hotmail.fr), Yau Center, Tsinghua University, and **Trajan Hammonds** (trajanh@princeton.edu), Princeton University. *The functional equation for completed standard L-function of modular forms on G_2*

Gan, Gross, and Savin introduced modular forms on the split exceptional group $G_2/\mathbb{Q}$. Suppose π is a cuspidal automorphic representation of G_2 corresponding to a level one, even weight modular form φ on G_2. We consider an associated completed standard L-function, $\Lambda(\pi, Std, s)$. Using refined analysis of the Rankin integral due to Gurevich and Segal, we obtain the functional equation $\Lambda(\pi, Std, s) = \Lambda(\pi, Std, 1-s)$ assuming some conditions on Fourier coefficients of φ. This work is joint with Fatma Çiçek, Giuliana Davidoff, Sarah Dijols, Trajan Hammonds, and Aaron Pollack. It was initiated as part of the Rethinking Number Theory 2020 workshop. (Received September 19, 2021)

1174-11-8431 **Maxie Dion Schmidt*** (maxieds@gmail.com), Georgia Institute of Technology. *New characterizations of partial sums of the Möbius function*

The Mertens function, $M(x) := \sum_{n\leq x}\mu(n)$, is defined as the summatory function of the classical Möbius function for $x \geq 1$. The inverse function $g^{-1}(n) := (\omega+1)^{-1}(n)$ taken with respect to Dirichlet convolution is defined in terms of the strongly additive function $\omega(n)$ that counts the number of distinct prime factors of the integers $n \geq 2$ without multiplicity. For large x and $n \leq x$, we associate a natural combinatorial significance to the magnitude of the distinct values of $g^{-1}(n)$ that depends directly on the exponent patterns in the prime factorizations of the integers in $\{2, 3, \ldots, x\}$ viewed as multisets. We have an Erdős-Kac theorem analog for the distribution of the unsigned sequence $|g^{-1}(n)|$ over $n \leq x$ as $x \to \infty$. The key connection of the partial sums of the auxiliary function $C_{\Omega(n)}(n) := (\Omega(n))! \times \prod_{p^\alpha || n}(\alpha!)^{-1}$ to $|g^{-1}(n)|$ is proved using assumptions on the independence of the completely additive function $\Omega(n)$ and the distribution of the exponents of the distinct prime factors of $2 \leq n \leq x$ when x is large. Discrete convolutions of the summatory function $G^{-1}(x) := \sum_{n\leq x}\lambda(n)|g^{-1}(n)|$ with the prime counting function $\pi(x)$ determine exact formulas and new characterizations of asymptotic approaches to $M(x)$. (Received September 19, 2021)

1174-11-8441 **Alexandra Hoey*** (ahoey@mit.edu), Massachusetts Institute of Technology, **Jonas Iskander** (jonasiskander@college.harvard.edu), Harvard University, **Fernando Trejos Suarez** (fernando.trejos@yale.edu), Yale University, and **Steven Jin** (stevenjin118@gmail.com), University of Maryland. *An unconditional explicit bound on the error term in the Sato-Tate conjecture*

Let $f(z) = \sum_{n=1}^{\infty} a_f(n)q^n$ be a holomorphic cuspidal newform with even integral weight $k \geq 2$, level N, trivial nebentypus, and no complex multiplication (CM). For all primes p, we may define $\theta_p \in [0, \pi]$ such that $a_f(p) = 2p^{(k-1)/2}\cos\theta_p$. The Sato-Tate conjecture states that the angles θ_p are equidistributed with respect to the probability measure $\mu_{\text{ST}}(I) = \frac{2}{\pi}\int_I \sin^2\theta\, d\theta$, where $I \subseteq [0, \pi]$. Using recent results on the automorphy of symmetric-power L-functions due to Newton and Thorne, we construct the first unconditional explicit bound on the error term in the Sato-Tate conjecture, which applies when N is squarefree as well as when f corresponds to an elliptic curve with arbitrary conductor. In particular, if $\pi_{f,I}(x) := \#\{p \leq x : p \nmid N, \theta_p \in I\}$, and $\pi(x) := \#\{p \leq x\}$, we show the following bound:

$$\left|\frac{\pi_{f,I}(x)}{\pi(x)} - \mu_{\text{ST}}(I)\right| \leq 58.1\frac{\log((k-1)N\log x)}{\sqrt{\log x}} \qquad \text{for} \quad x \geq 3.$$

As an application, we give an explicit bound for the number of primes up to x that violate the Atkin-Serre conjecture for f. This research was conducted at the University of Virginia REU. (Received September 19, 2021)

1174-11-8471 **Akshat Mudgal*** (am16393@bristol.ac.uk), University of Oxford. *On solutions to additive equations in sets of lattice points on spheres*

Given $d, m \in \mathbb{N}$, we are interested in studying additive equations over the set of lattice points

$$S_{d,m} = \{(x_1, \dots, x_d) \in \mathbb{Z}^d \mid x_1^2 + \dots + x_d^2 = m\}.$$

Thus, given a subset A of $S_{d,m}$, we wish to estimate the number of solutions $E(A)$ to the equation

$$x + y = z + w,$$

such that $x, y, z, w \in A$. The problem of obtaining such bounds is closely related to various other questions in analytic number theory and harmonic analysis, including the discrete restriction problem for lattice points on spheres. In this talk, we will focus on the case when $d = 4$, where the previous best known bound was a result by Bourgain–Demeter, who showed that $E(A) \ll_\epsilon m^\epsilon |A|^{2+1/3}$. We improve upon this by proving that

$$E(A) \ll_\epsilon m^\epsilon |A|^{2+1/3-c},$$

for some $c > 0$. This involves applications of incidence geometric techniques combined with the higher energy method from arithmetic combinatorics. (Received September 19, 2021)

1174-11-8496 **Bruce W McOsker*** (mcoskerbruce@gmail.com), Texas A&M University - San Antonio. *A New Proof of Legendre's Theorem*

As part of our research into the representation theory of quadratic forms, we take a look at the famous Legendre's Theorem in number theory regarding ternary quadratic forms. Namely, the Legendre equation $ax^2+by^2+cz^2 = 0$, with a, b and c being nonzero, square-free and pairwise coprime integers, has non-trivial integer solutions if and only if the following criteria are met: (1) a, b and c do not all have the same sign; (2) $-bc$, $-ac$ and $-ab$ are quadratic residues of a, b and c, respectively.

As the non-trivial solutions of Legendre equation are closely related to the non-trivial rational points on a rational conic, this result has piqued the interest of many mathematicians for centuries, including Gauss, who either reproved it using new techniques, or found efficient algorithms of having solutions within small bounds. In our talk, we give an alternate proof of this theorem using Local-Global Principle, Hasse invariant, and Jacobi symbol. (Received September 19, 2021)

1174-11-8506 **Benjamin Breen*** (Benjamin.k.breen.gr@dartmouth.edu), Clemson. *Computing Hilbert modular forms via a trace formula*

The Jacquet-Langlands correspondence relates spaces of Hilbert modular forms to spaces of quaternionic modular forms; the latter being far more amenable to computations. We present a magma implementation of a trace formula based on quaternionic modular forms which allows us to compute the Fourier coefficients of Hilbert modular forms. We conclude with computations and a comparison between the different methods for producing Hilbert modular forms. (Received September 19, 2021)

1174-11-8602 **Eleanor McSpirit*** (egm3zq@virginia.edu), University of Virginia, and **Kristen Scheckelhoff** (qpz4ex@virginia.edu), University of Virginia. *On the Number of 2-Hooks and 3-Hooks of Integer Partitions*

Let $p_t(a, b; n)$ denote the number of partitions of n such that the number of t hooks is congruent to a mod b. For $t \in \{2, 3\}$, arithmetic progressions r_1 mod m_1 and r_2 mod m_2 on which $p_t(r_1, m_1; m_2n + r_2)$ vanishes were established in recent work by Bringmann, Craig, Males, and Ono using the theory of modular forms. Here we offer a direct combinatorial proof of this result using abaci and the theory of t-cores and t-quotients. (Received September 19, 2021)

1174-11-8609 **Samuel Mundy*** (mundy@math.columbia.edu), Princeton University. *Eisenstein series and automorphic cohomology: The case of G_2, Part 1*

Appearing in the cohomology of the locally symmetric spaces attached to a reductive group G are automorphic representations which are often interesting from an arithmetic point of view. Given a specific automorphic representation π of G, it is an interesting question to ask how many times and in what degrees π may appear in this cohomology. When π arises from Eisenstein series, this can be a tricky problem whose solution depends on many interesting phenomena in the world of automorphic forms. The purpose of this first talk is to discuss this problem, its motivation, and the automorphic phenomena on which it depends. (Received September 19, 2021)

1174-11-8653 **Margaret Kepner*** (renpek1010@yahoo.com), Independent Artist. *Packing Circles, Packing Squares* Preliminary report.

Shape-packing has the objective of fitting a specified group of shapes into a larger envelope shape in the most efficient way. One example is packing a number of circles into a larger circle with a minimum amount of residual space left over. Similarly, the packing of squares within a larger square, subject to certain constraints, can generate a family of images. In particular, the subject of Simple Perfect Squared Squares (SPSSs) yields a set of solutions, on which I have based recent artwork. I will demonstrate how certain packings have served as starting points for more complex designs involving number theory concepts and color systems. (Received September 19, 2021)

1174-11-8690 **Brandt Kronholm*** (brandt.kronholm@utrgv.edu), University of Texas - Rio Grande Valley. *The Unimodality of Gaussian polynomials* $\begin{bmatrix} N+m \\ m \end{bmatrix}$ *for a few small values of m.*

In this presentation I will give new proofs of the unimodality of Gaussian polynomials $\begin{bmatrix} N+4 \\ 4 \end{bmatrix}$, $\begin{bmatrix} N+3 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} N+2 \\ 2 \end{bmatrix}$. The proofs are completed by examining a novel collection of generating functions for these Gaussian polynomials.

Surprisingly, the partitions of n into at most three parts plays an important role in the proof for the unimodality of $\begin{bmatrix} N+4 \\ 4 \end{bmatrix}$. (Received September 19, 2021)

1174-11-8693 **Amy DeCelles*** (amy.decelles@betheluniversity.edu), Bethel University, Indiana. *The Automorphic Heat Kernel and Ladders of Zeta Functions*

Heat kernels arise in many contexts in physics and mathematics; the automorphic heat kernel is particularly important in string theory and number theory. For example, it plays a key role in Jorgenson and Lang's construction of a Selberg-type zeta function on a compact quotient of hyperbolic 3-space, their general prescription for constructing zeta-type functions from theta-type functions, and their conjectural "ladder" of zeta functions corresponding to a "ladder" of symmetric spaces of increasingly higher rank. While existence, uniqueness, and smoothness of the automorphic heat kernel are expected (sometimes even presumed) in great generality, these are nontrivial to prove in higher rank, especially in the non-compact quotient case. As has been shown in a previous paper, the analytic framework of global automorphic Sobolev theory and operator semigroup theory allows decisive treatment of these issues. However, the issue of convergence of the "periodicized" heat kernel is also nontrivial. In this paper, we prove convergence by invoking Garrett's theorem, which relies the theory of norms (or gauges) on groups to prove convergence of Poincaré series whose data is of sufficiently rapid decay, combined with a global bound for the heat kernel on a manifold due to Debiard, Gaveau, and Mazet. As an application, we reframe and extend Jorgenson and Lang's construction of a zeta-type function and discuss possible interpretations of their "ladder" of zeta functions. (Received September 19, 2021)

1174-11-8699 **Alexander Berkovich*** (alexb@ufl.edu), University of Florida. *New Polynomial Identities associated with the Capparelli Partition Theore* Preliminary report.

I discuss new polynomial identities implying Capparelli partition theorem. These identities involve Jacobi symbol modulo 3. These identities lead to new infinite family of multi sum= product identities. This talk is based on a joint work with Ali Kemal Uncu (Received September 19, 2021)

1174-11-8716 **Rishabh Sarma** (rishabh.sarma@ufl.edu), University of Florida. *New symmetries for Dyson's rank function* Preliminary report.

Let $R(z, q)$ be the two-variable generating function for Dyson's rank function. In his lost notebook Ramanujan gives the 5-dissection of $R(\zeta_p, q)$ where ζ_p is a primitive p-th root of unity and $p = 5$. This result is related to Dyson's famous rank conjecture which was proved by Atkin and Swinnerton-Dyer. In 2016 one of us showed that there is an analogous result for the p-dissection of $R(\zeta_p, q)$ when p is any prime greater than 3, by extending work of Bringmann and Ono, and Ahlgren and Treneer. It was also shown how the group $\Gamma_1(p)$ acts on the elements of the p-dissection of $R(\zeta_p, q)$. We extend this to the group $\Gamma_0(p)$, thus revealing new and surprising symmetries for Dyson's rank function. (Received September 19, 2021)

1174-11-8725 **Rong Chen** (rongchen20@tongji.edu.cn), Tongji University, Shanghai. *Ramanujan's mock theta functions and the mod* 4 *unimodal sequence conjectures*

In 2012 Bryson, Ono, Pitman and Rhoades showed how the generating functions for certain strongly unimodal sequences are related to quantum modular and mock modular forms. They proved some parity results and conjectured some mod 4 congruences for the coefficients of these generating functions. In 2016 Kim, Lim and Lovejoy obtained similar results for odd-balanced unimodal sequences and made similar mod 4 conjectures. We sketch the idea of proof of the mod 4 conjectures. Our method of proof involves Ramanujan's mock theta

functions, new Hecke-Rogers type identities for indefinite binary quadratic forms and the Hurwitz class number.
(Received September 19, 2021)

1174-11-8786 **Byungchul Cha*** (cha@muhlenberg.edu), Muhlenberg College. *Class Number of Almost Pythagorean Triples* Preliminary report.

In a profoundly influential textbook *Disquisitiones Arithmeticae*, Gauss introduced the notion of *class numbers*. For a fixed nonzero integer D, the class number $h(D)$ is defined to be the number of $\mathrm{SL}_2(\mathbb{Z})$-equivalence classes in the set of integral binary quadratic forms of discriminant D. In this talk, we study the number of certain equivalence classes of *almost Pythagorean triples*, which are by definition integer triples (a, b, c) satisfying $a^2 + b^2 = c^2 + D$ for a fixed nonzero integer D. The easiest special case is $D = -1$. Barning's results in 1963, which were motivated by a somewhat different problem, showed that the class number of almost Pythagorean triples for $D = -1$ is one. We will present some partial results on Barning's class numbers for other values of D. (Received September 19, 2021)

1174-11-8797 **Edray Herber Goins*** (ehgoins@mac.com), Pomona College, **Tesfa Asmara** (tgac2020@mymail.pomona.edu), Pomona College, **Erik M. Imathiu-Jones** (eimathiu@caltech.edu), California Institute of Technology, **Maria Maalouf** (maria.maalouf@student.csulb.edu), California State University at Long Beach, **Isaac Robinson** (isaac_robinson@college.harvard.edu), Harvard University, and **Sharon Sneha Spaulding** (sharon.spaulding@uconn.edu), University of Connecticut. *Critical Points of Toroidal Belyǐ Maps*

A Belyǐ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; we may assume these values are $\{0, 1, \infty\}$. Replacing $\mathbb{P}^1$ with an elliptic curve $E : y^2 = x^3 + Ax + B$, there is a similar definition of a Belyǐ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$. Since $E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$ is a torus, we call (E, β) a Toroidal Belyǐ pair.

There are many examples of Belyǐ maps $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ associated to elliptic curves; several can be found online at LMFDB. Given such a Toroidal Belyǐ map of degree N, the inverse image $G = \beta^{-1}\big(\{0, 1, \infty\}\big)$ is a set of N elements which contains the critical points of the Belyǐ map. In this project, we investigate when G is contained in $E(\mathbb{C})_{\text{tors}}$.

This was work done as part of the Pomona Research in Mathematics Experience (NSA H98230-21-1-0015).
(Received September 19, 2021)

1174-11-8819 **Ae Ja Yee*** (yee@psu.edu), Pennsylvania State University. *New truncated theorems for three classical theta function identities*

In 2012, Andrews and Merca derived a truncated version of Euler's pentagonal number theorem. Their work inspired several mathematicians to work on truncated theta series including Guo and Zeng, who examined two other classical theta series identities of Gauss. In this talk, revisiting these three theta series identities of Euler and Gauss, we obtain new truncated theorems. As corollaries of our results, we obtain infinite families of linear inequalities involving the partition function, the overpartition function and the pod function. These inequalities yield the positive result of Andrews and Merca on the partition function as well as a conjecture on the overpartition function, which was posed by Andrews–Merca and Guo–Zeng, and proved independently by Mao and Yee. We will also provide a unified combinatorial treatment for our results. This talk is based on joint work with Ernest Xia and Xiang Zhao. (Received September 19, 2021)

1174-11-8832 **Olena Kozhushkina*** (okozhushkina@ursinus.edu), Western Washington University, Ursinus College, **Stephen Hu** (stephen.hu@rutgers.edu), Rutgers University-New Brunswick, **Olena Kozhushkina** (okozhushkina@ursinus.edu), Ursinus College, **Will Boultinghouse** (will.boultinghouse@kwc.edu), Kentucky Wesleyan College, **Emily Hammett** (ehammett@ursinus.edu), Rowan University, and **Justin Trulen** (jtrulen@kwc.edu), Kentucky Wesleyan College. *p-adic Valuations of Quadratic Polynomials*

For prime p, the p-adic valuation $\nu_p(n)$ of a non-zero integer n is the highest power of p that divides n. In this paper, we examine the behavior of the p-adic valuations of quadratic polynomials with integer coefficients for certain values of prime p. We explore this behavior through tree representations of these sequences. For instance, the 2-adic valuation tree for $f(n) = n^2 + 1$ is finite, while the 2-adic valuation tree for $g(n) = n^2 + 2n$ has two infinite branches. We consider some properties associated with p-adic valuation trees such as finiteness, symmetry, and number of levels, along with their relationship to the corresponding sequences. This work was part of the REU at Ursinus College in Summer 2021. (Received September 20, 2021)

1174-11-8860 **Shiva Chidambaram*** (shivac@mit.edu), Massachusetts Institute of Technology. *Modularity of typical abelian surfaces over $\mathbb{Q}$ 1*

The modularity lifting theorem of Boxer-Calegari-Gee-Pilloni established for the first time the existence of infinitely many modular abelian surfaces $A/\mathbb{Q}$ upto twist with $\mathrm{End}_{\mathbb{C}}(A) = \mathbb{Z}$. We render this explicit by first finding some abelian surfaces whose associated mod-p representation is residually modular and for which the modularity lifting theorem is applicable, and then transferring modularity in a family of abelian surfaces with fixed 3-torsion representation. (Received September 20, 2021)

1174-11-8906 **Ling Long** (llong@math.lsu.edu), Louisiana State University, **Jenny G. Fuselier*** (jfuselie@highpoint.edu), High Point University, and **Ravi Ramakrishna** (rkr5@cornell.edu), Cornell University. *Translating hypergeometric identities to the finite field setting: a systematic approach*

We introduce hypergeometric functions over finite fields, originally due to Greene, Katz, and McCarthy. These functions have been shown to relate to point-counting, traces of Hecke operators, and supercongruences. After a brief overview, we present a systematic approach for translating some classical hypergeometric identities and evaluations to the finite field setting by an explicit dictionary. (Received September 20, 2021)

1174-11-8962 **Thomas Morrill*** (morrillt@trine.edu), Trine University. *Quadimodularity of the k-th Residual Cranks*

The moment generating functions for the Andrews-Garvan crank of a partition are known to demonstrate quasimodular behavior. The coefficients of these series also admit combinatoric interpretation in terms of partition counts. We establish similar quasimodularity for a family of residual cranks defined on the set of overpartitions. We also show that the second moments of these kth residual cranks admit a interpretation as weighted overpartition counts. Joint work with Aleksander Simonic. (Received September 20, 2021)

1174-11-8974 **Ricardo Conceicao*** (rconceic@gettysburg.edu), Gettysburg College. *On integral points on isotrivial elliptic curves over function fields*

Let k be a finite field and L be the function field of a curve C/k. In this talk, we discuss certain arithmetical properties satisfied by integral points on elliptic curves over L with j-invariant in k. One particular result that we prove is that the number of separable S-integral points on a constant elliptic curve E/L is bounded solely in terms of the size of S and the genus of C, and does not depend on the Mordell-Weil rank of $E(L)$. (Received September 20, 2021)

1174-11-8989 **William Craig*** (wlc3vf@virginia.edu), University of Virginia. *On the Distribution of t-hooks in Partitions*

The hook numbers of partitions play an important role in number theory, representation theory, and combinatorics. In the spirit of Dirichlet's theorem on primes in arithmetic progressions, it is natural to study the partition function $p_t(a, b; n)$, which counts the number of partitions of n whose total number of t-hooks is equivalent to a modulo b. We compute precise distribution properties of such partition functions, which in particular show that t-hooks are rarely equidistributed. (Received September 20, 2021)

1174-11-8997 **Nicolas Allen Smoot*** (nsmoot@risc.jku.at), Research Institute for Symbolic Computation, JKU Linz. *Computing Partition Identities Arising From Generalized Eta Quotients* Preliminary report.

An important subject in the theory of integer partitions is the computation of identities which relate a given partition function in various arithmetic progressions to linear combinations of eta quotients. Such identities, called Ramanujan-Kolberg identities, play a significant role in studying individual congruences, infinite congruence families, vanishing conditions, and equivalences between different partition functions. Silviu Radu was the first to realize that the theory underlying such identities is so well-understood that it can be formalized into an algorithm, which has since been implemented in Mathematica. More recent work by Chen, Du, and Zhao has extended Radu's algorithmic framework to include generalized eta quotients, thereby significantly extending the class of identities that can be studied. We will demonstrate an implementation of the Chen-Du-Zhao algorithm. (Received September 20, 2021)

1174-11-9000 **David P. Roberts*** (roberts@morris.umn.edu), University of Minnesota, Morris. *Modularity problems for hypergeometric motives I*

For every motive over the rational numbers, one can ask to find a conjecturally existing automorphic form that has the same L-function. This general problem is particularly explicit for hypergeometric motives: one has explicit formulas for vectors $(h^{w,0}, \ldots, h^{0,w})$ of Hodge numbers giving infinity types, explicit formulas for good factors of L-functions, and relatively good control over bad factors and conductors. This talk summarizes how one can compute directly with hypergeometric L-functions. It gives examples of modularity corresponding to the Hodge vectors $(1,1)$, $(1,0,0,1)$, and $(1,0,0,0,0,1)$ by exhibiting corresponding classical modular forms of weight 2, 4, and 6. (Received September 20, 2021)

1174-11-9055 **Kimball L. Martin*** (kimball.martin@ou.edu), University of Oklahoma, and **Alexander Cowan** (cowan@math.harvard.edu), Harvard University. *Counting abelian surfaces with RM 1* Preliminary report.

Classical modularity gives a correspondence between rational elliptic curves and rational modular forms of weight 2. In particular, it is instrumental in enumerating elliptic curves up to a given conductor. More generally, modularity relates rational abelian varieties with sufficient symmetry (of GL(2) type) to weight 2 modular forms. In Part 1, we will give an introduction to these ideas with a focus on attempting to count rational abelian surfaces with real multplication (RM). (Received September 20, 2021)

1174-11-9141 **Katherine Thompson*** (kthomps@usna.edu), United States Naval Academy, and **Jeremy Rouse** (rouseja@wfu.edu), Wake Forest University. *Quaternary Quadratic Forms with Prime Discriminant* Preliminary report.

Let Q be a positive-definite quaternary quadratic form with prime discriminant. We give an explicit lower on the number of representations of a positive integer n by Q. This problem is connected with deriving an upper bound on the Petersson norm $\langle C, C\rangle$ of the cuspidal part of the theta series of Q. We derive an upper bound on $\langle C, C\rangle$ that depends on the smallest positive integer not represented by the dual form Q^* and is within a constant factor of the true size in all cases. In addition, we give a non-trivial upper bound on the sum of the integer n excepted by Q. (Received September 20, 2021)

1174-11-9152 **Olivia Beckwith*** (obeckwith@tulane.edu), Tulane University, **Scott Ahlgren** (sahlgren@illinois.edu), University of Illinois at Urbana-Champaign, and **Martin Raum** (raum@chalmers.se), Chalmers Technical University. *Scarcity of congruences for the partition function* Preliminary report.

The arithmetic properties of the ordinary partition function $p(n)$ have been the topic of intensive study ever since Ramanujan proved that for all integers n, $p(24n+t) \equiv 0 \pmod{Q}$ for the primes $Q = 5$, 7, 11, where $24t = 1 \pmod{Q}$. Today it is known that, there are many congruences of the form $p(Qmn+t) \equiv 0 \pmod{Q}$ for all n, where Q is prime and $m \geq 5$. Here we prove that such congruences are scarce when m is prime unless a certain nonzero cusp form has an unexpectedly large number of coefficients which are divisible by Q. The proofs rely on a variety of tools from the theory of modular forms and analytic number theory. This is joint work with Scott Ahlgren and Martin Raum. I will also briefly discuss an investigation with Jack Chen, Maddie Diluia, Oscar Gonzalez, and Jamie Su, in which we observe that there appear to be infinitely many congruences of this form for the colored partition functions. (Received September 20, 2021)

1174-11-9237 **Lindsay Dever*** (lmdever@brynmawr.edu), Bryn Mawr College. *Distribution of Holonomy on Compact Hyperbolic 3-Manifolds*

The study of hyperbolic 3-manifolds draws deep connections between number theory, geometry, topology, and quantum mechanics. Specifically, the closed geodesics on a manifold are intrinsically related to the eigenvalues of Maass forms via the Selberg trace formula and are parametrized by their length and holonomy, which describes the angle of rotation by parallel transport along the geodesic. The trace formula for spherical Maass forms can be used to prove the Prime Geodesic Theorem, which provides an asymptotic count of geodesics up to a certain length. I will present an asymptotic count of geodesics (obtained via the non-spherical trace formula) by length and holonomy in prescribed intervals which are allowed to shrink independently. This count implies effective equidistribution of holonomy and substantially sharpens the result of Sarnak and Wakayama in the context of compact hyperbolic 3-manifolds. I will then discuss new results regarding biases in the finer distribution of holonomy. (Received September 20, 2021)

1174-11-9239 **Emily Anible*** (eeanible@mtu.edu), Pennsylvania State University. *Major Index over Descent Distributions of Standard Young Tableaux*

A general formula for the generating function $f_{\lambda,k}(q)$ of Standard Young Tableaux of shape λ with precisely k descents was found through the work of Kirillov and Reshetikhin in 1988. Extending prior work by William Keith, we give families of $f_{\lambda,k}(q)$ for low numbers of descents, yielding several nice combinatorial symmetries. We also investigate a potential relation among these polynomials, the existence of which may lead to a greater insight to the overall structure of these distribution statistics. (Received September 20, 2021)

1174-11-9260 **Gene S. Kopp** (gene.s.kopp@gmail.com), Purdue University. *Polyharmonic Maass Forms and Hecke series* Preliminary report.

Polyharmonic Maass forms were defined by Lagarias and Rhoades as a generalization of both holomorphic modular forms and harmonic Maass forms. We construct bases for spaces of polyharmonic Maass forms on the group $\Gamma(N)$, generalizing a result of Lagarias and Rhoades for $N = 1$. Combining this with a generalization of the Gauss composition law, we describe leading coefficients of Hecke series for real quadratic fields as twisted traces of cycle integrals of biharmonic Maass forms. This is ongoing joint work with Gene Kopp. (Received September 20, 2021)

1174-11-9261 **Frank Patane*** (fpatane@samford.edu), Samford University. *Special Cases of an Identity Connecting Theta Series Associated with Discriminants Δ and Δp^2.*

In the 2016 paper *On a generalized identity connecting theta series associated with discriminants Δ and Δp^2*, the author gives a formula which relates theta series of binary quadratic forms of discriminant Δ and Δp^2. In this talk, we classify all cases where this identity yields a formula for a theta series corresponding to a single class of binary quadratic forms. We discuss applications of these cases as well as give a proof of a stated (but unproven) Lemma from the mentioned paper. (Received September 20, 2021)

1174-11-9275 **Daniel White*** (dwhite@gettysburg.edu), Bryn Mawr College. *Extreme values of L-functions* Preliminary report.

Soundararajan's resonance method provides insight on the extremal behavior of the zeta function along the critical line and the last decade has seen a number of adaptations of this method for application to various families of L-functions. In this talk, a conceptual illustration of the resonance method will be given and new preliminary results on L-functions over number fields will be presented. (Received September 20, 2021)

1174-11-9296 **Steven T Flammia*** (sflammi@amazon.com), AWS Center for Quantum Computing, and **Marcus Appleby** (marcus.appleby@gmail.com), University of Sydney. *Ghosts, Necromancy, and Zauner's Conjecture*

Sets of d^2 equiangular lines in $\mathbb{C}^d$, known as SICs, are closely related to certain dual objects known as ghost SICs. Starting from a conjectural formula for ghost SICs in dimension d (see the talk by G. Kopp in this session), we devise a procedure that we call necromancy to reconstruct a SIC from a (numerical approximation of) a ghost. Using necromancy, we reanimate the ghosts in $d = 100$ and construct four (numerical) SICs, three of which are new. We further establish a unified understanding of many previously mysterious properties of the known SICs, including the orders of the symmetry groups and the number of unitarily inequivalent SICs in each dimension. (Received September 20, 2021)

1174-11-9308 **Olav Richter** (Olav.Richter@unt.edu), University of North Texas. *Ramanujan-type congruences for Hurwitz class numbers*

Hurwitz class numbers $H(N)$ count weighted classes of quadratic forms with discriminant $-N$, and they are the Fourier coefficients of a weight 3/2 mock modular form. Their close connection with class numbers of imaginary quadratic number fields and to mock modular forms makes the divisibility properties of Hurwitz class numbers an area of interest in classical number theory and the study of real-analytic modular forms. We investigate congruence relations of the form $H(mN + t) = 0 \pmod{p}$ for all integers N, where p is prime. We prove that congruence relations of this type exist on square classes modulo m, and we prove certain restrictions on the m and t appearing in such congruences. This is joint work with Martin Raum and Olav Richter. (Received September 20, 2021)

1174-11-9363 **Mary Wootters** (marykw@stanford.edu), Stanford, **Gretchen Matthews** (gmatthews@vt.edu), Virginia Tech, and **Hiram H. Lopez** (h.lopezvaldez@csuohio.edu), Cleveland State University. *Hermitian Lifted Codes*

In recent work of Lopez, Malmskog, Matthews, Pinero-Gonzales, and Wootters, we constructed codes for local recovery of erasures with high availability and constant-bounded rate from the Hermitian curve. These new codes, called Hermitian-lifted codes, are evaluation codes with evaluation set being the set of F_{q^2}-rational points on the affine curve. The novelty is in terms of the functions to be evaluated; they are a special set of monomials which restrict to low degree polynomials on lines intersected with the Hermitian curve. The resulting codes are neither punctured traditional lifted codes, nor subcodes of previously defined locally recoverable codes on the Hermitian curve. This talk will introduce the codes and bounds on their parameters, and discuss questions for further research. (Received September 20, 2021)

1174-11-9365 **Tian An Wong** (tiananw@umich.edu), University of Michigan-Dearborn, **Robert Lemke Oliver** (robert.lemke_oliver@tufts.edu), Tufts University, and **Allechar Serrano Lopez*** (serrano@math.harvard.edu), Harvard University. *How do points on plane curves generate fields? Let me count the ways.* Preliminary report.

In their program on diophantine stability, Mazur and Rubin suggest studying a curve C over $\mathbb{Q}$ by understanding the field extensions of $\mathbb{Q}$ generated by a single point of $C(\overline{\mathbb{Q}})$; in particular, they ask to what extent the set of such field extensions determines the curve C. A natural question in arithmetic statistics along these lines concerns the size of this set: for a smooth projective curve $C/\mathbb{Q}$ how many field extensions of $\mathbb{Q}$ — of given degree and bounded discriminant — arise from adjoining a point of $C(\overline{\mathbb{Q}})$? Can we further count the number of such extensions with specified Galois group? Asymptotic lower bounds for these quantities have been found for elliptic curves by Lemke Oliver and Thorne, for hyperelliptic curves by Keyes, and for superelliptic curves by Beneish and Keyes. We discuss similar asymptotic lower bounds that hold for all smooth plane curves C, using tools such as geometry of numbers, Hilbert irreducibility, Newton polygons, and linear optimization. (Received September 20, 2021)

1174-11-9381 **Ayan Maiti*** (ayan.maiti@okstate.edu), Oklahoma State University. *Weyl's Law for Automorphic cusp forms of arbitrary K_∞-type* Preliminary report.

Let M be a compact Riemannian manifold. It was proved by Weyl that number of Laplacian eigenvalues less than T, is asymptotic to $C(M)T^{\dim(M)/2}$, where $C(M)$ is the product of the volume of M, volume of the unit ball and $(2\pi)^{-\dim(M)}$. Let Γ be an Arithmetic subgroup of $SL_2(\mathbb{Z})$, $\mathbb{H}^2$ be an upper-half plane. When $M = \Gamma\backslash\mathbb{H}^2$, Weyl's asymptotic estimate holds true for the discrete spectrum of Laplacian. It was proved by Selberg, who used his celebrated trace formula.

Let $\mathbf{G}$ be a semisimple algebraic group of Adjoint and split type over $\mathbb{Q}$. Let $G(\mathbb{R})$ be the set of $\mathbb{R}$-points of $\mathbf{G}$. For simplicity of this exposition we will assume that $\Gamma \subset G(\mathbb{R})$ be an torsion free arithmetic subgroup. Let K_∞ be the maximal compact subgroup. Let $L^2_{\text{cusp}}(\Gamma\backslash G(\mathbb{R}))$ be the cuspidal subspace of $L^2(\Gamma\backslash G(\mathbb{R}))$. Suppose $d = \dim(\Gamma\backslash G/K_\infty)$. Then it was proved by Lindenstrauss and Venkatesh, that number of spherical, i.e. bi-K_∞ invariant cuspidal Laplacian eigenfunctions, whose eigenvalues are less than T is asymptotic to $C(M)T^{\dim(M)/2}$, where $C(M)$ is the same constant as above.

We are going to prove the same Weyl's asymptotic estimate (main term) for cusp forms of arbitrary K_∞-type, generalizing their method. (Received September 21, 2021)

1174-11-9403 **Felice Manganiello** (manganm@clemson.edu), Clemson University, **Jessalyn Bolkema*** (jbolkema@csudh.edu), California State University, Dominguez Hills, **Emma Lee Andrade** (eandrade4@mail.fresnostate.edu), California State University, Fresno, **Harrison Eggers** (heggers@g.clemson.edu), Clemson University, **Victoria Luongo** (toriluongo@gmail.com), Clemson University, and **Thomas Dexter** (t.dextrous@gmail.com), Texas A&M University. *CSS-T Codes from Reed-Muller Codes For Quantum Fault-Tolerance*

Fault-tolerant quantum computation is a critical step in the development of practical quantum computers. Unfortunately, not every quantum error-correcting code can be used for fault-tolerant computation. Rengeswamy et. al. define CSS-T codes, which are CSS codes that admit a physical transversal T-gate. In this talk we give a comprehensive study of CSS-T codes from Reed-Muller codes. These codes allow for the construction of CSS-T code families with non-vanishing asymptotic rate and possibly diverging minimum distance, desirable properties for fault-tolerant quantum computation. This work was partially supported by the National Science Foundation under grant DMS-1547399. (Received September 20, 2021)

1174-11-9441 **Maria De Los Angeles Chara*** (charamaria@gmail.com), Universidad Nacional del Litoral, CONICET. *Cyclic algebraic geometry codes*

In this talk we will introduce cyclic algebraic geometry codes and give conditions to construct such codes in the context of algebraic function fields over a finite field by using their group of automorphisms. We will show that cyclic algebraic geometry codes constructed in this way are closely related to cyclic extensions. With this method, in the case of a rational function field, there is only one code of a fixed length and dimension, up to monomial equivalence.

This a joint work with Gustavo Cabaña (Universidad Nacional del Litoral, Argentina), Ricardo Podestá (CONICET and Universidad Nacional de Córdoba, Argentina) and Ricardo Toledano (Universidad Nacional del Litoral, Argentina). (Received September 20, 2021)

1174-11-9470 **Edna Luo Jones*** (elj44@math.rutgers.edu), Rutgers University. *A strong asymptotic local-global principle for integral Kleinian sphere packings* Preliminary report.

We will discuss a strong asymptotic local-global principle for certain integral Kleinian sphere packings. Examples of Kleinian sphere packings include Apollonian circle packings and Soddy sphere packings. Sometimes each sphere in a Kleinian sphere packing has a bend (1/radius) that is an integer. When all the bends are integral, which integers appear as bends? For certain Kleinian sphere packings, we expect that every sufficiently large integer locally represented as a bend of the packing is a bend of the packing. We will discuss ongoing work towards proving this for certain Kleinian sphere packings. This work uses orientation-preserving isometries of $(n+1)$-dimensional hyperbolic space, quadratic polynomials, the circle method, spectral theory, and expander graphs. (Received September 20, 2021)

1174-11-9493 **Hanneke Wiersema** (hannekeWiersema@hotmail.com), Kings College London, **Nandita Sahajpal** (simplynandita@gmail.com), Nevada State College, and **Darwin Xavier Tallana Chimarro** (darwin12563xtch@gmail.com), n/a. *Local data of elliptic curves with non-trivial torsion* Preliminary report.

Given a rational elliptic curve, we can use Tate's algorithm to determine the following local data of the elliptic curve at each prime: the conductor, Néron-Kodaira type, and local Tamagawa number. It is well understood how this local data is related for isogenous elliptic curves if the curve does not have a 2- or 3-isogeny. Using parameterized families of elliptic curves, we look to complete the picture by investigating how the local data varies over isogenous elliptic curves with a 2- or 3-isogeny, and how the data depends on the parameters. Specifically, we explicitly show how the local data changes for isogeny graphs. (Received September 20, 2021)

1174-11-9495 **Nayma E Rodriguez-Huerta*** (nrodri22@stedwards.edu), Author. *Musical Polybius Cipher: Encryption and Composition* Preliminary report.

A Polybius cipher uses a $n \times n$ square to encode secret messages. In a basic Polybius square, the array may be filled in with letters as well as symbols. Each letter is encrypted by assigning a pair of numbers representing the coordinates of its array. In a musically extended Polybius square, the coordinates now are pairs of notes. A character is then encrypted by the two notes, where the first note is then assigned to the treble clef of a music sheet, while the second note to the bass clef. Further, we may incorporate beat and degree substitutions to enhance the encryption. The result is a message encrypted in a composition represented by sheet music that can be read and performed.
(Received September 20, 2021)

1174-11-9523 **Joselyne Aniceto*** (Joselyne.rodriguez01@utrgv.edu), University of Texas - Rio Grande Valley. *Gupta, Ramanujan, Dyson and Ehrhart: Formulas for Partition Functions, Congruences, Cranks, and Polyhedral Geometry*

We will prove properties of an infinite family of congruences for certain restricted partition functions. we find and prove combinatorial witnesses for these congruences using polyhedral geometry. We then explore the possibility of expanding our restricted partition function and finding a more general witness for any new congruences. (Received September 20, 2021)

1174-11-9541 **Larry Rolen*** (larryrolen@gmail.com), Vanderbilt University. *Recent problems in partitions and other combinatorial functions*

In this talk, I will summarize some recent work with a number of collaborators on conjectured analytic and combinatorial properties of partitions and related functions. In particular, I will look at recent conjectures of Stanton, which ultimately aim to give a deeper understanding into the workings of the rank and crank functions in explaining Ramanujan's congruences, as well as recent progress in producing such functions explaining congruences for combinatorial functions using Gritsenko-Skoruppa-Zagier's theory of theta blocks (special products built out of Dedekind eta and Jacobi theta functions with ties to Lie theory). I will also discuss how analytic questions about partitions can be used to study Stanton's conjectures, as well as recent conjectures of Chern-Fu-Tang and Heim-Neuhauser, which are related to the Nekrasov-Okounkov formula. (Received September 20, 2021)

1174-11-9619 **Dominic Lanphier** (dominic.lanphier@wku.edu), Western Kentucky University. *Values and Properties of Dirichlet Series* Preliminary report.

We study several classes of Dirichlet series, some of which were studied by Shimura, Choi, and others. We extend this Dirichlet series to multiple variables and prove the meromorphic continuation and locations of the poles of the series. We also study the values of the series at certain integers. We generalize results of Choi by relating values of some series to values of other, related series. (Received September 20, 2021)

1174-11-9650 **Kristin A Camenga*** (camenga@juniata.edu), Juniata College, **Patrick X. Rault** (prault@unomaha.edu), University of Nebraska Omaha, **Brandon Nicholas Collins** (brandoncollins@unomaha.edu), University of Nebraska At Omaha, **Gage Joseph Hoefer** (ghoefer@udel.edu), University of Delaware, **James Willson** (jwillson@unomaha.edu), University of Nebraska Omaha, and **Rebekah Yates** (Rebekah.Yates@houghton.edu), Houghton College. *The geometry of numerical ranges over finite fields*

The numerical range of a matrix has been classically studied over the complex numbers for over a hundred years; interesting properties include that the classical numerical range is a convex set containing the eigenvalues of the matrix. Numerical ranges over finite fields were first studied in 2016 by a team of four undergraduates at the State University of New York's College at Geneseo, led by the presenter. In that first paper, the numerical range of certain types of matrices were classified over finite fields with a prime number of elements. Soon afterward this classification was generalized to all finite fields, with many new results about the cardinality of the finite field numerical range. We now present results that have been worked on or inspired by teams of undergraduates at three institutions, the University of Nebraska at Omaha, Houghton College, and Juniata College. In this paper we study the geometry of these finite field numerical ranges using the boundary generating curve, first introduced by Kippenhahn in 1951. This allows us to classify the set of points in the numerical range as a line; a collection of scalings of a boundary generating curve (which itself is a hermitian circle or hyperbola); or in a singular case, as the complement of a boundary line. We restrict our study to square matrices of dimension 2, with at least one eigenvalue in the finite field. (Received September 20, 2021)

1174-11-9816 **Dan Fretwell*** (daniel.fretwell@bristol.ac.uk), University of Bristol, and **Eran Assaf** (eran.assaf@dartmouth.edu), Dartmouth. *Orthogonal modular forms and Eisenstein congruences* Preliminary report.

Orthogonal modular forms are automorphic forms attached to orthogonal groups. When working with a positive definite quadratic space these forms become very explicit. . . they are certain functions on global isometry classes of lattices in a genus.

In this talk we will compute some of these spaces, as Hecke modules, and see how one can often use this information to prove examples of interesting Eisenstein congruences. In particular, we will see known congruences arise (such as the Ramanujan 691 congruence) as well as new unknown congruences (of Katsurada-Mizumoto type). (Received September 20, 2021)

1174-11-9839 **Alyson Deines** (aly.deines@gmail.com), Center for Communications Research, La Jolla. *Isogenous discriminant twins over totally real number fields*

Let K be a number field with class number one. We say that two elliptic curves E and E' defined over K are isogenous discriminant twins if E and E' are isogenous and non-isomorphic curves such that their minimal discriminants are equal. When $K = \mathbb{Q}$, Deines showed there are finitely many (up to twist) isogenous discriminant twins. In this talk, we consider the case when K is a totally real number field and show that there are finitely many (up to twist) isogenous discriminant twins separated by an isogeny of prime degree p where $p \neq 11$, 17, 19. When $p = 3$, 5, 7, and 13, we explicitly find all discriminant twins separated by a p-isogeny. (Received September 20, 2021)

1174-11-9885 **Rachel Snyder** (snyderra@comcast.net), Western Washington University. *The p-adic Valuations of Quadratic Integer Sequences* Preliminary report.

Let p be an odd prime. The p-adic valuation of an integer x is the highest power of p which divides x, denoted by $\nu_p(x)$. We will consider the p-adic valuations of sequences generated by quadratic polynomials $f(n)$ with integer coefficients. The sequences $\{\nu_p(f(n))\}$ can be represented by p-nary trees, that might be finite, infinite, or even look like a single dot. For example, for $p = 3$ the polynomial $f(n) = 2n^2 + 81$ has a 3-adic valuation tree with two infinite branches, while the tree of $g(n) = 2n^2 + 3n + 18$ is a finite two-level tree. On the other hand, if $f(n) = an^2 + bn + c$ where both a and b are congruent to 1 (mod 3) and c is congruent to 2 (mod 3), then its 3-adic valuation tree is a dot tree with one node. In this talk, we will focus on the properties of p-adic valuation trees, including classifying whether for a given quadratic polynomial with integer coefficients the tree is finite, infinite, or a dot tree depending on the coefficients of the polynomial. This is a joint project done in part during the REU at Ursinus College in Summer 2021. (Received September 21, 2021)

1174-11-9888 **Chandra Copes*** (clc0100@mcdaniel.edu), McDaniel College, and **Kevin Rabidou** (kmr0100@mcdaniel.edu), McDaniel College. *Waring's Problem in Ramified p-adic Rings* Preliminary report.

Generalization of Waring's Problem - that for every natural number k there exists an integer $g(k)$ such that every natural number can be written as the sum of at most $g(k)$ k-th powers - have been studied in a variety

of contexts from algebraic number fields to non-commutative groups. We will examine values of $g(k)$ for certain ramified extensions of $\mathbf{Z}_p$, with specific focus on the case where p divides k and wildly ramified extensions. (Received September 21, 2021)

1174-11-9958 **Caroline Sujin Choi** (cchoi1@stanford.edu), Stanford University, **Mateo Attanasio*** (mateoattanasio09@gmail.com), Stanford University, and **Andrei Mandelshtam** (andreim1828@gmail.com), Stanford University. *Symbol Length in Brauer Groups of Elliptic Curves*

Let ℓ be an odd prime integer, and let K be a field of characteristic not $2, 3$ and coprime to ℓ containing a primitive ℓ-th root of unity. For an elliptic curve E over K, we consider the standard Galois representation

$$\rho_{E,\ell} : \mathrm{Gal}(\overline{K}/K) \to \mathrm{GL}_2(\mathbb{F}_\ell),$$

and denote the fixed field of its kernel by L. The Brauer group $\mathrm{Br}(E)$ of an elliptic curve E is an important invariant. A theorem by Merkurjev and Suslin implies that every element of the ℓ-torsion ${}_\ell\mathrm{Br}(E)$ can be written as a product of symbol algebras. We use an explicit computation of the Brauer group to show that if $\ell \nmid [L : K]$ and $[L : K] > 2$, then the symbol length of ${}_\ell\mathrm{Br}(E)/{}_\ell\mathrm{Br}(K)$ is bounded above by $[L : K]-1$. Under the additional assumption that $\mathrm{Gal}(L/K)$ contains an element of order $d > 1$, the symbol length of ${}_\ell\mathrm{Br}(E)/{}_\ell\mathrm{Br}(K)$ is bounded above by $(1 - \frac{1}{d})[L : K]$. In particular, these bounds hold for all CM elliptic curves, in which case we have an upper bound of $\ell + 1$ on the symbol length. We provide an algorithm implemented in SageMath to compute these symbols explicitly over number fields. (Received September 21, 2021)

1174-11-10046 **Travis Morrison** (tmo@vt.edu), Virginia Tech, **Changningphaabi Namoijam** (namoijam@math.nthu.edu.tw), National Tsing Hua University, **Mark Kozek** (mkozek@whittier.edu), Whittier College, and **Annamaria Iezzi** (annamaria.iezzi@upf.pf), Université de la Polynésie Française. *Computing the endomorphism ring of a supersingular elliptic curve* Preliminary report.

In recent years, isogeny-based cryptosystems have drawn attention in academic and government circles, in part due to their potential resilience to quantum algorithms. Central to these cryptosystems are supersingular elliptic curves. Efficient computation of isogenies $\phi : E \to E'$ where E and E' are supersingular can happen if one can efficiently compute the rings of endomorphisms of E and E'. In this talk, we report on an improved algorithm for computing $\mathrm{End}(E)$, where E is a supersingular elliptic curve. (Received September 21, 2021)

1174-11-10063 **Armand Brumer** (brumer@fordham.edu), Fordham University, **Hyun Jong Kim** (hyunjong.kim@math.wisc.edu), University of Wisconsin at Madison, **Jacob Mayle** (jmayle2@uic.edu), University of Illinois at Chicago, **Zev David Klagsbrun** (zdklags@ccrwest.org), CCR, and **Barinder Banwait** (b.s.banwait86@gmail.com), Harish-Chandra Research Institute. *Explicit determination of nonsurjective primes for abelian surfaces* Preliminary report.

In this talk I will discuss the implementation of an algorithm of Dieulefait to compute the nonsurjective primes of an abelian surface with typical endomorphism ring. This algorithm crucially uses Serre's modularity conjecture. (Received September 21, 2021)

1174-11-10071 **Ursula Whitcher** (uaw@umich.edu), Mathematical Reviews (AMS), and **Adriana Salerno*** (asalerno@bates.edu), Bates College. *Hypergeometric congruences and K3 surface pencils*

In this talk, we describe our work counting points on K3 surfaces obtained as hypersurfaces in weighted projective Gorenstein Fano threefolds. Our approach involves realizing the threefolds as toric varieties obtained from reflexive polytopes and applying a generalization of the work of Katz developed by Beukers-Vlasenko and Huang-Lian-Yau-Yu. We show the point counts over finite fields are related to hypergeometric series in a geometrically and arithmetically natural way. (Received September 21, 2021)

1174-11-10076 **Jaebum Sohn** (jsohn@yonsei.ac.kr), Yonsei University, **Hayan Nam*** (hnam@duksung.ac.kr), Duksung Women's University, **Jisun Huh** (hyunyjia@ajou.ac.kr), Ajou University, and **Hyunsoo Cho** (hyunsoo@ewha.ac.kr), Ehwa Womans University. *Results on bar-core partitions, core shifted Young diagrams, and doubled distinct cores* Preliminary report.

Simultaneous bar-cores, core shifted Young diagrams (or CSYDs), and doubled distinct cores have been studied since Morris and Yaseen introduced the concept of bar-cores. In this talk, we give a formula for the number of these core partitions on (s, t)-cores and $(s, s + d, s + 2d)$-cores for the remaining cases that are not covered yet. In order to get this formula, we observe a characterization of $\overline{s}$-core partitions to obtain characterizations of doubled distinct s-core partitions and s-CSYDs. By using them, we construct NE lattice path interpretations of these core partitions on (s,t)-cores. Also, we give free Motzkin path interpretations of these core partitions on $(s, s+d, s+2d)$-cores. (Received September 21, 2021)

1174-11-10099 **Kalani Thalagoda** (kmthalag@uncg.edu), University of North Carolina At Greensboro, and **Kimberly Klinger-Logan** (kim.klingerlogan@rutgers.edu), Rutgers University, Kansas State University. *A Dedekind-Rademacher homomorphism for Bianchi groups* Preliminary report.

The classical Dedekind-Rademacher homomorphism is a map from the modular group to the integers, and has many important arithmetic applications. We report on work in progress on a construction generalizing the homomorphism to Bianchi groups, with potential applications to the Eisenstein ideal. (Received September 21, 2021)

1174-11-10244 **Leanne D. Robertson*** (robertle@seattleu.edu), Seattle University. *Number theory on Gaussian lines*

How similar is the study of the rational integers on the real line to that of the Gaussian integers that lie on a different line in the complex plane? This question provides a wealth of problems appropriate for collaboration with undergraduate students. In this spirit, we give a Gaussian analog of Bertrand's postulate, the Chinese remainder theorem, the periodicity of divisibility, Newman's coprime mapping problem, and a problem of Pillai, and suggest several open problems for further study. (Received September 21, 2021)

1174-11-10287 **aBa Mbirika*** (mbirika@uwec.edu), University of Wisconsin-Eau Claire. *Two tantalizing formulas for the GCD of sums of k consecutive generalized Fibonacci numbers*

In the inaugural issue of the *Fibonacci Quarterly* in 1963, I. D. Ruggles proposed the following problem: "Show that the sum of twenty consecutive Fibonacci numbers is divisible by the 10^{th} Fibonacci number $F_{10} = 55$." We observed that not only are these sums divisible by 55, but also that 55 is the greatest common divisor (GCD) of these sums. This became a main motivation for us to explore sums of any finite length of consecutive Fibonacci numbers, then for Lucas numbers, and then for generalized Fibonacci sequences $(G_n)_{n\geq 0}$ given by the recurrence $G_n = G_{n-1} + G_{n-2}$ for all $n \geq 2$ with integral initial conditions G_0 and G_1. Denoting the GCD of all sums of k consecutive generalized Fibonacci numbers by the symbol $\mathcal{G}_{G_0,G_1}(k)$, we give two tantalizing characterizations for these values, one involving a simple formula in k and another involving generalized Pisano periods:

$$\mathcal{G}_{G_0,G_1}(k) = \gcd(G_{k+1} - G_1,\, G_{k+2} - G_2)$$

$$\mathcal{G}_{G_0,G_1}(k) = \operatorname{lcm}\{m \mid \pi_{G_0,G_1}(m) \text{ divides } k\}$$

where $\pi_{G_0,G_1}(m)$ denotes the generalized Pisano period of the generalized Fibonacci sequence modulo m. The fact that these vastly different-looking formulas coincide leads to some surprising and delightful new understandings of the Fibonacci and Lucas numbers. (Received September 21, 2021)

1174-11-10443 **Nathan Kaplan** (nckaplan@math.uci.edu), University of California Irvine, and **Kelly Isham*** (kisham@colgate.edu), Colgate University. *The growth rate of subrings in $\mathbb{Z}^n$ of corank at most k* Preliminary report.

Subgroups in $\mathbb{Z}^n$ are well-understood. For example, the growth rate of the number of subgroups in $\mathbb{Z}^n$ is known, and it is known that for any $k > 0$, a positive proportion of subgroups have corank at most k. Much less is known about subrings in $\mathbb{Z}^n$. There is not even a conjecture about what the growth rate of the number of subrings in $\mathbb{Z}^n$ should be, but several authors have been able to give upper and lower bounds. The main idea to prove these bounds is to define the subring zeta function which carries information about the number of subrings of a fixed index. In this talk, we consider subrings of corank k and define a similar zeta function that describes information about the number of corank at most k subrings in $\mathbb{Z}^n$ of a fixed index. We express the corank zeta function for $\mathbb{Z}^n$ when $n \leq 4$ in terms of simpler Euler products. We also show that, for n sufficiently large, the proportion of corank at most k subrings in $\mathbb{Z}^n$ is 0. This is joint work with Nathan Kaplan. (Received September 21, 2021)

1174-11-10458 **Spencer Martin*** (sm5ve@virginia.edu), University of Virginia. *On the Mertens conjecture over number Fields*

We introduce a number field analogue of the Mertens conjecture and demonstrate its falsity for all but finitely many number fields of any given degree. We establish the existence of a logarithmic limiting distribution for the analogous Mertens function, expanding upon work of Ng. Finally, we explore properties of the generalized Mertens function of certain dicyclic number fields as consequences of Artin factorization. (Received September 21, 2021)

1174-11-10459 **Haydee Lindo*** (hlindo@hmc.edu), Harvey Mudd College, **Santiago Estupiñán** (s.estupinan10@uniandes.edu.co), -, and **Joshua Carlson** (joshua.carlson@drake.edu), Drake University. *Quasi-prime matrices and b-visibility* Preliminary report.

If r and s are positive integers, we say that the lattice point (r,s) is visible from the origin if there are no lattice points on the line segment connecting (0,0) and (r,s). In this talk we build on the work of Mbirika et al. and discuss the relationship between quasi-prime matrices and invisible (or b-invisible) arrays to determine bounds for their distances from the origin. (Received September 21, 2021)

1174-11-10463 **Caroline L. Turnage-Butterbaugh*** (cturnagebutterbaugh@gmail.com), Carleton College. *Twisted Moments of Dirichlet polynomial approximations of Dirichlet L-functions* Preliminary report.

The recipe of Conrey, Farmer, Keating, Rubinstein, and Snaith gives precise conjectures for moments of families of L-functions. In all cases, only low moments can be handled rigorously using current techniques. A new approach, developed by Conrey and Keating to study high moments of the Riemann zeta-function, considers moments of Dirichlet polynomial approximations and outlines how the prediction of the recipe may be verified. In this talk, I will discuss progress made on studying twisted moments of the family of Dirichlet L-functions, averaged over bounded moduli q, via this new approach. This is joint work with Sieg Baluyot. (Received September 21, 2021)

1174-11-10539 **Jennifer Berg** (jsb047@bucknell.edu), Bucknell University, **Stephanie Treneer*** (stephanie.treneer@wwu.edu), Western Washington University, and **Hussain Kadhem** (hmk46@cam.ac.uk), University of Cambridge. *Connections between linear codes, lattices, and vertex operator algebras* Preliminary report.

Recent work towards characterizing linear codes, lattices and vertex operator algebras (VOAs) and, in certain cases, the sporadic simple groups which act naturally on them, suggests a correspondence between VOAs, even positive definite lattices, and doubly-even linear binary codes. We aim to further develop this correspondence by studying fixed point sublattices of a lattice under an automorphism of an associated binary code, starting with the case of the E_8 lattice and the extended binary Hamming code H. We explore connections between the theta functions of these sublattices and certain sporadic groups and between the theta functions of these sublattices and the characters of certain fixed point sub-VOAs. (Received September 21, 2021)

1174-11-10562 **Andrei Mandelshtam*** (andreim1828@gmail.com), Stanford University, and **Daniil Kalinov** (dkalinov1396@gmail.com), MIT. *The Structure of the Positive Monoid of Integer-Valued Polynomials Evaluated at an Algebraic Number*

In the ring $\mathbb{Q}[x]$ of polynomials with coefficients in the rational numbers, it is interesting to consider the subring of all integer-valued polynomials, i.e. polynomials $p(x)$ such that $p(n)$ is an integer for every integer n. This ring is known as the most natural and simple example of a non-Noetherian ring. One may wonder whether this is not just the set of all polynomials with integer coefficients. However, e.g. the polynomial $(x^2+x)/2$ is integer-valued. It turns out that this ring consists of exactly the polynomials with integer coefficients in the basis of binomial coefficients $\binom{x}{n}$. Motivated by the characterization of symmetric monoidal functors between Deligne categories, we examine the set $R_+(x)$ of polynomials which have nonnegative integer coefficients in this basis. More precisely, we study the set of values of these polynomials at a fixed number α. It turns out that this set has a fascinating algebraic structure, explicitly determined by the p-adic roots of the minimal polynomial of α, which we will fully describe in this talk. (Received September 21, 2021)

1174-11-10577 **Li-An Chen*** (lianchen@udel.edu), University of Delaware, and **Robert S Coulter** (coulter@udel.edu), University of Delaware. *Algorithms for computing the permutation resemblance of functions on finite groups*

Given two functions f, g on a finite group $(G,+)$, the resemblance of f to g is defined by the number of distinct images of the difference $f-g$. For $f: G \to G$, the permutation resemblance of f is defined by the minimum resemblance of f to any permutation of G. In this talk, we introduce a linear programming algorithm which gives the permutation resemblance of any function. We shall also discuss an upper bound for specific functions obtained by analyzing this algorithm. (Received September 21, 2021)

1174-11-10588 **Kevin Gomez** (kevin.j.gomez@vanderbilt.edu), Vanderbilt University. *Bounds for Coefficients of the $f(q)$ Mock Theta Function and Applications to Partition Ranks*

We compute effective bounds for $\alpha(n)$, the Fourier coefficients of Ramunujan's mock theta function $f(q)$ utilizing a finite algebraic formula due to Bruinier and Schwagenscheidt. We then use these bounds to prove two conjectures of Hou and Jagadeesan on the convexity and maximal multiplicative properties of the even and odd partition rank counting functions. (Received September 21, 2021)

1174-11-10658 **Sean Li*** (seanjli@mit.edu), Massachusetts Institute of Technology, **Casia Siegel** (casiaphonemail@gmail.com), University of Virginia, and **Apoorva Panidapu** (apoorva.panidapu@sjsu.edu), San Jose State University. *Tamagawa Products for Elliptic Curves Over Number Fields*

In recent work, Griffin, Ono, and Tsai constructs an $L-$series to prove that the proportion of short Weierstrass elliptic curves over $\mathbb{Q}$ with trivial Tamagawa product is $0.5054\ldots$ and that the average Tamagawa product is $1.8183\ldots$. Following their work, we generalize their $L-$series over arbitrary number fields K to be

$$L_{\text{Tam}}(K;s) := \sum_{m=1}^{\infty} \frac{P_{\text{Tam}}(K;m)}{m^s},$$

where $P_{\text{Tam}}(K;m)$ is the proportion of short Weierstrass elliptic curves over K with Tamagawa product m. We then construct Markov chains to compute the exact values of $P_{\text{Tam}}(K;m)$ for all number fields K and positive integers m. As a corollary, we also compute the average Tamagawa product $L_{\text{Tam}}(K;-1)$. We then use these results to uniformly bound $P_{\text{Tam}}(K;1)$ and $L_{\text{Tam}}(K,-1)$ in terms of the degree of K. Finally, we show that there exist sequences of K for which $P_{\text{Tam}}(K;1)$ tends to 0 and $L_{\text{Tam}}(K;-1)$ to ∞, as well as sequences of K for which $P_{\text{Tam}}(K;1)$ and $L_{\text{Tam}}(K;-1)$ tend to 1. (Received September 21, 2021)

1174-11-10684 **Jeffrey Hatley*** (hatleyj@union.edu), Union College, **Anwesh Ray** (ar2222@cornell.edu), University of British Columbia, and **Debanjana Kundu** (dkundu@math.ubc.ca), University of British Columbia. *Arithmetic Statistics for Iwasawa Invariants of Elliptic Curves*

We study the average behavior of the Iwasawa invariants for Selmer groups of elliptic curves, considered over anticyclotomic $\mathbb{Z}_p$-extensions in both the definite and indefinite settings. (Received September 21, 2021)

1174-11-10686 **Kaya Malika Lakein** (epi2@stanford.edu), Stanford University, and **Anne Larsen** (larsen@college.harvard.edu), Harvard University. *Supersingular Loci from Traces of Hecke Operators*

A classical observation of Deligne shows that, for any prime $p \geq 5$, the divisor polynomial of the Eisenstein series $E_{p-1}(z)$ mod p is closely related to the supersingular polynomial at p,

$$S_p(x) := \prod_{E/\overline{\mathbb{F}}_p \text{ supersingular}} (x - j(E)) \in \mathbb{F}_p[x].$$

Deuring, Hasse, and Kaneko and Zagier found other families of modular forms which also give the supersingular polynomial at p. In a new approach, we prove an analogue of Deligne's result for the Hecke trace forms $T_k(z)$ defined by the Hecke action on the space of cusp forms S_k. We use the Eichler-Selberg trace formula to identify congruences between trace forms of different weights mod p, and then relate their divisor polynomials to $S_p(x)$ using Deligne's observation. (Received September 21, 2021)

1174-11-10717 **Marie Jameson*** (mjameso2@utk.edu), University of Tennessee Knoxville. *Wronskians of graded dimensions* Preliminary report.

In the study of vertex operator algebras, one finds that the graded dimensions of irreducible modules are q-series that can often be understood as modular forms. Here we follow work of Milas and others to study the Wronskians of these and related q-series. (Received September 21, 2021)

1174-11-10796 **Santanu Chakraborty*** (santanu.chakraborty@utrgv.edu), University of Texas Rio Grande valley. *A study on binary representation of odd numbers in a Collatz sequence* Preliminary report.

In the 80 year old Collatz Conjecture, one talks about a sequence consisting of positive integers where if a term in the sequence is even, the next term is obtained by dividing it by 2 and if a term is odd, we multiply it by 3 and add 1 to get the next term in the sequence. The conjecture states that such a sequence is a terminating sequence with the last term in the sequence being 1. Here we do not claim to prove the conjecture, but we prove some interesting results that may help future researchers. For this, we classified the odd positive integers in to two broad groups, $\mathcal{G}_1$ and $\mathcal{G}_2$: $\mathcal{G}_1$ consists of numbers of the form $4m + 1$ and $\mathcal{G}_2$ consists of numbers of the form $4m + 3$. One preliminary observation about these two groups is, if an odd number in the Collatz sequence belongs to $\mathcal{G}_1$ ($\mathcal{G}_2$), the next odd number in the sequence is smaller (bigger). One of our initial results studies that if an odd number in a Collatz sequence belongs to $\mathcal{G}_2$, then assuming that number of consecutive ones starting from the units' position in the binary representation is l ($l \geq 2$), the next $l - 2$ odd numbers in the sequence also belong to $\mathcal{G}_2$ but the $(l-1)$st number must belong to $\mathcal{G}_1$. And our main result thoroughly investigates how the binary representation of an odd number in a Collatz sequence determines the group for the the next odd number in the sequence - $\mathcal{G}_1$ or $\mathcal{G}_2$. (Received September 21, 2021)

1174-11-10811 **Tomas Guardia** (guardia@gonzaga.edu), Gonzaga University, and **Lauren Spensiero*** (lspensiero@zagmail.gonzaga.edu), Gonzaga University. *Pellquadratic and Jacobsthalquadratic Numbers* Preliminary report.

With relation to the introductory research by Guardia, Jimenez, and McCurdy on the Fiboquadratic numbers and their properties in the medieval board game, Rithmomachia, this new research creates an alternative extension of the Rithmomachia board in terms of Pellquadratic and Jacobsthalquadratic numbers. We have found that the new extensions uphold the definitions of Greek Number Theory, and have similar properties as the Fiboquadratic numbers, such as the boards extending to positive and negative infinity, they each reach their respective alpha and beta values. In addition, the superpartients for the Pell and Jacobsthal numbers have been proven to hold the same qualities as the Fibonacci numbers. (Received September 21, 2021)

1174-11-10817 **Kevin J. McGown*** (kmcgown@csuchico.edu), California State University, Chico, and **Daniel Vallieres** (dvallieres@csuchico.edu), California State University, Chico. *Abelian p-towers of multigraphs*

In the 1950s, Iwasawa proved his celebrated theorem on the growth of the p-part of the class number in $\mathbb{Z}_p$ extensions of number fields. We prove an analogous result in graph theory involving the p-part of the number of spanning trees in certain infinite towers of graphs. The proof involves p-adically interpolating shifted Chebyshev polynomials and the value of the Artin–Ihara L-function at $s = 1$. This work is joint with Daniel Vallières. (Received September 21, 2021)

1174-11-10924 **Yanan Jiang*** (yanan_jiang22@milton.edu), Milton Academy, MIT PRIMES. *Generalizing Ruth-Aaron Numbers*

Let $f(n)$ be the sum of the prime divisors of n, counted with multiplicity; thus, $f(2020) = f(2^2 \cdot 5 \cdot 101) = 110$. *Ruth-Aaron* numbers, or integer n with $f(n) = f(n+1)$, have been an interest of many number theorists since the famous 1974 baseball game gave them the elegant name after two baseball stars. Many of their properties were first discussed by Erdös and Pomerance in 1978. In this paper, we generalize their results in two directions: by raising prime factors to a power and allowing a small difference between $f(n)$ and $f(n+1)$. We prove that the number of integers up to x with $f_r(n) = f_r(n+1)$ is $O\left(\frac{x(\log\log x)^3 \log\log\log x}{(\log x)^2}\right)$, where $f_r(n)$ is the *Ruth-Aaron* function replacing each prime factor with its $r--$th power. We also prove the density of n remains 0 if $|f_r(n) - f_r(n+1)| \leq k(x)$, where $k(x)$ is a function of x with relatively low rate of growth. Moreover, we further the discussion of the infinitude of *Ruth-Aaron* numbers and provide a few possible directions for future study. (Received September 21, 2021)

1174-11-10959 **Thomas Wright*** (wrighttj@wofford.edu), Wofford College. *Prime gaps and siegel zeroes* Preliminary report.

For an L-function $L(s, \chi_D)$, we call β a Siegel zero if $L(\beta, \chi_D) = 0$, β is real, and $1 - \beta > \frac{1}{\eta \log D}$ for some $\eta > 0$. In 1983, Roger Heath-Brown proved that if there are infinitely many D for which $L(s, \chi_D)$ has a Siegel zero then there are infinitely many twin primes. In this talk, we prove a generalization of this result to larger prime tuples as follows. Fix a large r, and assume that there are infinitely many D for which $L(s, \chi_D)$ has a zero β with $1-\beta > \frac{1}{(\log D)^r}$. Then there are infinitely many m-tuples of primes $(p_1, \cdots, p_m)$ where $p_m - p_1 \ll e^{1.9828m}$. This "improves" (in some sense) on the bounds of Maynard-Tao, Baker-Irving, and Polymath 8b, who found bounds of $e^{3.815m}$ unconditionally and me^{2m} assuming the Elliott-Halberstam conjecture. (Received September 21, 2021)

1174-11-11007 **Carrie E. Finch-Smith*** (finchc@wlu.edu), Washington & Lee University. *Searching for Special Sierpiński numbers* Preliminary report.

A Sierpiński number is an odd positive integer k with the property that $k \cdot 2^n + 1$ is composite for all natural numbers n. In this talk, we discuss Sierpiński numbers that can be found in other special integer sequences. (Received September 21, 2021)

1174-11-11008 **Alyssa Brasse** (brassealyssa@gmail.com), Hunter College of City University of New York, **Nevin Etter** (ettern23@mail.wlu.edu), Washington and Lee University, **Gustavo Flores** (floresg9055@gmail.com), Carleton College, **Drew Miller** (anmiller@ucsb.edu), University of California Santa Barbara, Santa Barbara, and **Summer Soller*** (summersoller@icloud.com), University of Utah. *Minimal discriminants of rational elliptic curves with prescribed isogeny degree*

For a positive integer n, we say that an elliptic curve E exhibits a degree n isogeny if E has a cyclic subgroup of order n. By studying parameterized families of elliptic curves, we explicitly classify the minimal discriminants of rational elliptic curves whose isogeny degree is $n = 6, 8, 9$. This work is part of PRiME (Pomona Research in Mathematics Experience, H98230-21-1-0015). (Received September 21, 2021)

1174-11-11012 **Ayse Alaca*** (aysealaca@cunet.carleton.ca), Carleton University. *Some Eisentein series in terms of Ramanujan's theta functions*

We express some Eisenstein series, for example, $\sum_{n=1}^{\infty}\Big(\sum_{d|n}\Big(\frac{5}{d}\Big)\Big(\frac{8}{n/d}\Big)d\Big)q^n$ and $\sum_{n=1}^{\infty}\Big(\sum_{d|n}\Big(\frac{5}{n/d}\Big)\Big(\frac{8}{d}\Big)d\Big)q^n$, in terms of Ramanujan's theta functions. (Received September 21, 2021)

1174-11-11039 **Pavel Coupek*** (pcoupek@purdue.edu), Purdue University. *Ramification bounds for mod p étale cohomology via prismatic cohomology*

I will discuss new upper bounds on ramification of Galois representations obtained as the mod p étale cohomology of a proper smooth formal scheme over $\mathcal{O}_K$ where K is a local number field. These bounds are expressed in terms of the prime p, the cohomological degree i, and the absolute ramification index e of K, and work for arbitrarily large i and e. A crucial tool in obtaining the bounds is the recently developed prismatic cohomology in its Breuil–Kisin and A_{inf}–instances, and a series of conditions $(\mathrm{Cr}_s)_{s\geq 0}$ whose purpose is to control the Galois action on the elements of the Breuil–Kisin cohomology inside the A_{inf}–cohomology. (Received September 21, 2021)

1174-11-11054 **Matthew Ronald Just*** (mrjust@emory.edu), Emory University. *Semi-modular forms*

We identify a class of "semi-modular" forms invariant on special subgroups of $GL_2(\mathbb{Z})$ which includes classical modular forms together with complementary classes of functions that are also nice in a specific sense. We then discuss advances towards a classification of semi-modular forms, along with generalizations. This is joint work with R. Schneider. (Received September 21, 2021)

1174-11-11067 **Rachel Davis** (rachel.davis@wisc.edu), University of Wisconsin-Madison, **Edmond Anderson*** (edmond.anderson@morehouse.edu), Morehouse College, **Aurora Hiveley** (ahiveley@macalester.edu), Macalester College, **Cyna Nguyen** (cyna.nguyen@student.csulb.edu), Cal State Long Beach, and **Daniel Tedeschi** (tedeschi@grinnell.edu), Grinnell College. *Monodromy of Compositions of Belyĭ* Preliminary report.

Say that $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a Dynamical Belyĭ map. Given any Toroidal Belyĭ map $\gamma : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$, the composition $\beta \circ \gamma : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is also a Toroidal Belyĭ map. There is a group $\mathrm{Mon}(\beta)$, the monodromy group, which contains information about the symmetries of a Belyĭ map β. It is well-known that, for any Toroidal Belyĭ map γ,

(1) there is always a surjective group homomorphism $\mathrm{Mon}(\beta\circ\gamma) \twoheadrightarrow \mathrm{Mon}(\beta)$, and

(2) the monodromy group $\mathrm{Mon}(\beta\circ\gamma)$ is contained in the $\mathrm{Mon}(\gamma) \wr \mathrm{Mon}(\beta)$.

In this project, we study how the three groups $\mathrm{Mon}(\beta)$, $\mathrm{Mon}(\beta\circ\gamma)$, and $\mathrm{Mon}(\gamma) \wr \mathrm{Mon}(\beta)$ compare as we vary over Dynamical Belyĭ maps β. This is work done as part of the Pomona Research in Mathematics Experience (NSA H98230-21-1-0015). (Received September 21, 2021)

1174-11-11070 **Karen E Taylor*** (karen.taylor@bcc.cuny.edu), Bronx Community College, Cuny. *Hyperbolic Shintani Lift* Preliminary report.

The Shintani lift on $\mathrm{G}_0(4)$ is constructed using cycle integrals which are 0th order hyperbolic coefficients of a modular form In this talk we will discuss constructing a Shintani type map attached to the nth hyperbolic coefficient of a modular form. This is work in progress with Larry Rolen. (Received September 21, 2021)

1174-11-11079 **Leo Goldmakher** (lg5@williams.edu), Williams College, **Huy Tuan Pham** (huypham@stanford.edu), Stanford University, and **Gal Gross** (g.gross@mail.utoronto.ca), University of Toronto. *Large Sets are Sumsets*

Let $[n] := \{0, 1, 2, ..., n\}$. Intuitively, all large subsets of $[n]$ have additive structure, and Roth famously made this precise by finding constants $c, N > 0$ such that for $n \geq N$, any subset of $[n]$ containing more than $cn/\log\log n$ elements *must contain* an arithmetic progression of length 3. We establish a different interpretation of the intuition by finding explicit constants α and β such that:

(i) any subset of $[n]$ with more than $n - \alpha$ log n elements has a nontrivial decomposition as the sum of two sets, and

(ii) there exists a subset of $[n]$ of size at least $n - \beta \log n$ that has no such decomposition.

Our methods allow us to prove a higher-dimensional analogue of this result as well, which we will discuss. (Received September 21, 2021)

1174-11-11081 **Saban Alaca*** (sabanalaca@cunet.carleton.ca), Carleton University. *Modular Forms and Convolution Sums with Applications to Representations of Integers by Certain Quadratic Forms*

We first evaluate certain convolution sums using modular forms. We then use our evaluations together with known evaluations of other convolution sums to determine the number of representations of a positive integer n by certain quadratic forms. (Received September 21, 2021)

1174-11-11087 **George D. Hauser*** (gdh43@math.rutgers.edu), Rutgers University. *Eisenstein Series on Higher Covers of SL(2,R)* Preliminary report.

Eisenstein series on covers of SL(2) are the subject of much interest in the theory of higher theta functions, although it is necessary to extend the base field to contain roots of unity. In this talk, I will explore a construction of some Eisenstein series on high covers of SL(2,R) without extending the base field. The main result is the computation of the poles and residues of this series, which are automorphic forms on the universal cover of SL(2,R). For technical convenience we will use language of automorphic distributions. (Received September 21, 2021)

1174-11-11158 **Ajmain A Yamin*** (ayamin@gradcenter.cuny.edu), CUNY, The Graduate Center. *Complete Regular Dessins* Preliminary report.

A complete regular map is an embedding of a complete graph K_n on n vertices into an oriented surface which is "as symmetric as possible", i.e. its automorphism group has order $n(n-1)$. Examples include the regular tetrahedron, which is a regular embedding of the complete graph on four vertices in the sphere, and the dual Heawood map, which is a regular embedding of K_7 in the torus. In 1985, Lynne D James and Gareth A Jones, building on the 1971 work of Norman Biggs, classified all complete regular maps. In this talk, I will view complete regular maps from an algebro-geometric viewpoint. Namely, I will study the dessins d'enfants associated to complete regular maps and produce some explicit affine models of their Belyi pairs using the Weierstrass p function, ideas from Galois theory and number theory. (Received September 21, 2021)

1174-11-11229 **Louis M Gaudet*** (lmg289@math.rutgers.edu), Rutgers University. *Counting Zeros of Indefinite Forms* Preliminary report.

In 1926, H. Kloosterman developed a refinement of the Hardy-Littlewood circle method that allowed him to produce asymptotics for the representation numbers of quaternary quadratic forms. We show how the same method can be used to count representations of 0 by indefinite quaternary forms. In fact, we are able to give results that display the dependence on the coefficients of the form, which is important for applications. (Received September 21, 2021)

1174-11-11233 **Benjamin Toomey*** (toomeyb@oregonstate.edu), Oregon State University. *Zeros of Modular Functions For Some Genus Zero Groups* Preliminary report.

We demonstrate a method to locate the zeros of a basis for the space of weakly holomorphic modular functions for certain genus zero groups having one cusp (the normalizers of $\Gamma_0(m)$, $m \leq 10$). This method uses the theory of twisted Hecke Operators, which are closely associated with Monstrous Moonshine, and exploits relationships between groups of the form $\Gamma_0(mh|h) + e, f, g, \ldots$. We give a geometric construction for approximating the modular functions of the form $F_n(q) = q^{-n} + O(q)$ for these groups, and sketch how this may be used to locate the zeros of F_n. (Received September 21, 2021)

1174-11-11275 **Benjamin Peet** (bpeet@stmartin.edu), Saint Martin's University, **Rebekah Linh Kuss*** (rebekah.kuss@gmail.com), Saint Martin's University, **Jamie Bishop** (Jamie.Bishop@stmartin.edu), Saint Martin's University, and **Abigail Bozarth** (Abigail.Bozarth@stmartin.edu), Saint Martin's University. *The Abundancy Index and Feebly Amicable Numbers* Preliminary report.

This research explores the sum of divisors - $\sigma(n)$ - and the abundancy index given by the function $\frac{\sigma(n)}{n}$. We give a generalization of amicable pairs - feebly amicable pairs (also known as harmonious pairs), that is m, n such that $\frac{n}{\sigma(n)} + \frac{m}{\sigma(m)} = 1$. We first give some groundwork in introductory number theory, then the goal of the paper is to determine if all numbers are feebly amicable with at least one other number by using known results about the abundancy index. We establish that not all numbers are feebly amicable with at least one other number. We generate data using the R programming language and give some questions and conjectures. (Received September 22, 2021)

1174-11-11288 **Claire Frechette*** (frech014@umn.edu), University of Minnesota, Twin Cities. *Yang-Baxter Equations for General Metaplectic Ice*

We extend results connecting quantum groups to spherical Whittaker functions on metaplectic covers of $GL_r(F)$, for F a nonarchimedean local field. Brubaker, Buciumas, and Bump showed that for a certain metaplectic n-fold cover of $GL_r(F)$ a set of Yang-Baxter equations model the action of standard intertwiners on principal series Whittaker functions. These equations arise from a Drinfeld twist of the quantum affine Lie superalgebra $U_{\sqrt{v}}(\widehat{\mathfrak{gl}}(n))$ where $v = q^{-1}$ for q the cardinality of the residue field. We extend their results to all metaplectic covers of $GL_r(F)$, providing new solutions to Yang-Baxter equations matching the scattering matrix for the associated Whittaker functions. Each cover has an associated integer invariant n_Q and the resulting solutions are connected to the quantum group $U_{\sqrt{v}}(\widehat{\mathfrak{gl}}(n_Q))$ and quantum superalgebra $U_{\sqrt{v}}(\widehat{\mathfrak{gl}}(1|n_Q))$. (Received September 28, 2021)

1174-11-11768 **Naiomi T. Cameron** (naiomi.cameron@spelman.edu), Spelman College, **Caleb Ashley** (ashley.cj@gmail.com), Boston College, and **Karoline Pershell** (pershell@utm.edu), University of Tennessee at Martin. *Monodromy Groups of Compositions of Belyĭ Maps.* Preliminary report.

Given a Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ of degree n, it is well known that its monodromy group $\mathrm{Mon}(\beta)$ is a subgroup of the symmetric group S_n. In fact, this group can be viewed as the "Galois closure" of the automorphism group $\mathrm{Aut}(\beta) \subseteq S_n$. Given a composition $\beta \circ \phi : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ of two Belyĭ maps β and γ, it is known that the monodromy group $\mathrm{Mon}(\beta \circ \phi) \subseteq \mathrm{Mon}(\phi) \wr \mathrm{Mon}(\beta)$ is contained in the wreath product of the monodromy groups of each of the maps. However, when do we have equality? And what exactly is the relationship between these three groups?

In the 2018 doctoral thesis of Jacob Bond, there was an explicit description of the relationship between these three groups. Explicitly, $\mathrm{Mon}(\beta \circ \gamma) \simeq \rho_\gamma(A) \wr \mathrm{Mon}(\beta)$ where $\rho_\gamma(A)$ is a subgroup of the collection of maps $E_\beta \to \pi_1(X) \to \mathrm{Mon}(\gamma)$ from the edges of the Dessin d'Enfant of β to the Fundamental Group of the thrice punctured sphere $X = \mathbb{P}^1(\mathbb{C}) \backslash \{0, 1, \infty\}$ to the monodromy group of ϕ. In this talk, we explain the details, and provide some examples.

This is based on work done as part of ADJOINT with Caleb Ashley (Boston College), Naiomi Cameron (Spelman College), Emille Davie Lawrence (University of San Francisco), Theo McKenzie (UC Berkeley), and Karoline Pershell (Service Robotics & Technologies). (Received October 20, 2021)

1174-11-12232 **Alex Cohen*** (alex.cohen@yale.edu), Yale University. *An optimal inverse theorem for polynomials over large fields.*

A polynomial f over a finite field has a large bias if its output distribution is far from uniform. It has rank r if we can write f as a function of polynomials $g_1, \ldots, g_r$ that have smaller degree. Bias measures the amount of randomness, and rank measures the amount of structure. It is known that if f has small rank, it must have large bias—or in other words, if f is highly structured, it does not behave randomly. Green and Tao proved an inverse theorem stating that if f is significantly biased, its rank cannot be too large. Their bound was qualitative, however, and several authors gave quantitative improvements, including a recent breakthrough of Milicevic and independently Janzer who proved a polynomial relationship between the rank and the log of the bias. We prove an optimal inverse theorem: the rank and the log of the bias are equivalent up to linear factors (over large enough fields). Our techniques are very different from the usual methods in this area, we rely on algebraic geometry rather than additive combinatorics. Heuristically, our theorem states that for polynomials, all of the bias can be explained by the rank. (Received December 2, 2021)

12 ► Field theory and polynomials

1174-12-9168 **Jeffery Yu*** (jeffery@mit.edu), Massachusetts Institute of Technology, and **Guy Moshkovitz** (guymoshkov@gmail.com), Department of Mathematics, City University of New York (Baruch College). *Sharp Effective Nullstellensatz over Finite Fields* Preliminary report.

Hilbert's Nullstellensatz is a central theorem in classical algebraic geometry, relating ideals of polynomials over algebraically closed fields to their locus of common zeros. Kollar gave sharp effective bounds for this in 1988. We study finite field analogues of the Nullstellensatz. Green and Tao proved an effective version, showing that the degrees of the polynomials can be made independent of the number of variables, but with an Ackermann-type dependence on the other parameters. We give a new simple proof of the finite field Nullstellensatz that achieves linear bounds. The linear dependence on each parameter is tight up to an absolute constant. (Received September 21, 2021)

1174-12-9754 **Neil P. Sigmon*** (npsigmon@radford.edu), Radford University, **Rick E. Klima** (klimare@appstate.edu), Appalachian State University, and **Adam S. Downs** (asdowns@radford.edu), Radford University. *Integrating Reed-Solomon codes into QR codes* Preliminary report.

Since their description in a 1960 paper by Irving Reed and Gustave Solomon, Reed-Solomon codes have been widely used to ensure reliable transmission of data. Reed-Solomon codes involve sending information in the form of polynomial coefficients using finite field arithmetic. Among their many applications, Reed-Solomon codes are used to correct errors when scanning Quick Response (QR) codes, which are two-dimensional bar codes that have become a common medium for easily accessing information such as URLs, phone numbers, and small amounts of text. Utilizing Reed-Solomon codes allows logos and emblems to be embedded within QR codes to advertise their purpose. In this presentation we will present how QR codes are constructed, and how Reed-Solomon codes are incorporated into them to provide error correction. Technology involving Maplets will be used to demonstrate examples of how QR codes can be generated. This information provides a nice means for demonstrating a hands-on method for the use of mathematics in a practical real-life application for students. (Received September 20, 2021)

1174-12-10288 **Hadley Vaughn*** (Hvaughn@stedwards.edu), St. Edward's University. *Lill Paths and Beloch Squares*

Lill's method entails constructing a path from polynomial coefficients to find polynomial roots. We examine the properties of Lill paths and how they relate to the Beloch Square, a geometric technique for solving cubic roots through paper folding and point identification. (Received September 21, 2021)

1174-12-11772 **Daniel Reuben Krashen** (daniel.krashen@gmail.com), Rutgers University, **Cory Colbert*** (ccolbert@wlu.edu), Washington and Lee University, **Edinah K. Gnang** (egnang1@jhu.edu), Johns Hopkins University, **Lori D Watson** (Lori.d.watson@gmail.com), Wake Forest University, and **Ulrica Wilson** (Ulrica.wilson@morehouse.edu), Morehouse College. *Parametrizing Special Galois Extensions.* Preliminary report.

It is a difficult open problem in number theory to determine which finite groups occur as the Galois groups of number fields. In this talk we tackle a related problem of parametrizing Galois extensions for a given group which satisfy extra constraints such as fixing their discriminant, or fixing a resultant. We do this by using a variation on Noether's classical method of constructing generic Galois extensions, and the technology of algebraic tori and torsors for finite group schemes. (Received October 20, 2021)

13 ► Commutative algebra

1174-13-6505 **Aleksandra C Sobieska*** (asobieska@math.wisc.edu), University of Wisconsin - Madison, and **Maya Banks** (mdbanks@wisc.edu), University of Wisconsin - Madison. *Subcomplexes of the Koszul Complex*

What are the subcomplexes of the Koszul complex? In this talk, I will explain how the BGG correspondence and the Kruskal–Katona theorem provide an answer to this seemingly elementary question, as well as insight to the parallel question for the Eagon–Northcott complex. (Received September 9, 2021)

1174-13-6845 **Jay Yang** (yangj306@mcmaster.ca), McMaster University, **Patricia Klein** (klein847@umn.edu), University of Minnesota, **Michael Loper*** (michael.loper@uwrf.edu), University of Wisconsin-River Falls, and **Christine Berkesch** (cberkesc@umn.edu), University of Minnesota. *Virtually Cohen-Macaulay Stanley-Reisner rings*

In 2017, Berkesch, Erman, and Smith introduced virtual resolutions for toric varieties as an analogue of minimal free resolutions for projective varieties. A module over the Cox ring of a smooth projective toric variety is virtually Cohen–Macaulay if it has a virtual resolution whose length is equal to the module's codimension. In this talk we will discuss a class of virtually Cohen–Macaulay rings that come from simplicial complexes. (Received September 9, 2021)

1174-13-7093 **Gradmar E. Maldonado Marti*** (gradmar.maldonado@upr.edu), UPR Mayagüez, and **Reyes M. Ortiz Albino** (reyes.ortiz@upr.edu), UPR Mayagüez. *Actions and Factorizations* Preliminary report.

In 2006, Anderson and Frazier defined the notion of generalized factorizations using a restriction of the multiplicative operation with respect to a symmetric relation τ on the non-zero and non-unit elements of an integral domain. The idea is to allow only the elements that are related with respect to τ to multiply. This notion has been studied since 2003 through 2014 by Anderson's PhD students, but very few results have been published since. Relations play an important role, as they provide information crucial to new ideas for research, and their study will look for several additional properties of relations. Anderson and Frazier defined three relations not seen before. However, the nature of the properties of relations need further study. The proposed research project hopes to define a type of action of the domain D on a relation τ, coordinate-wise. We expect to expand our toolset to characterize properties of relations and the potential implications they might have in the theory of generalized factorizations. (Received September 12, 2021)

1174-13-7121 **Thai Thanh Nguyen*** (tnguyen11@tulane.edu), Tulane University. *Newton-Okounkov Bodies and Analytic Spread of Graded Systems of Monomial Ideals*

Newton-Okounkov bodies are convex sets associated to algebro-geometric objects, that was first introduced by Okounkov in order to show the log-concavity of the degrees of algebraic varieties. In special cases, Newton-Okounkov bodies associated to graded systems of ordinary powers and symbolic powers of a monomial ideal are Newton polyhedron and symbolic polyhedron of the ideal. Studying these polyhedra can be beneficial to the study of relation between ordinary powers, integral closure powers and symbolic powers of a monomial ideal as well as its algebraic invariants. In this talk, I will survey some known results in this subject and present our results on computing and bounding the analytic spread of a graded system of monomial ideals and some related invariants through the associated Newton-Okounkov body.

This is joint work with Tài Huy Hà. (Received September 12, 2021)

1174-13-7152 **Karl Schwede*** (schwede@math.utah.edu), University of Utah, **Alicia Mae Lamarche** (alicial@math.sc.edu), University of Utah, and **Christopher Hacon** (hacon@math.utah.edu), University of Utah. *Properties of mixed characteristic test ideals*

We discuss a definition of mixed characteristic test ideals for rings of of finite type over a complete local Noetherian ring that commutes with localization. We prove Skoda-type theorems for this test ideal and also describe a restriction theorem. (Received September 13, 2021)

1174-13-7628 **Daniel Max Erman*** (derman@math.wisc.edu), University of Wisconsin-Madison, and **Michael Brown** (mkb0096@auburn.edu), Auburn University. *Results and questions in multigraded commutative algebra*

Just like graded rings have more structure than local rings, we should expect facts about standard graded polynomial rings to have richer analogues when we extend them to a multigraded setting. I will discuss some recent results in this vein, with a focus on syzygies and homological algebra for multigraded polynomial rings. (Received September 15, 2021)

1174-13-7651 **Vi Anh Nguyen*** (nguyenvi@grinnell.edu), Grinnell College, and **Zion Hefty** (heftyzio@grinnell.edu), Grinnell College. *Minimal Generating Sets of Determinantal Ideals in Alternating Matrices* Preliminary report.

In commutative algebra and algebraic geometry, a topic of interest is ideals of minors of matrices. In this project, we consider the problem of finding a minimal generating set for the ideal of t-minors in the $n \times n$ generic alternating matrix. We conjecture different such sets for matrices over fields depending on characteristic and prove that they are generating sets. Further, we discuss various attempts at proving minimality, including adapting Aldo Conca's proof for symmetric matrices in the characteristic 2 case. (Received September 15, 2021)

1174-13-7655 **Sara Faridi** (faridi@dal.ca), Dalhousie University, **Sabine El Khoury** (se24@aub.edu.lb), American University of Beirut, **Susan Marie Cooper** (susan.cooper@umanitoba.ca), University of Manitoba, **Sarah Mayes-Tang** (smt@math.toronto.edu), University of Toronto, **Liana M. Sega** (segal@umkc.edu), University of Missouri Kansas City, **Susan Morey*** (morey@txstate.edu), Texas State University, and **Sandra Spiroff** (spiroff@olemiss.edu), University of Mississippi. *Resolutions of Powers of Square-free Monomial Ideals* Preliminary report.

Using combinatorial structures to obtain resolutions of monomial ideals traces back to Diana Taylor's thesis, where a simplex associated to the generators of a monomial ideal was used to construct a free resolution of the ideal. This concept has been expanded, with various authors determining conditions under which simplicial or cellular complexes can be associated to monomial ideals to produce a free resolution. This talk will focus on examining structures that produce free resolutions of powers of square-free monomial ideals. The properties of the original monomial ideal determine which structures are used and whether the resulting resolution is cellular or simplicial. This work is part of a research project initiated at a BIRS workshop "Women in Commutative Algebra" in Fall 2019. (Received September 15, 2021)

1174-13-7679 **Vinh Nguyen*** (vinhngn@gmail.com), Purdue University, and **Hunter Simper** (hsimper@purdue.edu), Purdue University. *Heights of ideals of minors of matrices with a given rank.* Preliminary report.

A well-known result is Krull's principal ideal theorem, which bounds the height of an ideal by the number of generators of that ideal. There has been multiple generalization of this result, one of which is the Eagon-Northcott bound for the ideals of minors of a matrix. The Eagon-Northcott bound is sharp in general, in particular it is sharp for generic matrices. However if we know more about the matrix, such as it being symmetric, then the Eagon-Northcott bound is far from being sharp. A better bound can be proven, using the same technique in Eagon-Northcott's original paper, for symmetric matrices. As a separate consideration, if we have information on the rank of the matrix a better bound can also be produced, which is a result by Bruns. Now, we can combine the two conditions of being symmetric and having a specified rank and ask for the sharpest bound for the ideals of minors. This is a conjecture by Eisenbud, Huneke, and Ulrich.

There is a general program that one can follow to come up with natural bounds for a whole family of conditions on the matrix. In this talk I'll present some background on the topic and then talk about the program. I'll conclude with some known results on the conjecture by Eisenbud, Huneke, and Ulrich, and then some small results on the conjecture from my recent work with Hunter Simper. (Received September 15, 2021)

1174-13-7717 **Sarasij Maitra*** (sm3vg@virginia.edu), University of Virginia. *Reflexive and I-Ulrich Modules*

In this talk, we will discuss the notion of an I-Ulrich module in dimension one Cohen-Macaulay local rings. In a joint work with Hailong Dao and Prashanth Sridhar, we explored this tool to study the class of reflexive modules (even ideals) and birational extensions over a one-dimensional reduced Cohen-Macaulay local ring. If time permits, we will briefly venture into those results as well. (Received September 15, 2021)

1174-13-7782 **Anton Dochtermann** (anton.dochtermann@gmail.com), Texas State University, and **Andrew Newman*** (andrew.newman.775@gmail.com), Carnegie Mellon University. *Random subcomplexes and Betti numbers of random edge ideals*

The coedge ideal of an Erdős–Rényi random graph is a model for random squarefree monomial ideals. Using Hochster's formula one can study and interpret properties of the resulting random monomial ideal in terms of the topology of the flag complex of the random graph. By applying methods from stochastic topology we prove sharp bounds on the regularity and projective dimension of random coedge ideals in a probability regime where the Krull dimension is bounded. (Received September 16, 2021)

1174-13-7887 **Luigi Ferraro*** (lferraro@ttu.edu), Texas Tech University, **Lars Winther Christensen** (lars.w.christensen@ttu.edu), Texas Tech University, and **Peder Thompson** (thompson@niagara.edu), Niagara University. *Rigidity of Ext and Tor via flat-cotorsion theory*

Let $\mathfrak{p}$ be a prime ideal in a commutative noetherian ring R. We prove that if an R-module M satisfies $\operatorname{Ext}_R^{n+1}(k(\mathfrak{p}), M) = 0$ for some $n \geqslant \dim R$, where $k(\mathfrak{p})$ is the residue field at $\mathfrak{p}$, then $\operatorname{Ext}_R^i(k(\mathfrak{p}), M) = 0$ holds for all $i > n$. This is an improvement of a result of Christensen, Iyengar and Marley. Similar improvements concerning homological dimensions and the rigidity of Tors are proved. The main tool that we use to provide these improvements is the existence of minimal semi-flat-cotorsion replacements. (Received September 16, 2021)

1174-13-7893 **Michael Debellevue*** (michael.debellevue@huskers.unl.edu), University of Nebraska Lincoln. *Graded Deviations and the Koszul Property* Preliminary report.

The graded deviations $\varepsilon_{ij}(R)$ of a graded ring R record the vector space dimensions of the graded pieces of a certain Lie algebra attached to the minimal resolution of the quotient of R by its homogeneous maximal ideal. Vanishing of deviations encodes properties of the ring: for example, $\varepsilon_{ij}(R) = 0$ for $i \geq 3$ if and only if R is complete intersection and, provided R is standard graded, $\varepsilon_{ij}(R)$ whenever i is not equal to j implies R is Koszul. We extend this fact by showing that if $\varepsilon_{ij}(R) = 0$ whenever j and $i \geq 3$, then R is a quotient of a Koszul algebra by a regular sequence. This answers a conjecture by Ferraro. (Received September 16, 2021)

1174-13-7911 **Kuei-Nuan Lin*** (Kueinuanlin@gmail.com), The Penn State University, Greater Allegheny. *Symbolic powers of generalized star configurations of hypersurfaces*

We introduce the class of sparse symmetric shifted monomial ideals. These ideals have linear quotients and their Betti numbers are computed. Using this, we prove that the symbolic powers of the generalized star configuration ideal are sequentially Cohen–Macaulay under some mild genericness assumption. With respect to these symbolic powers, we also consider the Harbourne–Huneke containment problem and establish the Demailly-like bound. (Received September 16, 2021)

1174-13-7918 **Vaibhav Pandey*** (pandey@math.utah.edu), University of Utah. *Are classical determinantal rings direct summands of polynomial rings?*

Over an infinite field, the classical determinantal rings are known to be fixed subrings of the action of a linearly reductive group on a polynomial ring. It then follows from a theorem of Hochster and Roberts that these determinantal rings are direct summands of polynomial rings (in characteristic zero).

In this talk we investigate if these determinantal rings continue to be direct summands of the polynomial rings in which they naturally embed into by the above group action in characteristic p¿0. These groups are known to be not linearly reductive in characteristic p¿0! This is joint work with Mel Hochster, Jack Jeffries, and Anurag Singh. (Received September 16, 2021)

1174-13-7921 **Mohamed Essam Lotfi Topala*** (mohamed.lotfi@wagner.edu), Wagner College, and **Zachary Lihn** (zachlihn@gmail.com), Columbia University. *Betti numbers and Axial constants of homogenous ideals* Preliminary report.

Given a homogeneous ideal I of a polynomial ring, two numerical invariants may be defined: the Betti numbers β_i (I) and the axial constants $a_i(I)$. The Betti numbers, defined for each i as the rank of the i-th differential matrix of a minimal free resolution for I, are studied for characteristic = 2. Using the software Macaulay2, we calculate $\{\beta_i\}_{i\geq 0}$ for all ideals generated by quadratic homogeneous polynomials over $\mathbb{F}_2$. We also obtain a classification for the structure of minimal free resolutions for those ideals with up to 3 generators. The second invariant is the i-th axial constant, $a_i(I)$, of a homogeneous ideal I, which is defined as the smallest power of the variable x_i that belongs to the generic initial ideal of I. We investigate when $a_i(n)$ is defined for certain families of ideals. Furthermore, we examine the linearity of $a_i(n)$, the i-th axial constant of the ideal I^n, viewed as a function of n, and show that it is linear for all i for sufficiently large n. We also study different classes of monomial ideals, including those generated by a set of powers of variables, and the behavior of their axial constants. Finally, we establish different relations between the axial constants and other invariants such as regularity. (Received September 16, 2021)

1174-13-7980 **Todd Morra*** (tmorra@clemson.edu), Clemson University. *Minimal Differential Graded Algebra Resolutions of Certain Stanley-Reisner Rings* Preliminary report.

Stanley-Reisner rings possess rich algebraic information about the rings encoded as geometric information found in simplicial complexes. Using independence complexes of K_1-coronas of simple graphs, we find a more general class of simplicial complex that is always Cohen-Macaulay, and the Stanley-Reisner rings they determine each have an easily computed type, and admit a canonical module that is likewise easy to write down. Based on work by D'Alì, Fløystad, Nematbakhsh, these Stanley-Reisner rings also have an explicit finite free resolution. In this talk we share progress towards explicitly defining a differential graded algebra structure for these resolutions. (Received September 17, 2021)

1174-13-8056 **John Machacek*** (jmachacek.math@gmail.com), University of Oregon, and **Nicholas Ovenhouse** (ovenh001@umn.edu), University of Minnesota. *Discrete Dynamical Systems From Real Valued Mutation*

We introduce a family of discrete dynamical systems which includes, and generalizes, the mutation dynamics of rank two cluster algebras. These systems exhibit behavior associated with integrability, namely preservation of a symplectic form, and in the tropical case, the existence of a conserved quantity. We show in certain cases that the orbits are unbounded. The tropical dynamics are related to matrix mutation, from the theory of cluster algebras. We are able to show that in certain special cases, the tropical map is periodic. We also explain how our dynamics imply the asymptotic sign-coherence observed by Gekhtman and Nakanishi in the 2-dimensional situation. (Received September 17, 2021)

1174-13-8216 **Yevgeniya Tarasova*** (ytarasov@purdue.edu), Purdue University. *Residual Intersections of Determinantal Ideals of* $2 \times n$ *Matrices* Preliminary report.

In this talk we prove that n-residual intersections of ideals generated by 2×2 minors of generic $2 \times n$ matrices can be written as a sum of links. (Received September 18, 2021)

1174-13-8330 **Kevin S Harris*** (kevin.harris@mavs.uta.edu), University of Texas at Arlington. *Decompositions of Modules over Subalgebras of Truncated Polynomial Rings*

We investigate how modules decompose over principal subalgebras of certain truncated polynomial rings. In particular, we investigate how module decompositions may (or may not) change when we decompose over different principal subalgebras. Varying decompositions are related to the notion of rank varieties. Finally, we will examine how one might extend the notion of rank varieties to more general truncated polynomial rings. (Received September 18, 2021)

1174-13-8376 **Alapan Mukhopadhyay*** (alapanm@umich.edu), University of Michigan. *Frobenius-Poincare Function and Hilbert-Kunz Multiplicity* Preliminary report.

Given a graded ring with a prime characteristic and a homogeneous ideal of finite co-length, we show that a naturally defined sequence of 'Poincare series' converges to a complex valued function. This limiting function captures the corresponding Hilbert-Kunz multiplicity. We name this limiting function the Frobenius-Poincare function. Our result shows that Frobenius-Poincare functions are holomorphic everywhere on the complex plane. We shall discuss properties of Frobenius-Poincare functions, compute examples and describe these functions in terms of data of homological flavour. (Received September 18, 2021)

1174-13-8411 **Alessandra Costantini*** (alecost@okstate.edu), Oklahoma State University, **Kyle Logan Maddox** (maddox@ku.edu), University of Kansas, and **Lance Miller** (lem016@uark.edu), University of Arkansas. *Rees algebras of ideals in a weakly F-nilpotent ring* Preliminary report.

When a ring R has positive characteristic, the nature of the singularities of R can be often understood by analyzing the Frobenius structure of the local cohomology modules of R. For instance, a local ring R of dimension d is said to be weakly F-nilpotent if its local cohomology modules $H^i_{\mathfrak{m}}(R)$ are nilpotent with respect to the canonical Frobenius action for $i < d$. Recent work of Kyle Maddox and Lance E. Miller gives sufficient conditions so that weakly F-nilpotent singularities are preserved under various geometric constructions, like gluing, Segre products and Veronese subrings. In this talk, I will present our current understanding of how weakly F-nilpotent singularities behave under blow-ups. This is part of ongoing joint work with Kyle Maddox and Lance E. Miller. (Received September 18, 2021)

1174-13-8415 **Louiza Fouli** (lfouli@nmsu.edu), New Mexico State University, and **Jooyoun Hong** (hongj2@southernct.edu), Southern Connecticut State University. *Residual intersections and core of modules* Preliminary report.

Let I be an ideal defining an algebraic variety X. A residual intersection of I is an ideal K whose corresponding algebraic variety Y is so that $X \cup Y$ is generated by the smallest possible number of equations. From an algebraic perspective, the residual intersections of I are a powerful tool in order to understand the powers of I and have been used, for instance, to study the Rees algebra $\mathcal{R}(I)$, the multiplicity of I and the core of I. The latter is a fundamental object that allows to relate the powers of I with the powers of its integral closure $\bar{I}$. However, it is very difficult to determine without any knowledge of the residual intersections of I, being a priori an infinite intersection of ideals.

With geometric motivations coming from equisingularity theory, one is interested in studying integral dependence of modules, and in particular the structure of the core. The problem is technically challenging, since several tools used to study the core of ideals do not have a well-understood counterpart for modules. In this talk, I will introduce residual intersections of modules, and discuss our current understanding of the core of modules. This is part of ongoing joint work with Louiza Fouli and Jooyoun Hong. (Received September 19, 2021)

1174-13-8599 **Anna Weigandt** (weigandt@mit.edu), Massachusetts Institute of Technology. *Bumpless pipe dreams encode Gröbner geometry of Schubert polynomials*

Knutson and Miller established a connection between the anti-diagonal Gröbner degenerations of matrix Schubert varieties and the pre-existing combinatorics of pipe dreams. They used this correspondence to give a geometrically-natural explanation for the appearance of the combinatorially-defined Schubert polynomials as representatives of Schubert classes. Recently, Hamaker, Pechenik, and Weigandt conjectured a similar connection between diagonal degenerations of matrix Schubert varieties and bumpless pipe dreams, newer combinatorial objects introduced by Lam, Lee, and Shimozono.

We prove this conjecture in full generality. The proof provides tools for assessing the Cohen–Macaulayness of equidimensional unions of matrix Schubert varieties, of which alternating sign matrix varieties are an important example. (Received September 19, 2021)

1174-13-8669 **Josh Pollitz*** (pollitz@math.utah.edu), University of Utah, **Eloísa Grifo** (grifo@unl.edu), University of Nebraska—Lincoln, and **Daniel McCormick** (mccormic@math.utah.edu), University of Utah. *Cohomological supports in local algebra* Preliminary report.

The theory of cohomological supports has been an integral tool in revealing structural information in local algebra. Their use in commutative algebra can be traced back to foundational work of Avramov, and they were put on centerstage by Avramov and Buchweitz in 2000 in their investigations of complete intersection rings. In the past twenty years this theory has been further developed, extended and applied by Avramov-Iyengar, Burke-Walker, Jorgensen, and the presenting author. In this talk recent applications of this support theory will be surveyed, and I will discuss two new support theories that have been developed in two separate collaborations: one project is joint with Briggs and Grifo and the second is joint with Briggs and McCormick. The two support theories generalize the existing support theories in distinct ways, and each contains interesting homological data not readily available from the previous versions of support. (Received September 19, 2021)

1174-13-8715 **Ethan Roy** (ethanroy@utexas.edu), University of Texas At Austin, and **William Frendreiss*** (wfrendreiss@gmail.com), Texas A&M University. *Monomials, Convex Bodies, and Optimization*

In this talk, we generalize the exponentiation of monomial ideals by associating every monomial ideal to a corresponding convex polyhedron, called the Newton polyhedron. We explore connections between the geometry of Newton polyhedra and the algebraic properties of the corresponding monomial ideals. Knowing these algebraic properties allow us to prove results about two other exponentiation operations, the symbolic power and the real power, and rephrase a particular algebraic invariant of monomial ideals. (Received September 19, 2021)

1174-13-8743 **Sophie Zhu*** (sophieiriszhu@gmail.com), MIT-PRIMES Program. *Factorizations in evaluation monoids of Laurent semirings*

For $\alpha \in \mathbb{R}_{>0}$, let $\mathbb{N}_0[\alpha, \alpha^{-1}]$ be the semiring of real numbers $f(\alpha)$ with all $f(x) \in \mathbb{N}_0[x, x^{-1}]$, where $\mathbb{N}_0$ is the set of nonnegative integers and $\mathbb{N}_0[x, x^{-1}]$ is the semiring of Laurent polynomials with coefficients in $\mathbb{N}_0$. In this paper, we study various factorization properties of the additive structure of $\mathbb{N}_0[\alpha, \alpha^{-1}]$. We characterize when $\mathbb{N}_0[\alpha, \alpha^{-1}]$ is atomic. Then we characterize when $\mathbb{N}_0[\alpha, \alpha^{-1}]$ satisfies the ascending chain condition on principal ideals in terms of certain well-studied factorization properties. Finally, we characterize when $\mathbb{N}_0[\alpha, \alpha^{-1}]$ satisfies the unique factorization property and show that, when this is not the case, $\mathbb{N}_0[\alpha, \alpha^{-1}]$ has infinite elasticity. (Received September 19, 2021)

1174-13-8984 **Caitlyn Booms*** (cbooms@wisc.edu), University of Wisconsin-Madison. *Characteristic dependence of syzygies of random monomial ideals* Preliminary report.

When do syzygies depend on the characteristic of the field? Even for well-studied families of examples, very little is known. For a family of random monomial ideals, namely the Stanley–Reisner ideals of random flag complexes, we prove that the Betti numbers asymptotically almost always depend on the characteristic. Using this result, we also develop a heuristic for characteristic dependence of asymptotic syzygies of algebraic varieties. (Received September 20, 2021)

1174-13-9043 **Neil Epstein** (nepstei2@gmu.edu), George Mason University, **Rebecca R.G.*** (rrebhuhn@gmu.edu), George Mason University, and **Janet Vassilev** (jvassil@unm.edu), University of New Mexico. *The dual of a closure operation*

The relationship between tight closure and its test ideal can be described in terms of a duality between a closure operation and a corresponding interior operation. In previous work, Epstein and the speaker described this duality when the closure operation is residual, meaning that the closure is preserved by quotients. Residual closures include tight closure, module closures, and Frobenius closure, but not integral closure. In this talk, I will present results extending the duality to non-residual closures such as integral closure and basically full closure, and discuss some of the useful applications. This work is joint with Neil Epstein and Janet Vassilev. (Received September 20, 2021)

1174-13-9086 **Jonathan Montano*** (jmon@nmsu.edu), New Mexico State University, **Fatemeh Mohammadi** (Fatemeh.mohammadi@ugent.be), Ghent University, **Federico Castillo** (federico.castillo@mat.uc.cl), Universidad Católica de Chile, and **Yairon Cid Ruiz** (Yairon.CidRuiz@UGent.be), Ghent University. *Double Schubert polynomials do have saturated Newton polytopes*

We prove that double Schubert polynomials have the Saturated Newton Polytope property. This settles a conjecture by Monical, Tokcan and Yong. In fact, we confirm the conjecture by proving a stronger result that the support of each double Schubert polynomial is a discrete polymatroid. Our proof is based on the theory of multidegrees. We also introduce a notion of standardization of ideals that enables us to study non-standard multigradings. This allows us to show that the support of the multidegree polynomial of a Cohen-Macaulay prime ideal, and in particular of a Schubert determinantal ideal, is a discrete polymatroid. (Received September 20, 2021)

1174-13-9107 **Lauren Cranton Heller*** (lch@math.berkeley.edu), University of California, Berkeley. *Multigraded regularity on products of projective spaces* Preliminary report.

Eisenbud and Goto described the Castelnuovo-Mumford regularity of a sheaf on projective space in terms of three different properties of the corresponding graded module: its betti numbers, its local cohomology, and its truncations. For the multigraded generalization of regularity defined by Maclagan and Smith, these three conditions are no longer equivalent. I will characterize each of them for sheaves on products of projective spaces. (Received September 20, 2021)

1174-13-9267 **Thomas M Polstra*** (tp2tt@virginia.edu), University of Virginia, and **Craig L Huneke** (huneke@virginia.edu), University of Virginia. *Annihilating Local Cohomology Modules* Preliminary report.

Let $(R, \mathfrak{m}, k)$ be local normal Cohen-Macaulay domain and $I \subseteq R$ an ideal of pure height 1. For each natural number N let $I^{(N)}$ denote the Nth symbolic power of I. We consider annihilators of the local cohomology modules $H^i_{\mathfrak{m}}(R/I^{(N)})$. When R is of prime characteristic $p > 0$ and I is a multiple of an anticanonical ideal of R then understanding the annihilators $H^i_{\mathfrak{m}}(R/I^{(p^e)})$ as e varies through the natural numbers sheds light on the weak implies strong conjecture from tight closure theory. This talk is based on joint work with Craig Huneke. (Received September 20, 2021)

1174-13-9509 **Ashley K. Wheeler*** (wheeler@math.gatech.edu), Georgia Tech, **Milena Hering** (m.hering@ed.ac.uk), The University of Edinburgh, **Diane Maclagan** (D.Maclagan@warwick.ac.uk), University of Warwick, **Jenna Rajchgot** (rajchgot@math.mcmaster.ca), McMaster University, and **Josephine Yu** (jyu@math.gatech.edu), Georgia Tech. *Toric and tropical Bertini theorems in positive characteristic* Preliminary report.

We generalize the toric Bertini theorem of Fuchs, Mantova, and Zannier to positive characteristic. A key part of the proof is a new algebraically closed field containing the field $K(t_1, \ldots, t_d)$ of rational functions over an algebraically closed field K of prime characteristic. As a corollary, we extend the tropical Bertini theorem of Maclagan and Yu to arbitrary characteristic, which removes the characteristic dependence from the d-connectivity result for tropical varieties from that paper. (Received September 20, 2021)

1174-13-9570 **Felix Gotti** (fgotti@mit.edu), MIT, and **Bangzheng Li*** (libz2003@outlook.com), Christian Heritage School. *A New Class of Atomic Monoid Algebras without the Ascending Chain Condition on Principal Ideals* Preliminary report.

An integral domain R is atomic if every nonzero nonunit factors into irreducibles. On the other hand, R satisfies the ACCP if every ascending chain of principal ideals eventually terminates. Every integral domain satisfying the ACCP is known to be atomic. Although the converse does not hold in general, atomic domains without the ACCP are notorious for being very elusive and only a few classes have been discovered since A. Grams constructed the first example back in the seventies. We will discuss how to construct one of such integral domains using polynomial expressions with real exponents. This is a recent construction, and presumably one of the most elementary. This construction is part of a joint work with Felix Gotti. (Received September 21, 2021)

1174-13-9595 **Michael Shible*** (shima-22@rhodes.edu), Rhodes College Department of Mathematics and Computer Science. *Hilbert Series of Invariants of* $\mathbb{T}^2$ Preliminary report.

Let f be a function on $\mathbb{C}^n$, and L be a group of linear transformations mapping $\mathbb{C}^n$ to $\mathbb{C}^n$. Then f is invariant under L if, for every transformation l in L, f is unchanged when l acts upon it. The Hilbert series of a group of transformations gives information about the ring of invariant polynomials at each degree. Given a term of the series of the form Ct^d, the coefficient C is the dimension of the vector space of invariant polynomials of degree d under the transformation group. In this presentation, we give progress on our investigation into the Hilbert series of invariant polynomials under the 2-torus. Given a faithful representation of $\mathbb{T}^2$ on $\mathbb{C}^n$, we give a $2 \times n$ weight matrix A. We then express the torus action in terms of the weight matrix, use the Molien-Weyl theorem to express the Hilbert series as an iterated contour integral over the complex unit circle, and evaluate the integral using the Cauchy Residue Theorem. (Received September 21, 2021)

1174-13-9618 **Shuai Wei*** (wei6@g.clemson.edu), Clemson University. *Proceedings, conferences, collections, etc. pertaining to commutative algebra* Preliminary report.

The Stanley-Reisner correspondence allows one to use ideas from combinatorics and other fields to understand properties of square-free monomial ideals. In this talk, we consider a weighted version of this construction which allows us to investigate certain non-square-free monomial ideals. In particular, we discuss a variation of E. Miller's Alexander duality in this context which is slightly better behaved in some respects. (Received September 20, 2021)

1174-13-9644 **Srikanth B. Iyengar** (iyengar@math.utah.edu), University of Utah. *Complete intersections and the cotangent modules*

The cotangent complex is an important invariant of a commutative ring, and it's been known since the beginning that it is closely connected with the complete intersection property. For an ideal I of finite projective dimension in a commutative noetherian ring R, Quillen conjectured that I is complete intersection if and only if the cotangent complex of $R \to R/I$ has finite projective dimension over R/I, and Avramov proved this in 1999. Vasconcelos made a similar sounding conjecture: I is complete intersection if and only if the conormal module I/I^2 has finite projective dimension over R/I. I'll explain how there is a natural common generalisation of these two conjectures involving the "cotangent modules", which come in an infinite sequence: the zeroth one the module of differentials, the first one is I/I^2, and the third one is the Koszul homology of I. This is joint work with Srikanth Iyengar. (Received September 20, 2021)

1174-13-9646 **Laura Felicia Matusevich** (laura@math.tamu.edu), Texas A&M University, **Janet Page*** (jrpage@umich.edu), University of Michigan, and **Chin-Yi Jean Chan** (chan1cj@cmich.edu), Central Michigan University. *Rings of differential operators via retracts* Preliminary report.

In this talk, I will introduce a class of k-algebras which generalize both Stanley-Reisner rings and toric face rings. In particular, they are characterized by certain homomorphisms which are algebra retracts. I will show how we can compute their rings of differential operators (when k is an algebraically closed field of characteristic 0) using these retracts, and discuss some applications. (Received September 20, 2021)

1174-13-9679 **Swapnil Garg*** (swapnil.garg.3@gmail.com), MIT. *F-polynomial Ratios in the r-Kronecker* Preliminary report.

We provide a new algebraic proof for the limit of the ratio of consecutive F-polynomials of the 2-Kronecker. Our method follows Reading's proof but bypasses the need for scattering diagrams. We also define a new limit of ratios of F-polynomial powers in the r-Kronecker that generalizes the above 2-Kronecker limit. Our algebraic method gives a general formula for this limit in the 2-Kronecker case, and a functional equation in the r-Kronecker case. (Received September 20, 2021)

1174-13-9771 **Hugh Roberts Geller*** (Hrgeller@sewanee.edu), Sewanee: The University of the South.
Minimal DG Algebras for Families of Edge Ideals Preliminary report.

Within the study of squarefree monomial ideals of standard graded polynomial rings, there is particular interest in studying edge ideals of finite graphs. In this talk we consider a finite graph G and its edge ideal $I(G) \subseteq R$. Using a minimal free resolution of $R/I(G)$ over R, we give an algorithm for minimally resolving the edge ideal of the join of G and the complete graph K_n for any n. Moreover, we give sufficient conditions on G such that the resulting minimal resolution yields the structure of a differential graded R-algebra for any choice of n. (Received September 20, 2021)

1174-13-9936 **Fuxiang Yang** (fuxiangyang0411@gmail.com), University of California, San Diego, **Lillian Elizabeth McPherson** (mcphelil@umich.edu), University of Michigan, and **Monroe Ame Stephenson*** (mostephen@reed.edu), Reed College. *Characterization of Simple $\mathcal{D}-$Modules by the Differential Closure Operator*

In commutative algebra, closure operations can be defined using definitions of powers of an ideal. In this poster, we discuss the differential power defined in the 2017 paper "Symbolic powers of ideals" (Dao, De Stefani, Grifo, Huneke, Núñez-Betancourt) and our resulting definition of differential closure. Using polynomial rings as a starting point, we generalize our results to characterize simple $\mathcal{D}-$modules. Specifically, the relationship between the differential closure of an ideal and the radical of that ideal. (Received September 21, 2021)

1174-13-9973 **Ranthony A C Edmonds*** (edmonds.110@osu.edu), The Ohio State University, and **J. R. Juett** (jjuett@dbq.edu), University of Dubuque. *Factorization in Monoid Rings with Zero Divisors*

Given a commutative ring R and a monoid $(S,+)$, we can think of elements of a monoid ring $R[S]$ as polynomials with coefficients in R and exponents in S. Note that in the case $(S,+) = \mathbb{N}$, then $R[S]$ is just the polynomial ring $R[X]$ over R. In this talk we focus on factorization in monoid rings with zero divisors, a setting which simultaneously generalizes monoid domains and polynomial rings with zero divisors. We will focus on the topics of associates, irreducibility, and factorization length in monoid rings with zero divisors, with the goal of understanding how they can illuminate how certain factorization properties may pass between a monoid ring and its ring of coefficients. (Received September 21, 2021)

1174-13-10032 **Tim Tribone*** (ttribone@syr.edu), Syracuse University, and **Graham J. Leuschke** (gjleusch@syr.edu), Syracuse University. *Branched covers and matrix factorizations* Preliminary report.

A matrix factorization of an element f in a regular local ring S consists of a pair of square matrices whose product is f times an identity matrix of the appropriate size. These objects, introduced by Eisenbud, correspond to maximal Cohen-Macaulay modules over $R = S/(f)$, the hypersurface ring defined by f. In this talk, we will consider a natural generalization where the factorizations consist of more than two matrices. Our focus will be on the connection between this generalization and the d-fold branched cover of R, that is, the hypersurface ring defined by the equation $f + z^d$, where z is a new indeterminate. Portions of this work are joint with Graham Leuschke. (Received September 21, 2021)

1174-13-10050 **Kevin Tucker** (kftucker@uic.edu), University of Illinois At Chicago, and **Austyn Simpson*** (awsimps2@uic.edu), University of Illinois At Chicago. *Inversion of Adjunction for F-purity* Preliminary report.

The phenomenon of inversion of adjunction (very roughly) describes limitations that the singularities of a hyperplane impose on the singularities of the ambient variety. This problem has a complicated history in the realm of Frobenius singularities including F-purity, and removing divisibility conditions on the $\mathbb{Q}$-Gorenstein index often poses a significant challenge. In this talk, I'll describe how to reformulate inversion of adjunction for F-purity into a statement about Frobenius actions on local cohomology, and present some recent progress on this problem. This is part of forthcoming joint work with Thomas Polstra and Kevin Tucker. (Received September 21, 2021)

1174-13-10294 **Augustine O'Keefe*** (aokeefe@conncoll.edu), Connecticut College, **Jennifer Biermann** (biermann@hws.edu), Hobart and William Smith Colleges, **Kuei-Nuan Lin** (kul20@psu.edu), Pennsylvania State University, Greater Allegheny, and **Selvi Kara** (selvikara@gmail.com), University of Utah. *Ideals defined by vertex-weighted oriented graphs* Preliminary report.

One can naturally extend the notions of edge ideals and toric edge rings of simple graphs to those of vertex-weighted oriented graphs via monomials that are not square-free. In this talk we consider how this added complication manifests in the dictionary between the structure of the vertex-weighted oriented graph and the algebraic structure of the associated ideals. (Received September 21, 2021)

1174-13-10442 **Hailong Dao** (hdao@ku.edu), University of Kansas, and **Haydee M Lindo*** (hlindo@g.hmc.edu), Harvey Mudd College. *On Stable Trace Ideals* Preliminary report.

In this talk we will explore the intersection of trace ideals and stable ideals, that is, ideals that are stable under homomorphisms to the ring and ideals that are isomorphic to their endomorphism rings. We apply our results to study of Arf rings. This is ongoing joint work with Hailong Dao. (Received September 21, 2021)

1174-13-10694 **Uwe Nagel** (uwe.nagel@uky.edu), University of Kentucky, and **Jenny Kenkel*** (jkenkel@umich.edu), University of Michigan. *Local Cohomology of Thickenings on Sequences of Rings* Preliminary report.

Let R be a standard graded polynomial ring and let I be a homogenous prime ideal of R. Bhatt, Blickle, Lyubeznik, Singh, and Zhang examined the local cohomology of R/I^t as t grows arbitrarily large. I will discuss their results and give an explicit description of the transition maps between these local cohomology modules in a particular example. I will also consider asymptotic structure in a different direction: as the number of variables of R grows. This study of families of modules over compatible varying rings hints at the existence of FI structures. (Received September 21, 2021)

1174-13-10758 **Kayla Wright*** (kaylaw@umn.edu), University of Minnesota, **Gregg Joseph Musiker** (musiker@math.umn.edu), University of Minnesota, and **Libby Farrell** (farre423@umn.edu), University of Minnesota, Twin Cities. *Mixed Dimer Model for Cluster Algebras*

In this talk, we will discuss a combinatorial model for cluster algebras in type D. This combinatorial model consists of graph theoretic objects called mixed dimer configurations that we organize into the data of a poset. In order to prove this model and motivate it beyond the world of cluster algebras, we will discuss why these mixed dimer configurations are rich algebraically. We will see that these graphs represent submodules of an indecomposable module of the associated Jacobian algebra. (Received September 21, 2021)

14 ► Algebraic geometry

1174-14-5233 **Lauren Kiyomi Williams*** (williams@math.harvard.edu), Harvard University. *Tropical geometry and shallow water waves.*

The field of (algebraic) geometry studies geometric objects associated to polynomials; for example, a parabola is associated to the polynomial $y - x^2$. Tropical geometry is a new field which studies geometric objects associated to "tropical" polynomials, in which the operations of multiplication and addition are replaced by addition and the minimum operation. Surprisingly, tropical geometry arises in some real life applications. In particular, I will explain how tropical curves can be used to model shallow water waves, such as waves at the beach. (Received November 16, 2021)

1174-14-5471 **Shanna Dobson*** (Shanna.Dobson@calstatela.edu), California State University, Los Angeles. *Universality of Diamonds in Langlands Local Functoriality* Preliminary report.

Motivated by Scholze's 'etale cohomology of diamonds and Scholze and Fargues' geometrization of the local Langlands correspondence, we conjecture a universal construction of spatial diamonds in Langlands Local Functoriality. We then extend this universal construction to an $(\infty, 1)$-Grothendieck construction on our $(\infty, 1)$-category of spatial diamonds. A diamond $\mathcal{D}$ is a certain pro-'etale sheaf on the category of perfectoid spaces of characteristic p. A perfectoid space is an adic space covered by adic spaces of the form $Spa(R, R^+)$ for R a perfectoid ring. A spatial diamond is a small v-sheaf in the v-topology, which is a Grothendieck topology. Constructing quotients of diamonds by a diamond equivalence relation yields v-sheaves and constructing quotients of small v-sheaves by a small v-sheaf equivalence relation produces v-stacks. (Received August 20, 2021)

1174-14-5616 **Frank Sottile** (sottile@math.tamu.edu), Texas A&M University, **Jose Israel Rodriguez*** (jrodriguez43@wisc.edu), University of Wisconsin Madison, **Taylor Brysiewicz** (tbrysiew@nd.edu), Max Planck Institute For Mathematics In the Sciences, and **Thomas Yahl** (thomasjyahl@math.tamu.edu), Texas A&M. *Decomposable sparse systems*

Améndola et al. proposed a method for solving systems of polynomial equations lying in a family which exploits a recursive decomposition into smaller systems. A family of systems admits such a decomposition if and only if the corresponding Galois group is imprimitive. When the Galois group is imprimitive we consider the problem of computing an explicit decomposition. A consequence of Esterov's classification of sparse polynomial systems with imprimitive Galois groups is that this decomposition is obtained by inspection. In this talk, I will discuss how this leads to a recursive algorithm to solve decomposable sparse systems and the combinatorics behind it. (Received August 23, 2021)

1174-14-5636 **Timothy Duff*** (tduff3@gatech.edu), University of Washington, Georgia Tech. *Structured polynomial constraints from computer vision* Preliminary report.

Many problems in computer vision are inextricably linked with polynomial constraints. These problems may relate to reconstruction (solving for cameras and a scene given image-to-image data), resectioning (solving for cameras given world-to-image data), triangulation (minimizing the distance from noisy image data to an idealized model), and many other practical tasks. I will describe ongoing work that attempts to answer fundamental questions about such problems with effective tools from algebraic geometry. (Received August 23, 2021)

1174-14-5645 **Minyoung Jeon*** (jeon.163@buckeyemail.osu.edu), The Ohio State University. *Mather Classes of Schubert Varieties via Small Resolutions*

We study Chern-Mahter (CM) classes of Schubert varieties in even Orthogonal Grassmannians. As opposed to the Nash blowup of singular varieties that is used to define the CM class by MacPherson, we compute the CM class of Grassmannian Schubert varieties in type D through small resolutions by Sankaran and Vanchinathan. We also present Kazhdan-Lusztig classes associated to Schubert varieties in Lagrangian Grassmannians (type C). These are analogous to Jones result for ordinary Grassmannians (type A), under the existence of the small resolutions. It is expected that the similar technique is applied to find the Kazhdan-Lusztig class of Schubert varieties in the odd Orthogonal Grassmannians (type B). (Received August 24, 2021)

1174-14-5985 **Juliette Emmy Bruce*** (juliette.bruce@berkeley.edu), University of California, Berkeley, **Madeline Brandt** (madelinevbrandt@gmail.com), Brown University, **Corey Wolfe** (cwolfe@tulane.edu), Tulane University, **Melody Chan** (melody_chan@brown.edu), Brown University, **Margarida Melo** (melo@mat.uniroma3.it), Università Roma Tre, and **Gwyeneth Moreland** (gwynm@math.harvard.edu), Harvard University. *The top weight cohomology of $\mathcal{A}_g$*

I will discuss recent work calculating the top weight cohomology of the moduli space $\mathcal{A}_g$ of principally polarized abelian varieties of dimension g for small values of g. The key idea is that this piece of cohomology is encoded combinatorially via the relationship between the boundary complex of a compactification of $\mathcal{A}_g$ and the moduli space of tropical abelian varieties. This is joint work with Madeline Brandt, Melody Chan, Margarida Melo, Gwyneth Moreland, and Corey Wolfe. (Received September 1, 2021)

1174-14-6003 **Nida K Obatake*** (nobatake@ucsd.edu), University of California, San Diego, Institute for Defense Analyses. *Newton-Okounkov bodies of chemical reaction networks* Preliminary report.

An important invariant of a chemical reaction network is its maximum number of positive steady states. This number, however, is in general difficult to compute. Indeed, computing the maximum number of positive steady states is equivalent to counting the maximum number of positive real roots of a parameterized polynomial system. Accordingly, in this talk, we introduce a new upper bound on this number, namely the normalized volume of a Newton-Okounkov body associated to a chemical network. The theory of Newton-Okounkov bodies is an exciting modern research area that was originally developed in pure mathematics but has potential in applications. In particular, volumes of Newton-Okounkov bodies give bounds on root counts of systems of polynomial equations. Despite their noted potential, there are very few concrete examples of Newton-Okounkov bodies for polynomial systems arising from applications. Through explicit examples, we show in this talk that the volume of a Newton-Okounkov body associated to a network gives a good upper bound on its maximum number of positive steady states. In our examples, we also compare this 'Newton-Okounkov body bound' to another related upper bound, namely the mixed volume of a chemical reaction network. (Received September 13, 2021)

1174-14-6067 **Hannah Larson*** (hlarson@stanford.edu), Stanford University. *On an equivalence of divisors on $\overline{M}_{0,n}$ from Gromov-Witten theory and conformal blocks*

In this talk, I will define two types of divisors on the moduli space of stable pointed genus 0 curves. The first type comes from Gromov-Witten theory, while the second comes from vector bundles of conformal blocks. Despite their different origins, certain families of these divisors are conjectured to be numerically equivalent. I'll present a proof of the conjecture in a special case (which includes many new infinite families). This is joint work with L. Chen, A. Gibney, L. Heller, E. Kalashnikov, and W. Xu. (Received September 3, 2021)

1174-14-6327 **Eugene Gorsky*** (egorskiy@math.ucdavis.edu), University of California, Davis, **Matthew Hogancamp** (m.hogancamp@northeastern.edu), Northeastern University, and **Anton Mellit** (anton.mellit@univie.ac.at), University of Vienna. *Tautological classes and symmetry in Khovanov-Rozansky homology*

We define a new family of commuting operators F_k in Khovanov-Rozansky link homology, similar to the action of tautological classes in cohomology of character varieties. We prove that F_2 satisfies "hard Lefshetz property" and hence exhibits the symmetry in Khovanov-Rozansky homology conjectured by Dunfield, Gukov and Rasmussen. (Received September 8, 2021)

1174-14-6917 **Elise Walker*** (walkere@math.tamu.edu), Texas A&M University. *Numerical homotopies from Khovanskii bases*

Homotopies are useful numerical methods for solving systems of polynomial equations. Embedded toric degenerations are one source for optimal homotopy algorithms. In particular, if a projective variety has a toric degeneration, then linear sections of that variety can be optimally computed using the polyhedral homotopy. Any variety whose coordinate ring has a finite Khovanskii basis is known to have a toric degeneration. We provide embeddings for this Khovanskii toric degeneration and use the resulting homotopy to compute general linear sections of the variety. (Received September 10, 2021)

1174-14-6946 **Mahrud Sayrafi** (mahrud@umn.edu), University of Minnesota, Twin Cities, and **Lauren Cranton Heller** (lch@math.berkeley.edu), University of California - Berkeley. *Characterizing multigraded regularity on products of projective spaces*

On projective space the Castelnuovo-Mumford regularity of a module can be characterized in terms of the minimal graded free resolutions of the module and its truncations. On more general toric varieties the analogous relationship between multigraded Castelnuovo-Mumford regularity and truncations does not hold. I will discuss recent work exploring the relationship between multigraded Castelnuovo-Mumford, truncations, Betti numbers, and virtual resolutions for modules on products of projective spaces. (Received September 10, 2021)

1174-14-7255 **Ana-Maria Castravet** (ana-maria.castravet@uvsq.fr), University of Versailles, France, **Libby Taylor*** (lt691@stanford.edu), Stanford University, **Carolina Araujo** (caraujo@impa.br), Instituto Nacional de Matemática Pura e Aplicada, **Roya Beheshti** (beheshti@wustl.edu), Washington University in St. Louis, **Svetlana Makarova** (murmuno@yandex.ru), University of Pennsylvania, **Kelly Jabbusch** (jabbusch@umd.umich.edu), University of Michigan Dearborn, **Nivedita Viswanathan** (nivedita.viswanathan@ed.ac.uk), University of Edinburgh, and **Enrica Mazzon** (mazzon.enr@gmail.com), University of Michigan. *Higher Fano manifolds*

In this paper we address higher Fano manifolds. These are Fano manifolds with positive higher Chern characters. They were introduced as potential higher analogs of Fano manifolds, and are expected to enjoy stronger versions of several of the nice properties of Fano manifolds. For instance, they should be covered by higher dimensional rational varieties, and families of higher Fano manifolds over higher dimensional bases should admit meromorphic sections (modulo the Brauer obstruction). Aiming at finding new examples of higher Fano manifolds, we investigate positivity of higher Chern characters of rational homogeneous spaces. We determine which rational homogeneous spaces of Picard rank 1 have positive second Chern character, and show that the only rational homogeneous spaces of Picard rank 1 having positive second and third Chern characters are projective spaces and quadric hypersurfaces. We also classify Fano manifolds of large index having positive second and third Chern characters. We conclude by discussing conjectural characterizations of projective spaces and complete intersections in terms of these higher Fano conditions. (Received September 13, 2021)

1174-14-7281 **Dhruv Bhatia*** (dhruv_bhatia@brown.edu), Brown University, **Maximillian Paul Watson** (mpwats22@g.holycross.edu), College of the Holy Cross, and **Kara Fagerstrom** (kfagerstro@brynmawr.edu), Bryn Mawr College. *Probability Distributions for Elliptic Curves in the CGL Hash Function* Preliminary report.

Hash functions map data of arbitrary length to data of predetermined length. Good hash functions are hard to predict, making them useful in cryptography. We are interested in the elliptic curve CGL hash function, which maps a bitstring to an elliptic curve by traversing an input-determined path through an isogeny graph. The nodes of an isogeny graph are elliptic curves, and the edges are special maps betwixt elliptic curves called isogenies. Knowing which hash values are most likely informs us of potential security weaknesses in the hash function. We use stochastic matrices to compute the expected probability distributions of the hash values. We generalize our experimental data into a theorem that completely describes all possible probability distributions of the CGL hash function. We use this theorem to evaluate the collision resistance of the CGL hash function and compare this to the collision resistance of an "ideal" hash function. (Received September 13, 2021)

1174-14-7633 **Fernando Luis Piñero** (fernando.pinero1@upr.edu), University of Puerto Rico In Ponce, **Doel Alexander Rivera** (driveralaboy@pucpr.edu), PCUPR, **Lani Southern*** (lmsouthern@willamette.edu), Willamette University, and **Sarah Gregory** (sarah.gregory@richmond.edu), University of Richmond. *On the Parameters of Polar Hermitian Grassmann Codes* Preliminary report.

The Grassmannian, $\mathcal{G}_{\ell,m}$, is the collection of all ℓ dimensional subspaces of a vector space of length m. It is one of the most widely studied objects in Algebraic Geometry. It has several algebraic, geometric and combinatorial properties. We use a class of linear codes, known as Grassmann codes, to study the Grassmannian. The polar Hermitian Grassmannian is a subvariety of the Grassmannian consisting of all spaces isotropic under a sesquilinear form. The minimum distance of the corresponding linear code is related to the intersection of the Grassmannian (or its subvariety) with a hyperplane. Cardinali and Giuzzi determined the minimum distance for the case where $\ell = 2$. For the case where $\ell = 3$ and $m = 6$ we employ an elementary counting argument to prove a lower bound on the minimum distance for the polar Hermitian Grassmann codes. (Received September 15, 2021)

1174-14-7904 **Xindong Tang** (xit039@ucsd.edu), UC San Diego, and **Kisun Lee*** (kil004@ucsd.edu), UC San Diego. *Polyhedral homotopy method for Nash equilibrium problems* Preliminary report.

In this talk, we discuss the problem of finding generalized Nash equilibria (GNE) in the viewpoint of sparse polynomials. To obtain optimality conditions for GNE, we consider the Karush-Kuhn-Tucker (KKT) system using the Lagrange multiplier. We discuss that if all objectives and constraints polynomials are generic, the number of solutions of the KKT system equals its mixed volume, and so the polyhedral homotopy method can be optimal for finding GNEs. Lastly, comparisons with existing methods will be given. (Received September 16, 2021)

1174-14-7970 **Saugata Basu** (sbasu@purdue.edu), Purdue University, and **Sarah Percival*** (perciva9@msu.edu), Michigan State University. *Efficient Computation of a Semi-Algebraic Basis of the First Homology Group of a Semi-Algebraic Set*

We give an algorithm for computing a semi-algebraic basis for the first homology group, $H_1(S, \mathbb{F})$, with coefficients in a field $\mathbb{F}$, of any given semi-algebraic set $S \subset R^k$ defined by a closed formula. The complexity of the algorithm is bounded singly exponentially. More precisely, if the given quantifier-free formula involves s polynomials whose degrees are bounded by d, the complexity of the algorithm is bounded by $(sd)^{k^{O(1)}}$. This algorithm generalizes well known algorithms having singly exponential complexity for computing a semi-algebraic basis of the zero-th homology group of a semi-algebraic set, which is equivalent to the problem of computing a set of points meeting every semi-algebraically connected component of the given semi-algebraic set at a unique point. (Received September 17, 2021)

1174-14-8034 **Montserrat Teixidor i Bigas*** (mteixido@tufts.edu), Tufts University. *Ranks of multiplication maps and application* Preliminary report.

Tangent spaces to varieties can often be interpreted as spaces of sections of vector bundles. Properties of the maps among these spaces have important consequences for the geometry of the varieties in question. For example, in Brill-Noether theory, the injectivity of the Petri map guarantees that Brill-Noether loci are well behaved. Traditionally, a lot of effort has been devoted to showing that the kernel of the map is small. We propose that in some cases it is more efficient to show that the image is large. We will see some examples and applications to the theory of moduli spaces of curves. (Received September 17, 2021)

1174-14-8272 **Renee Bell** (rhbell@math.upenn.edu), University of Pennsylvania, **Jennifer Li*** (jenniferli@princeton.edu), Princeton University, **Paola Comparin** (paola.comparin@ufrontera.cl), Universidad de la Frontera UFRO, **Alessandra Sarti** (Alessandra.Sarti@math.univ-poitiers.fr), Université de Poitiers, **Alejandra Rincón Hidalgo** (arincon@ictp.it), ICTP, and **Aline Zanardini** (a.zanardini@math.leidenuniv.nl), Leiden University. *Automorphisms of $K3$ surfaces* Preliminary report.

A $K3$ surface is a two-dimensional compact complex manifold which is simply connected and has a nowhere zero global holomorphic 2-form ω. We study automorphisms of $K3$ surfaces which are purely non-symplectic, meaning the pullback of the holomorphic 2-form ω is $\zeta_n\omega$, where ζ_n is a primitive n-th root of unity. Our focus is on classifying such automorphisms with finite orders n, using previous classification results by Artebani, Comparin, Sarti, Taki, Valdés, and others. In particular, we have studied orders 14, 21, 28, and 42. In this talk, I will present an example, along with an update of our classification progress. This is joint work with Renee Bell, Paola Comparin, Alejandra Rincón Hidalgo, Alessandra Sarti, Aline Zanardini. (Received September 18, 2021)

1174-14-8766 **Rebecca F Goldin*** (rgoldin@gmu.edu), George Mason University, and **Brent Gorbutt** (bgorbutt@gmu.edu), George Mason University. *Peterson Schubert calculus and positivity*

From one perspective, Schubert calculus counts the intersections of a set of T-invariant subvarieties called Schubert varieties within the flag manifold G/B. These intersections also compute quantities important in representation theory, combinatorics and geometry, and the term 'Schubert calculus' is now used to describe an interrelated set of questions in a more general geometric, cohomological, or combinatorial context. In this talk, we answer a question in Peterson Schubert calculus. The Peterson variety is a subvariety of G/B consisting of flags satisfying specified conditions related to a regular nilpotent linear operator, and is a special case of regular nilpotent Hessenberg varieties. Petersons are not T-invariant but are invariant under a one-dimensional torus $S \subset T$. In joint work with Brent Gorbutt, we proved the positivity of S-equivariant Schubert calculus for the Peterson variety in type A_{n-1}, resolving a conjecture of Tymoczko and Harada. To do this, we found an explicit, combinatorially-positive formula for the coefficients of the products. Along the way, we also discovered an unexpected combinatorial identity of binomial and multinomial coefficients. (Received September 19, 2021)

1174-14-9047 **Jean Kieffer*** (kieffer@math.harvard.edu), Harvard University. *Asymptotically faster point counting on abelian surfaces*

Given an elliptic curve E over a finite field $\mathbb{F}_p$, the *point counting problem* is to compute the order its group of rational points $E(\mathbb{F}_p)$. The Schoof–Elkies–Atkin (SEA) algorithm provides an answer to this question when p is very large, and has important applications in cryptography. In this talk, we describe recent generalizations of the SEA algorithm to the case of genus 2 curves, or rather their Jacobians: these are abelian varieties of dimension 2, also known as abelian surfaces. Our tools are a new algorithm to compute isogenies between these abelian surfaces based on *modular equations*, and fast analytic methods for evaluating Siegel modular forms in dimension 2. (Received September 20, 2021)

1174-14-9116 **Bianca Viray*** (bviray@math.washington.edu), University of Washington, and **Brendan Creutz** (b.creutz@canterbury.ac.nz), University of Canterbury. *Quadratic points on intersections of quadrics*

A projective degree d variety always has a point defined over a degree d field extension. For many degree d varieties, this is the best possible statement, that is, there exist classes of degree d varieties that never have points over extensions of degree less than d (nor over extensions whose degree is nonzero modulo d). However, there are some classes of degree d varieties that obtain points over extensions of smaller degree, for example, degree 9 surfaces in $\mathbb{P}^9$, and 6-dimensional intersections of quadrics over local fields. In this talk, we explore this question for intersections of quadrics. In particular, we prove that a smooth complete intersection of two quadrics of dimension at least 2 over a number field has index dividing 2, i.e., that it possesses a rational 0-cycle of degree 2. This is joint work with Brendan Creutz. (Received September 20, 2021)

1174-14-9150 **Simon Telen** (simon.telen@mis.mpg.de), Max Planck Institute For Mathematics In the Sciences. *Parameter Homotopies in Cox Coordinates*

Duff et al. introduced Cox homotopies for solving sparse polynomial systems with solutions lying in a compact toric variety. This takes advantage of the Cox construction of a compact toric variety as a quotient of a quasi-affine space by a reductive group. These Cox homotopies generalize parameter homotopies in projective space and products of projective space while giving more refined information about solutions lying at infinity. (Received September 20, 2021)

1174-14-9179 **Cigole Thomas*** (cthoma40@gmu.edu), George Mason University. *Dynamics of the outer automorphism group action on* $\mathrm{SL}_2(\mathbb{F}_q)$*- character variety of* $\mathbb{Z}^r$ Preliminary report.

If G is a reductive algebraic group over $\mathbb{Z}$, the G-character variety of a finitely presented group, Γ parametrizes the set of closed conjugation orbits in $\mathrm{Hom}(\Gamma, G)$. The group of outer automorphisms, $Out(\Gamma)$ naturally acts on the character variety. The dynamics of mapping class group action on the finite field points of character variety is particularly interesting. Specifically, the $\mathrm{SL}_2(\mathbb{F}_q)$-character variety of $\mathbb{Z}^r$, determined by the set of all r-tuples of matrices that are pairwise commute, is being acted upon by the group $\mathrm{GL}_r(\mathbb{Z})$. After stratifying the character variety, the structure of orbits under this action is explored by stratum to determine if the action is transitive or asympotically transitive. A visual representation of the orbit structures when $q = 2$ and $r = 2$ will be presented. (Received September 20, 2021)

1174-14-9249 **Michael Burr** (burr2@clemson.edu), Clemson University, and **Elise Walker*** (elise.walker@tamu.edu), Texas A&M University. *Numerically solving polynomial systems using Khovanskii bases*

Homotopies are useful numerical methods for computing the solutions of a system of polynomial equations. Such solution sets are known as varieties. Homotopies compute varieties by tracking paths from solutions of a similar, pre-solved system. Generally, homotopies may track extraneous paths, which wastes computational resources. A homotopy is optimal if paths are smooth and there are no extraneous paths. Embedded toric degenerations are one source for optimal homotopy algorithms. In particular, if a projective variety has a toric degeneration, then there is an optimal homotopy for computing linear sections of that variety. There is a toric degeneration for any variety whose coordinate ring has a finite Khovanskii basis. We provide embeddings for this Khovanskii toric degeneration and give the corresponding optimal homotopy algorithm for computing a general linear section of the variety. This is joint work with Michael Burr and Frank Sottile. (Received September 20, 2021)

1174-14-9259 **Shiyue Li*** (shiyue_li@brown.edu), Brown University, **Emily Clader** (eclader@sfsu.edu), San Francisco State University, **Rohini Ramadas** (Rohini.Ramadas@warwick.ac.uk), University of Warwick, **Daoji Huang** (huan0664@umn.edu), University of Minnesota, and **Chiara Damiolini** (chiara.damiolini@gmail.com), University of Pennsylvania. *Permutohedral complexes and rational curves with cyclic action*

We define a moduli space of rational curves with finite-order automorphism and weighted orbits, and we prove that the combinatorics of its boundary strata are encoded by a particular polytopal complex that also captures the algebraic structure of a complex reflection group acting on the moduli space. This generalizes the situation for Losev–Manin's moduli space of curves (whose boundary strata are encoded by the permutohedron and related to the symmetric group) as well as the situation for Batyrev–Blume's moduli space of curves with involution, and it extends that work beyond the toric context. (Received September 20, 2021)

1174-14-9602 **Margaret Regan*** (mregan@math.duke.edu), Duke University, and **Timothy H Duff** (tduff3@gatech.edu), Duke University. *Galois/monodromy groups for decomposing minimal problems in 3D reconstruction*

Many problems in computer vision are represented using a parameterized polynomial system, where the solution set is critical for 3D reconstruction. Minimal problems can be of special interest as these polynomial systems are well-constrained and generically have finitely many solutions. Computing the Galois/monodromy group of the associated branched cover can yield information and understanding about the underlying structure of the minimal problem. One potential outcome of this computation is identifying possible decompositions or symmetries that enable more efficient solution solving. These Galois/monodromy groups can be computed using numerical homotopy continuation via a multitude of softwares. Examples such as the classical homography estimation and five-point relative pose problems will be provided to show these computations and the structure they can help identify. This is joint work with Timothy Duff, Viktor Korotynskiy, and Tomas Pajdla. (Received September 20, 2021)

1174-14-9882 **Giulia Saccà** (giulia@math.columbia.edu), Columbia University, **Emma Brakkee** (e.l.brakkee@uva.nl), University of Amsterdam, **Chiara Camere** (chiara.camere@unimi.it), University of Milan, **Laura Pertusi** (laura.pertusi@unimi.it), University of Milan, **Alexandra Viktorova*** (alexandra.viktorova@stonybrook.edu), Stony Brook University, and **Annalisa Grossi** (annalisa.grossi@math.tu-chemnitz.de), Chemnitz University of Technology. *Singular symplectic varieties, moduli spaces on K3 surfaces, and Prym varieties* Preliminary report.

Let S be a K3 surface, i an antisymplectic involution on S, and $C \subset S/i$ a smooth ample curve. After choosing a polarization on S, we can construct the corresponding relative Prym variety $\mathcal{P}$ together with a natural Lagrangian fibration $\mathcal{P} \to |C|$. By the work of Arbarello, Saccà, and Ferretti, we know that if S/i is a general Enriques surface, then $\mathcal{P}$ is an irreducible symplectic variety that admits a symplectic resolution when $|C|$ is hyperelliptic. Inspired by these results, we investigate the situation in which S/i is a rational surface and find sufficient conditions to ensure that $\mathcal{P}$ is an irreducible symplectic variety. We start our analysis with the rational elliptic surface case and give some other concrete examples of this construction. (Received September 21, 2021)

1174-14-9994 **Sarah Frei*** (sarah.frei@rice.edu), Rice University, **Isabel Vogt** (ivogt.math@gmail.com), Brown University, **Lena Ji** (lenaji@umich.edu), University of Michigan, and **Soumya Sankar** (ssankar3@wisc.edu), Ohio State University. *Conic bundle threefolds over non-algebraically closed fields* Preliminary report.

In this talk, I will give a progress report on a project started at the 2020 ICERM workshop "Women in Algebraic Geometry." Over algebraically closed fields, it is completely understood when conic bundles over $\mathbb{P}^2$ are rational. Joint with L. Ji, S. Sankar, B. Viray, and I. Vogt, we are determining necessary and sufficient conditions for rationality of certain geometrically rational conic bundles over $\mathbb{P}^2$ over non-algebraically closed fields. We have partial results over finite fields and the real numbers. (Received September 21, 2021)

1174-14-9998 **Taylor Brysiewicz** (tbrysiew@nd.edu), University of Notre Dame. *Traces of Zero-dimensional Polynomial Systems*

The trace of a zero-dimensional polynomial system is the coordinate-wise sum of its zeros. In applications, the trace may be used to decide if a subset of the zeros is complete. In this talk, I will present theoretical properties of the trace for structured polynomial systems. I will also illustrate how to apply these properties to compute the trace without explicitly computing the zeros of the system. (Received September 21, 2021)

1174-14-10039 **Silviana Amethyst** (amethyst@uwec.edu), University of Wisconsin—Eau Claire, **Samantha Maurer*** (maurersl5880@uwec.edu), University of Wisconsin—Eau Claire, and **William O'Brien** (obrienwt5773@uwec.edu), University of Wisconsin—Eau Claire. *A 3D printed Arduino powered electronic Barth Sextic* Preliminary report.

During the summer of 2020, two undergraduate students at the University of Wisconsin—Eau Claire researched icosahedron symmetries found within the Barth Sextic, an algebraic surface featuring 65 double points. The Barth Sextic light fixture used in the study combines art and mathematics; this is most noticeable with the physical model of the surface utilizing 3D modeling and printing, Arduino, and Neopixel Jewel chips. In addition, the students contributed to the code that powers both the controller and fixture. With the controller, individuals can interactively explore the symmetries, namely rotations and reflections, by influencing the presence of colored light within each of the double points. (Received September 21, 2021)

1174-14-10078 **Michael Mumm*** (mummmj6824@uwec.edu), UW Eau Claire. *Implementing Straight-Line Programs in Bertini 2* Preliminary report.

Numerical algebraic geometry studies the solution sets of polynomial systems using numerical techniques. Bertini2 is a C++ and Python implementation of this mathematical theory. In this project, we improved Bertini's core C++ polynomial evaluation routine, making it more efficient. In particular, we replaced a tree-like structure which used polymorphism with straight-line programs (SLP's).

In a straight-line program, data for evaluation is stored alongside instructions for each operation, in two linear objects. To produce these, we used visitor function, which are functions that can grab data from certain points in a program's execution without having to recompile the entire program. We implemented these visitor functions for each different type of sign/evaluation to grab that data, and store it in memory for future use in our evaluation. After the program gathers up all the evaluations we need for the SLP in memory via the visitor functions, we need to execute all of these evaluations. In our SLP evaluation routine, we wrote the function to loop through the memory we have saved and send instructions to the program to evaluate every part of the SLP. (Received September 21, 2021)

1174-14-10106 **Frank Sottile*** (sottile@tamu.edu), Texas A&M University. *Algebraic Varieties at Infinity* Preliminary report.

Algebraic geometry has long been interested in the behavior at infinity of solutions to polynomial equations or varieties. It is known that asymptotically, the structure of a variety is governed by its tropical variety, an object from geometric combinatorics. This in turn leads to natural compactifications, complete with coordinate charts in which to study these points at infinity. My talk will give a sketch of this technology suitable for a general audience. (Received September 21, 2021)

1174-14-10156 **Kristin DeVleming*** (kdevleming@umass.edu), UMass Amherst. *The Noether-Lefschetz locus in families of singular Fano threefolds* Preliminary report.

We will summarize joint work in progress with M. Cheung, A. Grassi, and J. Rana on the Noether-Lefschtez locus in singular Fano threefolds and its behavior in families. The Noether-Lefschetz locus parametrizes families of surfaces whose Picard rank is higher than that of the ambient space, and we will discuss modularity of components of both maximal and minimal codimension and present many examples. (Received September 21, 2021)

1174-14-10339 **Yixian Wu*** (yixianwu@math.utexas.edu), University of Texas at Austin. *Splitting of Gromov-Witten Invariants with Toric Gluing Strata*

The computations of Gromov-Witten invariants are rarely easy. Luckily, under nice degeneration situations, relative Gromov-Witten theory provides us with a degeneration formula allowing us to compute the invariants from two easier-to-compute components.

As a generalization of relative Gromov-Witten invariants, logarithmic Gromov-Witten theory provides a beautiful framework to study Gromov-Witten invariants of simple normal crossing varieties. I will introduce the development of logarithmic Gromov-Witten theory and present a numerical formula under the assumption that the gluing strata are toric varieties. (Received September 21, 2021)

1174-14-10735 **Zonia Karina Menendez*** (zmenendez@wesleyan.edu), Wesleyan University. *Sporadic Points on $X_0(n)$* Preliminary report.

We say a closed point x on a curve C is sporadic if C has only finitely many closed points of degree at most $\deg(x)$. CM elliptic curves provide a natural class of examples of sporadic points and in fact , thanks to Sutherland, there exist sporadic points corresponding to CM elliptic curves on $X_1(n)$ for all sufficiently large integers n. *On the Level of Modular Curves that give rise to Sporadic j-invariant* (by Bourdon, Ejder, Liu, Odumodu, and Viray i.e. BELOV) studies sporadic points of arbitrary degree on the modular curves $X_1(n)$, focusing particularly on sporadic points corresponding to non-CM elliptic curves. In this talk we will be extending ideas from the article by BELOV for non-cuspidal non-CM sporadic points on $X_0(n)$. (Received September 21, 2021)

1174-14-10953 **Yelena Mandelshtam*** (yelena@math.berkeley.edu), UC Berkeley. *KP Solitons from Tropical Limits*

We study solutions to the Kadomtsev-Petviashvili equation whose underlying algebraic curves undergo tropical degenerations. Riemann's theta function becomes a finite exponential sum that is supported on a Delaunay polytope. We introduce the Hirota variety which parametrizes all tau functions arising from such a sum. We compute tau functions from points on the Sato Grassmannian that represent Riemann-Roch spaces and we present an algorithm that finds a soliton solution from a rational nodal curve. This is joint work with Daniele Agostini, Claudia Fevola, and Bernd Sturmfels. (Received September 21, 2021)

1174-14-10956 **Alex Nash*** (alexpnash@gmail.com), Dickinson College, **Joe Miller** (miller1374@uwlax.edu), University of Wisconsin-Madison, and **Hani Samamah** (hanisamamah@ufl.edu), University of Florida. *Classifying 2-Dimensional Real Algebraically Defined Graphs by Diameter* Preliminary report.

A 2-dimensional algebraically defined graph $\Gamma_{\mathcal{R}}(f(X,Y))$ is a bipartite graph, constructed using a ring $\mathcal{R}$ and a bivariate function f, where each partite set is a copy of $\mathcal{R}^2$. In this graph, two vertices (a_1,a_2) and (x_1,x_2) are adjacent if and only if $a_2+x_2=f(a_1,x_1)$. The study of algebraically defined graphs can be motivated by incidence geometry, as every graph $\Gamma_{\mathbb{F}_q}(f)$ with girth 6 can be

Previously, all girth 6 graphs $\Gamma_{\mathbb{C}}(f)$ were proved to be isomorphic; however, it was unknown whether all girth 6 graphs $\Gamma_{\mathbb{R}}(f)$ were isomorphic. To address this question, we first prove that whenever p is a polynomial, $\Gamma_{\mathbb{R}}(p)$ has diameter 4 or 5. Moreover, we classify infinite families of such graphs by diameter, including a proof that all known girth 6 graphs $\Gamma_{\mathbb{R}}(p)$ have diameter 4. Ultimately, we use these tools to prove that the two girth 6 graphs $\Gamma_{\mathbb{R}}(XY)$ and $\Gamma_{\mathbb{R}}(X^3Y^3+XY)$ are non-isomorphic. (Received September 21, 2021)

1174-14-11237 **Xiaorun Wu*** (xiaorunw@princeton.edu), Princeton University. *The Picard Group of a General Toric Variety In Higher Dimensions* Preliminary report.

Suppose $\{v_1,\cdots,v_r\}$ is a finite set of vectors in $\mathbb{R}^d$. Denote

$$\sigma=\{x\in\mathbb{R}^d : x=\sum\lambda_i v_i, \lambda_i\in\mathbb{R}, \lambda_i\geq 0\}$$

to be the polyhedral cone. We say σ is a rational if all generators $v_i\in\mathbb{Z}^d$, and σ is strongly convex if $\sigma\cap(-\sigma)=\{0\}$.

Next, define a rational fan Δ in $\mathbb{R}^d$ as a finite union of strongly convex, polyhedral, rational cones such that each pair of cones $\in\delta$ shares a common face.

Additionally, we say Δ is complete if $\bigcup_{\sigma\in\Delta}\sigma=\mathbb{R}^d$. For each complete rational Δ, we may associate a toric variety $T_N(\Delta)$, denoted as X, and denote the Picard group of X as $\mathrm{Pic}(X)$.

The relationship between $\mathrm{Pic}(X)$ and combinatoric property of Δ have been extensively studied. Ford and Stimets(2002) showed if $\Delta\in\mathbb{R}^3$ is a complete rational fan such that every 3-dimensional cone in Δ is non-simplicial, then in any non-empty open neighborhood of Δ exists a fan Δ' such that every Δ'-linear support function is linear and the Picard group of the associated toric variety is zero.

In this paper, we first present a simpler proof on Ford and Stimets's original result. Then extending their results, we study the case in when Δ is a complete rational fan in $\mathbb{R}^d$ such that every d-dimensional cone in Δ is non-simplicial for some $d \geq 4$. (Received September 22, 2021)

1174-14-12241 **Thomas Warren Scanlon*** (scanlon@math.berkeley.edu), University of California, Berkeley. *Tame geometry for Hodge Theory*

Hodge theory brings the methods of complex analysis and differential geometry to algebraic geometry. As such, highly transcendental constructions, such as those of period mappings produced through integration, are used to study problems of an algebraic nature. Some fundamental conjectures in the subject, most notably the Hodge Conjecture itself, predict that certain objects defined using these transcendental methods are in fact algebraic. In 1994, Cattani, Deligne, and Kaplan proved one of the strongest theorems in this vein on the algebraicity of the so-called Hodge locus.

In a paper published in 2020, Bakker, Klingler, and Tsimerman gave a simplified proof of the Cattani-Deligne-Kaplan theorem by showing that the period mappings appearing in that theorem are definable in an o-minimal structure. Here, "definable" carries its precise meaning in the sense of first-order logic and o-minimality is a technical, tameness condition on structures (again in the sense of first-order logic) on the real numbers. The Bakker-Klingler-Tsimerman theorem and a string of subsequent results tying o-minimal to Hodge theory exhibit once more that o-minimality may serve as tame geometry.

In this lecture, I will discuss o-minimality in concrete terms, recall some of the basics of Hodge theory, state the Bakker-Klingler-Tsimerman theorem in a simplified form, and explain the relevance of o-minimality to this theorem and its generalizations. (Received December 5, 2021)

15 ► *Linear and multilinear algebra; matrix theory*

1174-15-5427 **Derya Asaner*** (deryaasaner@csus.edu), CSU-Sacramento, and **Jackson Leaman** (jleaman@g.clemson.edu), Clemson University. *Totally Positive Completion of Partial Matrices with Two Unspecified Entries*

A matrix is *totally positive* if all the square submatrices have positive determinant. A matrix with some unspecified entries is called a *partial matrix*, and a partial matrix is *partial TP* if all square submatrices containing only specified entries have positive determinant. A *completion* of a partial matrix is a matrix obtained by replacing unspecified entries with real values. A partial TP matrix is *TP-completable* if it has a completion which is a totally positive matrix. The TP-completion problem queries when a partial TP matrix is TP-completable. In this talk, we discuss the TP-completion problem for $4 \times n, n \geq 5$ partial matrices with exactly two unspecified entries. In particular, we emphasis on the case in which both unspecified entries lie in the same column. (Received August 20, 2021)

1174-15-5487 **Luyining Gan*** (lgan@unr.edu), University of Nevada Reno, and **Tin-Yau Tam** (ttam@unr.edu), University of Nevada Reno, University of Nevada, Reno. *Metric and spectral geometric means of positive definite matrices* Preliminary report.

We prove log-majorization relations between t-metric geometric mean and t-spectral (geometric) mean, $\left(B^{ts/2}A^{(1-t)s}B^{ts/2}\right)^{1/s}$ and the t-spectral mean, respectively. In particular, the t-spectral mean is the dominant one. The limit involving t-spectral mean is also studied. The results are then extended in the context of symmetric space associated with a noncompact semisimple Lie group. (Received August 20, 2021)

1174-15-5514 **Aleks Kleyn*** (aleks_kleyn@mailaps.org), AMS. *Eigenvector in Non-Commutative Algebra*

Let $\overline{\overline{e}}$ be a basis of vector space V over non-commutative D-algebra A. Endomorhism $\overline{\overline{eb}}$ of vector space V whose matrix with respect to given basis $\overline{\overline{e}}$ has form Eb where E is identity matrix is called similarity transformation with respect to the basis $\overline{\overline{e}}$.

Let V be a left A-vector space and $\overline{\overline{e}}$ be basis of left A-vector space V. The vector $v \in V$ is called eigenvector of the endomorphism

$$\overline{f} : V \to V$$

with respect to the basis $\overline{\overline{e}}$, if there exists $b \in A$ such that

$$\overline{f} \circ v = \overline{\overline{eb}} \circ v$$

A-number b is called eigenvalue of the endomorphism $\overline{f}$ with respect to the basis $\overline{\overline{e}}$.

There are two products of matrices: ${}_*{}^*$ (row column: $(ab)^i_j = a^i_k b^k_j$) and ${}^*{}_*$ (column row: $(ab)^i_j = a^k_j b^i_k$).

A-number b is called ${}_*{}^*$-eigenvalue of the matrix f if the matrix $f - bE$ is ${}_*{}^*$-singular matrix.

A-number b is called right ${}_*{}^*$-eigenvalue if there exists the column vector u which satisfies to the equality

$$a_*{}^*u = ub$$

Column vector u is called eigencolumn for right ${}_*{}^*$ eigenvalue b.

A-number b is called left ${}_*{}^*$-eigenvalue if there exists the row vector u which satisfies to the equality

$$u_*{}^*a = bu$$

Row vector u is called eigenrow for left ${}_*{}^*$-eigenvalue b.

The set ${}_*{}^* - \operatorname{spec}(a)$ of all left and right ${}_*{}^*$-eigenvalues is called ${}_*{}^*$-spectrum of the matrix a. (Received August 21, 2021)

1174-15-5593 **Michael J. Tsatsomeros*** (tsat@wsu.edu), Washington State University. *The Fiber of P-matrices: The Recursive Construction of All Matrices with Positive Principal Minors*

P-matrices have positive principal minors and include many well-known matrix classes (positive definite, totally positive, M-matrices etc.) How does one construct a generic P-matrix? Specifically, is there a characterization of P-matrices that lends itself to the tractable construction of every P-matrix? To answer these questions positively, a recursive method is employed that is based on a characterization of rank-one perturbations that preserve the class of P-matrices. (Received August 23, 2021)

1174-15-5786 **Philip Chodrow*** (pchodrow@gmail.com), Department of Mathematics, UCLA, **Jamie Haddock** (jhaddock@g.hmc.edu), Department of Mathematics, Harvey Mudd College, and **Nicole Eikmeier** (eikmeier@grinnell.edu), Department of Computer Science, Grinnell College. *Spectral Clustering of Nonuniform Hypergraphs*

Hypergraphs encode sets of relationships or interactions between two or more entities. Matrix representations are fundamental to our graph theory toolkit, but the presence of multiway interactions poses challenges for representing hypergraphs using standard matrices such as the adjacency matrix. We study the Hashimoto matrix on hypergraphs with edges of varying size. This matrix encodes multiway interactions by extending the concept of nonbacktracking walks to the hypergraph setting. We then apply the Hashimoto matrix to the community detection problem, which asks us to partition a hypergraph into densely-interconnected subsets. The leading eigenvectors of the Hashimoto matrix reflect community structure in a hypergraph. We prove a theorem of Ihara-Bass type, which considerably enhances the scalability of spectral methods. We then propose an algorithm for aggregating eigenvector information into labeled communities. We also describe in closed form the conditions under which this algorithm succeeds in detecting true labels in synthetic data, and conjecture information-theoretic thresholds for the hypergraph community detection problem. (Received August 27, 2021)

1174-15-6093 **Javad Mashreghi** (Javad.Mashreghi@mat.ulaval.ca), Université Laval, **Ludovick Bouthat*** (ludovick.bouthat.lb@gmail.com), Université Laval, and **Frédéric Morneau-Guérin** (Frederic.Morneau-Guerin@teluq.ca), Université Teluq. *The p-norm of circulant matrices*

In this talk, we'll present our study of the induced p-norm of circulant matrices $A(n, \pm a, b)$, acting as operators on the Euclidean space $\mathbb{R}^n$. For circulant matrices whose entries are nonnegative real numbers, in particular for $A(n, a, b)$, we provide an explicit formula for the p-norm, $1 \leq p \leq \infty$. The calculation for $A(n, -a, b)$ is more complex. The 2-norm is precisely determined. As for the other values of p, two different categories of upper and lower bounds are obtained. These bounds are optimal at the end points (i.e. $p = 1$ and $p = \infty$) as well as at $p = 2$. (Received September 4, 2021)

1174-15-6182 **Caleb X Bugg*** (caleb_bugg@berkeley.edu), UC Berkeley, **Anil Aswani** (aaswani@berkeley.edu), UC Berkeley, and **Chen Chen** (chen.8018@osu.edu), Ohio State University. *Nonnegative Tensor Completion via Integer Optimization* Preliminary report.

There is an unresolved tension in the literature on tensor completion. One set of approaches has polynomial-time computation but requires exponentially more samples than another set of approaches that require solving optimization problems, for which there are no known numerical algorithms to compute global minima.

This paper resolves this tension for nonnegative tensors by developing a numerical algorithm that provably converges to a global minima in a linear (in numerical tolerance) number of oracle steps, while achieving the exponentially-faster sample rate. Our approach is to define a new norm for nonnegative tensors using the gauge of a specific 0-1 polytope that we construct. Because the norm is defined using a 0-1 polytope, this means we can use integer linear programming to solve linear separation problems over the polytope. We combine this insight with a variant of the Frank-Wolfe algorithm to construct our numerical algorithm, and we demonstrate its effectiveness and scalability through numerical experiments. (Received September 6, 2021)

1174-15-6251 **Xiucai Ding*** (xcading@ucdavis.edu), UC Davis. *Fluctuations of eigenvectors of deformed random matrices* Preliminary report.

In this talk, I will report some recent results of the eigenvectors of several deformed random matrix models, including signal-plus-noise model, spiked covariance matrix model and deformed invariant model which are commonly used objects in statistics and applied math. The first and second order fluctuations of the generalized components, i.e., the projections of the eigenvectors onto arbitrary given direction will be reported and some applications will be discussed. This talk is based on joint works with Zhigang Bao, Hong Chang Ji, Jingming Wang, Ke Wang and Fan Yang. (Received September 7, 2021)

1174-15-6429 **Rachel Lawrence*** (rslawrence95@gmail.com), University of California, Berkeley, **Jamie Haddock** (jhaddock@math.ucla.edu), University of California, Los Angeles, **Sam Spiro** (sspiro@ucsd.edu), University of California, Berkeley, and **Sam Spiro** (sspiro@ucsd.edu), University of California, San Diego. *Zero Forcing with Random Sets* Preliminary report.

In the zero forcing procedure, an "infection" propagates through a graph G such that any infected vertex with a unique unaffected neighbor infects that neighbor. While traditional zero forcing problems ask for the minimum cardinality of an initial set which infects all vertices, we ask the related question: Given a randomly chosen initial set, what is the probability that it infects the entire graph? We construct an initial set $B_p(G)$ by selecting vertices independently and with probability p, and determine thresholds for p under which $B_p(G)$ is zero forcing with high probability. We find analogs to known results from the deterministic setting, and derive upper and lower bounds on the probability of selecting a zero forcing set for several classes of graphs. In particular, we show that the unique value $p^*(G)$ at which $\Pr[B_p(G)$ is zero forcing$] = \frac{1}{2}$ for an n-vertex graph satisfies $p^*(G) = \Omega(n^{-1/2})$, and make progress towards a new "randomized path conjecture" proposing that the path graph P_n satisfies $Pr[B_p(G)$ is zero forcing$] \leq Pr[B_p(P_n)$ is zero forcing$]$ for all n-vertex graphs. (Received September 21, 2021)

1174-15-6454 **Franklin Kenter*** (kenter@usna.edu), United States Naval Academy, and **Jephian C.-H. Lin** (jephianlin@gmail.com), National Sun Yat-sen University. *Zero forcing parameters, the ordered multiplicity inverse eigenvalue sequence problem for graphs and powers of graphs*

Given a graph G, one may ask: "What sets of eigenvalues are possible over all weighted adjacency matrices of G?" (Here, negative and diagonal weights are allowed). This is known as the Inverse Eigenvalue Problem for Graphs (IEPG) A mild relaxation of this question considers the multiplicity sequence instead of the exact eigenvalues themselves. For instance, given a graph G on n vertices and an ordered partition $\mathbf{m} = (m_1, \ldots, m_\ell)$ of n, is there a weighted adjacency matrix where the i-th distinct eigenvalue has multiplicity m_i? This is known as the ordered multiplicity inverse eigenvalue sequence problem. Recent work has solved this problem for all graphs on 6 vertices.

In this talk, we develop zero forcing methods for the ordered multiplicity IEPG in a multitude of different contexts. Namely, we utilize zero forcing parameters on powers of graphs to achieve bounds on sums of various multiplicities. Not only can we verify the above result in a more straight-forward manner, but we apply our techniques to skew-symmetric matrices, nonnegative matrices, among others. (Received September 9, 2021)

1174-15-6850 **Michael L Overton*** (overton@cs.nyu.edu), Courant Institute, NYU. *Local Minimizers of the Crouzeix Ratio: A Nonsmooth Optimization Case Study*

Given a square matrix A and a polynomial p, the Crouzeix ratio is the norm of the polynomial on the field of values of A divided by the 2-norm of the matrix $p(A)$. Crouzeix's conjecture states that the globally minimal value of the Crouzeix ratio is 0.5, regardless of the matrix order and polynomial degree, and it is known that 1 is a frequently occurring locally minimal value. Making use of a heavy-tailed distribution to initialize our optimization computations, we demonstrate for the first time that the Crouzeix ratio has many other locally minimal values between 0.5 and 1. Besides showing that the same function values are repeatedly obtained for many different starting points, we also verify that an approximate nonsmooth stationarity condition holds at computed candidate local minimizers. We also find that the same locally minimal values are often obtained both when optimizing over real matrices and polynomials, and over complex matrices and polynomials. We argue that minimization of the Crouzeix ratio makes a very interesting nonsmooth optimization case study, illustrating among other things how effective the BFGS method is for nonsmooth, nonconvex optimization. Our method for verifying approximate nonsmooth stationarity is based on what may be a novel approach to finding approximate subgradients of max functions on an interval. Our extensive computations strongly support Crouzeix's conjecture: in all cases, we find that the smallest locally minimal value is 0.5. (Received September 9, 2021)

1174-15-6871 **I. M Spitkovsky*** (imspitkovsky@gmail.com), NYUAD. *Kippenhahn curves and numerical ranges of some structured matrices*

The numerical range $W(A)$ (a.k.a. the field of values, or the Hausdorff set) of an n-by-n matrix A is defined as the image of the unit sphere of $\mathbb{C}^n$ under the mapping $f_A : x \mapsto x^*Ax$. It is a compact subset of $\mathbb{C}$, which is also convex due to the celebrated *Toeplitz-Hausdorff theorem*. Moreover, as was first observed by R. Kippenhahn, $W(A)$ is the convex hull of a certain algebraic curve $C(A)$ of class n, thus nowadays called the *Kippenhahn curve* of A. This provides an insight into the *Elliptical range theorem*: for $n = 2$ the numerical range is an elliptical disk (degenerating into the line segment connecting the eigenvalues of A when A is normal).

As n increases, there is more variety in possible shapes of $W(A)$. Surprisingly though, for some classes of matrices $W(A)$ stays elliptical (or ends up being the convex hull of a small, compared to n, number of ellipses). It became clear recently that the phenomenon at hand is caused by $C(A)$ consisting of several components, the "exposed" of them being ellipses.

In this talk, we describe several such classes. It is based on joint work with N. Bebiano, J. Providéncia, K. Vazquez, and M. Jiang. (Received September 10, 2021)

1174-15-7064 **Dibyajyoti Deb*** (dibyajyoti.deb@oit.edu), Oregon Institute of Technology. *Integrating code into your Linear Algebra class - Teaching data science and engineering majors at an engineering focused undergraduate university.* Preliminary report.

In this talk, we will look into some ways I have incorporated application problems and solving them using Python into my linear algebra class. The audience in this class are various engineering and data science majors, hence incorporating various applications and simulations are of utmost importance. Students in this class have often reiterated that while they "understand" what a matrix is, they often fail to relate it to a real-world problem. This has allowed me to include various application oriented problems, that relate to the standard topics in a linear algebra class. In particular, we will look at using linear transformations, matrices and Python code to break ciphers and few other examples. (Received September 11, 2021)

1174-15-7080 **Pan-Shun Lau*** (panlau@connect.hku.hk), Univerisity of Nevada Reno, **Chi-Kwong Li** (ckli@math.wm.edu), College of William and Mary, and **Yiu-Tung Poon** (ytpoon@iastate.edu), Iowa State University. *The Joint Numerical Range of Commuting Matrices*

In the talk, we show that for $n \geq 3$ the joint numerical range of a family of commuting $n \times n$ complex matrices is always convex; for $n \geq 4$ there are two commuting matrices whose joint numerical range is not convex. (Received September 12, 2021)

1174-15-7226 **Daryl Q Granario** (daryl.granario@dlsu.edu.ph), De La Salle University. *Products of positive definite symplectic matrices*

In the last 1960 Ballantine showed that every matrix with positive determinant is a product of five positive definite matrices. We consider the complex symplectic group $\mathrm{Sp}(2n, \mathbb{C})$:

$$\mathrm{Sp}(2n, \mathbb{C}) = \left\{ A \in \mathrm{GL}(2n, \mathbb{C}) : A^\top J_n A = J_n \right\},$$

where

$$J_n = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}.$$

The symplectic group is a classical group defined as the set of linear transformations of a $2n$-dimensional vector space over $\mathbb{C}$, which preserve the non-degenerate skew-symmetric bilinear form that is defined by J_n. We show that every symplectic matrix is a product of five positive definite symplectic matrices. We also show that five is the best in the sense that there are symplectic matrices which are not product of less. (Received September 13, 2021)

1174-15-7289 **Eduardo Oregon Reyes*** (eoregon@berkeley.edu), University of California, Berkeley. *Matrix products inequalities, the Berger-Wang identity, and non-positive curvature* Preliminary report.

The joint spectral radius of a bounded set $\mathcal{M}$ of $d \times d$ matrices was introduced by Rota and Strang, and represents the maximal exponential growth rate of the partial sequence of products $(A_1 \cdots A_n)_n$ of a sequence of matrices $A_1, A_2, \ldots$ with each A_i in $\mathcal{M}$. In 2003, Jairo Bochi gave an upper bound for the joint spectral radius in terms of the spectral radii of products of at most $2^d - 1$ matrices, implying a new proof of the Berger-Wang identity, which allows us to recover the joint spectral radius from a suitable sequence of spectral radii of matrix products. Since then, several versions of Bochi's inequality have appeared in literature, with applications including regularity of pressure-like functions and uniform exponential growth for isometries of some non-positively curved metric spaces. In this talk, I will introduce a new Bochi-type inequality relating the singular values of a product of matrices and the eigenvalues of appropriate subproducts, and show how this implies a geometric Berger-Wang identity for symmetric spaces of non-compact type, recovering a result of Breuillard-Fujiwara. (Received September 14, 2021)

1174-15-7372 **Samuel Cole*** (smpcole@gmail.com), University of Missouri. *Clusters in Markov Chains via the SVD of the Laplacian Matrix* Preliminary report.

Suppose that T is a stochastic matrix. We wish to identify clusters in the Markov chain associated with T—i.e., to partition the state space into subsets within which the transition density is high, while the transition density between different subsets is low. We present two algorithms which can be used for this task, both based on the singular value decomposition of the *Laplacian matrix* $I - T$. The first algorithm is recursive in nature and uses the sign pattern of a left singular vector associated with the second smallest singular value to isolate one cluster at a time. The second uses the orthogonal projection matrix onto the left or right singular subspace corresponding to the smallest k singular values to simultaneously identify k clusters. (Received September 14, 2021)

1174-15-7639 **Xiangxiang Wang*** (xxw@nevada.unr.edu), University of Nevada, Reno. *Arnoldi method for right eigenvalue problem of the large-scale quaternion matrices* Preliminary report.

Abstract: In this talk, we will discuss the right eigenvalue problem of the quaternion matrices. In particular, we will provide Arnoldi method to compute the right eigenvalues of the large-scale quaternion matrices. We finally show the application of the quaternion matrices in color image processing. (Received September 15, 2021)

1174-15-7712 **Hayden Julius*** (hjulius@ysu.edu), Youngstown State University, and **Louisa Catalano** (catalanl@union.edu), Union College. *Product preserving mappings in matrix algebras* Preliminary report.

Let $M_n(\mathbb{C})$ denote the algebra of $n \times n$ complex matrices. Every automorphism of $M_n(\mathbb{C})$ is an inner automorphism (that is, of the form $X \mapsto UXU^{-1}$ for some invertible matrix U) by the classical Skolem-Noether theorem. Linear preserver problems study linear mappings on matrix algebras with an invariant subset, function, or relation, etc., and measure how close these mappings are to automorphisms. LPPs have a varied and rich history dating back to pioneering work of Frobenius, Hua, and Dieudonné. In the contemporary, LPPs are actively studied both on finite-dimensional and infinite-dimensional algebras of matrices/linear operators. One well-studied LPP is the *zero-product* preserving problem, which asks for a description of linear maps ϕ satisfying $\phi(A)\phi(B) = 0$ whenever $AB = 0$. In many cases, the solution to the zero-product preserver problem is that the linear map ϕ must, in fact, be a scalar times a homomorphism.

A nonzero modification of this problem is the following: if ϕ is a bijective linear map satisfying $\phi(A)\phi(B) = D$ whenever $AB = C$, must ϕ also be a scalar times a homomorphism? We provide a complete answer to this problem for $M_n(\mathbb{C})$ and discuss recent results and generalizations to other spaces. This talk is largely accessible to audiences familiar with linear algebra. The presentation is based on the recent paper "On maps preserving products equal to fixed elements" appearing in Journal of Algebra. (Received September 15, 2021)

1174-15-7898 **Aelita Klausmeier** (aelita@umich.edu), University of Michigan, **Joey Veltri*** (jveltri@purdue.edu), Purdue University, and **Maribel Bueno** (mbueno@ucsb.edu), University of California, Santa Barbara. *Linear Preservers of Eigenvalues Induced by the Two-Dimensional Ice Cream Cone* Preliminary report.

Given a real $n \times n$ matrix A and a closed convex cone $K \subseteq \mathbb{R}^n$, the eigenvalue complementarity problem generalizes the eigenvalue problem by finding $\lambda \in \mathbb{R}$ and nonzero $x \in \mathbb{R}^n$ such that $K \ni x \perp (A - \lambda I)x \in K^*$, where K^* denotes the dual cone of K. We say λ is a Lorentz eigenvalue of A when K is the Lorentz cone, also called the ice cream cone. Linear maps on the space M_n of real $n \times n$ matrices and the space S_n of real symmetric $n \times n$ matrices preserving Lorentz eigenvalues are well studied when $n \geq 3$, assuming the map is standard, i.e., of the form $A \mapsto PAQ$ or $A \mapsto PA^TQ$ for some matrices P and Q. Because the Lorentz cone for $n = 2$ is polyhedral, this case has been left out of the literature and turns out to be more complex than $n \geq 3$ in certain ways. In our research, we fully characterize the standard maps preserving the Lorentz spectrum on M_2 and S_2 and show that it is precisely these maps that preserve the nature of the Lorentz eigenvalues, i.e., whether they correspond to eigenvectors on the interior or boundary of the Lorentz cone. In the case of M_2, we show that different kinds of linear preservers are possible compared to M_n for $n \geq 3$. Moreover, we prove that all linear maps preserving the Lorentz spectrum on S_2 are standard and conjecture that the same holds for M_2. (Received September 19, 2021)

1174-15-7998 **Yuqiao Li*** (yli53@email.wm.edu), College of William and Mary. *Copositive Matrices, their Dual, and the Recognition Problem*

Copositivity is a generalization of positive semidefiniteness. It has applications in economics, operations research, and statistics. An n-by-n real matrix A is copositive (CoP) if $x^T Ax \geq 0$ for any nonnegative vector $x \geq 0$. The CoP matrices form a proper cone. A CoP matrix is ordinary if it can be written as the the sum of a positive semidefinite (PSD) matrix and a symmetric nonnegative (sN) matrix. When $n < 5$, all copositive matrices are ordinary. However, recognition that a given CoP matrix is ordinary and the determination of an ordinary decomposition is an unresolved issue. Here, we make observations about CoP-preserving operations, make progress about the recognition problem, and discuss the relationship between the recognition problem and the PSD completion problem. We also mention the problem of copositive spectra and its relation to the symmetric nonnegative inverse eigenvalue problem. (Received September 19, 2021)

1174-15-8140 **Joel Louwsma*** (jlouwsma@niagara.edu), Niagara University. *Divisibility properties of minors of matrices* Preliminary report.

Let A be an $m \times n$ matrix with integer entries, and let k be an integer satisfying $1 \leq k \leq \min\{m, n\} - 1$. It is well known and easy to see that the greatest common divisor of all $k \times k$ minors of A divides the greatest common divisor of all $(k+1) \times (k+1)$ minors of A. We discuss an analogue of this statement for a certain subset of minors of A. (Received September 17, 2021)

1174-15-8314 **Elizabeth Newman*** (elizabeth.newman@emory.edu), Emory University, **Misha Kilmer** (misha.kilmer@tufts.edu), Tufts University, **Lior Horesh** (lhoresh@us.ibm.com), IBM Research, and **Haim Avron** (haimav@tauex.tau.ac.il), Tel Aviv University.
Tensor-tensor algebra for optimal representation and compression of multiway data

With the advent of machine learning and its overarching pervasiveness it is imperative to devise ways to represent large datasets efficiently while distilling intrinsic features necessary for subsequent analysis. The primary workhorse used in data dimensionality reduction and feature extraction has been the matrix singular value decomposition (SVD), which presupposes that data have been arranged in matrix format. In this talk, we will show that high-dimensional datasets are more compressible when treated as tensors (i.e., multiway arrays) and compressed via tensor-SVDs under the tensor-tensor product constructs and its generalizations. We will demonstrate that the tensor-SVD under the tensor-tensor product satisfies Eckart-Young-like optimality results. Moreover, we will show that the optimal representation under the tensor-tensor product is provably better than its matrix counterpart and two tensor-based analogs. We will support these theoretical findings with some empirical studies. (Received September 20, 2021)

1174-15-8333 **Yihua Xu*** (yxu604@gatech.edu), Georgia Institute of Technology, **Katherine Keegan** (keegank1624@marybaldwin.edu), Mary Baldwin University, and **Tanvi Vishwanath** (tanvivishwanath@tamu.edu), Texas A&M University. *Tensor-Based Approaches to fMRI Classification*

To analyze the abundance of multidimensional data, tensor-based frameworks have been developed. Traditional matrix-based frameworks extract the most relevant features of vectorized data using the matrix-SVD. However, we may lose crucial high-dimensional relationships in this process. To facilitate efficient multidimensional feature extraction, we propose a projection-based classification algorithm using the t-SVDM, a tensor-based extension of the matrix-SVD. We apply our algorithm to the StarPlus functional Magnetic Resonance Imaging (fMRI) dataset. Through our numerical experiments, we conclude that there exists a more accurate tensor-based approach to fMRI classification than the best possible equivalent matrix-based approach. Our research showcases the potential of tensor-based classification frameworks, and justifies further research into the usage of tensors for numerous other classification tasks. (Received September 21, 2021)

1174-15-8364 **Maverick Lara*** (mzlara@calpoly.edu), Cal Poly San Luis Obispo, Bill and Linda Frost Fund, and **Brooke Randell** (brandell@calpoly.edu), Cal Poly San Luis Obispo, Bill and Linda Frost Fund. *Characterizing the Numerical Range of Block Toeplitz Operators*

The numerical range of an operator A on a complex Hilbert space $\mathcal{H}$ is the subset of the complex plane defined by $W(A) = \{\langle Av, v\rangle : v \in \mathcal{H}, ||v|| = 1\}$. Let ϕ be a bounded analytic matrix-valued function on the unit disc $\mathbb{D}$. The block Toeplitz operator of symbol ϕ, denoted T_ϕ, is defined by $T_\phi f(z) = \phi(z)f(z)$ for a function f on the vector-valued Hardy Hilbert space. A theorem of Bebiano and Spitkovsky states that the closure of the numerical range of a general analytic block Toeplitz operator is the convex hull of the set of numerical ranges $\{W(\phi(z)) : |z| = 1\}$. We have restricted our research to $W(T_\phi)$ where $\phi(z) = A_0 + zA_1$. Even in this case, a wide variety of shapes is possible. We have several results describing the numerical range of block Toeplitz operators when the blocks are 2x2 matrices. These include separate results about $W(T_\phi)$ when A_0 is the zero matrix, when the $W(\phi(z))$ sets are nested, and when the boundary of $W(T_\phi)$ has flat portions. (Received September 18, 2021)

1174-15-8608 **Judi J McDonald*** (judijmcdonald@gmail.com), Washington State University, and **Faizah Alanazi** (shine-star2008@hotmail.com), Washington State University. *The Symmetric Nonnegative Inverse Eigenvalue Problem for 6x6 Matrices*

The Symmetric Inverse Eigenvalue Problem asks when an n-tuple of numbers can be the eigenvalues of a nonnegative symmetric matrix. This problem is unsolved for $n \geq 5$. In this talk, we will discuss results for 6x6 matrices. (Received September 19, 2021)

1174-15-8663 **Paula Kimmerling*** (paula.kimmerling@wsu.edu), Washington State University. *Average Mixing Matrices of Dutch Windmills*

Let A be the adjacency matrix of a graph. We may associate this graph with a continuous-time quantum walk by using a transition matrix $U(t) = \exp itA$. This allows us to also create another matrix $\hat{M}$ which is independent of time and gives some measure of average probability values and long-term behavior. We will call $\hat{M}$ the average mixing matrix. In our research, we've shown what happens to the rank of $\hat{M}$ if we restrict our graphs to Dutch Windmill graphs, a type of cactus graph. In this talk we will show that $\hat{M}$ for Dutch Windmill has no better than half-rank and why. (Received September 19, 2021)

1174-15-8934 **Shaun M Fallat** (shaun.fallat@uregina.ca), University of Regina, **H Tracy Hall** (h.tracy@gmail.com), Hall Labs, LLC (Provo, UT), and **Bryan L Shader*** (bshader@uwyo.edu), University of Wyoming. *Matrix Liberation, Bifurcation and other Techniques Related to Strong Properties*

Recently strong properties for various matrix problems (e.g. nullity, spectra, multiplicity list, allowing orthogonality) have been developed. These strong properties are conditions on a matrix A with graph G that guarantee that every supergraph H of G contains a matrix B with the same matrix property as A.

Sometimes one is interested in just some supergraphs H of G having a matrix B with the same matrix property as A, or in G itself having a matrix B with a slightly different matrix property than that of A. We will discuss some recent techniques and results that allow one to study such questions. (Received September 20, 2021)

1174-15-9521 **Kevin Vander Meulen** (kvanderm@redeemer.ca), Redeemer University, **Minerva Catral*** (catralm@xavier.edu), Xavier University, **Jane Breen** (jane.breen@ontariotechu.ca), Ontario Tech University, **Carraugh Brouwer** (cbrouwer@redeemer.ca), Redeemer University, **Michael Cavers** (mscavers@gmail.com), University of Toronto, and **Pauline van den Driessche** (vandendr@uvic.ca), University of Victoria, B.C., Canada. *On the number of distinct eigenvalues allowed by a sign pattern* Preliminary report.

A sign pattern (matrix) $\mathcal{A}$ has entries in $\{+, -, 0\}$. For a real $n \times n$ matrix A, we define $q(A)$ to be the number of distinct eigenvalues of A. The minimum number of distinct eigenvalues allowed by sign pattern $\mathcal{A}$, denoted $q(\mathcal{A})$, is the minimum value of $q(A)$ over all A having the sign pattern of $\mathcal{A}$. We explore $q(\mathcal{A})$ using digraph properties of the sign pattern and give characterizations for patterns of small order. (Received September 20, 2021)

1174-15-9859 **Thomas R Cameron** (trc5475@psu.edu), PSU Behrend, and **Ehssan Khanmohammadi*** (ehssan@pm.me), Bowdoin College. *The Strong Restricted Interlacing Property of Matrices*

In a sense that can be made precise, the most natural compression of a symmetric matrix is onto the orthogonal complement of the all-ones vector. Interlacing of eigenvalues of a matrix and its compression with respect to this particular choice of vector is called *restricted interlacing.* We define the Strong Restricted Interlacing Property (SRIP) which is equivalent to the transversality of certain pairs of manifolds and investigate general results on graph families and particular results for small graphs. (Joint with T. R. Cameron, B. Curtis, H. T. Hall, M. Hunnell, B. Shader, and B. Small) (Received September 21, 2021)

1174-15-10261 **Sara Abdali*** (sabda005@ucr.edu), University of California, Riverside, Georgia Institute of Technology. *Tensor embedding for misinformation and fake news detection* Preliminary report.

Distinguishing between misinformation and real information is one of the most challenging problems in today's interconnected world. The vast majority of the state-of-the-art in detecting misinformation are fully supervised, requiring a large number of high-quality human annotations. In such scenarios, we leverage a multi-linear algebra framework a.k.a. "tensor" which has been shown to be a proper tool for semi-supervised and unsupervised settings where there is a few or no annotation. We propose the following techniques:

- Content-based detection of misinformation: We propose a novel strategy mixing tensor-based modeling of article content and semi-supervised learning on article embeddings.
- Tensor-based techniques for multi-aspect detection of mis-information: We propose HiJoD a 2-level decomposition pipeline that characterize a news article as a manifold of multiple "aspects" to compensate for the lack of labels.
- Identifying misinformation from website screenshots: we propose using the overall look of the domain web page for detecting misinformation. To capture this overall look, we leverage a tensor decomposition based classification technique on screenshots of serving domains.
- Tensor embedding for Deepfake detection: we develop a multi-linear framework for Deepfake video detection as a newly emerging source of misinformation.

Based on the aforementioned techniques, we have already made a huge progress in detecting misinformation in label scare settings.
(Received September 21, 2021)

1174-15-11073 **Kouakou Francois Domagni*** (domagnifran@gmail.com), AMS. *Supersaturated designs for $N = 10$.* Preliminary report.

We provide a list of $E(s^2)$-optimal supersaturated designs for $N = 10$ and $10 \leq m \leq 126 = \frac{1}{2}\binom{10}{5}$. In the process we revisit and provide simpler proofs for some known result and point out a combinatorial restriction that has led to an improved lower bound. (Received September 21, 2021)

1174-15-11083 **Luc Telemaque*** (luc.telemaque@mail.citytech.cuny.edu), New York City College of Technology. *The Combinatorial Laplacian* Preliminary report.

The Laplacian matrix of a graph, denoted L, can be defined as $L = D - A$, where D is the diagonal matrix and A is the adjacency matrix of the graph. The Laplacian matrix is often referred to as the graph Laplacian or combinatorial Laplacian and it is the discrete analogue to the Laplacian operator in multivariable calculus. The Laplacian matrix and its eigenvalues provide important information on the graph. The Laplacian matrix has numerous applications such as in data mining, pathfinding algorithms and electrical network theory. The Laplacian matrix is also the key idea in Kirchhoff's matrix tree theorem. We will study the Laplacian matrix and its eigenvalues for different graphs and we will investigate some of the applications. (Received September 21, 2021)

16 ► Associative rings and algebras

1174-16-6863 **Xingting Wang*** (xingting.wang@howard.edu), Howard University, **Van C. Nguyen** (vnguyen@usna.edu), United States Naval Academy, **Charlotte Ure** (cu9da@virginia.edu), University of Virginia, **Padmini Veerapen** (pveerapen@tntech.edu), Tennessee Tech University, **Hongdi Huang** (h237huan@rice.edu), Rice University, and **Kent B. Vashaw** (kentv@mit.edu), MIT. *A cogroupoid associated to preregular forms* Preliminary report.

We construct a cogroupoid given by preregular forms, which yields Morita-Takeuchi equivalence between Manin's universal quantum groups associated to superpotential algebras. (Received September 9, 2021)

1174-16-6943 **Manuel L. Reyes*** (mreyes57@uci.edu), University of California, Irvine. *Quantizing the maximal spectrum*

Integrating perspectives from work of Takeuchi, Batchelor, Kontsevich-Soibelman, and Le Bruyn, we view the Sweedler dual coalgebra A° of an algebra A with "many" finite-dimensional representations as a quantization of its maximal spectrum. We will explain a sense in which the Sweedler dual is the minimal contravariant functor from algebras to coalgebras that performs as expected on finite-dimensional semisimple algebras, even though it yields a larger invariant than expected for commutative algebras. We will also discuss methods to compute the dual coalgebra in a variety of situations. (Received September 10, 2021)

1174-16-7000 **Peter Goetz** (pdg11@humboldt.edu), Humboldt State University, and **Andrew B Conner*** (abc12@stmarys-ca.edu), Saint Mary's College of California. *Noncommutative projective geometry of certain 3-dimensional, non-regular algebras* Preliminary report.

Let $\Bbbk$ be an algebraically closed field. Let $\mathcal{Z} = \{Z_d \subset (\mathbb{P}^n)^{\times d}\}$ be a sequence of subschemes having the property that $\pi_1(Z_d), \pi_2(Z_d) \subset Z_{d-1}$, where π_1, π_2 project onto the first and last $d-1$ factors, respectively. Artin-Tate-van den Bergh described a canonical way to associate a graded algebra B to the sequence $\mathcal{Z}$. Of principal interest in noncommutative projective geometry is the following special case: Let T be a free associative $\Bbbk$-algebra, and $I \subset T$ a homogeneous ideal. Let Z_d be the subscheme of zeros of elements of I_d, viewed as multilinear forms on d copies of T_1^*.

If A is a quadratic Artin-Schelter (AS) regular algebra on three generators, $Z_d \cong Z_2$ for all $d \geq 2$, and B is isomorphic to the twisted homogeneous coordinate ring of A. If A is not AS-regular, the sequence $\{Z_d\}$ need not stabilize and computing B can be difficult. In this talk we will present new examples, and discuss the ring structure of B, when A is a non-AS-regular algebra on three generators. (Received September 11, 2021)

1174-16-7287 **Hongdi Huang** (hh40@rice.edu), Rice University, **Elizabeth Wicks*** (elizabeth.wicks@microsoft.com), Microsoft, and **Robert Won** (robertwon@email.gwu.edu), George Washington University. *Universal Quantum Semigroupoids*

We introduce the concept of a universal quantum linear semigroupoid (UQSGd), which is a weak bialgebra that coacts on a (not necessarily connected) graded algebra A universally while preserving grading. We restrict our attention to algebraic structures with a commutative base so that the UQSGds under investigation are face algebras (due to Hayashi). The UQSGd construction generalizes the universal quantum linear semigroups introduced by Manin in 1988, which are bialgebras that coact on a connected graded algebra universally while preserving grading. Our main result is that when A is the path algebra kQ of a finite quiver Q, each of the various UQSGds introduced here is isomorphic to the face algebra attached to Q. The UQSGds of preprojective algebras and of other algebras attached to quivers are also investigated. (Received September 14, 2021)

1174-16-7630 **Xin Tang*** (xtang@uncfsu.edu), Math & Computer Science, Fayetteville State University, and **James J. Zhang** (zhang@math.washington.edu), University of Washington. *Cancellation Properties for Algebras with Respect to Non-standard Factors* Preliminary report.

Motivated by a question of Zariski, the cancellation property has been extensively investigated for commutative algebras. Recently, there has been much progress in the study of cancellation and skew cancellation property for noncommutative algebras and Poisson algebras. In these works, the cancellation factor has been mainly assumed to be either a polynomial algebra or a skew polynomial algebra of finite rank. In this talk, we will present some preliminary results on the cancellation problem for algebras with respect to some non-standard factors such as a torus or a polynomial algebra or a skew polynomial algebra of infinite rank. This is joint work with James Zhang. (Received September 15, 2021)

1174-16-7637 **Kayla Orlinsky*** (KaylaOrlinsky@gmail.com), University of Southern California. *Computing indicators of simple modules of a bismash product*

For any factorizable group $L = FG$, the bismash product $H_n = \mathbb{C}^F \# \mathbb{C}G$ and its dual $J_n = \mathbb{C}^G \# \mathbb{C}F$ form semisimple Hopf algebras. When $L = S_n = C_n S_{n-1}$, the indicators of all simple modules of $H_n = \mathbb{C}^{C_n} \# \mathbb{C}S_{n-1}$ are known to be $+1$ or 0 [JM]. In this talk, we discuss the far more elusive dual $J_n = \mathbb{C}^{S_{n-1}} \# \mathbb{C}C_n$ and describe how a Python program showed the indicators of all simple J_n-modules for $n \leq 10$ are also $+1$ or 0 as well as describe how it provided insight into larger n. (Received September 15, 2021)

1174-16-8025 **Fabio Calderon*** (facalderonm@unal.edu.co), Universidad Nacional de Colombia. *On the study of algebraic properties of universal quantum semigroupoids*

In the last few decades there has been an open inquiry on whether a coacting Hopf-type structure H and an algebra A that is coacted upon share algebraic properties. In this talk I will address the progress that has been made for the case when H is an universal quantum semigroupoid, which are weak bialgebras arising when studying quantum symmetries over non-connected graded algebras. Also, I will present some motivations behind this research and the following steps of our study. (Received September 17, 2021)

1174-16-8072 **Ellen E Kirkman** (kirkman@wfu.edu), Wake Forest University, and **W. Frank Moore** (moorewf@wfu.edu), Wake Forest University. *A certain 4-dimensional Artin-Schelter regular algebra from non-commutative invariant theory* Preliminary report.

According to Kirkman-Kuzmanovich-Zhang a finite group G is a *dual reflection group* if the Hopf algebra $H = \urcorner^G$ acts homogeneously and inner faithfully on a noetherian AS regular domain A such that the fixed ring A^H is also AS regular. For G the semidihedral group of order 16, Vashaw constructed an algebra A that would show G is a dual reflection group, but, at that time, it was unknown if A was AS regular. In this talk I will discuss proofs that A is AS regular, and other algebraic and geometric properties of A. The algebra exhibits some new behavior, especially regarding incidence relations among point and line modules. This is joint work with Ellen Kirkman, Frank Moore and Kent Vashaw. (Received September 17, 2021)

1174-16-8074 **Alan Koch*** (akoch@agnesscott.edu), Agnes Scott College. *Commutator-central maps, Hopf-Galois structures, braces, and systems of solutions to the Yang-Baxter equation* Preliminary report.

Let L/K be a nonabelian Galois extension, and let $G = \mathrm{Gal}(L/K)$. Let $C, Z \leq G$ denote the centralizer and center of G respectively, and let $\psi : G \to G$ be an endomorphism such that $\psi(C) \leq G$. It is known that ψ gives rise to a Hopf-Galois structure on L/K, a skew left brace, and a pair of non-degenerate, involutive set-theoretic solutions to the Yang-Baxter equation. We will show how ψ can be used to generate multiple Hopf-Galois structures, a system of interconnected skew left braces called a *brace block*, as well as numerous solutions to the Yang-Baxter equation. (Received September 17, 2021)

1174-16-8231 **Tolulope Oke** (oket@wfu.edu), Wake Forest University, **Benjamin Briggs** (briggs@math.utah.edu), MSRI, **Sarah J. Witherspoon*** (sjw@tamu.edu), Texas A&M University, and **Pablo Sanchez Ocal** (socal@ucla.edu), UCLA. *Twisted tensor product algebras and Hochschild cohomology*

Many algebras of interest can be viewed as twisted tensor products of two subalgebras. To what extent can the Hochschild cohomology of a twisted tensor product algebra be described as a corresponding twisted tensor product? We describe some recent results in answer to this question for tensor products of algebras twisted by a bicharacter. (Received September 18, 2021)

1174-16-8351 **James Jian Zhang** (zhang@math.washington.edu), University of Washington. *The Jacobian, Reflection Arrangement, and Discriminant for Reflection Hopf Algebras*

Let $\Bbbk$ be an algebraically closed field of characteristic zero. When H is a semisimple Hopf algebra that acts inner faithfully and homogenously on an Artin-Schelter algebra A so that A^H is also Artin-Schelter regular, we call H a reflection Hopf algebra for A; when $H = \Bbbk[G]$ and $A = \Bbbk[x_1, \ldots, x_n]$ then H is a reflection Hopf algebra for A if and only if G is a reflection group. We show that there exist notions of the Jacobian, reflection arrangement, and discriminant that extend the definitions used for reflection groups actions on polynomial algebras to this noncommutative setting. (Received September 18, 2021)

1174-16-8444 **Birge Huisgen-Zimmermann*** (birge@math.ucsb.edu), University of California at Santa Barbara, **Zahra Nazemian** (z_nazemian@yahoo.com), University of Graz, and **Manuel Saorin** (msaorinc@um.es), University of Murcia. *Dualities from iterated tilting* Preliminary report.

We will start with an overview of dualities induced by tilting objects of a finite dimensional algebra Λ; the most comprehensive dualities of this type correspond to *strong tilting modules* (= tilting modules which are relatively injective in the category of Λ-modules of finite projective dimension). Then, we will explore the situation in which the process of strongly tilting Λ-mod allows for arbitrary repeats. This turns out out be a frequent event, which occurs precisely when the initial process of strongly tilting Λ-mod allows for at least one repetition; in this case the resulting sequence of strongly tilted categories eventually turns periodic with period two. These findings are part of recent joint work with Z. Nazemian and M. Saorín. (Received September 19, 2021)

1174-16-8751 **Anne Virginia Shepler*** (ashepler@unt.edu), University of North Texas. *Drinfeld Hecke algebras and quantum polynomial rings*

We consider automorphisms of quantum polynomial rings (i.e., skew polynomial rings) from the viewpoint of a quantum determinant. This approach aids exploration of deformations of skew group algebras for reflection groups acting on quantum polynomial rings over fields of positive characteristic. In the nonquantum setting, a combinatorial approach is still helpful for studying deformations in positive characteristic, and we consider quadratic algebras built on relations that deform both the commutativity of the polynomial ring (Drinfeld-type) and the action of the group (Lusztig-type). (Received September 19, 2021)

1174-16-8755 **Sarah Witherspoon** (sjw@math.tamu.edu), Texas A&M University. *Gerstenhaber brackets for skew group algebras in positive characteristic*

The Gerstenhaber bracket of an algebra controls its deformation theory. We discuss techniques for investigating this graded Lie bracket in the setting of groups acting on algebras. The twisted product resolution provides a tool for analyzing the case when the characteristic divides the order of the acting group. (Received September 19, 2021)

1174-16-8919 **Marcelo Aguiar** (maguiar@math.cornell.edu), Cornell University, and **Jose Bastidas*** (jdb394@cornell.edu), LaCIM, Université du Québec à Montréal (UQÀM). *Type B Hopf monoids* Preliminary report.

The theory of Hopf monoids in the category of species provides a powerful tool to study families of combinatorial structures with natural merge and break operations. To name a few, linear orders, matroids, and generalized permutahedra give rise to different Hopf monoids. In this talk, we introduce a novel definition of type-B Hopf monoids. Following the ideas of Bergeron and Choquette, we consider $\mathcal{H}$-species. This type of species studies structures over finite sets with a fixed-point free involution, where the hyperoctahedral group naturally acts. However, instead of directly defining a monoidal structure on $\mathcal{H}$-species, we endow some $\mathcal{H}$-species with a module and comodule structure over a (standard) Hopf monoid. We present the basic definitions of type-B bimonoids, their antipode map, and some examples arising from type-B linear orders and type-B generalized permutahedra. This is a work in progress with M. Aguiar. (Received September 20, 2021)

1174-16-9017 **Kathryn McCormick*** (kathryn.mccormick@csulb.edu), California State University, Long Beach. *Twisted Steinberg Algebras*

Twisted Steinberg algebras are a generalization of Steinberg algebras, and a purely algebraic notion of twisted groupoid C^*-algebras. Given a sufficiently nice commutative ring R, one builds a certain algebra of R-valued functions $A_R(G;\Sigma)$ on a discrete twist Σ over an ample Hausdorff groupoid. In this talk, we will describe the relationship between some properties of the algebra $A_R(G;\Sigma)$ and the groupoid G, from the work of Armstrong-Clark-Courtney-Lin-M-Ramagge. Time permitting, we will also discuss some results of A-C-C-de Castro-L-M-R-Sims-Steinberg on the reconstruction of twisted Steinberg algebras from an algebraic quasi-Cartan pair. (Received September 20, 2021)

1174-16-9262 **Danny Lara*** (Danny.Lara@cwu.edu), Central Washington University, and **Ryan Kinser** (ryan-kinser@uiowa.edu), The University of Iowa. *General Representation Type of Algebras*

A fundamental result from representation theory states that any finite dimensional $\Bbbk$-Algebra A is either of tame or wild representation type. Informally defined, a finite dimensional algebra is of wild representation type if for each $n \in \mathbb{N}_0$, there exist a dimension $\mathbf{d}$ such that A has a family of isomorphism classes of indecomposable representations that depend on at least n parameters from $\Bbbk$. Therefore, it is infeasible to classify the representations of wild algebras. We will construct a family of wild algebras and show that we obtain a finite number of indecomposable representations, up to isomorphism, if we restrict to indecomposable representations whose orbits in $\mathrm{rep}_A(\mathbf{d})$ are open sub-varieties. We call these algebra finite general representation type. There are few wild algebras in the literature of this type and we aim to broaden the example base from which a broader understanding can be pursued. (Received September 20, 2021)

1174-16-9354 **Kurt Trey Trampel*** (ktrampel@nd.edu), University of Notre Dame, **Michael Gekhtman** (mgekhtma@nd.edu), Notre Dame, and **Chris Fraser** (fraserc4@msu.edu), Michigan State University. *Biinfinite band matrices and quantum affine algebras* Preliminary report.

We will review the representation theory of quantum affine $\mathfrak{sl}_n$ at roots of unity. We will then shift to describing the generalized cluster structure of periodic, biinfinite band matrices. We bring these two stories together and describe their connections via cluster theory. (Received September 20, 2021)

1174-16-9406 **Julia Pevtsova*** (julia@math.washington.edu), University of Washington, and **Cris Negron** (cnegron@usc.edu), University of Southern California. *Support and realizations for integrable Hopf algebras* Preliminary report.

I shall report on an ongoing work with Cris Negron on support theories for finite dimensional integrable Hopf algebras. Examples include small quantum groups and their Borels, quantum linear planes, restricted Lie algebras and their Drinfeld doubles. (Received September 20, 2021)

1174-16-9676 **Noha Abdelghany*** (nabdelgh@colby.edu), Colby College, and **Jay A. Wood** (jay.wood@wmich.edu), Western Michigan University. *Failure of the MacWilliams Identities for the Lee Weight Enumerator over $\mathbb{Z}_m$, $m \geq 5$*

The MacWilliams identities give a relation between the Hamming weight enumerator of a linear code and the Hamming weight enumerator of its dual. We are interested in the question of whether there is some version of the MacWilliams identities for the Lee weight over $\mathbb{Z}_m$, the integers modulo m. The Lee weight and the Hamming weight are equal when $m = 2$ or $m = 3$; thus the MacWilliams identities are valid in those cases. It is also known that the MacWilliams identities are valid for the Lee weight over $\mathbb{Z}_4$. In this talk we show the nonexistence of any version of the MacWilliams identities for Lee weight enumerators over $\mathbb{Z}_m$, $m \geq 5$. (Received September 20, 2021)

1174-16-9919 **Ian Le*** (iantuanle@gmail.com), Australian National University. *Skein algebras from quantum character stacks*

I will explain recent work with David Jordan, Gus Schrader and Alexander Shapiro in which we construct quantum (decorated) character stacks. In the cases where $G = SL_2$ or PGL_2, we give a hands-on description of these quantum stacks using, on the one hand, cluster algebras and, on the other, skein algebras. (Received September 21, 2021)

1174-16-10013 **Steve Szabo*** (steve.szabo@eku.edu), Eastern Kentucky University. *Codes over RIngs and Their Images* Preliminary report.

Linear codes over $\mathbb{Z}_4$ were used to produce nonlinear binary codes as their images with favorable coding theoretic properties. The most well known are the Kerdock and Preparata codes. In this talk, the general notion of taking images of linear codes over rings will be discussed. (Received September 21, 2021)

1174-16-10189 **Yorck Sommerhauser*** (sommerh@mun.ca), Memorial University of Newfoundland. *New Semisimple Hopf Algebras via the Biproduct Construction*

In joint work with Yevgenia Kashina, the speaker recently constructed two families of Yetter-Drinfel'd Hopf algebras over the Klein four-group in which certain subspaces, the so-called cores, have special properties. The algebras in these families have dimension 8 and depend on a not necessarily primitive fourth root of unity. The algebras in the first family are commutative, while the algebras in the second family are noncommutative. Via the Radford biproduct construction, these Yetter-Drinfel'd Hopf algebras give rise to semisimple Hopf algebras of dimension 32 that can also be constructed as extensions, although not as abelian extensions that are central or cocentral.

In this talk, we introduce these examples and discuss their basic properties from both the biproduct and the extension perspective. The talk is also based on joint work with Yevgenia Kashina, who will give the subsequent talk and discuss how many of these algebras there really are. (Received September 21, 2021)

1174-16-10500 **Zajj B Daugherty*** (zdaugherty@gc.cuny.edu), The City College of New York & The CUNY Graduate Center, and **Arun Ram** (aram@unimelb.edu.au), The University of Melbourne. *Calibrated representations of the two-boundary Temperley-Lieb algebras*

The two boundary Temperley-Lieb algebras arise in the transfer matrix formulation of lattice models in Statistical Mechanics, but they also interconnect beautifully with combinatorial representation theory. In this talk, we will see some of the ways in which the rich combinatorial structure of Hecke algebras informs the representation theory of these infinite-dimensional Temperley-Lieb algebras. In particular, we consider the case where certain operators that play important physical roles become easily diagonalizable from the combinatorial perspective. (Received September 21, 2021)

1174-16-10523 **Yevgenia Kashina*** (ykashina@depaul.edu), Depaul University. *Semisimple Hopf Algebras of Dimension 32: Isomorphisms* Preliminary report.

In joint work with Yorck Sommerhäuser, the speaker recently constructed new semisimple Hopf algebras of dimension 32 as biproducts of eight-dimensional Yetter-Drinfel'd Hopf algebras over the Klein four-group. These Yetter-Drinfel'd Hopf algebras come in two families, the first one commutative, the second one noncommutative. The algebras in both families depend on a not necessarily primitive fourth root of unity.

It is not easy to see which of these Yetter-Drinfel'd Hopf algebras and which of these biproducts are isomorphic, so that it is not clear how many such algebras we really have. In the talk, which is also based on joint work with Yorck Sommerhäuser and continues his preceding presentation, we describe our recently found answer to this question. Furthermore, we address the question of the self-duality of these algebras. (Received September 21, 2021)

1174-16-10762 **Guillermo Sanmarco*** (sanmarco@iastate.edu), Iowa State University. *Some non-semisimple modular categories constructed from super quantum groups* Preliminary report.

We will report on joint work in progress with Robert Laugwitz. We construct a class of super quantum groups associated to Nichols algebras of super type A and prove that their representation categories provide examples of non-semisimple modular tensor categories. The main tools come from recent work by Laugwitz-Walton, where braided Drinfeld doubles and relative monoidal centers are employed to construct such modular categories. (Received September 21, 2021)

1174-16-10843 **Thomas Lamkin*** (lamkintd@miamioh.edu), Miami University. *The Center of the Quantum Matrix Algebra at Roots of Unity* Preliminary report.

The quantum matrix algebra $M_q(n)$ arises as the coordinate ring of the quantum analogue of the matrix algebra $M(n)$. In the case where q is not a root of unity, the center of $M_q(n)$ is generated by an element known as the quantum determinant and is isomorphic to the polynomial ring in one variable. When q is a root of unity however, the center is far more complicated having several families of generators in addition to the quantum determinant. We find the relations between these generators when q is an odd root of unity, thus determining a presentation of the center. In addition, we discuss the application of this presentation in determining the automorphism group of $M_q(n)$. (Received September 21, 2021)

17 ► Nonassociative rings and algebras

1174-17-5798 **Nicholas William Mayers*** (nwm215@lehigh.edu), Lehigh University. *The breadth of Lie poset algebras* Preliminary report.

The breadth of a Lie algebra L is defined to be the maximal dimension of the image of $ad_x = [x, -] : L \to L$, for $x \in L$. In this talk, we discuss recent results concerning the breadth of certain Lie algebras which are defined by posets. Such results include explicit combinatorial breadth formulas. (Received August 27, 2021)

1174-17-6038 **Nick Russoniello*** (nvr217@lehigh.edu), Lehigh University. *Lie path algebras* Preliminary report.

A Lie path algebra is a subalgebra of a semisimple Lie algebra $\mathfrak{g}$ containing a Cartan subalgebra of $\mathfrak{g}$. In this talk, we define Lie path algebras and their associated quivers, present some recent results regarding their breadth (a Lie-algebraic invariant), and discuss directions for future work relating to two important subclasses: seaweed algebras and Lie poset algebras. (Received September 3, 2021)

1174-17-6281 **Meighan Irene Dillon*** (mdillon1@kennesaw.edu), Kennesaw State University. *Octonions and Lie Algebras* Preliminary report.

Deep connections between the octonions and the exceptional Lie algebras are well-known. Our goal is to understand how these connections carry over to the case of infinite-dimensional affine Lie algebras that we can construct using certain representations of semi-simple Lie algebras. The talk should be accessible to mathematicians without specialized knowledge. (Received September 8, 2021)

1174-17-6916 **Jason Gaddis*** (gaddisj@miamioh.edu), Miami University, and **Daniel Yee** (dyee@fsmail.bradley.edu), Bradley University. *Cancellation and skew cancellation of Poisson algebras*

In its algebraic form, the Zariski cancellation problem asks whether an isomorphism of algebras $A[x] \cong B[x]$ implies $A \cong B$. In this talk, I will discuss this problem in the context of Poisson algebras. Our results show that quadratic Poisson algebras in three variables are cancellative. I will also discuss skew Poisson cancellation for Poisson Ore extensions and invariants related to this problem, including the divisor Poisson subalgebra and Poisson stratiform length. This is joint work with Xingting Wang and Dan Yee. (Received September 10, 2021)

1174-17-7179 **Samuel H Chamberlin** (samuel.chamberlin@park.edu), Park University, and **Jagrit Niraula*** (1613771@park.edu), Park University. *Straightening Identities in the Universal Enveloping Algebras of the Twisted Multiloop Algebras of $\mathfrak{sl}_4$ with a Chevalley Involution Twist* Preliminary report.

We have formulated and proven some straightening identities in the universal enveloping algebra of the twisted multiloop algebra of $\mathfrak{sl}_4$ with a Chevalley involution twist. These identities involved reordering certain products of divided powers of basis elements as integer linear combinations of products of divided powers of basis elements, which were in a preferred order. (Received September 13, 2021)

1174-17-7294 **K. R. Goodearl*** (goodearl@math.ucsb.edu), University of California at Santa Barbara, and **S. Launois** (s.launois@kent.ac.uk), University of Kent at Canterbury. *Poisson catenarity in Poisson nilpotent algebras*

Many Poisson algebras appear as semiclassical limits of quantized coordinate rings, and these Poisson algebras are expected to display properties that parallel those of the corresponding quantized coordinate rings, just as the latter are expected to display properties that parallel those of their classical counterparts. We will focus on the important property of *catenarity.* In the quantum and classical cases, this condition asks that all saturated chains of prime ideals between any two fixed primes have the same length; the parallel in the Poisson setting is *Poisson catenarity*, which asks for the analogous chain property in the poset of Poisson-prime ideals.

Catenarity is known to hold in classical coordinate rings of affine varieties and has been proved for many quantized coordinate rings. The parallel Poisson catenarity has recently been established by Launois and the speaker for the large class of Poisson algebras called *Poisson nilpotent.* On the other hand, it does *not* hold for *all* affine Poisson algebras. (Received September 14, 2021)

1174-17-7333 **Caleb J Fernelius*** (1680528@park.edu), Park University. *Closed formulas for the structure constants for the universal enveloping algebras of types A_1 and D_2* Preliminary report.

Gourley and Kennedy give a recursive formula for the structure constants of the universal enveloping algebra of $\mathfrak{sl}_2$. Using Kostant's formula we give a closed formula for these structure constants and then extend this result to the Lie algebra of type D_2. (Received September 14, 2021)

1174-17-8546 **Michael Penn*** (mpenn@randolphcollege.edu), Randolph College. *Permutation Orbifolds of Vertex Operator Algebras:*

Vertex operator algebras have played an important role to understanding q-series and modular forms since their origin in the late 1980's. Famously, they were an instrumental part of the proof of the Moonshine conjecture. In more recent years, they have been show to be linked to Rogers-Ramanujan, Gordon-Andrews, and other similar combinatorial identities. In this talk, we investigate permutation orbifolds of affine and Virasoro vertex algebras via their internal structure. As an application, we look at some q-series identities that follow. (Received September 19, 2021)

1174-17-9059 **Lilit Martirosyan*** (martirosyanl@uncw.edu), University of North Carolina, Wilmington. *BRAID RIGIDITY FOR PATH ALGEBRAS*

Path algebras are a convenient way of describing decompositions of tensor powers of an object in a tensor category. If the category is braided, one obtains representations of the braid groups B_n for all n in N. We say that such representations are rigid if they are determined by the path algebra and the representations of B_2. We show that besides the known classical cases also the braid representations for the path algebra for the 7-dimensional representation of G_2 satisfies the rigidity condition, provided B_3 generates $\mathrm{End}(V \otimes 3)$. We obtain a complete classification of ribbon tensor categories with the fusion rules of $g(G_2)$ if this condition is satisfied. (Received September 20, 2021)

1174-17-9343 **Noah Thomas Carney*** (ncarney@randolphcollege.edu), Randolph College. *Permutation orbifolds of rank 4 Heisenberg vertex algebras*

Vertex operator algebras have played an important role to understanding q-series and modular forms since their origin in the late 1980's. Famously, they were an instrumental part of the proof of the Moonshine conjecture. Orbifold algebra of vertex algebras have been studied extensively in mathematics and physics. In this talk we provide explicit constructions of several different permutation orbifolds of the rank 4 Heisenberg algebra, associated to different subgroups of S_4. Along the way we explore an orbifold of the universal W(2,4) algebra at arbitrary central charge. (Received September 20, 2021)

1174-17-9520 **John Lin*** (Jlin211@binghamton.edu), Binghamton University, **Christopher Sadowski** (csadowski@ursinus.edu), Ursinus College, **Zachary Couvillion** (zacharycouvillion@gmail.com), Dartmouth College, and **Patrick McCourt** (pcm239@gmail.com), Kent State University. *Recursions, q-series and intertwining operators*

The notion of principal subspace of an irreducible highest weight module for an affine Lie algebra was introduced by Feigin and Stoyanovsky and later studied by many other authors. Of particular interest is the graded dimension of the principal subspace, which is written as a q-series and in some cases tied to integer partition identities. In our work, we examine a certain filtration of subspaces of the irreducible highest weight modules from the work of Feigin and Stoyanovsky and construct exact sequences using all modes of an intertwining operator which generalize those found by Capparelli, Lepowsky, and Milas among principal subspaces of basic $\widehat{\mathfrak{sl}(2)}$-modules. We obtain recursions satisfied by these subspaces and examine corresponding q-series one can obtain from these recursions. (Received September 20, 2021)

1174-17-11294 **Matthew Krauel*** (krauel@csus.edu), Sacramento State University. *Some recent applications of modular and Jacobi forms in studying vertex operator superalgebras* Preliminary report.

This talk will briefly discuss how modular and Jacobi forms arise in the theory of vertex operator superalgebras (VOSAs). We will then describe some recent uses of these functions to study VOSAs and vertex operator algebras, their representations, and their characters. (Received September 29, 2021)

18 ▶ Category theory; homological algebra

1174-18-5752 **Michael Niemeier*** (meniemeier@bsu.edu), The Indiana Academy for Science, Mathematics and Humanities, Ball State University. *CENTRAL EXTENSIONS OF SIMPLICIAL GROUPS AND PRESHEAVES OF SIMPLICIAL GROUPS* Preliminary report.

We show that path components of central extensions of a simplicial group G by a simplicial abelian group A are in bijection with the homotopy classes of maps between classifying spaces $[\overline{W}G, \overline{W}^2 A]$. Generalizing the classical correspondence for central extensions of a group G by an abelian group A. We then prove an analogous theorem for central extensions of presheaves of simplicial groups. Finally, we use the correspondence to show that the cup product factors through a Heisenberg central extension. (Received September 13, 2021)

1174-18-6218 **Pablo Sanchez Ocal*** (pablosanchezocal@gmail.com), UCLA. *The Relative Künneth Theorem*

The Künneth Theorem is a well known tool to control the homology of the tensor product of complexes. In this talk I will present an analogous result in the context of relative homological algebra. This will include an unpretentious introduction to relative homological algebra, and some of its derived functors. To achieve our goal we described a long exact sequence for relative Tor that splits in 2 out of 3 terms, which can also be used to characterize relative flat modules using relative Tor groups in a very familiar way. Our work reconciles several interpretations of relative free and flat modules. (Received September 7, 2021)

1174-18-7195 **Harshit Yadav*** (hy39@rice.edu), Rice University, and **Chelsea Walton** (notlaw@rice.edu), Rice University. *Filtered Frobenius algebras in monoidal categories*

We develop filtered-graded techniques for algebras in monoidal categories with the main goal of establishing a categorical version of Bongale's 1967 result: A filtered deformation of a Frobenius algebra over a field is Frobenius as well. Towards the goal, we first construct a monoidal associated graded functor, building on prior works of Ardizzoni-Menini, of Galatius et al., and of Gwillian-Pavlov. Next, we produce equivalent conditions for an algebra in a rigid monoidal category to be Frobenius in terms of the existence of categorical Frobenius form; this builds on work of Fuchs-Stigner. These two results of independent interest are then used to achieve our goal. We illustrate these results by discussing braided Clifford algebras, which are filtered deformations of Bespalov et al.'s braided exterior algebras, and show that these are Frobenius algebras in symmetric rigid monoidal categories. Several directions for further investigation are proposed. (Received September 13, 2021)

1174-18-7315 **Daniel K Nakano*** (nakano@math.uga.edu), University of Georgia, and **Milen T. Yakimov** (milenyakimov@gmail.com), Northeastern University. *Noncommutative Tensor Triangular Geometry and Finite Tensor Categories*

In this talk, I will first review the general noncommutative version of Balmer's tensor triangular geometry due to Nakano, Vashaw and Yakimov. Insights from noncommutative ring theory are used to obtain a framework for prime, semiprime, and completely prime (thick) ideals of a monoidal triangulated category, $\mathbf{K}$, and then to associate to $\mathbf{K}$ a topological space–the Balmer spectrum Spc $\mathbf{K}$.

Later in the talk, I will describe new ideas for applying this machinery to address questions involving finite tensor categories. Our new results will be expounded in more detail in a talk presented later in the meeting by Kent Vashaw.

This work represents joint work with Kent Vashaw and Milen Yakimov. (Received September 14, 2021)

1174-18-8430 **Julia Plavnik** (jplavnik@iu.edu), Indiana University, Indiana University, Bloomington, **Colleen Delaney** (crdelane@iu.edu), Indiana University, and **Sung Kim*** (skim2261@usc.edu), University of Southern California. *Zesting Produces Modular Isotopes*

The modular data S and T are important invariants in the study of modular tensor categories. We say that inequivalent modular categories are *modular isotopes* if they have the same modular data up to relabeling. In this talk, I will discuss about how the ribbon zesting construction transforms Reshetikhin-Turaev link invariants and produces modular isotopes. (Received September 19, 2021)

1174-18-8597 **Siu-Hung Ng** (rng@math.lsu.edu), Louisiana State University, **Samuel N Wilson*** (swil311@lsu.edu), Louisiana State University, and **Yilong Wang** (wyl@bimsa.cn), Yanqi Lake Beijing Institute of Mathematical Sciences and Applications. *Representations of* $\mathrm{SL}_2(\mathbb{Z})$ *and applications for modular tensor categories.*

Every modular category gives rise to a family of representations of $\mathrm{SL}_2(\mathbb{Z})$. Such representations have been completely characterized and may be explicitly constructed by means of quadratic forms; they may therefore be used to differentiate and classify modular tensor categories. As one application of this technique, we classify certain modular categories whose simple objects fall into precisely two orbits of the same size under Galois action. (Received September 19, 2021)

1174-18-8772 **Agustina Mercedes Czenky** (aczenky@uoregon.edu), University of Oregon. *On odd-dimensional modular tensor categories*

Modular categories arise naturally in many areas of mathematics, such as conformal field theory, representations of braid groups, quantum groups, and Hopf algebras, low dimensional topology, and they have important applications in condensed matter physics.

One interesting class of modular categories is the one of odd-dimensional ones, which is closely related to maximally non-self dual (MNSD) modular tensor categories. In this talk, we will present some properties of odd-dimensional and MNSD modular categories and we will show some advances on their classification by rank. (Received September 19, 2021)

1174-18-8783 **Julia Yael Plavnik** (juliaplavnik@gmail.com), Indiana University, **Sean Sanford*** (scsanfor@iu.edu), Indiana University, and **Dalton Sconce** (dsconce@iu.edu), Indiana University. *Non-Split Tambara Yamagami Categories over the Reals* Preliminary report.

In 1998, Daisuke Tambara and Shigeru Yamagami investigated a simple set of fusion rules, and proved under which circumstances those rules could be given a coherent associator. This elementary yet difficult computation remains an outlier in the subject, as such associators are rarely computed by hand.

In this project, we are investigating a generalization of such fusion rules to the setting where the simple objects are no longer required to be absolutely simple. Over the real numbers, this means that objects are either real, complex or quaternionic. This flexibility has interesting implications that we will explore through a series of examples.

This is joint work with Julia Plavnik and Dalton Sconce. (Received September 19, 2021)

1174-18-9289 **Qing Zhang** (zhan4169@purdue.edu), Purdue University, **Eric C. Rowell*** (rowell@math.tamu.edu), Texas A&M University, and **Cesar N. Galindo** (cn.galindo1116@uniandes.edu.co), Universidad de los Andes. *Physics and Zesting* Preliminary report.

I will discuss physical interpretations and applications of the construction known as zesting. Originally developed as a method for categorifying fusion rules reminiscent of those of $SU(3)_3$, recently it has seen use in a number of physical settings. (Received September 20, 2021)

1174-18-9334 **Corey Jones*** (cormjones88@gmail.com), North Carolina State University. *Computing fusion rules for G-extensions of fusion categories*

A G-graded extension of a fusion category C yields a categorical action of G on the center Z(C). If the extension admits a spherical structure, we provide a method for recovering its fusion rules in terms of the action. We then apply this to find closed formulas for the fusion rules of extensions of some group theoretical categories and of cyclic permutation crossed extensions of modular categories. (Received September 20, 2021)

1174-18-9503 **Christina Knapp*** (chknapp@microsoft.com), Microsoft Station Q, **David Aasen** (david@aasen.ca), Microsoft Station Q, and **Parsa Bonderson** (parsab@microsoft.com), Microsoft Station Q. *Characterization and Classification of Fermionic Symmetry Enriched Topological Phases* Preliminary report.

We examine the interplay of symmetry and topological order in 2+1 dimensional fermionic topological phases of matter. We define fermionic topological symmetries acting on the emergent topological effective theory described using braided tensor category theory. Connecting this to the Gf fermionic symmetry of the microscopic physical system, we characterize and classify symmetry fractionalization in fermionic topological phases. We develop an algebraic theory of Gf symmetry defects for fermionic topological phases using G-crossed braided tensor category theory, which describes the fusion, braiding, symmetry action, and fractionalization patterns of quasiparticles, vortices, and defects. This formalism allows us to fully characterize and classify 2+1 dimensional fermionic symmetry enriched topological phases with on-site unitary fermionic symmetry group Gf. We first apply this formalism to extract the minimal data specifying a general fermionic symmetry protected topological phase, and demonstrate that such phases with fixed Gf form a group under fermionic stacking, i.e. pairing Gf-symmetric phases and condensing the bound pair of their physical fermions. Then we analyze general fermionic symmetry enriched topological phases and find their classification is given torsorially by the classification of the symmetry fractionalization of quasiparticles combined with the classification of fermionic symmetry protected topological phases. (Received September 20, 2021)

1174-18-9537 **Rina Anno*** (ranno@ksu.edu), Kansas State University. *Derived categories of modules over monads* Preliminary report.

For an adjoint pair of enhanceable functors (F,R) between DG enhanced triangulated categories (e.g. any adjunction in algebraic geometry) we construct the derived category of modules over the monad RF. It comes with a fully faithful functor to the target category of F, where its image is the triangulated hull of the image of F. This category is also equivalent to the derived category of comodules over the comonad LF which we also define. This construction leads to a version of descent for triangulated categories. An example of that is an affine covering $f : U \to X$ and $F = f_*$ between the derived categories of quasicoherent sheaves. Then the category in question is equivalent to the derived category of comodules over a certain coalgebra built out of R_i where $U = \bigcup Spec\, R_i$.

This is joint work in progress with Timothy Logvinenko (Cardiff University). (Received September 20, 2021)

1174-18-9627 **Shawn Xingshan Cui** (cui177@purdue.edu), Purdue University, and **Paul Gustafson** (paul.gustafson@gmail.com), University of Pennsylvania. *From torus bundles to particle-hole equivariantization*

We continue the program of constructing (pre)modular categories from 3-manifolds first initiated by Cho-Gang-Kim using M theory in physics and then mathematically studied by Cui-Qiu-Wang. An important structure involved is a collection of certain $\mathrm{SL}(2,\mathbb{C})$ characters on a given manifold which serves as the simple object types in the corresponding category. Chern-Simons invariants and adjoint Reidemeister torsions play a key role in the construction, and they are related to topological twists and quantum dimensions, respectively, of simple objects. The modular S-matrix is computed from local operators and follows a trial-and-error procedure. It is currently unknown how to produce data beyond the modular S- and T-matrices. There are also a number of subtleties in the construction which remain to be solved. In this talk, we consider an infinite family of 3-manifolds, that is, torus bundles over the circle. We show that the modular data produced by such manifolds are realized by the $\mathbb{Z}_2$-equivariantization of certain pointed premodular categories. (Received September 20, 2021)

1174-18-9832 **David Eric Weisbart** (dweisbart@gmail.com), University of California Riverside, and **Adam Yassine*** (a.yassine@bowdoin.edu), Bowdoin College. *A Compositional Framework for Classical Mechanical Systems*

We build on the recent work of Weisbart and Yassine on generalized span categories and on the application of this work, by Baez, Weisbart, and Yassine, to the description of open classical mechanical systems. The extended work is used to study a wide variety of classical mechanical systems and explore their composition. This extended work is joint with David Weisbart (University of California, Riverside). (Received September 20, 2021)

1174-18-10016 **David Penneys*** (penneys.2@osu.edu), The Ohio State University. *Classification of Z/2Z-quadratic unitary fusion categories*

A unitary fusion category is called quadratic if there is a group of simple objects and one other orbit of simple objects under the group action. We give a complete classification of Z/2-quadratic unitary fusion categories. As

a corollary, we complete the classification of rank 4 unitary fusion categories with a dual pair of simple objects. This is joint work with Cain Edie-Michell and Masaki Izumi (Received September 21, 2021)

1174-18-10066 **Sarah Klanderman*** (sklanderman@marian.edu), Marian University. *Loday constructions on twisted products and on tori*

The Loday construction $\mathcal{L}_X(A)$ with respect to a simplicial set X of a commutative algebra A gives a simplicial commutative ring whose homology generalizes the Hochschild homology of A. In this talk we will discuss computational tools for studying Loday constructions of twisted products. Further we will build on work of Dundas-Tenti and investigate stability results via the Loday construction of tori. This is joint work with Alice Hedenlund, Ayelet Lindenstrauss, Birgit Richter, and Foling Zou. (Received September 21, 2021)

1174-18-10341 **Daniel S Rogalski*** (drogalski@ucsd.edu), UCSD. *Closed subcategories of noncommutative projective schemes*

A noncommutative scheme is a Grothendieck category. The concept of a closed subscheme of a noncommutative scheme has been studied in detail by S. Paul Smith. We are especially interested in projective noncommutative schemes, which arise from noncommutative graded rings. We explain formally how the closed subschemes of such a noncommutative projective scheme can be understood in terms of the graded ring. For example, this allows one to describe the spectrum of closed subschemes in the noncommutative projective planes, which arise from Artin-Schelter regular algebras of dimension 3. (Received September 21, 2021)

1174-18-10491 **Amrei Oswald*** (amrei-oswald@uiowa.edu), University of Iowa. *Decomposing tensor products of bimodules in pointed fusion categories*

I determine how bimodules tensored over algebras in pointed fusion categories decompose into indecomposable bimodules based on the classification of such bimodules by Ostrik and Natale. This has applications in determining fusion rules in group-theoretical fusion categories. I will also discuss how this can be used to classify tensor algebras in pointed fusion categories. (Received September 21, 2021)

1174-18-10673 **Joshua Robert Edge*** (jedge@austincollege.edu), Austin College. *Classification of symmetric trivalent planar algebras of small dimension*

A trivalent category is a planar algebra generated by a symmetric trivalent vertex. This planar algebra can be thought of as a quotient of the planar algebra of planar graphs containing only trivalent and univalent vertices. Trivalent categories of small dimension were classified by Morrison, Peters, and Snyder. In this talk, we will expand on this classification by classifying symmetric trivalent categories, planar algebras generated both by a symmetric trivalent vertex and a symmetric crossing. A symmetric trivalent category essentially removes the planarity requirement in previous work. We will discuss the relationship between planar algebras and monoidal categories. We will also classify which planar algebras have sub-braidings. (Received September 21, 2021)

1174-18-11210 **Aaron R Bagheri*** (bagheri@math.ucsb.edu), University of California, Santa Barbara. *Condensation of Anyons and Applications*

Anyon behavior is modeled by modular tensor categories. We discuss modular tensor categories and a process known as condensation, which is equivalent to taking the category of modules over an algebra. We discuss some results about the modular data of the condensed category and talk about an interesting application of error correcting codes and near group categories. (Received September 21, 2021)

19 ► K-theory

1174-19-5406 **Ariana Chin** (ariana.g.chin@gmail.com), University of California, Berkeley, **Noah Caplinger** (nccaplinger@gmail.com), Georgia Institute of Technology, **Nyah Davis*** (nyah-davis@uiowa.edu), University of Iowa, **Elisabeth Bullock** (edb22@mit.edu), Massachusetts Institute of Technology, and **Gahl Shemy** (gahlshemy@ucsb.edu), University of California, Santa Barbara. *Lattice Models and Puzzles for Dual Weak Symmetric Grothendieck Polynomials* Preliminary report.

We construct a solvable lattice model for the dual weak symmetric Grothendieck polynomials given by Pylyavskyy and Lam in hopes of using such a model to prove related properties of these polynomials, including Cauchy identities and branching rules. We also consider a similar lattice model construction for the weak symmetric Grothendieck polynomials in hopes of proving a Cauchy identity given by Yeliussizov, concluding with a negative result. Moreover, we expand on the work done by Pylyavskyy and Yang and by Zinn-Justin by giving boundary conditions for a proposed lattice model for the Littlewood Richardson coefficients of the dual weak symmetric Grothendieck polynomials, via an MS puzzle construction. (Received August 25, 2021)

1174-19-6260 **Cheyne J Glass*** (cglass@sjcny.edu), St. Joseph's College, New York, **Micah Miller** (mmiller@bmcc.cuny.edu), Borough of Manhattan Community College, The City University of New York, **Thomas Tradler** (ttradler@citytech.cuny.edu), New York City College of Technology, The City University of New York, and **Mahmoud Zeinalian** (mahmoud.zeinalian@lehman.cuny.edu), Lehman College, The City University of New York. *Chern Simons Forms for Coherent Sheaves via Sheafification* Preliminary report.

In this talk we improve upon the Chern character for coherent sheaves defined by O'Brian, Toledo, and Tong (1980). Coherent sheaves on general complex manifolds do not necessarily have resolutions by finite complexes of vector bundles. However, O'Brian, Toledo, and Tong showed that one can resolve coherent sheaves by objects analogous to chain complexes of holomorphic vector bundles, whose cocycle relations are governed by an infinite system of homotopies. In the modern language such objects are obtained by the infinity-sheafification of the simplicial presheaf of chain complexes of holomorphic vector bundles. On this simplicial presheaf we will define a Chern character, and then sheafify the map to recover their Chern character on π_0. As a consequence of our construction the induced maps on higher homotopy groups provide new Chern-Simons, and higher Chern-Simons, invariants for coherent sheaves. Additionally we extend these constructions to the equivariant setting. (Received September 7, 2021)

20 ► Group theory and generalizations

1174-20-5503 **Michael Gintz*** (m.gintz@icloud.com), Princeton University, **Matthew Kortje** (mkortje@cedarville.edu), Cedarville University, **Megan Laurence** (mlauren2@nd.edu), Notre Dame University, and **Zili Wang** (ziliwang271@berkeley.edu), Berkeley University. *Square Factors in Projective Character Degrees*

While the relationship between the irreducible character degrees of a group and the structure of the group itself has been studied extensively, a relatively small number of attempts have been made to extend these results to the more general projective characters. We progress towards generalizing such a result regarding the structure of a finite group whose character degrees are all square-free. We show that a minimal-order exception to our proposed result would be an extension of a finite simple group by a solvable group, and outline progress toward showing that such an exception would be simple. We also show that a finite simple exception would have to be of Lie type. We use a weighted pigeonhole argument using Deligne-Lusztig theory to handle some cases, and express how it may be used to solve others. We also discuss the merit of considering representations such as the Weyl representation. (Received August 21, 2021)

1174-20-5744 **Radu Balan** (rvbalan@math.umd.edu), University of Maryland, and **Daniel Richard Levy*** (drlevy@terpmail.umd.edu), University of Maryland. *Permutation Invariant Representations: Lower-Bounding the Redundancy of Universal Keys*

In this paper, we study a special Euclidean embedding of the quotient space of matrices, where two matrices are said equivalent if one is a row permutation of the other. The embedding $\alpha_R(X)$ is obtained by applying the sorting operator columnwise on the product of matrix X with the given key R. We call R a universal key if α_R is injective on the quotient space. We restructure the problem into a computationally tractable form and prove the guaranteed existence of universal keys of low redundancy. We then explore several open questions with strong connections to combinatorial linear algebra and algebraic geometry. Notable applications of our results include graph deep learning tasks, such as graph classifications and graph regressions. (Received September 15, 2021)

1174-20-6000 **Ayush Kumar*** (akumar@patriots.uttyler.edu), The University of Texas-Tyler. *On Morita Equivalence of Inverse Hulls of Markov Shifts*

We study the relationship between stable isomorphism of Cuntz-Krieger C^*-algebras and "Morita equivalence" of shift inverse semigroups (inverse hulls). In the known C^*-algebras theory stable isomorphism corresponds to flow equivalence of shift spaces. In the construction of the C^*-algebras of a semigroup S, a quotient $S/\leftrightarrow$ is instrumental. In the case where S is the inverse hull of a Markov shift we show, under a mild condition, that $S/\leftrightarrow$ is isomorphic to the inverse hull of a flow equivalent Markov shift. In doing so we initiate the study of a weaker notion of Morita equivalence of inverse hulls that corresponds to stable isomorphism of their C^*-algebras. (Received September 3, 2021)

1174-20-6087 **Ruth Charney** (charney@brandeis.edu), Brandeis University, **Karen Vogtmann*** (kvogtmann@gmail.com), University of Warwick, and **Corey Bregman** (corey.bregman1@gmail.com), University of South Maine. *Gamma-complexes* Preliminary report.

The right-angled Artin group A_Γ based on a graph Γ acts properly on a contractible space of "marked Γ-complexes" called $\mathcal{O}_\Gamma$. For Γ a complete graph on n vertices $\mathcal{O}_\Gamma$ is the symmetric space $SL(n,R)/O(n)$ and a Γ-complex is a flat n-torus. For Γ a discrete graph $\mathcal{O}_\Gamma$ is Outer space and a Γ-complex is a finite graph with fundamental group F_n. We describe Γ-complexes for general Γ, and explain how to recognize whether a CAT(0) cube complex with fundamental group A_Γ is indeed a Γ-complex. This is joint work with Corey Bregman and Ruth Charney. (Received September 4, 2021)

1174-20-6185 **Robert Fitzgerald Morse** (rm43@evansville.edu), University of Evansville, **Luise-Charlotte Kappe*** (Menger@math.binghamton.edu), Binghamton University, and **Matthew P Visscher** (matt@nyoss.com), Binghamton University. *The nonabelian tensor product of cyclic groups of p-power order, p an odd prime* Preliminary report.

The nonabelian tensor product of two groups G and H was introduced by R. Brown and J.–L. Loday in connection with applications in homotopy theory of a generalized Van Kampen theorem. The nonabelian tensor product $G \otimes H$ is the group generated by the symbols $g \otimes h$ with relations $gg' \otimes h = ({}^g g' \otimes {}^g h)(g \otimes h)$ and $g \otimes hh' = (g \otimes h)({}^h g \otimes {}^h h')$, where G and H act on each other via automorphism in a compatible way and on themselves via conjugation. Good progress has been made in the computation of large classes of groups in the case of the nonabelian tensor square $G \otimes G$, where the actions are conjugation which are always compatible.

Much less is known for the nonabelian tensor product even in the case that the groups are cyclic. We show that the nonabelian tensor product of two cyclic groups is an abelian group of at most rank 2. In case G and H are cyclic groups of p-power order, p an odd prime, we classify all compatible actions and show that $G \otimes H$ is cyclic with $|G \otimes H| = \min(|G|, |H|)$. In case G and H are cyclic 2-groups we give examples such that $G \otimes H$ is an abelian group of rank 2. (Received September 6, 2021)

1174-20-6478 **Andrew Peña*** (apena13@msudenver.edu), Metropolitan State University of Denver, and **Frank Pryor** (fpryor@msudenver.edu), Metropolitan State University of Denver. *Galois Automorphisms and Characters of Symplectic Groups* Preliminary report.

The study of groups gives us a way to describe the structure of symmetries, and for a given group, a matrix representation gives us a way to represent each of its elements as an invertible matrix. Such representations are a particularly useful tool for studying groups since matrix multiplication is well understood, but also because the structure of a matrix allows us the ability to compress the information we care most about into smaller, more manageable pieces. We do this by taking the trace of a matrix representation; this encoding function is known as a character. We call a character "irreducible" if it cannot be written as the sum of two other characters. By studying the irreducible characters, we can discover important properties of the group and its subgroups. Here, we will look at a conjecture that proposes a correlation between the makeup of a group's irreducible characters, with particular attention to the values they take, and the properties of its subgroups. In particular, we prove the case for the finite symplectic groups $Sp_6(q)$ with q even. (Received September 9, 2021)

1174-20-6864 **Ben Elias** (belias@uoregon.edu), University of Oregon, **Elijah Bodish*** (ebodish@uoregon.edu), University of Oregon, **David Rose** (davidrose@unc.edu), University of North Carolina at Chapel Hill, and **Logan Tatham** (ltatham@live.unc.edu), University of North Carolina at Chapel Hill. *Type C Webs*

Let $Rep(G)$ be the monoidal category of finite dimensional representations of a semisimple Lie group. Kuperberg's 1996 paper "Spiders for rank 2 Lie algebras" proposed studying $Rep(G)$ by first studying $Fund(G)$, the full monoidal subcategory generated by finite dimensional irreducible modules with highest weight a fundamental weight. Kuperberg went on to give generators and relations for $Fund(G)$ when $G = SL_3, Spin_5, Sp_4$, and G_2. The problem of giving analogous generators and relations for $Fund(SL_n)$ was solved in 2012 by Cautis-Kamnitzer-Morrison.

In joint work with Elias, Rose, and Tatham (arXiv:2103.14997) we define a category by generators and relations and argue it is equivalent to $Fund(Sp_{2n})$, solving Kuperberg's problem in type C.

Joint work with Ben Elias (University of Oregon), David Rose (UNC-Chapel Hill) and Logan Tatham (UNC-Chapel Hill). (Received September 9, 2021)

1174-20-7664 **Ivan Levcovitz*** (ivan.levcovitz@tufts.edu), Tufts University. *Random BMW Groups* Preliminary report.

BMW (Burger-Mozes-Wise) groups are a class of groups that act geometrically on the product of two infinite, regular trees. For large enough trees, these groups admit the same coarse geometry (they are all pairwise quasi-isometric), yet they can be algebraically quite different. In fact, by a celebrated result of Burger-Mozes, there are even BMW groups which are virtually simple. In this talk, I will discuss a random model for generating BMW groups and how we use ideas from this model to solve counting problems in these groups. This is joint work with Nir Lazarovich and Alex Margolis. (Received September 15, 2021)

1174-20-7692 **Mark Hunnell** (hunnellm@wssu.edu), Winston-Salem State University, **John Hutchens*** (johnd.hutchens@gmail.com), Winston-Salem State University, and **Nathaniel Schwartz** (gnat79@gmail.com), DoD. *Involutions of orthogonal groups over characteristic 2*

An orthogonal group defined over a field of characteristic 2 is the group of invertible linear operators preserving a quadratic form on a vector space over the field. We extend previous results on the classification of involutions of these groups to cases where the vector space is singular or defective, and determine their conjugacy classes. The involutions of these groups are important for the study of the associated symmetric k-varieties. (Received September 15, 2021)

1174-20-7852 **Jordan Christopher Bounds*** (jordan.bounds@furman.edu), Furman University. *Some applications of translation-like actions*

An action of a group H on a metric space (X, d) is *translation-like* if it is free and the set $\{d(x, x * h) : x \in X\}$ is bounded for every $h \in H$. The concept of translation-like actions was introduced by Whyte in 1999 as a geometric analog for subgroup containment. In his work, Whyte noted that there are numerous conjectures of the form "A finitely generated group has property... iff it contains a subgroup isomorphic to..." which are known to be false, such as the Burnside Problem and the von Neumann Conjecture. Further, he hypothesized that if the property is geometric in nature, then there is a chance that a geometric reformulation of such conjectures in terms of translation-like actions could be true. In this talk, we will examine the concept of a translation-like action, discuss some applications to well-known problems, and explore a few areas which are still open for discovery. (Received September 16, 2021)

1174-20-7978 **Sami Douba*** (doubasami@gmail.com), McGill University. *Proper CAT(0) actions of unipotent-free linear groups* Preliminary report.

Button observed that finitely generated linear groups containing no nontrivial unipotent matrices behave much like groups admitting proper actions by semisimple isometries on complete CAT(0) spaces. It turns out that any finitely generated $\mathbb{C}$-linear group possesses an action on such a space whose restrictions to unipotent-free subgroups are in some sense tame. We discuss this phenomenon and some of its implications for the representation theory of graph manifold groups. (Received September 17, 2021)

1174-20-8117 **Heejoung Kim*** (hkim404@illinois.edu), The Ohio State University. *Detection algorithms for closed hyperbolic 3-manifold groups*

In geometric group theory, finding algorithms for detection and decidability of various properties of groups is a fundamental question. For a finitely generated group G, we can study not only algorithmic problems for G itself but also algorithms related to a particular class of subgroups. For word-hyperbolic groups, quasiconvex subgroups have been studied widely and there are algorithmic results. For example, Kapovich provided a partial algorithm which, for a finite set S of G, only halts if S generates a quasiconvex subgroup of G. On the other hand, Kapovich and Weidmann gave a complete algorithm which, for a finitely generated subgroup of a closed hyperbolic 3-manifold group G, decides whether or not the subgroup is quasiconvex in G. In this talk, we discuss detecting algorithms for finitely generated groups focusing on closed hyperbolic 3-manifold groups and their limit groups. (Received September 17, 2021)

1174-20-8401 **Radhika Gupta*** (radhikagupta.maths@gmail.com), Temple University. *Relating polynomial invariants of free-by-cyclic groups*

In this talk, we will relate a McMullen polynomial of a free-by-cyclic group to its Alexander polynomial. In order to do so we introduce 'orientable' fully irreducible outer automorphisms, motivated by pseudo-Anosov homeomorphisms that admit transversely oriented stable foliation. We also relate the homological stretch factor of such automorphisms to their geometric stretch factor. This is joint work with Dowdall and Taylor. (Received September 19, 2021)

1174-20-8432 **Matthias Merzenich*** (matthias@oakbasin.com), Oregon State University. *Results from a Computational Method for Building Spherical Pictures Preliminary Report* Preliminary report.

Attempts have been made to classify cyclically-presented groups by their status as 3-manifold groups, and the asphericity status of their presentations. For groups with relators of the form $x_i x_{i+m} x_{i+k}^{-1}$, these classifications are nearly complete. Among these "groups of Fibonacci type", only the Gilbert-Howie groups $H(9,4)$ and $H(9,7)$ represent outstanding cases.

We develop and apply a simple computational method to build reduced spherical pictures over presentations of split extensions of $H(9,4)$ and $H(9,7)$. In each case, a resulting symmetric picture reveals useful relations in the group. In particular, we can complete the asphericity classification for the presentations of these two Gilbert-Howie groups (neither is aspherical) and show that $H(9,4)$ is not a 3-manifold group. These results indicate the potential for further research in the application of computational methods to asphericity questions. (Received September 19, 2021)

1174-20-8727 **Marissa Chesser*** (marissa9@illinois.edu), University of Illinois At Urbana-Champaign. *Stable subgroups of handlebody groups*

In a paper examining convex cocompact subgroups of the mapping class group, Durham and Taylor introduce the notion of a stable subgroup. These subgroups, which can be defined for any finitely generated group, can be thought of as a generalization of quasi-convex subgroups of hyperbolic groups. In the setting of mapping class groups, by combining work of Durham-Taylor, Kent-Leininger, and Hamenstädt, one finds that stable subgroups of mapping class groups are precisely those whose orbit maps quasi-isometrically embed into the curve graph.

There are many groups closely related to mapping class groups that also come with associated hyperbolic graphs, so one can ask if there is a similar characterization of stable subgroups in those settings as well. In this presentation, I will discuss the characterization of stable subgroups in the setting of handlebody groups, which are mapping class groups of three dimensional handlebodies. We find that for genus two, there is an analogous characterization using orbit maps to the disk graph, but for all higher genus, we can find counter examples to this characterization. (Received September 19, 2021)

1174-20-8729 **Joshua Guo*** (duckeagle1280@gmail.com), MIT PRIMES. *On the Gauss-Epple homomorphism of B_n, and generalizations to Artin groups of finite type*

In this paper, we introduce the Gauss-Epple invariant of braids, originally defined by Epple based on a note of Gauss, which we prove is well defined. We consider the Gauss-Epple invariant as an action of the braid group B_n on the set $\{1,\ldots,n\}\times\mathbb{Z}$ and as a group homomorphism from B_n to the symmetric group $\mathrm{Sym}(\{1,\ldots,n\}\times\mathbb{Z})$. We prove that this homomorphism factors through $\mathbb{Z}^n \rtimes S_n$ (in fact, its image is an order 2 subgroup of the previous group). We also describe the kernel of the homomorphism, including the probability that it contains a given random braid. Furthermore, we discuss the *super-Gauss-Epple homomorphism*, a homomorphism which extends the generalization of the Gauss-Epple homomorphism and describe a related 1-cocycle of the symmetric group S_n on the set of antisymmetric $n\times n$ matrices over the integers. We then generalize these notions to Artin groups of finite type. For future work, we suggest studying possible generalizations to complex reflection groups and computing the vector spaces of Gauss-Epple analogues. (Received September 19, 2021)

1174-20-8864 **Casey Donoven*** (casey.donoven@msun.edu), Montana State University Northern. *3/2-Generated Semigroups* Preliminary report.

A group G is $\frac{3}{2}$-generated if every non-trivial element of G is contained in a two-element generating set. It was recently shown that a finite group is $\frac{3}{2}$-generated if and only if it only has cyclic quotients, solving an open conjecture. In this talk, I will present several definitions that generalize $\frac{3}{2}$-generated to semigroups and monoids and give complete classifications thereof. (Received September 20, 2021)

1174-20-8945 **Genevieve Walsh** (genevieve.walsh@tufts.edu), Tufts University, **Rebecca Barry*** (barry_rebe@bentley.edu), Bentley/Brandeis University, **Kim Ruane** (Kim.Ruane@tufts.edu), Tufts University, **Lorenzo Ruffoni** (lorenzo.ruffoni@tufts.edu), Tufts University, **Mackenzie McPike** (Mackenzie.McPike@tufts.edu), Tufts University, **Darien Farnham** (darien.farnham@tufts.edu), Tufts University, **Sarah Hayward** (sarahhay@sas.upenn.edu), University of Pennsylvania, and **Thomas Sachen** (tsachen@princeton.edu), Princeton University. *Classifying Finitely Presented Infinite Groups* Preliminary report.

Measurements of similarity between groups include the existence of an isomorphism (a bijective homomorphism), isomorphic finite index subgroups (commensurability), and a quasi-isometry, i.e., quasi-isometric embedding and quasi-surjectivity or quasi-density. We compared groups with two generators and one relation, and we determined all possible relators r of length n such that $0 \leq n \leq 4$ that a group of this nature could possess. We defined the notation: $G_r =$ such that r is a trivial word in a and b. We proved that the normal closure of r is equivalent to both the normal closure of the conjugation of r by a word c in a, b and the normal closure of the inverse R of r, i.e., $\langle\langle r\rangle\rangle = \langle\langle crC\rangle\rangle = \langle\langle R\rangle\rangle$. This result allowed us to begin constructing the equivalence classes of groups G_r up to isomorphism by cyclic permutation and inversion of relators. Let $x \in S := \{a, b, A, B\}$, $y \in S - \{x, X\}$, and $n \in \mathbb{Z}_{\geq 0}$. We found that by Tietze transformations any group of the form G_{xy^n} is isomorphic to the group $\mathbb{Z}$. Also, there exists an isomorphism between any group G_{x^n} and $\mathbb{Z} * \mathbb{Z}/n\mathbb{Z}$; G_{abab} and G_{AbAb}; as well as G_{ABab} and $\mathbb{Z} \oplus \mathbb{Z}$. (Received September 20, 2021)

1174-20-9056 **Jason Manning** (jfmanning@cornell.edu), Cornell University, and **Kathryn Mann*** (k.mann@cornell.edu), Cornell University. *Hyperbolic groups acting on their boundaries*

A hyperbolic group acts on its Gromov boundary by homeomorphisms. In recent joint work with Jason Manning, we show these actions are topologically stable whenever the boundary is a sphere: any small perturbation of the action is semi-conjugate to the original action. This is also true for free groups, with cantor set boundary, and in ongoing work we are investigating the general case. In my talk, I will explain some of the strategy of the proof and motivation for the problem. (Received September 20, 2021)

1174-20-9222 **Christopher Adam Perez*** (caperez@loyno.edu), Loyola University New Orleans. *Towers and elementary embeddings in total relatively hyperbolic groups*

In a remarkable series of papers Zlil Sela classified the first-order theories of free groups and torsion-free hyperbolic groups using geometric structures he called towers, and independently Olga Kharlampovich and Alexei Myasnikov did the same using equivalent structures they called regular NTQ groups. It was later proved by Chloé Perin that if H is an elementarily embedded subgroup (or elementary submodel) of a torsion-free hyperbolic group G, then G is a tower over H. We prove a generalization of Perin's result to toral relatively hyperbolic groups using JSJ and shortening techniques. (Received September 20, 2021)

1174-20-9385 **Alexander J. Rasmussen*** (rasmussen@math.utah.edu), University of Utah. *Hyperbolic actions of metabelian groups*

Due to the importance of hyperbolic metric spaces in geometric group theory, classifying the actions of a fixed group on hyperbolic metric spaces is a natural, but typically very difficult problem. The speaker and his collaborators have solved this problem for certain classically-studied solvable groups. Tools from commutative algebra turn out to be very useful in these studies. In this talk, I will describe the classification theorems for groups including Baumslag-Solitar groups (due to Abbott-Rasmussen) and lamplighter groups (due to Balasubramanya) and some of the relevant tools. (Received September 20, 2021)

1174-20-9400 **Grayson Jorgenson** (grayson.jorgenson@pnnl.gov), Pacific Northwest National Laboratory, and **Scott Vasquez** (scott.vasquez@pnnl.gov), Pacific Northwest National Laboratory. *When Diagrams Don't Quite Commute: Quantifying Approximate Invariance and Equivariance in Machine Learning Models*

Equivariance is a concept that is ubiquitous in mathematics and science. In mathematics we usually only distinguish between maps that are either equivariant or are not equivariant. On the other hand, the real world is noisy, and measurements are always imperfect. In this talk we discuss how we might begin to quantify approximate equivariance. This includes describing different ways that a map can fail to be equivariant "on the nose". We focus on the case of machine learning models that either through learning or by design can achieve some degree of equivariance. Finally, we show that ultimately the kind of "approximate equivariance" that one desires is dependent on the problem that one is interested in studying. (Received September 20, 2021)

1174-20-9612 **Lorenzo Ruffoni*** (lorenzo.ruffoni2@gmail.com), Tufts University. *Graphical splittings of Artin kernels*

A main feature of the theory of right-angled Artin groups (RAAGs) consists in the fact that the algebraic properties of the group can be described in terms of the combinatorial properties of its defining graph. This idea carries over to the study of Artin kernels, i.e. subgroups of RAAGs obtained as kernels of maps to the integers. For some specific classes of chordal graphs, we obtain a sharp structural dichotomy for the Artin kernels. We will discuss some applications to the study of splittings, fibrations, and BNS invariants of these groups. This talk is based on joint work with M. Barquinero and K. Ye. (Received September 20, 2021)

1174-20-9785 **Spencer Dowdall** (spencer.dowdall@vanderbilt.edu), Vanderbilt, **Matthew Durham** (mdurham@ucr.edu), UC Riverside, and **Alessandro Sisto** (a.sisto@hw.ac.uk), Hariot Watt University. *Extensions of Veech groups*

A lattice Veech subgroup of the mapping class group is the stabilizer of a hyperbolic plane isometrically embedded in Teichmueller space, acting with finite coarea. There is a natural extension group which is a surface bundle over the associated hyperbolic orbifold. In this talk I'll describe both the fine and coarse geometry of such groups, explaining why they are HHG's, they are quasiisometrically rigid, and how they act with finite volume on singular locally homogeneous geometries. (Received September 20, 2021)

1174-20-9845 **Kasia Jankiewicz*** (kasia@ucsc.edu), University of California Santa Cruz, **Bakul Sathaye** (bakul.sathaye@gmail.com), WWU Munster, and **Annette Karrer** (annettek@campus.technion.ac.il), Technion Israel Institute of Technology. *Boundary rigidity of lattices in products of trees*

A CAT(0) group G is boundary rigid if the visual boundaries of any two CAT(0) spaces that admit geometric actions of G are homeomorphic. Not all of CAT(0) groups are boundary rigid, by the famous example of Croke-Kleiner. We prove that the groups that act geometrically and vertex-transitively on the product of two regular finite valence trees are boundary rigid. That includes the virtually simple Burger-Mozes groups. (Received September 20, 2021)

1174-20-9921 **George Domat*** (domat@math.utah.edu), University of Utah, and **Sanghoon Kwak** (kwak@math.utah.edu), University of Utah. *Coarse Geometry of Pure Mapping Class Groups of Infinite Graphs* Preliminary report.

The mapping class group of a finite-type surface and the outer automorphism group of a finite-rank free group, $\mathrm{Out}(F_n)$, share many similarities and these groups are often studied in parallel. The mapping class group of an infinite graph, Γ, is the group of proper homotopy classes of proper homotopy equivalences of Γ. These groups are uncountable, Polish, topological groups and serve as an $\mathrm{Out}(F_n)$-analog of mapping class groups of infinite-type surfaces. In work with Hannah Hoganson and Sanghoon Kwak, we investigate the coarse geometry of these groups using the framework of Rosendal. (Received September 21, 2021)

1174-20-10067 **Chloe Isabella Avery*** (chloe@math.uchicago.edu), University of Chicago, and **Lvzhou Chen** (lvzhou.chen@math.utexas.edu), University of Texas at Austin. *Stable T*

The stable torsion length in a group is the stable word length with respect to the set of all torsion elements. We show that the stable torsion length vanishes in crystallographic groups. We then give a linear programming algorithm to compute a lower bound for stable torsion length in free products of groups. Moreover, we obtain an algorithm that exactly computes stable torsion length in free products of finite abelian groups. The nature of the algorithm shows that stable torsion length is rational in this case. As applications, we give the first exact computations of stable torsion length for nontrivial examples. (Received September 21, 2021)

1174-20-10233 **Arturo Magidin*** (avmagidin@gmail.com), University of Louisiana at Lafayettee. *Generalizing the Chermak-Delgado lattice of a group* Preliminary report.

Joint work with William Cocke, Luise-Charlotte Kappe, and Elizabeth Wilcox. The Chermak-Delgado lattice of a group G consists of all subgroups H for which $|H||C_G(H)|$ achieves its maximum possible value. We are exploring possible generalizations, where $C_G(H)$ is replaced by a "relative margin" of H in G with respect to a group word w. We discuss the obstacles that the generalization encounters, and some situations in which they can be surmounted. (Received September 21, 2021)

1174-20-10272 **Hannah Park-Kaufmann*** (hk9622@bard.edu), Bard College, **Ceyhun Elmacioglu** (elmacioglu.ceyhun@gmail.com), Lafayette College, and **Melin Okandan** (melin.okandan@ug.bilkent.edu.tr), Bilkent University. *Minimal Presentations of Numerical Semigroups*

A numerical semigroup S is a subset of integers closed under addition. A minimal presentation of S is a choice of minimal relations between generators of S. It is a well-known fact that if m is the smallest positive element of S, then the size of the minimal presentation is at most $\binom{m}{2}$. Finding the possible minimal presentation sizes of numerical semigroups given a fixed m has been a long-standing open problem. In this talk, we introduce a combinatorial approach involving posets to determine the attainable minimal presentation sizes. (Received September 21, 2021)

1174-20-10415 **Scott Mahan*** (scmahan@ucsd.edu), University of California, San Diego. *DNA: Dynamic Network Augmentation* Preliminary report.

In most cases, a human can identify a dog in a picture regardless of the orientation and pose in which it appears (for example, standing right-side up or laying upside-down on its back). The human visual system's remarkable invariance to group actions such as translation, rotation, and illumination is not generally shared with deep learning models. These models are commonly 'taught' geometric invariances via data augmentation by having random elements of the underlying symmetry group G act on input during the process of training. Without a deep understanding of the data itself, the best strategy for sampling from G is not generally clear. Determining an effective data augmentation policy can require domain expertise or extensive data pre-processing. Recent efforts like AutoAugment optimize over a parameterized search space of data augmentation policies to automate the augmentation process. While AutoAugment and similar methods achieve state-of-the-art classification accuracy on several common datasets, they are limited to learning one data augmentation policy. Often different features call for different geometric invariances. We introduce Dynamic Network Augmentation (DNA), which learns input-conditional augmentation policies. Augmentation parameters in our model are outputs of a neural network and are implicitly learned as the network weights are updated. Our model allows for dynamic augmentation policies and performs well on data with geometric invariances conditional on input features. (Received September 21, 2021)

1174-20-10423 **Chloe Bishop*** (cbbish1233@ung.edu), University of North Georgia. *An Investigation of Hamiltonian Cayley Graphs* Preliminary report.

Cayley graphs are graphs associated to a group and a set of generators for that group (there is also an associated directed graph). The purpose of this study was to examine multiple examples of Cayley graphs through group theory, graph theory, and applications. We gave background material on groups and graphs and gave numerous examples of Cayley graphs and digraphs. We have established a necessary and sufficient condition for a Cayley graph to be path connected. This helped investigate the conjecture that the Cayley graph of any group (except finite Abelian group of two elements) is Hamiltonian. We found the conjecture to still be open. We found Cayley graphs and Hamiltonian cycles could be applied to campanology (in particular, to the change ringing of bells). We conclude this talk by citing the example of the Lamp Lighter Group and how its Cayley graph based on the two generators helps us view the group as acting on a doubly infinite sequence of street lamps each of which may be on or off and a lamp lighter standing at some lamp. (Received September 21, 2021)

1174-20-10486 **Shannon Talbott** (talbotts@moravian.edu), Moravian College, **Bethany Kubik** (bakubik@d.umn.edu), University of Minnesota Duluth, and **Austin Antoniou** (austinantoniou@gmail.com), Nationwide Hospital. *Factorization in Numerical Semigroup Algebras*

A numerical semigroup S is a subset of the nonnegative integers $\mathbb{N}$ that is closed under addition, has finite complement in $\mathbb{N}$, and contains 0. In this talk, we take a journey through visual storytelling to understand factorization in numerical semigroup algebras, and our initiation of the study of atomic density, an asymptotic measure of the proportion of irreducible elements in a given numerical semigroup algebra. (Received September 21, 2021)

1174-20-10687 **Luke Robitaille*** (tapir1729@gmail.com), Robitaille Homeschool, and **Minh-Tam Quang Trinh** (mqt@mit.edu), Massachusetts Institute of Technology. *Topological Entropy of Simple Braids* Preliminary report.

We investigate the proportion of permutation braids on n strands that have positive topological entropy, as n grows large. We do this by finding many long cycles in almost any permutation (using some results from enumerative combinatorics), raising the braid to some power to make it a pure braid, and then deleting all but three strands from the same cycle, which does not increase the topological entropy of the braid. We also present some conjectures regarding the images of permutation braids under the Burau representation; these matrices are related to the topological entropy of the braid by a result of Fried and Kolev. (Received September 21, 2021)

1174-20-10763 **William A. Bogley*** (bill.bogley@oregonstate.edu), Oregon State University, and **Kirk McDermott** (kirk.mcdermott@gmail.com), Slippery Rock University. *Cyclic spines and Brieskorn-Pham 3-manifolds* Preliminary report.

We describe a family of cyclic 3-manifold spines that generalize the Sieradski manifolds and which are in many cases commensurable with the Brieskorn-Pham manifolds as studied by Milnor in 1975. These arise from a family of non-aspherical relative presentations for centrally extended triangle groups. We identify those that are spherical through our solution to the finiteness problem for the fundamental groups and exhibit examples with Nil and SL2(R) geometries. (Received September 21, 2021)

1174-20-10927 **Amrita Acharyya*** (amrita.acharyya@utoledo.edu), University of Toledo. *Coverings of Profinite Graphs* Preliminary report.

We define a covering of a profinite graph to be a projective limit of a system of covering maps of finite graphs. With this notion of covering, we develop a covering theory for profinite graphs which is in many ways analogous to the classical theory of coverings of abstract graphs. For example, it makes sense to talk about the universal cover of a profinite graph and we show that it always exists and is unique. We define the profinite fundamental group of a profinite graph and show that a connected cover of a connected profinite graph is the universal cover if and only if its profinite fundamental group is trivial. (Received September 21, 2021)

1174-20-12231 **Sophie Kriz*** (Krizsophie@gmail.com), University of Michigan. *Some results on modular representation stability of symmetric groups*

Many questions of representation stability of symmetric groups can be encoded in FI-modules, which are representations of the category of finite sets and injections. While FI-modules are relatively well understood in characteristic 0, less is known in the case of positive characteristic. In this talk, I will describe the simple objects of the category of generic FI-modules (i.e. modulo torsion) in positive characteristic, and discuss some surprising facts about their structure. I will also show a computation of local cohomology of certain examples of FI-modules over the integers. (Received December 2, 2021)

22 ► *Topological groups, Lie groups*

1174-22-7402 **Benjamin Gammage*** (gammage@math.harvard.edu), Harvard University. *2-categorical 3d mirror symmetry* Preliminary report.

3d mirror symmetry can be understood as a duality relating a pair of 2-categories, one algebraic and one symplectic, associated to a pair of hyperkähler varieties (or Artin stacks). The first successful formulations in this fashion can be found in Teleman's ICM address. In this talk, we discuss some work in progress, joint with Justin Hilburn and Aaron Mazel-Gee, to extend this strategy beyond Teleman's ICM address, including hypertoric stacks and (in rank 1) a "B to A" result for pure gauge theory. (Received September 14, 2021)

1174-22-8817 **Judith A. Packer*** (Judith.Jesudason@colorado.edu), University of Colorado, Boulder. *Cohomology related to commuting k-tuples of local homeomorphisms* Preliminary report.

Cohomology groups for the locally compact étale groupoid G constructed from k commuting surjective local homeomorphisms acting on a compact metric space X are considered. One aim is to construct all continuous 1-cocycles on the groupoid G taking values in a locally compact abelian group, in terms of k-tuples of continuous functions on the unit space of G satisfying certain canonical identities. We determine when such k-tuples of functions give 1-cocycles and 1-coboundaries on the groupoid. Restricting our study to the case where the locally compact abelian group is the additive group of real numbers, we discuss the construction of a one-parameter automorphism group acting on the groupoid C^*-algebra $C^*(G)$ corresponding to the continuous 1-cocycle on G. The result specializes to give a result for higher-rank graph C^*-algebras. Possibilities for generalizations are explored.

This work is joint with C. Farsi, L. Huang, and A. Kumjian. (Received September 19, 2021)

1174-22-8890 **Milen Tchernev Yakimov*** (m.yakimov@northeastern.edu), Northeastern University, and **Stefan Kolb** (stefan.kolb@newcastle.ac.uk), Newcastle University. *Quantum symmetric pairs and star products*

We will present solutions to two problems on quantum symmetric pairs: the description of their defining relations and the construction of universal quantum K-matrices via quantum doubles. Both of them are based on star products on graded noncommutative algebras. (Received September 20, 2021)

1174-22-9131 **Abby Brauer** (abrauer@lclark.edu), Lewis & Clark College, and **Oderico-Benjamin Buran*** (oburan@sas.upenn.edu), University of Pennsylvania. *Invariant Geometric Structures on Complex Almost Abelian Groups*

An almost Abelian group is a non-Abelian Lie group with a codimension 1 Abelian subgroup. This project investigates invariant Hermitian and Kähler structures on complex almost Abelian groups. In doing so, we find explicit formulas for the left and right Haar measures, the modular function, and left and right generator vector fields on simply connected complex almost Abelian groups. From the generator fields, we obtain invariant vector and tensor field frames, allowing us to find an explicit form for all invariant tensor fields. Namely, all such invariant tensor fields have constant coefficients in the invariant frame. From this, we classify all invariant Hermitian forms on complex simply connected almost Abelian groups, and we prove the nonexistence of invariant Kähler forms on all such groups. Via constructions involving the pullback of the quotient map, we extend the explicit description of invariant Hermitian structures and the nonexistence of Kähler structures to quotients of complex simply connected almost Abelian groups. (Received September 20, 2021)

1174-22-9693 **Alan Peng*** (apeng1@mit.edu), Massachusetts Institute of Technology. *Convolution-exact perverse sheaves on the affine flag variety* Preliminary report.

An unpublished result by Mirković states that convolution-exact sheaves on the flag variety of a simple linear algebraic group G over $\overline{\mathbb{F}_p}$ are tilting, that is, they admit both standard and costandard filtrations. The analogous statement for convolution-exact sheaves on the affine flag variety is false, but Arkhipov and Bezrukavnikov noted that it is still not known whether the projections of such sheaves to a different category, which they called the Iwahori–Whittaker category, are tilting. We make partial progress toward this question by considering reductions to the combinatorics of the extended affine Weyl group of G. In particular, we demonstrate an obstruction to a direct generalization of Mirković's original proof, even in the $G = \mathrm{SL}_2$ case. We also investigate Wakimoto filtrations, as introduced by Arkhipov and Bezrukavnikov, and their variants. (Received September 20, 2021)

1174-22-9726 **Eric Nofziger*** (enofzige@butler.edu), Butler University. *Valid Extended Sets for Standard and Row-Strict Young Tableaux* Preliminary report.

Young tableaux are combinatorial objects related to the partitions of an integer and have various applications in representation theory. They are particularly useful in the study of the fibers arising from the Springer resolution. The study of this map was instrumental in the classification of irreducible representations of finite groups of Lie type, and it is still a vibrant source of ongoing research. In recent work of Graham-Precup-Russell, an association has been made between a given row-strict tableau and three disjoint subsets I, J, and K, also called extended sets. These subsets are then used in the study of extended Springer fibers. In this project, we begin to classify which extended sets correlate to a valid row-strict or standard tableau. We are able to identify several global properties of these valid sets, and we further find an algorithm that produces a valid tableau given only the extended sets in special cases. (Received September 20, 2021)

26 ► Real functions

1174-26-5991 **Ali Pirhadi*** (apirhadi@gsu.edu), Georgia State University. *Real zeros of random trigonometric polynomials with ℓ-periodic coefficients* Preliminary report.

The large degree asymptotics of the expected number of real zeros of a random trigonometric polynomial

$$T_n(x) = \sum_{j=0}^{n} a_j \cos(jx) + b_j \sin(jx),\ x \in (0, 2\pi),$$

with i.i.d. real-valued standard Gaussian coefficients is known to be $2n/\sqrt{3}$. We consider quite a different and extreme setting on the set of the coefficients of T_n and show that a random trigonometric polynomial of degree n with ℓ-periodic coefficients is expected to have significantly more real zeros compared to the classical case with i.i.d. Gaussian coefficients. (Received September 2, 2021)

1174-26-10657 **Malgorzata Marciniak** (mmarciniak@lagcc.cuny.edu), City University of New York. *Introduction to Nonstandard Analysis, Part 1*

The session organizers will speak about the foundations and intuition and practice of nonstandard methods. This talk is suitable for students and participants less familiar with nonstandard analysis. (Received September 21, 2021)

28 ► Measure and integration

1174-28-5213 **Marianna Csörnyei*** (csornyei@math.uchicago.edu), University of Chicago. *The Kakeya needle problem for rectifiable sets.*

A planar set admits the "Kakeya property" if it can be moved continuously to any other position covering arbitrary small area during the movement. It was known for more than 100 years that line segments have this property, but until recently there were only very few other known examples.

In the talk we will study two variants of this problem, the geometric and the analytic version. In the classical, geometric version, we find all connected closed sets with the Kakeya property. In the analytic version, where we are allowed to delete a null set at each time moment, we will show that every rectifiable set admits the Kakeya property, moreover, they can be moved to any other position covering not only arbitrary small but zero area. (Received November 15, 2021)

1174-28-6139 **Timothy Ira Myers*** (tim-myer@hotmail.com), Howard University. *A Constructive Definition of the Fourier Transform over a Separable Banach Space*

Gill and Myers proved that every separable Banach space, denoted $\mathcal{B}$, has an isomorphic, isometric embedding in $\mathbb{R}^\infty = \mathbb{R} \times \mathbb{R} \times \cdots$. They used this result and a method due to Yamasaki to construct a sigma-finite Lebesgue measure $\lambda_\mathcal{B}$ for $\mathcal{B}$ and defined the associated integral $\int_\mathcal{B} \cdot \, d\lambda_\mathcal{B}$ in a way that equals a limit of finite-dimensional Lebesgue integrals.

The objective of this talk is to apply this theory to developing a constructive definition of the Fourier transform on $L^1[\mathcal{B}]$. Our approach is constructive in the sense that this Fourier transform is defined as an integral on $\mathcal{B}$, which, by the aforementioned definition, equals a limit of Lebesgue integrals on Euclidean space as the dimension $n \to \infty$. Thus with this theory we may evaluate infinite-dimensional quantities, such as the Fourier transform on $\mathcal{B}$, by means of finite-dimensional approximation. As an application, we will apply the familiar properties of the transform to solving the heat equation on $\mathcal{B}$. (Received September 5, 2021)

1174-28-6161 **Dylan King*** (dylank1029@gmail.com), University of Cambridge, **Nate Gillman** (nategillman1@gmail.com), Brown University, **Jennifer Zhu** (jzhu42@gmail.com), University of California, Berkely, **Tamás Keleti** (tamas.keleti@gmail.com), Institute of Mathematics, Eötvös Loránd University, and **Frank Coen** (fcoen@villanova.edu), Department of Mathematics & Statistics, Villanova University. *Large Sets with Small Injective Projections*

Let $\ell_1, \ell_2, \ldots$ be a countable collection of lines in $\mathbb{R}^d$. For any $t \in [0, 1]$ we construct a compact set $\Gamma \subset \mathbb{R}^d$ with Hausdorff dimension $d - 1 + t$ which projects injectively into each ℓ_i, such that the image of each projection has dimension t. This immediately implies the existence of homeomorphisms between certain Cantor-type sets whose graphs have large dimensions. As an application, we construct a collection E of disjoint, non-parallel k-planes in $\mathbb{R}^d$, for $d \geq k + 2$, whose union is a small subset of $\mathbb{R}^d$, either in Hausdorff dimension or Lebesgue measure, while E itself has large dimension. As a second application, for any countable collection of vertical lines w_i in the plane we construct a collection of nonvertical lines H, so that F, the union of lines in H, has positive Lebesgue measure, but each point of each line w_i intersects at most one $h \in H$ and, for each w_i, the Hausdorff dimension of $F \cap w_i$ is zero. (Received September 5, 2021)

1174-28-6389 **Matthew Badger*** (matthew.badger@uconn.edu), University of Connecticut. *Hausdorff dimension of caloric measure*

Caloric measure is a probability measure supported on the boundary of a domain in $R^{n+1} = R^n \times R$ (space $\times$ time) that is related to the Dirichlet problem for the heat equation in a fundamental way. Equipped with the parabolic distance, R^{n+1} has Hausdorff dimension $n + 2$. We prove that (even on domains with geometrically very large boundary), the caloric measure is carried by a set of Hausdorff dimension at most $n + 2 - \beta_n$ for some $\beta_n > 0$. The corresponding theorem for harmonic measure is due to Bourgain (1987), but the proof in that paper contains a gap. I will give an overview of the theorem and its proof, focusing on the geometric measure theory techniques used to prove an upper bound on the dimension of a measure. I will also indicate what is needed to fix the original proof that the dimension of harmonic measure in R^n is at most $n - b_n$ for some $b_n > 0$. This is joint work with Alyssa Genschaw. (Received September 8, 2021)

1174-28-6999 **Guy C. David*** (gcdavid@bsu.edu), Ball State University, and **Raanan Schul** (schul@math.stonybrook.edu), Stony Brook University. *Decompositions and factorizations of Lipschitz mappings*

We will discuss recent quantitative theorems on the structure of Lipschitz maps. One result gives a simple condition implying that a Lipschitz map has large pieces of its domain on which it behaves like simple orthogonal projections. Another says that the converse of this condition in low dimensions forces the mapping to be close to factoring through a tree, like a classic example of Kaufman. The talk will be based on 2020 and 2021 results with Raanan Schul and will also discuss underlying work of Azzam–Schul and Esmayli–Hajlasz. (Received September 11, 2021)

1174-28-7139 **Lisa Naples*** (lnaples@macalester.edu), Macalester College. *Lipschitz graphs and the geometry of measures*

In order to understand the geometric structure of a measure, we explore the measure's interaction with a class of sets defined by a geometric property. If we find a countable collection of sets within the class such that almost all of the mass of the measure is contained within the collection, we say the measure is carried by the class. We will survey some results related to the characterization of measures carried by sets satisfying Lipschitz properties. Then we will discuss an undergraduate research experience in the field, and share a characterization of doubling measures carried by Lipschitz graphs established by participants. (Received September 12, 2021)

1174-28-7299 **Eva Aigner*** (ebneraigner@web.de), MJS. *On differentiable Loeb measures*

We present some results on differentiable Loeb measures and discuss applications. (Received September 14, 2021)

1174-28-7477 **Zichen Zhang*** (zzhang4@macalester.edu), Macalester College, and **Yutong Wu** (ywu1@macalester.edu), Macalester College. *Characterization of Rectifiable Measures that are Carried by Lipschitz Graphs*

The Analysts' Traveling Salesman Problem asks for necessary and sufficient conditions under which a set is contained inside of a Lipschtiz image. One direction for further study is to find a characterization of measures carried by Lipschitz graphs. In previous work, balls centered at each point in the support are used to give a characterization of doubling measures that are carried by Lipschitz graphs. To further extend that work, we develop and prove sufficient and necessary conditions for doubling measures carried by Lipschitz graphs in terms of dyadic cubes. Along the way, we prove a doubling measure property and a geometric lemma for measures that hold under the dyadic cube regime. These new results provide a characterization of measures carried by Lipschitz graphs that is more discrete in nature. (Received September 14, 2021)

1174-28-8283 **Iqra Altaf*** (iqra@uchicago.edu), University of Chicago, **Marianna Csörnyei** (csornyei@math.uchicago.edu), University of Chicago, and **Bobby LaRue Edward Wilson** (blwilson@uw.edu), The University of Washington. *Scaled Oscillation and Level Sets*

For $x \in A$ and $A \subset \mathbb{R}^m$, the upper-scaled and lower-scaled oscillation of f are defined as

$$L_f(x) = \limsup_{r\to 0} \frac{\sup_{d(x,y)\le r}|f(y)-f(x)|}{r}$$

and

$$l_f(x) = \liminf_{r\to 0} \frac{\sup_{d(x,y)\le r}|f(y)-f(x)|}{r}.$$

We study the size and regularity properties of level sets of continuous functions with bounded upper-scaled and lower-scaled oscillation. (Received September 18, 2021)

1174-28-8517 **Kevin Ren*** (kevinren@mit.edu), MIT, and **Yuqiu Fu** (yuqiufu@mit.edu), MIT. *Incidence estimates for α-dimensional tubes and β-dimensional balls in $\mathbb{R}^2$*

We prove an essentially sharp incidence estimate for a collection of δ-tubes and δ-balls in the plane, where the δ-tubes satisfy an α-dimensional spacing condition and the δ-balls satisfy a β-dimensional spacing condition. Our approach combines a combinatorial argument for small α, β and a Fourier analytic argument for large α, β. (Received September 19, 2021)

1174-28-8800 **Maxwell David Auerbach** (maxwelld.auerbach@gmail.com), Emory University, **Ethan J Berkove*** (berkovee@lafayette.edu), Lafayette College, **Rebecca Whitman** (rebecca_whitman@berkeley.edu), UC Berkeley, **Derek Smith** (smithder@lafayette.edu), Lafayette College, and **Adam Hodapp** (hodap054@umn.edu), University of Minnesota. *Shining a Light on the Menger Sponge* Preliminary report.

When C is the ternary Cantor set built from $[0,1]$, it is a classic result that C + C = [0,2]. A beautiful proof of this fact is that the lines $y = -x + b$ intersect $C \times C$ for $b \in [0,2]$. That is, "no light gets through" that has slope -1. In this talk we will investigate a similar question involving the three-dimensional Menger sponge, finding how much light shines through this fractal from different directions. We will do this without written words and equations on our slides, and the Cantor set will be a prominent player. (Received September 19, 2021)

1174-28-8896 **Annina Iseli*** (annina.iseli@math.ucla.edu), University of Fribourg, Switzerland, and **Anton Lukyanenko** (alukyane@gmu.edu), George Mason University, Fairfax. *Projection theorems in conformal and projective settings*

Marstrand's theorem (1954) states that given a Borel set $A \subset \mathbb{R}^2$, the Hausdorff dimension of the image of A under the orthogonal projection onto a line L equals $\min\{1, \dim A\}$, for almost every line L that contains the origin. This theorem has since been generalized to higher dimensions and to various different spaces.

From a geometric perspective, Marstrand's result can be phrased as follows: given a Borel set $A \subset \mathbb{R}^2$, for almost all $g \in O(2)$, the orthogonal projection of $g(A)$ onto the x-axis is a set of dimension $\min\{1, \dim A\}$. This phrasing naturally raises the question of whether projection theorems hold if $O(2)$ is replaced by another group G that admits a natural action on $\mathbb{R}^2$.

From a viewpoint of conformal geometry, $O(2)$ naturally is a subgroup of $\mathrm{Mob}(\mathbb{R}^2)$; in a projective setting, $O(2)$ naturally is a subgroup of $\mathrm{PSL}(3, \mathbb{R})$. Employing a C^2-smooth version of a transversality property (Peres-Schlag, 2000), we establish Marstrand-type as well as Besicovich-Federer-type projection theorems for the projection families induced by $\mathrm{Mob}(\mathbb{R}^2)$ and $\mathrm{PSL}(3, \mathbb{R})$. We also prove that these results generalize to all compact 1-dimensional subgroups G of both $\mathrm{Mob}(\mathbb{R}^2)$ and $\mathrm{PSL}(3, \mathbb{R})$, whenever A avoids a curve in $\mathbb{R}^2$ that we explicitly describe in terms of G. (Received September 20, 2021)

1174-28-10534 **Lan Mai*** (lhm004@mcdaniel.edu), McDaniel College. *Hausdorff Dimension of $k-$Fibonnaci Word Fractals* Preliminary report.

k-Fibonacci fractals are generated by applying a simple drawing rule to the k-Fibonacci words, which are strings of 0 and 1 with k as a positive, integer parameter. A scaling limit is taken of a sequence of finite drawn curves resulting in the fractal. k-Fibonacci fractals possess an interesting geometry; we focus on their Hausdorff dimension. We present a generalized calculation of the Hausdorff dimension of k-Fibonacci fractals for all values of k. We utilized the fractals' symmetry and their associated iterated function systems. (Received September 21, 2021)

1174-28-10552 **Benjamin Steinhurst*** (bsteinhurst@mcdaniel.edu), McDaniel College. *From Fibonacci-like words to Iterated Function Systems and Hausdorff Dimension* Preliminary report.

We have studied several variations of the classical Fibonacci word (a prototype of a Sturmian word) by viewing them as strings of 0's and 1's drawn as curves using a particular rule. The scaling limit of these curves generate interesting fractals. We provide for these fractals an Iterated Function System (IFS) that also generates the same fractal that is more amenable to geometric analysis. We overview the process by which we accomplish this and state the central theorem that correlates the features of a Sturmian word to the existence of a scaling limit. (Received September 21, 2021)

30 ► Functions of a complex variable

1174-30-5440 **Loredana Lanzani** (llanzani@syr.edu), Syracuse University, **Jue Xiong** (xiong.147@osu.edu), University Of Colorado Boulder, **William E Gryc*** (wgryc@muhlenberg.edu), Muhlenberg College, and **Yuan Zhang** (zhangyu@pfw.edu), Purdue University Fort Wayne. *Solution and Data Spaces for the Holomorphic Neumann Problem*

Let D be a bounded N-connected domain in $\mathbb{C}$ with Lipschitz boundary bD equipped with the arc length measure σ and suppose $1 < p < \infty$. The Hardy space corresponding to D can be thought of in two isomorphic ways: the set $H^p(D)$ of all holomorphic functions on D (that is, $\bar{\partial} F = 0$) whose nontangential maximal function lies in $L^p(bD, \sigma)$ and the set $h^p(bD)$ of all nontangential limits of functions in $H^p(D)$. Denote the nontangential limit of $F \in H^p(D)$ as $\dot{F}$. If $f \in L^p(bD, \sigma)$ and $2 - \delta < p < \infty$ (where $\delta \geq 1$ depends on D), the Dirichlet problem of $\bar{\partial} F(z) = 0$ for all $z \in D$ and $\dot{F}(\zeta) - f(\zeta)$ for almost every $\zeta \in bD$ is solvable if and only if $f \in h^p(bD)$ (if D is simply connected, then $\delta = 1$), and in this case the unique solution F lies in $H^p(D)$. Thus $H^p(D)$ is the solution space and $h^p(D)$ is the data space for the holomorphic Dirichlet problem on D. With this point of view, is natural to ask if there is a similar solution space and data space for the holomorphic Neumann problem. In this talk we will define such spaces, prove they have the desired properties of a solution and data space, and note other properties of them. (Received August 20, 2021)

1174-30-6365 **Erik Lundberg*** (elundber@fau.edu), Florida Atlantic University, and **Dmitry Khavinson** (dkhavins@usf.edu), University of South Florida. *The classification problem for arclength null quadrature domains*

A planar domain Ω is referred to as an *arclength null quadrature domain* if the integral along the boundary $\partial\Omega$ of any analytic function in the Smirnov space $E^1(\Omega)$ vanishes. We use classical results of Havinson-Tumarkin and Denjoy-Carleman-Ahlfors in order to prove the existence of a roof function (a positive harmonic function whose gradient coincides with the inward pointing normal along $\partial\Omega$) for arclength null quadrature domains having finitely many boundary components. This bridges a gap toward classification of arclength null quadrature domains by removing an a priori assumption from previous classification results. This result also strengthens an existing connection to free boundary problems for Laplace's equation and the hollow vortex problem in fluid dynamics. We conclude by discussing the current status of the classification problem for arclength null quadrature domains. (Received September 8, 2021)

1174-30-6376 **Aaron Yeager*** (ayeager@ccga.edu), College of Coastal Georgia. *Random polynomials and their zeros* Preliminary report.

We investigate the distribution of zeros of random polynomials with independent and identically distributed standard normal coefficients in the complex domain, obtain explicit formulas for the density and mean distribution of the zeros and level-crossings, and inquire into the consequences of their asymptotical evaluations for a variety of orthogonal polynomials. In addition, we bridge a small gap in the method of proof devised by Shepp and Vanderbei. Our approach makes use of the Jacobians of functions of several complex variables and the mean ratio of complex normal random variables. This is a joint work with Christopher Corley and Andrew Ledoan. (Received September 8, 2021)

1174-30-7061 **Raymond Centner*** (rcentner@usf.edu), University of South Florida. *Optimal Polynomial Approximants in L^p* Preliminary report.

Over the past several years, optimal polynomial approximants (OPAs) have been studied extensively in the Dirichlet-type spaces $\mathcal{D}_\alpha$, $\alpha \in \mathbb{R}$. In this context, many papers have recently been published which investigate topics such as boundary behavior, connections to reproducing kernel functions, and location of zeros. In this talk, I will introduce the notion of optimal polynomial approximant to the space L^p, $1 < p < \infty$. As one of the highlights of this talk, I will present a relationship which generalizes the orthogonality relationship from the Dirichlet-type spaces. Interestingly enough, this relationship can be used to relate OPAs in L^p to OPAs in the more structured space L^2. I'll later show how this relationship can be used to compute some OPAs in L^p. Throughout this talk, I will pose open questions to inspire further research in this topic. (Received September 11, 2021)

1174-30-7192 **Alea L Wittig*** (awittig@albany.edu), University at Albany SUNY. *Wolff's Ideal Problem on the Multiplier Algebra of the Dirichlet Space* Preliminary report.

In 1962, Carleson proved the classical Corona Theorem, which provides a sufficient condition on a finite set $\{f_j\}_{j=1}^n \in H^\infty(\mathbb{D})$ so that $1 \in \mathcal{I}(\{f_1\}_{j=1}^n)$, where $\mathcal{I}(\{f_1\}_{j=1}^n)$ is the ideal generated by $\{f_j\}_{j=1}^n$. With the goal of classifying ideal membership, Wolff found a sufficient condition on $\{f_j\}_{j=1}^n \in H^\infty(\mathbb{D})$ and $H \in H^\infty(\mathbb{D})$ to conclude that $H^3 \in \mathcal{I}(\{f_j\}_{j=1}^n)$.

Recently, an analogue to Wolff's ideal problem for the multiplier algebra of the Dirichlet space was proved by Banjade and Trent. It gives sufficient conditions on H and finite set $\{f_j\}_{j=1}^n$ in the multiplier algebra of the Dirichlet space so that $H^3 \in \mathcal{I}(\{f_j\}_{j=1}^n)$. I will discuss the problem of classifying ideal membership of $H \in \mathcal{I}(\{f_j\}_{j=1}^n)$ in the multiplier algebra of the Dirichlet space. (Received September 17, 2021)

1174-30-7330 **Pamela Gorkin*** (pgorkin@bucknell.edu), Bucknell University, and **Brett Wick** (wick@math.wustl.edu), Washington University in Saint Louis. *Interpolating sequences in model spaces*

Interpolating sequences for the space of bounded analytic functions, H^∞, are well understood. In a recent paper, Dyakonov studied the interpolation problem in the model space setting. In this talk we discuss what happens to such sequences under large and small perturbations, and apply these results to Frostman sequences; that is, sequences (a_j) in $\mathbb{D}$ such that

$$\sup\left\{\sum_j \frac{1-|a_j|}{|\zeta - a_j|} : \zeta \in \mathbb{T}\right\} < \infty.$$

(Received September 14, 2021)

1174-30-7523 **Kelly Bickel** (kelly.bickel@bucknell.edu), Bucknell University, and **J. E. Pascoe*** (pascoej@ufl.edu), University of Florida. *Relaxations of the Laguerre-Pólya class and zero free regions near a line*

The Laguerre-Pólya class is the set of analytic functions which are locally the uniform limit of real-rooted polynomials. Such functions have nice Hadamard factorizations with real roots. We showed that functions in the "closed radical" of the Laguerre-Pólya class possess a continuous version of these nice Hadamard factorizations and are characterized by a certain determinantal inequality. We discuss the relationship between relaxations of Laguerre-Pólya class, matrix monotonicity, and the zero sets of general Hadamard products.

Some of this talk represents joint work with Kelly Bickel and Meredith Sargent. (Received September 14, 2021)

1174-30-7794 **Jonathan Rehmert*** (rehmert@math.ksu.edu), Kansas State University. *Quasisymmetric Koebe Uniformization via Transboundary Modulus*

A circle domain is an open and connected subset of the plane whose complementary components are points or round disks. Given a metric space, X, which is homeomorphic to a countably connected circle domain (and satisfies some mild conditions), we obtain a characterization of when X is quasisymmetric to a circle domain. This characterization is in terms of Schramm's transboundary modulus and Heinonen-Koskela's Loewner property and is inspired by Bonk's celebrated uniformization result for planar carpets. This is joint work with Hrant Hakobyan. (Received September 16, 2021)

1174-30-7987 **Ritu Dhankhar*** (ritu.dhankhar26@gmail.com), Birla Institute of Technology and Science Pilani, India, and **Prasanna Kumar** (prasannak@goa.bits-pilani.ac.in), Birla Institute of Technology and Science Pilani, India. *On an Inequality due to Rivlin*

An inequality due to Rivlin states that if $P(z) = \sum_{k=0}^{n} a_k z^k$ is a polynomial of degree n having all its zeros in $|z| \geq 1$, then

$$|P(re^{i\theta})| \geq \left(\frac{r+1}{2}\right)^n |P(e^{i\theta})|, \quad \text{if } 0 \leq r < 1, 0 \leq \theta < 2\pi.$$

In this paper, a sharpening of the above inequality is obtained. Our result gives a bound sharper than the bound in Rivlin's inequality except when $|a_0| = |a_n|$. Some important consequences of the obtained result are also discussed. (Received September 17, 2021)

1174-30-8076 **Matthew Hohertz*** (matt.hohertz@rutgers.edu), Rutgers University. *Extending the Geometric Modulus Principle*

In a 2011 paper, Kalantari observes that complex polynomials exhibit a certain radial symmetry at every point. To wit, if the polynomial $p(z)$ has order m as an analytic function centered at the point $z = z_0$, then the complex plane near z_0 splits into $2m$ sectors of equal angle, alternating radially between sectors in which $|p(z)| > |p(z_0)|$ and sectors in which $|p(z)| < |p(z_0)|$. In our subsequent research, we found that this symmetry, which Kalantari formalizes as the Geometric Modulus Principle, is retained for much larger classes of functions, in particular holomorphic functions as well as harmonic functions with respect to two real variables. This paper continues our work in several respects. First, we expand the Geometric Modulus Principle to the so-called approximately harmonic functions, which behave much like the class of harmonic functions with respect to ring operations. We then define the geometric modulus property, enjoyed by a larger-still class of functions, which formalizes the idea of the sign of a function "undulating" in sectors around a point of its domain. If a function has the geometric modulus property and is of order m as an analytic function at a point, then its m^{th} derivative with respect to radius is a trigonometric polynomial that behaves like a sine function; we call such trigonometric polynomials sinusoidal, devoting a section of the appendix to their properties. We conclude with some applications of our work and questions for future study. (Received September 17, 2021)

1174-30-8794 **Baichuan Chen*** (bchen@reed.edu), Reed College. *Finding Ellipses: Polymath Jr. 2021 Progress Report*

In this talk, we report on the work completed by the Finding Ellipses subgroup of the Polymath Jr. 2021. We explored a diverse range of topics related to the finite Blaschke products and the linear algebra and geometries. We investigated subprojects such as Sendov's conjecture, and topics with emphasis on linear algebra and geometries, including forming conjectures on the connection between Blaschke curves and Kippenhahn curve, doing expository work on foci of algebraic curves, learning about isogonal conjugates, and also remembering the Chapple-Euler and Fuss formula. We made efforts to make numerous codes of visualization along the way, and many inspirations and conjectures that we pursued owe the inspiring images.

Significant outcomes include discovering a neat formula calculating the coordinate of the isogonal conjugate of a point with respect to a triangle where we used degree three Blaschke products. We also constructed a slightly more general result on Sendov's conjecture in the case of finite Blaschke products and with some conditions of the vanishing points. Other efforts are made on giving new conjectures on the correspondence between algebraic curves given by the dual of Linfield function and the Kippenhahn Curve, and thinking of a generalization of the Linfield function to introduce more ellipses and algebraic curves into future consideration. (Received September 20, 2021)

1174-30-9109 **Robert Xin Dong** (dong@uconn.edu), University of Connecticut. *Extremal properties of kernels and capacities in one-complex variable*

In this talk, using the analytic and logarithmic capacity we will investigate extremal properties of several kernel functions including the Bergman, Szegö, higher-order Bergman kernels. We will then explore rigidity results concerning the logarithmic capacity and the sublevel sets of the Green's function. This is joint work with Xin Dong (University of Connecticut) and Yuan Zhang (Purdue University Fort Wayne) (Received September 20, 2021)

1174-30-9118 **Stefan Llewellyn Smith** (sgls@ucsd.edu), UCSD, and **Elena Luca** (e.louca@ucl.ac.uk), U. College London. *The Desingularization of the Cauchy Kernel in Bounded Convex Domains* Preliminary report.

One physical application of the Cauchy integral formula appears in fluid dynamics. However, the Cauchy kernel becomes numerically unstable near the boundary of the given domain. We will present a new technique to desingularize the Cauchy kernel for bounded convex polygons which is inspired by Fokas' celebrated Unified Transform method for convex polygons. Time permitting, we will show one or two applications. This is joint work with L. Lanzani (Syracuse U.), S. Lewellyn Smith (UCSD) and E. Luca (U. College London). (Received September 20, 2021)

1174-30-9809 **Elaine Kibby Danielson*** (elaine.danielson@ufl.edu), University of Florida, **Hamilton Ji Wan** (hamilton.wan@yale.edu), Yale University, **Aidan Mager** (aidanmager@gmail.com), University of Washington, and **Paul Apisa** (paul.apisa@gmail.com), University of Michigan. *Constructing Translation Surfaces from Hecke Eigenforms* Preliminary report.

A translation surface can be represented both as a collection of polygons with sides identified and as a Riemann surface with a holomorphic one-form. Examples of the latter representation can be obtained by equipping a modular curve for the congruence subgroup $\Gamma_0(N)$ with a weight two Hecke eigenform. These translation surfaces in particular are of interest because we suspect that some of these surfaces have novel $\mathrm{GL}_2(\mathbb{R})^+$ orbit closures. However, these orbit closures are computed from the polygonal representation of the translation surface, which requires building a flat atlas from the eigenform as an intermediate step. We discuss our work towards an algorithm in Sage to produce the flat atlas for a given eigenform. (Received September 21, 2021)

1174-30-10284 **Ryan Alvarado*** (rjalvarado@amherst.edu), Amherst College. *Optimal embeddings and extensions for Triebel-Lizorkin spaces in spaces of homogeneous type*

Embedding and extension theorems for certain classes of function spaces in $\mathbb{R}^n$ (such as Sobolev spaces) have played a fundamental role in the area of partial differential equations. In this talk, we will discuss some recent work which builds upon such results and identifies necessary and sufficient conditions guaranteeing that certain Sobolev-type inequalities and extension results hold for the scale of Triebel-Lizorkin spaces ($M^s_{p,q}$ spaces) in the general context of spaces of homogeneous type. An interesting facet of this work is how the range of s (the smoothness parameter) for which these inequalities and extension results hold is intimately linked to the geometric makeup of the underlying space. (Received September 21, 2021)

1174-30-11267 **Jane M McDougall*** (jmcdougall@coloradocollege.edu), Colorado College. *Rosette Harmonic Mappings and Minimal Surfaces* Preliminary report.

The familiar plane-curve hypocycloids can be modified using hypergeometric functions, leading to the rosette harmonic mappings. A fundamental feature of a rosette harmonic mapping $f = h + \overline{g}$, is a mirror image property of the analytic functions g and h on the boundary of the unit disk, where h and g are the analytic functions in the canonical decomposition of f. This gives rise to boundary values of the harmonic mappings that sometimes include arcs of constancy. The corresponding minimal surfaces however are not Jenkins-Serrin surfaces, and instead the bounding curves lie in planes; certain configurations produce saddle towers, and even an embedded triply periodic minimal surface. (Received September 22, 2021)

31 ► Potential theory

1174-31-8780 **Cosmas Kravaris*** (cosmaskravaris@tamu.edu), Texas A&M University. *On the density of eigenvalues on periodic graphs*

Using the Hilbert basis theorem, we show that the discrete Laplacian on $\mathbb{Z}^d$-periodic graphs have finitely many finite support eigenfunctions up to translations and linear combinations. We show that this property can be used to calculate the density of eigenvalues (i.e. elements of the pure point spectrum) and illustrate the techniques on the Kagome lattice. Next, we provide a formula for the von Neumann dimension (i.e. density) of eigenvalues on $\mathbb{Z}^d$-periodic graphs using syzygy modules. If time permits, we generalize the claims to G-periodic graphs whose acting group G is virtually polycyclic. (Received September 19, 2021)

1174-31-11047 **Rebekah Y Jones*** (ryjones@ncf.edu), New College of Florida, and **Panu Lahti** (panulahti@amss.ac.cn), Chinese Academy of Sciences. *Quasiconformal Invariance* Preliminary report.

It is well known that quasiconformal maps on metric measure spaces quasi-preserve the modulus of curves. In this talk, I will discuss known results regarding the preservation of surface moduli as well as recent progress regarding quasiconformal invariance of other potential theoretic quantities. (Received September 21, 2021)

32 ► Several complex variables and analytic spaces

1174-32-5490 **Xiaojun Huang*** (xhuangj@yahoo.com), Department of Mathematics, Rutgers University. *Revisit of a non-degeneracy property of extremal maps*

We give a generalization of an old result on the small extremal maps near a strongly pseudo-convex boundary and also propose new open questions. (Received August 20, 2021)

1174-32-5587 **Andrew S. Raich** (araich@uark.edu), University of Arkansas. *The Fundamental Solution to $\Box_b$ on Quadric Manifolds with Nonvanishing Levi-Form* Preliminary report.

We report on recent results regarding the computation and estimates of the relative fundamental solution, N, to the $\Box_b$ operator on quadric submanifolds of complex Euclidean space of real codimension greater than one with nonvanishing scalar Levi-form. In this case, we compute an explicit convolution kernel for N and provide explicit pointwise estimates of this kernel that are analogous to the classical estimates for the fundamental solution to $\Box_b$ for the hypersurface Heisenberg group. In particular, if D_I is a differential operator on the quadric of weighted order two, then we show $D_I N$ extends to a continuous operator on the Sobolev spaces, $W^{k,p}$, for $k \geq 0$ and $1 < p < \infty$.
(Received August 23, 2021)

1174-32-5594 **Nathan Wagner*** (nathanawagner@wustl.edu), Washington University In St. Louis, and **Cody B. Stockdale** (stockdalecody@gmail.com), Clemson University. *Weighted theory of Toeplitz operators on the Bergman space*

We study the weighted compactness and boundedness properties of Toeplitz operators on the Bergman space with respect to Békollè-Bonami type weights. Let T_u denote the Toeplitz operator on the (unweighted) Bergman space of the unit ball in $\mathbb{C}^n$ with symbol $u \in L^\infty$. We give sufficient conditions on u that imply the compactness of T_u on L^p_σ for $p \in (1,\infty)$ and all weights σ in the Békollè-Bonami class B_p and from L^1_σ to $L^{1,\infty}_\sigma$ for all $\sigma \in B_1$. Additionally, using an extrapolation result, we characterize the compact Toeplitz operators on the weighted Bergman space $\mathcal{A}^p_\sigma$ for all σ belonging to a nontrivial subclass of B_p. Concerning boundedness, we show that T_u extends boundedly on L^p_σ for $p \in (1,\infty)$ and weights σ in a u-adapted class of weights containing B_p. Finally, we establish an analogous weighted endpoint weak-type $(1,1)$ bound for weights beyond B_1. (Received August 23, 2021)

1174-32-5595 **Albert Boggess** (boggess@asu.edu), Arizona State University. *The $\Box_b$ equation on quadric submanifolds and applications*

In this talk, I will discuss the Kohn Laplacian $\Box_b$ on quadric submanifolds of $\mathbb{C}^n \times \mathbb{C}^m$. I will show you how to solve the $\Box_b$-equation through the $\Box_b$-heat equation and discuss several applications of the formulas we derive, shedding light on some old problems as well as solving new ones. All work is joint with Albert Boggess of Arizona State University. (Received August 23, 2021)

1174-32-5606 **Walton Green** (awgreen@wustl.edu), Washington University In St. Louis. *Dominating Sets in Bergman Spaces on Strongly Pseudoconvex Domains*

We obtain local estimates, also called propagation of smallness or Remez-type inequalities, for analytic functions in several variables. Using Carleman estimates, we obtain a three sphere-type inequality, where the outer two spheres can be any sets satisfying a boundary separation property, and the inner sphere can be any set of positive Lebesgue measure. We apply this local result to characterize the dominating sets for Bergman spaces on strongly pseudoconvex domains in terms of a density condition or a testing condition on the reproducing kernels. Our methods also yield a sufficient condition for arbitrary domains and lower-dimensional sets. (Received August 23, 2021)

1174-32-5707 **Shiferaw Berhanu*** (berhanu@temple.edu), Temple University. *A generalization of a microlocal version of Bochner's tube theorem.* Preliminary report.

Abstract. We prove a generalization of the microlocal version of Bochner's tube theorem due to Baouendi and Treves. The results provide a class of CR structures where CR functions extend holomorphically to a full neighborhood of a point which may be of infinite type. (Received August 25, 2021)

1174-32-5898 **Debraj Chakrabarti*** (debraj.chakrabarti@gmail.com), Central Michigan University, and **Luke Edholm** (edholm@umich.edu), University of Michigan/University of Vienna, University of Vienna. *Projection operators onto Bergman spaces* Preliminary report.

The Bergman projection of a domain $\Omega \subset \mathbb{C}^n$ is the orthogonal projection from $L^2(\Omega)$ to the Bergman space $A^2(\Omega)$. It is natural to ask whether there is a similar natural linear projection operator from the space $L^p(\Omega)$ to $A^p(\Omega)$ when $p \neq 2$. Traditionally, one answers this question by extending the Bergman projection by continuity from $L^2(\Omega) \cap L^p(\Omega)$ to $L^p(\Omega)$. However, this answer is unsatisfactory in general domains since the density and boundedness properties needed for this extension do not hold in many interesting domains. In this talk we will suggest an alternative construction of a projection operator from $L^p(\Omega)$ to $A^p(\Omega)$ when Ω is a bounded Reinhardt pseudoconvex domain, and show that this construction yields a better projection operator on L^p than the extended Bergman projection. (Received August 31, 2021)

1174-32-6088 **Steven George Krantz*** (sk@math.wustl.edu), Washington University, and **Marco Peloso** (marco.peloso@unimi.it), Università degli Studi di Milano. *THE WORM DOMAIN AND THE BERGMAN KERNEL*

In joint work with Marco Peloso we analyze the Bergman kernel on a version of the Diederich-Fornaess worm domain that was developed by Christer Kiselman. We are able to obtain an asymptotic expansion for the kernel. And we can study the mapping properties of the Bergman projection. They are quite pathological. We also discuss more recent work with Caterina Stoppato. (Received September 4, 2021)

1174-32-6197 **Laszlo Lempert*** (lempert@purdue.edu), Purdue University. *Bergman kernels in holomorphic vector bundles* Preliminary report.

Consider a holomorphic vector bundle $E \to X$ over a compact complex manifold, and the space $\mathcal{O}(E)$ of its holomorphic sections. Somewhat traditionally, any positive definite inner product on $\mathcal{O}(E)$ induces a Bergman–type kernel function, a holomorphic section of a certain vector bundle over $X \times \overline{X}$. Here $\overline{X}$ is X with the opposite complex structure. In this talk we will argue that it is more natural to associate kernel functions with inner products on the dual $\mathcal{O}(E)^*$, not necessarily positive definite ones, and we will describe various properties of this association. (Received September 6, 2021)

1174-32-6272 **Qi Han*** (qhan@tamusa.edu), Texas A&M University-San Antonio. *Complex analytic solutions to eikonal-type equations* Preliminary report.

In the N-dimensional Euclidean space $\mathbf{R}^N$ with $N \geq 2$, the eikonal equation reads $u_{x_1}^2(x) + u_{x_2}^2(x) + \cdots + u_{x_N}^2(x) = 1$. Caffarelli and Crandall [CommPDE, 2010] proved, in a simpler form, that: Let $u(x) \in C^1(\mathbf{R}^N \setminus \mathcal{S})$ be a solution to the eikonal equation with its singularity set $\mathcal{S}$ closed and of Hausdorff 1-measure 0; then $u(x)$ is either an affine function or a cone function. As noted by Caffarelli and Crandall in their Remark 2.3, the affine-solution result when $\mathcal{S} = \emptyset$ is attributed back to Khavinson [AmerMathMonthly, 1995] but only in $\mathbf{C}^2$. Define $u(z) := \frac{2}{7}z_1 + \frac{3}{7}z_2 + \frac{6}{7}z_3 + e^{\frac{12-21i}{13}z_1 + \frac{18+14i}{13}z_2 - z_3}$ to observe that $u(z)$ is an entire function on $\mathbf{C}^N$ when $N \geq 3$ and solves the eikonal equation; yet it is nonlinear. We aim to characterize this phenomenon for holomorphic solutions to the eikonal equation in higher complex dimensions. On the other hand, meromorphic solutions to a general form of the so-called Fermat Diophantine equation $u_{z_1}^p(z) + u_{z_2}^q(z) = 1$ when $p, q \geq 2$ in $\mathbf{C}^2$ are also characterized, which turn out all to be affine. (Received September 7, 2021)

1174-32-6841 **Timothy George Clos*** (tclos@kent.edu), Kent State University, and **Alexander J. Izzo** (aizzo@math.bgsu.edu), Bowling Green State University. *A uniform algebra approach to an approximation theorem of Şahutoğlu and Tikaradze*

In a recent paper, Şahutoğlu and Tikaradze prove a Bishop's type approximation theorem on bounded pseudoconvex domains in $\mathbb{C}^n$ for which the $\overline{\partial}$ problem is solvable in L^∞. Motivated by this result, we prove a similar approximation theorem on a different class of bounded pseudoconvex domains. The domains we consider are open (i.e., can be identified with an open set via evaluation functionals) in the maximal ideal space of bounded holomorphic functions. We use techniques from uniform algebras. (Received September 9, 2021)

1174-32-6872 **Elena Kim** (elenakim@mit.edu), Massachusetts Institute of Technology, **Colin Fan** (colin.fan@rutgers.edu), Rutgers University, **Ian Shors*** (ishors@hmc.edu), Harvey Mudd College, **Zoe Plzak** (zplzak@bu.edu), Boston University, and **Samuel Sottile** (sottile1@msu.edu), Michigan State University. *Spectral Analysis of the Kohn Laplacian on Lens Spaces*

A key problem in spectral geometry is the relationship between the spectrum of the Laplacian and the geometry of the underlying manifold. The celebrated Weyl's law relates the asymptotics of the spectrum to the volume of the manifold. Analogous to the Laplace-Beltrami operator on Riemannian manifolds, we study the Kohn Laplacian for functions on CR manifolds. Lens spaces are manifolds obtained as quotients of the sphere $S^{2n-1} \subset \mathbb{C}^n$ by cyclic unitary actions with the natural CR structure inherited from the spheres. Folland computes the eigenvalues of the Kohn Laplacian on S^{2n-1} and these eigenvalues descend on the quotient with different multiplicities. We investigate these multiplicities on lens spaces by analyzing a system of Diophantine equations. Using this, we prove an analog of Weyl's law for the Kohn Laplacian on lens spaces.

While Ikeda and Yamamoto classify isospectral lens spaces as Riemannian manifolds, an analog for the Kohn Laplacian on lens spaces does not exist in the literature. We show that two 3-dimensional lens spaces with fundamental groups of equal prime order are isospectral if and only if they are CR isometric. Moreover, we present lens spaces that are isospectral with respect to the Laplace-Beltrami operator, but have different spectra with respect to the Kohn Laplacian, reflecting the fact that information on the CR structure is contained in the Kohn spectrum. (Received September 10, 2021)

1174-32-7292 **Yunus E. Zeytuncu** (zeytuncu@umich.edu), University of Michigan - Dearborn. *Weyl's Law on Compact Heisenberg Manifolds*

Let $M = \Gamma \setminus \mathbb{H}_d$ be a compact quotient of the d-dimensional Heisenberg group $\mathbb{H}_d$ by a lattice subgroup Γ. We show that the eigenvalue counting function $N(\lambda)$ for any fixed element of a family of second order differential operators $\{\mathcal{L}_\alpha\}$ on M has asymptotic behavior $N(\lambda) \sim C_{d,\alpha} \operatorname{vol}(M) \lambda^{d+1}$, where $C_{d,\alpha}$ is a constant that only depends on the dimension d and the parameter α. As a consequence, we obtain an analog of Weyl's law (both on functions and forms) for the Kohn Laplacian on M. Our main tools are Folland's description of the spectrum of $\mathcal{L}_\alpha$ and Karamata's Tauberian theorem. (Received September 14, 2021)

1174-32-7488 **Jeet Sampat*** (sampatjeet@wustl.edu), Washington University in St. Louis. *Cyclicity preserving operators*

Let $\mathcal{X}$ and $\mathcal{Y}$ be spaces of analytic functions in several complex variables. It is an important problem to characterize all shift-cyclic functions of any analytic function space. A common way of obtaining cyclic functions is through cyclicity preserving operators, i.e. a bounded operator $T : \mathcal{X} \to \mathcal{Y}$ such that Tf is cyclic, whenever f is cyclic. We will show that under certain reasonable conditions for the spaces $\mathcal{X}$ and $\mathcal{Y}$, all such operators have to be weighted composition operators. In other words, there exist analytic functions a and b, defined on the appropriate domains, such that $Tf = a \cdot (f \circ b)$ for all $f \in \mathcal{Y}$. We will also show that whenever $\mathcal{Y}$ is the Hardy space on the unit polydisk $\mathbb{D}^n$, $H^p(\mathbb{D}^n)$ for $p \geq 1$, the converse is true. That is, all bounded weighted composition operators $T : \mathcal{X} \to H^p(\mathbb{D}^n)$ also preserve cyclicity. (Received September 14, 2021)

1174-32-7519 **Miklós Pálfia*** (miklos.palfia@uni-corvinus.hu), Corvinus University of Budapest. *Free functions preserving certain partial orders of operators* Preliminary report.

Recently free analysis has been a very active topic of study in operator and function theory. In particular free functions that preserve partial orders of operators have been studied by a number of authors, in connection to Loewner's theorem. Also operator concave and convex free functions naturally get into the picture as we study the positive definite order preserving free functions. We will go through recent results and open problems in this field, and we will cover some recent joint work with M. Gaál on real operator monotone functions. (Received September 14, 2021)

1174-32-7660 **Dusty Grundmeier** (deg@math.harvard.edu), Harvard University, and **David E Barrett*** (barrett@umich.edu), University of Michigan. *Sums of CR and projective dual CR functions*

A real hypersurface in projective space $\mathbb{CP}^n$ that is locally projectively equivalent to a strongly convex hypersurface admits, in addition to the standard CR structure, a secondary "projective dual" structure obtained from a CR version of the Gauss map. The problem of characterizing functions that are locally the sum of a standard CR function and a projective dual CR function is analogous to the classical problem of characterizing boundary values of pluriharmonic functions.

The talk will show how methods of J. Lee can be adapted to characterize this class of functions using triple products of vector fields in dimension two and by double products of vector fields in higher dimension. The two-dimensional results extend earlier work by the authors in the special case of circular hypersurfaces. (Received September 15, 2021)

1174-32-7716 **Ming Xiao*** (m3xiao@ucsd.edu), University of California, San Diego. *Holomorphic isometric maps from the unit ball to bounded symmetric domains*

We will discuss some recent results on the rigidity of holomorphic isometric maps from the unit ball to bounded symmetric domains. (Received September 15, 2021)

1174-32-7968 **Alberto Dayan*** (alberto.dayan@wustl.edu), Norwegian University of Science and Technology (NTNU). *Random Interpolating Sequences in the Polydisc and the Unit Ball*

A random sequence Z in the unit disc is determined by a sequence of deterministic radii and a sequence of i.i.d. random variables uniformly distributed on the unit circle. Chochran and Rudowicz found the 0-1 Kolmogorov law for Z to be interpolating, that is, the cut-off condition on the a-priori fixed radii in order for Z to be interpolating almost surely. In this talk, we will extend their work to random interpolating sequences for bounded analytic functions in the d-dimensional polydisc and for the Besov-Sobolev spaces on the unit ball. The case of the Besov-Sobolev spaces is more treatable, since such spaces have their interpolating sequences well understood and characterized in the deterministic setting. This is not the case for interpolating sequences in the polydisc: in this second case our necessary and sufficient conditions for almost sure interpolation do not coincide, and they are obtained by looking at the decay of related random Grammians off their diagonals.

This is a joint work with Brett Wick and Shengkun Wu (Received September 17, 2021)

1174-32-8133 **Emil Straube*** (straube@math.tamu.edu), Texas A&M University. *A sufficient condition for regularity in the $\overline{\partial}$-Neumann problem revisited* Preliminary report.

I plan to present an improved version of a sufficient condition for global regularity in the $\overline{\partial}$–Neumann problem, and to show how it gives a new result on the connection between Diederich–Fornæss index and regularity. Comparing this result to existing ones raises new questions. (Received September 17, 2021)

1174-32-8232 **Yuliy M Baryshnikov*** (ymb@illinois.edu), University of Illinois. *Jumping across amoeba* Preliminary report.

Consider a rational multivariate function. The complements to amoeba of its denominator correspond to different Laurent series expansions. How these expansions relate to each other? In this talk I will present several results addressing this question. (Received September 18, 2021)

1174-32-8323 **Michael R. Pilla*** (mpilla@iu.edu), Indiana University, Bloomington. *Which Subsets of the Boundary of the Unit Ball in $\mathbb{C}^N$ Can Be Mapped Into the Boundary Anyway?* Preliminary report.

For holomorphic self maps of the disk analytic on its closure, one gets a lot of mileage out of studying the dynamical properties of iteration. Boundary behavior is particularly important. For maps that are not inner, at most a finite amount of boundary points can be mapped into the boundary. In this talk we investigate generalizations of this result in higher dimensions. In particular, we observe the behavior of holomorphic maps of the ball into itself and discuss which subsets of the boundary can map into the boundary. We then give applications to the spectral properties of composition operators. (Received September 18, 2021)

1174-32-8331 **Blake Joseph Boudreaux*** (bboudre7@uwo.ca), University of Western Ontario. *Equivalent Bergman Spaces with Inequivalent Weights*

A space of square-integrable holomorphic functions on a domain $\Omega \subseteq \mathbb{C}^n$ with respect to a weight function μ is known as a weighted Bergman space. If the weight function is sufficiently well-behaved, then the space admits a unique reproducing kernel known as the weighted Bergman kernel. More precisely, the weighted Bergman kernel function is a function $K_\mu : \Omega \times \Omega \to \mathbb{C}$ with the property that

$$f(z) = \int_\Omega f(\zeta) K_\mu(z,\zeta)\mu(\zeta) \mathrm{d}V(\zeta)$$

for each f in the weighted Bergman space.

We give a proof that every weighted Bergman space admits an equivalent weight $\tilde{\mu}$ whose Bergman kernel with respect to $\tilde{\mu}$ has zeroes. Here the weights are equivalent in the sense that they determine the same space of holomorphic functions.

Also, if time allows, we will exhibit a family of radial weights in $L^1(\mathbb{C})$ whose associated Bergman kernel functions have infinitely many zeroes, contrary to the case in the unit disc $\mathbb{D} \subseteq \mathbb{C}$. (Received September 18, 2021)

1174-32-8350 **James J Heffers*** (heffers@umich.edu), University of Michigan. *On Lelong Numbers of Currents*

Let T be a positive closed bidimension (p,p) current in $\mathbb{P}^n$. In this talk we will look at the geometric properties of sets of points where a current T has "large" Lelong numbers, and observe that the set of points where our current has a large Lelong number can be contained in a small subspace of $\mathbb{P}^n$. We will then show that some of these results can be extended into multiprojective spaces $\mathbb{P}^{n_1} \times \ldots \times \mathbb{P}^{n_k}$. (Received September 18, 2021)

1174-32-8593 **Jesse Hulse** (jjhulse@syr.edu), Syracuse University, **Sean N. Curry** (sean.curry@okstate.edu), Oklahoma State University, **Purvi Gupta** (purvigupta@iisc.ac.in), Indian Institute of Science, **Valentin Daniel Kunz** (valentin.kunz@manchester.ac.uk), University of Manchester, **Alan R Legg*** (leggar01@pfw.edu), Purdue University Fort Wayne, and **John N Treuer** (jtreuer@tamu.edu), Texas A&M University. *Analytic Continuation of Solutions to Diffraction Problems*

Recently, R. Assier and A. Shanin developed methods to analytically continue solutions of a wide class of diffraction problems (which are certain boundary value problem for the Helmholtz operator $\Delta + k^2$) in $\mathbb{R}^2$ and studied the continuation's singularities. This heavily relied on the concept of Sommerfeld surfaces. For the past months, the MRC group 'SCV in Applied Mathematics' studied the mathematical methods involved, aiming to generalise the results to more general geometries/differential operators. In this talk, we will introduce the core concepts to our research, explain how different areas of theoretical mathematics are related to diffraction theory, and why we hope to be able to generalise Assier's and Shanin's results in a more abstract setting. Finally, if time permits, we will also present the results we have obtained thus far. (Received September 19, 2021)

1174-32-8842 **Jiri Lebl*** (lebl@okstate.edu), Oklahoma State University, **Alan Noell** (alan.noell@okstate.edu), Oklahoma State University, and **Sivaguru Ravisankar** (sivaguru@tifrbng.res.in), Tata Institute of Fundamental Research. *CR singular submanifolds and CR functions* Preliminary report.

We study the properties of CR functions and mappings on CR singular submanifolds, in particular an analogue of Cartan's theorem and its relation to extension of CR functions (that is, functions that are CR at CR points). (Received September 20, 2021)

1174-32-8859 **Emily Rudman** (walthere@indiana.edu), Indiana University, **Stephen E Gillen*** (stepe@sas.upenn.edu), University of Pennsylvania, **Terrence George** (terrence_george1@brown.edu), University of Michigan, **Alperen Ergur** (alperenali@gmail.com), University of Texas at San Antonio, and **Benjamin Branman** (branman@wisc.edu), University of Wisconsin-Madison. *Critical Points at Infinity using Toric Varieties* Preliminary report.

Analytic combinatorics in several variables (ACSV) uses Morse theory to decompose the poly-torus from the multivariate Cauchy Integral Formula into cycles around critical points of the height function. This method can fail to find coefficient asymptotics if there is a "critical point at infinity" (CPAI) that causes the poly-torus homotopy to go off to infinity at a finite height. Given that distinct CPAI can merge to the same point in ordinary complex projective space, we seek to extend the height function to the toric variety corresponding to the Newton polytope, with the goal of eliminating such collisions. (Received September 20, 2021)

1174-32-8871 **Jennifer Brooks** (jbrooks@mathematics.byu.edu), Brigham Young University, **Purvi Gupta** (purvi.gpt@gmail.com), Indian Institute of Science, **Alekzander Malcom** (alekmalcom@gmail.com), Oklahoma State University, and **Kevin Palencia Infante*** (palencia@niu.edu), Northern Illinois University. *Invariant CR mappings between spheres*

Motivated by the extensive study of CR mappings between spheres, we study whether there exist non-constant, minimal (in N) rational CR mappings from S^{2n-1}/Γ to S^{2N-1}, where $n, N > 2$, and Γ is a finite subgroup of $U(n)$. Using a tensoring procedure of D'Angelo, we construct invariant, monomial CR mappings and study the possible target dimensions. (Received September 20, 2021)

1174-32-8936 **Zhenghui Huo*** (huo198936@gmail.com), Duke Kunshan University, and **Bingyang Hu** (hu776@purdue.edu), Purdue University. *Sparse domination of weighted composition operators on strictly pseudoconvex domains*

In this talk, we will present our results on the behavior of the weighted composition operators acting on Bergman spaces over strictly pseudoconvex domains. We achieve a sparse domination for the weighted composition operator by constructing dyadic systems within the domain and using known estimates for the Bergman kernel function. As a byproduct, we also obtain a weighted type estimate for the weighted composition operators which is adapted to Sawyer-testing conditions. This talk is based on joint work with Bingyang Hu. (Received September 20, 2021)

1174-32-9010 **Beth Branman** (branman@wisc.edu), University of Wisconsin-Madison. *Critical Points at Infinity using Toric Varieties II* Preliminary report.

Building on Gillen's talk, we suggest a new definition for critical points at infinity (CPAI) that is adapted to the toric compactification. We show that our definition of CPAI is equivalent to that of Baryshnikov, Melczer and Pemantle, and discuss the theoretical and computational merits of the new definition. (Received September 20, 2021)

1174-32-9192 **Yang Hong*** (yh013@bucknell.edu), Bucknell University. *Graphs, Adjacency Matrices and Stable Polynomials*

Our research concerns the interplay of undirected graphs and stable polynomials. Stable polynomials, which are polynomials with restricted zero sets, are used in a variety of mathematical fields. Here, a stable polynomial p is defined as a two-variable polynomial that satisfies $p(z_1, z_2) \neq 0$ if $(z_1, z_2) \in \mathbb{D}^2$, where $\mathbb{D}^2 = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|, |z_2| < 1\}$. In this research, we use adjacency matrices of undirected graphs to construct stable polynomials and investigate the relationships between the shapes of the graphs and the zeros of the polynomials. In a variety of situations, we establish the existence and location of polynomial zeros on the boundary of $\mathbb{D}^2$ and characterize how the zero set of a stable polynomial could approach those boundary zeros. We also pose and examine conjectures about more generalized and complicated cases. Using our results, one can build polynomials with specific boundary zeros and identify the boundary zeros implied by given polynomial properties. (Received September 20, 2021)

1174-32-9378 **Peter Ebenfelt** (pebenfel@math.ucsd.edu), University of California, San Diego. *The local CR embedding problem* Preliminary report.

We will discuss recent work (in progress) on the local embedding problem for strictly pseudoconvex CR manifolds. (Received September 20, 2021)

1174-32-9447 **Chloe Urbanski Wawrzyniak*** (ChloeUWawrzyniak@uky.edu), University of Kentucky. *The Hull(s) of an n-sphere in $\mathbb{C}^n$*

This talk presents a result on the (global) Bishop problem for small perturbations of the unit sphere of $\mathbb{C} \times \mathbb{R}^{n-1}$ in $\mathbb{C}^n$. We show that if $S \subset \mathbb{C}^n$ is a sufficiently-small perturbation of this sphere (in the C^3-norm), then S bounds an $(n+1)$-dimensional ball $M \subset \mathbb{C}^n$ that is foliated by analytic disks attached to S. Furthermore, if S is either smooth or real analytic, then so is M (up to its boundary). Finally, if S is real analytic (and satisfies a mild condition), then M is both the envelope of holomorphy and the polynomially convex hull of S. This generalizes the previously known case of $n = 2$ (CR singularities are isolated) to higher dimensions (CR singularities are nonisolated). This work is from a joint paper with Purvi Gupta. (Received September 20, 2021)

1174-32-9974 **Eric Nathan Stucky*** (enstucky@gmail.com), Univerzita Karlova, and **Sam Simon** (ssimon@sfu.ca), Simon Fraser University. *Asymptotics of Walks in Weyl Chambres* Preliminary report.

Many classical enumeration problems concern walks of a given length, constrained to lie in a quarter-plane and use only a specified collection of steps. Unfortunately, even relatively simple collections often give rise to unwieldy exact formulas, and so asymptotic results are often more desirable. Historically, many step sets have been analyzed in this way, and recently attempts have been made to unify these results.

Following a broad framework proposed by Melczer and Mishna, we consider collections of steps which are invariant under the action of a finite reflection group. In this talk we discuss our recent work on weighted versions of these models, including determining the asymptotics for all such models in two dimensions, and progress toward their higher-dimensional cousins. (Received September 21, 2021)

1174-32-10335 **Joel Coacalle** (jopcrat@gmail.com), Universidad Nacional de San Antonio Abad del Cusco. *Compactness of the complex Green operator on non-pseudoconvex CR manifolds*

In this talk, I will discuss the compactness theory of the complex Green operator not necessarily pseudoconvex CR manifolds of hypersurface type. I will motivate the sufficient geometric and potential theoretic conditions that we require and provide the first non-pseudoconvex example (and a level q) for which the complex Green operator is compact. The results are joint with Joel Coacalle, currently at Universidad Nacional de San Antonio Abad del Cusco. (Received September 21, 2021)

1174-32-10436 **Max Reinhold Jahnke*** (max.jahnke@gmail.com), Federal University of São Carlos. *Levi-flat CR structures on compact Lie groups*

In this talk, I will discuss how to compute the cohomology of a left-invariant CR structure on a compact Lie group. This is a generalization of a result of Pittie [CK04].

Pittie proved that the Dolbeault cohomology of all left-invariant complex structures on compact Lie groups can be computed by looking at the Dolbeault cohomology induced on a conveniently chosen maximal torus. We use the algebraic classification of left-invariant CR structures of maximal rank on compact Lie groups [Pit88] to generalize Pittie's result to left-invariant Levi-flat CR structures of maximal rank on compact Lie groups.

This is a joint work with Howard Jacobowitz from Rutgers University.

[CK04] J-Y. Charbonnel and H. O. Khalgui. Classification des structures CR invariantes pour les groupes de Lie compacts. J. Lie Theory.

[Pit88] H. V. Pittie. The Dolbeault-cohomology ring of a compact, even-dimensional Lie group. Proc. Indian Acad. Sci. Math. Sci.

(Received September 21, 2021)

1174-32-10497 **Bingyuan Liu*** (bingyuan.liu@utrgv.edu), The University of Texas Rio Grande Valley. *On the global regularity of $\bar{\partial}$-Neumann problem*

The $\bar{\partial}$-Neumann problem can be dated back to the 1960s. Among others, the problem on global regularity is the most challenging open question. The global regularity is, roughly speaking, given a data smooth up to boundary, the solution is also smooth up to boundary.

In this talk, we will give a brief review on this fascinate topic and introduce some recent progress. (Received September 21, 2021)

1174-32-10897 **Song Ying Li*** (sli@math.uci.edu), University of California, Irvine. *Hrmonic Functions and Harmonic Mappings in Bergman metrics* Preliminary report.

In this talk, I will present some joint work with R-Y. Chen on the harmonic functions in Bergman metric in a classical bounded symmetric domains. I will also present some joint work with R-Y. Chen and J. Luo on the Asymptotic expansion on proper harmonic maps between the unit balls with Bergman metrics. (Received September 21, 2021)

33 ► Special functions (33-XX deals with the properties of functions as functions)

1174-33-5709 **James G. Mc Laughlin*** (jmclaughlin2@wcupa.edu), West Chester University. *m-Dissections of some infinite products and related identities*

For integers t and m with $\gcd(m,6)=1$, $1 \le tn_0$, the signs of the c_n are periodic with period m. In addition, a formula is given for a polynomial $f_{t,m}(q)$ (deriving from the m-dissection of $Q(t,m)$) from which a second polynomial $g_{t,m}(q)$ of degree $m-1$ is derived, such that the coefficient of q^d (either 1 or -1) in $g_{t,m}(q)$ indicates the sign of all the coefficients in the arithmetic progression c_{mk+d}, for $0 \le d \le m-1$ and $mk+d > n_0$.

By using methods like those used to prove the results above, other similar results are proved. We re-derive the result of Evans and Ramanathan giving the m-dissection of $(q;q)_\infty$ in terms of quintuple products for any integer $m \equiv \pm 1 \pmod 6$.

Other results include various Lambert series identities, such as the following. Define

$$A = A(q) := \frac{(q^2, q^5; q^7)_\infty}{(q, q^6; q^7)_\infty},\ B = B(q) := \frac{(q^3, q^4; q^7)_\infty}{(q^2, q^5; q^7)_\infty}.$$

Then

$$1 + 14\sum_{n=1}^{\infty} \frac{q^{3n}}{1-q^{7n}} - 14\sum_{n=1}^{\infty} \frac{q^{4n}}{1-q^{7n}} = \left(A^3 + \frac{3Bq}{A} - \frac{6q}{B}\right) \frac{f_7^2}{(q^3, q^4; q^7)_\infty},$$

where $f_i := (q^i; q^i)_\infty$. (Received August 25, 2021)

1174-33-7498 **Michael Allen*** (allenm3@math.oregonstate.edu), Oregon State University. *On supercongruence conjectures of Long*

In 2003, Rodriguez Villegas conjectured 14 supercongruences between hypergeometric functions arising as periods of certain families of rigid Calabi-Yau threefolds and the Fourier coefficients of weight 4 modular forms. Uniform proofs of these supercongruences were given in 2019 by Long, Tu, Yui, and Zudilin, and in 2020 Long conjectured a number of further such supercongruences for hypergeometric functions of a similar shape. In this talk, we generalize the approach of Long, Tu, Yui, and Zudilin to establish six of Long's conjectured supercongruences. (Received September 14, 2021)

1174-33-8523 **Tom Cuchta*** (tcuchta@fairmontstate.edu), Fairmont State University. *Discrete matrix hypergeometric functions*

We present the matrix-parameter generalization of discrete hypergeometric series. Their properties will be presented, including their connection to continuous matrix hypergeometric series, divergence criteria, difference equations, and integral representations. Some open questions about these functions will be presented at the end. (Received September 19, 2021)

1174-33-9373 **Jonathan Gabriel Bradley-Thrush*** (j.bradleythrush@ufl.edu), University of Florida. *Telescoping q-series* Preliminary report.

One of the simplest instances of a telescoping q-series is the expansion of the generating function for integer partitions in the form $1 + \sum_{n=1}^{\infty} q^n/(q)_n$, where $(q)_n = (1-q)(1-q^2)\ldots(1-q^n)$. Motivated by this example, I will consider other q-series which can be expressed as infinite products in a similar way. In particular, it will be shown that the q-binomial theorem and the q-Gauss identity can both be established by means of telescoping series. Some formulae related to the theory of plane partitions will also be described. (Received September 21, 2021)

1174-33-11124 **Vaishavi Sharma** (vsharma1@tulane.edu), Tulane University, **Victor H Moll** (vhm@tulane.edu), Tulane University, **Blaine Larson*** (blainelarson@uvic.ca), Department of Mathematics, University of Victoria, and **Qiusu "emily" Miao** (qmiao@berkeley.edu), Department of Mathematics, University of California at Berkeley. *A selection of integrals in Gradshteyn and Ryzhik. Part 1*

The classical table of integrals by Gradshteyn and Ryzhik is one of the most used by the scientific community. As part of a global effort to validate and verify the entries in it, we have developed some new techniques. A variety of examples will be presented. (Received September 21, 2021)

34 ► Ordinary differential equations

1174-34-5210 **Gaston Mandata N'Guerekata*** (Gaston.NGuerekata@morgan.edu), Morgan State University. *An invitation to periodicity.*

Periodicity is everywhere, every day. Considering some periodic phenomena, we will revisit the mathematical concept of periodicity and its recent generalizations up to almost automorphy. We will study their applications to some differential equations. An elementary proof of the celebrated Massera Theorem will be presented. We will also show that an almost periodic second order semilinear elliptic equation may not have almost periodic solutions, but many almost automorphic solutions in the envelop of the equation. An application to almost periodically forced pendulum will be given. (Received November 16, 2021)

1174-34-5423 **Viktoria Savatorova*** (vsavatorova@gmail.com), Central Connecticut State University.
Modeling nonlinear oscillations in the plane: from Van der Pol oscillator to mathematics of heartbeat and nerve impulse transmission

The generalizations of the Van der Pol and Lienard equations are used to model the heartbeat and nerve impulse transmission. In the case of the heart, we use Zeeman's model to explain how some complexities of the beat can be deduced from the behavior of the muscle fibre. In case of the nerve impulse, we consider the FirzHugh-Nagumo system to model the neuronal action potential. Students prove the existence and uniqueness of the fixed point of the system and the existence of a stable limit cycle within a certain range of values of the governing parameter of the nonlinear system of ODEs. They use software to plot the vector field and nullclines for the model. Analytical reasoning and numerical computations are used to show that, starting from some value of the governing parameter, the fix point becomes unstable, and a stable limit cycle arises because of supercritical Hopf bifurcation. Students are asked to vary the values of the governing parameter and the initial conditions and provide an interpretation for each of the cases. (Received August 20, 2021)

1174-34-5465 **Gang George Yin** (gyin@uconn.edu), University of Connecticut, and **Nhu N. Nguyen*** (nguyen.nhu@uconn.edu), University of Connecticut. *Biological and ecological models under stochastic perturbation, past dependent and spatial inhomogeneity: Modeling and longtime characterization*

The dynamics of many models in Biology and Ecology such as: epidemic models, tumor-immune models, chemostat models, prey-predator models, competitive models, and among others can be mathematically described. The earliest and simplest mathematical models are given by ordinary differential equations (ODE). Long-standing and important questions in mathematical biology are that: How is the long-time behavior of the system? Does one group of populations come extinct or persistent? Under which condition, the disease will be controlled in the epidemic systems?. In longtime, which species dominates the others? and among others. This talk focuses on modeling these above biological and ecological systems and answering such problems when the random factors (leading to stochastic system), past-dependence (leading to delay system), spatial inhomogeneity (leading to reaction-diffusion models) are taken into consideration, which are described under stochastic differential equations (SDEs), stochastic functional differential equations (SFDEs) and stochastic partial differential equations (SPDEs) framework. (Received August 20, 2021)

1174-34-5824 **Toka Diagana*** (tokadiag@gmail.com), University of Alabama in Huntsville.
Well-posedness and stability results for some non-autonomous abstract linear hyperbolic equations Preliminary report.

In this talk we study a class of second-order abstract linear hyperbolic equations with infinite memory which involves time-dependent unbounded linear operators. Under some suitable assumptions, we obtain the well-posedness and the stability of solutions to those nonautonomous second-order evolution equations. Our results generalize numerous known results in the autonomous case. To illustrate our abstract results, some specific examples are given including the nonautonomous Petrovsky type and wave equations. (Received August 29, 2021)

1174-34-5826 **Romario Gildas Foko Tiomela*** (rofok1@morgan.edu), Morgan State University.
(ω, c)-asymptotically periodic solutions to some fractional integro-differential equation Preliminary report.

In this paper, we establish a new composition theorem for (ω, c)-asymptotically periodic functions. Then, we use the Banach contraction principle to investigate the existence and uniqueness of (ω, c)-asymptotically periodic mild solutions to the fractional integro-differential equation $u'(t) = \dfrac{1}{\Gamma(\alpha-1)}\displaystyle\int_0^t (t-\tau)^{\alpha-2}Au(\tau)d\tau + F(t, u_t),\ t \geq 0$ and $u_0 = \phi \in \mathcal{B}(\mathbb{X})$, where $\mathcal{B}(\mathbb{X})$ is a linear space of functions defined from $(-\infty, 0] \longrightarrow \mathbb{X}$ and A is a closed but not necessarily bounded linear operator of sectorial type $\varpi < 0$. (Received August 29, 2021)

1174-34-5846 **Hamid Mofidi*** (hamid-mofidi@uiowa.edu), University of Iowa. *Local Hard-Sphere Poisson-Nernst-Planck Models: Reversal Potential and Zero-Current Fluxes*

This talk investigates reversal potential and examines ion size effects on the flow rate of matter through a cross-section by treating the ion sizes as small parameters. We consider a one-dimensional version of a Poisson-Nernst-Planck-type system with a local hard-sphere potential model for ionic flow through a membrane channel with fixed boundary ion concentrations (charges) and electric potentials. The research aims to set up a simple structure defined by permanent charges with two mobile ion species. A local hard-sphere potential that depends pointwise on ion concentrations is incorporated in the model to evaluate ion-size influences on the ionic flow. The analysis of our BVP is based on the geometric singular perturbation theory. (Received August 29, 2021)

1174-34-6120 **James Larrouy*** (james.larrouy@univ-antilles.fr), Université des Antilles, LAMIA.
On asymptotically (ω, c)-periodic mild solutions to fractional Cauchy problems with delay.

In this work we establish some new properties of a new class of functions called "asymptotically (ω, c)-periodic". Then we apply them to study the existence and uniqueness of mild solutions of this type to the following semilinear fractional differential equation: ${}^cD_t^\alpha u(t) = Au(t) +^c D_t^{\alpha-1} f(t, u(t-h))$ with $u(0) = 0$ and where $1 < \alpha < 2$ and $t, h \in \mathbb{R}_+$. Here, ${}^cD_t^\alpha(\cdot)$ $(1 < \alpha < 2)$ stands for the Caputo derivative and A is a linear densely defined operator of sectorial type on a complex Banach space $\mathbb{X}$ and the function $f(t, x)$ is asymptotically (ω, c)-periodic with respect to the first variable. Our results are obtained using the Leray-Schauder alternative theorem and the Banach fixed point principle. In addition, we give an Holder-type condition in order to ensure the existence of an asymptotically (ω, c)-periodic mild solution to previous Equation and we illustrate our main results with an application to fractional diffusion-wave equations. (Received September 4, 2021)

1174-34-6842 **Sunaina Butler*** (butlersu@grinnell.edu), Grinnell College. *Mathematical Model of Algal Bloom with Time Delay* Preliminary report.

Algal blooms have become a major threat to natural environments, and hence, predicting and monitoring them are essential for mitigating the damage they may cause. Eutrophication via surface runoff from agricultural lands greatly affects the increase in density of algae. The growth of algae can be controlled by methods that have a direct impact on algae and by reducing the nutrients in water, which ultimately results in a decrease in the density of algae. In recent years, some mathematical models have been proposed to study the effects of various control measures on the reduction in algal blooms in lakes. We propose and analyze a mathematical model using odeint and ddeint MATLAB with time delay for nutrient density, algae density, and budget available to assess the effect of applying efforts for removal of algae and detritus from a lake on the occurrence of algal bloom. We found that if nutrients and budget are constant, we can meet a certain condition based on rates of algae growth, death, and removal to send algae to zero. However, the parameter nutrient input rate had the most effect on the system overall. We also found the conditions and delay for which the system becomes unstable and oscillates. To expand to further work, we intend to train a neural network with the data available on NOAA website to predict required chlorophyll-a concentrations in a system (measure for algae density). (Received September 9, 2021)

1174-34-6856 **Thomas J Clark*** (Tom.Clark@dordt.edu), Dordt University. *Draining Tanks: Modeling, Data, and Separable Equations* Preliminary report.

Can you describe the height over time of a draining column of water? Students encounter modeling with an in-class demonstration of draining a cylindrical tank. Students gather data and apply Torricelli's law to derive a model to experience firsthand how mathematical models can be employed to investigate and understand not just abstract/theoretical situations, but real physical problems. The model returns later as an example problem motivating solving separable equations. Students then tackle a group project wherein they use geometric thinking to upgrade their model to a conical tank. Real collected data allows for parameter fitting; both explicit and implicit solutions arise when solving the model. (Received September 9, 2021)

1174-34-7063 **Blessing Emerenini*** (mailstoblessing@gmail.com), Rochester Institute of Technology.
Role of environmental and demographic factors in the superspreading of COVID19 Preliminary report.

Superspreading of infectious disease is an event in which an infectious disease is spread much more than usual. Key role player in superspreading event is the superspreader. COVID-19 is one of the infectious diseases that are potentially driven by superspreaders. This infection can be spread by asymptomatic and symptomatic carriers. The success of the spread of COVID19 is also linked to certain environmental and demographic factors. The airports, health care centers, shopping malls and social gathering spots are hot spots for supersreading. Better understanding of the transmission dynamics associated with superspreading events, identifying and mitigating high risk settings, strict adherence to health care infection prevention and control measures can help prevent and control COVID19 as well as future infectious disease outbreaks. We develop deterministic models with optimal control theory to reveal underlying effects of these factors and optimal controls to mitigate disease prevalence.
(Received September 11, 2021)

1174-34-8143 **Christopher Boyette** (cboyette@elon.edu), Elon University. *Using a network model to control the spread of an infectious disease on a college campus with contact tracing*

The control of infectious diseases has been a topic of recent discussion due to the emergence of COVID-19. To study this issue, we use an SIR-type model on a dynamic network to analyze the effect of contact tracing, quarantining, and asymptomatic testing on disease spread. Our model emulates a college campus environment with

a special emphasis on interactions between cliques, mirroring the dormitory and extracurricular environment. This study mimics the measures that were utilized by our own campuses to contain the transmission of the virus in a close living situation. We also examine different types of mandatory COVID-19 testing and identify which are the most efficient at determining the accurate number of infected individuals. Lastly, we analyze our findings using Monte Carlo simulations to determine effectiveness of different control measures in a randomized setting. (Received September 19, 2021)

1174-34-8240 **Roshini Samanthi Gallage*** (roshi@siu.edu), Southern Illinois University, Carbondale, and **Harry Randolph Hughes** (hrhughes@siu.edu), Southern Illinois University, Carbondale. *Analysis and Numerical Approximation of Nonlinear Stochastic Differential Equations with Continuously Distributed Delay* Preliminary report.

There are many nonlinear stochastic delay differential equations where a linear growth condition is not satisfied, for example, the stochastic delay Lotka-Volterra model of food chain [X. Mao and M.J. Rassias, 2005]. We prove the existence of a unique solution of certain nonlinear stochastic differential delay equations (SDDEs) with continuously distributed delay under local Lipschitz and generalized Khasminskii-type conditions. Further, we show that Euler-Maruyama numerical approximations of such nonlinear SDDEs converge in probability to their exact solutions. (Received September 19, 2021)

1174-34-8355 **Yulong Li*** (yulongl@unr.edu), University of Nevada, Reno. *On the regularity and simplicity of a class of fractional elliptic operators*

This work studies the spectral problem of a class of fractional elliptic operators

$$\begin{cases} -\alpha D_a D_x^{-(1-\mu)} Du - \beta D_x D_b^{-(1-\mu)} Du = \lambda u, x \in (a,b) \\ u(a) = u(b) = 0, \\ 0 < \mu < 1, 0 < \alpha, \beta < 1, \alpha + \beta = 1. \end{cases}$$

With regularity results, we mainly prove that each eigenvalue $\lambda \in \mathbb{C}$ must have $\Re(\lambda) \neq 0$ and if $\Im(\lambda) = 0$, then it must be simple. In particular, we point out that, when $\alpha = \beta = \frac{1}{2}$, the problem is equivalent to the spectral problem of 1-D fractional Laplacian, and with this connection, we provide a promising approach to link the way of calculating analytic eigenfunctions of the 1-D fractional Laplacian to a new type of singular integral equation.
(Received September 18, 2021)

1174-34-8658 **Govinda Pageni*** (govinda.pageni@louisiana.edu), University of Louisiana at Lafayette, and **Aghalaya Vatsala** (vatsala@louisiana.edu), University of Louisiana at Lafayette. *Laplace transform Method to solve Three system of Caputo Fractional Differential Equations with Application to SIR Models* Preliminary report.

In this work, we have used Laplace transform method to solve three systems of Caputo fractional differential equations of order ‘q’ with 0 ¡ q ¡1, with initial conditions. This will be useful to study the asymptotic stability of the Caputo fractional SIR and SIRS model relative to infectious diseases. We have also extended the Laplace transform method to solve the nonlinear SIR model using recurrence relations relative to the nonlinear terms. We have presented some numerical and graphical solutions of the infectious disease model. (Received September 19, 2021)

1174-34-8744 **Zhilan Feng*** (zfeng@nsf.gov), National Science Foundation, and **John Glasser** (jwg3@cdc.gov), The US Centers for Disease Control and Prevention. *Assessing the burden of congenital rubella syndrome in China and evaluating mitigation strategies: a metapopulation modelling study*

Abstract: A rubella vaccine was licensed in China in 1993 and added to the Expanded Programme on Immunization in 2008, but a national cross-sectional serological survey during 2014 indicates that many adolescents remain susceptible. Using an age-structured population model we aimed to evaluate possible supplemental immunisation activities (SIAs) to accelerate elimination of rubella and congenital rubella syndrome. Our findings suggest that SIAs among adolescents would most effectively reduce congenital rubella syndrome as well as eliminate rubella, owing both to fewer infections in the immunised population and absence of infections that those immunised would otherwise have caused. Metapopulation models with realistic mixing are uniquely capable of assessing such indirect effects. This talk is based on the following paper: Su Q, Feng Z, Hao L, Ma C, Hagan JE, Grant GB, Wen N, Fan C, Yang H, Rodewald LE,Wang H, Glasser JW, Assessing the burden of Congenital Rubella Syndrome in China and evaluating mitigation strategies: a meta-population modeling study, The Lancet ID, https://doi.org/10.1016/S1473- 3099(20)30475-8, 2021. (Received September 19, 2021)

1174-34-8815 **Rosa Flores** (D00351877@dmail.dixie.edu), Dixie State University, **Vinodh Kumar Chellamuthu*** (Vinodh.Chellamuthu@dixie.edu), Dixie State University, and **Christian Riordan** (christian.riordan1@dmail.dixie.edu), Dixie State University. *Assessing the Efficiency of Predator-Prey Control Strategies in the Persistence of Dengue with Wolbachia Transinfection* Preliminary report.

Dengue is a mosquito-borne viral infection found in tropical/subtropical regions causing millions of infections each year. Dengue is caused by any of the four serotypes of the dengue virus. Several studies have shown that secondary infections have increased morbidity rates. Dengue is mainly transmitted by the Aedes aegypti mosquitoes (AEM). The transinfection of AEM with the bacterium Wolbachia pipientis has emerged as a "successful" method to reduce dengue virus transmission while minimizing environmental harm. Despite this control method in place the persistence of dengue virus is continuing to infect individuals worldwide. Studies indicate that incorporating a natural predator of the AE larvae such as the Toxorhynchites Splendens (TxS) larvae can help control the population of the AEM and aid in the control of Dengue. We have developed a mathematical model by incorporating both primary and secondary infection of two serotypes of the virus and utilizing two control strategies: Wolbachia-carrying mosquitoes and the Introduction of TxS larvae. Furthermore, our simulations show intervention strategies that might be useful in controlling Dengue based on the temperature dependency. (Received September 19, 2021)

1174-34-9032 **Timothy Lucas*** (timothy.lucas@pepperdine.edu), Pepperdine University, and **Krista L Lucas** (krista.lucas@pepperdine.edu), Pepperdine University. *Using Mobile Apps to Enhance Learning in Differential Equations*

In a differential equations course it is crucial for mathematics students to engage in active learning along with discussion of the material with their peers. A key for these students to understand mathematical models that incorporate differential equations is visualizing slopefields, phase planes and solutions. *Slopes* is a mobile application with an intuitive interface that is designed to visualize solutions to differential equations and support active learning in the classroom. *Slopes* is currently available for iPads, iPhones, and Android phones, which are highly portable and feature larger touch screens that allow students to view and manipulate content easily. To study the possible benefits of the app, we implemented group activities using *Slopes* into an ordinary differential equations class, conducted observations and focus groups, and examined final poster projects on modeling topics. We found that students used *Slopes* to visualize solutions, aid in discussion and cooperation, and demonstrate understanding of differential equations concepts. (Received September 20, 2021)

1174-34-9309 **Minato Hiraoka*** (hiram19@wfu.edu), Wake Forest University, **Malindi Whyte** (whytmg19@wfu.edu), Wake Forest University, and **Danielle DaSilva** (ddasilva@elon.edu), Elon University. *Optimal intervention strategies to minimize spread of infectious diseases and economic impact on a dynamic small-world network* Preliminary report.

We investigate the economic impacts of pharmaceutical and non-pharmaceutical interventions on a labor force during an epidemic. Specifically, we study an optimal control problem on a dynamic SIV-type small world network model with the controls corresponding to vaccinations, lockdown orders, and other intervention strategies. The cost functional utilized is a Cobb-Douglas production function measuring labor productivity as well as a functional measuring the cost of treating the disease. Using Pontryagin's maximum principle we can numerically approximate the optimal control strategy which allows us to determine the optimal level of intervention. These methods illustrate the usefulness of this approach to inform policymakers and better equip society for emerging epidemics. (Received September 20, 2021)

1174-34-9321 **Chris Ahrendt*** (ahrendcr@uwec.edu), University of Wisconsin-Eau Claire. *Exploration of Solutions to a Discrete Analog of Clairaut's Equation on Time Scales* Preliminary report.

Using the framework of the time scale calculus, we focus on a discrete analog of the classic Clairaut differential equation. We briefly describe the derivation of the discrete equation. We then explore a specific example of the Clairaut equation to compare and contrast with the well known behavior of the solutions of the classic Clairaut differential equation. In particular, we examine the general solution and a singular solution using phase portraits and a strategic substitution.

The time scale calculus generalizes and unifies differential and difference equations, but also is a source of many interesting results that do not have a direct analog to these classic cases. A brief introduction to the key results of the time scale calculus that are used in this work will be provided. (Received September 20, 2021)

1174-34-9388 **Scott C. Gensler*** (scott.gensler@gmail.com), University of Nebraska-Kearney. *Higher Order Differences on Arbitrary Discrete Time Scales*

In his thesis in 1988 Stephan Hilger introduced the notion of a derivative on a time scale $\mathbb{T}$, where $\mathbb{T}$ is simply a closed subset of $\mathbb{R}$. Hilger's derivative is a generalization that reduces to the usual derivative for a function on a closed interval and to the forward difference for a function on $\mathbb{Z}$. During this talk we look at discrete but otherwise *arbitrary time scales* and identify some: recurrence relations, explicit formulas, and even generating functions that describe the coefficients one needs to calculate the 1st, 2nd, 3rd, . . ., nth Hilger derivative of a function on such a time scale. As a check on the theory we show that, as expected, in the special case where the time scale under consideration is either the well-studied q-calculus (quantum calculus) or h-calculus the general formulas developed for arbitrary discrete time scales simplify to formulas that are well-known. (Received September 20, 2021)

1174-34-9476 **Gisele Adelie Mophou*** (gisele.mophou@univ-antilles.fr), Universite des Antilles en Guadeloupe. *An existence result of (ω,c)-periodic mild solutions to some fractional differential equation* Preliminary report.

We first investigate in this paper further properties of the new concept of (ω,c)-periodic functions; then we apply the results to study the existence of (ω,c)-periodic mild solutions of the fractional differential equations

$$D_t^{\alpha}(u(t) - F_1(t, u(t))) = A(u(t) - F_1(t, u(t))) + D_t^{\alpha-1}F_2(t, u(t)),\ t \in \mathbb{R},$$

where $1 < \alpha < 2$; $A : D(A) \subset \mathbb{X} \to \mathbb{X}$ Banach space $\mathbb{X}$, F_1, $F_2 : \mathbb{R} \times \mathbb{X} \to \mathbb{X}$ are two (ω,c)-periodic functions satisfying suitable conditions in the second variable. The fractional derivative is understood in the sense of Riemann-Liouville. (Received September 20, 2021)

1174-34-9742 **Roger Thelwell*** (thelwerj@jmu.edu), James Madison University. *Stability via computed Taylor series* Preliminary report.

We exploit the power of an iterative symbolic computation to efficiently compute the Lyapunov spectrum of highly nonlinear systems of ODEs. The technique constructs a truncated Taylor series representation (a jet of degree k) at a generic point of the separation of the flow. An approximate spectrum is then recovered and analyzed. Several examples will be discussed. The methods and theory are accessible to undergraduate students at the end of a first course in ODE, and the technique is a useful extension of the powerful series based iterative solution methods. (Received September 20, 2021)

1174-34-9823 **Masilamani Sambandham*** (msambandham@yahoo.com), Morehouse College. *Approximate solutions to Ordinary Fractional Differential Equations* Preliminary report.

By an application of various numerical and analytical methods we develop approximate solutions to Ordinary Fractional Differential Equations. Based on the numerical solutions we discuss and compare the methods used. (Received September 20, 2021)

1174-34-10082 **Casey J Mills*** (casey.j.mills@ttu.edu), Texas Tech University, and **Raegan J Higgins** (raegan.higgins@ttu.edu), Texas Tech University. *Modeling Intermittent Androgen Deprivation Therapy using Fractional Calculus*

Aside from non-melanoma skin cancer, prostate cancer is the most common cancer among men in the United States. A common and effective treatment for patients with prostate cancer is intermittent androgen deprivation therapy. This therapy requires a patient to alternate between periods of androgen suppression treatment and no treatment. Androgen is any type of male reproductive hormone. Prostate cancer is typically estimated using prostate-specific antigen levels in men; these levels are used to track relative changes in tumor volume of prostate cancer patients undergoing intermittent androgen deprivation therapy. In this work, we explore the use of fractional calculus to model intermittent androgen deprivation therapy. Current theory in fractional calculus only allows for strictly continuous or strictly discrete domains (in particular the natural numbers shifted and/or scaled) to be used. We create a strictly discrete time scale and construct a dynamic equation on this time scale. We then develop theory that allows us to calculate the fractional difference of this dynamic equation. We model intermittent androgen deprivation therapy using this fractional difference and find that an improved fit is achieved for the majority of the patients tested. (Received September 21, 2021)

1174-34-10342 **Basiru Usman*** (busman@ncsu.edu), North Carolina State University. *A Hopfield neural lattice model is developed as the infinite dimensional extension of the classical finite dimensional Hopfield model*

A Hopfield neural lattice model is developed as the infinite dimensional extension of the classical finite dimensional Hopfield model. In addition, random external inputs are considered to incorporate environmental noise. The resulting random lattice dynamical system is first formulated as a random ordinary differential equation on the space of square summable bi- infinite sequences. Then the existence and uniqueness of solutions, as well as long term dynamics of solutions are investigated. (Received September 21, 2021)

1174-34-10435 **Calistus N Ngonghala*** (calistusnn@ufl.edu), University of Florida. *Integrating disease epidemiology with socio-economic and game-theoretic approaches: implications for disease control*

A dynamic model framework that incorporates socio-economic and game theoretic approaches into disease epidemiology is presented and analyzed in order to assess the impact of one control measure (social-distancing) on disease transmission. The model exhibits a variety of dynamical behaviors ranging from globally stable states through bistable states or sustained bounded periodic oscillations. Simulations of the model suggest that the disease under consideration can be contained if a sizable proportion of the population embrace strict social-distancing and that prospects of containing the disease are enhanced by the strength of the economy. (Received September 21, 2021)

1174-34-10453 **Kobra Rabiei*** (kr1667@msstate.edu), Mississippi State University, and **Mohsen Razzaghi** (razzaghi@math.msstate.edu), Mississippi State University, Prof.. *A new numerical method for solving problems in fractional differential equations by using Mott wavelets.* Preliminary report.

In this paper, a novel numerical method is suggested for solving the fractional pantograph delay differential equations. An exact formula of the Riemann-Liouville fractional integral operator (RLFIO) of the Mott wavelets is given by using the regularized beta function. We have solved some examples and compared the numerical findings with either the exact solutions or the existing results. (Received September 21, 2021)

1174-34-10602 **Sundar Tamang*** (sundar.tamang@wnmu.edu), UAB PhD Thesis. *A Model For Currency Exchange Rates*

Catastrophic economic events like the oil price shock in 1973, the 9/11 event in 2001, the stock market crash in October, 1987, the financial crash of late 2008 and the Covid-19 epidemic impact the US economy without warning, with varying effect, from sharp downturns to actual market crashes. Current economic theory and the associated statistical models are not able to predict these events. The development of predictive models for the US economy needs at the very least prediction models for its major components: commodities, stocks, bonds and currencies. I am going to model the currency exchange rates using a stochastic differential equation model by predicting the exchange rate trend using a system of delay differential equations, and recovering the future volatility of exchange rates by solving an inverse problem obtained from the Garman-Kohlhagen partial differential equation for foreign exchange options. (Received September 21, 2021)

1174-34-10610 **Douglas B Meade*** (meade@math.sc.edu), University of South Carolina. *Approximate Solutions for Differential Equations: Past, Present, and Future*

Students, hopefully, quickly learn that most differential equations cannot be solved analytically. While graphical representations of solutions are often possible, this talk will focus on approximate solutions. For many students, this mean “Euler’s method” (or another numerical method). But, for this talk, we will focus on other approximate solution methods: linearization, Picard iterates, and series solutions. Each of these ideas is very old, but has fallen somewhat out of fashion as new technologies have developed more powerful numerical and graphical tools. The presentation will attempt to demonstrate how modern technologies make these classical tools more effective for the future. (Received September 21, 2021)

1174-34-10691 **Seddik M Djouadi** (mdjouadi@utk.edu), University of Tennessee, **Fatima Z Taousser*** (taoussfatima@gmail.com), University of Tennessee, and **Kevin Tomsovic** (ktomsovi@tennessee.edu), University of Tennessee. *A Dwell Time Approach for the Stabilization of Continuous/Discrete Switched Systems With Application to Consensus for Multi-agent Systems With Intermittent Information Transmission*

A wide range of physical and engineering systems involve coupling between continuous dynamics and discrete events called switched systems. The stability issue is the main concern in the study of such systems. A switched system can be stabilized under an appropriate switching law called the dwell-time, such that the duration between any consecutive switching have to be no smaller than a constant $\tau > 0$. This concept has been studied for continuous-time switched systems and discrete-time switched systems separately. However, in several areas of engineering applications, there are many situations when the switched system is composed of continuous-time and discrete-time subsystems simultaneously, as modeling of intermittent hormone therapy for prostate cancer. In this situation the time domain is neither continuous nor uniformly discrete. To study such complex systems, we will introduce, in this work, the time scales theory to derive a new stabilizing dwell time conditions for a class of switched systems between continuous-time subsystems with variable time interval length and discrete-time subsystems with variable discrete steps-size. Notice that, when the discrete step size is variable in time, the stability of the discrete-time subsystem depends strongly on the size of the discrete steps. In this case, the existing results of dwell time approaches cannot be applied. The consensus problem for multi-agent systems with intermittent information transmission is studied to show the result. (Received September 21, 2021)

1174-34-10882 **Catherine Elizabeth Cavagnaro*** (ccavagna@sewanee.edu), Sewanee: The University of the South. *Modeling Aircraft Takeoffs* Preliminary report.

Aviation offers a treasure trove of opportunities for mathematical applications. We consider a general aviation aircraft on its takeoff roll and construct a model for this motion. Such a model can provide thumb rules that a pilot may employ to determine the distance along the runway at which she can, assuming insufficient aircraft performance, still safely abort the takeoff and stop on the remaining runway. (Received September 21, 2021)

1174-34-10905 **Christopher S Goodrich*** (c.goodrich@unsw.edu.au), University of New South Wales. *A Topological Approach to Kirchhoff-type Differential Equations with Convolution Coefficients*

I will consider nonlocal problems both of the form

$$-A\Big(\big(b*(g\circ u)\big)(1)\Big)u''(t)=\lambda f\big(t,u(t)\big),\ t\in(0,1)$$

and of the form

$$-A\big((b*(u')^q)(1)\big)u''(t)=\lambda f\big(t,u(t)\big),\ t\in(0,1),$$

where $*$ denotes a finite convolution, A, b, and g are given functions, and both $q\geq 1$ and $\lambda>0$ are constants. By means of a nonstandard cone, together with a specially tailored open set, I will demonstrate the existence of at least one positive solution to these types of problems under given boundary conditions. It will be shown that this approach can improve existing results, which rely on a more standard cone. (Received September 21, 2021)

1174-34-10915 **Feng Tao** (tfeng.math@gmail.com), School of Mathematical Science, Yangzhou University, and **Zhipeng Qiu** (nustqzp@njust.edu.cn), Nanjing University of Science and Technology. *Recruitment Dynamics of Social Insect Colonies*

Recruitment plays a vital role in the ecological and evolutionary successes of social insect colonies. In this paper, we formulate a four-compartment model and its simplified version to explore how we should model the recruitment dynamics of workers in social insect colonies properly. Our four-compartment model has the components of the unalarmed patrollers, the alarmed patrollers, the alarmed recruiters, and the available workers, while its simplified version has three components where we combine the unalarmed patrollers and the alarmed patrollers into the patrollers. We perform complete mathematical and bifurcation analyses on both the full system and its simplified system. We have many interesting findings, including that (i) the simplified three-compartment system has only simple equilibrium dynamics, i.e., no periodic and chaotic dynamics; (ii) the four-compartment system has very complex dynamics; for example, it can have up to three subcritical Hopf bifurcations, two supercritical Hopf bifurcations, two limit point bifurcations, and a fold bifurcation of the limit cycle. Those important results provide theoretical guidance for modeling and studying recruitment dynamics of social insect colonies: It is critical to have proper compartments for biological systems as the number of compartments could lead to totally different dynamics, and hence affect policy-making. (Received September 21, 2021)

1174-34-10930 **Michelle L Ghrist*** (ghrist@gonzaga.edu), Gonzaga University. *Pivoting Towards Developing Student Writing in a Differential Equations Course*

As we pivoted to online assessments in Spring 2020, I observed colleagues attempting to securely proctor remote exams as well as a sharp uptick in academic integrity violations. I spent the summer pondering my assessments, ultimately choosing to reduce dependence on exams in an effort to find ways to do assessment that disincentivized cheating and focused attention on learning. As a result, I chose to seek a "writing enriched" core designation for my Differential Equations courses for Fall 2020 and Spring 2021.

Writing assignments were a major component of these courses, consisting of both informal writings as well as mini-projects. In the latter, students explored in-depth applications of differential equations and formally wrote up their results. In this talk, I discuss five original applied assignments in the areas of the environment, space, circuit design, and carnival attractions, the last of which was used successfully as a group modeling assignment (administered remotely) in place of a final exam.

I saw significant improvement in students' writing throughout both semesters, and many students' submissions reflected a deep level of understanding. I discuss my observations, to include some of my successes and struggles plus what I would change if I were to do this again. (Received September 21, 2021)

1174-34-11038 **Ami Radunskaya*** (ami.radunskaya@pomona.edu), Pomona College. *The immune response to cancer: a great way to teach differential equations by example.*

Studies show that students are most engaged and learn the most in mathematics courses when the topic is socially relevant. According to the American Cancer Society, over 600,000 people in the US died of cancer in 2020, approximately, 1,600 deaths each day. The good news is that this number has decreased by circa 30% in the past 10 years, partly due to advances in immunotherapy: the harnessing of an individual's immune system to mount a response against the tumor. In this talk I will present a relatively simple model of the tumor-immune response that can explain mysterious phenomena such as "creeping through", when the tumor seems to disappear but then reappears some time later, and an asynchronous response to chemotherapy. By exploring this model, students can learn fundamental techniques in differential equations, such as phase-plane analysis, bifurcation theory, numerical methods and data fitting.

This work is in collaboration with Prof. Lisette de Pillis from Harvey Mudd College, Claremont CA (Received September 21, 2021)

1174-34-11262 **Indika Rajapakse*** (indikar@umich.edu), University of Michigan. *Reprogramming On Demand*

In 2007, Shinya Yamanaka (Nobel Prize, 2011) changed the paradigm further by reprogramming human skin cells into embryonic stem cell-like cells using four transcription factors. This shifted our understanding of the genome. I will discuss how this understanding can guide us to general principles of how to reprogram normal and abnormal cells like cancer. We have been developing "Data-Guided Control" to investigate the form/function dynamics of the genome and to find intelligent reprogramming strategies. Our work highlights the advantages of bringing together techniques from mathematics, biology, and engineering. (Received September 22, 2021)

1174-34-11281 **Seshadev Padhi*** (ses_2312@yahoo.co.in), Birla Institute of Technology, Mesra, Ranchi-835215, India, **Alexander Domoshnitsky** (adom@ariel.ac.il), Ariel University, Ariel, Israel, and **Satyam Narayan Srivastava** (satyamsrivastava983@gmail.com), Ariel University, Ariel, Israel. *Vallee-Poussin Theorem for a Fractional Differential Equation with Deviating Arguments* Preliminary report.

In this paper, we prove the existence of solutions and their positivity to the fractional differential equation with delay

$$D^{\alpha}_{0+}x(t) + \sum_{i=1}^{m} q_i(t)x(t-\tau_i(t)) = f(t),\ t \in [0,1],$$

subject to the boundary condition

$$x(0) = x'(0) = \cdots = x^{(n-2)}(0) = 0, x'(1) = 0$$

where $n \geq 3$, $n-1 < \alpha \leq n$, $D^{\alpha}_{0+}x$ is the standard Riemann-Liouville fractional derivatives of x of order α, $f, q : [0,1] \to R$ are measurable essentially bounded continuous functions with f nonpositive and $q_i, i = 1, 2, \cdots, m$ are positive, and $x(\xi) = 0$ for $\xi < 0$. We obtain Vallee-Poussin type theorem for the considered fractional delay differential equation. The result are completely new in the literature of fractional differential equations. (Received September 22, 2021)

1174-34-12197 **Elamin Elbasha*** (elamin_elbasha@merck.com), Merck & Co., Inc.. *Mathematics and the quest for vaccine-induced herd immunity threshold.*

Mathematics plays a major role in providing realistic insights into the spread and control of infectious diseases, dating back to the pioneering works of the likes of Daniel Bernoulli (on smallpox immunization modeling) in the 1870s. For example, mathematics provides answers to pertinent questions relating to the control and mitigation of vaccine-preventable diseases such as: what is the minimum fraction of the unvaccinated susceptible population that needs to be vaccinated to achieve disease elimination (in a local setting) or end pandemics (globally)? This minimum fraction is called herd immunity threshold. In this talk, I will discuss the mathematical theories and modeling methodologies associated with the derivation of vaccine-induced herd immunity thresholds for eliminating or eradicating infections, highlight the properties and assumptions behind the derived thresholds, discuss common misconceptions, and outline areas for future research. Examples of a few vaccine-preventable diseases, such as the COVID-19 pandemic, will be used for illustrative purposes. (Received November 15, 2021)

35 ► Partial differential equations

1174-35-5211 **Jill Pipher*** (Jill_pipher@brown.edu), Brown University. *Regularity of solutions to elliptic operators and elliptic systems.*

The celebrated De Giorgi-Nash-Moser theory, developed in the middle of the last century, showed that a structural condition on the matrix of coefficients of a second order PDE implied the Holder continuous regularity of its solutions, even for rough (measurable, bounded) coefficients. The structural condition is called ellipticity. This theory had a big impact for the study of non-linear equations and opened the door to a better understanding of how to quantify the connection between regularity (smoothness) of the coefficients and that of solutions. In this talk, we will review progress towards that understanding, and introduce a recently discovered structural condition, generalizing ellipticity, that has sparked new results for complex coefficient operators and real/complex systems. (Received November 16, 2021)

1174-35-5215 **Qiang Du*** (qd2125@columbia.edu), Columbia University. *Analysis and applications of nonlocal models.*

Nonlocality has become increasingly noticeable in nature. The modeling and simulation of its presence and impact motivate new development of mathematical theory. In this lecture, we focus on nonlocal models with a finite horizon of interactions, and illustrate their roles in the understanding of various phenomena involving anomalies, singularities and other effects due to nonlocal interactions. We also present some recent analytical studies concerning nonlocal operators and nonlocal function spaces. The theoretical advances are making nonlocal modeling and simulations more reliable, effective and robust for applications ranging from classical mechanics to traffic flows of autonomous and connected vehicles. (Received November 15, 2021)

1174-35-5473 **Jincheng Yang** (jcyang@math.utexas.edu), The University of Texas At Austin, and **Alexis F Vasseur*** (vasseur@math.utexas.edu), The University of Texas At Austin.
Boundary vorticity estimate for the Navier-Stokes equation and control of layer separation in the inviscid limit

Consider the steady solution to the incompressible Euler equation $U = Ae_1$ in the periodic tunnel $\Omega = [0,1] \times \mathbb{T}^2$. Consider now the family of solutions U_ν to the associated Navier-Stokes equation with no-slip condition on the flat boundaries, for small viscosities $\nu = 1/Re$, and initial values close in L^2 to Ae_1. Under a conditional assumption on the energy dissipation close to the boundary, Kato showed in 1984 that U_ν converges to Ae_1 when both the viscosity converges to 0 and the initial value converge to Ae_1. It is still unknown whether this inviscid limit is unconditionally true. Actually, the convex integration method predicts the possibility of a layer separation with energy at time T up to:

$$\|\bar{U}(T) - Ae_1\|_{L^2}^2 \approx A^3 T.$$

In this work we prove that at the double limit for the inviscid asymptotic, where both the Reynolds number Re converges to infinity and the initial value U_ν converges to Ae_1 in L^2, the energy of layer separation cannot be more than:

$$\|U_\nu(T) - Ae_1\|_{L^2}^2 \lesssim A^3 T \ln(Re).$$

Especially, it shows that, even if if the limit is not unique, the shear flow pattern is observable up to time $1/(A\ln(Re))$. The result relies on a new boundary vorticity estimate for the Navier-Stokes equation. This new estimate, inspired by previous work on higher regularity estimates for Navier-Stokes, provides a non-linear control scalable through the inviscid limit. (Received September 4, 2021)

1174-35-5657 **Barbara Lee Keyfitz** (bkeyfitz@math.ohio-state.edu), The Ohio State University, **John Holmes*** (holmesj@wfu.edu), Wake Forest University, and **Feride Tiglay** (trigly.1@math.osu.edu), The Ohio State University. *WEAK DIFFEOMORPHISMS AND SOLUTIONS TO CONSERVATION LAWS*

Evolution equations which describe the changes in a velocity field over time have been classically studied within the Eulerian or Lagrangian frame of reference. Classically, these frameworks are equivalent descriptions of the same problem, and the equivalence can be demonstrated by constructing particle paths. For hyperbolic conservation laws, we extend the equivalence between these frameworks to weak solutions for a broad class of problems. Our main contribution in this paper is that we develop a new framework to extend the idea of a particle path to scalar equations and to systems in one dimension which do not explicitly include velocity fields. For systems, we use Riemann invariants as the tool to develop an analog to particle paths. (Received August 24, 2021)

1174-35-5668 **Jin Liang*** (jinliang@sjtu.edu.cn), Shanghai Jiao Tong University. *Uniform decay estimates for integro-differential evolution equations* Preliminary report.

This talk is concerned with integro-differential evolution equations with indirect memory-damping. We first discuss the global existence and uniqueness of the solutions of initial-boundary value problem for the integro-differential evolution equations. Then we present some basic and recent results of the uniform decay estimates for this class of evolution equations and talk about how to obtain the decay rates of the energy for the integro-differential evolution equations under some suitable conditions on the memory kernels. (Received August 24, 2021)

1174-35-5671 **Ti Jun Xiao*** (tjxiao@fudan.edu.cn), Fudan University. *Asymptotic stability of evolution equations with Wentzell type boundary conditions* Preliminary report.

Evolution equations with Wentzell type boundary conditions are useful mathematical models in various practical problems, for instance, in those modelling the dynamic vibrations of linear viscoelastic rods and beams with tip masses attached at their free ends. In recent decades, the behaviour of differential operator with Wentzell type boundary conditions and the asymptotic behaviour of solutions for evolution equations with Wentzell boundary conditions, have been investigated extensively. In this talk, we present some developments in the study of wellposedness and asymptotic stability of evolution equations with Wentzell type boundary conditions. (Received August 24, 2021)

1174-35-5770 **Lorena Bociu** (lvbociu@ncsu.edu), North Carolina State University, NC State University, and **Sarah Strikwerda*** (slstrikw@ncsu.edu), North Carolina State University. *Optimal Control of Fluid Flows through Poro-Visco Elastic Media*

We consider an optimal control problem subject to a nonlinear poro-visco-elastic model with applications to fluid flows through biological tissues. In particular, our goal is to optimize the fluid pressure and solid displacement using either distributed or boundary control. We present results on the existence and uniqueness of optimal control. Additionally, we present the associated necessary optimality conditions which were used to approximate optimal controls in the linear setting of the problem. (Received September 3, 2021)

1174-35-5797 **Guofang Wei** (wei@math.ucsb.edu), UC Santa Barbara. *The fundamental gap of horoconvex domains in* $\mathbb{H}^n$

We introduce the fundamental gap problem and its characteristics in hyperbolic space. We show that, unlike in the Euclidean or spherical cases, the product of their fundamental gap with the square of their diameter has no positive lower bound. The result follows from the study of the fundamental gap of geodesic balls as the radius goes to infinity, then comparing horoconvex domains to balls. (Received August 27, 2021)

1174-35-5840 **Hamidou Toure*** (toureh98@gmail.com), Université Joseph KI-ZERBO, ANSAL-BF. *Weighted Stepanov-like pseudo almost automorphic solutions of class rfor some partial differential equations*

In this work, we study the existence and uniqueness of Stepanov-like pseudo almost automorphic solutions of class r for a neutral partial functional differential equation governed by A a linear operator on a Banach space X satisfying the Hille-Yosida condition.

We make some assumptions on the linear operator, for which the semigroup is compact on the closure of the domain of the operator. We then introuce the notion of pseudo-almost automorphic fucntions of class r. Assuming that the semigroup is hyperbolic, if the data are pseudo-almost automorphic functions of class r, then we establish that the neutral partial functional differential equation has an unique solution which is pseudo-almost automorphic functions of class r. (Received August 29, 2021)

1174-35-5889 **Hong-Ming Yin*** (hyin@wsu.edu), Washington State University. *On the stability analysis for an infectious disease model without lifetime immunization* Preliminary report.

In this talk I will discuss an infectious disease model caused by a bacteria without a lifetime immunization. The mathematical model is governed by a nonlinear reaction-diffusion system. I will discuss the recent progress about the global existence, uniqueness and long-time behavior of the system. Particularly, we obtain a global bound for the system with a general diffusion coefficients. One of the crucial technique is based on a Morry-John-Nirenberg-Campanato estimate for parabolic equations. (Received August 30, 2021)

1174-35-5915 **Alejandro Sarria*** (asarria@ung.edu), University of North Georgia, and **Ikechukwu Obi-Okoye** (ivobio3570@ung.edu), University of North Georgia. *Breakdown of Solutions to a Damped Fluid Equation with Homogeneous Three-Point Boundary Condition* Preliminary report.

The inviscid Proudman-Johnson equation is a partial differential equation with applications to two-dimensional fluid dynamics. It is known that solutions to this equation subject to a homogeneous three-point boundary condition can develop singularities in finite time. In this presentation, we incorporate damping effects into an n-dimensional analogue of this system and consider the possibility of finite time breakdown of solutions subject to the same homogeneous three-point boundary condition. In particular, we will derive conditions the initial data must satisfy in order for solutions to blowup in finite time with either bounded or unbounded smooth damping term. (Received August 31, 2021)

1174-35-5925 **Stefan Steinerberger*** (steinerb@uw.edu), University of Washington, Seattle. *A maximum principle for the first nontrivial Neumann eigenfunction of the Laplacian*

The Hot Spots conjecture states that, under suitable assumptions on the domain, the maximum of the first nontrivial eigenfunction of the Laplacian subject to Neumann boundary conditions is assumed on the boundary. The conjecture is widely assumed to be true in convex domain but known to fail in general (Burdzy & Werner, 1999). We prove a substitute result for all (connected) domains in all dimensions: the maximum inside the domain is at most 60 times as large as the maximum on the boundary. One would assume that the sharp constant is a lot closer to 1. (Received August 31, 2021)

1174-35-5944 **Yidan Yang*** (yy2513@rit.edu), Rochester Institute of Technology. *Coefficient Inverse Problem in a Stochastic Partial Differential Equation*

The problem we study in this project is the coefficient identification (an inverse problem) in a model that is governed by a stochastic partial differential equation (SPDE). One of the common approaches for identification of a coefficient in a PDE is to formulate the problem is an optimization problem with the PDE as a constraint. Stochastic approximation method for solving the optimization problem is explored where the objective function is formulated using output least squares (OLS) and modified output least squares (MOLS) methods. Section 1 contains a description of an application problem. Section 2 discusses weak formulation and finite element discretization of elliptic PDEs with homogeneous/nonhomogeneous Dirichlet boundary conditions, nonhomogeneous Neumann boundary condition, and mixed boundary conditions. Section 3 has an introduction to the stochastic PDE we consider and a brief summary of solution methods such as Monte Carlo FEM, stochastic Galerkin, and stochastic collocation. Sections 4 and 5 contain some optimization formulations of the inverse problem as well as discrete formulas useful for numerical implementations. Main ideas of the stochastic approximation method and numerical results are included in Sections 6. (Received August 31, 2021)

1174-35-6113 **Michail E. Filippakis*** (mfilip@unipi.gr), Department of Digital Systems, Univercity of Piraeus, Greece. *Nodal and Multiple solutions for nonlinear equations driven by a nonhomogeneous differential operator* Preliminary report.

In this paper we consider a nonlinear parametric Dirichlet problem driven by a nonhomogeneous differential operator (special cases are the p-Laplacian and the (p, q)-differential operator) and with a reaction which has the combined effects of concave ($(p-1)$-sublinear) and convex ($(p-1)$-superlinear) terms. Using variational methods based on critical point theory, together with truncation and comparison techniques and Morse theory (critical groups), we show that for all small $\lambda > 0$ (λ is a parameter), the problem has at least five nontrivial smooth solutions (two positive, two negative and the fifth nodal). We also prove two auxiliary results of independent interest. The first is a strong comparison principle and the second relates Sobolev and Hőlder local minimizers for C^1 functionals.Then we consider a nonlinear nonhomogeneous Robin problem and with Morse Theory and variational methods we prove the existence of nontrivial smooth solutions.

The author would like to thank the "Bioinformatics-Computational Biology" Master's Program at the Department of Biology of the National and Kapodistrian University of Athens for generous support (Project Code: 70/3/15427). (Received September 4, 2021)

1174-35-6124 **Mahmud M Alam*** (gmalam@alaska.edu), University of Alaska Fairbanks. *Forward and Control Problems for Telegrapher's Equations on Metric Graphs* Preliminary report.

In this talk, we address forward and control problems for telegrapher's equations on metric graphs. The forward problem is considered on general graphs, and an efficient algorithm for solving the equations with variable coefficients is presented. The control problem is considered on tree graphs, i.e. graphs without cycles, for

equations with constant coefficients. The necessary and sufficient conditions of the exact controllability are presented and the minimal control time is indicated.

The talk is based on joint work with S. Avdonin and N. Avdonina. (Received September 4, 2021)

1174-35-6126 **Jichun Li*** (jichun.li@unlv.edu), University of Nevada Las Vegas. *A DG method and analysis for the Cohen-Monk PML model*

In 1994, Berenger introduced the concept of Perfectly Matched Layer (PML) in order to simulate the electromagnetic wave propagation phenomenon by solving the time-dependent Maxwell's equations in unbounded domains. Since 1994, the PML idea has been extended to various of wave propagation problems. In 1999, Cohen and Monk developed a PML model for 3-D Maxwell's equations and solved it using mapped mass-lumped edge elements without deep mathematical analysis of the model. Here we first establish the stability analysis for the model, then develop and analyze a DG method for the 2-D Cohen-Monk PML model. Numerical results are presented to show its wave absorbing effectiveness of our DG method. Finally, we will present a preliminary result of using the Deep Learning library DeepXDE to solve another PML model. (Received September 5, 2021)

1174-35-6172 **Pablo Raul Stinga*** (stinga@iastate.edu), Iowa State University, and **Mary Vaughan** (maryv@utexas.edu), The University of Texas at Austin. *Harnack inequality for fractional nondivergence form elliptic equations*

We prove the interior Harnack inequality for nonnegative solutions to fractional elliptic equations driven by fractional powers of nondivergence form elliptic operators. Such equations appear in several applications, including the analysis of fractional Monge–Ampère equations. This is joint work with Mary Vaughan (UT Austin). (Received September 6, 2021)

1174-35-6369 **Diaraf Seck*** (diaraf.seck@ucad.edu.sn), University Cheikh Anta Diop of Dakar, SENEGAL. *Evolution system of PDE: regular and singular limits cases for parabolic and hyperbolic equations*

In this talk, first we aim to study the existence of solution for a model of semi linear para- bolic equation by using fixed point tools. In the second part we study a couple of Parabolic and Hyperbolic system of PDE with singular perturbations. In that cases considered, degeneracy situations may occur when a parameter (called ϵ) goes to zero. Existence and uniqueness results in the case of a two dimensional torus are proved and an asymptotic analysis is done in order to derive a limit system of PDE. (Received September 8, 2021)

1174-35-6405 **Landon Gauthier*** (gauthierl@uky.edu), University of Kentucky, and **Russell Brown** (russell.brown@uky.edu), University of Kentucky. *Inverse boundary value problems for polyharmonic operators with non-smooth coefficients*

We consider the polyharmonic operator with lower order coefficients of the form $\mathcal{L} = (-\Delta)^m + Q \cdot D + q$. The main interest of ours is to lower the regularity of Q and q to establish uniqueness of the operator. The key component we used to lower the regularity is by adapting an averaging argument first introduced by Haberman and Tataru. This technique allows us to find a sequence of solutions that give an estimate better than the pointwise bound one may first attempt. (Received September 13, 2021)

1174-35-6406 **Mamadou abdoul Diop*** (ordydiop@gmail.com), GASTON BERGER UNIVERSITY. *Existence results for some stochastic functional integrodierential systems driven by Rosenblatt process* Preliminary report.

We discuss the existence and uniqueness of mild solutions for a class of stochastic evolution equations with varying-time delays driven by Rosenblatt process via resolvent operator theory in the sens of Grimmer, stochastic analysis, Kuratowski measure of noncompactness and fixed-point technique. Mild solutions existence and uniqueness are discussed for two cases. An example is provided to illustrate the applicability of the obtained results. (Received September 8, 2021)

1174-35-6867 **Maria Amarakristi Onyido*** (mao0021@auburn.edu), Auburn University. *Dynamics of Solutions of Nonlocal Dispersal Equations with Almost Periodic Dependence.*

We discuss the dynamics of solutions of nonlocal Fisher KPP equations in almost periodic environment. In particular, using the generalized principal eigenvalue and top Lyapunov exponent, we establish the existence, uniqueness and stability of strictly positive entire almost periodic solutions. We also discuss the spreading speeds of solutions with front-like initials. (Joint work with Dr. Wenxian Shen) (Received September 10, 2021)

1174-35-6870 **Wenxian Shen** (wenxish@auburn.edu), Auburn University, and **Tahir Bachar Issa*** (tahirbachar.issa@sjsu.edu), San Jose State University. *Pointwise persistence in full chemotaxis models with logistic source on bounded heterogeneous environments*

In this talk, I will present our work on pointwise persistence in full chemotaxis models with local as well as nonlocal time and space dependent logistic source in bounded domains. We first prove the global existence and boundedness of nonnegative classical solutions under some conditions on the coefficients in the models. Next, under the same conditions on the coefficients, we show that pointwise persistence occurs, that is, any globally defined positive solution is bounded below by a positive constant independent of its initial condition when the time is large enough. It should be pointed out that Tao and Winkler established the persistence of mass for globally defined positive solutions, which indicates that any extinction phenomenon, if occurring at all, necessarily must be spatially local in nature, whereas the population as a whole always persists. The pointwise persistence proved in the current paper implies that not only the population as a whole persists, but also it persists at any location eventually. It also implies the existence of strictly positive entire solutions. (Received September 10, 2021)

1174-35-6918 **Joshua L Flynn*** (joshua.flynn@uconn.edu), University of Connecticut, **Guozhen Lu** (guozhen.lu@uconn.edu), University of Connecticut, and **Qiaohua Yang** (qhyang.math@whu.edu.cn), Wuhan University. *Sharp Hardy-Sobolev-Maz'ya, Adams and Hardy-Adams inequalities on quaternionic hyperbolic spaces and the Cayley hyperbolic plane*

The main goal of this talk is to establish the Poincaré-Sobolev and Hardy-Sobolev-Maz'ya inequalities on quaternionic hyperbolic spaces and the Cayley hyperbolic plane. A crucial part of our work is to establish appropriate factorization theorems on these spaces which are of their independent interests. Combining the factorization theorems and the Geller type operators with the Helgason-Fourier analysis on symmetric spaces, the precise heat and Bessel-Green-Riesz kernel estimates and the Kunze-Stein phenomenon for connected real simple groups of real rank one with finite center, we succeed to establish the higher order Poincaré-Sobolev and Hardy-Sobolev-Maz'ya inequalities on quaternionic hyperbolic spaces and the Cayley hyperbolic plane. The kernel estimates required to prove these inequalities are also sufficient for us to establish, as a byproduct, the Adams and Hardy-Adams inequalities on these spaces. This paper, together with our earlier works, completes our study of the factorization theorems, higher order Poincaré-Sobolev, Hardy-Sobolev-Maz'ya, Adams and Hardy-Adams inequalities on all rank one symmetric spaces of noncompact type. (Received September 10, 2021)

1174-35-6939 **Ahmet Ozkan Ozer** (ozkan.ozer@wku.edu), Western Kentucky University, and **Ahmet K Aydin*** (ahmetkaan.aydin288@topper.wku.edu), Western Kentucky University. *Robust-filtering of sensor data for the approximation of the multi-layer Mead-Marcus beam equation* Preliminary report.

A space-discretized Finite Difference approximation for the corresponding abstract multi-layer Mead-Marcus beam model (PDE) with hinged boundary conditions is investigated. The PDE model describes transverse vibrations for a multi-layer sandwich beam whose outer layers are piezoelectric or elastic, and the constrained layers are viscoelastic. Even though the PDE model is known exactly observable with a single observation on its boundary, the discretized model is not able to retain exact observability uniformly with respect to the mesh parameter. This is mainly due to the loss of the uniform gap among the eigenvalues as the mesh parameter goes to zero. To obtain a uniform gap, and therefore, an exact observability result with the same observed quantity, the spurious (computer-generated) high frequency eigenvalues of the approximated model are filtered by the Direct Fourier filtering method. After filtering, the approximated solution space covers the whole infinite-dimensional solution space as the mesh parameter goes to zero. Filtering space strongly depends on the material parameters of each layer. (Received September 10, 2021)

1174-35-7005 **Yuan Gao*** (gao662@purdue.edu), Purdue University. *Rigidity and global stability for vectorial dislocation system*

Dislocations are important line defects in crystalline materials and play essential roles in understanding materials properties like plastic deformation. In this talk, I will first talk about the static vectorial Peierls-Nabarro (PN) models with a single straight/curved dislocation line, which can be reduced to a Ginzburg-Landau equation/systems involving "anisotropic half-Laplacian". The existence of global minimizers, uniqueness of the stable solution with a 1D profile are proved. In a 2D simplified bulk-interface interaction dynamics, the uniform convergence of dynamic solution to the uniquely determined global minimizer will be discussed. (Received September 11, 2021)

1174-35-7091 **Mimi Dai*** (mdai@ias.edu), Institute for Advanced Study, Princeton. *Dyadic models for magnetohydrodynamics*

We propose some dyadic models for magnetohydrodynamics. The dyadic models include the intermittency dimension as a parameter in the nonlinear scales. For intermittency dimension below certain threshold value, finite time blow-up solutions and non-unique Leray-Hopf solutions are constructed. (Received September 12, 2021)

1174-35-7114 **Rachidi Salako*** (rachidi.salako@unlv.edu), University of Nevada, Las Vegas. *Asymptotic dynamics of solutions of a multi-strain PDE epidemic model in spatial heterogeneous environment*

We study the large time behavior of solutions of a multi-strain PDE epidemic model in spatially heterogeneous media. In particular, the stability of equilibrium solutions are investigated. In the case of a total lock down of the susceptible subgroup of the population, our results indicate that the impact of the disease will be significantly minimized in the long run if there is a common safety area against all its strains. Numerical simulations will be presented to illustrate our theoretical findings. (Received September 12, 2021)

1174-35-7220 **Ananta Acharya*** (a_achary@uncg.edu), UNC Greensboro. *Sigma Shaped Bifurcation Curves*

We study positive solutions of steady state reaction diffusion equation in a bounded domain where a parameter λ influences the equation as well as the boundary condition. We show that there exist multiple positive solutions for certain range of λ such that we get a sigma shaped bifurcation diagram. We prove our results using the method of sub-super solutions. (Received September 13, 2021)

1174-35-7265 **Rajan Puri*** (purir@wfu.edu), Wake Forest University. *Non-uniqueness for the ab-family of equations with peakon traveling waves on the circle.*

Peakon traveling wave solutions on the circle are derived for the cubic *ab*-family of equations, which includes both the Fokas-Olver-Rosebau-Qiao (FORQ) and Novikov(NE) equations. For $a \neq 0$, it is proved that there exists an initial data in the Sobolev space H^s, $s < 3/2$, with non-unique solutions on the circle. We construct a two-peakon solution with an asymmetric peakon-antipeakon initial profile that collides at a finite time. At the time of collision, the two-peakon solution reduces to a single peakon solution, which will complete the proof of nonuniqueness. (Received September 13, 2021)

1174-35-7278 **Ruoyu P. T. Wang*** (rptwang@math.northwestern.edu), Northwestern University. *Boundary Stabilisation of Waves in Cylindrical Waveguides*

Consider the wave equation in a finite cylinder, with the dissipative boundary condition on the cylindrical wall and the homogeneous Neumann boundary condition on two cylindrical caps: it describes the waves whose energy could be dissipated when they interact with the cylindrical wall. We will discuss our recent result that the energy of those waves must uniformly decay no faster than $t^{-1/2}$, and no slower than that. We will also survey related results in this field. (Received September 13, 2021)

1174-35-7286 **Hongjie Dong** (hongjie_dong@brown.edu), Brown University, and **Seick Kim*** (kimseick@yonsei.ac.kr), Yonsei University. *Green's Function for Nondivergence Elliptic Operators in Two Dimensions*

We construct the Green function for second-order elliptic equations in nondivergence form when the mean oscillations of the coefficients satisfy the Dini condition. We show that Green's function is a bounded mean oscillation in the domain and establish logarithmic pointwise bounds. We also obtain pointwise bounds for first and second derivatives of Green's function. (Received September 13, 2021)

1174-35-7297 **Sungjin Lee*** (sungjinlee@yonsei.ac.kr), Yonsei University. *Estimates for Green's functions of elliptic equations in non-divergence form with continuous coefficients*

We present a new method for the existence and pointwise estimates of a Green's function of non-divergence form elliptic operator with Dini mean oscillation coefficients. We also present a sharp comparison with the corresponding Green's function for constant coefficients equations. (Received September 14, 2021)

1174-35-7318 **Madelyne Brown*** (madelyne@live.unc.edu), University of North Carolina Chapel Hill. *Growth of Fourier coefficients of restricted eigenfunctions*

In this talk, we will discuss the growth of Laplace eigenfunctions on a compact, Riemannian manifold when restricted to a submanifold. We analyze the behavior of the restricted eigenfunctions by studying their Fourier coefficients with respect to an arbitrary orthonormal basis for the submanifold. Assuming the eigenfunctions and the basis have defect measures associated to them, we obtain a bound on the Fourier coefficients In the high-energy limit. (Received September 14, 2021)

1174-35-7494 **Arunima Bhattacharya*** (arunimab@uw.edu), University of Washington. *Hessian Estimates for the Lagrangian mean curvature equation*

In this talk, we will derive a priori interior Hessian estimates for the Lagrangian mean curvature equation under certain natural restrictions on the Lagrangian phase. As an application, we will use these estimates to solve the Dirichlet problem for the Lagrangian mean curvature equation with continuous boundary data on a uniformly convex, bounded domain. (Received September 14, 2021)

1174-35-7536 **Na Eun Hanna Kim*** (nekim2@illinois.edu), University of Illinois. *Maximization of the second Laplacian eigenvalue on the sphere* Preliminary report.

Isoperimetric problem for eigenvalues of the Laplacian on a given manifold is concerned with finding an upper bound and understands for which metrics the upper bound is attained. On the sphere, the first nonzero eigenvalue can be maximized by the standard "round" metric. For 2-dimensional sphere, the second eigenvalue becomes maximal as the surface degenerates to two disjoint spheres by a result of Nadirashvili. It was conjectured by Girouard, Nadirashvili and Polterovich that for higher dimensional spheres, the second eigenvalue has an analogous upper bound. In this talk, we will establish a proof by construction of trial functions confirming the conjecture. (Received September 14, 2021)

1174-35-7594 **Simon Bortz*** (sbortz@ua.edu), University of Alabama. *Recent progress on parabolic free boundary problems*

It is known that parabolic Lipschitz graphs are not 'good' for parabolic singular integrals, nor are the regions above such graphs 'good' for caloric measure. It was shown by Hofmann (in '95) that an additional non-local regularity condition is necessary and sufficient for the L^2 boundedness of parabolic singular integrals. This non-local condition has also been known (due to Lewis and Murray in '95) to be sufficient for the caloric measure for the region above the graph to be quantitatively absolutely continuous with respect to parabolic surface measure; however, it is still unknown if this condition is necessary. We report on some recent progress towards showing the necessity of this condition and some future directions in rougher, non-graphical settings. (Received September 15, 2021)

1174-35-7650 **Jeffrey Langford** (Jeffrey.Langford@Bucknell.edu), Bucknell University, and **L. Mercredi Chasman*** (chasmanm@morris.umn.edu), University of Minnesota Morris. *Low Eigenvalues of the Robin Plate*

Inspired by recent results by Freitas and Laugesen for the Robin Laplacian, we consider the Robin bi-Laplacian (plate) under tension $(\Delta^2 - \tau\Delta)$. Using similar methodology and a careful treatment of special functions, we are able to obtain a similar result for the second eigenvalue of the Robin plate for certain parameter ranges. (Received September 15, 2021)

1174-35-7667 **Mariana Smit Vega Garcia*** (smitvem@wwu.edu), Western Washington University. *On a Bernoulli-type overdetermined free boundary problem*

We study a Bernoulli-type free boundary problem in the context of A-harmonic PDEs. In particular, we show that if K is a bounded convex set satisfying the interior ball condition and $c > 0$ is a given constant, then there exists a unique convex domain U containing K and a function u which is A-harmonic in $U \setminus K$, has continuous boundary values 1 on ∂K and 0 on ∂U, such that $|\nabla u| = c$ on ∂U. Moreover, ∂U is $C^{1,\gamma}$, for some $\gamma > 0$, and it is smooth provided A is smooth in $\mathbb{R}^n \setminus \{0\}$. This is joint work with M. Akman and A Banerjee. (Received September 15, 2021)

1174-35-7721 **Zhaosheng Feng*** (zhaosheng.feng@utrgv.edu), University of Texas Rio Grande Valley. *A nonconvex dissipative system and its applications*

This talk presents a class of solutions in terms of implicit functions and elliptic functions to a nonconvex dissipative system. Applications to the complex Ginzburg-Landau equation are illustrated and several classes of uniformly translating solutions are obtained. (Received September 15, 2021)

1174-35-7773 **Chen-Chih Lai*** (cl4205@columbia.edu), Columbia University, **Tai-Peng Tsai** (ttsai@math.ubc.ca), University of British Columbia, **Kyungkeun Kang** (kkang@yonsei.ac.kr), Yonsei University, and **Baishun Lai** (laibaishun@henu.edu.cn), Hunan Normal University. *The Green tensor of the nonstationary Stokes system in the half-space and finite energy Navier-Stokes flows with unbounded gradients near boundary*

Back in 1977 Solonnikov derived an explicit formula and pointwise estimates of a restricted Green tensor of the nonstationary Stokes system in the half-space under the assumption that the force field is solenoidal.

In this talk we will present the first ever pointwise estimates of the "unrestricted" Green tensor and the associated pressure tensor of the nonstationary Stokes system in the half-space, for every space dimension greater than one. We will also talk about the symmetry of the Green tensor as a byproduct of the proof of the pointwise estimates and discuss some applications of the pointwise estimates. In particular, we will show the existence of Navier-Stokes flows with finite global energy and unbounded velocity derivative near the boundary, caused by Holder continuous boundary fluxes with compact support.

This talk is based on joint works with Kyungkeun Kang, Baishun Lai and Tai-Peng Tsai (arXiv:2011.00134 and arXiv:2107.00810). (Received September 17, 2021)

1174-35-7801 **Anna L. Mazzucato*** (alm24@psu.edu), Pennsylvania State University, **Gianluca Crippa** (gianluca.crippa@unibas.ch), University of Basel, **Gautam Iyer** (gautam@math.cmu.edu), Carnegie Mellon University, and **Tarek Elgindi** (tarek.elgindi@duke.edu), Duke University. *Growth of Sobolev norms and loss of regularity for transport equations*

We discuss recent results concerning growth of Sobolev norms for solutions of transport equations by incompressible flows, and how this growth leads to instantaneous loss of regularity for all non-constant initial data with square integrable derivative. (Received September 16, 2021)

1174-35-7846 **Aghalaya S. Vatsala** (Vatsala@Louisiana.edu), University of Louisiana at Lafayette, and **V A Vijesh*** (antonyvijesh@gmail.com), IIT Indore. *An accelerated technique for coupled system of reaction-diffusion-transport equations arising from catalytic converter*

This paper presents a study on accelerated monotone iterative methods for two systems of nonlinear partial differential equations arising from the catalytic converter models. Two mathematical models were considered in this work. The first mathematical model consists of a semi linear parabolic partial differential equation and two integral equations, whereas the second mathematical model for the catalytic converter model consists a semi linear parabolic partial differential equation and an integral equation. The proposed monotone iteration methods converge to the solution of these mathematical models faster than the existing monotone iteration schemes available in the literature. Interesting theoretical justification has been provided for the accelerated convergence as well as the monotone behaviour of the proposed iterative schemes. (Received September 16, 2021)

1174-35-7875 **Linhan Li*** (li001711@umn.edu), University of Minnesota, **Svitlana Mayboroda** (svitlana@umn.edu), University of Minnesota, and **Guy David** (guy.david@universite-paris-saclay.fr), University of Paris Sud. *Carleson measure estimates for the Green function with applications to elliptic measures*

We are interested in the relations between an elliptic operator $L = -div(A\nabla)$ on a domain, the geometry of the domain, and the boundary behavior of the Green function. In joint works with Guy David and Svitlana Mayboroda, we show that if the coefficients of the operator satisfy a quadratic Carleson condition, then the Green function on the half-space is almost affine, in the sense that the normalized difference between the Green function with a sufficiently far away pole and a suitable affine function at every scale satisfies a Carleson measure estimate. We demonstrate with counterexamples that our results are optimal, in the sense that the class of the operators considered are essentially the best possible. Analogous results can be obtained on sets with lower-dimensional boundaries. In this talk, I will also talk about applications of these quantitative results to elliptic measures and characterizations of uniform rectifiable sets of higher co-dimension. (Received September 16, 2021)

1174-35-7882 **William E Fitzgibbon*** (fitz@uh.edu), University of Houston. *Highly Heterogeneous FKPP Equations and Epidemic Models* Preliminary report.

Abstract: Our interest is a highly heterogeneous advective diffusive KFPP Equation and a class of advective diffusive systems describing the spread of infectious disease across a geographic region. The differential operators of the KFPP equation are allowed to have discontinuous coefficients, the growth terms, and the logistic damping terms. The reaction diffusion systems comprising the epidemic terms can feature discontinuity in the disease kinetics as well as in the differential operators and demographic terms. We first obtain global existence of weak

solution to the FKPP Equation and ascertain its asymptotic profile. These results are then applied to the epidemic models. We also consider a class of partially dissipative epidemic models where the susceptible class is a parametrically evolving ordinary differential equation. (Received September 16, 2021)

1174-35-7907 **Hugo Sanchez*** (hsanchez@trinity.edu), Trinity University, **Annemily Gammie Hoganson** (hogansona@carleton.edu), Carleton College, **Helen Dai** (hdai@college.harvard.edu), Harvard University, **Zoe Markman** (zmarkma1@swarthmore.edu), Swarthmore College, **Tess Harvey** (T.Harvey@sms.ed.ac.uk), University of Edinburgh, and **River Newman** (river.newman@yale.edu), Yale University. *Existence of global solutions to the discrete one-phase Bernoulli problem.* Preliminary report.

The problem of minimizing the continuous Alt-Caffarelli functional given by $\mathcal{E}[f] = \int_D |\nabla f|^2 + |\{f > 0\} \cap D|$, where $D \subset \mathbb{R}^d$ is a connected open set, has been thoroughly investigated and much is known about the existence of solutions. Here, we analyze a discrete version of the one-phase free boundary problem, restricting ourselves to locally finite graphs where every vertex has finite degree. Specifically, we adapt the direct method to prove there always exists a global minimizer f on a locally finite graph G to the discrete analogous functional $J_G[f] = \sum_{\{x,y\} \text{ s.t. } x \sim y}(f(x) - f(y))^2 + \#\{f > 0\}$. (Received September 16, 2021)

1174-35-7940 **Nancy Rodriguez*** (rodrign@colorado.edu), University of Colorado At Boulder. *Directed movement and the Allee effect*

It is well known that relocation strategies in ecology and in economics can make the difference between extinction and persistence. In this talk I present a unifying model for the dynamics of ecological populations and street vendors, an important part of many informal economies. I discuss the effects of chemotactic movement of populations subject to the Allee Effect by discussing the existence of equilibrium solutions subject to various boundary conditions and the evolution problem when the chemotaxis effect is small. On an interesting note, I present numerical simulations, which show that in fact chemotaxis can help overcome the Allee effect as well as some partial analytical results in this direction. I will conclude by making a connection to the Ideal Free Distribution and analyze what happens under competition. (Received September 16, 2021)

1174-35-8043 **Ummugul Bulut*** (ubulut@tamusa.edu), Texas A&M University San Antonio, and **Tamer Oraby** (tamer.oraby@utrgv.edu), University of Texas - Rio Grande Valley. *A Stochastic Model of Avian Influenza in the Migratory Birds* Preliminary report.

Avian Influenza (AI) is a zootonic disease with a 53% case fatality rate (H5N1 strain) since 2003. The number of humans infected with AI has increased in the recent epidemic wave during the period 2016-2017. In this talk, the stochastic fisher equation is used, with the time-dependent white noise, to investigate the dynamics of the susceptible-infected (SI) compartmental model. Our goal is to see whether applying the random perturbation to the migration speed of the wild bird population can eradicate the disease or not. We prove that the disease-free equilibrium is exponentially asymptotically stable almost surely for the stochastic model. The computational graphs support the theoretical results. (Received September 17, 2021)

1174-35-8082 **Christopher Henderson*** (ckhenderson@math.arizona.edu), University of Arizona. *Invasion fronts of bacteria*

The invasion of a species in a new environment is based on the balance of two behaviors – reaction (i.e. reproduction up to carrying capacity) and diffusion. Basic PDE models for this date back many decades and are well-understood. We know that the invasion happens at a fixed speed that can be explicitly computed. On the other hand, some species, such as some bacteria and slime molds, are influenced additionally by intraspecies aggregation or repulsion. This talk will be focused on when and how the intraspecific interactions changes the speed. (Received September 17, 2021)

1174-35-8153 **Vlad C Vicol** (vicol@cims.nyu.edu), New York University, **Igor Kukavica** (kukavica@usc.edu), University of Southern California, **Trinh Tien Nguyen*** (tnguyen5@usc.edu), University of Southern California, and **Fei Wang** (fwang256@sjtu.edu.cn), Shanghai Jiao Tong University. *On the Euler+Prandtl expansion for the Navier-Stokes equations*

We establish the validity of the Euler+Prandtl approximation for solutions of the Navier-Stokes equations in the half plane with Dirichlet boundary conditions, in the vanishing viscosity limit, for initial data which are analytic only near the boundary, and Sobolev smooth away from the boundary. Our proof does not require higher order correctors, and works directly by estimating an L1-type norm for the vorticity of the error term in the expansion

Navier-Stokes-(Euler+Prandtl). An important ingredient in the proof is the propagation of local analyticity for the Euler equation, a result of independent interest. (Received September 17, 2021)

1174-35-8168 **Ayman Rimah Said*** (aymanrimah@gmail.com), Duke University. *On the dispersive Burgers equation* Preliminary report.

In this talk we will give a review of the Cauchy problem of the dispersive Burgers equation:

$$\partial_t u + u\partial_x u + \partial_x |D|^\alpha u = 0, \alpha \geq 0.$$

The problem is now known to exhibit wave breaking for $\alpha \leq 0$ and to be globally well-posed for $\alpha \geq \frac{6}{7}$. We will give a quick an overview of the literature, discuss numerical conjectures by Klein et Saut describing the behavior for $0 \leq \alpha \leq 1$, and present the recent techniques developed to answer them. More precisely we will discuss a para-differential generalization of a complex Cole-Hopf gauge transform, normal form transforms and the use of Bourgain type spaces and the interplay between those three techniques and present some recent results obtained by them in the regime $\alpha \geq \frac{1}{2}$. (Received September 17, 2021)

1174-35-8242 **Dallas Albritton*** (dallas.albritton@gmail.com), Institute for Advanced Study. *On self-similarity and non-uniqueness in fluid models* Preliminary report.

A decade ago, Jia and Sverak introduced a connection between self-similar solutions of the Navier-Stokes equations and (conjectural) non-uniqueness of Leray-Hopf solutions. We describe an analogous connection in a variety of fluid models, including the surface quasi-geostrophic equation with critical dissipation. (Received September 18, 2021)

1174-35-8255 **Seungly Oh*** (seungly.oh@wne.edu), Western New England University, and **Atanas Stefanov** (stefanov@uab.edu), University of Alabama Birmingham. *Smoothing and growth bound of periodic generalized Korteweg-de Vries equation*

For generalized KdV models with polynomial nonlinearity, we establish a local smoothing property in H^s for $s > \frac{1}{2}$. Such smoothing effect persists globally, provided that the H^1 norm does not blow up in finite time. More specifically, we show that a translate of the nonlinear part of the solution gains $\min(2s-1,1)-$ derivatives for $s > \frac{1}{2}$. Following a new simple method, which is of independent interest, we establish that, for $s > 1$, H^s norm of a solution grows at most by $\langle t \rangle^{s-1+}$ if H^1 norm is a priori controlled. (Received September 18, 2021)

1174-35-8378 **Irina A Kogan** (iakogan@ncsu.edu), North Carolina State University, and **John P Revelle*** (jprevelle3@gmail.com), NC State University. *A Modified Picard Iteration Scheme for a Boundary Value Problem of $u_{xy} = u$* Preliminary report.

Consider the PDE $u_{xy} = u$ with data for u given locally along a line M and data for u_x given along a line N, with M and N located in the first quadrant of the (x, y)-plane, intersecting only at the origin. It is known that if M is located above N, then there is a unique local solution. When M lies below N, a strong non-uniqueness of solutions was proved by Jenssen and Kogan (2020), in the sense that for given boundary data, there are two solutions that differ arbitrarily close to the origin. By providing additional data for u along some curve connecting M and N, we are able to implement a modified Picard iteration scheme and obtain concrete examples of the strong non-uniqueness phenomenon. (Received September 18, 2021)

1174-35-8454 **Changhui Tan*** (tan@math.sc.edu), University of South Carolina. *Nonlocal models in collective dynamics*

In this talk, I will introduce a class of nonlocal models in collective dynamics, and discuss some recent progress on the analysis of these models. (Received September 19, 2021)

1174-35-8809 **Fauziya Ado Yakasai*** (fay0002@auburn.edu), Auburn University, Department of Mathematics and Statistics.. *Hybrid Sampling for Uncertainty Quantification in Systems with High Dimensional Parameter Spaces* Preliminary report.

Spatial models in mathematical biology, formulated as partial differential equations, often include space-dependent parameters that are not readily estimable and are therefore uncertain. There are two broad classes of methods for computing statistical quantities of interest related to the model solution: Spectral methods, such as the generalized polynomial chaos and stochastic collocation methods, are well-suited for systems with low parameter complexity. However, their convergence rates deteriorate as the dimension of the parameter space increases, and hence for systems with high parameter complexity, methods whose convergence rates are independent of the stochastic dimension, such as the Monte Carlo method, are more appropriate. In this work, we propose a hybrid sampling scheme which uses conditional sampling to combine sparse grid quadrature rules on a low-dimensional projection of the parameter space with a Monte Carlo scheme to compensate in the remaining dimensions. Using complexity arguments, we show that our method is more efficient than either of its constituents. We include some numerical examples to illustrate our results. (Received September 19, 2021)

1174-35-8849 **Katerina Nik*** (katerina.nik@univie.ac.at), University of Vienna. *On a three-dimensional model for MEMS with a hinged top plate*

An idealized electrostatic microelectromechanical system (MEMS) consists of a rigid ground plate above which a thin elastic plate is suspended. The elastic plate is assumed to be hinged on its boundary. Applying a voltage difference between the two plates induces a Coulomb force that deforms the elastic plate. The corresponding mathematical model couples a fourth-order parabolic equation for the vertical deformation of the elastic plate to the harmonic electrostatic potential in the free domain between the two plates.

In this talk, I will present some recent results on local and global well-posedness of the model as well as on existence and non-existence of stationary solutions. (Received September 20, 2021)

1174-35-8946 **Sung-Jin Oh** (sjoh@math.berkeley.edu), UC Berkeley, and **Federico Pasqualotto*** (fp54@duke.edu), Duke University. *Gradient blow-up for dispersive and dissipative perturbations of the Burgers equation*

In this talk, I will discuss a construction of "shock forming" solutions to a class of dispersive and dissipative perturbations of the Burgers equation. This class includes the fractional KdV equation with dispersive term of order $\alpha \in [0, 1)$, the Whitham equation arising in water waves, and the fractal Burgers equation with dissipation term of order $\beta \in [0, 1)$.

Our result seems to be the first construction of gradient blow-up for fractional KdV in the range $\alpha \in [2/3, 1)$. We construct blow-up solutions by a self-similar approach, treating the dispersive term as perturbative.

The blow-up considered is stable for $\alpha < 2/3$, and it is unstable for $\alpha \geq 2/3$. Moreover, the construction is carried out by means of a weighted L^2 approach (in lieu of the characteristic method), which may be of independent interest.

This is joint work with Sung-Jin Oh (UC Berkeley). (Received September 20, 2021)

1174-35-8964 **King-Yeung Lam** (lam.184@osu.edu), The Ohio State University, and **Stephen Cantrell*** (rsc@math.miami.edu), The University of Miami. *On the evolution of slow dispersal in multi-species communities*

For any $N \geq 2$, we show that there are choices of diffusion rates $(d_1, \ldots, d_N)$ such that for N competing species which are ecologically identical and have distinct diffusion rates, the slowest disperser is able to competitively exclude the remainder of the species. In fact, the choices of such diffusion rates are open in the Hausdorff topology. Our result provides some evidence in the affirmative direction regarding the conjecture by Dockery et al. in 1998. The main tools include Morse decomposition of the semi-flow and the theory of normalized Floquet principal bundle for linear parabolic equations. A critical step in the proof is to establish the smooth dependence of the Floquet bundle on diffusion rate and other coefficients, which may be of independent interest. (Received September 20, 2021)

1174-35-9169 **Senwei Liang*** (liang339@purdue.edu), Purdue University. *Solving PDEs on unknown manifolds with machine learning*

Solving high-dimensional PDEs on unknown manifolds is a challenging computational problem that commands a wide variety of applications. In this talk, I will introduce a mesh-free computational framework and machine learning theory for solving elliptic PDEs on unknown manifolds, identified with point clouds, based on diffusion maps and deep learning. The PDE solver is formulated as a supervised learning task to solve a least-squares regression problem that imposes an algebraic equation approximating a PDE (and boundary conditions if applicable). The resulting numerical method is to solve a highly non-convex empirical risk minimization problem subjected to a solution from a hypothesis space of neural networks. In a well-posed elliptic PDE setting, when the hypothesis space consists of neural networks with either infinite width or depth, we show that the global minimizer of the empirical loss function is a consistent solution in the limit of large training data. When the hypothesis space is a two-layer neural network, we show that for a sufficiently large width, gradient descent can identify a global minimizer of the empirical loss function. Supporting numerical examples demonstrate the convergence of the solutions and the effectiveness of the proposed solver in avoiding numerical issues that hampers the traditional approach when a large data set becomes available, e.g., large matrix inversion. (Received September 20, 2021)

1174-35-9171 **Eduardo V. Teixeira*** (eduardo.teixeira@ucf.edu), University of Central Florida. *On a new class of free boundary problems*

I will discuss a new class of free boundary problems arising by non-locality considerations. An interesting feature of such problems is that diffusion is naturally governed by degenerate/singular weights. Different geometric behaviors of solutions are noted depending on analytic properties of the medium. In particular solutions may be smoother than the medium along "singular" free boundary points. Several prospective lines of investigation will be discussed. (Received September 20, 2021)

1174-35-9209 **Abba Ramadan*** (aramadan@ku.edu), university of Kansas. *On the standing waves of the Schrödinger equation with concentrated nonlinearity*

We study the concentrated NLS on $\mathbb{R}^n$, with power non-linearities, driven by the fractional Laplacian, $(-\Delta)^s, s > \frac{n}{2}$. We construct the solitary waves explicitly, in an optimal range of the parameters, so that they belong to the natural energy space H^s. Next, we provide a complete classification of their spectral stability. Finally, we show that the waves are non-degenerate and consequently orbitally stable, whenever they are spectrally stable.

Incidentally, our construction shows that the soliton profiles for the concentrated NLS are in fact exact minimizers of the Sobolev embedding $H^s(\mathbb{R}^n) \hookrightarrow L^\infty(\mathbb{R}^n)$, which provides an alternative calculation and justification of the sharp constants in these inequalities. (Received September 20, 2021)

1174-35-9245 **Ratnasingham Shivaji** (shivaji102@gmail.com), University of North Carolina Greensboro, **Byungjae Son*** (byungjae.son@maine.edu), University of Maine, and **Nalin Fonseka** (g_fonsek@uncg.edu), Carolina University. *A diffusive weak Allee effect model with U-shaped emigration and matrix hostility*

We study positive solutions to steady state reaction diffusion equations

$$\begin{cases} -\Delta u = \lambda f(u),\ x \in \Omega, \\ \alpha(u)\frac{\partial u}{\partial \eta} + \gamma\sqrt{\lambda}[1-\alpha(u)]u = 0,\ x \in \partial\Omega, \end{cases}$$

where u is the population density, $f(u) = \frac{1}{a}u(u+a)(1-u)$ represents a weak Allee effect type growth of the population with $a \in (0,1)$, $\alpha(u)$ is the probability of the population staying in the habitat Ω when it reaches the boundary, and positive parameters λ and γ represent the domain scaling and effective exterior matrix hostility, respectively. In particular, we analyze the case when $\alpha(s) = \frac{1}{[1+(A-s)^2+\epsilon]}$, where $A \in (0,1)$ and $\epsilon \geq 0$. In this case, $1 - \alpha(s)$ represents a U-shaped relationship between density and emigration. (Received September 20, 2021)

1174-35-9301 **Amila Muthunayake*** (axm7861@miami.edu), University of Miami. *Modeling the effects of trait-mediated dispersal on coexistence of mutualists*

We analyse positive solutions (u, v) to the steady state reaction diffusion system:

$$\begin{cases} -\Delta u = \lambda u(1-u); & \Omega \\ -\Delta v = \lambda r v(1-v); & \Omega \\ \dfrac{\partial u}{\partial \eta} + \sqrt{\lambda} g(v) u = 0; & \partial\Omega \\ \dfrac{\partial v}{\partial \eta} + \sqrt{\lambda} h(u) v = 0; & \partial\Omega \end{cases}$$

where $\lambda > 0$, $r > 0$ are parameters and $g, h \in C^1([0,\infty),(0,\infty))$ are decreasing functions. This system models the steady states of two species living in a habitat where the interaction is limited to the boundary. Here, λ is directly proportional to the size of the habitat and we will study the ranges of λ where coexistence and nonexistence occurs. Namely, we will consider three cases: (a) $E_1(1,g(0)) = E_1(r,h(0))$, (b) $E_1(1,g(0)) > E_1(r,h(0))$, (c) $E_1(1,g(0)) < E_1(r,h(0))$. Here $E_1(r,K)$ denotes the principal eigenvalue of: $-\Delta z = rEz;\ \Omega$, $\frac{\partial z}{\partial \eta} + K\sqrt{E}z = 0;\ \partial\Omega$.

*co-authors: J. T. Cronin, , J. Goddard II and R. Shivaji. (Received September 20, 2021)

1174-35-9311 **Nicola Garofalo*** (rembrandt54@gmail.com), Department of Civil and Environmental Engineering/University of Padova, ITALY, and **Giulio Tralli** (giulio.tralli@unipd.it), Department of Civil and Environmental Engineering/University of Padova. *Heat kernels for a class of hybrid evolution equations* Preliminary report.

In this talk I discuss the construction of explicit heat kernels for a class of hybrid evolution equations of interest in conformal CR geometry and in the applied sciences. For instance, these equations arise in a certain extension procedure for the conformal fractional powers of horizontal Laplacians in groups of Heisenberg type. By hybrid I mean that the relevant partial differential operator appears in the form $L_1 + L_2 - \partial_t$, but the variables cannot be decoupled. Consequently, the relative heat kernel cannot be written as the product of the heat kernels of the operators $L_1 - \partial_t$ and $L_2 - \partial_t$. This is joint work with Giulio Tralli. (Received September 20, 2021)

1174-35-9344 **Jacob Shapiro*** (jzshapiro@gmail.com), University of Dayton, and **Jeffrey Galkowski** (j.galkowski@ucl.ac.uk), University College London. *Semiclassical resolvent bounds for long range Lipschitz potentials*

We give an elementary proof of weighted resolvent estimates for the semiclassical Schrödinger operator $-h^2\Delta + V(x) - E$ in dimension $n \neq 2$, where h, $E > 0$. The potential is real-valued, V and $\partial_r V$ exhibit long range decay at infinity, and may grow like a sufficiently small negative power of r as $r \to 0$. The resolvent norm grows exponentially in h^{-1}, but near infinity it grows linearly. When V is compactly supported, we obtain linear growth if the resolvent is multiplied by weights supported outside a ball of radius $CE^{-1/2}$ for some $C > 0$. The E-dependence is sharp and answers a question of Datchev and Jin. This is joint work with Jeffrey Galkowski. (Received September 20, 2021)

1174-35-9375 **James M. Scott*** (jms2555@columbia.edu), Columbia University. *The Fractional Lamé-Navier Operator in Local and Nonlocal Continuum Mechanics*

We find an explicit formulation for fractional powers of the Lamé-Navier operator of linear elasticity. We show that this "fractional Lamé-Navier" operator appears in several models in continuum mechanics. The operator coincides with the equations of motion in state-based peridynamics for a particular choice of parameters. The half-power of the Lamé-Navier operator also appears as the Dirichlet-to-Neumann map associated to the constitutive equations of an isotropic homogeneous elastic solid occupying a half-space. We establish basic calculus properties of the operators, and derive fundamental solutions in well-known function space classes. Finally, we derive a Caffarelli-Silvestre extension problem associated to the fractional Lamé-Navier equation. We show that mean-value properties, regularity, and calculus identities for solutions to the fractional Lamé-Navier equations can be derived from this degenerate elliptic system. (Received September 20, 2021)

1174-35-9481 **Lyudmila Korobenko*** (korobenko@reed.edu), Reed College, and **Eric Sawyer** (sawyer@mcmaster.ca), McMaster University. *Hypoellipticity via sums of squares*

Many results on hypoellipticity of second order operators rely on the assumption that the operator can be written as a sum of squares of vector fields (e.g. Hörmander's bracket condition, and Christ's hypoellipticity theorem for infinitely degenerate operators). For operators that are not subelliptic and not sums of squares, hypoellipticity have been only proved in some very special cases, for example, when $L = L_1 + g(x)L_2$ and L_1 and L_2 are subelliptic.

In this talk I will address the question of hypoellipticity for a general divergence form operator, whose matrix is comparable, but not necessarily equal, to a diagonal matrix of a special form. The idea is to find sharp sufficient conditions which guarantee that a smooth positive matrix can be written as a sum of squares of positive dyads with sufficient degree of smoothness. Interestingly, this question have not been completely resolved even for scalar positive functions. (Received September 20, 2021)

1174-35-9870 **Evan Patrick Davis*** (epdavis4@wisc.edu), University of Wisconsin-Madison, **Guangming Yao** (gyao@clarkson.edu), Clarkson University, **Kalani Rubasinghe** (rubasik@clarkson.edu), Clarkson University, **Elizabeth Javor** (lrj8881@rit.edu), Rochester Institute of Technology, and **Luis Antonio Topete Galván** (to358433@uaeh.edu.mx), Universidad Autónoma del Estado de Hidalgo. *Deep Learning Techniques for Solving Semilinear Parabolic Partial Differential Equations* Preliminary report.

Problems in physics, engineering, finance, and many other fields require solving partial differential equations (PDE). It is preferable to get an analytical solution to PDEs; however, many cannot be solved exactly using analytical methods. This has led to the creation of many numerical algorithms to solve the problems for which analytical solutions cannot be derived. Deep learning techniques can be useful for high dimensional PDEs where traditional numerical methods such as finite difference or finite element methods fail due to the curse of dimensionality. A backwards stochastic differential equation (BSDE) solver, as introduced by Jiequn Han, Arnulf Jentzen, and Weinan E, can be used to solve semilinear parabolic PDEs in both low and high dimensions. Euler's method is used to discretize the time space, and deep learning techniques are used to approximate changes in spatial variables. For semilinear parabolic PDEs, this approach enables the PDE to be solved numerically. Such an approach was used to investigate examples including the heat equation, a simple reaction-diffusion equation, and the Black-Scholes equation. The results were compared to analytical solutions and other numerical methods. The numerical results show that the BSDE solver is capable of numerically solving PDEs in high dimensions, and also comparable to traditional numerical techniques in low dimensions in terms of accuracy. (Received September 21, 2021)

1174-35-9914 **Charis Tsikkou*** (tsikkou@math.wvu.edu), West Virginia University, and **Helge Jenssen** (jenssen@math.psu.edu), The Pennsylvania State University. *Amplitude Blowup in Radial Non-Isentropic Euler Flow*

We show that the full compressible Euler system admits unbounded solutions. The examples are radial flows of similarity type and describe a spherically symmetric and continuous wave moving toward the origin. At time of focusing, the primary flow variables suffer amplitude blowup at the origin. The flow is continued beyond collapse and gives rise to an expanding shock wave. We verify that the resulting flow provides a genuine weak solution to the full, multi-d compressible Euler system. While unbounded radial Euler flows have been known since the work of Guderley (1942), those are at the borderline of the regime covered by the Euler model: the upstream pressure field vanishes identically (either because of vanishing temperature or vanishing density there). In contrast, the solutions we build exhibit an everywhere strictly positive pressure field, demonstrating that the geometric effect of wave focusing is strong enough on its own to generate unbounded values of primary flow variables. (Received September 21, 2021)

1174-35-10069 **Luke McClennan*** (luke.mclennan@cameron.edu), Cameron University, and **Janak Joshi** (jjoshi@cameron.edu), Cameron University. *Numerical Simulation of one dimensional Allen Cahn Equation*

In this project, we numerically solve the one dimensional Allen-Cahn equation. The Allen-Cahn equation is a nonlinear reaction-diffusion equation of the type $u_t = \delta u(x,t) - 1/\epsilon^2 f(u)$, where the parameter ϵ is a small positive constant. We use finite differences in our numerical method. We assume that the reaction term $f(u)$ is an odd, locally Lipschitz, nonlinear function whose primitive $F(u)$ is a double-well potential. We use various finite difference schemes, including the Crank-Nicolson method to simulate the Allen-Cahn equation with Dirichlet boundary conditions. We demonstrate the accuracy of our method with several numerical experiments. (Received September 21, 2021)

1174-35-10207 **Jerome A Goldstein*** (jgoldste@memphis.edu), University of Memphis. *Equipartition of Energy for an Ill-posed system with Cross Friction*

Of concern is the cross friction system

$$u_{tt} + 2F(S)\, v_t + S^2 u = 0,$$
$$v_{tt} + 2F(S)\, u_t + S^2 v = 0,$$

where $S = S^* \geq 0$ on a complex Hilbert space $\mathcal{H}$ and $F(S)$ is the linear damping operator, F being a nonnegative real function on $\mathbb{R}$. The initial value problem for this cross friction system is well-posed on $\mathbb{R}$ if and only if $F(S)$ is a bounded linear operator. We allow F to be unbounded and assume (hyperbolicity)

$$\mathcal{D}(F(S)) \supset \mathcal{D}\left(S^{1-\varepsilon}\right)$$

for some $\varepsilon > 0$. Then various forms of weighted energy (potential and kinetic) can be defined and asymptotic equipartition of energy holds when A is spectrally absolutely continuous. Connections with Brownian motion and Poisson processes are explained.

This is joint work with Guillermo Reyes. (Received September 21, 2021)

1174-35-10319 **Sara Jane Lynn*** (slynn@zagmail.gonzaga.edu), Gonzaga University, and **Max A Pansegrau** (mpansegrau@zagmail.gonzaga.edu), Gonzaga University. *Modeling the Movement of a Tree Blowing in the Wind* Preliminary report.

We have mathematically modeled the movement of a tree via the partial differential equation

$$u_{tt} = -c^2 u_{xxxx} - ku_t + f(x),$$

where $u(x,t)$ describes the displacement of the tree from equilibrium as a function of the distance from the ground x and time t.

The tree was assumed to be a cantilever beam with the top end free and bottom end embedded. We included a damping term, and the forcing function was modeled as a wind profile load which is dependent on the distance from the ground.

Separation of variables was used to find an analytic solution; we then used MATLAB to simulate and visualize the effects of the forcing function and other parameters. We are currently exploring allowing the forcing function to also be a function of time. (Received September 21, 2021)

1174-35-10347 **Derek Kielty*** (dkielty2@illinois.edu), University of Illinois Urbana-Champaign.
Spectral gaps of Robin eigenvalue problems

In this talk I will discuss a number of recent results concerning estimates on spectral gaps of the Laplacian and Schrodinger operators with Robin boundary conditions. These will include sharp lower bounds on the spectral gap of 1-dimensional Schrodinger operators, as well as upper bounds in higher dimensions that demonstrate the spectral gap can be arbitrarily small among domains of a given diameter when the Robin parameter is negative. (Received September 21, 2021)

1174-35-10461 **Gisèle Ruiz Goldstein*** (ggoldste@memphis.edu), Department of Mathematical Sciences.
Recent Results in Mathematical Finance

Major developments in mathematical finance have come from the study of two deterministic parabolic partial differential equations, the Nobel Prize winning Black-Scholes equation for stock options

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2} x^2 \frac{\partial^2 u}{\partial x^2} + rx\frac{\partial u}{\partial x} - ru,$$

and the Cox-Ingersoll-Ross equation for zero coupon bonds

$$\frac{\partial u}{\partial t} = \frac{\sigma^2}{2} x \frac{\partial^2 u}{\partial x^2} + (\beta x + \gamma)\frac{\partial u}{\partial x} - xu,$$

where $(x,t) \in (0,\infty) \times [0,\infty)$. Each has a particular initial condition $u(x,0) = u_0(x)$ of relevance in economics.

We study these problems in weighted sup norm Banach spaces whose functions are unbounded near infinity and possibly also near 0. The Black-Scholes equation is governed by a semigroup that is strongly continuous, quasicontractive, and chaotic. New extentions to time dependent coefficients will be given for this model. The Cox-Ingersoll-Ross equation is governed by a strongly continuous quasicontractive semigroup, and the solution is given by a new type of Feynman-Kac formula. New extensions to more general potential terms will be explained. (Received September 21, 2021)

1174-35-10492 **Suyi Gao*** (sg032@mcdaniel.edu), McDaniel College. *Laplacian on the 2-Fibnacci Fractal at the Critical Angle* Preliminary report.

We studied the 2-Fibonacci Word Fractal generated by an iterated function system and then constructed a finitely ramified structure for 2-Fibonacci Word Fractal at a critical angle where the fractal overlaps itself at only a set of discrete points. We also wrote an algorithm that generates graph approximations of the 2-Fibonacci Word Fractal and calculated the eigenvalues and eigenfunctions of the associated Laplacian and numerically approximated the spectral dimension of the 2-Fibonacci Word Fractal. (Received September 21, 2021)

1174-35-10541 **Hayley Olson*** (hayley.olson@huskers.unl.edu), University of Nebraska-Lincoln.
Tempered Fractional Models: Integration Into Nonlocal Vector Calculus and Comparison to a Computationally Cheaper Alternative Preliminary report.

Tempered fractional calculus has use modeling phenomena in hydrology and geophysics. However, the tempered fractional models are computationally expensive, which limits their usefulness in applications. Here, we integrate the tempered fractional Laplacian into the broader nonlocal calculus framework, then compare it to another variation of fractional operators – the truncated fractional Laplacian – which is computationally cheaper and thus more efficient to use in modeling and computations. In particular, we provide an equivalence of the energies of the tempered and truncated fractional Laplacians. Then, we investigate training parameters of the truncated fractional Laplacian using neural networks in order to make the actions of the two operators equivalent. This research was done as part of the NSF Mathematical Sciences Graduate Internship Program (MSGI) hosted by Sandia National Laboratories. (Received September 21, 2021)

1174-35-10712 **Changyou Wang*** (wang2482@purdue.edu), Purdue University. *Analysis of Phase Transition Problems in Nematic Liquid Crystals*

Liquid crystal droplets are of great interest from physics and applications. Rigorous mathematical analysis is challenging as the problem involves harmonic maps, free interfaces and topological defects which can occur either inside the droplet or on its surface along with some intriguing boundary anchoring conditions for the orientation configurations. In this talk, I will present some recent result (joint with Fanghua Lin) on the phase transition between the isotropic and nematic states of minimizers in the Ericksen model of liquid crystal via the framework of Γ-convergence theory. This Γ-limit is a sharp interface limit for the phase transition between the isotropic and nematic regions when the small parameter, corresponding to the transition layer width, goes to zero. This limiting process provides a geometric description of the shape of the droplet as well anchoring conditions for the liquid crystal orientations on the surface of the droplet, which can be either homeotropic, planar, or free boundary type, which has been assumed in phenomenological modelings. (Received September 21, 2021)

1174-35-10769 **Luis Caicedo Torres*** (lcaic005@fiu.edu), Florida International University, and **Ciprian G. Gal** (cgal@fiu.edu), Florida International University. *Superdiffusive fractional in time nonlinear Schrodinger equations: A unifying approach to superdiffusive waves* Preliminary report.

We consider the fractional in time initial value problem in what follows:

$$\begin{cases} i^\beta \partial_t^\beta u = Au + g(u), & (t,x) \in (0,\infty) \times X, \\ u\,|_{t=0} = u_0, & \partial_t u.\,|_{t=0} = u_1. \end{cases}$$

or $1 < \beta < 2$. The fractional in time derivative is the Caputo derivative. Such equations are sometimes dubbed superdiffusive fractional in time wave equations and will serve as the proper interpolation between the classical nonlinear Schrödinger equation and the classical wave equation. Here, A is assumed to be a nonnegative self-adjoint operator on $L^2(X)$ where X is to be a locally compact metric space. We provide examples where X could be a smooth Riemannian manifold, a quantum graph, d-set, etc. We obtain results on existence and regularity of integral wave solutions and energy wave solutions for the nonlinear equation in this context. (Received September 21, 2021)

1174-35-10790 **John Eoghan O'Keefe*** (okeeje18@wfu.edu), Wake Forest University. *Finite Time Blowup of the Nonlinear Schrödinger Equation with a Delta Potential*

The Nonlinear Schrödinger equation (NLS) with a delta potential (NLS-Delta) is: $iu_t + \Delta u + |u|^{2\sigma} u + c\delta u = 0$, with $u(0,t) = u_0$, where $u(\mathbf{x}, t)$ is a complex valued function of an n-dimensional vector $\mathbf{x}$, and time t. The delta potential δ is the Dirac delta functional. We started with proving a variance identity for the NLS-Delta and showed that the Hamiltonian is conserved. Using these facts, we then used arguments from analysis to show that given certain conditions, which depended on the Hamiltonian, the NLS-Delta would blowup in finite time. We proved these facts initially for the NLS without the delta potential by reading and reproving some of the Theorems presented in *The Nonlinear Schrödinger Equation: Self-Focusing and Wave Collapse* (Sulem and Sulem, 1999) that argued for the blowup of the NLS. We then expanded those arguments to the NLS-Delta. (Received September 21, 2021)

1174-35-10938 **Enrico Valdinoci*** (enrico.valdinoci@uwa.edu.au), University of Western Australia. *Perspectives on nonlocal minimal surfaces*

Minimizers of nonlocal perimeter functionals arise naturally both as limit interfaces of long-range phase transitions and as numerically convenient approximations of classical minimal surfaces. In this talk we review some recent results and discuss some open problems. (Received September 21, 2021)

1174-35-11017 **Alex David Rodriguez*** (arodr1128@fiu.edu), Florida International University. *Nonlinear Schrödinger equation with combined nonlinearities in 1D* Preliminary report.

We consider the nonlinear Schrödinger equation with finitely many combined nonlinearities $iu_t + \partial_x^2 u \pm \sum_{j=1}^N \epsilon_j |u|^{\alpha_j} u = 0$ with $\alpha_j > 0$, $x \in \mathbb{R}$. We investigate the local well-posedness of this equation for any positive powers of α for a certain class of initial data, subset in H^1. Using the pseudo-conformal transformation, we obtain global solutions (in the focusing case for $\alpha_j < 4$). Solutions with the quadratic phase $e^{ib|x|^2}$ and a sufficiently large positive b scatter in H^1. In the focusing case we also investigate scattering and blow-up numerically and show examples. (Based on the joint work with G. Azcotia, O. Riaño, S. Roudenko and H. Wubben) (Received September 21, 2021)

1174-35-11749 **Mustafa Aggul** (mustafaggul@hacettepe.edu.tr), Hacettepe University. *The NS-Omega Model for Fluid-Fluid Interaction at High Reynolds Numbers*

This report introduces the NS-ω turbulence model for a fluid-fluid interaction problem, in which two different fluids exchange energy across a fixed boundary. Prior work has shown that, in order to obtain a reliable model that decouples the problem at high Reynolds numbers, one needs to be careful in the treatment of the joint interface. We consider the NS-ω model, coupled with the geometric averaging approach for the stable decoupling of the flows. The new model is shown to be numerically stable and it has optimal convergence properties. We also validate the model numerically, using existing and novel test problems for fluid-fluid interaction. (Received October 1, 2021)

37 ► Dynamical systems and ergodic theory

1174-37-5562 **Sungju Moon*** (sjmoon90@snu.ac.kr), Seoul National University, and **Jong-Jin Baik** (jjbaik@snu.ac.kr), Seoul National University. *The* $(3N)$*- and* $(3N+2)$*-dimensional generalizations of the Lorenz system, chaos synchronization, and their applications as a testbed model for data assimilation algorithms*

Lorenz's 1963 system is generalized to $(3N)$ and $(3N+2)$ dimensions, N a positive integer. Motivated by their chaos synchronization properties, we explore the feasibility of using these systems to test data assimilation algorithms. The generalizations extend the Lorenz system to higher dimensions by including additional wavenumber modes from the governing equations for Rayleigh-Bénard convection. It can be shown that additional nonlinear terms in the generalizations are completely determined by a set of polynomials, allowing for computational generation of higher-dimensional Lorenz systems for any given N. Numerical experiments reveal that, like the original Lorenz system, these generalized systems exhibit chaos self-synchronization. Furthermore, the closer the dimensions between the two systems, the better their chaotic solutions tend to synchronize. For data assimilation experiments, we fix a certain high-dimensional Lorenz system as the true system and sample observations from it. Model imperfections are simulated by taking a lower-dimensional system as the model system. The ensemble Kalman filter is used to generate initial benchmark results considering a number of scenarios with different ensemble sizes, model dimensions, and observation accuracies. (Received August 23, 2021)

1174-37-5619 **Feng Fu*** (feng.fu@dartmouth.edu), Dartmouth College. *Social Dilemma of Disease Control*

The global siege of SARS-CoV-2, the pathogen causing COVID-19 infections, has upended everyone's normal life, and caused health crises, lockdowns, and economic percussions at an unprecedented pace and scale. Top-down intervention approaches, like drastic lockdowns and public health mandates of mask wearing and social distancing, consider the society as a whole and attempt to optimize pandemic mitigation measures from the perspective of central planners. On the contrary, adopting personal intervention measures such as face covering and vaccine uptake incurs a cost to oneself, and thus the tragedy of the commons can arise as a result of 'free-riding' in this important context. The multiple waves of infections fueled by the virus evolution and compliance and adherence issues underscore the importance of integrating both top-down and bottom-up modeling approaches. This talk will focus on the social dilemma aspect of disease control and unravel the previously unforeseen "hysteresis" effect in bottom-up public health behavior dynamics that is responsible for resistance to top-down public health recommendations, ranging from mask hysteria, distancing disobedience, and vaccine hesitancy. Synergistic integration of top-down and bottom-up perspectives is needed to increase the awareness and preparedness of epidemics of disease. (Received August 23, 2021)

1174-37-5626 **Rachel Kuske** (rachel@math.gatech.edu), Georgia Tech, and **Ruofeng Liu*** (rl79@rice.edu), Rice University. *Stability and Bifurcation Analysis of a Vibro-Impact Nonlinear Energy Sink With Poincare Map* Preliminary report.

A nonlinear energy sink (NES) is a nonlinear oscillator with variable stiffness which can suppress vibration over a wide range of frequencies. We study a vibro-impact NES (VI-NES), based on an impact pair of masses, where a ball moves freely in a cavity inside a primary mass, impacting both ends of the cavity. Previous studies of this VI-NES focus on the method of multiple scales to approximate the dynamics of 1:1 internal resonance, assuming a small ball to primary mass ratio.

Recent studies have used Poincare maps to analyse the stability and bifurcation of vibro-impact energy harvesters based on a similar impact pair. Adapting this methodology for the VI-NES system allows for an exact analysis of periodic solutions and asymmetric states that capture more complicated resonance scenarios without the need for the small mass ratio assumption. The maps are derived using both the ODEs for the dynamics between impacts and the non-smooth impact conditions. From the composition of these maps we obtain semi-analytical expressions for periodic solutions and perform the linear stability analysis of these states, all of which agree with numerical experiments. Finally, these results are used to compare the energy transfer efficiency for different states that vary with system parameters. Not only does this work show the applicability of Poincare maps to more complicated vibro-impact systems, but it also provides an exact, analytical approach for studying targeted energy transfer parametrically and efficiently. (Received August 29, 2021)

1174-37-5687 **Brian Harry Marcus*** (marcus@math.ubc.ca), UBC, Vancouver. *Examples of polynomial time approximation of topological entropy and Gibbs free energy*

The topological entropy of a typical Z^2 shift of finite type (SFT) is notoriously difficult to compute. The same can be said for the more general problem of computing Gibbs free energy (a.k.a. topological pressure) of a nearest neighbour interaction for a Z^2 model in statistical physics. Aside from some very specific instances, such as the Z^2 3-colored checkerboard SFT and the Z^2 ferromagnetic Ising model with no external field, there are no known closed form expressions. On the other hand, there are many approaches that have yielded excellent approximations, and these continue to improve with more powerful commputation methods and devices. This begs the question: is there a rigorous sense of a "good" convergent sequence of approximations that can yield the topological entropy or Gibbs free energy? One possible answer comes from the theory of approximation algorithms, where one asks for a convergent sequence of approximations with polynomial accuracy computed in polynomial time. In this talk, we give examples of such polynomial approximation schemes from joint work with Stefan Adams, Raimundo Briceno, and Ronnie Pavlov. (Received August 24, 2021)

1174-37-5974 **Maleafisha Joseph Pekwa Stephen Tladi*** (`Maleafisha.tladi@gmail.com`), African Scientific Institute. *Well-Posedness and Long-Time Dynamics of the Rotating Boussinesq and Quasigeostrophic equations*

The author elucidates in a concrete way dynamical challenges concerning approximate inertial manifolds (AIMS), i.e., globally invariant, exponentially attracting, finite-dimensional smooth manifolds, for nonlinear dynamical systems on Hilbert spaces. The goal of this theory is to prove the basic theorem of approximation dynamics, wherein it is shown that there is a fundamental connection between the order of the approximating manifold and the well-posedness and long-time dynamics of the rotating Boussinesq and quasigeostrophic equations describing the motion of rotating stratified fluid flows. (Received September 1, 2021)

1174-37-6865 **Alan A Sola** (`sola@math.su.se`), Stockholm University, Sweden, and **Ryan K. Tully-Doyle*** (`rtullydo@calpoly.edu`), Cal Poly SLO. *Dynamics of low-degree rational inner skew-products on* $\mathbb{T}^2$

We consider a family of two-variable maps of the form $\Phi(z,w) = (\phi_1(z,w), \phi_2(w))$, where ϕ_1 is an analytic maps $\mathbb{D}^{\nvDash} \to \mathbb{D}$ and $\mathbb{T}^{\nvDash} \to \mathbb{T}$. Such a map Φ is an example of a rational inner skew-product, or RISP. We will examine the the dynamics of such RISPs on the 2-torus, which show features not present in the related two-variable Blaschke products.

We'll also look at a family of maps arising from a Nevanlinna-type representation applied to matrices associated with simple graphs, which provides a rich family of examples. (Received September 9, 2021)

1174-37-6967 **David M. McClendon*** (`mcclend2@ferris.edu`), Ferris State University, and **Aimee S.A. Johnson** (`aimee@swarthmore.edu`), Swarthmore College. *Bounded speedups of* $\mathbb{Z}^d$*-odometers* Preliminary report.

Given a dynamical system (X,T), a speedup of (X,T) is another dynamical system (X,T^p) where $p : X \to \{1,2,3,\dots\}$. We are interested two big-picture questions involving speedups: (1) to what degree must (X,T^p) be "equivalent" to (X,T)? (2) given two dynamical systems, when does there exist a speedup of the first that is "equivalent" to the second? In this talk, we will briefly review results addressing both questions.

Then, we will discuss what is meant by a speedup of an action of $\mathbb{Z}^d$. In particular, we will give some general results for minimal $\mathbb{Z}^d$-Cantor systems, focusing on speedups of $\mathbb{Z}^d$-odometers. One notable result generalizes work of Alvin, Ash and Ormes: we will show is that a bounded speedup of a $\mathbb{Z}^d$-odometer must be an odometer (as in the $\mathbb{Z}$ case), but unlike the $\mathbb{Z}$ case, the speedup need not be topologically conjugate to the original odometer. (Received September 10, 2021)

1174-37-7049 **Kimberly Ayers*** (`kayers@csusm.edu`), Cal State San Marcos. *Harris Irreducibility of the Stochastic Logistic Map*

In this presentation, we will consider the stochastic logistic map, given by the function $f(x) = \lambda x(1-x)$, where λ takes values according to a uniform distribution on $[3.87, 4]$. This map takes the closed unit interval $[0,1]$ to itself. When considering logistic maps, limiting behavior is given by considering asymptotically stable invariant distributions rather than sets. Here we will show that the stochastic logistic map demonstrates Harris irreducibility, a key component of showing that the stochastic logistic map demonstrates asymptotic stability on the entire interval [0,1]. This work builds on the work of Krishna Athreya et. al. (Received September 11, 2021)

1174-37-7269 **Stephanie Dodson*** (`sadodson@ucdavis.edu`), University of California, Davis, **Emily Meyer** (`eemeyer@arizona.edu`), University of Arizona, and **Timothy J Lewis** (`tjlewis@ucdavis.edu`), University of California, Davis. *When curvature promotes or obstructs the ability of a pacemaking region to drive activity in excitable tissue*

In cardiac tissue, the sinoatrial node (SAN) is responsible for initiating the periodic electrical pulses underlying heart beats. However, other regions of local heterogeneous tissue (e.g., ischemic regions) can act as pacemakers and produce oscillations in neighboring tissue that compete with the natural pacemaking of the SAN and cause potentially life-threatening arrhythmias. Thus, it is important to understand the physiological conditions that enable the SAN to robustly act as the cardiac pacemaker and for local depolarized regions of tissue to form pathological rhythms. It is well known that small heterogeneities (sources) should not be able to easily activate a large area of excitable tissue (sink). On a local level, this source-sink balance implies that positive curvature of a pacemaking region reduces the ability to drive the neighboring tissue. However, while numerous studies provide evidence that supports the source-sink balance relationship in which high curvature deters oscillations, other studies have shown that for some depolarized heterogeneities, oscillations tend to emerge from corners and other areas of high curvature. Here, we use an idealized two-domain reaction-diffusion system and corresponding two-cell model to bridge the gap between these seemingly opposing viewpoints. In doing so, we identify the conditions for which curvature of a pacemaking region promotes or obstructs the production of oscillations in the neighboring tissue. (Received September 13, 2021)

1174-37-7416 **Maxwell Joseph Fox*** (maxwelljfox@gmail.com), University of Alabama in Huntsville, and **Shangbing Ai** (ais@uah.edu), University of Alabama in Huntsville. *Four positive equilibria in a model for sterile and wild mosquito populations*

A well proposed ODE system that models the interactive dynamics of sterile and wild mosquito populations with a saturated release rate of sterile mosquitoes can have up to four positive equilibria, but only two positive equilibria have been confirmed to exist in the literature. In this note we establish that for a range of parameters this system does have four positive equilibria. We also obtain the stability of these equilibria. (Received September 14, 2021)

1174-37-7666 **Jasper Weinburd*** (jweinburd@hmc.edu), Harvey Mudd College. *Assessing Locust Interactions within a Swarm*

Locust swarms pose a serious threat to agriculture and present a quintessential example of animal collective motion. In a precursor to the most devastating swarms, flightless juvenile locusts crawl and hop across the ground in directed groups. These *hopper bands* are composed of tens of thousands of insects and have been well-documented in the field; meanwhile, individual locust interactions have been studied in much smaller groups in laboratory settings. We present the first trajectory data that details the movement of individual locusts within a hopper band in a natural setting. We find evidence of distinct motion states, a repulsive length scale, and anisotropic interaction with nearby neighbors. These findings contribute to the biological basis for swarming models that illustrate the effect of the individual rules and the emergent behavior of the swarm. (Received September 15, 2021)

1174-37-7796 **Ethan Brady** (brady54@purdue.edu), Purdue University, **John Hood*** (jhood3@ncsu.edu), Bowdoin College, **Anna Asch** (aasch@ncsu.edu), Cornell University, **Hugo A Gallardo** (hgallar@ncsu.edu), University of Texas Rio Grande Valley, **Bryan Chu** (bchu@ncsu.edu), North Carolina State University, and **Mohammad Farazmand** (farazmand@ncsu.edu), North Carolina State University. *Model-assisted deep learning of extreme events from incomplete observations*

Extreme events such as rogue waves, earthquakes, epileptic seizures, and stock market crashes occur sporadically and have devastating humanitarian, environmental, and financial consequences. Real-time prediction of these events would help with response management and mitigating their most undesirable effects. In practice, a lack of complete measurements of the system state hinders our predictive capabilities. Although deep learning has proven largely effective in predicting chaotic systems, prior work studying extreme events rarely considers the constraint of incomplete information. We use deep neural networks to predict extreme events from sparse and incomplete observations, using feedforward, long-short term memory, and reservoir computing networks. For feedforward networks, augmenting our input data with time delay embeddings improves the prediction accuracy. All networks display comparable predictive power, as measured by mean squared error, precision, and recall. However, long short-term memory networks are most robust to noise in the observational data. We demonstrate our results on two chaotic dynamical systems that exhibit extreme events: a FitzHugh-Nagumo model of the nervous system and a turbulent fluid flow. (Received September 19, 2021)

1174-37-7850 **May Mei*** (meim@denison.edu), Denison University, **David Damanik** (damanik@rice.edu), Rice University, **Mark Embree** (embree@vt.edu), Virginia Tech, and **Jacob D. Fillman** (fillman@txstate.edu), Texas State University. *Boats and Stars and Kites and Darts, Oh My!*

Two tilings are *mutually locally derivable* (mld) if each can be obtained from the other using local rules. In many settings, all tilings in the same mld-class are treated as "the same" tiling. However, local derivability greatly impacts the adjacency relationship between tiles, which in turn results in a vastly different graph Laplacian. We consider four tilings from the Penrose mld-class, arguably the most popular example of an aperiodic tiling: boats and stars, kites and darts, rhombi, and triangles. We show explicitly that each tiling supports a compactly supported eigenfunction. Moreover, we estimate the size of the resulting discontinuity in the integrated density of states from below. (Received September 16, 2021)

1174-37-7927 **Salisu M Garba*** (Salisu.Garba@up.ac.za), Fred Hutchinson Cancer Research Center. *Mathematical Model for assessing the impact of quarantine and isolation on the transmission dynamics of COVID-19*

In this presentation, a new mathematical model for the transmission dynamics of Coronavirus (COVID- 19) is designed and analyzed. The model is used to assess the impact of non-pharmaceutical interventions in controlling the disease in South Africa. By fitting the existing daily mortality incidence with the model plot, we estimated the cumulative epidemic trajectories and future mortality rate. The associated effective and control reproduction numbers are computed. The impact of social distancing, face mask use, quarantine and isolation of individuals infected with COVID-19 is assessed via a threshold analysis approach. Analysis of the model demonstrates that COVID-19 can be controlled effectively in the republic by the combined use of non-pharmaceutical interventions (notably effective face mask use, social distancing, monitoring close contacts, self-isolation and quarantining of suspected asymptomatic individuals). (Received September 16, 2021)

1174-37-8081 **Anush Tserunyan** (anush.tserunyan@gmail.com), McGill University, and **Jenna Zomback*** (zomback2@illinois.edu), University of Illinois at Urbana Champaign. *Ergodic theorems along trees*

In the classical pointwise ergodic theorem for a probability measure preserving (pmp) transformation T, one takes averages of a given integrable function over the intervals $\{x, T(x), T^2(x), \ldots, T^n(x)\}$ in front of the point x. We discuss a new "backward" ergodic theorem for a countable-to-one pmp T, where the averages are taken over subtrees of the graph of T that are rooted at x and lie behind x (in the direction of T^{-1}). Surprisingly, this theorem yields forward ergodic theorems for countable groups, in particular, one for pmp actions of free groups of finite rank, where the averages are taken along subtrees of the standard Cayley graph rooted at the identity. This strengthens Bufetov's theorem from 2000, which was the most general result in this vein. This is joint work with Anush Tserunyan. (Received September 17, 2021)

1174-37-8270 **Rian Boutin*** (rian.boutin@student.fairfield.edu), Fairfield University. *Hyperbolicity in Asymmetrical Lemon Billiards*

Mathematical billiards are important models of dynamical systems from statistical mechanics. This project concerns a class of billiard tables called asymmetrical lemon billiards, which are convex tables formed by two circles of different radii. These billiards exhibit hyperbolicity despite violating the usual defocusing rule for billiards with concave boundaries. By varying two parameters, the radius of the larger circle and the distance between the centers of the circles, we classify different regimes which exhibit nonergodic behavior by proving the existence of elliptic islands. In addition, using a MATLAB simulation, the shape of the elliptic islands and strength of the Lyapunov exponents are analyzed numerically as a function of the parameters. This project was conducted at Fairfield University during Summer 2021 with support from the National Science Foundation. (Received September 18, 2021)

1174-37-8375 **Junze Yin*** (junze@bu.edu), Boston University. *Fractal Analysis of the Urbanization Development in Boston: 2000-2020*

The existing research about fractal analysis focuses on analyzing urbanization via the contour of cities based on their satellite images. The cities' contour resembles a coastline, which does not have a regular shape. Fractal analysis is an indicator of urbanization because it measures the urban sprawl, which is the degree of fragmentation and transition from monocentric to polycentric structures. The fractal dimension is an index that describes the complexity of a geometric object: if when measuring the object we scale the unit by ϵ and the measure gets scaled by N, then the fractal dimension D is defined by

$$\log_\epsilon N = -D.$$

Our goal is to create a visual representation for the changes of Boston's metropolitan area and its growth rate, and to do so we analyze its fractal dimension. Two methods, box-counting and radial analysis are utilized to estimate Boston's fractal dimension, based on census data. A longitudinal study is lacking. We estimate the fractal dimension of Boston over the past 20 years by a time series, and by non-linear regression we project the future extent of Boston's urbanization. By an exclusion algorithm, we also analyze the fractal dimension of Boston boroughs, to bring out the fractal (self-similar) nature of the city.

We compare planimetric maps with satellite images and comment on superior features of one or the other. Lastly, the logistic equation is postulated as a model to fit the data of the change of fractal dimension throughout these years. (Received September 18, 2021)

1174-37-8479 **Chandrika Sadanand** (sadanand@illinois.edu), Bowdoin College, **Felipe Ramírez** (framirez@wesleyan.edu), Wesleyan University, and **Andre P Oliveira** (aoliveira@wesleyan.edu), Wesleyan University. *Straight line flows and the Veech group of the mucube.* Preliminary report.

The dynamics of straight-line flows on compact translation surfaces (surfaces formed by gluing Euclidean polygons edge-to- edge via translations) has been widely studied due to connections to polygonal billiards and Teichmüller theory. However, much less is known regarding straight line flows on non-compact infinite translation surfaces. In this talk we will review work on straight line flows on infinite translation surfaces and consider such a flow on the Mucube – an infinite $\mathbb{Z}^3$ periodic half-translation square-tiled surface – first discovered by Coxeter and Petrie and more recently studied by Athreya-Lee. We will give a complete characterization of the periodic directions for the straight line flow on the Mucube – first in terms of a genus one quotient and secondly in terms of an infinitely generated subgroup of $SL_2(\mathbb{Z})$. Finally we will use the latter characterization to obtain the Veech group of the Mucube. (Received September 19, 2021)

1174-37-8623 **Anca Radulescu*** (radulesa@newpaltz.edu), SUNY New Paltz. *Complex dynamics in templates and mutated systems*

Copying errors which occur during mechanisms like cell differentiation, or tumor growth, are highly local, affecting specific genetic loci in the genome structure. If we use the complex plane to symbolically represent a cell's DNA, each point in the plane corresponds to a single gene. The folding of the plane through a complex map can be seen as a theoretical representation of how the emphasis of each gene is recomputed in the replication process, along the specialization path of the cell. This results each time into a new copy of the plane, used as template for subsequent replications.

To study this process, we insert local mutations into the iteration of a replication map, in the form of errors that act on a small mutation disk, around a mutation focus. We analyze asymptotic trajectories under iterations of the resulting mutated system. We extend the concept of Julia set from its traditional definition in the context of single map iterationsto to the broader context of mutated iterations. We then study the dependence of the Julia set topology on the position, size and timing of mutations.

We interpret the prisoner set as the set of chromosomes that persist along the cell differentiation process, and end up in the stable profile of the cell. This ties our results to potential applications to genetics and modeling of tumor growth processes. (Received September 19, 2021)

1174-37-8765 **Victor Ginsburg*** (victor.ginsburg888@gmail.com), Pennsylvania State University, **Jane Wang** (wangjan@iu.edu), Indiana University Bloomington, **Veronica Kirgios** (vkirgios@nd.edu), University of Notre Dame, **Catherine Cui** (ccui@college.harvard.edu), Harvard University, and **Vanessa Lin** (vllin@live.unc.edu), University of North Carolina at Chapel Hill. *Geodesics on Dilation Surfaces* Preliminary report.

A translation surface is a collection of polygons with sides identified by translation. The geometry and dynamics of translation surfaces is an active research area in Teichmüller theory. Geodesics on translation surfaces have been well-studied, but much less is known about the geodesics on dilation surfaces, a generalization of translation surfaces. We demonstrate that the geodesics on a dilation surface can have dramatically different behavior compared to geodesics on a translation surface. (Received September 19, 2021)

1174-37-8905 **Camille Jane Herman*** (cami.tj@gmail.com), United States Naval Academy, and **Kostya Medynets** (medynets@usna.edu), United States Naval Academy. *Swarm Dynamics on General Riemannian Manifolds* Preliminary report.

Swarms are a collection of self propelled agents that interact to organize and behave as a coherent group. Examples of swarms can be found extensively in nature, such as schools of fish, but their novel application to robotics is highly desirable for the maneuvering of unmanned vehicles. There is an increased interest in developing general theory for swarm dynamics on general Riemannian manifolds. The goal of our project is to simulate swarm dynamics on various Riemannian surfaces, including the hyperbolic plane, sphere, and the torus. We use the Euler-Lagrange approach to incorporate geodesic motion in swarming models. (Received September 20, 2021)

1174-37-9229 **Mari Kawakatsu*** (mari.kawakatsu@princeton.edu), Program in Applied and Computational Mathematics, Princeton University, and **Daniel Larremore** (daniel.larremore@colorado.edu), Department of Computer Science & BioFrontiers Institute, University of Colorado Boulder. *Emergence of hierarchy in networked endorsement dynamics*

Many social and biological systems are characterized by enduring hierarchies, including those organized around prestige in academia, dominance in animal groups, and desirability in online dating. Despite their ubiquity, the general mechanisms that explain the creation and endurance of such hierarchies are not well understood. We introduce a generative model for the dynamics of hierarchies using time-varying networks, in which new links are formed based on the preferences of nodes in the current network and old links are forgotten over time. The model produces a range of hierarchical structures, ranging from egalitarianism to bistable hierarchies, and we derive critical points that separate these regimes in the limit of long system memory. Importantly, our model supports statistical inference, allowing for a principled comparison of generative mechanisms using data. We apply the model to study hierarchical structures in empirical data on hiring patterns among mathematicians, dominance relations among parakeets, and friendships among members of a fraternity, observing several persistent patterns as well as interpretable differences in the generative mechanisms favored by each. Our work contributes to the growing literature on statistically grounded models of time-varying networks. (Received September 20, 2021)

1174-37-9328 **Punit Gandhi** (gandhipr@vcu.edu), Virginia Commonwealth University, and **Mary Silber*** (msilber@uchicago.edu), University of Chicago. *Response of Dryland Vegetation Bands to Stochastic Rain Pulses*

In certain dryland ecosystems, vegetation organizes in bands that alternate with bare soil. Due to increased water infiltration in vegetated zones, compared to barren ones, a redistribution of water resource occurs that sustains biomass at an appropriate coverage fraction. The large-scale bands, spaced 100 meters apart, are oriented transverse to a gentle elevation grade, with slow upslope colonization. This spontaneous pattern formation is often modeled on long ecological timescales, using a reaction-advection-diffusion framework, with bands represented as traveling periodic wave solutions and with mean annual precipitation serving as a bifurcation parameter. However, it is known that the critical pattern formation feedbacks between water resource and biomass density occur on fast hydrological timescales of rain events. Gandhi et al. (Physica D 2020) developed a fast-slow modeling framework that resolves these feedbacks. We modify that to a pulse-response setting, in which rain storms are instantaneous. This speeds up simulations, allowing us to investigate pattern characteristics under stochastic precipitation; rain pulses, during a rainy season, are treated as a Poisson point process, with their intensity drawn from an exponential distribution. We investigate how changes to the mean rain pulse size impacts the patterns and their possible collapse. This suggests critical ways that mean annual precipitation is not all that matters to these fragile dryland ecosystem (Received September 20, 2021)

1174-37-9411 **Jonathan Krueger*** (kruegerj1@csp.edu), Concordia University, St. Paul, **Bianca Teves** (bteves@haverford.edu), Haverford College, and **Emma Anderson** (emmalovespi@gmail.com), Ithaca College. *Asymmetric Fractal Trees: Koch Canopies and Dragon Curves* Preliminary report.

In 1999, Michael Frame and Benoit Mandelbrot wrote a paper discussing self-contacting symmetric fractal trees. Given a certain branching angle, one can find the scaling ratio such that two branches converge to the same point, creating an inverted Koch curve by self-similarity. We then looked at asymmetric fractal trees, formed in one case by shifting one branch down the trunk, and in another case by choosing different left and right angles. These differences yielded unexpected results, from canopies constructed via an affine transformation to space filling dragon curves. (Received September 20, 2021)

1174-37-9421 **Artiom Bic*** (abic01@manhattan.edu), Author, and **Andrew Cirincione** (acirincione01@manhattan.edu), Co-Author. *A Mathematical Analysis of the Inhibition Stabilized Network* Preliminary report.

In the present project we present a mathematical analysis of the mechanisms of neurons in the visual pathway. We focus specifically on neurons in the primary visual cortex (V1), which is responsible for the first stage of visual processing in the brain. The Wilson-Cowan model is commonly implemented to generalize the dynamics of a neural population by classifying neurons into either an excitatory or inhibitory subpopulation. An excitatory neuron is one that increases (excites) the activity of nearby connected neurons, whereas an inhibitory neuron reduces (inhibits) the activity of nearby connected neurons. Neurons are connected via synapses, the junction between two nearby neurons. An Inhibition Stabilized Network (ISN) is a network of neurons stabilized by strong synaptic connections between the inhibitory and excitatory subpopulations. The presence of an ISN has been theorized to allow for important biological phenomena such as surround suppression to be explained. This paper explores the mathematical background of an ISN and attempts to obtain specific sets of pa- rameters to create an ISN. It additionally attempts to expand on the topic of synaptic connections and identifies the relationships between certain network parameters. (Received September 20, 2021)

1174-37-9440 **Cara Sulyok*** (csulyok@lewisu.edu), Lewis University. *A Mathematical Framework to Augment the Q-MARSH Score in the Diagnosis of Celiac Disease*

Celiac disease is a hereditary autoimmune disease that affects approximately 1 in 133 Americans. It is caused by a reaction to the protein gluten found in wheat, rye, and barley. After ingesting gluten, a patient with celiac disease may experience a range of unpleasant symptoms while small intestinal villi, essential to nutrient absorption, are destroyed in an immune-mediated process. The only known treatment for this disease is a lifelong gluten-free diet and there is currently no drug treatment. This work provides a mathematical model to better understand the effects of immune activation on gut health using a system of ordinary differential equations to track changes in small intestinal cell densities and relates them to the Q-MARSH score, a criterion used in the diagnosis of celiac disease. The model can be used to investigate and analyze the immune response and various theories behind the progression of this disease by focusing on understanding the dynamics of the small intestine in situations mirroring healthy function and celiac disease. By doing so, we can assist in further quantifying and augmenting diagnostic measures and investigate potential therapies to mitigate the negative effects of celiac disease. (Received September 20, 2021)

1174-37-9513 **María Isabel Sánchez Muñiz*** (sanch522@umn.edu), University of Minnesota, Twin Cities. *Permafrost response to Climate Change via Budyko's Model* Preliminary report.

Climate change is affecting permafrost thaw depth and the extent of permafrost. As permafrost degrades, stored soil organic carbon gets released into the atmosphere as carbon dioxide and methane. This feedback cycle increases climate change. We will approximate carbon emissions from permafrost using a simple planetary energy balance model coupled with a heat equation. Under certain boundary conditions and parameters, we describe the degradation at an example latitude. (Received September 20, 2021)

1174-37-9542 **Christopher Michael McKay*** (christopher.mckay@montana.edu), Montana State University. *A Reclassification of Critically Fixed Anti-Rational Maps and their Applications*

Given a critically fixed anti-rational map there is a natural construction of a finite, invariant, connected graph, the Tischler graph. Is there a natural way of generating new Tischler graphs from old ones? I show that like critically fixed rational maps, critically fixed anti-rational maps are characterized by "blowing-up" arcs. The main tool for this construction is Thurston's characterization of anti-rational maps and its realization on Tischler graphs. Time allowing, I will outline some open questions surrounding critically fixed anti-rational maps that I am interested in. (Received September 20, 2021)

1174-37-9629 **Mei Rose Connor*** (meirose.connor@stonybrook.edu), Stony Brook University, Stony Brook, NY, **Zachary Steinberg** (zsteinberg23@amherst.edu), Amherst College, and **Paige Helms** (phelm001@ucr.edu), University of Washington. *Hyperbolic Staircases: Periodic Paths on $2g+1$-gons*

The problem of the dynamics of a ball bouncing on a polygon–shaped billiards table, particularly those in the regular pentagon, is a well–studied and beautiful field that has won its proponents two Fields Medals. One recent paper in this field approaches the problem of discovering the periodic trajectories on the pentagon by identifying slopes of periodic directions with points in the Poincaré disk generated by hyperbolic isometric transformations. The other approach, coming from another paper, transforms the double pentagon into a rectilinear translation surface called the 'golden L', where periodic directions are generated by a set of matrices associated to this surface in a special way. We connect and unify these two approaches, and use our unification of these results to generalize them to arbitrary $2g+1$–sided regular polygons. (Received September 20, 2021)

1174-37-9770 **Addie Mae McCurdy** (addiemccurdy@gmail.com), University of St. Thomas, **Nagaprasad Rudrapatna** (nr131@duke.edu), Duke University, **Sharadiant Turner** (sturne43@spelman.edu), Spelman College, and **Ben Maloy*** (Benjamin.Maloy@tufts.edu), Tufts University. *Deconvolution of Temporally Under-Resolved Image Sequences for Coupled Dynamical Systems*

There are many physical systems that can be effectively modeled as dynamical systems with a strange attractor. This fact allows the use of powerful results from dynamical systems theory, namely Takens' Embedding Theorem, to analyze different component variables in the system. Namely, if the system state is given by $(X(t), Y(t))$ and

$Y(t)$ is corrupted by noise while $X(t)$ remains comparatively clear, $X(t)$ can be used to retrieve the denoised $Y(t)$ signal. Recently, a method was proposed by Araki and colleagues (2021) to denoise corrupted signals and reconstruct underlying signals from temporally under-resolved time series (deconvolution). Both denoising and deconvolution involve mapping a clean reference signal to phase space and averaging the corrupted signal within partitions of phase space. We successfully decreased the runtime of the deconvolution algorithm so that it can be applied to a system where many corrupted signals are coupled to the same clean reference signal. We demonstrate the algorithm's effectiveness at deconvoluting time-averaged solutions to the Lorenz system and electrical current signal readings from a Hall-Effect Thruster. (Received September 20, 2021)

1174-37-9773 **Samuel Everett*** (saev1252@colorado.edu), University of Colorado, Boulder, **Zawad Chowdhury** (zawadx@mit.edu), Massachusetts Institute of Technology, and **Destine Lee** (dll2141@columbia.edu), Columbia University. *Computing periodic points on Veech surfaces*

It is natural to ask which points on a translation surface are periodic under the action of the affine automorphism group. It is known that there are finitely many such points in the general case of non-square-tiled surfaces; these points have been exactly determined for all surfaces in genus two and for special cases in higher genus. We present an algorithm that takes a polygonal representation of a non-square-tiled Veech surface and returns its periodic points. The results from this algorithm provide experimental evidence for a classification of periodic points in the genus three Prym eigenforms. (Received September 20, 2021)

1174-37-9828 **Kelly Chen*** (kjchen@mit.edu), MIT. *Periodic Orbits of Affine Interval Exchange Transformations* Preliminary report.

Dilation surfaces are a relatively new class of surfaces which can be represented as collections of polygons in the plane with pairs of parallel opposite edges identified by translation and dilation (scaling). When a path on a dilation surface enters an edge, it emerges in the same direction from the edge identified with the original one. Straight-line paths in dilation surfaces exhibit interesting dynamics, for example attraction onto periodic paths or Cantor-like sets. Straight-line paths on dilation surfaces are closely related to affine interval exchange transformations (AIETs): functions from an interval to itself which cut the interval into subintervals, then permute and rescale the subintervals. In particular, periodic paths on dilation surfaces correspond to periodic orbits of AIETs. We show that almost every AIET has finitely many periodic orbits, and we present computational and theoretical evidence towards our conjecture that almost every AIET has at least one periodic orbit. (Received September 20, 2021)

1174-37-9837 **Anindya Chanda*** (achanda@math.fsu.edu), PhD Student, Department of Mathematics, Florida State University. *Quasigeodesic Anosov flows in Dimension 3*

An Anosov flow on a Riemannian Manifold M is called *Quasigeodesic* if all of its flowlines are *quasigeodesics* in the supporting manifold M. In this talk we will survey old and new results on quasigeodesity of Anosov flows on different classes of 3-manifold. We will also talk about *leafwise quasigeodesity*, i.e, quasigeodesity of flowlines w.r.t the metric restricted to the leaves of a flow saturated two foliation. (Received September 20, 2021)

1174-37-9854 **Kaitlin Hill*** (kaitlin.m.s.hill@gmail.com), Wake Forest University, **John Gemmer** (gemmerj@wfu.edu), Wake Forest University, and **Jessica Zanetell** (jzanetell@math.arizona.edu), Wake Forest University. *Most probable transition paths in piecewise-smooth stochastic differential equations* Preliminary report.

We develop a path integral framework for determining most probable paths in a class of systems of stochastic differential equations with piecewise-smooth drift and additive noise. This approach extends the Freidlin-Wentzell theory of large deviations to cases where the system is piecewise-smooth and may be non-autonomous. In particular, we consider an $n-$dimensional system with a switching manifold in the drift that forms an $(n-1)-$dimensional hyperplane and investigate noise-induced transitions between metastable states on either side of the switching manifold. We mollify the drift and use $\Gamma-$convergence to derive an appropriate rate functional for the system in the piecewise-smooth limit. This functional is the $\Gamma-$limit of the Freidlin-Wentzell rate functional for the mollified system, with an additional contribution due to times when the most probable path slides in a crossing region of the switching manifold. We explore implications of the derived functional through two case studies, which exhibit notable phenomena such as non-unique most probable paths and noise-induced sliding in a crossing region. (Received September 20, 2021)

1174-37-10073 **Christopher L Cox*** (clcox@udel.edu), University of Delaware, and **Jan Ahmed** (janahmed@udel.edu), University of Delaware. *The Dynamics of Billiards with Particles of Variable Mass Distribution*

Billiards systems have proved to be a useful tool in generally understanding dynamical systems, in part because of the flexibility in creating a variety of dynamics from regular to chaotic by varying the geometry of the billiard table. We show that a similar range of behaviors may arise from varying the mass distribution of the colliding particle; indeed, new dynamics arise which are unattainable in standard billiard systems. (Received September 21, 2021)

1174-37-10077 **Hanna Yang** (hannay@mit.edu), Massachusetts Institute of Technology, and **Brin Harper** (brinh@mit.edu), Massachusetts Institute of Technology. *Towards a Classification of Veech 12-Gons and Trapezoidal Unfoldings*

Veech surfaces are translation surfaces with closed $GL(2,\mathbb{R})$ orbits and optimal dynamics. The classification of Veech surfaces is an open question even in low genus. We study the power of the Kenyon-Smillie J-invariant (a 2-dimensional analogue of the Dehn invariant used to solve Hilbert's third problem) as a tool for distinguishing between Veech and non-Veech surfaces. Making use of the J-invariant, we approach the classification problem in the stratum $\mathcal{H}(4)$. We explore the power of the J-invariant in comparison with newer classification tools from geometric topology introduced by Mirzakhani-Wright. Finally, we use these tools in tandem to present a new classification theorem for convex 12-gons with $\pi/3$-rotational symmetry as well as unfoldings of right trapezoids with smallest angle $\pi/2n$. This work was conducted as part of the Summer@ICERM REU 2021. (Received September 21, 2021)

1174-37-10157 **Alexander Berliner*** (aiberliner@email.wm.edu), Department of Mathematics, College of William and Mary, Williamsburg, Virginia, USA. *Period Doubling Cascades from Models and from Data* Preliminary report.

Orbit diagrams of period doubling cascades represent systems going from periodicity to chaos. Here, we investigate whether a Gaussian process can be used to reconstruct a system from data through asymptotic dynamics in the orbit diagrams for period doubling cascades. To compare the orbits of a system to the Gaussian process reconstruction, we compute the Wasserstein metric between the point clouds of their obits for varying bifurcation parameter values. Visually comparing the period doubling cascades, we note that the exact bifurcation values may shift, which is confirmed in the plots of the Wasserstein distance. This has implications for studying dynamics from time series data since a reconstruction of a system's period doubling cascade may lead to unpredictable model behavior in a neighborhood of the true bifurcation parameter value. (Received September 21, 2021)

1174-37-10299 **Cameron Gregory Bundy*** (bundyc@cwu.edu), Central Washington University. *Optimal Lockdown Policies during the COVID-19 Pandemic* Preliminary report.

The COVID-19 virus has had a substantial impact on public and economic health in countries around the world. In order to bolster the economy and maintain human life, continued economic and epidemiological research is vital. Global nations have implemented lockdown policies with the purpose of slowing the spread of the novel corona virus. This research analyzes how lockdown parameters can help control a nation's fatalities. The study incorporated an SIRD disease model into a minimization function that analysis dynamics that best produce minimal loss of GDP as well as low loss of life during a lockdown. To solve this minimization function, a Hamilton-Jacobi-Bellman equation was used, and the results are compared to similar studies. The goal in recreating this epidemiological model was to further investigate the outcomes of parameter adjustment within a model that works to minimize fatalities and economic loss in a nation, in hopes of preparing for future disease outbreaks. (Received September 21, 2021)

1174-37-10333 **Paul Hurtado*** (phurtado@unr.edu), University of Nevada, Reno. *A new technique for ODE model derivation and interpretation: The generalized linear chain trick*

I will provide an overview of a relatively new technique for ODE model derivation and interpretation: the generalized linear chain trick (GLCT). The many benefits of this approach will be discussed using multiple examples.

Many ODE models used in applications are state transition models. These include SIR-type contagion models, ecological models, chemical reaction models, among others. More specifically, such models can be considered "mean field" models intended to capture the average behavior of some (often unspecified) stochastic model. Existing techniques for deriving mean field ODEs from first principles often include intermediate steps like the derivation, and subsequent differentiation, of mean field integral equations. Such approaches have proven to be too cumbersome for many mathematical modelers, who instead often rely on “rule of thumb” to construct such models.

The GLCT allows one to quickly derive a mean field ODE model from first principles by framing the underlying stochastic model assumptions in a standard continuous time Markov chain (CTMC) framework. The resulting matrix-vector parameters of that CTMC can then be used to immediately write down the corresponding mean field ODEs via the GLCT. GLCT-based ODE model formulations are also very useful for recognizing the underlying assumptions implicit in an existing ODE model, and there are computational and analytical benefits to working with ODE models in their GLCT form. Importantly, the GLCT is accessible to myriad practitioners. (Received September 21, 2021)

1174-37-10516 **Brooks Emerick*** (bemerick@kutztown.edu), Kutztown University. *Ecological modeling of hyperparasitoids in host-parasitoid population dynamics* Preliminary report.

A parasitoid is an organism that spends most of its life cycle attached to or inside the host. Unlike an actual parasite, the parasitoid eventually kills the host. During a particular season of the year, known as the vulnerable period, the parasitoid injects eggs into the host larvae. Parasitoid larvae, then, emerge from the host, effectively killing it. Early models of this phenomenon include the discrete-time Nicholson-Bailey model, which is known to be unstable i.e. coexistence is impossible. In this paper, we implement relevant changes during the vulnerable period using a semi-discrete framework that includes an obligate hyperparasitoid population. Oviposition by the parasitoid and hyperparasitoid populations is assumed to be asynchronous. We consider several forms of parasitism including combinations of constant and functional response attack rates as well as variable rates of attack. We find that a feasible coexistence regime exists for when the hyperparasitoid attack is more efficient than that of the parasitoid. We also explore the addition of a host refuge and its influence on the coexistence regimes of all three species. (Received September 21, 2021)

1174-37-10542 **Jessica Bennett*** (jessica_bennett@brown.edu), Brown University. *The Finite Blocking Problem on Cyclic Covers of the Regular Octagon* Preliminary report.

Finding when two points on a translation surface are finitely blocked under straight line flow is an open question for many translation surfaces. We investigated a specific instance of this problem: finite blocking on cyclic covers of the regular octagon. In this presentation, we answer the finite blocking problem in the case of double covers of the octagon and propose an approach for degree 3 covers of the octagon. (Received September 21, 2021)

1174-37-10572 **Hemaho Beaugard Taboe*** (h.taboe@gmail.com), Laboratoire de Biomathématiques et d'Estimations Forestières, University of Abomey-Calavi, Cotonou, Benin, **Kolawolé V. Salako** (salakovalere@gmail.com), Laboratoire de Biomathématiques et d'Estimations Forestières, University of Abomey-Calavi, Cotonou, Benin, **James M. Tison** (jamestison@ufl.edu), Department of Computer and Information Science and Engineering, University of Florida, Gainesville, FL 32611, USA, and **Romain Glele Kakai** (romain.glelekakai@fsa.uac.bj), Laboratoire de Biomathématiques et d'Estimations Forestières, University of Abomey-Calavi, Cotonou, Benin. *Predicting COVID-19 spread in the face of control measures in West Africa* Preliminary report.

The COVID-19 pandemic has been causing devastating health and economic damage globally since its emergence in 2019. Understanding current patterns of the pandemic spread and forecasting its long-term trajectory is essential in guiding policies aimed at curtailing the pandemic. We formulate and use a deterministic compartmental model to (i) assess the current patterns of COVID-19 spread in West Africa, (ii) evaluate the impact of currently implemented control measures, and (iii) predict the future course of the pandemic with and without currently implemented and additional control measures in West Africa. Numerical simulations of the model using baseline parameter values estimated from West-African COVID-19 case data project a 67% reduction in the daily number of cases when the first wave of the pandemic in West Africa attains its peak. In addition, we found that disease elimination is difficult when asymptomatic individuals are not identified and isolated in a timely manner. We showed that the currently implemented measures triggered a 33% reduction in the time-dependent effective reproduction number. (Received September 21, 2021)

1174-37-10824 **Zoran Sunic** (zoran.sunic@hofstra.edu), Hofstra, **Sanjit Dandapanthula*** (sanjitd@ucla.edu), UCLA, **Ibrahim Abdelaziz** (ibrahim66010@gmail.com), AU Cairo, **Alexander Cernei** (alexander.cernei@macaulay.cuny.edu), CUNY, **John Connell** (johnconnelltutor@gmail.com), South Florida, **Gabriel Diraviam** (gabriel.a.diraviam@vanderbilt.edu), Vanderbilt, **Arthur Daniel Dressenwall** (arthurdanieldw@gmail.com), Macalester, **Shalini N. Hewage** (h.shalini.hewage@gmail.com), Colombo, **Risitha Randima Nambuwasam** (risitha_nambuwasam@icloud.com), Colombo, **Micah Pietraho** (micah.pietraho@gmail.com), Harvard, and **Jack Stade** (js2519@cam.ac.uk), Cambridge. *Schreier graphs of iterated monodriomy groups of conservative polynomials* Preliminary report.

We studied the Schreier graphs of iterated monodromy groups (IMG) of conservative polynomials. A polynomial is conservative if all of its critical points are fixed. The IMG of a polynomial acts on the rooted tree of iterated preimages of a point in the complex plane and the sequence of Schreier graphs represents the action of the group on the levels of this tree (the graph on level n has d^n vertices, where d is the degree of the polynomial). We calculated or estimated the diameter and other parameters of these graphs. In particular, for a subclass of conservative polynomials we calculated the spectrum of the graph on each level, which enabled us to construct a Laplacian on the Julia set of the polynomial as a limit of the Laplacians on the Schreier graphs. The spectrum of the Schreier graphs is obtained by backward iteration of a polynomial of degree equal to the maximal local degree of a critical point. On the algebraic side, we established that the IMG of a conservative polynomial is always a branch group. Moreover, whenever the multiplicities of all critical points are even, the group is branching over itself (its structure is the semidirect product $G = \mathrm{Alt}(d) \ltimes \underbrace{G \times G \times \cdots \times G}_{d}$). We calculated presentations for each of the groups in the class (they are nor finitely presented, but have recursive presentations). On the other hand, the submonoid generated by the group generators is finitely presented, and is a free partially commutative monoid. (Received September 21, 2021)

1174-37-10861 **Kevin Manogue*** (manoguek@lafayette.edu), Lafayette College, **Shanshan Cao** (scao3@bu.edu), Boston University, and **Jack Klawitter** (jklawitter@middlebury.edu), Middlebury College. *Periodic Billiard Orbits on Surfaces of Revolution* Preliminary report.

Billiard dynamics have been studied extensively on planes and surfaces with constant curvature, but billiard systems on surfaces with variable curvature are relatively unexplored. Our work focuses on surfaces of revolution, a small subset of surfaces with variable curvature. We study "rectangular" billiard boards on such surfaces–that is, boards whose boundaries are defined by two parallels and two meridians. Using a method analogous to unfolding on the plane, we identify a sufficient condition for orbits on such billiard boards to be periodic and determine their period based on the orbit's initial angle. Finally, we explore the implications and applications of our periodicity condition, including the construction of a new unilluminable room on the sphere. (Received September 21, 2021)

1174-37-10985 **Sabina Adhikari** (Sabina.adhikari@colorado.edu), University of Colorado at Boulder. *Synchronization in hypergraphs* Preliminary report.

We study the synchronization of phase oscillators on heterogeneous hypergraphs. By combining dimension reduction techniques with mean field approaches and generative models, we are able to derive analytically conditions for the onset of bistable behavior. We support our results with numerical simulations. (Received September 21, 2021)

1174-37-11053 **Lily Qiao Li*** (lilyli@berkeley.edu), University of California, Berkeley, and **Caleb T Partin** (ctpartin@gmail.com), Georgia Institute of Technology. *Extending the Twisted Rabbit Problem* Preliminary report.

The Twisted Rabbit problem in complex dynamics was first solved by Bartholdi-Nekrashevych, and it was recently reformulated and solved topologically by Belk-Lanier-Margalit-Winarski through a lifting algorithm. This topological viewpoint supplies a virtual endomorphism of the pure mapping class group, which enables us to extend the problem to the n-eared rabbit and other topological polynomials. In this talk, we will fully describe the virtual endomorphism for all n-eared rabbit polynomials and discuss algebraic structures relating families of topological polynomials, with the n-eared rabbit polynomial as the prototype. (Received September 21, 2021)

1174-37-11222 **Iris Dina Glogic*** (iglogi2@uic.edu), Research Collaborator, **Skyler Moomey** (skyler_moomey-1@ou.edu), Research Collaborator, and **Quinn Stoddard-O'Neill** (qstoddar@skidmore.edu), Research Collaborator. *Modelling Perceptual Machinery and its Effects on Dispersal* Preliminary report.

Lattice-based models of ecological dispersal have generally been developed with the assumption that patches in an environment are homogeneous. The impact of this assumption is that organisms are modeled to disperse with equal frequency to every patch within the lattice, as in models of plant dispersal in the work of Sato and Iwasa. Our work aims to advance this model by creating heterogeneity within the lattice, allowing for each patch to be either desirable or undesirable, and modeling the ability of organisms to discriminate between desirable and undesirable habitats. In our analysis of the model, we use the moment-closure method described by Sato and Iwasa, pair approximation, in order to come to a closed system of differential equations describing the proportion of occupied spaces of various arrangements in the lattice with respect to time. We additionally describe the clustering behavior of organisms using densities of pairs. After Monte Carlo analysis and the collection of numerical solutions, we conclude that the model contains implicit competition costs dependent on a population's degree of perceptual accuracy. We also perform preliminary analysis of a model in which more explicit competition costs to settling in an undesirable habitat were defined. Future work on this problem may include the application or development of methods to collect algebraic equilibria to examine this competition cost more quantitatively as well as assuming and modeling further heterogeneity within the environment. (Received September 21, 2021)

39 ► Difference and functional equations

1174-39-6309 **Marta Lewicka*** (lewicka@pitt.edu), University of Pittsburgh. *Non-local Tug-of-War with noise for the geometric fractional p-Laplacian*

This talk concerns the fractional p-Laplace operator Δ_p^s in non-divergence form. For any $p \in [2, \infty)$ and $s \in (\frac{1}{2}, 1)$ we first define two families of non-local, non-linear averaging operators, parametrised by ϵ and defined for all bounded, Borel functions $u : \mathbb{R}^N \to \mathbb{R}$. We prove that $\Delta_p^s u(x)$ emerges as the ϵ^{2s}-order coefficient in the expansion of the deviation of each ϵ-average from the value $u(x)$, in the limit of the domain of averaging exhausting an appropriate cone in $\mathbb{R}^N$ at the rate $\epsilon \to 0$.

Second, we consider the ϵ-dynamic programming principles modeled on the first average, and show that their solutions converge uniformly as $\epsilon \to 0$, to viscosity solutions of the homogeneous non-local Dirichlet problem for Δ_p^s, when posed in a domain D that satisfies the external cone condition and subject to bounded, uniformly continuous data on $\mathbb{R}^N \setminus D$.

Finally, we interpret such ϵ-approximating solutions as values to the non-local Tug-of-War game with noise. In this game, players choose directions while the game position is updated randomly within the infinite cone that aligns with the specified direction, whose aperture angle depends on p and N, and whose ϵ-tip has been removed. (Received September 7, 2021)

1174-39-8035 **Saber Elaydi*** (selaydi@trinity.edu), Trinity University. *Global Dynamics of discrete-time Evolutionary competition Models* Preliminary report.

We investigate discrete-time competition models of two species, where both or one of the species have an evolving trait with time. The model used her is an evolutionary Ricker competition model of two species. Our model is based on the evolutionary game theory and Darwinian evolution. Conditions under which the two species persist are given. Moreover, we also provide conditions undef which the competitive exclusion principle holds, where one of the species would survive on the expense of the other species. Global stability results are obtained by utilizing three theories, perturbation theory, singularity theory and the theory of the limiting equations of non-autonomous discrete dynamical systems. Finally, we show that persistence of the two species is enhanced by evolution. (Received September 17, 2021)

1174-39-8203 **Yu Jin*** (yjin6@unl.edu), University of Nebraska-Lincoln, and **Xiao-Qiang Zhao** (zhao@mun.ca), Memorial University of Newfoundland. *The spatial dynamics of a zebra mussel model in river environments*

Huang et al. developed a hybrid continuous/discrete-time model to describe the persistence and invasion dynamics of Zebra mussels in rivers (see SIAM Journal on Applied Mathematics, 77 (2017), 854-880). They used a net reproductive rate R0 to determine population persistence in a bounded domain and estimated spreading speeds in a unbounded domain. Since the associated solution operator is non-monotonic and non-compact, it is nontrivial to rigorously establish these quantities. In this work, we analyze the spatial dynamics of this model

mathematically. We first solve the parabolic equation and rewrite the model into a fully discrete-time model. In a bounded domain, we show that the spectral radius of the linearized operator can be used to determine population persistence and also confirm that R0 defined Huang et al's work can be used to determine population persistence. In an unbounded domain, we construct two monotonic operators to control the model operator from above and from below and obtain upper and lower bounds of the spreading speeds of the model. (Received September 18, 2021)

1174-39-9928 **Wei Hu*** (whu@tusculum.edu), Tusculum University, and **Feifei Du** (qinjin65@126.com), Shanghai Jiaotong University, China. *Halanay Inequalities of Variable Order with Applications* Preliminary report.

In this presentation, we discuss Halanay inequalities of variable order based on the Atangana-Baleanu-Caputo fractional derivative and applications to the stability of time-delayed systems. (Received September 21, 2021)

1174-39-10014 **Ronald E Mickens** (rmickens@cau.edu), Clark Atlanta University, and **Talitha M Washington*** (twashington@cau.edu), Clark Atlanta University and the Atlanta University Center. *Positivity Preserving Hyperbolic NSFD Scheme for Heat Transfer* Preliminary report.

In this talk, we construct a new nonstandard finite difference (NSFD) scheme for a hyperbolic partial differential equation (PDE) modeling heat transfer. This discretization satisfies a positivity condition for its solutions and is consistent in its general features with the experimental data. A brief discussion will also be provided as to how the "additional" initial condition, $u_t(x;0) = 0$, should be implemented on the computational grid. Further, we examine the details of the mathematical structure of the scheme and their implications for systems involving heat transfer. (Received September 21, 2021)

1174-39-10478 **Arzu Bilgin*** (arzu.bilgin@erdogan.edu.tr), Recep Tayyip Erdogan University. *Single Species Two-Generation Population Model* Preliminary report.

In this paper, I consider the cooperative system

$$x_{n+1} = ax_n + \frac{by_n}{1+y_n} \quad y_{n+1} = \frac{cx_n}{1+x_n} + dy_n, \quad n = 0, 1, \ldots,$$

where all parameters a, b, c, d are positive numbers and the initial conditions x_0, y_0 are nonnegative numbers. This system can be considered as cooperative Leslie two-generation model, where each generation helps growth of the other. I describe the global dynamics of this system in a number of cases. All global dynamic results for this system can be extended to the general cooperative discrete system in the plane. (Received September 21, 2021)

1174-39-10565 **Eddy Kwessi*** (ekwessi@trinity.edu), Trinity University. *Strong Allee Effect-type plasticity rule in unsupervised learning environment* Preliminary report.

The Allee effect was introduced by Allee (1949) and characterizes a phenomenon in population dynamics where there is a positive correlation between a population density or size and its per capita growth rate.The strong Allee effect occurs when a population has a "critical density" below which it declines to extinction while the weak Allee effect occurs when a population lacks such a "critical density", but at lower densities, the population growth rate arises with increasing densities. Brain plasticity can be thought of as the ability of the brain to adapt to external activities by modifying some of its synaptic structure. Synapses play an important role in the brain because they constitute junctions between nerve cells and therefore facilitate diffusion of chemical substances called neurotransmitters from the brain to other parts of the body. To understand these modifications at the functional and behavioral levels, one must understand how experience and training modify synapses and how these modifications change patterns of neuronal firings to affect behavior. In this talk, I will propose a discrete time model of brain plasticity based on the strong Allee effect that captures synaptic modifications at the functional level. Stability analysis of the model will be discussed and simulations will be given (Received September 21, 2021)

40 ► Sequences, series, summability

1174-40-5399 **Edward Huynh*** (bluesymoony@gmail.com), University of Nevada, Las Vegas. *The Second Raabe's Test and Other Series Tests* Preliminary report.

The classical D'Alembert's Ratio Test is a powerful test that we learn from calculus to determine convergence for a series of positive terms. Its range of applicability and ease of computation make this test extremely appealing. However, when the limiting ratio of the terms equals 1, then the test is inconclusive. Several series tests like

Raabe's and Gauss' Tests have been proposed in order to address this case. These tests were also generalized by Kummer through Kummer's Test. More recently, a Second Ratio Test was constructed that also possessed an inconclusive case. This presentation conducts a survey of existing series tests, introduces an extension of Raabe's Test to the Second Ratio Test, and proposes extensions of classical tests such as Gauss's Test and Kummer's Test. It also offers proofs of the aforementioned tests and a brief application of the Second Raabe's Test. (Received August 19, 2021)

1174-40-7144 **Sukhdev Singh*** (singh.sukhdev01@gmail.com), Department of Mathematics, Lovely Professional University, Jalandhar-Delhi G.T Road (NH-44), Phagwara 144411, Punjab, INDIA. *On the mixed paranormed Norlund sequence space of nonabsolute type using Orlicz function* Preliminary report.

In this paper, we introduced the difference Norlund sequence space, $N_t(\mathcal{M}, \Delta, p)$ using the Orlicz functions and Mixed paranomed. The relation between $N_t(\mathcal{M}, \Delta, p)$ and $\ell(p)$, the space defined by Maddox has been studied. Here, we discussed about the Norlund matrix transformation of sequences from $N_t(\mathcal{M}, \Delta, p)$ into the spaces of bounded, convergent, absolutely convergent and null sequences. Some, algebraic and topological properties of this new space has also been studied in detail. Further, the α-, β- and γ-duals of the space $N_t(\mathcal{M}, \Delta, p)$ are computed. (Received September 13, 2021)

1174-40-7489 **Pierre-Olivier Parise*** (parisepo@hawaii.edu), University of Hawai'i at Manoa. *Summability in Banach spaces of holomorphic functions*

In this talk, I will discuss an abstract result showing that, if one regular summability method includes another for scalar-valued sequences, then it automatically does so for Banach-space-valued sequences too. I will also discuss some consequences of this result on the strong summability of the Taylor expansion of functions in Banach spaces of holomorphic functions.

Joint work with Javad Mashreghi and Thomas Ransford. (Received September 14, 2021)

1174-40-9833 **Nikola Milicevic** (nqm5625@psu.edu), Pennsylvania State University, and **Owen Y Mireles Briones*** (owen.mirelesbriones@stonybrook.edu), Stony Brook University. *Crossing the Amoeba* Preliminary report.

Any rational function f can be expressed as a convergent power series in any domain U that has no poles of f. By varying the localization, the same function will yield several, different power series. If $f(z) = 1/P(z)$ – where $P(z)$ is a polynomial of several variables, and if $f(z) = \sum a_n z^n$ in U and $f(z) = \sum b_n z^n$ in V, we give an expression for $a_n - b_n$. This difference is related to some monodromy groups. We use amoebas, contour integrals, and some formulas to distill as much information as possible. (Received September 20, 2021)

1174-40-11751 **Vardayani Ratti*** (vratti@csuchico.edu), Department of Mathematics and Statistics, California State University Chico, CA, **Jonathan M Winter** (jonathan.m.winter@dartmouth.edu), Dartmouth College, Hanover NH, and **Dorothy I Wallace** (dwallace@math.dartmouth.edu), Dartmouth College, Hanover NH. *Dilution and amplification effects in Lyme disease: Modeling the effects of reservoir-incompetent hosts on Borrelia burgdorferi sensu stricto transmission* Preliminary report.

The literature on Lyme disease includes a lively debate about the paradoxical role of changing deer populations. A decrease in the number of deer will both 1) reduce the incidence of Lyme disease by decreasing the host populations for ticks and therefore tick populations, and 2) enhance the incidence of Lyme disease by lowering fewer reservoir-incompetent hosts for ticks, forcing the vector to choose reservoir-competent, and therefore possibly diseased, hosts to feed on. A review of field studies exploring the net impact of changing deer populations shows mixed results. In this talk, we investigate the hypothesis that the balance of these two responses to changing deer populations depends on the relative population sizes of reservoir-competent versus reservoir-incompetent hosts and the presence of host preference in larval and adult stages. A temperature driven seasonal model of Borrelia burgdorferi sensu stricto (cause of Lyme disease) transmission among three host types (reservoir-competent infected and uninfected hosts, and reservoir-incompetent hosts) is constructed as a system of nonlinear ordinary differential equations. The model, which produces biologically reasonable results for both the tick vector (Ixodes scapularis(Say)) and the hosts, is used to investigate the effects of reservoir-incompetent host removal on both tick populations and disease prevalence for various relative population sizes of reservoir-competent hosts versus reservoir-incompetent hosts. Our numerical simulations show that the model with host preference appears to be more accurate than the one with no host preference. Given these results, we found that removal of adult I. scapularis (Say) hosts is likely to reduce questing nymph populations. At very low levels questing adult abundance may rise with lack of adult hosts. There is a dilution effect at low reservoir-competent host populations and there is an amplification effect at high reservoir-competent host populations. (Received October 6, 2021)

41 ► Approximations and expansions

1174-41-5858 **Alexander Geoffrey Strang*** (alexstrang@uchicago.edu), The University of Chicago, **Peter J Thomas** (pjthomas@case.edu), Case Western Reserve University, and **Michael Hinczewski** (mxh605@case.edu), CWRU, Physics, Dept. of Physics, Case Western Reserve University. *Applications of the Combinatorial Helmholtz-Hodge Decomposition to Near Equilibrium Systems*

The combinatorial Helmholtz-Hodge Decomposition (HHD) expands an edge flow on a network into a conservative component associated with the gradient of a potential, and a rotational component. An edge flow can be defined for reversible continuous-time Markov processes via the log ratio of forward and backward transition rates. In statistical thermodynamics, this edge flow is associated with the work required for a system to perform the transition. The process is an equilibrium process if the edge flow can be written as the gradient of a potential. Equilibrium processes are time-reversible, satisfy detailed balance, and are energetically closed. At equilibrium probability fluxes balance on all pairs of forward and backward edges, and the steady state is a Boltzmann distribution. In contrast, non-equilibrium systems exchange energy with their environment, and maintain nonvanishing steady state fluxes driven by a nonzero rotational edge flow. Most interesting processes in molecular biology are not at equilibrium since they use chemical energy to perform their life function. The HHD offers a natural framework for simultaneously expanding the steady state distribution and steady state fluxes of reversible Markov chains near equilibrium. The expansion generalizes classical reciprocity results from linear thermodynamics, while providing an intuitive framework for analyzing the relationships between perturbations to the Boltzmann distribution, thermal efficiency, and entropy production. (Received August 30, 2021)

1174-41-5953 **George A. Anastassiou*** (ganastss@memphis.edu), University of Memphis. *Simultaneous high order abstract fractional polynomial spline monotone approximation and applications* Preliminary report.

Here we extended our earlier high order simultaneous fractional polynomial spline monotone approximation theory to abstract high order simultaneous fractional polynomial spline monotone approximation, with applications to Prabhakar fractional calculus and non-singular kernel fractional calculi. We cover both the left and right sides of this constrained approximation. Let f in $C^s([-1,1])$, s in N and L be a linear left or right side fractional differential operator such that $L(f) >_0$ over $[0,1]$ or $[-1,0]$, respectively. Then there exists a sequence Q_n; n in N of polynomial splines with equally spaced knots of given fixed order such that $L(Q_n) \geq 0$ on $[0,1]$ or $[-1,0]$, respectively. Furthermore f is approximated with rates fractionally and simultaneously by Q_n in the uniform norm. This constrained fractional approximation on $[-1,1]$ is given via inequalities involving a higher modulus of smoothness of $f^{(s)}$: (Received September 1, 2021)

1174-41-6243 **Jeffrey Parks Ledford** (ledfordjp@longwood.edu), Longwood University, and **Keaton Hamm*** (keaton.hamm@uta.edu), University of Texas at Arlington. *Quasi-shift Invariant Spaces and Interpolation*

We discuss generalizations of shift-invariant spaces called quasi-shift invariant spaces. These are defined to be the L_p closure of the shift of a given L_p generator by nonuniform points:

$$V_p(\phi, \mathcal{X}) := \overline{\text{span}}^{L_p}\{\phi(\cdot - x) : x \in \mathcal{X}\}.$$

We discuss the structure of such spaces, when they are closed subspaces of L_p, and when the set of shifts is equivalent to the unit vector basis of ℓ_p. Additionally, we consider when functions from one such space $V_p(\phi, \mathcal{X})$ can be interpolated by another space $V_p(\psi, \mathcal{Y})$, as well as some convergence criteria for a sequence of such space $V_p(\psi_\alpha, \mathcal{Y})$ where α is some limiting parameter making ψ more flat. (Received September 7, 2021)

1174-41-6326 **Karen Abbott** (kca27@case.edu), Case Western Reserve University. *The Weak Rotation Expansion: Using the Helmholtz-Hodge Decomposition to Explain Markov Chains Near Equilibrium*

In statistical thermodynamics, a Markov chain models an equilibrium system if all probability fluxes across pairs of forward and backward edges vanish at the steady state. Equilibrium systems are energetically closed. Most life processes are not energetically closed. The continual use of energy to perform a function is the defining hallmark of active matter. As such, there is great interest in non-equilibrium systems where energy is exchanges maintain nonvanishing, cyclic probability currents. The Helmholtz-Hodge decomposition (HHD) can be used to separate the work to cross each edge in a discrete-state Markov chain into a conservative component associated with internal energy, and a rotational component associated with external energy sources. We consider the near equilibrium case, when driving rotation is weak, and introduce a formal expansion of the steady state distribution and currents as a Taylor series. Each term satisfies a recursively defined HHD. We derive a nonzero lower bound

on the radius of convergence of the expansion. We use the expansion to prove classical results from linear thermodynamics such as reciprocity. We also use the expansion to explore situations in which the steady state is independent of driving rotation at all orders, or up to a given order, to show that some reciprocity extends past first order, and to relate the thermal efficiency of the system and the amount the steady state is perturbed. Our approach applies to any Markov chain with microscopic reversibility. (Received September 8, 2021)

1174-41-6895 **Alexander M Powell*** (alexander.m.powell@vanderbilt.edu), Vanderbilt University, and **Jonathan Ashbrock** (jdashbrock@gmail.com), The MITRE Corporation. *Dynamical dual frames with applications to quantization*

Frames are types of overcomplete signal expansions that arise naturally in the study of dynamical sampling. A frame is said to be dynamical if the frame vectors are generated by the iterates of a linear operator. Aceska and Kim proved that if a frame is dynamical then its canonical dual frame is also dynamical. However, not all frames are dynamical. Given a frame which is not necessarily dynamical, we study the dynamical structure of its noncanonical dual frames. We prove that every redundant frame for R^d has infinitely dual frames that are dynamical. We then show an application of dynamical dual frames to error diffusion algorithms for signal quantization. (Received September 10, 2021)

1174-41-11120 **Armenak Petrosyan*** (arm.petros@gmail.com), Georgia Institute of Technology. *Frame representations with non convex penalization*

The challenge we address in this work is the following. We want to find small size dictionary that can be trained algorithmically and which achieve guaranteed approximation speed and precision. To maintain the small size we apply penalties on the coefficients of the dictionary. We show that under minimal requirements, all local minima of the resulting problem are well behaved and possess a desirable small size without sacrificing precision. We use techniques from optimization theory and harmonic analysis to prove our results. In this talk, we will discuss our existing work and possible future promising areas of interest where this approach can potentially be adopted (Received September 21, 2021)

42 ► Harmonic analysis on Euclidean spaces

1174-42-5909 **Matthew Fickus*** (Matthew.Fickus@gmail.com), Air Force Institute of Technology. *Harmonic Grassmannian codes*

An equi-isoclinic tight fusion frame (EITFF) is a type of Grassmannian code, being a sequence of subspaces of a finite-dimensional Hilbert space of a given dimension with the property that the smallest spectral distance between any pair of them is as large as possible. EITFFs arise in compressed sensing, yielding dictionaries with minimal block coherence. Despite this, they remain poorly understood, with known constructions falling far short of known necessary conditions on their existence. We construct new infinite families of EITFFs in a way that generalizes the known equivalence between harmonic equiangular tight frames and difference sets for finite abelian groups. (Received August 31, 2021)

1174-42-6014 **Akram Aldroubi*** (akram.aldroubi@vanderbilt.edu), Vanderbilt University. *Dynamical sampling: An overview and some open problems*

Dynamical sampling is a term describing an emerging set of problems related to recovering signals and evolution operators from space-time samples. For example, consider the abstract IVP in a separable Hilbert space $\mathcal{H}$:

$$\begin{cases} \dot{u}(t) = Au(t) + F(t) \\ u(0) = u_0, \end{cases} \qquad t \in \mathbb{R}_+,\ u_0 \in \mathcal{H}, \tag{1}$$

where $t \in [0, \infty)$, $u : \mathbb{R}_+ \to \mathcal{H}$, $\dot{u} : \mathbb{R}_+ \to \mathcal{H}$ is the time derivative of u, and u_0 is an initial condition. When, $F = 0$, A is a known (or unknown) operator, and the goal is to recover u_0 from the samples $\{u(t_i, x_j)\}$ on a sampling set $\{(t_i, x_j)\}$, we get the so called *space-time sampling* problems. If the goal is to identify the operator A, or some of its characteristics, we get the *system identification* problems. If instead we wish to recover F, we get the *source term* problems. In this talk, I will present an overview of dynamical sampling, and some open problems. (Received September 2, 2021)

1174-42-6227 **Ilya A. Krishtal*** (ikrishtal@niu.edu), Northern Illinois University. *A few problems in dynamical sampling of bandlimited functions*

We discuss a few results related to reconstruction of a bandlimited function f from dynamical samples, i.e. from samples of its states $f_t = \phi_t * f$ resulting from the convolution with a kernel in a special class. Even though uniform space-time samples are typically not sufficient for stable reconstruction of any such f, we show that a portion of f that is away from certain blind spots can nevertheless be stably reconstructed. We also show how, in case of non-uniform spacial sampling, quantitative bounds on stability of reconstruction control the maximal gaps between spacial samples. Presented results are based on joint work with A. Aldroubi, K. Gröchenig, L. X. Huang, P. Jaming, and J. L. Romero. (Received September 7, 2021)

1174-42-7442 **Demetrio Labate*** (dlabate@math.uh.edu), University of Houston, **Jenny Schmalfuss** (jenny.schmalfuss@vis.uni-stuttgart.de), University of Stuttgart, and **Nikolaos Karantzas** (nickoskarantzas@gmail.com), Baylor College of Medicine. *Image inpainting using sparse multiscale representations and Convolutional Neural Networks* Preliminary report.

Image inpainting aims at recovering missing blocks of data in an image or video. To derive performance guarantees in the image inpainting problem, I will assume a simplified image model consisting of distributions supported on curvilinear singularities and consider the problem of recovering a missing edge fragment. Within this setting, it is known that certain classes of direction-sensitive multiscale representations such as shearlets are remarkably efficient at approximating discontinuities occurring along curves. I will prove that the theoretical performance of image inpainting depends critically on the microlocal properties of the image representation system, namely, exact image recovery is achieved if the size of the missing edge fragment is smaller than the size of the structure elements of the representation system. As a result, a shearlet-based image inpainting algorithm - exploiting their microlocal properties - significantly outperforms a similar approach based on more traditional multiscale methods. This result extends a previous result by King, Kutyniok, and Zhuang. Based on this theoretical observation, I will next propose an improved method for blind image inpainting that combines Convolutional Neural Networks and sparse representations. (Received September 14, 2021)

1174-42-7836 **Kasso A. Okoudjou** (kasso.okoudjou@tufts.edu), Tufts University, **Shujie Kang*** (shujie.kang@uta.edu), University of Texas at Arlington, and **Assaf Goldberger** (assafg@tauex.tau.ac.il), Tel-Aviv University. *Non-informationally complete Gabor POVMs*

A POVM (positive operator valued measure) is informationally complete if the d^2 corresponding rank 1 matrices form a basis for the space of $d \times d$ matrices. Generically Gabor POVMs are informationally complete. We classify non-informationally complete Gabor POVMs and describe some properties of them. We also use the result to construct sets of of d^2 unit vectors in $\mathbb{C}^d$ with a relatively smaller number of distinct inner products. (Received September 16, 2021)

1174-42-8244 **Roza Aceska*** (raceska@bsu.edu), Ball State University, **Shidong Li** (shidong@sfsu.edu), San Francisco State University, and **Jean-Luc Bouchot** (jlbouchot@gmail.com), Titus Tech. *A Compressed Sensing Framework for Fusion Frame Structured Signals*

We solve the problem of recovering signals of high complexity from low quality sensing devices via a combination of tools from signal processing and harmonic analysis. While the computational complexity remains relatively low, we achieve stronger recovery performance compared to usual single-device compressed sensing systems.

We introduce a compressed sensing framework in which we split the dense information into subchannels and fuse the local estimations by utilizing a fusion frame representation. Each piece of information is measured by linear sensors, and recovered via compressed sensing in each subspace. Finally, after a fusion process, we are able to recover the original signal with high accuracy. We verify that by increasing the size of the fusion frame, a certain robustness to noise is also achieved. Our techniques are applicable in various signal processing tasks such as Doppler signal denoising, natural scene scanning and reconstruction, and MR Image reconstruction. (Received September 18, 2021)

1174-42-8275 **John Oliver Maclellan*** (jomaclellan@crimson.ua.edu), University of Alabama. *Necessary Conditions for Two Weight Weak Type Norm Inequalities for Multilinear Singular Integral Operators* Preliminary report.

The goal of this paper is to establish necessary conditions for two weight weak type norm inequalities for multilinear Calderón Zygmund operators. We generalize results derived by the authors in a recent paper and earlier work by Lacey, Sawyer and Uriarte-Tuero and by Stein in the linear case. We prove results assuming a non-degeneracy condition similar to that introduced by Stein in the linear case, both assuming doubling conditions and without them, only assuming that our measures do not have common point masses. (Received September 18, 2021)

1174-42-8522 **Malabika Pramanik*** (malabika@math.ubc.ca), University of British Columbia/BIRS.
Configurations determined by distances: a survey

Many patterns of interest in analysis, number theory, combinatorics and geometric measure theory are dictated by distances between their constituent points. Examples include arithmetic and geometric progressions, equilateral triangles, polygons and chains. While there are natural conjectures concerning distance problems that are still open, many results have been found that indicate a richness of phenomena in this area. We will provide a short survey of a few such results. (Received September 19, 2021)

1174-42-8576 **Neil Lyall*** (lyall@math.uga.edu), University of Georgia. *Maximal functions and pinned configurations in both $\mathbb{R}^d$ and $\mathbb{Z}^d$*

We plan to discuss maximal functions estimates in both $\mathbb{R}^d$ and $\mathbb{Z}^d$ that can be used to determine the existence of certain pinned configurations in sets of positive upper density. (Received September 19, 2021)

1174-42-8639 **Dominique Kemp*** (ddkemp@math.wisc.edu), University of Wisconsin-Madison.
Decoupling and Approximation of Surfaces

Decoupling is an attempt to decompose multidimensional waves $\Upsilon \subset \mathbb{R}^n$ having curved frequencies into the composite parts υ whose frequencies are each approximately collections of line segments. The idea is that curvature within the set of frequency points greatly enhances destructive interference among the υ resulting in a more subdued composite wave Υ. In this talk, we shall explicitly define what decoupling is and the initial, seminal results in the theory, followed by a brief exploration into possible future developments of the field. (Received September 19, 2021)

1174-42-8891 **Sean Sovine*** (sovine5@vt.edu), Virginia Tech. *Lp Bounds for the Triangle Averaging Operator*

The triangle averaging operator is a bilinear analogue of the spherical averaging operator, which takes two input functions and computes the average of their tensor product over all pairs of vertices of an equilateral triangles with one vertex fixed at a given point. This operator has been used as a tool to solve continuous point configuration problems, such as a generalization of the Falconer problem to counting (in a measure-theoretic sense) distinct triangles in a thin set. We apply several methods to obtain Lp bounds for this operator and its maximal variants. (Received September 20, 2021)

1174-42-9108 **Alan Chang*** (alan.chang.math@gmail.com), Princeton University, **Georgios Dosidis** (dosidis@karlin.mff.cuni.cz), Charles University, and **Jongchon Kim** (jongchon.kim.work@gmail.com), City University of Hong Kong. *Nikodym-type spherical maximal functions*

We study L^p bounds on Nikodym maximal functions associated to spheres. In contrast to the spherical maximal functions studied by Stein and Bourgain, our maximal functions are uncentered: for each point in $\mathbb{R}^n$, we take the supremum over a family of spheres containing that point. (Received September 20, 2021)

1174-42-9244 **Galia D Dafni*** (galia.dafni@concordia.ca), Concordia University. *Recent results on BMO, VMO and their nonhomogeneous counterparts*

We present recent results on the space of functions of bounded mean oscillation and its subspace of functions of vanishing mean oscillation, as well as the corresponding nonhomogeneous ("localized") space bmo introduced by Goldberg and the analoguous vanishing mean oscillation subspaces. (Received September 20, 2021)

1174-42-9774 **Arian Nadjimzadah*** (arianaddress@gmail.com), University of Rochester. *Similar Volume Configurations in Thin Subsets of Euclidean Space*

How large must a set be for it to contain some given geometric structure? T. Ziegler, building on the work of Furstenburg, Katznelson, and Weiss, showed that sets in $\mathbb{R}^d$ with positive upper density contain many isometric copies of a given simplex. Greenleaf, Iosevich, and Mkrtchyan proved an analogous result for sets of fractional Hausdorff dimension, leveraging the action of the isometry group.

We extend this study to *volume configurations* in $\mathbb{R}^d$, where the action of $\mathrm{SL}(d)$ is most natural. Let $k \geq d \geq 2$ and $E \subset \mathbb{R}^d$ compact. The $\binom{k}{d}$*-volume configuration* set of E is the set of tuples of volumes of parallelopipeds in $\mathbb{R}^d$ which can be formed from k points in E, namely

$$\mathcal{V}_{k,d}(E) = \{(\det(x_{i_1}, \ldots, x_{i_d}))_{i_1,\ldots,i_d \in \binom{k}{d}} : x_1, \ldots, x_k \in E\}.$$

Galo and McDonald established a nontrivial threshold such that the measure of $\mathcal{V}_{k,d}(E)$ is positive for the natural Hausdorff measure on the space of volume configurations. We show that under the same threshold and for any $r > 0$, the *r-similar volume set* $\mathcal{V}^r_{k,d}(E) = \{t \in \mathcal{V}_{k,d}(E) : rt \in \mathcal{V}_{k,d}(E)\}$ has positive measure. (Received September 21, 2021)

1174-42-9820 **Gino Angelo Velasco*** (gamvelasco@math.upd.edu.ph), University of the Philippines Diliman. *Quilted local time-frequency frames - Dual frames and approximately dual frames* Preliminary report.

We consider frames for $L^2(\mathbb{R})$ from certain quilted local time-frequency systems, particularly local Gabor systems and systems from subspaces of eigenfunctions of certain time-frequency localization operators. We investigate dual frames and approximately dual frames of such quilted local time-frequency systems. We also illustrate the results with numerical examples. (Received September 20, 2021)

1174-42-9869 **David Weed*** (david@davidweed.net), California State University Fullerton, **William Riley Casper** (williamrileycasper@gmail.com), California State University Fullerton, and **Daniel Bustamante** (darkrailegend5@csu.fullerton.edu), California State University Fullerton. *Chebyshev-type Orthogonal Matrix Polynomials*

Orthogonal polynomials play a pivotal role in numerous areas of both pure and applied mathematics, from numerical methods for differential equations to harmonic analysis. As analogs of their scalar-valued counterparts, matrix-valued orthogonal polynomials (MVOP's) are the focus of numerous research articles. A certain class of MVOP's, those of Chebyshev-type, arise in connection with spherical functions on certain Lie groups. Motivated by this, we explore ways of obtaining explicit formulas for certain families of MVOP's of Chebyshev type by leveraging Birkhoff decompositions and quasi-determinants of Slater-type block matrices. (Received September 21, 2021)

1174-42-10054 **Daniel Spector** (spectda@gmail.com), National Taiwan Normal University, and **Felipe Hernandez*** (felipehernandezbarroso@gmail.com), Stanford University. *Fractional integration estimates for divergence free vector fields in L^1*

The Hardy-Littlewood-Sobolev inequality $\|I_\alpha f\|_{L^{dp/(d-\alpha p)}} \leq C\|f\|_{L^p}$ fails at the endpoint $p=1$, as can be seen for example by testing against a positive bump function. However, this inequality can be recovered for certain classes of functions that preclude concentration of mass at a point. For example, Jean van Schaftingen showed in 2007 that $\|I_\alpha F\|_{L^{d/(d-\alpha)}} \leq C\|F\|_{L^1}$ holds for vector fields $F \in L^1(\mathbb{R}^d;\mathbb{R}^d)$ satisfying $\operatorname{div} F = 0$. I will describe the proof of an improvement of this result; namely that $I_\alpha F$ belongs to the Lorentz space $L^{d/(d-\alpha),1}$. This answers a question of Bourgain and Brezis from 2007. There are two simple ingredients to the proof. The first is a decomposition of divergence-free vector fields into measures supported on well behaved curves. The second ingredient is an elegant nonlinear interpolation argument. This is joint work with Daniel Spector. (Received September 21, 2021)

1174-42-10278 **Krystal Taylor*** (taylor.2952@osu.edu), The Ohio State University, Department of Mathematics. *Distances and Projections in a Fractal Setting*

We look at some notions of largeness which guarantee that a given subset of Euclidean space is guaranteed to generate many distances. In particular, we consider the notion of Newhouse thickness in finding paths within Cantor sets. Nonlinear projection theory and the Fourier transform play a role. The first known result on the interior of pinned distance sets will make an appearance. (Received September 21, 2021)

1174-42-11259 **Gil Kalai*** (gil.kalai@gmail.com), Hebrew University of Jerusalem. *Fourier methods for some problems in the theory of computing*

I will try to explain some applications (and hoped for applications) of Fourier methods in the study of influences, collective coin-flipping, threshold behavior, noise, and error-correction. (Received September 22, 2021)

43 ► Abstract harmonic analysis

1174-43-7068 **Noah Kravitz*** (nbkravitz@gmail.com), Department of Mathematics, Princeton University. *The smoothest average: Dirichlet, Fejér, and Chebyshev*

In the real world, it is often desirable to obtain "smoothed" local averages of a function $f:\mathbb{Z}\to\mathbb{R}$ (for instance, new covid cases per day). But there are many possible ways to average, and it is not clear which one is best! Inspired by the axiomatic approach in game theory, we propose several criteria for "good" averaging schemes; these make it possible to show that some averages are better than others. In particular, we consider averaging f by convolving it with a kernel on a fixed scale, and we measure the "smoothness" of the result by the ℓ^2-norm of either the first or second discrete derivative. The determination of the optimal kernels involves the extremizing properties of Chebyshev polynomials and is related to new uncertainty principles for the Fourier transform. We also address a continuous analog of this problem where we work over $\mathbb{R}$ instead of $\mathbb{Z}$. This talk is based on joint work with Stefan Steinerberger. (Received September 11, 2021)

1174-43-10017 **Sarah Wolff*** (wolffs@denison.edu), Denison University. *Computational Bounds for Doing Harmonic Analysis on Permutation Modules of Finite Groups*

We develop an approach to finding upper bounds for the number of arithmetic operations necessary for doing harmonic analysis on permutation modules of finite groups. The approach takes advantage of the intrinsic orbital structure of permutation modules and it uses the multiplicities of irreducible submodules within individual orbital spaces to express the resulting computational bounds. We conclude by showing that these bounds are surprisingly small when dealing with certain permutation modules arising from the action of the symmetric group on tabloids. (Received September 21, 2021)

44 ► Integral transforms, operational calculus

1174-44-9147 **Lance W Nielsen*** (lnielsen@creighton.edu), Creighton University. *Feynman's Operational Calculus using Arbitrary Time-Ordering Measures and an Evolution Equation*

We discuss the abstract setting of Feynman's operational calculus in the most general case, that when we allow arbitrary time-ordering measures to be used. We will also present an evolution equation obeyed by this most general version of Feynman's operational calculus in the presence of a (C_0) semigroup of linear operators. (Received September 20, 2021)

1174-44-10148 **Gustavo Rohde*** (gustavo.rohde@gmail.com), University of Virginia. *Linearizing data science problems using transport and other Lagrangian embeddings*

Data science problems such as detection, estimation, clustering, and classification using data emanating from physical sensors (e.g. signals and images) often present difficult mathematical challenges due to the complex nonlinearities present when modeling physical phenomena. As a consequence, the solutions (algorithms) often used to solve such problems can be sub-optimal, computationally expensive, require large amounts of data, and have no known mathematical performance guarantees. In this talk we will present an emerging mathematical signal and image transformation (embedding) framework especially tailored to represent data measurements obtained from physical processes related to transport. Amongst other interesting properties, the transport signal transformation framework converts signal displacements in the independent variable domain (transport phenomena) to amplitude (dependent variable) modulations, and can thus "linearize" certain problems, leading to computationally efficient, closed form (non-iterative), and with known mathematical properties. Example applications related to computer vision, signal processing, and machine learning will be demonstrated. (Received September 21, 2021)

45 ► Integral equations

1174-45-5844 **Wenchuan Tian*** (tian.wenchuan@gmail.com), UC Santa Barbara. *On A Family Of Integral Operators On The Ball*

In this work, we transform the equation in the upper half space first studied by Caffarelli and Silvestre to an equation in the Euclidean unit ball $\mathbb{B}^n$. We identify the Poisson kernel for the equation in the unit ball. Using the Poisson kernel, we define the extension operator. We prove an extension inequality in the limit case and prove the uniqueness of the extremal functions in the limit case using the method of moving spheres. In addition we offer an interpretation of the limit case inequality as a conformally invariant generalization of Carleman's inequality. (Received August 29, 2021)

1174-45-6076 **Yunyi Mu*** (849824272@qq.com), School of Arts and Sciences, Shanghai Dianji University. *Two types of integro-differential equations in Banach spaces* Preliminary report.

This work concerns two types of integro-differential equations in Banach spaces: (1) impulsive delay equations with general conformable fractional derivatives on unbounded intervals; (2) equations with operator pairs without/with impulsive conditions involving ψ-Caputo fractional derivatives. With the help of the theories of C_0-semigroups, the generalized Laplace transforms, probability density functions and fractional order derivatives, we give proper definitions of mild solutions for the above equations. Using our newly derived Gronwall-type inequalities and combining with operator semigroup theory, new sequence estimations and other tools, we investigate the existence, uniqueness, (ω, c)-periodicity, asymptotic stability, continuous dependence on initial values, boundedness and some other properties of mild solutions of the above equations, and established some new results.

This is a joint work with Jin Liang and Ti-Jun Xiao. (Received September 3, 2021)

1174-45-6947 **Marta D'Elia*** (mdelia@sandia.gov), Sandia National Laboratories. *Data-driven learning of nonlocal models*

Nonlocal models are characterized by integral operators that embed lengthscales in their definition. As such, they are preferable to classical partial differential equation models in situations where the dynamics of a system is affected by the small scale behavior, yet the small scales would require prohibitive computational cost to be treated explicitly. In this sense, nonlocal models can be considered as coarse-grained, homogenized models that, without resolving the small scales, are still able to accurately capture the system's global behavior. However, nonlocal models depend on "kernel functions" that are often hand tuned. We propose to learn optimal kernel functions from high fidelity data by combining machine learning algorithms, known physics, and nonlocal theory. This combination guarantees that the resulting model is mathematically well-posed and physically consistent. Furthermore, by learning the operator rather than a surrogate for the solution, these models generalize well to settings that are different from the ones used during training, hence enabling transfer learning. We apply this learning technique to find homogenized nonlocal models for molecular dynamics. Here, the machine-learned nonlocal operator embeds material properties in the kernel function and allows for accurate predictions at much coarser scales than the molecular scale. (Received September 10, 2021)

1174-45-7142 **Bacim Alali*** (bacimalali@math.ksu.edu), Kansas State University, and **Nathan Albin** (albin@ksu.edu), Kansas State University. *Eigenvalues of linear peridynamic operators*

A characterization for the eigenvalues of linear peridynamic operators is provided. The analysis is presented for state-based peridynamic operators for isotropic homogeneous media in any spatial dimension. We provide explicit formulas for the eigenvalues in terms of the space dimension, the nonlocal parameters, and the material properties.

The approach we follow is based on the Fourier multiplier analysis developed for nonlocal Laplace operators in [1]. It is shown that the Fourier multipliers of linear peridynamic operators are second-order tensor fields, which are given through integral representations. The eigenvalues of the peridynamic operators can be derived directly from the eigenvalues of the Fourier multiplier tensors. We reveal a simple structure for the Fourier multipliers in terms of hypergeometric functions, which allows for providing integral representations as well as hypergeometric representations for the eigenvalues. These representations are utilized to show the convergence of the eigenvalues of linear peridynamics to the eigenvalues of the Navier operator of linear elasticity in the limit of vanishing nonlocality. Moreover, the hypergeometric representation of the eigenvalues is utilized to compute the spectrum of linear peridynamic operators.

[1] Alali, B. and Albin, N., 2021. Fourier multipliers for nonlocal Laplace operators. Applicable Analysis, 100(12), pp.2526-2546. (Received September 13, 2021)

1174-45-8445 **Yue Yu*** (yuy214@lehigh.edu), Lehigh University, **Huaiqian You** (huy316@lehigh.edu), Lehigh University, and **Lu Zhang** (luz319@lehigh.edu), Lehigh University. *Meta-NOR: A Provable Meta-Learnt Nonlocal Operator Regression Approach* Preliminary report.

In this talk, we propose a meta-learnt approach for transfer-learning mappings between function spaces (operators), Meta-NOR, based on the nonlocal operator regression. The overall goal is to efficiently provide accurate operator surrogates in new and unknown PDE-learning tasks, such that each task is governed by a different operator (e.g., with different constitutive laws or material parameters). The proposed sample-efficient meta-learning algorithm uses a multi-task nonlocal operator regression model in the kernel space, which consists of two phases: (1) learning a common nonlocal kernel representation from existing tasks; and (2) transferring the learned knowledge on the nonlocal kernel representation and rapidly learning surrogate operators for new tasks with different governing PDEs, where the governing PDEs could be possibly unknown and only a few test samples are provided. Under the linear kernel regression setting, a provable optimization-based approach is provided, with theoretically guaranteed transfer-learning error bounds. We apply the proposed technique to model the wave propagation within 1D meta-materials with both periodic and random microstructures, showing that the meta-learnt kernel representation would greatly improve the sampling efficiency in new and unseen microstructures, compared to existing baselines. (Received September 19, 2021)

1174-45-8700 **Richard Gustavson** (rgustavson01@manhattan.edu), Manhattan College, and **Sarah Rosen*** (srosen01@manhattan.edu), Manhattan College. *Reduction Algorithms in Volterra Integral Equations*

An integral equation is a way to encapsulate the relationships between a function and its integrals. We develop a systematic way of describing Volterra integral equations – specifically an algorithm that reduces any integral equation into an equivalent one in operator-linear form, i.e. one that only contains iterated integrals. This serves to formalize a method of simplifying integral equations to a form that has a more direct process of being solved. We use the algebraic object of the integral operator, the twisted Rota-Baxter identity, and vertex-edge decorated trees to construct and prove our algorithm. (Received September 19, 2021)

1174-45-9298 **Tadele Mengesha*** (mengesha@utk.edu), University of Tennessee. *Fractional Hardy-type and trace theorems for nonlocal function spaces with heterogeneous localization*

I will present a recent work that proves a Hardy-type inequality as well as a trace theorem for a class of function spaces on smooth domains with a nonlocal character. Functions in these spaces are allowed to be as rough as an L^p-function inside the domain of definition but as smooth as a $W^{s,p}$-function near the boundary. This feature is captured by a norm that is characterized by a nonlocal interaction kernel defined heterogeneously with a special localization feature on the boundary. As the spaces properly include the classical fractional Sobolev space, the results we obtain can be viewed as a refinement of the classical trace theorems and Hardy inequalities for Sobolev spaces. A Poincaré-type inequality we establish for the function space under consideration together with the new trace theorem allow formulating and proving well-posedness of a nonlinear nonlocal variational problem with conventional local boundary condition. This is joint work with Q. Du and X. Tian. (Received September 20, 2021)

1174-45-9835 **Petronela Radu*** (pradu@unl.edu), University of Nebraska-Lincoln. *Operators in nonlocal frameworks: properties, decompositions, and convergence*

The emergence of nonlocal theories as successful models for studying phenomena and behaviors in different areas of science (continuum mechanics, biology, image processing) has led the mathematical community to introduce and conduct investigations of integral operators and associated systems of equations. In this talk I will present some recent results on nonlocal frameworks systems based on some existing, as well as newly introduced, nonlocal operators. An in-depth study of properties of the operators includes a series of results on nonlocal versions of integration by parts theorems, Helmholtz-Hodge type decompositions, as well as convergence of operators to their classical equivalents as the interaction horizon vanishes. (Received September 20, 2021)

1174-45-9930 **James Scott** (jms652@pitt.edu), Columbia University, and **Mamikon Gulian*** (mamikon.armen.gulian@gmail.com), Sandia National Laboratories. *Connections between nonlocal operators: from vector calculus identities to a fractional Helmholtz decomposition* Preliminary report.

Nonlocal vector calculus, based on the nonlocal forms of gradient, divergence, and diffusion operators in multiple dimensions, has promising applications in fields such as subsurface transport and turbulence modeling. We provide a rigorous treatment of compositions of nonlocal operators, which allows proofs of important nonlocal vector calculus identities and unification of weighted and unweighted variational frameworks. We combine several of these result to obtain a weighted fractional-order Helmholtz decomposition. Our rigorous approach allows for identifying function spaces in which the stated identities and decompositions hold. This work provides useful and intuitive analogous of classical vector calculus for nonlocal modeling. (Received September 21, 2021)

1174-45-11098 **Nicole Buczkowski*** (nbuczkowski@huskers.unl.edu), University of Nebraska-Lincoln, **Michael L Parks** (mlparks@sandia.gov), Sandia National Laboratories, and **Jeremy Trageser** (jtrages@sandia.gov), Sandia National Laboratories. *Solution sensitivity on data parameters for nonlocal systems*

Nonlocal models have gained interest due to their flexibility in handling discontinuities. Continuous dependence on data and stability is an important feature for solutions that model physical phenomena, since measurements of physical parameters can never be exact. In this talk we consider the stability of the solution with regards to changes in given data for both the boundary value problem for the nonlocal Possion's equation and the hinged problem for the nonlocal biharmonic. This research was done as part of the NSF Non-Academic Research Internships for Graduate Students (INTERN) hosted by Sandia National Laboratories. (Received September 21, 2021)

46 ► Functional analysis

1174-46-5933 **Jack Spielberg*** (jack.spielberg@asu.edu), Arizona State University. *Categories of paths that define the Effros-Shen algebras*

The approximately finite dimensional (or AF) C^*-algebras were introduced by Bratteli, who described them by means of certain directed graphs (called Bratteli diagrams). Five years later Cuntz studied the algebras

O_n, and this led to a different construction of C^*-algebras from arbitrary directed graphs. This construction could produce all AF algebras (up to explicit Morita equivalence) and many other algebras. This construction was generalized by Kumjian and Pask with the notion of *higher rank graph*, which may be thought of as a directed graph with certain identifications among the finite paths. These identifications are notoriously difficult to produce (with certain exceptions). A more flexible generalization of directed graphs was given by the notion of *category of paths*. These are also directed graphs with identifications, but the requirements are much more relaxed than those for higher rank graphs. In this talk I will describe a family of categories of paths whose C^*-algebras give the continued fraction AF algebras of Effros and Shen. This demonstrates an unusual internal structure of these algebras. This is joint work with Ian Mitscher. (Received August 31, 2021)

1174-46-6407 **Lauren Chase Ruth** (LRuth@mercy.edu), Mercy College, **Sarah Browne*** (slbrowne@ku.edu), University of Kansas, **Sara Azzali** (sara.azzali@uni-greifswald.de), Universitat Greifswald, **Maria Paula Gomez Aparicio** (maria-paula.gomez-aparicio@universite-paris-saclay.fr), Universite Paris-Saclay, and **Hang Wang** (wanghang@math.ecnu.edu.cn), East China Normal University. *K-homology and K-theory of pure Braid groups* Preliminary report.

The Baum-Connes conjecture is known to be true for the pure braid groups. We compute the left and right hand side of the Baum-Connes correspondence for the pure braid groups on n strands, by providing explicit calculations for both sides. In this talk I will concentrate on the right hand side for the pure Braid group on 4 strands. If time permits I will discuss the left hand side computation. (Received September 14, 2021)

1174-46-6960 **John Quigg*** (quigg@asu.edu), Arizona State University. *Covariant Stone-von Neumann Theorems*

We use nonabelian duality to generalize the Stone-von Neumann Theorem to "modular" representations of actions and coactions of locally compact groups on elementary C^*-algebras. This significantly extends a special case for actions of abelian groups recently proven by Huang and Ismert. Our approach is based on a new result about Hilbert C^*-modules that is simple to state yet is widely applicable and can be used to streamline many previous arguments, and could be regarded as a next step in a long line of results in quantum physics that goes back to von Neumann's proof of the classical Stone-von Neumann Theorem. This is joint work with Lucas Hall and Leonard Huang. (Received September 10, 2021)

1174-46-7129 **Hiroyuki Osaka*** (osaka@se.ritsumei.ac.jp), Ritsumeikan University. *Factorization property of positive maps on C*-algebras*

The purpose of this report is to clarify and present a general version of an interesting observation by Piani and Mora (Physic. Rev. A 75, 012305 (2007)).

Let A_i, C_i be unital C*-algebras and let α_i be positive linear maps from A_i to C_i, $i = 1, 2$. We study conditions under which any positive map β from the minimal C*-tensor product $A_1 \otimes_{min} A_2$ to $C_1 \otimes_{min} C_2$, such that $\alpha_1 \otimes \alpha_2 \geq \beta$, factorizes as $\beta = \gamma \otimes \alpha_2$ for some positive map γ. We obtain that when $\alpha_i \colon A_i \to B(\mathcal{H}_i)$ are completely positive (CP) maps for some Hilbert spaces $\mathcal{H}_i$ $(i = 1, 2)$, and α_2 is a pure CP map and β is a CP map so that $\alpha_1 \otimes \alpha_2 - \beta$ is also CP, then $\beta = \gamma \otimes \alpha_2$ for some CP map γ. We show that a similar result holds in the context of positive linear maps when $A_2 = C_2 = B(\mathcal{H})$ and $\alpha_2 = id$.

As an application we extend the observation by Piani and Mora (revisited recently by Huber et al. in 2018) to show that for any linear map τ from a unital C*-algebra A to a C*-algebra C, if $id_k \otimes \tau$ is decomposable for some $k \geq 2$, where id_k is the identity map on the algebra $M_k(\mathbb{C})$ of $k \times k$ matrices, then τ is completely positive. This report is mainly depend on the joint work of B. V. Rajarama Bhat (Int. J. Quantum Inf. 18 (2020), no. 5, 2050019). (Received September 12, 2021)

1174-46-7164 **Meric Augat** (maugat@wustl.edu), Washington University in St. Louis, **Meredith Sargent*** (Meredith.Sargent@umanitoba.ca), University of Manitoba, University of Arkansas, **Michael T. Jury** (mjury@ufl.edu), University of Florida, and **Palak Arora** (palak.arora@ufl.edu), University of Florida. *Non commutative optimal approximants*

Optimal polynomial approximants have recently been studied in the several variable commutative case on both the polydisk and the unit ball. In this talk, we discuss the framework for studying optimal approximants in the non commutative case, as well as a relation to orthogonal polynomials. (Received September 13, 2021)

1174-46-7222 **Samuel Harris*** (sharris@tamu.edu), Texas A&M University. *Unitary error bases and embeddings of certain universal quantum groups* Preliminary report.

A unitary error basis in the space of $n \times n$ matrices is an orthonormal basis of unitaries for M_n with respect to the normalized trace. These objects have been well-studied and arise frequently in different fields of mathematics.

For example, in quantum information theory, the protocols of quantum teleportation and super-dense coding can be succinctly described using a certain unitary error basis in M_n, along with the maximally entangled Bell state in $\mathbb{C}^n \otimes \mathbb{C}^n$. In this talk, we will look at a generalized version of this unitary error basis and obtain surprising embeddings at the level of matrix algebras over full C^*-algebras of certain quantum automorphism groups of finite-dimensional spaces. As a result, many open questions about these quantum automorphism groups can be reduced to, or solved by, the same problem in the setting of quantum permutation groups.

*This talk is based on joint work with Michael Brannan. (Received September 13, 2021)

1174-46-7261 **Jesse Gabriel Sautel*** (jsautel@vols.utk.edu), University of Tennessee. *Characterization of Multiplicative Shifts on de Branges-Rovynak Spaces* Preliminary report.

In this talk, we will consider de Branges-Rovnyak spaces of analytic functions of d variables in which the row operator of multiplication by z, $M_z = [M_{z_1}, \ldots, M_{z_d}]$, is bounded. In investigating the properties of such an operator, we will present a result that characterizes which row operator tuples $T = [T_1, \ldots, T_d]$, acting on an arbitrary Hilbert space $\mathcal{H}$, are unitarily equivalent to M_z on some de Branges-Rovnyak space $H(B)$. (Received September 13, 2021)

1174-46-7841 **Ishan Ishan*** (ishan.ishan@vanderbilt.edu), Vanderbilt University. *Von Neumann equivalence and weak forms of amenability*

The notion of von Neumann equivalence, which generalizes both measure equivalence and W*-equivalence, was introduced recently by Jesse Peterson, Lauren Ruth, and myself. We showed that many analytic properties, such as amenability, property (T), the Haagerup property, and proper proximality are preserved under von Neumann equivalence. In this talk, I will present my new work expanding the list of properties stable under von Neumann equivalence. In particular, we will discuss the stability of weak amenability, weak Haagerup property, and the approximation property (of Haagerup and Kraus) under von Neumann equivalence. (Received September 16, 2021)

1174-46-7981 **Chi N.Y. Huynh*** (nyhuynh2@illinois.edu), University of Illinois Urbana-Champaign. *Frequency of Besov Mapping Distortion*

Let $f : \Omega \to \mathbb{R}^k$ where $n, k \in \mathbb{N}$, $\Omega \subseteq \mathbb{R}^n$. With additional assumptions on the space f belongs to, many mathematicians have studied Hausdorff dimension distortion of subsets and subspaces under such a mapping. Especially in the case of subspace dimension distortion for Sobolev mappings, the history is rich with results from Gehring and Väisälä, Kaufman, as well as Balogh, Tyson and Wildrick. Using ideas from results proven for f Sobolev mappings, Hencl and Honzík extended the study into mappings of the Triebel-Lizorkin type $\dot{F}^s_{p,q}$ for $1 < p, q < \infty$, a generelization of the Sobolev scale. As the Besov scale $\dot{B}^s_{p,q}$ is another generalization of the Sobolev scale, it is natural to expect similar results for mappings in such space. In this talk, we present our results regarding the dimension distortion of subsets and subspaces under $f \in \dot{B}^s_{p,q}(\Omega, \mathbb{R}^k)$.
(Received September 17, 2021)

1174-46-7989 **Travis Russell*** (travisbrussell@gmail.com), United States Military Academy, **Mark Tomforde** (mtomford@uccs.edu), University of Colorado at Colorado Springs, and **Roy M Araiza** (raraiza@illinois.edu), University of Illinois at Urbana-Champaign. *Matricial Archimedean Order Unit Spaces and Quantum Correlations*

We introduce a matricial analogue of an Archimedean order unit space called a k-AOU space. We develop the category of k-AOU spaces and k-positive maps and exhibit functors from this category to the category of Operator Systems and completely positive maps. We demonstrate the existence of injective envelopes and C*-envelopes in the category of k-AOU spaces as well. Finally, we show that finite-dimensional quantum correlations can be characterized in terms of states on finite-dimensional k-AOU spaces. Combined with previous work, this yields a reformulation of Tsirelson's conjecture in terms of operator systems and k-AOU spaces. (Received September 17, 2021)

1174-46-7995 **Eden Prywes*** (eprywes@princeton.edu), Princeton University, and **Ary Shaviv** (ashaviv@princeton.edu), Princeton University. *On Schwartz Equivalence of Quasidiscs and Other Planar Domains*

Two open subsets of $\mathbb{R}^n$ are Schwartz equivalent if there exists a diffeomorphism between them that induces an isomorphism between the space of Schwartz functions defined on those subsets. In this talk I will discuss the Schwartz space of certain planar domains. I will present a result that shows that all quasidiscs are Schwartz equivalent. Quasidiscs are a subclass of simply connected domains that includes simply connected domains with smooth boundaries as well as domains like the van Koch snowflake. I will also discuss a generalization to non-simply connected domains. The talk is based on joint work with Ary Shaviv. (Received September 17, 2021)

1174-46-8098 **Srivatsav Kunnawalkam Elayavalli** (srivatsav.kunnawalkam.elayavalli@vanderbilt.edu), Vanderbilt University, **Benjamin Richard Hayes*** (brh5c@virginia.edu), University of Virginia, and **David Jekel** (djekel@ucsd.edu), University of California, San Diego. *Property (T) and strong 1-boundedness for von Neumann algebras*

The notion of strong 1-boundedness for finite von Neumann algebras was introduced by Jung in 2007. This framework provided a free probabilistic approach to study rigidity properties and classification of finite von Neumann algebras. I will discuss recent work, joint with Jekel and Kunnawalkam Elayavalli, proving that if M is either a Property (T) von Neumann algebra (in the sense of Connes-Jones, Popa) with finite dimensional center or a group von Neumann algebras of a Property (T) group, then M is strongly 1-bounded. This result generalizes all the previous results in this direction due to Voiculescu, Ge, Ge-Shen, Connes-Shlyakhtenko, Jung-Shlyakhtenko, Jung, and Shlyakhtenko. Prior knowledge of Property (T) will not be assumed. (Received September 17, 2021)

1174-46-8103 **Konrad Aguilar*** (konrad.aguilar@pomona.edu), Pomona College, **Jens Kaad** (kaad@imada.sdu.dk), University of Southern Denmark, and **David Kyed** (dkyed@imada.sdu.dk), University of Southern Denmark. *The Podleś spheres converge to the sphere*

We prove that the Podleś spheres S^2_q converge in quantum Gromov-Hausdorff distance to the classical 2-sphere as the deformation parameter q tends to 1. Moreover, we construct a q-deformed analogue of the fuzzy spheres, and prove that they converge to S^2_q as their linear dimension tends to infinity, thus providing a quantum counterpart to a classical result of Rieffel. (Received September 17, 2021)

1174-46-8184 **Brent Andrew Nelson*** (brent@math.msu.edu), Michigan State University. *Kazhdan sets for stacial von Neumann algebras*

In 1985 Connes and Jones studied a notion of property (T) for a von Neumann algebra M, which says that the standard correspondence $L^2(M)$ is isolated in the space of all correspondences of M when equipped with a topology determined by matrix coefficients. This definition is inspired by Kazhdan's property (T) for groups, and indeed they show that a countable discrete i.c.c. group G has property (T) if and only if its group von Neumann algebra $L(G)$ has property (T). An important characterization of property (T) for type II_1 *factors* obtained by Connes and Jones is the existence of a *Kazhdan set*. A Kazhdan set in M is a finite subset F that admits a constant κ >0 so that for any correspondence $\mathcal{H}$ over M and any $\xi \in \mathcal{H}$

$$\|\xi - P\xi\|^2 \leq \kappa \sum_{x\in F} \|x\xi - \xi x\|^2,$$

where P is the projection onto the M-central vectors in $\mathcal{H}$. In particular, a vector that is *almost* M-central is necessarily close to an honest M-central vector. In this talk I will discuss a generalization of this result to a class of factors which includes the type III_λ factors for $0 < \lambda < 1$. This class consists of factors M admitting faithful normal states φ whose modular automorphism group σ^φ is sufficiently "tame," in which case property (T) is equivalent to the existence of a finite set satisfying a Kazhdan condition twisted by σ^φ. (Received September 17, 2021)

1174-46-8246 **Lara Ismert** (ismertl@erau.edu), Embry-Riddle Aeronautical University, **Mitch Hamidi*** (hamidim@erau.edu), Embry-Riddle Aeronautical University, **Michael Brannan** (michael.p.brannan@gmail.com), University of Waterloo, and **Mateusz Wasilewski** (wasilewskimat@gmail.com), KU Leuven. *The Quantum Cuntz-Krieger Algebra and Edge Correspondence of a Quantum Graph*

A quantum graph is a triple consisting of a finite-dimensional C*-algebra, a state, and a quantum adjacency matrix and can be thought of as a noncommutative analogue of a finite directed graph. The *quantum Cuntz-Krieger* (*QCK*) *algebra* of a quantum graph is defined as the universal C*-algebra generated by the operator coefficients of matrix partial isometries subject to relations that encode information from the quantum graph analogous to the classical Cuntz-Krieger relations. In this talk, we will discuss the connections between a quantum graph's QCK algebra and two natural C*-algebras arising from the graph's *quantum edge correspondence*. (Received September 18, 2021)

1174-46-8488 **Himanshu Singh*** (himanshu4@usf.edu), University of South Florida. *On the theory of Mittag-Leffler Reproducing Kernel Hilbert Space* Preliminary report.

The Mittag-Leffler space of entire functions is a reproducing kernel Hilbert space; was introduced by Rosenfeld et. al. in 2018 whose reproducing kernel function is given by the classical Mittag-Leffler function. We intend to augment the study of this reproducing kernel Hilbert space on the classical mathematical aspects such as general theory pertaining to the equivalent norm representations, isometry, translations, point evaluation estimates, interaction on certain Integral Operators, duality and complex interpolation. These results that we obtain in our study generalizes the analogous results in the Fock space. The primary references for this study are "The Mittag-Leffler reproducing kernel Hilbert spaces of entire and analytic functions" by Rosenfeld, Russo, and Dixon and "Hankel Forms and the Fock space" by Janson, Peetre, and Rochberg.

Research supported by 2021 AFOSR YIP FA9550-21-1-0134 and NATIONAL SCIENCE FOUNDATION-2027976. Research funds are attributed to Dr. Joel A. Rosenfeld, University of South Florida, Tampa, FL, USA. (Received September 19, 2021)

1174-46-8498 **Sofya S Masharipova*** (ssm57@pitt.edu), University of Pittsburgh at Johnstown, and **Shukhrat M Usmanov** (shukhratusmanov@yahoo.com), Slippery Rock University of PA. *Classification of real non-commutative L_p-spaces, $1 \leq p < \infty$, associated with real semifinite hyper-finite W^*-algebras.* Preliminary report.

We consider non-commutative real valued L_p-spaces, $1 \leq p < \infty$, associated with real hyperfinite semi-finite W^*-algebras. We construct some non-trivial examples of real valued non-commutative L_p-spaces. The theorem of classification of such real non-commutative L_p-spaces is proven. This result is a real valued analogy of the remarkable result of U. Haagerup, H. P. Rosenthal, and F. A. Sukochev (Theorem 6.1 from "Banach embedding properties of non-commutative L_p-spaces", Memoirs of AMS, 2003; Volume 163, Number 776). (Received September 19, 2021)

1174-46-9156 **Samantha Pilgrim*** (spilgrim@hawaii.edu), The University of Hawai'I At Manoa. *Isometric Actions and Finite Approximations*

We demonstrate relationships between group actions which fix metrics and those which admit certain finite approximations (definitions and some elementary theory of such approximations for group actions will also be explained). In particular we show that every isometric action on a Cantor set is conjugate to an inverse limit of actions on finite sets; and that every isometric action by a finitely generated, amenable group is residually finite. We also discuss the implications for C^*-algebras arising from such actions, proving in a more direct way than previously known that crossed products of the isometric actions mentioned earlier are quasi-diagonal. (Received September 20, 2021)

1174-46-9172 **Marcelo E Laca*** (laca@uvic.ca), University of Victoria. *Boundary quotients and equilibrium states of the right Toeplitz algebra of the ax+b semigroup over the natural numbers*

We consider the $ax + b$ semigroup over the natural numbers and we study the C^*-algebra generated by its right-regular representation, which we call the right Toeplitz algebra. We analyze the structure of this C^*-algebra by focusing on three ideals and their associated quotients, which we interpret as an additive boundary, a multiplicative boundary and a minimal boundary. We also describe the simplex of equilibrium states at each inverse temperature for a natural time evolution on the right Toeplitz algebra. This is based on joint work with A. an Huef and I. Raeburn (NZJM 2021) and with T. Schulz (in progress). (Received September 20, 2021)

1174-46-9181 **Igor V. Nikolaev*** (igor.v.nikolaev@gmail.com), St. John's University, Staten Island, NY. *Langlands reciprocity for C^*-algebras*

We introduce a C^*-algebra $\mathcal{A}_V$ of a variety V over the number field K and a C^*-algebra $\mathcal{A}_G$ of a reductive group G over the ring of adeles of K. Using Pimsner's Theorem we construct an embedding $\mathcal{A}_V \hookrightarrow \mathcal{A}_G$, where V is a G-coherent variety, e.g. the Shimura variety of G. The embedding is an analog of the Langlands reciprocity for C^*-algebras. It follows from the K-theory of the inclusion $\mathcal{A}_V \subset \mathcal{A}_G$ that the Hasse-Weil L-function of V is a product of the automorphic L-functions corresponding to irreducible representations of the group G. Reference: arXiv:1505.03054 (Received September 20, 2021)

1174-46-9775 **Mikil Foss*** (mfoss3@unl.edu), University of Nebraska-Lincoln. *A Trace Theory for Some Nonlocal Function Spaces*

Trace theorems typically require some regularity away from the boundary of the function's domain. I will present some new conditions under which a well-defined trace can be identified on a set with strictly positive codimension. The conditions involve an oscillation constraint that does not require regularity in the domain's interior but ensures some regularity at the boundary itself. The assumptions allow domains with very rough boundaries that possess cusp-like features and are compatible with nonlocal problems where convolution-like operators with integrable kernels are employed. (Received September 20, 2021)

1174-46-9813 **Michel L Lapidus** (lapidus@math.ucr.edu), University of California Riverside/AMS Associate Secretary, and **Therese Landry*** (tland004@ucr.edu), University of California, Riverside. *The Spectral Propinquity, the Stretched Sierpinski Gasket, and other Almost Piecewise C^1-Fractal Curves* Preliminary report.

The stretched Sierpinski gasket of parameter α, $0 < \alpha < 1/3$, or SSG_α, is a self-affine fractal that can be viewed as a variation of the Sierpinski gasket, SG. Although both fractals are each used to model fundamentally different media, SSG_α can be shown to converge to SG for the Hausdorff distance as α goes to zero. In the setting of noncommutative geometry, spectral triples are a generalization of differential structure due to Connes. Via a construction due to Michel Lapidus and Jonathan Sarhad, there also exist spectral triples on SSG_α and SG that capture important aspects of their geometries. Building on earlier work by Marc Rieffel, Frederic Latremoliere formulated the spectral propinquity metric, which can be viewed as an extension of Hausdorff distance to a certain class of spectral triples that includes the Lapidus-Sarhad spectral triples. Spectral propinquity metric convergence of the Lapidus-Sarhad spectral triple on SSG_α to that on SG as α goes to zero will be used as a motivating example for the definition of the class of almost piecewise C^1-fractal curves, thereby extending earlier work by Lapidus, Latremoliere, and the presenter on metric approximation of spectral triples on fractal curves. (Received September 21, 2021)

1174-46-10302 **Roberto Hernandez Palomares*** (rhp@tamu.edu), The Ohio State University. *Q-System Completion for C^* 2-categories*

Q-systems were introduced by Longo to describe the canonical endomorphism of a finite index inclusion of infinite von Neumann factors $N \subset M$. Q-systems in C^*Alg characterize finite Watatani index extensions of a unital C^*-algebra $A \subset B$ equipped with a faithful conditional expectation $E : B \to A$. Following work of Douglas-Reutter, a Q-system is also a unitary version of a higher idempotent, and Q-system completion is a unitary version of a higher idempotent completion for C^* 2-categories.

We will focus on the C^* 2-category C^*Alg with objects unital C^*-algebras, 1-morphisms right C^*-correspondences, and 2-morphisms adjointable intertwiners. We will show C^*Alg is Q-system complete, i.e. Q-system completion is a †-2-equivalence. We prove this by constructing an inverse †-2-functor called realization which turns Q-systems back into unital C^*-algebras. These techniques allow for the straightforward adaptation of subfactor techniques to studying actions of unitary tensor categories on C^*-algebras.

This is joint work with Quan Chen, Corey Jones, and David Penneys. (Received September 21, 2021)

1174-46-10431 **Lucas Hall*** (lhall10@asu.edu), Arizona State University. *Skew Products for Topological Quivers* Preliminary report.

We introduce the skew product construction to topological quivers, the broadest topological analogs of discrete directed graphs. We give a characterization of the topological quivers which arise from this construction. As in the discrete setting, skew products for topological quivers successfully model the C^*-algebraic notion of coactions. (Received September 21, 2021)

1174-46-10437 **Daniel W van Wyk*** (daniel.w.van.wyk@dartmouth.edu), Dartmouth College, and **Dana P Williams** (dana@math.dartmouth.edu), Dartmouth College. *The Primitive Ideal Space of Groupoid C^*-Algebras for Groupoids with Abelian Isotropy* Preliminary report.

We study the topology of the primitive ideal space of groupoid C^*-algebras for groupoids with abelian isotropy. Our results include the known results for action groupoids with abelian stabilizers. Furthermore, we obtain complete results when the isotropy map is continuous except for jump discontinuities, and also when G is a unit space fixing extension of a proper groupoid by an abelian group bundle. We hope that our methods will be a springboard to further results of this type. (Received September 21, 2021)

1174-46-11291 **Dorsa Ghoreishi*** (dorsa.ghoreishi@slu.edu), Saint Louis University, and **Daniel Freeman** (daniel.freeman@slu.edu), Saint Louis University. *Discretizing L_p Norms and Frame Theory* Preliminary report.

Given an N-dimensional subspace X of $L_p([0,1])$, we consider the problem of choosing M-sampling points which may be used to discretely approximate the L_p norm on the subspace. We are particularly interested in knowing when the number of sampling points M can be chosen on the order of the dimension N. For the case $p = 2$ it is known that M may always be chosen on the order of N as long as the subspace X satisfies a natural L_∞ bound, and for the case $p = \infty$ there are examples where M may not be chosen on the order of N. We show for all $1 \leq p < 2$, there exist classes of subspaces of $L_p([0,1])$ which satisfy the L_∞ bound, but where the number of sampling points M cannot be chosen on the order of N. We show as well that the problem of discretizing the L_p norm of subspaces is directly connected with frame theory. In particular, we show that discretizing a continuous frame to obtain a discrete frame which does stable phase retrieval requires discretizing both the L_2 norm and the L_1 norm on the range of the analysis operator of the continuous frame. This is joint work with Daniel Freeman. (Received September 29, 2021)

47 ▶ Operator theory

1174-47-5610 **Roger Nichols** (roger-nichols@utc.edu), University of Tennessee at Chattanooga, **Minsung Cho*** (minsungc@andrew.cmu.edu), Carnegie Mellon University, **Brian Udall** (briandudall@gmail.com), Rice University, and **Seth Hoisington** (seh9gb@virginia.edu), University of Virginia. *The Krein-von Neumann Extension of a Regular Even Order Quasi-Differential Operator*

We characterize by boundary conditions the Krein–von Neumann extension of a strictly positive minimal operator corresponding to a regular even order quasi-differential expression of Shin–Zettl type. The characterization is stated in terms of a specially chosen basis for the kernel of the maximal operator and employs a characterization of the Friedrichs extension due to Möller and Zettl. Using our result, we recover known characterizations of the Krein–von Neumann extension for special cases. (Received August 23, 2021)

1174-47-6041 **Henry Adams** (henry.adams@colostate.edu), Colorado State University, and **Mark Blumstein** (mark.blumstein@gmail.com), Colorado State University. *Multidimensional Scaling on Metric Measure Spaces*

Multidimensional scaling (MDS) is a popular technique for mapping a finite metric space into a low-dimensional Euclidean space in a way that best preserves pairwise distances. We overview the theory of classical MDS, along with its optimality properties and goodness of fit. Further, we present a notion of MDS on infinite metric measure spaces that generalizes these optimality properties. As a consequence we can study the MDS embeddings of the geodesic circle S^1 into $\mathbb{R}^m$ for all m, and ask questions about the MDS embeddings of the geodesic n-spheres S^n into $\mathbb{R}^m$. Finally, we address questions on convergence of MDS. For instance, if a sequence of metric measure spaces converges to a fixed metric measure space X, then in what sense do the MDS embeddings of these spaces converge to the MDS embedding of X? (Received September 3, 2021)

1174-47-7006 **Robert T.W. Martin** (rtwmartin@gmail.com), University of Manitoba, **Edward John Timko*** (etimko6@gatech.edu), Georgia Institute of Technology, and **Raphael Clouatre** (raphael.clouatre@umanitoba.ca), University of Manitoba. *An F. and M. Riesz Theorem for the Free Disc Algebra*

We explore non-commutative operator-theoretic analogues of the F. and M. Riesz Theorem. By the classical theorem, measures on the unit circle in $\mathbb{C}$ that annihilate the disc algebra are absolutely continuous with respect to arc-length measure, with the Radon-Nikodym derivatives possessing certain analytic structure. We develop generalizations of this univariate result, finding that the GNS representation naturally associated to an 'analytic' functional on the Cuntz algebra cannot have any singular summand. In contrast to the univariate result, we find that weak*-continuous extensions of analytic functionals on the free disc operator system do not always exist, and we identify the obstruction in terms of the so-called universal structure projection. This is joint work with R. Clouâtre and R. T. W. Martin. (Received September 11, 2021)

1174-47-7136 **Anne Greenbaum*** (greenbau@uw.edu), University of Washington. *K-Spectral Sets* Preliminary report.

Let A be an n by n matrix or a bounded linear operator on a Hilbert space $(H, \langle\cdot,\cdot\rangle, \|\cdot\|)$. A closed subset $\Omega \subset \mathbb{C}$ is a K-spectral set for A if the spectrum of A is contained in Ω and if the following inequality,

$$\|f(A)\| \leq K \sup_{z\in\Omega} |f(z)|,$$

holds for all rational functions f bounded in Ω. For a normal matrix A, one can simply take Ω to be the set of eigenvalues and $K = 1$, but for nonnormal matrices and operators things are not so straightforward.

Von Neumann's inequality states that if A is a contraction, i.e., $\|A\| \leq 1$, then the unit disk is a K-spectral set with $K = 1$. Crouzeix and Palencia showed that the numerical range, $W(A) := \{\langle Aq, q\rangle : \|q\| = 1\}$, or its closure in the case of operators, is a $(1+\sqrt{2})$-spectral set, and Crouzeix's conjecture is that it is actually a 2-spectral set. Okubo and Ando showed that any disk containing $W(A)$ is a 2-spectral set.

Here we derive other K-spectral sets and discuss applications. We show that if $\Omega = W(A)\backslash\mathbb{D}(\omega, r)$, where $r = 1/\|(A - \omega I)^{-1}\|$, then Ω is a $(2 + \sqrt{11})$-spectral set for A. We extend this to show that if m such disks $\mathcal{D}(\omega_j, r_j)$, $r_j = 1/\|(A - \omega_j I)^{-1}\|$, $j = 1, \ldots, m$, are removed from $W(A)$, then the resulting region is a $(1 + m + \sqrt{(2m + 1)^2 + 1 + m})$-spectral set for A. (Received September 12, 2021)

1174-47-7227 **Chi-Kwong Li*** (ckli@math.wm.edu), College of William & Mary, and **Jor-Ting Chan** (jtchan@hku.hk), University of Hong Kong. *Commuting normal operators and joint numerical range*

Let $\mathcal{B}(\mathcal{H})$ be the algebra of all bounded linear operators acting on a Hilbert space $\mathcal{H}$. For a positive integer k and $\mathbf{A} = (A_1, \ldots, A_m) \in \mathcal{B}(\mathcal{H})^m$, the joint k-numerical range $W_k(\mathbf{A})$ is the set of $(\alpha_1, \ldots, \alpha_m) \in \mathbb{C}^m$ such that $\alpha_i = \sum_{j=1}^{k} \langle A_i x_j, x_j\rangle$ for an orthonormal set $\{x_1, \ldots, x_k\}$ in $\mathcal{H}$. It is shown that there is $k \in \mathbb{N}$ such that $W_k(\mathbf{A})$ is a polyhedral set if and only if $A_1, \ldots, A_m$ have a common reducing subspace $\mathbf{V}$ of finite dimension such that the compression of $A_1, \ldots, A_m$ on the subspace $\mathbf{V}$ are diagonal operators $D_1, \ldots, D_m$ and $W_k(\mathbf{A}) = W_k(D_1, \ldots, D_m)$. Characterization is also given to $\mathbf{A}$ such that the closure of $W_k(\mathbf{A})$ is polyhedral. The conditions are related to the joint essential numerical range of $\mathbf{A}$. These results are used to study $\mathbf{A}$ such that (a) $\{A_1, \ldots, A_m\}$ is a commuting family of normal perators, or (b) $W_k(A_1, \ldots, A_m)$ is polyhedral for every positive integer k. It is shown that conditions (a) and (b) are equivalent for finite rank operators but it is no longer true for compact operators. Characterizations are given for compact operators $A_1, \ldots, A_m$ satisfying (a) and (b), respectively. Results are also obtained for general non-compact operators. (Received September 13, 2021)

1174-47-7256 **Christoph Fischbacher*** (fischbac@uci.edu), University of California, Irvine. *Complete Non-Selfadjointness for Schrödinger Operators on the Semi-Axis*

We investigate complete non-selfadjointness for all maximally dissipative extensions of a Schrödinger operator on a half-line with dissipative bounded potential and dissipative boundary condition. We show that all maximally dissipative extensions that preserve the differential expression are completely non-selfadjoint. However, it is possible for maximally dissipative extensions to have a one-dimensional reducing subspace on which the operator is selfadjoint. We give a characterization of these extensions and the corresponding subspaces and present a specific example. (Received September 13, 2021)

1174-47-7327 **William Thomas Ross** (wross@richmond.edu), University of Richmond, **Raymond Cheng** (rcheng@odu.edu), Old Dominion University, and **Daniel Seco*** (dsf_cm@yahoo.es), Universidad Carlos III de Madrid. *Zeros of optimal polynomial approximants in ℓ^p_A*

The study of inner and cyclic functions in ℓ^p_A spaces requires a better understanding of the zeros of so-called optimal polynomial approximants. We determine that a point of the complex plane is the zero of an optimal polynomial approximant for some element of ℓ^p_A if and only if it lies outside of a closed disk (centered at the origin) of a particular radius which depends on the value of p. We find the value of this radius for $p \neq 2$. In addition, for each positive integer d there is a polynomial f_d of degree at most d that minimizes the modulus of the root of its optimal linear polynomial approximant. We develop a method for finding these extremal functions f_d and discuss their properties. The method involves the Lagrange multiplier method and a resulting dynamical system. (Received September 14, 2021)

1174-47-7396 **Robert T.W. Martin*** (Robert.Martin@umanitoba.ca), University of Manitoba, and **Eli Shamovich** (shamovic@bgu.ac.il), Ben-Gurion University of the Negev. *Non-commutative rational Clark measures*

Any contractive analytic function in the complex unit disk corresponds uniquely to a positive measure on the unit circle, its *Aleksandrov–Clark measure*. We completely characterize the non-commutative (NC) Clark measures and the minimal realizations of the NC rational functions which are contractive on the NC unit row–ball of all finite–dimensional strict row contractions. In particular, we identify the NC Clark measures and minimal realizations of the inner, *i.e.* isometric, multipliers of the Fock space of square–summable power series in several NC variables.

Given a contractive NC rational multiplier, $\mathfrak{b}$, of the Fock space, we further extend a classical result of Aronszajn–Donoghue to show that the singular parts of the one–parameter family of Clark measures of $\zeta \cdot \mathfrak{b}$, where ζ belongs to the unit circle, are mutually singular in an appropriate operator-algebraic sense. (Received September 14, 2021)

1174-47-7459 **David Kimsey*** (david.kimsey@ncl.ac.uk), Newcastle University. *The matricial subnormal completion problem*

Given a truncated sequence of positive numbers $a = (a_j)_{j=0}^m$, the subnormal completion problem (originally posed and solved via a geometric approach by J. Stampfli) asks whether or not there exists a subnormal weighted shift operator on ℓ_2 whose initial weights are given by a. Subsequently a concrete solution based on a concrete solution of the truncated Stieltjes moment problem was discovered by Curto and Fialkow.

In this talk, we will consider a matricial analogue of the subnormal completion problem, where the truncated sequence of positive numbers $(a_j)_{j=0}^m$ is replaced by a truncated sequence of positive definite matrices. We will provide concrete conditions for a solution based on the parity of m by making a connection with a truncated matrix-valued Stieltjes moment problem and zeros of matrix polynomials which describe the rank preserving extensions. (Received September 14, 2021)

1174-47-7521 **Linda J. Patton*** (lpatton@calpoly.edu), Cal Poly San Luis Obispo. *Toeplitz operators with matrix numerical ranges* Preliminary report.

If A is an n by n matrix such that the interior of the numerical range $W(A)$ is nonempty, then $W(A)$ is the closure of the numerical range of the Toeplitz operator T_ϕ, where ϕ is the conformal map from the open unit disk onto the interior of $W(A)$. The converse problem seems to be interesting: given a Toeplitz operator T_ϕ, when does there exist a finite matrix A such that the closure of $W(T_\phi)$ is $W(A)$? We'll discuss some examples. (Received September 14, 2021)

1174-47-7527 **Benjamin Peter Russo*** (russo588@gmail.com), Oak Ridge National Laboratory. *Applications of Reproducing Kernels to Dynamical Systems in the Sciences*

In this talk we will discuss applications of reproducing kernel Hilbert spaces to dynamical systems problems appearing in the science and engineering. We will cover currently used techniques, extensions, and new theoretical directions. (Received September 14, 2021)

1174-47-7533 **Cynthia Vinzant*** (vinzant@uw.edu), University of Washington, **Lillian Pasley Simon** (lfpasley@ncsu.edu), North Carolina State University, **Daniel Plaumann** (daniel.plaumann@math.tu-dortmund.de), TU Dortmund, and **Abeer Al-Ahmadieh** (aka2222@uw.edu), University of Washington. *Invariant determinantal representations and numerical ranges*

The numerical range of a matrix is a convex body in the complex plane capturing its values as a Hermitian bilinear form on the sphere. By a classical theorem of Kippenhahn, its dual convex body is given by a linear matrix inequality and bounded by the corresponding determinantal plane curve. One can show that the numerical range of a block cyclic weighted shift matrix is invariant under the action of a dihedral group. In this talk, I will talk about the converse, namely that any invariant numerical range comes from such a matrix, as well as the set of matrices all of whose numerical ranges are invariant under arbitrary rotation. This relies on understanding invariance of definite determinantal representations of plane curves. This is based on joint works with Abeer Al Ahmadieh, Daniel Plaumann and Faye Pasley Simon. (Received September 14, 2021)

1174-47-7550 **Priyanga Ganesan*** (priyanga.g@tamu.edu), Texas A&M University, College Station. *Spectral bounds on chromatic number of quantum graphs*

Quantum graphs are an operator space generalization of classical graphs that have appeared in different branches of mathematics including operator systems theory, non-commutative topology and quantum information theory. In this talk, I will review the different perspectives to quantum graphs and introduce a chromatic number for quantum graphs using a non-local game with quantum inputs and classical outputs. I will then show that many spectral lower bounds for chromatic number of classical graphs (such as Hoffman's bound) also hold in the setting of quantum graphs. This is achieved using an algebraic formulation of quantum graph coloring and an interplay between matrix analysis and operator theory. (Received September 14, 2021)

1174-47-7593 **Christopher Felder** (cfelder@wustl.edu), Washington University In St. Louis, and **Douglas T. Pfeffer*** (dpfeffer@berry.edu), Berry College. *Spectra for Toeplitz operators associated with finite codimensional subalgebras of H^∞*

Given a symbol $\phi \in L^\infty$, its Toeplitz operator $T_\phi\colon H^2 \to H^2$ is defined by $T_\phi f = P_{H^2}\phi f$, where $P_{H^2}: L^2 \to H^2$ is the orthogonal projection. Here, H^∞ is viewed as the multiplier algebra for H^2. Investigations into the spectrum of these Toeplitz operators have been carried out for a variety of symbols ϕ. Recently, similar investigations have taken place for constrained subalgebras of H^∞ such as the 2-point and Neil algebras. In this talk, we talk about our work with the 2-point algebra, and discuss our progress on generalizing to finite codimensional subalgebras of H^∞. This is joint work with C. Felder and B. Russo. (Received September 15, 2021)

1174-47-7643 **Tapesh Yadav*** (tapeshyadav@ufl.edu), University of Florida. *On Asymptotic Moments and Freeness of Patterned Random Matrices* Preliminary report.

For a pattern (that is, a subset of unit square with zero Lebesgue measure boundary), we define its approximating matrix as a random matrix with iid entries arranged according to the given pattern. We will show that such matrices converge in distribution and almost surely asymptotically. We will also discuss the results for several independent patterned matrices including asymptotic freeness of such matrices. The results are based on our preliminary findings. (Received September 15, 2021)

1174-47-8101 **Maria Tjani*** (mtjani@uark.edu), University of Arkansas. *Closed range composition operators on $BMOA$*

Let φ be an analytic self-map of the unit disk $\mathbb{D}$. We find necessary and also sufficient conditions for the composition operator C_φ to be closed range on $BMOA$. Important ingredients are a new sampling condition and a new reverse type Carleson condition. We show that a composition operator is closed range as an operator from the Bloch space to BMOA if and only if it is bounded below on all Möbius transformations. This is joint work with Kevser Erdem. (Received September 17, 2021)

1174-47-8114 **Jonathan Stanfill*** (jonathan_stanfill@baylor.edu), Baylor University. *Spectral ζ-functions for singular Sturm-Liouville operators and the generalized Bessel equation*

We employ a recently developed unified approach to the computation of traces of resolvents and ζ-functions to compute spectral ζ-functions associated to singular (three-coefficient) self-adjoint Sturm–Liouville differential expressions τ. We discuss how this approach can be used to efficiently compute positive integer values of the spectral zeta function. As an application we end by discussing the generalized Bessel equation on a finite interval including computing the sum of the inverse even powers of certain Bessel function zeros.

This is based on joint work with Guglielmo Fucci, Fritz Gesztesy, and Klaus Kirsten. (Received September 17, 2021)

1174-47-8221 **Raul E Curto*** (raul-curto@uiowa.edu), University of Iowa. *The Extended Aluthge Transform*

Given a bounded linear operator T with canonical polar decomposition $T = V|T|$, the Aluthge transform of T is the operator $\Delta(T) := \sqrt{|T|}V\sqrt{|T|}$. For P an arbitrary positive operator such that $VP = T$, we define the *extended* Aluthge transform of T associated with P by

$$\Delta_P(T) := \sqrt{P}V\sqrt{P}.$$

First, we establish some basic properties of Δ_P; second, we study the fixed points of the extended Aluthge transform; third, we consider the case when T is an idempotent. Next, we discuss whether Δ_P leaves invariant the class of complex symmetric operators. We also study how Δ_P transforms the numerical radius and numerical range.

As a key application, we prove that the spherical Aluthge transform of a commuting pair of operators corresponds to the extended Aluthge transform of a 2×2 operator matrix built from the pair. Thus, the theory of extended Aluthge transforms yields results for spherical Aluthge transforms.

This talk is based on joint work with Chafiq Benhida (Université de Lille, France). (Received September 18, 2021)

1174-47-8422 **Bhupendra Paudyal*** (bpaudyal@centralstate.edu), Central State University. *Invariant subspaces of composition operators on $\mathcal{S}^2$* Preliminary report.

Suppose φ is an analytic self map of the unit disk $\mathcal{D}$ and $\mathcal{S}^2$ is the space of the holomorphic functions such that their first derivative belong the Hardy space $\mathcal{H}^2$. The composition operator C_φ and the shift operator M_z on $\mathcal{S}^2$ are defined by $C_\varphi(f) = f \circ \varphi$ and the multiplication by z, $M_z(f) = zf$, respectively. In this work, the invariant subspaces of composition operators on $\mathcal{S}^2$ are studied. Additionally, the conditions for some shift-invariant subspaces to be invariant for composition operators are discussed. (Received September 19, 2021)

1174-47-8774 **Matthew H Faust*** (mfaust@math.tamu.edu), Texas A&M University. *The number of critical points of discrete periodic operators* Preliminary report.

The spectral gap conjecture is a well known and widely believed conjecture in mathematical physics concerning the structure of the Bloch variety (dispersion relation) of periodic operators. The Bloch variety of a discrete operator is algebraic, inviting methods from algebraic geometry to their study.

Motivated by this conjecture, this talk will introduce a bound on the number of critical points of the dispersion relation for discrete periodic operators, and provide a general criterion for when this bound is achieved. We also present a class of periodic graphs for when this criteria is satisfied for Laplace-Beltrami operators. This is joint work with Frank Sottile. (Received September 19, 2021)

1174-47-8807 **John Kyei*** (kyei@usf.edu), University of South Florida. *Interaction Between Occupation Kernels and Multiplication Operators and their Induced Transforms.* Preliminary report.

The Occupation kernel was introduced in a joint work by Joel A. Rosenfeld, Rushikesh Kamalapurkar, and Benjamin Russo. Coupled with densely defined Liouville operators, occupation kernel was used for system Identification and it has been demonstrated through their work to provide a more efficient way for computing dynamic modes of a dynamical system. In this talk, we discuss an analogous notion on the interaction of the multiplication operator with an occupation kernel.

We will investigate properties of the Hilbert Space generated through the interaction between the operator and the occupation kernel. Introducing a bilinear map between symbols of densely defined multiplication operators and the occupation kernel, a kernelized Fourier Transform will be constructed. We will discuss Properties of the transform born out of action of the adjoint of the multiplication operator on the kernel function. (Received September 20, 2021)

1174-47-8921 **Michael Bush*** (mikebush@udel.edu), University of Delaware, **Constanze Liaw** (liaw@udel.edu), University of Delaware and National Science Foundation, and **Josh Padgett** (padgett@uark.edu), University of Arkansas. *Applications of Matrix-Valued Clark Theory* Preliminary report.

The theory of finite-rank perturbations allows for the determination of spectral information for broad classes of operators using the tools of analytic function theory. In this talk, we will explore the implementation of Clark theory in the process of obtaining matrix-valued spectral measures for finite-rank perturbations of operators. Our discussion will contain a variety of settings, ranging from specific results relating to ordinary differential expressions to the broader setups for elliptic partial differential expressions, graph Laplacians, and Toeplitz operators. (Received September 20, 2021)

1174-47-9099 **Boyu Li*** (b32li@uwaterloo.ca), University of Waterloo. *Dilation theory for right LCM semigroup dynamical systems*

We consider the dilation theory for a certain type of semigroup dynamical systems that encode the right LCM structure of the semigroup. This leads to a generalized Naimark dilation theorem and Stinespring's dilation theorem for these semigroup dynamical systems. As an application, we prove a dilation result for contractive representations of the boundary quotient. This is a joint work with Marcelo Laca. (Received September 20, 2021)

1174-47-9149 **Krishnendu Khan*** (krishnendu-khan@uiowa.edu), The University of Iowa. *On fundamental group of type* II_1 *factors*

The fundamental group is an important invariant of finite factors introduced by Murray and von Neumann. Calculation of fundamental group is in general a very difficult and challenging problem. In this talk I shall give the first examples of property (T) factors with trivial fundamental group based on a joint work with Ionut Chifan and Sayan Das. This can be considered as the first step towards Connes' rigidity conjecture. If time permits, I shall talk about some recent development about calculation of fundamental groups of some finite factors based on an ongoing work with Sayan Das. (Received September 20, 2021)

1174-47-9196 **Clare Michelle Bassano*** (cmb071@bucknell.edu), Bucknell University. *Investigations of Inequalities Related to Spectral Set Problems*

This research is motivated by spectral set problems. Spectral set problems are a type of optimization problem, in which one fixes a square matrix A and a domain Ω in $\mathbb{C}$. The goal is, for all polynomials p, to control the norm of $p(A)$ with the supremum of p over Ω. In such problems, objects called extremal functions and extremal vectors demonstrate the extremal cases. In this research, we investigated problems related to these extremal functions and vectors. For example, in the paper "Crouziex's conjecture and related problems" by Bickel et al., it was found that, in most cases, extremal vectors satisfy a surprising inequality. Part of this research was an in-depth study of this inequality and the set of vectors that satisfy it. Our results include the identification of other classes of vectors that satisfy this inequality, and, in certain cases, the complete characterization of the set of vectors that satisfy this inequality. (Received September 20, 2021)

1174-47-9485 **Se-Jin Kim*** (soundsflyout@gmail.com), University of Glasgow. *The Ideal Intersection Property for Essential Groupoid C^*-Algebras* Preliminary report.

This talk concerns a simplicity criterion for essential groupoid C^*-algebras of etale groupoids with locally compact and hausdorff unit space. This is an extension of the work of Breuillard-Kalantar-Kennedy-Ozawa, who classified simplicity of reduced group C^* algebras using the dynamics of its subgroups as well as the work of Kawabe, who described a similar characterization for crossed products of the form $C(X) \rtimes_r G$. We give a purely groupoid theoretic characterization of when the essential C^*-algebra of an etale groupoid with locally hausdorff unit space admits the ideal intersection property. If our groupoid is hausdorff and our unit space minimal, then this gives us a characterization of simplicity of the reduced groupoid C^*-algebra using the dynamics of its isotropy subgroupoid. This is joint work with Matthew Kennedy, Xin Li, Sven Raum, and Dan Ursu. (Received September 20, 2021)

49 ► Calculus of variations and optimal control; optimization

1174-49-5642 **Suzanne Lenhart** (slenhart@tennessee.edu), University of Tennessee, University of Tennessee, Knoxville, and **Wencel W Valega-Mackenzie*** (wenvalegam@gmail.com), University of Tennessee, Knoxville. *Resource Allocation in a PDE Ecosystem Model* Preliminary report.

The effect caused by habitat heterogeneity on a diffusing population is a topic of great importance to understand population dynamics. In this talk, we present a reaction-diffusion population model to analyze the effect of resource allocation in an ecosystem with resources having their own dynamics in space and time. This approach is more realistic than simply assuming the resource level does not change as the population changes. In addition, we solve an optimal control problem of our ecosystem model such that the abundance of a single species is maximized while minimizing the cost of inflow resource allocation. (Received September 15, 2021)

1174-49-6024 **Vinicio Rafael Rios*** (vrios3@lsu.edu), Louisiana State University. *The Hukuhara's theorem for a differential inclusion with time delay*

The celebrated Hukuhara's theorem states that the trajectories of a system of ordinary differential equations cannot leave the interior of their reachable cone once they penetrate it. This observation has played an essential role in engineering applications, specifically in the design of positional and optimal control mechanisms. The aim of this talk is to outline the main aspects of the extension of Hukuhara's theorem to an important class of dynamical systems that is governed by a differential inclusion with a time delay component. We show that under very mild hypotheses, the trajectories of our delayed systems behave similarly to those of the non-delayed case. (Received September 21, 2021)

1174-49-6039 **Jeffrey Jauregui*** (jaureguj@union.edu), Union College. *Capacity, intrinsic flat convergence, and connections with scalar curvature*

The classical Newtonian capacity of a compact set in Euclidean space generalizes readily to asymptotically flat Riemannian manifolds, where it has some interesting interactions with scalar curvature. Motivated from both general relativity and geometric analysis, we investigate the continuity behavior of the capacity functional as the background space varies (and is not necessarily smooth). Specifically, in joint work with Raquel Perales and Jim Portegies, we show the capacity is upper semicontinuous under Sormani–Wenger intrinsic flat convergence, and discuss some possible applications. (Received September 3, 2021)

1174-49-6170 **Andrew Warren*** (awarren1@andrew.cmu.edu), Carnegie Mellon University. *Ultralimits of Wasserstein spaces and metric measure spaces with Ricci curvature bounded from below*

Abstract. We investigate the stability of the Wasserstein distance under metric ultralimits. We first show that if $(X_i, d_i)_{i\in\mathbb{N}}$ is a sequence of metric spaces with metric ultralimit $(\hat{X}, \hat{d})$, then the p-Wasserstein space $(\mathcal{P}_p(\hat{X}), W_p)$ embeds isometrically in a canonical fashion into the metric ultralimit of the sequence of p-Wasserstein spaces $(\mathcal{P}_p(X_i), W_p)$. Second, using a notion of ultralimit of metric measure spaces modeled on the one introduced by Elek, and which corresponds roughly to "metric ultralimit of the underlying metric spaces, together with a Loeb measure coming from the ultraproduct of the reference measures", we show that an ultralimit of $\mathrm{CD}(K,\infty)$ spaces is a $\mathrm{CD}(K,\infty)$ space. In other words, the synthetic notion of lower Ricci curvature bounds due to Lott-Sturm-Villani is stable under metric measure ultralimits. This complements both existing $\mathrm{CD}(K,\infty)$ stability results for Gromov-Hausdorff type convergence of metric measure spaces, and known results on the stability of the synthetic sectional curvature bounds CAT(κ) and CBB(κ) under metric ultralimits. (Received September 6, 2021)

1174-49-6948 **Kayla D Davie*** (kdavie@umd.edu), University of Maryland College Park. *A New Approach to Model Order Reduction for the Stochastic Optimal Control Problem* Preliminary report.

Stochastic optimal control problems have been studied and solved reliably using the reduced basis (RB) method. We look specifically at the diffusion control problem where the parameterized diffusion equation is the PDE constraint. In the deterministic setting, discretizing the optimal control problem using Galerkin FEM results in a 3×3 saddle-point system. This problem can be expressed as an equivalent, reduced 2×2 system. We propose a new approach to implementing reduced basis methods for the stochastic optimal control problem and implement this approach for both the full 3×3 and reduced 2×2 problem. In this approach, we replace the traditional block-diagonal form of the reduced basis matrix built in the offline stage and instead implement a stacked version of this matrix which is of smaller order and requires less computational work to use in solving the online problem. We present numerical results to compare this approach to the standard way of implementing the RB method and prove the efficiency of the proposed approach. (Received September 10, 2021)

1174-49-7544 **Henry Wolkowicz*** (hwolkowicz@uwaterloo.ca), University of Waterloo, and **Jiyoung Im** (j5im@uwaterloo.ca), University of Waterloo. *A Strengthened Barvinok-Pataki Bound on SDP Rank*

The Barvinok-Pataki bound provides an upper bound on the rank of extreme points of a spectrahedron, the intersection of the positive semidefinite cone and a linear manifold. This bound depends solely on the number of affine constraints of the problem, i.e., on the algebra of the problem. Specifically, the triangular number of the rank r is upper bounded by the number of affine constraints. We revisit this bound and provide a strengthened upper bound on the rank using the singularity degree of the spectrahedron. Thus we bring in the geometry and stability of the spectrahedron, i.e., increased instability as seen by higher singularity degree, yields a lower, strengthened rank bound. (Received September 14, 2021)

1174-49-7706 **Nick Edelen*** (nedelen@nd.edu), University of Notre Dame. *Degeneration of 7-dimensional minimal hypersurfaces with bounded index*

A 7D area-minimizing hypersurface M can in general have a discrete singular set. The same is true if M is only locally-stable for the area-functional, provided $\mathcal{H}^6(singM) = 0$. In this paper we show that if M_i is a sequence of 7D minimal hypersurfaces with discrete singular set which are minimizing, stable, or have bounded index, and varifold-converge to some M, then the geometry, topology, and singular set of the M_i can degenerate in only a very precise manner. We show that one can always "parameterize" a subsequence i' by ambient, controlled bi-Lipschitz maps taking $\phi_{i'}(M_{1'}) = M_{i'}$. As a consequence, we prove that the space of closed, C^2 embedded minimal hypersurfaces in a closed 8-manifold (N, g) with a priori bounds $\mathcal{H}^7(M) \leq \Lambda$ and $index(M) \leq I$ divides into finitely-many diffeomorphism types, and this finiteness continues to hold if one allows the metric g to vary, or M to be singular. (Received September 15, 2021)

1174-49-7884 **Alessio Figalli*** (alessio.figalli@math.ethz.ch), ETH Zurich. *Optimal transport: from nature to machine learning*

At the end of the 18th century, Gaspard Monge introduced the optimal transport problem to understand the most efficient way of transporting a distribution of material from one place to another to build fortifications. In the last 30 years, this theory has found various applications in many areas. In this short talk, we will briefly review some of these applications. (Received September 16, 2021)

1174-49-7937 **Michael Novack*** (mnovack99@gmail.com), The University of Texas at Austin. *Large-volume relative isoperimetry in exterior domains*

We study the relative isoperimetric problem in the large volume regime and posed in an exterior domain $\mathbb{R}^n \setminus W$. We provide a quantitative description of minimizers and identify the second order term in the asymptotic expansion of the minimum energy as the volume approaches infinity, which is given in terms of a geometric variational problem involving W. This is joint work Francesco Maggi (UT Austin). (Received September 16, 2021)

1174-49-8287 **Joshua Kline*** (klinejp@mail.uc.edu), University of Cincinnati. *Non-locality, non-linearity, and existence of solutions for the Dirichlet problem for least gradient functions in metric measure spaces*

We study the Dirichlet problem for least gradient functions in metric spaces equipped with a doubling measure and supporting a (1,1)-Poincaré inequality, specifically when the boundary of the domain satisfies a positive mean curvature condition. In this setting, it was shown by Malý, Lahti, Shanmugalingam, and Speight that existence of solutions is guaranteed for continuous boundary data. We extend these results, showing existence of solutions for boundary data that is approximable from above and below by continuous functions. We also show that for arbitrary $f \in L^1(\partial\Omega)$, there exists a least gradient function in Ω whose trace agrees with f at continuity points of f, and so we obtain existence of solutions for boundary data which is continuous almost everywhere. This is in contrast to the result of Spradlin and Tamasan, in which it was shown that there exists an L^1 function on the unit circle which has no least gradient solution in the unit disk in $\mathbb{R}^2$. Modifying the example of Spradlin and Tamasan, we show that the space of solvable L^1 functions on the unit circle is non-linear, even though the unit disk satisfies the positive mean curvature condition. (Received September 18, 2021)

1174-49-9153 **John M Turnage*** (turnjm20@wfu.edu), Wake Forest University. *The Brachistochrone Problem for a Rolling Disk on a Surface of Revolution* Preliminary report.

In this talk we consider the problem of computing paths of minimal time of flight for a disk rolling without slipping on a surface of revolution in a uniform gravitational field. The dynamics of the disk are computed using the Lagrange-D'Alembert principle with nonholonomic constraints. We will use geometric mechanics, specifically Lie brackets, Chow's theorem, and Frobenius' theorem to establish the controllability of the disk. We then use Pontryagin's Maximum Principle to establish the existence of time-optimal paths as well as construct numerical techniques for computing the paths. (Received September 20, 2021)

1174-49-9449 **Camillo De Lellis** (camillo.delellis@ias.edu), Institute For Advanced Study, Princeton, and **Zihui Zhao*** (zhaozh@uchicago.edu), University of Chicago. *Boundary regularity of area-minimizing currents*

Given a curve Γ, what is the surface T that has least area among all surfaces spanning Γ? This classical problem and its generalizations are called Plateau's problem. In this talk we consider area minimizers among the class of integral currents, or roughly speaking, orientable manifolds. Since the 1960s a lot of work has been done by De Giorgi, Almgren, et al to study the regularity of these minimizers at the interior. Much less is known about regularity at the boundary (in the case of codimension greater than 1). Recently, De Lellis et al. have found surprising examples of boundary singularity even when the prescribed curve Γ is smooth. I will speak about some recent progress in this direction and my joint work with C. De Lellis. (Received September 20, 2021)

1174-49-9508 **David Simmons*** (simmons3@uw.edu), University of Washington. *Regularity of Almost-Minimizers of Hölder-Coefficient Surface Energies*

We share recent work studying almost-minimizers of anisotropic surface energies defined by a Hölder-continuous matrix of coefficients. These coefficients act on the unit normal direction to the surface to give a weighted generalization of surface area. In this talk we shall discuss some of the key ideas to showing that almost-minimizers are locally Hölder continuously differentiable at regular points with estimates on the size of the singular set. Our work is done in the framework of sets of locally finite perimeter and follows an excess-decay type argument. (Received September 20, 2021)

1174-49-10048 **Ryan Christopher Unger*** (runger@princeton.edu), Princeton University. *Positive scalar curvature on noncompact manifolds*

We prove a Riemannian positive mass theorem which allows for incompleteness and negative scalar curvature. The manifolds are assumed to have one asymptotically Schwarzschild end, but the complement of this end is otherwise arbitrary. The incompleteness and negativity is compensated for by large positive scalar curvature on an annulus, in a quantitative fashion. In the complete noncompact case with nonnegative scalar curvature, we have no extra assumption and hence prove a long-standing conjecture of Schoen and Yau. This is joint work with Lesourd and Yau. (Received September 21, 2021)

1174-49-10524 **Mark S Squillante*** (mss@us.ibm.com), IBM Research. *Optimal Transport and Stochastic Optimal Control*

Information processing and decision making are the two basic processes that underlie all intelligent systems (living and nonliving) ranging from a single cell to the brain and to artificial intelligent systems. In this talk, we consider some recent progress on fundamental problems at the intersection of optimal transport and stochastic optimal control, and discuss the use of this progress to establish a mathematical foundation for understanding the general properties of stochastic information processing and decision making processes. (Received September 21, 2021)

1174-49-10709 **Ghodsieh Ghanbari*** (gg646@msstate.edu), Mississippi State University. *A novel numerical approach for solving problems in fractional calculus by applying Chebyshev wavelets.* Preliminary report.

A new numerical approach is proposed for the time delay fractional optimal control problems using generalized Chebyshev wavelets. We give an exact formula of the Riemann-Liouville fractional integral operator of the generalized Chebyshev wavelets by using the incomplete beta function. Several examples are solved. Our numerical results are compared with either the exact solutions or the existing results. (Received September 21, 2021)

1174-49-10748 **Luke Kiernan** (rubikscubepro@gmail.com), Stony Brook University, **Keigo Kusumegi** (kusukun7@gmail.com), University of Tsukaba, and **Keitaro Okochi** (okochiwork@gmail.com), Waseda University. *Two Different Approaches to Optimizing Coverage with Elliptical Regions* Preliminary report.

Motivated by the signal regions of directional wireless transmitters, we consider covering a 2D region via ellipses.

One approach is to use homological methods. De Silvia and Ghrist proved a persistent homology criterion for coverage by circles; with a few changes, a similar result holds for ellipses. We can then reframe the problem of achieving coverage with few transmitters and low power as a combinatorial optimization one. Randomly place excess transmitters, and consider selecting a subset with the desired properties. Assign a cost to each 2-simplex, reflecting the power required to cover the triangle by the transmitters at the corners. Then, compute a minimal cost generator of the 2nd homology; via the persistent homology criterion, the transmitters corresponding to the vertices in the minimal generator cover the region, and do so efficiently.

Another approach that focuses on the elliptical shape of the coverage is a modified version of an algorithm called Gaussian Mixture Model Expectation Maximization (GMMEM). GMMEM is used to provide a best fitting model of normal distributions so that the confidence regions of each distribution collectively approximate the area to be covered. We modify GMMEM to accommodate sensors that generate elliptical coverage areas with a fixed eccentricity, and sensors that penetrate the walls poorly. (Received September 21, 2021)

1174-49-10876 **Emily Burns*** (burnse@southwestern.edu), Southwestern University, and **Jesse Stovall** (stovallj@southwestern.edu), Southwestern University. *Isoperimetric 4-bubble Regions on the Real Number Line with Density —x—*

We examine a modified isoperimetric problem that takes place on the real number line, imbued with a density function given by the absolute value of x. Specifically, we look at sets of N regions, in which each region is a collection of closed intervals. Each interval $I = (a, b)$ has weighted volume and perimeter that is affected by the density function, so that volume is given by $\int_I |x|\, dx$ and perimeter is given by $|a| + |b|$. Volume and perimeter of regions is computed additively. In this framework, we study configuration of N regions with fixed (weighted) volumes $V_1, \dots, V_n$, and identify configurations that have minimal total weighted perimeter. Our results extend previous known work to include an optimal configuration for $N = 4$, as well as progress towards a argument for general N. (Received September 21, 2021)

1174-49-11004 **Soumik Pal*** (soumikpal@gmail.com), University of Washington. *Optimal Transport and Information Geometry*

The theory of Monge-Kantorovich optimal transport has had a profound impact in many areas of mathematics, including geometry, optimization, and probability, with more recent applications being found in statistics, control theory, and computer science. What is underappreciated is the deep connection of the geometry of optimal transport with that of the classical information geometry, a theory originally conceived to view exponential families of statistical models as manifolds and has had a long history of successful application over the last half century. In this talk we will survey some of the recent progress developing this connection and the new avenues of research this has opened up. (Received September 21, 2021)

1174-49-11179 **Silvia Ghinassi*** (ghinassi@uw.edu), University of Washington, **Camillo De Lellis** (camillo.delellis@ias.edu), Institute for Advanced Study, and **Matteo Focardi** (matteo.focardi@unifi.it), Università degli Studi di Firenze. *Regularity of 2d Mumford-Shah minimizers*

The Mumford-Shah functional was introduced by Mumford and Shah in 1989 as a variational model for image reconstruction. The regularity theory has seen several contributions, both in two and several space dimensions.

The most important regularity problem is the famous Mumford-Shah conjecture, which states that (in 2 dimensions) the closure of the jump set can be described as the union of a locally finite collection of injective C^1 arcs that can meet only at the endpoints, in which case they have to form triple junctions. If a point is an endpoint of one (and only one) of such arcs, it is called a *cracktip*. We give a proof (based on previous work of Anderssonn and Mikayelyan) of the regularity, up to the loose end, of minimizers of the Mumford-Shah functional when they are sufficiently close to the cracktip. (Received September 21, 2021)

51 ► Geometry

1174-51-7464 **Dami Lee*** (damilee@uw.edu), University of Washington, and **Charles Camacho** (camachoc@uw.edu), University of Washington. *On infinite octavalent polyhedral surfaces*

In this talk, we discuss octavalent triply periodic polyhedral surfaces that have identifiable underlying Riemann structures. We focus our study to eightfold cyclic covers over the thrice punctured sphere. There are three such surfaces up to equivalency, where one of them was studied as the cover of Fermat's quartic by the second author. We discuss the remaining two cases in this talk. Results include a polyhedral surrogate of Schwarz minimal CLP surface and a polyhedral surface that is not a cover of the Bolza surface despite its construction. (Received September 14, 2021)

1174-51-7860 **Tegan Emerson*** (tegan.emerson@pnnl.gov), Pacific Northwest National Laboratory, Colorado State University, University of Texas El Paso. *A Tale of Two Shapes: The Circle and the Sphere in Natural Language Processing*

In this talk we will present an exploration of how both geometry and topology can be leveraged within natural language processing tasks. We will begin by surveying some of the geometric- and manifold-based observations around word-embedding models from the last several years. We will then show how word-embedding models can be improved by constraining the model to have a known geometric structure, specifically a sphere, during training. By imposing such a manifold structure we are afforded a different collection of insights and tools for analysis which helps build a more robust intuition for the meaning of other structures in the space. Inspired by the question "can one find a circle in a circular argument?" we will discuss the interpretation of a circle in a spherically constrained word-embedding and describe an alternate topological notion of circularity based on probabilistic periodicity which does not rely on a learned word-embedding. Finally, we will demonstrate how this topological approach can be applied to classify research papers with a common first author as valid or fraudulent based on their title and abstract. (Received September 16, 2021)

1174-51-8257 **Annalisa Crannell*** (annalisa.crannell@fandm.edu), Franklin and Marshall College. *Perspectives through a Two-Slit Camera*

What is the picture of a cube in a two-slit camera? That is the question this talk explores. A two-slit camera is a mathematically interesting variation of the pin-hole camera. It has two barriers between the film/canvas and the objects in the world; each one of these barriers contains a linear slit that allows light to pass through. If these slits are skew to one another, then for any point in the object world, there is exactly one light ray passing through that point and each of the two slits, and so the image of that point is again a point on the canvas. Two-slit cameras have been of some interest in the field of algebraic vision, which use Grassmannians to describe the mapping that takes points from three-dimensional space to a two-dimensional canvas. In this talk, in contrast, we will focus on the geometry of the images of lines rather than images of points. (In general, the images of lines are hyperbolas.) Our approach mimics the traditional notion of perspective drawing: what does it mean for a cube to be in n-point perspective, as viewed by a two-slit camera, and how can we use ruler-and-compass geometry to draw these images? (Received September 18, 2021)

1174-51-9446 **Emily Autumn Windes*** (ewindes@u.rochester.edu), University of Rochester. *Morse ThoeLagrange Multipliers* Preliminary report.

In this talk, we consider a Lagrange multipliers problem where the constraint is a section s of a bundle $E \xrightarrow{\pi} M$. We relate the Morse homology of a function restricted to $s^{-1}(0)$ to the Morse homology of the associated Lagrange function on the total space E^*. The motivation for studying this problem is a similar, infinite-dimensional Lagrange multipliers problem appearing in Donaldson and Segal's paper *Gauge Theory in Higher Dimensions II.* The long term goal is to apply Floer theory to a functional whose critical points are a generalization of three dimensional, special Lagrangian submanifolds. (Received September 20, 2021)

1174-51-9516 **Daniel Perry*** (daniel.perry@augie.edu), Augustana University. *The universal Lipschitz path space of* $\mathbb{H}^1$ Preliminary report.

Inspired by the definition of a universal covering of a topological space, we define the universal Lipschitz path space over the first Heisenberg group $\mathbb{H}^1$. The universal Lipschitz path space is a metric space with a metric map onto $\mathbb{H}^1$, the fibers of which are totally disconnected and perfect. As is the case with a universal cover of a topological space, the universal Lipschitz path space supports a unique lifting property and is Lipschitz simply connected. (Received September 21, 2021)

1174-51-9557 **Allen Lin*** (allen.lin101@topper.wku.edu), Carol Martin Gatton Academy, Western Kentucky University. *Properties of Pólya's Circular Symmetrization*

Circular symmetrization, a procedure first introduced by Pólya and Szegő, has applications to isoperimetric problems in various settings because it is known to reduce perimeter of a region. The original proof that this symmetrization procedure reduces perimeter uses the calculus of variations. We present an elementary proof (without using calculus of variations) that circular symmetrization decreases the perimeter of a region in $\mathbb{R}^2$. We also present the images of certain shapes after circular symmetrization. This research was conducted at the Research Science Institute (RSI) during the summer of 2021. (Received September 20, 2021)

1174-51-9563 **Sarah E Tammen** (setammen@mit.edu), Massachusetts Institute of Technology, and **Allen Lin*** (allen.lin101@topper.wku.edu), Western Kentucky University, Carol Martin Gatton Academy. *Properties of Circular Symmetrization*

Circular symmetrization, a procedure first introduced by Pólya and Szegő, has applications to isoperimetric problems in various settings because it is known to reduce perimeter of a region. The original proof that this symmetrization procedure reduces perimeter uses the calculus of variations. We present an elementary proof (without using calculus of variations) that circular symmetrization decreases the perimeter of a region in $\mathbb{R}^2$. We also present the images of certain shapes after circular symmetrization. This research was conducted at the Research Science Institute (RSI) during the summer of 2021. (Received September 20, 2021)

1174-51-9783 **Angela Wu*** (angelawu0312@gmail.com), Indiana University Bloomington. *Non-quasispheres with Euclidean weak tangents* Preliminary report.

In this talk we discuss the problem of local characterization of the quasispheres, or metric spaces that are quasisymmetrically equivalent to S^n. Contrary to previous result for $n = 1$, we show that for all $n \geq 2$, there exists a doubling linearly locally contractible metric space X that is topologically a n-sphere such that every weak tangent is isometric to $\mathbb{R}^n$ but X is not quasisymmetrically equivalent to the standard n-sphere. Moreover, our construction gives spaces of Hausdorff dimension n. When $n = 2$, our result shows that Ahlfors 2-regularity condition in Bonk and Kleiner's result is necessary and optimal. (Received September 20, 2021)

1174-51-10540 **Jack Mealy** (jmealy@austincollege.edu), Austin College, and **Justus Fagan*** (jfagan20@austincollege.edu), Austin College. *Closed orbits and curvature terms in staircase metric geometries*

After a brief introduction to the staircase metric (SCM) geometry category and its accompanying methodology, this general framework is utilized to analyze and define various curvatures related to SCM surfaces. Specific manifolds defined and studied include tori with a.e.-positive Gaussian curvature, and others with a.e.-negative Gaussian curvature. Also, closed geodesics in newly defined non-compact SCM cylinders are exhibited, and that further involve df-boundaries that are scaled normal distribution curves; in the space-time case, non-trivial time-like closed orbits result; the corresponding inherent 'gravity effect' in these manifolds is then illustrated. (Received September 21, 2021)

1174-51-10561 **Theodore Gonzales** (Theodore.Gonzales@colorado.edu), University of Colorado Boulder, and **Apollo Albright** (aalbright@reed.edu), Reed College. *The Gromov-Hausdorff distance for interval graphs* Preliminary report.

The Gromov-Hausdorff distances measures shape differences between objects representable as compact metric spaces, e.g. point clouds, manifolds, or graphs. Computing the Gromov-Hausdorff distance for general graphs is equivalent to solving an NP-hard optimization problem, deeming the notion impractical for applications. However various approximations and relaxations to the Gromov-Hausdorff distance have been shown in previous works to be performative on many real world networks from a variety of fields. In this work we completely determine the Gromov-Hausdorff distance between interval graphs, which have been used to model collections of events underlying complex processes. (Received September 21, 2021)

1174-51-11180 **Viridiana Jasmin Neri** (vjn2108@columbia.edu), Columbia University, **Dane Kealii Gollero** (3rundane@gmail.com), University of Utah, **Izah Tahir** (itahir3@gatech.edu), Georgia Tech, **Len White*** (len_white@live.com), Cal Poly Pamona, and **Siddhi Krishna** (siddhi@math.columbia.edu), Columbia University. *Computing the Braid Index of 1-Bridge Braids* Preliminary report.

A knot is a potentially knotted circle in 3-dimensional space. Knots provide a powerful probe into studying 3- and 4-dimensional phenomena, which is why they are a central object of study within low-dimensional topology (a subfield of pure mathematics). It is known that every knot is realized as the closure of a braid. An important invariant of a knot, K, is the braid index: a numerical value that represents the minimum number of strands required to present K as the closure of some braid. In general, this knot invariant is very hard to compute! In this work, we determine the braid index of all 1-bridge braids, an infinite family of well-studied knots, which are determined by three parameters. We also present some avenues for future directions generalizing this work. (Received September 21, 2021)

1174-51-11188 **Tobey Michael Mathis*** (tobeymmathis@gmail.com), Kiko Kawamura. *Revolving Sequences and Terdragon*

In 1970, Davis and Knuth introduced the concept of revolving sequences to represent Gaussian integers. Much later, Kawamura and Allen recently generalized this idea to a wider class of revolving sequences that parametrize certain self-similar fractals including the Levy Dragon and Tiling Dragon, which are the unique compact solution of certain families of Iterated Function Systems. In this paper, we build on the work of Kawamura and Allen to include a wider collection of Iterated Function Systems and introduce a new type of revolving sequence which parametrizes a different family of self-similar fractals including the Terdragon. (Received September 21, 2021)

1174-51-11265 **Olivia Bouthot*** (o_bouthot@coloradocollege.edu), Colorado College. *Extension of McDougall's Circle Theorem* Preliminary report.

Ptolemy's Theorem states that for a cyclic quadrilateral, the sum of the products of opposite side lengths is equal to the product of the diagonal lengths. A ratio form of this property extends to McDougall's Circle Theorem, which shows that a similar property is true for all cyclic n-gons where n is even. McDougall's Circle Theorem extends further. Arbitrary point(s) are added within the circle, and their squared distances from the vertices of the cyclic polygon are used as weights. This project aims to use elementary methods to prove this further extension of McDougall's Circle Theorem. We find a set of n points in the complex plane that fall in a balanced alignment. Additionally, we describe an alternate elementary proof of McDougall's Circle Theorem from the balanced alignment. (Received September 22, 2021)

52 ► Convex and discrete geometry

1174-52-5536 **Abdul Basit*** (basit.abdul@gmail.com), Iowa State University, **Terence Tao** (tao@math.ucla.edu), UCLA, **Artem Chernikov** (chernikov@math.ucla.edu), University of California Los Angeles, **Sergei Starchenko** (sstarche@nd.edu), University of Notre Dame, and **Chieu-Minh Tran** (mtran6@nd.edu), University of Notre Dame. *Zarankiewicz's problem for semilinear hypergraphs*

Zarankiewicz's problem in extremal graph theory asks for the maximum number of edges in a bipartite graph on n vertices which does not contain a copy of $K_{k,k}$, the complete bipartite with k vertices in both classes. We will consider this question for incidence graphs of geometric objects. Significantly better bounds are known in this setting, in particular when the geometric objects are defined by systems of algebraic inequalities. We show even stronger bounds under the additional constraint that the defining inequalities are linear. We will also discuss connections of these results to combinatorial geometry and model theory. (Received August 21, 2021)

1174-52-5677 **Matthew Steven Harvey*** (msh3e@uvawise.edu), University of Virginia, Wise. *Nets of Regular Polytopes* Preliminary report.

The study of unfolding polyhedra was popularized by Albrecht Dürer in the early 16th century, motivating the conjecture that every convex polyhedron has an edge unfolding that admits a net: it can be cut along certain edges and unfolded flat without overlap. This claim remains tantalizingly open and has resulted in numerous areas of exploration. In the past decade, it has been shown that *every* edge unfolding of Platonic solids yields a net. This talk explores the higher-dimensional analog for regular polytopes, with cuts along codimension 2 ridges. We show that *every* ridge unfolding of the n-simplex and n-cube yields valid nets, but not the n-orthoplex. The combinatorial and computational underpinnings of these results are presented. (Received August 24, 2021)

1174-52-6017 **Joshua Zahl*** (jzahl@math.ubc.ca), University of British Columbia. *Sphere tangencies and unit distances in F^3*

I will discuss how the problem of counting sphere tangencies in F^3 can be transformed using Lie's line-sphere correspondence into the problem of counting line-line intersections in E^3, where E is a certain field extension of F. As an application, if F is a field in which -1 is not a square, then n points in F^3 determine $O(n^{3/2})$ unit distances, provided n is not too large compared to the characteristic of F. (Received September 2, 2021)

1174-52-6909 **Steven Simon*** (ssimon@bard.edu), Bard College. *Inscribed polytopal partitions of a Tverberg-type*

Tverberg's theorem states that any set of $T(r,d)=(r-1)(d+1)+1$ points in $\mathbb{R}^d$ can be partitioned into r subsets with overlapping convex hulls. While almost any collection of fewer points cannot be so divided, we show that in many such circumstances one can nonetheless guarantee tight inscribed "polytopal partitions" with prescribed symmetry conditions. Namely, given any group G of order r and any faithful and full–dimensional orthogonal d–dimensional representation, we show that any generic set of $T(r,d)-d$ points in $\mathbb{R}^d$ can be partitioned by r subsets so that there are r points, one from each of the r resulting convex hulls, which are the vertex set of a d–dimensional polytope whose isometry group contains G via a free and transitive action afforded by the representation. At one extreme, this gives polytopal partitions for all regular r–gons in the plane [ ] and three of the six regular 4–polytopes in $\mathbb{R}^4$. At the other, one has polytopal partitions for polytopes on $|G|$ vertices with isometry group G whenever G is the isometry group of a vertex–transitive polytope. As with Tverberg's theorem, our results admit topological extensions when G is elementary abelian of prime power order. (Received September 10, 2021)

1174-52-7090 **Adam Sheffer*** (adamsh@gmail.com), Baruch College. *Distinct Distances in the Complex Plane*

We discuss the distinct distances theorem in the complex plane C^2. This will be a basic introduction to this topic, which does not explain the long and technical proof of this result.

Joint work with Joshua Zahl. (Received September 12, 2021)

1174-52-7099 **Isabella Novik*** (novik@math.washington.edu), University of Washington, and **Hailun Zheng** (hz@math.ku.dk), University of Copenhagen. *Reconstructing simplicial polytopes from their graphs and affine 2-stresses*

What partial information about a convex simplicial polytope P of dimension d is enough to uniquely determine the combinatorial type of P? By a result of Perles and Dancis, the $\lfloor d/2\rfloor$-skeleton of P determines the entire face lattice of P. Another piece of information that allows one to reconstruct the face lattice of P is the space of affine dependencies of vertices of P, which can also be thought of as the space of affine 1-stresses. Motivated by these results, Kalai conjectured that the 1-skeleton of P (also known as the graph of P) given as an abstract simplicial complex and the space of affine 2-stresses of P uniquely determine the combinatorial type of P. We discuss the proof of this conjecture. In fact, we show that it is enough to know the graph of P and the set of sign vectors of stresses. (Received September 12, 2021)

1174-52-7126 **Carina Curto** (ccurto@psu.edu), Penn State, **Caitlin Lienkaemper*** (clienkaemper@psu.edu), Pennsylvania State University, **Juliana Londono-Alvarez** (jbl5958@psu.edu), Pennsylvania State University, and **Hannah Rocio Santa Cruz** (hqs5441@psu.edu), Pennsylvania State University. *Finding hidden low dimensional structure in data distorted by monotone transformations*

Measurements of biological data are often distorted by unknown monotone transformations. While these non-linear distortions make detecting low-dimensional structure using traditional matrix analysis impossible, we can recover some of this hidden structure using combinatorial and topological techniques. In this talk, we explore the *underlying rank* of a matrix A, which we define as the smallest value of r such that there is a rank r matrix B and monotone function f such that $A_{ij}=f(B_{ij})$. We give several methods for estimating underlying rank. To derive these bounds, we decompose matrices as pairs of point configurations and use the order of matrix entries to extract geometric information about these point configurations. In particular, motivated by results from the theory of random polytopes, we define the minimal nodes of a matrix and show that it is possible to estimate the underlying rank of a random matrix by counting minimal nodes. We also derive lower bounds on monotone rank via Radon's theorem, and show that it is possible for the underlying rank of a matrix to exceed these bounds using ideas from oriented matroid theory. (Received September 12, 2021)

1174-52-7373 **Matthew L Wright** (wright5@stolaf.edu), St. Olaf College, **Mutiara Sondjaja*** (sondjaja@nyu.edu), New York University, **Kristen Mazur** (kmazur@elon.edu), Elon University, and **Carolyn Yarnall** (cyarnall@csudh.edu), California State University Dominguez Hills. *Bounds for Piercing Numbers in Approval Voting with Circular Societies* Preliminary report.

In the system of approval voting, individuals vote for all candidates they find acceptable. Many approval voting situations can be modeled geometrically and thus, geometric concepts such as the piercing number—the size of the smallest set that intersects every set in a collection— have natural interpretations. In this talk, we explore piercing numbers in the setting where voting preferences can be modeled by arcs on a circle – i.e., in circular societies. We give upper bounds on the piercing numbers in circular societies in which all approval sets have the same length. We also present probabilistic results about the average piercing number for such circular societies. (Received September 14, 2021)

1174-52-7541 **Henry Fleischmann*** (henryfl@umich.edu), University of Michigan, **Charles Isaac Wolf** (charlesyitziwolf@gmail.com), University of Rochester, **Eyvindur Ari Palsson** (palsson@vt.edu), Virginia Tech, **Steven J. Miller** (sjm1@williams.edu), Williams College, **Ethan Pesikoff** (epesikoff@gmail.com), Yale University, **Hongyi Hu** (hongyih@andrew.cmu.edu), Carnegie Mellon University, and **Faye Jackson** (alephnil@umich.edu), University of Michigan. *Angle Variants of the Erdős Distinct Distance Problem* Preliminary report.

The Erdős distinct distance problem is a ubiquitous problem in discrete geometry. Less well known is Erdős' distinct angle problem, the problem of finding the minimum number of distinct angles between n non-collinear points in the plane.

We provide brand new upper and lower bounds on a broad class of distinct angle problems. We show that the number of distinct angles formed by n points in general position is $O(n^{\log_2(7)})$, providing the first non-trivial bound for this quantity. We introduce several new asymptotically optimal configurations and analyze their sensitivity to perturbation. In higher dimensions we show that a variant of Lenz's construction admits fewer distinct angles than the previously known optimal configurations in two dimensions.

We also show that the minimum size of a maximal subset of n points in general position admitting only unique angles is $\Omega(n^{1/5})$ and $O(n^{\log_2(7)/3})$. Finally, we provide asymptotically tight bounds on the partite variants of the standard distinct angle problem. (Received September 15, 2021)

1174-52-7570 **Nets Katz*** (nets@caltech.edu), California Institute of Technology. *A discrete model for the Carbery rectangle problem* Preliminary report.

We discuss a discrete model for the Carbery rectangle problem for which techniques from extremal graph theory may be applicable. (Received September 15, 2021)

1174-52-7588 **Oleg R Musin*** (omusin@gmail.com), University of Texas Rio Grande Valley. *The uniqueness problem of kissing arrangements using SDP* Preliminary report.

We present a new extension of known semidefinite and linear programming upper bounds for spherical codes and consider a version of this bound for distance graphs. We apply the main result for the distance distribution of a spherical code and discuss reasonable approaches for solutions of the uniqueness of maximum kissing arrangements in four dimensions. (Received September 15, 2021)

1174-52-7631 **Christopher O'Neil** (cdoneill@sdsu.edu), San Diego State University, **Jiajie Ma*** (jma2@haverford.edu), Haverford College, **Shuhang Xue** (xues@carleton.edu), Carleton College, and **Rosa Stolk** (rosasofiestolk@gmail.com), Maastricht University. *Embedding Dimension of Numerical Semigroups and Face Dimension of Kunz Polyhedra Faces* Preliminary report.

A *numerical semigroup* is a set of nonnegative integers that contains 0, is closed under addition (meaning that the addition of two elements in the set remains in the set), and whose complement in $\mathbb{Z}_{\geq 0}$ is finite. Despite their algebraic nature, it was recently found that there is a one-to-one correspondence between numerical semigroups and the integer points of a class of polyhedra in Euclidean space, namely the *Kunz polyhedra.* This provides a geometric angle to study properties of numerical semigroups. For example, numerical semigroups corresponding to points on the same face of a Kunz polyhedron are minimally generated by the same number of integers (*generate* means every element in the semigroup can be written in terms of the specified integers with addition). This implies that besides the geometric *face dimension*, each face of a Kunz polyhedron also carries an *embedding dimension* that denotes the number of minimal generators of the semigroups corresponding to points in that face. In this work, we characterize all possible pairs of face dimension and embedding dimension appearing in a Kunz polyhedron. Our approach develops a new construction that reflects the face structure of a Kunz polyhedron, namely the *DED lattice*, and locates some special classes of numerical semigroups such as *arithmetical numerical semigroups* in the DED lattice. (Received September 15, 2021)

1174-52-7906 **Steven Michael Senger*** (stevensenger@missouristate.edu), Missouri State University. *Dot product variants of Erdős and Falconer type distance problems*

The Erdős unit distance problem asks how often a particular distance can occur in a large finite set of points in the plane. While this problem is still open, the corresponding problem for a fixed dot product has tight upper and lower bounds. In this talk, we will survey dot product variants of several distance problems. In some cases, more is known about distances, and in other cases, more is known about dot products. We will also discuss a hard problem in additive combinatorics that arises naturally from this study. (Received September 16, 2021)

1174-52-8205 **Zilin Jiang*** (zilinj@asu.edu), Arizona State University. *Forbidden subgraphs and spherical two-distance sets*

A set of unit vectors in a Euclidean space is called a spherical two-distance set if the pairwise inner products of these vectors assume only two values $\alpha > \beta$. It is known that the maximum size of a spherical two-distance grows quadratically as the dimension of the Euclidean space grows. However when the values α and β are held fixed, a very intricate behavior of the maximum size emerges. Building on our recent resolution in the equiangular case, that is $\alpha + \beta = 0$, we make a plausible conjecture which connects this behavior with spectral theory of signed graphs in the regime $\beta < 0 < \alpha$, and we confirm this conjecture when $\alpha + 2\beta < 0$ or $(1-\alpha)/(\beta-\alpha) < 2.0198$. Joint work with Alexandr Polyanskii, Jonathan Tidor, Yuan Yao, Shengtong Zhang and Yufei Zhao. (Received September 18, 2021)

1174-52-8218 **Guido Francisco Montufar*** (montufar@math.ucla.edu), UCLA, MPI MiS, **Yue Ren** (yue.ren2@durham.ac.uk), Durham University, and **Leon Zhang** (leonyz@berkeley.edu), University of California, Berkeley. *Sharp bounds for the number of regions of maxout networks and vertices of Minkowski sums* Preliminary report.

We present results on the number of linear regions of the functions that can be represented by artificial feed-forward neural networks with maxout units. A rank-k maxout unit is a function computing the maximum of k linear functions. For networks with a single layer of maxout units, the linear regions correspond to the upper vertices of a Minkowski sum of polytopes. We obtain face counting formulas in terms of the intersection posets of tropical hypersurfaces or the number of upper faces of partial Minkowski sums, along with explicit sharp upper bounds for the number of regions for any input dimension, any number of units, and any ranks, in the cases with and without biases. Based on these results we also obtain asymptotically sharp upper bounds for networks with multiple layers. (Received September 18, 2021)

1174-52-8311 **Samuel Speas** (samuel0@berkeley.edu), University of California, Berkeley, **Moaaz AlQady*** (malqady@aucegypt.edu), University of Oregon, and **William Dudarov** (wdudarov@uw.edu), University of Washington. *Distinct Distances with ℓ_p Metrics*

We study Erdős's distinct distances problem under ℓ_p metrics with integer p. We prove that, for every $\varepsilon > 0$ and n points in $\mathbb{R}^2$, there exists a point that spans $\Omega(n^{6/7-\varepsilon})$ distinct distances with the other $n-1$ points. This improves upon the previous best bound of $\Omega(n^{4/5})$. We also characterize the sets that span an asymptotically minimal number of distinct distances under the ℓ_1 and ℓ_∞ metrics. (Received September 18, 2021)

1174-52-8572 **Jessica Wang*** (jwang22@wpi.edu), Worcester Polytechnic Institute, **Harper Jacob Niergarth** (nierg001@umn.edu), University of Minnesota, **Levi Y Borevitz** (lyb1@williams.edu), Williams College, and **Daniel Pocklington** (Pockling@grinnell.edu), Grinnell College. *Existence of Numerical Semigroups on Faces of Kunz Polyhedra*

A numerical semigroup is a cofinite subset of the non-negative integers that is closed under addition and contains zero. The Kunz Polyhedra, P_m, are a family of rational polyhedra whose integer points are in bijection with numerical semigroups with fixed smallest nonzero element m. Each face of P_m corresponds to a poset that describes the structure of the numerical semigroups found on that face. Interestingly, some faces of P_m do not have any integer points on them. In this work, we present conditions to classify which faces of P_m contain integer points by examining their poset. Based on these conditions, we provide an algorithm that locates an integer point on the face if any exist. (Received September 19, 2021)

1174-52-9001 **Benjamin Braun*** (benjamin.braun@uky.edu), University of Kentucky, **Khrystyna Serhiyenko** (khrystyna.serhiyenko@uky.edu), University of Kentucky, **Matias von Bell** (matias.vonbell@uky.edu), University of Kentucky, **Martha Yip** (martha.yip@uky.edu), University of Kentucky, **Katie Bruegge** (kaitlin_bruegge@uky.edu), University of Kentucky, and **Zachery Peterson** (Zachery.Peterson@uky.edu), University of Kentucky. *Triangulations of Flow Polytopes, Ample Framings, and Path Algebras* Preliminary report.

Given a finite directed acyclic graph G, a framing on the edges at inner vertices determines a regular unimodular triangulation of the flow cone for G. When G has what is called an ample framing, then this triangulation quotients to a complete fan. We classify the graphs that admit ample framings and investigate combinatorial properties of their triangulations induced by ample framings, motivated by connections to representations of path algebras. (Received September 20, 2021)

1174-52-9040 **Hans Parshall*** (parshah@wwu.edu), Western Washington University. *Spherical configurations over finite fields*

Euclidean Ramsey theory studies "Ramsey" sets; that is, finite point configurations that must appear monochromatically in finite colorings of sufficiently high-dimensional Euclidean space. Erdős et al. (1973) established that all Ramsey sets must be spherical, and Graham has conjectured that the converse is true. We will discuss some recent evidence (joint with Neil Lyall and Ákos Magyar) for Graham's conjecture in the model setting of vector spaces over finite fields. (Received September 20, 2021)

1174-52-9054 **Ilani Axelrod-Freed** (ilani_af@mit.edu), Massachusetts Institute of Technology, and **Pablo Soberon*** (pablo.soberon-bravo@baruch.cuny.edu), Baruch College. *Bisection of mass assignments using flags of affine subspaces*

Mass partition results, such as the ham sandwich theorem, describe how we can split measures or finite sets of points in Euclidean spaces given geometric constraints on the partition. They provide a direct link between discrete geometry and topological combinatorics. In this talk we present generalizations of the ham sandwich theorem for mass assignments in Euclidean spaces. A k-dimensional mass assignment in $\mathbb{R}^d$ is a way to assign continuously a measure to each affine k-dimensional subspace of $\mathbb{R}^d$. Given mass assignments of different dimensions, we seek affine subspaces $S_k \subset S_{k+1} \subset \ldots \subset S_d = \mathbb{R}^d$ such that the dimension of S_i is i and S_i halves all the mass assignments of dimension $i+1$ on S_{i+1} for $i = k, \ldots, d-1$. We give bounds for the number of mass assignments for which this is possible and use the results to confirm a conjecture of Schnider on mass assignments and to generalize the central transversal theorem. (Received September 20, 2021)

1174-52-9236 **Michael L Bruner*** (Mbruner@mtech.edu), Montana Tech, **Heidi S Steiger** (hsteiger@mtech.edu), Montana Tech, and **Marie A Steiger** (mariesteigerdesigns@yahoo.com), Montana Tech. *Non-face-to-face tilings of the Euclidean plane and their duals* Preliminary report.

We extend the classical combinatorial encoding of face-to-face mosaics (tilings) of the Euclidean plane to include non-face-to-face tilings. This encoding allows us to extend the natural concept of duality to such tilings, and to give an algorithm for constructing such duals. We provide a simple geometric proof of the Jordan-Schönflies Theorem for our class of tilings.

We discuss our theory in studying the average properties of tilings. In particular, we give possible applications of our theory in analyzing metallic grain structure. (Received September 20, 2021)

1174-52-9251 **Dora Woodruff*** (dorawoodruff@college.harvard.edu), Harvard University. *On Variants of the Unit Distance Problem* Preliminary report.

The Erdős Unit Distance Problem, which asks for the maximum number of times that the unit distance can occur among n points (in the plane or $\mathbb{R}^3$), is one of the most well-known problems in Discrete Geometry. Many variants have been studied in the work of Elekes, Pach, and others. Our work is on bounds on the maximum number of isomorphic copies of a given subgraph in a unit distance graph, the graph whose edges represent unit distances between the n points. Recently, Palsson et al. asked about the case of paths of fixed length. We continue this line of research, proving sharp results on the maximum number of paths, cycles, or copies of a given tree in the unit distance graph on a specific sphere. Interestingly, while for the planar path problem the answer depends on the length of the path mod 3, on the sphere it depends on the length mod 5. We also find bounds on the number of copies of any given 3-regular graph in $\mathbb{R}^3$ by analyzing a linear optimization problem we derive from the Cutting Lemma. Our proof is inspired by a paper of Agarwal and Sharir on unit simplices. (Received September 20, 2021)

1174-52-9460 **Xinyu Tan*** (xinyu.tan@duke.edu), Duke University, **Robert Calderbank** (robert.calderbank@duke.edu), Duke University, **Jingzhen Hu** (jingzhen.hu@duke.edu), Duke University, and **Qingzhong Liang** (qingzhong.liang@duke.edu), Duke University. *Quadratic Form Diagonal Gates and Grassmannian Packings* Preliminary report.

We analyze Grassmannian packings where the individual subspaces are obtained by applying a family of diagonal operators to a quantum error correcting code. The diagonal operators are drawn from the Clifford hierarchy of unitary operators. The first level is the Pauli group. The second level is the Clifford group which consists of unitary operators that normalize the Pauli group. The l-th level consists of unitary operators that map Pauli operators to the $(l-1)$-th level under conjugation. Quadratic form diagonal (QFD) gates are a family of diagonal gates associated with quadratic forms connected to classical error correcting codes over rings. We focus on the Grassmannian packings obtained by conjugating a single Pauli projector by a group of QFD gates in the 3rd level of the Clifford hierarchy. We describe how each subspace is associated with a classical bent function and derive a formula for the distances between individual subspaces. More specifically, each QFD gate is determined by a symmetric matrix R over ring $\mathbb{Z}_{2^l}$. When $l = 3$, we can expand $R = R_0 + 2R_1 + 4R_2$ to investigate how each R_i contributes to the distance analysis. We show that when R_0 is fixed, the possible distances between the subspaces obtained by conjugating the 3rd level QFD gates only take values from 0, 2^{n-1}, and 2^{n-2} where n is the number of physical qubits. (Received September 20, 2021)

1174-52-9469 **Tianran Chen** (ti@nranchen.org), Auburn University at Montgomery, **Robert Davis*** (rdavis@colgate.edu), Colgate University, and **Evgeniia Korchevskaia** (ekorchev@gatech.edu), Georgia Institute of Technology. *Facets and facet subgraphs of adjacency polytopes*

Adjacency polytopes, a.k.a. symmetric edge polytopes, associated with undirected graphs have been defined and studied in several seemingly-independent areas including number theory, discrete geometry, and dynamical systems. In particular, we are motivated by the study of the algebraic Kuramoto equations of unmixed form whose Newton polytopes are the adjacency polytopes.

The interplay between the geometric structure of adjacency polytopes and the topological structure of the underlying graphs is a recurring theme in recent studies. In particular, "facet/face subgraphs" emerged as one of the central concepts in describing this symmetry. Continuing along this line of inquiry we provide a complete description of the correspondence between facets/faces of an adjacency polytope and maximal bipartite subgraphs of the underlying connected graph. (Received September 20, 2021)

1174-52-9749 **Philip L. Bowers*** (bowers@math.fsu.edu), Florida State University. *Noncompact Complex Projective Surfaces and Circle Packings*

After reviewing the current status of the Kojima-Mizushima-Tan conjecture concerning circle packings on compact complex projective surfaces, we pivot to the non-compact setting and examine circle packings on complex projective surfaces with punctures. For the thrice punctured sphere we describe a three-dimensional slice of the infinite dimensional moduli space that parameterizes the elementary triangular surfaces and ones obtained from these by grafting. We then construct circle packings on all those of hyperbolic and parabolic type. This falsifies the Kojima-Mizushima-Tan conjecture in the non-compact setting. This is joint work with Sam Ballas, Alex Casella, and Lorenzo Ruffoni. (Received September 20, 2021)

1174-52-9913 **Jason Todd Callahan** (jasonc@stedwards.edu), St. Edward's University, Advisor, and **Peyton John Slepekis*** (pslepeki@stedwards.edu), St. Edward's University, Author. *Agreement proportion and boxicity for* $(2,3)$*-agreeable box societies*

We use an upper bound on piercing numbers for axis-parallel boxes with the $(3,2)$-property to improve the lower bound on minimum agreement proportion for $(2,3)$-agreeable box societies. We also improve the lower bound on boxicity for such societies and show that it is sharp when the associated piercing number is less than six. (Received September 21, 2021)

1174-52-10061 **Nora Youngs*** (nora.youngs@colby.edu), Colby College, and **Robert Amzi Jeffs** (amzij@cmu.edu), Carnegie Mellon University. *Order-forcing in Neural Codes*

Convex neural codes are subsets of the Boolean lattice that record the intersection patterns of convex sets in Euclidean space. Much work in recent years has focused on finding combinatorial criteria on codes that can be used to classify whether or not a code is convex. In this paper we introduce order-forcing, a combinatorial tool which recognizes when certain regions in a realization of a code must appear along a line segment between other regions. We use order-forcing to construct novel examples of non-convex codes, and to expand existing families of examples. We also construct a family of codes which shows that a dimension bound of Cruz, Giusti, Itskov,

and Kronholm (referred to as monotonicity of open convexity) is tight in all dimensions. (Received September 21, 2021)

1174-52-10070 **Federico Ardila** (federico@sfsu.edu), San Francisco State University, San Francisco State University and Los Andes University, **Anastasia Chavez** (amc59@stmarys-ca.edu), St. Mary's College of California, and **Jose Luis Herrera Bravo** (joseherrera@unicauca.edu.co), Universidad del Cauca. *The valuation polytope of the zig-zag poset 1* Preliminary report.

The summer 2021 Latinx Mathematical Research Community (LMRC) served as a catalyst for several research projects in various areas of mathematics. This talk will introduce the research of one such project studying combinatorial properties of the valuation polytope. Geissinger defined the valuation polytope as the set of all $[0,1]$-valuations on a finite distributive lattice. A linear translation of Geissinger's valuation polytope can be defined on the order ideals of a poset. We highlight what is known about valuation polytopes of posets and further narrow our scope to studying that of the height 2 up-down poset, referred to as the zig-zag poset. This is joint work with Federico Ardila, Anastasia Chavez, Pamela E. Harris, Jose Luis Herrera Bravo, and Andrés R. Vindas-Meléndez. (Received September 21, 2021)

1174-52-10100 **John Christopher Bowers*** (bowersjc@jmu.edu), James Madison University. *Obtaining Koebe-Andre'ev-Thurston packings via flow from tangency packings*

Recently, Connelly and Gortler gave a novel proof of the circle packing theorem for tangency packings by introducing a hybrid combinatorial-geometric operation, flip-and-flow, that allows two tangency packings whose contact graphs differ by a combinatorial edge flip to be continuously deformed from one to the other while maintaining tangencies across all of their common edges. Starting from a canonical tangency circle packing with the desired number of circles a finite sequence of flip-and-flow operations may be applied to obtain a circle packing for any desired (proper) contact graph with the same number of circles.

In this talk, I will show how to extend the Connelly-Gortler method to allow circles to overlap by angles up to $\pi/2$. This results in a new proof of the general Koebe-Andre'ev-Thurston theorem for disk packings on $\mathbb{S}^2$ with overlaps and a numerical algorithm for computing them. The development makes use of the correspondence between circles and disks on $\mathbb{S}^2$ and hyperplanes and half-spaces in the 4-dimensional Minkowski spacetime $\mathbb{R}^{(1,3)}$. Along the way I will generalize a notion of convexity of circle polyhedra that has recently been used to prove the global rigidity of certain circle packings and use this view to show that all convex circle polyhedra are infinitesimally rigid, generalizing a recent related result. (Received September 21, 2021)

1174-52-10254 **Anastasia Chavez*** (amc59@stmarys-ca.edu), Saint Mary's College of California. *The valuation polytope of the zig-zag poset 2* Preliminary report.

The preceding talk introduced the valuation polytope of the zig-zag poset of n elements, $\mathcal{V}(Z_n)$. Dobbertin showed that the valuation polytope of any poset can be described as the convex hull of vertices characterized by all the chains of that poset. It follows that the dimension of the valuation polytope is the number of elements of the corresponding poset. In this talk we discuss recent combinatorial results of $\mathcal{V}(Z_n)$ such as its normalized volume, the existence of a unimodular triangulation, and facet enumeration. We also share results on the Ehrhart positivity and a recursive inequality description of $\mathcal{V}(Z_n)$. (Received September 21, 2021)

1174-52-10256 **Nayan Rajesh** (nayan0rajesh@gmail.com), Krea University, **Gautham Sathish** (gautham_s.sias19@krea.ac.in), Krea University, **Santiago Morales** (s.morales30@uniandes.edu.co), Universidad de los Andes, and **Simon Foss*** (safoss@uchicago.edu), University of Chicago. *Characterizing Posets Associated to the Faces of the Kunz Polyhedron* Preliminary report.

Recent papers have explored combinatorial descriptions of the Kunz polyhedron, a rational polyhedron whose integer points are in bijection with numerical semigroups with fixed smallest non-zero element. Each face of the Kunz polyhedron can be identified with a finite poset. In this poster, we introduce a formal game which we conjecture provides us with the necessary and sufficient conditions to determine whether a poset corresponds to a face. We also explicitly identify a bijection between ridges of the Kunz polyhedron and a certain class of posets. (Received September 21, 2021)

1174-52-10503 **Matthias Beck** (beck@math.sfsu.edu), San Francisco State University, and **Magda Hlavacek*** (magda-hlavacek@berkeley.edu), UC Berkeley. *Signed Poset Polytopes* Preliminary report.

Posets can be viewed as subsets of the type-A root system that satisfy certain properties. Geometric objects arising from posets, such as order cones, order polytopes, and chain polytopes, have been widely studied. In

1993, Vic Reiner introduced signed posets, which are subsets of the type-B root system that satisfy the same properties. In this talk, we will explore the analogue of order and chain polytopes in this setting, focusing on the Ehrhart theory of these objects. (Received September 21, 2021)

1174-52-10530 **Brandon Hanson*** (brandon.w.hanson@gmail.com), University of Maine. *Superquadratic expansion and applications*

We will discuss recent results on expanders and sum-product type problems. The results imply quantitative improvements on Erdős-type problems in combinatorial geometry. Joint with O. Roche-Newton and S. Senger. (Received September 21, 2021)

1174-52-10634 **Deane Yang*** (deane.yang@nyu.edu), New York University. *Moment-entropy inequalities for random vectors via convex geometry*

The Shannon moment-entropy inequality shows that the Shannon entropy of a random variable with given variance is maximized by the Gaussian random variable. We discuss generalizations of this inequality to random vectors, p-th moments, and Renyi entropy using convex geometric analysis. (Received September 21, 2021)

1174-52-10666 **Kaylee Weatherspoon** (skw4@email.sc.edu), University of South Carolina, **Alan Li*** (ali24@amherst.edu), Amherst College, and **Amelia Shapiro** (amelia_shapiro@brown.edu), Brown University. *A Generalization of Szlam's Lemma*

The best-known form of Szlam's Lemma asserts that if $\mathbb{R}^n$ is colored with red and blue so that no two blue points are at Euclidean distance 1 from each other, and there is a non-empty set $F \subseteq \mathbb{R}^n$ so that no translate of F is all red, then the chromatic number $\chi(\mathbb{R}^n, 1)$ of the Euclidean unit distance graph on $\mathbb{R}^n$ is no greater than $|F|$. Here we prove a theorem that contains Szlam's Lemma as a special case: Suppose D is a set of positive real distances and (S, ρ) is a metric space. Then if (S, ρ) can be colored with red and k other colors, $c_1, \ldots, c_k$, so that every distance in D is forbidden for every color c_i, and for some set $F \subseteq S$, no translate $b + F$, $b \in S$, is all red, then the smallest number such that we can color (S, ρ) with that number of colors so that every distance in D is forbidden for every color in the coloring is $\leq k|F|$. An exciting possibility for this generalization is to provide new upper bounds for the chromatic number of Euclidean spaces $\mathbb{R}^n$: if we can find such a coloring of $\mathbb{R}^n$ with red and k other colors where each of the k colors forbids distance 1, which forbids translates of some set $F \subseteq \mathbb{R}^n$ from being all red, then we have an upper bound of $k|F| \geq \chi(\mathbb{R}^n, 1)$. (Received September 21, 2021)

1174-52-10841 **Daishi Kiyohara*** (kiyohara@mit.edu), Massachusetts Institute of Technology. *A new approach to the upper estimate of lattice points on a curve via ℓ^2 decoupling*

In this paper we examine the number of $\frac{1}{N}$-integral points on a fixed planar curve Γ. We prove that, if the curve Γ is $C^{n,\alpha}$ for some $\alpha > 0$ and satisfies a certain analytic condition, then we have $|\Gamma \cap (\frac{1}{N}\mathbb{Z})^2| \lesssim N^{e(n)+\epsilon}$ for any $\epsilon > 0$ with a function $e(n)$ which is asymptotically $n^{-1/2}$, where the implicit constant depends on ϵ. This gives an extension of Bombieri and Pila's result to curves of any regularity. Our approach is to lift up a planar curve into higher dimensional spaces and apply ℓ^2 decoupling inequalities there. (Received September 21, 2021)

1174-52-10918 **Lei Xue*** (lxue@uw.edu), University of Washington, Seattle. *A Proof of Grünbaum's Lower Bound Conjecture for polytopes, lattices, and strongly regular pseudomanifolds.* Preliminary report.

In 1967, Grünbaum conjectured that any d-dimensional polytope with $d + s \leq 2d$ vertices has at least

$$\phi_k(d+s, d) = \binom{d+1}{k+1} + \binom{d}{k+1} - \binom{d+1-s}{k+1}$$

k-faces. In the talk, we will prove this conjecture and discuss equality cases. We will then extend our results to lattices with diamond property (the inequality part) and to strongly regular normal pseudomanifolds (the equality part). We will also talk about recent results on d-dimensional polytopes with $2d+1$ or $2d+2$ vertices. (Received September 21, 2021)

1174-52-10973 **Galen Dorpalen-Barry** (dorpa003@umn.edu), University of Minnesota. *On the Ehrhart positivity of polytopes arising from unit interval positroids* Preliminary report.

A fundamental question asked of any polytope is what is its volume. For an integral polytope, an equally desirable result is to describe its Ehrhart polynomial which encodes the volume of dilates of a polytope. It was conjectured by De Loera-Haws-Köppe that the Ehrhart polynomial of any matroid polytope has positive coefficients. Recent work of Ferroni provides counter examples to this conjecture. Yet, there is evidence showing that Ehrhart positivity may still hold for a family of matroid polytopes called positroid polytopes. Thus, a refinement of this conjecture still awaits a verdict: all positroid polytopes are Ehrhart positive. To this aim, in this talk we discuss preliminary results on the Ehrhart positivity of unit interval positroid polytopes, introduced by Chavez and Gotti, and similar results for the related completion unit interval positroid polytope. We also explore their volume and existence of a unimodular triangulation. (Received September 21, 2021)

1174-52-12233 **Travis Dillon*** (dillont@mit.edu), Massachusetts Institute of Technology. *Boxing up Helly's theorem*

Helly's theorem is a classical result in convex geometry that tells you that the intersection of a family of convex sets is nonempty if and only if the intersection of all of its small subfamilies is nonempty. Quantitative versions of Helly's theorem extend this with some measure of the size of an intersection—for example, volume, diameter, or the number of integer lattice points it contains—telling you that if all the small subfamilies have "big" intersections, then the intersection of the entire family is big, as well. In this talk, I will describe quantitative Helly-type theorems where the small intersections contain not just many integer points, but a box that "witnesses" this fact. After motivating this setup, I will describe the novel techniques used to prove these theorems and discuss the surprising way they differ from other quantitative Helly-type theorems. (Received December 2, 2021)

53 ► Differential geometry

1174-53-5523 **Zhiqin Lu*** (zlu@uci.edu), UC Irvine. *The Spectrum of the Laplacian on forms over open manifolds*

In this talk, we present the proof of the following theorem: let M be a complete non-compact Riemannian manifold whose curvature goes to zero at infinity, then its spectrum of the Laplacian on differential forms is a connected set. The result under different special cases was obtained before, but we recently prove the full general case. This is joint with Nelia Charalambous. (Received August 21, 2021)

1174-53-5649 **Alina Stancu** (stancu@mathstat.concordia.ca), Concordia University, **Theodora Bourni*** (tbourni@utk.edu), University of Tennessee/Knoxville, **Julie Clutterbuck Clutterbuck** (Julie.Clutterbuck@monash.edu), Monash University, **Xuan Hien Nguyen** (xhnguyen@iastate.edu), Iowa State University, and **Valentina-Mira Wheeler** (vwheeler@uow.edu.au), University of Wollongong. *Ancient solutions for flow by powers of the the curvature in the plane*

We construct a new compact convex embedded ancient solution of the κ^α flow in $\mathbb{R}^2$, $\alpha \in (1/2, 1)$, that lies between two parallel lines and additionally, for $\alpha \in (2/3, 1)$, we provide a uniqueness result. Moreover, we show that any non-compact convex embedded ancient solution of the κ^α flow in $\mathbb{R}^2$, $\alpha \in (1/2, 1)$ must be a translating solution. (Received August 24, 2021)

1174-53-5755 **Gunhee Cho*** (gunhee.cho@math.ucsb.edu), University of California, Santa Barbara. *The lower bound of the integrated Carathéodory-Reiffen metric and Invariant metrics on complete noncompact Kähler manifolds* Preliminary report.

We seek to gain progress on the following long-standing conjectures in hyperbolic complex geometry: prove that a simply connected complete Kähler manifold with negatively pinched sectional curvature is biholomorphic to a bounded domain and the Carathéodory-Reiffen metric does not vanish everywhere. As the next development of the important recent results of D. Wu and S.T. Yau in obtaining uniformly equivalence of the base Kähler metric with the Bergman metric, the Kobayashi-Royden metric, and the complete Kähler-Einstein metric in the conjecture class but missing of the Carathéodory-Reiffen metric, we provide an integrated gradient estimate of the bounded holomorphic function which becomes a quantitative lower bound of the integrated Carathéodory-Reiffen metric. Also, without requiring the negatively pinched holomorphic sectional curvature condition of the Bergman metric, we establish the equivalence of the Bergman metric, the Kobayashi-Royden metric, and the complete Kähler-Einstein metric of negative scalar curvature under a bounded curvature condition of the Bergman metric on an n-dimensional complete noncompact Kähler manifold with some reasonable conditions which also imply non-vanishing Carathédoroy-Reiffen metric. This is a joint work with Kyu-Hwan Lee. (Received August 26, 2021)

1174-53-5821 **Catherine Searle** (searle.catherine@gmail.com), Wichita State University, **Fred Wilhelm*** (fred@math.ucr.edu), University of California-Riverside, and **Pedro Solorzano** (pedro.antonio.solorzano@gmail.com), Instituto de Matemáticas, UNAM. *Smoothing Quotients with Lower Curvature Bounds* Preliminary report.

It is natural to ask whether a given n-dimensional Alexandrov space X is smoothable, that is, whether X is the limit of a sequence of Riemannian n-manifolds with a uniform lower curvature bound. By Perelman's Stability

Theorem, such an X must be a topological manifold. V. Kapovitch showed the converse is false, by proving that if X is smoothable, then all of its spaces of directions are smoothable. This motivates the following question.

Question: (Kapovitch) Is X smoothable if all of its spaces of directions are smoothable?

We show the answer is "yes", provided X is an orbit space all of whose spaces of directions are metric joins of round spheres. This generalizes an earlier result of Dyatlov. (Received August 28, 2021)

1174-53-5874 **Ella Pavlechko*** (epavlec@ncsu.edu), North Carolina State University, and **Teemu Saksala** (tssaksal@ncsu.edu), North Carolina State University. *Determination of a Strictly Convex and Non-trapping Riemannian Manifold from Partial Travel Time Data*

Let (M, g) be a smooth, compact, strictly convex, and non-trapping Riemannian manifold with smooth boundary ∂M. Let Γ be a nonempty open subset of the boundary. For each p in M we obtain $r_p(z) = d(p, z)$, where z is in Γ, called the boundary distance function. Given Γ and $\{r_p|_\Gamma \in C(\Gamma) \ : \ p \in M\}$ we determine (M, g) up to Riemannian isometry. The problem has applications in geophysical imaging.

Previous works consider the full data case $\Gamma = \partial M$, where the key result is the smoothness of $d(p, z)$ in a neighborhood near the closest boundary point. In the partial data case we do not have access to the closest boundary point, so further work must be done in characterising when $d(p, z)$ is smooth. Our characterization focuses on the behavior of the distance function outside of each point's cut locus. From the smoothness of $d(p, z)$ it follows the data distinguishes points from one other, allowing for the reconstruction of the topological, smooth, and metric structures on the manifold M. (Received August 30, 2021)

1174-53-5923 **Jiayin Pan*** (jypan10@gmail.com), The Fields Institute. *Examples of Ricci limit spaces with non-integer Hausdorff dimension*

We give the first examples of collapsing Ricci limit spaces on which the Hausdorff dimension of the singular set exceeds that of the regular set; moreover, the Hausdorff dimension of these spaces can be non-integers. This answers a question of Cheeger-Colding about collapsing Ricci limit spaces. (Received August 31, 2021)

1174-53-5924 **Xi Sisi Shen*** (xss@math.northwestern.edu), Columbia University. *Estimates for metrics of constant Chern scalar curvature*

We discuss the existence problem of constant Chern scalar curvature metrics on a compact complex manifold. We prove a priori estimates for these metrics conditional on an upper bound on the entropy, extending a recent result by Chen-Cheng in the Kähler setting. (Received August 31, 2021)

1174-53-6094 **Demetre Kazaras*** (dkazaras@gmail.com), Duke University. *If Ricci is bounded below, then mass is in control*

The ADM mass of an isolated gravitational system is a geometric invariant measuring the total mass due to matter and other fields. In a previous work, we showed how to compute this invariant (in 3 spatial dimensions) by studying harmonic functions. Now I'll use this formula to consider the question: How flat is an asymptotically flat manifold with very little total mass? We make progress on this problem and confirm special cases of conjectures made by Huisken-Ilmanen and Sormani. The main results asserts that in the class of reasonably-behaved asymptotically flat manifolds satisfying a uniform lower bound on Ricci curvature, small mass implies Gromov-Hausdorff closeness to flat space. (Received September 21, 2021)

1174-53-6265 **Da Rong Cheng*** (darong.daren.cheng@gmail.com), University of Waterloo. *Existence of constant mean curvature 2-spheres*

This talk is based on joint work with Xin Zhou (Cornell). I'll describe our work (arXiv:2012.13379) on the existence of constant mean curvature 2-spheres. In particular, we show that in a 3-sphere equipped with a Riemannian metric of positive Ricci curvature, for all H there exists a non-trivial, branched immersed 2-sphere with constant mean curvature H. Time permitting, I'll also talk about our more recent work (arXiv:2106.12374) applying some of the ideas involved in the proof of the above result to study the existence of curves with constant geodesic curvature in Riemannian 2-spheres. (Received September 12, 2021)

1174-53-6462 **Lawrence Mouillé*** (lawrence.mouille@gmail.com), Rice University. *Torus actions on manifolds with positive intermediate Ricci curvature*

On an n-dimensional manifold, positive k^{th}-intermediate Ricci curvature ($\mathrm{Ric}_k > 0$) is a condition that interpolates between positive sectional curvature ($k = 1$) and positive Ricci curvature ($k = n - 1$). In this talk, we will present new tools for studying isometric actions on closed manifolds with $\mathrm{Ric}_k > 0$. Namely, we will discuss generalizations of the isotropy rank lemma and the connectedness principle from the setting of positive sectional curvature. Using these new tools, we study closed, simply connected manifolds with $\mathrm{Ric}_2 > 0$ that admit isometric actions by tori of large dimension. In odd dimensions, we show that these manifolds are spheres.

In even dimensions other than 6, we show that they have positive Euler characteristic, and their cohomology is weakly periodic. Consequently, if the second Betti number is no greater than 1, then these spaces are either spheres or complex projective spaces. (Received September 9, 2021)

1174-53-6492 **Boris I Botvinnik*** (botvinn@uoregon.edu), University of Oregon. Spin^c *manifolds, positive scalar curvature and manifolds with fibered singularities*

I will discuss a problem of existence of positive scalar curvature on manifolds with fibered singularities. It turns out there are necessary and sufficient conditions for a psc-metric to exist on such objects. There is a particular case of manifolds with fibered singularities when the fiber is a circle. This case leads to psc-metrics on spin^c manifolds with special conditions near the singular locus. In particular, I describe some results concerning metrics on spin^c manifolds with positive "twisted scalar curvature", where the twisting comes from the curvature of the spin line bundle. This work is joint with Jonathan Rosenberg. (Received September 9, 2021)

1174-53-6851 **Raquel Perales*** (raquel.peralesaguilar@gmail.com), CONACyT-UNAM. *Upper bound on the revised first Betti number and torus stability for RCD spaces*

It was shown by Gromov and Gallot that for a fixed dimension n there exists a positive number $\varepsilon(n)$ so that any n-dimensional riemannian manifold satisfying $\mathrm{Ric}_g \mathrm{diam}(M,g)^2 \geq -\varepsilon(n)$ must have first Betti number smaller than or equal to n. Later on, Cheeger and Colding showed that if the first Betti number equals n then M has to be bi-Hölder homeomorphic to a flat torus.

In this talk we will generalize the previous results to the case of $\mathrm{RCD}(K,N)$ spaces, which is the synthetic notion of riemannian manifolds satisfying $\mathrm{Ric} \geq K$ and $\dim \leq N$. This class of spaces include ricci limit spaces and Alexandrov spaces.

Joint work with I. Mondello and A. Mondino. (Received September 10, 2021)

1174-53-6866 **Man-Wai Cheung*** (mandywai24@gmail.com), Harvard University, **Yu-Shen Lin** (yslin@bu.edu), Boston University, **Sam Alexander Bardwell-Evans** (sambe@bu.edu), Boston University, and **Hansol Hong** (hansol84@gmail.com), Yonsei University. *Mirror symmetry for rank 2 cluster varieties* Preliminary report.

The Gross-Hacking-Keel mirror is constructed in terms of scattering diagrams and theta functions. The ground of the construction is that scattering diagrams inherit the algebro-geometric analogue of the holomorphic disks counting. With Sam Bardwell-Evans, Hansol Hong, and Yu-shen LIn, we construct a special Lagrangian fibration on the non-toric blowups of toric surfaces that contains nodal fibres, and prove that the fibres bounding Maslov 0 discs reproduce the scattering diagrams. As a consequence, we can then illustrate the mirror duality between the A and X cluster varieties. (Received September 9, 2021)

1174-53-6998 **Xianzhe Dai** (dai@math.ucsb.edu), UC Santa Barbara, and **Junrong Yan*** (j_yan@math.ucsb.edu), UC, Santa Barbara. *Analytic torsion for Witten deformation on noncompact manifolds*

The Landau-Ginzburg (LG) B-model in mirror symmetry highlights the study of Witten deformation on noncompact manifolds. To understand the genus one term of the LG B-model, we develop the notion of analytic torsion for Witten deformation on noncompact manifolds following our previous works on cohomological theory and heat kernel expansions (for Witten deformation on noncompact manifolds). We establish the anomaly formula and Cheeger-Müller/Bismut-Zhang theorem for the analytic torsion in this setting. Moreover, we find an interesting connection between the analytic torsion of the Witten deformation on a noncompact manifold and the analytic torsion of its compact core with absolute/relative boundary conditions. This is joint work with Xianzhe Dai. (Received September 11, 2021)

1174-53-7008 **Maxim Jeffs*** (jeffs@math.harvard.edu), Harvard University. *Mirror symmetry and Fukaya categories of singular varieties*

In this talk I will explain Auroux' definition of the Fukaya category of a singular hypersurface and two results about this definition, illustrated with some examples, as well as ongoing work on generalizations to Fukaya-Seidel categories. The first result is that Auroux' category is equivalent to the Fukaya-Seidel category of a Landau-Ginzburg model on a smooth variety; the second result is a homological mirror symmetry equivalence at certain large complex structure limits. (Received September 11, 2021)

1174-53-7062 **Sema Salur*** (semasalur@gmail.com), University of Rochester. *Geometric Structures on G_2 manifolds*

One promising direction for future investigation is the study of almost contact structures on G_2 manifolds. In this talk, we review the properties of special holonomy geometries and almost contact structures. We show that every closed, oriented, smooth 3 manifold has a trivial normal bundle inside a Riemannian 7 manifold with spin structure. We also study the 3-dimensional associative and Harvey-Lawson submanifolds of G_2 manifolds and show that their almost contact structures can be extended to the ambient space. (Received September 11, 2021)

1174-53-7140 **Bradley Burdick*** (bburdick@ucr.edu), University of California, Riverside. *Gluing constructions for positive σ_2-scalar curvature* Preliminary report.

For simply connected manifolds of dimension $n \geq 9$, the existence question for positive σ_2-scalar curvature metrics has been treated similarly to the case of positive scalar curvature metrics. In dimension $n \leq 3$, positive σ_2-scalar curvature implies positive sectional curvature, so it is known which manifolds admit such metrics. In this talk we introduce a gluing construction for positive σ_2-scalar curvature that may produce new examples of manifolds admitting positive σ_2-scalar curvature metrics in dimensions $4 \leq n \leq 8$. (Received September 12, 2021)

1174-53-7332 **Ryad Ghanam*** (raghanam@vcu.edu), Virginia Commonwealth University in Qatar, **Hassan Almusawa** (haalmusawa@jazanu.edu.sa), Jazan University, and **Gerard Thompson** (gerard.thompson@utoledo.edu), University of Toledo. *Lie Symmetries of the Canonical Connection on Lie groups of Codimension One Abelian Nilradical*

In this talk, we will present our results about Lie symmetries of the geodesic equations of the canonical connection on Lie groups. We focus on the case of codimension one abelian nilradical. In this case we are able to explicitly write the conditions and integrate the system of PDE's. (Received September 14, 2021)

1174-53-7591 **Nicole Magill*** (nm627@cornell.edu), Cornell University. *Infinite Staircases in Symplectic Embeddings* Preliminary report.

McDuff and Schlenk determined the ellipsoidal embedding capacity function for a four-dimensional ball. They found that infinitely many obstructions given by Fibonacci numbers affect the capacity function. This talk will focus on determining whether ellipsoidal embeddings into other four-dimensional targets are given by infinitely or finitely many obstructions. The main targets I will talk about are one- and two-fold blow ups of the complex projective plane. One of the major proof strategies used in this work is almost toric fibrations. Some of this is joint work with Tara Holm, Dusa McDuff, Ana Rita Pires, and Morgan Weiler. (Received September 16, 2021)

1174-53-7723 **Yueh-Ju Lin*** (lin@math.wichita.edu), Wichita State University, and **Wei Yuan** (yuanw9@mail.sysu.edu.cn), Sun Yat-Sen University, China. *Volume comparison of Q-curvature*

Classical volume comparison for Ricci curvature is a fundamental result in Riemannian geometry. In general, scalar curvature is too weak to control the volume. However, with the additional stability assumption on the closed Einstein manifold, one can obtain a volume comparison for scalar curvature. In this talk, we investigate a similar phenomenon for Q-curvature, a fourth-order analogue of scalar curvature. In particular, we prove a volume comparison result of Q-curvature for metrics near strictly stable Einstein metrics by variational techniques and a Morse lemma. This is a joint work with Wei Yuan. (Received September 15, 2021)

1174-53-7835 **Audrey Rosevear*** (arosevear22@amherst.edu), Amherst College, and **Willie Wong** (wongwwy@math.msu.edu), Michigan State University. *Geodesic Motion on $SL(n)$ with the Hilbert-Schmidt Metric*

We study the geometry of geodesics on $\mathrm{SL}(n)$, equipped with the Hilbert-Schmidt metric which makes it a Riemannian manifold. These geodesics are known to be related to affine motions of incompressible ideal fluids. The $n = 2$ case is special and completely integrable, and the geodesic motion was completely described by Roberts, Shkoller, and Sideris; when $n = 3$, Sideris demonstrated some interesting features of the dynamics and analyzed several classes of explicit solutions. Our analysis shows that the geodesics in higher dimensions exhibit much more complex dynamics. We generalize the Virial-identity-based criterion of unboundedness of geodesic given by Sideris, and use it to give an alternative proof of the classification of geodesics in 2D obtained by Roberts-Shkoller-Sideris. We study several explicit families of solutions in general dimensions that generalize those found by Sideris in 3D. We additionally classify all "exponential type" geodesics, and use it to demonstrate the existence of purely rotational solutions whenever n is even. Finally, we study "block diagonal" solutions using a new formulation of the geodesic equation in first order form, that isolates the conserved angular momenta. This reveals the existence of a rich family of bounded geodesic motions in even dimension $n \geq 4$. This in particular allows us to conclude that the generalization of the swirling and shear flows of Sideris to even dimensions $n \geq 4$ are in fact dynamically unstable. (Received September 16, 2021)

1174-53-7931 **Luya Wang*** (luya_wang@berkeley.edu), UC Berkeley. *Connected sum formula of embedded contact homology* Preliminary report.

I will discuss work in progress on computing the connected sum formula of embedded contact homology. (Received September 16, 2021)

1174-53-8018 **Diego Corro** (diego.corro.math@gmail.com), Mathematisches Institut, Universität zu Köln, and **Adam J Moreno*** (amoreno@amherst.edu), Amherst College. *Locating Boundary in Leaf Spaces with Pre-Sections*

We expand upon the notion of a pre-section for a singular Riemannian foliation $(M, \mathcal{F})$, i.e. a proper submanifold $N \subset M$ retaining all the tranverse geometry of the foliation. This generalization of a polar section provides a similar reduction, allowing one to recognize certain geometric or topological properties of $(M, \mathcal{F})$ and the leaf space $M/\mathcal{F}$ by studying a "smaller" foliation $(N, \mathcal{F}')$. In particular, we show that if a foliated manifold M has positive sectional curvature and contains a nontrivial pre-section $N \neq M$, then the leaf space $M/\mathcal{F}$ has nonempty boundary. We recover as corollaries the known result for the special case of polar foliations as well as the unwritten analogue for isometric group actions. (Received September 19, 2021)

1174-53-8065 **Paula Burkhardt-Guim*** (pb2682@nyu.edu), New York University. *Pointwise lower scalar curvature bounds for C^0 metrics via regularizing Ricci flow*

We propose a class of local definitions of weak lower scalar curvature bounds that is well defined for C^0 metrics. We show the following: that our definitions are stable under greater-than-second-order perturbation of the metric, that there exists a reasonable notion of a Ricci flow starting from C^0 initial data which is smooth for positive times, and that the weak lower scalar curvature bounds are preserved under evolution by the Ricci flow from C^0 initial data. (Received September 17, 2021)

1174-53-8070 **Paula Burkhardt-Guim*** (pb2682@nyu.edu), New York University. *A C^0 mass with well-controlled distortion under Ricci-DeTurck flow*

We introduce a notion of mass that depends only on C^0 data of the metric, is well controlled under evolution by Ricci-DeTurck flow, and agrees with the usual ADM mass at infinity in the classical setting of the Positive Mass Theorem. We show that the mass at infinity exists for C^0 metrics with nonnegative scalar curvature in the sense of Ricci flow, and is well-defined. (Received September 17, 2021)

1174-53-8178 **Andrew Hanlon*** (andrew.hanlon@stonybrook.edu), Stony Brook University. *Cocores and linking disks in toric mirror symmetry*

Cocores and linking disks generate the wrapped Fukaya category of a stopped Weinstein domain. We will discuss the computation, joint with Jeff Hicks, of the mirror objects to these generators in the toric setting. In addition, we will speculate on some further properties of these objects and applications to understanding the categories appearing in homological mirror symmetry for toric varieties. (Received September 17, 2021)

1174-53-8266 **Morgan Weiler*** (mocowe@gmail.com), Cornell University. *Fractals, ECH capacities, and symplectic embeddings of four-dimensional ellipsoids into Hirzebruch surfaces*

In 2012, McDuff and Schlenk proved that the optimal symplectic embeddings of four-dimensional ellipsoids into complex projective space are governed by an "infinite staircase" of pseudoholomorphic curve obstructions. Recently, Cristofaro-Gardiner–Holm–Mandini–Pires conjectured that there is only a finite list of integrally-weighted blowups of complex projective space whose ellipsoid embeddings share this same property. We explain work towards this conjecture, which can be reinterpreted in the language of ECH capacities, and relies on a Cantor set structure of the obstructions as the sizes of the blowups vary. This is joint work with Dusa McDuff and Nicole Magill. (Received September 18, 2021)

1174-53-8503 **Yusuf Baris Kartal*** (ykartal@math.princeton.edu), Princeton University, and **Laurent Cote** (lcote@math.harvard.edu), Harvard University. *Categorical action filtrations and the growth as a symplectic invariant*

Vector spaces appearing in algebraic geometry and symplectic geometry are often filtered. For instance, the ring of functions on an open variety can be filtered by "the order of pole at infinity", and the wrapped Floer cohomology and symplectic cohomology can be filtered by action. One goal of this work is to categorify these filtrations to be able to compare them through mirror symmetry, and also to allow techniques of non-commutative geometry to work with them. More precisely, we show how to obtain filtrations, "equivalent" to the geometrically defined ones, by using the categorical compactifications. We study the filtrations through their associated growth functions. As major applications, we relate growth invariants from Hamiltonian dynamics to the dimensions of the support of mirror coherent sheaves. We also prove an at most exponential growth property for the wrapped

Floer cohomology and the symplectic cohomology by applying techniques from categorical dynamics. This is joint work with Laurent Cote. (Received September 20, 2021)

1174-53-8533 **Yuan Gao*** (yuangaohl@gmail.com), University of Georgia. *The formal neighborhood of a Liouville divisor* Preliminary report.

For a stopped Liouville manifold coming from a Liouville sector, we introduce a symplectic analogue of the formal neighborhood of the stop. This construction can be performed in two ways: 1) via new types of Fukaya categories using variants of Floer theory on a Liouville sector, or 2) via a purely categorical construction of the formal neighborhood from the partially wrapped Fukaya category. Our main result shows that the two approaches are equivalent. Under homological mirror symmetry for pairs, this is mirror to the formal neighborhood of a divisor in an ambient projective variety. (Received September 19, 2021)

1174-53-8595 **Lan-Hsuan Huang** (lan-hsuan.huang@uconn.edu), University of Connecticut, and **Hyun Chul Jang*** (hcjangmath@gmail.com), University of Miami. *Mass rigidity for asymptotically locally hyperbolic manifolds with boundary*

Asymptotically locally hyperbolic (ALH) manifolds are a class of manifolds whose sectional curvature converges to -1 at infinity. If a given ALH manifold is asymptotic to a static reference manifold, the Wang-Chruściel-Herzlich mass integrals are well-defined for it, which essentially measure the difference from the reference manifold. In this talk, I will present a recent result with L. -H. Huang, which characterizes ALH manifolds that minimize the mass integrals. The proof uses scalar curvature deformation results for ALH manifolds with nonempty compact boundary. Specifically, we show the scalar curvature map is locally surjective among (1) the space of ALH metrics that coincide exponentially toward the boundary or (2) the space of ALH metrics with arbitrarily prescribed nearby Bartnik boundary data. As a direct consequence, we establish the rigidity of the known positive mass theorems. (Received September 19, 2021)

1174-53-8596 **Lee Kennard*** (ltkennar@syr.edu), Syracuse University, **Michael Wiemeler** (wiemelerm@uni-muenster.de), Universität Münster, and **Burkhard Wilking** (wilking@uni-muenster.de), Universität Münster. *Systoles of graphs and torus representations* Preliminary report.

A classical graph invariant is the girth, which is the length of the shortest cycle. In the presence of weights or distances assigned to the edges, one can similarly define the weighted girth or systole of a graph. I will review known asymptotic bounds on this quantity where the first Betti number of the graph goes to infinity and applications to a question of Gromov on surfaces of large genus. I will also discuss bounds for small first Betti number proved recently in joint work with Michael Wiemeler and Burkhard Wilking. I will also discuss implications for torus representations and applications to the problem of classifying Riemannian manifolds with positive curvature and large isometry groups. (Received September 19, 2021)

1174-53-8698 **Lea Kenigsberg*** (lk2720@columbia.edu), Columbia University. *A co-product structure on symplectic cohomology* Preliminary report.

Symplectic Cohomology is a Floer theoretic invariant of Liouville domains. It carries a rich structure, such as a graded commutative product, and a Batalin Vilkovisky (BV) operator. I will use the BV structure to define a graded co-commutative coproduct and will briefly indicate how to compute it for T^*S^3 to show that it does not vanish. (Received September 19, 2021)

1174-53-8761 **Erin Griffin*** (griffine@spu.edu), Seattle Pacific University. *Gradient solitons of a general q-flow and the consequences for ambient obstruction solitons*

Consider a geometric flow by a general tensor, q, and the resulting solitons. We will see that by prescribing various properties to q (e.g. being divergence-free) that we are able to draw a number of conclusions about solitons of the q-flow.

Focusing on a specific q, we consider the ambient obstruction tensor, $\mathcal{O}$. We will see how our general results improve our understanding of ambient obstruction solitons and why this method is necessary for such an investigation. (Received September 19, 2021)

1174-53-8824 **McFeely Jackson Goodman*** (mjgoodman@berkeley.edu), University of California, Berkeley. *Moduli spaces of nonnegatively curved metrics on exotic spheres*

We show that the moduli space of nonnegatively curved metrics on each manifold homeomorphic to S^7 has infinitely many path components. The components are distinguished using the Kreck-Stolz s-invariant computed for metrics constructed by Goette, Kerin and Shankar. The invariant is computed by extending each metric to the total space of an orbifold disc bundle and applying generalizations of the Atiyah-Patodi-Singer index theorem for orbifolds with boundary. (Received September 19, 2021)

1174-53-8833 **Meera G. Mainkar*** (maink1m@cmich.edu), Central Michigan University. *Preserve one, preserve all*

The classical Beckman-Quarles theorem states that a self map on a Euclidean n-space ($n \geq 2$) which preserves the length 1 must be an isometry. We formulate and prove cases of a conjecture stating that if X is a complete Riemannian manifold (dim $X \geq 2$), then a self map on X which preserves sufficiently small length is an isometry. This is joint work with Benjamin Schmidt. (Received September 20, 2021)

1174-53-8943 **Chao Li*** (chaoli@nyu.edu), New York University. *Metrics with $\lambda_1(-\Delta + kR) \geq 0$ and flexibility in the Riemannian Penrose Inequality*

On a closed manifold, consider the space of all Riemannian metrics for which $-\Delta + kR$ is positive (nonnegative) definite, where $k > 0$ and R is the scalar curvature. This spectral generalization of positive (nonnegative) scalar curvature arises naturally for different values of k in the study of scalar curvature in dimension via minimal hypersurfaces, the Yamabe problem, and Perelman's Ricci flow with surgery. When $k = 1/2$, the space models apparent horizons in time-symmetric initial data to the Einstein equations. We study these spaces in unison and generalize Codá Marques's path-connectedness theorem. Applying this with $k = 1/2$, we compute the Bartnik mass of 3-dimensional apparent horizons and the Bartnik–Bray mass of their outer-minimizing generalizations in all dimensions. Our methods also yield efficient constructions for the scalar-nonnegative fill-in problem. (Received September 20, 2021)

1174-53-8952 **Andrew Clickard*** (ac24869@huskies.bloomu.edu), Bloomsburg University of Pennsylvania, and **Barry Minemyer** (bminemyer@bloomu.edu), Bloomsburg University of Pennsylvania. *Synthetic Geometry in Hyperbolic Simplices*

Let τ be an n-simplex and let g be a metric on τ with constant curvature κ. The lengths that g assigns to the edges of τ, along with the value of κ, uniquely determine all of the geometry of (τ, g). In this paper we focus on hyperbolic simplices ($\kappa = -1$) and develop geometric formulas which rely only on the edge lengths of τ. Our main results are distance and projection formulas in hyperbolic simplices, as well as a projection formula in Euclidean simplices. We also provide analogous formulas in simplices with arbitrary constant curvature κ. (Received September 20, 2021)

1174-53-8957 **Ruth Gornet*** (rgornet@uta.edu), University of Texas at Arlington, **Maura B Mast** (mmast@fordham.edu), Fordham, and **Jonathan Epstein** (jepstein@ou.edu), McDaniel College. *Periodic magnetic geodesics on Heisenberg manifolds.*

We study the dynamics of magnetic flows on Heisenberg groups, investigating the extent to which properties of the underlying Riemannian geometry are reflected in the magnetic flow. Much of the analysis, including a calculation of the Mañé critical value, is carried out for $(2n + 1)$-dimensional Heisenberg groups endowed with any left invariant metric and any exact, left-invariant magnetic field. In the three-dimensional Heisenberg case, we obtain a complete analysis of leftinvariant, exact magnetic flows. This is interesting in and of itself, because of the difficulty of determining geodesic information on manifolds in general. We use this analysis to establish two primary results. We first show that the vectors tangent to periodic magnetic geodesics are dense for sufficiently large energy levels and that the lower bound for these energy levels coincides with the Mañé critical value. We then show that the marked magnetic length spectrum of left-invariant magnetic systems on compact quotients of the Heisenberg group determines the Reimannian metric. Both results confirm that this class of magnetic flows carries significant information about the underlying geometry. Finally, we provide an example to show that extending this analysis of magnetic flows to the Heisenberg type setting is considerably more difficult. (Received September 20, 2021)

1174-53-8968 **Sabetta Matsumoto*** (sabetta@gatech.edu), Georgia Institute of Technology, **Rémi Coulon** (remi.coulon@univ-rennes1.fr), CNRS Rennes, and **Steve Trettel** (trettel@stanford.edu), Stanford University. *The Inside View: Raymarching and the Thurston geometries* Preliminary report.

The properties of euclidean space seem natural and obvious to us, to the point that it took mathematicians over two thousand years to see an alternative to Euclid's parallel postulate. The eventual discovery of hyperbolic geometry in the 19th century shook our assumptions, revealing just how strongly our native experience of the world blinded us from consistent alternatives, even in a field that many see as purely theoretical. Non-euclidean spaces are still seen as unintuitive and exotic, but with direct immersive experiences we can get a better intuitive feel for them. The latest wave of virtual reality hardware, in particular the HTC Vive, tracks both the orientation and the position of the headset within a room-sized volume, allowing for such an experience. We create realtime rendering to explore the three-dimensional geometries of the Thurston/Perelman geometrization theorem. In this talk, we use the "inside view" of each manifold to try to understand its geometry and what life might be like on the inside. Joint work with Rémi Coulon, Henry Segerman and Steve Trettel. (Received September 20, 2021)

1174-53-8973 **Jiajun Yan*** (jy4au@virginia.edu), University of Virginia. *A new construction of ALE spaces via gauge theory* Preliminary report.

In this talk, we first introduce the background needed for the construction of ALE spaces given by Peter Kronheimer in his PhD thesis. In particular, we will review moment maps, symplectic reduction, and hyperkähler reduction. Then we will describe Kronheimer's construction.

In the second part of the talk, we give a new construction of the same spaces via gauge theory. We realize each ALE space as a moduli space of solutions to a system of equations for a pair consisting of a connection and a section of a vector bundle over an orbifold Riemann surface, modulo a hyperkähler gauge group action. (Received September 20, 2021)

1174-53-8983 **Man-Chun Lee*** (mclee@math.cuhk.edu.hk), The Chinese University of Hong Kong. *Some compactness Theorem using Ricci flows* Preliminary report.

By Gromov compactness theorem, it is known that non-collapsed Riemannian manifold with Ricci bounded from below always admit a convergent subsequence in the pointed Gromov-Hausdorff topology. And thanks to the work of Cheeger, Colding, Naber and others, this class of manifolds is now well-understood as a metric space. It is natural to ask two types of questions. Firstly if the Ricci Curvature is further strengthened, what additional structure on the Gromov-Hausdorff limit can be said? Secondly, if the Ricci Curvature is weakened, can we still conclude some weak convergence behavior? In this talk, we will discuss some recent progress of the questions using Ricci Flow smoothing method. In particular, we will discuss the compactness on manifolds with i) curvature operator bounded from below; ii) scalar curvature bounded from below. (Received September 20, 2021)

1174-53-8986 **Daniel L Stern*** (dstern@uchicago.edu), University of Chicago. *The U(1)-Higgs model and the codimension-two area functional*

The self-dual U(1)-Higgs functionals on hermitian line bundles are a natural family of energies, whose minimizers and critical points are a long-standing subject of interest in the gauge theory community. In this talk, we'll present a series of results (based on joint works with Alessandro Pigati and Davide Parise) revealing a correspondence between the critical points, variational theory, and gradient flows for these functionals and those of the area functional on closed codimension-two submanifolds in a given ambient space. Though the geometric settings are quite different, the correspondence has many striking similarities with the well-known relationship between the Allen-Cahn energies and the codimension-one area functional first observed by De Giorgi. (Received September 20, 2021)

1174-53-8996 **Benjamin Schmidt*** (schmidt@math.msu.edu), Michigan State University, **Ralf J. Spatzier** (spatzier@umich.edu), University of Michigan, and **Krishnan Shankar** (Krishnan.Shankar-1@ou.edu), Oklahoma University. *Growth competitions in some homogeneous spaces.* Preliminary report.

A growth competition between competing subsets of a geodesic metric space is an expansion of the sets (starting as singletons) over time at different speeds. The problem of determining the shapes of the competing sets was first proposed by Benjamini in the discrete context of infinite graphs and was recently studied in the continuous context by Assouline.

I'll discuss ongoing work with Ravi Shankar and Ralf Spatzier where we study growth competitions in non-compact symmetric spaces of higher rank, Riemannian products with a pole, and some nilpotent Lie groups with an invariant metric. (Received September 20, 2021)

1174-53-9005 **Rachelle C DeCoste** (decoste_rachelle@wheatoncollege.edu), Wheaton College, **Allie Ray*** (adray@bsc.edu), Birmingham-Southern College, and **Lisa DeMeyer** (demey1la@cmich.edu), Central Michigan University. *Geometry of Nilpotent Lie Algebras Constructed From Graphs* Preliminary report.

Starting with a directed edge-labelled graph, we introduce two methods for constructing a two-step nilpotent metric Lie algebra. We will see how the singularity of these Lie algebras, as well as other aspects of their geometry, can be determined by properties of the graphs. (Received September 20, 2021)

1174-53-9095 **Craig J. Sutton** (craig.j.sutton@dartmouth.edu), Dartmouth College, and **Samuel Lin*** (samuel.z.lin-1@ou.edu), University of Oklahoma. *Three-dimensional Elliptic Manifolds and the Laplace Spectrum*

The Laplace spectrum of a closed Riemannian manifold is defined as the set of all eigenvalues of the corresponding Laplace operator. Inverse spectral geometry studies how the Laplace spectrum, a set of analytic data, relates to the underlying geometry of the manifold.

We say that two closed Riemannian manifolds are isospectral if they share the same Laplace spectrum. Around 1980, Ikeda-Yamamoto and Ikeda proved that isopectral three-dimensional spherical space forms are in fact isometric. This means that the Laplace spectrum completely determines the geometry of three-dimensional spherical space forms. It is natural to ask the following generalized question: Given two Riemannian manifolds M and N, both Riemannian covered by homogeneous three-spheres, can we conclude that M and N are isometric? This problem seems to be difficult, and is still open. In this talk, I will present out progress in this direction. This is based on projects with Ben Schmidt and Craig Sutton, and another project with Craig Sutton. (Received September 20, 2021)

1174-53-9139 **Marco Radeschi*** (marco.radeschi@gmail.com), University of Notre Dame. *Rational ellipticity of G-manifolds from their quotients.* Preliminary report.

A major conjecture in Riemannian geometry is that simply connected, non-negatively curved manifolds are rationally elliptic (roughly speaking, they have tame topology). Following the Grove program, about adding symmetry assumptions and then relaxing them, a number of results proved that a non-negatively curved manifold is rationally elliptic if it admits either a cohomogeneity 1, a cohomogeneity 2, a hyperpolar or a spherical polar action.

We generalize all these results at once, and remove the curvature assumptions, by proving that if a simply connected G-manifold M (with G compact) has orbit space M/G isometric to a quotient N/H with N of zero topological entropy, then M is rationally elliptic. In fact, the result also holds by replacing the group action with a singular Riemannian foliation with rationally elliptic leaves.

This is a joint work with Elahe Khalili Samani (University of Notre Dame). (Received September 20, 2021)

1174-53-9263 **Henry Kvinge** (henry.kvinge@pnnl.gov), Pacific Northwest National Laboratory, and **Nicolas Courts*** (ncourts@uw.edu), University of Washington, Pacific Northwest National Laboratory. *Fiber Bundles and Local Trivializations as a Framework for Modeling Many-to-one Processes In Machine Learning*

In real-world datasets, a single "class" can have many dimensions of variation. To take a classic example, cats can appear in a range of poses, orientations, and lighting conditions in an image while remaining a cat. Humans can disentangle orientation, illumination, and pose from "catness" but this is a nontrivial task for a machine learning model. In this work we discuss an approach to the problem using the mathematical framework of fiber bundles and local trivializations. Using this insight, we propose a novel generative deep learning architecture, Bundle Networks, that exploit the locally trivial nature of data in the modeling process. We apply our model to both synthetic and real-world datasets to demonstrate how it can be used to explore the local geometry of the data distribution as well as performing nontrivial tasks such as sampling from the inverse image ("fiber") of a model over a point. (Received September 20, 2021)

1174-53-9272 **Megan M Kerr*** (mkerr@wellesley.edu), Wellesley College, and **Tracy L. Payne** (payntrac@isu.edu), Idaho State University. *Submanifolds of Noncompact Homogeneous Spaces with Special Curvature Properties* Preliminary report.

The Ricci curvature form of a submanifold is not, in general, the restriction of the Ricci curvature of the ambient space. Therefore, classes of manifolds and submanifolds where the Ricci curvatures are aligned are very special. Indeed, Tamaru exploited this idea in the setting of noncompact symmetric spaces to construct new examples of Einstein solvmanifolds via special subalgebras. We characterize the largest category in which Tamaru's construction can be extended, identifying two crucial algebraic/metric conditions. We explore a new class of solvmanifolds defined by Kac-Moody algebras that are generalizations of symmetric spaces for which our crucial extra conditions hold. And furthermore, in current work in progress, we investigate other metric properties of these spaces. (Received September 20, 2021)

1174-53-9422 **Jordan Weaver*** (jeweaver@uw.edu), University of Washington. *A Grassmannian Walk Through a Dataset: Improving Data Visualization with Geometry*

As datasets grow in size and complexity, there is often a need to use visualizations to explore and better understand dataset composition and structure. For example, one may wish to identify several features characterizing the elements in the set and the corresponding ways to group the data according to those features. Because datasets often reside in high dimensional spaces where direct visualization is impossible, exploration is often easier after projecting into 2 or 3 dimensions where humans have visual intuition. However, applying such drastic dimension reduction can eliminate information about the structure of the data. The goal of this project is to study visualization and dimension reduction techniques that preserve the geometry of the set in a way that highlights multiple facets of the data. In the case of linear projections, the structure of a dataset can best be recovered by observing projections onto many different subspaces. Furthermore, these subspaces should be carefully chosen to illuminate properties of the original data. To obtain an appropriate set of subspaces, we pass to the Grassmannian manifold, which parametrizes the set of linear projections. Specifically, we investigate how the geometry of the Grassmannian can be used to provide useful visualizations of the data. (Received September 20, 2021)

1174-53-9532 **Zhongshan An*** (zhshan.an@gmail.com), University of Connecticut. *Static vacuum extensions with prescribed Bartnik data near the flat metric.*

The study of static vacuum Riemannian metrics arises naturally in differential geometry and general relativity. It plays an important role in scalar curvature deformation, as well as in constructing Einstein spacetimes. Existence of static vacuum Riemannian metrics with prescribed Bartnik data is one of the most fundamental problems in Riemannian geometry related to general relativity. It is also a very interesting problem on the global solvability of a natural geometric boundary value problem. In this talk I will first discuss the basic properties of static vacuum metrics and their boundary geometry. Then I will present some recent progress towards the existence problem based on a joint work with Lan-Hsuan Huang. (Received September 20, 2021)

1174-53-9604 **Catherine Cannizzo*** (ccannizzo@scgp.stonybrook.edu), Simons Center for Geometry and Physics, and **Sara Venkatesh** (sjvenkat@stanford.edu), Stanford University.
Semi-orthogonal decompositions in Fukaya-Seidel mirrors to blowups of abelian varieties

Homological mirror symmetry (HMS) is an equivalence of categories on pairs of complex and symplectic manifolds: the bounded derived category of coherent sheaves on a complex manifold is equivalent to a symplectic invariant, the Fukaya category, on the mirror symplectic manifold. Three cases where mirrors and HMS have been studied include Calabi-Yau manifolds, Fano manifolds, and those of general type (negative first Chern class). In finding a geometric mirror to the genus 2 curve (which is of general type), there are interesting intermediary manifolds with which to check HMS on. We will see an example of this, which makes use of semi-orthogonal decompositions and wrapping in the Fukaya category. This is joint work with S. Venkatesh. (Received September 20, 2021)

1174-53-9795 **Elahe Khalili Samani*** (ekhalili@nd.edu), University of Notre Dame, and **Catherine Searle** (searle@math.wichita.edu), Wichita State University. *Positively curved Riemannian manifolds with discrete abelian symmetry*

Classification of positively curved manifolds is a long-standing and important problem in Riemannian geometry. While there is no complete classification of such manifolds, there are many results in the presence of torus symmetry. In this talk, we discuss some of these results and their generalizations to the case of discrete symmetry. This is based on joint work with Lee Kennard and Catherine Searle. (Received September 20, 2021)

1174-53-9939 **Marina Meila*** (mmp@stat.washington.edu), University of Washington, and **Yu-Chia Chen** (yuchaz@uw.edu), Facebook Inc. *Short homologous loop detection and decomposing the homology embedding of the k-Laplacian*

The null space of the k-th order Laplacian, L_k, known as the *k-th homology vector space*, encodes the non-trivial topology of a manifold, a network, or a complex. The study of the null-space of the graph Laplacia L_0 has spurred new research and applications, such as spectral clustering algorithms. In this work, we investigate the the geometry of the k-th homology embedding of a sample from a manifold and focus on "loosely connected" cases reminiscent of spectral clustering. Namely, we analyze the *connected sum* of manifolds as a perturbation of the *direct sum* of their homology embeddings. We propose an algorithm to decompose the homology embedding into subspaces corresponding to the manifold prime decomposition. The proposed method is applied to the *shortest homologous loop detection* problem, a problem known to be NP-hard in general. Our spectral loop detection algorithm is effective on diverse data such as images and point clouds. (Received September 21, 2021)

1174-53-9950 **Elizabeth Stanhope*** (stanhope@lclark.edu), Lewis & Clark College, **Ben Lattes** (bglattes@lclark.edu), Lewis & Clark College, and **Andrew Ferris** (aferris@lclark.edu), aferris@lclark.edu. *Computing & applying heat invariants of 3-orbifolds* Preliminary report.

We address the question "Can you hear the shape of a Riemannian orbifold?" by examining the heat invariants of 3-dimensional Riemannian orbifolds. An orbifold is a mildly singular generalization of a manifold in which singular points arise as the fixed points of finite groups of isometries acting locally on the orbifold. Of particular interest are isolated singular points. In dimension two there are only two types of isolated singularities, those corresponding to a point fixed by rotations, and those corresponding to a point fixed by two reflections. In dimension three, however, the wider variety of finite subgroups of the orthogonal group $O(3)$ yield a longer list of isolated singularities. We examine how these isolated singularities contribute to the heat invariants of a 3-orbifold, seeking to use them to "hear" differences between various 3-orbifolds. (Received September 21, 2021)

1174-53-10274 **James Thomas Pascaleff*** (jpascale@illinois.edu), University of Illinois at Urbana-Champaign, and **Nicolò Sibilla** (nsibilla@sissa.it), Scuola Internazionale Superiore di Studi Avanzati, Trieste. *Fukaya categories and pants decompositions of surfaces*

Extending earlier work of Heather Lee and the present authors, we study the way that the Fukaya category of a (possibly compact) Riemann surface may be reconstructed from a pants decomposition. The freedom of choice of the pants decomposition allows us to match this category with several geometrically different mirror partners (derived categories of singularities). This theory allows us to introduce twisted versions of the Fukaya category of a surface and prove mirror theorems for them, and it allows us to apply ideas from the noncompact case to study the geometricity question on compact surfaces. (Received September 21, 2021)

1174-53-10439 **Carolyn Gordon** (carolyn.s.gordon@dartmouth.edu), Dartmouth College, **LIz Stanhope*** (stanhope@lclark.edu), Lewis & Clark College, **Asma Hassannezhad** (asma.hassannezhad@bristol.ac.uk), University of Bristol, **Emily Dryden** (ed012@bucknell.edu), Bucknell University, and **Teresa Arias Marco** (teamar3@gmail.com), Universidad de Extremadura. *Eigenvalue bounds for the mixed Steklov problem* Preliminary report.

In the mixed Steklov-Neumann eigenvalue problem, a portion of the boundary of a manifold is given Steklov boundary conditions and the remainder is given Neumann boundary conditions. Recently extensions of Steklov eigenvalue inequalities, such as Weinstock's inequality and Hersch-Payne-Schiffer inequality, to the mixed eigenvalue problem setting have appeared. We will discuss our work in this area, focusing on mixed eigenvalues of compact surfaces with Lipschitz boundary. (Received September 21, 2021)

1174-53-10440 **Heather Lee** (heatherlee.math@gmail.com), N/A, **Chiu-Chu Melissa Liu** (ccliu@math.columbia.edu), Columbia University, and **Haniya Azam** (haniya.azam@lums.edu.pk), Lahore University of Management Sciences. *Homological Mirror Symmetry for Theta Divisors* Preliminary report.

Mirror symmetry relates complex and symplectic manifolds which come in mirror pairs, and homological mirror symmetry is an equivalence of categories on each. In forthcoming joint work [ACLL] with Haniya Azam, Heather Lee, and Chiu-Chu Melissa Liu, we generalize arXiv:1908.04227 to *global* homological mirror symmetry for genus 2 curves. We consider genus 2 curves as hypersurfaces of abelian surfaces, on the complex side. In a follow-up paper [ACLL2], we allow the abelian variety to have arbitrary dimension, and hypersurfaces are now theta divisors. [ACLL2] will be the content of this talk. (Received September 21, 2021)

1174-53-10590 **Hannah Alpert** (hcalpert@gmail.com), Auburn University, USA, **Adriana Haydeé Contreras Peruyero** (haydeeperuyero@gmail.com), Department of Mathematics, UNAM, and **Regina Rotman** (rina@math.toronto.edu), University of Toronto. *Positive Curvature and Cohomogeneity-two, v.1* Preliminary report.

The search for new examples of manifolds with positive sectional curvature is a fundamental problem in global Riemannian geometry. When cohomogeneity two actions on positively curved manifolds are polar actions, Fang, Grove, and Thorbergsson have shown that all such closed, simply connected manifolds are equivariantly diffeomorphic to CROSSes. The class of simply connected positively curved manifolds admitting a non-polar cohomogeneity two action remains unclassified. We prove a structure theorem for non-polar actions of cohomogeneity two on closed, simply connected positively curved manifolds and using this result obtain a classification of such manifolds in low dimensions. (Received September 21, 2021)

1174-53-10651 **Junqing Qian*** (jqian20@unm.edu), Independent. *The Kahler-Einstein metric on punctured Riemann sphere and an algebraic perspective on curvature* Preliminary report.

We will talk about using modular functions in number theory to derive a precise asymptotic expansion of the complete Kähler-Einstein metric for the punctured Riemann spheres with a certain number of omitting points. On the other side, inspired by its negative constant curvature, we investigate the algebraic independency of functions related to the metric as well as the uniqueness of the highest order differential equation satisfied by the metric (this is a joint work with A. Buium). (Received September 21, 2021)

1174-53-10755 **Abigail Ward*** (arward@mit.edu), Massachusetts Institute of Technology. *Symplectomorphisms mirror to automorphisms of open log Calabi-Yau surfaces*

We construct a certain non-finite type four-dimensional Liouville manifold, describe symplectomorphisms of this manifold, and exhibit a mirror correspondence between these symplectomorphisms and birational maps from $(\mathbb{C}^*)^2$ to itself which preserve the standard complex volume form. From this we recover a correspondence between the automorphism group of any open Calabi-Yau surface U and the symplectomorphism group of its mirror space M. The correspondence respects the action of each group on its respective side of the isomorphism between the derived category of U and the wrapped Fukaya category of M. This is joint work in progress with Ailsa Keating. (Received September 21, 2021)

1174-53-10812 **Jonathan Epstein*** (jepstein@mcdaniel.edu), McDaniel College. *Symmetry Groups of Solvmanifolds* Preliminary report.

Although it is generally difficult to determine the full isometry group of a solvmanifold S, partial knowledge of its symmetries can yield useful information. For example, the existence of a maximally symmetric metric is related to the existence of extensions of the Lie algebra $\mathfrak{s}$ of S which admit a nontrivial Levi decomposition. Motivated by this, we define pre-Levi decompositions $\mathfrak{s} = \mathfrak{s}_1 \ltimes \mathfrak{s}_2$ which yield such extensions. We discuss some of the representation theoretic aspects involved in determining the structure of pre-Levi decompositions when the step-size of the nilradical of $\mathfrak{s}$ is bounded. (Received September 21, 2021)

1174-53-10814 **John Ross*** (rossjo@southwestern.edu), Southwestern University. *Remarks on immersed, symmetric lambda-hypersurfaces* Preliminary report.

Lambda-hypersurfaces are generalizations of self-shrinking solitons that are critical to the study of mean curvature flow. While self-shrinkers serve as critical points for the (Gaussian weighted) area functional under all variations, lambda-hypersurfaces serve as critical points for the weighted area functional only with respect to variations that preserve the (Gaussian weighted) volume. As a result, lambda-hypersurfaces behave more similarly to constant-mean-curvature surfaces while self-shrinkers are akin to minimal surfaces. In this report, we examine the existence of immersed, rotationally symmetric lambda-hypersurfaces. (Received September 21, 2021)

1174-53-10998 **Aleksander Doan** (aleksander.doan@columbia.edu), Columbia University and Trinity College, Cambridge, and **Semen Rezchikov*** (semonr@gmail.com), Harvard University. *Holomorphic Fukaya Categories via Fueter Maps*

The 3-dimensional Fueter equation is studied from a Fukaya-categorical perspective, as arising from a complexified symplectic action functional on the space of paths between holomorphic Lagrangians (L_0, L_1) in a hyper-Kähler manifold X. We describe a proposal to define an A_∞-category $Fuk_{\mathbb{C}}(L_0, L_1)$ where the Fueter maps define A_∞-operations. We discuss the Fredholm theory and differential geometry needed to allow for perturbation of otherwise rigid holomorphic Lagrangians to order to achieve transversality. We establish a complexification of Floer's theorem, namely that Fueter maps between nearby holomorphic Lagrangians in are in bijection with pseudoholomorphic maps. We analyze the special role of convexity of critical values of the complexified symplectic action functional and discuss the relation, relying on the complexification of Floer's theorem, to Fukaya-Seidel categories, as well as other categorical structures expected to arise from the Fueter equation.
(Received September 21, 2021)

1174-53-11022 **Daniel Pomerleano*** (daniel.pomerleano@umb.edu), University of Massachusetts, Boston. *The Frobenius property and HMS for log CY varieties*

I will discuss a symplectic version of the "Frobenius structure conjecture," recently proven by Keel and Yu. I will then explain how this result together with recent finiteness results for wrapped invariants of affine log Calabi-Yau varieties allows one to prove HMS for a wide class of examples. (Received September 21, 2021)

54 ► General topology

1174-54-6467 **Hannah Leopold-Brandt*** (leopoldbranh@merrimack.edu), Merrimack College. *Finding Cycles in the Cone of Book Representations of Complete Graphs*

A book representation is a way to embed graphs in three dimensional space. A book representation is divided into sheets where each edge must be contained in a sheet. Edges within a sheet must not intersect one another. We can cone a book representation by adding a new vertex and creating edges between the new vertex and all the previously existing vertices. We prove that using the knowledge of m cycles in a book representation of K_n, we can find some of the $m+1$ cycles of the same knot type in the cone of that book representation. We prove that this method will produce at least two $m+1$ cycles of the same knot type in the cone of the book representation of K_n for every m cycle in the book representation of K_n. (Received September 9, 2021)

1174-54-7411 **Yusu Wang*** (yusuwang@ucsd.edu), University of California - San Diego, **Facundo Mémoli** (facundo.memoli@gmail.com), The Ohio State University, and **Zhengchao Wan** (wan.252@buckeyemail.osu.edu), The Ohio State University. *Persistent Laplacian: Properties and algorithms* Preliminary report.

The combinatorial graph Laplacian, as an operator on functions defined on the vertex set of a graph, is a fundamental object in the analysis of and optimization on graphs. There is also an algebraic topology view of the graph Laplacian which arises through considering boundary operators and specific inner products defined on simplicial (co)chain groups. This permits extending the graph Laplacian to a more general operator, the q-th combinatorial Laplacian to a given simplicial complex. An extension of this combinatorial Laplacian to the setting of pairs (or more generally, a sequence of) simplicial complexes was recently introduced by (R.) Wang, Nguyen and Wei. In this talk, I will present serveral results (including a persistent version of the Cheeger inequality) from our recent study of the theoretical properties for the persistence Laplacian, as well as efficient algorithms to compute it. This is joint work with Facundo Memoli and Zhengchao Wan. (Received September 14, 2021)

1174-54-8138 **P. A. Loeb*** (PeterA3@AOL.com), University of Illinois at Urbana-Champaign. *Hausdorff compactifications and the Martin Compactification*

Classical Hausdorff compactifications continuously extend a family of bounded continuous functions using an embedding of the original space X into a product space. The boundary is a quotient of the Stone-Cech boundary. A quite different, more intuitive construction employed by Insall, Marciniak and the speaker embeds X in a very natural way into a quotient of its nonstandard extension *X. The boundary is a quotient of the set of remote points. These remote points are the points not infinitely close in terms of the metric or topology to any standard point of X. The boundary points of the compactification are the equivalence classes determined by an equivalence relation on the remote points. An important compactification in probability and potential theory is due to Robert Martin. The talk proposes a probabilistic equivalence relation on the remote points of *X that "looks inside" the domain. It produces the Martin compactification for important examples. (The speaker is indebted to Renming Song for helpful conversations.) (Received September 17, 2021)

1174-54-8249 **Barbara A. Shipman*** (bshipman@uta.edu), University of Texas at Arlington, and **Elizabeth R. Stephenson** (elizabeth.stephenson@ist.ac.at), IST Austria. *From donuts and coffee cups to owls and more!*

Starting with the simple definition of a topology on a set of points and what it means for a sequence to have a limit, we quickly come to new questions that are simple to state but whose outcomes are elusive and strangely unintuitive. We look at digressive convergence through the eyes of the winking owl topology. We explore the utility of minimal neighborhoods through stalagmite and stalactite topologies, and we arrive at scarce limit topologies, where our most basic question is our challenge to you! Students and faculty alike will find the questions intriguing and some of them perplexing. (Received September 18, 2021)

1174-54-8872 **Matt Insall*** (insall@mst.edu), Missouri University of Science and Technology. *The Fixed Point Property for Some Planar Continua - Update* Preliminary report.

We provide new arguments for some fixed point theorems related to the Plane Fixed Point Problem, using nonstandard methods applied to continua in the real plane. (Received September 20, 2021)

1174-54-8898 **Austin Benson** (arb@cs.cornell.edu), Cornell University, and **Ilya Amburg*** (ia244@cornell.edu), Cornell University. *Localization in harmonic vectors of simplicial complexes* Preliminary report.

Simplicial complexes allow for modeling higher-order interactions beyond the standard pairwise graph paradigm, and naturally lend themselves to analysis of edge flows through the spectra of higher-order Laplacians. The study of these and related operators has led to a host of developments in signal processing and nullspace-based spectral clustering. These techniques rely on analysis of harmonic spaces associated with the Laplacians. However, theoretical foundations and principled tools to justify and unify the various methods that have been introduced to study harmonic flows are sorely missing. In particular, at the heart of many edge signal analysis algorithms lie implicit assumptions regarding localization of harmonic vectors around "holes" in the complex. Here, we begin the principled study of localization properties of harmonic vectors. We provide theoretical insights for why localization of harmonics is expected given some assumptions about the underlying simplicial complex, and demonstrate that it occurs in empirical networks from a wide range of disciplines. We leverage these results to construct a hole localization algorithm that performs many orders of magnitude faster than state of the art algorithms while yielding solutions with more desirable properties. The resulting localized harmonics also provide rich features, which we demonstrate increase performance in various machine learning tasks. (Received September 20, 2021)

1174-54-9189 **Hannah Schwartz*** (hs25@princeton.edu), Princeton University. *Applications and Extensions of the Light Bulb Theorem*

The "light bulb theorem" in dimension four, due to Gabai in 2017 and later extended by Schneiderman and Teichner, gives suitable conditions under which homotopic spheres S and T are smoothly isotopic in a 4-manifold. Required in particular is the presence of a *dual* in common to S and T, i.e. a smoothly embedded sphere with trivial normal bundle that intersects both S and T transversally in a single point. The presence of such a dual is incredibly helpful for many geometric constructions, which we will outline during the talk. We will also discuss various extensions and applications of the light bulb theorem in more recent literature. (Received September 20, 2021)

1174-54-9669 **Karthik Vinay Seetharaman*** (kvseetharaman2@gmail.com), MIT PRIMES/Massachusetts Academy of Math and Science at WPI, **William Yue** (williamy330@gmail.com), MIT PRIMES/Phillips Academy Andover, and **Isaac Zhu** (isaaczhu@gmail.com), MIT PRIMES/North Carolina School of Science and Mathematics. *Patterns in the Lattice Homology of Seifert Homology Spheres*

Seifert fibered integral homology spheres are an important class of 3-dimensional manifolds. There are several numerological invariants and properties of these manifolds that are studied, such as the d-invariant and the lattice homology, which have been of intense interest in the last few decades, as well as the maximal monotone subroot, which is a homology cobordism invariant recently introduced by Dai and Manolescu in 2017. In this paper, we prove that the d-invariants of the Seifert homology spheres $\Sigma(a_1, a_2, \ldots, a_n)$ and $\Sigma(a_1, a_2, \ldots, a_{n-1}, a_n + a_1a_2\cdots a_{n-1})$ are equal using a novel method which analyzes their τ-sequences and the behavior of the numerical semigroup minimally generated by $a_1a_2\cdots a_n/a_i$ for $i \in [1, n]$. Then, we prove the new result that the maximal monotone subroots of the lattice homologies of $\Sigma(a_1, a_2, \ldots, a_n)$ and $\Sigma(a_1, a_2, \ldots, a_{n-1}, a_n + 2a_1a_2\cdots a_{n-1})$ are equal. However, we note that the maximal monotone subroots of the lattice homologies of $\Sigma(a_1, a_2, \ldots, a_n)$ and $\Sigma(a_1, a_2, \ldots, a_{n-1}, a_n + a_1a_2\cdots a_{n-1})$ are in general *not* always equal. (Received September 20, 2021)

1174-54-9778 **Anna Schenfisch*** (annaschenfisch@montana.edu), Montana State University, **Brittany Terese Fasy** (brittany.fasy@gmail.com), Montana State University, and **David Millman** (david.millman@montana.edu), Montana State University. *Posets of Topological Descriptors* Preliminary report.

Given a general simplicial complex embedded in $\mathbb{R}^d$, there exist finite sets of topological descriptors (persistence diagrams, etc.) that fully represent the complex. By comparing the cardinalities of such sets for different topological descriptor types, we build a framework through which descriptor types can be ordered. We discuss a case study of ordering six common descriptor types and then describe the construction of a simplicial complex that requires surprisingly many augmented descriptors to form a representative set. (Received September 20, 2021)

1174-54-10300 **Jack Burkart*** (burkart2@wisc.edu), University of Wisconsin-Madison. *A Jordan Curve that cannot be Crossed by a Rectifiable Arc on a Set of Zero Length*

In this talk, we will illustrate how to construct a Jordan curve $\Gamma \subset \mathbb{C}$ with the following pathological property: suppose that $\sigma : [0, 1] \to \mathbb{C}$ is a rectifiable arc with $\sigma(0) \in \Gamma_{int}$ and $\sigma(1) \in \Gamma_{ext}$ where Γ_{int} and Γ_{ext} are the

bounded and unbounded complementary components of Γ. Then $H^1(\sigma \cap \Gamma) > 0$, where H^1 is the Hausdorff 1-measure. The existence of such a curve answers a question of Sauter. We will also briefly discuss the connection of this example to conformal welding and questions of Pugh and Wu regarding ordinary differential equations. (Received September 21, 2021)

1174-54-10576 **Lu Li*** (lli1@macalester.edu), Macalester College. *Minimal Cycle Representatives in Persistent Homology using Linear Programming: an Empirical Study with User's Guide*

Cycle representatives of persistent homology classes can be used to provide descriptions of topological features in data. However, the non-uniqueness of these representatives creates ambiguity and can lead to different interpretations of the same set of classes. One approach to solving this problem is to optimize the choice of representative against some measure. In this work, we provide a study of the effectiveness and computational cost of several ℓ_1-minimization optimization procedures for constructing homological cycle bases for persistent homology in dimension one, including uniform-weighted and length-weighted edge-loss algorithms as well as uniform-weighted and area-weighted triangle-loss algorithms.

Our key findings are: (i) optimization is effective in reducing the size of cycle representatives, though the extent of the reduction varies according to the dimension and distribution of the underlying data, (ii) the computational cost of optimizing a basis of cycle representatives exceeds the cost of computing such a basis, in most data sets we consider, (iii) strikingly, whether requiring integer solutions or not, we almost always obtain a solution with the same cost and almost all solutions found have entries in $\{-1, 0, 1\}$ and therefore, are also solutions to a restricted ℓ_0 optimization problem, and (iv) we obtain qualitatively different results for generators in Erdős-Rényi random clique complexes than in real-world and synthetic point cloud data. (Received September 21, 2021)

1174-54-11021 **Matthew Kvalheim** (kvalheim@seas.upenn.edu), University of Pennsylvania, and **Samuel A. Burden** (sburden@uw.edu), University of Washington. *A pasting lemma for Lipschitz functions*

We give a necessary and sufficient condition ensuring that any function which is separately Lipschitz on two fixed compact sets is Lipschitz on their union. We also show that this condition is local. As an example application we show that this condition is satisfied by a transverse pair of closed Riemannian submanifolds. (Received September 21, 2021)

1174-54-11207 **Julien Chaput*** (jachaput@utep.edu), UTEP/PNNL. *Word2Sphere: Toward geometrically interpretable language models*

The rapid architectural advances in natural language processing combined with exponential increases of computational resources and corresponding data sets have resulted in language models (LM) increasingly capable of both broad and specific task competence. The unconstrained nature of word vectors in N-dimensional space results in LMs that are difficult to interpret in terms of structure, and even more so in terms of dimensional reduction and visualization, given that complex manifold structures must be estimated without uniform sampling of the space. Here, we present a novel framework utilizing a skip-gram architecture that seeks to force the LM onto an N-dimensional sphere via regularization, thus relating geodesic distances along a known spherical manifold to simple computations of cosine similarity. We show that our model outperforms an unregularized version of the skip-gram model trained on the same data in terms of semantic similarity and other metrics. (Received September 21, 2021)

55 ► Algebraic topology

1174-55-5400 **Steve Huntsman*** (sch213@nyu.edu), independent. *Some applications of path homology*

The notion of *path homology* introduced by Grigor'yan, Lin, Muranov, and Yau (strictly) generalizes simplicial homology to digraphs. It is therefore well suited for topologically analyzing data that are more naturally represented by digraphs than by simplicial complexes, which are the usual arena for topological data analysis. We review the basics of path homology and give an exhaustive description of path homologies for small digraphs and a family of DAGs to build some intuition for the theory. Finally, using a MATLAB implementation, we briefly outline analyses of transportation backbones, control flow graphs of computer programs, neural network architectures, and temporal networks. (Received August 19, 2021)

1174-55-5640 **Wako Bungula*** (wbungula@uwlax.edu), UW-La Crosse. *Quantifying Ecological States of the Upper Mississippi River System using Topological Data Analysis* Preliminary report.

Topological Data Analysis (TDA) gives understanding of the "shape" of the underlying space of a given data. Data analysis from Topological perspective is beneficial in visualizing the data, and this is useful specifically when the data is high dimensional and complex. The Upper Mississippi River Restoration Program has been collecting data from the Upper Mississippi River (UMR) for over 25 years, and these data include water quality, vegetation, fish, time-series, and more. In my talk, I will present how we applied TDA to quantify ecological states in the UMR. Furthermore, I will discuss the driver variables of the ecological states. (Received August 25, 2021)

1174-55-6162 **Jiahui Chen*** (chenj159@msu.edu), Michigan State University. *Evolutionary de Rham-Hodge method*

The de Rham-Hodge theory is a landmark of the 20th Century's mathematics and has had a great impact on mathematics, physics, computer science, and engineering. This work introduces an evolutionary de Rham-Hodge method to provide a unified paradigm for the multiscale geometric and topological analysis of evolving manifolds constructed from a filtration, which induces a family of evolutionary de Rham complexes. While the present method can be easily applied to close manifolds, the emphasis is given to more challenging compact manifolds with 2-manifold boundaries, which require appropriate analysis and treatment of boundary conditions on differential forms to maintain proper topological properties. Three sets of unique evolutionary Hodge Laplacians are proposed to generate three sets of topology-preserving singular spectra, for which the multiplicities of zero eigenvalues correspond to exactly the persistent β numbers of dimensions 0, 1, and 2. Additionally, three sets of non-zero eigenvalues further reveal both topological persistence and geometric progression during the manifold evolution. Extensive numerical experiments are carried out via the discrete exterior calculus to demonstrate the potential of the proposed paradigm for data representation and shape analysis of both point cloud data and density maps. To demonstrate the utility of the proposed method, application is considered to the protein B-factor predictions of a few challenging cases for which biophysical models break down. (Received September 5, 2021)

1174-55-6515 **Taekgeun Jung*** (xorrms78@korea.ac.kr), Korea University, School of Industrial and Management Engineering, and **Hong Seo Ryoo** (hsryoo@korea.ac.kr), Korea University, School of Industrial and Management Engineering. *Property of n-Simplex Covered-ness and n-Spherical Complex for Topological Data Clustering via Persistent Homology* Preliminary report.

Topological data analysis (henceforth, TDA) studies data via persistent homology by computing β_n, the number of n-dimensional rooms/holes in their simplicial complex representations. Note that each n-hole ($n \in Z_+$) in a simplicial complex must be enclosed in a blanket of n-simplices; hence, there exists a minimal set of n-simplices that surrounds each n-hole. Calling this 'the property of n-simplex covered-ness,' this paper first presents a method for extracting an 'n-spherical complex,' a sub-simplicial complex with the aforementioned property. Next, we establish necessary and sufficient conditions for the formation of an n-hole in a Rips complex and geometrically show that $\beta_n \simeq 0$ for $n \geq 3$. This provides a firm, theoretical ground that the computation of β_n, $n \leq 2$, suffices in the practice and applications of TDA. When applied to fMRI data of brain activity of 18 participants performing Video, Memory and Math tasks, our results clearly identified those brain regions (ROIs) that are commonly and also distinctly activated for performing different tasks. This illustrates practical utilities of new results in this paper for TDA, both in supervised as well as unsupervised learning setups. (Received September 19, 2021)

1174-55-7004 **Inbar Klang*** (inbarklang@gmail.com), Columbia University, and **Sarah Yeakel** (sarah.yeakel@ucr.edu), University of California Riverside. *Isovariant fixed point theory*

If X and Y are spaces with an action of a group G, an isovariant map between them is an equivariant map that preserves isotropy groups. In this talk, I will discuss joint work with Sarah Yeakel, in which we study the homotopy theory of isovariant maps, and use this to provide complete obstructions to eliminating fixed points of isovariant maps between manifolds. (Received September 11, 2021)

1174-55-7150 **Laura Scull*** (scull_l@fortlewis.edu), Fort Lewis College. *WIT Project Update on Orbifold Mapping Spaces*

Orbifolds, and more generally orbispaces, are spaces which have well-behaved singularities. They can be defined using atlases analogously to manifolds, with charts having a finite group action defining the local singularity structure. More recently, orbispaces have been modeled using topological groupoids. It is shown by Moerdijk-Pronk that orbispaces can be represented by topological groupoids with etale structure maps and proper diagonal. This representation is not unique, however, as two Morita equivalent groupoids represent the same orbispace. Thus to represent orbispaces, we use a bicategory of fractions where the Morita equivalences have been inverted. This gives a definition of a map between orbispaces $G \to H$ defined as a span of maps between groupoids $G \leftarrow K \to H$ where K is Morita equivalent to G, giving an alternate representation of the domain orbispace.

I will discuss how to create a topological mapping groupoid for orbispaces, $OMap(G, H)$, which encodes these spans and satisfies the properties of a mapping object. I will show how to define an etale proper groupoid $OMap(G, H)$ which is the exponential object for orbigroupoids and gives orbispaces the structure of an enriched bicategory, so that composition induces a continuous functor $OMap(G, H) \times OMap(H, K) \to OMap(G, K)$.

This is joint work with D. Pronk at Dalhousie University. (Received September 13, 2021)

1174-55-7661 **Aurora Clark** (auclark@wsu.edu), Washington State University, **Brittany Story*** (brittany.carr@colostate.edu), Colorado State University, and **Biswajit Sadhu** (biswajit.sadhu@wsu.edu), Washington State University. *Sublevelset persistent homology and energy landscapes*

The energy of a molecule is determined by a variety of parameters, such as bond length, bond angle, and bond type. Chemists capture changes in these parameters with something called an energy landscape. But, the energy landscape can grow to be quite high-dimensional due to the large number of degrees of freedom in a chemical system. Sublevelset persistent homology is a tool that can capture relevant features of an energy landscape, including all transition paths (whereas other tools, such as merge trees, detect at most one transition path between minima). In this talk, we derive a complete characterization of the sublevelset persistent homology of the energy function on any branched alkane: a tree-like molecule consisting of only carbons and hydrogens. More generally, we explain how the sublevelset persistent homology of an additive energy landscape can be computed from the individual terms comprising that landscape. (Received September 15, 2021)

1174-55-7668 **William Y. Zhang*** (william_zhang1@brown.edu), Brown University, **Dhananjay Bhaskar** (dbhaskar92@gmail.com), Yale University, and **Ian Y. Wong** (ian_wong@brown.edu), Brown University. *Topological Analysis of Self-Organized Patterns in Heterogeneous Interacting Cell Populations* Preliminary report.

Heterogeneous cell populations exhibit coordinated motion, self-organization, and phase transitions during embryo formation, skin pigmentation, wound healing, and cancer metastasis. Interestingly, such complex behavior can be simulated with a minimal agent-based model (ABM) consisting of random polarization, differential adhesion, and cell division. Parameter sweeps of the ABM generate patterns of cell sorting, engulfment, and self-assembly into radially symmetric, spotted, and stripe patterns. Using simulation data, we demonstrate that a combination of topological data analysis (TDA) and machine learning can automatically identify distinct cell arrangements and delineate phase boundaries, thus uncovering the relationship between geometry at the tissue scale and cell-cell interactions at the local level. We test the robustness of our approach by performing ablation studies, finite-size scaling, and systematic perturbations to pattern size, frequency, population ratios, etc. Our technical contributions include distributed computation of persistent homology and overcoming the challenges associated with comparing non-constant population sizes. Additionally, our unsupervised and model-agnostic approach can be used to investigate a variety of phenomena in active and condensed matter. (Received September 21, 2021)

1174-55-7821 **Michael Hull*** (mbhull@uncg.edu), University of North Carolina at Greensboro. *Persitent homology detects curvature*

Given a collection of points randomly sampled from an unkown space X, persistent homology has become a standard tool for detecting aspects of the topology of X. We show that persistent homology, in particular the "short bars" in the bar code, can also detect aspects of the geometry of X. Specifically, we show the curvature of the space can be recovered by encoding the persistent homology as a persistence landscape and applying standard tools from statistical and machine learning. This is joint work with P. Bubenik, D. Patel, and B. Whittle. (Received September 16, 2021)

1174-55-8050 **Francis Motta** (fmotta@fau.edu), Florida Atlantic University, and **Amish Mishra*** (mishra.amish@gmail.com), Florida Atlantic University. *Computation of Persistent Homology Using the Delaunay-Rips Complex: An efficient family of simplicial complexes for topological data analysis* Preliminary report.

Topological Data Analysis is an emerging area rooted in theories from Algebraic Topology, which enables researchers to extract discriminating geometric and topological features from data. We give an overview of some of the popular methods of extracting features from point-cloud data, which first requires one to construct a 1-parameter family of spaces on the data using the geometry of the cloud. We demonstrate their benefits and shortcomings and introduce a new, more efficient construction that we name the Delaunay-Rips Complex. We justify conditions on the data that gurantee stability of our method when computing persistent homology. Aided by intuitive examples, we also provide an empirical run-time comparison of the two existing methods with our new algorithm on the computation of the persistence diagrams of some synthetic data sets. (Received September 17, 2021)

1174-55-8234 **Zifan Wang*** (zifanawang04@gmail.com), Princeton International School of Mathematics and Science, and **Arun S Kannan** (akannan@mit.edu), Massachusetts Institute of Technology. *Representation Stability of Orthogonal Groups* Preliminary report.

We proved stability results about orthogonal groups over finite commutative rings where 2 is a unit. Analogous to past studies of FI-, SI-, and VIC-modules, we constructed a category of OrI-modules and proved that local Noetherianity holds for this category. Via an argument similar to Sam–Snowden's, this implies a representation stability result for orthogonal groups. We also showed homological stability results with twisted coefficients for these orthogonal groups. (Received September 21, 2021)

1174-55-8245 **Jasna Urbancic*** (j.urbancic@qmul.ac.uk), Queen Mary University of London. *Optimizing Embedding using Persistence* Preliminary report.

We look to optimize Takens-type embeddings of a time series using persistent (co)homology. The motivation is to assume that the input time series exhibits periodic, quasi-periodic, or recurrent behavior, and then use continuous optimization on the persistence diagram to find good embeddings. We construct the embedding by applying covolution with a higher dimensional kernel to the time series and then use gradient descent on a loss function defined on the persistence diagram of the embedding. We provide a practical approach to finding good embeddings and investigate the space of embeddings with indications as to possible theoretical questions and directions that arise. This is joint work with Primož Škraba. (Received September 18, 2021)

1174-55-9201 **Cliff Joslyn** (cliff.joslyn@pnnl.gov), Pacific Northwest National Laboratory, **Bei Wang** (wang.bei@gmail.com), University of Utah, **Brenda Praggastis** (brenda.praggastis@pnnl.gov), Pacific Northwest National Laboratory, **Brett Jefferson** (brett.jefferson@pnnl.gov), Pacific Northwest National Laboratory, **Emilie Purvine*** (emilie.purvine@pnnl.gov), Pacific Northwest National Laboratory, **Madelyn Shapiro** (madelyn.shapiro@pnnl.gov), Pacific Northwest National Laboratory, and **Youjia Zhou** (zhou325@sci.utah.edu), University of Utah. *Topology of machine learning activations* Preliminary report.

Neural networks (NNs), intricate networks of thousands of artificial neurons and synaptic connections, are popular tools to tackle classification tasks involving large amounts of data. But it can be difficult to grasp how groups of neurons influence final classifications. As NN models continue to be developed and improved for a wide variety of tasks there is increasing recognition that mathematical methods are needed to understand and interpret how these models, once trained, internally represent data features for classification. The techniques of topological data analysis (TDA) are particularly effective at representing the complexities of data in hidden layers because of their ability to compactly summarize complex, high-dimensional data while preserving relevant global structures. Recent work has used TDA to explore NN performance, improve the training process, and study the topological structure of the trained weights of an NN. Our work explores the topology (mapper and persistent homology) of the layer-wise activations of fully trained NNs generated by the input data with the eventual goal of understanding if and how NN models use the topology present in data for classification. This talk will survey the use of TDA to explore NN models and show some of our most recent work. (Received September 20, 2021)

1174-55-9227 **Gregory Dungan*** (Ivan.dungan@fmarion.edu), Francis Marion University. *Using n-Topodesics to Enhance Homological Information* Preliminary report.

A lot of intensive research has been in topological methods like homology to better characterize data. Although it has been successful, using homotopy instead could provide better results if it wasn't for its computational limitations. There is a lot of current work in this area in hope that the limitations will be manageable in the future. We take a different approach and try to leverage more information from homology. In particular, we will introduce an invariant called the n-topodesics number which further extends the information of Betti numbers. This discussion will be accompanied with some prototypical examples to provide insight. (Received September 20, 2021)

1174-55-9254 **Hans Matthew Riess*** (hmr@seas.upenn.edu), University of Pennsylvania. *Cellular Sheaves of Lattices and the Tarski Laplacian*

Our paper initiates a discrete Hodge theory for cellular sheaves taking data in the category of complete lattices and Galois connections. The key development is the Tarski Laplacian, an endomorphism on the cochain complex whose fixed points agree with global sections in degree zero. After laying the foundation for the basic theory, we, then, explore ongoing work (with Paige Randall-North and Miguel Lopez) on distributed concept analysis in both a crisp and fuzzy setting with potential applications to opinion dynamics and data mining. (Received September 20, 2021)

1174-55-9293 **Foling Zou*** (folingz@umich.edu), University of Michigan, **Jia Kong** (hana.jia.kong@gmail.com), IAS, and **Peter May** (may@math.uchicago.edu), University of Chicago. *Fixed set systems of G-operads and monads* Preliminary report.

The operadic infinite loop space machine is a way to produce spectra. Equivariantly, G-spaces that are algebras over genuine G-E_∞-operads are input of this machine. Costnoble-Waner constructed a monad on the fixed set presheaves of G-spaces to extend the machine to presheaves. We present a reinterpretation and simplification of their machine. As an application, the machine produces units of equivariant ring spectra and Eilenberg-MacLane spectra. (Received September 20, 2021)

1174-55-9326 **Daniel C Isaksen** (isaksen@wayne.edu), Wayne State University, **Eva Belmont** (ebelmont@ucsd.edu), University of California San Diego, and **Jia Kong*** (hana.jia.kong@gmail.com), Institute for Advanced Study. *The homotopy of $\mathbb{R}$-motivic image-of-j spectrum*

Bachmann–Hopkins defines the motivic "image of orthogonal j" spectrum over base fields with characteristic not 2. In this talk, I will talk about the effective slice computation of this spectrum over the real numbers. Analogous to the classical story, the result captures a regular pattern that appears in the $\mathbb{R}$-motivic stable stems. This is joint work with Eva Belmont and Dan Isaksen. (Received September 20, 2021)

1174-55-9364 **Maru Sarazola*** (mesarazola@jhu.edu), Johns Hopkins University, **Lyne Moser** (moser@mpim-bonn.mpg.de), Max Planck Institute for Mathematics, **Viktoriya Ozornova** (viktoriya.ozornova@mpim-bonn.mpg.de), Max Planck Institute for Mathematics, **Simona Paoli** (simona.paoli@abdn.ac.uk), University of Aberdeen, and **Paula Verdugo** (paula.verdugo@hdr.mq.edu.au), Macquarie University. *The stable homotopy hypothesis*

The homotopy hypothesis is a well-known bridge between topology and category theory. Its most general formulation, due to Grothendieck, asserts that topological spaces should be "the same" as infinity-groupoids. In the stable version of the homotopy hypothesis, topological spaces are replaced with spectra.

In this talk we will review the classical homotopy hypothesis, and then focus on the stable version. After discussing what the stable homotopy hypothesis should look like on the categorical side, we will sketch how the Tamsamani model of higher categories provides a proof. This is based on joint work with Moser, Ozornova, Paoli and Verdugo. (Received September 20, 2021)

1174-55-9390 **Anna Marie Bohmann** (am.bohmann@vanderbilt.edu), Vanderbilt University, **Jocelyne Ishak*** (jocelyne.ishak@vanderbilt.edu), Vanderbilt University, **Christy Hazel** (chazel@math.ucla.edu), UCLA, **Magdalena Kędziorek** (m.kedziorek@math.ru.nl), Radboud University, and **Clover May** (clover.may@ntnu.no), Norwegian University of Science and Technology. *Uniqueness of commutative structures on rational equivariant K-theory*

Modeling rational spectra via algebraic data has a long and fruitful history in homotopy theory. More precisely, rational spectra are equivalent to rational chain complexes, and this algebraic data is called an algebraic model for rational spectra. Our goal is to understand rational equivariant K-theory as a naive & genuine commutative ring spectrum when G is a finite group. We do this by calculating its image in the algebraic model for naive-commutative ring G-spectra given by Barnes, Greenlees and Kędziorek. As for the genuine-commutativity structure, we use the algebraic model constructed by Wimmer. Our calculations show that these spectra are unique as naive & genuine-commutative ring spectra in the sense that they are determined up to weak equivalence by their homotopy groups.

This work is joint with Anna Marie Bohmann, Christy Hazel, Magdalena Kędziorek, and Clover May. It was the result of the WIT 2019 workshop. (Received September 20, 2021)

1174-55-9554 **Maximilien Péroux*** (mperoux@sas.upenn.edu), University of Pennsylvania. *Fibrantly generated model categories*

Cofibrantly generated model categories provide a generalization of the CW-approximation for topological spaces: one can build a cofibrant replacement built out of pushouts and transfinite colimits of certain generating cells. Similarly, one can introduce fibrantly generated model categories by generalizing Postnikov towers. We obtain an inductive fibrant replacement built out of pullbacks and towers of certain layers. Unfortunately, in context of interests, we lack of cosmall object arguments, and one has to build an ad-hoc factorization. In such context, we say that we provide a Postnikov presentation for the model category.

The technique was introduced by Kathryn Hess in order to induce model structures from certain left adjoints, typically the forgetful functor from coalgebras/comodules to their underlying model categories. Later work of Bayeh, Hess, Karpova, Kedziorek, Riehl, Shipley provided certain conditions that ensured left-induced model categories via the Postnikov presentations. However, the techniques turned out to be complicated. It was later fixed by work of Bayeh, Hess, Kedziorek, Riehl, Shipley in a second paper where Postnikov presentations were not used.

I introduce another application of Postnikov presentations. I will show how they can provide inductive arguments for the rigidification of higher homotopy coalgebraic objects and provide a certain right derived symmetric monoidal product on comodules. The techniques still provide powerful tools in ∞-categories. (Received September 20, 2021)

1174-55-9623 **Shelley Kandola*** (kandola@umn.edu), University of Minnesota. *Motion planning algorithms on categorical subsets of finite topological spaces*

The unreduced topological complexity (TC) of a space is the minimal number of continuous motion planning rules required to move from one point in the space to another. Typically, the space will represent the space of configurations of some mechanical system, such as a robot. In this way, TC is useful for determining algorithms for robot motion planning problems. Restricting the input to finite topological spaces might allow for more accurate models of real-life mechanical systems. There is no known algorithm for computing the TC of a space, however, there are upper bounds that can be estimated and in some cases calculated using algorithms. Two such upper bounds are called the Lusternik-Schnirelmann category and the geometric category, respectively. In this paper, we present a heuristic for computing the Lusternik-Schnirelmann category of a finite space, an algorithm for computing the geometric category of a finite space, and an algorithm for determining an explicit motion planning rule defined on each of these categorical sets. (Received September 20, 2021)

1174-55-9652 **Prashant Gupta** (pragpt100@gmail.com), Natural Intelligence. *Box Filtration* Preliminary report.

We propose a new filtration for the topological data analysis of points in high-dimensional Euclidean space where the convex sets are hyperrectangles or boxes. Rather than center them at each point, we grow the boxes non-uniformly in each dimension or direction based on the distribution of points so as to better capture the topology of the point set. We present efficient algorithms to construct the box filtration. We also prove stability results for the boxes constructed in the filtration under small changes of input parameters. We compare topological summaries of standard data sets in the form of persistence diagrams produced by box filtration to those produced by the standard distance-to-measure (DTM) filtration. (Received September 20, 2021)

1174-55-9892 **Nithin Chalapathi** (nithin.ch10@gmail.com), University of California, Berkeley. *Adaptive Covers for Mapper Graphs Using Information Criteria* Preliminary report.

The mapper construction is a widely used tool from topological data analysis that has enjoyed great success in data science, including cancer research, sports analytics, and visualization. However, developing practical and automatic parameter selection for the mapper construction remains a challenging open problem for both topological analysis and visualization communities. In this paper, we focus on parameter selection for the 1-dimensional skeleton of the mapper construction, called the mapper graph. Specifically, we explore how information criteria can inform and generate adaptive covers for mapper graphs. Our approach makes novel progress towards automatic parameter selection for the mapper construction using information theory. (Received September 21, 2021)

1174-55-9918 **Rachel Neville*** (Rachel.Neville@nau.edu), Northern Arizona University, **Patrick Shipman** (shipman@math.colostate.edu), Colorado State University, and **Steven Fassnacht** (steven.fassnacht@colostate.edu), Colorado State University. *Topological and Geometric Methods in the Study of the Snow Surface Roughness* Preliminary report.

Roughness of the snowpack surface exhibits a high degree of spatiotemporal variance. It proves difficult to estimate numerically from real data. Accurate estimation of this parameter is of fundamental importance for estimating turbulent fluxes and is an input into all existing numerical models of surface-atmosphere interactions. We discuss challenges and describe geometric and topological techniques to estimate roughness from airborne LIDAR measurements and compare them to current techniques. (Received September 21, 2021)

1174-55-10068 **Radmila Sazdanovic*** (rsazdan@ncsu.edu), NC State University, **Javier Arsuaga** (jarsuaga@ucdavis.edu), UC Davis, University of California,Davis, **Jai Aslam** (jkaslam@ncsu.edu), NC State, and **Sergio Ardanza-Trevijano** (sardanza@unav.es), Universidad de Navarra. *Topology applied to cancer genomics* Preliminary report.

Cancer is a polygenic disease in which genomic events are selected in order to produce a sophisticated and coordinated outcome. Determining when two events are co-occurring is an important open question in data science. This work focuses on addressing that question in breast cancer genomics by developing topology-based methods for analyzing how Copy Number Aberrations (CNAs) relate to the breast cancer types and prognosis. (Received September 21, 2021)

1174-55-10408 **Samantha C. Moore*** (scasya@live.unc.edu), University of North Carolina at Chapel Hill. *Hyperplane Restrictions of Indecomposable n-Parameter Persistence Modules*

Indecomposable n-parameter persistence modules are the basic building blocks for n-parameter persistence modules (which are tools used in Topological Data Analysis). Understanding the structure of indecomposable n-parameter persistence modules is thus foundational for studying persistent homology, but Carlsson and Zomorodian proved that this is a measurably difficult problem. We extend a result of Buchet and Escolar to prove the following: If M is any finitely presented $(n-1)$-parameter persistence module with finite support, then there exists an indecomposable n-parameter persistence module M' such that M is the restriction of M' to a hyperplane. The result is a deeper understanding of how complicated the structure of n-parameter persistence modules can be. We also show that any finite zigzag persistence module is the restriction of some indecomposable 3-parameter persistence module to a path. (Received September 21, 2021)

1174-55-10419 **Adam Brown*** (abrown@ist.ac.at), Institute of Science and Technology Austria, and **Ondrej Draganov** (ondrej.draganov@ist.ac.at), Institute of Science and Technology Austria. *Injective Resolutions of Cellular Sheaves* Preliminary report.

Sheaf theory models relationships between local structures and global properties of a topological space. To apply this theory to data, it is necessary to describe theoretical concepts in a computationally tractable form. In this talk we will discuss methods for computing injective resolutions of sheaves on simplicial complexes. Injective resolutions, a fundamental ingredient of homological algebra, are used to study sheaves from the 'derived' perspective, i.e. as objects in a derived category. Explicitly (and efficiently) constructed injective resolutions allow for new applications of homological algebra in computational topology. From injective resolutions, we can compute many cohomology theories, such as simplicial cohomology, local cohomology, persistent (co)homology, etc. We will discuss the existence and uniqueness of 'minimal' injective resolutions and outline an algorithm for their construction. We will illustrate these calculations through examples and show applications to level-set persistence. (Received September 21, 2021)

1174-55-10517 **Iva Halacheva*** (i.halacheva@northeastern.edu), Northeastern University, **Marcy Danielle Robertson** (marcy.robertson@unimelb.edu.au), University of Melbourne, and **Zsuzsanna Dancso** (zsuzsanna.dancso@sydney.edu.au), University of Sydney. *Tensor categories, tangled surfaces, and Lie algebras*

Circuit algebras were first introduced by Bar-Natan and Dancso in the study of welded tangles and more generally welded foams—an extension of usual tangles capturing a family of knotted surfaces in 4-space. I will discuss recent work with Dancso and Robertson, where we show that circuit algebras are equivalent to a class of strict symmetric tensor categories with duals, thus opening the door to their exploration from a categorical perspective. We further prove that the automorphisms of welded foams are realizable as the Kashiwara-Vergne group, which emerged in Lie theory through the study of the Baker-Campbell-Hausdorff series. (Received September 21, 2021)

1174-55-11036 **Rebecca E Field*** (fieldre@jmu.edu), James Madison University. *A gentle introduction to equivariant cohomology*

What does it mean for object to have symmetries? I'll explain what a classifying space is, and equivariant cohomology, and elucidate some of the more subtle things that we can mean by 'the shape of a group'. (Received September 21, 2021)

1174-55-11108 **Sushanth Sathish Kumar*** (brokard81517@gmail.com), PRIMES USA Program, and **Adela Zhang** (adelayyz@mit.edu), MIT. *The Top Dyer-Lashof Operation on the Bar Spectral Sequence of an iterated loop space* Preliminary report.

Given a space X with a chosen basepoint $*$, one can consider the loop space ΩX. Iterating the loop space construction, we obtain the n-fold loop space $\Omega^n X$. The mod p homology of the n-fold loop space has a rich algebraic structure. For instance, $H_*(\Omega^n X;\mathbb{F}_2)$ is a Poisson Hopf algebra with a shifted Lie bracket that increases degree by $n-1$. It also supports Dyer-Lashof operations $Q_0,\ldots,Q_{n-1}$ compatible with the bracket. The top Dyer-Lashof operation Q_{n-1} gives $H_*(\Omega^n X;\mathbb{F}_2)$ a restricted Poisson Hopf algebra structure.

In the unstable case where n is finite, Ni shows in his paper that the bar spectral sequence

$$E^2_{s,t} = \mathrm{Tor}^{H_*(\Omega^n X)}_{s,t}(\mathbb{F}_2,\mathbb{F}_2) \Rightarrow H_*(\Omega^{n-1}X)$$

is a spectral sequence of Poisson Hopf algebras. He constructed a bracket on the E^1 page and showed it passed to a bracket on $H_*(\Omega^{N-1}X)$. The goal of this project is to further investigate Ni's result by taking into account the restriction on the Possion algebra $H_*(\Omega^2 X;\mathbb{F}_2)$, given by Q_1 operation. We establish an extension of the restriction $\xi : E^1_{s,t} \to E^1_{2s-1,2t+1}$ on the E^1-page $E^1_{s,t} = B_{s,t}(H_*(\Omega^2 X))$ of the bar spectral sequence. (Received September 21, 2021)

1174-55-11190 **Preston Cranford*** (prestonc@mit.edu), Massachusetts Institute of Technology, and **Peter Rowley** (rowley@mit.edu), Massachusetts Institute of Technology. *Finding Bounded Simplicial Sets with Finite Homology* Preliminary report.

The central problem in computational algebraic topology is the computation of the homotopy groups of a given space, represented as a simplicial set. Algorithms have been found which achieve this, but the running times depend on the size of the input simplicial set. In order to reduce this dependence on the simplicial set chosen, we describe in this paper a procedure which, given a prime p and a finite, simply-connected simplicial set with finite integral homology, finds a p-locally equivalent simplicial set with size upper bounded by a function of dimension and homology. Using this in conjunction with the above algorithm, the p-local homology can be calculated such that the running time dependence on the size of the initial simplicial set is contained in a separate preprocessing step. (Received September 21, 2021)

57 ► Manifolds and cell complexes

1174-57-5212 **Dan Margalit*** (margalit@math.gatech.edu), Georgia Institute of Technology. *Mixing surfaces, algebra, and geometry.*

Taffy pullers, lab stirrers, and paint mixers are complicated dynamical systems. To any such system we can ascribe a real number, called the entropy, which describes the amount of mixing being achieved. Which real numbers arise, and what do they say about the dynamics of the system? We will explore this question through the lens of topological surfaces, making unexpected connections to algebra and number theory. Our tour will take us from the work of Max Dehn and Jakob Nielsen a century ago, to the revelations of the Fields medalist William Thurston in the 1970s, to the breakthroughs of Fields medalist Maryam Mirzakhani in the 21st century. (Received November 16, 2021)

1174-57-5422 **Anthony Conway*** (anthonyyconway@gmail.com), MIT, and **Mark Powell** (mark.a.powell@durham.ac.uk), Durham University. *Knotted surfaces with infinite cyclic knot group*

This talk will concern embedded surfaces in 4-manifolds for which the fundamental group of the complement is infinite cyclic. Working in the topological category, necessary and sufficient conditions will be given for two such surfaces to be isotopic. Emphasis will be placed on surfaces with boundary. (Received August 20, 2021)

1174-57-5426 **Patrick Orson** (patrickorson@gmail.com), MPIM Bonn. *Abelian invariants of doubly slice links*

We provide obstructions to a link in S^3 arising as the cross section of any number of unlinked spheres in S^4. Our obstructions arise from the multivariable signature, the Blanchfield form and generalised Seifert matrices, all of which will be reviewed. (Received August 20, 2021)

1174-57-5443 **Hugo Zhou*** (hzhou92@gatech.edu), Georgia Institute of Technology. *Homology Concordance and an Infinite Rank Free Subgroup*

Two knots are homology concordant if they are smoothly concordant in a homology cobordism. The group $\widehat{\mathcal{C}_{\mathbb{Z}}}$ (resp. $\mathcal{C}_{\mathbb{Z}}$) was previously defined as the set of knots in homology spheres that bound homology balls (resp. in S^3), modulo homology concordance. We prove the quotient $\widehat{\mathcal{C}_{\mathbb{Z}}}/\mathcal{C}_{\mathbb{Z}}$ contains an infinite rank free subgroup. We construct our family of examples by applying the filtered mapping cone formula to L-space knots, and prove linear independence with the help of the connected knot complex. (Received August 20, 2021)

1174-57-5604 **Ian Agol*** (ianagol@math.berkeley.edu), UC Berkeley. *Foams and invariants of planar graphs* Preliminary report.

Kronheimer and Mrowka defined vector space invariants $J^{\#}$ of spatIal cubic graphs, thought of as orbifolds with 2-torsion or "bifolds", where the graph represents the singular locus (also called a "web"). These invariants have functorial properties with respect to orbifold cobordism, represented by foams (the singular locus of a 4-dimensional bifold). For planar webs, we will describe how this invariant with appropriate twisted coefficients is spanned by foams in 3 dimensions. A version of this result was proved by Khovanov-Robert, but our description seems to be a bit more direct with an explicit basis parameterized by 3-colorings of the edges of the graph in a decomposition of the invariant described by Kronheimer-Mrowka in their paper "A deformation of instanton homology for webs". (Received August 23, 2021)

1174-57-5659 **Emily Shinkle*** (emilyss2@illinois.edu), University of Illinois At Urbana-Champaign. *Finite Rigid Sets for Combinatorial Complexes Associated to Surfaces*

The arc complex and flip graph are simplicial complexes associated to a given surface, each describing combinatorial information about the surface. These complexes were used by Harer in his study of homological properties of the mapping class group, and have connections to Penner's decorated Teichmuller theory and Fomin-Shapiro-Thurston's theory of cluster algebras, among others. Irmak-McCarthy, Korkmaz-Papadopoulos, and Aramayona-Koberda-Parlier have shown correspondences between simplicial maps of these complexes and homeomorphisms of the associated surface. The complexes are infinite, but I show that these results can be extended for simplicial maps of certain finite subcomplexes, which we call finite rigid sets. (Received August 24, 2021)

1174-57-5734 **Sujoy Mukherjee** (sujoymukherjee.math@gmail.com), The Ohio State University, **Jozef Henryk Przytycki** (przytyck@gwu.edu), George Washington University, **Rhea Palak Bakshi** (rhea_palak@gwmail.gwu.edu), George Washington University, and **Dionne Ibarra*** (Dfkunkel@gwmail.gwu.edu), The George Washington University. *Gram Determinants Motivated by Knot Theory*

In the 1990's, a general formula for the Gram determinant of Type A was formulated in order to prove the existence and uniqueness of Lickorish's construction of the Witten-Reshetikhin-Turaev invariants of 3-manifolds. In the early 2000's a closed formula was formulated for the Gram Determinant of Type B. Using skein modules we present a general definition of a Gram determinant motivated by knot theory. We also introduce the Gram determinant of Type MB along with its conjectured closed formula and Type Generalized A with its closed formula. (Received August 26, 2021)

1174-57-5845 **Vincent Longo** (vincent.longo@huskers.unl.edu), College of Saint Benedict & Saint John's University, **Michael R. Klug** (michael.r.klug@gmail.com), University of California, Berkeley / Max Planck Institute for Mathematics, University of Chicago, **Sarah Blackwell*** (seblackwell@uga.edu), University of Georgia, **Robion Kirby** (rckirby@berkeley.edu), University of California, Berkeley, and **Benjamin Ruppik** (benjamin.ruppik@gmail.com), Max Planck Institute for Mathematics. *Group Trisections and Smoothly Knotted Surfaces* Preliminary report.

A trisection of a (smooth, connected, closed, oriented) 4-manifold induces a Van Kampen cube of fundamental groups coming from the pieces of the trisection, and more surprisingly, vice versa. That is, a cube of groups satisfying a few simple requirements produces a trisection of a 4-manifold [Abrams-Gay-Kirby]. One natural question to ask is whether the same holds for bridge trisections of smoothly knotted surfaces in 4-manifolds. In this talk I will show how algorithmically producing a trivial tangle from a surjection between free groups allows us to answer this question in the affirmative. Consequently, although smoothly knotted surfaces in the 4-sphere cannot be distinguished by fundamental groups, they can be distinguished by group trisections. Stallings folding, a technique that translates between surjections between free groups and directed graphs, guides the proof. This is joint work with Robion Kirby, Michael Klug, Vincent Longo, and Benjamin Ruppik. (Received August 29, 2021)

1174-57-6246 **Yitzchak Elchanan Solomon*** (elchanansolomon@gmail.com), Duke University, **Alexander Wagner** (alexander.wagner@duke.edu), Duke University, and **Paul Bendich** (pbendich@gmail.com), Duke University. *DIPOLE: Dimensionality Reduction via Distributed Persistent Homology*

Dimensionality reduction is the task of replacing a high-dimensional data set with a low-dimensional proxy of the same "shape". We argue that a dimensionality reduction method should preserve the geometry of the input data on small scales and the topology of the input data on large scales. To that end, we define a loss function on embeddings with a local geometric term (using nearest neighbor distances) and a global topological term (using persistent homology). For a host of theoretical and computational reasons, we use a variant of persistent homology called "distributed persistence" that computes the topology of many random subsamples of the input data. Performing gradient descent on this loss gives rise to a new dimensionality reduction scheme, DIPOLE, which can qualitatively and quantitatively outperform state-of-the-art methods like UMAP on a number of classical data sets. (Received September 7, 2021)

1174-57-6283 **Christian Millichap*** (christian.millichap@furman.edu), Furman University, and **Rolland Trapp** (RTrapp@csusb.edu), California State University, San Bernardino. *Flat fully augmented links are determined by their complements* Preliminary report.

The Gordon-Luecke Theorem states that knots are determined by their complements. While this fact does not hold when we expand to links, it is natural to ask: are there certain infinite classes of links that are determined by their complements? In this talk, we will discuss how the geometry of flat fully augmented links can be exploited to show that such links are determined by their complements (within the class of flat fully augmented links). This work requires a careful analysis of embedded totally geodesic surfaces inside these (hyperbolic) link complements which greatly restricts the behavior of homeomorphisms between such link complements. (Received September 7, 2021)

1174-57-6373 **Jeffrey Meier*** (jeffrey.meier@wwu.edu), Western Washington University, and **Alexander Zupan** (zupan@unl.edu), University of Nebraska-Lincoln. *Classifying fibered, homotopy-ribbon disks*

I will discuss the classification of fibered, homotopy-ribbon disks bounded by generalized square knots. Up to diffeomorphism rel-boundary, such disks are in bijection with the rational numbers with even numerator. The classification up to diffeomorphism (unrestricted on the boundary) remains open, and it is possible there is only one such disk. This is joint work with Alex Zupan. (Received September 8, 2021)

1174-57-6388 **Ryan D. Budney*** (rybu@uvic.ca), University of Victoria, Department of Mathematics and Statistics. *A Vassiliev invariant for everyone.*

There is a technique called scanning that provides an often non-trivial map from the diffeomorphism group of a manifold to certain iterated loop spaces of embedding spaces in that manifold. Using Vassiliev invariants of these embedding spaces we can identify non-trivial diffeomorphisms of some manifolds. By "Vassiliev invariant" I am referring to any invariant of the homotopy or homology coming from a comparison between an embedding space and configuration spaces, such as in Vassiliev's work or using the Embedding Calculus of Goodwillie, Weiss and Klein. I will sketch how scanning could be viewed as a standard "evaluation map" to a non-standard version of the Embedding Calculus that compares embedding spaces not to configuration spaces, but to spaces of string links. At present our work gives non-trivial elements of the low-dimensional homotopy groups of the diffeomorphism groups of $S^1 \times D^n$ and $S^1 \times S^n$, for $n \geq 3$. Specifically our elements are in the k-th homotopy group of these diffeomorphism groups for $k = 0$ and $k = n-3$, for all $n \geq 3$. Given that computations with these tools are rather lengthy, I will describe the kind of invariants we are using, in the simplified context of knotted arcs in dimension 3. (Received September 8, 2021)

1174-57-6838 **Nicholas Scoville*** (nscoville@ursinus.edu), Ursinus College, **Maxwell Lin** (mxlin@berkeley.edu), University of California Berkeley, and **Connor Donovan** (codonovan@ursinus.edu), Ursinus College. *The homotopy type of the Morse complex for some collections of trees* Preliminary report.

The Morse complex is the simplicial complex consisting of all gradient vector fields on a fixed simplicial complex K. The homotopy type of this complex is in general not well understood. In this talk, we give some recent results on the homotopy type of the Morse complex for certain collections of trees. We show that if K contains two leaves that share a common vertex, then the Morse complex is strongly collapsible and hence has the homotopy type of a point. We also use the result that the Morse complex of a disjoint union $K \sqcup L$ is the Morse complex of the join $K * L$ to compute the homotopy type of the Morse complex of some families of graphs, including Caterpillar graphs. (Received September 9, 2021)

1174-57-7026 **John B Etnyre** (etnyre@math.gatech.edu), Georgia Institute of Technology, **Lisa Traynor*** (Ltraynor@brynmawr.edu), Bryn Mawr College, and **Jennifer Dalton** (dalton@bancroftschool.org), The Bancroft School. *Legendrian torus and cable links* Preliminary report.

In contact topology, an important problem is to understand Legendrian submanifolds; these submanifolds are always tangent to the plane field given by the contact structure. In fact, every smooth knot type will have an infinite number of different Legendrian representatives. A basic problem is to give the "Legendrian mountain range" of a smooth knot, which records all Legendrian representatives of the knot type.

In topology, torus knots, torus links, and cable links form important families of knots and links. The mountain range classifications of all Legendrian torus knots have been established by Etnyre and Honda. I will explain the classification of all Legendrian torus links. In the process, we will see that there are interesting patterns on which tuples of points on the Legendrian mountain range of a torus knot can be realized as the components of a Legendrian torus link. We will also see that there are some Legendrian torus links that have smooth symmetries that cannot be realized by a Legendrian isotopy. I will also explain how these torus link statements have extensions to Legendrian cable links. These results are applications of convex surface theory. (Received September 11, 2021)

1174-57-7029 **Orsola Capovilla-Searle** (ocapovillasearle@ucdavis.edu), UC-Davis, UC Davis, **Yu Pan** (paulinenk65@gmail.com), Tianjin University, **Noémie Legout** (noemie.legout@math.uu.se), Uppsala University, **Maÿlis Limouzineau** (lim.maylis@gmail.com), University of Cologne, and **Emmy Murphy** (e.murphy.math@gmail.com), Princeton University. *Obstructions to reversing surgery for immersed Lagrangian fillings* Preliminary report.

An important problem in smooth topology is to understand the 4-ball genus and the 4-ball crossing number of a smooth knot. More generally, one can study the "geography" question of which combinations of genus and double points can be realized by a smooth, orientable surface in the 4-ball that has the given knot as its boundary. I will discuss analogous problems when the knot and surface satisfy additional geometric conditions imposed by symplectic geometry: the surface is Lagrangian and the boundary knot is Legendrian. It is natural to study a "mutation" problem for Lagrangian fillings: when does the existence of a filling with a particular combination of genus and double points give rise to another filling with a different combination of genus and double points. There is a well-established procedure to resolve certain types of double points at the cost of increasing the genus. We show that that it is not always possible to reverse the surgery, meaning that there exist immersed Lagrangian fillings that cannot be obtained from surgery on a Lagrangian filling with less genus and more double points. To prove this, we develop theory to obstruct the existence of immersed Lagrangian fillings. (Received September 11, 2021)

1174-57-7067 **Christine Ruey Shan Lee*** (christine.rs.lee@gmail.com), University of South Alabama, and **Effie Kalfagianni** (kalfagia@math.msu.edu), Michigan State University. *Crossing numbers of Whitehead doubles*

Determining the minimal crossing number of a knot is typically a difficult problem. In particular, not much is known about the behavior of crossing numbers under the operation of taking Whitehead doubles. In this talk, we will discuss how the colored Jones polynomial can be used to study the crossing numbers of Whitehead doubles. We use the connection between the asymptotics of the degrees of the colored Jones polynomial and crossing numbers to determine the minimum crossing numbers of an infinitely family of satellite knots. This is joint work with Efstratia Kalfagianni. (Received September 11, 2021)

1174-57-7092 **Eleni Panagiotou*** (eleni-panagiotou@utc.edu), University of Tennessee At Chattanooga, and **Louis H. Kauffman** (kauffman@uic.edu), UIC, University of Illinois at Chicago. *Knot polynomials and Vassiliev measures of open and closed curves in 3-space and their applications*

Many physical systems are composed by entangled filamentous structures whose entanglement greatly affects their mechanical properties. Measuring entanglement complexity of physical systems has been a challenge because these can be seen as open curves in 3-space for which the conventional measures of topological complexity do not apply. In this talk we introduce the Jones polynomial for open curves in 3-space. We show it is a continuous function of the curve coordinates that, as the ends tend to coincide, it tends to the Jones polynomial of the resulting knot. The calculation of the Jones polynomial for real physical filaments can be intractable. In this talk we also introduce Vassiliev measures of open curves in 3-space. These are also continuous functions of the curve coordinates and tend to the Vassiliev invariant of the resulting knot as the endpoints coincide. We show that the second Vassiliev measure can provide a measure of entanglement complexity of open curves in 3-space that may be preferable for applications. (Received September 12, 2021)

1174-57-7096 **Julia V Shneidman*** (julshn16@gmail.com), Rutgers University, and **Sarah Ruth Nicholls** (sarahruthnicholls@gmail.com), Wake Forest University. *Large 1-systems of Curves in non-orientable surfaces*

A longstanding avenue of research in orientable surface topology is to create and enumerate collections of curves in surfaces with certain intersection properties. We look for similar collections of curves in non-orientable surfaces. A surface is *non-orientable* if and only if it contains a Möbius band. We generalize a construction of Malestein-Rivin-Theran to non-orientable surfaces to exhibit a lower bound for the maximum number of curves that pairwise intersect 0 or 1 times in a generic non-orientable surface. (Received September 12, 2021)

1174-57-7097 **Kai Foley Nakamura*** (kainakamura@utexas.edu), University of Texas Austin. *Trace Embeddings From Zero-Surgery Homeomorphisms*

Manolescu and Piccirillo recently proposed a construction of exotic homotopy 4-spheres using 0-surgery homeomorphisms and rasmussens s-invariant. They produced a family of knots that if any were smoothly slice, one could construct an exotic 4-sphere. I will discuss this and recent work where I show that these knots are not slice. Hence they cannot be used to construct an exotic 4-sphere. Sliceness is obstructed by showing that if any of these knots were slice, another knot which has homeomorphic 0-surgery to one of the knots would be null homologously slice in $\#n\mathbb{CP}^2$ by constructing a trace embedding (Received September 12, 2021)

1174-57-7171 **Matthew Kahle*** (mkahle@gmail.com), Ohio State University. *Configuration spaces of particles: homological solid, liquid, and gas*

Configuration spaces of points in the plane are well studied and the topology of such spaces is well understood. But what if you replace points by particles with some positive thickness, and put them in a container with boundaries? It seems like not much is known. To mathematicians, this is a natural generalization of the configuration space of points, perhaps interesting for its own sake. But is also important from the point of view of physics—physicists might call such a space the "phase space" or "energy landscape" for a hard-spheres system. Since hard-spheres systems are observed experimentally to undergo phase transitions (analogous to water changing into ice), it would be quite interesting to understand topological underpinnings of such transitions.

We have just started to understand the homology of these configuration spaces, and based on our results so far we suggest working definitions of "homological solid, liquid, and gas". This is joint work with a number of collaborators, including Hannah Alpert, Ulrich Bauer, Kelly Spendlove, and Robert MacPherson. (Received September 13, 2021)

1174-57-7290 **Hee Jung Kim*** (heejungorama@gmail.com), Western Washington University. *Knotted configurations of surfaces in 4-manifolds and stabilization* Preliminary report.

Infinite families of configurations of surfaces that are topologically isotopic but smoothly distinct are known to be constructed in 4-manifolds, using a operation, double point surgery introduced by Ruberman and the presenting author, on immersed surfaces. In this talk, we will discuss these examples and show that they are all 1-stably diffeomorphic. (Received September 14, 2021)

1174-57-7375 **Michael Robinson*** (michaelr@american.edu), American University. *Topological Representation of Signals*

Traditional signal processing studies techniques for manipulating spaces of functions, using primarily algebraic, geometric, and analytic tools. These tools work by lifting algebraic, geometric, and analytic properties of the codomain into the space of functions. This approach is very effective since it supports various decompositions of the function spaces involved. Signals of interest can be confined to one part of the space, while unwanted signals can be isolated in other parts of the space. Unfortunately, the resulting decompositions of signals are necessarily tied to their common domain, and are often brittle to changes in the domain as a result.

What if we changed the approach, and instead lifted properties of the domain into the function space? This talk will explore what happens if we lift combinatorial and/or differential topological properties from the domain space into the function space of signals. The resulting function spaces still support decompositions, after a fashion, but are more tolerant to disruptions to the domain. To bring time-honored signal processing tasks, like filtering and detection, into this new framework requires some new tools, which I'll present. (Received September 14, 2021)

1174-57-7415 **Elaina Aceves*** (elaina-aceves@uiowa.edu), University of Iowa. *Flypes and Agol cycles of 3-braids*

In 1993, Birman and Menasco proved that two 3-braids related by a non-degenerate flype must live in distinct conjugacy classes. In this talk, I compare the Agol cycles of 3-braids related by a non-degenerate flype to see how effective the cycles are as a conjugacy class invariant. This is joint work with Keiko Kawamuro. (Received September 14, 2021)

1174-57-7443 **Laufey Jörgensdóttir*** (ljorgens@stedwards.edu), St. Edward's University, Author. *Component-Preserving Amphicheiral Links*

Knot theory is an exciting field of mathematical research that has many applications. We address some important properties of knots and links and what it means for a link to be amphicheiral. In particular, we address the question suggested by Kadokami and Kobataki: Is there a prime, component-preserving amphicheiral link of odd crossing number less than 21? (Received September 14, 2021)

1174-57-7560 **Anubhav Mukherjee** (anubhavmaths@gatech.edu), Georgia Institute of Technology, **Rima Chatterjee*** (rchatt1@lsu.edu), University of Cologne, and **Hyunki Min** (hkmin@mit.edu), MIT. *Cabling of knots in overtwisted manifolds* Preliminary report.

Knots associated to overtwisted manifolds are less explored. There are two types on knots in an overtwisted manifold - loose and non-loose. While we understand loose knots, non-loose knots remain a mystery. The classification and structure problems of these knots vary greatly compared to the knots in tight manifolds. Especially we are interested in how satellite operations on a knot in overtwisted manifold changes the geometric property of the knot. In this talk, I will discuss under what conditions cabling operation on a non-loose knot preserves non-looseness. This is a joint work with Etnyre, Min and Mukherjee. (Received September 15, 2021)

1174-57-7669 **Autumn E Kent*** (kent@math.wisc.edu), University of Wisconsin - Madison, **Kenneth Bromberg** (bromberg@math.utah.edu), University of Utah, and **Yair Minsky** (yair.minsky@yale.edu), Yale University. *The* 0π*-Theorem.* Preliminary report.

Thurston's Hyperbolic Dehn Filling Theorem tells us that all but finitely many Dehn fillings on the cusp of a hyperbolic 3–manifold are in fact hyperbolic Dehn fillings. The Universal Hyperbolic Dehn Filling Theorem of Hodgson and Kerckhoff tells us that if the normalized length of a filling slope is greater than 8, then the resulting filling is a hyperbolic Dehn filling. Hodgson and Kerckhoff suspected that the same theorem should hold, with perhaps a different constant, provided that the actual length of the slope, in a cusp cross section, is long. We provide evidence for this by showing that this is indeed the case provided the volume growth of the cusp is suitably controlled. Our techniques allow us to perform hyperbolic Dehn fillings on rank–1 cusps, in which case the normalized length of the filling slope is zero. (Received September 15, 2021)

1174-57-7703 **Tian Yang*** (tianyang@math.tamu.edu), Texas A&M University, and **Giulio Belletti** (gbelletti@mathi.uni-heidelberg.de), Heidelberg University. *Discrete Fourier transforms, quantum* $6j$*-symbols and deeply truncated tetrahedra*

The asymptotic behavior of quantum $6j$-symbols is closely related to the volume of truncated hyperideal tetrahedra, and plays a central role in understanding the asymptotics of the TuraevViro invariants of 3-manifolds. In this talk, I will introduce a conjecture relating the asymptotics of the discrete Fourier transforms of quantum $6j$-symbols on one hand, and the volume of deeply truncated tetrahedra of various types on the other; and as supporting evidence prove this conjecture in the case that the dihedral angles are sufficiently small. This is a joint work with Giulio Belletti. (Received September 15, 2021)

1174-57-7704 **Dan Margalit** (margalit@math.gatech.edu), Georgia Institute of Technology, **Thi Phuong Anh Pham*** (ptp170001@utdallas.edu), The University of Texas at Dallas, **Wenxi Yao** (wxyao10@uchicago.edu), The University of Chicago, **Adele Long** (allong@smith.edu), Smith College, and **Yvon Verberne** (yvon.verberne@mail.utoronto.ca), Georgia Tech. *Automorphisms of the fine curve graph*

The fine curve graph of a surface, recently defined by Bowden-Hensel-Webb, is the graph whose vertices are essential simple closed curves on the surface and edges are given by pairs of disjoint curves. Building on the work of Farb and Margalit, we show that the group of automorphisms of the fine curve graph for a surface is isomorphic to the group of homeomorphisms of the surface. Because of this isomorphism, we can think of the fine curve graph as a combinatorial model of the surface. Our theorem is analogous to the seminal result of Ivanov that the group of automorphisms of the (classical) curve graph is isomorphic to the extended mapping class group of the corresponding surface. This work was completed as part of the 2021 REU program at Georgia Institute of Technology under the mentorship of Dan Margalit and Yvon Verberne. (Received September 15, 2021)

1174-57-7780 **Jennifer Hom*** (hom@math.gatech.edu), Georgia Tech, Georgia Institute of Technology, **Kristen Hendricks** (kristen.hendricks@rutgers.edu), Rutgers University, **Ian Zemke** (ian.zemke@gmail.com), Princeton University, and **Matthew Stoffregen** (stoffre1@msu.edu), Michigan State University. *Homology cobordism and Heegaard Floer homology*

The homology cobordism group consists of integer homology spheres under connected sum, modulo an equivalence relation called homology cobordism. We discuss applications of Heegaard Floer homology to the homology cobordism group. In particular, we will show that the homology cobordism group is not generated by Seifert fibered spaces. This is joint work with Kristen Hendricks, Matthew Stoffregen, and Ian Zemke. (Received September 16, 2021)

1174-57-7781 **Samuel J. Taylor*** (samuel.taylor@temple.edu), Temple University, **Giulio Tiozzo** (XXX@aol.com), University of Toronto, and **Ilya Gekhtman** (YYY@aol.com), Technion. *Central limit theorems for counting measures in coarse negative curvature*

We establish general central limit theorems for an action of a group on a hyperbolic space with respect to counting for the word length in the group. In 2013, Chas, Li, and Maskit produced numerical experiments on random closed geodesics on a hyperbolic pair of pants. Namely, they drew uniformly at random conjugacy classes of a given word length, and considered the hyperbolic length of the corresponding closed geodesic on the pair of pants. Their experiments lead to the conjecture that the length of these closed geodesics satisfies a central limit theorem, and we proved this conjecture in 2018.

In our new work, we remove the assumptions of properness and smoothness of the space, or cocompactness of the action, thus proving a general central limit theorem for group actions on hyperbolic spaces. We will see how our techniques replace the classical thermodynamic formalism and allow us to provide new applications, including to lengths of geodesics in geometrically finite manifolds and to intersection numbers with submanifolds. (Received September 16, 2021)

1174-57-7795 **Robert E Gompf*** (gompf@math.utexas.edu), The University of Texas at Austin. *Exotic planes in* $\mathbb{R}^4$

The first step from 2-knot theory into the noncompact world should be to study proper embeddings of $\mathbb{R}^2$ into $\mathbb{R}^4$. To avoid a morass of topological difficulties, we focus on such embeddings that are smoothly knotted but topologically ambiently isotopic to the standard plane, i.e., *exotic planes.* While analogous embeddings of closed, orientable surfaces in $\mathbb{R}^4$ are not known to exist, exotic planes have been known (but not widely) since the early days of Freedman theory. We present the first detailed study of exotic planes. There are several constructions, yielding uncountably many distinct examples that fall into two types with very different properties. One type is simple enough to admit level diagrams without local maxima, and we can draw such diagrams explicitly. The other type is more complicated, requiring infinitely many local maxima in any such diagram. We will discuss these examples and their properties, and perhaps applications and open questions. (Received September 16, 2021)

1174-57-7848 **Jeffrey Meier** (jeffrey.meier@wwu.edu), Western Washington University, **Jason Joseph** (jason.joseph@rice.edu), Rice University, **Alexander Zupan*** (zupan@unl.edu), University of Nebraska-Lincoln, and **Maggie Miller** (maggie.miller.math@gmail.com), Stanford University. *Seifert solids via tri-plane diagrams*

Whereas every knot in S^3 bounds a Seifert surface, every orientable knotted surface in S^4 bounds an embedded 3-manifold called a Seifert solid. Recently, Meier and Zupan showed that every knotted surface can be represented by a tri-plane diagram, a triple of trivial tangle diagrams that join in pairs to form unlink diagrams. We describe a procedure that uses a tri-plane diagram for a knotted surface K to produce a Seifert solid for K. Notably, this procedure restricts to the classical Seifert's algorithm on each paired unlink diagram. (Received September 16, 2021)

1174-57-7971 **Michelle Chu*** (michu@uic.edu), University of Illinois at Chicago, and **Alan Reid** (alan.reid@rice.edu), Rice University. *Totally geodesic 3-manifolds in hyperbolic 4-manifolds of small volume*

In this talk we discuss the existence of closed embedded totally geodesic hyperbolic 3-manifolds in hyperbolic 4-manifolds of small volume. In particular, we show that certain hyperbolic link complements of 2-tori in S^4 do not contain closed embedded totally geodesic hyperbolic 3-manifolds. (Received September 17, 2021)

1174-57-7996 **Justin Bryant*** (jabryant@wesleyan.edu), Wesleyan University. *A Hedden-Style Conjecture for String Links* Preliminary report.

A conjecture (attributed to Hedden) states that the only endomorphisms of the knot concordance group which are induced by satellite operations are the zero homomorphism, the identity homomorphism, and the involution that takes each class to its inverse. In this talk I will discuss a generalization of this conjecture for maps from the string link concordance group to the knot concordance group which are induced by string link infection. In particular, I will focus on the new homomorphisms that can be defined in this setting and homomorphism obstructions which utilize linking information that was not available in the original conjecture. (Received September 17, 2021)

1174-57-8002 **Seppo M Niemi-Colvin*** (seppo.niemi.colvin@duke.edu), Duke University. *Invariance of Knot Lattice Homology*

Links of singularity and generalized algebraic links are ways of constructing three-manifolds and smooth links inside them from complex algebraic surfaces and complex curves inside them. Némethi created lattice homology as an invariant for links of normal surface singularities which developed out of computations for Heegaard Floer homology. Later Ozsváth, Stipsicz, and Szabó defined knot lattice homology for generalized algebraic knots in rational homology spheres, which is known to play a similar role to knot Floer homology and is known to compute knot Floer homology in some cases. I discuss a proof that knot lattice homology is an invariant of the smooth knot type, which had been previously suspected but not confirmed. (Received September 17, 2021)

1174-57-8130 **Michael Landry*** (mlandry@wustl.edu), Washington University. *Flows, growth rates, and the veering polynomial*

I will explain the following statement: the limit points of the pseudo-Anosov stretch factors in a single fibered cone of a hyperbolic three-manifold can be naturally interpreted as stretch factors of automorphisms of infinite type surfaces. These surfaces are leaves of depth one foliations of the original manifold. If there is time, I will discuss connections with the Handel-Miller theory of endperiodic automorphisms and with the Cantwell-Conlon theory of foliation cones. This is joint work with Yair Minsky and Samuel Taylor. (Received September 17, 2021)

1174-57-8144 **Seungwon Kim** (math751@gmail.com), Center for Geometry and Physics, IBS, Center For Geometry and Physics, Institute For Basic Science, and **Mark Hughes** (hughes@mathematics.byu.edu), Brigham Young University. *Diagrams of immersed surfaces and homotopies of surfaces*

I will discuss a diagrammatic theory for immersed surfaces in 4-manifolds. These diagrams, called singular banded unlink diagrams (SBUDs), consist of singular links with bands attached contained in Kirby diagrams. Any two SBUDs of smoothly equivalent or isotopic (when that notion makes sense) surfaces are related by a finite set of explicit moves; any two SBUDs of homotopic surfaces are related by a slightly larger finite set of explicit moves. In particular, I will give some sample applications of how to use these diagrams to compute surface invariants or construct explicit surface homotopies (making this talk arguably about immersed 3-manifolds that are traces of such homotopies). (Received September 17, 2021)

1174-57-8247 **Kasturi Barkataki*** (mfh936@mocs.utc.edu), University of Tennessee at Chattanooga.
The Jones polynomial in systems with Periodic Boundary Conditions

The entanglement of collections of filaments is a problem that arises in many contexts, such as polymers and textiles. Measuring entanglement in such systems is a challenge. In addition, many systems of filaments are modelled using Periodic Boundary Conditions (PBC). Studying entanglement in such systems is even more complex. In this paper we propose a definition for the Jones polynomial of open or closed curves in systems employing periodic boundary conditions. This is a one variable Laurent polynomial of a finite link in 3-space. For closed curves, this gives a topological invariant that captures the grain of entanglement in this infinite periodic system. In fact, we show that for systems of closed chains in 1 PBC, the periodic Jones polynomial is a repetitive factor of the Jones polynomial of the infinite component link. For open curves, this gives a polynomial with real coefficients which are continuous functions of the chain coordinates. We show with some illustrative examples that the periodic Jones polynomial is a useful tool for measuring knotting in periodic systems. (Received September 18, 2021)

1174-57-8414 **Nicholas Cazet*** (nccazet@ucdavis.edu), UC Davis, **Marion Campisi** (marion.campisi@sjsu.edu), San Jose State University, **Luis Torres** (luistorres@utexas.edu), UT Austin, **Todd Fellman** (fellmt@rpi.edu), Rensselaer Polytechnic Institute, **Vatsal Srivastava** (18d180029@iitb.ac.in), Indian Institute of Technology Bombay, **David Crncevic** (dcrncevi@u.rochester.edu), University of Rochester, and **Nikolas Rieke** (nikolasrieke@kentucky.edu), University of Kentucky.
Vertex Distortion Detects the Unknot Preliminary report.

Gromov distortion measures the ratio between distance along the knot and distance in the ambient space. No nontrivial knot class has its Gromov distortion invariant calculated because of its complexity. While upper bounds on numerical knot invariants are easily given by examples, the Gromov distortion calculation of even the most contrived diagrams has required significant proof.

Vertex distortion of lattice knots was introduced as a combinatorial approach to understand the enigmatic Gromov distortion of smooth knots. We show that vertex distortion detects the unknot and that vertex distortion is bounded above and below be a constant multiple of Gromov distortion. This allows for the port of Pardon's well-known result on Gromov distortion to vertex distortion.

Lattice knots are used in DNA Topology since arbitrary smooth knots do not model the rigidity of the molecule well. We give a vertex distortion calculator that can detect the unknot, if a small value is returned. (Received September 18, 2021)

1174-57-8475 **Kyle Hayden*** (hayden@math.columbia.edu), Columbia University, and **Isaac Sundberg** (icraig@brynmawr.edu), Bryn Mawr College. *Khovanov homology and exotic surfaces in the 4-ball*

We show that the cobordism maps on Khovanov homology can distinguish between smooth surfaces in the 4-ball that are exotically knotted (i.e., isotopic through ambient homeomorphisms but not ambient diffeomorphisms). This provides a direct, elementary, and combinatorial approach to the detection of exotically knotted surfaces. (Received September 19, 2021)

1174-57-8490 **Beibei Liu*** (bbliumath@gmail.com), Georgia Institute of Technology. *Triple linking numbers and Heegaard Floer homology*

It is known that every closed oriented 3-manifold is obtained by surgery on some link in the three sphere. It is natural to ask whether the Milnor invariants, which contain both link homotopy and concordance invariants of links, place algebraic restrictions on some 3-manifold invariant of the surgery manifolds. In this talk, we show that the Milnor triple linking number places restrictions on the d-invariant of the surgery manifold, which is a 3-manifold invariant extracted from Heegaard Floer homology. This is a joint work with Gorsky, Lidman and Moore. (Received September 19, 2021)

1174-57-8591 **Eli Pinchas Meyers*** (elipimeyers@gmail.com), Research Science Institute, St. Stephen's Episcopal School. *Equivalence in Discrete Morse Functions on Graphs*

The set of discrete Morse functions on a simplicial complex can be studied up to different notions of equivalence. Two of the most commonly used classifications are Forman equivalence and homological equivalence. Recently, Rand and Scoville produced an algorithm which, given a pair of discrete Morse functions f and g on a tree, generates a new discrete Morse function h which is homologically equivalent to f and Forman equivalent to g. We build on their work by finding, for any graph, necessary and sufficient conditions on f, g so that such an h exists. We also show that the class of graphs for which we can always find h for any f, g is exactly graphs which are either trees or cycles. To prove these results, we introduce the notions of critical graphs and critical matrices, which describe the CW decomposition of a graph under a discrete Morse function. (Received September 19, 2021)

1174-57-8697 **Feride Ceren Kose*** (fkose@math.utexas.edu), The University of Texas At Austin. *Composite ribbon knots with symmetric union presentations*

A symmetric union of a knot is an aesthetically appealing construction which generalizes the connected sum of a knot and its mirror image. As the connected sum of a knot and its mirror image is always ribbon, hence smoothly slice, symmetric unions too are ribbon. Like the slice-ribbon question, one may ask whether every ribbon knot is a symmetric union. This is the case for a high number of prime ribbon knots with up to 12 crossings as well as some infinite families such as 2-bridge ribbon knots. However, for some composite ribbon knots no such presentation has yet been found. In my talk, I will describe these composite ribbon knots and show that they indeed do not admit the simplest type of symmetric union presentation. (Received September 19, 2021)

1174-57-8708 **Nicholas Rouse*** (nicholas.rouse@rice.edu), Rice University. *Integrality for hyperbolic knots and links in S^3.*

A finite volume orientable hyperbolic 3-manifold can be thought of as the quotient of hyperbolic 3-space by a discrete subgroup of $\mathrm{PSL}(2, \mathbf{C})$. By Mostow-Prasad rigidity, the traces of elements of this subgroup are algebraic numbers. These traces are often algebraic integers (for example when the manifold is arithmetic), but there are many examples of subgroups with non-integral trace. We say the manifold has non-integral trace if there is an element of the corresponding group that has non-integral trace. I will discuss some questions and observations about integrality in knots and joint work with Alan Reid constructing infinitely many hyperbolic knot complements in S^3 that have non-integral trace. (Received September 19, 2021)

1174-57-8718 **Daniel Charles Douglas*** (dcdougla@usc.edu), Yale University. *Quantum higher Teichmüller theory* Preliminary report.

We survey some recent developments in quantum higher Teichmüller theory. One of the key players is the *quantum trace map*, which provides a bridge between certain quantum topological and quantum geometric aspects. The quantum trace map was conjectured by Chekhov-Fock in the 90s, constructed for SL_2 around 2010 by Bonahon-Wong, and more recently has been generalized to the case of SL_n by multiple groups. One of the fascinating connections is to the quantum duality conjectures of Fock-Goncharov. (Received September 19, 2021)

1174-57-8749 **Rylee Alanza Lyman*** (rylee.lyman@rutgers.edu), Rutgers University—Newark. *Nielsen realization for infinite-type surfaces*

Given a finite subgroup G of the mapping class group of a surface S, the Nielsen realization problem asks whether G can be realized as a finite group of homeomorphisms of S. In 1983, Kerckhoff showed that for S a finite-type surface, any finite subgroup G may be realized as a group of isometries of some hyperbolic metric on S. We extend Kerckhoff's result to orientable, infinite-type surfaces. As an application, we show that topological groups containing sequences of torsion elements limiting to the identity do not embed continuously into the mapping class group of S. We show that compact subgroups of the mapping class group of S are finite, and locally compact subgroups are discrete. This represents joint work with S. Afton, D. Calegari, and L. Chen. (Received September 19, 2021)

1174-57-8750 **Sarah Collins*** (sallycollins@gatech.edu), Georgia Institute of Technology. *The figure-eight knot and its image under the Mazur pattern* Preliminary report.

Given two knots K_1 and K_2, their 0-surgery manifolds $S^3_0(K_1)$ and $S^3_0(K_2)$ are homology cobordant rel meridian if they are homology cobordant preserving the homology class of the positively oriented meridian. It is known that if $K_1 \sim K_2$, then $S^3_0(K_1)$ and $S^3_0(K_2)$ are homology cobordant rel meridian. The converse of this statement was first disproved by Cochran-Franklin-Hedden-Horn using concordance invariants τ and s. In this talk we will provide a new counterexample, the pair of knots 4_1 and $M(4_1)$ where M is the Mazur satellite operator. $S^3_0(4_1)$ and $S^3_0(M(4_1))$, are homology cobordant rel meridian, and 4_1 and $M(4_1)$ are not only non-concordant but have concordance orders two and infinity, respectively. (Received September 19, 2021)

1174-57-8778 **Moshe Cohen*** (cohenm@newpaltz.edu), State University of New York At New Paltz, and **Adam M. Lowrance** (adlowrance@vassar.edu), Vassar College. *The average genus of a 2-bridge knot grows linearly with respect to crossing number* Preliminary report.

Dunfield et al provide experimental data to suggest that the Seifert genus of a knot grows linearly with respect to crossing number. We prove this holds among 2-bridge knots using Chebyshev billiard table diagrams developed by Koseleff and Pecker. This work builds on results by the first author with Krishnan and Even-Zohar and Krishnan on a random model using these diagrams. This work also uses and improves upon results by the first author demonstrating a lower bound for the average genus among a weighted collection of 2-bridge knots. (Received September 19, 2021)

1174-57-8788 **Melissa Zhang*** (melissa.zhang@uga.edu), University of Georgia. *Plamenevskaya's invariant, braids, and stability*

Given a braid closure representative $\hat{\beta}$ of a transverse knot, Plamenevskaya's invariant picks out a class ψ in the Khovanov homology of the (smooth) knot $\hat{\beta}$. It is still open whether ψ is effective (i.e. distinguishes better than self-linking number), but its construction has nevertheless provided the basis for many interesting constructions. In this talk, I describe two structural (algebraic) phenomena in Khovanov homology that play well with Plamenevskaya's construction: (1) canonical generators in Khovanov-Robert's $U(1) \times U(1)$ equivariant link homology and (2) Rozansky's discovery that the Khovanov homology of torus braids stabilizes for each quantum grading. This talk is partially based on ongoing joint work with Carmen Caprau, Christine Ruey Shan Lee, Nicolle González, and Radmila Sazdanović. (Received September 19, 2021)

1174-57-8789 **Yang Qiu*** (yangqiu@math.ucsb.edu), University of California, Santa Barbara, and **Zhenghan Wang** (zhenghwa@microsoft.com), Microsoft Station Q, UC Santa Barbara. *From Three Dimensional Manifolds to Premodular Tensor Categories*

Using M-theory in physics, Cho, Gang, and Kim (2020) outlined a program that connects two parallel subjects of three dimensional manifolds, namely, geometric topology and quantum topology. They suggest that classical topological invariants such as Chern-Simons invariants of $\mathrm{SL}(2,\mathbb{C})$-flat connections and $\mathrm{SL}(2,\mathbb{C})$-adjoint Reidemeister torsions of a three manifold can be packaged together to produce a modular tensor category. We study the program mathematically, provide strong support for such a program and make a number of improvements to the program based on our examples. The program produces an algorithm to generate the potential modular T-matrix and the quantum dimensions of a candidate modular data. The modular S-matrix follows from essentially a trial-and-error procedure. In this talk, we will introduce the program, give a mathematical construction of the modular data of a premodular category from each Seifert fibered space with three singular fibers and torus bundles over the circle with Thurston SOL geometry, and show that the modular data from Seifert fibered spaces and torus bundles can be realized by Temperley-Lieb-Jones categories and $\mathbb{Z}_2$-equivariantization of certain pointed premodular categories respectively. In addition, we show that a resulting premodular category from our examples is modular if and only if the three manifold is a $\mathbb{Z}_2$-homology sphere. (Received September 21, 2021)

1174-57-8798 **Gabriel Islambouli*** (gabrielIslambouli@gmail.com), University of California, Davis. *Knotted surfaces in bridge 4-sections*

We give a gentle, picture heavy introduction to bridge 4-sections. We will highlight what the extra piece buys you when moving from trisections to 4-sections. We also show how to get efficient bridge 4-sections of surfaces smoothly isotopic to each complex curve in S2xS2. This is joint work with Homayun Karimi, Peter Lambert-Cole, and Jeffrey Meier. (Received September 19, 2021)

1174-57-8808 **Diana Hubbard** (Diana.hubbard@brooklyn.cuny.edu), Brooklyn College, and **Hannah Turner*** (hannah.turner@math.gatech.edu), Georgia Institute of Technology. *Fractional Dehn twists and left-orders on mapping class groups* Preliminary report.

Three-manifolds admit descriptions called open book decompositions; in this setting a surface with boundary and a mapping class describe the 3-manifold. One invariant of an open book is the fractional Dehn twist coefficient (FDTC). The FDTC is a real number invariant of a mapping class of a surface with boundary, which has connections to contact topology and foliation theory. I'll show that the FDTC of a given surface can be computed using a multitude of geometrically defined left-orders on the mapping class group due to Thurston. This is joint work with Diana Hubbard. (Received September 19, 2021)

1174-57-8960 **Zachary Winkeler*** (zachary.j.winkeler.gr@dartmouth.edu), Dartmouth College. *Khovanov homology for links in thickened multipunctured disks*

Khovanov homology is a knot invariant that categorifies the Jones polynomial. For knots in a thickened annulus, we can define a filtration on the Khovanov chain complex that gives us an invariant called annular Khovanov homology. In this talk, we will discuss a generalized notion of filtration that allows us to construct an analogous invariant for knots in thickened disks with any number of punctures. (Received September 20, 2021)

1174-57-8967 **Gage Martin*** (gage.martin@bc.edu), Boston College. *Annular Khovanov homology and meridional disks*

The wrapping conjecture of Hoste-Przytycki suggests a relationship between the maximum non-zero annular Khovanov grading and the minimal geometric intersection number with a meridional disk. Inspired by this conjecture, we exhibit infinite families of annular links for which the maximum non-zero annular Khovanov grading grows infinitely large but the maximum non-zero annular Floer-theoretic gradings are bounded. We also show this phenomenon exists at the decategorified level for some of the infinite families. Our computations provide further evidence for the wrapping conjecture and its categorified analogue. Additionally, we show that certain satellite operations cannot be used to construct counterexamples to the categorified wrapping conjecture. (Received September 20, 2021)

1174-57-9052 **Hannah Lynn Hoganson*** (hlhoganson@gmail.com), University of Utah. *The L^p metrics on Teichmüller Space* Preliminary report.

In this talk, we will discuss completeness properties and some of the local geometry of Teichmüller space with the L^p metrics. Teichmüller space parameterizes the possible complex structures on a differentiable surface. By defining this space analytically, we can equip it with the L^p metrics. These metrics provide a natural interpolation between the Teichmüller metric and the Weil-Petersson metric, which correspond to $p = \infty$ and $p = 2$ respectively. (Received September 20, 2021)

1174-57-9058 **Shawn Williams*** (shawn.williams@rice.edu), Rice University. *A Fox-Milnor Condition for 1-Solvable Links* Preliminary report.

A well known result of Fox and Milnor states that the Alexander polynomial of slice knots factors as $f(t)f(t^{-1})$, providing us with a useful obstruction to a knot being slice. In 1978 Kawauchi extended this result to the case of the multivariable Alexander polynomial of slice links. In this talk, we will present a generalization of this result for the multivariable Alexander polynomial of 1-solvable boundary links. This extends a previous result for certain localizations of the Alexander module. (Received September 20, 2021)

1174-57-9073 **Christopher Jay Leininger*** (cjl12@rice.edu), Rice University, **Viveka Erlandsson** (v.erlandsson@bristol.ac.uk), University of Bristol, and **Chandrika Sadanand** (chandrika.v.sadanand@gmail.com), Bowdoin College. *Billiards and hyperbolic cone surfaces*

Billiard trajectories on hyperbolic polygons determine geodesics on hyperbolic cone surfaces. In this talk, I will begin by describing these objects and this interaction. Then I will discuss the problem of determining the shape of a hyperbolic polygon from the "symbolic coding" of its billiard trajectories. I will then explain our main theorem that precisely states when this is and isn't possible, and connect it to the resolution of an associated problem of determining a hyperbolic cone metric from some data involving geodesics. (Received September 20, 2021)

1174-57-9091 **Leah Katherine Mork*** (lmork@cord.edu), Concordia College - Moorhead MN. *Enumerating b-prime Fully Augmented Links*

In a effort to enumerate fully augmented links that are not belted-sum decomposable (b-prime), this work will present an operation, called complete augmentation, on prime links that will produce all b-prime fully augmented links (FAL). A lemma proven by Jorge Calvo in 1985 will be vital in order to prove all completely augmented prime links will result in a b-prime FAL. (Received September 21, 2021)

1174-57-9114 **Sungwoon Kim*** (sungwoon@jejunu.ac.kr), Jeju National University. *Primitive stable representations of a rank 2 free group in SL(3,R)*

We study primitive stable groups acting on projective space in order to find new examples. In this talk, we present a way to construct primitive stable representations of a rank 2 free group in $\mathbf{SL}(3, \mathbb{R})$. It enables us to construct large family of primitive stable groups which have been unknown. (Received September 20, 2021)

1174-57-9125 **Philip Smith*** (cph196@mocs.utc.edu), The University of Tennessee at Chattanooga. *The second Vassiliev measure of uniform random walks in confined space*

Polymer chains and other filamentous structures can be represented by polygonal curves in 3-space. Biopolymers, like chromatin, are often confined in small volumes. In this manuscript, we examine the entanglement complexity of polygonal chains in 3-space and in confinement by studying equilateral random walks in 3-space and uniform random walks in confinement. We use a new measure of entanglement complexity of random walks, the second Vassiliev measure. For uniform random walks in confined space, we prove that the average value of the Vassiliev measure in the space of configurations increases as $O(n^2)$ with the length of the walks or polygons. We verify this result numerically and we show that for equilateral random walks, the mean value of the second Vassiliev measure increases as $O(n)$. These results reveal the rate at which knotting and not simply entanglement are affected by confinement. (Received September 20, 2021)

1174-57-9137 **Christopher William Davis** (Daviscw@uwec.edu), University Of Wisconsin - Eau Claire, **Carolyn Otto** (OTTOA@uwec.edu), University of Wisconsin-Eau Claire, and **Taylor E Martin*** (taylor.martin@shsu.edu), Sam Houston State University. *Moves relating C-complexes for links*

In 2004, Cimasoni gave a geometric computation of the multivariable Conway potential function through a generalization of Seifert surfaces for links called C-complexes. C-complexes have been used to compute a variety of link invariants, including Alexander invariants, signatures, and nullities. Drawing on work of Cooper, Cimasoni gave a set of C-complex moves relating two C-complexes for isotopic links. In this talk, we will introduce C-complexes and then show that the 2004 C-complex moves are insufficient for relating C-complexes for isotopic links. We then introduce a new move that completes the set, giving a family of C-complex moves that relate any two C-complexes for isotopic links. (Received September 20, 2021)

1174-57-9157 **Samuel Tripp*** (samuel.tripp1@gmail.com), Dartmouth College. *Integral Grid Homology for Links in Lens Spaces*

Knot Floer homology is a powerful invariant of knots and links introduced independently by Ozsváth and Szabó and by Rasmussen, as a refinement of Heegaard Floer homology. It was given a combinatorial description for links in S^3 by Manolescu, Ozsváth, Szabó, and Thurston, and for links in lens spaces by Baker, Grigsby, and Hedden. The combinatorial description gives a chain complex generated by certain intersection points on a grid diagram, whose differential counts embedded parallelograms in that grid diagram. We prove combinatorially that the homology of this chain complex is a link invariant for links in lens spaces. Using a sign assignment on parallelograms on the grid, Celoria generalized this differential to count embedded parallelograms with sign. We show that our combinatorial proof of invariance can be extended over $\mathbb{Z}$, allowing us to define a version of grid homology over the integers for links in lens spaces. (Received September 20, 2021)

1174-57-9186 **Donghao Wang*** (dwang@scgp.stonybrook.edu), Simons Center for Geometry and Physics. *Monopole Floer Homology for 3-manifolds with toroidal boundary*

The monopole Floer homology of an oriented closed 3-manifold was defined by Kronheimer-Mrowka around 2007 and has greatly influenced the study of 3-manifold topology since its inception.

In this talk, we will generalize their construction and define the monopole Floer homology for any pair (Y, ω), where Y is a compact oriented 3-manifold with toroidal boundary and ω is a suitable closed 2-form. The graded Euler characteristic of this Floer homology recovers the Milnor-Turaev torsion invariant by a classical theorem of Meng-Taubes. In the end, we will explain its relation with Landau-Ginzburg models and discuss some analytic problems in this construction. (Received September 20, 2021)

1174-57-9230 **Nur Saglam*** (nurmath7@gmail.com), Koc University, **Anar Akhmedov** (akhmedov@umn.edu), University of Minnesota, and **Mohan Bhupal** (bhupal@metu.edu.tr), Middle East Technical University. *Families of Lefschetz Fibrations via Cyclic Group Actions and Applications*

Using various diagonal cyclic group actions on the product manifolds $\Sigma_g \times \Sigma_g$ for $g > 0$, we obtain some families of Lefschetz fibrations over S^2. Then, we study the monodromies of these families applying the resolution of cyclic quotient singularities. We also realize some patterns of singular fibers and study deformations of these Lefschetz fibrations. Some cases give rise to nice applications using rational blow-down operation. (Received September 20, 2021)

1174-57-9247 **Miriam Kuzbary*** (mkuzbary3@gatech.edu), Georgia Institute of Technology, and **Santana Afton** (santana.afton@gatech.edu), Georgia Institute of Technology. *Asymptotics of the Casson invariant for Rational Homology Spheres* Preliminary report.

In this ongoing project with Santana Afton, we are working to bound the Casson invariant of rational homology spheres in terms of word length in the mapping class group inspired by work of Broaddus, Farb, and Putman in the case of integral homology spheres. Along the way, we prove a result on the Casson invariant of a product of mapping classes which, when used as gluing maps for a Heegaard splitting, give rational homology spheres. (Received September 20, 2021)

1174-57-9253 **Allison H Moore** (moorea14@vcu.edu), Virginia Commonwealth University. *Topological Symmetry Groups of Algebraically Trivial Theta Graphs* Preliminary report.

The orientation preserving topological symmetry group of a graph embedded in the three-sphere is the subgroup of the automorphism group of the underlying abstract graph that encodes symmetries of the spatial graph up to isotopy. In this ongoing project with Allison H. Moore, we consider the problem of studying symmetries of spatial theta graphs that are algebraically trivial in the sense that their constituent knots and links are comprised of unknots of linking number zero. We present a method for analyzing their topological symmetry groups that generalizes work of Cochran-Ruberman, and is based on Cochran's higher order linking numbers via formal Massey products. Along the way, we show that certain higher order linking numbers of three-component links are link concordance invariants. (Received September 20, 2021)

1174-57-9280 **Joseph M Melby** (melbyjos@msu.edu), Michigan State University, and **Sanjay Kumar*** (sanjay_kumar@ucsb.edu), The University of California, Santa Barbara. *Asymptotic additivity of the Turaev-Viro invariants for a family of* 3*-manifolds*

In 2015, Chen and Yang conjectured and provided computational evidence that the asymptotics of the Turaev-Viro invariants of a hyperbolic 3-manifold evaluated at the root of unity $\exp(\frac{2\pi i}{r})$ have growth rates given by the hyperbolic volume. The conjecture was later extended in terms of the simplicial volume to manifolds which are not necessarily hyperbolic. Using elementary hyperbolic pieces obtained from a construction of Agol, we will build a family of 3-manifolds with arbitrarily large simplicial volume such that the simplicial volume is additive with respect to the elementary pieces. We will then show that the asymptotics of the Turaev-Viro invariants preserve the additivity under these gluings and satisfy the conjecture. This is joint work with Joseph Melby.
(Received September 20, 2021)

1174-57-9417 **Isaiah Alfred Martinez*** (martineziam217@gmail.com), University of California, Santa Barbara, **Zack Dooley** (dooleyz@reed.edu), Reed College, **Alexander Goldman** (agoldma1@skidmore.edu), Skidmore College, and **Wolfgang Joachim Allred** (wolfgang.allred@gmail.com), University of Utah. *Tri-plane diagrams for simple surface knots* Preliminary report.

Classical knot theorists consider one dimensional knots embedded in three-space and are typically concerned with distinguishing knots from one another by examining their invariants. Unfortunately, every one-dimensional knot in four-space is isotopic to an unknotted loop. Thankfully, a collection of two-dimensional surfaces, which we call surface knots, can be embedded in four-space and yield a rich field of study. We represent these surface knots via tri-plane diagrams, and have provided a list of the simplest knotted surfaces along with some of their invariants such as Euler characteristic, normal Euler number, and orientability. Additionally, we provide some preliminary results regarding some lower complexity surface knots, as well as determining the tri-plane crossing numbers of all non-orientable surface knots. (Received September 20, 2021)

1174-57-9438 **Michael N Johnson*** (johnmino@iu.edu), Indiana University, **Sireesh Vinnakota** (sireesh.vinnakota@cgu.edu), Claremont Graduate University, **Jeremy Ethan Gordon** (jegordon@hamilton.edu), Hamilton College, **Erik Beserra** (erikbeserra2@gmail.com), Reed College, **Aryan Bhatt** (aryan.bhatt06@myhunter.cuny.edu), Hunter College (CUNY), and **Aljohara Ahmed** (hea252@nyu.edu), New York University. *Stick index of n-component Brunnian Links* Preliminary report.

A Brunnian link is a nontrivial link that becomes a set of trivial unlinked circles if any one component is removed. The stick index, i.e. minimal number of sticks needed to construct a link, was given by Adams et. al for Brunnian links of 2 and 3 components as 8 and 12 respectively. We conjecture that the stick index of a 4 component Brunnian link is 20. We provide an upper bound of 20 by exhibiting two conformations and lower bounds under certain hypotheses. We indicate the ways in which the techniques we use can shed light on the n-component case. (Received September 20, 2021)

1174-57-9451 **Scott A. Taylor** (sataylor@colby.edu), Colby College, **Roman Aranda*** (romanaranda123@gmail.com), Binghamton University, **Puttipong Pongtanapaisan** (puttipong@usask.ca), University of Saskatchewan, and **Cindy Zhang** (szhang22@colby.edu), Colby College. *L-invariant for spun knots* Preliminary report.

One can think of trisections of 4-manifolds as the higher dimensional analog of Heegaard splittings in dimension 3. Inspired by the notion of distance for links in S^3, Blair, Campisi, Taylor, and Tomova introduced the L-invariant in 2020. This new invariant is a measure of complexity for embedded surfaces in S^4. This talk will describe estimates for the L-invariant for spun knots in S^4. This project is the result of an Undergraduate Research Experience at Colby College. (Received September 20, 2021)

1174-57-9452 **Lei Chen*** (chenlei@caltech.edu), Caltech. *Topological actions of diffeomorphism groups on manifolds* Preliminary report.

In this talk, I will discuss the conjecture that when M is an n-dim manifold, then the group of diffeomorphisms Diff(M) of M cannot act on (n-1)-dimensional manifolds. If we assume that the action is continuous, then we have a way to prove. However, we will focus on the case when the action is not necessarily continuous. This is partly a joint work with Kathryn Mann. (Received September 20, 2021)

1174-57-9458 **Nathaniel Ferguson** (nrferg21@colby.edu), Colby College. *Finiteness conjectures for the Kauffman bracket skein module*

The Kauffman bracket skein module (KBSM) of a 3-manifold M captures the space of all links in M. It is defined as a vector space over $\mathbb{Q}(A)$ generated by all links modulo the skein relations. The purpose of this talk is to discuss upper bounds for the dimension of the KBSM, particularly for an infinite family of Seifert fibered spaces. (Received September 20, 2021)

1174-57-9475 **Joshua Charles Pankau*** (joshuapankau@ou.edu), University of Oklahoma.
Pseudo-Anosov Stretch Factors Coming From Thurston's Construction

The Nielsen-Thurston classification of mapping classes tells us that, up to isotopy, every automorphism of a closed orientable surface S is either periodic, reducible, or pseudo-Anosov. The latter case has lead to a rich theory with applications in (among others) dynamical systems, number theory, and low dimensional topology. Associated to every pseudo-Anosov mapping class f is a real number $\lambda > 1$, called the *stretch factor* of f. Fried proved that every stretch factor is a bi-Perron unit and conjectured that every bi-Perron unit has a power that is a stretch factor of some pseudo-Anosov homeomorphism. In this talk, I will discuss my work on showing that Fried's conjecture is true for the class of Salem numbers, as well as classifying, up to power, the stretch factors coming from a construction of pseudo-Anosov maps due to Thurston, known colloquially as *Thurston's Construction.* (Received September 20, 2021)

1174-57-9565 **Avalon Vanis*** (avanis@brynmawr.edu), Bryn Mawr College, **Katherine Betts** (kbetts@furman.edu), Furman University, **Troy Larsen** (larsent22@mail.wlu.edu), Washington and Lee University, and **Jeffery Utley** (jutley8@vols.utk.edu), University of Tennessee, Knoxville. *The Tri-Pants Graph of the Twice-Punctured Torus*

We investigate the structure of the tri-pants graph, a simplicial graph introduced by Maloni and Palesi, whose vertices correspond to particular collections of homotopy classes of simple closed curves of the twice-punctured torus, called tri-pants, and whose edges connect two vertices whenever the corresponding pants differ by suitable elementary moves. In particular, by examining the relationship between the tri-pants graph and the dual of the Farey complex, we prove that the tri-pants graph is connected and it has infinite diameter. (Received September 20, 2021)

1174-57-9654 **Juanita Pinzon-Caicedo*** (jpinzonc@nd.edu), University of Notre Dame. *Trisections, contact structures, and covers.* Preliminary report.

Relative trisections induce open book decompositions on the bounding 3-manifolds and can thus be regarded as fillings of open book decompositions. A theorem of Thurston-Winkelnkemper shows that every open book decomposition supports a contact structure. In this talk I will describe an ongoing project that explores the connection between the topology of trisections and the geometry of the contact structure from the perspective of branched covers. (Received September 20, 2021)

1174-57-9723 **Linh Truong*** (tlinh@umich.edu), University of Michigan, and **Irving Dai** (ifdai@stanford.edu), Stanford University. *Homology concordance homomorphisms in knot Floer homology*

Two knots in homology 3-spheres are homology concordant if they are smoothly concordant in a homology cobordism. I will explain how to construct integer-valued homomorphisms from this group of knots up to homology concordance. This construction uses knot Floer homology and generalizes concordance homomorphisms for knots in the 3-sphere. (Received September 20, 2021)

1174-57-9811 **Elizabeth Aurora Vidaurre*** (elizabeth.vidaurre@gmail.com), Molloy College.
Combinatorial Conditions for Directed Collapsing

While collapsibility of CW complexes dates back to the 1930s, collapsibility of *directed* Euclidean cubical complexes has not been well studied to date. The classical definition of collapsibility involves certain conditions on pairs of cells of the complex. The direction of the space can be taken into account by requiring that the past links of vertices remain homotopy equivalent after collapsing. We call this type of collapse a *link-preserving* *directed collapse*. In the undirected setting, pairs of cells are removed that create a deformation retract. In the directed setting, topological properties—in particular, properties of spaces of directed paths—are not always preserved. We show that there are computationally simple conditions which preserve the topology of past links. Furthermore, we give conditions for when link-preserving directed collapses preserve the contractability and connectedness of spaces of directed paths. This is joint work with Robin Belton, Robyn Brooks, Stefania Ebli, Lisbeth Fajstrup, Brittany Terese Fasy, and Nicole Sanderson. (Received September 20, 2021)

1174-57-9893 **Cara Bennett*** (cara.bennett@gatech.edu), Georgia Institute of Technology, **Sarah Clarke** (sclarke@bowdoin.edu), Bowdoin College, **Jonah Mendel** (jonah_mendel@brown.edu), Brown University, and **Evan Bell** (belleva1@msu.edu), Michigan State University. *Knot Colorability and Maximum Knot Determinants* Preliminary report.

Knot theory is the study of embeddings of circles in $\mathbb{R}^3$, where such embeddings are called knots and links. Invariants are a means of differentiating knots and links of different types. We focus on two closely related such invariants, called p-colorability and the knot determinant. Grid diagrams are rectilinear representations of knots and links which provide novel methods for understanding and computing these invariants. The grid diagram perspective on determinants raises the natural question of determining the maximal knot or link determinant amongst links represented by grids of a fixed size. We connect this problem to the long-standing Hadamard's maximal determinant problem regarding determinants of matrices for which the entries are restricted to 0 or 1. After exploring these connections, we present our findings on lower and upper bounds for the maximal determinants of knots and links on grid diagrams of a fixed size. (Received September 21, 2021)

1174-57-9948 **Ryan Dickmann*** (dickmann@math.utah.edu), University of Utah. *Mapping class groups of surfaces with noncompact boundary components* Preliminary report.

We will discuss the widely unknown classification of orientable surfaces with noncompact boundary due to Brown and Messer and a new method for decomposing a surface with noncompact boundary components into simpler pieces. Then we will mention how this can be used to get general statements about the pure mapping class groups. (Received September 21, 2021)

1174-57-9997 **Priyam Patel** (patelp@math.utah.edu), University of Utah, and **Elizabeth Field*** (field@math.utah.edu), University of Utah. *Stable commutator length on big mapping class groups* Preliminary report.

In this talk, we will study stable commutator length on mapping class groups of certain infinite-type surfaces. In particular, we will show that stable commutator length defines a continuous function on the commutator subgroups of such infinite-type mapping class groups, and we will discuss other properties of the commutator subgroups as well as the abelianizations of certain big mapping class groups. These results apply to many popular infinite-type surfaces with locally coarsely bounded mapping class groups. (Received September 21, 2021)

1174-57-10003 **Marissa Loving** (mloving6@gatech.edu), Georgia Institute of Technology, Georgia Tech. *End-periodic homeomorphisms and volumes of mapping tori* Preliminary report.

In this talk, we will discuss mapping tori associated to irreducible, end-periodic homeomorphisms of certain infinite-type surfaces. Inspired by a theorem of Brock in the finite-type setting, we will relate the minimal convex core volume of such a mapping torus to the translation distance of its monodromy on (a certain subgraph of) the pants graph. (Received September 21, 2021)

1174-57-10033 **Samantha Allen*** (samantha.allen1@uga.edu), University of Georgia. *Obstructions for concordances between L-space knots and connected sums of torus knots*

Krcatovich showed that all L–space knots are prime. Thus no nontrivial connected sum of knots is an L–space knot. We consider instead concordances from L–space knots to connected sums. In particular, we examine concordances from L–space knots to connected sums of (possibly many) positive and negative torus knots. In this talk, I will discuss various obstructions for such concordances involving both classical invariants and those arising from Heegaard Floer theory. (Received September 21, 2021)

1174-57-10040 **Vinicius Ambrosi*** (vambrosi@indiana.edu), Indiana University. *Sutured Floer Homology of Handlebodies*

Sutured Floer homology was developed by Juhász as an extension of Heegaard Floer homology to manifolds with boundary. This theory can be used to understand sutured decompositions, foliations, and contact structures in 3-manifolds. However, we lack practical and fast tools to compute this invariant, especially when the boundary has genus greater than 1. I address this problem in a representative class of manifolds by describing an algorithm that computes the sutured Floer homology of handlebodies. This algorithm takes the Dehn-Thurston parameters for a suture as input and uses bordered sutured Floer homology to compute its result. In this talk, I will introduce this algorithm and discuss its applications. (Received September 21, 2021)

1174-57-10092 **Yvon Verberne*** (verberne.math@gmail.com), Georgia Institute of Technology, and **Assaf Bar-Natan** (assaf.bar.natan@mail.utoronto.ca), University of Toronto. *The grand arc graph* Preliminary report.

Abstract: One of the key tools to study surfaces of finite-type is the curve graph. Masur and Minsky showed that the curve graph is both infinite diameter and Gromov hyperbolic. Additionally, Masur and Minsky showed the curve graph's utility by using it to study the geometry of the mapping class group for surfaces of finite-type. Unfortunately, for surfaces of infinite-type the curve graph has diameter 2. In this talk, we introduce the grand arc graph and show that for large collections of infinite-type surfaces, the grand arc graph has infinite diameter and is Gromov hyperbolic. This work is joint with Assaf Bar-Natan. (Received September 21, 2021)

1174-57-10151 **Daniel Guyer** (guyerdm7106@uwec.edu), University of Wisconsin-Eau Claire. *Concordances of linear combinations of Torus knots to L-space knots* Preliminary report.

The study of classical knot theory was extended to four dimensions, via the knot concordance group, by Fox and Milnor in the 1960's. In the knot concordance group, we examine connected sums of torus knots and their mirrors, which we understand as a "linear combination" of torus knots. We continue the work of Allen and Livingston by determining when a linear combination of torus knots is concordant to an L-space knot. Using knot determinants and other tools, we exhibit more instances when a linear combination of torus knots is not concordant to any L-space knot. (Received September 21, 2021)

1174-57-10179 **Robyn Kaye Brooks*** (robyn.brooks@bc.edu), Boston College. *Generalized Arrow Polynomial Formulas for Finite Type Knot Invariants*

Finite type knot invariants stem from Chern–Simons theory, and were extensively studied by Kontsevich, Polyak and Viro, Bott and Taubes and many others. In this talk, I will explain how one may compute finite type invariants from a regular knot plane diagram. This is done by using Arrow polynomial formulation; i.e., a formula which counts certain sub-diagrams of crossings within a plane diagram, as originally developed in the work of Polyak and Viro. Further, I will discuss, how to extend these formulas to plane diagrams which admit triple and higher multiple crossings. I will illustrate these ideas in the case of the second coefficient of the Conway polynomial, also known as the Casson knot invariant. (Received September 21, 2021)

1174-57-10242 **Biji Wong*** (wongb@mpim-bonn.mpg.de), Max Planck Institute For Mathematics. *d-invariants of double branched covers of links*

Using Heegaard Floer homology, one can associate to a rational homology 3-sphere Y, equipped with a spin^c-structure $\mathfrak{s}$, a rational number, commonly referred to as the d-invariant of $(R,\mathfrak{s})$. d-invariants have been useful in answering a range of questions in low-dimensional topology. A nice source of rational homology 3-spheres comes from considering double branched covers $\Sigma 2(K)$ of knots K in S^3. If $\Sigma 2(K)$ is an L-space, then the d-invariant of $\Sigma 2(K)$, at the unique spin-structure, is well-understood: Lin-Ruberman-Saveliev in 2020 showed that it's a multiple of the signature of K. When the branch set is a quasi-alternating link L, the double branched cover $\Sigma 2(K)$ is an L-space and its d-invariants (at the various spin structures) can be recovered from the various signatures of L in a similar way; this is due to Lisca-Owens in 2015. In this talk, we show that a similar phenomenon holds for branching over certain Montesinos links, many of which are not quasi-alternating. This is work in progress with M. Marengon. (Received September 21, 2021)

1174-57-10343 **Francis Bonahon** (fbonahon@usc.edu), University of Southern California, and **Helen M Wong*** (hwong@cmc.edu), Claremont McKenna College. *A Volume Conjecture for a new invariant for mapping tori* Preliminary report.

We'll discuss a new quantum invariant for mapping tori for punctures surfaces that is based on the representation theory of the Kauffman bracket skein algebra. We conjecture that the asymptotic behavior of the quantum invariant detects the hyperbolic volume of the mapping torus, and present some experimental evidence for it. This is joint work with Francis Bonahon and Tian Yang. (Received September 21, 2021)

1174-57-10376 **Thomas Koberda*** (thomas.koberda@gmail.com), University of Virginia. *Critical regularity of group actions*

If G is a group of homeomorphisms of a smooth manifold M, it is natural to wonder the highest degree of regularity with which that group can act on M, i.e. the critical regularity of the group. I will survey some of what is known about critical regularity for groups acting on compact manifolds in dimension one, and give some perspectives on approaches to understanding critical regularity in higher dimensions. Much of this talk will represent joint work with Sang-hyun Kim. (Received September 21, 2021)

1174-57-10385 **Jared Tristan Miller*** (jmiller@math.fsu.edu), Florida State University. *Conjugating subgroups of $PGL(2,\mathbb{C})$ into $PGL(2,\mathbb{R})$* Preliminary report.

Projective transformations acting on $\mathbb{C}P^1$ may or may not be conjugate into $PGL(2,\mathbb{R})$. Since real transformations will preserve the standard copy of $\mathbb{R}P^1$, or equivalently a circle in $\mathbb{C}P^1$, and matrix similarity preserves eigenvalues, only matrices whose ratio of eigenvalues is real or of magnitude 1 have any chance to be conjugate into $PGL(2,\mathbb{R})$. In this talk we discuss when a finitely generated subgroup in $PGL(2,\mathbb{C})$ is conjugate into $PGL(2,\mathbb{R})$. (Received September 21, 2021)

1174-57-10412 **Christopher Cericola*** (cericola@seattleu.edu), Seattle University. *A New Unknotting Operation: The Arc Crossing Change* Preliminary report.

Any knot diagram can be unknotted if we are allowed to perform crossing changes, i.e. alter crossings to swap the under- and over-strands. Shimizu proved a rather more surprising result: that any diagram of any knot can be unknotted if we are allowed to perform region crossing changes, where all crossings along the boundary of a region are changed. In our work, we've developed a 1-dimensional analog—called the arc crossing change—to these 0- and 2-dimensional unknotting operations. In this talk, we present the arc crossing change and discuss its behavior along with the proof that it can unknot any knot. (Received September 21, 2021)

1174-57-10417 **Daniel López Neumann*** (dlopezne@indiana.edu), Indiana University. *Fox-calculus-twisted quantum invariants from G-crossed ribbon categories* Preliminary report.

Reidemeister torsion is a topological invariant of 3-manifolds that captures deep topological and geometric information. This invariant is defined through covering space theory and is usually computed from a presentation of the fundamental group by using Fox's free differential calculus. In this talk, we will describe a natural quantum topology setting for Reidemeister torsion. More generally, we explain how a certain G-crossed extension of the Drinfeld center of $\mathrm{Rep}(H)$ (where H is a finite dimensional Hopf algebra and $G = \mathrm{Aut}(H)$) leads to quantum invariants of knots which are "twisted" via Fox calculus. This invariant specializes to the (relative) twisted Reidemeister torsion of the knot complement when H is an exterior algebra. (Received September 21, 2021)

1174-57-10422 **Nancy Scherich*** (nancy.scherich@gmail.com), University of Toronto. *Homomorphisms of Braid Groups*

Much work has been done in the last few years using totally symmetric sets to study homomorphisms of braid groups. In this talk I will introduce totally symmetric sets and discuss how they are used to study finite quotients of braid groups as well as classify homomorphisms between braid groups. (Received September 21, 2021)

1174-57-10430 **Hemanth Kumar Mandya Nagaiah*** (ylx381@mocs.utc.edu), University of Tennessee at Chattanooga. *The Topology of the Meiotic Spindle* Preliminary report.

The meiotic spindle undergoes significant changes during cell division, using a complex mechanism that involves changes in the conformations of microtubules. In this talk will use tools from Topology to rigorously characterize the 3-dimensional conformation of microtubules in 3 stages of meiosis using experimental data obtained through electron tomography. Our results show that the geometry/topology and entanglement of microtubules changes throughout cell division and it depends on the location of the microtubules from the centrosomes. (Received September 21, 2021)

1174-57-10473 **Patricia Cahn*** (patricia.cahn@gmail.com), Smith College, **Jack Kendrick** (jkendrick@smith.edu), University of Washington, **Elise Catania** (emcatania19@gmail.com), University of Minnesota, **Sarangoo Chimgee** (schimgee@smith.edu), Smith College, and **Olivia Del Guercio** (delgur@gmail.com), Rice University. *Dihedral Linking Invariants*

Given a branched cover of 3-manifolds $f : M \to S^3$ with branching set a knot K, the set of linking numbers of the components of $f^{-1}(K)$ is an invariant of K. Linking numbers in branched covers arise in the Cappell-Shaneson formula for the Rokhlin invariant, Litherland's formula for the Casson-Gordon invariants, and Kjuchukova's Ξ invariants, via 4-dimensional constructions. We give an explicit algorithm for computing this linking invariant for dihedral covers of S^3 along K, and tabulate its values for knots up to 12 crossings, generalizing work of Perko on 3-fold dihedral covers. If time permits, we will discuss potential 4-dimensional analogs of the invariant. (Received September 21, 2021)

1174-57-10501 **Thang Tu Quoc Le** (letu@math.gatech.edu), Georgia Insitute of Technology, **Wade Bloomquist*** (wbloomquist3@gatech.edu), Georgia Tech, and **Hiroaki Karuo** (karu@kurims.kyoto-u.ac.jp), Research Institution for Mathematical Sciences, Kyoto University. *The quantum trace for LRY skein algebras*

The stated skein algebra of a punctured bordered surface is an $\mathcal{O}_{q^2}(SL(2))$ comodule algebra built out of stated tangles on the surface. We introduce a topologically motivated extension of stated skein algebras to punctured bordered surfaces with interior marked points which also generalizes the Roger-Yang skein algebras. We explore the structure and properties of this generalization while attempting to highlight the relevant interplay between topology and algebra. We will keep an eye on lessons that can be carried over to the study of skein algebras more generally. (Received September 21, 2021)

1174-57-10536 **Thomas Kindred** (kindret@wfu.edu), Wake Forest University, and **Nickolas Andres Castro*** (ncastro.math@rice.edu), Rice University. *Obstructing Relative Stabilizations of Trisected 4-manifolds with Boundary* Preliminary report.

A relative trisection of a compact, oriented, smooth 4-manifold with boundary X is a decomposition of X into three codimension-0 submanifolds which are all diffeomorphic to $\natural S^1 \times B^3$ and have certain nice intersection properties. Every compact, smooth 4-manifold admits such a decomposotion. An interesting feature of a relative trisection is that it induces an open book decomposition of ∂X. There is an operation of relative trisections called a *relative stabilization* which induces a Hopf plumbing of the induced open book decomposition. Any two relative trisections of a fixed X can be made isotopic after finitely many relative stabilizations of each. However, even if the induced open book decompositions of two relative trisections are related via a single Hopf plumbing, it is unlikely that the trisections are related via a single relative stabilization. In this talk, we will search for an obstruction to relatively destabilizing a trisected 4-manifold. (Received September 21, 2021)

1174-57-10616 **Vyacheslav Krushkal*** (krushkal@virginia.edu), University of Virginia, **Rostislav Akhmechet** (ra5aq@virginia.edu), University of Virginia, and **Peter K Johnson** (pkj4vj@virginia.edu), University of Virginia. *q-series invariants and lattice cohomology of 3-manifolds*

This talk will define an invariant of negative definite plumbed 3-manifolds, equipped with a spin^c-structure. Its relation will be discussed to two theories with rather different origins and structures, lattice cohomology and certain q-series which recover SU(2) quantum invariants of 3-manifolds at roots of unity. (Received September 21, 2021)

1174-57-10632 **Hongtaek Jung** (htjung1905@gmail.com), Center for Geometry and Physics, Institute for Basic Science, and **Shelly Harvey** (shelly@rice.edu), Rice University. *Knotted spheres and standard projective planes* Preliminary report.

Viro provided an example of a knotted sphere K with the property that its connected sum with a standard projective plane is isotopic to a standard projective plane. We prove that this property holds for any ribbon 2-knot of 1-fusion with determinant 1, and more generally that the determinant completely classifies when 1-fusion 2-knots become isotopic upon connected sum with a standard projective plane. Conversely, we show that Suciu's examples, an infinite collection of ribbon 2-knots of 2-fusion, all with the trefoil group, remain distinct after connected sum with a standard projective plane. This is joint work with Shelly Harvey, Hongtaek Jung, and Seungwon Kim. (Received September 21, 2021)

1174-57-10678 **Jenya Sapir*** (esapir@binghamton.edu), Binghamton University, and **Sebastian Hensel** (hensel@math.lmu.de), Universitat Muenchen (Munich). *A projection from geodesic currents to Teichmuller space*

Given a genus g surface S, we consider the space of projective geodesic currents $\mathbb{P}\mathcal{C}(S)$. This space contains many objects of interest in low dimensional topology, such as the set of all closed curves on S up to homotopy, the set of all marked, negatively curved metrics on S, as well as some higher Teichmuller spaces. We show that there is a mapping class group invariant, length minimizing projection from the space of filling projective currents onto Teichmuller space, and that this projection is continuous and proper. This is joint work with Sebastian Hensel. (Received September 21, 2021)

1174-57-10679 **Andrew Paul*** (anp004@ucsd.edu), University of California, San Diego. *Topological Properties of Almost Abelian Lie Groups* Preliminary report.

The central object of Lie theory is the Lie group, which is a group that is also a smooth manifold such that the group operation and inversion map are both smooth functions on the Lie group. Our work focuses on the topological properties of a certain family of Lie groups. If a Lie group is non-Abelian but has a codimension 1

Abelian subgroup, we say that the Lie group is almost Abelian. We show that all discrete subgroups of complex simply connected almost Abelian groups are finitely generated. We then prove that no complex connected almost Abelian group is compact and give conditions for the compactness of connected subgroups of such groups. It is a known result in Lie theory that there is an isomorphism between the homotopy groups of a Lie group and those of its maximal compact subgroup, so we study the maximal compact subgroup of complex connected almost Abelian groups as a first step towards determining their homotopy type.

As Lie groups are both algebraic and geometric objects, it is a recurring theme in Lie theory to use algebraic techniques to study the geometry of Lie groups. In our work, the topology of connected almost Abelian Lie groups is studied by expressing each connected almost Abelian Lie group as a quotient of its universal covering group by a discrete normal subgroup. This proves to be a fertile approach and allows us to probe the various topological questions we ask of almost Abelian Lie groups. (Received September 21, 2021)

1174-57-10695 **Casandra D. Monroe*** (cmonroe@utexas.edu), University of Texas - Austin.
Deformations of hyperbolic structures on manifolds

Hyperbolic geometry has proved to be an incredibly useful tool for understanding manifolds, and especially surfaces and knot complements. But hyperbolic structures on manifolds are just one example of how we can use geometry to explore topology. In this talk, we'll deform complete $\mathbb{H}^n$ structures to arrive at other interesting (G, X)-structures. We'll go over a few different kinds of these deformations, such as bending and flexing, and examine the resulting structures. (Received September 21, 2021)

1174-57-10696 **Kate Bernklau Halvor** (kbernklauhalvor@smith.edu), Smith College, **Hannah Gong** (hgong@smith.edu), Smith College, **Annika Gonzalez-Zugasti** (agonzalezzugasti@smith.edu), Smith College, **Anna Lowery** (alowery@smith.edu), Smith College, and **Stephanie Oh*** (soh27@smith.edu), Smith College. *The Dihedral Genus of p-Colorable Knots* Preliminary report.

Given a Fox p-colorable knot K, we study the minimum genus of an orientable surface in B^4 with boundary K, over which a coloring of K extends. We find bounds on this genus for knots with low crossing number. We also describe families of knots for which this genus is equal to the ordinary 4-genus of K. (Received September 21, 2021)

1174-57-10713 **David Ben-Zvi** (benzvi@math.utexas.edu), University of Texas, Austin, **David Jordan** (djordan@ed.ac.uk), University of Edinburgh, **Pavel Safronov** (p.safronov@ed.ac.uk), University of Edinburgh, and **Sam Gunningham*** (sam.gunningham@montana.edu), Montana State University. *Langlands Duality for Skein Modules* Preliminary report.

Skein modules are linear spaces spanned by knots and links in a 3-manifold, modulo certain skein relations. They were defined about 30 years ago independently by Przytycki and Turaev and have been extensively studied in the subsequent years. In this talk I will propose a new role for skein modules: as (a component of) the state space of a certain 4-dimensional topological quantum field theory, which according to the work of Kapustin and Witten, encodes the mathematical features of the geometric Langlands program. This realization leads to some surprising conjectures (which can be directly verified in some key cases), relating two quite different flavors of skein module on a given closed 3-manifold. (Received September 21, 2021)

1174-57-10839 **Abhishek Mallick** (mallick@mpim-bonn.mpg.de), Max Planck Institute for Mathematics.
Equivariant concordance and knot Floer homology

Following the work of Juhász-Zemke and Hendricks-Manolescu-Zemke, we define several equivariant concordance invariants using knot Floer homology. We show that our invariants provide a bound for the equivariant genus and use these to give a family of strongly invertible slice knots whose equivariant genus grows arbitrarily large, answering a question of Boyle-Issa. We also apply our formalism to some non-equivariant questions, and recover some recent results by Miller-Powell regarding stabilization distance and Hayden regarding exotic slice disks. (Received September 21, 2021)

1174-57-10999 **Clayton McDonald*** (clayton@math.ucdavis.edu), University of California, Davis.
Doubly slice Montesinos links

A knot in the 3-sphere is said to be slice if it is the cross section of a sphere in the 4-sphere, and it is said to be doubly slice if that sphere can be made to be unknotted in the 4-sphere. In this talk, we will discuss generalizations of this notion to links, as well as some relevant constructions and obstructions. This is joint work with Duncan McCoy. (Received September 21, 2021)

1174-57-11023 **Julia Elisenda Grigsby*** (grigsbyj@bc.edu), Boston College, **Kathryn Anne Lindsey** (kathryn.a.lindsey@gmail.com), Boston College, and **Marissa Masden** (mmasden@uoregon.edu), University of Oregon. *Local and Global Topological Complexity Measures of ReLU neural network functions*

It is well-known that the class of functions $\mathbb{R}^n \to \mathbb{R}$ realizable by a parameterized feedforward ReLU neural network is precisely the class of piecewise linear functions with finitely many pieces. What is less well-understood is how network architecture impacts expressiveness.

We introduce some natural local and global topological complexity measures associated to a feedforward ReLU neural network function and initiate a theoretical investigation of how these complexity measures behave with respect to the network architecture. We focus on explicit qualitative and quantitative differences between shallow and deep networks. (Received September 21, 2021)

1174-57-11031 **Siddhi Krishna*** (siddhi@math.columbia.edu), Columbia University. *Taut foliations and braid positivity* Preliminary report.

The L-space conjecture predicts a surprising relationship between the algebraic, geometric, and Floer-homological properties of a 3–manifold Y. In particular, it predicts exactly which 3-manifolds admit a "taut foliation". In this talk, I'll discuss work investigating these connections. In particular, I'll discuss a strategy for building taut foliations manifolds obtained by Dehn surgery along knots realized as closures of "positive braids". As an application, I will show how taut foliations can be used to obstruct braid positivity for cable knots, and produce a new unknot detector along the way. No background in foliation or Floer homology theories will be assumed. All are welcome! (Received September 21, 2021)

1174-57-11051 **Alexandra Kjuchukova** (Akjuchuk@nd.edu), University of Notre Dame, and **Nathan Sunukjian** (nss@calvin.edu), Calvin University. *Brunnian exotic surface links in the 4-ball*

One of the motivating principles of 4-dimensional topology is to understand exotic phenomena, namely differences between the smooth and topological categories. I'll present a strategy for constructing, for any positive integer n, pairs of n-component surface links in the 4-ball which are exotic in the strongest possible sense: they are topologically ambiently isotopic rel. boundary, but there is no ambient diffeomorphism carrying one to the other. These examples are also "Brunnian" – every proper sublink is an unlink – thereby providing an example of subtle exotic behavior. This is joint work with Kyle Hayden, Alexandra Kjuchukova, Maggie Miller, Mark Powell, and Nathan Sunukjian. (Received September 21, 2021)

1174-57-11080 **Ben Bruncati** (kbruncati@smith.edu), Smith College, **Page Wilson*** (pmwilson@smith.edu), Smith College, and **Mariem Snoussi** (msnoussi@smith.edu), Smith College. *Knots and Permutations* Preliminary report.

Given a diagram of a knot K, we label its arcs by permutations according to the Wirtinger relations. We determine all such labellings for certain families of knots, with restrictions on the types of permutations considered. (Received September 21, 2021)

1174-57-11189 **Sherry Gong*** (sgongli@tamu.edu), Texas A&M University, and **Marco Marengon** (marengon@renyi.hu), Alfréd Rényi Institute for Mathematics. *Non-orientable link cobordisms and torsion order in Floer homologies*

In a recent paper, Juhasz, Miller and Zemke proved an inequality involving the number of local maxima and the genus appearing in an oriented knot cobordism using a version of knot Floer homology. In this talk I will be discussing some similar inequalities for non-orientable knot cobordisms using the torsion orders of unoriented versions of knot Floer homology and instanton Floer homology. This is a joint work with Marco Marengon. (Received September 21, 2021)

1174-57-11260 **Viridiana Jasmin Neri*** (vjn2108@columbia.edu), Columbia University, and **Siddhi Krishna** (iddhi@math.columbia.edu), Columbia University. *Quantitative properties of 1-bridge braids* Preliminary report.

Low-dimensional topology, a subfield of pure mathematics, studies 1-, 2-, 3-, and 4-dimensional spaces. One way to study these spaces is by using knots and braids, which are one dimensional objects. In general, it is difficult to show that two knots are the same. In order to tackle this task, topologists often use knot invariants to instead decide when two knots are different. One example of a knot invariant is the braid index, the minimal number of strands required to represent a knot as the closure of a braid on that many strands. We determined a knot invariant called the braid index for a special family of knots known as 1-bridge braids, denoted K(w,b,t). Our main theorem gives a formula for the braid index of 1-bridge braids in terms of their defining parameters, w,b, and t. We also present some directions for future work. (Received September 22, 2021)

1174-57-11747 **Constantin Teleman*** (teleman@berkeley.edu), UC Berkeley. *Character theory for categorical group representations* Preliminary report.

I describe a variant of the category proposed by Kapustin, Rozansky, and Saulina for complex integral elements systems which, applied to the Toda system, describes a calculus of categories with topological action of compact groups. Applications to gauge theory in dimensions 2 and 3 will be reviewed quickly. (Received October 1, 2021)

58 ► Global analysis, analysis on manifolds

1174-58-5739 **Mary R Sandoval*** (mary.sandoval@trincoll.edu), Trinity College. *Do the Hodge spectra distinguish orbifolds from manifolds? Part 1* Preliminary report.

Given a compact Riemannian orbifold, we consider the spectrum of the Hodge Laplacian on p-forms. We compute the heat invariants of the p-spectrum, which depend on the codimension of the singular sets of the orbifold. We show that by comparing the heat invariants for the 0-spectrum and the 1-spectrum together these spectral data are enough to distinguish orbifolds from manifolds whenever the codimension of the singular sets is less than or equal to 3. This is enough to detect the presence of singular sets in orbifolds of dimension less than or equal to 3. This is based on joint work with Katie Gittens, Carolyn Gordon, Magda Khalile, Ingrid Membrilllo-Solis, and Elizabeth Stanhope. (Received August 26, 2021)

1174-58-7537 **Jing Wang*** (jingwang@purdue.edu), Purdue University, and **Jeremy Tyson** (tyson@illinois.edu), University of Illinois at Urbana Champaign. *A sub-Riemannian Steiner's formula and the heat content problem on $SU(2)$ and $SL(2)$* Preliminary report.

In this talk we will discuss small time asymptotic expansion of the heat content for a smoothly bounded, non-characteristic domain in $SU(2)$ or $SL(2)$. Such expansion captures geometric information of the domain including perimeter and the total horizontal mean curvature of its boundary. The key step is to obtaining a sub-Riemannian Steiner's formula for the domain. We then use a probabilistic method that involves the escaping probability of a horizontal Brownian motion on the underlying space. This is a joint work with J. Tyson. (Received September 14, 2021)

1174-58-7559 **Luca Spolaor*** (lspolaor@ucsd.edu), University of California, San Diego. *Minimizing currents mod p*

In this talk I will introduce area minimizing currents mod p, explain why they are useful and describe some recent development on their regularity theory. This is based on joint work with C. De Lellis, J. Hirsch, A. Marchese and S. Stuvard (Received September 15, 2021)

1174-58-8268 **Ingrid Membrillo Solis*** (i.membrillo-solis@soton.ac.uk), University of Southampton. *Do the Hodge spectra distinguish orbifolds from manifolds? Part 2.*

Orbifolds can be thought of as generalisations of manifolds which have well-structured singularities. As these singularities are defining characteristics of the class of orbifolds, one might ask:

(1) Do spectral data distinguish orbifolds with singularities from manifolds?

(2) What geometric and topological information on the set of singularities can be obtained from spectral data?

Using the heat invariants discussed in Part 1, we address both questions (1) and (2) for individual p-spectra. For example, we give conditions on the codimension of the set of orbifold singularities which guarantee that the volume of the singular set is determined by spectra. This is based on joint work with Katie Gittins, Carolyn Gordon, Magda Khalile, Juan Pablo Rossetti, Mary Sandoval, and Elizabeth Stanhope. (Received September 18, 2021)

1174-58-8985 **Daniel Solano*** (daniel_solano@brown.edu), Brown University, and **Laurent Younes** (laurent.younes@jhu.edu), Johns Hopkins University. *METRIC REGISTRATION FOR CURVES AND SURFACES OF DIFFERENT TOPOLOGIES*

This project presents an important step in shape analysis of submersed curves and surfaces to extend shape comparison to shapes with different topologies. The usual construction of shape space metrics for submersed curves and surfaces are induced from Riemannian metrics defined on the ambient space. These Riemannian distances are computed by solving geodesic matching problems, which cannot be completed in the case of shapes of different topology. This presentation focuses on a relaxation of the matching problem by flowing both shapes in question into auxiliary shapes that are close to each other up to a certain distance, whether that be a varifold, current, or Hausdorff metric. The resulting problem is an optimal control theory problem which balances the energy spent to make the two auxiliary shapes and the size of the topological jump, also called a surgery, as measured by the chosen metric. Shape analysis has already found applications in medical imaging in the field of computational anatomy, where patient anatomical data is compared to healthy templates; allowing for topological differences will expand the scope of the comparative analysis in this application. Future directions include expanding this model to account for more than one topological jump and additional auxiliary shapes as well as modelling the continuous change of topology numerically and theoretically. (Received September 20, 2021)

1174-58-9340 **Christine Guenther*** (guenther@pacificu.edu), Pacific University. *Sectoriality of the Laplacian on Asymptotically Hyperbolic Manifolds* Preliminary report.

We derive a new estimate for the resolvent of the Laplacian on asymptotically hyperbolic manifolds. As an application we prove that if a solution to the Ricci flow starting at an asymptotically hyperbolic metric exists for all time and converges to the standard hyperbolic metric, then solutions that start at nearby geometries must also converge. (Received September 20, 2021)

1174-58-9575 **Magdalena Czubak** (czubak@math.colorado.edu), University of Colorado Boulder, **Padi Fuster Aguilera*** (padi.fuster@colorado.edu), University of Colorado Boulder, and **Chi Han Chan** (cchan@math.nctu.edu.tw), National Yang Ming Chiao Tung University. *Stokes's Paradox on the Hyperbolic Plane* Preliminary report.

For a long cylinder moving slowly through an incompressible, viscous fluid in the direction perpendicular to the axis of the cylinder, there is no non-trivial solution for the Stokes equation. This is known as the Stokes' Paradox. Chan and Czubak showed that when formulating the problem in the hyperbolic plane there exists a non-trivial solution of the problem. We will discuss the different ways of posing this problem on manifolds, as well as the decomposition of the function spaces of the solution on non-compact manifolds. (Received September 20, 2021)

1174-58-10498 **Jie Qing*** (qing@ucsc.edu), University of California, Santa Cruz, and **Shiguang Ma** (msgdyx8741@nankai.edu.cn), Nankai University. *On n-superharmonic functions and geometric applications*

In this talk we will report our recent work on n-superharmonic functions and geometric applications. We first will introduce the n-Laplace equations in conformal geometry. Then we will report our results that are inspired by Arsove-Huber and Huber on asymptotic behaviors of n-superharmonic functions and finite point conformal compactifications. These results are presented in our papers that have posted on arXiv. (Received September 21, 2021)

1174-58-10920 **Kamryn Spinelli** (kamrynspinelli@gmail.com), Worcester Polytechnic Institute, Brandeis University, **Connor Charles Anderson*** (anderson.con@northeastern.edu), Worcester Polytechnic Institute, Northeastern University, and **Xavier Ramos Olivé** (xramosolive@wpi.edu), Worcester Polytechnic Institute. *Manifolds with bounded integral curvature and no positive eigenvalue lower bounds*

We provide an explicit construction of a sequence of closed surfaces with uniform bounds on the diameter and on L^p norms of the curvature, but without a positive lower bound on the first non-zero eigenvalue of the Laplacian λ_1. This example shows that the assumption of smallness of the L^p norm of the curvature is a necessary condition to derive Lichnerowicz and Zhong-Yang type estimates under integral curvature conditions. (Received September 21, 2021)

1174-58-11230 **Laura Mora*** (Lmora@smith.edu), Smith College. *Flow of quasiconformal maps*

A map u is conformal if $du^t du$ is a scalar multiple of the identity matrix. Intuitively, conformal maps preserve angles and infinitesimal circles, meaning that the image of a curve doesn't get perturbed "too much". We consider deformations of a map $u : \Omega \to \mathbb{R}^2$ from a region Ω in the plane that make u more and more conformal. We discuss computational results and also special cases in which we can define and analyze a flow with a partial solution. (Received September 22, 2021)

1174-58-11771 **Nalini Anantharaman** (anantharaman@math.unistra.fr), Université de Strasbourg, and **Mostafa Sabri** (mmsabri@sci.cu.edu.eg), Cairo University. *Quantum ergodicity on graphs.*

Quantum ergodicity is an expression of the notion that for well behaved Riemannian manifolds, eigenfunctions become equidistributed in the high energy limit. We introduce the history of the equivalent notion on graphs, and how to show equidistribution of eigenvectors. (Received October 20, 2021)

60 ► Probability theory and stochastic processes

1174-60-5573 **Shusen Pu*** (shusen.pu@vanderbilt.edu), Vanderbilt University. *Stochastic Hodgkin-Huxley Models* Preliminary report.

The Hodgkin-Huxley (HH) model offers a continuous-time dynamical system to approximate the electrical characteristics of excitable cells such as neurons and cardiac myocytes. The HH framework suggested an underlying system of gating variables consistent with a multistate Markov process description. While Markov chain (MC) methods are computationally expensive, simulations based on Langevin approximations can capture the effects of stochastic ion channel fluctuations with reasonable accuracy and excellent computational efficiency. Here we discuss various ways of incorporating noise based on the HH framework and efficient approximations to the MC models. Particularly, we advocate for a class of models combining the best features of Langevin models with the recently developed stochastic shielding approximation. We proved that, given identical boundary conditions, our 14×28D model is pathwise equivalent both to Fox and Lu's original Langevin model and its several variants. Based on the stochastic HH framework, we proved that the numerically calculated variance in inter-spike intervals (ISIs) can be quantified from the molecular-level fluctuations of the stochastic HH kinetics. We prove in theory, and confirm via numerical simulations, that in the limit of small noise, the variance of the iso-phase intervals decomposes linearly into a sum of contributions from each edge. We show numerically that the same decomposition affords an efficient and accurate estimation procedure for the ISIs. (Received September 21, 2021)

1174-60-5667 **Steven Michael Heilman*** (stevenmheilman@gmail.com), University of Southern California Mathematics, and **Alex Tarter** (atarter@usc.edu), University of Southern California Mathematics. *Three Candidate Plurality is Stablest for Small Correlations*

Using the calculus of variations, we prove the following structure theorem for noise stable partitions: a partition of n-dimensional Euclidean space into m disjoint sets of fixed Gaussian volumes that maximize their noise stability must be $(m-1)$-dimensional, if $m-1 \leq n$. In particular, the maximum noise stability of a partition of m sets in $\mathbb{R}^n$ of fixed Gaussian volumes is constant for all n satisfying $n \geq m-1$. From this result, we obtain:

i A proof of the Plurality is Stablest Conjecture for 3 candidate elections, for all correlation parameters ρ satisfying $0 < \rho < \rho_0$, where $\rho_0 > 0$ is a fixed constant (that does not depend on the dimension n), when each candidate has an equal chance of winning.

ii A variational proof of Borell's Inequality (corresponding to the case $m = 2$).

The structure theorem answers a question of De-Mossel-Neeman and of Ghazi-Kamath-Raghavendra. Item (i) is the first proof of any case of the Plurality is Stablest Conjecture of Khot-Kindler-Mossel-O'Donnell (2005) for fixed ρ, with the case $\rho \to 1^-$ being solved recently. Item (i) is also the first evidence for the optimality of the Frieze-Jerrum semidefinite program for solving MAX-3-CUT, assuming the Unique Games Conjecture. Without the assumption that each candidate has an equal chance of winning in (i), the Plurality is Stablest Conjecture is known to be false. (Received August 24, 2021)

1174-60-5673 **Yizhe Zhu*** (yiz084@ucsd.edu), University of California Irvine. *Community detection in sparse random hypergraphs*

The stochastic block model has been one of the most fruitful research topics in community detection and clustering. We consider the community detection problem in a sparse random tensor model called the hypergraph stochastic block model. Angelini et al. (2015) conjectured a threshold for detecting the community structure in this model, and we confirmed the positive part of the phase transition in the 2-block case. We introduced a matrix that counts self-avoiding walks on random hypergraphs, whose leading eigenvectors give a correlated reconstruction of the community. (Received August 24, 2021)

1174-60-6048 **Ioannis Karatzas*** (ik@math.columbia.edu), Columbia University. *TRAJECTORIAL APPROACH TO ENTROPIC GRADIENT FLOWS FOR CONSERVATIVE DIFFUSIONS*

We provide a detailed, probabilistic interpretation for the variational characterization of conservative diffusion as entropic gradient flow. Using a numerical scheme, Jordan, Kinderlehrer, and Otto showed in 1998 that, for diffusions of Langevin-Smoluchowski type, the Fokker-Planck probability density flow minimizes the rate of relative entropy dissipation, as measured by the distance traveled in terms of the quadratic Wasserstein metric in the ambient space of configurations. We use a very direct perturbation analysis to obtain novel, stochastic-process versions of such features; these are valid along almost every trajectory of the motion in both the forward and, most transparently, the backward, directions of time. The original results follow then simply by taking expectations. As a bonus, a version of the HWI inequality of Otto & Villani relating relative entropy, Fisher information, and Wasserstein distance, literally falls in our lap.

(Joint work with W. Schachermayer and B. Tschiderer, from Vienna. Available as arXiv:2008.09220) (Received September 3, 2021)

1174-60-6096 **Noga Alon** (nogaa@post.tau.ac.il), Princeton University, **Colin Defant*** (cdefant@princeton.edu), Princeton University, and **Noah Kravitz** (nbkravitz@gmail.com), Princeton University. *The Runsort Permuton*

Suppose we choose a permutation π uniformly at random from S_n. Let $\mathsf{runsort}(\pi)$ be the permutation obtained by sorting the ascending runs of π into lexicographic order. Alexandersson and Nabawanda recently asked if the plot of $\mathsf{runsort}(\pi)$, when scaled to the unit square $[0,1]^2$, converges to a limit shape as $n \to \infty$. We answer their question by showing that the measures corresponding to the scaled plots of these permutations $\mathsf{runsort}(\pi)$ converge with probability 1 to a permuton (limiting probability distribution) that we describe explicitly. In particular, the support of this permuton is $\{(x,y) \in [0,1]^2 : x \leq ye^{1-y}\}$. (Received September 4, 2021)

1174-60-6361 **Jeremy Seeman*** (jhs5496@psu.edu), Penn State University, **Matthew Reimherr** (mlr36@psu.edu), Penn State University, and **Aleksandra Slavkovic** (abs12@psu.edu), Penn State University. *Exact Privacy Guarantees for Markov Chain Implementations of the Exponential Mechanism*

Implementations of the exponential mechanism in differential privacy often require sampling from intractable distributions. When approximate procedures like Markov chain Monte Carlo (MCMC) are used, the end result incurs costs to both privacy and accuracy. Existing work has examined these effects asymptotically, but implementable finite sample results are needed in practice so that users can specify privacy budgets in advance and implement samplers with exact privacy guarantees. In this paper, we use tools from ergodic theory and perfect simulation to design exact finite runtime sampling algorithms for the exponential mechanism by introducing an intermediate modified target distribution using artificial atoms. We propose an additional modification of this sampling algorithm that maintains its ϵ-DP guarantee and has improved runtime at the cost of some utility. We then compare these methods in scenarios where we can explicitly calculate a δ cost (as in (ϵ, δ)-DP) incurred when using standard MCMC techniques. Much as there is a well known trade-off between privacy and utility, we demonstrate that there is also a trade-off between privacy guarantees and runtime. (Received September 8, 2021)

1174-60-6370 **Marcus Michelen*** (michelen.math@gmail.com), University of Illinois at Chicago. *Roots of random polynomials near the unit circle*

It is a well-known (but perhaps surprising) fact that a polynomial with independent random coefficients has most of its roots very close to the unit circle. Using a probabilistic perspective, we understand the behavior of roots of random polynomials exceptionally close to the unit circle and prove several limit theorems; these results resolve conjectures of Shepp and Vanderbei. We will also discuss how our techniques provide a heuristic, probabilistic explanation for why random polynomials tend to have most roots near the unit circle. Based on joint work with Julian Sahasrabudhe (Cambridge). (Received September 8, 2021)

1174-60-6414 **Hong Qian*** (hqian@uw.edu), University of Washington. *A New Mathematical Foundation for Gibbs' Theory of Statistical Thermodynamics?* Preliminary report.

Like classical mechanics (CM), quantum mechanics (QM), and the theory of relativity (ToR), statistical thermodynamics is one of the greatest theories of physics. It is the branch that is the most relevant to our complex daily lives, with wide applications in chemistry and biology. Yet, while it is well understood that the mathematical foundations for CM, QM, and ToR are, respectively, calculus and differential equations, linear algebra and its infinite-dimensional counterpart functional analysis, and differential geometry, the mathematics for Gibbs'

theory of statistical thermodynamics has not been clearly presented to the applied mathematics community. In this talk, I shall attempt to present a coherent applied probability approach to this important subject. We shall discuss the theory of large deviations and how the concept of "entropy" arises through an asymptotic theorem. We show that Gibbs thermodynamic entropy is to Shannon's information entropy what Cramer's theorem (1938) is to Sanov's theorem (1957). The large deviations theory implies a duality symmetry via a Legendre-Fenchel transform (LFT); applying this idea shows that the Helmholtz free energy, the most important concept in Gibbs' theory, when viewed as a function of inverse temperature is dual to Gibbs entropy, and when viewed as a functional of the energy function is dual to Shannon-Sanov entropy. The key concept of an equation of state in thermodynamics can be understood as a mathematical relation between conjugate variables of the LFT. (Received September 8, 2021)

1174-60-7101 **David Thomas Renfrew*** (renfrew@math.binghamton.edu), Binghamton University. *Singularities in the spectrum of random block matrices*

We consider the density of states of structured Hermitian random matrices with a variance profile. As the dimension tends to infinity the associated eigenvalue density can develop a singularity at the origin. The severity of this singularity depends on the relative positions of the zero submatrices. We provide a classification of all possible singularities and determine the exponent in the density blow-up. (Received September 12, 2021)

1174-60-7111 **Stewart N Ethier*** (ethier@math.utah.edu), University of Utah, and **Persi Diaconis** (diaconis@math.stanford.edu), Stanford University. *Three-Player Gambler's Ruin and the ICM*

Consider gambler's ruin with three players, 1, 2, and 3, having initial positive-integer capitals A, B, and C units. At each round a pair of players is chosen (uniformly at random) and a fair coin flip is made resulting in the transfer of one unit between these two players. Eventually, one of the players is eliminated and play continues with the remaining two. This model dates back to Bachelier (1912). Let $\sigma \in S_3$ be the elimination order (e.g., $\sigma = 132$ means player 1 is eliminated first and player 3 is eliminated second, leaving player 2 with $A+B+C$ units). We seek approximations for the elimination order probabilities $P_{A,B,C}(\sigma)$. Exact, as well as arbitrarily precise, computation of these probabilities is possible when $N := A+B+C$ is not too large. Linear interpolation can then give reasonable approximations for large N. One frequently used approximation, the independent chip model (ICM), is shown to be inadequate, but it may suffice in the context of poker tournaments. (Received September 12, 2021)

1174-60-7141 **James Allen Fill*** (jimfill@jhu.edu), The Johns Hopkins University, Department of Applied Mathematics and Statistics. *Breaking Multivariate Records*

For general dimension d, we identify, with proof, the asymptotic conditional distribution of the number of (Pareto) records broken by an observation given that the observation sets a record.

Fix d, and let $\mathcal{K}(d)$ be a random variable with this distribution. We show that the (right) tail of $\mathcal{K}(d)$ satisfies

$$\mathbb{P}(\mathcal{K}(d) \geq k) \leq \exp\left[-\Omega\left(k^{(d-1)/(d^2-2)}\right)\right]$$

and

$$\mathbb{P}(\mathcal{K}(d) \geq k) \geq \exp\left[-O\left(k^{1/(d-1)}\right)\right],$$

in each case as $k \to \infty$.

When $d = 2$, the description of $\mathcal{K}(2)$ in terms of a Poisson process agrees with the main result from Fill [*Comb. Probab. Comput.* 30 (2021) 105–123] that the distribution of $\mathcal{K}(2)$ is Geometric(1/2) with support $\{0, 1, \ldots\}$.

We show that $\mathbb{P}(\mathcal{K}(d) \geq 1) = \exp[-\Theta(d)]$ as $d \to \infty$; in particular, $\mathcal{K}(d) \to 0$ in probability as $d \to \infty$. (Received September 13, 2021)

1174-60-7156 **Anant Godbole*** (godbolea@etsu.edu), ETSU, and **Aradhana Soni** (aradhanasoni04@gmail.com), University of Tennessee, Knoxville. *An Analysis of the First Passage to the Origin Distribution* Preliminary report.

What is the probability that in a fair coin toss game (a simple random walk) we go bankrupt in n steps when there is an initial lead of some known or unknown quantity \$$m$? What is the distribution of the number of steps N that it takes for the lead to vanish? This talk explores some of the features of this first passage to the origin (or gambler's ruin) distribution, including the mode and quantiles. First, we explore the distribution of N when m is known, from an analytic/distribution theoretic (rather than combinatorial) viewpoint. Next, we compute the maximum likelihood estimator of m for a fixed n and also the posterior distribution of m when we are given that m follows some known prior distribution. (Received September 13, 2021)

1174-60-7239 **Louigi Addario-Berry*** (louigi.addario@mcgill.ca), McGill University, **Jessica Lin** (jessica.lin@mcgill.ca), McGill University, and **Erin Beckman** (beckman.erin@gmail.com), Concordia University. *A connection between RDEs and PDEs, via tree-indexed random processes.*

I will present a connection between a family of recursive distributional equations (RDEs) we call *cooperative motion*, and several PDEs: the Hamilton–Jacobi equation

$$u_t = -\sigma |u_x|^{m+1}\,,$$

the scalar conservation law

$$v_t = -\sigma (v^{m+1})_x\,,$$

the porous medium equation

$$u_t = \Delta(u^{m+1})\,,$$

and the parabolic p-Laplace equation

$$v_t - \frac{1}{2}\left(|v_x|^m v_x\right)_x = 0.$$

We establish distributional limits for the RDEs, proving distributional convergence to a variety of different Beta distributions (depending on the parameters of the process).

Our arguments proceed by applying numerical stability results for the corresponding PDEs (in particular using results of Crandall and Lions (1984) and of Barles and Souganidis (1991)). This sort of connection between numerical PDEs and probability limit theorems seems powerful; we hope that our results only scratch the surface of its usefulness. (Received September 13, 2021)

1174-60-7271 **Reza Gheissari*** (gheissari@berkeley.edu), UC Berkeley. *Sampling from Potts on random graphs via random-cluster dynamics*

We consider the problem of sampling from the Potts model on sparse random graphs in their high-temperature regime. The approach we take is via the random-cluster model, itself a well-studied model of dependent bond percolation that is coupled in a precise manner to the Potts model. We show that the Glauber dynamics of the random-cluster model on a general family of random graphs of bounded average degree (but possibly unbounded maximal degree) are fast mixing throughout their uniqueness regime. In particular, this gives the first polynomial-time sampler for the Potts model on the Erdos–Renyi random graph $G(n, d/n)$, up to the uniqueness point of the infinite d-ary tree. Interestingly, the presence of a few high-degree vertices, even ones whose degree grows polynomially with n, has no effect on the mixing time of random-cluster based sampling methods. Based on joint work with A. Blanca. (Received September 13, 2021)

1174-60-7331 **Andrew Campbell*** (andrew.j.campbell@colorado.edu), University of Colorado. *Spectrum of Heavy-Tailed Elliptic Random Matrices*

An elliptic random matrix X is a square matrix whose (i, j)-entry X_{ij} is a random variable independent of every other entry except possibly X_{ji}. Elliptic random matrices generalize Wigner matrices and non-Hermitian random matrices with independent entries. When the entries of an elliptic random matrix have mean zero and unit variance, the empirical spectral distribution is known to converge to the uniform distribution on the interior of an ellipse determined by the covariance of the mirrored entries.

In this talk we consider elliptic random matrices whose entries fail to have two finite moments, but are in the domain of attraction of an α-stable random variable, for $0 < \alpha < 2$. We will discuss the heavy-tailed analogue of covariance for the mirrored entries which gives the empirical spectral measure converging, in probability, to a deterministic limit. This is joint work with Sean O'Rourke. (Received September 14, 2021)

1174-60-7453 **Nicolas Fraiman** (fraiman@email.unc.edu), University of North Carolina at Chapel Hill, **Mariana Olvera-Cravioto*** (molvera@email.unc.edu), University of North Carolina at Chapel Hill, and **Tzu-Chi Lin** (tzlin@live.unc.edu), University of North Carolina at Chapel Hill. *Opinion dynamics on random digraphs*

We study a general version of the classical DeGroot-Friedkin model on sparse random digraphs whose local weak limit is a marked Galton-Watson process. This model has been extensively used in the social sciences to study opinion dynamics on social networks. The type of graphs in our analysis include the directed versions of the Erdos-Renyi graph, the Chung-Lu model, the Norros-Reittu model, and the configuration model, among others. We show that the stationary distribution of a typical vertex in the graph converges in a Wasserstein metric to a random variable that can be constructed on the limiting tree. Furthermore, our approach allows us to provide exact formulas for the mean and variance of the limiting stationary distribution, which can be used to characterize the conditions that lead to either "consensus" or the lack of it. In particular, we explain the role that stubborn agents (vertices that cannot be influenced) have on the existence of consensus. (Received September 14, 2021)

1174-60-7532 **Hoi Nguyen*** (nguyen.1261@osu.edu), The Ohio State University. *Universality of Poisson limits for moduli of roots of Kac polynomials*

We give a new proof of a recent resolution by Michelen and Sahasrabudhe of a conjecture of Shepp and Vanderbei that the moduli of roots of Gaussian Kac polynomials of degree n, centered at 1 and rescaled by n^2, should form a Poisson point process. We use this new approach to verify a conjecture from that the Poisson statistics are in fact universal.

Based on joint work with Nicholas Cook (Duke University), Oren Yakir (Tel Aviv University), and Ofer Zeitouni (Weizmann Institute) (Received September 14, 2021)

1174-60-7577 **Teresa Marie Dunn*** (mtdunn@ucdavis.edu), University of California-Davis, **Alexander Shashkov** (aes7@williams.edu), Williams College, **Luke Reifenberg** (lreifenb@nd.edu), University of Notre Dame, Notre Dame University, and **Stephen Willis** (sdw2@williams.edu), Williams College. *Closed form densities for the limiting spectral measure of random block matrix ensembles* Preliminary report.

We introduce a new matrix operation,

$$\text{swirl}(A, X) := \begin{pmatrix} AX & A \\ XAX & XA \end{pmatrix}$$

for A, X both $n \times n$ matrices. This operation was inspired by the behavior of radially symmetric matrix ensembles. We reduce analysis of swirl ensembles to that of matrix product ensembles, AX. We show that for A a random circulant Toeplitz matrix and X an exchange matrix, the limiting spectral distribution of the ensemble converges almost surely to the symmetrized Rayleigh distribution,

$$f(x) = |x|e^{-x^2}.$$

In showing this, we provide a new combinatorial proof that the limiting spectral distribution of the circulant Hankel matrix ensemble converges almost surely to the Rayleigh distribution. (Received September 15, 2021)

1174-60-7604 **Berit Nilsen Givens*** (bngivens@cpp.edu), Cal Poly Pomona, and **Jennifer M Switkes** (jmswitkes@cpp.edu), Cal Poly Pomona. *Spread of Infection in a Network: the benefits of masks, social distancing, and vaccines* Preliminary report.

The spread of an infection across a network depends on the degree of interconnectedness of the network and on the level of contagiousness of the infection. We explore what happens as networks grow or shrink in size, along with how the infection rate p affects the spread. We first consider a complete graph as a model of a "bubble" of people who interact freely. Then we explore a variant of star graphs, which could model a teacher interacting with several disjoint classes of students. Using graph theory and combinatorial reasoning, we show that limiting the size of gatherings and reducing the value of p can dramatically slow the spread of a highly contagious disease like COVID-19. (Received September 16, 2021)

1174-60-7689 **Phanuel Mariano*** (marianop@union.edu), Union College. *Spectral bounds and large time asymptotics for exit times on metric measure Dirichlet spaces with applications* Preliminary report.

Let (M, d, μ) be a metric measure space where μ satisfies a volume doubling property. We consider $\left(\{X_t\}_{t>0}, \{\mathbb{P}_x\}_{x\in M}\right)$ to be a diffusion process on (M, d, μ) associated with a local regular Dirichlet form $(\mathcal{E}, \mathcal{F})$ on $L^2(M, \mu)$. Now let τ_D be the first exit time of X_t from a domain $D \subset M$ and let $\lambda_1(D)$ be the bottom of the spectrum for the related diffusion operator L acting on $L^2(D)$. Assuming the heat kernel exists and has sub-Gaussian bounds we prove a sharp spectral bound on $\mathbb{P}_x(\tau_D > t)$ and show that $\lim_{t\to\infty} t^{-1} \log \mathbb{P}_x(\tau_D > t) = -\lambda_1(D)$. Moreover we can show that $\sup_{x\in D} \mathbb{E}_x[\tau_D] < \infty$ if and only if $\lambda_1(D) > 0$ by proving that their product is uniformly bounded from above and below for all domains $D \subset M$. One application of our results is a new characterization of sub-Gaussian heat kernel bounds that gives a partial answer to a conjecture of Grigor'yan, Hu and Lau. Our results also apply to many interesting examples that include sub-Riemannian manifolds and fractals. This talk is based on joint work with Jing Wang. (Received September 15, 2021)

1174-60-7691 **Vishesh Jain*** (visheshj@stanford.edu), Stanford University, **Ashwin Sah** (asah@mit.edu), Massachusetts Institute of Technology, and **Mehtaab Sawhney** (msawhney@mit.edu), Massachusetts Institute of Technology. *On the smoothed analysis of the smallest singular value with discrete noise*

Let A be an $n \times n$ matrix and let M be an $n \times n$ random matrix with i.i.d entries. In this talk, I will discuss some recent results and open problems concerning the behavior of the lower tail of the smallest singular value of $A + M$. (Received September 15, 2021)

1174-60-7744 **Pawel Lorek*** (pawel.lorek@math.uni.wroc.pl), University of Wrocław. *Dualities in Markov chains: from ruin probabilities, through absorption time to rate of convergence to stationarity*

We will discuss three types of dualities in Markov chains: i) Siegmund duality – enables computing winning probabilities in gambler models by computing the stationary distribution of another (ergodic) chain; ii) Strong stationary duality (SSD) – enables studying the rate of convergence to stationarity by studying the absorption time in another chain; iii) Intertwining – enables studying absorption time via studying absorption time of another chain (usually easier to handle). We will present results on construction of such duals for Möbius monotone Markov chains defined on partially ordered state spaces and some relations between these dualities (e.g., SSD can be computed via a Doob's transform of a Siegmund dual). As an example, we will present the formula for the winning probability in some multidimensional gambler's ruin problem. We will also present recent formulas for the expectation of a game duration in the one-dimensional gambler's ruin problem with arbitrary winning and losing probabilities. Similarly, we will present formulas for expectation of a conditional game duration (noticing some interesting symmetries in such models – extending results of Stern, Beyer and Waterman obtained for constant winning/losing rates). Finally, we will use all three dualities to provide a result on the fastest strong stationary time (which gives the exact rate of convergence to stationarity measured in the separation distance) for a random walk on a circle, improving the results of Diaconis and Fill. (Received September 16, 2021)

1174-60-7885 **Barbara Haas Margolius*** (b.margolius@csuohio.edu), Cleveland State University. *Asymptotic periodic analysis of cyclic stochastic fluid flows with time-varying transition rates* Preliminary report.

We consider a cyclic stochastic fluid model $\{(\widehat{X}(t), J(t)) : t \geq 0\}$ driven by a continuous-time Markov chain $\{J(t), t \geq 0\}$ with a time-varying generator $T(t)$ and cycle of length 1 such that $T(t) = T(t+1)$ for all $t \geq 0$. We derive theoretical expressions and algorithms for the transient distribution and for the asymptotic periodic distribution of the model, and illustrate the theory with numerical examples. This work is an extension of the results in Margolius and O'Reilly 2016, where the model was introduced. (Received September 16, 2021)

1174-60-7890 **Koby F Robles*** (roblesk2@wwu.edu), Western Washington University. *A General Algorithm for Generating Memoryless Distributions* Preliminary report.

The memoryless property is a common distributional property discussed in introductory probability and statistics. The most explored and well-known memoryless distributions are the exponential and geometric distributions, having supports in the non-negative reals and the non-negative integers, respectively. Noting that the geometric distributions are the discretized versions of the exponential distributions, we present a general algorithm for generating memoryless distributions. Finally, we show a few interesting examples to demonstrate how this algorithm works. (Received September 19, 2021)

1174-60-8007 **Max Jon Martin*** (max.martin@westpoint.edu), United States Military Academy, **Emily Rhodes** (emilyrhodes@ou.edu), University of Oklahoma, and **Jason Vogel** (jason.vogel@ou.edu), University of Oklahoma. *Optimization of Sewage Sampling for Wastewater-based Epidemiology through Stochastic Modeling*

The proliferation of the SARS-CoV-2 global pandemic has brought to attention the need for epidemiological tools that can detect diseases in specific geographical areas through non-contact means. Such methods may protect those potentially infected and public health officials while allowing quarantining to prevent the spread of the disease. Wastewater sampling analysis has been previously studied as a plausible solution that can detect pathogens, even from asymptomatic patients. However, many challenges exist in wastewater sampling such as identifying a representative sample for a population, determining the appropriate sample size, and establishing the right time and place for samples. This research involves the development of a stochastic model for optimizing various wastewater sampling parameters. Different and distinct scenarios are evaluated using Monte Carlo simulation; thereafter, a sensitivity analysis is performed to provide information on the robustness of model outcomes. The results of this study allow us to continue evaluating how efficient wastewater sampling

may provide an earlier warning of infection and prevent the further spread of viruses. (Received September 17, 2021)

1174-60-8077 **James M. Murphy*** (jm.murphy@tufts.edu), Tufts University. *Fermat distances, percolation, and multimanifold clustering*

Fermat distances (FD) are a class of data-driven metrics based on geodesic paths that penalize passing through low density regions. We develop new computational algorithms that enable FD for fast clustering of potentially intersecting manifolds. Continuum limits of discrete FD are analyzed using percolation theory, allowing for a subsequent continuum analysis of operators constructed using this metric. The tension between geometry and density in FD is elucidated and compared to the tensions in a range of data-driven metrics. Applications to big, high-dimensional synthetic and real data are emphasized throughout, demonstrating state-of-the-art performance in unsupervised clustering. Portions of this work are joint with Anna Little (Utah) and Daniel McKenzie (UCLA). (Received September 17, 2021)

1174-60-8080 **Tobias Johnson** (tobias.johnson@csi.cuny.edu), College of Staten Island (CUNY), **Jean Carlos Pulla** (jean.pulla@baruchmail.cuny.edu), CUNY Baruch College, **Zoe Ann McDonald*** (zmcd@bu.edu), Boston University, **Matthew Junge** (Matthew.Junge@baruch.cuny.edu), CUNY Baruch College, and **Lily Reeves** (zw477@cornell.edu), Cornell University. *Recurrence on the Threshold Frog Model* Preliminary report.

Frog models are random two-type particle systems that feature rapid activation. They are used as toy models to understand basic mechanisms in epidemics, rumor spread, combustion, and nuclear fission. We study the frog model on infinite trees with initially an independent Poisson-m-distributed number of sleeping frogs at each site. Awake frogs jump at random between adjacent vertices and wake any sleeping frogs they visit. The main question of interest, which went unsolved for over a decade, was whether or not the model is recurrent i.e., whether or not every frog eventually wakes up. We answer a generalization of this question for the threshold variant in which sleeping frogs require multiple visits, rather than a single visit, in order to wake up. Using a new technique, we prove that the threshold frog model can be made recurrent so long as m is large enough.
(Received September 17, 2021)

1174-60-8188 **Dawit Denu*** (ddenu@georgiasouthern.edu), Georgia Southern University. *Vector-host epidemic model with direct transmission in random environment* Preliminary report.

In this talk, we will discuss a stochastic vector-host epidemic model with direct transmission in random environment, governed by a system of stochastic differential equations with regime-switching diffusion. We first examine the existence and uniqueness of a positive global solution. Then we investigate stability properties of the solution, including almost sure and pth moment exponential stability and stochastic asymptotic stability. Moreover, we study conditions for the existence and uniqueness of a stationary distribution. Numerical simulations are presented to illustrate some of the theoretical results. (Received September 17, 2021)

1174-60-8449 **Alan C Krinik** (ackrinik@cpp.edu), Cal Poly Pomona, and **Gerardo Rubino*** (gerardo.rubino@inria.fr), INRIA. *The power-dual and the exponential-dual matrices; applications to the computation of matrix powers and matrix exponentials* Preliminary report.

In a recent paper, we proposed a matrix transformation that we call the exponential-dual, which we used to find closed-forms of some matrix exponentials. The initial motivation of this work was the transient analysis of basic Markovian models, mainly in queueing. Using this transformation in the particular case of transition rate matrices, corresponding to these queuing models, allowed to efficiently apply combinatorial techniques and derive expressions of the transient distributions of the number of customers in the system at an arbitrary point in time. This could also be done for some infinite models (infinite matrices).

In this talk, we will present the discrete counterpart of this idea, the power-dual of a matrix, which completes the picture and which is strongly related to the exponential-dual concept. The power-dual appears when we work with the power of a given matrix. These ideas are basically extensions (to arbitrary linear systems of Ordinary Differential Equations) of Siegmund's duality, defined on continuous time Markov processes, and of some of its developments as they appear in Anderson's book on Markov chains. (Received September 19, 2021)

1174-60-8732 **Nicolas Lanchier** (nlanchie@asu.edu), Arizona State University, and **Stephanie Reed*** (sjreed@fullerton.edu), California State University Fullerton. *Distribution of money on social networks with multiple banks*

We consider a spatially explicit agent-based model for the dynamics of money with debt. In this model, the underlying structure is a finite connected graph $G = (V, E)$ where each vertex $x \in V$ represents an individual that is characterized by the amount of money in her possession at time t, which can be negative in case she is in debt, and each monetary transaction results in one coin moving from one agent to one of her neighbors. We consider the scenario with multiple banks such that an individual with no coin or who is in debt at time t may borrow a coin from their bank in order to complete an exchange of money with one of their neighbors as long as their bank is not empty. Of particular interest is the distribution of wealth in the long run. The case with one bank has been studied previously and based on numerical simulations on the complete graph, it was conjectured that in the large population/temperature limits, the distribution of coins converges to an asymmetric Laplace distribution. Here we prove and extend this conjecture to any finite connected graph and also present results for the distribution of wealth when there are multiple banks. (Received September 21, 2021)

1174-60-8818 **Louis Fan*** (waifan@iu.edu), Indiana University. *Long-time behaviors of stochastic reaction networks* Preliminary report.

Chemical reaction processes and cellular processes are often modeled by reaction networks that describe the interactions between the constituent molecules. The evolution of the number of molecules are then described by a dynamical system. Such dynamical systems can be deterministic or stochastic, and they are usually challenging to analyse due to the complexity of the underlying reaction network. In this talk, I will present recent results about mixing time of Markov models associated with stochastic reaction networks and their diffusion approximations, explain how stochastic analysis can help clarify the subtle discrepancies among different modeling approaches, and discuss about the insights into the dynamical or stationary behavior of the system offered by these analysis.
(Received September 19, 2021)

1174-60-8878 **David Beecher** (dbeecher@cpp.edu), Cal Poly Pomona, **Heba Ayeda*** (hayeda@cpp.edu), Cal Poly Pomona, **Xiaoxiao Cui** (xiaoxiaocui@cpp.edu), Cal Poly Pomona, **Jeremy Lin** (jeremylinardi90@gmail.com), University of California, Irvine, **Thuy Vu Dieu Lu** (fthuyvlu@outlook.com), University of California, Irvine, **Weizhong Wong** (weizhongwang@cpp.edu), Cal Poly Pomona, **David Perez** (davidp@cpp.edu), Cal Poly Pomona, and **Zackary Muraca** (zmmuraca@cpp.edu), Cal Poly Pomona. *GAMBLER'S RUIN PROBABILITIES FOR FINITE BIRTH-DEATH CHAINS WITH ALTERNATING PROBABILITIES*

We consider a class of finite, linear, alternating birth-death chains on states: $0, 1, 2, \ldots, H-1, H$ having absorbing states at 0 and H, alternating ascending probabilities, alternating descending probabilities and alternating return probabilities at states $1, 2, 3, \ldots, H-1$. Then for each state i, $0 < i < H$ and for each $n \in \mathbb{N}$, we determine explicit eigenvalue expressions for the n-step ruin transition probabilities $P(n)(i, 0)$. This result follows from duality theory and eigenvalue results for tridiagonal matrices having alternating entries (Kouachi, S. 2008). These results generalize to alternating birth-death chains also having catastrophe transitions. Analogous results hold for finite birth-death processes with similar alternating rates. (Received September 20, 2021)

1174-60-8908 **Emily Marie Shoemaker*** (emshoemaker99@gmail.com), Butler University, and **Rasitha Jayasekare** (rjayasek@butler.edu), Butler University. *Identifying the Effect of Home-Court Advantage on Efficiency in Basketball Using the Stochastic Frontier Approach* Preliminary report.

Butler University's basketball team has been in the Big East Conference since 2013 and is known for having a distinct home-court advantage named 'Hinkle Magic' by fans. It is of interest to identify how home-court advantage affects a Big East team's ability to play to its full potential for the sake of generating wins and what factors are most accurate in measuring this potential. This project will utilize the Stochastic Frontier Approach (SFA) model to identify a team's efficiency when playing at home vs. away in order to identify when teams meet their potential and if the game location affects this result. The model will compute each team's maximum attainable wins given the strength of their program and compare this threshold to the team's actual success during the season. Panel data from the Men's Basketball Big East 2013-2014 to 2019-2020 seasons will be used in modeling SFA. Measures such as the Likelihood ratio test and pseudo-R-squared will be used for model evaluation. (Received September 20, 2021)

1174-60-9049 **William Alfred Massey*** (wmassey@princeton.edu), Princeton University. *Group Symmetries and Bike Sharing for M/M/1/k Queueing Transience* Preliminary report.

Fundamental stochastic models for studying the dynamics of bike sharing systems are found within the transient behavior of the M/M/1/k queue and related stopped processes. We develop new techniques to obtain exact solutions for their transition probabilities by using group symmetries and complex analysis. These methods are intrinsic to the underlying Markovian structure of the random processes. We do not use any indirect analysis from generating functions or Laplace transforms. These results complement and extend the previous M/M/1/k transient solutions given by Takacs.

This is joint work with Robert Hampshire, Jamol Pender, and Emmanuel Ekwedike. (Received September 20, 2021)

1174-60-9103 **Bala Krishnamoorthy** (kbala@wsu.edu), Washington State University, **Kevin Vixie** (vixie@speakeasy.net), Washington State University, and **Enrique G Alvarado*** (ealvarado@math.ucdavis.edu), University of California Davis. *Geometry of Sets and its Random Covers*

Let E be a bounded open subset of $\mathbb{R}^n$. We are interested in the following question: if we take i.i.d. samples $X_1, \ldots, X_N \sim \text{Unif}(E)$ of E, what is the probability that δ-balls centered at the samples $\cup_i \mathbf{B}(X_i, \delta)$ cover E? We study geometric conditions of E that allow us to derive lower bounds to the probability. The basic tool that we use is a *good* partition of E, i.e., one whose partition elements have diameters that are uniformly bounded from above and volumes that are uniformly bounded from below.

We show that E^c having positive *reach* allows us to construct a good partition of E that is motivated by the Whitney decomposition of E. We identify a class of bounded open subsets of $\mathbb{R}^n$ that do not have positive reach, but do have good partitions.

In the case that $E^c \subset \mathbb{R}^2$ does not have positive reach, we show that the *mutliscale flat norm* may be used to approximate E with a set that has a good partition under certain conditions. In this case, we provide a lower bound on the probability that a union of balls covers a large subset of E. (Received September 20, 2021)

1174-60-9160 **Oanh Nguyen*** (oanh_nguyen1@brown.edu), Brown University. *The number of limit cycles bifurcating from a randomly perturbed center*

We consider the average number of limit cycles that bifurcate from a randomly perturbed linear center where the perturbation consists of random (bivariate) polynomials with i.i.d. coefficients. We reduce this problem to the number of real roots of the random polynomial

$$f(x) = \sum_{k=0}^{n} k^\rho \xi_k x^k$$

where the ξ_k are independent with mean 0 and variance 1 and $\rho \le -1/2$ is a constant. In earlier work, Do, Vu, and myself established this number for $\rho > -1/2$ via the universality method which naturally breaks down for $\rho \le -1/2$. In this talk, we discuss the solution for the $\rho \le -1/2$. Joint work with Erik Lundberg (Received September 20, 2021)

1174-60-9175 **Marco Carfagnini*** (marco.carfagnini@uconn.edu), University of Connecticut. *Small deviations and Chung's laws of iterated logarithm for hypoelliptic diffusions* Preliminary report.

In this talk we discuss the small ball problem and its connection to Chung's laws of iterated logarithm for hypoelliptic diffusions. Most of existing results on small deviations concern Gaussian processes that satisfy a scaling property. In this talk, we focus on the hypoellitic Brownian motion on the Heisenberg group $\mathbb{H}$, and on the Kolmogorov diffusion in $\mathbb{R}^2$. The former is not a Gaussian diffusion but it satisfies a scaling property, and the latter satisfies a scaling property but is not Gaussian. (Received September 20, 2021)

1174-60-9292 **Amber L Puha*** (apuha@csusm.edu), California State University San Marcos, and **Ruth J Williams** (rjwilliams@ucsd.edu), University of California San Diego. *Diffusion Limits for Multiclass Processor Sharing Queues* Preliminary report.

Consider a single server queue that serves a finite number heterogeneous job types according to the processor sharing service discipline. Measure valued processes that keeps track of the residual service times of all jobs in the system at any given time are a natural descriptor of the system state. Under appropriate asymptotic assumptions, including standard heavy traffic assumptions, we show that (suitably rescaled) measure valued processes corresponding to a sequence of such queues converge in distribution to certain measure valued diffusion processes. An important contribution of this work is to devise a new methodology for establishing state-space collapse via the use of a certain relative entropy functional. (Received September 20, 2021)

1174-60-9323 **Hezhong Zhang*** (wzhang1@colgate.edu), Colgate University. *Differentially Private Sampling Via Langevin Monte Carlo* Preliminary report.

Differential Privacy is a concept which enables computer scientists to evaluate how a substitution of one record in a database would withhold information about the individual. There is a growing trend in applying the concept to machine learning algorithms; here we focus on the gradient descent algorithm with added Gaussian noise, also known as the Langevin algorithm. We are interested in studying sampling from a target distribution $f = e^{-U}$ where U is any Lipschitz gradient, convex cost function. Since the time discretization introduces error, we wish to quantify the divergence of the running distribution from the target distribution, and eventually give a non-asymptotic rate of divergence from the target distribution. Using α-order Renyi divergence as a proxy for quantifying differential privacy, we study the rate of Renyi divergence, which is the sum of two terms: a negative energy term and an error term. We claim that under a set of constraints, we are able to bound the error linearly in α and use a Log-Sobolev inequality in order to achieve a closed-form bound for the Renyi divergence. Our work is built upon recent works from Erdogdu-Hosseinzadeh-Zhang, Ganesh-Talwar, and Chourasia-Ye-Shokri. (Received September 20, 2021)

1174-60-9418 **Andrey Sarantsev*** (asarantsev@unr.edu), University of Nevada, Reno. *Exponential convergence of ruin probabilities*

We find explicit estimates for the exponential rate of long-term convergence for the ruin probability in a level-dependent Lévy-driven risk model, as time goes to infinity. The duality allows us to reduce the problem to long-term convergence of a reflected jump-diffusion to its stationary distribution, which is handled via Lyapunov functions. (Received September 20, 2021)

1174-60-9500 **Clayton Shonkwiler*** (clayton.shonkwiler@colostate.edu), Colorado State University. *Finding Good Coordinates for Sampling Configuration Spaces*

Many spaces of configurations of physical systems are typically presented as measure-zero subsets of some simple space: think of the unit sphere in 3-space, or energy isosurfaces in phase space, or the configurations of linkages with fixed distances between joints. Sampling points from the configuration space (that is, simulating a configuration) can then be quite challenging, even if sampling the ambient space is easy. I will discuss some examples that demonstrate the importance of finding good coordinates, which might come from linear algebra, symplectic geometry, or elsewhere, and illustrate how these ideas connect both to Archimedes' Theorem and models of topological polymers. (Received September 20, 2021)

1174-60-9529 **Dario Cruzado*** (dario.cruzado@upr.edu), UPR Mayagüez. *Universality in Coalescing Ballistic Annihilation* Preliminary report.

During the 1980's, the statistical physics literature introduced a model that caricatures chemical reactions called ballistic annihilation. Particles in this model are placed randomly throughout the real line and then proceed to move at preassigned discrete velocities. Upon collision, the particles involved are removed from the system. Many results were inferred by physicists, but it wasn't until recently that the process received renewed attention from mathematicians who saw a probabilistic and combinatorial interest beyond physics applications. We study the three-velocity system in which collisions result in coalescence instead of mutual annihilation and generalize a result from Haslegrave, Sidoravicius, and Tournier from 2019. For many symmetric coalescing reactions, we prove that the index of the first particle that visits 0 does not depend on the law of the (random) distances between particles. (Received September 20, 2021)

1174-60-9805 **Christopher Wang*** (cyw2124@columbia.edu), Columbia University, **Clarice Pertel** (cep87@cornell.edu), Cornell University, and **Alex Negrón** (AlexNegron18@gmail.com), Illinois Institute of Technology. *Extensions of True Skewness for Unimodal Distributions* Preliminary report.

In a 2020 paper, Y. Kovchegov introduced the notion of true positive and negative skewness for the distributions of continuous random variables via Fréchet p-means and stochastic dominance criteria. True skewness ensures consistency in sign of widely used but sometimes discrepant measures of the degree of skewness, including Pearson's moment coefficient of skewness, Pearson mode skewness, and Pearson median skewness. Moreover, unlike Pearson's measures of skewness, true skewness is well-defined for distributions with infinite integer moments. In this work, we find novel criteria for true skewness, identify a parameter region for true positive skewness of the Weibull distribution, and formally establish positive skewness of the Lévy distribution for the first time. Additionally, we consider the application of true skewness to discrete and multivariate settings and discuss the complications in doing so. An analogous stochastic dominance criterion is proved for the discrete case. In the multivariate case, we propose a notion of a trajectory of true skewness. Finally, we offer a new measure of skewness that is defined for a larger class of distributions based on the notion of true skewness. Several properties of the p-means of random variables are established. (Received September 20, 2021)

1174-60-9817 **Isaiah Milbank*** (Isaiah.milbank@gmail.com), University of Minnesota Twin Cities, **Enrique Treviño** (trevino@mx.lakeforest.edu), Lake Forest College, **Lisa Lin** (lhl1@rice.edu), Rice University, **Rachael Ren** (rren@uw.edu), University of Washington, **Julian Burden** (burdju02@gettysburg.edu), Gettysburg College, **Chandramauli Chakraborty** (bs1901@isical.ac.in), Indian Statistical Institute, Kolkata, **Qizhou Fang** (qsfang@uci.edu), University of California Irvine, **Nasser Malibari** (nasser.malibari@student.unsw.edu.au), University of New South Wales, **Sammi Matoush** (smatoush@wustl.edu), Washington University in St. Louis, **Zahan Parekh** (zahan.parekh@yale.edu), Yale University, **Martín Prado** (m.prado@uniandes.edu.co), Universidad de los Andes, **Qizhao Rong** (qizhao.rong@baruchmail.cuny.edu), Baruch College, **Maximiliano Sánchez Garza** (maxsanchez_99@hotmail.com), Universidad Autonoma de Nuevo Leon, **Eli Sun** (esun4@u.rochester.edu), University of Rochester, and **Daisuke Yamada** (yamadad@carleton.edu), Carleton College. *Selecting Balls from Urns* Preliminary report.

The study of probability often involves sampling from a heterogeneous population either with or without replacement. Sampling with replacement is modeled by the binomial distribution, and sampling without replacement is modeled by the hypergeometric distribution. However, both of these sampling methods do not allow for preferential treatment of different items within the population. Thus, in this presentation, we will explore two such preferential cases. We define our population as red and white balls placed in an urn. First, we consider the case where the white balls are strictly preferred over the red balls, meaning that a randomly selected white ball will not be replaced into the urn, while a randomly selected red ball will be replaced. In this case, we attempt to bound the expected value or long-run average of white balls selected. Second, we consider a case when preference for red balls and white balls are variable. Specifically, the probability of not replacing a red or white ball is some number in the interval $[0, 1]$. We endeavor to define reasonably simple expressions for the distribution and expectation of white balls for this sampling method. Presented with Lisa Lin and Rachael Ren. (Received September 20, 2021)

1174-60-9922 **Irfan Alam*** (irfanalamisi@gmail.com), Department of Mathematics, University of Pennsylvania. *Generalizing de Finetti's theorem using nonstandard methods* Preliminary report.

In its classical form, de Finetti's theorem provides a representation of any exchangeable sequence of Bernoulli random variables as a mixture of sequences of iid random variables. Following the work of Hewitt and Savage, such a representation was known for exchangeable random variables taking values in any Polish space. In a recent work, the author has used nonstandard analysis to show that such a representation holds for a sequence of exchangeable random variables taking values in any Hausdorff space as long as their underlying distribution is Radon (in fact, tightness and outer regularity on compact sets are also sufficient). The arguments have topological measure theoretic and combinatorial flavors, with nonstandard analysis serving as a bridge between these themes. This talk will give an overview of this work. (Received September 21, 2021)

1174-60-9943 **Randall J. Swift*** (rjswift@cpp.edu), Cal Poly Pomona, and **Heba Ayeda** (hayeda@cpp.edu), California State Polytechnic University, Pomona. *Exact transition probabilities of Markov chains with steps* $0, \pm 1, \pm 2$ *and Generalized Ballot Box Problem for couples*

We are interested in the following problems and their connections.

(1) Determine an explicit eigenvalue description of the n-step transient probabilities of finite Markov chains on a State Space $S = \{1, 2, 3, \ldots H-1, H\}$ that have one-step transition probabilities of step size 0, 1 or 2 only. For simplicity, we assume that the one-step transition matrix is symmetric and penta-diagonal. We are also assuming that transitions that would normally take us outside the boundaries of the matrix are deleted.

(2) The traditional two candidate Ballot Box Problem is generalized to electorates consisting of couple voters.

(3) The linear algebraic search is ongoing for explicit eigenvalue formulas of matrices H that are symmetric, penta-diagonal and Toeplitz.

During our talk the following results and conclusions will be discussed.

(1) The solution of problems 1, 2 and 3 are related by duality theory.
(2) Computer Algebra Systems can give explicit eigenvalue expressions for H having dimension 8×8 or less.
(3) In general, the explicit solution of problem 3 for H having dimension 8×8 or higher seems to be unknown.

(Received September 21, 2021)

1174-60-10018 **Steven Joel Miller** (sjm1@williams.edu), Williams College. *A Stochastic Central Limit Theorem and Applications to Integer Decompositions*

Base-b decompositions have long been known and valued. Extended to non-integer bases in 1953 by George Bergman when he showed that every positive integer n can be expressed as a sum of integer powers of the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$ using at most one copy of each power. If we add the additional rule that no two consecutive powers can be used, each integer has a *unique* representation. This gives rise to a number system with an irrational base, where for example $4 = \varphi^2 + \varphi^0 + \varphi^{-2} = 101.01$.

In our paper, we consider the number of 1's in the base-φ representation of an integer n. We show that the distribution of the number of 1's to represent n, where n is in the Lucas interval $[L_k, L_{k+1})$, approaches a Gaussian in probability and we compute the kth mean and variance. We prove these results by deriving a new central limit theorem for stochastic processes which satisfy a certain condition similar to regenerativity, which is of independent interest.

Our result generalizes earlier work which was confined to the distribution of the number of summands in Zeckendorf decompositions of n in the Fibonacci interval $[F_k, F_{k+1})$, and more generally to Positive Linear Recurrence Sequences. Our proof allows us to more easily derive earlier results, bypassing difficult combinatorics and the necessity to analyze numerous special cases of associated characteristic polynomials. (Received September 21, 2021)

1174-60-10094 **Jonathan Touboul** (jtouboul@brandeis.edu), Brandeis University. *Mean-field spatial models via stochastic coupling theory* Preliminary report.

One of the central and longstanding challenges in multi-scale modeling is how to derive tractable models from large systems of stochastically interacting agents. Many applications call for explicit spatial extent, heterogeneity of the agents, and even movement of agents. These complications, and the associated mathematical overhead, often make such models inaccessible to applied researchers. We address this problem by using stochastic coupling theory to provide elementary proofs of convergence to a mean-field limit for spatial interacting particle systems. Moreover, these systems may have discrete, continuous, or mixed state spaces (i.e., jump-diffusion processes); this affords significant flexibility in the range of possible applications. In addition, we derive and analyze Fokker-Planck-Kolmogorov equations (FPKEs) governing the evolution of the probability measure of the mean-field limit. Finally, we present applications to vegetation and cell modeling where analysis of the FPKEs reveals rich dynamics in the original stochastic systems. (Received September 21, 2021)

1174-60-10104 **Sean O'Rourke** (sean.d.orourke@colorado.edu), University of Colorado Boulder, and **Noah Williams*** (williamsnn@appstate.edu), Appalachian State University. *Partial linear eigenvalue statistics for non-Hermitian random matrices*

For an $n \times n$ independent-entry random matrix X_n with eigenvalues $\lambda_1, \ldots, \lambda_n$, it is well known (due to Rider and Silverstein among others) that the fluctuations of the linear eigenvalue statistics $\sum_{i=1}^{n} f(\lambda_i)$ converge to a Gaussian distribution for sufficiently nice test functions f. We study the fluctuations of $\sum_{i=1}^{n-K} f(\lambda_i)$, where K randomly chosen eigenvalues have been removed from the sum. In this case, we identify the limiting distribution and show that it need not be Gaussian. Our results hold for the case when K is fixed as well as the case when K tends to infinity with n. Our proof utilizes the predicted locations of the eigenvalues introduced by E. Meckes and M. Meckes. As a consequence of our methods, we obtain a rate of convergence for the empirical spectral distribution of X_n to the circular law in Wasserstein distance, which may be of independent interest.

This article is joint work with Sean O'Rourke (sean.d.orourke@colorado.edu) from the University of Colorado Boulder. (Received September 21, 2021)

1174-60-10120 **Sefika Kuzgun*** (sefika.kuzgun@ku.edu), University of Kansas. *Central Limit Theorems for Stochastic Heat Equation*

In this poster, we will present recent results on normal approximations for spatial averages of the solution to stochastic heat equation in dimension one driven by a space-time white noise. (Received September 21, 2021)

1174-60-10346 **Kavita Ramanan** (Kavita_Ramanan@brown.edu), Brown University, and **Ankan Ganguly*** (ankan_ganguly@brown.edu), Brown University. *Marginal Distributions of Interacting Particle Systems on Unimodular Galton-Watson Trees* Preliminary report.

We consider (possibly non-Markovian) interacting particle systems (IPS) on unimodular Galton-Watson trees, which have recently been shown by Ganguly and Ramanan to arise as local weak limits of IPS on many sequences of sparse random graphs. We obtain an autonomous characterization of the "marginal dynamics" of the neighborhood of the root particle in terms of a local equation and show that it describes a flow. Our proof is predicated on a general dimensional reduction property of a class of graphical models, as well as a (quenched and annealed) second-order Markov random field property for the trajectories of IPS on UGW trees. Our results complement results for diffusions by Lacker, Ramanan and Wu, but notably go beyond i.i.d. assumptions on the initial conditions which makes the analysis significantly more complicated. (Received September 21, 2021)

1174-60-10413 **Mudassar Razzaq** (mudassar.razzaq@lums.edu.pk), TU Dortmund, Germany, Syed Babar Ali School of Science and Engineering - LUMS, **Hafiz Tamoor Shehzad** (19070012@lums.edu.pk), Syed Babar Ali School of Science and Engineering - LUMS, **Fazal Abbas** (drfabbas@gmail.com), Mathematics and Computer Sciences, Stetson University,, **Muhammad Arif Ayoub** (19070006@lums.edu.pk), Syed Babar Ali School of Science and Engineering - LUMS, and **Adnan Anwar*** (adnan.anwar@lums.edu.pk), Syed Babar Ali School of Science and Engineering - LUMS. *Predicting Stock Prices using Heston and Geometric Brownian Motion Models* Preliminary report.

Stock market prediction allows great profit avenues and is a natural inducement for most researchers in this area. To predict the stock price, most researchers practice either technical or fundamental analysis. Technical analysis centers on analyzing the direction of prices to predict future prices, while fundamental analysis depends on studying economic and financial factors. This paper based on technical analysis, introduces Ito's lemma and Euler-Maruyama methods to solve Heston and Geometric Brownian Motion Models which arises in the modeling of such phenomenon. We include volatility, interest rate, and historical stock prices as the key variables for time-dependent coefficients in the predictive models. Later, we use solutions of these differential equations to predict/forecast stock prices. It is observed that our model equations are competitive for price prediction in comparison to leading statistical indicators in the stock market. (Received September 21, 2021)

1174-60-10519 **Yin-Ting Liao*** (yin-ting_liao@brown.edu), Brown University. *Quenched random projections for high-dimensional ℓ_p^n balls and an important sampling scheme*

Random projections of high-dimensional probability measures have gained much attention recently in asymptotic convex geometry, high-dimensional statistics and data science. Accurate estimation of tail probabilities is of importance in applications. Large deviation principles for such projections have only been recently studied. In this talk, I will describe refine large deviation estimates for quenched random projections of ℓ_p^n balls and their norms. These provide refinements to the large deviation estimates that provide the prefactor in addition to the exponential decay of the probabilities, which can be viewed as a significant extension of the classical work of Bahadur and Ranga-Rao. Both theoretical and computational results would be provided and compared. This talk is based on several joint works with Kavita Ramanan. (Received September 21, 2021)

1174-60-10543 **Tomoyuki Ichiba*** (ichiba@pstat.ucsb.edu), University of California Santa Barbara. *Cauchy problem, mean-field game and relative arbitrage among investors*

The relative arbitrage portfolio, formulated in Stochastic Portfolio Theory (SPT), outperforms a market portfolio over a given time-horizon with probability one under some conditions on the volatilities in the market, where the optimal relative arbitrage can be characterized by the strict local martingales and the smallest nonnegative continuous solution of a Cauchy problem. In this talk, we consider a mean-field game, and the corresponding Nash equilibrium of investors who compete with a benchmark determined by the market portfolio and other investors' performance. With the market price of risk processes depending on the market portfolio and total volumes invested, we solve the multi-agent optimization problem under the framework of SPT. A part of this talk is joint work with Tianjiao Yang. (Received September 21, 2021)

1174-60-10594 **James Normand Maclaurin*** (james.n.maclaurin@njit.edu), New Jersey Institute of Technology. *Phase Reduction of Stochastic Waves and Patterns*

In this paper we present a general framework in which one can rigorously study the effect of spatio-temporal noise on traveling waves, stationary patterns and oscillations that are invariant under the action of a finite-dimensional set of continuous isometries (such as translation or rotation). This formalism can accommodate patterns, waves and oscillations in reaction- diffusion systems. We define the phase by precisely projecting the infinite-dimensional system onto the manifold of isometries. Two differing types of stochastic phase dynamics are

defined: (i) a variational phase, and (ii) an isochronal phase, defined as the limiting point on manifold obtained by taking $t \to \infty$ in the absence of noise. We outline precise stochastic differential equations for both types of phase. The variational phase SDE is then used to show that the probability of the system leaving the attracting basin of the manifold after an exponentially long period of time (in the magnitude of the noise) is exponentially unlikely. In the case that the manifold is periodic (such as for spiral waves, spatially-distributed oscillations, or neural-field phenomena on a compact domain), the isochronal phase SDE is used to determine asymptotic limits for the average occupation times of the phase as it wanders in the basin of attraction of the manifold over very long times. (Received September 21, 2021)

1174-60-10596 **Emily Beatrice Crawford Das*** (ebc60213@uga.edu), University of Georiga, **Jingzhi Tie** (jtie@uga.edu), University of Georgia, and **Qing Zhang** (qz@uga.edu), University of Georgia. *AN OPTIMAL STRATEGY FOR ROUND-TRIP PAIRS TRADING UNDER GEOMETRIC BROWNIAN MOTION* Preliminary report.

This poster is concerned with an optimal strategy for simultaneously trading a pair of stocks. The idea of pairs trading is to monitor their price movements and compare their relative strength over time. A pairs trade is triggered by the divergence of their prices and consists of a pair of positions to short the strong stock and to long the weak one. Such a strategy bets on the reversal of their price strengths. A round-trip trading strategy refers to opening and closing such a pair of security positions. Typical pairs-trading models usually assume a difference of the stock prices satisfies a mean-reversion equation. However, we consider the optimal pairs-trading problem by allowing the stock prices to follow general geometric Brownian motions. The objective is to trade the pairs over time to maximize an overall return with a fixed commission cost for each transaction. The optimal policy is characterized by threshold curves obtained by solving the associated HJB equations. (Received September 21, 2021)

1174-60-10702 **Na'Ama Nevo** (n_nevo@coloradocollege.edu), Colorado College, **Connor Bass*** (cbass@macalester.edu), Macalester College, and **Caitlyn Powell** (cnpowell2@crimson.ua.edu), University of Alabama Tuscaloosa. *Investigating the Growth of the Ballistic Deposition Model on Connected Finite Graphs*

The discrete-time ballistic deposition process on a finite connected graph is a Markov chain on the space of integer-valued functions on the vertex set, representing the "height" of the process at each vertex. Each unit of time a vertex is sampled uniformly and independently of the past, and its height is updated to one plus the maximum among the current heights of vertex and of all of its neighbors. It follows from subadditive theory that the maximal height grows asymptotically linearly as a function of time, with a deterministic growth rate. The growth rate is typically hard to compute explicitly, even for relatively simple graphs like the N-cycle. In this work we present some numerical and rigorous bounds on the growth rate for several classes of graphs. (Received September 21, 2021)

1174-60-10721 **Amir Dembo*** (adembo@stanford.edu), Stanford University. *Universality for diffusions interacting through a random matrix*

Consider a system of N stochastic differential equations interacting through an N-dimensional matrix J of independent random entries (starting at an initial state whose law is independent of J). We show that the trajectories of a large class of observables which are averaged over the N coordinates of the solution, are universal. That is, for a fixed time interval the limit of such observables as N grows, essentially depends only on the first two moments of the marginal distributions of entries of J.

Concrete settings for which such universality holds include aging in the spherical Sherrington-Kirkpatrick spin-glass and Langevin dynamics for a certain collection of Hopfield networks.

This talk is based on joint works with Reza Gheissari, Alice Guionnet, Eyal Lubetzky and Ofer Zeitouni. (Received September 21, 2021)

1174-60-10933 **Bikram Bhusal*** (bbhusal2014@fau.edu), Bikram Bhusal. *Stability of the solution of a stochastic logistic growth model with α-stable Lévy noise.* Preliminary report.

In this talk, we discuss one dimensional stochastic logistic jump-diffusion processes driven by Brownian motion and symmetric α-stable Lévy motion. Existence of unique global positive solution of the model under certain conditions are discussed. Then sufficient conditions for the stability in probability and almost sure exponential stability of the trivial solution are established. (Received September 21, 2021)

1174-60-10946 **Sarah Cannon*** (scannon@cmc.edu), Claremont McKenna College. *Properties of Redistricting Markov Chains*

Markov chains have become widely-used to generate random political districting plans. These random districting plans can be used to form a baseline for comparison, and any proposed districting plans that differ significantly from this baseline can be flagged as potentially gerrymandered. However, very little is know about these Markov chains - Are they irreducible? What is their mixing time? For some, even the stationary distribution remains unknown. I will present recent work that answers some of these questions. (Received September 21, 2021)

1174-60-10958 **Eliza O'Reilly*** (eoreilly@caltech.edu), Caltech. *Stochastic Geometry for Machine Learning*

The Mondrian process in machine learning is a recursive partition of space with random axis-aligned cuts used to build random forests and Laplace kernel approximations, and it can be viewed as a special case of the stable under iterated (STIT) process in stochastic geometry. We utilize this viewpoint to resolve open questions on the generalization of cut directions in the Mondrian process. Our results generalize the use of random partitions for kernel approximation and show minimax rates of convergence for random forest estimators built from STIT tessellations. This work calls for further developments at the novel intersection of stochastic geometry and machine learning. Based on joint work with Ngoc Tran. (Received September 21, 2021)

1174-60-10979 **Jess Banks*** (jess.m.banks@berkeley.edu), UC Berkeley. *Gaussian Regularization of Pseudospectrum, Eigenvalue Gaps, and Overlaps*

Unlike their normal counterparts, the spectra of non-normal matrices can be highly unstable under small additive perturbations; indeed there are non-diagonalizable $n \times n$ matrices whose eigenvalues move by $O(\epsilon^{1/n})$ after a perturbation of size ϵ. This issue is especially concerning in numerical linear algebra (where routine machine noise in even a modestly-sized matrix can dramatically alter the spectrum) and is mitigated only by the classical fact that the non-diagonalizable matrices of any dimension have measure zero.

In this talk I'll quantify this fact by showing the following: for any $n \times n$ matrix A, a small, complex, entry-wise Gaussian perturbation $A + \epsilon \boldsymbol{G}$ has (with high probability) inverse polynomial eigenvalue gaps and a basis of eigenvectors with condition number $O(n^{3/2}/\epsilon)$. Our main technical innovation is to exploit the relationship between pseudospectrum and eigenvector condition number, reducing the problem to the proof of tail bounds on small singular values of the matrices $zI - A - \epsilon \boldsymbol{G}$, for generic $z \in \mathbb{C}$. I'll finally discuss an application to numerical linear algebra, where this Gaussian regularization is used as a preconditioning step in an algorithm to approximately diagonalize any matrix in nearly matrix-multiplication time. Joint work with J. Garza-Vargas, A. Kulkarni, S. Mukherjee, and N. Srivastava. (Received September 21, 2021)

1174-60-11015 **Kaviita Ramanan*** (kavita_ramanan@brown.edu), Brown University. *Extracting information about high-dimensional measures from lower-dimensional projections*

The study of high-dimensional measures by looking at their lower-dimensional projections is a common theme in high-dimensional statistics and asymptotic convex geometry. While the central limit theorem for convex sets and related universality results imply the negative result that fluctuations of lower-dimensional projections do not provide much information on high-dimensional measures, in this talk we describe how tail behavior can be used to extract important information about high-dimensional measures, and discuss the ramifications for asymptotic convex geometry. This talk will be based on joint works with Steven Soojin Kim and Yin-Ting Liao. (Received September 21, 2021)

1174-60-11290 **Ram Sharan Adhikari*** (radhikari@rsu.edu), Rogers State University. *Mean square stability analysis of a weak Simpson method based on Simpson rule for a class of stochastic differential equations*

The proposed weak Simpson method has the potential to overcome some of the numerical instabilities that are often experienced when using explicit Euler method. This work also aims to determine the mean-square stability region of the weak Simpson method for linear stochastic differential equations with multiplicative noises. In this work, a mean-square stability region of the weak Simpson scheme is identified, and step-sizes for the numerical method where errors propagation are under control in a well-defined sense are given. The main results are illustrated with numerical examples (Received September 29, 2021)

1174-60-11775 **Isabelle Kemajou-Brown** (Elisabeth.Brown@morgan.edu), Morgan State University, **Erica Graham** (ejgraham@brynmawr.edu), Bryn Mawr College, **Oyita Udiani*** (obudiani@gmail.com), Virginia Commonwealth University, and **Julie Ivy** (jsivy@ncsu.edu), North Carolina State University. *Decision Modeling of Complex Human-Centered Dynamics under Uncertainty.* Preliminary report.

Abstract body/text including any latex coding if necessary. We look at decision modeling that accounts for uncertainties in models of complex interactions in order to quantitatively capture the impact of different factors, objectives, system dynamics, intervention options and policies on model outcomes. Some of the methods to be utilized include Markov decision processes (MDPs), semi-Markov decision process (SMDPs) and partially observable Markov decision process (POMDP) modeling. Applications we are interested in, using the methodologies given, include mosquito control, traffic management, medical diagnosis/treatment of Polycystic ovary syndrome (PCOS), with the goal of providing decision makers with informed choices in addressing complex societal issues. This talk will focus on our discussion sessions in Baltimore and ideas under discussion towards a manuscript. (Received October 20, 2021)

62 ► Statistics

1174-62-5218 **Xihong Lin*** (xlin@hsph.harvard.edu), Harvard University, Broad Institute of MIT and Harvard. *Learning from COVID-19 Data on Transmission, Health Outcomes, Interventions and Vaccination*

COVID-19 is an emerging respiratory infectious disease that has become a pandemic. In this talk, I will first provide a historical overview of the epidemic in Wuhan. I will then provide the analysis results of 32,000 lab-confirmed COVID-19 cases in Wuhan to estimate the transmission rates using Poisson Partial Differential Equation based transmission dynamic models. This model is also used to evaluate the effects of different public health interventions on controlling the COVID-19 outbreak. I will next present transmission dynamic regression models for estimating transmission rates in USA and other countries, as well as factors including intervention effects that affect transmission rates. I will discuss estimation of the proportion of undetected cases and the disease prevalence. I will also present the analysis results of ¿500,000 participants of the HowWeFeel project on health outcomes and behaviors in US, and discuss the factors associated with infection, behavior, and vaccine hesitancy. To help plan safely reopen schools. I will discuss efficient pooled testing design using hypergraph factorization (Received November 16, 2021)

1174-62-5467 **Claire McKay Bowen*** (cbowen@urban.org), Urban Institute. *Introduction on Data Privacy and Confidentiality: Protecting Your Privacy in a Data-Driven World*

At what point does the sacrifice to our personal information outweigh the public good?

If public policymakers had access to our personal and confidential data, they could make more evidence-based, data-informed decisions that could accelerate economic recovery and improve COVID-19 vaccine distribution. However, access to personal data comes at a steep privacy cost for contributors, especially underrepresented groups. Revealing too much location information places people at risk such as empowering stalkers to more easily track people, but too little personal, location information will severely hinder the effectiveness of contact tracing. This talk introduces and surveys what is data privacy and confidentiality, why should you care, and what are the future challenges, preparing the audience for the remaining talks in the session. (Received August 20, 2021)

1174-62-5617 **Nathan Nicholson** (nlnicholson@wisc.edu), University of Wisconsin-Madison, **Julia Lindberg** (jrlindberg@wisc.edu), University of Wisconsin — Madison, and **Zinan Wang** (zwang894@math.wisc.edu), University of Wisconsin — Madison. *The maximum likelihood degree of sparse polynomial systems*

In this talk we consider statistical models arising from the common set of solutions to a sparse polynomial system with general coefficients. The maximum likelihood degree counts the number of critical points of the likelihood function restricted to the model. Our main result proves the maximum likelihood degree of a sparse polynomial system is determined by its Newton polytopes and equals the mixed volume of a related Lagrange system of equations. As a corollary, we find that the algebraic degree of several optimization problems is equal to a similar mixed volume. (Received August 23, 2021)

1174-62-5699 **Aida Maraj** (maraja@umich.edu), University of Michigan - Ann Arbor, **Jane Ivy Coons*** (jicoons@ncsu.edu), St John's College, University of Oxford, **Pratik Misra** (pmisra@ncsu.edu), KTH Royal Institute of Technology, and **Miruna-Stefana Sorea** (miruna.stefana.sorea2019@gmail.com), Scuola Internazionale Superiore di Studi Avanzati (SISSA). *Gaussian RCOP Models with Toric Vanishing Ideals*

Gaussian graphical models are frequently used to describe random vectors for which conditional independence relations among the random variables are encoded by a graph. Coloring this graph restricts the model so that the concentrations of vertices or edges with the same color are equal. These linear constraints on the space of concentration matrices yield interesting nonlinear restrictions on the space of covariance matrices of the model. In this talk, we consider a specific type of colored graphical model known as an RCOP model. We show that when the underlying graph is a one-clique sum of complete graphs, or "block graph", the vanishing ideal of the covariance matrices of the model is toric and generated in degrees one and two. We also give a Markov basis for the ideal in these cases based on the combinatorial structure of the colored graph. (Received August 25, 2021)

1174-62-5867 **Tarunima Agarwal*** (tarunimaagarwal@gmail.com), Student, Modern School, New Delhi, India, and **Stavelin AK** (stavelin.ak@jssuni.edu.in), Assistant Professor, Division of Medical Statistics, Faculty of Life Sciences, JSSAHER, Mysuru, Karnataka, India. *A Mathematical Model for COVID-19 to Predict Daily Cases using Time Series Auto Regressive Integrated Moving Average (ARIMA) Model in Delhi Region, India*

INTRODUCTION: Coronavirus disease (COVID-19) is a global pandemic caused by SARS COV-2. Various countries have adopted different strategies to control the spread of the disease. Many studies have adopted the mathematical modeling to predict the cases during the pandemic. In our study we have used Box- Jenskin's time series Auto Regressive Integrated Moving Average (ARIMA) mathematical model. MATERIALS AND METHODS: Publicly available data of daily COVID-19 confirmed cases along with Meteorological variables were considered using Expert Modeler in SPSS to Predict and forecast COVID-19 cases in Delhi region, India. RESULTS: Spearman's correlation was used to find the relationship between COVID-19 cases along with Meteorological variables. Humidity, rainy days and Average sunshine were found to be significant. ARIMA (0, 1, 14) model was found to be best fitted model for the given data with R square value of fitted model is 0.920. Ljung-Box test value is 39.368 with p value showing significant, indicating that the fitted model is adequate to predict and forecast COVID-19 cases. CONCLUSION: ARIMA (0, 1, 14) mathematical model was selected as a best suited model to predict and forecast the incidence of COVID-19 cases in Delhi region, which would be useful for the policymakers for better preparedness. (Received August 30, 2021)

1174-62-5935 **Michelle R. DeDeo*** (mdedeo@unf.edu), University of North Florida, Mayo Clinic - Jacksonville. *State-space Smoothing Models and the Opioid Epidemic: A Case Study*

In light of the increase in pill mills and opioid prescribing correlated to a rapid rise in mortality from prescription opioid overdoses, the DEA and the State of Florida adopted several measures to rein in the negligent practices of pill mills.

The first hurdle of this study was deciphering the recently released DEA's ARCOS database which tracks all opioids from manufacturer to retailer. We show how this was accomplished, describe issues with the 130GB raw ARCOS data, and provide verification that the database was properly cleaned. Then using overdose data obtained through an IRB, we apply the best of 19 possible state-space smoothing models to predict expected overdose deaths.

We conclude that although the total number of pills sold has decreased over time, it did not result in a corresponding reduction of overdose deaths when compared to the counterfactual time series data. According to the state-space smoothing models, overdose deaths should have declined. Instead, Northeast Florida saw 673 more deaths than expected. (Received August 31, 2021)

1174-62-5994 **Ellen Galantucci*** (galantucci.ellen@bls.gov), Bureau of Labor Statistics, **Alex Measure** (measure.alex@bls.gov), Bureau of Labor Statistics, and **David Oh** (oh.david@bls.gov), Bureau of Labor Statistics. *Differentially Private machine learning algorithms for data sharing* Preliminary report.

In order to distribute a machine learning model that is used to code occupation data as well as injury and illness data, the Survey of Occupational Injuries and Illnesses (SOII) program at the Bureau of Labor Statistics (BLS) applied differential privacy to the model. This required adding noise to the gradients during the training of the model and clipping the norms. While the accuracy of the model is still quite high and has not significantly decreased with the addition of noise, the macro f-1 drastically declined, which demonstrates that the primary result of the noise addition is that rare codes are not able to be coded with success while the common codes are

coded with nearly the exact same level of accuracy as in the original model with no noise added. (Received September 2, 2021)

1174-62-6135 **Sachith Eranga Dassanayaka*** (sachith-eranga.dassanayaka@ttu.edu), Texas Tech University, **Ori Swed** (ori.swed@ttu.edu), Texas Tech University, and **Dimitri Volchenkov** (dr.volchenkov@gmail.com), Texas Tech University. *A Model to differentiate Actors in order to Uncover the Artificial Legitimacy of the Russian Information Operation Networks*

The information operation networks manipulate public arguments on social media platforms. Thus, these networks pose a meaningful threat to democratic processes. Given the increased frequency of this type of threat, understanding those operations is paramount in the effort of combating their influence. Building on existing scholarship on the inner functions within those influence networks on social media, we suggest a new probabilistic approach to map those types of operations. Using Twitter content identified as part of the Russian influence network, we classify accounts type based on their authenticity function for a sub-sample of accounts and analyze their activities within the network. We trained a predictive model to map the activities and identify similar patterns of behavior across the network. Our model attains 88% prediction accuracy for the test set. We validate our predicted results set by comparing the similarities with the 3 million Russian troll tweets dataset and the result indicates a 90.7% similarity between the two datasets. Furthermore, we compare our model predictions on a Russian-language tweets dataset, and the result states that there is 90.5% correspondence between the predictions and the actual categories. According to the predictive and validation results, our predictive model is useful to identify the tweets actors in the information operation networks. (Received September 5, 2021)

1174-62-6312 **Min Shu*** (shum@uwstout.edu), University of Wisconsin Stout, and **Ruiqiang Song** (rsong1@mtu.edu), Michigan Technological University. *Shocks, Bubbles and Crashes in Bitcoin* Preliminary report.

We develop the novel drawdown and drawup methodology to systematically investigate the characteristics of shocks, bubbles and crashes of the Bitcoin. At each time point, the activeness of the current drawup (drawdown) phase will be checked by comparing the value of largest deviation with a predefined tolerance value, which quantifies the allowable level of price movement against the drawup (drawdown) trend. In addition, we apply the log-periodic power law singularity (LPPLS) methodology to classify the origins of bubbles and crashes into two types: endogenous and exogenous. Further, we employ the Lagrange regularisation of the normalised sum of the squared residuals to identify the optimal start time of Bitcoin bubbles. This paper creates a paradigm for future studies in bubble and crash detection and underlying mechanism dissection in not only the cryptocurrency market, but also other financial markets and economic indexes. (Received September 7, 2021)

1174-62-6315 **Lorin Crawford*** (lcrawford@microsoft.com), Microsoft Research, Brown University. *Statistical Frameworks for Discovering Biophysical Signatures in 3D Shapes and Images*

The recent curation of large-scale databases with 3D surface scans of shapes has motivated the development of tools that better detect global patterns in morphological variation. Studies which focus on identifying differences between shapes have been limited to simple pairwise comparisons and rely on pre-specified landmarks (that are often known). In this talk, we present SINATRA: a statistical pipeline for analyzing collections of shapes without requiring any correspondences. Our method takes in two classes of shapes and highlights the physical features that best describe the variation between them. We develop a rigorous simulation framework to assess our approach, which themselves are a novel contribution to 3D image and shape analyses. Lastly, as case studies with real data, we use SINATRA to (1) analyze mandibular molars from four different suborders of primates and (2) facilitate the visual identification of structural signatures differentiating between the trajectories of two protein ensembles resulting from molecular dynamics simulations. (Received September 7, 2021)

1174-62-6479 **Leah Woldemariam*** (lsw85@cornell.edu), Cornell University. *Glacial Cycle Attractors* Preliminary report.

Oftentimes, a system must be modeled, but there is a lack of information surrounding the system apart from observed data. Attractor reconstruction methods allow us to create a dynamical system model from a single observed time series. Takens theorem allows us to create a topologically equivalent attractor of the dynamical system from this single observed time series. The data used in the reconstruction and the reconstruction itself can be analyzed with various techniques such as computing fractal dimensions or measuring dynamical invariants. Using attractor reconstruction methods and time series analysis, we investigate the relationship between variables commonly used in conceptual glacial models and compare the ability of different data sets to accurately reconstruct the glacial cycle attractor. (Received September 9, 2021)

1174-62-7058 **Benjamin Wu*** (benwu200510@gmail.com), Western Connecticut State University, **Avi S Ray** (ray032@wcsu.edu), Western Connecticut State University, **Julia Lan Zhao** (julia.l.zhao1@gmail.com), Western Connecticut State University, and **Jiachen Xu** (xu007@wcsu.edu), Western Connecticut State University, Chinese University of Hong Kong. *American Option Pricing Based on Wavelet Denoised Machine Learning Methods* Preliminary report.

Financial markets are often volatile and offer high incentives to predict different option prices. Many algorithms have been proposed such as Black-Scholes model, Monte Carlo Simulation, Binomial Model, to price options. However, there are many drawbacks to each of these methods. In this research, we investigate separate methods and create a new algorithm for calculating American option pricing. To accomplish this, a variety of historical stock data was collected. Then, a subset of this data was taken, with dates ranging from 2020 to 2021, and was denoised using a 4-band wavelet transform. The data was then applied to several machine learning methods, which are Support Vector Regression (SVR) and Neural Networks; then we fitted these methods into a Least Squares Monte Carlo Simulation. The results of these tests are further compared and discussed in detail with other traditional methods. (Received September 11, 2021)

1174-62-7082 **Haoyu Du*** (haoyudu@umich.edu), University of Michigan, and **Ashley Tran** (ashlett5@uci.edu), University of California, Irvine. *Applications of Regression Tree and Linear Model Evaluation in Quantitative Trading* Preliminary report.

At the heart of automated quantitative stock trading is a model to forecast stock returns at a desired horizon. We developed a cutting-edge forecasting model that predicts stock returns 10 minutes into the future. We constructed one such model by using existing tools in statistical analyses and regression tree models. The algorithm was built to perform (1) data wrangling, (2) feature selection, (3) regression tree model training, and finally (4) future stock return predictions. All these processes require an extensive amount of computational power, and we utilized the supercomputer cluster provided by UCLA to perform these calculations. The poster will be aimed at a general audience. (Received September 12, 2021)

1174-62-7260 **Patrick Carroll*** (carropat@oregonstate.edu), Oregon State University, Western Washington University, **Alex Kuhn** (kuhnal@oregonstate.edu), Oregon State University, Western Washington University, and **Kimihiro Noguchi** (noguchk@wwu.edu), Western Washington University. *Wild Bootstrap Implementation of the Nonparametric Multiple Comparison Procedure with Log Odds Transformation on the Relative Effects in the One-Way Repeated Measures Setting* Preliminary report.

Many experiments in psychology, biology, medicine, etc. result in one-way repeated measures data. These experiments often end up having small sample sizes due to time or budget constraints. When the sample size is small, it is impossible to justify the underlying distributional assumptions, such as normality, for popular parametric tests (e.g., ANOVA F-test), making nonparametric tests an attractive option to properly assess treatment effects. However, common rank-based nonparametric tests utilize relative effects, which may not be as easily understandable as mean differences. To overcome the problem, a log odds transformation on the relative effect can be used as an interpretable effect size that researchers may choose to report in addition to the p-value. A wild bootstrap procedure, suggested recently for the untransformed nonparametric tests, may be suitably modified to improve the robustness of the log odds-transformed version in the small-sample setting. Thus, we investigate the performance of a rank-based nonparametric multiple comparison procedure with log odds transformation on the relative effects in the low-dimensional repeated-measures setting with small sample sizes. Our simulation results indicate that the proposed novel wild bootstrap-based procedure performs well relative to traditional procedures which rely on the asymptotic multivariate normal distribution or approximate multivariate t-distributions. (Received September 13, 2021)

1174-62-7266 **Hang J Kim*** (hang.kim@uc.edu), University of Cincinnati. *Synthetic Microdata for Official Statistics*

In preserving the privacy of data collected by statistical agencies, two main streams of research have been done: synthetic data approaches and differential privacy mechanisms. This talk covers the synthetic data approach to generate the microdata from the input data collected by government statistical agencies. We will review the seminal idea of synthetic data proposed initially by Rubin and introduce some modern synthetic data generators, for example, using nonparametric Bayesian models to capture the complex multivariate relations between items and handle defective measurements due to missing values or measurement errors. As an application example, synthetic data generation for the 2012 U.S. Economic Census and its performance is introduced. (Received September 13, 2021)

1174-62-7305 **Thomas Needham** (tneedham@fsu.edu), Florida State University, and **Thomas Weighill*** (t_weighill@uncg.edu), UNC Greensboro. *Geometric averages of redistricting plans and clustered datasets*

We consider the problem of extracting information from an ensemble of partitioned datasets. Some important examples of such datasets include clustered data and redistricting plans. We design a method for jointly registering partitioned datasets in a geometrically coherent way based on techniques from optimal transport. After establishing some basic theory, we will demonstrate our method on ensembles of political redistricting plans to extract and visualize basic properties of the space of plans for a particular state. (Received September 14, 2021)

1174-62-7601 **Michelle Nixon*** (map5672@psu.edu), Pennsylvania State University, **Steven Nixon** (sxn5077@arl.psu.edu), Pennsylvania State University, and **Roberto Molinari** (robmolinari@auburn.edu), Auburn University. *Data of the Defense and the Defense of Data*

The Department of Defense houses rich datasets that have the potential to transform the science and technology behind the defense sector. However, freely sharing these data sets often conflict with national security, leading to a status quo of restricted information. A special class of data, Controlled Unclassified Information (CUI), naturally lends itself to sharing, but special precautions are typically necessary. In this talk, we discuss the current state of the art of data sharing for the data of defense and several issues unique to disseminating this type of data. These data are often explicit in what exactly is sensitive, but the sensitive information may not conform to what is traditionally seen in the privacy literature. In addition, we discuss potential avenues for data privacy to address data sharing limitations in the defense sector moving forward. (Received September 15, 2021)

1174-62-7617 **Alex Lyford** (alyford@middlebury.edu), Middlebury College, and **Noah Whiting*** (nwhiting@middlebury.edu), Middlebury College. *Quantifying Racial Bias in Major League Baseball Commentary*

In this talk, we examine transcripts from Major League Baseball games to understand how commentators describe players of different races. Previous analyses of commentator bias in sports have yielded mixed results based on small samples. While some studies have asserted that commentators favor the cognitive abilities of white athletes and highlight the physical attributes of black athletes, others contend that this effect is not so clear. To contribute to this debate, we gather a novel dataset of thousands of baseball games from 2015-2019. This large sample establishes a more complete picture of commentator trends and allows us to analyze commentary patterns across four different racial categories: White, Black, Latino and Asian. Using an automated process, we extract phrases mentioning certain players from the transcripts and then characterize the nature of the comments by hand. The extensive performance data gathered in baseball games enables us to pair athletes of similar abilities, but different racial categories. As we look at the transcripts and compare players of different races, we strive to understand what causes a commentator to highlight attributes such as a player's speed: the player's maximum sprint speed or their race? (Received September 17, 2021)

1174-62-7619 **Breille Duncan*** (Bhduncan@cedarcrest.edu), Cedar Crest College. *The Effect of M-Shwari on Healthcare*

M-Shwari is a mobile banking service launched by Safaricom and the Commercial Bank of Kenya as an extension to their existing mobile money transferring service M-Pesa. M-Shwari allows users to save money and take out loans through the phone, making these services more accessible to those in rural areas. We will look at two different tests to determine how this service has affected household spending on healthcare. (Received September 15, 2021)

1174-62-7784 **Carlos Soto*** (cjs7363@psu.edu), Penn State University, and **Karthik Bharath** (Karthik.Bharath@nottingham.ac.uk), University of Nottingham. *Differential Privacy Over Riemannian Manifolds*

This paper considers the problem of releasing a differentially private statistical summary that resides on a Riemannian manifold. It presents an extension of the Laplace, or K-norm, mechanism that utilizes intrinsic distances and volumes while specifically considering the case where the summary is the Fréchet mean. The mechanism is shown to be rate optimal and depends only on the dimension of the manifold, not on the dimension of an ambient space, while also showing that ignoring the manifold structure can decrease the utility of the privatized summary. The proposed framework is illustrated in two examples of particular interest in statistics: the space of symmetric positive definite matrices, which is used for covariance matrices, and the sphere, which can be used as a space for modeling discrete distributions. (Received September 16, 2021)

1174-62-8085 **Ryan Shuman*** (shumanrs@dukes.jmu.edu), James Madison University, **Prabhashi Withana Gamage** (withanpw@jmu.edu), James Madison University, **Scott P Stevens** (stevensp@jmu.edu), James Madison University, and **Hasan Hamdan** (hamdanhx@jmu.edu), James Madison University. *Determining an Accurate Measure for the Standard Errors in the Cox PH Model* Preliminary report.

The Cox proportional hazards (PH) model is arguably one of the most popular models to analyze time-to-event data, which commonly arise in epidemiological studies and clinical trials. A good approximation of the standard errors for the maximum likelihood estimates of the regression parameters is crucial in making reliable inference. Therefore, this work focuses on evaluating several methods of approximating the standard errors for the regression coefficients in the Cox PH model. In particular, we compare four popular estimators; the inverse of the negative Hessian matrix, the so-called sandwich estimator, the Louis's method, and the outer product of the gradients estimator. We performed extensive simulation studies and offer general guidance on the selection of the best estimator to quantify the standard errors of the Cox PH model under different settings. The findings were further illustrated using real data sets. (Received September 17, 2021)

1174-62-8166 **Linda Ann Ness*** (nesslinda@gmail.com), Rutgers DIMACS. *Some Examples of Mathematical Tools for Computer Vision*

This talk will describe a set of multiscale mathematical tools that have been applied to images. The tools typically construct higher dimensional representations of the data which permit linear decision making and/or reveal patterns not easily visible in the original data. Several tools exploits multi-scale singular value decomposition; another tool exploits an easily computable theory for representing data as a multi-scale measure and then uses the parameters of the measure to re-represent the data. The talk will also include discussion of more recent work which uses a simple hidden layer neural network architecture to re-represent data as independent data points in a form which permits easy computation of a linear prediction rule and avoids used of gradient descent. The multiscale tools were developed in collaboration with a number of people: Devasis Bassu, Rauf Izmailov, Peter W. Jones, David Shallcross, Paul Bendic, Ellen Gasparovic, John Harer, Patricia Medina, Melanie Weber and Kara Yacoubou Djima. (Received September 17, 2021)

1174-62-8170 **Charlie Frazier** (ccfrazie@ncsu.edu), North Carolina State University, **Jordan Bramble*** (missylink@ku.edu), University of Kansas, **Frederick Donahey** (fdonahe@ncsu.edu), Lamar University, **Liam Hanson** (whhanson@ncsu.edu), University of North Carolina at Chapel Hill, **Aaron Marshall** (amarsha@ncsu.edu), Butler University, **Brian Reich** (bjreich@ncsu.edu), North Carolina State University, and **Mohamed Abdelkader Abba** (mabba@ncsu.edu), North Carolina State University. *COVID-19, Climate and Socioeconomic Status in the United States: A Bayesian Analysis of County-Level Case Data*

Since emerging in 2019, the novel coronavirus SARS-CoV-2 has infected over 41 million people in the United States, and its spread has varied by county. Differences may be partially explained by spatial and temporal variations in meteorological and sociological variables. Literature suggests temperature and humidity impact SARS-CoV-2 spread, but their influence remains unclear. Similar is true of various sociodemographic factors. PM 2.5 and mobility levels may also influence spread. We utilize discrete SIR models with county-dependent parameters to simulate SARS-CoV-2 spread across the U.S. Transmission rates are assumed to evolve temporally according to an AR(1) process. These models are fit to county-level COVID-19 case data via Markov Chain Monte Carlo sampling. We then explore the above factors' relative influence on SARS-CoV-2 spread using a Bayesian regression model with estimated county-level transmission rate means as the response and meteorological and sociological variables as covariates. This project was completed through an REU at NC State University. (Received September 17, 2021)

1174-62-8210 **Sarah Tymochko*** (tymochko@egr.msu.edu), Michigan State University, **Firas Khasawneh** (khasawn3@egr.msu.edu), Michigan State University, and **Audun Daniel Myers** (myersau3@msu.edu), Michigan State University. *Network Based Approaches to Topological Signal Processing*

While topological tools have been used to study networks from numerous applications, the use of these tools to study time series data is fairly new. Ordinal partition networks are a type of graph representation of a time series that can capture features indicative of periodic or chaotic behavior. In our work, we study modifications to the ordinal partition network method to determine if including additional features such as directionality captures more information about the underlying system than the original method. We investigate the use of persistent homology, including using directed graphs as inputs, or the use of zigzag persistence, to quantify changes in time series data. (Received September 18, 2021)

1174-62-8212 **Colin Olson** (colin.olson@nrl.navy.mil), U.S. Naval Research Laboratory, **Tim Doster** (timothy.doster@pnnl.gov), Pacific Northwest National Lab, and **Alexander Soloway** (alexander.soloway@pnnl.gov), Pacific Northwest National Laboratory. *I Spy in the Sky: a Stable Topological Approach for Aerial Tracking Data*

The task of classifying objects at range in low resolution imagery is challenging because of the lack of clarity of the object as well as the lack of spatial information around the object. While deep learning often fails in these cases, topological tools can often be used successfully. Using tracks of objects derived from video data, we can study the underlying dynamics of the objects motion to distinguish between target and objects that may be confused for targets (called confusers). We compare a topological approach using the time delay embedding, persistent homology and persistence images to more standard techniques and deep learning methods. We find the topological approach to be able to classify tracks more accurately, and more successfully generalize to new data collected under different circumstances. Further, we prove that the topological approach is stable, meaning that small perturbations to the input data such as noise will only cause small, bounded changes to the output persistence images. (Received September 18, 2021)

1174-62-8521 **Gracie Suzanna Johnson** (grace.johnson@my.wheaton.edu), Wheaton College (IL), **Austin Mathew Sibu** (austinsibu1@gmail.com), Texas A&M University, **Nicholas C Gawron*** (ncgawron@ncsu.edu), North Carolina State University, and **Aaron M Stapleton** (ams758@cornell.edu), Cornell University. *Decorrelation Detection in Financial Time Series Data*

For this NSF funded REU project, we partnered with Wellington Management Company and researched how dimension estimation can determine whether seemingly diversified index fund portfolios in fact have latent commonalities that increase risk for investors. Principal Component Analysis (PCA) is a widely used tool for dimension estimation, but during periods of financial crisis, PCA is unable to adequately perform dimension estimation, because it relies on linear relationships and is sensitive to outliers. We investigate robust and non-linear techniques including Robust PCA and autoencoders, assessing their usefulness for three tasks: dimension estimation, data reconstruction, and correlation visualization. We find several methods that are resistant to market shocks and develop a novel method to estimate dimension using autoencoders. We additionally provide insight into future time series modeling. (Received September 21, 2021)

1174-62-8548 **Jonathan Cuauhtemoc Garber*** (garberj2@wwu.edu), Western Washington University, **David Rice** (david.rice@wsu.edu), Washington State University, **Shea Frantz** (frantzs2@wwu.edu), Western Washington University, **Laura K. Fitzgibbon-Collins** (fitzgibbon.collins@gmail.com), University of Waterloo, **Mamiko Noguchi** (mnoguchi@uwaterloo.ca), University of Waterloo, and **Richard L. Hughson** (hughson@uwaterloo.ca), University of Waterloo. *Relating Gait Metrics and Cerebral Tissue Saturation Index Using Principal Component Analysis in Older Adults* Preliminary report.

We explore the relationship between different gait metrics and cerebral tissue saturation index (TSI) using data collected from Near Infrared Spectroscopy (NIRS) and tri-axial accelerometers. Specifically, using principal component analysis (PCA) and functional PCA (FPCA), we investigate how the standard deviations of several key gait variables, such as gait speed and step-step variability, are correlated with different functional shape features of their observed TSI, which measures the percent of oxygenated hemoglobin content relative to total hemoglobin content over time. Then, we discuss how these TSI features may be correlated with different gait metrics, focusing on those with slower gait speeds and more asymmetrical walking patterns, who are more at risk of falling. (Received September 19, 2021)

1174-62-8737 **Richard B Sowers*** (r-sowers@illinois.edu), University of Illinois. *The Topology of Traffic Congestion*

We study traffic congestion through the lens of persistent homology. We start with some estimates of traffic speeds in New York city derived from taxi data. We then develop the notion of H_0 persistence to understand robust patterns traffic gridlock. We then construct a Rips filtration of traffic speeds based on a symmetrization recently suggested by Turner. After reducing complexity based on a Louvain neighborhood-finding algorithm, we construct H_1 barcodes showing path structure through traffic. We briefly discuss comparisons between traffic in New York city and Chengdu, China.

This work is joint with others, and partly in collaboration with Daniel Carmody. (Received September 19, 2021)

1174-62-8740 **Venkateswara Rao Mudunuru** (vmudunur@mail.usf.edu), University of South Florida, **Vidya Bhargavi M*** (vidya.msc05@gmail.com), Stanley College of Engineering and Technology For, **Sireesha V** (vsirisha80@gmail.com), GITAM(Deemed to be University), and **Sampath Kalluri** (sampath.kalluri@gmail.com), Novartis Healthcare Private Limited. *A Comparison of Multilayer Neural Networks and Decision Trees for Stage Classification of Colon Cancer Data*

In the recent studies, machine learning methods such as Artificial Neural Networks (ANNs) and Decision Trees are applied as a predictor and classification techniques to several research areas including clinical, medical, and public health, business, fraud detection, time series prediction, among others. In this paper with the same set of attributable variables, we develop ANN and decision tree methods to classify the colon cancer stages. By measuring and evaluating the sensitivity, specificity and the area under the curve values, the best model for colon cancer stage classification is identified. (Received September 19, 2021)

1174-62-8754 **Elyse Borgert*** (elyseb@live.unc.edu), University of North Carolina at Chapel Hill. *Persistent topology of protein space*

Protein fold classification is a classic problem in structural biology and bioinformatics. We approach this problem using persistent homology. In particular, we use alpha shape filtrations to compare a topological representation of the data with a different representation that makes use of knot-theoretic ideas. We use the statistical method of Angle-based Joint and Individual Variation Explained (AJIVE) to understand similarities and differences between these representations. (Received September 20, 2021)

1174-62-8795 **Taylor Boatwright*** (Taylor.boatwright@g.fmarion.edu), Francis Marion University, and **Briana Monarca** (brianamonarca@yahoo.com), Francis Marion University. *Can you Belize What We Did?*

Water quality is an issue that can require intervention in order to protect the health of individuals. In particular, many communities in rural areas of Belize, Central America lack potable drinking water. In order to address remediation tactics for communities surrounding the Billy Barquedier National Park, we collected and tested water samples from the source in the park and the household sites in the Valley community in the Stann Creek District, Belize, C.A. We are interested in determining the source of contamination as well as correlations between microbial populations E. coli and chemical (or physical) components like nitrate levels. Testing involved taking samples as well as testing in the field with the Pro DSS water quality meter. Lab testing used a colorimeter and autoclave to test the microbial and chemical makeup. We discuss the results, their implications, and future work. (Received September 19, 2021)

1174-62-8805 **Hong Hu*** (honghu@g.harvard.edu), Harvard University. *Universality of Learning in Random Feature Model*

In this talk, I will present our work on studying a universality phenomenon, known as the Gaussian equivalence phenomenon, in learning under the random feature model (RFM). It has been conjectured and numerically verified that learning with the random features is equivalent to learning with certain linear Gaussian features. Concretely, the training error and test error are asymptotically the same under these two models in the large system limit. This conjecture first appeared in the earlier works studying spectrum of random kernel matrices and it is recently utilized to exactly characterize the learning performance in RFM. In particular, we prove this conjecture, under certain smoothness and convexity assumptions. (Received September 19, 2021)

1174-62-8830 **Gundeep Singh*** (Gundeepsingh12@yahoo.com), University of Houston. *Finding your way back in a random forest : Debias regression predictors* Preliminary report.

Random forest is one of the most popular and successful ensemble methods in machine learning used for classification and regression. It is well known that the random forest may reduce the variance of regression predictors through bagging while leaving the bias little changed. In general, the bias that arises from the random forest is not negligible and consequently bias correction is necessary. The default bias correction method implemented in the R package randomForest often does not work well. In this work, we develop and explore several bias correction methods as alternatives to the R default. We further offer guidance on the selection of different methods through a comprehensive comparison of these methods. We also propose a visualization technique to help users decide when bias correction is needed. (Received September 19, 2021)

1174-62-8846 **Vince Lyzinski*** (vlyzinsk@umd.edu), University of Maryland, College Park, **Carey Priebe** (cep@jhu.edu), Johns Hopkins University, **Jesus Arroyo** (jarroyo@tamu.edu), Texas A&M University, and **Daniel L Sussman** (sussman@bu.edu), Boston University. *Maximum Likelihood Estimation and Graph Matching in Errorfully Observed Networks*

Given a pair of graphs with the same number of vertices, the inexact graph matching problem consists in finding a correspondence between the vertices of these graphs that minimizes the total number of induced edge disagreements. We study this problem from a statistical framework in which one of the graphs is an errorfully observed copy of the other. We introduce a corrupting channel model, and show that in this model framework, the solution to the graph matching problem is a maximum likelihood estimator. Necessary and sufficient conditions for consistency of this MLE are presented, as well as a relaxed notion of consistency in which a negligible fraction of the vertices need not be matched correctly. The results are used to study matchability in several families of random graphs, including edge independent models, random regular graphs and small-world networks. We also use these results to introduce measures of matching feasibility, and experimentally validate the results on simulated and real-world networks. (Received September 20, 2021)

1174-62-8852 **David Siegmund*** (siegmund@stanford.edu), Stanford University. *Change-point Segmentation* Preliminary report.

The maximum score statistic is used to detect and estimate local signals in the form of change-points in the level, slope, or other property of a sequence of observations, and to segment the sequence when there appear to be multiple changes. Both fixed sample and sequential problems are discussed. False positive errors are controlled by approximations to the tail of the maximum of an appropriate stochastic process or random field. Modifications of a natural auto-covariance estimator to deal with the bias that arises when there are change-points are suggested. Applications to copy number variations, climate related time series, disease incidence (especially COVID-19), and vital statistics illustrate the general theory.

Contributors to aspects of this research include Xiao Fang, Jian Li, Benny Yakir, and Nancy Zhang. (Received September 20, 2021)

1174-62-8913 **Andres Felipe Barrientos*** (abarrientos@fsu.edu), Department of Statistics, Florida State University, and **Víctor Peña** (victor.pena@baruch.cuny.edu), Baruch College, The City University of New York. *Differentially private methods for managing model uncertainty in linear regression models*

Statistical methods for confidential data are in high demand due to an increase in computational power and changes in privacy law. This work introduces differentially private methods for handling model uncertainty in linear regression models. More precisely, we provide differentially private Bayes factors, posterior probabilities, likelihood ratio statistics, information criteria, and model-averaged estimates. Our methods are asymptotically consistent and easy to run with existing implementations of non-private methods. (Received September 20, 2021)

1174-62-8938 **Maxwell Lovig** (maxwelllovig@gmail.com), University of Louisiana - Lafayette, **Emiliano Planchon*** (eplanch@ncsu.edu), North Carolina State University, **Neil Dey** (ndey3@ncsu.edu), North Carolina State University, **Carolina Kapper** (ckapper@ncsu.edu), High Point University, **Jack Ferrell** (jferrel3@ncsu.edu), University of Florida, **Jing Ding** (jding7@ncsu.edu), North Carolina State University, and **Jonathan P Williams** (jwilli27@ncsu.edu), North Carolina State University. *Conformal prediction for text infill and part-of-speech prediction* Preliminary report.

Modern machine learning (ML) algorithms are capable of providing remarkably accurate predictions, however, questions remain about their statistical interpretability. These techniques are referred to as black box, where a single point estimate is generated, but it is unclear exactly how or with what degree of certainty the prediction is made. Unlike conventional ML models, the Conformal Prediction (CP) algorithm outputs predictive regions (i.e., set-valued predictions) that correspond to a given significance level. CP comes with the validity of finite sample control over type 1 error probabilities, allowing the practitioner to choose an acceptable error rate. The original CP method is prohibitively computationally expensive, so we explore computationally more efficient Inductive Conformal Prediction (ICP) algorithms on the tasks of text infil and part-of-speech prediction. We implement a state-of-the-art BiLSTM neural network and a BERT transformer-based model to perform the task of part-of-speech prediction, and extend our use of BERT to text infill prediction. We analyze the performance of our models and the ICP algorithm on the Brown Corpus from Brown University which contains over 57,000 sentences. The results show that the ICP implementation is able to produce valid set-valued predictions that are

small enough to be applicable in real-world situations. We also provide a real data example of how our proposed set-valued predictions can improve on machine generated audio transcriptions. (Received September 20, 2021)

1174-62-8978 **Evercita Cuevas Eugenio*** (eceugen@sandia.gov), Sandia National Laboratories, **Fang Liu** (Fang.Liu.131@nd.edu), University of Notre Dame, and **Ick Hoon Jin** (ijin@yonsei.ac.kr), Yonsei University. *Differentially Private Generation of Social Networks*

Social network analysis and research has grown tremendously in the last decade, especially with the emergence of social media (e.g., Facebook and Twitter). While the voluminousness and popularity of social network data have enabled new discoveries, they have also increased privacy risk of individuals, as many social networks contain sensitive relational information. This talk will provide insights to the current state of differential privacy when applied to social networks. Additionally, methods that protect the sensitive relational information while offering flexibility for social network research and analysis to release synthetic social networks at a pre-specified privacy risk level, given the original observed network, will be highlighted. A newly proposed method DP-ERGM procedure which synthesizes networks that satisfy the differential privacy (DP) via the exponential random graph model (EGRM) will be compared to two other approaches: differentially private DyadWise Randomized Response (DWRR) and Sanitization of the Conditional probability of Edge given Attribute classes (SCEA).

SNL is managed and operated by NTESS under DOE NNSA contract DE-NA0003525 (Received September 20, 2021)

1174-62-9016 **Ralph Smith*** (rsmith@ncsu.edu), North Carolina State University. *Active Subspace Techniques for Sensitivity Analysis and Uncertainty Quantification*

For many complex physical and biological models, the computational cost of high-fidelity simulation codes precludes their direct use for Bayesian model calibration and uncertainty propagation. For example, neutronics and nuclear thermal hydraulics codes can take hours to days for a single run. Furthermore, the models often have tens to thousands of inputs – comprised of parameters, initial conditions, or boundary conditions – many of which are unidentifiable in the sense that they cannot be uniquely determined using measured responses. In this presentation, we will discuss techniques to isolate influential inputs for subsequent surrogate model construction for Bayesian inference and uncertainty propagation. For input selection, we will discuss advantages and shortcomings of global sensitivity analysis to isolate influential inputs and detail the use of gradient-free active subspace techniques to determine low-dimensional input manifolds. We will also discuss the manner in which Bayesian calibration on active subspaces can be used to quantify uncertainties in physical parameters. These techniques will be illustrated for models arising in nuclear power plant design and quantum-informed material characterization. (Received September 20, 2021)

1174-62-9020 **Joshua Snoke*** (jsnoke@rand.org), RAND Corporation. *Statistical Data Privacy: Where Do We Go from Here?*

Statistical data privacy is a growing and pressing research field. As we see an increase in the need for access to high quality research data while protecting the privacy and confidentiality of participants in the data, there is a growing need for novel approaches to sharing data or results. Technological advances have made it both easier to collect data while also increasing the risk of unwanted disclosures. This talk will review the approaches to statistical data privacy from the session and the different models of data access, such as synthetic data, differential privacy, validation servers, or secure multiparty computation. I will review some practical applications and discuss the interplay of statistics data privacy with the required policy decisions by stakeholders. I will offer open questions for the field and future research areas that must be tackled with novel statistical approaches. (Received September 20, 2021)

1174-62-9033 **Emily Roberts*** (ekrobe@umich.edu), University of Michigan. *Causal inference methods to validate surrogate endpoints in clinical trials*

A common practice in clinical trials is to evaluate a treatment effect on an intermediate variable when the true outcome of interest would be difficult or costly to measure. We consider how to incorporate intermediate endpoints in a causally-valid way. Using counterfactual outcomes (those that would be observed if the counterfactual treatment had been given), the causal association paradigm assesses the relationship of the treatment effect on the surrogate S with the treatment effect on the true endpoint T. In our setting, we assume the surrogate under the placebo is zero-valued, which is an assumption that naturally arises in vaccine efficacy trials. We develop methods to incorporate conditional independence of the potential outcomes, and we assess the estimation properties of a Bayesian imputation method using Markov Chain Monte Carlo and corresponding impact of different prior distributions on non-identified model parameters. We demonstrate our approach via simulation of data that mimics an ongoing study of a muscular dystrophy gene therapy and consider a subsequent design where the outcomes are measured longitudinally. Further, it may be possible to measure additional T and S outcomes in a cross-over trial design where subjects who are first administered the placebo may be given the therapy mid-trial. We develop a mixed model approach that can potentially gain estimation efficiency by modeling these repeated measures. (Received September 20, 2021)

1174-62-9117 **Sui Tang*** (suitang@ucsb.edu), UCSB, **Yunxiang Ren** (renyunxiang@gmail.com), Harvard University, and **Jinchao Feng** (jcfeng1992@gmail.com), JHU. *Data-driven discovery of interacting particle system using Gaussian processe*

Interacting particle or agent systems are widely used to model complicated collective motions of animal groups in biological science, such as flocking of birds, milling of fish, and swarming of prey. A fundamental goal is to understand the link between individual interaction rules and collective behaviors. We consider second-order interacting agent systems and study an inverse problem: given observed data, can we discover the interaction rule and even the governing equation? For the interactions that only depend on pairwise distance, we propose a learning approach based on Gaussian processes that can simultaneously infer the interaction kernel without assuming a parametric form and learn other unknown parameters in the governing equations. The numerical results on prototype systems, including Cuker-Smale dynamics and fish milling dynamics, show that our approach produced faithful estimators from scarce and noisy trajectory data and made accurate predictions of collective behaviors. This talk is based on the joint work with Jinchao Feng, and Yunxiang Ren. (Received September 20, 2021)

1174-62-9210 **Jonah Lee Silverman*** (jsilverman@muhlenberg.edu), Muhlenberg College, and **James Russell** (jamesrussell@muhlenberg.edu), Muhlenberg College. *An Analysis of Classification Models Predicting Pancreatic Cancer Via Urinary Biomarker Panel* Preliminary report.

Pancreatic cancer is highly aggressive, with a survival rate less than 10% , 5 years following diagnosis. One issue the oncology community faces in treating this form of cancer is the difficulty in diagnosing the disease, due to both the hard to access position of the pancreas in the body, as well as the fact that commonly, symptoms only arise following dangerous amounts of malignant growth. These barriers to early detection mean that health care practitioners are in need of easy and effective diagnostic tools for pancreatic cancer. We set out to build a classifier which is able to distinguish between pancreatic cancer positive patients opposed to healthy controls, based on a panel of urinary biomarkers. The dataset we are employing in this project contains the patient diagnosis status, four biomarker measurements, the patient's age and their sex. In order to create the most effective model possible we plan to employ and compare three different machine learning/statistical techniques, these being Classification Trees, Support Vector Machines, and General Additive Models. The goal of the study is to evaluate which models are the most accurate in diagnosis, evaluated using k-fold cross validation, and how much benefit they would really garner in the clinic. (Received September 20, 2021)

1174-62-9216 **Justin R. Sims** (JSims12@utm.edu), University of Tennessee at Martin, and **Charles Austin Brown*** (chaabrow@ut.utm.edu), University of Tennessee at Martin. *Time-Series Approach for Forecasting Twitch Viewership*

Live streaming is a massively growing field where creators broadcast video games, music, art, and other activities for viewers to watch using platforms such as Twitch and Youtube. With millions of viewers everyday, accurate forecasting of viewership allows developers and external companies to target marketing toward periods of high viewership. It also allows creators to align their content with those areas that will be in high demand. We explore the use of seasonal autoregressive integrated moving average (ARIMA) models for forecasting the viewership of video games. We illustrate the model by describing and forecasting average monthly viewership for 15 video games on the Twitch platform. (Received September 20, 2021)

1174-62-9242 **Lamiae Taoudi*** (lt900@msstate.edu), Mississippi State University. *A Regime-Switching Synchronous-Jump Tempered Stable Levy Model for Structural Model for Credit Risk*

Hainout et.al [1] proposed Merton's structural model for credit risk modeling, where they assume that the value of the firm is modeled by a switching Lévy process. The volatility and mean value of the default probability are modeled using a Markov chain. However, they concentrate on finite activity lévy processes for modeling the default events. In this work, we broaden this model using tempered stable lévy processes including CGMY and KoBol. In this regard, we used econometric calibration to estimate the variables of these stochastic processes for the dynamic of two real-world firms. We fit two Markov modulated lévy processes with a synchronous jump to the return of their daily stock.

The data used in this work is daily data of these two firms for ten years including the financial crisis of 2008. We used the Hamilton filter to capture synchronous jumps and daily transition matrix. Parameters are obtained by numerically maximizing the loglikelihood function. The fitted parameters for the CGMY and KoBol process as well as their standard errors will be presented for both firms.

[1] Hainaut, Donatien, and David B. Colwell. "A structural model for credit risk with switching processes and synchronous jumps." The European Journal of Finance 22.11 (2016): 1040-1062. (Received September 20, 2021)

1174-62-9300 **Tim Zeitvogel*** (tim.zeitvogel@pepperdine.edu), Pepperdine University. *Understanding Momentum and Score Importance in Tennis Matches*

This research attempts to understand how in-match dynamics such as momentum or score pressure influence the outcomes of tennis points and matches. Due to the unique structure of the scoring system of tennis, we can construct a block-walk inspired stochastic match win probability model based on the service and return ability of each player. We will build upon a previous ordinary least squares regression model that describes the dependence of the service point win probability on pre-match quality measurements as well as in-match variables. In addition to information about the previous point, we will incorporate the current game score and a pressure index that compares the expected service and return game win probabilities with the actual outcomes. Our analysis of more than 2 million points, including men's and women's matches as well as best-of-three and best-of-five tournaments on three different surfaces from 2011–2015, gives us insights into the in-match patterns that develop based on the match setup. Finally, we incorporate pre-match estimations, the pressure index, and in-match trends into a dynamic in-match forecast model for match win probability. (Received September 20, 2021)

1174-62-9331 **Shayan Mukherjee*** (sayan@stat.duke.edu), Duke University. *Modeling shapes and fields* Preliminary report.

We will consider modeling shapes and fields via topological and lifted-topological transforms. Specifically, we show how the Euler Characteristic Transform and the Lifted Euler Characteristic Transform can be used in practice for statistical analysis of shape and field data. The Lifted Euler Characteristic is an alternative to the. Euler calculus developed by Ghrist and Baryshnikov for real valued functions. We also state a moduli space of shapes for which we can provide a complexity metric for the shapes. We also provide a sheaf theoretic construction of shape space that does not require diffeomorphisms or correspondence. A direct result of this sheaf theoretic construction is that in three dimensions for meshes, 0-dimensional homology is enough to characterize the shape. (Received September 20, 2021)

1174-62-9380 **Jonathan Wakefield*** (jonno@uw.edu), University of Washington, **William Msemburi** (msemburiw@who.int), WHO, **Victoria Knutson** (vknuts@uw.edu), University of Washington, and **Serge Aleshin-Guendel** (aleshing@uw.edu), University of Washington. *Modeling excess mortality at the country level* Preliminary report.

"Excess mortality" is defined as the difference between the total number of deaths recorded, and those expected, based on projections of what is "typical" from previous years. This talk will describe modeling work to produce estimates of excess mortality for all countries in the world for the period January, 2020 to June, 2021, for the World Health Organization. For 87 countries out of 194 we have the observed number of deaths, reported by the countries. For these countries we simple report the observed differences and there is no uncertainty. For the remaining 107 countries we need to predict the observed number of deaths, and to do this we build a regression model, based on covariates that are observed for all countries. The fundamental problem with estimating the excess mortality is that the between-country variation in the excess mortality rate is high, and only partially associated with country-specific covariates. We will describe how the model was developed, and highlight the challenges that were encountered. Priorities for future work will also be described. (Received September 20, 2021)

1174-62-9391 **Markus J Pflaum*** (markus.pflaum@colorado.edu), University of Colorado, **Ezzedine El Sai** (Ezzeddine.ElSai@colorado.edu), University of Colorado, and **Parker Gara** (parker.gara@colorado.edu), University of Colorado. *Algebraic Machine Learning and Applications*

A common assumption, which goes by the name *manifold hypothesis* asserts that in scientific models data sets with values in $\mathbb{R}^n$ occupy smooth manifolds of dimension less than n. The manifold hypothesis fails though in case the underlying geometry of the model contains singularities. Using methods from real algebraic geometry and statistics we present in this talk a method for *learning the underlying variety* of a data set in euclidean space. In a second step we explain numerical methods how to find singularities in this variety using Gröbner basis computations. We conclude the talk with several examples and an application in chemistry. (Received September 20, 2021)

1174-62-9649 **Jiwoo Lee** (leeji-22@rhodes.edu), Rhodes College, and **Abbey Gobble*** (abbey.gobble@yahoo.com), Rhodes College. *Using Persistent Homology to Analyze the Topological Signature of William Shakespeare*

Topological Data Analysis (TDA) is a recent and fast growing field that applies topological and geometric tools to infer relevant features of potentially complex data. TDA aims at providing mathematical and statistical methods to analyze the complex topological structures underlying data that are often represented as point clouds in Euclidean or general metric spaces. In this talk, we will discuss our ongoing attempt to use the methods of text mining (Gholizadeh, Seyeditabari, and Zadrozny, Topological Signature of 19th Century Novelists: Persistent Homology in Text Mining, BDCC, 2018) to explore the theory that William Shakespeare was a pseudonym employed by several different playwrights. Through the application of the cited paper's methods, we aim to determine unique topological structures for different Shakespearean plays to support the notion that their authors prove similar from a topological standpoint. (Received September 20, 2021)

1174-62-9662 **Sarah Elizabeth Harkins*** (sarah.harkins@g.fmarion.edu), Francis Marion University. *Comparative Study of Gaussian Mixtures and Clustering on Health Data* Preliminary report.

In this project, K-Means clustering and Gaussian Mixture Models (GMM) were compared and contrasted on their abilities to decipher clusters in 1-dimensional data sets. The K-Means is a popular clustering algorithm, but there are aspects that it fails to capture that possibly leave the GMM to be the superior algorithm. The abilities of the algorithms were compared and contrasted with a simulated data set. The algorithms were then further compared on a data set from the National Health and Nutrition Examination Survey. This data set includes 1,030 people ages 8 to 19 years old and their respective BMI. The goal of this application was to see if the algorithms would accurately discern between the group of males versus the group of females. GMM is typically used with higher-dimensional datasets. However, only one is used for this project, for example, features like BMI. The GMM delivers a more accurate distribution than K-Means due to the consideration of the standard deviation. (Received September 20, 2021)

1174-62-9686 **Mahmoud Affouf*** (maffouf@kean.edu), Kean University. *Mathematical Modeling of Sleep Dynamics* Preliminary report.

Using large data sets collected from sleep tracking applications, we apply statistical mixed-effect modeling techniques to identify the probability rates of various factors affecting sleep duration and quality. We also explore the dynamics of onset latency, bedtime variability, and sleep duration and modeling their dynamics. (Received September 20, 2021)

1174-62-9735 **Sam Reed Burnett*** (burnett98@me.com), Western Washington University, and **Fiona Cleary** (fiona.cleary.5@gmail.com), Western Washington University. *Analysis of the Wilcoxon-Mann-Whitney Test as a Test for the Equality of Central Tendencies* Preliminary report.

The Wilcoxon-Mann-Whitney (WMW) test checks for the equality of distributions of two independent populations by estimating the proportion of observations from the first population that are smaller than those from the second population. The quantity estimated is known as stochastic superiority or relative effect. A common mistake made when utilizing the WMW test is to consider it as a test for the equality of central tendencies. However, under variance heterogeneity, the probability of rejecting the null hypothesis after performing median centering on both samples is much closer to the nominal significance level than performing mean centering. To explain this phenomenon, we propose a set of equations which analytically determines how close the two populations are from achieving the stochastic equality and equality of central tendencies simultaneously. These equations can help expand our understanding of the conditions under which the WMW test may be used to determine the equality of central tendencies in addition to the stochastic equality. (Received September 20, 2021)

1174-62-9799 **Wanli Qiao*** (wqiao@gmu.edu), George Mason University. *Confidence Regions for Filamentary Structures* Preliminary report.

Filamentary structures, also called ridges, generalize the concept of modes of density functions and provide low-dimensional representations of point clouds. Using kernel type plug-in estimators, we give asymptotic confidence regions for filamentary structures based on two bootstrap approaches: multiplier bootstrap and empirical bootstrap. Our theoretical framework differs from many existing ones in the literature in that we allow the possible existence of intersections to emphasize the nontrivial topology of filamentary structures. Our confidence regions are asymptotically valid in different scenarios depending on how flat the filamentary structures are. (Received September 20, 2021)

1174-62-9822 **Dylan Ryan Way*** (wayd2@wwu.edu), Western Washington University, **Robert Bowen** (bowenr4@wwu.edu), Western Washington University, **Russell Goebel** (rgoebel@bu.edu), Boston University, and **Spencer Wood** (spwood@uw.edu), University of Washington. *Model Evaluation for Forecasting National Park Visitation Using Social Media* Preliminary report.

National Parks provide tremendous cultural, ecological, and economic value to societies. To properly maintain these public spaces, park managers rely on comprehensive on-site visitation data. For many parks, however, collecting such data can be an impractical, resource-intensive process which, consequently, leaves these managers with insufficient on-site visitation data for effective planning. A recent park visitation modeling approach provides a methodology for combining data about the popularity of parks on the photo-sharing website Flickr and partial on-site visitation data to forecast visitation in a short run. Here, we evaluate the forecast accuracy of the model using the National Park Service (NPS) visitation data, which are collected in full under its jurisdiction. Specifically, various partial monthly NPS visitation counts are used to train the model to evaluate its forecast performance under many different plausible scenarios. We find that only several monthly NPS visitation counts are needed to make the forecast performance of the model comparable to the one with full monthly NPS visitation counts, particularly when visitation counts from the most popular period of the year are used. (Received September 20, 2021)

1174-62-9883 **Jonathan Hehir*** (jh@psu.edu), Penn State University, **Siddharth Vishwanath** (suv87@psu.edu), Penn State University, and **Xiaoyue Niu** (xiaoyue@psu.edu), Penn State University. *Problems on Random Graphs under Local Differential Privacy*

The privacy of relationships (edges) in a binary graph can be preserved by performing randomized response on the edges. This privacy mechanism is compatible with a flavor of differential privacy that is local in nature, allowing us to produce a private synthetic network without any single party knowing the full set of true network relationships. We consider how to use these synthetic networks to perform two inference tasks on the original network: community detection of stochastic block models and topological data analysis of random dot product graphs. Through simple geometric arguments, we find appealing properties that enable theoretical guarantees for such inference. To our knowledge, these are the first theoretical guarantees for these forms of inference under differential privacy. (Received September 21, 2021)

1174-62-10060 **Jingchen Hu*** (jihu@vassar.edu), Vassar College, **Terrance Savitsky** (Savitsky.Terrance@bls.gov), Bureau of Labor Statistics, and **Matthew R Williams** (mrwillia@nsf.gov), National Center for Science and Engineering Statistics. *Private Tabular Survey Data Products through Synthetic Microdata Generation*

We propose two synthetic microdata approaches to generate private tabular survey data products for public release. We adapt a pseudo posterior mechanism that downweights by-record likelihood contributions with weights $\in [0,1]$ based on their identification disclosure risks to producing tabular products for survey data. Our method applied to an observed survey database achieves an asymptotic global probabilistic differential privacy guarantee. Our two approaches synthesize the observed sample distribution of the outcome and survey weights, jointly, such that both quantities together possess a privacy guarantee. The privacy-protected outcome and survey weights are used to construct tabular cell estimates (where the cell inclusion indicators are treated as known and public) and associated standard errors to correct for survey sampling bias. Through a real data application to the Survey of Doctorate Recipients public use file and simulation studies motivated by the application, we demonstrate that our two microdata synthesis approaches to construct tabular products provide superior utility preservation as compared to the additive-noise approach of the Laplace Mechanism. Moreover, our approaches allow the release of microdata to the public, enabling additional analyses at no extra privacy cost. (Received September 21, 2021)

1174-62-10145 **Curtis Tatsuoka*** (cmt66@case.edu), Case Western Reserve University, **Weicong Chen** (wxc326@case.edu), Case Western Reserve University, and **Xiaoyi Lu** (xiaoyi.lu@ucmerced.edu), University of California, Merced. *Bayesian Group Testing with Dilution Effects*

A Bayesian framework for group testing under dilution effects has been developed, using lattice-based models. This work has particular relevance given the pressing public health need to enhance testing capacity for COVID-19 and future pandemics, and the need for wide-scale and repeated testing for surveillance under constantly varying conditions. The proposed Bayesian approach allows for dilution effects in group testing and for general test response distributions beyond just binary outcomes. It is shown that even under strong dilution effects, an intuitive and simple-to-implement group testing selection rule that relies on the model order structure, referred to as the Bayesian halving algorithm, has attractive optimal convergence properties. Analogous look-ahead rules that can reduce the number of stages in classification by selecting several pooled tests at a time are proposed and evaluated as well. Group testing is demonstrated to provide great savings over individual testing in number of tests needed for accurate classification, even for moderately high prevalence levels. A trade-off is a higher number of testing stages, and increased variability in accuracy. High performance distributed computing methods have been implemented that accommodate analysis of larger pool sizes, when savings from group testing can be even more dramatic. The Bayesian probabilistic framework is used to reduce computational complexity. (Received September 21, 2021)

1174-62-10433 **Dan Han*** (dan.han@louisville.edu), University of Louisville, **Rajib Paul** (Rajib.Paul@uncc.edu), University of North Carolina at Charlotte, and **Sujit Ghosh** (sghosh2@ncsu.edu), North Carolina State University. *Constrained Estimation of Multivariate Density having Univariate Marginals using a Sieve of Bernstein Polynomials* Preliminary report.

This study proposed a novel approach on nonparametric estimation of multivariate density by using Bernstein polynomials. A flexible class of mixtures of Beta distributions is used and the mixing weights were constrained to preserve marginal unimodality. Simulation studies are conducted to assess the performance of the proposed method for distributions with different skewed types, which shows overall the proposed method provides more efficient estimates than the kde2d method of standard MASS R package and performs extremely well when modes are at the boundaries for skewed distributions. And the new proposed method is applied to a real dataset on gestational age and birthweight, where the underlying joint distribution of these variables are highly dependent and each one of them is marginally unimodal. (Received September 21, 2021)

1174-62-10482 **Addison Scanlon*** (ams0103@westminstercollege.edu), Westminster College, and **Kip Otuafi** (kmo0819@westminstercollege.edu), Westminster College. *Analysis of Bias by in Salt Lake City Police Department* Preliminary report.

Using information collected and provided by the Salt Lake City Police Department we analyzed statistical datasets to identify underlying biases in policing. This is an independent analysis of data surrounding police interactions with individuals from between 2014 and 2017.

In our pursuit to better understand the data, we utilized an online computing platform, R Studio, breaking substantial amounts of data into digestible, understandable, and shareable sets and visuals. Visuals include heat-maps demonstrating police interactions in Salt Lake City alongside statistically significant examples of police uses of force against communities of color.

The nuances of communicating with external departments became known through continuous attempts to understand the initial datasets. Through learning about communication with justice systems, building competency in online computing platforms and diving into Salt Lake City Police Department, our team increased individual competencies in statistical analyses. (Received September 21, 2021)

1174-62-10507 **David Schrittesser*** (david.schrittesser@univie.ac.at), University of Toronto, and **Daniel M Roy** (danroy@utoronto.ca), University of Toronto. *Admissibility is Bayes optimality with infinitesimals* Preliminary report.

The close connection between admissibility and Bayes optimality has been evident since the very first work on statistical decision theory by Wald, who first established this connection in the case of finite parameter spaces. One of the oldest open problems in statistical decision theory is to understand the extent to which this result can be generalized beyond finite parameter spaces. Generalizations to non-compact parameters spaces or problems with discontinuous risk function are already hard, and successive generalizations have been found by Wald, LeCam, Brown, Stein, Blackwell and Gershick, Berger, and others.

Recently, new light has been shed on these questions using tools from nonstandard analysis, with Haosui and Roy in their upcoming annals paper giving a general characterization of extended admissibility in decisions problems.

In this contribution, we present a complete solution to the long-standing problem of characterizing the admissible procedures in decision problems with strictly convex loss function (making no other regularizing assumptions, such as compactness or continuity). They are precisely the procedures which are uniquely Bayes among the standard decisions procedures with respect to a nonstandard prior. (Received September 21, 2021)

1174-62-10598 **John B Gonzalez*** (johngonz@alum.mit.edu), US Department of Defense, and **Janet Liu** (skatingjanet@gmail.com), Microsoft Corporation. *Inconsistencies in Olympic Figure Skating Judging* Preliminary report.

The International Skating Union implemented the International Judging System (IJS) in figure skating competitions beginning in 2004, in response to a judging scandal involving the pairs and ice dance events at the 2002 Winter Olympic Games. One of the stated reasons for developing a new judging system was to improve the consistency of the judges' ratings. This study uses three statistical measures of consistency to compare the judges' ratings within event segments at the 2002 Winter Olympic Games under the 6.0 judging system and at the 2018 Winter Olympic Games under the IJS. Results from our analysis show that ratings in the pairs and ice dance events under the IJS are not more consistent than ratings under the 6.0 judging system. (Received September 21, 2021)

1174-62-10656 **Khyam Paneru*** (kpaneru@ut.edu), The University of Tampa. *Bootstrap Confidence Intervals for the Mean of Zero-Inflated Population Having a Non-Zero Component with Normal Distribution*

An underlying population containing many zero values has a substantial spike at zero with a proportion of non-zero values and is thus referred to as a zero-inflated population (ZIP). For example, in an insurance industry where a large number of policyholders do not file a claim (zero claim amount is recorded), and the rest of the policyholders file claims (non-zero claim amount is recorded). A high proportion of zeros in such a population, causing the distribution to spike at zero, is known as zero inflation. The zero-inflated population can be considered as a mixture of two components, zero and non-zero. Existing methods using asymptotic distribution estimate the zero-inflated population means assuming multiple assumptions, and such methods are mathematically and computationally complex. This study extends the maximum pseudo-likelihood approach developed in the literature by applying the bootstrap resampling technique to estimate the confidence interval of the zero-inflated population mean. This study describes a mathematically and computationally simpler approach that avoids making assumptions for the asymptotic distribution. In addition to presenting bootstrap simulation results, this study applies the developed method to real-life data to estimate the mean of a population with a large proportion of zero values. (Received September 21, 2021)

1174-62-10893 **Md Sazib Hasan** (sazib25@gmail.com), Dixie State University, **Riley Morgan*** (rcmorgan1223@gmail.com), Dixie State University, and **Gregory Schmidt** (gregory.schmidt@dmail.dixie.edu), Dixie State University. *Statistical Modeling to Predict the Trend in Lung Cancer Data in Utah using Joinpoint Regression Analysis* Preliminary report.

Lung cancer is one of the deadliest cancers in the state of Utah. According to the National Cancer Society's estimates in the United States, there were about 228,820 new cases of lung cancer and about 135,720 deaths from lung cancer in the United States in 2020. We obtained data from the Surveillance Epidemiology and End Results (SEER) database of the National Cancer Institute (NCI) and utilized JoinPoint Regression to predict the future trend. We developed a hybrid joinpoint regression model to describe incidence and mortality trends in Utah, assuming that the observed counts are probabilistically characterized by the Poisson distribution. We will compare our results with the other traditional statistical techniques. Using these methods, we can identify common trends that will allow us to come to a conclusion about lung cancer mortality in Utah. Our simulation results predict the rise of new deaths per year and future implications to help the policymakers in decision making. (Received September 21, 2021)

1174-62-10936 **Julie Fucarino** (juliefucarino@yahoo.com), Wellesley College, **Justin Haenel** (justin.haenel@uvm.edu), The University of Vermont, **Victor Feagins*** (victorfeagins@att.net), The University of Texas at San Antonio, and **Melissa Adrian** (maadrian5@gmail.com), The University of Chicago. *Analyzing, Predicting, and Mitigating Defect Formation in Metal Additive Manufacturing* Preliminary report.

Recent advances in metal additive manufacturing have made this technique a promising direction in industry for printing metal parts, such as for biomedical devices and automobile machinery. Despite its ability to efficiently print highly specific parts, this process is prone to errors during printing, causing pores to form in the metal part. These pores weaken the metal by increasing the likelihood of internal cracks forming. With current techniques implemented by HRL Laboratories, these defects are only able to be detected after printing, which impedes any efforts to correct these errors in real time. In order to provide a real-time quality control framework, we developed a machine learning model that inputs the laser printing parameters and predicts whether a defect will form at a given location, which is informed by the X-ray images of the metal scanned after printing. This model provides an in-situ prediction for the formation of defects in real time so that corrections can immediately be made. In addition to building this predictive model, we provide a semi-automated process for preprocessing and registering the X-ray images with the in-situ laser data. (Received September 21, 2021)

1174-62-11060 **Sher Chhetri*** (schhetri@mailbox.sc.edu), University of South Carolina Sumter, **Cory Ball** (ballcbh@gmail.com), Oak Ridge National Lab, Tennessee, and **Nonhle Channon Mdziniso** (nonhlemdz@gmail.com), Bloomsburg University of PA. *Extended Lindley Distribution and Applications* Preliminary report.

In this work, we propose a three-parameter generalized Lindley distribution using the cubic rank transmutation map approach by Granzotto, Louzada & Balakrishnan (2017). We derive expressions for several mathematical properties including moments and moment generating function, mean deviation, probability weighted moments, quantile function, reliability analysis, and order statistics. We conducted a simulation study to assess the performance of the maximum likelihood estimation procedure for estimating model parameters. The flexibility of the proposed model is illustrated by analyzing two real data sets. (Received September 21, 2021)

1174-62-11178 **Allison Fisher*** (Allison.fisher@wsu.edu), Washington State University, **Macarena Sanz** (macarena@wsu.edu), Washington State University, **Warwick Bayly** (wmb@wsu.edu), Washington State University, **Sierra Jenice Shoemaker** (sierra.shoemaker@wsu.edu), Washington State University, **Julia Renee Bagshaw** (julia.bagshaw@wsu.edu), Washington State University, and **Clark Kogan** (clark.kogan@wsu.edu), Washington State University. *Modeling Exercise Induced Pulmonary Hemorrhage (EIPH) in Horses* Preliminary report.

Exercise Induced Pulmonary Hemorrhage (EIPH), is a condition commonly found in horses who are engaged in strenuous exercise. The extent to which EIPH affects race performance is not well understood, especially because race performance is affected by many contributing factors including the horse's speed index, horse's age, days between races and additional underlying conditions including Pharyngeal Lymphoid Hyperplasia and mucus. Beyond all these contributing factors, EIPH is associated with high speed exercise and its severity varies between and within individuals from race to race. Thus, we sought to identify the extent to which EIPH contributes to race performance. To accomplish this goal, we developed an ordinal logistic regression model based on disease severity of EIPH. Our model was used on 3 different groupings of horses: 1000 2-year-old horses running with and without furosemide, which is a drug that can attenuate EIPH severity, and 300 stakes horses. The analysis of the 2-year-old horses was particularly challenging because 2-year-olds horses are inexperienced racers and they do not have the same maturity as horses with more lifetime race starts. We relied on the horses' speed index to help explain the variable performance of the young horses. To accomplish this, we optimized the correlation between speed index, race performance variables and the number of observations. This analysis is the largest study of EIPH in 2-year-old horses to date and represents a major step in better understanding EIPH. (Received September 21, 2021)

1174-62-11198 **Lili Carr Donovan*** (lcarrdonovan@gmail.com), Western Washington University, and **Ramadha Piyadi Gamage** (piyadir@wwu.edu), Western Washington University. *Bootstrapping the Likelihood Ratio Test for Change Point Analysis* Preliminary report.

To detect mean changes in a sequence of independent observations from a certain distribution, the likelihood ratio (LR) test can be applied. Its p-value is typically computed using the asymptotic distribution of the test statistic, which may be unreliable when the number of observations is small. To overcome the problem, a simulation study is carried out to analyze the empirical Type I error rate and power of the LR test using the

parametric and nonparametric bootstrap methods. Under both normal and exponential distributions, it is found that the nonparametric bootstrap makes the test robust for small, moderate, and large sample sizes. (Received September 21, 2021)

1174-62-11203 **Benjamin Cooper Boniece*** (bcboniece@math.utah.edu), University of Utah. *Efficient truncated realized variance for Lévy processes with infinite-variation jumps*

The *integrated variance* of a semimartingale plays an important role in financial econometrics, and its estimation has received considerable attention over the past two decades.

Though integrated variance can be estimated in a straightforward manner when the underlying process is continuous, in the presence of jumps, naive estimation of integrated variance fails to be consistent, and accordingly several methods have been proposed to accommodate jump behavior. However, many of these methods are not efficient when the jump activity in the underlying semimartingale is too extreme. A fundamental and important estimator known as the *Truncated Realized Variance* (TRV), in particular, displays non-negligible bias that prevents efficiency when jumps are of infinite variation, even in the simple Lévy case.

In this talk, I will discuss an iterative debiasing procedure for TRV applied to a Lévy model that is efficient for arbitrarily high levels of jump activity. This is based on joint work with Yuchen Han and José E. Figueroa-López. (Received September 22, 2021)

1174-62-12243 **Holly Janes*** (hjanes@fredhutch.org), Fred Hutch. *Statistical design and analysis of COVID-19 vaccine efficacy trials.*

Rapid development and deployment of safe and effective COVID-19 vaccines for the global population is a public health imperative. I will overview the key statistical design elements of the US-government-funded vaccine efficacy trials, including the rationale and choice of endpoints, success criteria, triggers for analysis, and design adaptations following early evidence of efficacy. I will discuss open scientific questions around durability of vaccine efficacy, vaccine efficacy by SARS-CoV-2 variant and prevention of secondary transmission, and highlight statistical challenges in addressing these questions. (Received December 6, 2021)

1174-62-12244 **Bhramar Mukherjee*** (bhramar@umich.edu), University of Michigan. *Predictions, role of interventions and the crisis of virus in India: A data science call to arms*

India, world's largest democracy, had two very different surges of SARS-CoV-2 in 2020 and 2021 corresponding to the transmission of the ancestral strain and the rise of the Delta variant. The human behavior and public health intervention strategies were also very different during these two waves. In this presentation, we provide a brief chronicle of the modeling experience of our study team over the last two years, looking at the data from India, leading to the development of a tiered data-driven framework for public health interventions towards pandemic resilience. Through mathematical modeling we study the timing and duration of public health interventions with intervention effects estimated from actual data. We illustrate that early and sustained interventions can help us avoid harsh lockdowns and reduce COVID mortality drastically. This is joint work with many, with all supporting research materials and products available at covind19.org. (Received December 6, 2021)

65 ▶ Numerical analysis

1174-65-5531 **Chi-Wang Shu*** (chi-wang_shu@brown.edu), Brown University. *Controlling spurious oscillations for discontinuous Galerkin methods*

Discontinuous Galerkin (DG) methods are a class of finite element methods using completely discontinuous piecewise polynomials as basis functions. Since its inception in 1973, it has seen a sustained development, both in the computational mathematics community and in many scientific and engineering application communities. The DG methods are particularly effective for solving hyperbolic and convection-dominated problems. However, one drawback of the DG methods, especially when they are used to solve hyperbolic equations with strong shocks, is that spurious oscillations may appear, which may lead to nonlinear instability for nonlinear hyperbolic systems. In this talk we will survey several approaches in controlling spurious oscillations for the DG methods. One approach is the usage of total variation bounded (TVB) and other slope limiters, including our recent work, accepted for publication in the first issue of La Matematica, that uses machine learning to estimate the TVB constant in the TVB limiter. Recent development in weighted essentially non-oscillatory (WENO) type limiters will also be discussed. Finally, a recent strategy of using a carefully designed numerical damping term to obtain effectively oscillation-free DG methods while keeping the original high order accuracy and stability will be discussed. Different topics surveyed in this talk are joint work with different collaborators, including Xinyue Yu, Jun Zhu and Jianxian Qiu, and Jianfang Lu and Yong Liu. (Received August 21, 2021)

1174-65-5557 **Lili Ju*** (ju@math.sc.edu), University of South Carolina, **Rihui Lan** (rlan@mailbox.sc.edu), University of South Carolina, **Zhu Wang** (wangzhu@math.sc.edu), University of South Carolina, **Max Gunzburger** (mgunzburger@fsu.edu), Florida State University, and **Philip Jones** (pwjones@lanl.gov), Los Alamos National Laboratory.
High-Order Multirate Explicit Time-Stepping Schemes for the Baroclinic-Barotropic Split Dynamics in Primitive Equations

To treat the multiple time scales of ocean dynamics in an efficient manner, the baroclinic-barotropic splitting technique has been widely used for solving the primitive equations for ocean modeling. In this paper, we propose second and third-order multirate explicit time-stepping schemes for such split systems based on the strong stability-preserving Runge-Kutta (SSPRK) framework. Our method allows for a large time step to be used for advancing the three-dimensional (slow) baroclinic mode and a small time step for the two-dimensional (fast) barotropic mode, so that each of the two mode solves only need satisfy their respective CFL condition to maintain numerical stability. It is well known that the SSPRK method achieves high-order temporal accuracy by utilizing a convex combination of forward-Euler steps. At each time step of our method, the baroclinic velocity is first computed by using the SSPRK scheme to advance the baroclinic-barotropic system with the large time step, then the barotropic velocity is specially corrected by using the same SSPRK scheme with the small time step to advance the barotropic subsystem with a barotropic forcing interpolated based on values from the preceding baroclinic solves. Finally, the fluid thickness and the sea surface height perturbation is updated by coupling the predicted baroclinic and barotropic velocities. Two benchmark tests drawn from the "MPAS-Ocean" platform are used to numerically demonstrate the accuracy and parallel performance of the proposed schemes. (Received August 22, 2021)

1174-65-5598 **Xinyun Zhu*** (zhu_x@utpb.edu), University of Texas-Permian Basin. *Preconditioners based on matrix splitting for the structured systems from elliptic PDE constrained optimization problems* Preliminary report.

With regard to the structured systems of linear equations, which arises from the Galerkin finite element discretizations of elliptic PDE-constrained optimization problems, some new preconditioners based on coefficient matrix splitting are established to speed up the convergence rate of Krylov subspace methods such as GMRES. Additionally, eigenvalue distribution of the corresponding preconditioned matrices is deeply discussed. Numerical simulations are carried out, the results of which exhibit that the corresponding preconditioned GMRES methods perform very well with the theoretical analysis results and are superior to other newly devised preconditioners
(Received August 23, 2021)

1174-65-5615 **Juntao Huang*** (huangj75@msu.edu), Michigan State University, **Yingda Cheng** (ycheng@msu.edu), Michigan State University, **Andrew J. Christlieb** (christli@msu.edu), Michigan State University, **Luke F. Roberts** (robertsl@nscl.msu.edu), Michigan State University, and **Wen-An Yong** (wayong@tsinghua.edu.cn), Tsinghua University. *Enforcing hyperbolicity in gradient-based machine learning moment closures for the radiative transfer equation* Preliminary report.

In this talk, we take a data-driven approach and apply machine learning to the moment closure problem for radiative transfer equation in slab geometry. Instead of learning the unclosed high order moment, we propose to directly learn the gradient of the high order moment using neural networks, called the gradient-based moment closure. This new approach is consistent with the exact closure we derive for the free streaming limit and also provides a natural output normalization. Moreover, we introduce two approaches to enforce the hyperbolicity of our gradient-based machine learning moment closures. The main idea of the first one is to seek a symmetrizer for the closure system, and derive constraints such that the system is globally symmetrizable hyperbolic. For the second approach, motivated by the observation that the coefficient matrix of the closure system is a lower Hessenberg matrix, we relate its eigenvalues to the roots of some associated polynomial. Then, we design a new neural network architecture and the learned ML closure model is automatically hyperbolic. Moreover, the eigenvalues of the ML closure model are bounded by the speed of light, which inherits the physical characteristic speeds of the RTE. A variety of benchmark tests, including the variable scattering problem, the Gaussian source problem and the two material problem, show both good accuracy and generalizability of our machine learning closure model. (Received August 23, 2021)

1174-65-5651 **Jonathan D. Hauenstein*** (hauenstein@nd.edu), University of Notre Dame. *Structured polynomial systems arising in mechanism design*

Designing rigid mechanisms can be formulated as solving polynomial systems. These systems are highly structured and have many fewer solutions than traditional upper bounds on the root counts would suggest. This talk

will explore a collection of polynomial systems arising in the design of mechanisms and discuss both successes and current challenges for solving them. (Received August 24, 2021)

1174-65-5705 **Xinyue Yu*** (xinyue_yu@brown.edu), Brown University. *Multi-layer perceptron estimator for the total variation bounded constant in limiters for discontinuous Galerkin methods*

The discontinuous Galerkin (DG) method is widely used in numerical solution of partial differential equations, especially for hyperbolic equations. However, for problems containing strong shocks, the DG method often needs to be supplemented by a limiter to control spurious oscillations and to ensure nonlinear stability. The total variation bounded (TVB) limiter is a popular choice and can maintain the high order accuracy of the DG scheme in smooth regions and keep a non-oscillatory discontinuity transition, when a certain TVB constant M is chosen adequately. For scalar conservation laws, suitable choice of M can be based on solid mathematical analysis. However, for nonlinear hyperbolic systems, there is no rigorous mathematical guiding principle for the determination of this constant, and numerical experiments often use *ad hoc* choices based on experience. In this paper, we develop a TVB constant artificial neural network (ANN) based estimator by constructing a multi-layer perceptron (MLP) model. We generate the training data set by constructing piecewise smooth functions containing local extrema and discontinuities. By using the supervised learning strategy, the MLP model is trained offline. The proposed method gives the TVB constant M with robust performance to capture sharp and non-oscillatory shock transitions while maintaining the original high order accuracy in smooth regions.
(Received August 25, 2021)

1174-65-5765 **Qin Sheng*** (qin_sheng@baylor.edu), Baylor University. *A preliminary study of higher order approximations of the solution of a nonlinear quenching problem on arbitrary grids* Preliminary report.

Reaction-diffusion-advection equations provide precise interpretations for many important phenomena in complex interactions between natural and artificial systems. This report studies second order semi-discretizations for the numerical solution of reaction-diffusion-advection equations modeling quenching types of singularities occurring in numerous applications.

Our investigations particularly focus at cases where nonuniform spatial grids are utilized. Detailed derivations and analysis are accomplished. Easy-to-use and highly effective second order schemes are acquired. Computational experiments are presented to illustrate our results as well as to demonstrate the viability and capability of the new methods for solving singular quenching problems on arbitrary grid platforms. (Received August 26, 2021)

1174-65-5855 **Ke Chen*** (kechen1993@gmail.com), University of Texas at Austin, and **Ruhui Jin** (rhjin@math.utexas.edu), University of Texas at Austin. *Tensor-structured sketching for constrained least squares problems* Preliminary report.

Constrained least squares problems arise in many applications. The memory and com- putation cost are expensive in practice involving high-dimensional input data. We employ the so-called “sketching” strategy to project the least squares problem into a space of a much lower “sketching dimension” via a random sketching matrix. The key idea of sketching is to reduce the dimension of the problem as much as possible while maintaining the approximation accuracy.

Tensor structure is often present in the data matrices of least squares, including linearized inverse problems and tensor decompositions. In this work, we utilize a general class of row-wise tensorized sub-Gaussian matrices as sketching matrices in constrained optimization problems for their compatibility with tensor structures. We provide theoretical guarantees on the sketch- ing dimension in terms of error criterion and probability failure rate. For unconstrained linear regressions, we obtain an optimal estimate for the sketching dimension. For optimization problems with general constraint sets, we show that the sketching dimension depends on a statistical complexity that characterizes the geometry of the underlying problems. Our theories are demonstrated and validated in a few concrete examples, including unconstrained linear regression and sparse recovery problems.
(Received August 30, 2021)

1174-65-5971 **Isaac Sunseri*** (ipsunser@ncsu.edu), North Carolina State University, Sandia National Laboratories, **Alen Alexanderian** (alen@ices.utexas.edu), North Carolina State University, **Joseph Hart** (joshart@sandia.gov), Sandia National Laboratories, and **Bart Van Bloemen Waanders** (bartv@sandia.gov), Sandia National Laboratories. *Hyper-Differential Sensitivity Analysis of Inverse Problems Governed by PDEs*

Many applications and physical phenomena can be described by systems of partial differential equations (PDEs). Frequently, a researcher will want to understand an unknown quantity of interest that can be modeled as a

parameter in a governing system of PDEs. This gives rise to inverse problems, in which data measurements and knowledge of the governing model are used to infer the desired inversion parameter. In many cases, there are numerous unknown parameters that represent additional model uncertainties or experimental conditions that need to be specified to solve the inverse problem accurately. We refer to these as complementary parameters. We seek to quantify the sensitivity of the solution of the inverse problem to these additional uncertainties using hyper-differential sensitivity analysis (HDSA). To accomplish this, we compute the derivative of the inverse problem solution with respect to the complementary parameters and define sensitivity indices that measure their relative importance. We present a scalable mathematical framework for HDSA of deterministic PDE-constrained inverse problems and demonstrate its interpretability through a porous medium flow application, using pressure and concentration measurements to infer subsurface permeability. (Received September 2, 2021)

1174-65-6007 **Adityavikram Viswanathan** (adityavv@umich.edu), University of Michigan - Dearborn, **John Dalton Flynn** (jdflynn9@gmail.com), Georgia Institute of Technology, **Nicole Elizabeth Baker*** (Nicole.elizabeth650@gmail.com), Oakland University, **Jonathan Mousley** (jonathanmousley@gmail.com), Utah State University, and **Yulia Hristova** (yuliagh@umich.edu), University of Michigan-Dearborn. *Phase Retrieval from Local Measurements via Structured Lifting and Eigenvector-based Angular Synchronization* Preliminary report.

We study a computational approach to phase retrieval with application to ptychographic imaging. Phase retrieval is a challenging nonlinear inverse problem involving the recovery of a complex vector (sample), up to a single unimodular constant, from magnitude-only measurements. Ptychography refers to a computational imaging method where a sample of interest is recovered from a series of overlapping local magnitude only measurements. We present herein a reconstruction algorithm which intakes ptychographic measurements and outputs an estimate of the sample. Pre-computation converts the nonlinear system inherent in our measurement model into a higher-dimensional linear system via structured lifting. The solution to this linear system can then be represented by a matrix with a cascading block structure. Our algorithm performs eigenvector-based angular synchronization on the solution to this linear system, recovering an estimate for the sample in question. We present numerical results concerning the noise robustness and computational efficiency of said algorithm. (Received September 20, 2021)

1174-65-6095 **Zehua Lai*** (laizehua@uchicago.edu), University of Chicago, **Lek-Heng Lim** (lekheng@uchicago.edu), University of Chicago, and **Ke Ye** (keyk@amss.ac.cn), Chinese Academy of Sciences. *Simpler Grassmannian optimization*

Two widely used models for the Grassmannian $\mathrm{Gr}(k,n)$ are $\mathrm{O}(n)/(\mathrm{O}(k)\times\mathrm{O}(n-k))$ and $\{P\in\mathbb{R}^{n\times n} : P^{\mathsf{T}} = P = P^2,\ \mathrm{tr}(P)=k\}$. The former, standard in manifold optimization, has the advantage of giving numerically stable algorithms but the disadvantage of having to work with equivalence classes of matrices. The latter, widely used in coding theory and probability, has the advantage of using actual matrices but working with projection matrices is numerically unstable. We present an alternative that combines the best features of both: by representing k-dimensional subspaces as symmetric orthogonal matrices of trace $2k-n$, we obtain $\mathrm{Gr}(k,n)\cong\{Q\in\mathrm{O}(n) : Q^{\mathsf{T}}=Q,\ \mathrm{tr}(Q)=2k-n\}$. We show that differential geometric objects and operations — tangent vector, metric, normal vector, exponential map, geodesic, parallel transport, gradient, Hessian, etc — have closed-form analytic expressions that are computable with standard numerical linear algebra. Moreover, these expressions are considerably simpler, and can be computed with at most one QR decomposition and one exponential of a special skew-symmetric matrix that takes only $O(nk(n-k))$ time. In particular, we completely avoid eigen- and singular value decompositions in our steepest descent, conjugate gradient, quasi-Newton, and Newton methods. (Received September 4, 2021)

1174-65-6213 **Mikheil Tutberidze*** (mtutberidze@gmail.com), Institute of Applied Physics. *On the Discrete Analogue of Initial-Boundary Value Problem to One Nonlinear Parabolic Equation*

Investigation of some biological models bring us to the following initial-boundary value problem to nonlinear parabolic equation:

$$U_t = \left(k\left(x,\ t,\ U_x\right)U_x\right)_x + f\left(x,\ t,\ U,\ U_x\right),\quad (x,\ t)\in\Omega\times(0,\ T],$$
$$U\left(x,\ 0\right)=\varphi\left(x\right),\quad x\in\bar{\Omega},$$
$$U\left(0,\ t\right)=\phi_0\left(t\right),\quad U\left(1,\ t\right)=\phi_1\left(t\right),\quad t\in(0,\ T],$$

where $U=U\left(x,\ t\right)$ is unknown function, $k,\ f,\ \varphi,\ \phi_0$ and ϕ_1 are given functions, $T=const>0$, $\Omega=(0,\ 1)$. For this problem we construct the discrete analogue, for which under some restrictions on functions k, f, φ, ϕ_0 and ϕ_1 we prove the theorem of comparison, theorems of existence and uniqueness of the solution. Also, for the discrete analogue we construct the iteration scheme and prove convergence of the iteration scheme to the solution of discrete analogue. If solution U of the source problem is smooth enough, we also prove the convergence of the solution of discrete analogue to the solution of the source problem. (Received September 7, 2021)

1174-65-6273 **Eric Todd Quinto** (todd.quinto@tufts.edu), Tufts University, and **Elena S Martinez*** (elenasmartinez22@gmail.com), Loyola Marymount University. *A Mathematical Analysis of Reconstruction Artifacts in Radar Limited Data Tomography*

Tomography imaging systems produce cross-sectional images that are used to find solutions to a wide range of problems in varying fields, such as the biosciences and aeronautics. Sometimes, however, data values are missing and we find artifacts in the reconstructions of our data. This is referred to as limited data tomography. We study this problem in a bistatic radar system that has a radio transmitter in a fixed location and a receiver flying around the transmitter in a circular path. We secure our data by integrating over all ellipses in a given space that have the transmitter and receiver as foci. We reconstruct this data and analyze the artifacts that present themselves when we place objects within and outside of the receiver's path. Our research demonstrates how objects outside the receiver's path can create artifacts inside the receiver's path and vice versa. This shows an intrinsic limitation to a method that works well when the scanned region outside the receiver's path is clear. (Received September 7, 2021)

1174-65-6311 **Longxiu Huang*** (huangl3@math.ucla.edu), UCLA. *Robust Tensor Decomposition via Fiber CUR decomposition* Preliminary report.

In this talk, I would talk about my recent work with Hanqin Cai, Zehan Chao and Deanna Needell on the problem of tensor robust principal component analysis (TRPCA), which aims to separate an underlying low-multilinear-rank tensor and a sparse outlier tensor from their sum. In this talk, I would present a fast non-convex algorithm, coined Robust Tensor CUR (RTCUR), for large-scale TRPCA problems. RTCUR algorithm considers a framework of alternating projections and utilizes the recently developed tensor Fiber CUR decomposition to dramatically lower the computational complexity. The efficiency of our algorithm is empirically verified against the state-of-the-arts on the synthetic and real application datasets. (Received September 7, 2021)

1174-65-6363 **Yifei Lou*** (yifei.lou@utdallas.edu), The University of Texas at Dallas. *Seismic data completion by using low-rank tensor optimization* Preliminary report.

Seismic data are often incomplete due to equipment malfunction, limited source and receiver placement at near and far offsets, and missing cross-line data. Seismic data contain redundancies as they are repeatedly recorded over the same or adjacent subsurface regions, causing the data to have a low-rank structure. To recover missing data one can organize the data into a multidimensional array or tensor and apply a tensor completion method. We can increase the effectiveness and efficiency of low-rank data reconstruction based on the tensor singular value decomposition (tSVD) by analyzing the effect of tensor orientation and exploiting the conjugate symmetry of the multidimensional Fourier transform. Relating the singular values of the tSVD to those of a matrix leads to a simplified analysis, revealing that the most square orientation gives the best data structure for low-rank reconstruction. After the first step of the tSVD, a multidimensional Fourier transform, frontal slices of the tensor form conjugate pairs. For each pair a singular value decomposition can be replaced with a much cheaper conjugate calculation, allowing for faster computation of the tSVD. We consider synthetic and real seismic datasets to demonstrate the performance of the proposed approach in comparison to traditional methods, projection onto convex sets and multichannel singular spectrum analysis. (Received September 8, 2021)

1174-65-6427 **Caroline Hills** (carolinethills@gmail.com), University of Notre Dame. *Parallel Time Integration for Constrained Optimization*

Scientific computing has proven an essential tool in the modern world and historically has been driven by relentless gains in computational speed. While recent years have seen the number of transistors in an average processor continue to increase, individual clock speeds have plateaued, placing an emphasis on parallelizing computations. Many of the problems these computers tackle are under time-evolution constraints, and it can be difficult to solve such problems in parallel across the temporal domain. The Multi-Grid Reduction In Time (MGRIT) algorithm, developed at Lawrence Livermore National Laboratory (LLNL), solves differential equations with a method designed specifically to take advantage of extreme numbers of processors by parallelizing across time. The Tri-diagonal MGRIT (TriMGRIT) algorithm, also developed at LLNL, is a generalization of MGRIT which solves a larger class of tridiagonal systems. This is particularly effective for constrained optimization, where a control function is chosen to minimize an objective functional. We bring these theoretical algorithms of MGRIT and TriMGRIT to solve physical differential equations: applying torque to a pendulum to bring it to a gentle stop and moving a crowd of people from one distribution into another. (Received September 8, 2021)

1174-65-6928 **Kate E Gilbert*** (kgilbert721@g.rwu.edu), Roger Williams University, and **Yajni Warnapala** (ywarnapala@rwu.edu), Roger Williams University. *COVID-19 Pandemic Analysis by the Volterra Integral Equation Models: A Preliminary Study of Brazil and South Africa* Preliminary report.

Inspired by the COVID-19 pandemic, this research investigated the feasibility of obtaining good convergence results for a model of the non-homogeneous Volterra integral equation of the second kind for Brazil and South Africa. Volterra Integral Equations are widely used to model infection and recovery of disease in a population. Gaussian quadrature nodes were used to numerically approximate the integrals (the integrand-kernels were weakly singular). This model accounts for the number of initially infected individuals, susceptible individuals, removed individuals, number of contacts per person, the recovery rate, age, the total population, and an unknown function that is hypothesized to be a variable of combining age, preexisting health conditions, income, access to healthcare, etcetera. This model specifically looks at COVID-19 in Brazil and South Africa for the first 300 days of the pandemic. The numerical results of this research give good convergence (up to 10-4) for this model as well as limitations of the model. The model is built on the assumption that the infection curves are characteristic of cubic and inverse tangent functions. The pandemic has been shown to have two waves of increased infections, both with similar cubic and inverse tangent patterns. The Infected-Susceptible-Recovered (ISR) model was applied to both waves of the pandemic with modifications. (Received September 10, 2021)

1174-65-6964 **Anastasiia Minenkova*** (anastasiia.minenkova@uconn.edu), University of Connecticut. *Stability of Real Canonical Forms* Preliminary report.

In this presentation on our joint work with Vadim Olshevsky and Sahinde Dogruer-Akgul, we show the construction of a canonical Jordan basis, that is both flipped orthogonal and γ-conjugate symmetric (γ-FOCS), for real H-selfadjoint matrices. We discuss the existence of the γ-FOCS basis and its stability under small perturbation, preserving Jordan structure. This helped us to get the stability result for real canonical forms where all the matrices involved are real as well. (Received September 10, 2021)

1174-65-7094 **Michael Perlmutter** (perlmut6@msu.edu), UCLA, and **Mark Iwen** (iwenmark@msu.edu), MSU. *Modewise operators for low-rank tensor recovery* Preliminary report.

Recovery of sparse vectors and low-rank matrices from a small number of linear measurements is well-known to be possible under various model assumptions on the measurements. The key requirement on the measurement matrices is typically the restricted isometry property, that is, approximate orthonormality when acting on the subspace to be recovered. Among the most widely used random matrix measurement models are (a) independent sub-Gaussian models and (b) randomized Fourier-based models, allowing for the efficient computation of the measurements.

For the now ubiquitous tensor data, direct application of the known recovery algorithms to the vectorized or matricized tensor is awkward and memory-heavy because of the huge measurement matrices to be constructed. We propose modewise measurement schemes based on sub-Gaussian and randomized Fourier measurements. These modewise operators act on the pairs or other small subsets of the tensor modes separately. They are significantly smaller than the measurements working on the vectorized tensor, provably satisfy the restricted isometry property, and experimentally can recover the tensor data from fewer measurements. (Received September 13, 2021)

1174-65-7116 **Mariam Khachatryan*** (mzk0070@auburn.edu), Auburn University. *Blow-up Estimates for Fractional Reaction-Diffusion Equations*

This paper investigates the blow-up phenomena of the space-fractional reaction-diffusion equation $\partial_t u + (-\Delta)^{\alpha/2} u = f(u), x \in \Omega, t > 0$ with non-negative initial and Dirichlet boundary conditions. We first consider the full discretization of the fractional equation using already existing novel and accurate finite difference method for the fractional operator. Next, we implement an auxiliary function H in order to capture the blow-ups. The numerical blow-up times are computed for the fractional reaction-diffusion equation with the reaction term $f(u) = u^2$ and $f(u) = e^u$. Convergence results are proven. Moreover, the numerical blow-up time computed for the fractional reaction-diffusion equation with $\alpha \to 1.999$ is compared with the numerical blow-up time for the classical reaction-diffusion equation with $\alpha = 2$ and consistent results are obtained. (Received September 12, 2021)

1174-65-7124 **Xiaobing Henry Feng** (xfeng@math.utk.edu), The University of Tennessee, and **Mitchell Sutton*** (msutto11@vols.utk.edu), University of Tennessee, Knoxville. *A New Finite Element Method for One-side Fractional Differential Equations*

In this talk, we present a new finite element method for a class of one-side fractional boundary value problems. Unlike existing fractional differential equations, the problems considered in this talk rely on the notion of weak fractional derivatives and their associated fractional Sobolev spaces. The crux of the new method is a finite-dimensional fractional derivative operator; the so-called finite element fractional derivative. We will discuss the efficiency afforded by this construct and present convergence analysis and numerical examples. (Received September 12, 2021)

1174-65-7155 **Aravind Baskar*** (abaskar@nd.edu), Department of Aerospace and Mechanical Engineering, University of Notre Dame, **Mark Plecnik** (plecnikmark@nd.edu), Department of Aerospace and Mechanical Engineering, University of Notre Dame, and **Jonathan Hauenstein** (hauenstein@nd.edu), Department of Applied and Computational Mathematics and Statistics, University of Notre Dame. *Optimal Kinematic Synthesis of Mechanisms Using Numerical Polynomial Homotopy Continuation* Preliminary report.

Kinematic synthesis aims to find the dimensions of a mechanism after desired constraints have been posed on its motion. For exact synthesis, the number of constraints and dimensional design variables are equal. For approximate synthesis, the former exceeds the latter. In this work, approximate synthesis is tackled by an optimization formulation that is invariant to the number of constraint specifications. The objective function, which is polynomial in nature, is a sum of squares of the error in the kinematic equations at the task specifications. Using the first-order necessary conditions of optimality, solving the optimization problem boils down to that of finding the roots of a system of polynomial equations. Although these systems are of a larger total degree than their respective exact synthesis systems, homotopy continuation techniques based on multi-homogeneous structure and monodromy-based methods can be used to solve these systems. The roots of such systems comprise all the stationary points including minima, saddles, and maxima. Since secondary considerations further dictate whether a minimum is feasible for a practical design, some low index saddles are also found to lead to suitable designs. Investigation into the Morse-Smale complexes starting from these low index saddles into the adjoining minima leads to useful families of solutions. This provides the designer with parameterized sets of design candidates enabled by the computation of all possible critical points of the objective. (Received September 13, 2021)

1174-65-7199 **Phuong Nguyen*** (pnguye45@uncc.edu), University of North Carolina Charlotte, and **Xingjie Helen Li** (xli47@uncc.edu), University of North Carolina Charlotte. *A Study of Illumination Filtering and Movement Detection in Video Streaming Processing*

We study a movement detection scheme for video streaming data with time series analysis. We employ simple scalar quantities such as (i) the Frobenius norm trajectory of a forward difference matrix and (ii) the Auto-correlation function to analyze the time series and detect movements in the video sequences. To remove the background interruptions, such as the change of lights, we study the performance of two filters, (1) SVD filter and (2) SVD combined with the Butterworth filter, to repress data disruptions. From the example, we identify the best filter and the best statistical quantity to detect movement in the disrupted data. By using the background subtraction filters as well as the Frobenius norm trajectory of data, we can distinguish the movements and light disruptions at various time points in the video sequences. The combo filter is found to be the best as it completely removes the light interruption while maintaining the movements data unaffected and smooths other static artifacts (Received September 13, 2021)

1174-65-7205 **Abner J. Salgado*** (asalgad1@utk.edu), Department of Mathematics, University of Tennessee. *Numerical methods for spectral fractional diffusion*

We present three schemes for the numerical approximation of the spectral fractional Laplacian. The first method uses a rational approximation to approximate negative fractional powers of an operator. The second is a discretization of the so-called Balakrishnan formula, which expresses fractional powers of a positive operator. The third, is a PDE approach that exploits the extension to one higher dimension. We discuss pros and cons of each method, and error estimates. Time permitting, we extend some of these to more general problems, both steady, time dependent; linear and nonlinear. We show illustrative simulations, applications, and mention challenging open questions. (Received September 13, 2021)

1174-65-7549 **Jorge Garza-Vargas*** (jgarzavargas@berkeley.edu), UC Berkeley. *Numerical Diagonalization of Non-Normal Matrices*

In this talk I will present recent work obtaining rigorous guarantees on the convergence and stability of certain diagonalization algorithms. These results provide the best known running times for diagonalization of non-normal matrices. (Received September 15, 2021)

1174-65-7673 **E B Pitman*** (pitman@buffalo.edu), University at Buffalo. *Gaussian Process Emulators, Errors, and Properties of Solutions*

Gaussian Process emulators (GPs) are a non-parametric regression technique that provides the best, linear, unbiased estimator of a scalar output, given data. In the classical approach of emulation, predicting the output at, say, every point of a spatial grid in a simulation of PDEs requires each space point to be its own dimension, increasing the required computational work to make such a prediction impractical. Recent work by Gu, Berger, and colleagues has proposed an extension of GP emulation to the functional output of simulations within feasible computational work, by a clever sharing of certain parameters computed in the GP construction. In order to use this extended GP emulator in uncertainty analysis it is helpful to examine properties of the predicted solutions—How well do the predicted solutions satisfy the underlying differential equation? Are properties of the differential equation such as conservation satisfied by the predicted solutions? We examine these questions through simple analysis and computational studies. (Received September 15, 2021)

1174-65-7953 **Mirjeta Pasha*** (mpasha3@asu.edu), Arizona State University, **Julianne Chung** (jmchung@vt.edu), Virginia Tech, **Matthias Chung** (mcchung@vt.edu), Virginia Tech, and **Silvia Gazzola** (sg968@bath.ac.uk), University of Bath. *Efficient learning methods for large-scale optimal inversion design* Preliminary report.

In this talk, we discuss various approaches that use learning from training data to solve inverse problems, following a bi-level learning approach. We consider a general framework for optimal inversion design, where training data can be used to learn optimal regularization parameters, data fidelity terms, and regularizers, thereby resulting in superior variational regularization methods. In particular, we describe methods to learn optimal p and q norms for L^p-L^q regularization and methods to learn optimal parametric regularization matrices. We exploit efficient algorithms based on Krylov projection methods for solving the regularized problems, both at training and validation stages, making these methods well-suited for large-scale problems. We experimentally show that the learned regularization methods perform well even when the data are corrupted by noise coming from different distributions, or when there is some inexactness in the forward operator. (Received September 17, 2021)

1174-65-8356 **Scott Robert McIntyre** (scottmcintyre@berkeley.edu), University of California, Berkeley, and **Sam Dulin*** (swd7tw@virginia.edu), University of Virginia. *Numerical Simulations for Optimal Transport* Preliminary report.

In this project, we propose a numerical method to solve the classic L^2-optimal transport problem. Optimal transport has a wide range of applications, including economics, machine learning, image processing, and more. Following the fluid dynamics reformulation of this problem in Benamou and Brenier (2000), we apply spatial discretization to solve the underlying partial differential equation. Our algorithm is based on the use of multiple shooting, in combination with a continuation procedure, to solve the boundary value problem derived from spatially discretizing this partial differential equation. We present several numerical examples to illustrate the effectiveness of the method. (Received September 18, 2021)

1174-65-8406 **Laura Abigail Lyman*** (lymanla@stanford.edu), Stanford University, and **Gianluca Iaccarino** (jops@stanford.edu), Stanford University. *Triple Products of Multivariate Hermite Polynomials in Correlated Gaussian Random Variables* Preliminary report.

Polynomial chaos methods can be used to estimate the solutions of partial differential equations (PDEs) with uncertainty described by random variables. The stochastic solution is represented by a polynomial expansion, whose deterministic coefficient functions are recovered through Galerkin projections. When the governing PDEs have quadratic nonlinearity, such as in Burgers' equation, the projection step introduces triple products (third order moments) of the basis polynomials. In the presence of multiple *correlated* uncertainties, there is no closed-form expression for these triple products, even when the uncertainties are given a joint Gaussian distribution. As a result, the products are typically computed via sampling methods, which can: (a) become computationally expensive to implement, and (b) introduce errors if the sample count is insufficient. In recent work, a new expression was found for the simple and efficient evaluation of the double products (second order moments) of the basis polynomials (multivariate Hermite) with correlated Gaussian inputs. We present a formula for the triple product in terms of the double product computations. This formula will allow polynomial chaos methods to be applied more easily to stochastic PDEs with quadratic nonlinearity when the uncertainties exhibit correlation. (Received September 18, 2021)

1174-65-8427 **Xiaochuan Tian*** (xctian@ucsd.edu), University of California, San Diego. *A convergent monotone scheme for a nonlocal segregation model with free boundary*

We consider a free boundary problem arising from segregation of two species with high competition. One species moves according to the classical diffusion and the other adopts a nonlocal diffusion strategy. Being a fully nonlinear nonlocal model, it is challenging to design effective ways to compute the solution, especially to capture the free boundary well. We propose an iterative scheme that constructs a sequence of monotone viscosity supersolutions that is shown to converge to the viscosity solution (in the sense of Crandall-Lions). The numerical method applies to general domains in all dimensions. Moreover, for simple domains it can be shown that the sequence of supersolutions converges with a precise rate. We will shown numerical experiments in the end. (Received September 19, 2021)

1174-65-8469 **Leo G Rebholz** (rebholz@clemson.edu), Clemson University, **Muhammad Mohebujjaman*** (m.mohebujjaman@tamiu.edu), Texas A&M International University, **Hongwei Wang** (hongwei.wang@tamiu.edu), Texas A&M International University, and **Md.Abdullah Al Mahbub** (dipmahbub13@cou.ac.bd), Comilla University. *Simulation of parameterized MHD flow ensembles using an efficient algorithm* Preliminary report.

In this talk, we propose, analyze, and test an efficient algorithm for computing ensemble average of incompressible magnetohydrodynamics (MHD) flows, where instances/members correspond to varying kinematic viscosity, magnetic diffusivity, body forces, and initial conditions. The algorithm is decoupled in Elsässer variables and permits a shared coefficient matrix for all members at each time-step. Thus, the algorithm is much more computationally efficient than separately computing simulations for each member using usual MHD algorithms. We prove the proposed algorithm is unconditionally stable and convergent. Several numerical tests are given to support the predicted convergence rates. Finally, we test the proposed scheme and observe how the physical behavior changes as the coupling number increases in a lid-driven cavity problem with mean Reynolds number $Re \approx 15000$, and as the deviation of uncertainties in the initial and boundary conditions increases in a channel flow past a step problem. (Received September 19, 2021)

1174-65-8494 **Davood Damircheli*** (d.damirchi@gmail.com), MIssissippi state univ.. *Numerical Solution of Credit Rating Migration Problem Model with Galerkin Finite Element Method*

In this presentation, we propose a finite element method to study the free boundary value problem arisen from the credit rating migration problem. Indeed, the high and low rating region for a firm are separated by Free boundary and causes some difficulties including discontinuity of second-order derivative of the problem. Exploiting the weak formulation of the problem utilized in the Galerkin method, the discontinuity of the second-order derivative is averted. Some convergence and stability of the proposed method will be studying theoretically. Besides, Numerical results illustrate how derived convergence results are consistent in practice ones. (Received September 19, 2021)

1174-65-8594 **Levon Nurbekyan** (levon.nurbekyan@kaust.edu.sa), Department of Mathematics, UCLA, and **Wanzhou Lei*** (wl1860@nyu.edu), NYU. *Numerical Methods for Wasserstein Natural Gradient Descent in Inverse Problems*

We recast both the Euclidean and Wasserstein natural gradient descent methods from the conventional kernel formulation to an orthogonal projection perspective for solving constrained optimization problems. Instead of formulating and inverting the kernel matrix, the natural gradient descent direction can be seen as the solution to a least-squares problem. We propose a few efficient numerical schemes for scenarios where the derivative of the state variable with respect to the parameter is either known or unknown. For the latter, we utilize the adjoint-state method and techniques from randomized numerical linear algebra. Numerical results shed light on the qualitative differences among the standard gradient descent method, the Euclidean natural gradient method, and the Wasserstein natural gradient descent method in nonconvex optimization problems. (Received September 19, 2021)

1174-65-8806 **Bowen Zhu*** (bz1010@nyu.edu), New York University. *Sobolev Norm and Sobolev Gradient for Seismic Inverse Problems* Preliminary report.

Seismic inversion is the process of producing the subsurface velocity model from the seismic data. The problem is often formulated as a PDE-constrained optimization problem in which we aim to minimize the mismatch between the predicted wavefield and the observed data, where the two most common types are full-waveform inversion (FWI) and least-squares reverse time migration (LSRTM). It can then be solved using iterative methods such as gradient descent and quasi-Newton methods. The least-squares (L^2) norm has been the most widely used in both the misfit function and the metric space for the velocity model (unknown parameter). We propose using the class of Sobolev norm (H^s) in both the metric space for the wavefield (the data space) and the metric space for the velocity (the parameter space), aiming to improve the optimization landscape and accelerate the convergence as well as improving the stability with respect to noise. Numerical tests of FWI and LSRTM applied to the Marmousi model demonstrate the effectiveness of using the Sobolev norms. (Received September 20, 2021)

1174-65-8948 **Arundhati Bagchi Misra*** (abmisra@svsu.edu), Saginaw Valley State University. *Speckle denoising model using nonlocal similar neighborhoods* Preliminary report.

Image denoising is among the most fundamental problems in image processing. A large range of methods covering various fields of mathematics are available for denoising an image. The initial denoising models are derived from energy minimization using nonlinear partial differential equations (PDEs). The filtering based models have also been used for quite a long time where the denoising is done by smoothing operators. The most successful method among them was the nonlocal means proposed by Buades, Coll and Morel in 2005. Though the method is very accurate in removing noise, it is very slow and hence quite impractical. In 2008, Gilboa and Osher extended some known PDE and variational techniques in image processing to the nonlocal framework. The motivation behind this was to make any point interact with any other point in the image. Using nonlocal PDE operators, they proposed the nonlocal total variation method for Gaussian noise. Based on this, a nonlocal PDE based speckle denoising model has been developed earlier by the authors. The model is faster than nonlocal means but still much slower than the total variation based models. In this paper, we develop a faster version of the existing nonlocal PDE based speckle denoising model. We improve the existing model by using similar neighborhoods described by Mahmoudi and Sapiro in 2005. For faster convergence, we use the Split Bregman scheme to find the solution to this new model. The new model is compared with existing methods.
(Received September 20, 2021)

1174-65-9012 **Anthony Gruber*** (anthony.gruber@ttu.edu), Florida State University. *Comparing Autoencoder Architectures for Data Compression and Reduced-Order Modeling*

Advances in artificial neural network technology have let to recent breakthroughs in the reduced-order modeling (ROM) of parameterized PDEs. We discuss the role of autoencoder architectures in these results, and introduce a novel architecture for this purpose based on spectral graph convolution. A comparison of popular ROM autoencoders shows that each has noticeable strengths and weaknesses, with the proposed architecture producing a lightweight model which yields superior results on compression/reconstruction tasks involving PDEs on irregular domains. (Received September 20, 2021)

1174-65-9050 **Ziqiang Li*** (zli11@uwyo.edu), University of Wyoming, and **Victor Ginting** (vginting@uwyo.edu), University of Wyoming. *Spherical Diffusion with Spherical Finite Elements*

The time-dependent, isotropic diffusion PDE on the unit sphere $\mathbb{S}^2$, subject to a constant, zero-averaged force term and an initial condition, possesses a finite, closed-form solution derivable using harmonic analysis. This presentation introduces a finite element on $\mathbb{S}^2$ (and not on polyhedral approximations) as part of a fully-implicit time-stepping scheme that solves spherical diffusion, with specific application to the diffusion-based cartogram on spheres. Finally, a preliminary comparison of this finite element method to the band-limited harmonic approximation is made in terms of their computational complexity and numerical accuracy. (Received September 20, 2021)

1174-65-9177 **Rachel E Vincent-Finley** (rachel_finley@subr.edu), Southern University and A&M College, **Ron Buckmire** (ron@oxy.edu), Occidental College, **Sean D Brooks*** (sbrooks@coppin.edu), National Institute of Standards & Technology, **Sean Brooks** (sbrooks@coppin.edu), Coppin State University, **Franky Backeljauw** (franky.backeljauw@uantwerpen.be), University of Antwerp, Belgium, **Stefan Becuwe** (stefan.becuwe@uantwerpen.be), University of Antwerp, Belgium, **Bruce Miller** (bruce.miller@nist.gov), National Institute of Standards & Technology, **Marjorie McClain** (marj-mccl@verizon.net), National Institute of Standards & Technology, and **Annie Cuyt** (annie.cuyt@uantwerpen.be), University of Antwerp, Belgium. *Validated Computation of Special Mathematical Functions*

The advent of reliable computing machines, computer algebra systems, and multiple precision computational packages diminished the need for tables of reference values for computing function values by interpolation, but today's numerical analysts, scientific researchers, and software developers still need a way to confirm the accuracy of numerical algorithms that compute function values. The field of validated, or provably correct, computation of mathematical functions explores the development of multiple precision codes that compute certifiably accurate function values that can be used to test the accuracy of function data from personal, commercial, or publicly available codes. The National Institute of Standards and Technology (NIST) Applied and Computational Mathematics Division (ACMD) and the University of Antwerp Computational Mathematics Research Group (CMA) are collaborating on the development of an online system for generating validated tables of special function values computed to user-specified precision with a certified error bound. We will discuss the analysis used to obtain reliable error bounds and the implementation of the work in a publicly available beta site. (Received September 20, 2021)

1174-65-9180 **Federico Favali*** (federicofavali@gmail.com), Independent Researcher, and **Marta Riccardi** (M.Riccardi@liverpool.ac.uk), University of Liverpool. *Entropy and Chord Attraction. The First Movement of Haydn's String Quartet op.65 n.5 as a Case Study.* Preliminary report.

The concepts of entropy in music and chordal attraction have been thoroughly investigated for a long time by analysts. However, new perspectives for analysis continually emerge from the research. The aim of the present paper is to push forward what has been done until now, in order to open new perspectives in this field, combining mathematical methods with the analysis of music. To do this, this paper aims to relate these concepts together proposing two new methodologies for calculating both entropy and chord tension. With regards to entropy, it will be calculated using a formula already presented in a previous work by the composer and musicologist Federico Favali and already used several times. This formula calculates the entropy taking into account only the rhythmic parameter. Subsequently, a new methodology for the calculation of the chord tension will be presented. It is a development of the method developed by Fred Lerdahl in his foundational book "Tonal Pitch Space". The authors will present a new formula that takes into consideration the general musical space (called R) rather than an artificial space as it has been considered by Lerdahl. With this new method, the tension generated by each single chord and subsequently by each chord connection will be calculated. The results of the two analyses will be compared, illustrating differences and commonalities. The new approach presented offers a significant breakthrough, opening up the two-dimensional pitch-class space by including spatial considerations. (Received September 20, 2021)

1174-65-9243 **Tracey G Oellerich*** (toelleri@gmu.edu), George Mason University. *Inferring Dynamics of Biological Systems* Preliminary report.

Biological networks are complex and constructing the dynamics for large systems can prove difficult. We will explore utilizing data-driven techniques to uncover the dynamics of a biological network from data provided. In particular, we are interested in inferring the dynamics for biological networks containing conservation laws or other special structures which can result in a singularity. These properties will be discussed in detail and their effect of recovering the dynamics will be investigated. Several numerical dynamics identification algorithms will be presented and their strengths and weaknesses will be discussed. In addition, the question of whether reconstructed networks exhibit adaptation properties based on recently developed analytical criteria will be investigated. (Received September 20, 2021)

1174-65-9538 **Jiangguo Liu*** (liu@math.colostate.edu), Colorado State University. *New weak Galerkin finite elements on cuboidal hexahedra* Preliminary report.

In this talk, we present new weak Galerkin (WG) finite elements for 3-dim problems. Cuboidal hexahedral meshes are used since they are equally flexible as tetrahedral meshes for accommodating complicated geometry but involve much less elements/faces and unknowns. Piecewise constants or linear polynomials are used as approximants. Their discrete weak gradients are established in certain broken H(div) spaces, e.g., Arbogast-Tao spaces, for which cyclic symmetry is utilized for construction of local bases. Frames are also used in handling discrete weak shape functions. We will present numerical results on Darcy flow, for which these new WG solvers demonstrate optimal order convergence in all quantities (pressure, velocity, div of velocity, and normal flux). If time permits, we shall discuss techniques for efficient implementation in C++. This is a joint work with Zhuoran Wang at Sun Yat-sen University (China), Ruishu Wang at Jilin University (China), and Simon Tavener at Colorado State University (USA). (Received September 20, 2021)

1174-65-9621 **Wenbo Li*** (wli50@utk.edu), The University of Tennessee, Knoxville, **Ricardo H. Nochetto** (rhn@umd.edu), University of Maryland, College Park, and **Juan Pablo Borthagaray** (jpborthagaray@unorte.edu.uy), Universidad de la República. *Fractional p-Laplacian Equations: Regularity and Finite Element Approximation* Preliminary report.

We consider the fractional p-Laplacian problems for $1 < p < \infty$ with Dirichlet boundary conditions. Using ideas of G. Savaré [J. Funct. Anal., 152(1):176-201, 1998], we prove regularity for solutions to fractional p-Laplacian equations in general bounded Lipschitz domains. Proper modifications are made to overcome the difficulties of nonlocality and obtain optimal regularity. In addition, we study the finite element approximation of the equations and derive error estimates. Convergence rates are also established based on our regularity results. We present numerical experiments in two dimensional case to illustrate the properties of the scheme. (Received September 20, 2021)

1174-65-9672 **Tilsa Aryeni*** (taryeni@uwyo.edu), University of Wyoming. *Application of Stable Generalized Finite Element Method in Subsurface Flow and Transport Problems* Preliminary report.

We study the application of the Generalized Finite Element Method (GFEM) to approximate the solutions of quasilinear elliptic equations with multiple interfaces in one dimensional space. The problem is characterized by spatial discontinuity of the elliptic coefficient that depends on the unknown solution. It is known that unless the partition of the domain matches the discontinuity configuration, accuracy of standard finite element techniques significantly deteriorates and standard refinement of the partition may not suffice. The GFEM is a viable alternative to overcome this predicament. It is based on the construction of certain enrichment functions supplied to the standard space that capture effects of the discontinuity. This approach is called stable (SGFEM) if it maintains an optimal rate of convergence and the conditioning of GFEM is not worse than that of the standard FEM. A convergence analysis is derived and performance of the method is illustrated by several numerical examples.

Furthermore, we extend the discussion of the method to study Richards equation for the unsaturated flow in heterogeneous soil. In particular, we include the gravitational effect to the quasilinear elliptic equation and temporal dependent of the unknown solution. Some numerical examples are given to demonstrate the performance of the method. (Received September 20, 2021)

1174-65-9849 **Konstantinos Tsingas*** (ktsingas@sas.upenn.edu), University of Pennsylvania, Tufts University. *Numerical Study of Novel Reconstruction Artifacts from Limited Sonar Tomographic Data*

Two-dimensional sonar tomography uses indirect data to image the structural boundaries of an underwater object by inverting the circular Radon transform. For data over circles with centers and radii in a compact set, i.e., a limited data set, it has been shown that visual reconstruction artifacts arise from points on the boundary of the data, making it relatively difficult to image the object due to the ill-posedness of the inverse problem. Yet, reconstruction artifacts for other limited sonar data scenarios have not been fully characterized; similarly, numerical methods to analyze unique limited sonar data sets and algorithms to reduce artifacts in tomographic reconstructions vary in usability and effectiveness. As such, we develop a clear pipeline that generates limited sonar data and numerically evaluates the resulting 2-D reconstruction for transceivers on both linear and curved paths. Using this, we determine a unique case in which visual artifacts occur independent of the object of interest which could potentially inform the microlocal context of the inverse sonar problem. Lastly, we develop a convolution algorithm that minimizes artifacts by selectively targeting singularities in the boundary of the data set. (Received September 20, 2021)

1174-65-9873 **Dinh-Liem Nguyen** (dlnguyen@ksu.edu), Kansas State University, **Trung Thanh Truong** (trungt@ksu.edu), Kansas State University, and **Kale Joseph Stahl*** (Stahlkj@ksu.edu), Kansas State University. *A stable sampling method for inverse scattering from periodic structures* Preliminary report.

We consider the inverse scattering problem that aims to reconstruct periodic layers from measurements of scattered field data. This problem is known to be nonlinear and severely ill-posed. We propose a new imaging functional of sampling type to solve the inverse scattering problem. We analyze the behavior of the imaging functional as well as its stability. Compared with classical sampling methods that were studied for the inverse problem, the proposed imaging functional is more robust against noise in the data. Numerical results are presented to illustrate the efficiency of the method. This is joint work with Trung Truong and Dinh-Liem Nguyen. (Received September 21, 2021)

1174-65-9880 **Zhen Chao*** (zhench@umich.edu), University of Michigan-Ann Arbor. *An improved Poisson-Nernst-Planck ion channel model and numerical studies* Preliminary report.

In this talk, an improved Poisson-Nernst-Planck ion channel (PNPic) model is presented, along with its effective finite element solver and software package for an ion channel protein in a solution of multiple ionic species. Numerical studies are then done on the effects of boundary value conditions, membrane charges, and bulk concentrations on electrostatics and ionic concentrations for an ion channel protein, a gramicidin A (gA), and five different ionic solvents with up to four species. (Received September 21, 2021)

1174-65-10023 **Victor Churchill*** (churchill.77@osu.edu), The Ohio State University, **Dongbin Xiu** (xiu.16@osu.edu), The Ohio State University, **Kailiang Wu** (wukl@sustech.edu.cn), Southern University of Science and Technology, and **Zhen Chen** (zhen.chen@dartmouth.edu), Dartmouth College. *Deep Neural Network Modeling of Unknown PDEs in Nodal Space*

In this talk, we present a numerical framework for deep neural network (DNN) modeling of unknown time-dependent partial differential equations (PDEs) using their trajectory data. Unlike recent work where the learning takes place in modal/Fourier space, the current method conducts the learning and modeling in physical space and uses measurement data as nodal values. We present a DNN structure that has a direct correspondence to the evolution operator of the underlying PDE, thus establishing the existence of the DNN model. The DNN model also does not require any geometric information of the data nodes. Consequently, a trained DNN defines a predictive model for the underlying unknown PDE over structureless grids. A set of examples, including linear and nonlinear scalar PDE, system of PDEs, in both one dimension and two dimensions, over structured and unstructured grids, are presented to demonstrate the effectiveness of the proposed DNN modeling. Extensions to other equations such as differential-integral equations, are also discussed. (Received September 21, 2021)

1174-65-10171 **Chinedu Anthony Eleh*** (cae0027@auburn.edu), Auburn University, and **Hans-Werner van Wyk** (hzv0008@auburn.edu), Auburn University. *Optimal Parameter Upscaling for Partial Differential Equation Models in Mathematical Biology* Preliminary report.

Partial differential equation models in mathematical biology often involve space-dependent parameters, such as diffusion coefficients and advection fields, that cannot be measured explicitly and are therefore uncertain. In this work, we compute spatially adaptive, lower-dimensional approximations of these fields, using machine learning tools. Such parsimonious representations of the parameter space would greatly improve the efficiency of the resulting stochastic simulations, allow for more targeted use of reduced order models, and aid in the related design of interventions. Numerical examples demonstrate our theoretical results. (Received September 21, 2021)

1174-65-10177 **Tyler Jones*** (tyler.j.jones@me.com), Brigham Young University. *The applications and use of stochastic agent-based models in simulating localized fire spread* Preliminary report.

For simulating biological ecological systems, agent-based models are a powerful tool in understanding the behavior of these multi-agent systems. When considering fire spread in both urban and wild environments, we suggest that stochastic agent-based models are a powerful method to understand the lifetime of a fire. Sensitivity to initial conditions, uncertainty of fuel density, and other factors greatly impact the behavior of the fire and make it a difficult dynamic system to understand and model.

Through stochastic agent-based models based on the known behavior of fires, we factor in fuel density, ignition points, environmental factors, as well as include stochastic behavior. This model, paired with Monte Carlo methods, allow us to analyze the behavior of fire spread and its characteristics. We look at validating our approach through past known fires, the application of uncertainty quantification on model parameters, as well as the potential use of optimal control to better guide fire suppressing tactics and strategies. We believe these strategies can yield better insight into fire spread behavior in an environment, and how to best restrict it. (Received September 21, 2021)

1174-65-10238 **Cassidy Krause*** (ckrause@ku.edu), University of Kansas, and **Erik Van Vleck** (erikvv@ku.edu), University of Kansas. *Data Assimilation for PDEs Using Adaptive Moving Meshes*

Adaptive moving mesh methods are useful for solving physical models given by partial differential equations (PDEs), but they present both challenges and opportunities when combined with ensemble-based data assimilation (DA) procedures. We present a framework to combine DA with an adaptive moving mesh through the use of an adaptive common mesh, upon which the mean and covariance of an ensemble-based DA scheme can be computed at each observational timestep.

The adaptive common mesh is computed via the moving mesh PDE (MMPDE) method, which uses a metric tensor to define mesh movement. With this method, all elements of the mesh have the same size and are similar to a reference element when they are measured in a given metric. Allowing each ensemble member to evolve on its own independent mesh, we define the adaptive common mesh by taking the metric tensor intersection of the metric tensors of the ensemble meshes. We also present a new adaptive localization scheme and the ability to concentrate the common mesh near observation locations. We apply these techniques to the discontinuous Galerkin discretization of inviscid Burgers in one and two spatial dimensions. (Received September 21, 2021)

1174-65-10398 **Anna Ma*** (anna.ma@uci.edu), University of California, Irvine. *Solving Tensor Multilinear Systems* Preliminary report.

Machine learning tasks often utilize vast amounts of data in the form of multi-dimensional arrays. Standard techniques for processing this data have quickly become impractical due to the sheer size of the data sets at hand. In this talk we will discuss stochastic iterative approaches for solving multilinear systems under the t-product and connect these approaches to their matrix counterparts. Such approaches are well known to be beneficial in the big data setting when entire data sets are too large to fit into working memory. We highlight regimes under which t-product measurements are more computationally efficient than alternative measurement schemes and create a foundation for the design and analysis of future iterative approaches for solving multilinear systems. (Received September 21, 2021)

1174-65-10620 **Isaac Tate** (itate001@ucr.edu), University of California, Riverside, **Weitao Chen*** (weitaoc@ucr.edu), University of California, Riverside, and **Jolene Britton** (jolene.britton@pepperdine.edu), Pepperdine University. *Local discontinuous Galerkin Method for solving power-law beam equations* Preliminary report.

The dynamic power-law beam equations have important applications in design of mechanical systems. They are applied to estimate the natural frequencies and amplitudes of the structural elements while in vibration. Due to its high order and nonlinearity, no analytical solutions have been obtained so far for the dynamic power-law beam equations. It is necessary to develop a numerical method to provide an estimation of the solution with high precision and stability. Local Discontinuous Galerkin (LDG) method was designed to seek for numerical solutions of PDEs with high order of accuracy and stability even when the analytical solution developed discontinuities. However, applying the standard LDG method failed to estimate the numerical solutions of power-law beam equations with stability, due to a failure in conserving some energy. Here we developed a LDG method with the consideration of energy conservation derived analytically for power-law beam equations. Our preliminary results show that such method can capture the numerical solution with stability and high accuracy. (Received September 21, 2021)

1174-65-10774 **Tyler J. Jarvis** (jarvis@math.byu.edu), Brigham Young University, **Kate Wall*** (katejeanw@gmail.com), Brigham Young University, **Adrienne Russell** (adrienne32russell@gmail.com), Brigham Young University, **Michael Sullivan** (michael.tsullivan14@gmail.com), Brigham Young University, and **Erik Parkinson** (eparkinson@emergenttrading.com), Emergent Trading. *A Numerical Rootfinder for Multivariate Systems of Equations* Preliminary report.

We have developed methods for numerically finding the common roots of a multivariate system of equations, a difficult problem with many useful applications. Our algorithm uses a combination of Chebyshev approximation, properties of Chebyshev polynomials, Lipschitz bounds, and subdivision methods to identify the real roots in a given interval. We compare our performance to other root-finders such as Bertini and Chebfun2. These comparisons have been made using both a Python and a C++ implementation of our methods. (Received September 21, 2021)

1174-65-10850 **Benjamin Dennis Lombardi*** (blombardi20@outlook.com), Gonzaga University. *Stability of Explicit Free Parameter Multistep Methods for Second-Order ODE's* Preliminary report.

Multistep methods can be computationally efficient approaches to approximate solutions to differential equations (DE's). Unfortunately, increasing the accuracy (of order) of these approximations often results in worse stability properties. A stability domain provides a way to visually assess the stability of a method for various DE's; a stability domain is a graph in the complex plane that shows the combination of stepsizes and equations for which a given method is expected to produce a stable solution (i.e., roundoff error not growing).

This work extends previous work on free parameter methods for first-order DE's $y' = f(x, y)$ by investigating the stability of explicit free parameter multistep methods for second-order DE's of the form $y'' = f(x, y)$. Because Dahlquist's First Stability Barrier caps the maximum order of a stable method, we can add free parameters to a general multistep method and use these parameters to optimize different properties. In particular, varying these free parameters changes the size and shape of a method's stability domain, allowing for the construction of uniquely tailored methods for different problems. We benchmark our results against the typically used Störmer methods. (Received September 21, 2021)

1174-65-10912 **Brian Daniel Becsi*** (briandbecsi@gmail.com), Washington State University, and **Solomon Huang** (solomonhuang0703@csu.fullerton.edu), California State University Fullerton. *Rootfinding with Matrix Orthogonal Polynomials*

When faced with a function for which there is no known root-finding technique, we can use numerical methods to find approximate solutions. One common approach to approximation is through the use of orthogonal polynomials. The roots of an orthogonal polynomial expansion of a function are the eigenvalues of the associated companion matrix. This turns a root-finding problem into an eigenvalue problem which is easier to solve. This describes very generally the approach known as Boyd's Algorithm which is in use today.

By leveraging the theory of matrix-valued orthogonal polynomials, we adapt Boyd's Algorithm to make use of matrix-valued orthogonal polynomials. Results indicate that our matrix-valued model approximates these eigenvalues more accurately and efficiently for many commonly studied nonlinear eigenvalue problems. Therefore, we believe that our algorithm will be of use to professionals studying a variety on nonlinear eigenvalue problems. (Received September 21, 2021)

1174-65-10925 **Kaitlyn Roberts*** (roberts142@wcsu.edu), Western CT State University. *Principal Component Analysis Based Method to Predict Temperatures* Preliminary report.

Weather forecasting is an important tool in many industries around the world. While numerical models are useful in creating an accurate forecast, the accuracy decreases the further into the future the predictions go. Singular Value Decomposition (SVD) has increasingly been employed to assist in adjusting numerical model interpretations and reducing sampling errors. In this research, we use SVD to determine the overall error of historical model data and generate a basis of model errors and states to attempt to accurately interpret current weather data. We then confirm the accuracy with observed data after the fact. (Received September 21, 2021)

1174-65-10947 **Matthew James Rhilinger*** (mrhilinger@zagmail.gonzaga.edu), Gonzaga University. *Analyzing Störmer-Cowell Predictor-Corrector Methods* Preliminary report.

Multistep methods can provide a computationally efficient way to approximate solutions to differential equations (DE's). Two important properties of numerical methods for DE's are stability and accuracy. A stable method does not allow any roundoff error to grow unbounded with subsequent approximations. Accuracy is reflected by the order p of a method, which tells for which integer p the error of the method is proportional to h^p, where h is the stepsize used in the approximation.

The Störmer-Cowell family of multistep methods are used to approximate solutions to many second-order DE's of the form $y'' = f(x, y)$, particularly in celestial mechanics. In this project, we study the order of the combined Störmer-Cowell predictor-corrector methods. In particular, what order does combining an order p corrector with various orders of predictors give? After understanding which combined methods have the same order, we can then compare the stability properties amongst methods of the same order. (Received September 21, 2021)

1174-65-11076 **Kai Yang*** (yangk@fiu.edu), Florida International University. *Wiener Rational Basis functions for (fractional) KdV equations* Preliminary report.

We study the Wiener rational basis functions and its application to the fractional Korteweg-de-Vries (fKdV) equations on the whole real line $\mathbb{R}$. The coefficients for this kind of basis functions can be obtained by the fast Fourier transform, and thus, are on the same order of computational cost as the Fourier spectral method. The "bi-orthogonal process" allows us to approximate the fractional Laplacian by this kind of basis functions with the computational cost $O(N^2)$ in general cases. As a result, we use this kind of basis functions to solve the fKdV equations. The spatial error for the numerical solutions to the fKdV equation is analyzed. With a variety of the energy-conservative time integrators, the energy conservative schemes can be constructed for numerical study. (Received September 21, 2021)

1174-65-11115 **Junping Wang*** (jwang@nsf.gov), Division of Mathematical Sciences , National Science Foundation. *Generalized weak Galerkin finite element methods for PDEs* Preliminary report.

In this talk, the speaker will introduce a generalized weak Galerkin finite element framework for PDEs which permits the use of non-polynomials as approximating functions. A theory on solution existence and uniqueness will be described, and some numerical results will be discussed to demonstrate the performance of the numerical method. The talk should be accessible to graduate students specialized in the area of computational mathematics. The work is jointly conducted with Dan Li, Wenya Qi, and Yue Wang. (Received September 21, 2021)

1174-65-11144 **Liu Zhang** (zhangliu@u.yale-nus.edu.sg), Yale-NUS College, and **Aparna Gupte*** (agupte@mit.edu), Massachusetts Institute of Technology. *Improving Neural Network Training Using Sobolev Loss Functions* Preliminary report.

Training neural networks with gradient-based optimization is known to exhibit a frequency-biasing property: lower-frequency components of the target function are learned faster than higher-frequency components. Though believed to improve generalization, this property leads to slow convergence when higher-frequency components dominate the target function.

We propose a new class of loss functions based on the $\mathcal{H}^s$ Sobolev norms that provably improves training by enhancing or counterbalancing the inherent low-frequency bias of the neural network. The $\mathcal{H}^s$ norms allow us to strategically weigh the lower-frequency and higher-frequency components differently. We empirically show the benefits of our proposed loss function in two applications: fitting synthetic functions on $\mathbb{S}^1$, and solving Poisson equations on $\mathbb{S}^2$ in the Physics-Informed Neural Network (PINN) framework. For both, we derive the NTK and suggest how its spectrum informs the choice of appropriate $\mathcal{H}^s$ norms. (Received September 21, 2021)

1174-65-11183 **David Ketcheson** (dketch@gmail.com), KAUST University, **Manuel Quezada** (manuel.quezada@kaust.edu.sa), Kaust University, and **Otilio Rojas** (orojas@bsc.es), Barcelona Supercomputer Center. *High Order local Absorbing Boundary Condition for the Wave Equation* Preliminary report.

In this work, we derive *high order local time-dependent Farfield Expansions* absorbing boundary conditions (TDFE-ABC) for single acoustic scattering in two– and three–dimensions. A computational advantageous aspect of the TDFE-ABC is its local character. It means only few boundary points or elements are needed to compute the approximate solution at the different stages of the computation. This constitutes a significant improvement over well-known high order absorbing boundary conditions such as the Dirichlet to Neumann whose global nature requires computation over all the nodes or elements at the artificial boundary. We will present some numerical results by coupling a second order finite differences with the TDFE-ABC. (Received September 21, 2021)

1174-65-11224 **Vianey Villamizar Gonzalez** (vianey@mathematics.byu.edu), Brigham Young University, and **Jonathan Hale*** (hale@mathematics.byu.edu), Brigham Young University. *Iterative High Order Numerical Method for Multiple Scattering*

This paper outlines a new iterative method for determining the scattered wave for two dimensional multiple acoustic scattering problems. We seek to approximate the total wave as it is scattered off of multiple arbitrarily shaped obstacles. This is done by enclosing each obstacle in a circular artificial boundary and generating a curvilinear coordinate system for the computational region between the obstacles and the artificial boundary. We treat each obstacle as a separate singular acoustic scattering problem and use a finite difference method to create a linear system whose solution will approximate the scattered wave. The forcing vector in the linear system is determined from the total influence on the obstacle boundary from the incident wave and the scattered waves from the other obstacles. In each iteration, we solve the singular acoustic scattering problem for each obstacle by using the scattered wave approximations from the other obstacles obtained from the previous iteration. The iterations continue until the solutions converge. I will include numerical results which demonstrate the accuracy and advantages of our technique.
(Received September 21, 2021)

1174-65-11297 **Jean-Luc Guermond*** (guermond@tamu.edu), Texas A&M University, and **Alexandre Ern** (ern@cermcis.enpc.fr), Ecole des Ponts et Chaussees, Paris, France.
Invariant-domain-preserving high-order time stepping: Explicit Runge-Kutta schemes

Considering ODE systems with invariant-domain properties, the question addressed in the talk consists of constructing explicit and high-order accurate time stepping technique that preserve the invariant-domain property of these systems. One well-known class of methods meeting this goal is composed of the so-called strong stability preserving Runge-Kutta methods. In this work we go beyond the strong stability preserving barrier and develop a time stepping technique that makes every explicit Runge-Kutta method invariant-domain preserving. The technique is space discretization agnostic and can be combined with continuous finite elements, discontinuous finite elements, and finite volume discretizations in space. The key idea is that at each stage of the ERK scheme, one computes a low-order update, a high-order update, both defined from the same intermediate stage, and then one applies the nonlinear, mass conservative limiting operator. The main advantage with respect to the strong stability preserving (SSP) paradigm is more flexibility in the choice of the ERK scheme, thus allowing for less stringent restrictions on the time step. (Received September 30, 2021)

1174-65-11770 **Bonita V Saunders*** (bonita.saunders@nist.gov), National Institute of Standards and Technology. *Rounding error analysis for validated evaluation of special functions.*

Abstract body/text including any latex coding if necessary.

Special functions are mathematical functions that arise as solutions to problems in the mathematical and physical sciences. They include classical functions such as the Bessel, Airy function, and hypergeometric functions, but also include the gamma function, Riemann zeta function and others connected to the fields of number theory, probability, combinatorics, or statistics. Although there are many computer algebra systems and mathematical software libraries that contain special functions, most say little about the accuracy of the algorithms used. This talk will track the rounding error analysis of a numerical algorithm for a special function to demonstrate techniques being used to code software for computing special functions to user specified precision with a bound on the error. (Received October 20, 2021)

68 ► Computer science

1174-68-5458 **Lara Kassab*** (lara.kassab@colostate.edu), Colorado State University, **Hanbaek Lyu** (hlyu@math.ucla.edu), UCLA, **Alona Kryshchenko** (alona.kryshchenko@csuci.edu), California State University, Channel Islands, **Denali Molitor** (dmolitor@math.ucla.edu), UCLA, **Deanna Needell** (deanna@math.ucla.edu), UCLA, and **Elizaveta Rebrova** (elre@princeton.edu), Princeton University. *Detecting Short-Lasting Topics Using Nonnegative Tensor Decomposition*

Temporal data (such as, a news articles or twitter feeds) often consists of a mixture of long-term trends and popular but short-lasting topics of interest. A truly successful topic modeling strategy should be able to detect both types of topics and clearly locate them in time. We show that nonnegative CANDECOMP/PARAFAC tensor decomposition (NCPD) successfully detects such short-lasting topics that other popular methods such as Latent Dirichlet Allocation (LDA) and Nonnegative Matrix Factorization (NMF) fail to fully detect. We demonstrate the ability of NCPD to discover short and long-lasting temporal topics in synthetic and real-world data including news headlines and COVID-19 related tweets. Additionally, NCPD is considerably more accurate in discovering the points in time when topics appeared and disappeared. (Received August 20, 2021)

1174-68-5602 **Shuhao Cao*** (s.cao@wustl.edu), Washington University in St. Louis. *Galerkin Transformer*

Transformer in "Attention Is All You Need" is now the ubiquitous architecture in every state-of-the-art model in Natural Language Processing (NLP), such as BERT. At its heart and soul is the famous "attention mechanism". We apply the attention mechanism the first time to a data-driven operator learning problem related to parametric partial differential equations. Inspired by Fourier Neural Operator which showed a state-of-the-art performance in parametric PDE evaluation, we put together an effort to explain the heuristics of, and improve the efficacy of the self-attention. We have demonstrated that the widely-accepted indispensable softmax normalization in the scaled dot-product attention is sufficient but not necessary. Without softmax, the approximation capacity of a linear Transformer variant can be proved to be on par with a Petrov-Galerkin projection layer-wise. Some simple changes mimicking projections in Hilbert spaces are applied to the attention mechanism, and it helps the final model achieve remarkable accuracy in operator learning tasks with unnormalized data. Meanwhile in all experiments including the viscid Burgers' equation, an interface Darcy flow, and an inverse interface coefficient identification, the newly proposed simple attention-based operator learner, Galerkin Transformer, shows significant improvements in both speed and evaluation accuracy over its softmax-normalized counterparts, as well as other variants such as Random Feature Attention or FAVOR+ in Performer. (Received August 23, 2021)

1174-68-5949 **Nektarios A Valous*** (nek.valous@nct-heidelberg.de), Applied Tumor Immunity Clinical Cooperation Unit, National Center for Tumor Diseases (NCT), German Cancer Research Center (DKFZ), Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, **Niels Halama** (niels.halama@nct-heidelberg.de), Department of Translational Immunotherapy, German Cancer Research Center (DKFZ), Im Neuenheimer Feld 280, 69120, Heidelberg, Germany, Department of Medical Oncology, National Center for Tumor Diseases (NCT), Heidelberg University Hospital, Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, **Eckhard Hitzer** (hitzer@icu.ac.jp), College of Liberal Arts, International Christian University, Osawa 3-10-2, 181-8585 Mitaka, Tokyo, Japan, **Dragos Duşe** (dusedragos@gmail.com), Department of Medical Oncology, National Center for Tumor Diseases (NCT), Heidelberg University Hospital, Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, Department of Physics and Astronomy, Heidelberg University, Im Neuenheimer Feld 226, 69120, Heidelberg, Germany, **Rodrigo Rojas Moraleda** (rodrigorojasmoraleda@gmail.com), Applied Tumor Immunity Clinical Cooperation Unit, National Center for Tumor Diseases (NCT), German Cancer Research Center (DKFZ), Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, **Meggy Suarez-Carmona** (meggy.suarez-carmona@dkfz-heidelberg.de), Department of Translational Immunotherapy, German Cancer Research Center (DKFZ), Im Neuenheimer Feld 280, 69120, Heidelberg, Germany, **Anna Berthel** (anna.berthel@nct-heidelberg.de), Department of Translational Immunotherapy, German Cancer Research Center (DKFZ), Im Neuenheimer Feld 280, 69120, Heidelberg, Germany, **Bénédicte Lenoir** (benedicte.lenoir@nct-heidelberg.de), Applied Tumor Immunity Clinical Cooperation Unit, National Center for Tumor Diseases (NCT), German Cancer Research Center (DKFZ), Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, Department of Translational Immunotherapy, German Cancer Research Center (DKFZ), Im Neuenheimer Feld 280, 69120, Heidelberg, Germany, **Inka Zörnig** (inka.zoernig@nct-heidelberg.de), Department of Medical Oncology, National Center for Tumor Diseases (NCT), Heidelberg University Hospital, Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, and **Dirk Jäger** (dirk.jaeger@nct-heidelberg.de), Department of Medical Oncology, National Center for Tumor Diseases (NCT), Heidelberg University Hospital, Im Neuenheimer Feld 460, 69120, Heidelberg, Germany, Applied Tumor Immunity Clinical Cooperation Unit, National Center for Tumor Diseases (NCT), German Cancer Research Center (DKFZ), Im Neuenheimer Feld 460, 69120, Heidelberg, Germany. *Color image processing operations using quaternions*

Color pixels can be encoded by a linear combination of the three basis vectors in a hypercomplex algebra framework; this encoding provides the opportunity to process color images in a geometric way. The proposed approach is based on a rapid and flexible method, using quaternions, for color image processing operations in natural and biomedical images. This pixel-based approach is computationally efficient, thus taking advantage of parallel architectures in modern computing systems, and has applications either as a standalone tool or integrated in image processing pipelines. Essentially, the method demonstrates that feature-rich mathematical frameworks can provide efficient solutions for color image processing. (Received September 1, 2021)

1174-68-6112 **Nidhi Pai*** (npai9@gatech.edu), Georgia Institute of Technology, **Salvador Balkus** (sbalkus@umassd.edu), University of Massachusetts Dartmouth, **Tony Zeng** (txz@uw.edu), University of Washington, and **Eduardo Sosa** (ensosa22@colby.edu), Colby College. *Multi-Hypothesis Tracking of Space Objects and Targets*

Over 20,000 artificial satellites currently orbit the Earth, and thousands more are launched each year. As the sky becomes more cluttered, tracking the trajectories of objects in Earth's orbit is critical. Sensors such as radar are currently used to measure satellite locations; however, these measurements are inherently noisy. Dealing with this noise is especially challenging when tracking many objects in close proximity. Multi-Hypothesis Tracking (MHT) is a prevalent tracking algorithm for monitoring multiple objects based on a deferred decision-making approach. In this work, we empirically test a tree-based, track-oriented version of the MHT algorithm in difficult object tracking scenarios and develop improvements to overcome current shortcomings. These include a novel track scoring criteria based on the chi-squared test for variance. In addition, we develop heuristics for gating, thresholds for adding new tracks or incorporating missed measurements, and a modification to the traditional MHT algorithm that prevents new objects from being detected from false alarms prematurely. In light of simulation results, the benefits and drawbacks of MHT are discussed. (Received September 12, 2021)

1174-68-6423 **Harrison Quick*** (harryq@gmail.com), Drexel University. *Differentially private synthetic data for CDC WONDER*

CDC WONDER is a web-based tool for the dissemination of epidemiologic data collected by the National Vital Statistics System. While CDC WONDER has built-in privacy protections, they do not satisfy formal privacy protections such as differential privacy and are susceptible to targeted attacks. Given the importance of making high-quality public health data publicly available while preserving the data subjects' privacy, we aim to improve the utility of a recently developed approach for generating Poisson-distributed, differentially private synthetic data by using publicly available information to truncate the range of the synthetic data. Specifically, we utilize county-level population information from the U.S. Census Bureau and the CDC's national death reports to inform prior distributions on county-level death rates and infer reasonable ranges for Poisson-distributed, county-level death counts. In doing so, the requirements for satisfying differential privacy for a given privacy budget can be reduced by several orders of magnitude, thereby leading to substantial improvements in utility. To illustrate our proposed approach, we consider a dataset comprised of over 26,000 cancer-related deaths from Pennsylvania belonging to over 47,000 combinations of cause-of-death and demographic variables such as age, race, sex, and county-of-residence and demonstrate the proposed framework's ability to preserve features such as geographic, urban/rural, and racial disparities present in the true data. (Received September 8, 2021)

1174-68-6486 **Lingxi Gao*** (lindseygaox@gmail.com), University of Washington, **Ziqing Luo** (luoziqing@g.ucla.edu), University of California, Los Angeles, **Katheryn Lylybell Teran** (lylybellteran@mail.adelphi.edu), Adelphi University, and **Muhammad Abdullah** (mabdulla@bates.edu), Bates College. *Binary Classification: Predicting the Viewability of Digital Advertisements*

An estimated 40% of digital advertisements are not viewed by Internet users. To avoid ineffective branding campaigns, ad viewability is ideally determined prior to the ad being served on a webpage. We apply classical machine learning and deep learning techniques to predict the viewability of an ad before placement. We leveraged a large dataset from a reputable ad-tech corporation containing almost entirely categorical information on the user and publisher. To minimize false positives and maximize profit, we adjusted the classification threshold to emphasize precision over recall. We determined for our dataset that classical machine learning models offer the best predictability metrics. (Received September 9, 2021)

1174-68-6832 **Hannah Guan*** (hguan000@gmail.com), Basis San Antonio Shavano. *Interpreting Epigenetic Aging with Deep Neural Networks Using High Dimensional DNA Methylation Data*

Aging is a normal process linked to specific patterns and changes in the epigenome, e.g., DNA methylation. Multiple epigenetic aging clocks have been developed, but little research have focused on interpretation of these aging equations. Deep neural networks (DNN) are powerful prediction models which can model higher-order of interaction between biomarkers. We have previously developed a DNN-based prediction model for aging prediction. In this study, we use three model-agnostic interpretability methods to interpret how trained DNN predict human biological age by high-dimensional DNA methylation data. Specifically, we compare the Local Interpretable Model-agnostic Explanations, Shapley Values, and MMD-Critic by using the maximum mean discrepancy and large-scale submodular optimization. We discover that, for a large number of candidate features, such as genome-wide DNA methylation data, a key factor in improving prediction accuracy is to appropriately weight features that are highly correlated with the outcome of interest. We also show that explanations can mitigate the impact of misclassified features from the perspective of the end-user. We conclude that explanations are valuable for improving our understanding of epigenetics and what it means for aging. (Received September 9, 2021)

1174-68-6926 **Avi Wigderson*** (avi@ias.edu), Institute for Advanced Study, Princeton, NJ. *The Value of Errors in Proofs: a fascinating journey from Turing's seminal 1936* $\mathrm{R} \neq \mathrm{RE}$ *to the 2020 breakthrough of* $\mathrm{MIP}^* = \mathrm{RE}$

Last year, a group of theoretical computer scientists posted a paper on the Arxiv with the strange-looking title "$\mathrm{MIP}^* = \mathrm{RE}$", impacting and surprising not only complexity theory but also some areas of math and physics. Specifically, it resolved, in the negative, the "Connes' embedding conjecture" in the area of von-Neumann algebras, and the "Tsirelson problem" in quantum information theory. You can find the paper here: arXiv:2001.04383.

As it happens, both acronyms MIP^* and RE represent proof systems, of a very different nature. To explain them, we'll take a meandering journey through the classical and modern definitions of proof. I hope to explain how the methodology of computational complexity theory, especially modeling and classification (both problems

and proofs) by algorithmic efficiency, naturally leads to the generation of new such notions and results (and more acronyms, like NP). A special focus will be on notions of proof which allow interaction, randomness, and errors, and their surprising power and magical properties. The talk does not assume any special background. (Received September 10, 2021)

1174-68-6969 **Shuangping Li*** (`sl31@princeton.edu`), Princeton University. *On the Binary Perceptron*

We consider the binary perceptron model, a simple model of neural networks that has gathered significant attention in the statistical physics, information theory and probability theory communities, with recent connections made to the performance of learning algorithms.

For the symmetric binary perceptron model, we establish that the partition function of this model, normalized by its expected value, converges to a lognormal distribution. As a consequence, this allows us to establish several conjectures for this model: (i) it proves the contiguity conjecture of Aubin et al. '19 between the planted and unplanted models in the satisfiable regime; (ii) it establishes the sharp threshold conjecture; (iii) it proves the frozen 1-RSB conjecture in the symmetric case, conjectured first by Krauth-Mézard '89 in the asymmetric case. Our proof technique relies on a dense counter-part of the small graph conditioning method, which was developed for sparse models in the celebrated work of Robinson and Wormald.

Regarding the solution space structure, we show in the low constraint density regime, there is a cluster of solutions with the almost maximally possible diameter; further, a multiscale majority algorithm can find atypical solutions in these clusters with high probability. In addition, even close to the critical threshold, there exist linear sized clusters. We prove these results through algorithmic construction of a collection of solutions indexed by a tree, together with a scheme to interpolate solutions. (Received September 10, 2021)

1174-68-7336 **Zerotti Woods*** (`Zerotti.woods@jhuapl.edu`), Johns Hopkins University. *Explainable AI* Preliminary report.

The capabilities of Deep Convolutional Neural Networks (CNNs) to explore data in various fields has been documented extensively throughout the literature. One common challenge with adopting CNNs is the issue of trust. Decision makers are rightfully hesitant to make decisions just based on "the computer said so". In this study, we will explore a new paradigm that will leverage a suite of machine learning algorithms to provide a prediction based on a semantic understanding of the things being detected. This will enable a model capable of providing meaningful justifications for its predictions while still harnessing the capabilities of traditional CNNs or other AI/ML approaches. (Received September 14, 2021)

1174-68-7731 **Tanisha Saxena*** (`tanisha.saxena@gmail.com`), Lexington High School, MIT PRIMES (Massachusetts Institute of Technology). *A Compromise Between Synchronous and Asynchronous Systems* Preliminary report.

This paper aims to solve the issue of inaccurate modelling within distributed systems. Previous work introduced the idea of slightly synchronous or partially synchronous systems, however, these are insufficient as they either have too much structure to be realistic or do not have enough structure as user actions are not entirely independent. In this paper, we create a separate model that combines the structure of both synchronous and asynchronous systems into a model that is resilient yet also realistic. (Received September 15, 2021)

1174-68-8021 **Sheryl Hsu*** (`sherylhsu02@gmail.com`), MIT PRIMES-USA, Valley Christian High School, and **Laura P. Schaposnik** (`schapos@uic.edu`), University of Illinois at Chicago. *The Power of Many: A* Physarum *Swarm Steiner Tree Algorithm*

We present a novel *Physarum* swarm algorithm to solve the Euclidean Steiner tree problem. *Physarum* is a unicellular slime mold with the ability to form networks and fuse with other *Physarum* organisms. We use the simplicity and fusion of *Physarum* to create large swarms which independently operate to solve the Steiner problem. In order to create *Physarum* swarms, we develop a new cellular automaton model of the fusion of multiple *Physarum* organisms. The *Physarum* Steiner tree algorithm then utilizes a swarm of *Physarum* organisms which gradually find terminals and fuse with each other, sharing intelligence. The algorithm is also highly capable of solving the obstacle avoidance Steiner tree problem and is a strong alternative to the current leading algorithm. The algorithm is of particular interest due to its novel approach, time complexity, rectilinear properties, and ability to run on varying shapes and topological surfaces. (Received September 17, 2021)

1174-68-8088 **Lara Kallem*** (lara.kallem@my.simpson.edu), Simpson College, **Zach Geery** (zach.geery@my.simpson.edu), Simpson College, and **Sam McCoy** (sam.mccoy@my.simpson.edu), Simpson College. *Using a Speech Recognizer to help patients recover speech with the Well-Spoken System*

Speech therapy can be an intensive process for people with brain injuries, especially due to limited speech therapist availability. We have created a program to assist people during their speech recovery process. This system utilizes a automatic speech recognizer, allowing people to practice speaking without the presence of a speech therapist. To improve motivation, the program includes elements of gamification. The system logs the user's interactions and classifies their errors into different categories, so that immediate feedback can be given to the user. A statistical model of simulated patients is used to validate the accuracy of the error classification. The user interface is written in Python and uses the data analytics and graphing capabilities of R to display and highlight the user's progress. Based on the results of the error analysis, the patient and therapist can view different charts and graphs of the user's progress and their type of errors. The idea is not to replace traditional speech therapy but to supplement it. (Received September 17, 2021)

1174-68-8132 **Jeova Farias Sales Rocha Neto*** (jeova_farias@brown.edu), Haverford College. *Penalized Normalized Cuts* Preliminary report.

One of the most popular methods for spectral image segmentation and clustering is the Normalized Cuts algorithm, which generates segments by partitioning a graph that models the image. In this work, we generalize the its original framework to allow for the incorporation of prior knowledge about how pixels should be grouped into the segmentation pipeline. This additional information comes from two sources: (1) user data about pixels that should be assigned to certain regions (seeds) or about the expected region appearances and (2) segmentation cues concerning each region's color statistics. In order to accommodate these new features, a plug-and-play penalty function is added to the original Normalized Cuts formulation without requiring the explicit construction of a large, dense matrix. Although many strategies have been proposed to add seed information to the Normalized Cuts-based segmentation pipeline, this is, to the best of our knowledge, the first algorithm to add global appearance data and other segmentation cues to the original spectral framework. Our promising preliminary results show that the proposed methods out-perform the traditional spectral clustering algorithm and can be successfully used in image segmentation tasks. (Received September 17, 2021)

1174-68-8313 **Sergey Bravyi*** (sbravyi@us.ibm.com), IBM T. J. Watson Research Center. *Quantum advantage for computations with limited space*

The Holevo theorem gives a fundamental limit on the amount of information that can be stored in a quantum system: a single qubit cannot store more than one bit of information. Surprisingly, we show that a qubit can be vastly more powerful than a classical bit if used as scratch space for a computation. To this end we introduce a model of limited space computations, where input is a read-only memory and only one (qu)bit can be computed on. We show that any n-bit symmetric Boolean function can be implemented exactly through the use of quantum signal processing as a limited space quantum computation using $O(n^2)$ gates. Meanwhile, the analogously defined classical computation may only evaluate certain n-bit symmetric Boolean functions on a fraction of inputs approaching 50% for large n, no matter how long is the computation. (Received September 18, 2021)

1174-68-8545 **Srijanie Dey*** (srijanie.dey@wsu.edu), Department of Mathematics, Washington State University, and **Alexander Dimitrov** (alex.dimitrov@wsu.edu), Department of Mathematics, Washington State University. *Mapping and Validating a Point Neuron Model on Intel's Neuromorphic Hardware Loihi* Preliminary report.

Neuromorphic hardware is based on emulating the natural biological structure of the brain. Since its computational model is similar to standard neural models, it could serve as a computational acceleration for research projects including biomedical applications. However, in order to exploit this new generation of computer chips, rigorous simulation and consequent validation of brain-based experimental data is imperative. In this work, we investigate the potential of Intel's fifth generation neuromorphic chip - 'Loihi', which is based on the novel idea of Spiking Neural Networks (SNNs) emulating the neurons in the brain. The work is implemented in context of simulating the Leaky Integrate and Fire (LIF) models based on the mouse primary visual cortex matched to a rich data set of anatomical, physiological and behavioral constraints. Simulations on the classical hardware serve as the validation platform for the neuromorphic implementation. We find that Loihi replicates classical simulation very efficiently and scales fairly well in terms of both time and energy performance as the networks get larger. (Received September 19, 2021)

1174-68-8655 **Vincent Li*** (mail2vincentrli@gmail.com), Western Connecticut State University, and **Stacy Vazquez** (stacyvazquez97@gmail.com), Western Connecticut State University.
Transferability of the Fast Gradient Sign Attack on Quantum-Based Neural Networks

Quantum adversarial machine learning lies at the intersection of quantum computing and adversarial machine learning. As the attainment of quantum supremacy demonstrates, quantum computers have already outpaced classical computers in certain domains (Arute et al., 2019). The study of quantum computation is becoming increasingly relevant in today's world. A field in which quantum computing may be applied is adversarial machine learning. A step towards better understanding quantum computing applied to adversarial machine learning has been taken recently by Lu et al. (2020), who have shown that gradient-based adversarial attacks have transferred from classical to quantum neural networks. Inspired by Lu et al. (2020), in this research we investigate whether these attacks can transfer from quantum to other quantum or classical neural networks. We find that these attacks indeed do transfer, implying a deep similarity between these neural networks, despite their superficial differences. (Received September 19, 2021)

1174-68-8666 **Xiaodi Wang** (Wangx@wcsu.edu), Western Connecticut State University, and **Tyler Rust Wooldridge** (wooldridge002@wcsu.edu), Western Connecticut State University. *The Transferability of the Fast Gradient Sign Attack on Quantum Neural Networks*

Quantum adversarial machine learning lies at the intersection of quantum computing and adversarial machine learning. As the attainment of quantum supremacy demonstrates, quantum computers have already outpaced classical computers in certain domains (Arute et al., 2019). The study of quantum computation is becoming increasingly relevant in today's world. A field in which quantum computing may be applied is adversarial machine learning. A step towards better understanding quantum computing applied to adversarial machine learning has been taken recently by Lu et al. (2020), who have shown that gradient-based adversarial attacks have transferred from classical to quantum neural networks. Inspired by Lu et al. (2020), in this research we investigate whether these attacks can transfer from quantum to other quantum or classical neural networks. We find that these attacks indeed do transfer, implying a deep similarity between these neural networks, despite their superficial differences. (Received September 19, 2021)

1174-68-8706 **Anshul Vinay Rastogi*** (stellaranshul@gmail.com), MIT PRIMES Computer Science, and **Tanmay Gupta** (tanmaygupta2023@gmail.com), MIT PRIMES Computer Science.
Threshold-Based Inference of Dependencies in Distributed Systems Preliminary report.

Many current online services rely on the interaction between different machines that form a distributed system. Analyzing distributed systems is important in performance analysis (e.g. critical path analysis), debugging, and testing new features. However, the analysis of these systems can be difficult due to limited knowledge of how components work (referred to as black-box components) and the variety of services and applications present in the system. *The Mystery Machine* uses log events across many traces to generate and refine a causal model. We expand on *The Mystery Machine*'s algorithm, employed by Chow et al., by using thresholds to increase the tolerance to flawed trace data in the formation of causal relationships. (Received September 19, 2021)

1174-68-8922 **Donald J Liveoak*** (dliveoak@umich.edu), University of Michigan-Dearborn.
Schrödinger Bridges on Discrete Domains

Dynamical optimal transport is a field in mathematics and computer science involving interpolation between two probability measures. While dynamical optimal transport on continuous domains is well-understood, algorithms for solving the problem numerically struggle with both accuracy and efficiency. We apply existing theory surrounding Schrödinger bridges to arrive at a system of discrete dual variables which approximate the solutions to the dynamical optimal transport problem. We then propose a novel application of Sinkhorn's algorithm which can be used to numerically solve the dynamical optimal transport problem on discrete surfaces. We show empirically that this algorithm exhibits state-of-the-art performance on interpolation of probability measures defined on triangular meshes. We then propose an entropy-regularized variation of the semi-discrete optimal transport problem, in analogy to continuous Schrödinger bridges posed by Lavenant et al. and prove a result regarding the form of its solution. (Received September 20, 2021)

1174-68-8930 **Cherlin Zhu*** (czhu27@jhu.edu), Johns Hopkins University, **Bhargav Samineni** (bhargav.samineni@gmail.com), New Jersey Institute of Technology, **Benjamin James Thomas** (btho159@lsu.edu), Louisiana State University, **David Davini** (daviddavini@g.ucla.edu), University of California, Los Angeles, and **Amelia Tran** (tran.thuhuong248@gmail.com), Mount Holyoke College. *Using physics-informed regularization to improve extrapolation capabilities of neural networks* Preliminary report.

Neural-network-based surrogate models, which replace (parts of) a physics-based simulator, are attractive for their efficiency, yet they suffer from a lack of extrapolation capability. Focusing on the wave equation, we investigate the use of several physics-based regularization terms in the loss function as a way to increase the extrapolation accuracy, together with assessing the impact of a term that conditions the neural network to weakly satisfy boundary conditions. These regularization terms do not require any labeled data. By gradually incorporating the regularization terms while training, we achieve a more than $5\times$ reduction in extrapolation error compared to a baseline (i.e., physics-less) neural network that is trained with the same set of labeled data. We additionally develop and investigate a neuron saturation metric, which has potential to provide insight into the behavior of each layer and how this affects neural network extrapolation capabilities. (Received September 20, 2021)

1174-68-8971 **Elisa Negrini*** (enegrini@wpi.edu), Worcester Polytechnic Institute, **Giovanna Citti** (giovanna.citti@unibo.it), University of Bologna, and **Luca Capogna** (lcapogna@smith.edu), Smith College. *A Neural Network Ensemble Approach to System Identification*

We present a new algorithm for learning unknown governing equations from trajectory data, using and ensemble of neural networks. Given samples of solutions $x(t)$ to an unknown dynamical system $\dot{x}(t) = f(t, x(t))$, we approximate the function f using an ensemble of neural networks. We express the equation in integral form and use Euler method to predict the solution at every successive time step using at each iteration a different neural network as a prior for f. This procedure yields M-1 time-independent networks, where M is the number of time steps at which $x(t)$ is observed. Finally, we obtain a single function $f(t, x(t))$ by neural network interpolation. Unlike our earlier work, where we numerically computed the derivatives of data, and used them as target in a Lipschitz regularized neural network to approximate f, our new method avoids numerical differentiations, which are unstable in presence of noise. We test the new algorithm on multiple examples both with and without noise in the data. We empirically show that generalization and recovery of the governing equation improve by adding a Lipschitz regularization term in our loss function and that this method improves our previous one especially in presence of noise, when numerical differentiation provides low quality target data. Finally, we compare our results with the method proposed by Raissi, et al. arXiv:1801.01236 (2018). (Received September 20, 2021)

1174-68-8982 **Wojciech Czaja** (wojtek@math.umd.edu), University of Maryland, **Weilin Li*** (weilinli@cims.nyu.edu), Courant Institute, and **Ilya Kavalerov** (ilyak@terpmail.umd.edu), Google. *Clustering of radiological data via Fourier scattering* Preliminary report.

The detection of radioactive sources is an essential step in threat reduction and unmanned survey of high risk areas. Since the amount of available training data in such datasets is scarce, sophisticated mathematical tools that exploit the high dimensional geometry hidden within such datasets are required. In this talk, we present the Fourier scattering transform (FST) as a non-linear too for spectral analysis. The FST enjoys mathematical guarantees, changes the geometry of the raw data, and it generates features that are more interpretable by standard clustering algorithms such as k-means and DBSCAN. We test this method on a real radiological dataset collected at the Savannah River National Laboratory and analyze the FST feature space geometry. (Received September 20, 2021)

1174-68-9170 **Gunnar Erik Carlsson*** (carlsson@stanford.edu), Stanford University. *Topological Methods for Deep Learning*

Deep learning is a very powerful methdology for many problems in mahcine learning. However it faces certain problems, which include the fact that it is “data hungry”, that solutions often don’t generalize well from one data set to a seemingly very similar data set, and finally a general lack of transparency. In this talk we will demonstrate that by applying geometric constructions to collections of features, we may address all these problems. We will discuss theory as well as examples. (Received September 20, 2021)

1174-68-9214 **Ian McCallum*** (imccallum505@g.rwu.edu), Presenter, and **Hum Nath Bhandari** (hbhandari@rwu.edu), Advisor. *Implementation of LSTM Model in Financial Time Series Data Forecasting* Preliminary report.

Artificial Intelligence (AI) is the ability of a machine to process information such that it can perform complex tasks that typically require human intelligence. Machine Learning is a subset of AI that focuses on training the algorithm to learn from past experiences without being explicitly programmed. Deep learning is a type of machine learning technique based on neural networks which helps to automatically extract patterns in raw data to perform a task. In this study, we implement Long Short-Term Memory (LSTM), a particular type of deep learning model, to predict the next day's closing price of the Standard & Poor's 500 index. A carefully selected set of features is picked to aid in analyzing the behavior of the index, as well as provide further reliability in the prediction. Several LSTM models with different numbers of neurons and layers are considered, and their performances are compared to identify the best model. Various hyperparameter tuning and model selections strategies are used to test the effectiveness and resilience of the obtained model. (Received September 20, 2021)

1174-68-9567 **Andreas Mang** (andreas@math.uh.edu), Department of Mathematics, University of Houston, **Hossein Dabirian*** (h.dabirian@gmail.com), University of Michigan, **James Herring** (herrinj@gmail.com), Numerica Corporation, **Peng Zhang** (zhangzjcxpeng@gmail.com), Oliden Technology, **Jiwen He** (jiwenhe@math.uh.edu), Department of Mathematics, University of Houston, and **Robert Azencott** (razencott@uh.edu), Department of Mathematics, University of Houston. *Automatic classification of shapes and shape deformations in 3D* Preliminary report.

We present a novel approach for the classification of infinite-dimensional shapes embedded in $\mathbb{R}^3$. Our goal is to provide computational methods that allow us to automatically discriminate between clinically distinct patients groups through the lens of anatomical shape variability. In a Riemannian setting, we can express the similarity between two shapes s_0, s_1, in terms of an energy minimizing diffeomorphism f, where $f(s_0) = s_1$. We use an optimal control formulation, in which the diffeomorphism f is parameterized by a smooth, time-dependent velocity field v (the control variable of our problem). After computing an optimal $v^\star$ and the associated diffeomorphism $f^\star$, we derive the strain distribution of $f^\star$ as well as a Hilbert norm of $v^\star$ to characterize the dissimilarities between s_0 and s_1. We introduce two data augmentation methods to enrich our dataset. The first method is based on interpolation between neighboring 3D surfaces through the optimal matching. The second method is established through the perturbation of surfaces using Gaussian random fields. We implement these data augmentation methods for real-world shapes extracted from cardiac imaging (mitral valve data, to be precise). We report fully automatic classification results for the original and enriched dataset using random forest algorithm. (Received September 20, 2021)

1174-68-9603 **Vasileios Maroulas** (vmaroula@utk.edu), University of Tennessee Knoxville, and **Christopher Oballe*** (coballejr@gmail.com), University of Notre Dame. *ToFU: Topology Functional Units for Deep Learning*

We propose ToFU, a new trainable neural network unit with a persistence diagram dissimilarity function as its activation. Since persistence diagrams are topological summaries of structures, this new activation measures and learns the topology of data to leverage it in machine learning tasks. We showcase the utility of ToFU in two experiments: one involving the classification of discrete-time autoregressive signals, and another involving a variational autoencoder. In the former, ToFU yields competitive results with networks that use spectral features while outperforming CNN architectures. In the latter, ToFU produces topologically-interpretable latent space representations of inputs without sacrificing reconstruction fidelity. (Received September 20, 2021)

1174-68-9620 **Ying Zheng*** (zhengy3@my.erau.edu), Embry Riddle Aeronautical University, **Brian Danaher** (danaherb@my.erau.edu), Embry Riddle Aeronautical University, and **Matthew Brown** (matthew.t.brown.mtb@gmail.com), University of Toledo. *Evaluating the Variable Stride Algorithm in the Identification of Diabetic Retinopathy*

An experiment was performed to investigate an alternative pooling method—Variable Stride—for use in convolutional neural networks. Three variable stride methods were compared to Maxpool and Avgpool in three different network configurations tasked with classifying diabetic retinopathy images between healthy retinas and retinas with advanced retinopathy. Each combination of network structure and pooling method was run multiple times, and the AUCs, losses, accuracies, as well as the training speed of each run were all collected. Two-tailed t-tests were then run on the prior metrics to quantify the relative performance of each pooling method in each scenario. (Received September 20, 2021)

1174-68-9734 **Anna Grim*** (anna_grim@brown.edu), Brown University. *Belief Propagation with Homotopy Continuation*

Belief propagation is a message passing algorithm that is often used to perform approximate probabilistic inference in Markov random fields. One shortcoming of this algorithm is that it often fails to converge when the topology of the factor graph is complex. Our solution to this problem is an algorithm that utilizes numerical homotopy continuation between the traditional sum-product message passing operator and a similar operator that is contractive. This algorithm follows a path of fixed point that begins at the unique fixed point of the contractive operator and converges to a fixed point of loopy belief propagation. (Received September 20, 2021)

1174-68-9824 **Hum Nath Bhandari*** (hbhandari@rwu.edu), Roger Williams University, **Binod Rimal** (brimal2014@fau.edu), Florida Atlantic University, **Nawa Raj Pokhrel** (npokhrel@xula.edu), Xavier University of Louisiana, **Ramchandra Rimal** (ramchandra.rimal@mtsu.edu), Middle Tennessee State University, **Keshab R Dahal** (kdahal@truman.edu), Truman State University, and **Rajendra K C Khatri** (rajendraksee@utdallas.edu), University of Texas at Dallas. *Application of Deep Learning Models in Stock Market Index Prediction* Preliminary report.

The rapid advancement in artificial intelligence and machine learning techniques, availability of large-scale data, and increased computational capabilities of the machine opens the door to develop sophisticated methods in predicting stock price. In the meantime, easy access to investment opportunities has made the stock market more complex and volatile than ever. The world is looking for an accurate and reliable predictive model that can capture the market's highly volatile and nonlinear behavior in a holistic framework. This study uses different deep learning model architectures such as LSTM, CNN, and GRU, to predict the next day closing price of the S&P 500 index. Input dataset consists of carefully selected features from fundamental market data, macroeconomic data, and technical indicators. Both single layer and multilayer models are constructed using the chosen input variables, and their performances are compared using standard assessment metrics - Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), and Correlation Coefficient(R). Experimental results are promising with a varying degree of prediction accuracy depending on the model architecture. (Received September 21, 2021)

1174-68-9865 **Steven Luo** (contactstevenluo@gmail.com), Evergreen Valley High School, **Aayush Sheth** (aayush01@live.com), Tesla STEM High School, **Eric Zhu** (zhueric2004@gmail.com), Princeton High School, **Eric Guan** (ericguan2004@gmail.com), North Carolina School of Science and Mathematics, **Krish Jain** (krish.jain@outlook.com), Redmond High School, **Jansen Wong*** (JansenW95@gmail.com), Great Neck South High School, **Fan Yang** (yangfan41@gmail.com), Department of Materials Science and Chemical Engineering, Stony Brook University, **Pawel Polak** (pawel.polak@stonybrook.edu), Department of Applied Mathematics and Statistics, Stony Brook University, **Miriam Rafailovich** (miriam.rafailovich@stonybrook.edu), Department of Materials Science and Chemical Engineering, Stony Brook University, **Rena Max** (renamax6@gmail.com), Department of Computer Science, University of Maryland, and **Nilesh Chaturvedi** (nilesh.chaturvedi@stonybrook.edu), Department of Applied Mathematics and Statistics, Stony Brook University. *Deepfake Video Detection Using Biologically Inspired Geometric Deep Learning*

Deepfakes are synthetic media in which a person's appearance is altered to look like another's. In order to identify them we trace the face by employing a 3D manifold obtained from the open-source Google library—MediaPipe. It provides us with a local topology of the face that is invariant to any homeomorphic transformation across consecutive frames. We measure the local muscle movement in the face using a heatmap of correlations between pixels in the corresponding frames. These unique biological characteristics are projected to the canonical face mesh and used as features in a 3D Convolutional Neural Network to quantify the authenticity of the face on the video. The proposed model is trained on publicly available state-of-the-art databases of manipulated and real videos, including FaceForensics, FaceForensics++, and Deep Fakes Dataset, with more than 500 gigabytes of data available.

The paper is accompanied by a web application with the frontend written in React JS, and backend in Python using Flask with Nginx as the front-facing reverse proxy and Gunicorn to serve the Flask app. The trained model will be integrated with the web application and deployed on Stony Brook University servers as a publicly available application that allows users to analyze their own videos and label them as fake or real. (Received September 21, 2021)

1174-68-10015 **Juan Carlos Martinez Mori*** (jm2638@cornell.edu), Cornell University. *On the Request-Trip-Vehicle Assignment Problem*

The *request-trip-vehicle* assignment problem is at the heart of a popular decomposition strategy for online vehicle routing. In this framework, assignments are done in batches in order to exploit any shareability among vehicles and incoming travel requests. We study a natural ILP formulation and its LP relaxation. Our main result is an LP-based randomized rounding algorithm that, whenever the instance is feasible, leverages mild assumptions to return an assignment whose: *i*) expected cost is at most that of an optimal solution, and *ii*) expected fraction of unassigned requests is at most $1/e$. If trip-vehicle assignment costs are α-approximate, we pay an additional factor of α in the expected cost. We can relax the feasibility requirement by considering the penalty version of the problem, in which a penalty is paid for each unassigned request. We find that, whenever a request is repeatedly unassigned after a number of rounds, with high probability it is so in accordance with the sequence of LP solutions and not because of a rounding error. We additionally introduce a deterministic rounding heuristic inspired by our randomized technique. Our computational experiments show that our rounding algorithms achieve a performance similar to that of the ILP at a reduced computation time, far improving on our theoretical guarantee. The reason for this is that, although the assignment problem is hard in theory, the natural LP relaxation tends to be very tight in practice. (Received September 21, 2021)

1174-68-10081 **Samantha Erwin*** (erwinsh@ornl.gov), Oak Ridge National Laboratory. *Leveraging Deep Learning for Tabular Health Data* Preliminary report.

The Surveillance, Epidemiology, and End Results (SEER) database provides records on over 11 million cancer cases in the form of tabular data. These records include patient demographics, cancer diagnostics and biomarkers, and long-term patient outcomes. Using deep learning methodologies implemented within a high-performance computing paradigm, we can leverage this rich data set to understand leading features and factors in patient survival time. In this work, we develop a novel deep learning classification model to predict patient survival months. We implement hyperparameter tuning on our model to achieve prediction accuracy of 82.0% then compare our prediction metrics to eight traditional machine learning algorithms. Our results show that our deep learning model outperforms all other models examined in accuracy and speed. Finally, we analyze feature importance based on the weights in the trained model to understand which features drive the patient survival metrics in our predictions. We have implemented our deep learning pipeline for both prostate and breast cancer records with each data set containing over 1 million patients. Furthermore, our novel pipeline is readily applied to all unique data sets selected in the SEER database. (Received September 21, 2021)

1174-68-10194 **Rachel Aileen Stclair*** (rstclair2012@fau.edu), Florida Atlantic University. *Compressed Sensing Echo-State Network for Chaotic Time-Series Forecasting*

Echo-state networks (ESNs) are a type of reservoir computing model in which a stochastically connected recurrent hidden layer with fixed weights is connected to a trainable output layer. ESNs have shown specific utility in chaotic time-series inference, with applications such as energy monitoring and quantum computing. However, practical applications of ESNs require tuning three global parameters by hand in the form of two regularization techniques and reservoir capacity. All of these parameters have to be optimized jointly which can be inefficient. Here, we show that, for Lorenz-type time series, Ridge and state noise regularization can be avoided when passing the reservoir output through a compressed-sensing style random projection layer before sending to the output layer. The compressed-sensing layer acts as a compression of the sparse reservoir state which is smaller and thus more efficiently optimized by the learnable output weights. This approach allows for larger model capacity in the reservoir without suffering from a linear increase in computational complexity during learning. We found compressed-sensing ESN results in more accurate forecasting, less hyperparameter tuning, and increased model capacity without the computational costs. Our implementation is robust and easy to implement on high performance computing devices. (Received September 21, 2021)

1174-68-10267 **Juliet Whidden*** (jwhidden@vassar.edu), Vassar College, **Siddharth Berera** (s2020550@ed.ac.uk), The University of Edinburgh, **Alycia Doucette** (doucette.alycia@gmail.com), UC Santa Barbara, **Bridget Duah** (bduah@smith.edu), Smith College, **Bill Feng** (bill.feng@yale.edu), Yale University, **Andrés Gómez-Colunga** (andres.gomez-colunga@yale.edu), Yale University, **Luke Hammer** (luke_hammer@brown.edu), Brown University, **Joey Lakerdas-Gayle** (joey.lakerdasgayle@mail.utoronto.ca), University of Toronto, **Mary Olivia Liebig** (mol1@rice.edu), Rice University, **Alvedian Mauditra Aulia Matin** (a.mauditra@gmail.com), Bandung Institute of Technology, **Jacob Micheletti** (michelet@grinnell.edu), Grinnell College, **Daniel Roebuck** (dr211@st-andrews.ac.uk), University of St Andrews, and **Noam Scully** (noamscully166@gmail.com), Yale University. *Lexicographically least square-free words with a given prefix* Preliminary report.

A square-free word is a sequence of letters that does not contain any block appearing twice consecutively. Given a finite square-free word, we can ask about its lexicographically least (lex least) extension to an infinite square-free word over the alphabet of the non-negative integers. The lex least extension of the empty word was shown to be the ruler sequence by Guay-Paquet and Shallit in 2009. We will discuss how to build lexicographically least square-free extensions from different prefixes and show similarities and differences between the structures of different extensions. Finally, we talk about how we can work backwards by presenting an algorithm which lets us generate a prefix that will produce a desired sequence in its extension. (Received September 21, 2021)

1174-68-10467 **F. Patricia Medina*** (patricia.medina@yu.edu), Yeshiva University, and **Randy C Paffenroth** (rcpaffenroth@wpi.edu), Worcester Polytechnic Institute. *Machine Learning in LiDAR 3D point clouds*

LiDAR point clouds contain measurements of complicated natural scenes and can be used to update digital elevation models, glacial monitoring, detecting faults and measuring uplift detecting, forest inventory, detect shoreline and beach volume changes, landslide risk analysis, habitat mapping and urban development, among others. A very important application is the classification of the 3D cloud into elementary classes. For example, it can be used to differentiate between vegetation, man-made structures and water. Our goal is to present a preliminary comparison study for classification of 3D point cloud LiDAR data that includes several types of feature engineering. In particular, we demonstrate that providing context by augmenting each point in the LiDAR point cloud with information about its neighboring points can improve the performance of downstream learning algorithms. We also experiment with several dimension reduction strategies, ranging from Principal Component Analysis (PCA) to neural network based auto-encoders, and demonstrate how they affect classification performance in LiDAR point clouds. For instance, we observe that combining feature engineering with a dimension reduction method such as PCA, there is an improvement in the accuracy of the classification with respect to doing a straightforward classification with the raw data. (Received September 21, 2021)

1174-68-10593 **Luminita Vese** (lvese@math.ucla.edu), University of California Los Angeles, **Wen Li*** (wenli@math.ucla.edu), University of California, Los Angeles, **Joel Barnett** (jrbarnett@g.ucla.edu), University of California, Los Angeles, and **Elena Resmerita** (elena.resmerita@aau.at), Institute of Mathematics, The University of Klagenfurt (AAU). *Multiscale hierarchical image decomposition and refinements*

The multiscale hierarchical decomposition method (MHDM) proposed by Tadmor, Nezzar, Vese (2004, 2008) has been proven very appropriate for denoising images with features at different scales and for scale separation. Recently, a tighter version of the MHDM with better convergence properties has been introduced by Modin et al. (2019) and applied to nonlinear problems. The contribution of the present work is as follows. First, we derive novel error estimates for MHDM and its tighter version. Second, we provide rules for early stopping of the algorithms in the case of additive and multiplicative noise, while still ensuring stable approximations of the true image. Last but not least, we propose a refined version of the tighter MHDM, which allows recovering structured images and promotes different features of the components, as compared to the entire image. The theoretical results are validated by numerous numerical experiments for image denoising and deblurring, which also assess the analyzed methods in terms of rate of convergence, stopping rule, and quality of restoration. (Received September 21, 2021)

1174-68-10607 **Bill Kay** (bill.w.kay@gmail.com), Oak Ridge National Laboratory, **Kathleen Hamilton*** (hamiltonke@ornl.gov), Oak Ridge National Laboratory, **Catherine Schuman** (schumancd@ornl.gov), Oak Ridge National Laboratory, **Prasanna Date** (datepa@ornl.gov), Oak Ridge National Laboratory, **Travis Humble** (humblets@ornl.gov), Oak Ridge National Laboratory, and **Raphael Pooser** (pooserrc@ornl.gov), Oak Ridge National Laboratory. *Simulating network dynamics with neuromorphic hardware* Preliminary report.

Designing applications for next-generation computing platforms require algorithmic designs that leverage new methods of information processing. Neuromorphic hardware uses "brain-inspired" computational tasks, where information is transmitted in terms of discrete, temporal pulses (or "spikes'). These spikes travel between multi-level dynamical systems (neurons). Algorithmic tasks are implemented by tuning the weighted connections which connect neurons, as well as neuronal parameters. This interplay of neuronal and synaptic parameters, combined with external stimuli leads to complex spiking dynamics. The structural similarities between a weighted synaptic network, and general network models, has inspired a new set of applications for neuromorphic hardware, based on network dynamics. In this talk we will present a survey of several applications that have been developed to run on near-term neuromorphic processors. We focus on analysis of dynamical processes on undirected graphs (epidemic modeling and percolation modeling). (Received September 21, 2021)

1174-68-10675 **Drew Parker Worden*** (worden004@wcsu.edu), Western Connecticut State University. *Classifier-Resistant Video Steganography Using Homogeneous Block Selection*

Steganography, or the practice of concealing information within other information, is very useful when it comes to transferring information since it doesn't broadcast when or how information is being sent. This makes it very valuable for covert and intelligence operations, as well as file watermarking. With the rising need for privacy online and protection of intellectual property, the need for embedding messages within other files is at an all time high. Unfortunately past steganography techniques, such as the discrete cosine transform (DCT), have been becoming more obsolete due to machine learning counter-measures. These machine learning classifiers, given a large enough training set, can accurately predict if an image has hidden information. This research gives a theoretical algorithm and statistical analysis of a new stenographic technique that can resist today's machine learning classifiers using homogeneous block selection. (Received September 21, 2021)

1174-68-10733 **Abigail Thomas*** (abigailsusanthomas@gmail.com), MIT PRIMES. *The Implementation of Pruning to Optimize zk-SNARKs* Preliminary report.

Zero-Knowledge Succinct Non-Interactive Arguments of Knowledge (zk-SNARK)s are used to convince a verifier that a server possesses certain information without revealing these private inputs. Thus, zk-SNARKs can be useful when outsourcing computations for cloud computing. The proofs returned by the server must be less computationally intensive than the given task; otherwise, the device will not be able to verify the proof. However, the complexity of the task is directly proportional to the computational cost of the proof. We present a method that involves model pruning to decrease the complexity of the given task and consequently the proof to allow clients to outsource more complex programs. To test our method, we perform experiments using the task of training a neural network on the MNIST dataset. The proposed method harnesses the benefits of producing accurate results using a lower number of constraints while remaining secure. (Received September 21, 2021)

1174-68-10746 **Dylan Skinner*** (dylanskinner65@gmail.com), Brigham Young University, **Jackson Switzer** (jacksoncswitzer@gmail.com), Brigham Young University, **Dahlia Maxwell** (dahlialynne@gmail.com), Brigham Young University, and **Timothy Jay Keith** (timothykeith67@gmail.com), Brigham Young University. *Using deep reinforcement learning to generate small genus slice surfaces from knots in braid notation*

Deep reinforcement learning (DRL) has proven effective in recognizing patterns and finding solutions to problems that are difficult for humans. One problem in knot theory involves finding slice surfaces for knots with minimal genus. It is easy to find large genus slice surfaces bounded by a given knot, but in order to show that the slice genus of a knot is equal to a specified value, you must also prove that the knot does not bound a slice surface of a lesser genus. In this poster, I will outline an approach using DRL and braid notation of a knot to find small genus slice surfaces for a given knot, through a series of unknotted component addition/deletion, crossing addition/deletion, and relations in the braid group. (Received September 21, 2021)

1174-68-10752 **Claire A Wang*** (cwang23@andover.edu), Phillips Academy Andover, and **Yihao Huang** (yhuang23@andover.edu), Phillips Academy Andover. *Efficient Algorithms for Parallel Bi-core Decomposition*

Many real-world statistics and problems can be modeled by graphs, such as user-product networks, social networks, and biological networks. Identifying dense regions within these graphs is useful for product-recommendation, spam identification, and protein-function discovery. k-core decomposition is a fundamental graph theory problem that discovers dense substructures of a graph. However, k-core decomposition does not directly apply to bipartite graphs, which are graphs that model the connections between two disjoint sets of entities. Bipartite graphs are widely used to model authorship, affiliations, and gene-disease associations, to name a few.

In this paper, we solve the analog of the k-core decomposition problem, which is the bi-core decomposition problem. Existing sequential bi-core decomposition algorithms are not scalable to large-scale bipartite graphs with hundreds of millions of edges. Therefore, in this paper, we develop a theoretically efficient parallel bi-core decomposition algorithm. Compared to existing parallel algorithms, our algorithm reduces the length of the longest dependency path of the computational graph which measures the asymptotic bound of a parallel algorithm given sufficiently many threads. We provide an optimized parallel implementation that is scalable and fast. Using 30 threads, our parallel algorithm achieves up to 34.8x self-relative speedup. Our code achieves up to 4.1x speedup compared with the best existing parallel algorithm. (Received September 21, 2021)

1174-68-10756 **Lukasz W Zbroszczyk*** (zbroszczyk002@wcsu.edu), Western Connecticut State University, and **Peter Michael Bigica** (Bigica001@wcsu.edu), Western Connecticut State University. *Machine Learning-Based Algorithm for Projecting the Outcome of MLB At-Bats*

Over the last twenty years, there has been an explosion of the use of statistics and data science to inform tactical decisions in baseball. Baseball is uniquely suited to this kind of analysis due to the wealth of available statistics and the fact that the game is discretely broken up into individual plays and pitches. A traditional, naïve approach would have been to use observations of past performance to extrapolate into the future, but a more sophisticated approach is to take advantage of modern player performance metrics from MLB Statcast's database, such as pitch velocity, location, contact probability, and exit velocity, to model the outcome of at-bats. In this research, we develop a model using logistic regression, support vector machines, neural networks, decision trees, and random forests to model the outcome of MLB at-bats using physical data from Statcast. (Received September 21, 2021)

1174-68-10797 **Akhil Sai Kammila*** (akhilkammila@gmail.com), MIT PRIMES. *A Formal Security Analysis of the Tor Handshake Protocol* Preliminary report.

The Tor network is one of the most important anonymous services as it enables users to bypass censorship restrictions, avoid being traced, and access the internet with true privacy. Tor uses a unique handshake to establish connections in its circuit. The handshake attempts to avoid censorship by mimicking TLS, but practical methods to distinguish it from TLS may remain. The handshake also has high latency, which detracts from the user experience. There have been formal security analyses for TLS and QUIC, but Tor's handshake security has not been verified. Our work will present a security analysis of the Tor handshake protocol. The results apply to handshakes where mutual authentication between Tor relays is required and to handshakes where a relay authenticates to a client. We will also propose changes to the Tor handshake protocol to improve its censorship resistance and latency. (Received September 21, 2021)

1174-68-10991 **William Edward Hahn** (whahn@fau.edu), Florida Atlantic University. *Combining gradient free optimization with stochastic gradient descent* Preliminary report.

The agent consists of a deep learning model trained using our toolbox that combines gradient and non-gradient methods. In particular, we explore the utility of population-based models operating in conjunction with automatic differentiation-based gradient descent. Agents are optimized with Adam optimizer using distinct hyperparameters in the SGD step. We use the Simulated Annealing (SA) for a new solution with a probability that depends both on the new solution quality and the temperature that lowers the chance of accepting worsening moves.

Computational results for a virtual trading agent trained on stocks listed on S&P 500 index are reported. We trained a population of agents to predict appropriate trading behaviors such as buy, hold, or sell by optimizing the portfolio returns. Traditionally agents are trained on a single stock, and then inferences are made using that model about the future stock price. We have developed a novel validation method that splits the data set by time and by ticker symbol. The agent is trained on a set of tickers from a selected window in the historical data. The agent is then evaluated on new tickers for trading days after the initial training period. The proposed approach leads to an improvement in test accuracy. (Received September 21, 2021)

1174-68-11011 **Anna Katherine Vinnedge*** (anna.vinnedge@westpoint.edu), United States Military Academy. *DNA Data Storage and Cry*

Recent advances in the fields of chemical engineering, computer science, and mathematics have opened up a new means of data storage using DNA. An important component of data storage and computing is the ability to encrypt and decrypt data for protection and security purposes. Cryptographic algorithms will appear and act differently for data stored on DNA due to the unique structure of the molecule and the differing formats of standard data storage versus DNA storage. In this poster, we have reviewed and analyzed different encryption and decryption methods, to include both symmetric and asymmetric ciphers, and how they could be applied to DNA storage. We examine the structure of DNA as it relates to binary and XOR operations as well as how DNA cryptography may be more or less secure using some standard ciphers. We also explore some more complex cryptographic algorithms, to include DNA elliptic curve cryptography, to determine their potential for developing practical and secure methods of data encryption and decryption on a large scale. (Received September 21, 2021)

1174-68-11013 **Jacob Lee Ballington*** (Jacob.Ballington@g.fmarion.edu), Francis Marion University. *The Application of Probabilistic Greed to Predicting Market Trends with Q-Learning* Preliminary report.

The purpose of this study was to investigate the use of Reinforcement Learning as a tool for predicting market trends. Price and options data was sourced from the S&P 500 index from 2010-2019. P/C ratios were then compiled and used to establish a Q-Learning environment. Previous literature suggests that for the exploration/exploitation trade off, often called greed, a decaying function is standard. The development of this project has been a probabilistic approach to the greed function. At this time experiments suggest that a probabilistic greed function can outperform an industry standard benchmark by up to forty-four percent. Additionally probabilistic greed may outperform decaying greed functions currently used for Q-Learning by as much as sixteen percent. There remain further questions as to the generalizability of the model, but results are interesting. (Received September 21, 2021)

1174-68-11069 **Yavor Litchev*** (ylitchev@yahoo.com), Lexington High School. *Signature Scheme with Access Control* Preliminary report.

A wide variety of digital signature schemes currently exist, from RSA to El-Gamal to Schnorr. More recently, multi-party signature schemes have been developed, including distributed signature schemes and threshold signature schemes. In particular, threshold signature schemes provide useful functionality, in that they require the number of participating parties to pass a threshold in order to generate a valid signature. However, they are limited in their complexity, as they can only model a threshold function. The proposed signature scheme (monotonic signature scheme) allows for the modeling of complex functions, so long as they are monotonic. This would allow for a much greater degree of access control, all while security and correctness are preserved. (Received September 21, 2021)

1174-68-11086 **Chai Wah Wu*** (cwwu@us.ibm.com), IBM Research. *Hybrid analog digital algorithms*

We analyze hybrid digital analog algorithms for solving linear algebra problems and give conditions under which they converge and are stable. (Received September 21, 2021)

1174-68-11139 **Michael J Kirby*** (kirby@math.colostate.edu), Colorado State University. *Biomarker based predictive modeling of the host immune response to infection*

There is growing evidence that biological data, e.g., gene expression, and metabolomics data, capture detailed information related to a subjects health state. For example, it can be predicted with relatively high accuracy that a host will become contagious after exposure to respiratory pathogens well before the onset of symptoms. One challenge associated with the analysis these data sets is that they often consist of a relatively small number of points residing in very high-dimensional space. Hence the opportunities for overfitting and making erroneous predictions are abundant. This talk will present some examples of geometric machine learning algorithms for determining all discriminatory biomarkers associated with the host response to infectious disease. (Received September 21, 2021)

1174-68-12181 **Di Fang*** (difang@berkeley.edu), University of California, Berkeley, and **Dong An** (dong_an@berkeley.edu), University of Maryland. *Hamiltonian simulation of unbounded operators.*

Recent years have witnessed tremendous progress in developing and analyzing quantum algorithms for Hamiltonian simulation of bounded operators. However, many scientific and engineering problems require the efficient treatment of unbounded operators, which may frequently arise due to the discretization of differential operators. Such applications include molecular dynamics, electronic structure theory, quantum differential equation solver and quantum machine learning. We will introduce some recent progresses in quantum algorithms for efficient unbounded Hamiltonian simulation, including Trotter type splitting and the quantum highly oscillatory protocol (qHOP) in the interaction picture. (Received November 10, 2021)

1174-68-12251 **Robin Kothari*** (robin.kothari@microsoft.com), Microsoft Quantum and Microsoft Research. *Quantum algorithms and polynomials representing Boolean functions*

Much research on quantum algorithms is done in a model of computation called quantum query complexity. In this model, there is a close connection between real polynomials representing Boolean functions and quantum algorithms. Over the years this connection has led to new results in quantum query complexity using polynomials, and new results about such polynomials using quantum query complexity. I'll talk about this connection and survey some recent results in the area. (Received December 6, 2021)

70 ► Mechanics of particles and systems

1174-70-5796 **Dominique Patrice Zosso** (dominique.zosso@montana.edu), Montana State University, **Griffin Smith*** (griffinsmith@montana.edu), Montana State University, **Nathan Stouffer** (nathanstouffer1999@gmail.com), Montana State University, and **Scott McCalla** (scott.mccalla@montana.edu), Montana State University. *A PDE Model for Janus-Particle Swarming*

We consider a system of orientable Janus particles in the plane. Particles are subject to Brownian motion and can exhibit active forward drift. Forward drift is controlled by quorum sensing of a particle's peers in its conical field of view. It has been shown experimentally that this set of rules can lead to clustering behavior in certain perception configurations (Lavergne et al., 2018, 10.1126/science.aau5347). Here, we explore the group formation and cohesion of Janus particles using both an agent-based computer model and an advection-diffusion partial differential equation (PDE) model. Our PDE model can recreate the behavior from both physical experimentation and agent-based simulations. Indeed, agent-based simulations run with increasingly more particles approach the PDE results. Additionally, the PDE model highlights an annular structure which was not apparent in prior work. Through a histogramming process this structure can be verified in the agent-based computer simulations. Batches of simulations were run to produce a phase diagram offering insight into the group *formation* of particles at varying vision model parameter values. Allowing the initial condition to be a symmetrical group, a similar phase diagram can be constructed offering insight into the *cohesion* of groups at certain parameter values. Our simulations show that interesting group patterns can emerge from governing rules that are substantially different from classical swarming models. (Received August 31, 2021)

1174-70-8347 **Thomas Hull*** (thull@wne.edu), Western New England University. *Rigid origami applications in engineering: configuration spaces and self-folding*

The past ten years have witnessed an explosion of interest in origami, the art of paper folding, by the engineering, architecture, and materials science research communities. The reason for this is because rigid origami, where faces of the origami crease pattern remain planar as the creases flex, possesses key features that make it attractive, namely

(a) interesting mechanics (as rigid origami folds and unfolds, it can make the material move in surprisingly complex ways),

(b) independence of scale (rigid folding motions of a crease pattern are the same whether folded from nano-scale or building-sized materials), and

(c) ease of manufacturing (since origami starts with a flat sheet).

In this talk we will summarize rigid origami applications currently being pursued in industry ranging from biotech companies to NASA. We will also outline how rigid origami configuration spaces can be computed and how they are used in self-folding analysis, which is of particular interest in applications where the material needs to fold without the aid of human hands. (Received September 18, 2021)

1174-70-9940 **Gabriel Martins*** (g.martins@csus.edu), California State University, Sacramento.
Skateboard Tricks and Topological Flips

We study the motion of skateboard flip tricks by modeling them as continuous curves in the group $SO(3)$ of special orthogonal matrices. We show that up to continuous deformation there are only four flip tricks. The proof relies on an analysis of the lift of such curves to the unit 3-sphere. We are also able to use these lifted curves to visualize many of the tricks and deformations between them. (Received September 21, 2021)

74 ► Mechanics of deformable solids

1174-74-5549 **Olivia Lutterman*** (olutterman@carthage.edu), Carthage College, Los Alamos National Laboratory. *Engineering Magnet Lattices to Manipulate Guided Acoustic Waves in Plate Like Structures*

There have been several previous studies on acoustic wave propagation to be used in non-destructive evaluation of various engineered plate-like structures. Guided waves can travel long distances with little energy loss, so such waves are useful in these studies, however, guided waves have dispersion curves that allow multiple modes to travel through a medium. This study investigates how arrays of magnets attached on the surface of plates could be used to alter the propagation of guided acoustic waves in plates to achieve selective wave mode suppression. Although each individual attached magnet causes a small perturbation on the acoustic wave propagation, cumulative scattering effects are significant through magnet lattice engineering. For this, magnets are placed on a steel plate building two dimensional structures with variations in spacing. A linear chirp signal is excited on the plate, then a short-time Fourier transformation is applied to the guided wave and different wave modes are analyzed. Results show that within a chirp range of 80-140 kHz that selective wave suppression and enhancement are achieved via both absorption and reflection effects, with undefined relationship between lattice spacing and suppression effectiveness. Understanding the relationship between distributed surface scattering and the resulting cumulative effects leading to mode filtering and spatial control of propagation can be very useful in acoustic non-destructive evaluation of plate-like engineered structures. (Received August 22, 2021)

1174-74-6445 **Robert Lipton*** (lipton@lsu.edu), Louisiana State University. *Nonlocal Dynamics and Fracture*

The fracture of brittle solids is a particularly interesting collective interaction connecting both large and small length scales. Apply enough stress or strain to a sample of brittle material and one eventually snaps bonds at the atomistic scale leading to fracture of the macroscopic specimen.

In this talk a nonlocal mesoscopic fracture model is presented in which fractures emerge from the initial boundary problem as part of the solution. The nonlocal model carries details of the process zone seen at the mesoscopic length scale. In the limit of vanishing nonlocality, solutions of the model converge to solutions of the wave equation with evolving boundary formulated in Dal Maso and Toader (J. Diff. Equ. 2019). The classic Kinetic relation linking crack velocity to elastic energy flux flowing into the crack tip is recovered from the mesoscopic model in the limit of vanishing nonlocality.

The nonlocal dynamic initial value problem implicitly encodes the features of the classic model and delivers them in the limit of vanishing nonlocality. The mesoscopic model eleminates the need for separate mathematical treatment of crack and intact material seen in classic (macroscopic) fracture models. (Received September 9, 2021)

1174-74-6484 **Somayeh Mashayekhi*** (smashayekhi@fsu.edu), Kennesaw State University, and
William Oates (woates@eng.famu.fsu.edu), Florida State University. *Fractal Media and Fractional Viscoelasticity*

Fractional calculus has attracted considerable interest because of its ability to model complex phenomena such as continuum and statistical mechanics and viscoelastic materials. While the fractional integral has been used to describe materials' fractal structure, which leads to new thermodynamic relations, the fractional derivative could be used to describe viscoelasticity, thermal and chemical diffusion, and light-matter interactions in materials. This area opens up an application of fractional calculus, which may describe many materials' multiscale thermomechanical material behavior. In this talk, a physical connection between fractal media's fractional time derivative and fractal geometry will be described and applied to viscoelasticity and thermal diffusion in elastomers. (Received September 9, 2021)

76 ► Fluid mechanics

1174-76-7163 **Ryan Creedon*** (creedon@uw.edu), University of Washington, **Bernard Deconinck** (bernard@amath.washington.edu), University of Washington, and **Olga Trichtchenko** (otrichtc@uwo.ca), Western University. *High-Frequency Instabilities of Stokes Waves* Preliminary report.

Euler's equations govern the behavior of gravity waves on the surface of an incompressible, inviscid, and irrotational fluid of arbitrary depth. We investigate the spectral stability of sufficiently small-amplitude, one-dimensional Stokes waves, i.e., periodic gravity waves of permanent form and constant velocity, in both finite and infinite depth. Using a nonlocal formulation of Euler's equations developed by Ablowitz et al. (2006), we develop a perturbation method to describe the first few high-frequency instabilities away from the origin, present in the spectrum of the linearization about the small-amplitude Stokes waves. Asymptotic and numerical computations of these instabilities are compared for the first time to excellent agreement. (Received September 13, 2021)

1174-76-7405 **Linglai Chen*** (lc3879@nyu.edu), New York University, and **Yunan Yang** (yy837@cornell.edu), Cornell University. *Adjoint Direct Simulation Monte Carlo Method for Spacially Non-Homogeneous Boltzmann Equation* Preliminary report.

In this work, we extend the recently introduced adjoint DSMC method proposed in [D. Caflisch, D. Silantyev, Y. Yang, J. Comput. Phys. 439 (2021) 110404.], which is a particle-based method that efficiently computes the gradient of an objective function constrained by the spatially homogeneous Boltzmann equation. Under the discretize-then-optimize (DTO) framework, we add in the spatial domain. We derive the numerical adjoint of the forward direct simulation Monte Carlo (DSMC) method for both 1D and 2D spatial dimensions. Moreover, we discuss the variations to handle three spatial boundary conditions: periodic, reflection, and thermal conditions for the 1D interval and the 2D rectangular domains. Thirdly, we not only compare the accuracy of the gradient computed with respect to the initial temperature parameter but also consider situations where the unknown parameters are related to the spatial boundary condition and the spatial particle distribution in the initial condition. We use different parameters to demonstrate that the adjoint DSMC method, based on the adjoint-state method, can compute gradients independent of the dimension of the unknown parameter. In all cases, numerical tests have shown a consistently 2-3 significant digits matching, which demonstrates the accuracy of the proposed particle method in tackling Boltzmann equation constrained optimization problems.
(Received September 14, 2021)

1174-76-7713 **Jessica Radford*** (jessica.radford@temple.edu), Temple University, **Pratyush Potu** (psp693@utexas.edu), University of Texas at Austin, and **George Nakayama** (w4756677@stanford.edu), Stanford University. *Predicting Startup Behavior of Heat Pipes and Vapor Chambers from a Frozen State*

Heat pipes are devices that transfer heat via phase changes in electronic devices. They can be found in laptops, workstations, and satellites. The device is made of an outer metal shell lined with a porous wick, and filled with a working fluid. Heat pipe behavior is predictable and reliable at normal temperatures. However, if the heat pipe is exposed to very low temperatures, the working fluid may freeze, and change the behavior of the start-up. In particular, if the ingoing heat flux is too large, the heat pipe may overheat, which can result in permanent damage to the pipe and the device in which it is used. In this project, we attempted to numerically simulate the start-up behavior of a conventional heat pipe from a frozen state using transient finite element method within the MOOSE framework. Specifically, we successfully modeled liquid-phase fluid flow and solid-liquid phase change of the working fluid. Additionally, we explored potential methods for liquid-vapor phase change and created a heat pipe mesh which consists of all the parts common for a conventional heat pipe. (Received September 15, 2021)

1174-76-7722 **Chloe Elena Shiff*** (chloe@shiffty.com), Brandeis University. *Predicting Startup Behavior of Heat Pipes and Vapor Chambers from the Frozen State*

Heat pipes are devices that transfer heat via phase changes in electronic devices. They can be found in laptops, workstations, and satellites. The device is made of an outer metal shell lined with a porous wick, and filled with a working fluid. Heat pipe behavior is predictable and reliable at normal temperatures. However, if the heat pipe is exposed to very low temperatures, the working fluid may freeze, and change the behavior of the start-up. In particular, if the ingoing heat flux is too large, the heat pipe may overheat, which can result in permanent damage to the pipe and the device in which it is used. In this project, we attempted to numerically simulate the start-up behavior of a conventional heat pipe from a frozen state using transient finite element method within the

MOOSE framework. Specifically, we successfully modeled liquid-phase fluid flow and solid-liquid phase change of the working fluid. Additionally, we explored potential methods for liquid-vapor phase change and created a heat pipe mesh which consists of all the parts common for a conventional heat pipe. (Received September 19, 2021)

1174-76-8028 **Walter A Strauss*** (wstrauss@math.brown.edu), Brown University. *Steadily Rotating Stars*

I will present a brief survey of some recent mathematical theory of rotating stars. This is joint work with Yilun Wu and also partly with Juhi Jang. The rotating star is modeled as a compressible fluid subject to gravity. Under certain conditions there exists a curve of solutions (rotating around a fixed axis at varying speeds) on which the supports of the stars become unbounded. I will also mention more elaborate models that include a magnetic field or variable entropy. (Received September 17, 2021)

1174-76-8091 **Laurel Ohm*** (laurel.ohm@princeton.edu), Princeton University. *Pattern formation in active suspensions*

We consider the Saintillan–Shelley [2008] kinetic model of active rodlike particles in Stokes flow, for which the uniform, isotropic suspension of pusher particles is known to be unstable in certain settings. We determine exactly how the isotropic steady state loses stability in different parameter regimes. Through weakly nonlinear analysis accompanied by numerical simulations, we study each of the various types of bifurcations admitted by the system, including both subcritical and supercritical Hopf and pitchfork bifurcations. Elucidating this system's behavior near these bifurcations provides both a means of verifying the predictive power of the model against experimental observations as well as a theoretical means of comparing this model with other systems which transition to turbulence. (Received September 17, 2021)

1174-76-8281 **Fizay-Noah Lee*** (fl6@math.princeton.edu), Princeton University. *Well-posedness and Long Time Behavior of Nernst-Planck-Navier-Stokes systems*

We discuss electrodiffusion and electroconvection of ions (two or more species) in a fluid in the presence of boundaries, described by the Nernst-Planck-Navier-Stokes equations. We give an overview of various boundary conditions that model ion-selective membranes and applied electric potentials, and discuss questions of well-posedness and long time behavior of solutions. The answers to these questions depend heavily on both the boundary conditions and spatial dimension considered. We highlight some of the milestone results in the context of well-posedness theory, and in the case of stable configurations, we give a precise characterization of the asymptotic states for fixed Debye length and also in the limit of zero Debye length. (joint work with Peter Constantin and Mihaela Ignatova) (Received September 18, 2021)

1174-76-8456 **Liaosha Xu** (lx5af@virginia.edu), University of Virginia, and **Zoran Grujic*** (zg7c@virginia.edu), University of Virginia. *On criticality of the Navier-Stokes diffusion*

It has been known since the fundamental work of J.L. Lions in 1960s that the hyper-dissipative (HD) 3D Navier-Stokes (NS) system with the order of diffusion β (the power of the Laplacian) is regular for any $\beta \geq \frac{5}{4}$. The exponent $\frac{5}{4}$ is critical in the sense that in this case the unique scaling-invariance of the system takes place at the energy level; this allows for application of the classical regularity methods.

The objective of this talk is to argue the criticality of the Navier-Stokes diffusion ($\beta = 1$). A 3D HD NS flow near a potential spatiotemporal singularity is classified in two main categories, non-homogeneous (either higher or lower-order derivatives are dominant, indicating turbulent and laminar scenarios, respectively; it is sufficient that this domination property holds along a single direction and for a finite set of orders) and homogeneous (derivatives of all orders are comparable, along all directions). In the laminar scenario, the regularity holds for any $\beta \geq 1$, and with no assumptions. In the turbulent scenario, the regularity holds for any $\beta > 1$, and with no assumptions. In the homogeneous scenario, the regularity holds for any $\beta > 1$, with an assumption that the flow is 'focusing' (this is consistent with a focused build-up of the potential spatiotemporal singular profiles). (Received September 19, 2021)

1174-76-9531 **Hyunju Kwon*** (hkwon@ias.edu), Institute For Advanced Study. *The role of pressure in the regularity theory for the Navier-Stokes equations*

In this talk, I will discuss the ε-regularity theory for dissipative weak solutions, introduced by Duchon and Robert, and their short time regularity on a bounded domain. (Received September 20, 2021)

1174-76-10190 **Laura Ann Miller** (lauram9@math.arizona.edu), University of Arizona, **Nicholas Battista** (battistn@tcnj.edu), The College of New Jersey, and **Matea Santiago** (mateasantiago@math.arizona.edu), University of Arizona. *Simulations of fluttering leaves* Preliminary report.

Leaves may flutter in light winds for the purpose of convective and evaporative heat transfer. We will simulate the passive fluid-structure interaction of leaves fluttering using the immersed boundary method. We will discuss the models developed to appropriately study the heat dissipation and how we have incorporated these thermal dynamics coupled with the fluid flow into the computational simulations. We will present results of the simulations where we have studied the impact of varying Reynolds and Peclet numbers. (Received September 21, 2021)

1174-76-10285 **Ricardo Cortez** (rcortez@tulane.edu), Tulane University, **Lisa J. Fauci** (fauci@tulane.edu), Tulane University, and **Rudi Schuech*** (rudi.schuech@gmail.com), Tulane University. *Viscoelastic Network Traversal and Remodeling by Microswimmers* Preliminary report.

Microorganisms often navigate a complex environment composed of a viscous fluid with suspended microstructures such as elastic polymers and filamentous networks. These microstructures can have similar length scales to the microorganisms, leading to complex swimming dynamics. Some microorganisms are known to remodel the viscoelastic networks they move through. In order to gain insight into the coupling between swimming dynamics and network remodeling, we use a regularized Stokeslet boundary element method to compute the motion of a microswimmer consisting of a spherical body and rotating helical flagellum. The viscoelastic network is represented by a cloud of points with virtual Maxwell element links, whose properties (i.e., stiffness, relaxation time) can have non-obvious effects on the swimmer dynamics. We then consider two models of network remodeling in which (1) links break based on their distance to the microswimmer body, modeling enzymatic dissolution by bacteria or microrobots, or (2) links break based on a threshold strain. We compare the swimming performance of the microbes in each remodeling paradigm as they penetrate and move through the network. (Received September 21, 2021)

1174-76-10992 **Matthew McCurdy*** (matthew.mccurdy@trincoll.edu), Trinity College, and **M. Nicholas J. Moore** (moorem@usna.edu), United States Naval Academy. *Predicting convection configurations in coupled fluid-porous systems: from deep to shallow convection cells* Preliminary report.

A ubiquitous arrangement in nature is a free flowing fluid coupled to a porous medium, for example a river or lake lying above a porous bed. Depending on the environmental conditions, thermal convection can occur and may be either confined to the clear fluid region, forming shallow convection cells, or it can penetrate into the porous medium, forming deep cells. Here, we combine three complementary approaches– stability analyses, fully nonlinear numerical simulations, and a simple theory– to determine which configuration arises. The simple theory accurately predicts the transition between deep and shallow convection in the physically relevant limit of small Darcy number. (Received September 21, 2021)

1174-76-11748 **Yassaya Batugedara*** (bbatuged@mtu.edu), Michigan Technological University, **Alexander Labovsky** (aelabovs@mtu.edu), Michigan Technological University, and **Kyle Schwiebert** (kjschwie@mtu.edu), Michigan Technological University. *Higher Temporal Accuracy for LES-C Turbulent Models*

Large Eddy Simulations(LES) are widely used in modeling turbulent flows. A method called Large Eddy Simulation with Correction (LES-C) was proposed in Labovsky(2020), to reduce the modeling error. However, there was a need to reduce the Time discretization error of the LES-C models. Therefore, we propose a method that uses a defect-corrector scheme called Deferred correction to reduce the Time discretization error. Since the method was obtained by adding extra terms to the LES-C model, we can obtain extra accuracy with no additional computational cost. The method is tested for the ADC model (a member of the LES-C family) and the full numerical test is carried out where a clear reduction of errors can be observed. We also performed the full numerical analysis of the method. (Received October 1, 2021)

80 ► Classical thermodynamics, heat transfer

1174-80-11253 **Bob Palais*** (Bob.Palais@uvu.edu), Utah Valley University. *Mathematical considerations in modeling DNA denaturation, annealing, and extension.* Preliminary report.

We will discuss some mathematical issues that arise in two-state and cooperative nearest-neighbor Ising models, and kinetic theory used to predict behavior of PCR under various conditions, and for determining parameters of those models, that are used to in the development of some maximally rapid diagnostic assays. (Received September 22, 2021)

81 ► Quantum theory

1174-81-5609 **Ivan Contreras*** (icontreras85@gmail.com), Amherst College, **Nima Moshayedi** (nima.moshayedi@math.uzh.ch), University of California, Berkeley, and **Konstantin Wernli** (konstantinwernli@gmail.com), University of Southern Denmark. *Geometric Quantization and Groupoid Convolution* Preliminary report.

The Guillemin-Sternberg conjecture states that geometric quantization and symplectic reduction for certain geometric structure commute. In this talk, we approach this conjecture in the case of field theories, where we introduce a convolution algebra for the phase space of the Poisson sigma model (a 2-dimensional topological field theory), which is compatible with symplectic reduction. We give an overview of the Poisson sigma model and its connection with geometric quantization, and we also provide a finite dimensional interpretation of this construction in terms of Lie groupoids.

This talk is based on joint work with Nima Moshayedi (University of California, Berkeley) and Konstantin Wernli (University of Southern Denmark) (Received August 23, 2021)

1174-81-5891 **Zachary Stier*** (zstier@berkeley.edu), UC Berkeley. *On the best generators for PU(2), Part II*

We discuss recent algorithmic work in the design of universal single-qubit gate sets for quantum computing. Using the quaternionically-derived "super golden gates," we connect the problem of efficient approximate synthesis of given gates to arbitrary precision in quantum hardware design to "icosahedral gates" constructed using the symmetries of the icosahedron, which enjoy a form of optimality. Joint work with Terrence Blackman. (Received September 6, 2021)

1174-81-5897 **David W. Lyons*** (lyons@lvc.edu), Lebanon Valley College. *Qubits and Quaternions*

We compare two versions of rotation algebra: unitary evolution of states of quantum bits (or qubits), and a quaternion-based version that productively bridges gaps in the training of mathematics undergraduates—especially in the context of background study for research projects in quantum information science. (Received August 31, 2021)

1174-81-6217 **Terrence Richard Blackman*** (tblackman@mec.cuny.edu), Medgar Evers College - City University of New York. *On the best generators for PU(2), Part I* Preliminary report.

We discuss recent algorithmic work in the design of universal single-qubit gate sets for quantum computing. Using quaternionically–derived "super golden gates," we connect the problem of efficient approximate synthesis of given gates to arbitrary precision in quantum hardware design to "icosahedral gates" constructed using the symmetries of the icosahedron, which enjoy a form of optimality. This is joint work with Zachary Stier. (Received September 7, 2021)

1174-81-6930 **Nichol Furey*** (cohlfurey@gmail.com), Humboldt-Universität zu Berlin, and **Beth Romano** (beth.romano@maths.ox.ac.uk), University of Oxford. *The standard model's particle content as a Jordan algebraic mosaic* Preliminary report.

Our best understanding of fundamental particles is summarized in the Standard Model of Particle Physics. Loosely speaking, these particles are described by a long list of irreducible representations of the gauge group $SU(3) \times SU(2) \times U(1)/\mathbb{Z}_6$, and spacetime symmetries of the (universal cover of the) Poincare group.

On one hand, this description works remarkably well - to the extent that over the last few decades, experiment has succeeded little in luring the standard model from its originally proposed particle content. On the other hand, this choice of groups and representations continues to evade a logical explanation. Why these groups and representations, while not others?

In this talk, we demonstrate that a set of vector spaces, quite close to the standard model's particle content, can be embedded inside the 256-dimensional Euclidean Jordan algebra $\mathcal{H}_{16}(\mathbb{C})$. Eight dimensions within the algebra have yet to be identified. We close in pointing out how this particular embedding stems from the left- and right-multiplication algebras of $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, and $\mathbb{O}$.

This talk will not assume a background in particle physics, nor normed division algebras. Everyone is welcome. (Received September 10, 2021)

1174-81-6963 **Maxim Derevyagin** (maksym.derevyagin@uconn.edu), UConn, and **Nathan Sun** (nathan99sun@gmail.com), Harvard University. *Perfect Quantum State Transfer and Theorem of Joseph-Alfred Serret*

In this presentation we recast the Serret theorem about a characterization of palindromic continued fractions in the context of polynomial continued fractions. Then, using the relation between symmetric tridiagonal matrices and polynomial continued fractions we give a quick exposition of the mathematical aspect of the perfect quantum state transfer problem. (Received September 10, 2021)

1174-81-7355 **Madelyn Andersen*** (mandersen@hmc.edu), Harvey Mudd College. *Finite Rank Kernel Varieties: A Variant of Hilbert's Nullstellensatz for Graphons* Preliminary report.

Graphons are symmetric measurable functions that arise from a sequence of graphs. A graphon variety is the a set of all graphons defined by a condition of the form $t(g, W) = 0$ for a fixed quantum graph g, where $t(.,.)$ is the homomorphism density and a quantum graph is a formal linear combination of multigraphs. Using a method of representing graphs as polynomials, we construct a surjective homomorphism from the space of quantum graphs to a subring of the complex polynomial ring that is invariant under permutations of variables. When graphons are of finite rank, we demonstrate that the paralleled Algebraic Geometry "ideal" inverse of varieties is an ideal in our polynomial representation. Defining a kernel algebraic set using kernel varieties, we demonstrate that we can call such sets closed under the Zariski Topology. We determine several ties to Algebraic Geometry as a result of utilizing finite rank kernels and discover that a weaker version of Hilbert's Nullstellensatz applies to kernel zero-sets with respect to homomorphism density. Throughout, we examine the connection between Algebraic Geometry and Graphon Theory. (Received September 15, 2021)

1174-81-7879 **Jack T Rausch*** (jackrausch@creighton.edu), Creighton University. *Developing A Quantum Resource Theory for One-Way Information*

In quantum information theory, the one-way information of the joint evolution of a composite system quantifies the causal relationship between systems. Given two systems A and B, an algorithm is used to create a state $\rho^{A'ABB'}$ which quantifies the one-way information via the measure $R\left(\rho^{A'ABB'}\right) = I\left(\rho^{B} : \rho^{A'AB'}\right) - I\left(\rho^{B} : \rho^{B'}\right)$. A quantum resource theory offers a new perspective to view one-way information. A quantum resource theory examines a problem under a set of physically meaningful limitations which identify certain operations as free (can be used without limitations) and others as resources (operations with limitations or costs). We define a quantum resource theory for one-way information based on the measure $R\left(\rho^{A'ABB'}\right)$, showing that: R is an additive measure, all free states contain 0 one-way information, the free operations contain all unitary operators $U_{AB} = U_A \otimes U_B$, and R is monotonic under free operations, but not under the restricted operations. (Received September 16, 2021)

1174-81-8161 **Rachel Marie Kinard*** (navierstokes21@gmail.com), AFRL. *Algorithms for Skein Manipulation and Automation of Skein Computations in the Complement of All 2-bridge Knots*

Skein manipulations prove to be computationally intensive due to the exponential nature of skein relations. Resolving each crossing in a skein diagram produces 2 new diagrams; skein diagrams with over 5 crossings become increasingly difficult to work with. In this talk, I will introduce a method for automating these computations using algorithms developed to perform skein computations in knot complements. This method is developed for the complements of all 2-bridge knots, particularly twist knots and (2,2p+1)-torus knots, but can be extended to other families with modification. After showing these algorithms produce the desired result, I demonstrate their implementation in a Python program. This program is used to compute several known examples, demonstrating how results obtained through several months of work can now be obtained in less than 5 minutes. This program will be used for testing various hypotheses in $SU(2)$ Chern-Simons theory. (Received September 17, 2021)

1174-81-8284 **Nicholas Jared Forman*** (nforman@student.sjcny.edu), St. Josephs' College, NY, **Angela Sutton** (asutton@student.sjcny.edu), St. Josephs' College, NY, **Lotachukwu Isiofia** (lisiofia@student.sjcny.edu), St. Josephs' College, NY, and **Sambhav Shrestha** (sshrestha@student.sjcny.edu), St. Josephs' College, NY. *Visualizing the Mathematics of Quantum Computing Using Qiskit* Preliminary report.

This talk will explore the Deutsch algorithm which is one of the first algorithms in the world of quantum computers. We will observe the abstract mathematical foundations of this algorithm explicitly via IBM's Quiskit development package. Given a function from $\{0, 1\}$ to itself, it can be described as constant if we get an output of all 0's or all 1's from any input, and the function can be described as balanced if we get the same number of outputs for 0 and 1 given any input. In other words such a function is either bijective or constant. Classically, we would need at least two distinct queries about the function to determine if it is constant or balanced but with use of quantum gates we would only need one query to make this determination. This in turn makes the quantum algorithm more efficient than the classical. While any function from $\{0, 1\}$ to itself is relatively simple, the mathematics behind making queries about such a function using quantum gates is surprisingly not simple. In particular, we will show how both the tensor product of Hilbert spaces over complex numbers, and the partial trace of linear operators factor in one's understanding of algorithms in quantum computing. We will provide a better visualization of these mathematical foundations in action using the Quiskit package. (Received September 20, 2021)

1174-81-8883 **Jin Cheng Guu*** (jcguu95@gmail.com), Stony Brook University. *Categorical Center of Higher Genera and 4D Factorization Homology*

The discovery of Jones polynomials is a breakthrough in knot theory that connects mathematics and physics. They were fit into a 3D topological quantum field theory (TQFT) using Witten's insight in 1989. Later, it is known that such 3D TQFT fits into a 4D TQFT, the Crane-Yetter (CY) theory. In short, it is expected that CY vastly generalizes Jones polynomials, making CY a subject worth studying.

As an extended TQFT, CY assigns a category to each surface. To the cylinder, the assigned category is the celebrated Drinfeld categorical center, the idea of which led to the construction of a quantum group back in the 90s. What are attached to the general surfaces becomes the next natural question to ask. In this talk, we provide an answer which generalizes Drinfeld categorical center to higher genera (arXiv:2107.05914). This is a joint work with A. Kirillov and Ying Hong Tham. (Received September 20, 2021)

1174-81-9369 **Eric D'Hoker*** (dhoker@physics.ucla.edu), UCLA. *Modular Forms in Physics* Preliminary report.

Infinite discrete simple groups, such as the modular group SL(2,Z), play an increasingly important role in modern theoretical physics. They may arise either as symmetries of a given quantum field theory or string theory, or relate different theories to one another by duality. The role of modular invariance and modular forms in various physical problems will be reviewed. The physical motivation for the recent generalization to modular graph forms, which map certain graphs to modular forms, will be discussed and some of their mathematical properties will be summarized. (Received September 20, 2021)

1174-81-9484 **Elliot Samuel Kienzle*** (elliot.kienzle@gmail.com), University of Maryland, and **Steven Rayan** (rayan@math.usask.ca), University of Saskatchewan. *Hyperbolic Band Theory Through Higgs bundles* Preliminary report.

We study the band theory of quantum matter on a two-dimensional hyperbolic crystal using Higgs bundles. For a Hamiltonian with the symmetry of a hyperbolic lattice, an analog of Bloch's theorem sorts the eigenstates into irreducible representations of the lattice translation group ("crystal momenta"). By the Narasimhan-Seshadri theorem, each crystal momentum defines a stable holomorphic vector bundles on the quotient of the hyperbolic plane by the lattice. The Hamiltonian is then the twisted Dolbeault Laplacian plus potential on the vector bundle over the quotient Riemann surface. Higgs bundles play a couple of roles in this story. First, sending a Higgs bundle to its spectral data trades a high-rank crystal momentum with a rank one crystal momentum on a higher genus lattice. Then, the moduli of Higgs bundles packages the rank one crystal momenta of the whole family of lattices. Second, the Higgs field acts naturally as the complex part of crystal momentum, meaning the moduli of Higgs bundles parametrizes a family of non-self-adjoint Hamiltonians on the base curve. Both interpretations yield a band structure over the moduli of Higgs bundles. We demonstrate this framework for Euclidian crystals by describing their band theory using rank 2 Higgs bundles on $\mathbb{P}^1$ with 4 parabolic points. Finally, we speculate how Higgs bundles connect hyperbolic matter to other areas of mathematics and physics. (Received September 20, 2021)

1174-81-9514 **Muye Yang*** (willers@mit.edu), MIT, **Xinjie He** (xinjieh@andrew.cmu.edu), Carnegie Mellon University, **Drew Gao** (drewgao@stanford.edu), Stanford University, **James Woodcock** (jdubbs11@tamu.edu), Texas A&M University, and **Dmitri Maslov** (dmitri.maslov@ibm.com), IBM. *Improving Quantum Circuits of Toffoli Gates* Preliminary report.

Multiple control Toffoli gates (TOF^n) are a family of quantum gates which compute the bitwise product of $n-1$ control qubits and write the result onto a target qubit. These gates are used in various error correction schemes and function as an important primitive for decomposing arbitrary multiple qubit gates into physically implementable Clifford + T gates. However, despite their ubiquity, optimal decompositions of TOF^n are unknown in general. Our work focuses on optimizing Clifford + T implementations of TOF^n with respect to the number of controlled-not ($CNOT$) gates and T gates, as they are are by far the most difficult to implement gate of the Clifford + T set.

In this report, we show $2n-2$ and $\frac{3}{2}n-1$ lower bounds for the $CNOT$ cost of relative phase Toffoli gates ($RTOF^n$, a generalization of TOF^n), implemented with read only, and read-write memory respectively. These will be the first set of lower bounds known for the $CNOT$ cost of $RTOF^n$. We also provide a proof showing that the known implementation of $RTOF^4$ is $CNOT$-optimal. Finally, we give a systematic construction of TOF^n which improves on current best known implementations of TOF^n in terms of $CNOT$-cost, T-cost and Ancilla-count in a limited-space computation model. (Received September 20, 2021)

1174-81-9540 **Joseph Maciejko** (maciejko@ualberta.ca), University of Alberta. *Quantum matter and stable bundles*

The exciting and rapidly-growing field of topological materials has brought with it unexpected new connections between physics and pure mathematics. Algebraic topology in particular has played a significant role in classifying topological materials. In this talk, I will highlight another emerging chapter in this story in which algebraic geometry and number theory interact with geometric analysis to anticipate new forms of quantum matter associated with two-dimensional hyperbolic lattices. Key geometric objects appearing in this story are the moduli spaces of stable bundles on a Riemann surface, which provide countably-many distinct Brillouin zones for nonabelian Bloch states. (Received September 20, 2021)

1174-81-9769 **Shaikh Gohin Samad*** (shaikhgohin-samad@uiowa.edu), University of Toledo. *Reflection Positive Kernels arising in the study of Relativistic N-particle quantum system.*

Modern particle physics is built upon two fundamental pillars. They are:

1) Quantum mechanics.

2) The theory of special relativity.

The condition that enables us to incorporate these two theories in a Euclidean representation is known as Reflection Positivity.

In this presentation a Euclidean formulation of relativistic quantum mechanics for systems of a finite number of degrees of freedom will be discussed. Special relativity and quantum mechanics are most naturally combined using relativistic quantum field theory. Quantum field theory is ill defined, but can be used when perturbation theory is justified. Unfortunately the interactions in hadronic system are too strong for a perturbative treatment. Direct interaction approaches to relativistic quantum mechanics have two disadvantages. One is that cluster properties are difficult to realize for systems of more than two particles. The second is that the relation to quantum field theories is indirect. Euclidean formulations of relativistic quantum mechanics motivated by the Euclidean axioms of quantum field theory(Konrad Osterwalder and Robert Schrader 1974) provide an alternative representation that does not have these difficulties. In this talk Kernels for systems of N free particles of any spin are discussed. Reflection positivity are established for desired kernals. The structure of correlations that preserve both the Euclidean covariance and reflection positivity are discussed. (Received September 20, 2021)

1174-81-9902 **Anup Poudel*** (anup-poudel@uiowa.edu), University of Iowa. *Classifying the* SL_4 *state functions* Preliminary report.

We work with the notion of a “state function” introduced by Frohman. We start with a set of states for each marked disk colored with fundamental representations of SL_4. Then considering the vector space spanned by these states, one gets a projection onto it from the vector space spanned by all tangles. Our state function is topologically invariant and local. We discuss how one classifies all such state functions starting with some combinatorial data coming from the representation theory of quantum group associated to SL_4. In the process, we also explore the space of parameters that describe the SL_4 quantum invariants. Finally, we compare our results to some known results in the literature. (Received September 21, 2021)

1174-81-9944 **Lin Lin*** (linlin@math.berkeley.edu), University of California, Berkeley. *Quantum algorithms for unbounded operators*

The recently developed quantum singular value transformation (QSVT) [Gilyen, Su, Low, Wiebe, STOC 2019] provides a unified viewpoint of a large class of practically useful quantum algorithms. At the heart of QSVT is a perhaps new way of representing polynomials, called quantum signal processing (QSP), which encodes a polynomial using parameterized products of matrices in SU(2). For a given polynomial, finding the parameters (called "phase factors") is a challenging problem. I will introduce an optimization based fast algorithm, which is able to solve a large-scale QSP problem parameterized by more than 10,000 phase factors. I will then discuss recent progress in understanding the energy landscape of the optimization problem, which allows us to solve the open problem of finding phase factors using only standard double precision arithmetic operations. (Received September 21, 2021)

1174-81-10146 **Nathan Wiebe*** (nathanwiebe@gmail.com), University of Toronto.. *Quantum Algorithms for Simulation*

Quantum simulation has emerged as one of the most exciting fields in quantum algorithms within the last few years. It provides ways of translating the solution to the Schrodinger equation on an exponentially large Hilbert space into a polynomial number of quantum gates. In this presentation, I will review recent advances in quantum simulation including qubitization and interaction picture simulation methods can be used to solve such problems as well as solving more generic differential equations on quantum computers. Finally, I will discuss recent work that shows new methods for simulating constrained dynamics on quantum computers and show a new class of randomized simulation methods that can reduce the costs of simulation beneath the lowest costs that were previously known. (Received September 21, 2021)

1174-81-10243 **Peter Huston*** (huston.195@osu.edu), The Ohio State University, and **Fiona Burnell** (fburnell@umn.edu), University of Minnesota. *Particles with limited mobility from commuting condensates*

In recent work, Aasen, Bulmash, Prem, Slagle, and Williamson propose topological defect networks as a way to systematically describe phases exhibiting fracton order in (3+1)D. This motivates understanding the mathematical structure of such defect networks in more detail. Here, we take a first step in this direction, by systematically investigating a family of topological defect networks in (2+1)D. This family is constructed by condensing pairs of commuting étale algebras in a unitary modular tensor category periodically on the square lattice. The resulting (2+1)D topological defect networks are fracton-like: they host quasiparticle excitations with subdimensional mobility, as well as a subextensive ground state degeneracy. However, unlike in true fracton orders, this ground state degeneracy is not topologically stable. We give a detailed mathematical treatment of the theory of commuting condensates, including descriptions of the fully mobile particles and lineons. We also give families of Abelian and non-Abelian group-theoretical examples, together with lattice model realizations. (Received September 21, 2021)

1174-81-10704 **Sachin Jayesh Valera*** (sachin.valera@outlook.com), University of Bergen. *Fusion Structure from Exchange Symmetry in (2+1)-Dimensions*

Exchange symmetry is a cornerstone of quantum mechanics, first appearing when we study systems of identical particles. It indicates that effectively 2D quantum systems should play host to quasiparticles with exotic statistical behaviour called anyons.

Algebraically, anyonic systems are modelled using unitary ribbon fusion categories. Our goal will be to patch the gap between the fundamental principle of exchange symmetry and the categorical framework, using a minimal prescription of postulates.

We will show that the superselection sectors of anyonic systems form a fusion algebra, and emerge from a hierarchy of localised exchange symmetry mechanisms. Moreover, we prove that the superselection sectors of such systems are uniquely specified by the action of a particular braid, whose spectrum is related to the topological spins of the constituent particles. (Received September 21, 2021)

1174-81-11006 **Daniel Bulmash*** (dbulmash@umd.edu), University of Maryland College Park, and **Maissam Barkeshli** (maissam@umd.edu), University of Maryland College Park. *Anomaly cascade in (2+1)D fermionic topological phases*

We develop a theory of anomalies of fermionic topological phases of matter in (2+1)D with a general fermionic symmetry group G_f. In general, G_f can be a non-trivial central extension of the bosonic symmetry group G_b by fermion parity. We encounter four layers of obstructions to lifting a G_f symmetry action on a super-modular category $\mathcal{C}$ to a minimal modular extension $\breve{\mathcal{C}}$, which we dub the anomaly cascade: (i) An $\mathcal{H}^1(G_b, \mathbb{Z}_{\mathbf{T}})$

obstruction to extending autoequivalences of $\mathcal{C}$ to $\check{\mathcal{C}}$, (ii) An $\mathcal{H}^2(G_b, \ker r)$ obstruction to extending the G_b group structure of the symmetry action to $\check{\mathcal{C}}$, where r is a map that restricts autoequivalences of $\check{\mathcal{C}}$ to $\mathcal{C}$, (iii) An $\mathcal{H}^3(G_b, \mathbb{Z}_2)$ obstruction to extending the symmetry fractionalization class to $\check{\mathcal{C}}$, and (iv) the well-known $\mathcal{H}^4(G_b, \mathrm{U}(1))$ obstruction to developing a consistent G_b-crossed theory of symmetry defects for $\check{\mathcal{C}}$. A number of conjectures regarding symmetry actions on super-modular categories, guided by general expectations of anomalies in physics, are also presented. (Received September 21, 2021)

1174-81-11099 **Alberto Acevedo*** (`albertoacevedo@math.arizona.edu`), Brett Leroux, Micheal Curry, Shantanu H. Joshi, Nicholas Malaya.. *Vandermonde Wave Function Ansatz for Improved Variational Monte Carlo.*

Solutions to the Schrodinger equation (SE) can be used to predict the electronic structure of molecules and materials and therefore infer their physical and chemical properties. Variational Quantum Monte Carlo (VMC) is a technique that can be used to solve the weak form of the SE. Applying VMC to systems with N electrons involves evaluating the determinant of an N by N matrix. The evaluation of this determinant scales as $O(N^3)$ and is the main computational cost in the VMC process. In this work we investigate an alternative VMC technique based on the Vandermonde determinant (VT). VT is a product of pairwise differences and so evaluating it scales as $O(N^2)$. Therefore, our approach reduces the computational cost by a factor of N. We implemented VMC using the new low cost approach in PyTorch and compared its use in approximating the ground state energy of various quantum systems against existing techniques, starting with the one-dimensional particle in a box and moving on to more complicated atomic systems with multiple particles. We also implemented VT as a part of PauliNet, a deep-learning architecture for VMC. While the new method is computationally efficient and obtains a reasonable approximation for wavefunctions of atomic systems, it does not reach the accuracy of the Hartree-Fock method that relies on the Slater determinant. We observed that while the use of neural networks in VMC can result in highly accurate solutions, further new approaches are needed to best balance computational cost with accuracy. (Received September 21, 2021)

1174-81-11145 **Sonok Mahapatra*** (`sonokmahapatra@gmail.com`), Western Connecticut State University. *A post-quantum blockchain application in M-band Wavelet and Fresnel domain: A steganography based, decentralized, distributed ledger system* Preliminary report.

Following the Information Age, a new computational data structure known as blockchain has risen to prominence as an open, public, distributed ledger with a wide range of uses. With applications in secure sharing of medical data, voting mechanisms, cross-border payments, personal identity security, and most notably cryptocurrency exchange, blockchains seek to revolutionize how we handle our data fundamentally. On the other hand, the technological development of quantum computers has opened up several vulnerabilities to numerous blockchain applications. Improper methods of establishing privacy for blockchain can compromise large amounts of user data, making the development of high-level privacy-preserving mechanisms impervious to quantum computing of great importance. (Received September 21, 2021)

82 ► Statistical mechanics, structure of matter

1174-82-6840 **Ashesh Ghosh*** (`asheshg@stanford.edu`), Department of Chemical Engineering, **Andrew J. Spakowitz** (`ajspakow@stanford.edu`), Department of Chemical Engineering, Stanford University, and **Ariana Y. Tse** (`aytse@stanford.edu`), Department of Material Science and Engineering. *Active and Thermal Fluctuations in Biopolymer Dynamics*

One of the fundamental difficulties in biological physics is to establish a predictive time-dependent statistical mechanical theory of the out-of equilibrium processes that maintain and control virtually every biological event. We develop a theoretical statistical mechanical framework for predicting the combined influence of active and Brownian forces in biopolymer dynamics. In this talk, we will present our theoretical model, with a focus on the structural and dynamic behavior of polymers at various length and time scales. The active forces exhibit a temporal correlation in their statistical behavior, capturing the processivity associated with the characteristic time scale of biological fluctuations such as enzymatic activity. Based on a path-integral formalism, we demonstrate that the non-equilibrium fluctuations can be mapped onto an effective time-dependent temperature that depends on the active-force decorrelation rate. This theoretical picture suggests a hierarchy of behaviors, where local conformational fluctuations are unaffected by the presence of active forces whereas large length-scale conformational dynamics is significantly altered. These results suggest that active fluctuations have a varying impact on biological processes based on the time and length scale on which these events occur. Furthermore, the conceptualization of a time-dependent temperature provides a roadmap for the interpretation of *in vivo* measurements across the spectrum of time scales. (Received September 9, 2021)

1174-82-7089 **Tolson Hallauer Bell*** (tbell37@gatech.edu), Georgia Institute of Technology, Carnegie Mellon University, **Melita F Wiles** (mwiles22@wooster.edu), The College of Wooster, and **Clayton Mizgerd** (cmm12@williams.edu), Williams College. *Simulating the Random Cluster Model*

Modern understanding of the strong nuclear force comes from the field of quantum chromodynamics, which describes how quarks, gluons, and other subatomic particles interact. The random cluster model is a mathematical model that has been shown to describe important phase transitions in quantum chromodynamics and other physical systems through a probability distribution over graphs. To determine physics-relevant properties of the random cluster model, our team developed a Monte Carlo algorithm that simulates the model's probabilistic selection of graphs. In this presentation, we describe the relevance of the random cluster model to physics as well as the algorithm's implementation details, the results of its simulations, and other performance metrics. We also give theoretical explanations of these results and a discussion of the issues that the algorithm has with certain parameters. (Received September 12, 2021)

1174-82-9564 **Joseph Klobusicky*** (joseph.klobusicky@scranton.edu), The University of Scranton. *Markov models on foam with ruptures.*

We construct Markov processes for modeling the rupture of edges in a two-dimensional foam. We first describe a network model for tracking topological information of foam networks with a state space of combinatorial embeddings. Through a mean-field rule for randomly selecting neighboring cells of a rupturing edge, we consider a simplified version of the network model in the sequence space $l_1(\mathbb{N})$ which counts total numbers of cells with $n \geq 3$ sides (n-gons). Under a large cell limit, we show that number densities of n-gons in the mean field model are solutions of an infinite system of nonlinear kinetic equations. This system is comparable to the Smoluchowski coagulation equation for coalescing particles under a multiplicative collision kernel, suggesting gelation behavior. Numerical simulations reveal gelation in the mean-field model, and also comparable statistical behavior between the network and mean-field models. (Received September 20, 2021)

1174-82-10125 **Maximilian Pechmann*** (mpechmann@utk.edu), University of Tennessee, Knoxville. *The spectral gap of 1-D Schrödinger operators with Poisson random potentials*

We give an overview of recent results regarding the spectral gap of one-dimensional random Schrödinger operators $H = -\Delta + V$ where Δ is the Dirichlet Laplacian and V is a nonnegative Poisson random potential. We also discuss implications for the occurrence of Bose-Einstein condensation in random potentials. This talk is partly based on joint works with Joachim Kerner and Wolfgang Spitzer. (Received September 21, 2021)

83 ► Relativity and gravitational theory

1174-83-7079 **Patrick De Leenheer*** (deleenhp@math.oregonstate.edu), Oregon State University. *About a proof of Bertrand's Theorem* Preliminary report.

A cornerstone result in Newtonian mechanics is Bertrand's Theorem concerning the behavior of the solutions of the classical two-body problem. It states that among all possible gravitational laws there are only two exhibiting the property that all bounded orbits are closed. One of these is Newtonian gravitation, the other being Hookian gravitation. We present a proof of Bertrand's Theorem that is accessible to undergraduate students who are familiar with basic notions from advanced calculus and differential equations. (Received September 12, 2021)

1174-83-9781 **Douglas Sweetser*** (sweetser@alum.mit.edu), Quaternions.com. *Quaternion as space-time events and operators, a new symmetry for gravity, and analytic animation software* Preliminary report.

A light sketch of 3 deep areas of study will be provided.

Quaternions will be treated as events in space-time and as operators on those events. The origin in space-time is here-now, the present, where one is confined to be. Quaternions are famous for rotations in 3D space, $i * j = k$, but what does $i^2 = -1$ mean in space-time? A space-like separated source sends information to the observer at the origin one tick of a photon clock into the future, so here-future is a negative number. Unity does not change a thing, which would be consistent with a here-past operator.

The second subject is a new approach to gravity. If an event is squared, the Lorentz invariant interval sits next to three other terms,

$$(dt, dx, dy, dz)^2 = (dt^2 - (dx^2 + dy^2 + dz^2), 2dtdx, 2dtdy, 2dtdz)$$

If two observers at different heights in a gravitational field agree about the space-times-time terms, that symmetry could form a new approach to gravity that is distinct from general relativity. It would bend light.

Analytic geometry was Descartes' long-lasting contribution to math. It is time to modernize and develop analytic animations. The structure of the software should mimic physics, using base and tangent spaces to describe physics fields. There will be a demo of a few apps one can play with on a phone. (Received September 20, 2021)

1174-83-9970 **Arthur E Fischer*** (aef@ucsc.edu), Department of Mathematics, University of California, Santa Cruz. *The Friedmann-Schrödinger Equation and the Quantization of the Early Universe*

We combine the Friedmann equation with the Schrödinger equation to introduce the **Friedmann-Schrödinger equation**

$$\frac{d^2\psi(a)}{da^2} + \Big(\frac{\Omega_{m,0}}{a} + \frac{\Omega_{r,0}}{a^2} + \Omega_{\Lambda,0}\, a^2 + \Omega_{K,0} + \frac{2E}{E_P}\Big)\psi(a) = 0$$

where $\Omega_{x,0}$ are the cosmological density parameters at the current time and $E_P = \sqrt{\hbar c^5/G}$ is the Planck energy. Applying this equation to the **matter-radiation universe** ($\Lambda = 0$, $K = 0$), we find that except for the all-matter subcase, for **every** $E < 0$, the **global wave function of the universe** is the normalized wave function

$$\psi_E(a) = \frac{W_{\mu,\nu}(2\kappa a)}{\|W_{\mu,\nu}(2\kappa a)\|_{L^2}}$$

where $W_{\mu,\nu}$ is the Whittaker function that vanishes at infinity and $\kappa = \sqrt{\frac{-2E}{E_P}} > 0$. For the all-radiation universe, the Whittaker function reduces to

$$\psi_E(a) = \frac{a^{1/2}K_\nu(\kappa a)}{\|a^{1/2}K_\nu(\kappa a)\|_{L_2}}$$

where $K_\nu(\kappa a)$ is the modified Bessel function of the second kind of order ν. Consequently, in these cases, the energy spectrum of the Friedmann-Schrödinger equation is the continuum $(-\infty, 0)$ and the **energy levels of the matter-radiation universe are not quantizable**. Using these results and by examing $\psi_E(a)$ in the big-bang-near-zone $(0 < a \ll 1)$, we give a quantum mechanical conceptualization of the early universe. (Received September 21, 2021)

1174-83-12198 **Elena Giorgi*** (egiorgi@math.columbia.edu), Columbia University. *The stability of black holes with matter.*

Black holes are fundamental objects in our understanding of the universe. The mathematics behind them has surprising geometric properties, and their dynamics is governed by hyperbolic PDEs. A basic question one may ask is whether these solutions to the Einstein equation are stable under small perturbations, which is a typical requirement to be physically meaningful. We will see how the dispersion of gravitational waves plays a key role in the stability problem, illustrating the main conjectures and some recent theorems regarding the evolution of black holes and their interaction with matter fields. (Received November 15, 2021)

85 ► Astronomy and astrophysics

1174-85-6915 **Sean W Perry*** (Sperry9@fau.edu), Florida Atlantic University. *Random Polynomials in Gravitational Lensing*

The path of light through spacetime is bent by the matter therein, a phenomenon known as gravitational lensing. It is possible to observe multiple images of a single background source. The positions of lensed images occur at critical points of a time-delay function. This can be rephrased as the solution set of a system of rational equations. Here we will present some interesting problems from stochastic gravitational lensing, such as the expectation of the number of images, and image-counting simulations performed on models based on Weyl polynomials and polynomials uniformly-rooted in a disc. (Received September 10, 2021)

1174-85-8998 **Wendy Kaye Caldwell*** (wkcaldwell@lanl.gov), Los Alamos National Laboratory, **Abigail Hunter** (ahunter@lanl.gov), Los Alamos National Laboratory, **Catherine S Plesko** (plesko@lanl.gov), Los Alamos National Laboratory, and **Stephen Wirkus** (stephen.wirkus@asu.edu), Arizona State University, School of Mathematical and Natural Sciences. *Mathematical Modeling for Forthcoming NASA Missions*

Mathematical models are essential for many problems in planetary science because these problems often exceed experimental capabilities. Asteroids are of particular interest because they may provide information regarding the early formation of the solar system and, in some cases, may pose an eventual threat to life on Earth. Two forthcoming NASA missions will visit asteroids for two very different purposes.

Psyche: Journey to a Metal World, scheduled to launch in August 2022, will visit Asteroid 16 Psyche, the largest metallic Main Belt Asteroid. This mission is the first of its kind to explore a metallic asteroid, bodies often thought to be remnants of differentiated planetesimals that can provide insight into planetary formation. Estimates of Psyche's material composition, including porosity levels, vary considerably. Our models of impact structures on Psyche indicate which compositions and porosities are feasible and can rule out some compositions.

The Double Asteroid Redirection Test (DART) mission has a launch window from November 2021 to February 2022. The mission will visit asteroid Dimorphos, the smaller body of the binary Didymos system, and will test the planetary defense method of using a kinetic impactor to alter an asteroid's orbit. The DART spacecraft will make impact in October 2023. A key quantity in deflection by kinetic impactor is momentum enhancement. Our models of the DART impact, and of DART test problems, provide additional data points and predictions on the efficacy of this method. (Received September 20, 2021)

86 ► Geophysics

1174-86-9051 **Yash Agarwal*** (yash94404@gmail.com), Dougherty Valley High School, and **Sarah Y Greer** (sygreer@mit.edu), Massachusetts Institute of Technology. *Convolutional encoder decoder network for the removal of coherent seismic noise*

Seismologists often need to gather information about the subsurface structure of a location to determine if it is fit to be drilled for oil. In a seismic experiment, a wave propagates from a source location, interacts with the underlying discontinuities in the subsurface, and arrives back to the surface to be recorded by receivers. This data is used to produce an image of the subsurface, which aims to show geologic structure below the area of interest. However, there may be coherent electrical noise in these datasets which is most commonly removed by disregarding certain frequency bands of the data with the use of a notch filter. Instead, we look at using a convolutional encoder decoder network to remove such noise by training the network to take the noisy shot record produced by the receivers as input and to give the denoised or "clean" shot record as output. Our results reveal that the convolutional encoder decoder network structure works quite well, removing almost all the coherent noise while still retaining most of the characteristics of the shot record. This project was done as part of the MIT PRIMES-USA research program. (Received September 20, 2021)

90 ► Operations research, mathematical programming

1174-90-5943 **Drew Horton*** (drew.horton@ucdenver.edu), University of Colorado-Denver, **Emily Speakman** (emily.speakman@ucdenver.edu), University of Colorado-Denver, **Daphne Skipper** (skipper@usna.edu), United States Naval Academy, and **Tom Logan** (tom.logan@canterbury.ac.nz), University of Canterbury. *Hungry for Equality: Fighting Food Deserts with Optimization*

Food deserts are a form of food insecurity related to a lack of access to healthy, fresh, and affordable food. According to the United States Department of Agriculture (USDA), 13.7 million households in the U.S. experienced food insecurity in 2019. This problem has only been exacerbated by the ongoing COVID-19 pandemic, and disproportionately affects marginalized communities. In one traditional approach where we seek to minimize the expected distance of the population to grocery stores, the worst-off members in our communities tend to be ignored in the solution as outliers. To address these food insecurities, and the existing inequities, we demonstrate how the Kolm-Pollak equally-distributed equivalent function (EDE) can be minimized over a facility location integer program to minimize not only expected distance but also the inequality of the distribution. The EDE is a nonlinear function making the problem computationally significantly harder than the traditional model, therefore we discuss various ways to approach the optimization including a piecewise linear under-estimator of the model. We present results demonstrating how our model works on real-world data to produce an optimal

distribution of grocery store locations in New Orleans. In minimizing the inequality, we are ensuring that we are prioritizing relief in disproportionately affected communities. (Received August 31, 2021)

1174-90-7338 **David Papp** (dpapp@ncsu.edu), North Carolina State University, **Maria Macaulay** (mlmacaul@ncsu.edu), North Carolina State University, **Josiah Lim*** (josiah_lim@brown.edu), Brown University, **Nathan Rowan** (nate_rowan1@baylor.edu), Baylor University, **Sarah Halsey** (swhalsey@email.meredith.edu), Meredith College, and **Jessie Chen** (jchen49@students.kennesaw.edu), Kennesaw State University. *Optimizing Radiotherapy Treatments of Brain Metastases* Preliminary report.

Radiotherapy is a common form of cancer treatment for brain tumors, with the goal of irradiating tumors while minimizing damage to adjacent healthy tissue. Treatment plans are unique to each patient and are computed using large-scale optimization algorithms. Treatments are usually delivered over several consecutive days, allowing healthy cells to recover between sessions; this concept is known as fractionation. Today, treatment plans typically adhere to a uniform fractionation approach, delivering identical doses of radiation each day. We study an alternative treatment approach called non-uniform fractionation for patients with multiple metastatic lesions in their brain. For these patients, irradiating different subsets of lesions on each day may reduce harm to healthy brain tissue. We propose a new optimization model and computational approach to computing non-uniformly fractionated plans and quantifying the differences in healthy brain tissue damage between uniform and non-uniform fractionation. (Received September 16, 2021)

1174-90-8084 **Jesica Bauer*** (jesicabauer@outlook.com), Rensselaer Polytechnic Institute, **Kuran Abe** (krncln1013@gmail.com), University of Tsukuba, and **Teppei Tateda** (a.mail.address@icloud.com), Nagoya University. *Modeling Public Transportation Networks with Queues*

Planning and efficacy of new roads is becoming a significant issue due to rapid expansion and urban development. This has led to escalating traffic around city centers and necessitates the adoption of more efficient public transportation networks to help prevent congestion and negative environmental factors. Traditional transportation network problems capture traffic patterns and behaviors but fail to consider the impacts of these changes on the passenger population.

We propose a novel adaptation of a standard queuing model to analyze bus behavior and passenger interaction. This process allows us to suggest infrastructure changes that both benefit a bus company while avoiding negative impacts on current passenger loads. We apply our method to the existing bus system surrounding the University of Tsukuba in Japan and show that our method is able to accurately model the existing transportation network as well as measure the impact of proposed changes on passengers. (Received September 17, 2021)

1174-90-9978 **Dmitriy Bilyk** (dbilyk@math.umn.edu), University of Minnesota, **Josiah Park*** (j.park@tamu.edu), Texas A&M University, **Oleksandr Vlasiuk** (oleksandr.vlasiuk@gmail.com), Vanderbilt University, **Damir Ferizović** (damir.ferizovic@math.tugraz.at), Technische Universität Graz, and **Ryan Matzke** (matzk053@umn.edu), Technische Universität Graz. *Optimal energy for hard-spheres and equiangular lines* Preliminary report.

How does one spread lines (through the origin), or points on a sphere so as to minimize energy? We recently observed peculiarities in limiting problems of the above type, proving through linear programming methods that tight designs appear as discrete minimizers for "frame-like" continuous energies. These observations are an extension of a type of "universality" property of configurations now well studied. I will talk about recent developments in this area where we extend some of these universality properties to the setting where we require an energy minimizer to satisfy a degree of separation. The discussion will draw from work with D. Bilyk, D. Ferizović, A. Glazyrin, R. Matzke, and O. Vlasiuk. (Received September 21, 2021)

1174-90-10499 **Ali Mohammad Nezhad*** (alim@alum.lehigh.edu), Purdue University. *On the analytic reparametrization of the central path of semidefinite optimization*

It is well-known that the central path of semidefinite optimization, unlike linear optimization, may have no analytic extension to $\mu = 0$ in the absence of the strict complementarity condition. In this paper, we investigate the analyticity of the central path through the lens of real algebraic geometry. (Received September 21, 2021)

1174-90-10589 **Rob Thompson*** (rthompson@carleton.edu), Carleton College. *Optimization of level manufacturing*

Level manufacturing is a manufacturing paradigm that aims to decrease production waste by reducing variability in the manufacturing process. In this talk we introduce a mathematical model for a level manufacturing process based on a "loop" – a fixed sequence of quantities of products to be manufactured – and discuss how to make this loop as small as possible while still adhering to demand and cost constraints. This is a challenging optimization problem, and requires tools from linear programming and probabilistic optimization. The results of this research may be used to decrease costs and increase manufacturing output. (Received September 21, 2021)

1174-90-10889 **Yun Lu*** (lu@kutztown.edu), Kutztown University, and **Francis Vasko** (vasko@kutztown.edu), Kutztown University. *Methodology for Generating Bounded Solutions for Hard Combinatorial Optimization Problems* Preliminary report.

In this talk, we will present a simple methodology that iteratively solves COPs using CPLEX to generate solutions that are within a tight tolerance of the optimums, but require a reasonable amount of computer time. This robust procedure allows the user to specify two sequences of tolerance and maximum execution time. We will apply this methodology to solve hard combinatorial optimization problems including135 set K-covering problems (SKCP) commonly used by researchers to test the performance of metaheuristics for the SKCP. (Received September 21, 2021)

1174-90-10900 **Anna Hanlon*** (ah9313@desales.edu), DeSales University. *Research, Analysis & Topological Structures (R.A.T.S)*

The Research, Analysis, and Topological Analysis (R.A.T.S) project is focused on behavioral analysis of mice through machine learning and data analysis processes. The RATS pipeline is split into two main procedures, the first being the feature analysis. This section takes in the positional data of the mouse through a machine learning program and then identifies prominent features in the data through a process known as persistent homology. These features are not physical, like a face or tail, but more of an abstract concept that shows key behavioral features. These features are then narrowed down to the 12 most notable features (or about 30 percent of the sample size) through kernel density estimation (KDE). This feature data is then sent to the second phase of the pipeline, the machine learning phase. Here, we create and train a neural network by incorporating the given information about which mice are in pain and which are not. This produces a model tailored towards detecting mice in pain from their homological features alone, ultimately allowing new mice to be analyzed using this same model. (Received September 21, 2021)

91 ► *Game theory, economics, finance, and other social and behavioral sciences*

1174-91-5416 **Mason A Porter*** (mason@math.ucla.edu), University of California, Los Angeles, UCLA. *Bounded-Confidence Models of Opinion Dynamics on Networks*

From the spreading of diseases and memes to the evolution of opinions and social influence, dynamical processes are affected significantly by the networks on which they occur. In this talk, I will discuss bounded-confidence models of opinion dynamics on networks. In these models, the individuals in a network have continuous-valued opinions, which (if they are sufficiently close) they compromise by some amount when they interact with each other. I will also discuss some generalizations of bounded-confidence models, including ones that incorporate the affects of media, polyadic interactions (in which three or more individuals interact with each other at once), and opinions that coevolve with network structure. (Received August 19, 2021)

1174-91-5545 **Diana Cheng*** (dcheng@towson.edu), Towson University, and **Peter Coughlin** (pcoughlin@econ.umd.edu), University of Maryland College Park. *Competitive Ice Dancing and Power Values*

In adult competitive ice dancing, couples perform multiple dances within each judged event. This presentation illustrates how a weighted simple game can be formulated with a goal for a couple and the judges' evaluations of the couple's dance performances. We explain why couples and their coaches may consider a variety of goals. We also show how prominent power values can be used to measure the contributions of dance performances to achieving certain goals. We discuss power indices in two kinds of situations: 1) The weights vary, without varying the quota (as seen in literature on voting), 2) The quota varies, without varying the weights. As part of this analysis, we show visual representations of three-player situations that we developed to illustrate the Banzhaf and Shapley-Shubik index profiles for different thresholds. In addition, we show that the quota paradox is relevant in the context we are considering. (Received August 22, 2021)

1174-91-5689 **Subas Acharya*** (sa.subas@gmail.com), University of Texas at Dallas, **Alain Bensoussan** (axb046100@utdallas.edu), University of Texas at Dallas, **Dmitry Rachinskiy** (dmitry.rachinskiy@utdallas.edu), University of Texas at Dallas, and **Alejandro Rivera** (alejorivera1@gmail.com), University of Texas at Dallas. *Free Boundary Problem with Real Options*

We consider a Free Boundary Problem (FBP) associated with a real options problem. The real options problem is posed as a stochastic optimal control problem. The investment strategy (control), involves a one-time option to expand and a one-time option to terminate the project. The timing and amount of the investment are parameters to be optimized to maximize the profit. This stochastic optimization problem is reduced to a variational inequality (VI) and then to a FBP for a PDE. We use a specific substitution for the value function to reduce the PDE into a FBP for ODE. The focus of this work is on deriving sufficient conditions for solvability of the VI and providing a characterization of the solution (the value function), for a general form of the nonlinear investment cost. (Received August 24, 2021)

1174-91-5712 **Tuan MINH Pham*** (tuan.pham@meduniwien.ac.at), Complexity Science Hub Vienna, Medical University of Vienna, **Stefan Thurner** (stefan.thurner@meduniwien.ac.at), Medical University of Vienna, Complexity Science Hub Vienna, Santa Fe Institute, **Jan Korbel** (jan.korbel@meduniwien.ac.at), Medical University of Vienna, Complexity Science Hub Vienna, and **Rudolf Hanel** (rudalhan@gmail.com), Medical University of Vienna, Complexity Science Hub Vienna. *Empirical social triad statistics can be explained with dyadic homophylic interactions* Preliminary report.

The remarkable robustness of many human social systems has been associated with their peculiar triangular structure. Empirically, the so-called balanced state with either three or one positive link are strongly overrepresented. For almost a century, the mechanism that leads to this very specific ("balanced") statistics of triads remains elusive. Here we attempt to explain this phenomenon by a simple, however realistic, adaptive network model where every agent tends to minimize her individual social tension that arises from dyadic interactions. The novelty of the model arises from the fact that agents only need local information about their neighbors in the network, and do not need (often unrealistic) higher order information about the relations between these neighbors. The model exhibits a transition from unbalanced- to balanced society at a critical level of agents' rationality. As the relative strength of positive interactions to that of negative ones exceeds 1/2, a transition between the steady states with different fractions of balanced triads occurs. We also demonstrate the quality of the model on detailed temporal relation-data of a society of online game players. Not only can we successfully predict the distribution of triangle types but also obtain realistic group-size distributions. Finally, we discuss the possibility to extend the dyadic framework into the context of opinion formation and demonstrate the idea empirically for the "society" of voters in the United Nations General Assembly. (Received August 26, 2021)

1174-91-5993 **Graham Chambers-Wall*** (grahampchambers@gmail.com), William Jewell College, and **David McCune** (mccuned@william.jewell.edu), William Jewell College. *A Paradox of Guaranteed Representation*

The Arkansas Republican Party apportioned delegates in the 2016 presidential primary by first eliminating candidates (and their votes) who received less than 15% of the total vote, giving one delegate to each surviving candidate, and then distributing the remaining delegates to those candidates using Hamilton's apportionment method. Guaranteeing one delegate to each surviving candidate can lead to a paradox in which one of those candidates receives one less delegate. We use a geometric interpretation of apportionment to calculate the proportion of three-candidate elections that are susceptible to this paradox under the method of the Arkansas Republican Party. Additionally, we use simulation data to estimate the proportion of elections susceptible to this paradox for more than three candidates. (Received September 2, 2021)

1174-91-6264 **Joseph Davis Johnson*** (jdjmich@umich.edu), University of Michigan, **Daniel M Abrams** (dmabrams@northwestern.edu), Northwestern University, and **Adam Redlich** (adamredlich2020@u.northwestern.edu), Northwestern University. *A Mathematical Model for the Origin of Name Brands and Generics*

Firms in the U.S. spend over 200 billion dollars each year advertising their products to consumers, around one percent of the country's gross domestic product. It is of great interest to understand how that aggregate expenditure affects prices, market efficiency, and overall welfare. Here, we present a mathematical model for the dynamics of competition through advertising and find a surprising prediction: when advertising is relatively cheap compared to the maximum benefit advertising offers, rational firms split into two groups, one with significantly less advertising (a "generic" group) and one with significantly more advertising (a "name brand" group). Our

model predicts that this segmentation will also be reflected in price distributions; we use large consumer data sets to test this prediction and find good qualitative agreement. (Received September 7, 2021)

1174-91-6270 **Donald G. Saari*** (dsaari@uci.edu), Retired, Un of California Irvine. *Arrow's Theorem, Decision Theory, and the Traveling Salesperson*

Somewhat surprising is how the same general "paired comparison" structure explains and finds a resolution of Arrow's Theorem, identifies the power of the Borda Count along with the source of several issues in voting/decision theory, and, by identifying the inherent symmetry structure of the Traveling Salesperson Problem, reduces its degrees of freedom. (Received September 7, 2021)

1174-91-6354 **Michael A. Jones*** (maj@ams.org), Mathematical Reviews — AMS, and **Jennifer M. Wilson** (wilsonj@newschool.edu), Eugene Lang College, New School Univesrity, Eugene Lang College, The New School. *The Colley Method is an Extension of the Borda Count*

The Colley and Massey methods are two well-known methods to rate sports teams. We prove that when the Colley method is applied to election data, then it returns the same outcome as the Borda count, a well-known election procedure. Hence, the Colley method is an extension of the Borda count to partial orderings of candidates. We also show that the Colley method is not equivalent to the partial Borda count, another extension of the Borda count to partial orders. A similar relationship holds between the Massey method and range voting. Real data from Major League Baseball and sailing are used. (Received September 8, 2021)

1174-91-6362 **Adam Graham-Squire*** (agrahams@highpoint.edu), High Point University. *Conditions for Voting Anomalies in Ranked-Choice Voting*

Perhaps the most well-known result regarding conditions to create a voting fairness anomaly are for monotonicity anomalies in fully-ranked 3-candidate Instant-Runoff (IRV) elections. In that situation, there will be a monotonicity anomaly if the following necessary and sufficient conditions are present: the first-eliminated candidate (i) has greater than 25% of the first-place vote and (ii) beats the IRV winner in a head-to-head matchup. We generalize and expand that result by looking at situations where ballots are not fully-ranked and when there are 4 or more candidates. We also investigate necessary and sufficient conditions for No-show anomalies in IRV elections. (Received September 8, 2021)

1174-91-6836 **Ryka C. Chopra*** (rykachopra@gmail.com), William Hopkins Junior High School. *The Other Side of the Fence: Existing Obesity Rates & Predatory Location Choice of New Fast Food Chain Franchisees*

NIH reports that obesity is currently the second leading cause of death in the United States. Existing research suggests a positive correlation between obesity rates and fast-food access, but most studies focus on the demand side, consumers with easy access to fast-food branches substitute away from healthy options to fast foods resulting in obesity. In this project, we focus on the yet unexplored supply side. In particular, using detailed geographical data on fast food branch locations and historical obesity data, we ask if fast-food chains intentionally target regions with relatively high obesity rates and open branches to entice these vulnerable populations. Using Python geocoding and fixed effect regression techniques, we ask if current obesity rates in a region have any non-trivial impact on future locations of fast food branches, thus exhibiting a predatory behavior. We find that given a mean obesity rate of 31 percent in our sample, every one percent increase in obesity rate results in 3.8–4.3 additional branch openings in a region. The geographical dispersion is inverted U-shaped, with the highest concentration of new branches within a 2-mile radius of the city center. We also find more clustering in cities located in counties with existing high obesity rates compared to the sample mean, suggesting less competition for consumers in regions with existing obesity rates that exceed the sample mean. (Received September 9, 2021)

1174-91-7020 **Mehmet S Ismail*** (mehmet.s.ismail@gmail.com), King's College London, and **Steven J Brams** (steven.brams@nyu.edu), New York University. *Fairer Chess: A Reversal of Two Opening Moves in Chess Creates Balance Between White and Black*

Unlike tic-tac-toe or checkers, in which optimal play leads to a draw, it is not known whether optimal play in chess ends in a win for White, a win for Black, or a draw. But after White moves first in chess, if Black has a double move followed by a double move of White and then alternating play, play is more balanced because White does not always tie or lead in moves. Symbolically, Balanced Alternation gives the following move sequence: After White's (W) initial move, first Black (B) and then White each have two moves in a row (BBWW), followed by the alternating sequence, beginning with W, which altogether can be written as WB/BW/WB/WB/WB... (the slashes separate alternating pairs of moves). Except for reversal of the 3rd and 4th moves from WB to BW, this is the standard chess sequence. Because Balanced Alternation lies between the standard sequence, which favors White, and a comparable sequence that favors Black, it is highly likely to produce a draw with optimal play, rendering chess fairer. This conclusion is supported by a computer analysis of chess openings and how they would play out under Balanced Alternation. (Received September 11, 2021)

1174-91-7104 **Jessica Jiewei Shi*** (jjs2295@columbia.edu), Columbia University, **Abhishek Sivaram** (as5397@columbia.edu), Columbia University, and **Venkat Venkatasubramanian** (venkat@columbia.edu), Columbia University. *Statistical Teleodynamics Analysis of Emergent Equilibria in the Schelling Game* Preliminary report.

The behavior of a Schelling type agent-based model is studied. Here the space is divided into blocks and agents move between blocks to attempt to maximize their utility, and the agent utility function is dependent on block density and parameters. At equilibrium, all agents have equal utility and the system could exhibit segregation in which agents cluster in certain blocks. We programmed the agent-based simulation with NetLogo. We identify the parameter ranges for non-monotone utility functions which are necessary for the generation of segregation, and we show the asymptotic behavior of the system given initial conditions and verify it through simulation. The model can be used to explain the socioeconomic segregation occurring in many urban areas. (Received September 12, 2021)

1174-91-7190 **Steven J Brams*** (steven.brams@nyu.edu), Ben D. Mor, University of Haifa. *How Lies Induced Cooperation in "Golden Balls:" A Game-Theoretic Analysis* Preliminary report.

We analyze a well-known episode of a popular British TV game show, "Golden Balls," in which one of the two contestants lied about what he intended to do, which had the salutary effect of inducing both contestants to cooperate in what is normally a Prisoners' Dilemma (PD), wherein one or both contestants usually defected. This "solution" to PD assumes that the liar desired to be honorable in fulfilling his pledge to split the jackpot if he won but, surprisingly, he achieved this end without having to do so, astonishing the audience and receiving its acclaim. We note that this action has a biblical precedent in King Solomon's decision to cut a baby in two and, more generally, in resolving international conflicts, such as the 1962 Cuban missile crisis. (Received September 14, 2021)

1174-91-7516 **Longmei Shu*** (longmei.shu@dartmouth.edu), Dartmouth College. *Simple controls for evolutionary game dynamics*

Many population systems naturally have unstable mixed equilibriums where co-existence is impossible. We construct simple examples here to show that even if we just switch between two unstable systems, we can achieve a stable trapping of the system between the two unstable equilibriums. We give specific conditions on the initial condition and times for switching. One application of our results is to ensure desired mixed states in eco-evolutionary game dynamics where environmental feedback mediates the underlying population game dynamics. (Received September 20, 2021)

1174-91-7528 **Vivek Thatte*** (vthatt2@illinois.edu), University of Illinois at Urbana-Champaign. *Do Voting Paradoxes Occur in the Real World? The Case of AP College Football Polls* Preliminary report.

In the AP College Football Poll, each week during the college football season, around 65 sports writers and broadcasters submit a ranked list of their top 25 college football teams. The ballots are then aggregated using the Borda count method to produce an overall AP Top 25 ranking. In contrast to most other polls of this type, the individual ballots in the AP Poll are publicly available. In our research we compiled individual AP ballots from the last seven college football seasons and investigated the data for occurrence of well-known voting paradoxes, such as the Condorcet paradox (in which the plurality or Borda count winner is the Condorcet loser) and cyclical majorities (involving three teams A, B, C such that a majority of voters ranks A above B, a majority ranks B above C, and a majority ranks C above A). (Received September 14, 2021)

1174-91-7681 **D. Marc Kilgour*** (mkilgour@wlu.ca), Wilfrid Laurier University. *When to Stop Consulting* Preliminary report.

Suppose you must make a binary (Yes-No) decision, and you know that one choice is correct. Your utility will be 1 if you make the correct decision, and 0 otherwise. To gain information, you have the option of consulting independent experts. How many times should you consult? This problem is similar to the famous Secretary Problem, but different in that there are always two possible choices.

Our approach is to encode current information as a state, (h, k). When an expert recommends Yes {No}, the first {second} component of the state increases by 1. In state (h, k), the probability that Yes is correct is $\frac{h}{h+k}$. At state (h, k), you may choose to your best Yes-No guess, yielding utility $\frac{\max\{h,k\}}{h+k}$, or to consult again, leading to state $(h + 1, k)$ or $(h, k + 1)$. Usually, we assume (1, 1) as initial state.

Which decision is best, and when? We use several approaches to limit the number of consultations: impose either a maximum or a utility cost. We use both an updating model, where the probability that consultation at state (h, k) leads to state $(h+1, k)$ is $h/(h+k)$, and a non-updating model, where it is always $\frac{1}{2}$.

Paradoxically, we find that it is not always optimal to consult, even when it is free. But it is usually optimal near $h = k$. (Received September 15, 2021)

1174-91-7687 **Daewa Kim*** (kimd7@duq.edu), Duquesne University. *A kinetic theory approach to model crowd dynamics with disease contagion* Preliminary report.

We present some ideas on how to extend a kinetic type model for crowd dynamics to account for an infectious disease spreading. We focus on a medium size crowd occupying a confined environment where the disease is easily spread. The kinetic theory approach we choose uses tools of game theory to model the interactions of a person with the surrounding people and the environment and it features a parameter to represent the level of stress. It is known that people choose different walking strategies when subjected to fear or stressful situations. To demonstrate that our model for crowd dynamics could be used to reproduce realistic scenarios, we simulate passengers in one terminal of Hobby Airport in Houston. In order to model disease spreading in a walking crowd, we introduce a variable that denotes the level of exposure to people spreading the disease. In addition, we introduce a parameter that describes the contagion interaction strength and a kernel function that is a decreasing function of the distance between a person and a spreading individual. We test our contagion model on a problem involving a small crowd walking through a corridor. (Received September 15, 2021)

1174-91-8156 **Johanna Franklin** (johanna.n.franklin@hofstra.edu), Hofstra University, **Xiaoning Shi*** (xshi229@wisc.edu), University of Wisconsin - Madison, **Arnav Gangal** (arnavgangal@gmail.com), University of California, Los Angeles, **Luisa F. Estrada** (lf.estrada@uniandes.edu.co), University of the Andes, **Alexander G. Adams** (agadams@andrew.cmu.edu), Carnegie Mellon University, **Luke Philip Arceneaux** (luke.arceneaux@gmail.com), University of California, Irvine, **Ananyae Bhartari** (ananyae@iitk.ac.in), Indian Institute of Technology Kanpur, **Marcela Cruz-Larios** (marcelacruz@ciencias.unam.mx), Universidad Nacional Autónoma de México, **Qing Dong** (qdong@gatech.edu), Georgia Institute of Technology, **Kevin J. Martindale** (kevin.martindale99@gmail.com), Towson University, **Hope G. Neveux** (hope.neveux1@marist.edu), Marist College, **Yangjiayu Qiu** (yq12@rice.edu), Rice University, **Mingxuan S. Sun** (mingxuan.sun@pomona.edu), Pomona College, **Michael William Sunseri** (sunserm1@tcnj.edu), The College of New Jersey, **Harshit Tiwari** (harshitt@iitk.ac.in), Indian Institute of Technology Kanpur, **Himani Verma** (himaniverma@utexas.edu), The University of Texas at Austin, **Wenjun Wang** (wwang3024@gatech.edu), Georgia Institute of Technology, **Emerson Worrell** (e_worrell@coloradocollege.edu), Colorado College, **Jingyi Wu** (wu36j@mtholyoke.edu), Mount Holyoke College, **Zachary J. Hercher** (zjh5182@psu.edu), Pennsylvania State University, and **Heidi Benham** (heidi.benham@uconn.edu), University of Connecticut. *Analyses of Tapatan and Picaria*

We present analyses of two traditional abstract strategy games, Tapatan and Picaria, and some generalizations, primarily to polygonal boards. We determined formulas for the number of total board configurations and winning board configurations for these polygonal Tapatan and Picaria boards and identified a winning strategy for Player 1 for all polygonal Tapatan boards and all polygonal Picaria boards other than the square board, which had previously been shown to be undetermined. We then calculated the probability of each player winning these games under various combinations of memoryless strategies using a Markov chain analysis. We also constructed other variants of Tapatan and Picaria to obtain more balanced win rates and reduce the importance of the center node. These variants fell into three general categories: rule alterations, changes in the pieces, and changes in the board itself. (Received September 17, 2021)

1174-91-8223 **Zachariah Sinkala** (Zachariah.Sinkala@mtsu.edu), Middle Tennessee State University, **Vajira Asanka Manathunga*** (vmanathunga@mtsu.edu), Middle Tennessee State University, and **Bichaka Fayissa** (bichaka.fayissa@mtsu.edu), Middle Tennessee State University. *An Epidemic Compartment Model for Economic Policy Directions for Managing Future Pandemic* Preliminary report.

There is a renewed attention to the SIR (susceptible, infected, recovered) and other compartmentalized models in search of one that guides public policy to address the pandemic. In this research, we develop a framework to analyze the interaction between the economy and the disease dynamics. We assume there are two health related investments including general medical expenditures and the other for a direct investment for controlling the pandemic. When managing pandemic, learning would occur which contribute to the effective management of the pandemic in the future. Hence, we incorporate the learning dynamics associated with the management of the virus into our model. Given that the labor force in a society depends on the state of the epidemic, we allow birth, death, and vaccination to occur in our model and assume labor force consists of the susceptible, vaccinated, and recovered individuals. We also assume parameters in our epidemic compartmental model depend on investment amount earmarked for directly controlling the epidemic, the health stock of individual representative agents in the society, and the knowledge or learning about the epidemic in the community. By controlling consumption, the general medical expenditure, and the direct investment of funds for controlling the epidemic, we optimize the utility realized by the representative individuals because of consumption. This problem is nontrivial since the disease dynamics results in a non-convex optimization problem. (Received September 18, 2021)

1174-91-8603 **Fenghuan He*** (flindhe2005@gmail.com), MIT PRIMES. *A Topological Centrality Measure for Directed Networks*

Given a directed network G, we are interested in studying the qualitative features of G which govern how perturbations propagate across G. Various classical centrality measures have been already developed and proven useful to capture qualitative features and behavior for undirected networks. In this paper, we use topological data analysis (TDA) to adapt measures of centrality to capture both directedness and propagating effects. We introduce a new metric for computing centrality in directed networks, namely the *quasi-centrality* measure. We compute these metrics on the world trade production network to illustrate that our measure successfully captures propagating effects in the network and can also be used to identify sources of shocks that can disrupt directed networks. Moreover, we introduce a method that gives a hierarchical representation of the topological influences of nodes in a directed network using tools in TDA. (Received September 19, 2021)

1174-91-8745 **Chris Danforth*** (chris.danforth@uvm.edu), University of Vermont, **Thayer Alshaabi** (thayeralshaabi@gmail.com), UC Berkeley, **Jane Adams** (janelydiaadams@gmail.com), Northeastern University, **Michael Arnold** (michael.arnold@uvm.edu), University of Vermont, **Joshua Minot** (joshua.minot@uvm.edu), University of Vermont, **David Dewhurst** (david.dewhurst@uvm.edu), University of Vermont, **Andrew Reagan** (areagan@massmutual.com), MassMutual, and **Peter Sheridan Dodds** (Peter.Dodds@uvm.edu), University of Vermont. *Storywrangler: A massive exploratorium for sociolinguistic, cultural, socioeconomic, and political timelines using Twitter*

In real time, Twitter strongly imprints world events, popular culture, and the day-to-day, recording an ever-growing compendium of language change. Vitally, and absent from many standard corpora such as books and news archives, Twitter also encodes popularity and spreading through retweets. Here, we describe Storywrangler, an ongoing curation of over 100 billion tweets containing 1 trillion 1-grams from 2008 to 2021. For each day, we break tweets into 1-, 2-, and 3-grams across 100+ languages, generating frequencies for words, hashtags, handles, numerals, symbols, and emojis. We make the dataset available through an interactive time series viewer at http://storywrangling.org and as downloadable time series and daily distributions. Although Storywrangler leverages Twitter data, our method of tracking dynamic changes in n-grams can be extended to any temporally evolving corpus. Illustrating the instrument's potential, we present example use cases including social amplification, the sociotechnical dynamics of famous individuals, box office success, and social unrest. (Received September 19, 2021)

1174-91-9048 **Matthew I. Hudes*** (mhudes01@tufts.edu), Department of Mathematics, Tufts University, and **Bruce M. Boghosian** (bruce.boghosian@tufts.edu), Department of Mathematics, Tufts University. *Monte Carlo Simulations with Prospect Theory* Preliminary report.

Wealth inequality is a complex worldwide problem, the dynamics of which must first be understood before it can be mitigated. Prospect Theory is a known model of human decision making, which can be applied to studying the economics of wealth distribution. To do this, we made Monte Carlo simulations of an economy in which agents were randomly selected to transact based on the Prospect Theory framework. We observed the long-time dynamics of such economies by tracking the wealth of each ranked agent and elucidating its functional form. These simulations were computationally intensive, and we ran them in parallel on the Tufts High Performance Cluster (HPC). We will present results on the comparison of economies based on Prospect Theory with empirical wealth distribution data. (Received September 20, 2021)

1174-91-9096 **Emma R Zajdela*** (emmazajdela@u.northwestern.edu), Northwestern University, **Richard J Wiener** (rwiener@rescorp.org), Research Corporation for Science Advancement, and **Andrew Feig** (afeig@rescorp.org), Research Corporation for Science Advancement. *Catalyzing Collaborations: A Model for the Dynamics of Team Formation at Conferences*

The COVID-19 pandemic has brought to the fore the importance of scientific collaboration to address challenges of global significance. One of the main ways that new and innovative collaborations are catalyzed is by gathering scientists together at conferences. In the US alone, conferences amount to billions of dollars per year in terms of travel expenses, organizing costs, and loss of research time. In this talk, I present a dynamical model for predicting the formation of scientific collaborations at conferences, inspired by the process of catalysis. Specifically, the model tracks the probability that two participants at a conference will form a collaboration given their previous knowledge of each other and level of interaction throughout it. Model predictions are tested using data from four multi-year series of interactive conferences known as the Scialog Conferences, organized by the Research Corporation for Science Advancement over the period 2015-2020. We find that scientists who collaborated interacted significantly more than those who did not. Comparison between actual conferences and 2,500 alternate scenarios indicates that interaction plays a causal role on team assembly. Furthermore, this talk will provide insight into questions about virtual conferences based on data from 7 virtual Scialog Conferences which took place in 2020 and 2021. Our findings may have an impact on stakeholders who wish to optimize future conferences to promote new collaborations. (Received September 21, 2021)

1174-91-9122 **Christina Catlett*** (ccatlett@g.hmc.edu), Scripps College, and **Heather Zinn Brooks** (hzinnbrooks@hmc.edu), Harvey Mudd College. *Modeling media influence on expressed and private opinion discrepancies in online social networks* Preliminary report.

"Pluralistic ignorance" describes the development of discrepancies between expressed and privately-held opinions in a population. With the prevalence of information sharing on online social platforms, conversations about the radicalization of online opinion space due (in part) to pluralistic ignorance have become increasingly relevant. Both synchronous and asynchronous bounded-confidence models of opinion dynamics have been proposed to model the phenomenon on single-layer networks. However, the impact of media zealots on the emergence of opinion discrepancies and the mechanics by which users share content remain largely unexplored. To capture these phenomena, we propose a multilayer network model in which expressed opinions are broadcast synchronously according to a Hegselmann-Krause model in the presence of media, while private opinions are passed simultaneously among non-media users through asynchronous, user-to-user content-sharing according to a Deffuant-Weisbuch model. Through numerical simulation and sensitivity analyses, we interrogate the role of media in the divergence between expressed and private network layers, identifying the most influential parameters and structural elements in the emergence of pluralistic ignorance. (Received September 20, 2021)

1174-91-9151 **Ben G Fitzpatrick*** (bfitzpatrick@lmu.edu), Loyola Marymount University. *Modeling Uncertainty in Ecological Federalism*

Human-natural scale mismatches are common in ecology, and addressing them is important for sustainably managing social-ecological systems (SES). For example, wildlife, fish, invasive species and diseases are unaware of political borders that define their management. Under a federalist system of government, society can address human-natural scale mismatches by allocating management authority to federal or sub-national (e.g., state, province, territory) levels of government. Federalism is a system of government characterized by semiautonomous states in a regime with a common central government where governing authority is allocated between levels of government. Federal management is often characterized by one-size-fits-all policies that fail to account for local differences in habitats and economies. Conversely, state management is often criticized for placing undue weight on local economic growth over environmental protection. What aspects of SES management should be managed by a centralized federal government and which should be allocated to decentralized state governments?

In this talk, we consider a two-state plus federal dynamic invasive species problem, in which two eradication strategies are possible. All parties have uncertainty about their effectiveness, leading to some unusual stochastic control challenges. We give simulation and optimization results for some example scenarios. (Received September 20, 2021)

1174-91-9154 **Daniel I Rubenstein** (dir@princeton.edu), Princeton University, and **Pawel Romanczuk** (pawel.romanczuk@hu-berlin.de), Humboldt University of Berlin. *Social Dilemmas of Sociality due to Beneficial and Costly Contagion*

Levels of sociality in nature vary widely. Some species are solitary; others living in family groups; some form complex multi-family societies. Increased levels of social interaction can allow for the spread of useful innovations and beneficial social information, but can also facilitate the spread of harmful infectious disease. Given the selection pressures that infectious disease impose on a population, it is natural to explore the ways in which contagion processes may shape the evolution of complex social systems. We consider a model for the evolution of sociality strategies in the presence of both a beneficial and costly contagion. We study these dynamics at three timescales: using a susceptible-infectious-susceptible (SIS) model to describe contagion spread for given sociality strategies, a replicator equation approach to study the changing fractions of resident and mutant strategies, and an adaptive dynamics approach to study the long-time evolution of the resident level of sociality. For various assumptions on the benefits and costs of infection, we identify a social dilemma in which the evolutionarily-stable sociality strategy disagrees with the collective optimum. In particular, the ESS level of social interaction is greater (respectively less) that the social optimum when the good contagion spreads more (respectively less) readily than the bad contagion. (Received September 20, 2021)

1174-91-9162 **Todd Lee** (`tlee@elon.edu`), Elon University. *Measuring Gerrymandering: Investigating the distributions of party ratios for districting techniques.* Preliminary report.

The process of creating districts to be used in political elections in North Carolina has been a cause of concern for many years. This process is completed by hand by the legislature and recently has frequently resulted in maps that are later declared unconstitutional. These maps have been gerrymandered or designed specifically to benefit one political party over another, disenfranchising voters across the state. This project aims to investigate and compare multiple mathematical methods for computationally creating districts. These mathematical methods include the use of applied linear algebra, Euclidean geometry, and Voronoi diagrams. Each method is used to create a large number of theoretical districts which will be numerically represented by the ratio of Democrats to Republicans that would be elected from the set of districts based on previous election data. Using the distributions of the party ratio, we can compare the actual congressional districts to the computational methods and find indicators of unfair districting methods. (Received September 20, 2021)

1174-91-9339 **Micah Martin*** (`micah.martin@gordon.edu`), Gordon College. *Cyclic Voting: Some Theory and Applications* Preliminary report.

Cyclic orders can be thought of as people seated around a table, events occurring each year in a rotation, or any other such order where the position of a person, event, or object relative to the others in the cycle is more important than a starting or ending point. These orders can be thought of as permutations on the objects where the permutation $(abcd)$ is equivalent to $(bcda)$, $(cdab)$, and $(dabc)$.

This poster will cover research done on the structure of voting on such an order. Perhaps you are about to play a game of poker, but you care about who sits on your right due to the way that betting proceeds. How can we ensure that everyone who is playing has their preferences reflected in a fair voting system? Maybe you are coordinating a football team's schedule where you play the same team every four years. When would it be best for you to play them relative to playing other teams? Or more importantly, how can we reflect the preferences of everyone involved fairly?

Using representation theory, a combination of group theory and linear algebra, research into the structure of various systems for voting on cyclic orders was conducted and will be presented in this poster. Additionally, applications of cyclic orders and cyclic voting such as already mentioned will be discussed. (Received September 21, 2021)

1174-91-9733 **Nicholas Grabill*** (`grabilln@umich.edu`), University of Michigan, and **Ling Hu** (`linghu@andrew.cmu.edu`), Carnegie Mellon University. *Some Limiting Problems in Utility Maximization*

We consider limiting cases of utility maximization in binomial stock pricing models. We use 1-period models with n independent and identically distributed stocks and also n-period models with one stock. In both situations we're interested in letting $n \to \infty$. In all cases there is a bank account with risk-free interest rate r and we let p and q stand for the probability of an up and down movement in stock, respectively. We call $\tilde{p}$ and $\tilde{q}$ the risk-neutral probabilities.

Here are some of our results:

Theorem. Recall the Arrow-Pratt risk aversion coefficient $\alpha(x) := -\frac{U''(x)}{U'(x)}$. Assuming $\alpha''(x) \geq 0$ for all x in the domain of $U(x)$ and that $1 + r - d = u - 1 - r$, we found that if for each n the optimal portfolio has all its capital in stock, then $\frac{p}{q} \to 1$ as $n \to \infty$.

Conjecture. Keeping u, d, r fixed we found that if for each n the optimal portfolio has all of its capital in stock, then $\frac{p}{q} \to \frac{\tilde{p}}{\tilde{q}}$ as $n \to \infty$.

Theorem. Letting $p = q = 1/2, U(x) = ln(x)$, and x^* stand for the initial capital in stock for the optimal portfolio, we found that if $ud = (1+r)^2$, then $x^* = \Gamma/2$ and that if $ud < (1+r)^2$, then $x^* < \Gamma/2$, and that if $ud > (1+r)^2$, then $x^* > \Gamma/2$.

These results allow us to understand optimal investment strategy in the context of the up and down movements in the stock price as well as under what conditions it's optimal to invest everything (or nothing) in stock. (Received September 20, 2021)

1174-91-9855 **Brian Hopkins*** (bhopkins@saintpeters.edu), Saint Peter's University. *Organizing the 2 by 2 Games: The Goforth-Robinson and Jessie-Saari Systems* Preliminary report.

Two by two matrix games are a foundational concept in game theory, encompassing important examples such as Prisoners' Dilemma and Chicken. Considering such games with distinct ordinal ranks gives a collection that can be enumerated and interconnected. We compare two systems for organizing this important set. About 20 years ago, Goforth and Robinson devised a system based on neighborly transpositions from the study of permutations, e.g., swapping the worst and second worst of the four outcomes for one player produces an adjacent game. Recently, Jessie and Saari introduced a coordinate system on games which involves writing each one as a sum of what they call Nash and behavioral components. We will compare and contrast these two systems, and perhaps even reconcile them. (Received September 20, 2021)

1174-91-10123 **Ben Blum-Smith*** (blumsmib@newschool.edu), New York University, The New School, **Soledad Villar** (svillar3@jhu.edu), Johns Hopkins University, and **Irfan Jamil** (ijamil1@jhu.edu), Johns Hopkins University. *Fair-division approaches to redistricting* Preliminary report.

Prominent efforts to fight partisan gerrymandering in the US have sought the help of a (hopefully) neutral arbiter: they have aimed either to elicit intervention from the courts, or to delegate responsibility for redistricting to a formally nonpartisan body such as an independent commission. In this talk, we discuss mechanisms to allow partisan actors to produce a fair map without the involvement of such a neutral arbiter. Inspired by the field of game theory, and more specifically the study of fair-division procedures, the idea is to use the structured interplay of the parties' competing interests to produce a fair map. We survey the various mechanisms that have been proposed in this fairly young line of research, propose three new mechanisms with novel potentially-desirable features, and analyze them numerically.

This is joint work with Steven Brams, Irfan Jamil, and Soledad Villar. (Received September 21, 2021)

1174-91-10185 **Josephine Noonan** (josephine.noonan@gordon.edu), Gordon College, **Abraham E Holleran** (abraham.holleran@gordon.edu), Gordon College, and **Karl-Dieter Crisman*** (karl.crisman@gordon.edu), Gordon College. *Voting on Cyclic Orders, Group Theory, and Ballots* Preliminary report.

A *cyclic order* may be thought of informally as a way to seat people around a table, perhaps for a game of chance or for dinner. Given a set of agents such as $\{A, B, C\}$, we can formalize this by defining a cyclic order as a permutation on this finite set, under the equivalence relation where ABC is identified with both BCA and CAB.

Just as with other choices involving sets of people with some structure, such as tenure committees or officers in a club, we might want to aggregate preferences of a (possibly different) set of voters on the set of possible ways to make this choice. However, given the large number of possibilities as the cardinality of the set of agents increases, we may not wish to use the usual machinery of full ranked linear orders. This raises the question of what sort of ballots may be appropriate; a single cyclic order, a set of them, or some other ballot type?

Similarly, there is a natural action of the group of permutations on the set of agents. A reasonable requirement for a choice procedure would be to respect this symmetry (the equivalent of neutrality in normal voting theory), or even take advantage of it. This talk will discuss joint work with several students, using representation theory of the symmetric group, to analyze natural types of possible ballots for voting on cyclic orders, and what that tells us about all possible points-based voting methods for them. (Received September 21, 2021)

1174-91-10259 **Marion Campisi*** (marion.campisi@sjsu.edu), San Jose State University, **Ellen Veomett** (erv2@stmarys-ca.edu), Saint Mary's College of California, **Thomas Ratliff** (ratliff_thomas@wheatoncollege.edu), Wheaton College, and **Stephanie Somersille** (ssomersille@gmail.com), Somersille Math Education Services. *The Geography and Election Outcome (GEO) Metric: an Introduction*

We introduce the Geography and Election Outcome (GEO) metric, a new method for identifying potential partisan gerrymanders. In contrast with currently popular methods, the GEO metric uses both geographic information about a districting plan as well as election outcome data, rather than just one or the other. We motivate and define the GEO metric, which gives a count (a non-negative integer) to each political party. The count indicates the number of additional districts in which that party potentially could have been competitive, without losing any currently won districts, by making reasonable changes to the input map. We then analyze GEO metric scores for each party in several recent elections. (Received September 21, 2021)

1174-91-10283 **Solomon Vito Valore-Caplan*** (`svalorecaplan@gmail.com`), Harvey Mudd College. *Smoothed Approximations of Bounded-Confidence Models of Opinion Dynamics* Preliminary report.

One simple model of opinion dynamics is the repeated averaging model, where each agent updates toward the average of its neighbors. However, this model does not account for the fact that agents of dissimilar opinions are less likely to influence each other. In bounded-confidence models, this feature is integrated by introducing a threshold beyond which agents do not interact. We propose a new model that can be tuned via a parameter γ to interpolate between these two classical models to capture a wider variety of modeling conditions. Using a combination of analytical and computational methods, we investigate the stationary states and bifurcation points of our model at various points in our parameter space. We highlight similarities and differences between our smoothed model and the classical models at $\gamma = 0$ and as $\gamma \to \infty$, and we discuss new behavior that arises at various critical points for intermediate values of γ. (Received September 21, 2021)

1174-91-10349 **Phousawanh Peaungvongpakdy*** (`ppeaungvongpakdy@hmc.edu`), Harvey Mudd College. *In* Preliminary report.

Information spread and studying how opinions change over time within a population is an important topic to study due to the increase of polarized opinions and ideologies in the U.S. political landscape. We give an extension to the Hegselmann-Krause (HK) opinion dynamic model by adding an adaptive component to the model. We model a population through a graph $G(N, E)$, where the node set N represents the people in the population and the edge set E represents the connections between people. For each node i we assign a value $x_i \in [0, 1]$ that represents the ideology of that person. The original HK model utilizes a bounded confidence opinion update with a confidence parameter C. We update a node's opinion x_i with neighboring nodes x_j through an averaged sum if $|x_i - x_j| \leq C$. The adaptive component we introduce is modeled after the social phenomena of homophily, the tendency for people to interact with like minded people. We represent this idea by introducing a homophily parameter β and have people dissolve connections with their neighbors if their opinion is too different from their neighbor. The node then rewires to another node k with probability $\mathbb{P}(i \to k) \propto 1 - d(x_i, x_j)$ where $d(x_i, x_j)$ represents distance between the two opinions. As our model converges, we observe clusters—or disjoint complete graphs—appearing in our graphs. We explore how the number of clusters change with respect to different values of C and β through the graph Laplacian. (Received September 21, 2021)

1174-91-10378 **Bikash Das** (`bikash.das@ung.edu`), University of North Georgia, and **Valeria Carazas*** (`vacara3398@ung.edu`), University of North Georgia. *Discrete Analytic Study of the Traffic Light Problem* Preliminary report.

It is clear as technology and humanity continues to advance, the role of traffic also grows. As such, we continue to push our understanding of traffic. We derived an algorithm to predict an individual's most likely path of travel from point A to point B by solving a system of differential equations. Data were collected randomly on a local neighborhood of UNG's Gainesville Campus about the different attributes of paths in contrast to the varying driving conditions. Finally, we tested and compared the analytic model with a proposed ideal abstract graph theoretic model. (Received September 21, 2021)

1174-91-10692 **Corina Tarnita** (`ctarnita@princeton.edu`), Princeton University, **Olivia Jessica Chu*** (`olivia@math.dartmouth.edu`), Dartmouth College, and **Vítor V Vasconcelos** (`v.v.vasconcelos@uva.nl`), University of Amsterdam. *The role of loners in the evolution of cooperation in group-structured populations*

The evolution of cooperation has been studied in many systems, from bacterial communities to human populations. It is well known that population structure is crucial to a system's dynamics. In human populations, groups are fundamental – our group memberships strongly influence who we meet and interact with. From a theoretical perspective, there are network-based models to study human dynamics, but they generally do not allow for multiple group affiliations or incorporate barriers to group entry. In this work, we present a framework, based on evolutionary set theory, in which individuals interact with those who share their groups through an evolutionary game. The system updates stochastically, with strategy and group memberships subject to evolutionary updating. We impose realistic rules, including barriers to group entry and flexibility in the number of group memberships. With these rules, cooperation emerges, but it is most favored when we allow for the existence of "loners": a changing subset of individuals who spend a temporary "time-out" period not interacting with others. This work provides an analytical framework in which behavior in realistic population structures can be studied and adds to a growing body of literature that recognizes loners as vital parts of systems. (Received September 21, 2021)

1174-91-10720 **Michael Wise*** (Michael.Wise1@marist.edu), Marist College, **Ashley Mullan** (ashley.mullan@scranton.edu), University of Scranton, **Dylan Jamner** (dylanjamner@g.ucla.edu), UCLA, **Austin Biondi** (austin.biondi@yahoo.com), Gonzaga University, and **Sooie-Hoe Loke** (SooieHoe.Loke@cwu.edu), Central Washington University. *Deal or No Deal: Modeling a Game Show with Utility Theory and Machine Learning* Preliminary report.

Deal or No Deal, a game show that features nine rounds of high stakes player decisions, aired in the United States beginning in 2005. Utility theory helps to explain the rationale of the players at each stage of the game, as well as derive an optimal strategy for maximizing one's winnings. Using empirical data compiled by Post et al. (2008) and a simulation of 1000 players, one can develop models to predict what amount of money the banker will offer at the end of each round and in what conditions a player will take this offer, the deal. While other supervised learning algorithms were explored, regression and neural networks yielded accuracies of at least 80% at each stage of testing. A deep neural network yielding an R^2 value of 0.95 and comparatively low mean error was chosen as the best for predicting the banker offer. To further explore other avenues of intelligent gameplay, reinforcement learning techniques were used to implement a deep Q-network to play the game and be compared to simulated contestants. A path-dependent utility function was implemented to describe empirical and simulated player decisions, and players chose the option with higher utility over 85% of the time. Additional improvements were also implemented; such as nesting the utility function in models of stochastic choice, like the tremble model, to account for errors in player judgement. This report will summarize the results of the CC-REU NSF summer REU experience (DMS-2050692) where these questions were explored. (Received September 21, 2021)

1174-91-10919 **Nicholas O'Kelley** (nokelley1@catamount.wcu.edu), Western Carolina University, **Andrew Shelton*** (ashelt822@gmail.com), Western Carolina University, **Daniel Hammer** (dlhammer2@catamount.wcu.edu), Western Carolina University, and **Andrew Penland** (adpenland@email.wcu.edu), Western Carolina University. *Evolutionary Algorithms applied to Two Player Games in Search for Optimal Strategies*

Applications of evolutionary game theory concepts can be found in many fields. Emulating biological processes to simulate generational behavior on populations yields potentially informative results. We investigate applications of evolutionary game theory on two-player games attempting to identify game-independent optimal strategies. Our primary game of focus is the Graph coloring game and defining optimal strategies on various graphs. Analysis of the data from our simulations has led us to identify several optimized strategies for our games. We will present progress towards proofs that the strategies found are optimal. (Received September 21, 2021)

1174-91-11075 **Stanley R. Huddy** (srh@fdu.edu), Fairleigh Dickinson University. *Berge Decomposition of Bimatrix Games*

Motivated by Jessie and Saari whose decomposition breaks a game into Nash, behavioral, and kernel components, we offer the Berge decomposition for bimatrix games. Our decomposition breaks a game into Berge, Berge behavioral, and kernel components. We explain the relationship between the two decompositions. Further, we apply the decomposition to determine conditions for when symmetric two-player zero-sum games have pure or mixed Berge equilibria. (Received September 21, 2021)

1174-91-11227 **Wesley H. Holliday*** (wesholliday@berkeley.edu), University of California, Berkeley. *An approach to generalizing some impossibility theorems in social choice*

In social choice theory, voting methods can be classified by invariance properties: a voting method is said to be C1 if it selects the same winners for any two profiles of voter preferences that produce the same majority graph on the set of candidates; a voting method is said to be pairwise if it selects the same winners for any two preference profiles that produce the same weighted majority graph on the set of candidates; and other intermediate classifications are possible. As there are far fewer majority graphs or weighted majority graphs than there are preference profiles (for a bounded number of candidates and voters), computer-aided techniques such as satisfiability solving become practical for proving results about C1 and pairwise methods. In this talk, based on joint work with Eric Pacuit and Saam Zahedian, I will discuss an approach to generalizing impossibility theorems proved for C1 or pairwise voting methods to impossibility theorems covering all voting methods. We apply this approach to impossibility theorems involving "variable candidate" axioms, which concern what happens when a candidate is added or removed from an election. A key tool is a construction of preference profiles from majority graphs and weighted majority graphs that differs from the classic constructions of McGarvey and Debord, especially in better commutative behavior with respect to other operations on profiles. (Received September 21, 2021)

1174-91-11243 **Sylvia Bhattacharya*** (sbhatta6@kennesaw.edu), Kennesaw State University. *Multimodal Inference of Human State to Track Cognitive Processes in Risky Environments* Preliminary report.

In both military and civilian domains, there is a clear intent to replace human drivers with advanced automated systems. This is, of course, precipitating a shift towards humans experiencing vehicles from the primary perspective of being a passenger. While decades of research have provided vast amounts of data regarding vehicle drivers, very little is known about how people experience being a passenger in a vehicle that they do not control as well as how their changing states are manifest in both anticipatory and reactive responses at all levels from physiological to behavioral. Whether driven by another human or an automation, passengers have regular opportunities to make decisions regarding ongoing driving performance, and in many cases these decisions lead to both beneficial and harmful attempts at interventions (e.g. a verbal warning to a human driver or a take over from automated cruise control). Such passenger behaviors may critically impact vehicle behavior and, more importantly, when and how they occur is likely to be subject to a variety of factors such as individual decision-making style, driving preferences, level of fatigue, risk tolerance, and personality. In particular, human decisions regarding cooperative behaviors such as reliance and compliance, are believed to be significantly impacted by the amount of trust held in the paired agent, whether human or automated. (Received September 22, 2021)

92 ► Biology and other natural sciences

1174-92-5219 **Eitan Tadmor*** (tadmor@umd.edu), University of Maryland. *Emergent Behavior in Collective Dynamics*

A fascinating aspect of collective dynamics is self-organization, where small scale interactions lead to the emergence of high-order structures with larger-scale patterns. It is a characteristic feature in collective dynamics of "social particles" which actively probe the environment and aggregate into various forms of clusters. In different contexts these take the form of flocks, swarms, consensus, synchronized states etc. In this talk I will survey recent mathematical developments in collective dynamics, starting with the influential works of Reynolds, Krause, Vicsek and Cucker & Smale.

The dynamics is governed by different protocols of pairwise interactions, quantified in terms of proper communication kernels. Collisions are avoided. A main question of interest is how different classes of such kernels affect the large-time large-crowd dynamics. We will ask how short-range interactions can affect the emergence of large-scale patterns, what is the role of repulsion away thermal equilibrium, and how graph connectivity dictates the emergent behavior of multi-species dynamics. (Received November 16, 2021)

1174-92-5436 **Paige Knittel*** (plknit00@gmail.com), University of Michigan, and **Isabelle Stepler** (ifs5097@psu.edu), Pennsylvania State University. *Nonlinear Dynamical Modeling of the Transmission of Abnormal Beats in the Heart*

Abnormal cardiac beat patterns are responsible for serious rhythm disorders such as sinus node dysfunction in humans and other animals. We hypothesized that these patterns are produced by wave failure in or at the end of sinus node exit pathways. Related and unexplained irregular rhythm patterns have been observed in resting dogs due to increased vagal tone. Based on the Barkley nonlinear dynamical equations, we constructed a one dimensional computer model, which includes specialized regions representing the sinus node, exit pathway and the atrium, where heartbeats are formed and begin to propagate. While modifying a model parameter that functions similarly to variations in vagal tone, we discovered that increasing refractoriness with each beat caused patterns of isolated, missing beats in both the middle and end of the exit pathway. When this process occurred in the middle, it contributed to single and multiple beat failure patterns at the end of the exit pathway. All these patterns were similar to those observed in experimental data. This research may benefit our understanding of sinus node dysfunction in humans. (Received August 20, 2021)

1174-92-5513 **Alexandria Volkening*** (avolkening@purdue.edu), Purdue University. *Agent-based modeling and topological techniques for zebrafish patterns* Preliminary report.

Self-organization is present across biology, and the example that I will focus on today is exploring how brightly colored cells interact to form skin patterns in zebrafish. While wild-type zebrafish have black and gold stripes, mutants and other fish feature different (usually spotty and patchy) patterns. As each fish grows, tens of thousands of pigment cells interact in a stochastic environment to produce its pattern. Agent-based modeling is thus a natural approach for describing pattern development and helping to identify how genetic mutations alter cell behavior to create mutant zebrafish. Comparing in vivo and in silico patterns is often a qualitative process, however, especially because mutant patterns are messy and variable. Agent-based models of zebrafish also have many parameters, and together this makes identifying the parameters and cell behaviors involved in mutants challenging. In this talk, I will overview our models of zebrafish and discuss some of our approaches, including employing methods from topological data analysis, for comparing in vivo and in silico patterns quantitatively. (Received August 21, 2021)

1174-92-5650 **Sara M Clifton*** (sara.clifton43@gmail.com), St. Olaf College, and **Lucia M Wagner** (wagner10@stolaf.edu), St. Olaf College. *Modeling the Public Health Impact of E-Cigarettes on Adolescents and Adults*

Since the introduction of electronic cigarettes to the United States market in 2007, vaping prevalence has surged in both adult and adolescent populations. E-cigarettes are advertised as a safer alternative to traditional cigarettes and as a method of smoking cessation, but the U.S. government and health professionals are concerned that e-cigarettes attract young non-smokers. Here, we develop and analyze a dynamical systems model of competition between traditional and electronic cigarettes for users. With this model, we predict the change in smoking prevalence due to the introduction of vaping, and we determine the conditions under which e-cigarettes present a net public health benefit or harm to society. (Received August 24, 2021)

1174-92-5708 **Eleni Panagiotou** (eleni-pangiotou@utc.edu), University of Tennessee at Chattanooga, and **Jason Wang*** (wangj907@gmail.com), University of Pennsylvania. *Using topology and geometry to predict protein folding rates*

Many proteins fold in 3-dimensional conformations which are important for their function. To understand protein folding and function, it is necessary to characterize their 3-dimensional conformation. Using rigorous tools from knot theory, we examine the role of geometry and topology in protein folding, studying the correlation between the geometrical/topological complexity of the native states and the experimental folding rates. More precisely, as measures of geometrical/topological complexity, we use the Writhe, the ACN and the second Vassiliev measure. The second Vassiliev measure is the only function that can detect knottedness in a protein, even in the absence of well defined knots, and it is not affected by local secondary structure; the Writhe and the ACN are very sensitive to local geometry. We analyze a set of two-state and multi-state proteins, most of which contain no knots or slipknots. Our results suggests that both the local geometry and the topology of the proteins' native states correlate with folding rates, supporting the hypothesis that lower folding rates correlate with more complex native states. (Received August 28, 2021)

1174-92-5746 **Elizabeth Munch** (muncheli@msu.edu), Department of Computational Mathematics, Science & Engineering, Michigan State University, Department of Mathematics, Michigan State University, **Erik J Amezquita*** (amezqui3@msu.edu), Department of Computational Mathematics, Science & Engineering, Michigan State University, **Tim Ophelders** (t.a.e.ophelders@tue.nl), Department of Mathematics and Computer Science, TU Eindhoven, **Michelle Quigley** (quigle30@msu.edu), Department of Horticulture, Michigan State University, **Daniel Chitwood** (chitwoo9@msu.edu), Department of Horticulture, Michigan State University, Department of Computational Mathematics, Science & Engineering, Michigan State University, **Jacob B Landis** (jbl256@cornell.edu), School of Integrative Plant Science, Section of Plant Biology and the L.H. Bailey Hortorium, Cornell University, BTI Computational Biology Center, Boyce Thompson Institute, Department of Botany & Plant Sciences, University of California, Riverside, and **Daniel Koenig** (dkoenig@ucr.edu), Department of Botany & Plant Sciences, University of California, Riverside. *Measuring hidden phenotype: Quantifying barley morphology using the Euler Characteristic Transform*

Shape plays a fundamental role in biology. Traditional phenotypic analysis methods measure some features but fail to measure the information embedded in shape comprehensively. To extract, compare, and analyze this information embedded in a robust and concise way, we turn to Topological Data Analysis (TDA), specifically the Euler Characteristic Transform (ECT). To study its use, we compute both traditional and topological shape descriptors to quantify the morphology of 3121 barley seeds scanned with X-ray Computed Tomography (CT) technology at 127 micron resolution. The ECT measures shape by recording the Euler characteristic of an object at thresholds across a number of directional axes. A qualitative analysis of the information encoded by the ECT reveals that the it picks up the shape of the crease and bottom of the seeds. Moreover, while traditional shape descriptors can cluster the seeds based on their accession, topological shape descriptors can cluster them further based on their spike. We then successfully train a support vector machine (SVM) to classify 28 different accessions of barley based on the shape of their grains. We observe that combining both traditional and topological descriptors classifies barley seeds better than using just traditional descriptors alone. This improvement suggests that TDA is thus a powerful complement to traditional morphometrics to comprehensively describe a multitude of "hidden" shape nuances which are otherwise not detected. (Received August 26, 2021)

1174-92-5771 **Yvonne Niyonzima*** (yniyonz@ncsu.edu), 7011 Sandy Forks Rd, **Hien Tran** (tran@ncsu.edu), Center for Public Health and Environmental Assessment, U.S. Environmental Protection Agency, Research Triangle Park, NC, North Carolina State University, and **Hisham El-Masri** (El-Masri.Hisham@epa.gov), EPA. *A Quantitative Systems Toxicology (QST) Model for Hepatic Steatosis Induction by Carbon Tetrachloride* Preliminary report.

A Quantitative Systems Toxicology (QST) Model for Hepatic Steatosis Induction by Carbon Tetrachloride Yvonne Niyonzima1, Hien Tran1, and Hisham El-Masri2 1NC State University, Raleigh, NC, 2US EPA, CCET, RTP, NC,

Nonalcoholic fatty liver disease (NAFLD) disease impacts 25-30% of the US population. Hepatic steatosis, an increase of more than 5% of lipid content in the liver, is a biological manifestation of NAFLD. Hepatic steatosis can be initiated by lifestyle factors and exposure to many environmental chemicals such as carbon tetrachloride (CCl4). Research has shown that CCl4 decreased the levels of hepatic very low-density lipoprotein (VLDL) and increased the levels of sterol regulatory element binding protein 1c (SREBP-1c). VLDL and SREBP-1c were incorporated in a QST model describing CCl4 impact on hepatic steatosis. The QST model consisted of a physiologically based pharmacokinetic (PBPK) model for CCl4 and a mechanistic model describing the cascading biological events leading to formation of free fatty acids and triglycerides in the liver. The overall model was calibrated using literature in vivo data from rats exposed to CCl4. The calibrated model served as a quantitative method to establish dose-response relationships for CCl4 toxicity regarding hepatic steatosis. In humans, the CCl4 PBPK and the calibrated QST models can be quantitatively extrapolated to address health risk assessment of the chemical in inducing hepatic steatosis. This abstract does not necessarily reflect U.S. EPA policy. (Received August 28, 2021)

1174-92-5822 **Chris McCarthy*** (cmccarthy@bmcc.cuny.edu), BMCC City University of New York, and **Johannes Familton** (jfamilton@bmcc.cuny.edu), BMCC City University of New York. *Quasispecies, error catastrophe, origin of life: some of the mathematics, history, and implications.*

Quasispecies refers to the paradigm of viewing a species as a distribution of mutating genotypes. Error catastrophe refers to deleterious effects due to excessive mutation rates. We will discuss some of the interesting mathematics, implications (such as to the origin of life), and history of the quasispecies concept and the error catastrophe theory. (Received August 28, 2021)

1174-92-5828 **Maia Martcheva** (maia@ufl.edu), University of Florida, **Laura Nemeth** (knemeth1@fau.edu), Florida Atlantic University, and **Necibe Tuncer*** (ntuncer@fau.edu), Florida Atlantic University. *Structural and practical identifiability analysis of multiscale immuno-epidemiological model*

We perform the identifiability analysis of a multiscale model of seasonal influenza with multiscale data. We show that the well studied target cell limited within-host model is not structurally identifiable. So, we reformulate the model and work with a scaled within-host model which is structurally identifiable. We find that the scaled within-host model is practically identifiable with respect to two distinct viremia data sets while fitting with weighted or unweighted least squares. We introduce a methodology on how to study the structural identifiability of multiscale epidemic models specifically nested immuno-epidemiological models. All parameters of the multiscale model are practically identifiable. Furthermore, we find that the practical identifiability of the multiscale model is significantly better when fitted to viremia and incidence data as opposed to when fitted to viremia and cumulative incidence data. Comparing first and second order numerical methods for solving the partial differential equations suggests that using a higher order numerical method does not affect the identifiability of the parameters. Further simulations suggest that the choice of the linking functions has some impact on identifiability when viremia and incidences are fitted but no impact when viremia and cumulative incidences are fitted. (Received August 29, 2021)

1174-92-5833 **Ailene Macpherson*** (ailenem@sfu.ca), Simon Fraser University. *Limitations to inference in a general phylodynamic model*

A key aim of the field of Phylodynamics is to infer infectious disease dynamics from the evolutionary relationships between pathogen genome sequences. Recent results, however, have called into question the ability to uniquely identify the parameters of birth-death (BD) phylodynamic models over long, macroevolutionary, time scales. To characterize the robustness of BD inference within the epidemiological context, my coauthors and I first derived a general BD process which encompasses many existing epidemiological and macroevolutionary models as special cases. This general model allows us to 1) characterize the assumptions and connections between BD models across time scales, 2) derive novel model variants, and 3) asses the limitations to parameter inference within the epidemiological context. As in the macroevolutionary case, we find that BD model parameters are fundamentally unidentifiable from phylogenetic data alone. The robust application of these genomic epidemiology approaches will therefore require significant methodological advances, facilitated in part by the general BD model derived here. (Received August 29, 2021)

1174-92-5843 **Frederick Albert Matsen*** (ematsen@gmail.com), Fred Hutchinson Cancer Center. *Bayesian phylogenetics by systematic search and gradient ascent*

I will start by motivating Bayesian phylogenetics via the essential role it has played in understanding the origins, early establishment, and transmission dynamics of SARS-CoV-2.

However, Bayesian posterior distributions on phylogenetic trees remain difficult to sample despite decades of effort. The complex discrete and continuous model structure of trees means that recent inference methods developed for Euclidean space are not easily applicable to the phylogenetic case. Thus, we are left with random-walk Markov Chain Monte Carlo (MCMC) with uninformed tree modification proposals; these traverse tree space slowly because phylogenetic posteriors are concentrated on a small fraction of the very many possible trees.

I will then describe our work to design new scalable approaches to inferring the Bayesian posterior on phylogenetic trees. This includes establishing a new inferential structure, which we call the "subsplit directed acyclic graph," and a new al gorithm that will allow us to infer this structure using methods analogous to much faster maximum-likelihood (point-estimate) methods for phylogenetics. I will also describe how, once this structure is in hand, we can use stochastic gradient ascent to fit parameters to it using variational inference. (Received August 29, 2021)

1174-92-5847 **Zachary P Kilpatrick*** (zpkilpat@colorado.edu), University of Colorado Boulder, and **Heather Cihak** (Heather.Cihak@colorado.edu), University of Colorado Boulder.
Excitation/inhibition balance strongly shapes the stochastic dynamics of wandering bumps

Localized and reverberatory patterns of excitation in visual, executive, and decision-making areas of the brain have been shown to neurally represent remembered stimuli. Neural field models describe these dynamics via the activity patterns that emerge in integrodifferential equations whose weight kernels represent the strength and polarity of network architecture. Our analysis focuses on how localized pulses of activity called "bumps" wander in response to stochastic input, specifically when the effect of excitatory and inhibitory neurons are represented using separate population equations. Using two forms of local analysis, a linear perturbative method as well as a nonlinear interface method, we are able to approximate how much bumps wander as a function of architectural parameters and the network activation threshold. Interestingly, we find strongly non-monotonic relationships in the variance of bump position trajectories as a function of simple parameters like the strength of excitation to inhibitory populations. Our analysis allows us to identify specific network architectures that lead to the most robust storage of information and to suggest how short and long term plasticity may adapt networks to function more efficiently. This work was primarily driven by PhD student Heather Cihak. (Received August 30, 2021)

1174-92-5851 **Nicholas W Barendregt*** (nicholas.barendregt@colorado.edu), University of Colorado Boulder, **Kresimir Josic** (josic@math.uh.edu), University of Houston, and **Joshua I Gold** (jigold@pennmedicine.upenn.edu), University of Pennsylvania. *Adaptive Decision Rules are Optimal in Simple Environments*

Decision-making in uncertain environments often requires adaptive forms of evidence accumulation, but less is known about the decision rules needed to achieve optimal performance. While recent studies of decision models in stochastic and dynamic environments have resulted in several phenomenological models, such as the monotonically collapsing decision threshold of the "urgency-gating model" (UGM), we lack a general, normative description of decision rules and their relation to human decision-making. In this talk, we will study the prevalence of adaptive decision rules by developing a normative, Bayes-optimal framework for relatively simple two-alternative forced choice tasks. By allowing context variables, such as reward or difficulty of the decision, to vary in time, we find rich non-monotonic decision rules that vary throughout task parameter space. By comparing the performance of these strategies against simple heuristics, we show that these complex normative strategies significantly outperform alternative models such as the UGM. Finally, using subject data from the classic "tokens task", we perform rigorous model fitting and comparison which suggests humans may use adaptive normative strategies in such tasks. These results provide testable hypothesis for experimentalists to validate in future psychophysics tasks and give insights into the complexities of human decision strategies. (Received August 30, 2021)

1174-92-5913 **Rebecca A. Everett** (reverett@haverford.edu), Haverford College, **Angela Peace** (a.peace@ttu.edu), Texas Tech University, **Lale Asik*** (asik@uiwtx.edu), University of the Incarnate Word, and **Elizabeth T Borer** (borer@umn.edu), University of Minnesota. *Elements of disease in a changing world: modelling feedbacks between infectious disease and ecosystems*

An overlooked effect of ecosystem eutrophication is the potential to alter disease dynamics in primary producers, inducing disease-mediated feedbacks that alter net primary productivity and elemental recycling. Models in disease ecology rarely track organisms past death, yet death from infection can alter important ecosystem processes including elemental recycling rates and nutrient supply to living hosts. In contrast, models in ecosystem ecology rarely track disease dynamics, yet elemental nutrient pools can regulate important disease processes including pathogen reproduction and transmission. Thus, both disease and ecosystem ecology stand to grow as fields by exploring questions that arise at their intersection. However, we currently lack a framework explicitly linking these disciplines. We developed stoichiometric model using elemental currencies to track primary producer biomass (carbon) in vegetation and soil pools, and to track prevalence and the basic reproduction number of a directly transmitted pathogen. This model, parameterized for a deciduous forest, demonstrates that anthropogenic nutrient supply can interact with disease to qualitatively alter both ecosystem and disease dynamics. Using this element-focused approach, we identify knowledge gaps and generate predictions about the impact of anthropogenic nutrient supply rates on infectious disease and feedbacks to ecosystem carbon and nutrient cycling. (Received August 31, 2021)

1174-92-5931 **Susan Rogowski*** (sr19a@my.fsu.edu), Florida State University. *Finding the Source of a Foreign Pathogen Within a Water Supply Network*

If foreign bacteria are introduced into a water distribution network, the presence of a biofilm can lead to biofilm-assisted retention of the pathogens, affecting the potability of the water as well as increasing the chance of water-borne illnesses within a community. Given the presence of a pathogen within the water supply being reported at a home or apartment, it is unclear how to find the source of that pathogen within the water supply network. This is an inherently inverse problem. In this poster, I will present a model for the transport of pathogens downstream through a system of pipes. Then, I will present some data assimilation techniques that can be used to predict boundary and initial conditions of the system. (Received August 31, 2021)

1174-92-5954 **Xuming Xie*** (xuming.xie@morgan.edu), Morgan State University. *Well-posedness of a comprehensive mathematical model of diabetic atherosclerosis*

Atherosclerosis is a leading cause of death worldwide; it emerges as a result of multiple dynamical cell processes including hemodynamics, endothelial damage, innate immunity and sterol biochemistry. Making matters worse, nearly 463 million people have diabetes, which increases atherosclerosis-related inflammation, diabetic patients are twice as likely to have a heart attack or stroke. The pathophysiology of diabetic vascular disease is generally understood. Dyslipidemia with increased levels of atherogenic LDL, hyperglycemia, oxidative stress and increased inflammation are factors that increase the risk and accelerate development of atherosclerosis. In this paper, we propose a more comprehensive mathematical model for diabetic atherosclerosis which include more variables; in particular it includes the variable for Advanced Glycation End-Products (AGEs)concentration. Hyperglycemia trigger vascular damage by forming AGEs, which are not easily metabolized and may accelerate the progression of vascular disease in diabetic patients. The model is given by a system of partial differential equations with a free boundary. We also establish local existence and uniqueness of solution to the model. The methodology is to use Hanzawa transformation to reduce the free boundary to a fixed boundary and reduce the system of partial differential equations to an abstract evolution equation in Banach spaces, and apply the theory of analytic semigroup. (Received September 1, 2021)

1174-92-6040 **Scott McKinley** (scott.mckinley@tulane.edu), Tulane University, and **Veronica Ciocanel*** (ciocanel@math.duke.edu), Duke University. *Modeling microtubule polarity and organization in fruit fly neurons* Preliminary report.

The cellular cytoskeleton is essential in proper cell and neuronal function as well as in development. Microtubule filaments inside cells need to be stable to support healthy protein transport along them. On the other hand, microtubules also need to be dynamic and reorganize in response to injury events. These filaments have a polar structure (an orientation), which influences how cellular cargo gets transported. I will describe ongoing work on a spatially-explicit multiscale model that captures microtubule organization in branched dendrites of the fruit fly. The model aims to investigate how different experimentally validated mechanisms work together and contribute to the polarized filament organization observed in fruit fly dendrites. We are especially interested in identifying conditions that might explain the complete reversal of polarity in neuronal injury experiments. (Received September 3, 2021)

1174-92-6063 **Lauren M Childs** (lchilds@vt.edu), Virginia Tech, **Melody Walker*** (melody@vt.edu), Virginia Tech, **Michael A Robert** (robertma@vcu.edu), Virginia Commonwealth University, **Karthikeyan Chandrasegaran** (karthikeyan@vt.edu), Virginia Tech, Blacksburg, VA, and **Clement Vinauger** (vinauger@vt.edu), Virginia Tech, Blacksburg, VA. *Modeling the effects Aedes aegypti's larval environment on adult body mass at emergence and its implication on spread of disease* Preliminary report.

Millions of people are infected with deadly pathogens spread by mosquitoes, which make mosquitos an important public health issue. However, the success of many control measures is highly dependent upon ecological, physiological, and life history traits of mosquito species. One trait of interest is mosquito body mass, which depends upon many factors associated with the environment in which juvenile mosquitoes develop. Our experiments examined the impact of larval density on the body mass of *Aedes aegypti* mosquitoes, which are important vectors of dengue, Zika, and other pathogens. To investigate the interactions between the larval environment and mosquito body mass, we built a discrete time mathematical model that incorporates body mass, larval density, and food availability and fit the model to our experimental data. We considered three categories of model complexity informed by data, and selected the best model within each category. We found that the larval environment is an important determinant of the body mass of mosquitoes upon emergence. Furthermore, we found that inclusion of density dependence in the survival of female aquatic stages in models is important. From the larval model, we build an adult model that considers gonotrophic cycle stages as well as disease spread with a simple SEIR model for humans. We consider how mosquito body mass might effect the spread of diseases based on our model and differences in life cycle traits by mass found in literature. (Received September 16, 2021)

1174-92-6107 **Zhuolin Qu*** (zhuolin.qu@utsa.edu), University of Texas at San Antonio, **Tong Wu** (tong.wu@utsa.edu), University of Texas at San Antonio, and **James Macklin Hyman** (mhyman@tulane.edu), Tulane University. *Modeling Spatial Waves of Wolbachia Invasion for Controlling Mosquito-Borne Diseases*

Wolbachia is a natural bacterium that can infect mosquitoes and reduce their ability to transmit mosquito-borne diseases, such as dengue, Zika, and chikungunya. Field trials and modeling studies have shown that the fraction of infection among the mosquitoes must exceed a threshold for the infection to persist. To capture this threshold, it is critical to consider the spatial heterogeneity in the distributions of the infected and uninfected mosquitoes, which is created by the release of the infected mosquitoes. We develop and analyze partial differential equation models to study the invasion dynamics of Wolbachia infection among mosquitoes in the field. Our reaction-diffusion-type models account for both the complex vertical transmission and the spatial mosquito dispersion. We characterize the threshold for a successful invasion, which is a bubble-shaped profile, called the "critical bubble". The critical bubble is optimal in its release size compared to other spatial profiles in a one-dimensional landscape. The fraction of infection near the release center is higher than the threshold level for the corresponding homogeneously mixing ODE models. We show that the proposed spatial models give rise to the traveling waves of Wolbachia infection when above the threshold. We quantify how the threshold condition and traveling-wave velocity depend on the diffusion coefficients and other model parameters. Numerical studies for different scenarios are presented to inform the design of release strategies. (Received September 4, 2021)

1174-92-6109 **Asma Azizi** (aazizi@kennesaw.edu), Kennesaw State University, **Norine Schmidt** (nschmid1@tulane.edu), Tulane University, **Megan Clare Craig-Kuhn** (mcraigkuhn@tulane.edu), Tulane University, **Charles Stoecker** (cfstoecker@tulane.edu), Tulane University, and **Patricia Kissinger** (kissing@tulane.edu), Tulane University. *Modeling the Impact of Screening Men for Chlamydia Trachomatis on the Prevalence in Women*

Chlamydia trachomatis (Ct) is the most commonly reported sexually transmitted infection in the US and causes important reproductive morbidity in women. The Centers for Disease Control and Prevention recommend routine screening of sexually active women under age 25 but not among men. Despite three decades of screening women, Ct prevalence in women remains high. Untested and untreated men can serve as a reservoir of infection in women, and male-screening based intervention can be an effective strategy to reduce infection in women. We assessed the impact of screening men on the Ct prevalence in women. We created an individual-based network model to simulate a realistic chlamydia epidemic on sexual contact networks for a synthetic population. The model is calibrated to the ongoing routine screening among African American (AA) women in the US and detailed a male-screening program, Check It, that bundles best practices for Ct control. We estimate that by annually screening 7.5% of the AA male population in the age range, the chlamydia prevalence would be reduced relatively by 8.1% in AA women and 8.8% in AA men. Each man screened could prevent 0.062 cases in men and 0.204 cases in women. The model suggested the importance of intervention components ranked from high to low as venue-based screening, expedited index treatment, expedited partner treatment, and rescreening. The findings indicated that male-screening has the potential to substantially reduce the prevalence among women in high-prevalence communities. (Received September 4, 2021)

1174-92-6158 **Junping Shi*** (jxshix@wm.edu), College of William & Mary, **Wenjie Ni** (wni2@une.edu.au), University of New England, and **Ming Xin Wang** (mxwang@hit.edu.cn), Harbin Institute of Technology. *Global stability of nonhomogeneous equilibrium solution for the diffusive competition model*

A diffusive Lotka-Volterra competition model is considered and the combined effect of spatial dispersal and spatial variations of resource on the population persistence and exclusion is studied. A new Lyapunov functional method and a new integral inequality are developed to prove the global stability of non-constant equilibrium solutions in heterogeneous environment. The general result is applied to show that in a two-species system in which the diffusion coefficients, resource functions and competition rates are all spatially heterogeneous, the positive equilibrium solution is globally asymptotically stable when it exists, and it can also be applied to the system with arbitrary number of species under the assumption of spatially heterogeneous resource distribution, for which the monotone dynamical system theory is not applicable. (Received September 5, 2021)

1174-92-6159 **Marek Kimmel** (kimmel@rice.edu), Rice University, and **Kyung Hyun Lee*** (Kyung.Hyun.Lee@uth.tmc.edu), The University of Texas Health Science Center at Houston. *Stationary Distribution of Telomere Lengths in Cells with Telomere Length Maintenance and its Parametric Inference*

Telomeres are nucleotide caps located at the ends of each eukaryotic chromosome. Under normal physiological conditions as well as in culture, they shorten during each DNA replication round. Short telomeres initiate a proliferative arrest of cells termed 'replicative senescence'. However, cancer cells possessing limitless replication potential can avoid senescence by the telomere maintenance mechanism, which offsets telomeric loss. Therefore, cancer cells have sufficiently long telomeres even though their lengths are significantly shorter than their normal counterparts. This implies that the attrition and elongation rates play crucial roles in deciding whether and when cells ultimately become carcinogenic. In this research, we propose a concise mathematical model that shows the shortest telomere length at each cell division and prove mathematical conditions related to the attrition and elongation rates, which are necessary and sufficient for the existence of stationary distribution of telomere lengths. Moreover, we estimate the parameters of the telomere length maintenance process based on frequentist and Bayesian approaches. This study expands our knowledge of the mathematical relationship between the telomere attrition and elongation rates in cancer cells, which is important because the telomere length dynamics is a useful biomarker of cancer diagnosis and prognosis. (Received September 5, 2021)

1174-92-6160 **Hao Wang** (hao8@ualberta.ca), University of Alberta, **Chuncheng Wang** (wangchuncheng@hit.edu.cn), Harbin Institute of Technology, **Xiangping Yan** (xpyan72@163.com), Lanzhou Jiaotong University, and **Qingyan Shi** (qingyanshi@jiangnan.edu.cn), Jiangnan University. *Modeling animal movement with memory with partial differential equations with time-delay*

Diffusion has been widely applied to model animal movement that follows Brownian motion. However, animals typically move in non-Brownian ways due to their perceptual judgment. Spatial memory and cognition recently have received much attention in characterizing complicated animal movement behaviours. Explicit spatial memory is modeled via a distributed delayed diffusion term in this paper. The distributed time represents the memory growth and decay over time, and the spatial nonlocality reflects the dependence of spatial memory on location. When the temporal delay kernel is weak under the assumption that animals can immediately acquire knowledge and memory decays over time, the equation is equivalent to a Keller-Segel chemotaxis model. For the strong kernel with learning and memory decay stages, rich spatiotemporal dynamics, such as Turing and checker-board patterns, appear via spatially non-homogeneous steady-state and Hopf bifurcations. (Received September 5, 2021)

1174-92-6165 **Ashley V Schwartz** (ashleyvsch@gmail.com), San Diego State University's Computational Science Research Center, **Uduak Z George*** (ugeorge@sdsu.edu), San Diego State University's Department of Mathematics, and **Karilyn E Sant** (ksant@sdsu.edu), San Diego State University School of Public Health. *Using Complex Network Models to Characterize Deformities in Zebrafish Embryos Exposed to the Environmental Toxicant Tris(4-cholophenyl)methanol (TCPMOH)*

Tris(4-cholophenyl)methanol (TCPMOH) is a recently discovered environmental water contaminant with an unknown origin. It is highly persistent in the environment and bioaccumulates in marine species. It has been found in human breast milk, suggesting the potential for human developmental exposures. Zebrafish (*Danio rerio*) is a model organism that is often used to access the degree of toxicity of environmental substances. It has been recently found that zebrafish embryos exposed to TCPMOH have a significantly increased mortality and structural defects/deformities (such as yolk edema, paracardial edema, cranial malformations, spinal deformities, swim bladder inflation). With some of the embryos having more than one deformity. Deformity associations, i.e., the deformities that could fuel the onset of further deformities remains unknown. In this talk, I will show how we used multiple different weighted network models and spectral decomposition analysis to determine the deformities that could fuel the onset of further deformities. In particular, I will show that spectral decomposition analysis identifies yolk edema as being highly associated with the onset of further deformities. (Received September 6, 2021)

1174-92-6280 **Reinhard Laubenbacher*** (reinhard.laubenbacher@medicine.ufl.edu), University of Florida, **Henrique AL Ribeiro** (henrique.deassis@medicine.ufl.edu), University of Florida, **Luis Sordo Vieira** (luis.sordovieira@medicine.ufl.edu), University of Florida, **Yogesh Scindia** (Yogesh.Scindia@medicine.ufl.edu), University of Florida, **Bandita Adhikari** (bandita.adhikari@gmail.com), University of Pennsylvania, **Matthew Wheeler** (matthew.wheeler@medicine.ufl.edu), University of Florida, **Adam Knapp** (Adam.Knapp@medicine.ufl.edu), University of Florida, **William Schroeder** (will.schroeder@kitware.com), Kitware Inc, and **Borna Mehrad** (borna.mehrad@medicine.ufl.edu), University of Florida. *The innate immune response to respiratory fungal pathogens: A multi-scale modeling approach* Preliminary report.

Invasive aspergillosis is a fungal respiratory infection that poses an increasingly serious health risk with the rise in the number of immunocompromised patients and the emergence of fungal strains resistant to first-line anti-fungal drugs. The work presented in this talk is motivated by the need for host-centric therapeutics for this infection. Given the multi-scale nature of the immune response, computational models are a key technology for capturing the dynamics of the battle between the pathogen and the immune system. This talk will describe such a multi-scale computational model, focused on the mechanisms for iron regulation, a key element for fungal virulence in the pathogen *Aspergillus fumigatus*. The model has been extensively validated, and has served as a discovery tool for elucidating key drivers of the disease and potential therapeutic targets. (Received September 7, 2021)

1174-92-6282 **Thijs Frenken** (T.Frenken@nioo.knaw.nl), Netherlands Institute of Ecology, **Angélica González** (ag1271@camden.rutgers.edu), Rutgers University, **Alex Strauss** (atstrauss15@gmail.com), University of Georgia, **Dedmer van de Waal** (D.vandeWaal@nioo.knaw.nl), Netherlands Institute of Ecology, and **Eric Seabloom** (seabloom@umn.edu), University of Minnesota. *Modeling Frameworks that integrate Disease and Ecosystem Ecology* Preliminary report.

While there are well established modeling frameworks in both disease and ecosystem ecology, these processes are often decoupled in models. Despite repeated calls for stronger conceptual linkages, the theoretical lineages in disease and ecosystem ecology have remained largely separate. In part, this separation reflects the different views of death in these fields, and the resulting currencies and formulations of the core models used in fields rooted in population biology, such as disease ecology, and those arising from an ecosystem perspective. An obvious outcome of disease is death, and host death is integral to disease ecology theory. However, disease models typically assume dead hosts exit the system and disappear into a void. This assumption of a fully open system is rarely valid, because dead hosts can alter population dynamics and host-pathogen interactions through a variety of pathways. Here we extend disease ecology theory by explicitly accounting for the recycling of dead host biomass, and its impacts on pathogen transmission and host growth rates. We combine simple disease and ecosystem models to investigate the feedbacks between disease dynamics and ecosystem processes. (Received September 7, 2021)

1174-92-6313 **Yue Qin*** (y8qin@ucsd.edu), UCSD, and **Trey Ideker** (tideker@health.ucsd.edu), UCSD. *A multi-scale map of cell structure fusing protein images and interactions*

The cell is a multi-scale structure with modular organization across at least four orders of magnitude. Two central approaches for mapping this structure—protein fluorescent imaging and protein biophysical association—each generate extensive datasets, but of distinct qualities and resolutions that are typically treated separately. Here, we integrate immunofluorescence images in the Human Protein Atlas (HPA) with affinity purifications in BioPlex to create a unified hierarchical map of human cell architecture. Integration is achieved by configuring each approach as a general measure of protein distance, then calibrating the two measures using machine learning. The map, called the Multi-Scale Integrated Cell (MuSIC 1.0), resolves 69 subcellular systems of which approximately half are undocumented. Accordingly we perform 134 additional affinity purifications, validating subunit associations for the majority of systems. The map reveals a pre-ribosomal RNA processing assembly and accessory factors, which we show govern rRNA maturation, and functional roles for SRRM1 and FAM120C in chromatin and RPS3A in splicing. By integration across scales, MuSIC increases the resolution of imaging while giving protein interactions a spatial dimension, paving the way to incorporate diverse types of data in proteome-wide cell maps. (Received September 7, 2021)

1174-92-6351 **Doris Hartung*** (dh3187@ship.edu), Shippensburg University, and **Maya Williams** (mayaawilliamss@gmail.com), The College of New Jersey. *Effectiveness of Phage Therapy Against Bacteria Biofilm*

Biofilm formation in a host can be extremely problematic if left untreated, especially since antibiotics treatments sometimes can be ineffective. Certain lung diseases such as cystic fibrosis can cause the formation of biofilms in the lungs and can be fatal. With antibiotic-resistant bacteria, the use of phage therapy has been introduced as an alternative or an additive to the use of antibiotics in order to combat biofilm growth. In this study, we used both deterministic and stochastic framework to determine the effectiveness of phage therapy against bacteria biofilm; we also performed sensitivity analysis of the parameters. Our results show that phage migration and mortality rates can contribute to a balanced biofilm growth, which subsequently renders the biofilm vulnerable to antibiotics. (Received September 8, 2021)

1174-92-6367 **Morganne Igoe*** (migoe@utk.edu), University of Tennessee. *ZCTA-level Predictors of COVID-19 Hospitalization Risk in the St. Louis Area*

COVID-19 has overwhelmed U.S. healthcare systems, with over 40 million cases and 650,000 deaths as of September 2021. Older age, male gender, race, and underlying medical conditions have been identified as factors among hospitalized patients. There are also geographic disparities in COVID-19 hospitalization risk that are at least partly driven by geographic differences in sociodemographic, economic, and co-morbid factors. If these factors could be identified, they could help inform control efforts.

The aim of this study is to identify ZCTA-level predictors of COVID-19 hospitalization risk in the St. Louis Area using sociodemographic, economic, and chronic disease factors. These ZCTA-level factors were

evaluated for correlation with each other and univariable associations with the age-adjusted number of COVID-19 hospitalizations. The ZCTA population was used as the offset. A multivariable negative binomial regression model was then fit using a backwards elimination process.

Percent black population, percent of the population with some college education, number of diabetes discharges per 100 population, and population adjusted cases were all positively associated with COVID-19 hospitalization risk. A number of sociodemographic and chronic conditions are important determinants of disparities in COVID hospitalization risk. These findings will inform health care systems of where large numbers of patients may occur to reduce overburdening of hospitals and to guide vaccination efforts. (Received September 8, 2021)

1174-92-6404 **Garri Davydyan*** (garri.davydyan@gmail.com), Moscow State University named after M.V.Lomonosov (formerly), Appletree Medical Group, Appletree Medical Group, Ottawa, Canada. *Coquaternion self-similarity in regulations of biologic functions and its representation in functional structure of DNA.* Preliminary report.

Biologic evolution increases complexity of species by widening the spectrum of inherited functional properties. Comparison of embryos of high and low organized species suggests existence of similar DNA clusters released in early antenatal stages. It is presumed that inherited nucleotide sequences are embedded into one another providing stability of growing in complexity organisms. Functions of cells, organs are regulated by negative, positive and reciprocal feedback patterns (PNR). DNA releasing mechanism provides mapping of nucleotide sequences into "a pool" of proteins to form not only cells, organs, but also signaling agents for feedback circuits. Synthesized signaling substances are means to provide functional stability of systems through feedback. Three PNR patterns and identity element expressed in a matrix form represent basis elements of algebraically closed structure- coquaternion. Because PNR patterns exist on molecular, cell and organ levels, coquaternion structure provides functional self-similarity on different structural levels. Therefore, mapping of nucleotide sequences onto feedback carrier proteins should follow certain DNA markers to fit required hierarchical level. Feedback loops form regulatory network of functional triplets that may determine the primary "order" of functional organization of biologic systems. (Received September 8, 2021)

1174-92-6843 **Alvaro Fletcher*** (aarrospi@uci.edu), University of California, Irvine. *Non-cooperative mechanism for bounded and ultrasensitive chromatin remodeling*

Chromatin remodeling is an essential form of gene regulation that is involved in a variety of biological processes. We develop a theoretical model that takes advantage of percolation effects at the level of nucleosome interactions, which allows for ultrasensitive chromatin expansion. This model is non-cooperative and readily provides spatial bounds to the expansion region, preventing uncontrolled remodeling events. We explore different chromatin architectures and the ultrasensitivity of the chromatin density as a function of transcription factor concentration. We also compare our model with experimental data involving an inhibitor of nucleosome acetylation. These results suggest a novel mechanism for spatially-bounded chromatin remodeling and they provide means for quantitative comparisons between proposed models of chromatin architecture. (Received September 9, 2021)

1174-92-6852 **Nourridine Siewe*** (nxssma@rit.edu), Rochester Institute of Technology, and **Avner Friedman** (afriedman@math.osu.edu), The Ohio State University and Mathematical Biosciences Institute. *TGF-β inhibition can overcome cancer primary resistance to PD-1 blockade: a mathematical model*

Primary resistance to PD-1 blockade is reported to occur under conditions of immunosuppressive tumor environment, a condition caused by myeloid derived suppressor cells (MDSCs), and by T cells exclusion, due to increased level of T regulatory cells (Tregs). Since TGF-β activates Tregs, TGF-β inhibitor may overcome primary resistance to anti-PD-1. Indeed, recent mice experiments show that combining anti-PD-1 with anti-TGF-β yields significant therapeutic improvements compared to anti-TGF-β alone. The present paper introduces two cancer-specific parameters and, correspondingly, develops a mathematical model which explains how primary resistance to PD-1 blockade occurs, in terms of the two cancer-specific parameters, and how, in combination with anti-TGF-β, anti-PD-1 provides significant benefits. The model is represented by a system of partial differential equations and the simulations are in agreement with the recent mice experiments. In some cancer patients, treatment with anti-PD-1 results in rapid progression of the disease, known as hyperprogression disease (HPD). The mathematical model can also explain how this situation arises, and it predicts that HPD may be reversed by combining anti-TGF-β to anti-PD-1. The model is used to demonstrate how the two cancer-specific parameters may serve as biomarkers in predicting the efficacy of combination therapy with PD-1 and TGF-β inhibitors. (Received September 9, 2021)

1174-92-6876 **Elizabeth Zollinger*** (ezollinger@sjcny.edu), St. Joseph's College, Brooklyn, **Ellen Swanson** (ellen.swanson@centre.edu), Centre College, **Emek Kose** (ekose@smcm.edu), St. Mary's College of Maryland, and **Samantha Elliott** (slelliott@smcm.edu), St. Mary's College of Maryland. *Mathematical Modeling of Tumor and Cancer Stem Cells Treated with CAR-T Therapy and Inhibition of TGF-β*

The stem cell hypothesis suggests that there is a small group of malignant cells, the cancer stem cells, that initiate the development of tumors, encourage its growth, and may even be the cause of metastases. Traditional treatments, such as chemotherapy and radiation, primarily target the tumor cells leaving the stem cells to potentially cause a recurrence. Chimeric antigen receptor (CAR) T-cell therapy is a form of immunotherapy where the immune cells are genetically modified to fight the tumor cells.

Traditionally, the CAR T-cell therapy has been used to treat blood cancers and only recently has shown promising results against solid tumors. We create an ordinary differential equations model which allows for the infusion of trained CAR-T cells to specifically attack the cancer stem cells that are present in the solid tumor microenvironment. Additionally, we incorporate the influence of TGF-β which inhibits the CAR-T cells and thus promotes the growth of the tumor. We verify the model by comparing it to available data and then examine combinations of immunotherapy and targeted therapy treatments to determine the best treatment. (Received September 10, 2021)

1174-92-6906 **Emily Howerton*** (eah394@psu.edu), Pennsylvania State University, **Katriona Shea** (k-shea@psu.edu), Pennsylvania State University, **Matthew J. Ferrari** (mjf283@psu.edu), Pennsylvania State University, **Ottar N. Bjørnstad** (onb1@psu.edu), Pennsylvania State University, **Tiffany L. Bogich** (tiffany@psu.edu), Pennsylvania State University, **Rebecca K. Borchering** (rborchering@psu.edu), Pennsylvania State University, **Chris P. Jewell** (c.jewell@lancaster.ac.uk), Lancaster University, **James D. Nichols** (jnichols@usgs.gov), U.S. Geological Survey, **William J.M. Probert** (willprobert@gmail.com), University of Oxford, **Michael C. Runge** (mrunge@usgs.gov), U.S. Geological Survey, **Michael J. Tildesley** (m.j.tildesley@warwick.ac.uk), University of Warwick, and **Cécile Viboud** (viboudc@mail.nih.gov), National Institutes of Health. *Synergistic interventions to control COVID-19: mass testing and isolation miti-gates reliance on distancing*

Stay-at-home orders and shutdowns of non-essential businesses are powerful, but socially costly, tools to control the pandemic spread of SARS-CoV-2. Mass testing strategies, which rely on widely administered rapid diagnostics to identify and isolate infected individuals, could be a less disruptive management strategy. Here, we assess the extent to which mass testing and isolation can safely reduce reliance on socially costly non-pharmaceutical interventions (NPIs) such as distancing and shutdowns. We develop a multi-compartmental model of SARS-CoV-2 transmission incorporating both NPIs and mass testing and isolation to evaluate their combined effect on public health outcomes. We use stochastic processes to capture important realities of the testing system (non-random test allocation necessitated by constraints on test administration) that cannot easily be represented in a deterministic framework. We show how strategic changes in the characteristics of the testing system can reduce reliance on NPIs without compromising future public health outcomes. Our model is intended to be a tool to guide the ramp-up of testing capacity in outbreak settings, aid in flexible design of combined interventions, and inform cost-benefit analyses to identify efficient pandemic management strategies. (Received September 10, 2021)

1174-92-6935 **Jacob G Scott** (scottj10@ccf.org), Cleveland Clinic, Cleveland Clinic, Translational Hematology and Oncology Research, and **Steph J Owen*** (owens2@ccf.org), Cleveland Clinic Lerner Institute, Cleveland, OH. *Ecological frequency dependent fitness effects can mimic alternate or mask underlying genetic fitness landscapes, producing distinct population dynamics* Preliminary report.

Drug resistance models are often built upon the assumption that under treatment, cancer cells with the fittest genotype prevail. Many such models lack the complexity to predict tumor evolution in the clinic. In particular, the Wright-Fisher model from population genetics does not often predict the heterogeneity that we see in patients and cell-cell interactions are ignored. We develop a model of resistance evolution that incorporates average cell-cell interactions by adding game theoretic interactions to a Wright Fisher model with mutation. We show mathematically and through simulation how ecological game interactions between cell populations can modify evolutionary trajectories. With added game interactions, evolution on a genotypically selective landscape can mimic neutral landscapes and vice versa. We term the addition of game interactions to genotypic landscapes "ecological epistasis". We derive an expression for the payoff matrix required to produce any given evolutionary

outcome from a landscape and derive a "game signature" to test population data for the presence of games. We emphasize that the fitness benefits of resistant phenotypes can be ecological or genotypic. Due to the heterogeneous nature of tumors and their microenvironment, this result is of significance in planning treatment approaches and interpreting existing patient data. We highlight how game interactions may explain heterogeneous outcomes and contradicting results on cancer genotype fitness in the literature. (Received September 10, 2021)

1174-92-6959 **Lloyd T Elliott*** (lloyd_elliott@sfu.ca), Simon Fraser University. *Brain Imaging Genetics with ≈ 40,000 Subjects and ≈ 3,000 Phenotypes*

UK Biobank is a major prospective epidemiological study that is carrying out detailed multimodal brain imaging on 100,000 participants and includes genetics and ongoing health outcomes. In work with the Wellcome Centre for Integrative Neuroimaging, we present a new open resource of GWAS summary statistics, resulting from a greatly expanded set of genetic associations with brain phenotypes, using the 2020 UK Biobank imaging data release of approximately 40,000 subjects, 3,000 phenotypes, and 10 million variants with MAF $\geq$ 1%. We include associations on the X chromosome, and several new classes of image derived phenotypes (primarily, more fine-grained subcortical volumes, and cortical grey-white intensity contrast). We develop a method to identify clusters of associations across phenotypes (Peaks) and we find 692 replicating clusters of associations, including 12 on the X chromosome. Our novel associations implicate pathways involved in the rare X-linked syndrome STAR (syndactyly, telecanthus and anogenital and renal malformations), Alzheimer's disease and mitochondrial disorders. (Received September 10, 2021)

1174-92-6968 **David Murrugarra*** (murrugarra@uky.edu), University of Kentucky. *RNA secondary structure prediction via state inference with machine learning methods* Preliminary report.

RNA state inference is the task of determining which nucleotides of an RNA sequence are paired or unpaired in the secondary structure of an RNA, which can be studied by different machine learning techniques. The state inference is a binary classification task on each nucleotide which is different from determining the full secondary structure consisting of sets of nested base pairs. Successful state inference of RNA sequences can be used to generate auxiliary information for data-directed RNA secondary structure prediction. In this talk I will highlight the challenges from using methods such as hidden Markov models (HMM) and variational autoencoders (VAE) for state inference. I will also discuss how the imbalance of RNA families in training data sets can create prediction issues such as overfitting. Finally, I will discuss different approaches for improving RNA secondary structure prediction via the Nearest Neighbor Thermodynamic Model (NNTM) using machine learning techniques for state inference. (Received September 10, 2021)

1174-92-7085 **Asma Azizi*** (aazizi@kennesaw.edu), KSU, **Natalia Komarova** (komarova@uci.edu), UCI, and **Dominik Wodarz** (dwodarz@uci.edu), UCI. *Effect of human behavior on the evolution of viral strains during an epidemic*

It is well known in the literature that human behavior can change as a reaction to disease observed in others, and that such behavioral changes can be an important factor in the spread of an epidemic. It has been noted that human behavioral traits in disease avoidance are under selection in the presence of infectious diseases. Here we explore a complimentary trend: the pathogen itself might experience a force of selection to become less "visible", or less "symptomatic", in the presence of such human behavioral trends. Using a stochastic SIR agent-based model, we investigated the co-evolution of two viral strains with cross-immunity, where the resident strain is symptomatic while the mutant strain is asymptomatic. We assumed that individuals exercised self-regulated social distancing (SD) behavior if one of their neighbors was infected with a symptomatic strain. We observed that the proportion of asymptomatic carriers increased over time with a stronger effect corresponding to higher levels of self-regulated SD. Adding mandated SD made the effect more significant, while the existence of a time-delay between the onset of infection and the change of behavior reduced the advantage of the asymptomatic strain. These results were consistent under random geometric networks, scale-free networks, and a synthetic network that represented the social behavior of the residents of New Orleans. (Received September 12, 2021)

1174-92-7102 **Jesse Kreger*** (jessekre@usc.edu), University of Southern California, University of California, Irvine. *A hybrid stochastic-deterministic approach to explore multiple infection and evolution in HIV*

Mathematical models have been instrumental to study viral evolutionary processes in vivo. However, the need for stochastic simulations of minority mutant dynamics can pose computational challenges, especially in heterogeneous systems where very large and very small subpopulations coexist. In this talk, we will describe a hybrid stochastic-deterministic algorithm to simulate mutant evolution in large viral populations, such as acute HIV-1 infection, and further include the multiple infection of cells. We will demonstrate that the hybrid method can approximate the fully stochastic dynamics with sufficient accuracy at a fraction of the computational time, and can quantify evolutionary end points that cannot be expressed by deterministic models, such as the mutant distribution or the probability of mutant existence at a given infected cell population size. Finally, we will apply this method to study the role of multiple infection and intracellular interactions among different virus strains (such as complementation and interference) for mutant evolution, and show that synaptic transmission in HIV might promote virus evolvability. (Received September 12, 2021)

1174-92-7117 **David Chan** (dmchan@vcu.edu), VCU, and **Kelly Anne Reagan*** (reaganka2@vcu.edu), Virginia Commonwealth University. *Impact of Active Detection and Isolation on Annual* Clostridium difficule *Infections* Preliminary report.

Hundreds of thousands *Clostridium difficile* (*C. diff*) infections annually in the U.S. result in increases in mortality, morbidity and length of stay for patients in the hospital. *C. diff* is a bacteria that, when triggered by antibiotics, causes diarrhea and colitis by releasing toxins to attack the intestines. Patients in the Bone Marrow Transplant (BMT) Unit are particularly susceptible to *C. diff* infections due to their immune-compromised status and the high antibiotic use in the unit. A common practice is to test patients for the bacteria when they are symptomatic (having three or more loose stools within 24 hours). However, active detection and isolation (ADI) is a technique that identifies whether incoming patients are colonized with *C. diff* and allows for healthcare workers to use additional contact precautions to prevent the spread of the bacteria. We model two situations: the current practice of testing only symptomatic patients and the process of ADI. The goal of the research is to quantify the impact of ADI on the number of yearly *C. diff* infections in the BMT Unit. The results from our research inform healthcare leaders about the importance of applying contact precautions early to prevent infections from occurring. (Received September 12, 2021)

1174-92-7133 **Katherine Morrison** (Katherine.Morrison@unco.edu), University of Northern Colorado. *From structure to dynamics in combinatorial threshold linear networks*

Neural circuits display nonlinear dynamics. A network's structure is a key feature determining dynamics, but many questions remain as to how structure shapes activity. We study this relationship in a simple model of neural activity, combinatorial threshold linear networks (CTLNs), whose activity is governed by a system of threshold-linear ordinary differential equations determined by an underlying directed graph. Like real networks, CTLNs display the full spectrum of nonlinear behavior, including multistability, limit cycles, and chaos.

Much is known about fixed points of CTLNs, but less is known about their dynamic attractors. We give some of the first results which go beyond fixed points and relate the structure of a CTLN to its dynamics. We prove that if a CTLN's underlying graph is a directed acyclic graph, neural activity really must flow through the graph and to a stable fixed point. This is the first example of a proof guaranteeing convergence of the activity of a non-symmetric TLN to a fixed point.

We also construct a family of sequential memory networks. Each network consists of m layers of n neurons connected cyclically. The network has mn limit cycles, each corresponding to a sequence of neurons. These networks have a large capacity to encode dynamic patterns via limit cycles, giving a richer set of memory patterns than stable fixed points. Thus, these networks can model sequential memories, rhythms, or central pattern generators. (Received September 12, 2021)

1174-92-7145 **Saskia Haupt*** (saskia.haupt@uni-heidelberg.de), Engineering Mathematics and Computing Lab (EMCL), Interdisciplinary Center for Scientific Computing (IWR), Heidelberg University, Heidelberg, Germany, Data Mining and Uncertainty Quantification (DMQ) Group, Heidelberg Institute for Theoretical Studies (HITS), Heidelberg, Germany, **Alexander Zeilmann** (alexander.zeilmann@iwr.uni-heidelberg.de), Heidelberg University, **Nils Gleim** (nil.gleim@googlemail.com), Engineering Mathematics and Computing Lab (EMCL), Interdisciplinary Center for Scientific Computing (IWR), Heidelberg University, Heidelberg, Germany, **Aysel Ahadova** (Aysel.Ahadova@med.uni-heidelberg.de), Department of Applied Tumor Biology (ATB), Institute of Pathology, University Hospital Heidelberg, Heidelberg, Germany, **Hendrik Bläker** (hendrik.blaeker@medizin.uni-leipzig.de), Institute of Pathology, University Hospital Leipzig, Leipzig, Germany, **Magnus von Knebel Doeberitz** (magnus.knebel-doeberitz@med.uni-heidelberg.de), Department of Applied Tumor Biology (ATB), Institute of Pathology, University Hospital Heidelberg, Heidelberg, Germany, **Matthias Kloor** (Matthias.Kloor@med.uni-heidelberg.de), Department of Applied Tumor Biology (ATB), Institute of Pathology, University Hospital Heidelberg, Heidelberg, Germany, and **Vincent Heuveline** (vincent.heuveline@uni-heidelberg.de), Engineering Mathematics and Computing Lab (EMCL), Interdisciplinary Center for Scientific Computing (IWR), Heidelberg University, Heidelberg, Germany, Data Mining and Uncertainty Quantification (DMQ) Group, Heidelberg Institute for Theoretical Studies (HITS), Heidelberg, Germany. *Unraveling the black boxes in early cancer development using mathematical modeling at different scales*

Cancer development is a multi-scale process: Alterations on the DNA level may lead to changes on the cellular level which can alter the tissue and organ behavior and thus lead to cancer. However, long-term observational data at different scales are typically scarce. Thus, mathematical models are necessary to unravel this black box of early cancer development and to improve clinical management. Thus, we will present two models at the example of colon cancer: Firstly, an ordinary differential equation model for the development of different types of colon cancer based on the involved driver mutations. The model makes use of the Kronecker structure which allows for a detailed mathematical analysis and medical interpretation. Secondly, using a computational model and numerical simulations, we investigate the mutational dynamics within single crypts, which are believed to be the origin of colon cancer. The cells in the crypt are represented by a Voronoi tessellation and we model the cell cycle including cell division and possible mutations. In summary, with the models and parameter choices, we obtained results that are in concordance with clinical observations. Our models provide mathematical approaches towards a detailed multi-scale understanding of colon cancer development. (Received September 20, 2021)

1174-92-7206 **Megan A Greischar** (megan.greischar@cornell.edu), Cornell University. *Determining optimal transmission investment of malaria through re-analysis of human infection data* Preliminary report.

Plasmodium falciparum, the deadliest of the human malaria species, requires two hosts: humans and the *Anopheles* mosquito. While in the human, the parasite grows in numbers through asexual replication. During each replication event, a portion of parasites commit to terminal differentiation into a sexual state, which is the only form able to continue the infection cycle in the mosquito. Thus, there is a trade-off between further asexual growth within the human and transmission to the mosquito, and the level of commitment to transmission is thought to vary through the course of infection. Here, we evaluate a data set of 106 patients infected with malaria whose parasite numbers, both asexual and sexual, were tracked daily for over 100 days. We use a model developed by Eichner *et al.* (2001) and fit parameters including level of commitment to sexual state. In corroboration of previous work, we find parameters vary considerably across patients. In addition, we present a detailed examination of the variability in fitted parameters, showing the multi-modal aspects of the optimization space, which cannot be adequately described by the mean or median. Our detailed examination of a data-derived optimization space will improve understanding of models with issues in parameter identifiability. (Received September 13, 2021)

1174-92-7228 **Anna Sisk*** (asisk9@vols.utk.edu), University of Tennessee. *Connecting People to Food: A Network Approach to Alleviating Food Deserts*

In 2020, 13.8 million people in the United States struggled with food security, meaning they were unable to or uncertain that their food needs would be met. Whether a person struggles with food security can be influenced by where they live. Food deserts are census tracts that experience both high rates of poverty (20 percent or greater) and low access to grocery stores that offer fresh and nutritious foods. Unfortunately, food deserts and insecurity disproportionately affect minority and disadvantaged communities and can lead to serious health issues like diabetes, high blood pressure, and obesity. Clearly, public policies can be utilized to lessen the impact of

food deserts and one way city leaders can achieve this is through their public transit systems. In this talk we will use a networks approach to study the role public transit plays in combating food deserts. We will analyze these networks by looking at centrality measures, path length analysis, and edge densities. Finally, we will offer some suggestions on how city leaders can use these results to help eradicate food deserts. (Received September 13, 2021)

1174-92-7229 **Olivia Prosper** (oprosper@utk.edu), University of Tennessee/Knoxville, **Christina Edholm** (cedholm@scrippscollege.edu), Scripps College, **Denis Daniel Patterson*** (denispatterson@princeton.edu), Princeton University, **Joan Ponce** (joanponce90@gmail.com), UCLA, and **Lihong Zhao** (lzhao33@ucmerced.edu), UC Merced, University of California Merced. *Modeling Immunity to Malaria with an Age-Structured PDE Framework* Preliminary report.

Malaria is one of the deadliest infectious diseases globally, causing hundreds of thousands of deaths each year. It disproportionately affects young children, with two-thirds of fatalities occurring in under-fives. Individuals acquire protection from disease through repeated exposure, and this immunity plays a crucial role in the dynamics of malaria spread. We develop a novel age-structured PDE model of malaria specifically tracking acquisition and loss of immunity across the population. Using our analytical calculation of the basic reproduction number ($\mathcal{R}_0$), we study the role of vaccination and immunity feedback on severe disease and malaria incidence. Using demographic and immunological data, we parameterized our model to simulate realistic scenarios. Thus, via a combination of analytic and numerical investigations, our work sheds new light on the role of acquired immunity in malaria dynamics and the impact on vaccination strategies in the presence of demographic effects. (Received September 13, 2021)

1174-92-7325 **Simon Levin** (slevin@princeton.edu), Princeton University, **Daniel Brendan Cooney*** (dbcooney@sas.upenn.edu), University of Pennsylvania, **Fernando W. Rossine** (frossine@princeton.edu), Harvard University, and **Dylan H. Morris** (dhmorris@g.ucla.edu), University of California, Los Angeles. *A PDE Model for Protocell Evolution and the Origin of Chromosomes via Multilevel Selection*

The origin of chromosomes was a major transition in the evolution of complex cellular lie. In this talk, we model the origin of chromosomes by considering a simple protocell composed of two types of genes: a "fast gene" with an advantage for gene-level self-replication and a "slow gene" that replicates more slowly at the gene level, but which confers an advantage for protocell-level reproduction. Protocell-level replication capacity depends on cellular composition of fast and slow genes. Using a PDE to describe how the composition of genes within protocells evolves over time under within-cell and between-cell competition, we find that the gene-level advantage of fast replicators casts a long shadow on the multilevel dynamics of protocell evolution: no level of between-protocell competition can produce coexistence of the fast and slow replicators when the two genes are equally needed for protocell-level reproduction. By introducing a "dimer replicator", a linked pair of the slow and fast genes, we show that coexistence between the two genes can be promoted in pairwise multilevel competition between fast, slow, and dimer replicators. Our results suggest that dimerization, or the formation of a simple chromosome-like dimer replicator, can help to overcome the shadow of lower-level selection and work in concert with deterministic multilevel selection to allow for the coexistence of two genes that are complementary at the protocell level but compete at the level of individual gene-level replication. (Received September 14, 2021)

1174-92-7356 **Burt Simon*** (burt.simon@ucdenver.edu), University of Colorado Denver, and **Michael Doebeli** (doebeli@zoology.ubc.ca), University of British Columbia. *Evolutionary branching and diversification in group-structured populations* Preliminary report.

We study a model of group-structured populations featuring individual-level birth, death, and migration events, and group-level fission and extinction events. Individuals play games within their groups, and groups play games against other groups. Payoffs from individual-level games affect birth rates of individuals, and payoffs from group-level games affect group extinction rates. We focus on the interesting cases where individuals and groups play continuous prisoner's dilemma or snowdrift games. With respect to evolving game strategies, the resulting population dynamics can be very complex, including branching and type-diversification at both levels. Group-level branching like this has not been previously reported in the literature. (Received September 14, 2021)

1174-92-7451 **Benjamin Levy*** (blevy1@fitchburgstate.edu), Fitchburg State University. *A Vaccination Model for COVID-19 in South Africa*

The rate at which COVID-19 spread through South Africa varied over time as individuals responded to the ongoing pandemic and changing government policies. In this project we model the outbreak in the province of

Gauteng assuming that several parameters vary over time. We first determine the approximate dates of parameter changes and relate them to government policies. Unknown parameters are then estimated from available data and used to analyze the impact of each policy. We conclude by considering the implementation of two different vaccines at varying times. Our results quantify the impact of different government policies and demonstrate how vaccinations can alter infection dynamics. (Received September 16, 2021)

1174-92-7587 **Thi Thu Huong Vo*** (tvo2@angelo.edu), Angelo State University, and **Karl J Havlak** (karl.havlak@angelo.edu), Angelo State University. *Simulating a Predator-Prey System with a Lotka-Volterra Model Using Random Coefficients Estimated from a Dataset*

In this work, we study a Lotka-Volterra prey-predator differential equations model for a dataset on algae and rotifers from "Long-term cyclic persistence in an experimental predator-prey system" (Blasius et al., 2020). We study this model by allowing the coefficients to vary randomly over a range of values estimated from the dataset. This work summarizes the method used to estimate the mean and standard deviation for each of the coefficients for the various terms of the model from the data. Both uniformly and normally generated random numbers are used to analyze the model with varying coefficients. We found that the means and standard deviations we determined from the data generate results that vary significantly. Some simulations seem to capture the interaction seen in the data reasonably well and others result in "boom and bust" cycles in the populations. We noted that estimates on the standard deviations were likely not realistic given the limited data used to estimate them, so we analyzed the model further to determine the sensitivity of the model to the values of the standard deviations. We varied the deviations from the mean and determined the percentage of 10,000 simulations that had targeted population values that were no more than 25% larger or smaller than what is found in the dataset. We found that using randomly varying coefficients in the Lotka-Volterra model has the potential to produce results that capture predator-prey interactions better than simply using constant coefficients. (Received September 15, 2021)

1174-92-7600 **Margherita Maria Ferrari** (mmferrari@usf.edu), University of South Florida, **Lina Fajardo Gomez*** (lina_fajardo@hotmail.com), University of South Florida, **Masahico Saito** (saito@usf.edu), University of South Florida, and **Nataša Jonoska** (jonoska@mail.usf.edu), University of South Florida. *Directed Prodsimplicial Complexes Related to DNA Rearrangement* Preliminary report.

Double-occurrence words (DOWs), where every letter appears exactly twice, can be associated to genetic sequences where recombination happens. In this case, the insertion or deletion of certain subwords of DOWs models DNA rearrangement. A word graph is a graph which has DOWs as vertices, while edges indicate when a word can be obtained from another through the deletion of certain types of subwords. We study word graphs associated to the genome of a ciliate species and compute their homology groups using prodsimplicial complexes, which use Cartesian products of directed simplices as cells. The first homology group may represent pathways of recombination and be used to describe the complexity of these rearrangement processes. (Received September 15, 2021)

1174-92-7653 **Takao Sasaki*** (takao.sasaki@uga.edu), University of Georgia. *Psychology of animal groups: when and how collective intelligence emerges from individual cognition*

For over a century, researchers have been investigating collective cognition, in which a group of individuals together process information and act as a single cognitive unit. However, we still know little about circumstances under which groups achieve better (or worse) decisions than individuals. My research directly addressed this longstanding question, using the house-hunting ant, Temnothorax rugatulus, and the homing pigeon, Columba livia, as model systems. Here I applied concepts and methods developed in psychology not only to individuals but also to groups in order to investigate differences of their cognitive abilities. I combined experimental manipulations and models to elucidate the emergent processes of collective cognition. Our studies show that groups can achieve superior cognition by integrating or accumulating information from members using positive feedback. However, in certain circumstance, the same positive feedback can lock a group into a suboptimal choice. I will also briefly discuss my current projects about effects of individual differences on collective decision making. (Received September 15, 2021)

1174-92-7659 **James Burton Collins** (jbcolli2@gmail.com), University of Mary Washington, and **Madeline Slunt*** (maddyslunt@gmail.com), University of Mary Washington. *Herd Immunity Threshold in Small World Models* Preliminary report.

This study examines the mathematical definition of herd immunity threshold in contact networks, specifically, small-world networks. Herd immunity threshold has been defined in compartment-level models, disease transmission models that assume homogeneity of hosts and uniform mixing, as $HIT = 1 - \frac{1}{R_0}$, where R_0, is the number of secondary infections after a single infected case is introduced to a susceptible population. Small-world networks are graphed with nodes, representing people, and edges connecting two nodes. Transmission of disease between nodes is possible if and only if there is an edge connecting an infected node to a susceptible one. Each node is connected to its closest proximity neighbors and randomly selected nodes across the entirety of the graph to mimic different contacts people may have in everyday life. This poster will highlight experiments utilizing NetLogo where a single infected node was introduced into small-world networks of different parameters and analyzed to show consistencies between compartment-level models and these small-world networks. (Received September 21, 2021)

1174-92-7686 **Ian Murdock*** (idmurdock001@gmail.com), Western Connecticut State University. *Application of Wavelet Transforms for Optimized Molecular Space Modeling*

One of the main challenges facing traditional methods of generating chemical spaces to traverse has been the vast amounts of computing power needed to generate spaces with complex or lengthy molecules. The 3-dimensional input of the popular Long Short Term Memory cell architecture combined with explosive increases in vector length as molecules become more complex necessitates optimization. While it has a significant influence on the structure of the latent space, the SMILES representation was used due to its relatively simple nomenclature and widespread use. While methods such as principal component analysis (PCA) can reduce the dimension of the input data, it cannot be reconstructed and is not suitable for a one-hot vector, where each index corresponds to a token. Wavelets allow for the simple transformation and reconstruction of that data, even only if the approximation section is used, by utilizing an argmax function. While the data must be passed through the transform after it is vectorized and put through the inverse when it is to be read, the Coiflet 2-band transform, sharing similar properties to PCA, becomes more effective the longer each character vector becomes. This effectively "compresses" the representation of the SMILES into a significantly less intensive format, thus making an efficient method for training these models while preserving most of the properties of latent space of a model trained without this new method. (Received September 20, 2021)

1174-92-7702 **Linh Huynh*** (lxh390@case.edu), Case Western Reserve University. *Utilizing Stochasticity to Disambiguate Underlying Biological Mechanisms*

In studying the dynamics of drug resistance, many models have used net growth rates of cell populations. However, we have discovered that cell populations with the same net growth rate but different birth and death rates have dramatically different tendencies to escape extinction and develop drug resistance. Therefore, it is important to identify birth and death rates separately. We develop a method to parse out birth and death rates from cell count time series of populations that follow logistic birth-death processes. We validate our method on in-silico data generated using the tau-leaping approximation. With separate birth and death rates, we infer different underlying mechanisms and drug effects for the same observed population dynamics. From our results, we propose to replace a one-dimensional "fitness" phenotype (net growth rate) with a two-dimensional "fitness vector" phenotype (birth and death rates). (Received September 15, 2021)

1174-92-7768 **Briana Lynne Edwards** (bedwards2016@my.fit.edu), Florida Institute of Technology, **Vladislav Bukshtynov*** (vbukshtynov@fit.edu), Florida Institute of Technology, **Nick Huynh** (nhuynh2016@my.fit.edu), Florida Institute of Technology, and **Chris Bashur** (cbashur@fit.edu), Florida Institute of Technology. *Comprehensive Computational Framework for Modeling Physical and Biological Phenomena During Tissue Vascularization in Bioprinted Grafts* Preliminary report.

Tissue engineering of skin grafts is a promising tool to aid the body's healing process through its built-in regeneration mechanisms. A careful selection of graft parameters will ensure the restoration of healthy tissues. One of the promising methods consists of introducing template channels with controllable properties to provide faster microvasculature organization and transport of necessary nutrients to the cells outside these channels. It is critical, both for computational and experimental models, to understand how the biomaterial properties will impact angiogenesis and vasculogenesis in a graft since this situation is different than in native tissue. Essential parameters for modeling include the geometry of channels in the graft and its biomaterial properties, which can considerably alter the influence of swelling, the material degradation time, and the transport of dissolved substances through the grafts contributing positively to the cell viability. This collaboration between mathematicians and bioengineers aims to create a robust computational framework to simulate physical and biological phenomena in the graft material. We will present our first computational results obtained for multiple 2D models and discuss future applications to models using laboratory data. (Received September 16, 2021)

1174-92-7805 **Scott W Greenhalgh** (sgreenhalgh@siena.edu), Siena College, and **Ben Reale*** (bp11real@siena.edu), Siena College. *Simple stage-structured population models: An ODE approach based on reproductive potential* Preliminary report.

The mathematical modeling of population dynamics has provided researchers and ecologists with critical insights into the progression, control, and preservation of many species. Classically, these mathematical models are composed of systems of differential equations that abstractly categorize a species into distinct stages. However, these models have a major shortcoming in that their stage-structured formulations are typically unable to account for the variability in reproductive potential across a population. As the misspecification of such a biological characteristic is known to affect predictions, there is a need to develop new modeling approaches that capture this information. In this presentation, we illustrate a new class of stage-structured models that capture the variability in the reproductive potential of a species, while maintaining the convenient ODE model formulations that are the preferred choice of ecological modelers and population dynamics researchers. (Received September 16, 2021)

1174-92-7820 **Courtney Davis** (courtney.davis2@pepperdine.edu), Pepperdine University, and **Dev Patel*** (pate1281@purdue.edu), Purdue University. *A discrete mathematical model of the crayfish life cycle for predicting trapping efficacy* Preliminary report.

The red swamp crayfish (*Procambarus clarkii*) is an invasive species native to the southeastern United States that was introduced into multiple Santa Monica Mountain streams in California. Crayfish are decimating native species through predation, competition, and disease. To combat this invasive species, the Mountains Restoration Trust has regularly trapped crayfish for removal and recorded trapping data in Malibu Creek. We create a discrete mathematical model of the crayfish life cycle to explore how trapping different crayfish sizes and ages impact the effectiveness of crayfish removal. The mathematical model is a stage-and size-structured discrete model of crayfish eggs, juveniles, small adults, and large adults that takes into account cannibalism among crayfish. We simulate each life stage over time and examine the sensitivity of model predictions to crayfish survivorship and removal to understand which part of the life cycle most impacts crayfish population growth. Our modeling highlights that cannibalism of juveniles is a critical population control for crayfish. Model results also suggest that crayfish eradication may be possible with enough human effort. It also demonstrates that using trap mesh sizes small enough to trap juveniles may not be critical for population control. (Received September 16, 2021)

1174-92-7822 **Najat Ziyadi*** (najat.ziyadi@morgan.edu), Morgan State University. *A mathematical model of human papillomavirus (HPV) with vaccination* Preliminary report.

In this talk, we will introduce a mathematical model of human papillomavirus (HPV) with vaccination. Our model is a system of ordinary differential equations. We will use the next generation method to compute the basic reproduction number, and use sensitivity analysis to illustrate the impact of model parameters on HPV prevention and control. (Received September 16, 2021)

1174-92-7894 **Chris Cosner*** (gcc@math.miami.edu), University of Miami. *The ideal free distribution in temporally varying environments*

A population in a spatially heterogeneous environment is said to have an ideal free distribution if individuals distribute themselves so that the fitness of an individual is the same in all occupied locations. It has been shown that in various models for a single logistically growing population in a spatially varying but temporally constant environment, the dispersal strategies that are evolutionarily stable (a.k.a. evolutionarily steady) are those which produce an ideal free distribution. Recently such results have been extended to general periodic logistic reaction-advection-diffusion models and to integrodifference models with seasonal variation. For logistic reaction-advection-diffusion models and integrodifferential models in continuous time, it is possible for populations to achieve an ideal free distribution by using only local information about the environment, but in the time varying cases nonlocal information is necessary. This talk will describe results from a few papers related to these ideas. (Received September 16, 2021)

1174-92-7909 **Alexander Vladimir Novakovic*** (anovak97@bu.edu), Boston University, Department of Mathematics and Statistics, Boston University, Department of Biochemistry and Molecular Biology. *A Topological and Non-Euclidean Model of Biological and Viral Membranes* Preliminary report.

Membranes organize chemical reactions and form the boundary passageway of the most fundamental unit of life, the cell. This research presents novel mathematical models to evaluate the dynamic structure of cell and viral membranes, and describes the deformation and pinching of cell membranes during processes, such as cytokinesis, fertilization, and viral exocytosis. The membrane shape is determined by a variational principle, and life cycle properties and viral infection mechanism are evaluated physically and geometrically. The topological model of this work unveils the membrane dependence of the permeability and structural stability of membrane portions. Differential geometry is used to characterize the cell-level fusion and splitting, using Gaussian curvature. Bulk transport processes, understood using the fluid mosaic model of continuous membrane deformation, are examined in terms of geometric properties which remain invariant, especially topologically, under continuous changes. Laboratory digital media are analyzed computationally using Open Source Physics Tracker Software to extract dynamic geometrical properties of membranes and compare with the theoretical modeling. A case study proposal of COVID-19 unifies the deformation and differential geometry models is presented for the detectability of the exit of the prevalent virus from the host cell membrane. (Received September 16, 2021)

1174-92-7912 **Jerome Goddard*** (jgoddard@aum.edu), Auburn University Montgomery, and **Ratnasingham Shivaji** (r_shivaji@uncg.edu), University of North Carolina Greensboro. *Modeling density dependent dispersal and habitat fragmentation via reaction diffusion equations*

Dispersal is broadly defined as movement from one habitat patch to another and typically is considered to encompass three stages: 1) emigration, 2) inter-patch movement, & 3) immigration. Dispersal can have both beneficial and detrimental effects on the persistence of spatially structured systems. Recent empirical results indicate that certain organisms' emigration from a patch is dependent on density of their own species or even an interacting species—known as density dependent emigration. To date, little is known about the patch-level consequences of such dispersal strategies. In this talk, we will give a brief overview of density dependent emigration and its modeling history, discuss a framework built upon reaction diffusion equations designed to model patch-level effects of density dependent emigration, and share some recent advances. Several methods from nonlinear analysis will be employed such as time map analysis (quadrature method) and linearized stability analysis. (Received September 17, 2021)

1174-92-7949 **Ivana Bozic*** (ibozic@uw.edu), University of Washington. *Mathematical model of colorectal cancer initiation*

Cancer evolution cannot be observed directly in patients, and new methodologies are needed for obtaining a quantitative understanding of this obscure process. We developed and analyzed a stochastic model of malignant transformation in the colon that precisely quantifies the process of colorectal carcinogenesis in patients through loss of tumor suppressors APC and TP53 and gain of the KRAS oncogene. Our study employs experimentally measured mutation rates in the colon and growth advantages provided by driver mutations. We calculate the probability of a colorectal malignancy, the sizes of premalignant lesions, and the order of acquisition of driver mutations during colorectal tumor evolution. We demonstrate that the order of driver events in colorectal cancer is determined primarily by the fitness effects that they provide, rather than their mutation rates. (Received September 17, 2021)

1174-92-8030 **Kyle Dahlin*** (kyle.dahlin@uga.edu), University of Georgia, **Suzanne O'Regan** (smoregan@uga.edu), University of Georgia, **Barbara Han** (hanb@caryinstitute.org), Cary Institute of Ecosystem Studies, and **John Drake** (jdrake@uga.edu), University of Georgia. *Exploring the role of host traits on the transmission of mosquito-borne pathogens in wildlife populations* Preliminary report.

For over a century, mathematical models have been used to improve our understanding of the transmission of mosquito-borne diseases (MBDs) and to develop effective control methods for their elimination. Recently, the effect of temperature on MBD transmission has become a major focus as researchers contend with the impacts of global climate change. The emergence of infectious diseases out of wildlife populations and into human ones has also become a hot topic. However, such research has focused squarely on mosquitoes, their populations and traits, while mostly neglecting the role of the vertebrate host. Here we broadly discuss how host trait variation drives modeled transmission dynamics. In particular, we show that the predicted optimal temperature for transmission may shift with changes in host density when realistic contact rates are used. Furthermore,

incorporating realistic host trait variation can give us insights for targeting animal species for surveillance, to prevent future mosquito-borne disease epidemics. (Received September 17, 2021)

1174-92-8036 **Mya Austin*** (mya.austin@go.winona.edu), Winona State University, **Avery Kanel** (kanel.avery@uwlax.edu), University of Wisconsin - La Crosse, **James Peirce** (jpeirce@uwlax.edu), University of Wisconsin - La Crosse, River Studies Center, La Crosse, WI, and **Greg Sandland** (gsandland@uwlax.edu), University of Wisconsin - La Crosse, River Studies Center, La Crosse, WI. *Intergral Projection Model of the Gizzard Shad (*Dorosoma cependianum*) Incorporating Winter Temperature* Preliminary report.

The American gizzard shad (*Dorosoma cependianum*) is an important prey species for popular game fish. Winter kill is a survival bottleneck for shorter gizzard shad and can cause massive die-offs. Our goal was to incorporate winter temperature into an Integral Projection Model (IPM) to predict future population trends in gizzard shad. The survival function's inflection point for shad populations in pool four and twenty-six of the Mississippi River was found using the least square error method. A linear function was fitted to the pools and used to predict the inflection point of other pools based on early-spring average temperature. It was found that colder, more northern, pools had a higher inflection point than warmer, more southern, pools. Colder pools had a lower frequency of longer length fish suggesting that first year fish weren't able to survive the winter. Our results suggest gizzard shad in colder pools must reach longer lengths than individuals in warmer pools to survive winter. (Received September 17, 2021)

1174-92-8090 **Deena R. Schmidt*** (drschmidt@unr.edu), University of Nevada, Reno. *Contagion dynamics on an adaptive network with applications to norovirus and COVID-19* Preliminary report.

Classical contagion models, such as SIR, and other infectious disease models typically assume a well-mixed contact process. This may be unrealistic for infectious disease spread where the contact structure changes due to individuals' responses to the infectious disease. For instance, individuals showing symptoms might isolate themselves, or individuals that are aware of an ongoing epidemic in the population might reduce or change their contacts. Here we investigate contagion dynamics in an adaptive network context, meaning that the contact network is changing over time due to individuals responding to an infectious disease in the population. We generate the contact network from interaction data described in the well-known POLYMOD study. We consider norovirus as a specific example and investigate questions related to disease dynamics and applications to public health. We also briefly discuss how this framework applies to the recent spread of COVID-19. (Received September 17, 2021)

1174-92-8111 **Gideon Akumah Ngwa** (akumhed@yahoo.com), University of Buea, Cameroon, **Bime Markdonal Ghakanyuy** (bmarkdonal@gmail.com), University of Buea, Cameroon, University of Bamenda, Cameroon, and **Kristan A Schneider** (schneid2@hs-mittweida.de), Applied Computer and Bio-Sciences, University of Applied Sciences, Mittweida, Technikumplatz 17, 09648 Mittweida, Germany. *A Mathematical Study to Assess the Impact of Multiple Feeding Attempts on Mosquito Populations and Vector-Borne Disease Dynamics*

How do terrestrial female mosquitoes that fail in their attempt to draw blood from humans impact mosquito populations and vector-borne disease dynamics? When female mosquitoes interact with humans, they may succeed to obtain blood or fail in their attempt. For those waiting mosquitoes that failed but lived to try again and succeed, what impact do their second or later successful attempts have on mosquito populations and disease, and how can the results be exploited for control? We use a system of nonlinear differential equations derived based on the idea that the reproductive cycle of mosquitoes can be viewed as a set of alternating egg laying and blood feeding outcomes realized on a directed path called the gonotrophic cycle pathway, to investigate the aforementioned questions. Results from the model analyses reveal that waiting class mosquitoes contribute positively in sustaining mosquito populations as well as increase their interactions with humans via increased frequency and initial amplitude of oscillations. We identified a threshold parameter, the basic offspring number for mosquitoes, whose nature is affected by how we interpret the transitions involving the different classes on the gonotrophic cycle path. The trivial steady state for the system, which always exists, can be globally asymptomatically stable when the threshold parameter is less than unity. The non-trivial steady state, when it exists, is stable for a range of threshold values but can also be driven to instability via a Hopf bifurcation.
(Received September 17, 2021)

1174-92-8129 **Miranda Ijang Teboh-Ewungkem** (mit703@lehigh.edu), Lehigh University, Lehigh University, Bethlehem, PA, USA, and **Katharine F Gurski*** (kgurski@howard.edu), Howard University. *Seasonal Malaria Transmission: A Method to Connect Temperature and Mosquito Biology to Different Locales*

Changes in climatic temperatures observed over recent years have raised concerns over the potential effect on disease spread. Temperature is a well-known factor affecting mosquito population dynamics and malaria transmission disease dynamics. Our seasonal malaria model combines temperature data for mosquitos and parasites with regional temperature data to create a seasonal profile unique to each locale based on historical data without forcing a sinusoidal fit to the data. To do so, we introduce a spline methodology to incorporate temperature effects into our malaria model. We use the model to study the impact of seasonality on malaria transmission dynamics and burden in two regions, chosen because of their transmission setting - one a high transmission region and the other a low transmission region. We compute periodic and invasion threshold numbers for two strains of malaria parasites, one sensitive to drugs and one resistant, to shed light on disease dynamics. We present numerical simulations illustrating how climatic temperature changes will alter the monthly entomological inoculation rate and the number of malaria infections over the three years in these high and low transmission regions. (Received September 17, 2021)

1174-92-8148 **Junling Ma*** (junlingm@uvic.ca), University of Victoria. *The effect of harvesting adults on the evolution of reproduction age via density dependent juvenile mortality*

We used a generic two-stage population model to derive an adaptive dynamical system for the evolution of reproduction age, and studied how this evolution is driven by the harvest of adults. We considered the tradeoffs between maturation rate and fecundity, juvenile mortality, and adult mortality. We analyzed the benefit and cost of faster maturation under each tradeoff that drives the evolution. We found that harvesting adults affects the evolution of maturation by affecting the benefit. For the tradeoff between maturation and juvenile mortality, harvesting adults does not affect the benefit, and thus does not affect optimal maturation strategy. For the other two tradeoffs, harvesting adults affects the benefit through the equilibrium adult/juvenile ratio, which is determined by the density dependence of juveniles. Harvesting adults causes a slower maturation only if it significantly reduces this ratio, which can only happen with very strong adult protection to juveniles. Otherwise, harvesting adults always causes a faster maturation. (Received September 17, 2021)

1174-92-8171 **Bryan C. Daniels*** (bryan.daniels.1@asu.edu), Arizona State University. *Understanding the logic of adaptive collective behavior*

Neurons in a brain come together to make a decision; fish in a school efficiently alert one another about predators; proteins in a cell build information cascades that guide gene expression. We are impressed by the ability of collective living systems to produce coordinated, adaptive, and purposeful behavior at an aggregate scale while being controlled by a large number of noisy, unreliable components. What is the "logic" guiding their function?

With increasingly detailed data available across a number of living systems, we can now infer models that capture both the level of individual components and how their contributions scale up to affect aggregate function. Then, using insights from statistical physics, information theory, and dynamical systems theory, we ask: Are there patterns in these micro-macro maps?

Here, I connect three examples of collective behavior by arguing that the amplification of signals is a key collective variable that must be tuned to get functional behavior at the collective level. I will speculate about mechanisms that may carry out this tuning, from active regulatory feedback to longer-term evolutionary selection.
(Received September 17, 2021)

1174-92-8243 **Michael C Reed*** (reed@math.duke.edu), Duke University, **Janet Best** (best.82@osu.edu), The Ohio State University, and **H. Frederik Nijhout** (hfn@duke.edu), Duke University. *Mathematical Modeling for Public Health*

Each of us brings different variables to considerations of health and disease, our genotype, the environment that we live in, our diet, our patterns of exercise and the history of those variables (except genotype) in our lifetime, which influences the pattern of methylation on our genes. Statistical associations between these variables and disease outcomes can give some hints about what is important for health. But to really understand the causes of most diseases we need to understand the mechanisms by which the variables affect the cells of the body. This is a daunting task because of the complexity of physiological systems, the difficulty of experimentation, and large and significant differences between individuals. I will give two examples of current projects that use mathematics to understand physiological mechanisms. In the first, we will examine the connection between serotonin, depression, and histamine. In the second, we will discuss sex differences in one-carbon metabolism in the liver. (Received September 18, 2021)

1174-92-8251 **Necibe Tuncer** (ntuncer@fau.edu), Florida Atlantic University, **Maia Nenkova Martcheva** (maia@ufl.edu), University of Florida, and **Churni Gupta*** (churnibidisha@ufl.edu), University of Florida, University of Montreal. *A Multiscale Network model of HIV and Opioid coinfection*

This talk introduces a Network multi-scale immuno-epidemiological model of coinfection of HIV and opioid addiction. The within-host model gives the dynamics of HIV and the opioid in co-affected individuals. Analytical results suggest that besides the disease-free equilibrium, there is a unique equilibrium corresponding to HIV-only and a unique equilibrium corresponding to opioid-only. Each of the boundary equilibria is stable if the invasion number of the other disease is below one.Parameters were estimated using data on HIV and AIDS obtained from CDC, and data on Opioid overdosage, obtained from the National Center for Health Statistics. Different control strategies using fitted parameters to simulate the network model showed certain control strategies might be more beneficial than others. (Received September 18, 2021)

1174-92-8264 **Hyunjin Son*** (hson@swu.edu), Southern Wesleyan University. *Analysis and Optimal Control of a Deterministic Zika Virus Model*

In this paper, we consider a deterministic model explaining how Zika virus is transmitted between human and mosquito. The human population is divided into three groups as susceptible (x_1), infected (x_2), and treated (x_3). Similarly, the mosquito population is divided into susceptible (y_1) and infected (y_2) groups. First, we conduct the local and global stability of the disease-free and endemic equilibrium points in relation to the basic reproductive number. We also study the sensitivity of the basic reproductive number and the endemic equilibrium point with respect to each parameters used in the model. Furthermore, we apply optimal control theory to show that there are cost effective control methods with the prevention effort (u_1) of the contact between human and vector and the effort of treatment (u_2) for human. Finally, we provide numerical simulations to support and illustrate some of the theoretical results. (Received September 18, 2021)

1174-92-8267 **Elizabeth Balas** (ebalas04@gmail.com), Susquehanna University, and **Cameron Coles*** (colesc1@central.edu), Central College. *Using Integral Projection Models to Study Silver Carp Management Practices*

Invasive fishes have the potential to negatively alter native communities. One such species is the silver carp (*Hypophthalmichthys molitrix*), which has been spreading throughout the Mississippi River and its tributaries over the last few decades. Due to its ecological impacts, control methods, such as electric and carbon dioxide barriers and commercial harvesting, have been employed to reduce carp migration and prevent its introduction into the North American Great Lakes. Unfortunately, these control methods require substantial financial and labor investments, so it is critical that we understand the impacts of these measures before they are actually implemented .To do this, we established a meta-population integral projection model to evaluate the effectiveness of silver carp harvesting when fish have the potential to migrate between pools. We found that when silver carp were able to migrate, high harvesting rates were required in both pools to substantially decrease the population level within 50 years. However, when migration was constrained by barriers (such as carbon dioxide deterrents), harvesting in one pool was sufficient to substantially decrease carp numbers in that pool and a lower harvesting rate for both pools was needed to decrease carp numbers overall. We conclude that combining harvesting with migrational barriers may be the most effective method for controlling silver carp. (Received September 18, 2021)

1174-92-8321 **Shishi Luo*** (shishi.luo@gmail.com), Helix. *Evolution under multiple and opposing levels of selection*

The evolutionary dynamics of disease-causing pathogens occur both within each infected host and between hosts via transmission. A fast-replicating pathogen can outcompete other pathogens within the same host, but may be less transmissible due to the damage it causes the host. Similarly, in the evolution of cooperation, cheaters outcompete cooperators at the individual level while groups with fewer cheaters/more cooperators are favored at the group level. Here, I present the development and analysis of a Markov chain model of antagonistic natural selection acting at two scales. Simulations of this model as well as proofs of the scaling limit lead to general properties of this complex evolutionary phenomenon. We end by speculating on the implications these results have on the real world: i.e., what does this rigorous mathematical study tell us about the resilience of strategies that favor the greater good verus those that favor the individual? (Received September 18, 2021)

1174-92-8332 **Joan Ponce*** (joanponce90@gmail.com), UCLA. *An integrated framework for building trustworthy data-driven epidemiological models: Application to the COVID-19 outbreak in New York City*

The transmission dynamics of pandemics are often modeled by ordinary differential equations, which normally involve many undetermined parameters needed to be estimated from data. Often the available data are not sufficient to identify the model parameters and hence infer the unobserved dynamics. In this study, we provide a general framework, which includes identifiability analysis, sensitivity analysis, model robustness analysis, and uncertainty quantification, to examine the relationship between the model dynamics, data, and parameters. We find that we can uniquely estimate the model parameters and accurately project the daily new infection cases, hospitalizations, and deaths, in agreement with the available data from NYC's government's website. In addition, we employ the calibrated data-driven model to study the effects of vaccination and timing of reopening indoor dining in NYC. (Received September 18, 2021)

1174-92-8342 **Cvetelina Dimitrova Hill*** (cvasilev@kennesaw.edu), Kennesaw State University. *The steady-state degree and mixed volume of a chemical reaction network*

The steady-state degree of a chemical reaction network is the number of complex steady-states, which is a measure of the algebraic complexity of solving the steady-state system. In general, the steady-state degree may be difficult to compute. In this talk, we give an upper bound to the steady-state degree of a reaction network by utilizing the underlying polyhedral geometry associated with the corresponding polynomial system. We focus on three case studies of infinite families of networks, with emphasis on the multisite phosphorylation cycle, each generated by joining smaller networks to create larger ones. For each family, we give a formula for the steady-state degree and the mixed volume of the corresponding polynomial system. (Received September 18, 2021)

1174-92-8344 **Ashlee N. Ford Versypt*** (ashleefv@buffalo.edu), University at Buffalo, The State University of New York, **Mohammad Aminul Islam** (maislam4@buffalo.edu), University at Buffalo, The State University of New York, **Carley V. Cook** (carleyco@buffalo.edu), University at Buffalo, The State University of New York, and **Brenda J. Smith** (bjsmith@okstate.edu), Oklahoma State University. *Mathematical modeling of the gut-bone axis and implications of butyrate treatment on osteoimmunology*

Butyrate, a short-chain fatty acid produced by the gut microbiota, has pivotal roles in the regulation of the immune system. Recent studies have revealed that butyrate increases the differentiation of peripheral regulatory T cells in the gut-bone axis and promotes osteoblasts' bone forming activity. However, the mechanism of the therapeutic benefit of butyrate in bone remodeling remains incompletely understood. Here, we develop a multi-compartment mathematical model of the pharmacokinetics to quantitatively predict the contribution of butyrate on the expansion of regulatory T cells in the gut, blood, and bone compartments. We investigate the interplay between regulatory T cell-derived TGF-β and CD8+ T cell-derived Wnt-10b with changes in gut butyrate concentration. In addition, we connect our model to a detailed model of bone metabolism to study the impacts of butyrate and Wnt-10b on trabecular bone volume. Our results indicate both direct and indirect immune-mediated impacts of butyrate on bone metabolism. (Received September 18, 2021)

1174-92-8345 **Leif Zinn-Brooks*** (lzinnbrooks@hmc.edu), Harvey Mudd College, and **Marcus L Roper** (mroper@math.ucla.edu), UCLA. *Circadian rhythm shows potential for mRNA efficiency and self-organized division of labor in multinucleate cells*

Multinucleate cells occur in every biosphere and across the kingdoms of life. Data from filamentous fungi suggest that, even when bathed in a common cytoplasm, nuclei are capable of autonomous behaviors. How does this potential for autonomy affect the organization of cellular processes between nuclei? Here we analyze a simplified model of circadian rhythm in a mathematical model of the fungus *Neurospora crassa*. Our results highlight a potential role played by mRNA-protein phase separation to keep mRNAs close to the nuclei from which they originate, while allowing proteins to diffuse freely between nuclei. Our modeling shows that syncytism allows for extreme mRNA efficiency — we demonstrate assembly of a robust oscillator with a transcription rate 1000-fold less than in comparable uninucleate cells. We also show self-organized division of the labor of mRNA production, with one nucleus in a two-nucleus syncytium producing at least twice as many mRNAs as the other in 30% of cycles. This division can occur spontaneously, but division of labor can also be controlled by regulating the amount of cytoplasmic volume available to each nucleus. Taken together, our results show the intriguing richness and potential for emergent organization among nuclei in multinucleate cells. They also highlight the role of previously studied mechanisms of cellular organization, including nuclear space control and localization

of mRNAs through RNA-protein phase separation, in regulating nuclear coordination. (Received September 18, 2021)

1174-92-8426 **Duc Duy Nguyen*** (ducnguyen@uky.edu), University of Kentucky. *Topology, graph, and differential geometry-assisted AI for drug design*

Nowadays, machine learning, especially deep learning, is the main workhorse in many applications from speech recognition to molecular design. However, identifying the most promising drug candidates combating the deadly and newly emerging diseases such as COVID-19 still poses a big challenge for any AI-based models due to the high dimensionality of the biological datasets and the lack of properly encoded chemical and physical information. To address these issues, my lab has developed several powerful mathematical representations for the diverse biological datasets in the low-dimensional spaces, namely persistent graph theory, persistent homology, and differential geometry. By carefully designing cutting edge convolutional neural networks, graph neural networks, and recurrent neural networks integrating with attention and transformer gate mechanisms to tailor these mathematical features, we arrived at novel models not only perform well on virtual screening targeting important drug properties but also have the ability to design new drugs at an unprecedented speed. In recent years, our team has emerged as a top winner in D3R Grand Challenges, a worldwide annual competition series in computer-aided drug design. (Received September 19, 2021)

1174-92-8450 **Kristina Wicke*** (wicke.6@osu.edu), The Ohio State University, **Mareike Fischer** (mareike.fischer@uni-greifswald.de), University of Greifswald, and **Andrew Francis** (A.Francis@westernsydney.edu.au), Western Sydney University. *Phylogenetic Diversity Rankings in the Face of Extinctions: the Robustness of the Fair Proportion Index*

Planning for the protection of species often involves difficult choices about which species to prioritize, given constrained resources. One way of prioritizing species is to consider their "evolutionary distinctiveness", i.e. their relative evolutionary isolation on a phylogenetic tree. Several evolutionary isolation metrics or phylogenetic diversity indices have been introduced in the literature, among them the so-called Fair Proportion (FP) index. This index apportions the total diversity of a tree among all leaves, thereby providing a simple prioritization criterion for conservation.

Here, we focus on the prioritization order obtained from the FP index and analyze the effects of species extinction on this ranking. More precisely, we analyze the extent to which the ranking order may change when some species go extinct and the FP index is re-computed for the remaining taxa. We show that for each phylogenetic tree, there are edge lengths such that the extinction of one leaf per cherry completely reverses the ranking. Moreover, we show that even if only the lowest ranked species goes extinct, the ranking order may drastically change.

We end by analyzing the effects of these two extinction scenarios (extinction of the lowest ranked species and extinction of one leaf per cherry) for a collection of empirical and simulated trees. In both cases, we can observe significant changes in the prioritization orders, highlighting the empirical relevance of our theoretical findings. (Received September 19, 2021)

1174-92-8478 **Sedar Ngoma*** (ngoma@geneseo.edu), SUNY Geneseo. *Investigation of a HIV/AIDS epidemic model with a time delay and information campaigns*

We study a time-delayed HIV/AIDS epidemic model with education campaigns and investigate the asymptotic behavior of the endemic equilibrium with respect to the amount of information disseminated about the disease. We show that if the basic reproductive number is less than or equal to one, the disease will be eradicated in the long run. On the other hand, if the basic reproductive number is bigger than one, we show that the disease will be permanent but its impact on the population could be significantly minimized if the amount of information disseminated is increased. Moreover, we fit the model to real-world data on HIV and estimate the parameters of the model. We then use the estimated parameters to present numerical simulations. In particular, we show that the larger the time delay, the longer it takes for the infected population to stabilize at the endemic equilibrium. (Received September 19, 2021)

1174-92-8484 **Huijing Du*** (hdu5@unl.edu), University of Nebraska-Lincoln. *Multiscale modeling of epidermal-dermal interactions during skin wound healing*

Following injury, skin activates a complex wound healing programme. While cellular and signalling mechanisms of wound repair have been extensively studied, the principles of epidermal-dermal interactions and their effects on wound healing outcomes are only partially understood. To gain new insight into the effects of epidermal-dermal interactions, we developed a multiscale, hybrid mathematical model of skin wound healing. The model takes into consideration interactions between epidermis and dermis across the basement membrane via diffusible signals,

defined as activator and inhibitor. Simulations revealed that epidermal-dermal interactions are critical for proper extracellular matrix deposition in the dermis, suggesting these signals may influence how wound scars form. Our model predicts the important role of signalling across dermal-epidermal interface and the effect of fibrin clot density and wound geometry on scar formation. This hybrid modelling approach may be also applicable to other complex tissue systems, enabling the simulation of dynamic processes, otherwise computationally prohibitive with fully discrete models due to a large number of variables. (Received September 19, 2021)

1174-92-8485 **Seth Cowall*** (sethcowall@comcast.net), St. Mary's College of Maryland, **Matthew Oliver** (moliver@udel.edu), University of Delaware, and **Pamela Cook** (cook@udel.edu), University of Delaware. *Data-driven dynamics of phytoplankton blooms in a reaction-diffusion NPZ model*

Phytoplankton are the base of the marine food web. They are also responsible for much of the oxygen we breathe, and they remove carbon dioxide from the atmosphere. The mechanisms that govern the timing of seasonal phytoplankton blooms is one of the most debated topics in oceanography. Here, we present a macroscale plankton ecology model consisting of coupled, nonlinear reaction-diffusion equations with spatially and temporally changing coefficients to offer insight into the causes of phytoplankton blooms. This model simulates biological interactions between nutrients, phytoplankton and zooplankton. It also incorporates seasonally varying solar radiation, diffusion and depth of the ocean's upper mixed layer because of their impact on phytoplankton growth. The model's predictions are dependent on the dynamical behavior of the model. The model is analyzed using seasonal oceanic data with the goals of understanding the model's dependence on its parameters and of understanding seasonal changes in plankton biomass. A study of varying parameter values and the resulting effects on the solutions, the stability of the steady-states, and the timing of phytoplankton blooms is carried out. The model's simulated blooms result from a temporary attraction to one of the model's steady-states. (Received September 19, 2021)

1174-92-8486 **Sherli Koshy-Chenthittayil** (koshychenthittayil@uchc.edu), Center for Quantitative Medicine, University of Connecticut School of Medicine, **Jennifer Kim*** (jennifer.3.kim@uconn.edu), Undergraduate Program in Biomedical Engineering, University of Connecticut, and **Pedro Mendes** (pmendes@uhc.edu), Richard D. Berlin Center for Cell Analysis and Modeling, University of Connecticut School of Medicine. *Numerical Identification of Chaos in Dynamical Nonlinear Models by using COPASI* Preliminary report.

Chaos is a type of motion found in nonlinear dynamical systems that is highly sensitive to initial conditions. While many biological systems described in literature have been analyzed for the presence of chaotic dynamics, there are still many that are yet to be discovered. Models that have identified chaos can also be reassessed to explore various parameter ranges and initial conditions. The strategy adopted here is based on the Shimada-Benettin algorithm to numerically calculate the Lyapunov exponent (LE) spectrum of ODE models; which is then coupled with a numerical optimization algorithm to search for regions in parameter space that have positive LEs, indicating that they contain strange attractors. The open-source software COPASI allows us to couple the Shimada-Benettin algorithm to several optimization algorithms, such as genetic algorithm, simulated annealing, particle swarm, and many others. Often, the estimates of LEs have numerical errors that can dominate over small positive LEs. Thus, we explored several objective functions to determine which identify chaotic regions better, rather than falsely identify other attractors. These include a) maximization of the largest LE, b) minimization of the square of the second LE, c) maximizing the ratio of the largest LE over the absolute value of the second LE. This methodology was then applied to models which are known to contain strange attractors to assess the efficiency of various combinations of objective functions and optimization algorithms. (Received September 19, 2021)

1174-92-8524 **Santosh Manicka** (Santosh.Manicka@tufts.edu), Tufts University, and **Kathleen Johnson** (kgjo228@g.uky.edu), University of Kentucky. *A probabilistic logic generalization approach reveals that biological regulatory networks are less nonlinear than expected*

Nonlinearity is a characteristic of complex biological regulatory networks that has implications ranging from therapy to control. To better understand its nature, we analyzed a suite of published Boolean network models, containing a variety of complex nonlinear interactions, with an approach involving a probabilistic generalization of Boolean logic that George Boole himself had proposed. We leveraged the continuous-nature of this formulation using Taylor-decomposition methods to reveal the distinct layers of nonlinearity of the models. We then compared the resulting series of model-approximations with the corresponding sets of randomized ensembles and found that the biological networks are relatively more linearly approximable, suggesting that they may have been shaped by evolution to be linear perhaps for the purpose of controllability. (Received September 19, 2021)

1174-92-8551 **Nicole Pagane** (npagane@rockefeller.edu), The Rockefeller University, **Viviana I. Risca** (vrisca@rockefeller.edu), The Rockefeller University, **Ariana Brenner Clerkin*** (abrenner@rockefeller.edu), The Rockefeller University, **Devany West** (dwest@rockefeller.edu), The Rockefeller University, **Bruno Beltran** (brunobeltran0@gmail.com), Biophysics Program, Stanford University, and **Quinn MacPherson** (quinnmacp@gmail.com), Department of Physics, Stanford University. *Characterizing Chromatin Fiber Conformations Enriched in Epigenetic States* Preliminary report.

Sub-kilobase chromatin fiber structure determines the local fiber geometry and its accessibility to transcriptional machinery. Factors such as DNA linker length, biochemical histone modifications, and the degree of DNA wrapping around nucleosomes affect fiber geometries. We use a chromatin Monte Carlo simulation framework which approximates DNA as a stretchable-shearable worm-like chain as well as a Brownian Dynamics based chromatin structural model to generate chromatin structure ensembles for comparison against the spatially correlated cleavage data from RICC-seq, a method for measuring DNA-DNA contacts in mammalian cells. We simulate mesoscale chromatin fibers and predict RICC-Seq breakages to compute fragment length distributions, which permits comparison of simulated fibers to experimental data from cells. We then characterize features of the fiber geometries to identify epigenetic state-specific chromatin conformations. (Received September 20, 2021)

1174-92-8598 **Presley Kimball*** (presleykimball@creighton.edu), Creighton University, **Amy Elizabeth Moore** (amymoore1710@yahoo.com), Elon University, **Jacob Levenson** (levensonj22@mail.wlu.edu), Washington and Lee University, **Jan Rychtar** (rychtarj@vcu.edu), Virginia Commonwealth University, and **Dewey Taylor** (dttaylor2@vcu.edu), Virginia Commonwealth University. *An ODE model of yaws elimination in Lihir Island, Papua New Guinea*

Yaws is a chronic infection that affects mainly the skin, bone and cartilage and spreads mostly between children. The new approval of a medication as treatment in 2012 has revived eradication efforts and now only few localized foci of infection remain. The World Health Organization strategy mandates an initial round of total community treatment (TCT) with single-dose azithromycin followed either by further TCT or by total targeted treatment (TTT), an active case-finding and treatment of cases and their contacts. We develop the compartmental ODE model of yaws transmission and treatment for these scenarios. We solve for disease-free and endemic equilibria and also perform the stability analysis. We calibrate the model and validate its predictions on the data from Lihir Island in Papua New Guinea. We demonstrate that TTT strategy is efficient in preventing outbreaks but, due to the presence of asymptomatic latent cases, TTT will not eliminate yaws within a reasonable time frame. To achieve the 2030 eradication target, TCT should be applied instead. (Received September 19, 2021)

1174-92-8604 **Leah Shaw** (lbshaw@wm.edu), William & Mary, and **Wei Wang*** (wwang17@email.wm.edu), William & Mary. *Within-host Model of Poliovirus Control by Defective Interfering Particles and Interferon* Preliminary report.

Poliovirus is a single-stranded RNA virus which causes poliomyelitis. Although vaccines exist, poliovirus is still not fully eradicated in areas with limited health resources. In addition, in recent years there are increasing cases of vaccination-derived strains of poliovirus. We model an alternative therapeutic strategy to assist the polio eradication through the usage of defective interfering particles (DIPs), a defective variant of wild-type (WT) virus lacking a protein capsid essential for viral replication. Co-infection of DIPs and WT forms a competitive relationship which decreases the overall viral load. Previous modeling through ordinary differential equations shows an unrealistically small number of DIPs at the initial stage of infection. It also shows that viral load can be decreased through the introduction of DIPs, but a complete clearance of virus can not be achieved by DIPs alone. We conduct stochastic simulation through the Gillespie algorithm at the initial infection stage and conclude that DIPs are able to persist with enough initial DIPs. In addition, we introduce interferon as a representative of the immune system to examine the possibility of complete clearance. The implementation time and initial dosage of DIPs will be varied to find an optimal plan. (Received September 20, 2021)

1174-92-8620 **Annalisa Scimemi** (scimemia@gmail.com), SUNY Albany. *Estimating glutamate transporter surface density in mouse hippocampal astrocytes*

One of the main functions of astrocytes is to remove glutamate from the extracellular space, a task that is accomplished through the activity of glutamate transporters expressed in abundance in the plasma membrane.

This property allows astrocytes to limit glutamate diffusion out of the synaptic cleft, to limit extrasynaptic receptor activation and preserve the spatial specificity of synaptic transmission. The distribution of glutamate transporters on is known to be heterogeneous, as these molecules are enriched in astrocyte tip processes as opposed to the rest of the membrane. We investigate in depth the effect of this non-uniform distribution, while also evaluating how local crowding effects can limit the transporter expression in small astrocytic processes.

We generate a geometric model of astrocytes that capture statistically the main structural features of real astrocytes, to help estimate the proportion of the astrocyte cell membrane in different cellular compartments. We found stark differences in the density of expression of transporter molecules in different compartments, indicating that the extent to which astrocytes limit extrasynaptic glutamate diffusion depends not only on the level of astrocytic coverage, but also on the identity of the compartment in contact with the synapse. We discuss the potential long-range implications of these findings on the fields of synaptic plasticity and astrocyte physiology. (Received September 19, 2021)

1174-92-8634 **Adeethyia Shankar*** (adeethyia@gmail.com), Danbury Math Academy, Western Connecticut State University, **James Cao** (jamescaowork@gmail.com), Danbury Math Academy, Western Connecticut State University, and **Stephanie Chang** (stephanie.yien.chang@gmail.com), Danbury Math Academy, Western Connecticut State University. *Wavelet-Based Machine Learning Approaches Toward Precision Medicine in Diabetes*

It is estimated that 422 million people around the world have diabetes mellitus (DM)—a devastating, complex, and highly heterogeneous disease—requesting better interventions based on disease subtyping. In this research, we utilize the discrete wavelet transform (WT) to decompose and denoise DM data. Using WT, we enhance heart rate variability (HRV) based DM diagnosis, data visualization of the disparities in Human Microbiome Project (HMP) data (gut bacteria, metabolomics, proteomics, RNA sequencing, targeted proteomics, and transcriptomics data) using demographic features, and insulin resistance prediction. We also attempt to forecast continuous glucose monitoring (CGM) ahead by 90 minutes because CGM is unable to provide real-time blood glucose measurements. We achieve 91.9% diagnosis accuracy for Type 1 DM using Random Forest on data transformed with WT, holding the potential for usage in clinics. In addition, our WT-based t-SNE and UMAP explorative analysis of HMP data support subtypes of prediabetic patients stratified by sex, race, and age. Moreover, WT-based transformations on metabolomics, proteomics, RNA sequencing, targeted proteomics, and transcriptomics data outperform those without WT. Taken together, WT-based machine learning approaches enable a fine resolution of subtyping DM towards precision medicine. (Received September 19, 2021)

1174-92-8638 **Anthony Jake Cortez*** (acortez0042@mail.fresnostate.edu), California State University, Fresno. *A Two-Dimensional Model of Three Vital Cardiac Functions* Preliminary report.

The common understanding of the heart describes it as a pump that works to move blood around the body. While conceptually accurate, the function of the heart is far more complicated than this. The goal of this work is to add a level of detail to this everyday view by forming a two-dimensional numerical model to provide a deeper qualitative understanding of the heart. In its completed state, the model will depict the propagation of electrical waves that trigger the contraction of simulated muscle, which pump simulated fluid through the virtual heart. Electrical wave propagation, muscle contraction, and fluid flow, constitute the three parts of the model that we are proposing. These processes will be modeled with the FitzHugh-Nagumo (FHN) model, a mass-spring model, and Smoothed Particle Hydrodynamics (SPH), respectively. Thus far all components have been developed to run independently in real time. Future work will couple the methods to form a beating two dimensional heart. Coupling these popular methods should produce a real-time simulation that provides a good qualitative picture of the specified processes and gives a framework for scaling the program to a more quantitatively accurate three-dimensional version. (Received September 19, 2021)

1174-92-8733 **Somayeh Mashayekhi*** (smashayekhi@fsu.edu), Kennesaw State University, and **Peter Beerli** (pbeerli@fsu.edu), Florida State University. *Fractional coalescent*

Fractional calculus has attracted considerable interest because of its ability to model biological systems such as population genetics. The time-fractional generalizations of the Poisson process, which are based on the fractional Kolmogorov-Feller equation where a fractional-order derivative operator replaces the integer-order derivative operator, play a critical role in deriving the fractional coalescent in population genetics where the order of the fractional derivative shows the environmental heterogeneity in the population. In this talk, the fractional coalescent will be introduced, and the properties of this new model will be presented. (Received September 19, 2021)

1174-92-8827 **Erica Marie Rutter** (erutter2@ucmerced.edu), University of California Merced, **Suzanne Sindi** (ssindi@ucmerced.edu), University of California Merced, **Shilpa Khatri** (skhatri3@ucmerced.edu), University of California Merced, and **Fabian Santiago** (fsantiago@math.arizona.edu), University of Arizona. *Assessing Re-Opening Strategies for Mitigating COVID-19 Transmission Dynamics on A College Campus* Preliminary report.

Nearly every higher-education institute rapidly transitioned all courses from face-to-face instruction to online instruction in March 2020, shortly after the World Health Organization declared the 2019 novel coronavirus outbreak (COVID-19) as a pandemic. COVID-19 is still an ongoing public health emergency of international focus. Mathematical modeling can be used to analyze and predict the spread of COVID-19 as well as evaluate the effectiveness of disease mitigation strategies, which will help educational institution leaders with decisions of whether to reopen schools. In Summer 2020, we used a model for structured bubble-like institutions to evaluate Fall 2020 reopening strategies (e.g., class-size caps, mask-use, and housing) for University of California (UC) Merced. In this model individuals within the community have complex structured interactions defined by their roles but, rather than a bubble, the boundaries between the environment are porous and certain types of individuals intermix freely within a larger surrounding community. We seek to answer whether undertaking strong disease mitigation measures on campus alone would prevent COVID-19 to enter the UC Merced campus population, or both the campus and surrounding community should adhere to strict social distancing. Our model was also used to study the spread of COVID-19 on UC Merced campus under a Fall 2021 return with different vaccination rates among campus populations. (Received September 19, 2021)

1174-92-8853 **Jim Michael Cushing*** (cushing@math.arizona.edu), University of Arizona. *Does Darwinian evolution select against chaos?* Preliminary report.

Despite the fact that population models quite commonly predict the possibility of chaotic dynamics, unequivocal evidence for the occurrence of chaos in biological populations is very rare. There are several hypotheses as to why this is the case, one of which is that natural selection works against it. We investigate this hypothesis using Darwinian dynamic (evolutionary game theoretic) versions of the iconic Ricker equation. (Received September 20, 2021)

1174-92-8877 **Sarah Dianne Olson*** (sdolson@wpi.edu), Worcester Polytechnic Instituite. *Centrosome Movement During Mitosis*

Proper formation and maintenance of the mitotic spindle is required for faithful cell division. While much work has been done to understand the roles of the key molecular components of the mitotic spindle, identifying the consequences of force perturbations in the spindle remains a challenge. We develop a computational framework to account for centrosome movement within the cytoplasm and utilize live cell imaging to inform and validate the model. Specifically, we investigate the role of cortical dynein on spindle pole length fluctuations. (Received September 20, 2021)

1174-92-8959 **Adriana T Dawes*** (dawes.33@osu.edu), Ohio State University. *Uncovering the biochemical network regulating dynein activity in the early* C. elegans *embryo* Preliminary report.

Asymmetric cell division, where daughter cells inherit unequal amounts of specific factors, is critical for fundamental biological processes during development such as cell fate specification. In polarized cells, where specific factors are segregated to opposite ends of the cell, as seen in early embryos of the nematode worm *C. elegans*, asymmetric cell division occurs as a result of dynein-mediated centrosome positioning along the polarity axis. However, the biochemical signaling network regulating dynein activity is not fully resolved. Using a novel measure to characterize centrosome movement defects, we demonstrated that seemingly unrelated proteins give rise to similar centrosome movement patterns, despite no change in dynein localization. Predictions of the mathematical models are being used to direct experimental efforts to identify and characterize the biochemical signaling network responsible for dynein activity and centrosome positioning in the early *C. elegans* embryo. (Received September 20, 2021)

1174-92-8961 **Bin Xu*** (bxu@clarkson.edu), Clarkson University. *Modeling the oscillatory dynamics of Cdc42 in fission yeast* Preliminary report.

Regulation of polarized cell growth is essential for many cellular processes, including spatial coordination of cell morphology changes during growth and division. We present a mathematical model for the oscillatory dynamics of small GTPase Cdc42 in fission yeast. The model is based on the competition of growth zones of Cdc42 for a common substrate that diffuses in the cytosol and on potential positive and negative feedback between Cdc42 and its GEF. We analyze the bifurcations in a reduced ODE model as the cell length increases, and total amount of Cdc42 and GEF increase. We then use stochastic simulation algorithms to explore the effect of intrinsic noise on Cdc42 oscillations and compare the results with the dynamics in the deterministic counterpart. Finally, we explore how the Cdc42 and GEF diffusion rates affect the oscillatory dynamics. (Received September 20, 2021)

1174-92-8966 **Anita Layton** (anita.layton@uwaterloo.ca), University of Waterloo, and **Melissa Stadt*** (mstadt@uwaterloo.ca), University of Waterloo. *Adaptive Changes in GFR, Tubular Morphology and Transport in Pregnant Rat Kidneys: Modeling and Analysis*

Normal pregnancy is characterized by massive changes in plasma volume and electrolyte retention. Given kidneys regulate electrolytes and volume homeostasis, major adaptations in morphology, hemodynamics, and transport occur to achieve the volume and electrolyte retention required in pregnancy. These adaptations are complex, sometimes counterintuitive, and not fully understood. Furthermore, the demands of the developing fetus and placenta change throughout the pregnancy. Hence, renal adaptations are different during the various stages of pregnancy. The goal of this study is to analyze how gestational volume and electrolyte requirements are met by coordinated adaptive changes in the rat kidney using computational modeling. We developed epithelial cell-based computational models of solute and water transport in the nephrons of the kidney for a rat in mid- and late pregnancy to compare to a female-specific (non-pregnant) model. Subsequent simulations and sensitivity analysis were used to quantify which adaptations may be most essential to meet the increased volume and electrolyte demands during pregnancy. (Received September 20, 2021)

1174-92-8979 **Ciana Applegate*** (ciana.applegate@louisville.edu), University of Louisville, and **Dan Han** (dan.han@louisville.edu), University of Louisville. *Dynamical SIR Model with Migration* Preliminary report.

In this paper, an SIR (Susceptible- Infectious- Recovered) model with migration will be studied. A traditional SIR model used in epidemiology describes the transition of particles among states, such as susceptible, infected, and recovered states. However, the traditional model has no movement of particles. There are many variations of SIR models when it comes to the factor of mobility, the majority of studies use mobility intensity or population density as a measure of mobility. In this paper, a new dynamical SIR model, including the spatial migration of three-type particles, is constructed and the long-time behavior of the first and second moments of this dynamical system are analyzed. (Received September 20, 2021)

1174-92-8990 **Kelly Buch*** (kbuch@vols.utk.edu), University of Tennessee/ Knoxville. *Modeling the management of vector-borne tree diseases*

Non-native, lethal forests pathogens introduced via international trading routes pose a serious threat to forest ecosystems. After introduction, novel pathogens spread quickly and have devastating effects on susceptible tree populations. Pathogens which are introduced along with an insect vector have especially devastating effects since the mobility of the insect substantially increases the spread of the disease. To eradicate the forest pathogen and/or control the spread of the pathogen-vector duplex, rapid and effective management is necessary. Any management of a forest system is expensive and labor-intensive, so only methods with high chances of success should be used. Using Laurel Wilt as a model system, we consider various management strategies including application of fungicides on susceptible trees or size-based removal of infected or susceptible trees. We test these strategies for effectiveness in reducing disease, lowering vector population, and maintaining host population in a vector-borne disease system by integrating management strategies into our mathematical model for lethal, vector-borne tree diseases. We analyze the existence and stability of equilibrium the ODE system and use this analysis to make recommendations to forest managers on how to effectively eradicate an introduced vector-borne tree disease. (Received September 20, 2021)

1174-92-8995 **Gulsah Yeni*** (gzy5088@psu.edu), Pennsylvania State University, and **Jessica Conway** (jmconway@psu.edu), Pennsylvania State University. *Mechanistic modeling of PrEP on-demand* Preliminary report.

Prevention is one of the four pillars of the "Ending the HIV Epidemic in the U.S." plan to reduce new HIV transmissions using pre-exposure prophylaxis (PrEP). Daily adherence of Truvada for PrEP has been shown to reduce the risk of HIV infection effectively. Despite its advantages, daily adherence to Truvada can be straining and studies suggest that short-term use around the time of exposure may be just as effective at reducing HIV risk. However, the most effective on-demand Truvada dosing strategies have yet to be determined. Therefore, we investigate such "on-demand" or "event-based" use of PrEP strategies via a mathematical model. We incorporate previously developed models of pharmacokinetics and pharmacodynamics (PK/PD) of Truvada into a stochastic model of early HIV infection in-host. To evaluate PrEP dosing strategy effectiveness, we simulate strategies by sampling a virtual population and performing extensive sensitivity analyses. Our aim is to identify practical

dosing strategies that most effectively reduce risk of HIV infection in different contexts, such as exposure via sexual contact or infectious drug use, and repeated HIV exposure. (Received September 21, 2021)

1174-92-9041 **Suzanne Lenhart** (slenhart@utk.edu), University of Tennessee, Knoxville, **Lindsey Fox*** (foxl@eckerd.edu), Eckerd College, **Cara Sulyok** (csulyok@lewisu.edu), Lewis University, **Hannah Ritchie** (hjritchi@ncsu.edu), North Carolina State University, **Cristina Lanzas** (clanzas@ncsu.edu), North Carolina State University, and **Judy Day** (judyday@utk.edu), University of Tennessee, Knoxville. *Mathematically modeling the effect of touch frequency on the environmental transmission of Clostridioides difficile in healthcare settings*

Clostridioides difficile is the leading cause of infectious diarrhea and one of the most common healthcare acquired infections in United States hospitals. *C. difficile* persists well in healthcare environments because it forms spores that can survive for long periods of time and can be transmitted to susceptible patients through contact with contaminated fomites.

The mathematical model in this study investigates the relative contribution of high-touch and low-touch fomites on new cases of *C. difficile* colonization among patients of a hospital ward. The dynamics of transmission are described by a system of ordinary differential equations including two pathogen environmental reservoirs. Parameters that have a significant effect on incidence, as determined by a global sensitivity analysis, are varied in stochastic simulations of the system to identify feasible strategies to prevent disease transmission.

Results indicate that on average, over three-quarters of asymptomatically colonized patients are exposed to *C. difficile* via high-touch fomites, despite additional cleaning of high-touch fomites. Results also suggest that enhanced efficacy of disinfection upon discharge and extra cleaning of high-touch fomites, reduced contact with high-touch fomites, and higher discharge rates, among other control measures, could lead to a decrease in the incidence of colonized individuals. (Received September 20, 2021)

1174-92-9042 **Alexanderia Lacy*** (alacy4@vols.utk.edu), University of Tennessee/ Knoxville. *The Modeling and Control of an Invasive Species* Preliminary report.

The invasive species zebra mussels have caused significant damages and changes to the waters that host them. By using a hybrid population model with discrete-time equations and ordinary differential equations, we can represent the zebra mussel's life cycle and the spread of the species. The goal is to build a mechanistic model to investigate the effects of zebra mussel movement between different spatial locations and control their spread. This model will connect with feasible control actions, particular locations, and data. We design optimal control actions in our model to decrease zebra mussel population levels and the spatial spread. We will illustrate numerically the hybrid population model for particular scenarios. (Received September 20, 2021)

1174-92-9046 **Abigail Basener*** (basenerag24@mail.vmi.edu), Virginia Military Institute. *Identifying Images of Glycans with Neural Networks*

Gylcans are biomolecules that help regulate cell structure. Many researchers study them to better understand how the human body works using a database called GlyGen. GlyGen allows users to look up valuable information on glycans using an associated ID number. However, when looking at images of glycans in research papers there is not always an ID number listed. To spare the time-consuming process of finding the ID number, we created an application for Glygen that allows users to search the database using an image. This application uses image classification algorithms and neural networks. Building this application involved four main parts of machine learning: classes, data, features, and a method. For training the method we used known images from the glycan database. However, this set only contained a single image of each glycan. To work with this extremely small data set, we distorted the images to increase the data set. Finally, we applied a neural network to the distorted images of glycans and images of glycans from research papers. This application is currently on the Glygen website. https://beta-data.glygen.org/upload (Received September 20, 2021)

1174-92-9069 **Mario Banuelos*** (mbanuelos22@csufresno.edu), California State University, Fresno, and **Marissa Hernandez** (marissahernandez98@gmail.com), California State University, Fresno. *Machine Learning Approaches for Detecting Higher-Order Genomic Interactions*

An organism's genome, the complete set of genetic material, play a significant role in their phenotype, or observable traits. Various methods exist in identifying, predicting, and understanding how genomic changes may affect an organism. When more than one mutation, or single nucleotide polymorphisms (SNPs), affects an individual's quantitative or qualitative phenotype, the contribution is rarely additive. In most cases, one single SNP rarely completely describes the complexity of the measured phenotype. Relying on analyzing an exponential number of combinations of these mutations is often not possible and remains a challenge in this field. To address such computational bottlenecks, we propose a matrix-factorization and neighborhood-based collaborative filtering approach. We view this data with a recommender system formulation. As such, we are able to detect statistically significant higher order SNP interaction phenotypes and present our results with respect to muscle mice genomic variants. (Received September 20, 2021)

1174-92-9092 **Anne Joyce Shiu*** (annejls@gmail.com), Texas A&M University. *MSRI-UP 2022: Algebraic Methods in Mathematical Biology*

The MSRI Undergraduate Program (MSRI–UP) is a summer program for undergraduate students – especially those from groups that are underrepresented in mathematics – who wish to conduct research in the mathematical sciences. The theme of the upcoming 2022 MSRI-UP is "Algebraic Methods in Mathematical Biology", and this talk will highlight possible research projects that students will pursue. These research projects will be chosen from three areas of mathematical biology, with applications to biochemistry, neuroscience, and pharmacology. (No background in biology is required of undergraduate participants – just a willingness to learn!) (Received September 20, 2021)

1174-92-9120 **Chuntian Wang** (cwang27@ua.edu), The University of Alabama, **Kaiyan Peng*** (kaiyanpeng@gmail.com), UCLA, **Andrea Bertozzi** (bertozzi@math.ucla.edu), UCLA, **Zheng Lu** (zhenglu@math.wisc.edu), University of Wisconsin Madison, **Michael Lindstrom** (mikel@math.ucla.edu), UCLA, and **Christian Parkinson** (chparkin@math.arizona.edu), University of Arizona. *A Multilayer Network Model of the Coevolution of the Spread of a Disease and Competing Opinions*

During the COVID-19 pandemic, conflicting opinions on physical distancing swept across social media, affecting both human behavior and the spread of COVID-19. Inspired by such phenomena, we construct a two-layer multiplex network for the coupled spread of a disease and conflicting opinions. We model each process as a contagion. On one layer, we consider the concurrent evolution of two opinions—pro-physical-distancing and anti-physical-distancing—that compete with each other and have mutual immunity to each other. The disease evolves on the other layer, and individuals are less likely (respectively, more likely) to become infected when they adopt the pro-physical-distancing (respectively, anti-physical-distancing) opinion. We develop approximations of mean-field type by generalizing monolayer pair approximations to multilayer networks; these approximations agree well with Monte Carlo simulations for a broad range of parameters and several network structures. Through numerical simulations, we illustrate the influence of opinion dynamics on the spread of the disease from complex interactions both between the two conflicting opinions and between the opinions and the disease. We find that lengthening the duration that individuals hold an opinion may help suppress disease transmission, and we demonstrate that increasing the cross-layer correlations or intra-layer correlations of node degrees may lead to fewer individuals becoming infected with the disease. (Received September 20, 2021)

1174-92-9176 **Thomas G. Stojsavljevic Jr*** (stojsavljevictg@beloit.edu), Beloit College, **Yixin Guo** (yg48@drexel.edu), Drexel University, and **Dominick Macaluso** (djm486@drexel.edu), Drexel University. *Multi-Site Adaptive Stimulation in a Biophysical Network Model* Preliminary report.

In a normal state, the thalamacortical neurons (TC) in the thalamus serve to relay excitatory inputs from the sensorimotor cortex while they are targeted by the inhibitory output from the internal segment of the globus pallidus (GPi) in the basal ganglia. In parkinsonian conditions, a TC neuron fails to respond to excitatory cortical inputs in a one-to-one fashion. The TC cell either fires multiple spikes or no spikes at all in response to a single cortical excitatory signal. In the past two decades, DBS–through a surgically implanted electrode to the subthalamic nucleus (STN)–has become a widely used therapeutic option for the treatment of Parkinson's disease and other neurological disorders. Although the conventional DBS that delivers an ongoing stream of high frequency pulses to the stimulation target has shown remarkable therapeutic success, the mechanisms for the effectiveness of DBS and the potential for improvements on drawbacks of conventional DBS require further study and exploration. As experimental investigations of DBS mechanisms or development of new stimulation protocols are prohibited on humans, or are too costly to be performed on non-human primates, computational study is necessary to advance to clinical application. Here, we introduce a new class of protocols which delivers stimulation in an adaptive fashion. We stimulate an excitatory input train to a biophysically-detailed network models TC neurons and compare TC relay performance across various conditions. (Received September 20, 2021)

1174-92-9220 **Allison Lewis** (lewisall@lafayette.edu), Lafayette College, and **Anna C Zittle*** (zittlea@lafayette.edu), Lafayette College. *Analyzing the Trade-offs Between Model Complexity, Parameter Identifiability, and Data Availability in Math Modeling of Tumor Dynamics*

Mathematicians, biologists, and medical practitioners alike can benefit from modelling complex biological systems - such as describing tumor growth dynamics - using mathematical models. In fact, being able to accurately model these systems can inform clinical treatment decisions. However, models that capture the complex reality of the physical system can be difficult to uniquely parameterize given the sparsity of available data; some model simplification is often necessary. Therefore, we ask: how much complexity does a model need in order to accurately describe the given data while still maintaining parameter identifiability? In our research, we look at the trade offs between model complexity, parameter identifiability, and data availability. Using a Bayesian framework, we observe how credible intervals constructed about the model trajectory evolve as we sequentially add data points to inform our model. Additionally, we develop a procedure for performing the analysis required to ensure that all models are identifiable prior to using them for predictive purposes. (Received September 20, 2021)

1174-92-9234 **Kathryn G. Link*** (klink@math.ucdavis.edu), University of California, Davis. *Emergent Properties of Flagellar Waveforms in Viscoelastic Fluids* Preliminary report.

Eukaryotic cells move in rheologically complex environments via deformations of their flagella, which are slender threadlike structures that are powered by internal molecular motors. How flagellar beat emerges from the coordination of the mechanics of the flagella, the interactions with the external fluid environment, and the mechano-chemical feedback of the molecular motors is not entirely known. Existing theories have shed light on the origins of this behavior in a viscous fluid, however, due to the inherent nonlinearity and mathematical complexity involved in modeling viscoelastic fluids, both analytical and numerical predictions require nonstandard approaches. In this work we propose an extension to the current models to make a prediction about how viscoelasticity changes the beat frequency and amplitude of the emergent waveform. To compare these nonstandard models we use a prescribed kinematic computational swimmer model and published experimental data. (Received September 20, 2021)

1174-92-9252 **Lauren Michelle White** (laurenmwhite@ksu.edu), National Socio-Environmental Synthesis Cente, Kansas State University. *Implications of a nitrogen dependent host growth and immunity trade-off for infection dynamics in primary producers*

Pathogens rely on their host for reproduction, and changes in primary producer nutrition may alter their disease dynamics. Pathogens such as viruses, bacteria, and fungi, generally have higher nutritional demands and are stoichiometrically less flexible than primary producers. Increases in nutrient supply may thus more strongly enhance the performance of pathogens relative to their host. The consequences of shifts in nutrient availability for disease dynamics also will depend on changes in host immunity. For example, a growth-defense trade-off may lead to reduced resistance when increased nutrient availability enhances host growth and thus further increases disease prevalence. In contrast, host immunity may also rely on nutrients directly, for example, on nitrogen used to produce nitrogen-based compounds or build nitrogen-rich enzymatic machinery, which may lead to enhanced host resistance with increasing nitrogen availability. Here, we explored the role of nitrogen availability on infection dynamics of a primary producer host and its pathogen using a stoichiometric, based disease model. Specifically, we tested how changes in nitrogen investments in host immune response will alter host biomass build-up and pathogen infection rates. (Received September 20, 2021)

1174-92-9266 **Alexander Ruys De Perez*** (amrp3@gatech.edu), Georgia Institute of Technology. *Machine Learning and Topological Data Analysis for Pluripotent Stem Cells*

Pluripotent stem cells play a critical role in embryonic development, and studying their differentiation into the different dermal layers is an important task. Taking inspiration from McGuirl et al. (2020), we incorporate topological data analysis to quantify features of this process such as segregation and symmetry breaking. Furthermore, we show how a machine learning algorithm can be trained to identify the experimental conditions imposed on the embryo, such as introduction of BMP or spatial restrictions. (Received September 20, 2021)

1174-92-9291 **Kristen A Windoloski*** (kawindol@ncsu.edu), North Carolina State University, and **Mette S Olufsen** (msolufse@ncsu.edu), North Carolina State University. *A mathematical model of inflammation for a continuous infusion of endotoxin* Preliminary report.

Sepsis, a leading cause of death in U.S. hospitals, is a severe inflammatory illness characterized by sustained, abnormal inflammation. Several studies have explored short-term inflammation in mice and humans using a bolus administration of lipopolysaccharide (LPS), and mathematical models have been constructed and analyzed from these data sets. However, inflammatory stimuli are present in septic patients for a longer time than is seen in bolus studies. Thus, inflammation in conditions like sepsis is better represented by a continuous infusion of LPS over several hours. Investigation of cytokine dynamics during a continuous infusion reveal delayed peaks and larger peak magnitudes in the major pro and anti-inflammatory cytokines TNF-α and IL-10, respectively, compared to bolus data. To understand these differences, we extend a bolus mathematical model to a continuous infusion mathematical model that governs interactions between the endotoxin and major cells and cytokines in the immune system. By calibrating the model to experimental data and using sensitivity analysis and parameter estimation, we investigate the effect of a continuous infusion of LPS on inflammatory dynamics. This preliminary model is the framework for a mathematical model of sepsis calibrated to clinical data. (Received September 21, 2021)

1174-92-9294 **Jennifer Crodelle** (jcrodelle@middlebury.edu), Middlebury College, and **Bryan Currie*** (bcurrie@middlebury.edu), Middlebury College. *Synchronization of Electrically Coupled Neurons* Preliminary report.

Electrical coupling between neurons in the brain plays a significant role in synchronizing neuron activity, a behavior that underlies many regulatory and attentional processes. It has been shown experimentally that stronger electrical coupling between cells leads to more synchronous behavior. The contribution of changes in synaptic strength to synchrony remains still unclear. We have built a mathematical model of two Hodgkin-Huxley neurons connected electrically and synaptically with varying coupling strengths. We have used two different methods of measuring synchrony between neurons, the Van Rossum distance and the Jitter-Based Synchrony Index. Both of these methods synchrony measures verify the experimental finding that electrical coupling enhances synchrony between a pair of neurons, however, the effect of synaptic feedback between the two neurons on synchrony is a bit more complicated. Interestingly, our results show contradictory trends depending on which synchrony measure used, which calls into question the efficacy and implementation of different neuronal synchrony measures. (Received September 20, 2021)

1174-92-9303 **Naveen K. Vaidya*** (nvaidya@sdsu.edu), San Diego State University. *Choice of Antiretroviral Therapies to Mitigate HIV in the Brain*

It is known that the brain can play a role of viral reservoir causing HIV persistence under antiretroviral therapy, mainly because the blood-brain barrier (BBB) poses an obstacle to the entry of antiretroviral drugs into the brain. In this talk, I will present a novel mathematical model describing virus dynamics in the brain under antiretroviral therapy and use experimental data from SIV-infected macaques to identify key model parameters related to the brain infection, including transfer across BBB. Our model predicts that the brain can be an important reservoir causing long-term virus persistence, and the choice of drugs may play a critical role in mitigating HIV in the brain. (Received September 20, 2021)

1174-92-9335 **Tom Chou** (tomchou@ucla.edu), University of California at Los Angeles, **Mingtao Xia*** (xiamingtao97@g.ucla.edu), Dept. of Mathematics, UCLA, and **Lucas Boettcher** (l.boettcher@fs.de), Frankfurt School of Finance and Management, Dept. of Computational Medicine, UCLA. *Controlling epidemics through optimal allocation of test kits and vaccine doses across networks*

Efficient testing and vaccination protocols are critical aspects of epidemic management. To study the optimal allocation of limited testing and vaccination resources in a heterogeneous contact network of interacting susceptible, recovered, and infected individuals, we present a degree-based testing and vaccination model for which we use control-theoretic methods to derive optimal testing and vaccination policies using control-theoretic methods. Within our framework, we find that optimal intervention policies first target high-degree nodes before shifting to lower-degree nodes in a time-dependent manner. Using such optimal policies, it is possible to delay outbreaks and reduce incidence rates to a greater extent than uniform and reinforcement-learning-based interventions, particularly on certain scale-free networks. (Received September 20, 2021)

1174-92-9376 **Margaret Ann Grogan*** (margaret.grogan@westpoint.edu), United States Military Academy at West Point. *Modeling Governance in Managing Infectious Diseases* Preliminary report.

Disease outbreaks rarely respect sociopolitical borders. Governments overseeing multiple patches may decide to use a uniform approach to management actions, which may be different actions than those preferred at the local levels. We use optimal control theory to implement a uniform management strategy across two patches as well as a customized approach for each patch. We investigate two disease models with applications to ebola and cholera. This is a preliminary report.

This is joint work with Julie Blackwood, Kyle Dahlin, Christina Edholm, Lindsey Fox, Brandon Hollingsworth, Emily Howerton, Suzanne Lenhart, George Lytle, Katriona Shea, and Melody Walker. (Received September 20, 2021)

1174-92-9450 **Perry Beamer** (perry.beamer@gmail.com), University of Maryland, **Teresa Jones*** (tmjones2@vwu.edu), Virginia Wesleyan University, **Caroline Hammond** (cellham73@gmail.com), Delaware University, and **Nicole Gallegos** (gallegon@bu.edu), Boston University. *Characterizing POTS using mathematical modeling and machine learning* Preliminary report.

Postural Orthostatic Tachycardia Syndrome (POTS) causes autonomic nervous dysfunction resulting in an excessive heart rate increase due to postural change. POTS symptoms overlap with other conditions, making it difficult to diagnose and identify its underlying physiological causes. The head-up tilt test, used to diagnose POTS, requires special equipment, takes at least 20 minutes, and can result in fainting. As an alternative, the Valsalva maneuver (VM) is a fast and low-risk diagnostic test for autonomic function. The objective of this study is to use mathematical modeling, parameter estimation, and machine learning to test if the VM provides a reliable method to identify POTS in 700 patients who exhibited side effects of the human papillomavirus vaccine. Our patient-specific delay differential equations model uses blood pressure and electrocardiogram data as inputs to predict heart rate, estimating identifiable parameters that minimize the least squares error between the model predictions and data. Using machine learning, we analyze the estimated model parameters and quantities extracted from data to determine (i) if VM data and patient-specific model parameters can be used to identify POTS, and (ii) which markers best characterize the disease. (Received September 20, 2021)

1174-92-9490 **Sophia Grace Nelson*** (snelso18@gustavus.edu), Gustavus Adolphus College. *Effects of viscoelasticity on the oscillatory behavior of a two-link filament model* Preliminary report.

A subgroup of microscopic organisms, known as swimmers, use thin rod-like structures called cilia and flagella to propel themselves in various fluid environments. This locomotion is driven by both the dynamics of intracellular molecular motors within the flagella and the hydrostatic elastic forces exerted on the swimmer. Both swimmer gait and the stability of the swimmers' motion are subject to changes in both external and internal environments. In this work, we are interested in the 2D planar motion of a flagella driven by a follower force applied tangentially at the tail and pinned at the head. To characterize this phenomenon, we consider a discretized, two-link filament model that exhibits oscillatory behavior. We explore this motion in three different fluids: a viscous fluid, a Maxwell elastic fluid, and an Oldroyd-B viscoelastic fluid. Changes in the frequency, amplitude, and stability of the oscillations were observed as a result of variations in fluid properties. This result highlights the adaptive nature of swimmers in viscoelastic environments. (Received September 20, 2021)

1174-92-9510 **Bruce Pell*** (bpell@ltu.edu), Lawrence Technological University, and **Matthew D Johnston** (mjohnsto1@ltu.edu), Lawrence Technological University. *A Dynamical Framework for Modeling Fear of Infection and Social Distancing in COVID-19 Spread*

With the emergence of the COVID-19 pandemic and its unprecedented spread throughout the world, mitigation strategies such as social distancing have been used to help contain the spread of the disease. In this talk, I will introduce a novel modeling framework for incorporating the population's fear of infection and frustration with social distancing into disease dynamics, discuss some insights from the model analysis and its extensions.
(Received September 20, 2021)

1174-92-9512 **Rachel Grotheer** (grotheerre@wofford.edu), Wofford College, and **Stephen Chase Creamer*** (creamersc@email.wofford.edu), Wofford College. *Surviving Ragnarok: Modeling Humanity's Chance of Survival After a Major Disaster Event* Preliminary report.

Events such as natural disasters, disease epidemics, and wars heighten our collective awareness of the fragility of human life on Earth. Therefore, it comes as no surprise that researchers have sought to create mathematical models to predict the behavior of population growth following these events. Our research models the growth of a population from an initial number of survivors following a catastrophic event by using Markov population chains and differential equations while the population is living within a bunker. We then make use of stochastic models to study the dynamics of the population once it leaves the bunker after a set period of time. With these models, we establish a viable range for the initial population that ensures a steady growth rate. (Received September 20, 2021)

1174-92-9592 **Maiko Arichi*** (marichi@lincoln.edu), Lincoln University. *DIRECT MATHEMATICAL METHOD FOR REAL-TIME ISCHEMIC DETECTION FROM ELECTROCARDIOGRAMS USING THE DISCRETE HERMITE TRANSFORM*

An automated identification technique was developed for the detection of ischemic episodes in long term electrocardiographic (ECG) signals using mathematical expansions involving the discrete dilated Hermite Transform. The discrete Hermite functions are generated as eigenvectors of a symmetric tridiagonal matrix that commutes with the centered Fourier matrix. The Hermite transform values are computed from a simple dot product between an individual ECG complex extracted from the European Society of Cardiology (ESC) ST-T database and the corresponding discrete Hermite function. These values are found to contain information about the ECG shape, highlighting changes between ST-segment deviation and T-wave alterations which are the features of ischemic episodes. This information from the discrete Hermite transform, based on an orthonormal set of n-dimensional digital Hermite functions that serve as shape-identification functions, can be used to identify ischemic episodes from the ECG. (Received September 20, 2021)

1174-92-9597 **Zhisheng Shuai*** (shuai@ucf.edu), University of Central Florida. *Impact of Hotspot Arrangements on Disease Invasion*

We consider the spread of an infectious disease in a heterogeneous environment, modeled as a network of patches. We focus on the invasibility of the disease, as quantified by the basic reproduction number R_0, and investigate how the locations of disease hotspots and the changes in the network structure affect the value of R_0. These effects can be characterized using new indices for the network average and network heterogeneity, and provide both qualitative and quantitative information for mitigating disease spread among the patches. (Received September 20, 2021)

1174-92-9635 **Md Rafiul Islam*** (rafiul@iastate.edu), Iowa State University, **Claus Kadelka** (ckadelka@iastate.edu), Iowa State University, **Audrey Lamson McCombs** (amccombs@iastate.edu), Iowa State University, **Mohammad Mihrab Chowdhury** (MU.Chowdhury@ttu.edu), Texas Tech University, **Mohammad Al-Mamun** (mohammad.almamun@hsc.wvu.edu), West Virginia University, and **Michael Tyshenko** (mtyshenk@uottawa.ca), University of Ottawa. *Evaluation of the United States COVID-19 Vaccine Allocation Strategy*

Anticipating an initial shortage of vaccines for COVID-19, the Centers for Disease Control (CDC) in the United States developed priority vaccine allocations for specific demographic groups in the population. In this talk, I present our recent study that evaluates the performance of the CDC vaccine allocation strategy with respect to multiple potentially competing vaccination goals (minimizing mortality, cases, infections, and years of life lost (YLL)), under the same framework as the CDC allocation: four priority vaccination groups and population demographics stratified by age, comorbidities, occupation and living condition (congested or non-congested). We developed a compartmental disease model that incorporates key elements of the current pandemic including age-varying susceptibility to infection, age-varying clinical fraction, an active case-count dependent social distancing level, and time-varying infectivity (accounting for the emergence of more infectious virus strains). The CDC allocation strategy is compared to all other possibly optimal allocations that stagger vaccine roll-out in up to four phases (17.5 million strategies). The CDC allocation strategy performed well in all vaccination goals but never optimally. The developed global optimization approach can be used for future mass vaccination campaigns, and can be adapted for use by other countries seeking to evaluate and optimize their current prioritization strategies. (Received September 20, 2021)

1174-92-9681 **Timothy D Comar*** (tcomar@ben.edu), Benedictine University. *Long-Term Student Research Projects Involving Modeling with Agent-Based Models and Impulsive Differential Equations*

This presentation will discuss long-term student research projects, which involve modeling biological phenomena using both agent-based models and impulsive differential equations. In one of these projects, age-structured vaccination models for human papilloma virus HPV have been developed and analyzed. These models are used to investigate the consequences of a sufficient fraction of the population not completing the vaccination regimen. In another one of these projects, models were developed to analyze an age-structured integrated pest management system. We will also discuss the challenges and benefits for student researchers of approaching this complex problem via these two modeling paradigms. (Received September 20, 2021)

1174-92-9692 **Katherine Daftari*** (daftari@live.unc.edu), UNC Chapel Hill. *An information theoretic analysis of communication between golden shiners (Notemigonus crysoleucas)* Preliminary report.

Organisms live in groups for many evolutionarily advantageous reasons including protection from predators, breeding, and easier access to food sources. In many cases, such groups are leaderless but exhibit large scale, complex behaviors that are an emergent feature of individual interactions. Although there are a variety of agent-based models which can convincingly reproduce such phenomena as seen in nature, the exact method of inter-organism communication in many species is still unknown. To this end, we investigate the inverse problem of communication between fish by applying information theoretic tools to experimentally gathered movement data. From these direct measurements of system dynamics, we are able to statistically quantify the information shared between individuals and propose physical interpretations to explain communication between golden shiners. (Received September 20, 2021)

1174-92-9697 **Lynn G Schreyer*** (Lynn.Schreyer@wsu.edu), Washington State University, **Sergey Lapin** (slapin@wsu.edu), Washington State University, **Nikolaos Voulgarakis** (n.voulgarakis@wsu.edu), Washington State University, and **Zachary Hilliard** (zachary.hilliard@wsu.edu), Grove Christian School, Richmond, VA. *Deterministic Model for Migration of Herd of Ungulates accounting for Terrain, Predators, and Resources*

Here we discuss how a new deterministic model for mammal migration was developed by adopting a model for fluid flow through porous media. The resulting model is a generalized Cahn-Hilliard equation that captures terrain characteristics affecting speed, the group nature of herds preferring an "equilibrium density", and the preference to move where there are resources and avoid dangerous locations or move away from a threat. Numerical simulations of a herd moving under various scenarios will be presented. (Received September 20, 2021)

1174-92-9780 **Kellan Toman** (ktoman@wsu.edu), Washiington State University. *A Stochastic Pursuit-Evasion Model for Animal Foraging* Preliminary report.

Foraging, the process of locating hidden targets, is a fundamental function of living organisms. Here, a simple mathematical model that considers the mutual competition between the foragers and their environment is proposed. This approach introduces a new type of persistent random walk that describes the most prevalent properties of stochastic search patterns: power-law distributed random steps, superdiffusive dynamics, and directional persistence of the motion. Increasing competition leads to a transition from a Brownian motion to Levy-like motion. Interestingly, Levy movement emerges at the highest possible level of competition. At that critical point, the ability of the forager to exploit the available information is maximized. It is hypothesized that the system eventually relaxes to a forager-target interaction that prevents extinction. (Received September 20, 2021)

1174-92-9792 **Anish Mudide*** (amudide@gmail.com), Phillips Exeter Academy. *Predicting conservation status from genome summary statistics* Preliminary report.

The genome sequence of a species directly captures key markers of conservation status, such as regions of reduced genetic diversity and genetic load. Moreover, levels of inbreeding and historical population declines can be inferred via analyses of heterozygosity across the genome. While previous predictive models of conservation status have incorporated ecological variables, these results from conservation genetics suggest that such models will benefit greatly from the inclusion of genomic information. Using a whole-genome alignment of 240 eutherian mammal species created by the Zoonomia Project, we extracted relevant genomic features to include in machine learning classifiers. Our models can inform future conservation interventions by predicting the extinction risk of species classified as data deficient by the International Union for Conservation of Nature (IUCN). We find that genomic and ecological features provide comparable levels of predictive accuracy and that the best models integrate both types of features. Furthermore, our work highlights which genomic features have the greatest influence on conservation status and illustrates how ecological and genomic variables interact to produce extinction risk. (Received September 21, 2021)

1174-92-9894 **Manuel Lladser*** (manuel.lladser@colorado.edu), University of Colorado Boulder, and **Evan D. Gorman** (evan.gorman@colorado.edu), University of Colorado Boulder. *Euclidean Embeddability of a Phylogenetic Metric in Low Dimension* Preliminary report.

UniFrac is a phylogenetically informed distance between microbial environments, which regards environments with recently evolved microorganisms as more similar than those with more distantly evolved ones. In conjunction with ordination techniques, UniFrac has been very successful at visualizing metagenomics data in low-dimension and identifying clusters and gradients of biological significance in them. This talk aims to address UniFrac's suitability for ordination techniques by examining a related phylogenetic metric, which we call Euclidean UniFrac (EUniFrac). We present various of this metric's mathematical and computational properties and compare its performance to UniFrac on synthetic and real datasets. This work has been partially funded by the NSF grant 1836914. (Received September 21, 2021)

1174-92-9908 **Andras Balogh** (andras.balogh@utrgv.edu), THE UNIVERSITY OF TEXAS RIO GRANDE VALLEY. *Modeling the Effect of Social Learning on Vaccine Coverage* Preliminary report.

Network models are used in the literature to depict the social structure upon which the spread of opinions and diseases can take place. In this presentation, I will talk about the multiplex network model that our research team uses to study the effect of different representations of social learning has on vaccine coverage and so on the course of the epidemics. (Received September 21, 2021)

1174-92-9910 **Khanh Dao Duc*** (kdd@math.ubc.ca), University of British Columbia. *Impact of ribosome on translation across scales*

The ribosome is an RNA-protein complex that mediates translation from mRNA to protein. With the recent advances in imaging and sequencing techniques, the dynamics of translation and the regulatory role of the ribosome can be studied across scales, using various mathematical and computational approaches. I will present tools to extract and visualize the geometric properties of the ribosome at the atomic level, allowing to study how structural differences impact translation. At the mesoscopic scale, we combined RiboSeq data with a stochastic model of ribosome traffic to infer translation rates for a large set of genes, and determine which keys steps of translation are limiting. At the cell level, where most metabolic cost depends the maintenance of the ribosome population, I will finally present a deterministic model to study how ribosome production can be optimized for protein production. (Received September 21, 2021)

1174-92-9911 **Joel Dokmegang*** (joel.dokmegang@northwestern.edu), Northwestern University. *How does the zebrafish form its pectoral fin ?*

Understanding the mechanisms by which the zebrafish pectoral fin develops is expected to produce insights on how vertebrate limbs grow from a 2D cell layer to a 3D structure. Two mechanisms have been proposed to drive limb morphogenesis in tetrapods: a growth-based morphogenesis with a higher proliferation rate at the distal tip of the limb bud than at the proximal side, and directed cell behaviors that include elongation, division and migration in a non-random manner. Based on quantitative experimental biological data at the level of individual cells in the whole developing organ, we test the conditions for the dynamics of pectoral fin early morphogenesis. We found that during the development of the zebrafish pectoral fin, cells have a preferential elongation axis that gradually aligns along the proximodistal (PD) axis of the organ. Based on these quantitative observations, we build a center-based cell model enhanced with a polarity term and cell proliferation to simulate fin growth. Our simulations resulted in 3D fins similar in shape to the observed ones, suggesting that the existence of a preferential axis of cell polarization is essential to drive fin morphogenesis in zebrafish. (Received September 21, 2021)

1174-92-9957 **Jan Nagler** (j.nagler@fs.de), Frankfurt School of Finance & Management gGmbH. *Decisive Conditions for Strategic Vaccination against SARS-CoV-2*

While vaccines against SARS-CoV-2 are being administered, in many countries it may still take months until their supply can meet demand. The majority of available vaccines elicit strong immune responses when administered as prime-boost regimens. Since the immunological response to the first ("prime") injection may provide already a substantial reduction in infectiousness and protection against severe disease, it may be more effective—under certain immunological and epidemiological conditions—to vaccinate as many people as possible with only one dose, instead of administering a person a second ("boost") dose. Such a vaccination campaign may help to more effectively slow down the spread of SARS-CoV-2, and reduce hospitalizations and fatalities. The conditions which make prime-first vaccination favorable over prime-boost campaigns, however, are not well understood. By combining epidemiological modeling, random sampling techniques, and decision tree learning, we find that prime-first vaccination is robustly favored over prime-boost vaccination campaigns, even for low single-dose efficacies. For epidemiological parameters that describe the spread of COVID-19, recent data on new variants included, we show that the difference between prime-boost and single-shot waning rates is the only discriminative threshold, falling in the narrow range of 0.01–0.02 day^{-1} below which prime-first vaccination should be considered. (Received September 21, 2021)

1174-92-10144 **Jessica M Conway*** (jmc90@psu.edu), Pennsylvania State University. *Stochastic modeling of early events in SIV infection* Preliminary report.

Understanding the events that occur following HIV exposure is critical to the development of interventions, but these events are difficult to study experimentally. Nonhuman primate (NHP) experiments with SIV from the Keele Lab shed light on the dynamics of early viral replication and development of systemic dissemination following vaginal exposure. Importantly, the infecting inoculum was a synthetic viral swarm permitting distinction between genotypically separate, but phenotypically identical variants. In this talk we focus on events preceding systemic infection. The data reveals that SIV form foci of local infection in the female genital tract (FGT). Viral variants from multiple viral lineages were found within each focus of infection in the FGT. We will discuss results from our investigation, in which we model viral dynamics in the FGT via multitype continuous time branching processes, using observations of viral variants in foci to validate model predictions. We will explore in particular the role of the infected cell burst size, and use model-predicted time-dependent extinction probabilities, to examine the survival of viral lineages in the tissues during systemic dissemination. Early results suggest that the variability in the burst size can explain the variability in proportions of viral variants observed within foci of infection. (Received September 21, 2021)

1174-92-10150 **Maria G Emelianenko** (memelian@gmu.edu), George Mason University, **Lance Liotta** (lliotta@gmu.edu), George Mason University, and **Robyn P Araujo** (r.araujo@qut.edu.au), Queensland University of Technology. *Singular Jacobians and their effect on adaptation in biological networks*

In this talk we will discuss conditions under which a biological system will possess a singular Jacobian and its effect on adaptation criteria. First, we will develop the conditions under which a system will contain a singular Jacobian, including systems with monomial-type interaction terms and systems with linear or nonlinear conservation laws. Motivated by these occurrences, we then extend conditions for adaptation in biological networks to include systems with singular Jacobians. The proposed theoretical extension is compatible with the notions of homeostasis and robust perfect adaptation (RPA) and clarifies the relationship between the two. The new condition is derived using the notion of Moore-Penrose pseudoinverse and is implemented using a numerically efficient algorithm. The proposed approach is tested on several synthetic systems that are shown to exhibit homeostatic behavior yet lie outside of the scope of earlier work. (Received September 21, 2021)

1174-92-10197 **Amanda Lee Colunga*** (alcolung@ncsu.edu), North Carolina State University. *Classification of Pulmonary Hypertension via Computational Modeling* Preliminary report.

Cardiovascular (CV) disease is acknowledged as a leading cause of death globally, encompassing a wide array of functional and structural dysfunction in the CV system. The work presented here focuses on the analysis and discrimination of patients with pulmonary arterial hypertension (PAH) by integrating machine learning and data analysis techniques with computational modeling, to improve early detection and treatment protocols. Accounting for ventricular interaction (VVI) through the septal wall and left and right heart contraction we test if patient-specific model predictions via parameter inference can discriminate mild versus severe PAH. We use local and global sensitivity analysis to determine under what conditions VVI impacts left and right ventricular pressure and volume dynamics. We estimate posterior parameter distributions minimizing the least-squares error between the model and data. We determine what data best informs models characterizing PH dynamics using statistical criteria such as information criterion and adjusted coefficient of determination. Finally, we use machine learning to identify PH phenotypes examining if estimated model parameters and clinical data can distinguish mild versus severe disease outcomes. The latter will be done comparing predictions for pulmonary vascular resistance, ventricular afterload, power output, mean pulmonary arterial pressure, cardiac output, or stroke volume. (Received September 21, 2021)

1174-92-10200 **Danielle Middlebrooks*** (danielle.middlebrooks@nist.gov), National Institute of Standards and Technology, **Paul Patrone** (paul.patrone@nist.gov), National Institute of Standards and Technology, **Gregory Cooksey** (gregory.cooksey@nist.gov), National Institute of Standards and Technology, and **Geoffrey McFadden** (geoffrey.mcfadden@nist.gov), National Institute of Standards and Technology. *Separating Flow Cytometry Populations Based on Probabilistic Analysis* Preliminary report.

Flow cytometry (FC) is a technique used to measure biomarker abundance within populations of cells or attached to microparticles. FC experiments can produce on the order of 10^6 events, each comprising scalar measurements that correspond to a particle's relative size, complexity and fluorescence intensities. However, these experiments also contain background events including leftover particles from previous experiments and cells in the sample that

have broken apart into smaller pieces. One can also consider unstained cells as a population within the sample that can overlap with signals from the sample of interest. The traditional method for separating populations is a process called gating where a user selects regions on the scatter or histogram plot of events to distinguish between different populations. Gating is one of the most important parts in analyzing FC data but is both time-consuming and subjective to the user. We developed a methodology that identifies a population described by its probability density function (PDF) without the need for gating. Our method combines prior knowledge about the support of the PDFs with constrained optimization to subtract off one population from another within a given experiment. Our preliminary results suggests our method can efficiently separate populations while removing initial subjectivity in the data analysis. (Received September 21, 2021)

1174-92-10229 **Mikahl Banwarth-Kuhn*** (mbanwarth-kuhn@ucmerced.edu), UC MERCED. *Multiscale Modeling of Prion Aggregate Dynamics in Yeast* Preliminary report.

Prion proteins are responsible for a variety of fatal neurodegenerative diseases in mammals but are harmless to Baker's yeast (*S. cerevisiae*)- making it an ideal system for investigating the protein dynamics associated with prion diseases. Most mathematical approaches to modeling prion aggregate dynamics have focused on either protein dynamics in isolation, absent from a changing cellular environment, or modeling prion aggregate dynamics in a population of cells by considering the "average" behavior. However, such models have been unable to recapitulate *in vivo* properties of different yeast prion strains. In this talk, I will discuss results from recent cell-based model simulations where we study how factors across scales impact the emergence of population level phenotypes in growing yeast colonies. In particular, the rate of disease onset is related to both the replication and transmission kinetics of propagons, the infectious agents of prion diseases. In the model, each cell has their own configuration of prion aggregates governed by kinetic parameters previously estimated by our group for six different yeast prion strains. We use our model simulations to study how population level phenotypes are a natural consequence of the interplay between the cell cycle, budding cell division and prion aggregate dynamics. We then quantify how common experimentally observed outcomes depend on population heterogeneity. (Received September 21, 2021)

1174-92-10232 **Alexandria Lynn Ferrentino*** (lexieferrentino213@gmail.com), SUNY Geneseo, **Rachel E Brohead** (rbrodhead@zagmail.gonzaga.edu), Gonzaga University, **Emily B Evans** (Emily.Evans2@cwu.edu), Central Washington University, and **Vanessa Montano** (Vanessa.Montano@cwu.edu), Central Washington University. *Mathematical Modeling Polycystic Ovarian Syndrome* Preliminary report.

Almost half of the world's population menstruate, and about twelve percent of them have been diagnosed with Polycystic ovary syndrome. Polycystic ovarian syndrome is a hormonal disorder that affects the reproductive health of people who ovulate and is associated with infrequent menstrual cycles and hyperandrogenism. By examining current treatments for this disorder, such as birth control, we can determine how patients can be more effectively treated in the future. While the exact cause of this disorder is still unknown, this study was designed to build off these existing models and construct a new model for hormone levels in people diagnosed with PCOS to increase our understanding of the abnormal ovulation processes. By combining two separate systems of differential equations covering the menstrual cycle, one that incorporates the androgen testosterone and another that incorporates hormone control, we were able to create a more detailed model for normal ovulation and adjust hormone levels to fit expected PCOS levels. After creating a model that reflects PCOS, a comparative analysis of birth control (progesterone only, estrogen only, and combined) was done to determine the most effective hormone control for PCOS. This paper will summarize the results of the CC-REU NSF summer REU experience (DMS-2050692) where these questions were explored. (Received September 21, 2021)

1174-92-10237 **Marisabel Rodriguez Messan*** (marisabel.rodriguez@fda.hhs.gov), U.S. Food and Drug Administration. *Mathematical model of a personalized neoantigen cancer vaccine and the human immune system*

Cancer vaccines are an important component of the cancer immunotherapy toolkit enhancing immune response to malignant cells by activating $CD4^+$ and $CD8^+$ T cells. Multiple successful clinical applications of cancer vaccines have shown good safety and efficacy. However, significant challenges remain in obtaining consistent immune responses across heterogeneous patient populations, as well as various cancers. In this talk, I present a mathematical model describing key interactions of a personalized neoantigen cancer vaccine with an individual patient's immune system. This model considers the vaccine concentration of tumor-specific antigen peptides and adjuvant, the patient's MHC I and II copy numbers, tumor size, T cells, and antigen presenting cells. The model was parametrized using patient-specific data from a clinical study in which individualized cancer vaccines were used to treat six melanoma patients. Global sensitivity analysis was performed to identify key model parameters that may impact model outputs of interest (T and tumor cells) at specific time points. Model simulations predicted both immune responses to the vaccine as well as clinical outcome. Sensitivity analysis revealed that parameters involved in the proliferation/recruitment of immune cells show great influence on the efficiency of the immunotherapy in eliciting an immune response. This model can be used to describe, simulate, and predict the behavior of the human immune system to a personalized cancer vaccine. (Received September 21, 2021)

1174-92-10276 **Cornelia Pokalyuk*** (Pokalyuk@math.uni-frankfurt.de), Goethe University Frankfurt, **Irene Görzer** (Irene.goerzer@meduniwien.ac.at), Medical University Vienna, and **Anton Wakolbinger** (Wakolbinger@math.uni-frankfurt.de), Goethe University Frankfurt. *Diversity patterns in parasite populations capable for persistence and reinfection with a view towards the human cytomegalovirus* Preliminary report.

Many parasite like the cytomegalovirus, HIV or Escherichia coli are capable to persist in their host and reinfect it. We will investigate a host-parasite model with two parasite types for which persistence and reinfection are driving the evolution of the parasite population. We consider two variants of the model. In one variant parasite reproduction is directed by balancing selection, in the other variant parasite reproduction is neutral.

In the former scenario we identify parameter regimes for which reinfection and persistence sustain the maintenance of diversity in the parasite population and for which the evolution of the parasite population is asymptotically driven by a (deterministic) dynamical system.

In the neutral scenario reinfection may facilitate the coinfection of a host with both parasite types, but does not enhance the coexistence of both parasite types. Depending on the ratio of the reinfection, parasite reproduction and host replacement rate we determine the asymptotics distribution of frequencies at which hosts are infected with the two parasite types.

We evaluate the biological relevance of both model variants with respect to the human cytomegalovirus, an ancient herpesvirus that is carried by a substantial fraction of mankind and owns a high diversity in its coding regions. (Received September 21, 2021)

1174-92-10281 **Abdulmelik Mohammed*** (abdulmelik@usf.edu), University of South Florida, **Richard V Miller** (millrick10@gmail.com), Columbia University, **Jaspreet S Khurana** (jkhurana2311@gmail.com), Columbia University, **Yi Feng** (yf2342@columbia.edu), Columbia University, **Rafik Neme** (rneme@uninorte.edu.co), Universidad del Norte, and **Laura F Landweber** (laura.landweber@columbia.edu), Columbia University. *Clustering Analysis of Genome-Rearrangement Dynamics in* Oxytricha trifallax Preliminary report.

Somatic macronuclear (MAC) development from the micronucleus (MIC) in the ciliate *Oxytricha trifallax* involves massive genome rearrangement where genetic segments are reorganized through transposition, inversion and elimination. While the MAC and MIC genomes of *Oxytricha* have been sequenced and annotated, the architecture of the intermediates that are formed during the process of rearrangement are less well known. We study the dynamics of the removal of Internally Eliminated Sequences (IESs) from the MIC genome by selecting and aligning putative intermediate Illumina reads from 8 different time points against the MIC genome. The presence of an IES at a time point is captured by a retention score that represents the average of the number of reads that align at the two ends of the IES. Vectors of the retention scores of a filtered set of IESs were clustered using the K-means algorithm. Clustering reveals that conventional IESs tend to be amplified earlier than scrambled IESs, suggesting that conventional IESs are processed earlier. IESs from the same MIC chromosome, as well as those IESs whose flanking regions are destined to the same MAC, are significantly more likely to be in the same cluster, suggesting their concurrent processing. Moreover, proximity of IESs in the MIC chromosome leads to a significantly higher likelihood of concurrent processing. (Received September 21, 2021)

1174-92-10336 **Md Istiaq Hossain*** (mjh7277@psu.edu), Penn State Fayette, and **Amy Veprauskas** (aveprauskas@louisiana.edu), University of Louisiana at Lafayette. *Dynamics of a discrete-time predator-prey model with stage-structure in the prey* Preliminary report.

We develop and investigate a discrete-time predator-prey model in which the prey population is classified according to two developmental stages: juveniles and adults. We assume that the predators are capable of attacking and consuming the juvenile prey population with only the juvenile prey consumption regulating predator reproduction. Meanwhile, in the absence of an predators, prey fecundity is assumed to follow a Beverton-Holt nonlinearity. We thoroughly investigate the various dynamical behaviors of this discrete-time predator-prey model such as the existence and uniqueness of the extinction, predator-free, and interior equilibria as well as the local and global stability of the equilibria and the persistence of the system. We provide numerical simulations showing the various dynamical scenarios in support of our theoretical results. (Received September 21, 2021)

1174-92-10337 **Jessie D Jamieson*** (jessie.jamieson@jhuapl.edu), Johns Hopkins University Applied Physics Lab. *Directed Acyclic Graphs for Mission Impact: Transforming Risk into Reward*

One quarter of veterans returning from the 1990–1991 Persian Gulf War have developed Gulf War Illness (GWI) with chronic pain, fatigue, cognitive and gastrointestinal dysfunction. Exertion leads to characteristic, delayed onset exacerbations that are not relieved by sleep. We have modeled exertional exhaustion by comparing magnetic resonance images from before and after submaximal exercise. Here, the role of attention in cognitive dysfunction was assessed by seed region correlations during a simple 0-back stimulus matching task ("see a letter, push a button") performed before exercise. Analysis was analogous to resting state, but different from psychophysiological interactions (PPI). The patterns of correlations between nodes in task and default networks were significantly different between the three groups of subjects in our study. The three unique connectivity patterns during an attention task support the concept of Gulf War Disease with recognizable, objective patterns of cognitive dysfunction. We discuss applications of graph theoretical modeling to understanding these patterns of cognitive dysfunction. (Received September 21, 2021)

1174-92-10340 **Asja Radja*** (aradja@g.harvard.edu), Harvard University. *Pattern formation of cell surfaces*

Patterns are ubiquitous on biological cell surfaces, and we have only begun to develop an understanding of their formation. In this talk, I draw inspiration from rigid, extracellular surface patterns found on cells such as pollen grains and radiolaria (Ernst Haeckel's favorite marine protozoa), two incredibly morphologically varied systems with strikingly similar surface patterns, and discuss if there is a common mechanism in their pattern formation that can be described by a single, generalized theory. I highlight our experiments done in the more tractable system of the two (pollen) and expand upon how our modified Landau-Ginzburg theory of pollen wall development in this system may be applied to organisms in the entirely disparate kingdom, protozoa, expanding upon an idea first hinted towards in D'Arcy Wentworth Thompson's On Growth and Form. (Received September 21, 2021)

1174-92-10345 **Steven H Kleinstein** (steven.kleinstein@yale.edu), Department of Pathology and Program in Computational Biology and Bioinformatics, Yale School of Medicine, and **Anna Konstorum*** (anna.konstorum@yale.edu), Department of Pathology, Yale School of Medicine. *Tensor decomposition of time-course immune response data*

A central goal of biological experiments that generate intracellular time-course data is the discovery of patterns, or signatures, of response. A natural representation of such data is in the form of a third-order tensor. For example, if the dataset is from a bulk RNASeq experiment, which measures tissue-level gene expression, collected at multiple time points, the data can be structured into a gene-by-subject-by-time tensor $\mathcal{X}$. We consider the use of a non-negative CANDECOMP/PARAFAC (CP) decomposition (NCPD) on $\mathcal{X}$ to derive rank-one components that correspond to biologically meaningful signatures. While the NCPD does not suffer from degenerate solutions that may occur in a real-valued CP decomposition, we show that over-factoring in NCPD can lead to a phenomenon we term 'component splitting', which can make the analysis of individual components misleading. To assess whether component splitting has occurred in a model, we develop the maximum internal double cosine (mIDC) score. We use the mIDC, as well as the Frobenius norm and Similarity score (as defined by Kolda et al.), to choose a model rank for downstream analysis. We show that on time-course data profiling vaccination responses against the *Influenza* and *Bordetella Pertussis* pathogens, our NCPD pipeline yields novel and informative signatures of response. We finish with outstanding research challenges in the application of tensor decomposition to modern biological datasets. (Received September 21, 2021)

1174-92-10397 **Ruby Kim*** (ruby.kim@duke.edu), Duke University. *A mathematical model of circadian rhythms and dopamine*

The suprachiasmatic nucleus (SCN) serves as the primary circadian (24hr) clock in mammals, and is known to control important physiological functions such as the sleep-wake cycle, hormonal rhythms, and neurotransmitter regulation. Experimental results suggest that some of these functions reciprocally influence circadian rhythms, creating a complex and highly homeostatic network. Recent evidence shows that the activator BMAL1-CLOCK and downstream clock proteins REV-ERB and ROR are involved in dopamine (DA) synthesis and degradation. DA is involved in learning, motivation, and movement, and is linked to neurological conditions such as Parkinson's disease, schizophrenia, and addiction. We created a mathematical model consisting of differential equations that express how circadian variables are influenced by light, how REV-ERB and ROR feedback to the clock, and how REV-ERB, ROR, and BMAL1-CLOCK affect the dopaminergic system. We compare our model predictions to experimental data on clock components in different light-dark conditions and in the presence of genetic perturbations. Our model results allow us to predict circadian variations in tyrosine hydroxylase and monoamine oxidase. By connecting our model to an extant model of dopamine synthesis, release, and reuptake, we are able to predict circadian oscillations in extracellular DA and homovanillic acid that correspond well with experimental observations. (Received September 21, 2021)

1174-92-10424 **Julie Blackwood*** (jcb5@williams.edu), Williams College. *An Introduction to the Dynamics of Infectious Diseases: Ecological models Across Multiple Scales*

Dynamical models have become a central tool used to investigate a variety of problems related to the ecology of infectious diseases. For example, the classic Susceptible-Infected-Recovered (SIR) paradigm – a mathematical framework that describes the transition of individuals between compartments based on their disease status – provides a relatively simple yet powerful structure to describe the dynamics of infectious diseases. The utility of mathematical and computational approaches to simulate and numerically analyze infectious diseases within populations has also benefited from major advances in computational power over the past several decades. As a result, public health officials and policy makers have increasingly relied on a quantitative understanding of infectious disease dynamics. An area of growing interest, rife with mathematical and computational challenges, is the modeling of these processes at different scales, as well as the complex feedbacks between these scales. Here, we discuss a few aspects of infectious disease modeling across scales, including governance at different geographical scales and heterogeneity in contact structure. Finally, we discuss model validation using empirical data, which poses challenges for even the simple SIR model. (Received September 21, 2021)

1174-92-10425 **Andrea Arnold*** (anarnold@wpi.edu), Worcester Polytechnic Institute, and **Loris Fichera** (lfichera@wpi.edu), Worcester Polytechnic Institute. *Identification of Tissue Optical Properties During Thermal Laser-Tissue Interactions*

Successful use of lasers in modern medicine relies on the ability to understand and control the interactions that occur between the laser light and the tissue being treated. This talk will present a computational framework utilizing data assimilation techniques to estimate the physical tissue properties that govern the thermal response of laser-irradiated tissue. We focus in particular on two quantities, the absorption and scattering coefficients, which describe the optical absorption of light in the tissue and whose knowledge is vital to correctly plan medical laser treatments. With several numerical examples, we demonstrate the ability of the proposed approach to identify tissue optical properties and track their dynamic changes during laser exposure. (Received September 21, 2021)

1174-92-10449 **Kathryn J Montovan*** (kmontovan@bennington.edu), Bennington College, **Natasha Tigreros** (NTigreros13@gmail.com), University of Arizona, and **Jennifer Thaler** (thaler@cornell.edu), Cornell University. *Size and growth-dependent optimal foraging under predation pressures*

For many prey, there is a state-dependent fitness trade-off between growth and predation risk that shifts as the prey grows. Rapid growth can help the prey exit a high-risk size/growth phase and reduce the time to adulthood, but reducing or stopping foraging activity can reduce instantaneous predation risk. We explore the effects ontogenetic size and prey growth function have on prey behavior using a dynamic optimization model. The model is then applied as part of a holistic approach to investigated reduced foraging behavior in *Leptinotarsa decemlineata*, the Colorado potato beetle, that integrates experimental data and mathematical modeling. (Received September 21, 2021)

1174-92-10480 **Heiko Enderling** (heiko.enderling@moffitt.org), H. Lee Moffitt Cancer Center & Research Institute, **Mohammad Usama Zahid*** (mohammad.zahid@moffitt.org), H. Lee Moffitt Cancer Center & Research Institute, **Nuverah Mohsin** (nm1270@mynsu.nova.edu), Nova Southeastern University, **Abdallah Mohamed** (ASMohamed@mdanderson.org), The University of Texas MD Anderson Cancer Center, **Jimmy J Caudell** (jimmy.caudell@moffitt.org), H. Lee Moffitt Cancer Center & Research Institute, **Louis B Harrison** (louis.harrison@moffitt.org), H. Lee Moffitt Cancer Center & Research Institute, **Clifton D Fuller** (cdfuller@mdanderson.org), The University of Texas MD Anderson Cancer Center, and **Eduardo G Moros** (eduardo.moros@moffitt.org), H. Lee Moffitt Cancer Center & Research Institute. *Personalizing Radiotherapy Dose Using a Dynamic Carrying Capacity Model*

In order for radiotherapy (RT) to enter the realm of personalized medicine it will be necessary to model and predict individual patient responses to radiotherapy. Here we model tumor dynamics as logistic growth and the effect of radiation as a reduction in the tumor carrying capacity, motivated by the impact of radiation on the tumor microenvironment. The model was assessed on weekly tumor volume data collected using retrospective data from a combined cohort of n = 39 head and neck cancer patients from the Moffitt and MD Anderson

Cancer Centers that received 66-70 Gy RT in 2-2.12 Gy weekday fractions. To predict response to radiotherapy for individual patients, we developed a new forecasting framework that combined the learned tumor growth rate and carrying capacity reduction fraction (δ) distribution with weekly measurements of tumor volume reduction for a given test patient to estimate δ, which was used to predict patient-specific outcomes and to estimate personalized RT doses. This forecasting framework predicted patient-specific outcomes with 76% sensitivity and 83% specificity for locoregional control and 68% sensitivity and 85% specificity for disease-free survival with the inclusion of four on-treatment tumor volume measurements. Additionally, the framework estimated personalized RT to range from 8-186 Gy. These results demonstrate the potential to forecast the efficacy of an RT schedule and to estimate a personalized dose using a simple mathematical model of tumor volume dynamics. (Received September 21, 2021)

1174-92-10485 **Jason Rosenhouse*** (rosenhjd@jmu.edu), James Madison University. *The Use and Abuse of Probability Theory in Discussions of Evolutionary Biology* Preliminary report.

The teaching of evolutionary biology in high school science classes is a perennial flashpoint in American education. Misconceptions of mathematics, especially probability theory, play an important role in this controversy. Probability plays a central role in modern evolutionary biology, finding applications in both population genetics and in phylogenetic reconstruction. However, it is also easy for religious critics to abuse probability in the service of poor arguments against evolution. We will discuss the importance of basic quantitative literacy in helping the public distinguish proper from improper uses of probability in discussions of evolution. The talk will be framed around my own first-hand experiences interacting with anti-evolutionists at their conferences, from which I have learned just how rhetorically effective bad mathematics can be. (Received September 21, 2021)

1174-92-10493 **Hoa Nguyen** (hnguyen5@trinity.edu), Trinity University, **Mimi Koehl** (cnidaria@berkeley.edu), UC Berkeley, and **Emma Ross** (eross1@trinity.edu), Trinity University. *Effects of prey capture on the swimming and feeding performance of choanoflagellates.*

Choanoflagellates, eukaryotes that are important predators on bacteria in aquatic ecosystems, are used as a model system to study the evolution of animals from protozoan ancestors. The choanoflagellate *Salpingoeca rosetta*, has a complex life cycle that includes unicellular and multicellular stages, provides a model system to study the consequences of different cell morphologies, being free-swimming vs. sessile, or being a single cell vs. a multicellular colony. A unicellular *S. rosetta* has an ovoid cell body and a single flagellum surrounded by a collar of microvilli. The cell swims by waving its flagellum, which also creates a water current that brings bacteria to the collar of prey-capturing microvilli. One measure of the performance of a suspension-feeding organism is the volume of fluid that it can move into its collar during a beat cycle. The inward flux of fluid acts as a proxy for the rate of bacterial capture. While this is a good measure of uptake of dissolved nutrients, it is only an approximate measure of prey capture. Here we use a regularized Stokeslet framework to model the hydrodynamics of a unicellular choanoflagellate, the captured bacterial prey of non-zero volume, and their effect on swimming performance and clearance rate. We compare the model predictions with high-speed microvideography. Moreover, we will discuss current model assumptions, and future model improvements that, together with coordinated lab experiments, will help us probe this intriguing biophysical system. (Received September 21, 2021)

1174-92-10506 **Luoding Zhu** (luozhu@iupui.edu), Indiana University-Purdue University Indianapolis, **Kausik Das*** (kdas@hmc.edu), Harvey Mudd College, **Madison Albert** (madison.m.albert@vanderbilt.edu), Vanderbilt University, and **Jared Barber** (jarobarb@iupui.edu), Indiana University-Purdue University Indianapolis. *Image digitization and calculation of forces for osteocyte viscoelastic networks* Preliminary report.

Osteocytes are bone cells that respond to external mechanical forces by releasing signals (mechanotransduction) that cause other cells to create or degrade bone material. To better understand how macroscale external forces lead to osteocyte mechanotransduction, we are constructing a three-dimensional, multiscale model of an osteocyte in its natural environment. In this presentation, we focus on just the osteocyte's membrane and cytoskeleton, which are modeled as an interconnected network of viscoelastic elements (damped springs) that generates internal forces when subject to external loads. We use three-dimensional segmentation software (Mimics) to generate the viscoelastic networks from experimental images of osteocytes. We then use the resulting network geometries to compute cellular forces for a given cell/network configuration. To test the model's construction, we integrate the equations of motion for each node in the mesh by numerically solving a system of ordinary differential equations (ODEs). Tests examine the cellular response to compression using both a coarse and a fine viscoelastic network.

Results suggest reasonable behavior with the network recovering its original shape after compression, as is expected for a cell with a viscoelastic membrane and cytoskeleton. (Received September 21, 2021)

1174-92-10513 **Yeona Kang*** (yeona.kang@howard.edu), Howard University, **Artur Agaronyan** (aagarony@umd.edu), Children's National Hospital, Department of Radiology, Howard University, **Paul Wang** (pwang@howard.edu), Department of Radiology, Howard University, **Tsang-Wei Tu** (tsangwei.tu@howard.edu), Molecular Imaging Laboratory, Howard University, Department of Radiology, Howard University, **Nobuyuki Ishibashi** (nishibas@childrensnational.org), Children's National Hospital, Washington D.C, and **Chao-Hsiung Hsu** (chaohsuing.hsu@Howard.edu), 3Molecular Imaging Laboratory, Howard Universtiy. *3D baboon brain template for MRI and PET Image* Preliminary report.

We present a new three-dimensional atlas of the anatomical subdivisions of the olive baboon brain to complement the multimodal imaging analysis. The atlas was created on a population-representative template. 20 major anatomical subdivisions were delineated, including thalamus, cortex, putamen, corpus callosum and insula. The atlas was evaluated with four individual MRI images and twenty individual PET images employing the multi-radiotracers: [11C]benzamide, [11C]metergoline, [18F]FAHA, and [11C]rolipram. Results showed more than 80% and 70% agreement for MRI and PET compared with manual quantification. This new template atlas is intended for use as a reference standard for the baboon PET and MRI brain research. (Received September 21, 2021)

1174-92-10548 **Sebastian J. Schreiber*** (sschreiber@ucdavis.edu), University of California, Davis, **Alex Hening** (ahening@tamu.edu), Texas A&M, and **Dang Nguyen** (dangnh.maths@gmail.com), University of Alabama. *Coevolution of patch selection in spatially and temporally variable environments*

Species live and interact in landscapes where enviornmental conditions vary both in time and space. In the face of this spatial-temporal heterogeneity, species may co-evolve their habitat choices which determine their spatial distributions. To understand this coevolution, we analyze a general class of stochastic Lotka-Volterra models that account for space implicitly. We define a (stochastic) coevolutionarily stable strategy (coESS) as a set of habitat choice strategies for each species that, with high probability, resists invasion attempts from mutant subpopulations utilizing other habitat choice strategies. We show that the coESS is characterized by a system of second-order equations. This characterization shows that the stochastic per-capita growth rates are negative in all occupied patches despite the species persisting. Applying this characterization to the coevolution of habitat-choice of competitors and predator-prey systems identifies under what environmental condit ions, natural selection excorcises "the ghost of competition past" and banishes some predators to prey-free habitats. Collectively, our results highlight the importance of temporal fluctuations, spatial heterogeneity and species interactions on the evolution of species' spatial distributions. (Received September 21, 2021)

1174-92-10558 **Cathy Zhao*** (ziyzha23@colby.edu), Colby College, and **Ryan Thomas Curry** (rtcurr22@colby.edu), Colby College. *Characterizing Inductively-pierced Place Fields Arrangements* Preliminary report.

Neuroscientists have experimentally shown that there are neurons in many organisms' brains known as place cells that fire as a result of the animal's spatial location in its environment. The regions in the environment where each specific neuron tends to fire, which are called place fields, generally have a convex shape. We study the geometric patterns created when multiple place fields overlap; this can help us determine the limitations of what spatial information can be stored in the brain. In particular, we focus on detecting a special type of overlapping arrangement, known as a k-piercing, in which each place field intersects k other place fields. What are the possible values of k for which we can draw k-piercings in 2 dimensions? Does the shape of the place fields impact this number? We have shown that we cannot represent k-piercings with circular place fields for k greater than 3, while with oval place fields (more biologically realistic), we can represent 3- and 4-piercings but not larger k values. Finally, assuming the data reflects a k-piercing, can we draw a diagram of the place fields such that every new place field added is a piercing of those that already exist (a special type of diagram we call k-inductively pierced)? We have worked on an algorithm that gives an ordering to draw any k-inductively pierced diagram, and in future work, we hope to implement these algorithms on a computer to actually draw these overlapping place fields. (Received September 21, 2021)

1174-92-10574 **Olivia Keala Rae Tom*** (otom@bu.edu), Boston University, **Yael Yossefy** (yyossefy@wesleyan.edu), Wesleyan University, and **Veronica Berger** (vrb1@williams.edu), Williams College. *Modeling SARS-CoV-2 to Predict its Effect on Patients with End-Stage Renal Disease (Worcester Polytechnic Institute REU in Industrial Mathematics and Statistics 2021)*

Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) emerged in December of 2019, and subsequently greatly affected public health. Despite mitigation strategies, including effective vaccines, the virus is not under control. Using mathematical models can give us a better understanding of the virus, providing insight into its infection and spread, and allowing us to predict how it affects vulnerable populations, e.g., patients with End-Stage Renal Disease (ESRD). We therefore developed two models using nonlinear differential equations: one describing the population-level spread of the disease, and one describing the dynamics of the virus within an infected patient. The population-level model is a complex SEIR model fit to U.S. cumulative mortality data during the third wave (October 1, 2020 to February 1, 2021). It originally over-predicts mortality beyond these dates, but modifying it to account for mitigation strategies (e.g., social distancing, mask wearing) fixes this, allowing the model to successfully predict daily infections, and to capture the wave-like behavior of the disease. Our in-host model simulates the interaction between viral particles and immune response cells (e.g., T Cells, antibodies). The model fits viral load measurements as a function of time for infected patients from two different sources. Given the success of these models in fitting data of otherwise healthy individuals, fitting them to data specific to patients with ESRD can be used to inform treatment protocols. (Received September 21, 2021)

1174-92-10575 **Scott McKinley** (smckinl3@tulane.edu), Tulane University, and **Scott McKinley*** (scott.mckinley@tulane.edu), Clemson University. *Methods for quantifying lysosomal motion properties using changepoint analysis*

Live cell imaging and single particle tracking techniques have become increasingly popular amongst the mathematical biology community. Lysosomes, known for endocytosis, phagocytic destruction, and autophagy, move about the cell along microtubules. Intracellular transport of lysosomes is carried out in membrane-bound vesicles through the use of motor proteins. Single particle tracking methods utilize stochastic models to simulate intracellular transport and give rise to rigorous analysis of the resulting properties, specifically related to transitioning between inactive to active states. Results of statistical analysis show that the proportion of time lysosomes spend active is region-dependent. We expand on the regional analysis by computing the mean-squared displacement and by implementing methods that allow us to explore the distribution of the active velocities and their respective transitions. We find confidence in our methodology and develop simulations to capture these properties at multiple frames rates. (Received September 21, 2021)

1174-92-10584 **Keisha Cook*** (keisha@clemson.edu), Clemson University. *Qualitative analysis of lysosomal motion as a function of intracellular region*

Live cell imaging and single particle tracking techniques have become increasingly popular amongst the mathematical biology community. Lysosomes, known for endocytosis, phagocytic destruction, and autophagy, move about the cell along microtubules. Intracellular transport of lysosomes is carried out in membrane-bound vesicles through the use of motor proteins. Single particle tracking methods utilize stochastic models to simulate intracellular transport and give rise to rigorous analysis of the resulting properties, specifically related to transitioning between inactive to active states. Results of statistical analysis show that the proportion of time lysosomes spend active is region-dependent. We also determine an optimal frame rate for capturing live cell data is necessary in order to successfully infer properties or the underlying mechanisms, such as microtubules Results show that over the same length of time, data captured at faster frames provides more information than at slower frame rates. We explore this phenomena through a simulation study. Lastly, we explore transduction efficiency in control and sucrose treated cells to determine a relationship between size, speed, and gene delivery. (Received September 21, 2021)

1174-92-10685 **Breanna Shi** (shi00231@umn.edu), Univ of Minnesota, and **Fazal Abbas*** (drfabbas@gmail.com), Mathematics and Computer Sciences, Stetson University. *A Mathematical Study of Modeling the Bioaccumulation of Methyl-Mercury in Aquatic Systems*

Three models for the bioaccumulation of methyl-mercury in an aquatic ecosystem are described. each model combines predator-prey equations for interactions across a system of trophic levels with toxicokinetic equations for toxin elimination at each level. The models consider food preference and competition within trophic levels. A stability test and sesitivity analysis are conducted for each model. Using known elimination constants for methyl mercury in various fish species and known sources of input from literature, the model predicts toxin levels at

the three trophic levels that are compared to samples in the Tampa Bay. The data shows that the competition model is the most accurate of the models produced, and further that actual Methyl-Mercury concentrations are higher than predicted in all models. (Received September 21, 2021)

1174-92-10700 **Chris Kang*** (chris.kang@wsu.edu), Washington State University. *Emergent collective behaviors in coupled Boolean networks.* Preliminary report.

Exploring the spatio-temporal dynamics in coupled gene regulatory networks as a mathematical model brings us one step-closer to understanding the origin of life. In the paper, Emergence of diversity in homogeneous coupled Boolean networks, we showed through model simulations that complex behaviors such as phenotypic diversity in gene regulatory networks of isogenic cells can arise in simplistic, stochastic non-linear dynamical systems - specifically, coupled Boolean networks with perturbation. The work corroborated numerous past findings that complexity of information processing is maximized in systems when they operate near a phase transition between an ordered and a chaotic regime of dynamical stability. One of such observations in the model was the checkerboard pattern formation in long-term behaviors (characterized by the steady-state distributions of Boolean states) on a tissue of interacting cells at "near-critical" Lyapunov stability. In recent, we have improved and redefined the interactions of cells as an Ising model, where the phase transition of ferromagnetism is understood. Direct comparison with experimental results in collective cell-state transition in mouse embryonic stem cells will be discussed. (Received September 21, 2021)

1174-92-10734 **Yahe Yu** (yyu29@ncsu.edu), Dalian University of Technology, and **Ruian Ke** (rke@lanl.gov), Los Alamos National Laboratory. *A Machine Learning Approach for Detecting Exponential Virus Growth*

Being able to detect virus exponential growth is of paramount importance in infectious diseases. We can use this to predict an epidemic, or check a virus adaptation to the drug pressure, or check the immune escape within a person. In this talk we present the application of an ensemble method with UNet++ to detect the exponential growth subgroups in the virus sequencing data. Two UNet++ networks were built in this study, one is designed for large exponential block samples and the other is for small exponential block samples. Viral population changes would lead to differences in the pairwise difference matrices. By transferring the pairwise difference matrices of viral sequencing data into heat maps, the ensemble UNet++ can precisely detect the exponential growth subgroup virus of various sizes. The ensemble UNet++ achieves a very respectable 98.55% accuracy (mean IoU). (Received September 21, 2021)

1174-92-10747 **Joon Ha*** (joonha0733@gmail.com), Howard University, and **Arthur Sherman** (ArthurS@niddk.nih.gov), National Institutes of Health. *A mathematical model estimates relative beta-cell function during continuous glucose monitoring* Preliminary report.

A common form of diabetes, type 2 diabetes is a failure of insulin-secreting pancreatic beta-cells to increase insulin to the level demanded by the body to maintain normal blood glucose. Estimation of the secretory capacity of beta-cells is crucial to prevent and intervene in the disease. Oral glucose tolerance tests (OGTT) is currently used together with math models to estimate insulin requirements and insulin secretion capacity. During OGTTs, glucose and insulin measurements are required as inputs to estimate the metabolic parameters. Recently, continuous glucose monitoring (CGM) has been in the spotlight of diabetes management because it can track glucose excursions in free-living individuals. However, the CGM does not measure insulin, so that current math models are unable to estimate parameters. Beta-cell function and insulin requirement cannot be estimated without both glucose and insulin, but we have developed a new mathematical model to estimate beta-cell function relative to insulin requirements using CGM data. Validation was performed with patients who wore a CGM device during an OGTT. Relative beta-cell function from the OGTT was estimated with insulin, while relative beta-cell function from the CGM was estimated without insulin. The two relative beta-cell function estimates from OGTT and CGM were strongly correlated. These findings have a high potential to enhance diabetes research by allowing estimation of relative beta-cell function with data sets lacking insulin measurements. (Received September 21, 2021)

1174-92-10759 **Bard Ermentrout*** (bard@pitt.edu), University of Pittsburgh, and **Nour Riman** (nourr@andrew.cmu.edu), Carnegie Mallon University. *Noise as a strategy* Preliminary report.

Olfactory search is a necessary behavior used by animals to locate sources of food, mates, etc. A common strategy relies on two spatially separated sensors (such as antennae or even nostrils) where by the animal moves in the direction of higher concentration:

$$x' = v\cos(\theta), y' = v\sin(\theta), \theta' = f(C_L - C_R),$$

where C_L, C_R are the concentrations on the left and right sensors and θ is the heading direction. This algorithm fails of the concentrations are below detection and the animal can wander far off. Thus, a simple search strategy is needed. The easiest to analyze is to just add some noise to the heading, θ. In this talk, I will first discuss the deterministic dynamics in the presence of simple odor profiles such as a point source and a line. I will then show that success in finding the odor can be greatly improved with the addition of noise as long as it is in a limited range. I also show that various manipulations such as concentration dependent noise can greatly improve performance. (Received September 21, 2021)

1174-92-10810 **Chloe Brooks*** (cbrooks3@stetson.edu), Mathematics and Computer Sciences, Stetson University, and **Petko Kitanov** (pkitanov@wells.edu), Wells College. *Phenotypic Plasticity and Stability Analysis for Different Species in the Ecosystem*

The ability to follow various morphological trajectories in response to environmental and ecological conditions is called phenotypic plasticity. This research focuses on modeling populations of different species mathematically and performing the stability analysis given specific coefficients shown to model particular behaviors of the present species accurately. We consider four trophic levels to develop the system of equations for the phenomenon. Our study determines the long-term trends in population and proves the sustainability of the ecosystem under certain conditions. (Received September 21, 2021)

1174-92-10871 **Asia Wyatt*** (awyatt314@gmail.com), The Johns Hopkins University Applied Physics Laboratory, University of Maryland, College Park, and **Doron Levy** (dlevy@math.umd.edu), University of Maryland, College Park. *Modeling the Effect of Memory in the Adaptive Immune Response*

It is well understood that there are key differences between a primary immune response and subsequent responses. Specifically, memory T cells that remain after a primary response drive the clearance of antigen in later encounters. While the existence of memory T cells is widely accepted, the specific mechanisms that govern their function are generally debated. In this talk, we develop a mathematical model of the immune response. This model follows the creation, activation, and regulation of memory T cells, which allows us to explore the differences between the primary and secondary immune responses. Through the incorporation of memory T cells, we demonstrate how the immune system can mount a faster and more effective secondary response. This mathematical model provides a quantitative framework for studying chronic infections and auto-immune diseases.
(Received September 21, 2021)

1174-92-10880 **Christine Heitsch*** (heitsch@math.gatech.edu), Georgia Institute of Technology. *Further combinatorics of RNA branching*

Understanding the folding of RNA sequences into functional structures is a fundamental scientific challenge. For example, the branching of an RNA secondary structure is an important molecular characteristic yet difficult to predict correctly, especially at the length scale of RNA viral genomes. Efforts to understand the geometry of RNA configuration landscapes lead naturally to a local move transformation on plane trees, a combinatorial model for secondary structures. We give results illustrating how abstracting this challenging biomathematical question yields new combinatorial results as well as further biological insights. (Received September 21, 2021)

1174-92-10899 **Libin Rong*** (libinrong@ufl.edu), University of Florida. *Modeling HIV multiple infection*

Multiple HIV infection of target cells may lead to viral escape from host immune responses and drug resistance to antiretroviral therapy, posing more challenges to the control of infection. In this talk, I will present a model that includes sequential cell-free virus infection (i.e. one virus is transmitted each time in a sequential infection of target cells by virus) and cell-to-cell transmission (i.e. multiple viral genomes are transmitted simultaneously from infected to uninfected cells). By comparing model prediction with the distribution data of proviral genomes in HIV-infected spleen cells, we will evaluate the possible mechanisms that underlie HIV multiple infection and their relative contributions. The results suggest that cell-to-cell transmission may play a critical role in forming HIV multiple infection and thus has important implications for HIV evolution and pathogenesis. (Received September 21, 2021)

1174-92-10937 **Yun Kang** (yun.kang@asu.edu), Arizona State University, **Maria Gabriela Navas Zuloaga*** (mnavaszu@asu.edu), Arizona State University, **Theodore P Pavlic** (tpavlic@asu.edu), Arizona State University, **Kaitlin Mari Baudier** (kaitlin.baudier@usm.edu), Arizona State University, **Jennifer Fewell** (j.fewell@asu.edu), Arizona State University, and **Noam Ben-Asher** (noam.benasher@bostonfusion.com), Boston Fusion. *A Mathematical Model for Flexible Collective Defense: Crisis Response in Social-Insect Colonies* Preliminary report.

Fundamental to living systems, from cells to social-insect colonies, is an organizational distinction between inside and outside, and a need to allocate resources to defend this boundary. While a detailed study of cell-wall dynamics is difficult, the dynamics of defense adaptation in a social-insect colony are more conspicuous. This is particularly the case of *Tetragonisca angustula* stingless bees, which employ defensive strategies across three timescales: morphological specialization, where specialized, costly soldiers (majors) are produced over weeks; age-based polyethism, where young majors transition to guarding tasks over days; and task switching, where non-specialized small bees (minors) replace soldiers within minutes in crisis. To understand these multi-scale dynamics, we built a demographic ODE model of task allocation showing how reproduction, development, and behavior integrate to balance defensive demands and other colony needs. We study the effect of these three mechanisms on task allocation and colony size. Our analysis yields conditions for colony survival and shows that, with guard replacement and long maturation times, increasing major production can reduce the long-term allocation to defense. This work elucidates demographic factors constraining colony defense regulation and helps to establish stingless bees as model organisms for understanding other systems where transaction costs for component turnover are nontrivial, like manufacturing systems. (Received September 21, 2021)

1174-92-10939 **Imelda Trejo Lorenzo*** (imelda@lanl.gov), Los Alamos National Laboratory. *Metabolic dynamics within photoreceptors*

Metabolic dysfunction and nutrient deprivation in photoreceptors can lead a verity of retinal diseases, including age-related macular degeneration and retinitis pigmentosa. The two main causes of irreversible vision loss in the elderly and inherited blindness, respectively. We introduce a mathematical model of nonlinear ordinary differential equations to describe metabolic dynamics in a single cone photoreceptor, focusing on key metabolites, such as glucose, pyruvate and lactate. The bifurcation analysis revels that the model has a bistable regime, which corresponds to a healthy versus a pathological state. The sensitivity analysis shows that the processes influencing the cone's access to nutrients and its adaptation to nutrient deprivation have the largest impact on the metabolic system. These analyses provide insight into potential therapeutic targets and optimal treatment windows for metabolic photoreceptor diseases. (Received September 21, 2021)

1174-92-10940 **Raquel Castromonte*** (Rc757@cornell.edu), Cornell University, and **Gregory Sandland** (gsandland@uwlax.edu), University of Wisconsin - La Crosse, River Studies Center, La Crosse, WI. *Incorporating Chlorophyll-a Levels into an Integral Projection Model of Gizzard Shad (*Dorosoma cepedianum*) in the Upper Mississippi River*

A novel approach to acquiring time-saving management strategies for wild populations is with Integral Projection Models (IPM), a data-driven model that quantifies life history strategies. Gizzard Shad *(Dorosoma cepedianum)* are small native fish that play an important role in aquatic communities found throughout the central and eastern United States. In certain waterbodies, such as the Mississippi River, this species has been adversely affected by invasive carp, which compete with young shad for food resources (phytoplankton). To research the influence of food quality on Gizzard Shad populations, we aimed to (1) incorporate the consumption of plankton into juvenile Shad, and (2) determine how varying juvenile length distributions influence future adult stages. Using an IPM we utilized empirical information on plankton (chlorophyll-a) and juvenile length (at Age 1) to predict overall population patterns in two navigation pools of the Mississippi River (Pools 4 and 26) prior to carp invasion. There was an inverse relationship between chlorophyll-a levels and the size of juvenile shad. Incorporating this relationship into the model resulted in a higher frequency of adult fish in Pool 4, which was the pool with lower concentration of chlorophyll-a. This result can be explained by the life-history and density-dependent assumptions of our model. We will expand this model in the future to predict gizzard shad populations in additional pools of the Mississippi River both before and after carp invasion. (Received September 21, 2021)

1174-92-10957 **Siona Prasad*** (sionaprasad@college.harvard.edu), Harvard College. *Urban Inversions of Air Pollution Sources / Sinks and Uncertainty Quantification to Pinpoint Determinants of Poor Air Quality* Preliminary report.

Anthropogenic emissions of carbon dioxide (CO2) have resulted in the build-up of greenhouse gas concentrations responsible for climate change and worsening urban air quality. An accurate and efficient measurement strategy

to estimate CO2 emissions from large cities does not currently exist. Current atmospheric modeling techniques are computationally inefficient. In this project, I discuss the construction and calibration of an inexpensive CO2 sensor mounted on a low-powered drone for accurate measurements. I develop an atmospheric dispersion model and inversion techniques to estimate emission inventories for large cities using measurement data. Low-cost sensor data compared favorably with state-of-the-art instruments (correlation factor = 0.99) and exhibited expected diurnal cycles and traffic patterns. The plume dispersion model successfully combined sensor, meteorological data, and inversion tools. The model was able to predict emission inventories for Washington DC, and highlighted a large CO2 contribution from the transportation sector. Total CO2 emissions over the entire domain were 101,000 μ mol/s. Uncertainty reductions were as high as 90%, indicating the success of the overall methodology. We construct the tools for and demonstrate the efficiency and success of a comprehensive measurement system to enforce government-set carbon laws and mitigation strategies, taking the first step toward improving city-wide public health, reducing air pollution and combating climate change. (Received September 21, 2021)

1174-92-10971 **Hannah Callender Highlander*** (highland@up.edu), University of Portland, and **Anna Singley** (aesingley17@gmail.com), Central Catholic high school. *IDENTIFICATION OF TIPPING POINTS IN THE EMOTIONAL LANDSCAPE OF SUICIDE WITH SENTIMENT ANALYSIS* Preliminary report.

Suicide is a complex psychological process that can be mathematically modeled with nonlinear dynamics phenomena. Critical slowing down (CSD) is a proposed, but empirically unproven, indicator of increased risk for the actualization of suicidal ideation. Obtaining the data necessary for substantiating the efficacy of critical slowing down in this application is made difficult by a litany of ethical, legal, and logistical challenges. Additionally, data collected via survey tends toward drastic underreporting. Although applicable collection techniques exist, acquiring a functional dataset with a known the final outcome of suicidal behavior is fraught with barriers to practical application. The use of publicly available poetry and song lyrics from well-known suicidal and non-suicidal artists allows for analysis of the emotional and psychological landscape of suicide whilst avoiding ethical, legal, and logistical pitfalls. Moreover, the methodology described in this paper is applicable to the further modeling of other psychological processes. (Received September 21, 2021)

1174-92-11003 **Corey Beck*** (beckca@lafayette.edu), Lafayette College. *Coupling of Beating Filaments at Low Reynolds Number*

Microscopic organisms and biological systems at low Reynolds number frequently rely on filaments like flagella or cilia for movement. To understand these systems we must accurately describe the interactions between flagellar motion and the surrounding fluid. While single flagellar motion can be modeled, some instances involve interactions between multiple cilia. Coupling forces are modeled in the surrounding fluid using the method of regularized Stokeslets. In this model, the cilia are treated as simplified two-link filaments, with systems of ordinary differential equations solved numerically. Through this method, we observe varying phase differences between beating cilia for regimes dependent on coupling strength and initial phase difference. Unique results of this simplified model are then compared to past research to observe the similarities and differences between the discrete and continuous filament models.

This research was conducted as part of the 2021 UC Davis Pure and Applied Math REU, supported by the National Science Foundation under grant no. DMS 1950928. (Received September 21, 2021)

1174-92-11009 **Edward T Dougherty** (edougherty@rwu.edu), Roger Williams University, and **Elizabeth Marie Wexler*** (ewexler210@g.rwu.edu), Roger Williams University. *Variable Tissue Electrical Conductivities Within Computational Simulations of Transcranial Electrical Stimulation* Preliminary report.

Transcranial electrical stimulation (TES) is a form of neuromodulation that has demonstrated the ability to successfully address the symptoms associated with a variety of neurological disorders. The synthesis of neurology with applied mathematical modeling offers an approach to predict and enhance treatment parameters and settings for anatomies and medical conditions specific to individual patients. Computational simulations of TES has shown to facilitate treatment efficacy, however, exact brain tissue conductivities for particular patients are unknown and unmeasurable, and their variability impacts the utility of simulation findings. To help enhancing *in silico* TES experiments for patient-specific applications, we have implemented a stochastic partial differential equation based mathematical model of TES. This model utilizes Laplace's equation and is numerically solved using the finite element method. Monte Carlo iterative simulations of tissue electrical conductivity variability are run, using both an idealized two dimensional domain and an MRI-based head geometry. Results show expected simulation variance as well as the utility of incorporating head tissue electrical conductivity variability in TES numerical simulations. (Received September 21, 2021)

1174-92-11123 **Xiang-Sheng Wang*** (xswang@louisiana.edu), University of Louisiana at Lafayette. *Threshold dynamics of a nonlocal and delayed cholera model in a spatially heterogeneous environment*

A nonlocal and delayed cholera model with two transmission mechanisms in a spatially heterogeneous environment is derived. We introduce two basic reproduction numbers, one is for the bacterium in the environment and the other is for the cholera disease in the host population. If the basic reproduction number for the cholera bacterium in the environment is strictly less than one and the basic reproduction number of infection is no more than one, we prove globally asymptotically stability of the infection-free steady state. Otherwise, the infection will persist and there exists at least one endemic steady state. For the special homogeneous case, the endemic steady state is actually unique and globally asymptotically stable. Under some conditions, the basic reproduction number of infection is strictly decreasing with respect to the diffusion coefficients of cholera bacteria and infectious hosts. When these conditions are violated, numerical simulation suggests that spatial diffusion may not only spread the infection from high-risk region to low-risk region, but also increase the infection level in high-risk region. (Received September 21, 2021)

1174-92-11156 **Lucero Rodriguez Rodriguez*** (lrodri68@asu.edu), Arizona State University. *Mathematical modeling of two agents interaction dynamics with applications in Human-Automation* Preliminary report.

Modern interactions with technology have been moving away from simple human use of computers as tools to the establishment of human relationships with autonomous entities that carry out actions on our behalf, thus it is imperative to understand the human-autonomy interaction dynamics for best outcomes. In this project, we develop a two-agent interaction framework in discrete-time that could apply to human-autonomy interaction. We analyze our model and validate it through experimental data. Our study shows that the model has rich dynamics including multiple attractors. Combined with data, our work finds that in order to have a high-performing team, both team members need to have constant communication, and the importance of having one leader in the team. We investigate different member's and team's characteristics, such as personality and training, that would drive the team to succeed or fail in a given task. Our model could potentially help us to select team members that could work together efficiently, and train members in the established team to collaborate better. (Received September 21, 2021)

1174-92-11165 **Vivian Nguyen*** (viviantnguyen15@email.arizona.edu), The University of Arizona, Indiana University—Purdue University Indianapolis, and **McKenna Kaczanowski** (mskaczanowsk@bsu.edu), Ball State University, Indiana University—Purdue University Indianapolis. *A mathematical model of combination therapies for organ transplantation* Preliminary report.

Transplantation is a life-saving procedure that must be accompanied by immunosuppression to prevent transplant rejection. However, life-long use of immunosuppression compromises the health and quality of life of transplant patients. Thus, a new treatment strategy that combines the effectiveness of immunosuppression while preventing long-term side-effects is needed. Adoptive transfer of regulatory T cells (Tregs) has emerged as a promising method for promoting graft survival. In this study, we have adapted an ordinary differential equations model of the immune response to a mouse heart transplant to include the effects of both immunosuppression and adoptive transfer of Tregs with an ultimate goal of determining an effective combination therapy that minimizes the level of immunosuppression needed to promote survival of the graft. Ultimately, this study demonstrates the utility of mathematical modeling to optimize therapeutic treatment strategies and inform clinical studies. (Received September 21, 2021)

1174-92-11185 **Jake Snow*** (97jake@gmail.com), Brigham Young University. *Wildfire Detection and Segmentation* Preliminary report.

Leveraging machine learning mixed with various classical techniques of image segmentation, we consider the problem of determining fire boundaries from satellite images. In addition, we develop a mathematical model that assigns burn severity probabilities to regions post fire. Finally, we explore the potential impact of improved wildfire perimeter detection, active burn region classification , and post-fire burn analysis. (Received September 21, 2021)

1174-92-11223 **Lourdes Juan*** (lourdes.juan@ttu.edu), Texas Tech University, **Katharine Long** (katharine.long@ttu.edu), Texas Tech University, **Andrey Morozov** (am379@leicester.ac.uk), Sobolev Institute of Mathematics, University of Leicester, **Jackson Kulik** (jpk258@cornell.edu), Cornel University, and **Jacob Slocum** (jacob.slocum@ttu.edu), Texas Tech University. *Path integration and the structural sensitivity problem in partially specified biological models* Preliminary report.

It is known that the qualitative behavior of a model dynamical system can exhibit sensitivity not only to the numerical parameters in the model, but also to the choice of functional form used in the model. This is known as structural sensitivity. Mathematical models with unspecified functional forms are known as partially specified models. In such models, the conventional idea of revealing a concrete bifurcation structure becomes irrelevant: we can only describe bifurcations with a certain probability. We use a novel mathematical framework based on the probability measure on the response function space via path (or functional) integration, applying path integration to the study of structural sensitivity in partially specified models. We focus on a simple biological model which relies on the functional response of a predator. Previous work demonstrates that three convenient functional response forms yield different qualitative dynamics. We present a method for fairly sampling the space of reasonable functional responses given basic assumptions about continuity, differentiability, and monotonicity as well as quality of fit to data. Resulting from this fair sampling of functional responses is a fair sampling of reasonable models from which one can discern the proportion of various qualitative outcomes. These proportions can be interpreted as the likelihood of a specific qualitative behavior given the experimental data. (Received September 22, 2021)

1174-92-11241 **Carlos Enrique Bustamante** (cbustam3@asu.edu), Arizona State University, and **Jordy Jose Cevallos Chavez** (jcevall1@asu.edu), Arizona State University. *Mathematical Model of Trust in Automation Dynamics* Preliminary report.

Technological advancements have made self-driving cars available to the general population. As a result, current research has moved to understand Human-Automation interaction dynamics and the effects of human's trust to rely on the automation. Trust in Automation (TiA) is considered as one of the most important factors that influence human's reliance in automation. In this project, we define and build a mathematical model of TiA utilizing automation's quality, workload/environment, and risk that the operator faces. We analyze our model and validate it through experimental data of a simulated driving task. Our model help us further our understanding of the key factors that influence human's trust in automation and in the developing of an automation responsive to the operator's change in trust. (Received September 22, 2021)

1174-92-11247 **Chathuri Sandamali*** (chathuri.t.sandamali@ttu.edu), Texas Tech University. *Modelling and Analysis of Low Persistent ZIKV Dynamics with Sexual Transmission* Preliminary report.

Zika fever, caused by the Zika virus (ZIKV), becomes a global threat for birth deficiency. The limited understanding of ZIKV with its low but sufficient transmission to maintain itself makes the establishments of prevention techniques more challenging. To analyse the low persistence ZIKV with large periodic outbreak cycles, we use a simple SIR model with modeling of human-human encounter rate as a pure birth process by considering the infected human's searching distance as a Poisson point process. We derive a formula for basic reproduction number R_0. The proposed model shows at least one positive endemic equilibrium when $R_0 > 1$ and we prove disease-free equilibrium is locally asymptotically stable if $R_0 < 1$. The global stability analysis shows the disease-free equilibrium is globally asymptotically stable when $R_0 < 1$ under certain model parameter conditions. We find a closed-form formula for the occurrence of the backward bifurcation where the disease free equilibrium coexists with endemic equilibrium. This leads to Hopf bifurcation, which serves as an oscillation source. Furthermore, bifurcation analysis and numerical simulations show that the proposed simple model is sufficient to describe the large periodic epidemic cycles and low persistent ZIKV. The extended non-autonomous and stochastic models for the analysis of environmental and stochastic variability exhibit varying amplitudes and periods of the outbreaks. (Received September 22, 2021)

1174-92-11266 **Omar Saucedo** (osaucedo@vt.edu), Virginia Tech, **Tingting Tang*** (ttang2@sdsu.edu), San Diego State University, **Amanda Laubmeier** (amanda.laubmeier@ttu.edu), Texas Tech University, and **Tim Pollington** (timothy.pollington@gmail.com), Big Data Institute, University of Oxford. *Impact of data structure, availability and noise distribution on practical and structural identifiability of an SEIR model* Preliminary report.

Identifiability is an essential prerequisite for system identification, which can be approached from a structural or practical view. Practical identifiability analysis focuses on characterizing the uncertainty in parameter estimates considering the data deficiencies used to calibrate the model. Structural (theoretical) identifiability aims to establish whether the model parameters can be uniquely determined based on the model structure and from observation of the input-output behaviour of the model.

In this study, we explored both structural and practical identifiability of a Susceptible-Exposed-Infected-Recovered (SEIR) model. We examine the structural identifiability of the SEIR model with different combination of input data and compare them with the practical identifiablity of the model. We further investigate the practical identifiability of the SEIR model with respect to different observable data, data frequency, and noise distributions. The practical identifiability is performed by both Monte Carlo simulations (MC) and a correlation matrix approach (CM). Our results show that practical identifiability depends on the observable data and data frequency, and the region of the parameter space. In addition, we compare and distinguish the practical identifiablity from MC approach and the correlation index from CM approach. (Received September 22, 2021)

93 ► *Systems theory; control*

1174-93-5421 **Michael Malisoff*** (malisoff@lsu.edu), Louisiana State University. *Delay-Compensating Event-Triggered Control using Interval Observers*

We provide a new event-triggered feedback control method for a class of control systems with input delays. Our chain predictor approach makes it possible to prove uniform global exponential stability of the closed loop systems for any positive value of the constant input delay. Our exponential stability proof is based on positive systems and interval observers. We illustrate our work using a mathematical model for an underwater vehicle. This work is collaborative with Corina Barbalata and Frederic Mazenc. No prerequisite background in systems and controls will be needed to understand and appreciate this talk. (Received August 20, 2021)

1174-93-5850 **Said Hadd*** (s.hadd@uiz.ac.ma), Ibn Zohr University. *On perturbed semlinear stochastic systems*

In this talk, we will discuss the well-posedness of a class of semilinear stochastic equations on Hilbert spaces. Here, the nonlinear term is considered as the composition of a Lipschitz function with an unbounded linear operator. This case presents many difficulties in the application of the standard theory of nonlinear equations. To overcome this obstacle, we will use the concept of admissible observation operators for semigroups. Then will see that the solution is given by a new variation of constant formula using the concept of Yosida extensions of observation operators. In addition, we prove the Feller property for the transition semigroup associated with the stochastic equation. Finally, we give an application for neutral stochastic systems. (Received August 30, 2021)

1174-93-10570 **Hasala Senpathy Karunaratne Gallolu Kankanamalage** (hgallolu@rwu.edu), Roger Williams University, **Olivia Chesney*** (ochesney746@g.rwu.edu), Roger Williams University, and **Ezra Miller** (emiller626@g.rwu.edu), Roger Williams University. *Improved Energy Consumption Models for Automobile Systems*

Optimality in energy consumption in automobiles has become a significant matter in electric and green energy systems. Optimality in energy consumption in automobile systems can be impossible due to the stochastic nature of traffic conditions. However, deterministic models can lead to improvements in energy consumption. In this work, we primarily focus on external dynamical patterns of automobile systems. A continuation of current work leads to improvements in the internal dynamics of automobiles via specific control policies proposed for automobile platoons under urban driving conditions. (Received September 21, 2021)

1174-93-11209 **Saroj Aryal** (saryal@georgian.edu), Georgian Court University, and **Sarita Nemani*** (snemani@georgian.edu), Georgian Court University. *Robustness of Stable Polynomials under Uniform Perturbations* Preliminary report.

The problem of robustness under perturbations of coefficients of a polynomial representing a stable continuous-time linear system is an interesting one. In this work, we consider some variations of a conjecture on robustness of the stability of transfer polynomials. We establish some classes of stable polynomials that are robust under uniform perturbations by certain functions. (Received September 21, 2021)

94 ► Information and communication theory, circuits

1174-94-7243 **Nicholas Layman*** (laymann@mail.gvsu.edu), Grand Valley State University.
Continuous Guessing Games With Two Secret Numbers

A guessing game with two secret numbers is a game played between a questioner and a responder. The two players first agree upon the set, N, in which the game will be played as well as the number of questions, Q, which will be asked by the questioner. The responder first chooses two distinct numbers from N. The questioner then asks questions of the form "How many of your chosen numbers are in the set $A \subseteq N$?" to which the responder answers truthfully. The goal for the questioner is to determine the responder's two numbers using at most Q questions. We study a continuous version of this game where N is the closed interval of real numbers from 0 to 1. We introduce tools to study this game and use them to examine strategies for the questioner using a geometric approach. We establish a condition that must be satisfied by optimal strategies and give a strategy that can be made arbitrarily close to optimal. (Received September 16, 2021)

1174-94-7625 **Anthony Joseph Macula*** (macula@geneseo.edu), SUNY Geneseo. *Making Pooling Designs for COVID-19 Surveillance on a Mass Scale Deployable and Decodable in the Field* Preliminary report.

This talk describes practical, fieldable, pooled saliva COVID-19 surveillance and diagnostic testing designs that are more efficient than the Dorfman pooling designs widely in use. Symmetry, Geometry, Graph Theory, and Probability come into play. (Received September 15, 2021)

1174-94-7695 **David B Damiano** (ddamiano@holycross.edu), College of the Holy Cross, and **Sarah Vermette*** (srverm22@g.holycross.edu), College of the Holy Cross. *Discourse Sheaves to Model Opinion Dynamics on Watts-Strogatz Networks* Preliminary report.

Social networks can be modeled by mathematical networks which consist of nodes, representative of individuals, and edges, or the connections between these individuals. Our interest is in the development and changing of opinions across a network as individuals interact. A vector at a node represents an individual's set of opinions, while the interaction between two individuals takes place within a vector space associated with their discourse. We use the Laplacian, a matrix representation of the interactions, to develop a model for opinion dynamics. With this information we construct a discourse sheaf, an algebraic structure developed by Hansen and Ghrist, over the opinion space that reveals the details of the changing opinions. In particular, we apply this method to Watts-Strogatz ring networks and small world models to investigate how certain individuals can influence a collection of opinions. (Received September 17, 2021)

1174-94-7881 **Angela Robinson** (angela.robinson@nist.gov), NIST, **Sarah Arpin** (Sarah.Arpin@colorado.edu), University of Colorado Boulder, **Tyler Raven Billingsley*** (billin2@stolaf.edu), St. Olaf College of Northfield, MN, **Daniel Hast** (drhast@bu.edu), Boston University, and **Jun Lau** (jblau@ucsd.edu), University of California San Diego.
BIKE Decoders and Error Detection Preliminary report.

There is a class of public-key cryptography based on linear error-correcting codes. The decoder used during error correction directly affects the security of a code-based cryptosystem. Correlations between error patterns that lead to decoding failures and the private key of a scheme have been discovered, leading cryptographers to work diligently to minimize decoding failures. We look closely at the iterative decoders used by cryptosystem BIKE, a 3rd round proposal in the NIST Post Quantum Cryptography standardization process. The natures of these iterative, bit-flipping decoders make them difficult to analyze in a closed form. Cryptographers have thus relied on measurements of weakened versions of BIKE with a decryption failure rate that is high enough to measure with reasonable computational resources. These measurements have been extrapolated to design parameters for BIKE that are believed to provide a sufficiently low decryption failure rate for cryptographic purposes.

However, such extrapolations don't account for error floors caused by error vectors that are equidistant between two different codewords. While such error vectors are rare enough that they should not affect the extrapolations, it is unclear whether similar effects might occur with error vectors that are near-equidistant between two different codewords. By examining the performance of the BIKE decoder on this class of error vectors, we can determine the accuracy of the extrapolated decryption failure analysis of the BIKE decoder. (Received September 16, 2021)

1174-94-7984 **Hanson Hao*** (hhao@stanford.edu), Stanford University. *Investigations on Automorphism Groups of Quantum Stabilizer Codes* Preliminary report.

The stabilizer formalism for quantum error-correcting codes has been, without doubt, the most successful at producing examples of quantum codes with strong error-correcting properties. In this paper, we discuss *strong* automorphism groups of stabilizer codes, beginning with the analogous notion from the theory of classical codes. Two weakenings of this concept, the *weak* automorphism group and *Clifford-twisted* automorphism group, are also discussed, along with many examples highlighting the possible relationships between the types of "automorphism groups". In particular, we construct an example of a $[[10, 0, 4]]$ stabilizer code showing how the Clifford-twisted automorphism groups might be connected to the Mathieu groups. Finally, nonexistence results are proved regarding stabilizer codes with highly transitive strong and weak automorphism groups, suggesting a potential inverse relationship between the error-correcting properties of a quantum code and the transitivity of those automorphism groups. (Received September 17, 2021)

1174-94-8044 **Liam Gabriel Busch*** (buschl73@students.rowan.edu), Rowan University, and **Hieu D Nguyen** (nguyen@rowan.edu), Rowan University. *A New Decoding Algorithm for Correcting Both a Single Insertion and Single Deletion Error in Helberg codes* Preliminary report.

The corruption of binary codewords during data transfer can lead to devastating loss of information. As the world becomes increasingly digitized, finding methods to correct corrupted codewords become increasingly important. Helberg codebooks are sets of codewords constructed in such a way that transmitted codewords can be recovered given that a limited number of errors occurred either through insertions or deletions of bits. While it has been proven that Helberg codewords are recoverable, efficient algorithms are needed to recover them. Such decoding algorithms have already been found and proven for any number of deletion errors and up to two insertion errors. Presented here is the first known decoding algorithm for correcting both a single insertion and a single deletion bit error, which has been verified through extensive testing, but only special cases have been proven. (Received September 17, 2021)

1174-94-8149 **Eric Javier Pabon-Cancel** (eric.pabon1@upr.edu), University of Puerto Rico, Mayaguez. *Improving the Dimension of Trace Goppa Codes* Preliminary report.

Binary Goppa codes are one of the most widely studied classes of linear codes. They have a good decoder available and they resist cryptographic attacks. Several of the best known linear codes are Binary Goppa codes. In this talk we shall present an improvement on the minimum distance of Trace Goppa Codes, a class of linear codes where the Goppa polynomial is $\mathrm{Tr}_{F_q^m/F_q}$. P. Veron derived improvements on their dimension and minimum distance. We improve on the previous distance bound from $2q^2 + 2q$ to $2q^2 + 2q + 8$. This is a significant improvement for trace Goppa codes from non quadratic extensions. (Received September 17, 2021)

1174-94-8193 **Alexander Barg*** (abarg@umd.edu), University of Maryland, and **Gilles Zemor** (Gilles.Zemor@math.u-bordeaux.fr), University of Bordeaux. *High-rate storage codes on triangle-free graphs*

Consider an assignment of bits to the vertices of a connected graph $G(V, E)$ with the property that the value of each vertex is a function of the values of its neighbors. A collection of such assignments is called a *storage code* of length $|V|$ on G, and if it forms an F_2-linear space, it is called a linear storage code. If G contains many cliques, it is easy to construct codes of rate close to 1, so a natural problem is to construct high-rate codes on triangle-free graphs, where constructing codes of rate $> 1/2$ is a nontrivial task. Previously only isolated examples of storage codes of rate $\geq 1/2$ on triangle-free graphs were given in the literature.

The class of graphs that we consider is coset graphs of linear binary codes (Cayley graphs of the group F_2^r), which are naturally triangle-free if the code's distance is at least 4. One of the main results of this work is an infinite family of linear storage codes with rate approaching 3/4. We also give a group of necessary conditions for such codes to have rate potentially close to 1 and state a number of open problems.

The storage code problem can be equivalently formulated as maximizing the probability of success in *guessing games* on graphs, or constructing *linear index codes* of small rate, and it also exhibits a link with a construction of quantum codes from the CSS code family. (Received September 17, 2021)

1174-94-8199 **Peter G. Boyvalenkov** (peter@math.bas.bg), Bulgarian Academy of Sciences, and **Maya Stoyanova** (stoyanova@fmi.uni-sofia.bg), Sofia University. *Bounds for the sum of distances of spherical sets of small size*

We derive upper and lower bounds on the sum of distances of a spherical code of size N in n dimensions when $N \sim n^\alpha, 0 < \alpha \leq 2$. The bounds are derived by specializing recent general, universal bounds on energy of

spherical sets. We discuss asymptotic behavior of our bounds along with several examples of codes whose sum of distances closely follows the upper bound. (Received September 17, 2021)

1174-94-8318 **Nuh Aydin** (aydinn@kenyon.edu), Kenyon College, **Matthew Jia-Wei Harrington*** (harrington1@kenyon.edu), Kenyon College, **Dev Akre** (akre1@kenyon.edu), Kenyon College, and **Saurav Pandey** (pandey1@kenyon.edu), Kenyon College. *New Binary and Ternary Quasi-Cyclic Codes with Good Properties*

One of the most important and challenging problems in coding theory is to construct codes with best possible parameters and properties. The class of quasi-cyclic (QC) codes is known to be fertile to produce such codes. Focusing on QC codes over the binary field, we have found 113 binary QC codes that are new among the class of QC codes using an implementation of a fast cyclic partitioning algorithm and the highly effective ASR algorithm. Moreover, these codes have the following additional properties: a) they have the same parameters as best known linear codes, and b) many of the have additional desired properties such as being reversible, LCD, self-orthogonal or dual-containing. Additionally, we present an algorithm for the generation of new codes from QC codes using ConstructionX, and introduce 33 new record breaking linear codes produced from this method.
(Received September 18, 2021)

1174-94-8346 **Lesley Polanco** (lesleypolanco@gmail.com), Hudson County Community College. *Improving Bounds of Hermitian-Lifted Codes with their Automorphism Group* Preliminary report.

Locally Recoverable Codes (LRCs) are linear codes where the value of any single position may be recovered from a small subset of positions. There are several combinatorial and algebraic constructions of LRCs. For example, Lifted Reed-Solomon codes are LRCs whose recovery sets are point sets from the lines of the affine plane. In subsequent work, Lopez et. al. extended the same construction to Hermitian codes. In this work we improve previously known bounds on the parameters of Hermitian-Lifted Codes. We used their Automorphism Group to simplify our computations and easily check our LRCs. (Received September 18, 2021)

1174-94-8483 **Nuh Aydin*** (aydinn@kenyon.edu), Kenyon College, and **Saurav R Pandey** (pandey1@kenyon.edu), Kenyon College. *A Generalization of the ASR Search Algorithm to 2-Generator Quasi-Twisted Codes*

One of the main goals of coding theory is to construct codes with best possible parameters and properties. A special class of codes called quasi-twisted (QT) codes is well-known to produce codes with good parameters. Most of the work on QT codes has been over the 1-generator case. In this work, we focus on 2-generator QT codes and generalize the ASR algorithm that has been very effective to produce new linear codes from 1-generator QT codes. Moreover, making use of a recent algorithm to test equivalence of constacyclic codes, we make the generalized version the ASR search even more comprehensive and effective. As a result of implementing our algorithm, we have found 103 QT codes that are new among the class of QT codes. Additionally, most of these codes possess the following additional properties: a) they have the same parameters as best known linear codes, and b) many of them have additional desired properties such as being LCD and dual-containing. Further, we have also found a binary 2-generator QT code that is new (record breaking) among all binary linear codes and its extension yields another record breaking binary linear code. (Received September 19, 2021)

1174-94-8665 **Nuh Aydin** (aydinn@kenyon.edu), Kenyon College. *Good Codes From Quasi-Cyclic Codes Using ConstructionX*

One of the most important and challenging problems in coding theory is to construct codes with best possible parameters and properties. The class of quasi-cyclic (QC) codes is a generalization of cyclic codes known to be fertile to produce such codes. Using ConstructionX, a ternary operation on codes and some important properties of QC codes, we were able to find 33 codes with better parameters than any previously discovered code. In this work, we will explore what ConstructionX does, and how QC codes are well suited to it for use in producing new best known linear codes. (Received September 19, 2021)

1174-94-8769 **Emily McMillon*** (emily.mcmillon@huskers.unl.edu), University of Nebraska-Lincoln. *Spatially Coupled LDPC Codes with Cycle-Free Windows*

Low-density parity-check (LDPC) codes are a class of linear codes defined by sparse parity-check matrices and have corresponding sparse bipartite graph representations. They have been shown to be capacity-achieving over many channels using low complexity graph-based iterative decoders. Spatially coupled LDPC (SC-LDPC) codes are a special class of codes whose repetitive graph structure makes them amenable to window decoding, in which the nodes are decoded in groups from one end to the other. In this talk, I will introduce the notion of "window codes," the codes defined by restricting an SC-LDPC code to its decoding window. I will discuss designing

SC-LDPC codes whose window codes are cycle-free and therefore perform optimally under iterative decoding. Generally, cycle-free codes have poor distance properties, but, because these cycle-free window codes correspond to subgraphs of SC-LDPC codes with cycles, the overall code rates can be much higher than would normally be possible. I will present the optimal distance and rates of such codes under certain constraints and provide constructions that show these are achievable. (Received September 19, 2021)

1174-94-8810 **Dalton Gannon*** (dalton.gannon@louisville.edu), University of Louisville, and **Hamid Kulosman** (hamid.kulosman@louisville.edu), University of Louisville. *Binary and Ternary Linear LCD$[n,2]$ Codes with Biggest Minimal Distance that meet the Griesmer Bound* Preliminary report.

A linear code with complementary dual (LCD code) is a linear code C whose intersection with its dual code, $C^{\perp}$, is only the zero codeword (i.e. $C \cap C^{\perp} = \{0\}$). We will talk about binary and ternary linear LCD$[n,2]$ codes with biggest minimal distance that meet the Griesmer bound. (Received September 19, 2021)

1174-94-8931 **Reza Dastbasteh*** (rdastbas@sfu.ca), Simon Fraser University, and **Petr Lisonek** (plisonek@sfu.ca), Simon Fraser University. *On the equivalence of linear cyclic and constacyclic codes and applications in construction of new quantum codes* Preliminary report.

In this work, we first provide several new equivalence conditions for linear cyclic codes. These results are based on three recent conjectures on linear cyclic codes over finite fields proposed by Aydin et al. (2019) according to their computational results.

Next, these results will be generalized to the class of linear constacyclic codes. We also show that similar to linear cyclic codes, equivalent linear constacyclic codes of certain lengths can be completely determined by the action of multipliers on their defining sets.

Finally, the mentioned equivalence results will be applied to search more systematically for new binary quantum codes with better parameters than the previous best-known quantum codes. A list of such new quantum codes obtained from orthogonal and nearly self-orthogonal cyclic and constacyclic codes will also be provided.

References

[1] N. Aydin; J. Lambrinos; O. VandenBerg, *On equivalence of cyclic codes, generalization of a quasi-twisted search algorithm, and new linear codes*, Designs, Codes and Cryptography, 2019 Oct 1;87(10):2199-212.
[2] C. Bocong; F. Yun; L. Liren; L. Hongwei, *Constacyclic codes over finite fields*, Finite Fields and Their Applications 18(6); 2012; 1217-1231.
[3] PP. Palfy, *Isomorphism problem for relational structures with a cyclic automorphism*, European Journal of Combinatorics, 1987 Jan 1;8(1):35-43.

(Received September 20, 2021)

1174-94-9258 **Tyler Beauregard*** (trb4137@truman.edu), Truman State University, **Janabel Xia** (janabel@mit.edu), Massachusetts Institute of Technology, and **Mike Rosulek** (rosulekm@engr.orst.edu), Oregon State University. *Private Set Intersection: Problems on Sampling from the Intersection*

We explore the problem of Private Set Intersection (PSI), in which each participant of the protocol learns only the elements in the intersection of all of their sets, and nothing else. Though there exist protocols for several variations of this problem, including cardinality set intersection and threshold set intersection, we look at functions on the intersection that have not yet been explored outside of general secure circuit computation: sampling an element from the intersection. In particular, we propose new secure 2-party protocols for the following problems: randomly sampling an element from the intersection, sampling an element from the intersection by one party's ranking, and sampling an element from the intersection by some joint scoring function of elements in the input sets. We show our protocols are secure in the presence of semi-honest adversaries. (Received September 20, 2021)

1174-94-9670 **Kathryn Haymaker*** (kathryn.haymaker@villanova.edu), Villanova University, and **Justin O'Pella** (justin.opella@jefferson.edu), Thomas Jefferson University. *Pooled testing schemes for COVID-19 using coding theory* Preliminary report.

Group testing is a method for strategically pooling biological samples in order to use fewer tests than a one-per-sample approach. These methods have been revisited in view of the COVID-19 pandemic, particularly the need to efficiently test large groups of asymptomatic people for surveillance testing. A construction by Kautz and Singleton (1964) showed how error-correcting codes can be used to create group testing schemes. In this talk we will explain this connection and show a new result that implies flexibility in the number of samples being considered. (Received September 21, 2021)

1174-94-9730 **Ryan M Martinez*** (rmmartinez@hmc.edu), Harvey Mudd College, **Cordelia Horch** (chorch@oxy.edu), Occidental College, and **Mallory Dolorfino** (mallory.dolorfino19@kzoo.edu), Kalamazoo College. *On Good Infinite Families of Toric Codes or the Lack Thereof*

A toric code, introduced by Hansen to extend the Reed-Solomon code as a k-dimensional subspace of $\mathbb{F}_q^n$, is determined by a toric variety or its associated integral convex polytope $P \subseteq [0, q-2]^n$, where $k = |P \cap \mathbb{Z}^n|$ (the number of integer lattice points of P). There are two relevant parameters that determine the quality of a code: the information rate, which measures how much information is contained in a single bit of each codeword; and the relative minimum distance, which measures how many errors can be corrected relative to how many bits each codeword has. Soprunov and Soprunova defined a good infinite family of codes to be a sequence of codes of unbounded polytope dimension such that neither the corresponding information rates nor relative minimum distances go to 0 in the limit. We examine different ways of constructing families of codes by considering polytope operations such as the join and direct sum. In doing so, we give conditions under which no good family can exist and strong evidence that there is no such good family of codes. (Received September 20, 2021)

1174-94-10080 **Allison Beemer*** (beemera@uwec.edu), University of Wisconsin-Eau Claire, **Alberto Ravagnani** (a.ravagnani@tue.nl), Eindhoven University of Technology, and **Altan Kilic** (a@b.c), Eindhoven University of Technology. *One-shot Capacity in Networks with Restricted Adversaries*

Adversarial networks model scenarios in which a malicious outside actor can corrupt legitimate messages being sent between users in a communication network. The one-shot capacity gives a measure of how many symbols can be sent across the network reliably in a single transmission round. In this talk, we will discuss results on the one-shot capacity of communication networks with an adversary having access to only a proper subset of network edges. In particular, we will show that previously known cut-set bounds are not sharp in general, and that their non-sharpness comes precisely from restricting the action of the adversary to a region of the network. We will also discuss better bounding strategies and cases where we can compute exact capacities. (Received September 21, 2021)

1174-94-10348 **Heide Gluesing-Luerssen** (heide.gl@uky.edu), University of Kentucky, and **Hunter Lehmann*** (hunter.lehmann@uky.edu), University of Kentucky. *Weight distributions of cyclic orbit codes*

Cyclic orbit codes are the subspace code analogue of classical linear codes. They are generated by the cyclic action of a Singer cycle on a subspace of a finite vector space. Such codes have a weight distribution; the entries of which are the number of codewords with each possible distance to a fixed generating space for the code. In this talk, we discuss properties of this weight distribution for cyclic orbit codes with various parameters. (Received September 21, 2021)

1174-94-11000 **Aidan Murphy** (awmurphy@vt.edu), Virginia Tech, and **Welington Santos** (welington@vt.edu), Virginia Tech. *Fractional decoding of codes from curves*

Decoding algorithms for error-correcting codes typically take as input all symbols of a received word and attempt to determine the original codeword. In fractional decoding, only an α-proportion of symbols are used where $\alpha < 1$. In this talk, we consider how this framework may be applied to codes defined using curves. (Received September 21, 2021)

1174-94-11248 **Sreeram Kannan*** (k.sreeram@gmail.com), University of Washington, Seattle. *Coding theory for blockchains*

One important application of codes in blockchains is in information dispersal, where a node wants to disperse coded information to a set of nodes. However, in such applications it is possible that the data is inconsistently coded by the client dispersing the data. In this paper, we propose coded Merkle tree (CMT), a novel hash accumulator that provides the ability to show a short proof that the data was inconsistently coded. A CMT is constructed using a family of sparse erasure codes on each layer, and is recovered by iteratively applying a peeling-decoding technique that enables a compact proof for incorrect coding on any layer. We show that such structures have natural applications in guaranteeing data availability in blockchains. Our algorithm enables any node to verify the full availability of any data block generated by the system by just downloading a $\theta(1)$ byte block hash commitment and randomly sampling $\theta(\log b)$ bytes, where b is the size of the data block. With the help of only one connected honest node in the system, our method also allows any node to verify any tampering of the coded Merkle tree by just downloading $\theta(\log b)$ bytes. We provide a modular library for CMT in Rust and

Python and demonstrate its efficacy inside the Parity Bitcoin client. Finally, we point out other places where coded commitment can be used. (Received September 22, 2021)

97 ► Mathematics education

1174-97-5700 **Alicia Prieto Langarica** (aprietolangarica@ysu.edu), Youngstown State University.
Lessons Learned from Co-Mentoring Experiences

Mentoring students in undergraduate research can be challenging and for those who are new to mentoring it can be both overwhelming and scary. Having someone to go through that mentoring experience with is very helpful, a great way to build community, and an opportunity to learn from each other. In this talk, we share lessons learned from our experiences in co-mentoring students in undergraduate research. We highlight the benefits of having distinct areas of mathematical expertise as well as mentoring students at two very different institutions. We hope to inspire the audience to step outside their comfort zone where real learning can happen not only for the students, but also for the mentors. (Received August 25, 2021)

1174-97-5986 **Timothy J. Huber** (timothy.huber@utrgv.edu), University of Texas Rio Grande Valley, and **Josef Sifuentes*** (josef.sifuentes@utrgv.edu), University of Texas Rio Grande Valley. *Roadmap to glory: Scaffolding real analysis for deeper learning*

In this talk, we describe changes made in an introductory Real Analysis classroom that address common challenges and that allow for all levels of students to meet a high standard of learning and written work. We accomplish this by fusing elements of a hybrid course with scaffolded collaborative work to improve student learning. The hybrid structure allows students to engage new material at their pace, while the scaffolding allows for more challenging coursework and models typical arguments in analysis. As a result, students that initially submitted proofs needing significant improvement in coherence and organization show positive gains in self-confidence and are enabled to progress quickly to submitting mathematical writing of a high calibre. Our case study occurred at a large institution serving many underrepresented and non-traditional student populations. The approach may be effective at schools with similar goals and challenges and points to potential restructuring of upper level courses to increase active and collaborative learning. The work presented was done under the joint NSF Collaborative Research grant between UT Dallas and UT Rio Grande Valley. (Received September 1, 2021)

1174-97-6006 **Jim Fischer*** (jim.fischer@oit.edu), Oregon Institute of Technology. *Parameter Estimation Using Beam Deflection* Preliminary report.

Students estimate the Young's modulus of a small beam by using one or two measurements of deflection together with the analytic solution of the corresponding boundary value problem. Here we choose a beam and beam length so that when supported there is no measurable deflection. A single point mass is then suspended from the beam so that a measurable deflection occurs. Students solve the appropriate Euler-Bernoulli boundary value problem and use their solution to both estimate the Young's modulus and validate the mathematical model. The concentrated point mass is modeled using the Dirac delta function; students construct the model and find the analytic solution using the symbolic calculus of generalized functions. It should be noted that this project should be considered a work in progress as it has not been tested in the classroom. (Received September 2, 2021)

1174-97-6323 **Ahmad M Alhammouri*** (aalhammouri@jsu.edu), Jacksonville State University.
Advance Preservice Teacher Capacity to Teach Mathematical Modeling

During this session, we will discuss how a quantitative reasoning course was designed to advance pre-service mathematics teacher capacity to enact mathematical modeling effectively at the high school level. We will share the mathematical modeling framework that was used to design the activities of the course. Then, we will share some of these activities and some of student [preservice teacher] work samples. Finally, we will share student perspectives of the course. (Received September 7, 2021)

1174-97-6904 **Kathleen Kavanagh*** (kkavanag@clarkson.edu), Clarkson University. *Transitioning a Large-Scale STEM Outreach Program to Hybrid Mode* Preliminary report.

Clarkson University's Science Technology Entry Program (CU-STEP) serves roughly 185 rural students across northern New York state in STEM enrichment and academic support. Spread across 11 school districts in three counties, orchestration of events was already challenging for these geographically isolated participants. The pandemic was an opportunity to explore new methods of delivering content and engaging with students. We describe how the afterschool program, monthly campus workshops, and week-long summer roller coaster camp were redesigned for distance delivery. We discuss challenges, successes, and lessons learned. (Received September 10, 2021)

1174-97-6913 **Frank Uhlig*** (uhligfd@auburn.edu), Auburn University. *Taking our First Linear Algebra Course into the Third Millennium*

Current first Linear Algebra courses and textbooks overwhelmingly treat their subject matter on the knowledge base level of the 1920s. They teach abstract definitions, mathematical logic, proving and introduce students to rigorous mathematical thinking of outdated dead-end matrix methods.

Most students in first Linear Algebra are in science and engineering curricula where concrete matrix handling knowledge is needed nowadays.

Over time, this dichotomy has come to the attention of math educators who have tried to bridge this gap through elaborate didactic analyses, but without ever lifting the 'fog' in our students' minds when the logic based definition of linear dependence enters the classroom in week 3. And two centuries old and defunct concepts such as determinants, characteristic polynomials and polynomial root finders dominate the rest of the course.

Linear Algebra itself has made tremendous leaps and bounds over the last century with the advent of computers and their ubiquitous use today. Matrix based linear algebra governs all our computing today; the internet, AI, robotics, autonomous vehicles and...

The need to switch to phenomenon based teaching, away from abstraction to working with matrices concretely and learning computationally is urgent.

This talk looks at our complex Linear Algebra teaching problem from a holistic viewpoint in four dimensions. It proposes a matrix and discovery based syllabus instead that slowly fosters abstract understandings and does not start from there, foglessly. (Received September 10, 2021)

1174-97-6929 **Yvonne Lai*** (yvonnexlai@gmail.com), University of Nebraska-Lincoln. *From Graph Paper to Scissors Congruence to Riemann Sums* Preliminary report.

I will report on the design of a lesson taught to in-service middle and high school teachers on how progression of the concept of area, from kindergarten to high school, supports the learning of Riemann sums in Calculus. This lesson was well-received by the teachers in the course as giving a purpose to the mathematics they taught as well as understanding the purpose of mathematics taught prior to their grade level. (Received September 10, 2021)

1174-97-6984 **Janine Chua*** (mathwithjanine@gmail.com), Math with Janine. *Implementing Social Media as a Pedagogical Tool*

With the prevalence of social media in modern society comes the opportunity to create educational content to cultivate an interest in mathematics to a mainstream audience. In this talk, I discuss the use of social media to increase the accessibility of education, to promote diversity in STEM, and to instill an interest in mathematics in the next generation. I will also discuss strategies and best practices for creating educational content for social media. (Received September 11, 2021)

1174-97-7154 **Victor Piercey** (victorpiercey@ferris.edu), Ferris State University, and **Beverly L Wood*** (woodb14@erau.edu), Embry-Riddle Aeronautical University. *Adapting an in-person human trafficking group activity to an asynchonous online learning experience: A SUMMIT-P project*

By collaborating with faculty from social work, Ferris State University mathematics faculty developed a budgeting activity for a human trafficking shelter. Students assumed that the number of guests at the shelter grew monthly in a linear fashion. They developed models for each line item and applied them using an Excel spreadsheet.

Faculty at Ferris had students work on the activity in groups over the course of two days. Students individually submitted their completed budget along with a written justification. During the pandemic, it was not difficult to convert this activity to virtual breakout groups during online, synchronous class meetings. Faculty at Embry-Riddle Aeronautical University adapted this activity for use in an asynchronous quantitative reasoning course.

During this talk, we will discuss the activity, how it was modified for asynchronous delivery at Embry Riddle, and our reflections on the process and outcomes. Through this process, we noted that an activity designed for interactive in-person group work is adaptable to an asynchronous learning environment. (Received September 13, 2021)

1174-97-7585 **Natasha Gerstenschlager** (natasha.gerstenschlager@wku.edu), Western Kentucky University, and **Lukun Zheng** (lukun.zheng@wku.edu), Western Kentucky University. *Transforming College-level Calculus Courses with Cutting-edge Video recording and Editing technologies—A Case Study* Preliminary report.

Calculus I is the most fundamental courses for college level STEM-intended majors. In the fall of 2010, there were over 300,000 US college students taking Calculus I yet the universities are struggling with high attrition and failure rates in calculus. In fact, the recent National Study of College Calculus, conducted by the Mathematical Association of America with support from the National Science Foundation, observed that success rates in Calculus I are very low for traditional students, and even lower for non-traditional students. Students also express concerns about (i) the insufficiency of information covered in the course and the lack of opportunities to practice learned pedagogy, and (ii) lack of training of graduate teaching assistants (GTAs), mostly Ph.D. or M.Sc. students, assigned to the unique role of a recitation instructor. Therefore, students struggle in practicing complex steps of calculus.

Over the last three years, over 700 short instructional and tutorial videos have been developed. The production of these videos was the first phase of transitioning from traditional lectures and tutorials to a video-platform by the worked-out content and cutting-edge video recording & editing technologies. In this research, we exposed these videos to experimental and control groups. Using a mixed-methods approach, the analysis for the collected data revealed that there is a significant long-term effect (about four months) in student success. (Received September 15, 2021)

1174-97-7644 **Andrew G Bennett*** (bennett@math.ksu.edu), Kansas State University, and **Melea Roman** (mrroman@ksu.edu), Kansas State University. *Patterns in Student Usage of Online Videos* Preliminary report.

During 2020-2021, our large lecture undergraduate differential equations class was taught with pre-recorded videos in place of lecture and (mostly) synchronous online recitations and labs. Usage data on videos was recorded, and then analyzed to identify different approaches students adopted to studying from the posted videos. We will discuss the classification of types of students and videos and our preliminary findings on which approaches best support student learning. (Received September 15, 2021)

1174-97-7857 **Kathleen Snook*** (kgsnook@gmail.com), COMAP. *An Overview of COMAP and its Modeling Contests*

Founded in 1980 with a simple mission, "to improve mathematics education for students of all ages," the Consortium for Mathematics and Its Applications (COMAP) has produced modules, journals, newsletters, textbooks, videos, and national reports in support of mathematics teaching and learning. COMAP's focus is on applied problem solving and mathematical modeling. Since 1985, COMAP has annually held its international Mathematical Contest in Modeling (MCM) for undergraduate students, and in 1999 added its Interdisciplinary Contest in Modeling (ICM). After more than 40 years, COMAP continues to support the integration of mathematical modeling into the curriculum and has over 75,000 students from middle school to undergraduate level participating in modeling contests each year. During this introductory portion of this Special Session, we will overview COMAP's curricular materials and its international contests. (Received September 16, 2021)

1174-97-7876 **Sarah Ritchey Patterson*** (pattersonse@vmi.edu), Virginia Military Institute, and **Troy Siemers** (Siemerstj@vmi.edu), Virginia Military Institute. *Using COMAP as a senior capstone experience*

Every cadet at VMI is required to complete a capstone experience. In applied mathematics, we use a two course sequence MA490W (3-credits in the fall) and MA495 (1-credit in spring). In MA490W we work through 5-6 past COMAP problems. In January, the cadets participate in the MCM/ICM competition and then take MA495 where they revise their MCM/ICM paper and create a presentation/poster for the MD-DC-VA MAA section meeting. These courses also support institutional assessment in writing (W for writing-intensive), oral presentation (MAA section meeting), and Creative and Critical Thinking skills (final paper from MA495). (Received September 16, 2021)

1174-97-7883 **Blain Patterson*** (pattersonba@vmi.edu), Virginia Military Institute. *Mathematical Modeling for Secondary Teachers* Preliminary report.

In this talk, we will illustrate how mathematical modeling can help fulfill some of the endorsement competency requirements by the Virginia Department of Education. By mapping some of the endorsement competencies requires to the mathematical modeling process (as described by SIAM), we will show that engaging in mathematical modeling may help better prepare pre-service secondary mathematics teachers to be successful in their classrooms. Examples of specific topics/activities that could be done in a differential equation course to fulfill these requirements will be discussed. (Received September 16, 2021)

1174-97-7997 **Darryl Chamberlain Jr.** (chambd17@erau.edu), Embry-Riddle Aeronautical University, **Catherine Paolucci*** (cpaolucci@coe.ufl.edu), University of Florida, and **Sam Vancini** (svancini@ufl.edu), University of Florida. *Investigating alternatively-certified teachers' mathematical knowledge for teaching calculus*

This presentation will share results from a recent study on the content of materials used by aspiring teachers to prepare for mathematical content exams required for teacher certification. This study was conducted in a state where alternative routes to certification enable aspiring teachers with limited postsecondary mathematical studies to earn a teaching certification for secondary mathematics by passing a state content area exam. In this context, alternatively-certified teachers are more likely to rely on preparation materials for this certification exam to develop their mathematical knowledge for teaching. With this in mind, our research team analyzed the mathematical content of over 2,000 preparation items available to teachers to determine curricular alignment and cognitive demand. We will discuss findings specifically related to materials designed to prepare teachers for the calculus portion of the exam, with a focus on the nature of the mathematical knowledge and conceptions these materials promote and implications for alternatively-certified teachers' preparation for teaching calculus. (Received September 17, 2021)

1174-97-8000 **Jessica Ross*** (jross2@babson.edu), Babson College, **Sanskriti Chandak** (schandak1@babson.edu), Babson College, and **Salvatore P Giunta** (sgiunta@babson.edu), Babson College. *Project-Based Learning for Undergraduate Statistics through the use of R*

The 2016 American Statistician Association statement on the proper use of p-values in statistical analysis prompted a major overhaul of the statistical curriculum in higher education. New concepts taught in the redesigned statistics courses include entry level programming, meaningful interpretation of analytics, and industry-specific skills. This research project aims to help professors by creating an easy-to-use open source R-based pedagogical software repository. The lens of this archive will be through a project-based learning process that will include but not be limited to regression analysis, Bayesian statistics, clustering analysis, and neural network analytics. (Received September 17, 2021)

1174-97-8131 **Carlos William Castillo-Garsow*** (ccastillogarsow@ewu.edu), Eastern Washington University. *Lessons Learned in Mentoring Student-Led Projects in Mathematical Biology*

This study discusses a long running REU in mathematical biology. A characteristic of the program is that students choose their own topics. This creates a reversal of hierarchy, where students know more about their topic of interest than mentors, and mentors serve as consultants providing mathematical guidance and expertise. But how does this reversal of hierarchy really function? In this study four undergraduate research groups made presentations over six days while developing their research question and model. Analysis of the presentations' recordings, mentors' comments, and project final papers show that this reversal of hierarchy does not always function smoothly. Results identify the importance of respecting student choices and student expertise, and the critical role that identifying biological mechanisms plays in both model development and communication. (Received September 17, 2021)

1174-97-8142 **Nicole Infante*** (ninfante@unomaha.edu), University of Nebraska At Omaha, **Keith Gallagher** (keithgallagher@unomaha.edu), University of Nebraska at Omaha, **Lori Ogden** (lori.ogden@mail.wvu.edu), West Virginia University, and **Tim McEldowney** (tim.mceldowney@mail.wvu.edu), West Virginia University. *Fostering Student-Centered Online Tutoring Practices with OPTIMUM Interactions* Preliminary report.

Access to free, institutionally provided, high quality tutoring services can be critical to the success of students, especially those who are first-generation or from low socioeconomic status backgrounds. In early 2020, universities across the United States and around the world closed their campuses and swiftly moved their operations online, including tutoring services. We collected and analyzed data from 29 virtual tutoring sessions and found that only 4 (14%) of the sessions were student-centered with the remaining 25 (86%) being tutor-centered. With these observations we identified tutor practices that promote student engagement and those that are a barrier to learning. This led to a revision of how we train tutors to work in this new-to-them environment. Key aspects of training that we will cover are: building relationships, technology considerations, scheduling, and video case studies. Further, we will share the Online Practices for Tutoring In Mathematics Using Meaningful (OPTIMUM) Interactions training materials that have been developed so far. (Received September 17, 2021)

1174-97-8157 **Alvaro Alfredo Ortiz Lugo*** (aortizlugo@ggc.edu), Georgia Gwinnett College. *Active Learning Online: Methods and tools in the pandemic year.*

The pandemic year forced a transition to online classes in many universities and colleges. To accommodate the experience of regular classes and class times, the implementation of synchronous class sessions became a standard in many places, producing the challenge of keeping the students engaged in a Virtual Environment. In this talk, we discuss some strategies for fostering active learning in this type of setting, adapting summative and formative assessment strategies, and introducing the different technology tools and platforms used to implement these ideas. (Received September 19, 2021)

1174-97-8183 **Andrew Kercher*** (andrew.kercher@uta.edu), Simon Fraser University. *Mathematics in a Human Context: Integrating Approximations of Teaching Practice into Calculus I*

Teaching is both an application of mathematical knowledge and a deeply human activity. Approximations of practice (Grossman et al., 2009) can prepare prospective teachers for both aspects of their future work by simultaneously deepening their mathematical knowledge while allowing them to explore that knowledge in a human context. By inserting approximations of practice into Calculus I lessons, the Mathematical Education of Teachers as an Application of Undergraduate Mathematics (META Math) project seeks to meet the recommendations of the Conference Board of the Mathematical Sciences that prospective teachers experience meaningful connections between advanced mathematical content and the secondary school content they will eventually teach. Two META Math lessons were piloted in Calculus I courses at several universities. In this talk, I describe the development of the approximations of practices used by these lessons, as well as how the approximations of practice were perceived by both instructors and undergraduates as a tool for learning both mathematics and mathematical knowledge for teaching. (Received September 17, 2021)

1174-97-8379 **Rodi Steinig*** (rodi@mathrenaissance.com), Math Renaissance. *Engaging Math-Circle Topics that Have Attracted Girls*

Sixteen Math-Circle course topics have attracted a student demographic of over 60 percent girls to the Math Renaissance (formerly Talking Stick) Math Circle. All sixteen of these topics, including Sacred Geometry, The Eye of Horus, Gardner, Fermat, Proofs, Compass Art, and Embodied Mathematics, will be described in this session, with the goal that session participants can recreate the activities in their own Math Circles. In addition, there will be a quick overview of the commonalities in these topics, the trends in offered topics that did not attract many girls, and the other variable that may have influenced these demographics. (Received September 18, 2021)

1174-97-8482 **Sepideh Stewart*** (sepidehstewart@ou.edu), University of Oklahoma, and **Anthony Cronin** (anthony.cronin@ucd.ie), University College Dublin. *Students' Perspectives on Linear Algebra Proofs in Second Courses* Preliminary report.

Mathematics education research on teaching and learning the topics in second courses in linear algebra is scarce. To help fill this gap in the literature, in this study, we interviewed 18 students taking a second linear algebra course in both the USA and Ireland. The theoretical framework is based on Tall's (2008) formal world of mathematical thinking and Harel's (2008) ways of thinking and ways of understanding. In this talk, we will explore students' perspectives on the purpose and the nature of linear algebra proofs in two abstract second courses. (Received September 21, 2021)

1174-97-8487 **Kevin Charles Moore*** (kvcmoore@uga.edu), University of Georgia, **Cameron Byerley** (CBYERLEY@uga.edu), UGA, **Hyunkyoung Yoon** (hyoon14@asu.edu), ASU, **Surani Joshua** (sjoshua@asu.edu), ASU, **Laura Valaas** (valaasla@gmail.com), University of Wisconsin - Madison, **Minsook Park** (minsook@uwm.edu), University of Wisconsin - Milwaukee, **Heather Lavender** (heatherl@lsu.edu), LSU, **Sukjin You** (sukjinyou@gmail.com), UGA, **Mina Gong** (Mina.Gong@uga.edu), UGA, **James Drimalla** (James.Drimalla@uga.edu), UGA, **Anne Waswa** (Anne.Waswa@uga.edu), UGA, **Halil Tasova** (halil.tasova25@uga.edu), California State University, San Bernardino, **Stacy Musgrave** (smmusgrave@cpp.edu), California State Polytechnic University Pomona, and **Alexandra Yon** (Alexandra.Yon@uga.edu), UGA. *Using Mathematics Education Research to Design COVID-19 Data Representations*

The COVID-19 pandemic presents a unique opportunity for mathematics educators to conduct research that has important implications for people's lives and health. In our RAPID project, we have investigated how United States (U.S.) and South Korea (S.K.) citizens understand media-used COVID-19 quantitative data representations (QDRs). We have also leveraged extant mathematics education research constructs to design QDRs that better support individuals in understanding the COVID-19 pandemic including its health risks and vaccine benefits. In this presentation, we provide an outline of our project design to illustrate the different ways we have responded to the COVID-19 pandemic through empirical studies and incorporating available mathematics education research. We also narrow in on how we have extended mathematics education constructs to not only viably explain citizens' meanings for COVID-19 QDRs, but also design COVID-19 QDRs that are more sensitive to their meanings. In doing so, we underscore those meanings that enable citizens to make important and critical decisions in their lives inside and outside of the mathematics classroom. We also clarify important decisions for designing QDRs that better support citizens in understanding pandemic data including vaccine benefits and risks. (Received September 19, 2021)

1174-97-8748 **William McCallum*** (wmc@math.arizona.edu), The University of Arizona. *What is calculus about?*

There are many different answers to title question. Calculus is about rates of change and accumulated change; about a collection of algebraic techniques for calculating those; about infinitesimal analysis of the real number line; about modeling the physical world. A prospective calculus teacher would benefit from having all these perspectives, and the ability to coordinate and shift between them. In this talk we will attempt to lay out the topography of calculus as subject in school and college, and draw implications for the knowledge needed for teaching calculus. (Received September 19, 2021)

1174-97-8759 **Kamuela E. Yong*** (kamuela.yong@hawaii.edu), University of Hawaii West Oahu. *Using place-based education to attract students and promote equity in applied mathematics* Preliminary report.

Indigenous cultures have been using mathematics in their lives for generations. In this talk, we present methods of using place-based examples in a precalculus classroom using Native Hawaiian culture. Through these examples, we hope to increase student participation in the classroom from both Indigenous and non-Indigenous students as well as spread awareness of various cultures. (Received September 19, 2021)

1174-97-8944 **Mili Shah*** (mili@cooper.edu), The Cooper Union. *Incorporating Artistic Mathematical Installations into an Undergraduate Engineering Research Program* Preliminary report.

The Cooper Union is a small college in New York City consisting of just three schools - Architecture, Art, and Engineering. This talk will present how Vertically Integrated Projects (VIPs) are used at Cooper to encourage interdisciplinary undergraduate research between the three schools. In particular, this talk will showcase the artistic and robotic installations being created that are focused on mathematics and body tracking. (Received September 20, 2021)

1174-97-9130 **Shiv Smith Karunakaran*** (shivk1980@gmail.com), Michigan State University, **Mariana Levin** (mariana.levin@wmich.edu), Western Michigan University, and **John Smith III** (jsmith@msu.edu), Michigan State University. *Progressing Towards a Conceptualization of Mathematical Agency and Autonomy in Undergraduate Mathematics Courses*

This presentation describes an ongoing and iterative research effort aimed at conceptualizing the interrelated constructs of mathematical agency and autonomy in the context of mathematics teaching and learning at the undergraduate level. We describe the impetus for our focus on agency and autonomy in students' early work in proof and proving at the collegiate level and then illustrate the dialogue between data from student interviews and reflection logs and our emerging definitions of agency and autonomy. We close with our provisional working definitions and a discussion of questions for future investigation. We will also present on lessons learned from submitting an IUSE grant proposal, including modifications made to all aspects of the project due to the COVID-19 pandemic. (Received September 20, 2021)

1174-97-9138 **Erich McAlister*** (mcalister_e@fortlewis.edu), Fort Lewis College. *Using* Insightmaker *and Other Modeling Visualization Software in an Undergraduate ODE's Course.*

Modeling software such as *Insightmaker* and *Simulink* can be used to model dynamical systems as visual/conceptual maps. Because *Insightmaker* is free and open source, we will focus on its use in an undergraduate ODE course. Using this software increases students' symbolic literacy in translating from equation to visual model and vice versa. Several topics which regularly give students difficulty and may be clarified with *Insightmaker* models will be presented. Student feedback on usage will also be discussed. (Received September 20, 2021)

1174-97-9206 **Linda E Green*** (greenl@email.unc.edu), University of North Carolina at Chapel Hill, **Todd J Vision** (tjv@bio.unc.edu), University of North Carolina at Chapel Hill, **Jeff McLean** (mclean@unc.edu), University of North Carolina at Chapel Hill, and **Viji Sathy** (viji.sathy@unc.edu), University of North Carolina at Chapel Hill. *Cross-disciplinary approach for teaching data literacy: applying quantitative tools to societal issues like Covid, gerrymandering, and student debt* Preliminary report.

In an effort to improve student success, the University of North Carolina System embarked on a multi-year, multi-campus initiative to take a broad and thoughtful look at how students are moving through their quantitative courses and to provide modernized mathematics pathways. As part of this project, a cross-disciplinary team of faculty at the University of North Carolina at Chapel Hill developed a new data literacy course for students with no prior background in statistics or data analysis. The course highlights how quantitative tools are used in a variety of disciplines and applied to data sets to address societal issues. For example, we work with students in the course to develop criteria for detecting gerrymandering, to model the future of the Covid pandemic, and to evaluate the long term implications of student debt. Along the way, students learn how to use data analysis tools, including spreadsheet formulas and basic Python commands. The course also incorporates concepts that are broader than those traditionally found within terminal math courses, including geometric reasoning, mathematical modeling, probability, and statistical reasoning. To evaluate the impact of this novel course offering, we conducted pre- and post-test assessments of quantitative literacy and examined how students' attitudes towards math have shifted. Key findings from three semesters of the course will be discussed along with recommendations to campuses considering a similar course offering. (Received September 20, 2021)

1174-97-9404 **Luis Miguel Fernandez*** (Luis.fernandez01@utrgv.edu), The University of Texas Rio Grande Valley. *LATINX EL STUDENTS IN THE MATHEMATICS CLASSROOMS: AN (ONGOING) EDUCATIONAL INEQUITY IN AMERICA*

As we enter an era faced with increasingly unprecedented challenges, it has become clear that our society's prosperity will continue to depend on rapid and significant advances in the fields of STEM. It is no longer enough to only prepare a privileged portion of our students' population but rather all of our citizens. In the United States, such untapped repository lies within our bilingual and emergent bilingual K-12 student-population. However, our nation's educational history has long shown a significant underachievement by such population, especially in mathematics. This has resulted in the creation and adoption of theoretically and empirically driven frameworks aimed towards mathematics education, particularly Debora Ball's Mathematical Knowledge for Teaching (MKT) framework. Furthermore, I argue that in order for this to become accessible for teachers of linguistically diverse students, we must begin situating theory into practical and relevant contexts. For that reason, this study's objective was in beginning to situate Ball's MKT framework into a Latinx-emergent bilingual context via a qualitative analysis of pre-service teachers' knowledge and experiences located within a highly bilingual and ethnically diverse setting. This resulted in a collection of overarching themes grounded in data that demonstrate a high promise in the education of not only Latinx emergent bilingual students, but students from other cultures and languages as well. (Received September 20, 2021)

1174-97-9407 **Amanda I Beecher** (abeecher@ramapo.edu), Ramapo College of New Jersey, and **Kayla Blyman*** (kblyman@stmartin.edu), Saint Martin's University. *MCM/ICM Problem Immersion—Let's Model!*

COMAP's Mathematical Contest in Modeling (MCM) and Interdisciplinary Contest in Modeling (ICM) provide amazing opportunities and experiences for students from across the disciplines to demonstrate their modeling, problem solving, teamwork, and communication skills by solving a real-world problem. In 2021, over 26,000 teams of up to 3 students each solved one of six problems. What type of problems do they solve? In the past, students have addressed dozens of topics including, but not limited to, designing the longest lasting sandcastle, analyzing the influence of music, examining our food systems, exploring migration patterns of fish, investigating the spread of giant hornets, evaluating the health of higher education, and considering the relocations of environmentally displaced persons. Come to this session ready to dive into the modeling process and consider how your students might model some of these real-world problems. (Received September 20, 2021)

1174-97-9482 **Serena Jade Peterson*** (serena.peterson@wsu.edu), Washington State University. *Predicting Academic Performance in Second-Semester Calculus* Preliminary report.

This report explores second-semester undergraduate Calculus students' course performance as a linear function of four predictors: lab attendance, lecture participation, written homework submission, and online homework engagement. Using a multiple linear regression with $n = 50$ observations, the partial slopes of all four explanatory variables were found to be significant. With that said, the partial slope for written homework submission rate was only significant at the $\alpha = 0.1$ level, whereas all others were significant at $\alpha = 0.05$. The aforementioned model was chosen using model selection and had an adjusted R^2 of 69.22% (i.e., high for a study of human behavior). While most model assumptions were met, there were glaring issues of non-constant variance. Weighted least squares are recommended as a remedial measure moving forward. All factors considered; we conclude that the four explanatory variables are in fact positive predictors of student performance. Direction for future research and limitations of the data are discussed. (Received September 20, 2021)

1174-97-9594 **Erin Haller Martin*** (emartin@lindenwood.edu), Lindenwood University. *Developing Core Mathematical Concepts: The Role of Proof-Writing Strategies in Calculus II* Preliminary report.

Two of the most important yet notoriously challenging topics in Calculus II are sequences and series. Rather than being primarily conceptual or computational, these topics are analytically intensive and require students to read, understand, and appropriately apply complex definitions and theorems. In other words, students must learn to *prove* their claims. Explicitly teaching proof-writing strategies alongside sequences and series can be used to develop students' mathematical habits of mind and their ability to appropriately justify their claims, ultimately resulting in greater understanding of sequences and series and improved performance on related assessments. The presenter will share lessons and activities that couple proof-writing techniques with sequences and series to enhance students' understanding of both. (Received September 20, 2021)

1174-97-9772 **Kyle Evans*** (kyle.evans@trincoll.edu), Trinity College, and **Megan Staples** (megan.staples@uconn.edu), University of Connecticut. *MTC4SJ: A Circle Founded on Social Justice* Preliminary report.

To address the inequities in our world that are often isolated from mathematics education, we formed a Math Teachers' Circle with a theme of social justice to position mathematics as a tool to analyze injustices and engage in a democratic society. We are now in our second year and have run MTC workshops and a summer institute to compile lessons and activities that engage a wide variety of learners in understanding their community and world. In this session, we will engage participants in one of our activities and share our larger set of resources which include themes of fairness, equity, decision making, and representation through topics such as food, art, politics, and wealth distribution. (Received September 20, 2021)

1174-97-9857 **Alana Unfried*** (aunfried@csumb.edu), California State University, Monterey Bay, **Douglas Whitaker** (Douglas.Whitaker@msvu.ca), Mount Saint Vincent University, **Leyla Batakci** (batakcil@etown.edu), Elizabethtown College, and **Michael A Posner** (michael.posner@villanova.edu), Villanova University. *Motivational Attitudes in Statistics and Data Science*

Attitudes matter in mathematics education, especially in fields like statistics which sometimes suffer from a poor reputation; understanding the relationship between both student and instructor attitudes and student achievement is crucial for improving mathematics education. Through an NSF IUSE grant (Developing Validated Instruments to Measure Student/Faculty Attitudes in Undergraduate Statistics and Data Science Education; DUE-2013392), our research team is developing a new set of attitudinal instruments, the Student and Instructor Surveys of Motivational Attitudes toward Statistics (S- and I-SOMAS), to quantify these attitudes. Additionally, we are developing instruments to measure the learning environment and an analogous set of instruments to measure attitudes toward data science. We will share our rationale for developing these six instruments, and we will discuss the psychometric properties of the most recent S-SOMAS pilot survey administration. We will also present the theoretical framework for these instruments, Expectancy Value Theory (Eccles et al., 1983). Statistics and data science instructors and educational researchers who are interested in being involved with data collection in future phases of the project are encouraged to contact the authors. (Received September 21, 2021)

1174-97-10002 **Jonathan Herman*** (jon.herman@utoronto.ca), University of Toronto Mississauga. *Math Polls and Quizzes Using MathMatize*

This talk will give an overview of MathMatize, an EdTech platform that utilizes mastery based learning techniques. I will talk about the impact the learning materials have had on my online lectures, and how I believe this impact will last into the post-covid era.

MathMatize also offers live polling and online assessment tools built for math instruction. There are thousands of instructor created exercises available in the community catalog. I will give an overview of the polling, assessment, and community aspects of MathMatize. (Received September 21, 2021)

1174-97-10011 **Daniel Dotson*** (dotson.77@osu.edu), The Ohio State University. *A Librarian's Role in Developing Free Course Content for Statewide Use: The Ohio Open Ed Collaborative*

Seven mathematics courses (Calculus I and II, College Algebra, Pre-Calculus, Linear Algebra, Ordinary Differential Equations, and Abstract Algebra) have course content available for statewide use. This presentation will outline the librarian's role in working with mathematics instructors from across the state in developing this content. Also described is the overall process for the creation of content for these courses for statewide use. (Received September 21, 2021)

1174-97-10058 **Lauren Siegel*** (lsiegel@mathhappens.org), MathHappens Foundation. *Uses for Mathematics in the Real World*

In the process of developing math field trips, and creating mathematical experiences and conversations with the public we have researched math concepts and their relationship to historical events, usefulness in a given time period, and relevance to a variety of purposeful endeavors. Among the topics we have researched are tools of Navigation in the 1600s, methods of land survey, including some very unusual methods used early in Texas history, the founding of epidemiology and the contribution of voronoi regions to a critical insight (which is also relevant for locating franchise restaurants, and to sports analytics in soccer). Other compelling mathematical uses include the invention of pooled testing, mechanics of gerrymandering, graph theory to improve organ donor exchanges, mathematical origami for map folding and packing space ships, optimizing budget in a real estate portfolio with linear programming. We'll share some of the ways we've developed and presented these examples of useful mathematics in an informal education setting. We'd like to say a word as well about the value of relating mathematics to historical events, artifacts and museum exhibits that are accessible and meaningful to the local community. (Received September 21, 2021)

1174-97-10072 **Paul Christian Dawkins** (pcd27@txstate.edu), Texas State University, **Michael Duane Hicks*** (michaeldh@vt.edu), Virginia Tech, **Kate Melhuish** (melhuish@txstate.edu), Texas State University, and **Kristen Lew** (kristen.lew@txstate.edu), Texas State University. *Outcomes and Reflections on the Orchestrating Discussion Around Proof Project*

In this session, we share results from our design-based research project: Orchestrating Discussion Around Proof. Through a series of design cycles, we adapted high leverage teaching practices from K-12 contexts into a proof-based abstract algebra course. We found such practices were successful in engaging students in rich discussion and authentic mathematical proof activity. Further, we identified ways that instructors may maximize the potential of these practices by integrating teaching moves specific to the proof setting. We conclude with reflection on general challenges of converting tasks from a lab setting to the classroom setting as well as modifying the project to adjust to online instruction during the pandemic. (Received September 21, 2021)

1174-97-10241 **Erika L Ward*** (eward1@ju.edu), Jacksonville University, and **Daniel Franz** (dfranz1@ju.edu), Jacksonville University. *Supplementation and Collaboration: Just in Time Interventions in Mathematics Across Campus* Preliminary report.

While we celebrate the benefits that students get from a mathematical education, we also know that students often struggle to recall the particulars of skills they learned when needed in later classes. Inspired by conversations with colleagues in other departments, the Jacksonville University Mathematics department is developing interventions for courses that rely on students recalling previous mathematical knowledge. These brief modules consist of a video (covering the mathematical skill in the context of the course where it is needed) and interactive practice problems. Students will be encouraged to access these materials to refresh before they encounter the topic in class, or to use the resource to brush-up when they're struggling with homework or other assignments. We hope that these resources will improve student success and confidence in their mathematical skills, and we plan to study their efficacy as they are deployed. We've received an enthusiastic response from across campus as we begin to develop the modules. (Received September 21, 2021)

1174-97-10280 **Janice F Rech*** (jrech@unomaha.edu), University of Nebraska at Omaha. *Preparing Calculus Teachers to Integrate Active Learning and Enhance Understandings at the High School and College Levels* Preliminary report.

High school calculus teachers have frequently not been trained to teach the course. Their experiences have typically been that of students in the classroom. To address the need to increase the mathematical knowledge needed to teach calculus, current and future teachers attended workshops designed to improve their effectiveness. They engaged in hands-on activities, initially as learners, followed by discussions relevant to incorporation of the activities into classrooms as vehicles to advance understanding. Problem sets that integrated a variety of concepts taught in calculus were a focal point. Teachers worked individually, then in small groups, to develop deep understanding of the fundamental concepts in calculus, rather than focusing on a rote skill set. The goal of the workshop was to significantly increase student understanding and thus student success in calculus by increasing the mathematical content knowledge among teachers needed to effectively teach. Teachers participating in the workshop have shown evidence they increased their conceptual understanding of the content and increased their ability to engage students in mathematically rich activities and discussions. Several teachers have become adjuncts at the college level and their expertise was extended to the students at the University. (Received September 21, 2021)

1174-97-10305 **Ping Wang*** (pxw10@psu.edu), Penn State University. *Flipped Classroom in Math Education* Preliminary report.

Across the nation, students in all areas have been showing decline in performance due to the many distractions in the society and spent less and less time in doing the required school work to be successful. It is more serious in mathematics because a lot of practice is essential to learning and mastering subject materials and students are spending less and less time on studying in math classes. In the past few years, I have been experimenting the transition of Calculus classes from traditional classes to flipped classes. With specially designed activities and innovative approaches, I have seen significant improvement in students' engagement in learning and much better performance as demonstrated in students' grades. (Received September 21, 2021)

1174-97-10323 **Reza O Abbasian*** (rabbasian@tlu.edu), Texas Lutheran University, and **John T Sieben** (JSieben@tlu.edu), Texas Lutheran University. *Mathematics and Statistics Across The Curriculum: Modeling Projects for non-STEM majors for Improving attitudes toward quantitative analysis* Preliminary report.

In this talk, we will give a brief introduction to NSF funded grant titled "Mathematics and Statistics Across The Curriculum" and then discuss a series of projects for non-STEM disciplines that demonstrate the importance of quantitative modeling and analysis using real data. The projects are suitable for use in wide range of disciplines such as social sciences, theology, kinesiology, history, literature, and psychology . Through these projects we will pose exploratory questions that will challenge students to separate myth from facts through quantitative analysis. A common feature of these projects is use of real data, and commonly available technology ,in modeling current or historical events relevant to the specific disciplines. (Received September 21, 2021)

1174-97-10568 **Anna Marie Bergman*** (ambergma@sfu.ca), Simon Fraser Universtiy, and **Rina Zazkis** (zazkis@sfu.ca), Simon Fraser University. *Conditional Construals in Determining Continuity of a Function* Preliminary report.

Continuity is important in Calculus; however, prior work has shown that non-equivalent definitions of continuity lead to ambiguity. Furthermore, while the phrase "continuous function" is used in many places, there is a lack of explicit definitions, as standard definitions address continuity at a point rather than continuity of a function. Recent work focused on ambiguous situations introduced the concept of conditional construals as a form of pedagogical decision making. Conditional construals are defined as moments when teachers required additional context to judge whether a given teaching action was appropriate. We build on this idea by showing conditional construals may also be mathematical in nature. By asking pre-service teachers to determine whether a given function was a "continuous function," we gained deeper insight into the additional contexts and resources the participants required to form their arguments. Results show that students relied on a range of resources including personal experience, concept images, mathematical conventions, and definitions to argue both for and against the continuity of a given function. This work contributes to our understanding of how undergraduates think about continuity and the various authorities they rely on when faced with ambiguity. (Received September 21, 2021)

1174-97-10583 **Krishna P Pokharel*** (kppokharel@ung.edu), University of North Georgia, **Kedar Nepal** (nepal_k@mercer.edu), Mercer University, **Deepak Basyal** (dbasyal@coastal.edu), Coastal Carolina University, **Debendra Banjade** (dpbanjade@coastal.edu), Coastal Carolina University, and **Manoj Lamichhane** (manoj.lamichhane@uwc.edu), Florida Polytechnic University. *What are the Sources of Common and Persistent Student Errors in Algebra and Calculus?* Preliminary report.

Students in calculus courses often make numerous errors mostly related to calculus, algebra, and other prerequisite concepts and skills. Most of these errors are common, repetitive in nature, and are often observed in simple mathematical tasks. This qualitative study attempts to identify some sources of such student errors. We wrote a Precalculus and a Calculus test and administered them in twelve sections of four different undergraduate mathematics courses for which either Precalculus, Calculus I, or both were a prerequisite. The tests were announced on the first day of the class and given the following week. The tests included only True or False questions. Based on our experience as college mathematics instructors, we assumed that many students would perceive the True answers as False and the False as True. Therefore, if students selected a given answer, mathematical statement, process, or solution as True, they were asked to justify why that was not False, and vice-versa. Analysis of data using grounded theory approach resulted in the following four possible sources of common and persistent student errors: a) Difficulty with symbols and/or lack of attendance to their meanings, b) Memorized "rules", keywords or forms of expression, c) Instructional practices, and d) Lack of knowledge. (Received September 21, 2021)

1174-97-10642 **Natasha M Speer*** (speer@math.umaine.edu), The University of Maine, and **William Hall** (w.hall@wsu.edu), Washington State University. *Activities to Support Instructors' Development of Mathematical Knowledge for Teaching Definite Integrals*

Findings from education research demonstrate that definite integrals can be challenging for students to learn and challenging for instructors to teach. We have developed activities that provide instructors with opportunities to consider the conceptual foundations of definite integrals and to develop mathematical knowledge for teaching that is relevant for this topic. In this session, we will provide information about research findings related to teaching and learning of definite integrals and then we will share the activities we have developed for use with novice college mathematics instructors. In the activities, first instructors consider key ideas related to definite integrals and then they examine a set of sample student responses to a task that are based on research on student thinking about definite integrals. Following the examination of student work, instructors read a synthesis of research findings to support their development of knowledge of student thinking about definite integrals. Being able to make sense of student responses and understand the thinking behind them is valuable for instructors as they interact with students and interpret students' written and spoken responses to tasks. These activities are designed to promote instructors' engagement with, and reflection on, the teaching and learning of definite integrals. We will also share outcomes of our use of the activities in the context of teaching seminars for graduate student instructors. (Received September 21, 2021)

1174-97-10670 **Nancy Emerson Kress*** (Nancy.kress@colorado.edu), University of Colorado at Boulder. *Humanizing Mathematics Instruction: Students' Experiences and Perspectives*

Underrepresentation of students of color and women in Science, Technology, Engineering and Mathematics (STEM) fields is well-known as a complex and intractable challenge. Active learning has been proposed and implemented as one way to improve equity in mathematics among students who are members of groups that are persistently underrepresented in mathematics. It has become clear that, while active learning has an important role to play, it is not by itself sufficient to ensure equitable and inclusive learning experiences across demographic groups.

This talk will share results from two case studies conducted in undergraduate mathematics departments that were identified as cultivating especially positive experiences among students of color and women in their Precalculus and Calculus courses. The study centers the perspectives and experiences of instructors and their students at each institution. This talk will focus on the students' experiences and perspectives. The results shared provide new insights into the nature of equitable and inclusive mathematics instruction as evidenced by these students' experiences, interpretation of this instruction as emergent culturally sustaining pedagogy, and directions in which further improvement and growth could take place. (Received September 21, 2021)

1174-97-10754 **James Roger Valles*** (jrvalles@pvamu.edu), Prairie View A&M University, **Fred Bonner** (fabonner@PVAMU.EDU), Prairie View A&M University, **Orion Ciftja** (ogciftja@pvamu.edu), Prairie View A&M University, **Alphonso Keaton** (akkeaton@pvamu.edu), Prairie View A&M University, and **E. Gloria C. Regisford** (gcregisford@pvamu.edu), Prairie View A&M University. *Prairie View A&M University S-STEM Program: Report on the SUMS Multidisciplinary Project* Preliminary report.

The Prairie View A&M University Scholars in Undergraduate Math and Sciences (SUMS) project is funded by the NSF S-STEM program. This endeavor is a multidisciplinary effort, with Biology, Chemistry, Physics, and Mathematics working together to provide a support program for high-achieving students who demonstrate financial need and plan to enter the workforce or pursue a post-undergraduate degree or licensure in the aforementioned fields. Being housed at an HBCU (Historically Black College and University), this project seeks to provide a multiyear learning and mentoring experience designed to prepare students for undergraduate research, internship opportunities, and overall preparation for professional life after undergraduate studies.

This talk will present information about the SUMS project, especially in regards to mentoring opportunities made available to students. Successes, challenges, and areas of improvement that have been observed within

the program and among the participating student scholars will be discussed. In addition, lessons learned from student recruitment for the program will be shared with wider audiences. (Received September 21, 2021)

1174-97-10834 **Kristin Heysse*** (kheysse@macalester.edu), Macalester College. *AMC1: putting the "applied multivariable" in calculus 1* Preliminary report.

Applied Multivariable Calculus 1 (AMC1) is an introductory calculus course designed for students who utilize calculus concepts in their own chosen major outside of mathematics. These students are looking for immediate applications and computational tools rather than theoretical discussions. They also bring in a variety of mathematical backgrounds and often very little coding or modeling experience. How do we design a course that meets their needs? How can we interpret calculus content in a way that highlights the connections to their own fields? How do we bring in software for modeling as an effective learning tool?

We won't definitively answer those questions in this talk, but we'll explore some of the challenges of implementing such a course. This will include how modifying the topics list can make room less traditional content like Fermi estimation and Lagrange multipliers, how modeling and coding can empower exploration, and what choices in course design can create a learning environment for students to grow. (Received September 21, 2021)

1174-97-10836 **Melinda Dawn Lanius*** (melinda.lanius@auburn.edu), Auburn University, **Tiffany Frugé Jones** (tfjones@shsu.edu), Sam Houston State University, and **Nicole Fider** (nfider@uci.edu), University of Arizona. *Mute your mic if you need to cry: The capacity of remote educational modalities to both cause and cure math anxiety.*

In summer 2020 and in summer 2021, we invited any undergraduate student enrolled in a math course at the University of Arizona the previous spring to participate in a survey; roughly 7000 students were invited per summer, with 891 respondents in 2020 and 306 respondents in 2021. We asked students about their experience with remote learning and assessed changes in their math anxiety using the well-established Abbreviated Math Anxiety Scale (AMAS). We measured changes in their math anxiety from before the pandemic to after the initial transition to remote instruction, as well as changes in their math anxiety from the emergency remote semester to a planned remote semester.

While moving to remote instruction proved detrimental to many student's success - "Mute your mic if you need to cry" is the advice one undergraduate offered to other students taking a remote learning class - a substantial group of students greatly benefited from transitioning to remote modalities. Among our 2020 respondents, 20% experienced a decrease in math anxiety moving from face-to-face to online instruction. In this session we will share the aspects of remote learning that students found most beneficial and how our results can inform an expansion of online and hybrid modalities in university education. (Received September 21, 2021)

1174-97-10977 **Mary Beisiegel*** (mary.beisiegel@oregonstate.edu), Oregon State University. *Understanding the Definition and Meaning of the Derivative Through a Lens of Mathematical Knowledge for Teaching*

Mathematical Knowledge for Teaching (MKT) is described as having multiple facets that go beyond knowledge of mathematical content. Depending on the MKT theory, these facets include knowledge of content and students, knowledge at the mathematical horizon, and specialized content knowledge, OR transformation, connection, and contingency, among others. In this talk, I will describe Definition of the Derivative through the facets of MKT and how, through this work, we can expand our own and our students' knowledge of calculus concepts. (Received September 21, 2021)

1174-97-11109 **Brian J Lindaman*** (blindaman@csuchico.edu), California State University, Chico. *Wait Wait—Don't Use the Internet!... Adventures in Inquiry-Based Trigonometry* Preliminary report.

Do you feel like you are doing all the work in preparing rich lessons, beautiful, thoughtful examples, and assigning relevant problems in your trigonometry class – and yet your students are uninterested and/or passive in class? And, even worse, they reach for random internet videos, tutorials, or peers which arm them with quick tricks yet prevent their learning of just about every important big idea? Then hold the smartphone, Fellow Trig Lover, help is on the way!

Inquiry-based learning is a teaching format which works well for conceptual learning in a number of areas in mathematics. This presentation will delve into how IBL works especially well for conceptual learning of fundamental trigonometry ideas. In a nutshell, IBL positions students to be active participants in the classroom conversations by beginning each day with students' work on a carefully curated set of exercises that motivate key ideas. Trigonometry, because of its reliance upon a few fundamental ideas, such as the unit circle and radian measure, has been an amazing venue for inquiry-based learning to take hold. Presenters are excited to share data which demonstrates the efficacy of the IBL approach in trigonometry and is part of a larger course redesign effort underway at the host institution. A course action plan, sample daily lessons, syllabi, adapted problem banks, and other resources will all be shared freely for session participants, with active discussion of experiences, challenges, and collaborations to follow. (Received September 21, 2021)

1174-97-11132 **James A M Alvarez*** (james.alvarez@uta.edu), The University of Texas at Arlington. *Enhancing Our Toolbox of Tasks for Teaching Calculus*

Mathematical knowledge for teaching calculus includes an instructor's understanding of tasks that draw out multiple approaches and representations for solving problems, require students to construct important mathematical connections, or enhance student learning. In this talk, we explore the role good tasks take in an experienced calculus instructor's classroom and discuss ways to leverage good tasks in efforts to develop other instructors' mathematical knowledge for teaching calculus. (Received September 21, 2021)

101 ► Teaching and learning

1174-101-5208 **Hee Oh*** (hee.oh@yale.edu), Yale University. *Euclidean lines on hyperbolic manifolds.*

The classical Kronecker's theorem in 1884 says that the closure of a line on the torus is always a subtorus, depending on the direction of the line. A similar rigidity phenomenon persists for a Euclidean line living in the hyperbolic world, called a horocycle. On closed hyperbolic manifolds, the closure of any Euclidean line is a hyperbolic submanifold, up to translation. This was proved by Ratner (and by Shah independently) more than 30 years ago. What happens if we now venture into hyperbolic worlds of infinite volume? We now need to choose carefully which hyperbolic worlds are safe for rigidity, as not every world is safe. We present an infinite family of hyperbolic manifolds of infinite volume of any given dimension, where every Euclidean line is dense in some translate of a hyperbolic submanifold (based on joint works with McMullen and Mohammadi for dimension three and with Lee for higher dimensions). (Received December 20, 2021)

1174-101-5227 **Robert Q. Berry*** (robertberry@virginia.edu), University of Virginia. *Interest Convergence: An analytical viewpoint for examining how power dictates policies and reforms in mathematics.*

This Cox-Talbot talk uses a hybrid policy analysis-critical race theory lens informed largely by legal scholars like Derrick Bell to make the case that policies and reforms in mathematics education failed to address the needs of historically excluded learners. Rather, these policies and reforms are often designed and enacted to protect those in power's economic, technological, and social interests. This talk offers contrasting narratives between policy intentions and policy enactment, highlighting how the language of mathematics policies positions historically excluded learners as deficient within their cultures and communities. Finally, this talk considers features necessary in mathematics policies and reform documents when discussing the historically excluded learners. (Received November 15, 2021)

1174-101-5231 **Tyler Jarvis*** (jarvis@math.byu.edu), Brigham Young University. *Restoring confidence in the value of mathematics*

Ten years ago a group of my department's math majors told my colleague, Jeff Humpherys, and me, "We majored in math because we like it, but we know it won't get us a job unless we want to teach." That comment motivated us to create an entirely new program in applied and computational mathematics (ACME) at BYU—a program to teach students mathematics that is deep and beautiful and that employers are also eager to pay for, mathematics that students can use on the job to solve the problems of the 21st century.

Since we started the ACME program eight years ago, the number of majors in our department has almost doubled, ACME students account for two-thirds of all our majors, and resources have flowed to our department. Our graduates' starting salaries are substantially higher, and many of them are turning those big offers down to go to top graduate programs, where they are flourishing. Our alumni are fiercely loyal to ACME and eager to help the students that follow them.

In this presentation I'll talk about some of the problems we had to overcome to get ACME started, how we made ACME successful, and what we have learned along the way to help those of you wanting to do something similar for your students. (Received November 16, 2021)

1174-101-5628 **Itai Seggev*** (is+maa@cs.hmc.edu), Wolfram Research. *Rotational Motion: A Capstone for Linear Algebra and Differential Equations*

Rotional motion in three dimensions is a topic than can be explored at any of a number of levels with multiple different tools: elementary algebra, cross products, matrix or linear algebra, differential equations, Lie geometry, etc. In this talk we will describe a presentation of the material that encompasses key concepts of linear algebra and differential equations–first order system, eigendecomposition, and matrix exponentiation–that is a approachable for students at that level while pointing to more advanced topics such as Lie groups and Lie algebras. (Received August 23, 2021)

1174-101-5750 **Hortensia Soto*** (hortensia.soto@colostate.edu), Colorado State University/MAA Assoc. Secretary. *Dynamic Visualization of Complex Analysis Topics*

Algebraic and analytic concepts from complex analysis are generally omitted in most textbooks. In this presentation, I will depict how mathematicians and undergraduates depict dynamic aspects of complex analysis concepts via diagrams that they bring to life via gesture. I will also showcase how dynamic geometric environments can aid students to develop a geometric interpretation of analytic concepts. (Received August 26, 2021)

1174-101-5896 **Thomas W Judson*** (judsontw@sfasu.edu), Stephen F. Austin State University. *Teaching from an ODE Textbook that Integrates Sage, Reading Questions, and Activities*

The great majority of ordinary differential textbooks are written in a very traditional format. That is, each section consists of definitions, theorems, and examples followed by a set of exercises and possibly projects. Textbooks written with Sage cells, reading questions, and activities allow students to do work right in the textbook. Students can easily generate a phase plane, answer a reading question, or do a WeBWorK problem without ever leaving the textbook. Moreover, reading questions and activities provide opportunities for faculty to examine their own teaching. All of this is possible with an authoring language such as PreTeXt. With PreTeXt it is possible to include interactive Sage cells, WeBWorK problems, and links to other documents. With platforms such as Runestone, it is possible for students to enter answers to reading questions directly into the textbook, where the instructor can collect and grade student responses. Moreover, once a book has been created in PreTeXt, it is very easy to produce a textbook in different formats, including HTML, PDF, print, Jupiter notebooks, and even braille versions. We will demonstrate some of these devices and opportunities using an ordinary differential equations textbook that was written in PreTeXt. (Received August 31, 2021)

1174-101-5911 **Iordanka Panayotova** (iordanka.panayotova@cnu.edu), Christopher Newport University, and **Maila Hallare*** (mcbrucal-hallare@nsu.edu), Norfolk State University. *When Laplace Meets Tesla in a Differential Equations Class...*

Tesla's ambition of wirelessly transmitting electricity using radio frequency resonance was never realized. Now, more than a hundred years after Tesla's experiments, wireless transmission of electricity via magnetic induction is currently used in a humble household electronic item: the wireless toothbrush. The wireless toothbrush is one of many household applications of a 2-coil wireless power transfer (WPT). In this talk, we will look at a mathematical model of this WPT via magnetic induction. The system consists of two second-order differential equations coupled by a mutual inductance. We derive the model using Kirchoff's Voltage Law. Since the full model may be challenging to solve using elementary methods, we present some examples of WPT systems that can be used in class to supplement the learning of the Laplace Transform technique in solving systems of differential equations. Students appreciate learning about this engineering example from a mathematical perspective due to its applicability in daily life. Inspired by our students' curiosity and positive feedback on the topic, we have published a teaching resource on WPT systems in SIMIODE and we are currently supervising an undergraduate research project on analyzing a 3-coil WPT system. Our students' research experience is supported by NSF through the Center for Undergraduate Research in Mathematics. (Received August 31, 2021)

1174-101-5916 **Elizabeth Donovan*** (edonovan@murraystate.edu), Murray State University, and **Lesley Wiglesworth** (lesley.wiglesworth@centre.edu), Centre College. *Successes (and Challenges) in Mentoring Cross Institutional Undergradute Research Projects*

The positive effects of undergraduate research on students is well documented. Students who participate in undergraduate research experiences report improved communication skills, growth of self-confidence, improved focus on a career path, as well as positive improvements in student learning and attitude. In this talk, we will discuss our collaboration between undergraduate students at Centre College and Murray State University made possible by CURM. Successes and challenges throughout the mentoring process, as well as our own reflections, will be presented. (Received August 31, 2021)

1174-101-5929 **Peyam Ryan Tabrizian*** (drpeyam@gmail.com), Texas A&M University. *Spreading the Math: Using YouTube to Communicate with the Rest of the World* Preliminary report.
In this talk, I will describe my journey as a YouTuber, including the wonderful coincidence of how it all started back in 2017 at UC Irvine. I currently have a channel called Dr. Peyam which has over 104,000 subscribers and over 830 videos, on which I regularly post videos either related to my teaching, or others that are accessible to a wider audience, like "The Half Derivative of x" or "The Quadratic Formula that will change your life." These videos are for students and people around the world who are interested in math, and the advantage is that anyone can watch them online or on their phones, and ask questions and comments. (Received August 31, 2021)

1174-101-5979 **Nicholas Dwork** (ndwork@hotmail.com), UCSF, and **Gennifer Smith*** (gsmith5@usfca.edu), USF. *A Human Computer, Encoded Messages, A Locked Treasure Box: Engaging with Mathematics Through Adventure - Preliminary report* Preliminary report.
Across various classes, we send students on adventures where they discover the power and utility of math. By living through the processes, the students gain a visceral experience that ties theory to practice. Seeing these applications provides motivation and context. Often, even students that claim to loathe math will continue working after the class has ended.

* For the Manhattan project, a computer was purchased but delivered late. Meanwhile, people were used as components of the computer (e.g., adder, multiplier) to test and run algorithms. Students recreate that "person" computer and estimate the exponential function with a Taylor series approximation.

* Biology students are recruited to stop eco-terrorists and enlist the help of math students to decrypt messages from spies in the field.

* To encourage making progress in the face of uncertainty, students are presented with a locked bag containing, amongst other items, a locked treasure box. To get the treasure, students must wade through the items, identify the meaningful ones, and solve puzzles involving cellular automata, cryptography, optics, and probability. We then discuss the resources that were available, what went well, what went poorly, what expectations they arrived with, and what expectations were violated. (Received September 1, 2021)

1174-101-6142 **Padmanabhan Seshaiyer*** (pseshaiy@gmu.edu), George Mason University, and **Carmen Caiseda** (CCAISEDA@bayamon.inter.edu), Inter American University of Puerto Rico. *From Context to Habit: A CMATH framework for Solving Real-World Challenges in Mathematics* Preliminary report.
In this work, we present concrete examples of research problems in the mathematical sciences that engaged effectively students helping them to improve their understanding and appreciation of mathematics, and develop habits of mind to address real-world challenges. The talk will introduce a new framework called CMATH to build competencies for solving real-world global challenges from Brazil, Colombia, India, Tanzania and Puerto Rico using mathematical sciences. We will provide detailed ideas on how the constructs of CMATH have been used for a set of real-world projects involving mathematics. The talk will also provide some effective strategies that will help faculty to enhance their own pedagogical practices when mentoring and directing students on applying mathematics to solving such real-world challenges. Finally, we will share how such teaching and research practices in the mathematical sciences as described in this work can help create change agents at the student, faculty and institutional levels. (Received September 5, 2021)

1174-101-6279 **Christian Millichap*** (christian.millichap@furman.edu), Furman University. *Student homework helper videos: enhancing communication skills while fostering peer-assisted learning*
In mathematics courses, we often want our students to practice communicating mathematics in a variety of forms and see the value in hearing a peer's perspective on solving a problem. This can often become a challenge in calculus courses, where a few students frequently volunteer answers and share their solutions on the board, while many others are uncomfortable or unwilling to engage the class in this manner. To combat this challenge, we will discuss how student homework helper videos can be implemented to make sure every student had multiple opportunities to practice verbal and written communication skills in the context of calculus, while also providing a useful resource for their peers. Video creation guidelines will be discussed and available for sharing. Student feedback on creating such videos will be provided to support the effectiveness of these assignments. (Received September 7, 2021)

1174-101-6284 **Christopher Oehrlein*** (cdoehrlein@gmail.com), Oklahoma City Community College. *Physical Models and Guided Inquiry for Learning Linear Systems of ODE* Preliminary report.

Through collaborative guided inquiry activities, students explore simple physical models, trajectories in the phase planes, and finally the linear systems of differential equations that best represent the physical models. After thoroughly exploring the different representations of the models, students work through a guided development of the algorithmic or computational solution procedure and verify that the solutions produced reflect the prescribed behaviors of the physical models. The process of exploring physical and graphical models first allows students to appreciate the mathematical models and solutions. The students also develop the cognitive and affective process skills needed to acquire, interpret, and apply knowledge. (Received September 7, 2021)

1174-101-6285 **Md Sazib Hasan*** (mdsazib.hasan@dixie.edu), Dixie State University, and **Vinodh Chellamuthu** (d00323868@dixie.edu), Dixie State University. *Advancing Student Learning through Customized Open Education Resources*

The purpose of this presentation is to discuss the strategy to foster active learners through the creation of "customized" OERs (C-OERs)" curriculum that provide deeper conceptual understanding. In this presentation, we will share how we are developing C-OERs in collaboration with our students and a sample of student experiences in developing the C-OERs, along with sample artifacts. We will also discuss the rationale, challenges, and benefits of C-OERs and how it impacted their learning process through this authentic learning environment. (Received September 7, 2021)

1174-101-6424 **Kelly Gomez Johnson** (kgomezjohnson@unomaha.edu), University of Nebraska at Omaha, and **Paula Jakopovic*** (paulajakopovic@unomaha.edu), University of Nebraska Omaha. *Why COMMIT? Stories from Regional Math Faculty Leaders Engaged in Communities of Practice Centered Around Inquiry* Preliminary report.

The COMMIT (COMmunities for Mathematics Inquiry in Teaching) Network is a grant funded project aimed at connecting mathematics faculty engaged in regional communities of practice (CoPs) around teaching with inquiry. Understanding what motivates faculty involvement in CoPs is vital, as communities are interested in growing and sustaining membership. Further, examining why some faculty take on leadership roles to extend their involvement and ignite interest in others, is equally important in fostering long term and widespread change in mathematics teaching and learning. For faculty to effectively implement and promote the sustained use of teaching with inquiry practices in their mathematics courses, especially for historically underserved or underrepresented faculty and students, it is necessary for them to have a clear purpose and vision for their involvement. In this session we share takeaways from interviews with regional leaders about why they are COMMITed to collaborating with their CoPs as a means to connect with colleagues, share resources, and facilitate systemic changes in how undergraduate students engage with mathematics. (Received September 8, 2021)

1174-101-6932 **Samer S Habre*** (shabre@lau.edu.lb), Lebanese American University, and **Rami El Haber** (rami.elhabr@lau.edu), Lebanese American University. *Sociocultural Interaction: A Mathematical perspective*

Located on the eastern shore of the Mediterranean Sea, Lebanon has been historically a strategic land for almost every civilisation that graced the region. As a consequence, several religious sects spread over this small piece of land to total now 17. These are now part of "Modern Lebanon". After its independence in 1943, Lebanon's several religions did not live harmoniously at all times. Several conflicts mark the modern history of Lebanon, most important of which is the Lebanese Civil War (1975-1990). In our study, we take a scientific approach using systems of differential equations to model people of different religious backgrounds living together. We rely on their beliefs and behaviours to see if the result of their interaction is harmonious or not. We took the Lebanese American University (LAU) as a basis for our study because LAU can be perceived as a sample of the Lebanese population with its highly diversified student body, faculty, and staff. (Received September 10, 2021)

1174-101-7194 **Michael Janssen*** (mkjanssen@gmail.com), Dordt University. *Liberal Arts Mathematics for Human Flourishing*

In his retiring address as president of the MAA in 2017 (and the subsequent book released in 2020), Francis Su laid out a vision for mathematics as integral to a life of flourishing. In this talk, we will describe and share resources for an inquiry-oriented liberal arts math course informed by the vision Su describes. (Received September 13, 2021)

1174-101-7204 **Matthew Mauntel*** (mmauntel@fsu.edu), Florida State University, **Megan Wawro** (mwawro@vt.edu), Virginia Tech, and **David Plaxco** (davidplaxco@clayton.edu), Clayton State University. *Determining the Determinant: Using GeoGebra to visualize and measure spatial distortion*

Determinants are often presented in a formulaic way in the classroom that obscures their rich connections to graphical interpretations of linear transformations. The Inquiry-Oriented Linear Algebra (IOLA) curricular materials build from a set of experientially real tasks that allow for active student engagement in the guided reinvention of key mathematical ideas through student and instructor inquiry. During this session, we present a new task sequence from the IOLA curriculum on determinants using distortion of space as a realistic starting point. Students build from a conceptualization of matrix determinant as a measure of (signed) multiplicative change in the area and discover its formula for a general 2 x 2 matrix. GeoGebra applets allow students to actively explore the geometric effects of changing 2x2 and 3x3 matrix transformations, make conjectures, and note their relationship with the determinant. Students also link concepts such as linear independence, inverses, and column operations to changes in the determinant. (Received September 13, 2021)

1174-101-7362 **P. Gavin Larose*** (glarose@umich.edu), University of Michigan. *Applications and Writing in Calculus (and Beyond) Projects*

In this talk, we will discuss a number of interrelated ways that applications can be integrated into Calculus courses through assessments that share a thread of writing and mathematical communication. These assessments are broadly applicable to many undergraduate math courses, and we will explore some of the different ways they can be implemented depending on the context of a specific course. In particular, we will consider examples that include applied written homework assignments, formal written projects, and the assignments that require the explicit use of some technology to explore a problem or mathematical content (a mathematical "lab report"). We will examine how these assignments may be constructed, how they can integrate into courses, grading these types of assignments, and evidence of their effectiveness as a formative assessment instruments. (Received September 14, 2021)

1174-101-7368 **Mark Branson*** (mbranson@stevenson.edu), Stevenson University, and **Whitney George** (wgeorge@uwlax.edu), University of Wisconsin, LaCrosse. *Mathematics for the People: A Radical New Approach to Teaching Quantitative Literacy* Preliminary report.

Math for the People is a collaboratively written open educational resource designed to replace a classic textbook for a first year quantitative reasoning course. The text encourages students to explore how mathematics can be used to understand social justice concepts like generational wealth inequity, climate change, and racially-biased policing from a solutions-oriented perspective. Rather than learning a series of mathematical concepts, followed by applications of those concepts, *Math for the People* seeks to invert that structure, beginning with a problem that students are interested in and discovering the mathematics which can help to understand and even solve that problem. We will discuss the current state of the project and the status of the first edition, launching in Spring 2022. We will also talk about opportunities to review the first edition or author more modules for the second edition. (Received September 14, 2021)

1174-101-7445 **Dan May*** (daniel.may@bhsu.edu), Black Hills State University. *Mathematical Poetry for Educators, Mathematicians, Students, and Writers (And All Intersections Thereof)*

In this talk, we will introduce three categories of mathematical poetry: poems which are literally about mathematics, poems which employ mathematical language in metaphorical ways, and poems with mathematical structure. We will discuss the pedagogy of teaching mathematical poetry to both math students and creative writing students, and present benefits to both students and teachers. In particular, these techniques can help spark creative writing students' work, and also help students of mathematics understand certain mathematical concepts in innovative ways. Teachers of mathematics and creative writing will learn methods for teaching distinct mathematical poetic forms structured by each of the following topics: the Fibonacci sequence, the number pi, directed graphs, and the Fano plane. We also hope that creative writers of all stripes will find exciting new challenges to explore, and that the curious will be inspired to write some mathematical poetry even if they have never done so before. (Received September 14, 2021)

1174-101-7462 **Vladimir Dragovich*** (Vladimir.Dragovic@utdallas.edu), University of Texas Dallas. *Cross-Institutional Strategies for Enhancing Diversity in the Mathematics Graduate Applicant Pool*

We discuss implementation of cross-institutional strategies to address challenges faced by undergraduate students at minority serving institutions in their path toward graduate school. These challenges include access to and preparation for advanced courses that are prerequisites for success in PhD programs, access to internships and job opportunities, awareness of careers in the Mathematical Sciences, and familiarity with the graduate school admission process. We share outcomes and lessons learned from an NSF funded initiative between two large state schools to address these challenges. A fundamental innovation involves restructuring upper level courses to increase active and collaborative learning. The initiative provides a collaborative national model for building a sustainable and scalable pipeline of well-prepared applicants for PhD programs in the Mathematical Sciences. (Received September 14, 2021)

1174-101-7482 **Stephen Wang*** (`sswang@rice.edu`), Rice University. *Incorporating Inquiry-Based Learning into a Flipped Linear Algebra Class* Preliminary report.

This talk will report on a linear algebra course that ran partially flipped, using interactive online material from a MOOC. However, some content from the MOOC (e.g., certain propositions and theorems) was taken out so that students did not encounter it online. Instead, during class time students participated in small inquiry-based group activities to discover these facts on their own. The goal was to allow students to verbally process new terms and the logic behind their relationship with previously-learned concepts. Results of this class format, such as student response and a comparison of test scores, will also be discussed. (Received September 14, 2021)

1174-101-7490 **Kyle T Allaire*** (`kallaire@worcester.edu`), Worcester State University. *Transitioning to a Modeling-First Approach in Differential Equations* Preliminary report.

As an early career faculty member, I have taught differential equations courses in the traditional technique-driven manner. However, after attending a SIMIODE workshop (Systemic Initiative for Modeling Investigations & Opportunities with Differential Equations) in Summer 2021, I was excited by the idea of using a modeling-first approach to teaching differential equations. In this talk, we will discuss resources for and implementation of various modeling activities, as well as the challenges associated with transitioning to a more modeling focused pedagogy. Furthermore, we will discuss the effect of a modeling-first approach on student motivation and interest in the subject matter. (Received September 14, 2021)

1174-101-7573 **Paul F Zachlin*** (`pzachlin@lakelandcc.edu`), Lakeland Community College. *An Exploration of Elementary Matrices*

In 2018, the speaker worked on a team sponsored by the Ohio Department of Higher Education to develop free online educational resources (OERs) for a first course in linear algebra. The OER is housed on the XIMERA server, an interactive learning platform run by The Ohio State University. The authors created a number of Explorations for this OER. The speaker will share one of his favorites that allows students to learn about elementary matrices through discovery learning. The address for the OER is https://ximera.osu.edu/la/LinearAlgebra (Received September 15, 2021)

1174-101-7574 **Filippo Posta*** (`filippo.posta@estrellamountain.edu`), Estrella Mountain CC. *Course-based Undergraduate Research Experience (CURE) as a tool for Linear Algebra Pedagogy*

Linear Algebra is often the first abstract course that non-math majors encounter. It can provide quite a culture shock to the students. Therefore, it is important to provide applications that create connections with the many abstract concepts. In this talk, we will present the use of a course-long CURE that uses Crime Forecasting, Community Policing, and the Broken Window theory as a scenario for the application of Linear Algebra concepts. We will discuss implementation, lessons learned, and future directions. (Received September 15, 2021)

1174-101-7663 **Linda McGuire*** (`lindamcguire@muhlenberg.edu`), Muhlenberg College. *Chaucer's Mathematics: Modeling the Canterbury Tales* Preliminary report.

In 1907, English mathematician and puzzle-creator Henry Dudeney wrote The Canterbury Puzzles, a collection of over a hundred puzzles inspired by Geoffrey Chaucer's The Canterbury Tales. Taking the conceit of Chaucer's tales, that each pilgrim on the journey to Canterbury must tell one tale to entertain their fellow travelers, Dudeney proposes that each of these pilgrims should also propose a puzzle to pass the time. Noting that Chaucer himself was an excellent mathematician, Dudeney sees these puzzles as the logical corollary to the famously unfinished tales. Dudeney hit upon the mathematical potential of these tales, but did not extend his exploration of this potential beyond the realm of puzzles.

This presentaiton details a semester-long senior capstone project based on Chaucer and Dudeney's work. Students used the tales told by these pilgrims to create mathematical problem and solution sets for learners at various points of their mathematical journey. These problems use Chaucer's rich narratives to motivate problems from mathematical topics that include trigonometry, single-variable calculus, combinatorics, graph theory, and probability. The project also led to important conversations and considerations regarding issues of mathematical storytelling. In particular, the benefits of creating narratives that reflect, and often problematize, inclusion and equity in mathematical communities. (Received September 15, 2021)

1174-101-7688 **Claude Laflamme*** (laflamme@ucalgary.ca), University of Calgary and Lyryx Learning. *Engaging Students with the "Textbook"!* Preliminary report.

Educational software has made good progress in supporting online homework and examinations over the years in first year mathematics courses. Progress is also being made in developing software to encourage students to actually read (and learn) the material.

We will briefly review some of these emerging tools for linear algebra, including embedded exercise and computing activities (Sage, Geogebra, Jupyter, . . .), but also those that incorporate critical thinking activities in a way reminiscent to how mathematicians read a textbook (H5P, Mathmatize, Lyryx, . . .).

The better students will read the text without any assistance, but it is the weaker ones that require support or incentive to do so and we will conclude with a review of a pilot with over 2,000 first year linear algebra science and engineering students this year. (Received September 15, 2021)

1174-101-7865 **Kathryn Appenzeller Knowles*** (kknowles@tamusa.edu), Texas A&M-San Antonio, and **Ruby Daniels** (rdaniels@tamusa.edu), Texas A&M-San Antonio. *Medical Marijuana: An Asynchronous "Sniff Test" to Encourage Student Use of QR* Preliminary report.

Consumers are bombarded with thousands of marketing claims each day, many of which are allegedly supported by research. In this interactive session, we will explore how undergraduate students evaluated research about the social issue of medical marijuana in a discussion about a cannabidiol (CBD) product. After learning a QR "Sniff Test," college students critically evaluated CBD information from multiple sources (e.g., company advertising, customer product reviews, and peer reviewed research) and submitted a short video recording describing their conclusions, justifications, and evidence. Students also provided a brief written reflection of lessons learned from an asynchronous "sniff test" discussion. A qualitative content analysis of video and written responses provided strong evidence that college students:

- Tend to support conclusions with numerical evidence after learning a "sniff test"
- Are highly engaged in asynchronous QR video discussions

After watching some students' video responses and learning about our findings, attendees will break into small groups to reflect about how a QR "Sniff Test" might be used when analyzing other social issues. We will also discuss the pedagogical benefits of using an asynchronous approach (which provides students with a "safe space" to formulate and express their thoughts) when teaching QR. (Received September 16, 2021)

1174-101-8073 **Rebecca Garcia*** (rgarcia@shsu.edu), Sam Houston State University. *People Over Math: Building community at MSRI-UP*

For the last fifteen years, the Mathematical Sciences Research Institute has housed the Undergraduate Program (MSRI-UP), a six-week residential summer research and long-term mentorship program. In 2019, we worked together to direct a year that resulted in the most publications in MSRI-UP history. The key to this success was in centering students, in seeing students as genuine collaborators, and in building a strong sense of community for all participants and faculty. A couple of years later, we once again joined forces to take on the challenge of running MSRI-UP 2021, in the midst of the current pandemic. However, this time, the program ran virtually, which led us to our greatest challenge that summer: how can we build a strong sense of community in a virtual setting, particularly when "Zoom-fatigue" has set-in following a frustrating year of online learning? In this talk, we will share some of our responses to this question. (Received September 17, 2021)

1174-101-8083 **Katherine V Johnson*** (kjohnson@fgcu.edu), Florida Gulf Coast University. *Building Community with an LA Program* Preliminary report.

In 2016, we created a Learning Assistants program with the support of a Noyce Track 1 award, Noyce@FGCU. What began as a recruitment tool has now evolved into something much larger, bringing together diverse communities of faculty and students across campus. In this presentation, we'll share how we grew our program from a grassroots perspective, including the benefits and drawbacks of that route, as well as describe the current research and where further study is needed. Our goal is for you to leave with a better idea of how to start or sustain your own LA program, plus ideas of how to improve an existing program. (Received September 17, 2021)

1174-101-8201 **Jeffrey Stuart*** (jeffrey.stuart@plu.edu), Pacific Lutheran University. *When Matrices and Finance Collide!*

Our linear algebra course serves mathematics, physics, computer science and economics majors. I have used the following application problem to engage the economics majors, and to lure them into our senior level Mathematics of Risk course.

One of the fundamental problems for investors is how to balance risks versus potential rewards in selecting stocks, bonds and other assets for a portfolio. We briefly explore how investors assess risk and reward, and how the risks and rewards of individual investments combine to determine the risks and rewards of a portfolio. In its purely mathematical formulation, the mathematical problem we examine has two versions. The more general version is:

$$\min_{\vec{w} \in \mathbb{R}^n} \vec{w}^T C \vec{w} \text{ subject to } \vec{w}^T \vec{u} = 1 \text{ and } \vec{w}^T \vec{\mu} \geq \mu_{\text{Target}}$$

where $\vec{u}$ is the all 1's vector, $\vec{\mu}$ is the vector of asset returns, C is the asset covariance matrix, and μ_{Target} is the minimum acceptable portfolio return. (Received September 17, 2021)

1174-101-8413 **Jonathan Weisbrod** (jweisbrod@rcbc.edu), Rowan College At Burlington County. *Undergraduate Research in Math: Collaboration Between Two- and Four-Year Schools*

This presentation describes the experiences the speakers had as collaborative undergraduate research advisors at two differing types of institutions: a public research university and a public community college. Undergraduate research is considered a High-Impact Educational Practice (HIP) by the Association of American Colleges and Universities (AACU). Both two-year and four-year institutions should provide opportunities for students to conduct research both early on and late in their undergraduate years. While undergraduate research projects in an apprenticeship model are conducted similarly regardless of institutional type, the impact of research programs as part of an institution's strategic plan can vary widely. We plan to discuss our experience merging the differing strategic goals of two-year vs four-year institutions when conducting undergraduate research and share tips for success in recruiting and mentoring students, formulating research projects suitable for undergraduates, and obtaining seed funding. (Received September 18, 2021)

1174-101-8459 **Nathan Dalaklis*** (nathandalaklis@my.unt.edu), The University of North Texas, YouTube Content Creator. *"So You Want To Popularize Math?"*

With the rise of a plethora of social media and content creation platforms over the past 20 years, popularization of mathematics is, somehow, both easier and more difficult than ever before. That being said, the question of "where" or "how" to start popularizing mathematics may not have anything to do with the mathematics itself. In this talk, we will attempt to uncover some answers to these questions by revisiting the difference between popularization and teaching mathematics and what it means to have a culture around an academic subject before venturing forth and trying to learn something from both the speaker's personal experience communicating mathematics on the internet and the communication style of other more prominent STEM communicators that show up online today. (Received September 19, 2021)

1174-101-8508 **Deborah Hughes Hallett*** (dhh@math.arizona.edu), University of Arizona/Harvard Kennedy School. *Driven by Data: Calculus and Global Challenges* Preliminary report.

The world outside calculus courses is using more and more data. Students want to know about AI, big data, and machine learning—and how data can illuminate their fields of interest. To harness our students' enthusiasm, let us "let the data speak". In this talk, we consider how to include data in Calculus I and II. Since continuity is fundamental to calculus, using discrete data presents a challenge—but one of central importance to modeling. We will give examples of activities for calculus courses using data from the pandemic, sustainability, and climate change. (Received September 19, 2021)

1174-101-8514 **Katelyn Mei*** (XMEI@MIDDLEBURY.EDU), Middlebury College. *Where is data ethics in undergraduate data science education?*

Data has become a popular tool for individuals and organizations to tell stories and make convincing arguments. As data is being used more often, there is an increased need for broad data ethics education. As colleges and universities have been rapidly developing new data science courses and programs, it is worth exploring the prevalence of data-ethics-related content in the curriculum design of these courses. Our approach is to collect course descriptions and syllabus of several hundred data science courses from nearly one hundred undergraduate colleges and universities. In this talk, we will discuss the findings and challenges in carrying out this research. (Received September 19, 2021)

1174-101-8584 **Jennifer Clinkenbeard*** (jclinkenbeard@csumb.edu), California State University, Monterey Bay, and **Judith Canner** (jcanner@csumb.edu), California State University Monterey Bay. *Campus-wide Quantitative Reasoning Assessment Using A Modified VALUE Rubric* Preliminary report.

In 2009, the Association of American Colleges & Universities (AAC&U) developed a set of VALUE rubrics intended for institutional-level use in evaluating and discussing student learning. These rubrics "can and should be translated into the language of individual campuses, disciplines, and even courses" (Rhodes 2010). A team of faculty members at California State University Monterey Bay engaged in a faculty cooperative to adapt the original Quantitative Literacy VALUE rubric to reflect institutional priorities and curricular structures. The resulting rubric was intended to be robust enough to assess quantitative reasoning in any course, regardless of discipline or level. The modified rubric has been implemented in several large assessment projects, including assessing quantitative reasoning at different course levels (e.g., introductory level compared to senior level coursework) and across disciplines. We present the modified rubric and findings from the assessment work, as well as discuss strategies for engaging faculty in meaningful campus-wide assessment of quantitative literacy.

Rhodes, T. (2010). Quantitative Literacy VALUE Rubric. Washington, DC: Association of American Colleges and Universities. Accessed September 15, 2021 from https://www.aacu.org/value/rubrics/quantitative-literacy. (Received September 19, 2021)

1174-101-8648 **Ana T Castillo*** (ana.t.castillo.rivas@gmail.com), Horizon Montessori Public Schools. *Computed Tomography (CT) in High School Mathematics*

The motivation behind this activity comes from attending the Summer 2021 REU/RET at Emory University. Undergraduate students studied image reconstruction algorithms. I created an activity to introduce high school students to computed tomography (CT) and matrix algebra. In this activity, students will row reduce a matrix corresponding to an imaging diagram and draw conclusions. Two solution cases are considered: one where an infinite number of solutions exist and one where there is a unique solution. The calculations will be performed by hand and compared to those done in MATLAB or other software. Students will do the following: 1) talk about how math can save people's lives, 2) read an article about why to do math, 3) watch a video on computed tomography, 4) solve a kakuro puzzle and learn about connections to x-ray imaging, 5) discuss the connections between ill-posed and x-ray imaging problems, 6) play a game to determine if a system of equations has an infinite number of solutions, 7) create a system of equations from an x-ray imaging diagram, 8) solve a system of equations by row reduction and determine if it has an infinite number of solutions, 9) add data to the system of equations from the imaging diagram in Part 7 and show that the resulting system has a unique solution. The activity will highlight the importance of learning about math in high school and how it will apply to their future careers. The classroom experience will encourage them to continue their studies of image reconstruction algorithms after they graduate. (Received September 19, 2021)

1174-101-8903 **Kathryn Leonard*** (kleonard.ci@gmail.com), Occidental College. *Collaborative academic year research groups: The Center for Undergraduate Research in Mathematics*

The Center for Undergraduate Research in Mathematics (CURM), founded by Michael Dorff in 2007, supports students and faculty in academic year undergraduate research by providing funding and mentorship training. Research groups are funded by minigrants to support purchase of supplies, travel, stipends for students (or course credit), and a course buyout for the faculty mentor. CURM provides training for faculty on effective mentoring of groups of students during the summer preceding the award. Beginning in 2016, CURM applications require pairs of research groups to apply together, in the hope of fostering collaboration across institutions. During the funded year, the research groups work on open mathematical problems, present their work at regional conferences, and submit a final report. Groups are also asked to give back to the mathematical community through some form of public service. Throughout, CURM emphasizes the importance of positive collaboration and a strong sense of community. CURM is supported by the National Science Foundation and was recognized as a 2013 AMS Program That Makes a Difference. (Received September 20, 2021)

1174-101-8910 **José Garcia** (josejgarcia@unm.edu), University of New Mexico, **Ashley Oaks** (aoaks61@gmail.com), Yale University, **Isaac Ortega** (ortegai@seattleu.edu), Seattle University, **Cinnamon Hobbs** (hobbs_j@fortlewis.edu), Fort Lewis College, **Joyce Pechersky** (jp5762@uncw.edu), Univ of North Carolina Wilmington, and **Anibely Torres** (torreani@kean.edu), Kean University. *CURM: From the student's perspective, I*

This session is a student panel. Students from CURM groups past and present discuss their experiences. They will describe their challenges and successes, tips for effective collaboration, things to consider before starting research, and the overall impact of their CURM experience on their mathematical lives. (Received September 20, 2021)

1174-101-8917 **Michael John Dorff** (mdorff@math.byu.edu), Brigham Young University, and **Nancy Ann Neudauer** (nancy@pacificu.edu), Pacific University. *Open discussion: Academic year student research*

This session will be an open discussion about effective undergraduate research mentoring. All faculty and students with an interest in undergraduate research are invited to join the conversation (Received September 20, 2021)

1174-101-8929 **Vesna Kilibarda*** (vkilibar@iun.edu), Indiana University Northwest. *Mathematics in the World Active Collaborative Learning Course* Preliminary report.

Introductory mathematics courses have the greatest impact on strengthening students' quantitative and logical reasoning abilities needed for informed citizenship, the workplace, and in students' disciplines [CUPM, 2004]. Effective teaching and deep learning require student engagement with content both inside and outside the classroom [MAA Connect, 2020]. Active collaborative learning (ACL) research implies that learners in cooperative teams achieve higher levels of performance, retain information longer than learners who work individually [Webb, 1995], and disproportionately benefit members of minority groups in STEM fields [Treisman, 2009]. Guided with ACL research and our own encouraging results in the general education ACL course for health professions (which resulted in increase of scores on midterm and final exams and increased retention after a semester), we have designed and taught three times an ACL quantitative course for liberal arts and other non-science majors.

We have included colleagues and students in majors that we serve in selection of topics and adoption of ACL projects. Faculty Development ACL Workshops have been important in acceptance of the new model of teaching and learning in the Department and College. Collaborative projects have become a cornerstone of our design. Our students have found Brainteasers, GPA, Bar Codes, Gallup Polls and Voting Power Projects among most engaging. More details about these and other aspects of the course will be shared in our presentation. (Received September 20, 2021)

1174-101-9184 **Mihhail Berezovski*** (mihhail.berezovski@gmail.com), Embry-Riddle Aeronautical University. *Research Projects in Data-Enabled Industrial Mathematics*

In this talk, we will discuss our experience with several programs that support undergraduate research: MAA PicMath, MAA NREUP and NSF REU site at Embry-Riddle Aeronautical University. We will review challenges of bringing collaboration with real business, industry, or government (BIG) into the undergraduate research. We highlight differences and challenges compare to traditional undergraduate research. We will discuss our response to COVID19 situation and share ideas for successfully designing and mentoring such projects. (Received September 20, 2021)

1174-101-9348 **Tatiana Dezbah Shubin*** (tatiana.shubin@sjsu.edu), Alliance of Indigenous Math Circles, San Jose State University, **Mark Saul** (marksaul@earthlink.net), Alliance of Indigenous Math Circles, **Maria Droujkova** (maria@naturalmath.com), Natural Math, and **Craig Young** (cmyoung877@gmail.com), Tuba City Boarding School. *Bluebird Math Circle - a Virtual Math Community* Preliminary report.

Bluebird Math Circle https://aimathcircles.org/bluebird/ is a new project of the Alliance of Indigenous Math Circles (AIMC) launched in March, 2021. It is a supportive community of teachers, students, and their families - everyone who cares or is curious about mathematics, education and Indigenous young people's futures. Every week Bluebird MC publishes a flyer with fun and engaging activities that are accessible to everyone. People play with activities in class, at home, and at live online Circle meetings. After each meeting, we publish a recap where Circle members and friends share the ideas they developed in the meeting, ask additional questions, and suggest future topics. Bluebird Math Circle is a community-owned project, and it is an embodiment of the AIMC goal of creating a space where mathematical abilities of Indigenous students, their families and teachers are developed and nurtured while their cultural identity is fully recognized and celebrated. (Received September 20, 2021)

1174-101-9361 **Sarah A. Nelson*** (sarah.nelson@lr.edu), Lenoir-Rhyne University. *Innovative Assessment via Creative Final Projects in Calculus* Preliminary report.

Over the years, I have been trying to find alternate forms of assessment that afford my Calculus students a richer "exam" experience. I want my students to form deeper connections between something they know really well and/or enjoy very much and the Calculus concepts we are covering in class together. So I started transitioning my traditional final exam to a final project instead. During this talk, I will share my journey creating and adapting assessments that promote and encourage student creativity. I will share rubric(s) and samples of student work from Calculus I and II. Attendees will leave with ideas for what types of products students are capable of producing as well as suggestions for creating effective rubrics. (Received September 20, 2021)

1174-101-9423 **Francesca Gandini*** (fra.gandi.phd@gmail.com), Kalamazoo College, **Steven Craig Clontz** (steven.clontz@gmail.com), University of South Alabama, and **Drew Lewis** (drewlewis@southalabama.edu), University of South Alabama. *What is... Team-Based Inquiry Learning?*

Team-Based Learning is a highly structured form of collaborative learning usually implemented as flipped learning. But what happens when you want to introduce inquiry in the classroom? Team-Based Inquiry Learning was created to blend IBL with Team-Based Learning, with a particular eye towards lower division courses like calculus and linear algebra. Join us for this panel of TBIL implementers where instructors with different levels of experience will discuss challenges and advantages of this pedagogy. (Received September 20, 2021)

1174-101-9445 **Maiko Serizawa*** (mseri065@uottawa.ca), University of Ottawa. *Mathematics and Emotions* Preliminary report.

In this 20 minutes expository talk, I will explore how emotions play a vital role in a person's mathematical endeavour through a collection of personal stories. Considered as the most logical subject, mathematics is usually completely separated from emotions in our conscious practice. As a consequence, the emotional aspect of one's mathematical experiences is almost never discussed throughout formal education. The emotional aspect of a mathematical experience includes the joy of understanding something new, the excitement of having a new idea to try, the pain of not getting how certain things work, the fear of talking to someone about your ideas, the anxiety of not knowing if something is going to work or fail, and many more. I will argue that tuning into one's emotions and consciously sharing them with others while engaging in mathematical studies instead of neglecting them could be a key to an enhanced learning/research outcome and satisfaction. (Received September 20, 2021)

1174-101-9668 **Alex M. McAllister*** (alex.mcallister@centre.edu), Centre College, **Joel Kilty** (joel.kilty@centre.edu), Centre College, and **Prayat Poudel** (prayat.poudel@centre.edu), Centre College. *Redesigning Calculus - Incorporating Technology to Increase Accessibility and Persistence in STEM*

Amidst growing concerns that the Calculus sequence is posing considerable barriers for prospective STEM students, the mathematics program at Centre College has spent the past several years redesigning our calculus curriculum to ensure that we are better able to serve our students. In this talk we will give an overview of our work to improve the content and pedagogy of the Calculus sequence. In particular, we will talk about the goals for the curricular reform, our use of emerging technologies to teach Calculus, and how we have restructured the course to improve student persistence in Calculus. We will also talk about our attempts at incorporating mathematical modeling using real world data to promote students' innovative and real-world abilities as well as share some of the projects that were implemented in the redesigned sequence. (Received September 20, 2021)

1174-101-9727 **Istvan G Lauko** (iglauko@uwm.edu), University of Wisconsin-Milwaukee, and **Gabriella A. Pinter*** (gapinter@uwm.edu), University of Wisconsin-Milwaukee. *Attracting Students to Mathematics through Building a Community of Problem Solvers*

In this talk we highlight some efforts to establish a *Mathematics Journal for Students* with a featured problem solving competition. We present examples of successful journals from other countries and survey former student journals in the US. We envision a free online journal with 5-6 issues featuring articles, interviews, news, jokes, video and game recommendations, classic puzzles, open problems and most importantly, four-tiered problem sets encompassing grades 4-12, student solutions and photos of solvers. The goal is to attract students to problem solving early and keep them interested through building a community around the journal with teachers and mathematicians involved. In our vision such a journal could serve as a worthwhile complement to the many great math initiatives that exist today. A few fun problems that appeared in the Competition Corner of the Mathematics Student, a former student journal, will also be presented. (Received September 20, 2021)

1174-101-9738 **April Strom** (april.strom@cgc.edu), Chandler-Gilbert Community College, **Scott Adamson** (s.adamson@cgc.edu), Chandler-Gilbert Community College, **Karen Gaines** (karengainesedu@gmail.com), St. Louis Community College, **Anne Dudley** (annedudley@amatyc.org), AMATYC, **Julie Phelps** (jphelps@valenciacollege.edu), Valencia College, **Dennis Ebersole** (DEbersole@northampton.edu), Northampton Community College, **Ann Sitomer** (Ann.Sitomer@oregonstate.edu), Oregon State University, and **Scot Pruyn*** (scot.pruyn@clackamas.edu), Clackamas Community College. *Teaching for PROWESS (TfP): Increasing Student Success in Community College Mathematics through Facilitating Systemic Instructional Change* Preliminary report.

TfP serves the national need for community colleges to play a significant role in supporting students' pathways to college-level mathematics courses on the STEM pathway. Instructional standards from AMATYC seek to improve mathematics education in the first two years of college by presenting clear guidance on supporting students' mathematical PROWESS (PRoficiency, OWnership, Engagement, and Student Success). These standards intend to transform mathematical instructional practices that promote students' deep engagement in mathematical thinking, student-to-student interaction, instructors' interest in and use of student thinking, and instructors' attention to equitable and inclusive practices. TfP brings together college teams, consisting of faculty, administrators, and support services, from 8 community colleges to work towards a common vision of successfully transforming departments for implementing active learning in mathematics. This collaborative effort can lead to understanding the effects of sustained faculty development and participation in a researcher-practitioner partnership on student success rates and retention in community college mathematics, as well as how active learning improves students' engagement, knowledge, and ability especially for underrepresented groups. (Received September 21, 2021)

1174-101-9819 **Helmut Knaust*** (hknaust@utep.edu), The University of Texas at El Paso. *Constructing the Real Numbers IBL-Style*

"Topics in Advanced Calculus" is a graduate course in my institution's Master of Arts in Teaching Mathematics program, primarily taken by current high school teachers and those interested in teaching at the community college level. The topic of the construction of the real numbers à la Edmund Landau's "Grundlagen der Analysis" is well suited for such an audience: the subject is quite accessible for students with varied backgrounds, but requires substantial abstract thinking. Its subject matter is a main object of school mathematics, and last not least, it is a perfect vehicle to introduce inquiry-based learning to current teachers. We will present examples of the course notes developed during several iterations of the course, explain the instructional mode, and report on instructor and student experiences and feedback. An additional twist: In Spring 2021, the course was taught asynchronously online using a bulletin board for student contributions and discussions. (Received September 20, 2021)

1174-101-10007 **Mariah Birgen*** (mariah.birgen@wartburg.edu), Wartburg College, and **Brian Birgen** (brian.birgen@wartburg.edu), Wartburg College. *Modeling Calculus: Engaging the Students We Have With 21st Century Mathematics*

Once you acknowledge search engines, Computer Algebra Systems, PhotoMath, weak Algebra skills and Chegg, what belongs in a first term Calculus course taken by all students? What do you teach when you want these same students to be inspired to continue on in mathematics? And how do you account for the increasing numbers of your students who are taking AP Calculus, if not the test? It is certainly not the same first Calculus course I took as an undergraduate.

Modeling Calculus, which has been taught at Wartburg College for more than 10 years, is designed for first term college students of any discipline and nearly any background to challenge and engage them from the beginning and lead them to and understanding of Calculus that deep enough that no one asks why they "need" to take Calculus. The course uses models from a variety of disciplines and free modeling software to teach students the fundamentals of the definitions of the derivative, the integral and differential equations. By the end of the term students can read and interpret differential equations models published as research articles in their chosen field. Strong students will then improve on the published models. This course is followed by a second semester course on algebraic techniques of Calculus which are needed by a subset of students for their intended majors. This talk will quickly describe how the course works in order to focus on modeling scenarios used in the course. (Received September 21, 2021)

1174-101-10108 **Ann Patrice Gulley** (ann.gulley@auburn.edu), Auburn University, **Canek Phillips** (canek@rice.edu), Rice University, **Yvette E. Pearson*** (yepearson@utdallas.edu), University of Texas at Dallas, **Logan Prickett** (lprickett@aum.edu), Auburn University Montgomery, **Matthew Ragland** (mragland@aum.edu), Auburn University Montgomery, and **Luke Smith** (lsmith4@aum.edu), Auburn University Montgomery. *Process-Driven Math: Impacts, Innovations, and Insights for Prospective Investigators*

Mathematics courses are gateways to science, technology, engineering, and mathematics (STEM) degree programs. They often become gatekeepers, or barriers to success, of both intended STEM and non-STEM majors. This is especially true for students with disabilities, as lack of accessibility in mathematics teaching and assessment may present insurmountable barriers for some.

Presenters will introduce attendees to Process-Driven Math (PDM), a novel method for teaching and assessing mathematics. PDM was initially developed as a fully audio-based method to ensure accessibility for students who are blind or who have low vision and was later adapted for sighted learners. With funding from the National Science Foundation's Improving Undergraduate STEM Education (IUSE) program, the research team is studying the efficacy of PDM in foundational mathematics courses for students with and without disabilities. In addition to sharing outcomes from the project, presenters will discuss lessons learned in the process of applying for and in managing the IUSE award. (Received September 21, 2021)

1174-101-10188 **Anne E Yust*** (aey15@pitt.edu), University of Pittsburgh, and **Saharsh Talwar** (sat157@pitt.edu), University of Pittsburgh. *Agent-based modeling as an early introduction to mathematical and computational approaches to infectious disease research for pre-medical undergraduate students* Preliminary report.

In this talk, we will discuss the successful process and progress of an interdisciplinary undergraduate research project investigating the spread of COVID-19 with agent-based models. Led by a mathematics faculty member, the project is an ongoing independent research experience for a pre-medical student who began at the start of their first semester in an all-virtual environment. An outline of the facilitation of the project will be presented, followed by a description of the project – which is previewed below.

Due to the severe implications of widespread COVID-19 proliferation, public health officials rely on effective predictions of viral spread in order to establish appropriate precautionary procedures that promote public safety. Mathematical models are effective, viable tools in facing these pressing healthcare issues. Using the capabilities of agent-based models to encode simultaneous interactions and vary parameters easily, we are developing a model to track the viral spread of COVID-19, stemming from student interaction and environmental viral load, in a classroom throughout a semester. Our simulation shows the impact of various factors such as face mask use, the effectiveness of classroom sanitation measures, vaccination rate, and social distancing on transmission rate. (Received September 21, 2021)

1174-101-10196 **Carol J Overby** (overbyc@newschool.edu), The New School, and **Debasmita Basu*** (basud1@newschool.edu), The New School. *Exploring minimum wages and well-being* Preliminary report.

In our data-centric times, quantitative literacy is essential to assess important social issues and to make informed decisions. The Quantitative Reasoning program at The New School gives students tools to become critically-engaged citizens in a democratic society. It encourages students to explore data on socio-political and environmental issues, and shifts mathematics from abstraction to making it a potential tool for understanding society, promoting social engagement, and contributing to the public good. In this session, we will share a Quantitative Reasoning module on worldwide minimum wages. In this module, students explore minimum wages of different countries, adjusted for their cost of living, and compare the wage levels to measures of well-being. We conclude with some qualitative measures of the modules' effectiveness, and hope this will inspire our colleagues to help students engage critically with the world using quantitative frameworks. (Received September 21, 2021)

1174-101-10410 **David Patrick*** (patrick@aops.com), Art of Problem Solving, San Diego Math Teachers' Circle. *MathWalk: bringing beautiful math to the sidewalk*

What if some students never see the beauty in math? And what if we could change that? This summer, my colleagues and I worked to bring mathematical beauty directly to the neighborhoods of students in several locations around the country. Inspired by and in partnership with San Diego Math Teachers' Circle participant and original MathWalk creator Traci Jackson, we will describe our national MathWalk campaign, its impact, what makes a great sidewalk chalk math problem, and discuss ideas about how you can start a MathWalk in your own community. (Received September 21, 2021)

1174-101-10471 **Rachael M Lund*** (rlund@msu.edu), Michigan State Univeristy. *COVID-19 as a Catalyst for Re-Designing a Large QL Course with a Focus on Student Engagement*

Everyone, including us, had to scramble in Spring 2020 to move in-person courses to a virtual setting. In the chaos of the quick and forced transition, it became abundantly clear that instructors weren't necessarily bound to the traditional structures of course design and pedagogy. This talk will outline the continuing journey of a particular Quantitative Literacy course. We will discuss lessons learned from the iterative process of re-design: changes to assessment, content delivery, instructional staff qualifications, student feedback loop, etc. We will also look ahead on the transition to go back to more in-person modalities as we attempt to maintain fidelity to the lessons learned. (Received September 21, 2021)

1174-101-10566 **Kerry M Luse*** (lusek@trinitydc.edu), Trinity Washington University. *Growing our embedded tutoring program: creative solutions for a small college* Preliminary report.

The STEM departments at Trinity Washington University introduced an embedded tutor program as part of our Inclusive Excellence initiative. Embedded tutoring began with tutors placed in "gateway" courses as well as courses identified as common "stop-out" points in the STEM major curriculums. Preliminary results have shown improved student outcomes through course grades, retention, and sense of belonging. The program is growing to include tutors in gateway courses for non-STEM majors as well. This talk will include an overview of our program, challenges we've faced, creative ways our tutoring program has met a variety of student needs, and how we are assessing the program. Funding for our embedded tutoring program is partially supported by a Howard Hughes Medical Institute Inclusive Excellence grant. (Received September 21, 2021)

1174-101-10581 **Marc Steven Ordower*** (mordower@randolphcollege.edu), Randolph College. *Math in a Minute*

How do you convince someone scrolling through an unending feed of viral dances, talking pets, and demonstrations of artistic watermelon carving to pause and watch a video about math? And assuming you can, how can you say something meaningful about mathematics in under a minute? We explore the search for mathematical ideas that can be explained quickly and arouse sufficient curiosity to compete in the maelstrom of thoughts and emotions that is TikTok. (Received September 21, 2021)

1174-101-10591 **Terrance Pendleton*** (terrance.pendleton@drake.edu), Drake University. *The deep end of the pool: inviting early college students to be mathematicians* Preliminary report.

In this presentation, the author will share preliminary findings from an NSF project that investigates the ways in which mathematics research projects, conducted early in students' mathematical careers (as early as pre-Calculus courses) and around questions that the students themselves pose, can spark students' engagement and interest, with a particular focus on students who have been historically excluded from mathematics majors. (Received September 21, 2021)

1174-101-10641 **Jeffrey S Meyer*** (jeffrey.meyer@csusb.edu), California State University, San Bernardino. *Using Technology and Visualization to Enhance Students' Reasoning in Linear Algebra* Preliminary report.

Core concepts of linear algebra include: mixture, transformation, and intersection. In what ways can visualizations enhance a student's ability to productively reason about these concepts? In what ways can technology demonstrations and explorations strengthen these visualizations? Over the past several years, I have created a collection of dynamic, explorative demonstrations and activities in GeoGebra that I use to teach linear algebra. In this talk, I will share some of these activities and my experiences using them in the classroom. This is part of an ongoing project to better understand the use of technology and visualization in the teaching and learning of linear algebra. (Received September 21, 2021)

1174-101-10803 **Michael C Barg*** (mbarg@niagara.edu), Niagara University. *Giving Students Control...* Preliminary report.

...Theory in a first undergraduate ordinary differential equations (ODE) course is possible. With an aim toward promoting student-led activities, this talk will encourage participants to engage with control theory modeling projects in a student-first approach. Such projects might be drawn from disparate areas, e.g., the Vidale-Wolfe marketing model or insect colony ecology. Timely problems like determining an optimal vaccination schedule for infectious diseases might be appropriate to pursue. This talk will slide into athletics by considering a student project in which a shooting method is used to explore optimal running paths for softball hitters. All of the aforementioned topics can be cast in a control theory context, and the presenter believes that any of the topics could be suitable for a first ODE course. (Received September 21, 2021)

1174-101-10809 **Cody L Patterson*** (codypatterson@txstate.edu), Texas State University. *Assessing Teaching Assistants' Mathematical Knowledge for Teaching Calculus* Preliminary report.

Researchers in undergraduate mathematics education have investigated graduate teaching assistants' mathematical content knowledge (e.g., Firouzian & Speer, 2015) and pedagogical content knowledge (Judson & Leingang, 2016) for teaching calculus. To assist students in learning calculus, graduate students who conduct tutorials or "labs" for calculus classes must possess robust knowledge of the course material beyond the procedural fluency expected of undergraduate STEM students, along with some awareness of the development of key concepts (for example, understanding the connection between the idea of a definite integral as a limit of Riemann sums and the idea of an integral as an accumulation of a rate of change). I will share some questions that I have developed to assess incoming graduate students' mathematical knowledge for teaching calculus and some preliminary results from their implementation, and discuss how these questions can be used as a starting point for content-based professional development of teaching assistants. (Received September 21, 2021)

1174-101-10916 **James Rolf*** (jimrolf@yahoo.com), University of Virginia. *An Online Bridge Program for Incoming STEM Students* Preliminary report.

Cavalier Online Experiences (CONEX) is an online bridge program at the University of Virginia for STEM majors from "not-the usual-places". We report on the philosophy and structure of CONEX and the use of undergraduate coaches to help under-represented and first-generation students prepare for a STEM major. We will share data describing the impact of coaches on students and student growth over the summer based on pre/post exams and pre/post surveys. (Received September 21, 2021)

1174-101-10929 **Anna Burago*** (annab@pfmathcircle.org), Prime Factor Math Circle. *On Codes, Secrets, and Ciphers. Ciphers and cryptography for math circles and math festivals* Preliminary report.

Who would not like to learn how to create secret ciphers and break coded messages? And wouldn't everyone like to know more about the history of cryptography and an everlasting competition of codemakers and codebreakers?

Cryptography is a great topic that can be taught in many possible settings, including math festivals, math circles, and summer camps. It is also possible to adjust the scope and the difficulty to make it cool and accessible for a third-grader or challenging and exciting for a high-school kid.

In my talk, I will start by presenting an outline of a series of math circle lessons on cryptography, with samples of problems and activities, including a crypto battle. Next, I will talk about using cryptography in a setting with younger students or newcomers, and about presenting cryptography at a math festival.

Cryptography, being a cool math circles topic on its own, is also extremely important for modern computer science. This fact makes the topic especially interesting and motivating for students. (Received September 22, 2021)

1174-101-10948 **Therese Shelton** (shelton@southwestern.edu), Southwestern University, **Alison M. Marr*** (marra@southwestern.edu), Southwestern University, and **Fumiko Futamura** (futamurf@southwestern.edu), Southwestern University. *Using Visualizations and Applications to Understand Conceptual Ideas from Calculus* Preliminary report.

In our newly revised Calculus sequence (Modern Calculus), we have a guiding principle to approach all topics from multiple perspectives (numerical vs. analytical, discrete vs. continuous, etc.). A second guiding principle ensures that some concepts and applications are intentionally layered, building in complexity as students progress through the sequence. In this talk, we'll focus on some visualization and application modules that have developed from these guiding principles. The modules we mention have been used to help students gain conceptual understanding of big calculus ideas. Topics include the use of modeling data, thinking about trig functions through music and applets, and using visualizations to aid in understanding level curves, gradients, and constrained/unconstrained optimization. (Received September 21, 2021)

1174-101-10990 **Christina Duron** (duronc@math.arizona.edu), University of Arizona. *From Mirrors to Wallpapers: A Virtual Math Circle Module on Symmetry*

Symmetry is a natural property that every person sees in everyday life; it also has deep mathematical connections to areas like tiling and objects like wallpaper groups. In Spring 2021, the Tucson Math Circle led a 7-part module on symmetry that started with reflective symmetry and culminated in the deconstruction of wallpapers into their 'generating tiles'. This module utilized a scaffolded, hands-on approach to cover old and new mathematical topics with various interactive activities; all activities were made available through free web-based platforms. In this talk, we outline the module, highlight key moments and activities, and discuss our implementation using Zoom, Miro, and our custom-made TMC Wallpaper Widget. We touch on the benefits and disadvantages of the online environment, and provide suggestions for moving these activities to a live format or to different audiences. (Received September 21, 2021)

1174-101-11037 **Robin Cruz*** (rcruz@collegeofidaho.edu), The College of Idaho, and **Dave Rosoff** (drosoff@collegeofidaho.edu), The College of Idaho. *Calculus and Statistics with More Modeling and Less Algebra* Preliminary report.

The College of Idaho offers two introductory mathematics courses, *Data Analysis & Statistics* and *Applied Calculus*, which are taken by over half of our First-Year students. Many students enter the classes with weak algebra skills. These two courses are designed to give underprepared students experience with how statistics and calculus are used to solve realistic, open-ended problems that are more involved than a typical homework problem. Both courses have a significant modeling component woven into the content. We will explain how we use modeling in these courses and give examples of student work. (Received September 21, 2021)

1174-101-11157 **Barnabas Bede*** (bbede@digipen.edu), DigiPen Institute of Technology, Department of Mathematics, and **Emese M Bede** (ebede@nsd.org), Bothell High School, Northshore School District. *Engaging students in Calculus Classes by introducing Machine Learning concepts* Preliminary report.

Machine Learning has gained popularity in recent years, and students have become more and more familiar with its applications, ranging from autonomous vehicles to advanced video game AI. Due to increased popularity and media coverage, students show a strong interest in learning about concepts such as neural networks and deep learning. Since the backpropagation algorithm, at the basis of deep learning architectures is entirely rooted in the chain rule for multivariate functions, we propose an approach to include these concepts in the first calculus class. In this approach, a natural extension of the derivative concept is the partial derivative, while an immediate generalization of the chain rule for univariate case becomes the chain rule for partial derivatives. We will discuss the challenges of introducing these concepts early in the calculus sequence, in a rigorous, but engaging way, by linking them to deep learning algorithms. (Received September 21, 2021)

1174-101-11167 **Scott A Strong*** (sstrong@mines.edu), Colorado School of Mines. *Reflections on Differential Equations Education - Pandemic Edition* Preliminary report.

As the significant amplitude shift in teaching and learning brought about by the pandemic returns nearer to a face-to-face equilibrium, we reflect on our adaptations in teaching, learning, assessment, and support practices in compulsory introductory differential equations education at a STEM university. What worked, what didn't, and what methods do we retain? Specifically, we review issues encountered with the unsupervised assessments utilized during our pivot into a remote learning environment and how reflective/technical writing can be used to regain control of student assessment. While these techniques are amenable to implementation in individual courses, they fail to scale to large-lecture teaching assistant supported delivery models. Consequently, we discuss how specification and contract-based grading can be leveraged, in conjunction with computer-assisted formative assessments and plagiarism review, to support large-scale evaluation of student projects in place of high-stakes examinations. During the pandemic, where imperfection is the norm, opportunities to develop progressive curricular elements presented themselves, and though we find ourselves reaffirming much of the pre-2020 paradigm, the future of differential equations education seems more dynamic than ever. (Received September 21, 2021)

1174-101-11280 **Tingxiu Wang*** (tingxiu.wang@tamuc.edu), Texas A&M University-Commerce. *Integrate Modeling into a Traditional Class of Differential Equations* Preliminary report.

In this talk, the presenter will talk about his experience how he integrated modeling into his traditionally taught class of differential equations. Although he had used many examples of application in teaching differential equations, a full cycle of modeling is very different. He will discuss his observations of student learning and reactions. (Received September 22, 2021)

1174-101-11295 **Caitlin Esgana** (ce552@csus.edu), California State University, Sacramento, **Jennifer Lundmark** (lundmark@csus.edu), California State University, Sacramento, **Michelle Norris** (norris@csus.edu), California State University, Sacramento, **Vincent Pigno** (vincent.pigno@csus.edu), California State University, Sacramento, and **Corey Shanbrom** (corey.shanbrom@csus.edu), California State University, Sacramento. *Assessing a peer learning program using propensity score matching* Preliminary report.

We first briefly introduce an adapted peer-led learning team model, called the Peer Assisted Learning Program at California State University, Sacramento. The program is 'opt-in' for students, and therefore accounting for confounding factors in analyzing its success is more difficult. We then turn to discussing a recent analysis using

propensity score matching to study the program's effectiveness, and how this helps account for the voluntary enrollment of students. (Received September 29, 2021)

1174-101-11298 **Reena Tandon*** (reena.tandon2008@gmail.com), LOVELY PROFESSIONAL UNIVERSITY. *To Study the Bianchi type V Cosmological Model with Quadratic Equation of State in Modified theory of gravity*

In this paper, Bianchi type-V cosmological model in perfect fluid with quadratic equation of state $p = \alpha\rho^2 - \rho$, where $\alpha \neq 0$, has been studied in modified theory of gravity. The general solution of the (Received September 30, 2021)

1174-101-11303 **Katalin Bimbo*** (bimbo@ualberta.ca), University of Alberta. *Relational semantics for some classical relevance logics*

The framework called *generalized Galois logics* (or gaggle theory, for short) was introduced in [2] to encompass Kripke's semantics for modal and intuitionistic logics, Jónsson & Tarski's representation of BAO's and the Meyer–Routley semantics for relevance logics among others. In some cases, gaggle theory gives exactly the semantics defined earlier for a logic; in other cases, the semantics differ (cf. [3], [1]). Relational semantics for classical relevance logics such as **CR** and **CB** are usually defined as a modification of the Meyer–Routley semantics for $\mathbf{R}_+$ and $\mathbf{B}_+$, respectively (cf. [4]). In this talk, I compare the existing semantics for **CB** and **CR** to the semantics that results as an application of gaggle theory. (Received October 1, 2021)

1174-101-12201 **Heather Price*** (heather.price@seattlecolleges.edu), North Seattle College. *Climate Justice Integrated Learning in STEM.*

Our students learn about climate change from the news and in many of our classes, and they are hungry for what to do with that knowledge and how to connect it within their careers and communities. Climate touches and belongs in every subject we teach, from Humanities, business, and health sciences, to all areas of STEM, including mathematics and statistics. Dr. Price will share her work leading the Climate Justice Project at North Seattle College. This initiative seeks to build bridges between disciplines to help faculty incorporate climate justice and civic engagement into their core curriculum, in ways that empower students and encourage student retention and success. In today's talk Dr. Price will share ideas of how and why to integrate climate justice and civic engagement into STEM, with examples from mathematics courses. (Received November 16, 2021)

102 ► Recreational mathematics

1174-102-7606 **Megan Vesta*** (meganevesta@lewisu.edu), Lewis University. *Do win streaks predict college baseball end of season rankings?*

Ranking sports teams can be a challenging task, and using straight win percentage can be misleading at times. Among the many mathematically inspired sports ranking systems, linear algebra methods are among the most elegant and simple. In this research, we focus on applying a Markov chain method to predict the future results of NCAA Division 1 College Baseball. In particular we investigate whether win streaks can help predict the final standings for college baseball using data from multiple conferences and years. (Received September 15, 2021)

1174-102-8209 **Ian Robert Sammis*** (ian@nightmathradio.com), YouTube. *Braids and braided nets from distance functions, overlap rules, and conformal maps*

The distance to the nearest point on each of a set of curves $\mathbf{c}_i$ can be used to create a set of value ramps:

$$V_i(\mathbf{x}) = \begin{cases} 1 - \frac{d(\mathbf{x},\mathbf{c}_i)}{T} & d(\mathbf{x},\mathbf{c}_i) < T \\ 0 & d(\mathbf{x},\mathbf{c}_i) \geq T \end{cases},$$

where we take H_i to be a series of presupplied hues and $S_i = 1$. These can be combined into a single value map by supplying preset rules for which V_i should be selected at each point. By always choosing T and $\mathbf{c}_i$ to be such that no more than two V_i are nonzero at any given point, we can supply selection rules to specify which V_i to use. This transforms the curves into overlapping (or braided, with correct curve and rule choices) patterns specified by a function over $\mathbb{R}_2$ — suitable for mapping via conformal maps. An infinite braided cord, for example, can be transformed via $\frac{z^2}{2}$ to a good unit cell for a diagonal tiling with consistent braiding that forms an attractive initial source for other conformal maps. We will show some of the resulting lace-like braided networks. (Received September 18, 2021)

1174-102-8228 **Harvey Campos-Chavez*** (harveycamposchavez@lewisu.edu), Lewis University, **Will deBolt** (willpdebolt@lewisu.edu), Lewis University, **Jacob Prince** (jacobmprince@lewisu.edu), Lewis University, **Soren Thrawl** (sorenthrawl@comcast.net), Lewis University, **Robbie Dudzinski** (Robert_Dudzinski@ben.edu), Benedictine University, **Alia Alramahi** (Alia_Alramahi@ben.edu), Benedictine University, and **Anthony DeLegge** (adelegge@ben.edu), Benedictine University. *Analysis of NHL Hockey* Preliminary report.

Predicting the outcome of a hockey game can be challenging due to the fast paced and physical nature of the sport. In this talk, we share several approaches for determining the outcomes of NHL hockey games. One method involves a continuous time Markov process-based model that takes the certain state the home team is in at any point in the game and gives a winning probability statistic for that time. This state is based on the home team's shot and goal differential relative to the opposing team and approximates the probability that the home team would win depending on the state they are currently in at a given time in the game. Our second analysis technique uses hypothesis testing to determine whether variables (or combinations of these variables) like shot differential, manpower differential, face-off win percentages, the amount of time in powerplays, and the number of low, medium, or high-danger shots are predictive in determining the outcome of NHL hockey games. (Received September 18, 2021)

1174-102-8315 **Henry Segerman*** (segerman@math.okstate.edu), Oklahoma State University. *Designing holonomy mazes* Preliminary report.

Holonomy mazes are physical puzzles in which a piece moves along a network of rails on a surface. The piece is prevented from rotating other than by holonomy. Pegs alongside the rails block movement of the piece if it has the incorrect orientation, creating a maze.

I'll talk about the problems involved in choosing the peg locations to make an interesting maze. Since the orientation of the piece is important, the maze is best thought of as being embedded in the unit tangent space to the surface. If the surface is a sphere, this is real projective space, which is conveniently described with quaternions. (Received September 18, 2021)

1174-102-8340 **Nancy Blachman*** (nancy.blachman@jrmf.org), Julia Robinson Mathematics Festival. *Fold-and-Cut Challenge: Create your own Betsy Ross Flag*

"A...class of paper-cutting recreation, more familiar to magicians than mathematicians, involves folding a sheet of paper several times, giving it a single straight cut, then opening [it] up to reveal a regular geometric figure or various types of stars...and such complex patterns as a circular chain of stars..."—*New Mathematical Diversions*, by Martin Gardner

According to legend, George Washington asked Betsy Ross to sew a flag with six-pointed stars, thinking that five-pointed stars would be too difficult. But Besty Ross was an expert fold-and-cut puzzle solver! By folding a piece of fabric, she was able to produce a symmetric five-pointed star with a single cut.

Start by challenging participants to cut out squares, rectangles, triangles, other shapes, and eventually stars, by folding a piece of paper and making a single straight cut (no turning!). And I suggest ending with a star-spangled challenge—making your own version of a U.S. or other flag out of paper.

In my experience when leading this and other Julia Robinson Mathematics Festival (JRMF) activities, children and adults alike eagerly experiment, collaborate, and make discoveries. All the JRMF activities (jrmf.org/activities) appeal to diverse audiences, since they have low floors and high ceilings. (Received September 20, 2021)

1174-102-9317 **Laurie M Zack*** (lzack@highpoint.edu), High Point University. *A Math Teachers Circle and Card Games* Preliminary report.

The Triad Math Teachers Circle is a new MTC located in the Greensboro-Winston Salem-High Point area of North Carolina. We have kicked off our MTC by utilizing different games to attract a wide audience of participants. We first used the game Spot It! to identify different mathematical properties and structures of the game. More recently we incorporated the game EvenQuads, a new card game by the AMS consisting of five new games, featuring the biographies of women mathematicians through history. In this talk, I will briefly share the outline of our MTC events and the types of activities and mathematical questions we discuss. (Received September 20, 2021)

1174-102-9498 **Annette Rouleau*** (annette.rouleau@jrmf.org), Julia Robinson Mathematics Festival.
Königsberg for Kids - Come, draw a line with us!

From Euler comes from the famous Bridges of Königsberg problem in which he proved it was impossible to cross the seven bridges in Königsberg without retracing one's steps. This laid the foundation of graph theory which we use today to solve combinatorics problems such as city road and traffic planning or even for arranging who plays against who in round-robin tournaments.

In our activity, students are asked to trace simple paths before being challenged to consider multiple paths and starting points. Pedagogically, Königsberg is interesting because it requires the students to think about what impossible means mathematically. There's a difference between "I can't do it" and "No one can do it" that students can sometimes overlook.

The richness in this activity also lies in students being asked to draw their own Eulerian paths, which could be graphs of their own design such as their own communities or schools. Like all Julia Robinson Mathematics Festival (jrmf.org) activities, meaningful mathematics emerges as students engage with this play-based approach to problem solving and critical thinking. And it all begins by drawing a single, continuous line! (Received September 20, 2021)

1174-102-9507 **Allison Henrich** (henricha@seattleu.edu), Seattle University, **Justus Curry*** (jcurry@seattleu.edu), Seattle University, and **Mitchell Rask** (raskmitchell@seattleu.edu), Seattle University. *The Arc Crossing Change Game* Preliminary report.

One fundamental question for knots is: given a knot, how hard is it to unknot it if you are allowed to perform certain topology-changing local moves that incrementally simplify the knot? The tools that help us think about unknotting questions like this one can also be used to invent and play new games on knot diagrams. In the spirit of the Knotting-Unknotting Game (which uses crossing changes during game play) and the Region Unknotting Game (which uses the region crossing change), we introduce a new unknotting operation—called the arc crossing change—and develop a knotting-unknotting game that can be played using the move. We study winning strategies for the Arc Crossing Change Game on the family of twist knots. (Received September 20, 2021)

1174-102-9582 **Violeta Vasilevska*** (violeta.vasilevska@uvu.edu), Utah Valley University. *Staying "Sane" with the* Instant Insanity Puzzle Preliminary report.

This presentation highlights hands-on projects that the presenter has used in various outreach settings. First, we give an overview of the outreach programs/activities and the audience to which these projects have been presented. We then demonstrate the math hands-on projects (Solving *Instant Insanity* Puzzle). During the presentation, we describe the puzzle and the challenges it brings. In addition, we explain how the workshop participants are challenged to analyze the number of solutions, as well as discuss strategies for solving the puzzle. Furthermore, we discuss the two projects that provide two different approaches for solving this puzzle. The first approach uses only numbers and their properties to derive a solution to the puzzle, while the second project seeks a solution using graph theory. At the end, we share some of the workshop participants' feedback. (Received September 20, 2021)

1174-102-9626 **Edward J. Fuselier*** (edfuselier@gmail.com), High Point University. *A Mathematical Exploration of Enemy-Protector* Preliminary report.

In this talk we mathematically investigate the summer camp activity known as "Enemy-Protector." Participants of Enemy-Protector try to arrange themselves according to "enemy" and "protector" assignments, and these assignments can lead to interesting behavior as the players move. In an effort to explore the dynamics involved, we model the activity as a discontinuous dynamical system. We will share several observations from our simulations, discuss some of Enemy-Protector's underlying mathematical structures, pose a few questions, and explore avenues for future research. (Received September 20, 2021)

1174-102-9788 **Atharva Pathak*** (apathak3141592@gmail.com), MIT PRIMES-USA, and **Tanya Khovanova** (tanya@math.mit.edu), MIT. *Combinatorial Aspects of the Card Game War*

We study a single-suit version of the card game War on a deck of cards from 1 to n. There are varying methods of how players put cards they win back into their stacks, but we primarily consider randomly putting the cards back and always putting the winning card before the losing card. The concept of a *passthrough* is defined, which refers to a player playing through all cards in their stack from a particular point in the game. We consider games in which the second player wins during their first passthrough.

First, a bijection is established between forests of full binary trees and sequences of wins and losses for a player where wins correspond to non-leaves and losses correspond to leaves. Then, we find a recursive formula for the probability that given a random initial state, the first player wins within k passthroughs. We finish by counting how many initial states where the first player starts with a single card are guaranteed to follow a certain win-loss sequence. (Received September 20, 2021)

1174-102-10230 **Evelyn James Lamb*** (evel8yn@gmail.com), Freelance, and **Kevin P. Knudson** (kknudson@ufl.edu), University of Florida. *Tickling the Ears with Mathematical, Human Stories* Preliminary report.

On Emmy Noether's birthday in 2017, we recorded the first episode of our podcast My Favorite Theorem. Since then, we have published more than 70 episodes featuring mathematicians who come from a broad range of nationalities, races, ages, genders, geographical locations, job titles, career stages, and fields of study. In each episode, we ask a mathematician to tell us their favorite theorem and why they love it. Answers have ranged from theorems that are well-known among schoolchildren—for example, the Pythagorean theorem and the infinitude of primes—to gnarly technical theorems known to only a few experts. Whether or not the mathematics is down-to-earth or up in the clouds, our show invites mathematicians to share why they love their work and the unusual connections they make with other aspects of their lives. In this talk, I will describe how two podcasting novices started a diverse, accessible math podcast and share some of our favorite moments from the show. (Received September 21, 2021)

1174-102-10258 **Pavithra Devy Mohan*** (mohanpd2846@uwec.edu), UWEC, and **Danya Morman** (MORMANDR8396@uwec.edu), UWEC. *Knit Wallpaper Groups*

This project has aimed to study, analyze, and replicate the properties of the seventeen unique wallpaper groups through the medium of knitted works. Wallpaper groups are classes of tessellations made through translations designated by their reflexive and rotational symmetries, and are an excellent introduction to the study of groups which is a primary topic of abstract algebra. Most of our time has been focused on the development and construction of the knitted patterns, and these patterns will be distributed online for public use. These knitted works provide a physical representation of the groups which will allow us to further study the mathematics behind their classifications. (Received September 21, 2021)

1174-102-10365 **Anduriel Widmark*** (anduriel@andurielstudios.com), Anduriel Studios. *Modeling Hexastix: Creativity with Non-intersecting Cylinder Arrangements*

In this paper we explore several geometric and combinatorial problems encountered when modeling non-intersecting congruent cylinder packing arrangements known as polystix. Symmetric polystix models that are made with rods or sticks parallel to only 3 or 4 directions can be used to illustrate a verity of packing problems. Several Examples are given to emphasize both the mathematical and aesthetic qualities that polystix models exhibit. Building polystix is not only a fun, hands-on way to learn and promote mathematics, but also a way to make beautiful art. (Received September 21, 2021)

1174-102-10416 **Ellen Baker*** (ellie.baker@post.harvard.edu), Retired, **Charles Wampler** (charles.w.wampler@gm.com), University of Notre Dame, and **Daniel Baker** (danny.r.baker@comcast.net), Retired. *Infinitely Invertible Infinity: Crafts, Math, and the Joy of Turning Things Inside Out*

Crafting projects, and my design aspirations for them, have led me to a series of engaging puzzles. Although this is a story about learning some fascinating mathematics by solving crafting puzzles, my particular theme for this talk is about turning things inside out—or what we call inversions. Work with coauthors Charles Wampler and Daniel Baker on a series of projects, over multiple years, led to the design of a very unusual reversible infinity-style scarf with the entertaining feature that it can be turned inside out to expose not just two different surfaces... but three! I'll start by demonstrating it, and then describe the series of sewing and crafting puzzles, and their solutions, that led up to it.

Surprisingly, the scarf design also scales up without limit, so it is possible (at least theoretically) to make a version that is quadruply or quintuply invertible, or more. I'll explain how, and detail multiple sewing experiments involving knots, links, and other topological concepts related to the design. I'll also show samples of the scarf itself and other sewn objects for hands-on exploration of the mathematical concepts used. (Received September 21, 2021)

1174-102-10766 **Caleb Adams*** (cadams5@radford.edu), Radford University. *Analysis of Metrics Utilized in NCAA DI Women's Volleyball for Match Preparation* Preliminary report.

In this talk, the speaker will present current results of several metrics analyzed statistically from matches played in the Atlantic Coast Conference (NCAA Division I Women's Volleyball) that may be utilized by teams to determine both self and opponent strengths and weaknesses. Data gathered and analyzed examines (1) how points are scored, either offensively (ace or kill) or defensively (block or opponent error), and (2) whether there exist correlations between first ball offense and transition offense to winning a match. The analyses presented can be used by teams to enhance training and match preparation. (Received September 21, 2021)

1174-102-10896 **Eric Burkholder*** (eric.burkholder@valpo.edu), Valparaiso University. *Investigating a Graph Theory Game* Preliminary report.

We are investigating who has the winning strategy in a game in which two players take turns drawing arrows trying to complete cycle cells. The game boards are graphs, objects with dots and lines between them. A cycle cell looks like a polygon (triangle, square, pentagon, etc.). We examined game boards where the winning strategy was previously unknown. Starting with a pentagon and a heptagon glued by two sides, we worked to solve multiple classes of graphs involving stacked polygons. We also explored variations of the game where cycles, as defined in graph theory, are used in place of cycle cells, which opens the game up to non-planar graphs, such as complete graphs and gives the game a graph theory twist on top of topology. The original game was described by Francis Su in his book Mathematics for Human Flourishing. (Received September 21, 2021)

103 ► Professional development and professional concerns

1174-103-5546 **Brandy S Wiegers*** (brandy.wiegers@cwu.edu), Central Washington University. *Central Convergence Research Experience for Undergraduates (CC-REU)* Preliminary report.

The Central Convergence Research Experiences for Undergraduates (https://www.cwu.edu/reu/CCREU) provides a transformative research experience for student participants. Hosted by the Department of Mathematics at Central Washington University, the REU provides early training in mathematics research for future leaders in the mathematical sciences. Each research project, designed by research mentors who have training and expertise to produce high-quality research with undergraduate students, will be accessible to students with a calculus background. Through these well-designed research projects, problem-solving training, and other professional development support, students' tools and knowledge of the profession will be expanded to create a stronger sense of personal mathematical identity, including an individual understanding of diversity of the broader mathematical community. The 2021–2023 CC-REU is currently funded by the NSF (DMS-2050692) and will expand on the previous programs funded by the MAA National Research Experience for Undergraduates Program (NSF DMS-1652506).

This talk will focus on our experience in Summer 2021: starting an REU and then our experience guiding twelve students virtually through the three projects. We hope that you will learn from our experience and grow your own REU program. (Received August 22, 2021)

1174-103-5972 **sarah-marie belcastro*** (smbelcas@toroidalsnark.net), MathiLy, Smith College. *Secrets (well, not any more) of the MathILy-EST REU*

MathILy-EST is an early-college NSF-funded Research Experience for Undergraduates (REU) that is co-sited with MathILy, an intensive program for high-school students. We will describe the MathILy-EST REU in detail (it is very different from other REUs! and also very much the same!) and explicitly discuss how we get selected students working on open problems very quickly. Additionally, we will highlight successful practices in our admissions and program structure that are likely to generalize to other REUs. (Received September 1, 2021)

1174-103-8343 **Christine Ann Kelley*** (ckelley2@unl.edu), University of Nebraska-Lincoln. *The Nebraska Conference for Undergraduate Women in Mathematics (NCUWM)*

The math department at the University of Nebraska-Lincoln has been running the Nebraska Conference for Undergraduate Women in Mathematics (NCUWM) annually since 1999. Each conference brings around 260 undergraduates from around the country to Lincoln to present their research, meet and listen to plenary speakers and panelists, and participate in networking events and breakout discussions on specialized topics. The goal of NCUWM is to encourage women to pursue graduate study in mathematics, and/or math-related careers, and to give them tools to be successful should they choose to do so. In this talk we will share the model of this conference, how the conference has evolved to its current form, and the impact NCUWM has had on the mathematical community. (Received September 18, 2021)

1174-103-8601 **Rochelle Elaine Tractenberg** (rochelle.tractenberg@gmail.com), Georgetown University, **Victor Piercey*** (VictorPiercey@ferris.edu), Ferris State University Department of Mathematics, and **Catherine Buell** (cbuell1@fitchburgstate.edu), Fitchburg State University. *What Constitutes Ethical Mathematical Practice? Responses from the Mathematics Community on Ethics in Mathematics* Preliminary report.

Part of being quantitatively literate is recognizing the professional responsibilities and ethical obligations that go along with using and communicating mathematics and quantitative information. These responsibilities apply to those using mathematics from outside the profession as well as those who are members of the mathematics profession. But what are those obligations?

Many mathematics professional societies have ethics codes, but they are often incomplete and only address a narrow range of mathematical practices. Mathematics practitioners need ethical guidelines that are accessible, authentic, and usable, and which reflect the community's sense of professional identity formation and maintenance.

In this talk, we will describe the results of a survey regarding the applicability to mathematics of provisions from various mathematics-adjacent professions. We will highlight those items which displayed the broadest agreement, and disagreement, as touchstones for further discussion. (Received September 19, 2021)

1174-103-8969 **Edward Charles Keppelmann*** (keppelma@unr.edu), University of Nevada Reno. *A Math circle as an Elevator Speech* Preliminary report.

Wikepedia refers to an elevator speech as a short description of an idea, product or company that explains a concept in such a way that any listener can understand and appreciate in a very short time. At first glance this hardly seems like the place for an actual math circle but we beg to differ! Of course we could always advertise a real math circle in such a setting but we propose to do more by actually engaging the elevator rider (who was previously unaware of what a math circle even is) to think about some interesting, accessible, deeply engaging puzzle, topic or idea that, if revealed correctly, might actually make the person forget where they were originally going and quite possibly cause them to never view mathematics the same again! We will set up the framework for such outreach as well as providing a number of interesting concrete examples along with their associated takeaways that will keep the engaged coming back (virtually at least) for more and more. (Received September 20, 2021)

1174-103-8977 **Alexander Diaz-Lopez*** (alexander.diaz-lopez@villanova.edu), Villanova University. *Villanova Co-MaStER: Community of Mathematicians and Statisticians Exploring Research*

Co-MaStER (Community of Mathematicians and Statisticians Exploring Research) was created in 2019 at Villanova University with the purpose of encouraging undergraduate research activities at Villanova and to provide professional development sessions for the students. During this talk, I will provide an overview of the program, including what we think we have done well and where we have fell short. (Received September 20, 2021)

1174-103-9709 **Gizem Karaali*** (gizem.karaali@pomona.edu), Pomona College. *The Making of a Mathematician: Personal and Professional Growth Through Writing* Preliminary report.

It is standard practice for writing instructors to view and present writing as a way of thinking, and as a way to nurture ideas as well as the writers who practice holding them. In my mathematical career I have found that writing has often served these roles for my own personal and professional growth. Some writing projects helped me process and resolve teaching dilemmas, while others helped me enhance and expand my own perspective on mathematics. In this talk I share some examples of writing projects that have had significant impact on my path as a mathematician, and through their analysis converge to some (admittedly idiosyncratic) guidelines that might offer some hope for those who do not see themselves fit "the mathematical mold" and want to pave their own paths. (Received September 20, 2021)

1174-103-9782 **Courtney R. Gibbons** (crgibbon@hamilton.edu), Hamilton College, **Maria Mercedes Franco** (mfranco@qcc.cuny.edu), Queensborough Community College-Cuny, **Aris Winger** (aris.winger@gmail.com), Georgia Gwinnett College, **Brian Katz** (Brian.Katz@csulb.edu), California State University, Long Beach, and **David Kung*** (david.kung@austin.utexas.edu), Dana Center / MAA Project Next. *Promoting Anti-Racist Policies and Practices in Mathematics Departments* Preliminary report.

Following the brutal murder of George Floyd, a group of mathematicians gathered to brainstorm how to make the mathematics community a more equitable place, especially for Black students, staff, and faculty. Math Chairs 4 Racial Justice brought together over 130 department leaders to discuss how issues of race arise in departmental policies and practices—and to make plans to implement changes. Supported by the American

Institute of Mathematics, the groups met throughout the summer of 2020; some continue to interact over MAA Connect and elsewhere as they work through the difficult process of change. This session will feature stories and observations from the organizers and facilitators, as well as discussions of the NSF-supported follow-up program, Math Leaders 4 Racial Justice, to be held in the summer of 2022. (Received September 21, 2021)

1174-103-9867 **Tianxin Zhou*** (zhout22@mail.vmi.edu), Virginia Military Institute, and **Mariah Woods** (woodsmr@mail.vmi.edu), Virginia Military Institute. *Unpredictability in Football Playcalling* Preliminary report.

The objective of this project is to better understand football play calling using mathematical model building, partial dependence plots, data analytics and game theory. We have used and refined a data set encompassing all 32,000 plays (excluding special teams plays) from the 2018 NFL season. Our mission was to find and understand the inefficiencies in play calling, then show how unpredictability and game theory can be used to improve game strategies and generate more wins. Briefly, we begin by building a model using MATLAB's "fitrensemble" function which "returns the trained regression ensemble model object (Mdl) that contains the results of boosting 100 regression trees using LSBoost and the predictor and response data in the table Tbl" (MATLAB). Then we plot a partial dependence plot to give a visual interpretation of our data. A partial dependence plot is described by Matlab as a function that "computes and plots the partial dependence between the predictor variables listed in Vars and the responses predicted by using the regression model RegressionMdl, which contains predictor data" (MATLAB). We will be using these models to understand where coaches are inefficient in play calling, and how to correct play calling either situationally or across the board. (Received September 21, 2021)

1174-103-10051 **Rebekah Dupont*** (dupont@augsburg.edu), Augsburg University, **Yu-Ju Kuo** (yjkuo@iup.edu), Indiana University of Pennsylvania, **Perla Myers** (pmyers@sandiego.edu), University of San Diego, and **Ileana Vasu** (ivasu@hcc.edu), Holyoke Community College. *Theme in Variation: S-STEM Models of Faculty Mentorship* Preliminary report.

The NSF S-STEM Program aims to increase the number of academically talented low-income students who graduate with STEM degrees and contribute to the nation's innovation economy. Awards fund scholarships and provide resources to create, adapt, implement, and study support strategies and identify and understand effective evidence-based activities to advance recruitment, retention, and success of STEM students.

PIs of several S-STEM programs with mathematical connections created the Special Interest Group S-STEM to share ideas, experiences, and lines of inquiry. A core element of S-STEM projects is faculty mentorship. While a wealth of mentorship resources are available, each S-STEM program must adapt them to their own institutional contexts and needs.

In our session, we will share several models of faculty mentorship within S-STEM programs that draw from resources such as those of the Center for the Improvement of Mentored Experiences in Research (CIMER) and Becky Wai-Ling Packard's "Successful STEM Mentoring Initiatives for Underrepresented Students" (Packard, 2016). Strategies influenced by these resources and other emerging work on culturally responsive practices have been shown to have beneficial impacts on culturally diverse student achievement (Sleeter, 2012). We will highlight commonalities and distinctions in various S-STEM faculty mentorship models using a common infographic template, as well as discuss our successes and challenges in the implementation of our respective models. (Received September 21, 2021)

1174-103-10325 **Suzanne Weekes** (weekes@siam.org), SIAM. *Undergraduates working with BIG, communities, and non-profits to do research*

PIC Math is a national program that provides research experiences for undergraduate students using problems from industry, local communities, and non-profits, and prepares students for careers outside of academia. The program includes a 3-day faculty summer training workshop, a spring semester course in which students work on a research problem from outside academia, and an end-of-program research conference at which the students present. For the semester course, we have developed prepared materials for the course such as sample syllabi, a set of sample real-world research problems, sample student solutions to these research problems, and sample videos of students presenting their research. During the first five years of PIC Math, over 150 universities, 175 faculty members, 2000 undergraduate students and 150 industrial partners have participated. PIC Math is funded by NSF and NSA. (Received September 21, 2021)

1174-103-11116 **Elizabeth Carlson*** (ecarlson10@uvic.ca), University of Victoria. *Academia to Labs and Back Again: The Pros and Cons of Work Cultures and Research Opportunities in Universities and National Labs*

In this presentation, I will talk about my experience as a graduate student part time in academia and part time at a national lab. On the technical side, I will discuss how working on the "front lines" of applied research influences your own research directions, theoretically and in application, the assets (and drawbacks) of being a mathematician in an applied research environment, and how your research collaboration network can expand. On the administrative side, I will discuss the process of grant writing, the difference between a day to day academic and lab schedule, and the different responsibilities in a lab as compared to academia. (Received September 21, 2021)

1174-103-11293 **Gurcharan Singh Buttar*** (gurcharanbuttar@gmail.com), Department of Mathematics, Chandigarh University, Mohali. *Analysis of fuzzy queues by using pentagonal fuzzy number* Preliminary report.

This paper deals with the fact that pentagonal fuzzy numbers are pre-owned and systematic outcomes are discussed in real-life situations. The fuzzy set supposition is combined with well-established classical queuing theory but the classical queuing theory is far away from real-life situations. In this approach, we can use both fuzzy and probability theory to make this work more realistic with the help of the α-cut technique. Symmetric pentagonal fuzzy numbers are used to elaborate on the situation of the queue in linguistic terms (Received September 29, 2021)

104 ► Broader concerns

1174-104-5221 **Dave Kung*** (david.kung@austin.utexas.edu), Charles A. Dana Center, The University of Texas at Austin. *Why the math community struggles with Equity & Diversity - and why there's reason for hope.*

Numbers don't lie—we in the mathematical sciences are bad at equity and diversity. Our majors, graduate students, and faculty don't come close to reflecting our population—especially when it comes to gender, race, ethnicity, and disability status. And our community is far from creating equitable opportunities, despite the good will and extensive efforts of many. While we aren't the only field that struggles, some deeply-held beliefs, pervasive in the mathematical sciences, hold us back. From thinking that success requires "genius," to viewing our beloved subject as "pure," to being blind to structures that hold others back, aspects of our culture must be confronted—and changed—if we are to create a more just community. We must do better. We can do better. And thankfully, there are many people, programs and projects that are pushing us in exactly that direction. (Received December 7, 2021)

1174-104-5404 **Lloyd E Douglas*** (ledouglas@earthlink.net), Independent Consultant. *MAA's National Research Experience for Undergraduates Program*

The Mathematical Association of America (MAA) supports the participation of mathematics undergraduates from underrepresented groups in focused and challenging research experiences to increase their interest in advanced degrees and careers in mathematics. Each year mathematical sciences faculty are invited to apply for a grant to host National Research Experiences for Undergraduates Program at their own campus over the summer. NREUP is structured both to increase undergraduate completion rates and encourage more students to pursue graduate study by exposing them to research experiences after they complete their sophomore year (at this point in their studies, students typically have a strong background in calculus and a course such as linear algebra or differential equations with some degree of exposure to the proof-based mathematics needed prior to undertaking a research project). NREUP is designed to reach minority students at a critical point in their career path - midway through their undergraduate programs. This talk will give the history of NREUP as well as information on how to apply to the program. The audience will be able to discuss issues regarding running programs, providing research experiences for NREUP's target population and obtaining funding for such programs. (Received August 19, 2021)

1174-104-5591 **Jennifer L Mueller** (mueller@math.colostate.edu), Colorado State University, and **Emily Heavner*** (eheavner@colostate.edu), Colorado State Univeristy. *Estimation of Airway Resistance Throughout the Bronchial Tree using an Inverse Problem and an Asymmetric Multi-Compartment Lung Model* Preliminary report.

Ventilation is a vital treatment for patients with respiratory failure, but ventilated patients are also at risk of ventilator-induced lung injury. Optimal ventilator settings could be guided by knowledge of the airway resistance throughout the lung. While the ventilator provides a single value estimating the airway resistance of the patient, in reality the airway resistance varies along the bronchial tree. A multi-compartment asymmetric lung model based on an analogy between electric circuits and the human lungs is used to determine the role of airway resistance in the alveolar tree. The inverse problem of computing the vector of airway resistance values in the alveolar tree is solved using a linear least squares optimization approach. Comparing outputs of the model to electrical impedance tomography, a medical imaging technique, and real-world parameters collected from ventilation data of COVID-19 positive and negative patients allow for a better understanding of airway resistance throughout the lung. (Received August 23, 2021)

1174-104-5951 **Frederick Isadore Miller*** (fimiller@wpi.edu), Worcester Polytechnic Institute, and **Danelle Larson** (dmlarson@usgs.gov), United States Geological Survey. *Topological Data Analysis of the Upper Mississippi River System*

The United States Geological Survey (USGS) has been collecting LTRM data in the Upper Mississippi River for the last 25+ years. Since this dataset is highly dimensional, we need to couple classic statistical analysis techniques with topological data analysis (TDA) to gain insights about ecological states. The TDA mapper algorithm generates a simplicial complex, a graph consisting of nodes and edges that represents the shape of the data in two dimensions. In my poster, I will present 1.) the application of TDA in finding the (shape or) graph representation of the Upper Mississippi River Basin data, 2.) how density analysis is performed on the nodes to count the number of ecological states. Currently, when evaluating the entire dataset, three distinct ecological states are found. When narrowing the scope to pool 26 of the Upper Mississippi River, we find two different ecological states. We hypothesize that the major state variables are suspended solids, turbidity, chlorophyll, and total phosphorus. This information can help drive decision making for the Upper Mississippi Restoration Program, giving quantifiable goals for projects. (Received September 1, 2021)

1174-104-6415 **James Henderson*** (jrh66@psu.edu), Penn State Behrend. *Realism and Underdetermination in Mathematics and the Physical Sciences*

It is well known that, as a group, physicists who count themselves as scientific realists are completely untroubled by the existence and practice of different (and mutually incompatible) scientific theories. For example, general relativity and quantum mechanics are irreconcilable, yet brawls rarely break out between physicists specializing in the study of the very large and very small at APS conferences. This devil-may-care approach to the diversity of theoretical bases for physics doesn't exist because physicists are wholly incurious about things that are of great interest to philosophers (though there are those who aren't in the least interested in these matters); instead, there are solid reasons for this forbearance ranging from the practical to the theoretical. I will concentrate on the notion of underdetermination (strictly, the underdetermination of theories by experimental facts) and argue that, at least for some readings of 'realism', mathematicians of a realist orientation would be justified in adopting the same "Don't worry, be happy" attitude with respect to different (and incompatible) mathematical foundations for analogous reasons. (Received September 15, 2021)

1174-104-6518 **Amanda Harsy*** (harsyram@lewisu.edu), Lewis University. *Solving with Sherlock*

In this talk, I will share my experiences designing and co-teaching the general education interdisciplinary seminar course "Solving with Sherlock." In Solving with Sherlock, students examine selections from the genre of detective fiction and film, using the lenses of mathematical theory and principles of logic and syllogistic reasoning. In addition to learning the basic ideas of logic and reasoning techniques, students are introduced to basic approaches to film and literature study as they examine the portrayals of the reasoning and approaches to solving criminal cases of the legendary fictional detectives. Throughout the course, students imagine, present, and enact strategies for deduction and problem-solving. (Received September 9, 2021)

1174-104-6993 **Karl Schaffer*** (karl_schaffer@yahoo.com), De Anza College. *Dance and the Quaternions*

Swirling movements, popular among contemporary dancers and choreographers, often employ double rotations of the limbs that facilely embody how the group SU(2) double covers the rotation group SO(3), and that are efficiently modeled by the quaternions. These movements are also employed in a variety of performance forms, such as the Balinese candle dance, baton twirling, and poi. We will examine how this effect plays out in these performing arts, and how comprehending the embodiment of the quaternions helps the understanding of both the mathematics and the relevant movement arts. (Received September 11, 2021)

1174-104-7522 **Rona Gurkewitz*** (rgurkewitz@gmail.com), Western Connecticut State University. *Tiled Origami Quilts and Woven Paper Mats*

A tiled origami quilt is made up of square or triangle modules, attached using tabs, to a mat that is a grid of squares made from weaving paper strips. Each square or triangle module covers all or half of a grid square and the tabs don't show. If we take the covered and uncovered but visible portions of the mat as our quilt, we can consider a quilt a tiling of the mat. I show some quilts and tiles and my technique for making them. I will also discuss what tilings can be made on a rectangular grid made from paper strips with square and triangle tiles. These include American patchwork quilt designs, Truchet tilings, polyomino tilings and pixel art. Surprisingly not every tiling of a woven mat can be made from the square and triangle tiles without using an extra tile to provide an edge to insert a tab around for certain grid squares. (Received September 14, 2021)

1174-104-7774 **John T. Baldwin*** (baldwinj@sbcglobal.net), University of Illinois at Chicago. *Category theory and Model Theory: Symbiotic Scaffolds* Preliminary report.

A *scaffold* for mathematics includes both *local* foundations for various areas of mathematics and productive guidance in how to unify them. In a scaffold the unification does not take place by a common axiomatic basis but consists of a systematic ways of connecting results and proofs in various areas of mathematics. Two scaffolds, model theory and category theory, provide local foundations for many areas of mathematic including two flavors (material and structural) of set theory and different approaches to unification. We will discuss salient features of the two scaffolds including their contrasting but bi-interpretable set theories. We focus on the contrasting treatments of 'size' in each scaffold and the advantages/disadvantages of each for different problems. (Received September 16, 2021)

1174-104-7977 **Sarah Katherine Stengle*** (stenglesarah@cenuturylink.net), Stengle Studio. *Net Gain: an innovative body of three-dimensional artworks created by thirty visual artists affiliated with Central Booking Arts all based on two-dimensional mathematical nets* Preliminary report.

Net Gain is an art project developed by artists Sarah Stengle and Maddy Rosenberg in which a selection of two-dimensional mathematical nets were shared with three hundred book artists associated with Central Booking Arts. The artists were invited to experiment with producing artwork based on folding a piece of paper into a three-dimensional work of art. Thirty artists participated. The nets created included a dimpled dodecahedron, a sphericon, and two fractal folding patterns developed by Robert Fathauer. The participating artists produced a broad range of innovative work using diverse materials such as copper, steel, sand, and cyanotypes. Some of the Net Gain artists collaborated with mathematicians. Sarah Stengle worked with Robert Fathauer to produce an aluminum bound book, and artist Joan Lyon collaborated with mathematician Emmet Wyman to create a series of cyanotypes based on fractals. Other participating artists, such as Susan Happersett, had a longstanding interest in mathematical art. Most participating artists worked with mathematical nets for the first time and brought elements of their own practice to Net Gain. The resulting works were gathered into a sixty-page PDF which is viewable on Central Booking Arts website. The collected projects of Net Gain, together with newer pieces based on the same concept, will be exhibited at the Noyes Museum in Hammonton, New Jersey, in the spring of 2023. (Received September 17, 2021)

1174-104-8086 **Keara Schmitt** (keara.schmitt@my.simpson.edu), Simpson College, **Noah Nelsen*** (noah.nelsen@my.simpson.edu), Simpson College, and **Marc Medici** (marc.medici@my.simpson.edu), Simpson College. *Classifying Histological Images Through the Application of Multiparameter Persistent Homology*

Analyzing and classifying histological images of cells based on composition is an essential practice to diagnosing various types of cancer. However, medical professionals must contend with the time spent and variability introduced when classifying the images by hand. We use persistent homology, a tool in topology data analysis, to automate the classification process. Persistent homology methods of classification were explored in previous studies using stain normalization and single parameter persistent homology to classify histological images. Utilizing an open-source data set of histological images of colorectal cancer, this project evaluated six different forms of image preprocessing, single parameter persistent homology, and multiparameter persistent homology to understand pixel composition. Using k-nearest neighbor clustering on the outputs of the persistent homology functions, we were able to classify images of colorectal histological images with an accuracy of 80.58%. Forms of image preprocessing include stain normalization, deconvolution, grayscale, and gamma correction. Across all forms of preprocessing, classifying images through the application of multiparameter persistent homology outperformed the application of single parameter persistent homology. (Received September 21, 2021)

1174-104-8233 **Colin McLarty*** (colin.mclarty@case.edu), Case Western Reserve University. *Reality never has just one correct foundation*

No matter what foundation for mathematics you like—or even if you like none of them—mathematical reality today includes huge numbers of categories and functors, and huge numbers of variant models of Zermelo Fraenkel set theory. Algebraists, topologists, number theorists, and others routinely use categories and functors ranging from the intuitive (say, the category of Abelian groups) to the technical (say, the category of unfoldings of some singularity). Set theorists from Gödel through the latest work on determinacy, large cardinals, and inner models, work with many different models of Zermelo Fraenkel (say, models where the Continuum Hypothesis is true and others where it is false). All these things are mathematical realities. And because they are realities, every leading candidate foundation already interprets all of them. These different foundations do not describe different mathematical worlds. They offer different basic views of *the* mathematical world. Various examples show how one foundation can be the best choice in practice for one purpose, including an interesting axiomatization of set theory in Terence Tao's recent textbook *Analysis I.* (Received September 18, 2021)

1174-104-8279 **Amber Lee*** (apla2018@pomona.edu), Pomona College, and **Alaina Stockdill** (astockdill22@my.whitworth.edu), Whitworth University. *Topological Data Analysis for Identifying Ecosystem States in the Upper Mississippi River* Preliminary report.

Water quality data on the Upper Mississippi River System (UMRS) has been collected for over 25 years. We use topological data analysis to find ecological states, which are recurrent biological and physiochemical characteristics that define a river's ecology and guide restoration plans. We hypothesize that important state variables in the UMRS are suspended solids, turbidity, chlorophyll a, total phosphorus and dissolved oxygen. We use the Mapper algorithm to generate a simplicial complex of the data in fewer dimensions, with each node representing a cluster of data and each edge representing an intersection in the clusters. Using a kNN density method on the simplicial complex, we determine ecological states and state variables. We apply this method on two spatial scales, the entire UMRS and Pool 4, a 140 km sampling region in which at least two ecological states have been studied. The UMRS-wide analysis yields three potential states driven by one state variable, suspended solids. The Pool 4-specific analysis yields two states, one state of which has lower total suspended solids, lower chlorophyll a, and higher dissolved oxygen. We observe that both simplicial complices cluster nodes by sampling region, which can inform future management projects. (Received September 20, 2021)

1174-104-8380 **Oscar Vega** (ovega@csufresno.edu), California State University, Fresno, **John A. Rock** (jarock@cpp.edu), Cal Poly Pomona, **Robin T Wilson** (robinwilson@cpp.edu), Cal Poly Pomona, and **Kimberly Seashore*** (kimseash@sfsu.edu), San Francisco State University. *BAMM! Building a Virtual Mentorship Community with Math Masters' Students*

The BAMM! program (Bolstering the Advancement of Masters in Mathematics) is designed to provide scholarships and mentoring, and to build a supportive community of and for underserved and underrepresented master's students in the mathematical sciences as they make their way to PhD programs. This five-year NSF-funded project is a collaboration across math departments at three California State Universities: Fresno State, Cal Poly Pomona and San Francisco State University. The initial BAMM! cohort of scholars came on board amid the abrupt shift to remote and virtual learning due to COVID-19. This challenging start provided unexpected benefits. The PIs along with two BAMM! scholars will share the relationships fostered and the lessons we have learned about creating community and mentoring students across campuses in a virtual environment. (Received September 18, 2021)

1174-104-8588 **Martin E Flashman*** (flashman@humboldt.edu), Humboldt State University. *Choices and Commitment for the Philosophy of Mathematics* Preliminary report.

With many possible philosophical foundations for mathematics, choices are made for a variety of reasons which have ontological and epistemological consequences. The author will discuss some historical examples of such choices and their consequences. For a resolution (short or longer term), pragmatic criteria and flexible commitments are suggested that can enhance the continued development of the web of mathematics and a dynamic expanding framework for mathematical knowledge based on ontological and epistemological commitments. (Received September 19, 2021)

1174-104-8933 **Samuel Hansen*** (ams@acmescience.com), ACMEScience.com, University of Michigan. *Please leave a review, it really does make it easier for new listener to find us: The Landscape of Mathematical Podcast 7 Years after Serial*

Over the past few years podcasts have been enjoying their moment in the media sun. This is even true for mathematical podcasts, which less than a decade ago could be counted on the fingers of a single hand, and now have their own subcategory in the Apple podcasts ecosystem. In this presentation Sam Hansen, who has been producing mathematical podcasts since 2009 including the award winning Relatively Prime, will review the landscape of mathematical podcasts, examine the many, varied approaches used by mathematical podcasters, illustrated with examples, and discuss how different goals and audiences, e.g. connecting the non-mathematically minded with stories from the subject or strengthening connections with mathematicians of a specific positionality, lead podcasts to use contrasting production styles in order for the podcasts to reach those goals and to communicate to their intended audience. Additionally, Sam will lay out the questions any potential mathematical podcaster should ask themselves when starting their show. (Received September 20, 2021)

1174-104-9213 **Edward G. Dunne*** (egd@ams.org), American Mathematical Society — Mathematical Reviews, **Kathy Wolcott** (klw@ams.org), American Mathematical Society — Mathematical Reviews, and **Elizabeth Downie** (ead@ams.org), American Mathematical Society — Mathematical Reviews. *Mathematicians and Librarians at Mathematical Reviews*

The mathematicians and librarians at the Mathematical Reviews division of the AMS work in tandem to select, catalog, and classify the items that are in MathSciNet. An important element of what we do is the classification of books and articles using the Mathematics Subject Classification (MSC2020). These classes are verified and assigned by the mathematicians on our editorial staff, and are instrumental in matching papers to people, either as authors or as reviewers of those papers. This presentation will give an overview of MSC2020, how it is used at Mathematical Reviews, and how other cataloging skills and principles are essential to creating MathSciNet. (Received September 20, 2021)

1174-104-9240 **Tim Ryan** (rtimothy@umich.edu), University of Michigan, **Seth Greenfield** (sethjgre@umich.edu), University of Michigan, and **Christina Jiang** (chrjiang@umich.edu), University of Michigan. *Modeling Redistricting with Gerrychain* Preliminary report.

One of the reasons gerrymandering is such a threat to our democracy is the difficulty in detecting it. In recent decades there has been a massive push to effectively quantify gerrymandering. One exciting avenue is employing Markov Chain Monte Carlo (MCMC) methods. These methods generate a random sample of all possible redistricting plans and then compare proposed plans to the sample statistics of the ensemble. If the proposed plan differs dramatically in outcomes, there is reason to suspect map manipulation. We created a user friendly Python script which applies MCMC methods for the user. Our script was used to evaluate Ohio practice districts of the Michigan Independent Citizens Redistricting Commission.

Our third author's librarian expertise was invaluable during this project. They compiled much of the vast array of publicly available resources used in this work: federal precinct and census tract geographic information, state voting information, census results, compiled shapefiles, and Gerrychain documentation. Even running our script made heavy use of library computing resources. Our talk recounts the journey of compiling library-collected information to build a powerful tool in the fight for fair redistricting. (Received September 20, 2021)

1174-104-9341 **David Goldberg*** (goldberg@math.purdue.edu), Math Alliance/Purdue University, and **Stephanie Reed** (stephanie.reed.j@gmail.com), California State University Fullerton. *Math Alliance Scholar Doctorates Panel 1*

Math Alliance Scholars who have earned doctorates and are now in the mathematical science professions discuss their experience with the Math Alliance, their career development and experiences so far. (Received September 20, 2021)

1174-104-9496 **Piper Alexis H*** (paharron@gmail.com), University of Toronto. *Road to working group hell*

Part of Rethinking Number Theory is reimagining how we work together. Coming together with shared interests and even shared values is not necessarily enough to avoid all the pitfalls that normal working groups have. I was fortunate to be able to participate in RNT without the vulnerability that comes from caring (sorry, there's a whole pandemic happening which has tied up almost all of my concern) and I am taking this opportunity to share what I learned. (Received September 20, 2021)

1174-104-9678 **Felicia Yeung Tabing*** (feliciatabing@gmail.com), University of Southern California. *Combining Synesthesia, Mathematics, and Art* Preliminary report.

I have been exploring ways to express synesthesia mathematically and through art. I will describe my attempts at creating art informed by how I experience grapheme-color synesthesia and incorporating my favorite aspects of mathematics that I frequently work with due to my job as a mathematics educator. (Received September 20, 2021)

1174-104-9797 **Killian Davis*** (davisk462@gmail.com), Clemson University. *Topology and Ecology: Deducing States of the Upper Mississippi River System*

A large partnership systematically collected long-term water quality data from the Upper Mississippi River System, USA, for over 25 years. The river is thought to occur in multiple 'ecological states', which are recurring bundles of chemical and biological conditions. Identifying ecological states and the driving ecological variables that define the states would improve understanding of river status and help prioritize large restoration projects. We use new mathematical tools like topological data analysis (TDA) because of the complexity due to the large size (80,000+ sample sites) and high dimensionality of the data. We employed TDA Mapper, an algorithm that casts high dimensional data into lower dimensions, at a system scale (1200 km) and pool scale (80 km; La Grange Pool in Illinois) to determine ecological states and variables at different spatial scales. We found that the Upper Mississippi River System is dominated by three ecological states, which vary in turbidity and total suspended solids (measures of water clarity). La Grange, on the other hand, is dominated by a single ecological state and characterized by high turbidity and total suspended solids. Ultimately, topological data analysis can offer new insights into complex ecologies. (Received September 20, 2021)

1174-104-9945 **Hiroshi Udo*** (hiroorih@lal-lal.co.jp), LAL-LAL Inc.. *Torus Making Non-Developable Developable* Preliminary report.

It shows the structures resembling a Clifford torus, assembled from such panels, of which some tabs located inside of the structure. This has a non-developable surface and was approximated by introducing apertures to the panel. (Received September 21, 2021)

1174-104-9953 **Michelle Manes*** (mmanes@math.hawaii.edu), University of Hawaii at Manoa, **Donatella Danielli** (ddanielli@asu.edu), Arizona State University, and **Ami Radunskaya** (ami.radunskaya@pomona.edu), Pomona College. *Introducing La Matematica*

The Editors in Chief of *La Matematica* introduce the new journal, describing its history within AWM, its goals, and its publication process.

La Matematica is an international peer-reviewed journal featuring high-quality research from all areas of the mathematical sciences. It is dedicated to publishing research papers describing novel mathematical ideas and tools, survey articles on current trends that appeal to a wide range of readers, short communications and reviews. *La Matematica* seeks to encourage innovation, engagement and interdisciplinary research collaboration. As the flagship journal of the Association for Women in Mathematics, our goal is to support the flourishing of all mathematicians by adopting equitable practices in STEM publishing.

La Matematica seeks to publish a variety of article types in all fields of mathematics: pure, applied, and computational. We will include work on a wide spectrum of topics, ranging from mathematics education and the history of mathematics to mathematically-grounded work in data science, computer science, and statistics. Occasionally we will also publish special thematic issues.

La Matematica uses a doubly-anonymous review process, which research shows reduces the effect of initial impressions and implicit biases and leads to greater equity in outcomes. (Received September 21, 2021)

1174-104-9959 **Kathryn Leonard** (kathryn.leonard@csuci.edu), Occidental College. *Introducing La Matematica and Meet the Editors, 2*

The Editors in Chief of *La Matematica* introduce the new journal, describing its history within AWM, its goals, and its publication process.

La Matematica is an international peer-reviewed journal featuring high-quality research from all areas of the mathematical sciences. It is dedicated to publishing research papers describing novel mathematical ideas and tools, survey articles on current trends that appeal to a wide range of readers, short communications and reviews. *La Matematica* seeks to encourage innovation, engagement and interdisciplinary research collaboration. As the flagship journal of the Association for Women in Mathematics, our goal is to support the flourishing of all mathematicians by adopting equitable practices in STEM publishing.

La Matematica seeks to publish a variety of article types in all fields of mathematics: pure, applied, and computational. We will include work on a wide spectrum of topics, ranging from mathematics education and the history of mathematics to mathematically-grounded work in data science, computer science, and statistics. Occasionally we will also publish special thematic issues.

La Matematica uses a doubly-anonymous review process, which research shows reduces the effect of initial impressions and implicit biases and leads to greater equity in outcomes. (Received September 21, 2021)

1174-104-9961 **Marc Strauss*** (marc.strauss@springer.com), SPRINGERNATURE, and **Un Kim** (un.kim@springernature.com), Springer Nature. *Celebrate La Matematica!*

The Association for Women in Mathematics and Springer Nature are excited and proud to announce the inaugural issue of *La Matematica*. Join us to celebrate this exciting joint venture and the first issue. (Received September 21, 2021)

1174-104-9963 **Rachel Yovel*** (racheli.yovel@gmail.com), Lecturer in the Hebrew University. *Poetry of Matematical Definitions* Preliminary report.

Some mathematical definitions change the meaning attributed to words that we use regularly. For instance, a positive definite matrix sounds like an optimistic one, while a nilpotent matrix sounds like an insult. Some definitions gives us a very clear picture of the mathematical meaning, though. A dense set, for example, settles with our daily sense of density. This definition gives a crowded feeling, indeed. As a mathematician and a poet, I have a habit of searching poetic ideas that underlays behind all kinds of mathematical definitions. In this talk I will share with you my writing process, answering some of the following questions: What emotions does the definition enclose? How the mathematical word, that mathematicians chose to describe the new term, differs from its daily meaning? Finally, we will use personification of mathematical objects to extract some poetry from the definitions. (Received September 21, 2021)

1174-104-10093 **Phil Kutzko*** (philip-kutzko@uiowa.edu), Math Alliance. *Math Alliance Scholar Doctorates Panel 2*

Math Alliance Scholars who have earned doctorates and are now in the mathematical science professions discuss their experience with the Math Alliance, their career development and experiences so far. (Received September 21, 2021)

1174-104-10193 **Sam Hansen*** (hansensm@umich.edu), ACMEScience.com, University of Michigan. *A Lab of Geometry and its Embedded Librarian*

The Lab of Geometry Michigan [Log(M)] is a project based course which provides its undergraduate students with many unique opportunities from getting a taste of collaborative research with their faculty and graduate student mentors to public speaking practice to a workshop on mathematical storytelling. Log(M) was originally helmed by postdocs, who did amazing work but would then leave when their terms were up and would pass Log(M) onto someone new. Two years ago the instructor at that time, Harrison Bray, reached out to the mathematics & statistics librarian, Sam Hansen, so some support with a session or two which led to a deeper collaboration and Sam becoming an embedded librarian in the course. This relationship continued when Tim Ryan took over the course and for two years they have collaborated in the instruction of Log(M). In this talk Tim and Sam will discuss how they split up the instruction to match their personal skills, the usefulness of having a librarian in a mathematics classroom, and the impact of having a course's institutional memory held by someone who is not the instructor of record. (Received September 21, 2021)

1174-104-10199 **Tiina Katriina Kukkonen*** (tiina.kukkonen@iki.fi), University of Jyväskylä, Finland. *Difference in use of geometry and arithmetic in Aleksis Kivi's novel Seven Brothers* Preliminary report.

Aleksis Kivi's early Finnish novel - Seven Brothers (Seitsemän veljestä, 1870) - includes elementary descriptions of Euclidean geometry and numbers, simple arithmetic, and measurement units. The characters and the narrator use numbers, elementary arithmetic, and measurement units in similar ways. The narrator describes the brothers' positions in the landscape using geometric forms such as circles, radii and quadrangles seen from above. The narrator describes distances between the brothers and natural and settled places. Basic geometric forms provide the novel with structure in several scenes.(Kukkonen 2016, 2020, 2021)

The difference between the characters and the narrator consists in the characters not using geometry to describe events and settings. Unlike the narrator, even the narrating characters do not use geometry. The characters use some arithmetic knowledge and practical geometric knowledge in small-scale construction works, but they lack spoken geometric knowledge. As a result, the structures of the novel and its scenes remain unknown to them. The characters do not see the whole picture. The novel's world tends to be more ambiguous and chaotic for the characters than for the narrator and reader. This difference is also apparent when the narrator and a character memorize the same event. These findings characterize the importance of geometric forms for the novel's structure. (Received September 21, 2021)

1174-104-10338 **Ron Buckmire*** (ron@oxy.edu), Occidental College, **Omayra Ortega** (ortegao@sonoma.edu), Sonoma State University/ National Association of Mathematici, **Monica C Jackson** (monica@american.edu), American University, **Caleb Ashley** (caleb.ashley@bc.edu), Boston College, and **Duane Cooper** (dcooper@morehouse.edu), Morehouse College. *A Retrospective Look at MSRI's 2021 Math and Racial Justice Workshop*

The Mathematical Sciences Research Institute (MSRI) hosted the inaugural [Online] Workshop on Mathematics and Racial Justice (https://www.msri.org/workshops/1012) over six days in June 2021. The workshop included sessions on Bias in Algorithms and Technology; Fair Division, Allocation, and Representation; Public Health Disparities; and Racial Inequities in Mathematics Education. In this talk we will present a retrospective look back at the workshop in order to share lessons learned, to highlight and amplify its successes and to look forward to future activities that build upon these. (Received September 21, 2021)

1174-104-10420 **Mara Alagic*** (mara.alagic@wichita.edu), Professor @ Wichita State University, Visiting Professor @ Johannes Kepler University Linz. *Reflecting on current trends in mathematics and the arts* Preliminary report.

Conceptualizing mathematics and the arts with a deeper sense of possibilities surfaced in Gödel, Escher, Bach: An Eternal Golden Braid. The interplay of the music of Bach, the artwork of Escher, and the implications of Gödel's Incompleteness Theorem included Hofstadter's deliberations on the prospects of artificial intelligence emulating human thought. Many new thought-provoking examples of unexpected connections followed; connections from the world of mathematics to other worlds.

This meta-analysis reports on several paths that capture some trends in connecting mathematics and the arts. In this context, purposeful exploration of mathematical phenomena can be conceptualized through multiple categories/clusters leading to various ways of creative representations, from architectural, computational, and other types of designs to multi-modal artistic representations via music, dance, visual arts: embodied, nature-inspired, ethnocultural, computer-generated including machine learning and artificial intelligence. Focus on big data is generating innumerable representations with new illustrations of (mathematical) phenomena. Contrasting that with continuing interest in hands-on creation from fiber arts to Mathemalchemy and beyond, including additive manufacturing with various materials. (Received September 21, 2021)

1174-104-10429 **Johannah L Crandall*** (crandallj@gonzaga.edu), Gonzaga University. *Undergraduate engineering students' perceptions of broader impacts versus disciplinary relevance of differential equations problem-solving scenarios* Preliminary report.

Math and engineering educators seek differential equations (DE) curricula that reflect mathematical practices of the disciplines of its applications. Math educators have harnessed curricular innovations such as inquiry and modeling with real-world data to make DE knowledge more transferable to engineering contexts, yet evidence has not been gathered to show results of these efforts in students in later engineering contexts. Thus, this study sought to characterize the extent and nature of undergraduate chemical and mechanical engineering students' recall of methods taught in DE. 15 engineering students in 3 ABET-accredited engineering programs participated in task-based interviews during the spring of 2021. The preparation for future learning and actor-oriented transfer frameworks were used to cast problem-solving strategies from the perspectives of students and math and engineering instructors. Engineering students experienced varying recall of DE methods based on their perception of broader impacts of and disciplinary relationships with the tools. Counter to instructor expectations, students associated mathematical tool relevance more with broader impacts than with disciplinary relevance and reported little representation of their engineering disciplines in DE problems. This initial study has provided guidance for future work exploring effects of numerical methods in DE on later computational problem-solving of engineering students. (Received September 21, 2021)

1174-104-10629 **Gregory D Foley*** (foleyg@ohio.edu), Ohio University, **Deependra Budhathoki** (db497117@ohio.edu), Ohio University, and **Amrit B Thapa** (at737719@ohio.edu), Ohio University. *Quantitative Reasoning for Multifaceted Citizenship* Preliminary report.

Postsecondary education should prepare individuals to engage thoughtfully as citizens at home, at work, and in society. Understanding, interpreting, and using quantitative information are increasingly central to active engagement in the public sphere. A course in quantitative reasoning has the potential to develop key practical and intellectual skills for citizenship: critical thinking, inquiry, problem solving, quantitative literacy, media literacy, use of technology, and oral and written communication. In this paper, we report on the development and research of an entry-level postsecondary Quantitative Reasoning course across the state of Ohio, which is designed to improve the quantitative literacy of students enrolled in a wide variety of non-STEM programs. The course addresses numeracy, probability, statistics, and modeling. Students investigate mathematics and statistics in timely and timeless contexts such as household budgets, mortgages, food insecurity, redistricting, elections, and social security. At Ohio University, the course uses a student-centered, inquiry-based-learning approach. We will provide specifics on some course activities and projects, instructors' and students' perspectives, as well as variation across institutions and instructors. (Received September 21, 2021)

1174-104-10965 **Samuel Luke Tunstall*** (stunstal@trinity.edu), Trinity University - San Antonio, TX. *Infusing Quantitative Literacy into a Math for the Liberal Arts Course* Preliminary report.
Though distinct from a typical quantitative literacy (QL) course, a Liberal Arts Mathematics course nonetheless provides a ripe arena for fostering quantitative literacy skills and practices for our students. In this presentation, I distinguish between common aims for the two courses, then describe through examples how—in my context at Trinity University—I have infused QL into our Liberal Arts Mathematics course. In addition to presenting my approach to the course, I will allot time for active discussion and conversation about the topic with the audience. (Received September 21, 2021)

1174-104-11128 **William He*** (WilliamHe@u.northwestern.edu), Northwestern University, **Lu Cheng** (lucheng@g.ucla.edu), University of California, Los Angeles, **Girish Ganesan** (gg655@rutgers.edu), Rutgers University, **Daniel Silverston** (daniel_silverston@brown.edu), Brown University, **Harlin Lee** (harlin@math.ucla.edu), University of California, Los Angeles, and **Jacob G Foster** (foster@soc.ucla.edu), University of California, Los Angeles. *Joint Content-Context Analysis of Scientific Publications: Identifying Opportunities for Collaboration in Cognitive Science*
As scientific fields have grown larger and more specialized, researchers may be missing potentially-lucrative avenues of collaboration. It is thus important to be able to identify these missing links, which could help further our collective scientific knowledge faster than otherwise possible. We study publications in the field of cognitive science and utilize mathematical techniques to connect the analysis of the papers' content (abstracts) to their context (citations and journals). We apply hierarchical topic modeling on the abstracts and community detection algorithms on the citation network, and propose a content-context discrepancy measure to uncover academic fields that study similar topics but do not cite each other or publish in the same venues. These results show a promising, systematic framework that can be used to identify opportunities for scientific collaboration in highly interdisciplinary fields such as cognitive science and machine learning. (Received September 21, 2021)

1174-104-11231 **Takako Udo*** (takaakat@lal-lal.co.jp), LAL-LAL Inc.. *Explore Geometric Beauty!* Preliminary report.
We are surrounded by so many beautiful and unique shapes in this three-dimensional world. Even a pebble on the ground has a different look. When you were small, you might have given it a name by its resemblance. Polyhedra are abstract shapes in geometry. But, they seem to retain the essence of beauty or uniqueness. We may be able to gain some insights from them. An obstacle is that elements such as regular polygons can form relatively limited shapes. Most of polyhedra contain polygonal faces of different lengths or angles.

Cage-like molecules inspired us. Distances of atomic bonds are roughly similar, and each ring structure, a face, resembles a bent equilateral polygon or, more often, a bent regular polygon. Therefore, flexible panels would expand buildable shapes.

Then, just flex our minds to flex their faces, regarding polyhedra as softer solids in a topological sense. Shall we call them "Curved Regular-Faced Polyhedra" with flexible regular polygons? By curving faces, not all but many more shapes are formed.

RUPA Geometric Art provides geometric construction puzzles of dichroic-type panels. The flexible panels allow you to make not only regular-faced polyhedra but also some isomorphic shapes of non-regular faced polyhedra. Geometric shapes of your design will turn into beautiful ornaments in your room, which look differently day or night.

RUPA is here for you to explorer geometric wonderland! Your favorite shapes are to be discovered. (Received September 21, 2021)

1174-104-11284 **Paul R Bialek*** (pbialek@tiu.edu), Trinity International University. *Transformation behind bars: Teaching a college mathematics course in a maximum-security prison* Preliminary report.
College degree programs in penitentiaries can improve the prison environment by transforming the lives of prisoners. Consequently, I have found that teaching a college mathematics course in a maximum-security prison is worthwhile, redemptive, interesting, and rewarding. (Received September 22, 2021)

1174-104-11305 **Kaila Uyeda*** (kuyeda@haverford.edu), Haverford College. *Modeling Foodweb Interactions based on Predator Movements and Habitat Use* Preliminary report.

Insect and food web interactions are determined by behavior patterns, body size, non-competitive predator effects, and microhabitat use. Mathematical models have predicted locations of insect and arachnid predators in microhabitats that incorporate those parameters, but movements of predators over time are often not incorporated into these models. Data from laboratory observations of location of ground beetles, wolf spiders, pirate bugs, and ladybugs in cages containing aphid prey were used to determine movement patterns of predators. We create a mathematical model of the observations made to investigate the relationship between predators and their cage location (wall, ground, bean plant, barley plant, under leaves of bean plants) and their behavior (active, walking, resting, or feeding) over time. By gaining a stronger understanding of predator behavior and movements, food web dynamics can be better represented over longer periods of time, especially as food sources may change during those periods. (Received October 1, 2021)

1174-104-11752 **Amanda Gute*** (agute@uncc.edu), University of North Carolina at Charlotte. *Maximum Principle Preserving Finite Difference Scheme for 1-D Nonlocal-to-Local Diffusion Problems*

In a recent paper, a quasi-nonlocal coupling method was introduced to seamlessly bridge a nonlocal diffusion model with the classical local diffusion counterpart in a one-dimensional space. The proposed coupling framework removes interfacial inconsistency, preserves the balance of fluxes, and satisfies the maximum principle of diffusion problem. However, the numerical scheme proposed in that paper does not maintain all of these properties on a discrete level. In this paper we resolve this issue by proposing a new finite difference scheme that ensures the balance of fluxes and the discrete maximum principle. We rigorously prove these results and provide the stability and convergence analyses accordingly. In addition, we provide the Courant-Friedrichs-Lewy (CFL) condition for the new scheme and test a series of benchmark examples which confirm the theoretical findings. (Received October 6, 2021)

1174-104-11766 **Emille Davie Lawrence** (edlawrence@usfca.edu), University of San Francisco, **Anisah Nabilah Nu'Man*** (anisah.numan@spelman.edu), Spelman College, and **Nathan Broaddus** (broaddus@math.ohio-state.edu), Ohio State University. *Steinberg modules for low braid index.* Preliminary report.

I will introduce the braid group and discuss ongoing work on a presentation of its Steinberg module. While the motivation for our work comes from group cohomology of the braid group, the computations that I will discuss will be very concrete involving some fun surface combinatorics. This work is joint with N. Broaddus, L.-K. Lauderdale, E. Lawrence and R. Wilson and was supported by the 2021 ADJOINT program at MSRI. (Received October 20, 2021)

1174-104-11767 **Sherry Euvette Scott*** (sscott@msri.org), MSRI ADJOINT. *COVID-19: A follow up on previous work and a fluid flow perspective.* Preliminary report.

We'll discuss some follow up ideas from our paper "A closer look at the spreaders of COVID-19 in Wisconsin and the US" and then consider modeling the spread of COVID (and other airborne diseases) as a type of fluid flow. (Received October 20, 2021)

1174-104-12262 **Talithia Williams*** (twilliams@hmc.edu), Harvey Mudd College. *The Power of Talk: Engaging the Public in Mathematics*

When it comes to inspiring the future productivity and innovation of our nation, mathematicians are the on the front lines. In this talk, I will discuss the importance of engaging a wide range of audiences in conversations about the nature of our work and of scientific discovery. As we change the way readers, viewers, and audience members think about the natural world and the STEM disciplines, we can begin conversations that improve public perception of science and bring people from all backgrounds into this important work. (Received December 7, 2021)

1174-104-12306 **Imelda Trejo*** (imelda@uta.edu), Los Alamos National Laboratory, and **Nicolas Hengartner** (nickh@lanl.gov), Los Alamos National Laboratory. *A modified Susceptible-Infected-Recovered model for observed under-reported incidence data*

Fitting Susceptible-Infected-Recovered (SIR) models to incidence data is problematic when a fraction of the infected individuals are not reported. Assuming an underlying SIR model with general but known distribution for the time to recovery, we introduce a system of differential-integral equations to quantify the fraction of asymptomatic individuals during an epidemic outbreak. Using these differential equations, we develop a simple stochastic model for the observed incidence, and propose a Bayesian estimate of model parameters. We use our model to estimate the transmission rate and fraction of asymptomatic individuals for the current Coronavirus 2019 outbreak in eight American Countries: the United States of America, Brazil, Mexico, Argentina, Chile, Colombia, Peru, and Panama, from January 2020 to May 2021. Our analysis reveals that the fraction of reported cases varies across all countries. For example, the reported incidence fraction for the United States of America varies from 0.3 to 0.6, while for Brazil it varies from 0.2 to 0.4. (Received December 17, 2021)

2020 Mathematics Subject Classification System

Jointly produced by the editors of Mathematical Reviews and zbMATH, in consultation with the mathematical community

00 General and overarching topics; collections
01 History and biography
03 Mathematical logic and foundations
05 Combinatorics
06 Order, lattices, ordered algebraic structures
08 General algebraic systems
11 Number theory
12 Field theory and polynomials
13 Commutative algebra
14 Algebraic geometry
15 Linear and multilinear algebra; matrix theory
16 Associative rings and algebras
17 Nonassociative rings and algebras
18 Category theory; homological algebra
19 K-theory
20 Group theory and generalizations
22 Topological groups, Lie groups
26 Real functions
28 Measure and integration
30 Functions of a complex variable
31 Potential theory
32 Several complex variables and analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
37 Dynamical systems and ergodic theory
39 Difference and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Harmonic analysis on Euclidean spaces
43 Abstract harmonic analysis
44 Integral transforms, operational calculus
45 Integral equations
46 Functional Analysis
47 Operator theory
49 Calculus of variations and optimal control; optimization
51 Geometry
52 Convex and discrete geometry
53 Differential geometry
54 General topology
55 Algebraic topology
57 Manifolds and cell complexes
58 Global analysis, analysis on manifolds
60 Probability theory and stochastic processes
62 Statistics
65 Numerical analysis
68 Computer science
70 Mechanics of particles and systems
74 Mechanics of deformable solids
76 Fluid mechanics
78 Optics, electromagnetic theory
80 Classical thermodynamics, heat transfer
81 Quantum theory
82 Statistical mechanics, structure of matter
83 Relativity and gravitational theory
85 Astronomy and astrophysics
86 Geophysics
90 Operations research, mathematical programming
91 Game theory, economics, social and behavioral sciences
92 Biology and other natural sciences
93 Systems theory; control
94 Information and communication, circuits
97 Mathematics education

The expanded classification system, beyond classification 97 above, for sessions and talks at the Joint Mathematics Meetings is available at https://www.ams.org/meetings/national/session-classifications.

2689-4831(2022)43:1;1-D

AMS
www.ams.org